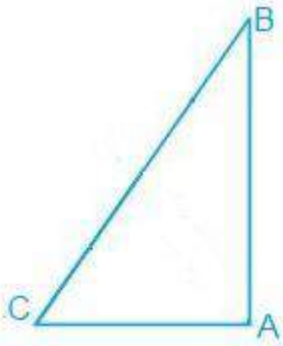
## EXERCISE: 7.4 (PAGE NO: 132)

**1. Show that in a right-angled triangle, the hypotenuse is the longest side.**




**Solution:**

It is known that ABC is a triangle right angled at B.

We know that,

∠A +∠B+∠C = 180°

Now, if ∠B+∠C = 90° then ∠A has to be 90°.

Since A is the largest angle of the triangle, the side opposite to it must be the largest.

So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e. ΔABC.

**2. In Fig. 7.48, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, ∠PBC < ∠QCB. Show that AC > AB.**

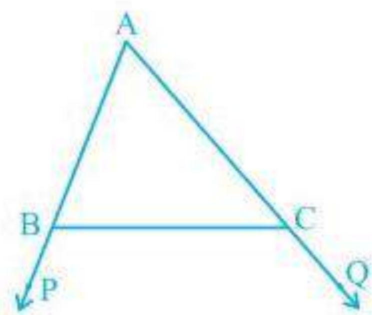



Fig. 7.48

**Solution:**

It is given that ∠PBC < ∠QCB

We know that ∠ABC + ∠PBC = 180°

So, ∠ABC = 180°-∠PBC

Also,

∠ACB +∠QCB = 180°

Therefore ∠ACB = 180° -∠QCB

Now, since ∠PBC < ∠QCB,

∴ ∠ABC > ∠ACB

Hence, AC > AB as sides opposite to the larger angle is always larger.

**3. In Fig. 7.49, ∠B < ∠A and ∠C < ∠D. Show that AD < BC.**

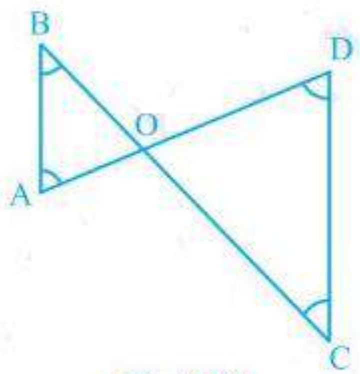



Fig. 7.49

**Solution:**

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e. ∠B < ∠A and ∠C < ∠D.

Now,

Since the side opposite to the smaller angle is always smaller

AO < BO — (i)

And OD < OC —(ii)

By adding equation (i) and equation (ii) we get

AO+OD < BO + OC

So, AD < BC

**4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).**

**Show that ∠A > ∠C and ∠B > ∠D.**

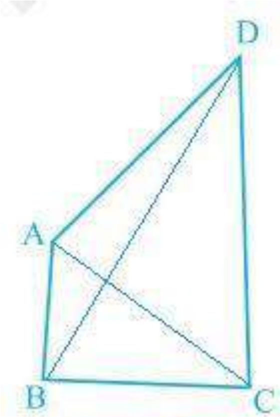



Fig. 7.50

**Solution:**

In ΔABD, we see that

AB < AD < BD

So, ∠ADB < ∠ABD — (i) (Since angle opposite to longer side is always larger)

Now, in ΔBCD,

BC < DC < BD

Hence, it can be concluded that

∠BDC < ∠CBD — (ii)

Now, by adding equation (i) and equation (ii) we get,

∠ADB + ∠BDC < ∠ABD + ∠CBD

∠ADC < ∠ABC

∠B > ∠D

Similarly, In triangle ABC,

∠ACB < ∠BAC — (iii) (Since the angle opposite to the longer side is always larger)

Now, In ΔADC,

∠DCA < ∠DAC — (iv)

By adding equation (iii) and equation (iv) we get,

∠ACB + ∠DCA < ∠BAC+∠DAC

⇒ ∠BCD < ∠BAD

∴ ∠A > ∠C

**5. In Fig 7.51, PR > PQ and PS bisect ∠QPR. Prove that ∠PSR > ∠PSQ.**

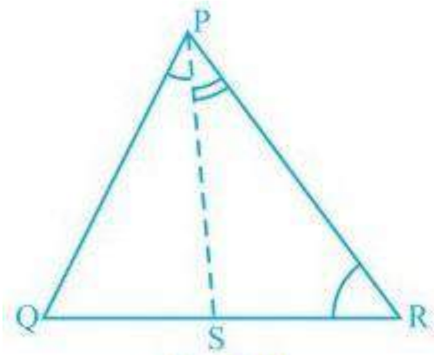



Fig. 7.51

**Solution:**

It is given that PR > PQ and PS bisects ∠QPR

Now we will have to prove that angle PSR is smaller than PSQ i.e. ∠PSR > ∠PSQ

**Proof:**

∠QPS = ∠RPS — (ii) (As PS bisects ∠QPR)

∠PQR > ∠PRQ — (i) (Since PR > PQ as angle opposite to the larger side is always larger)

∠PSR = ∠PQR + ∠QPS — (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)

∠PSQ = ∠PRQ + ∠RPS — (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles)

By adding (i) and (ii)

∠PQR +∠QPS > ∠PRQ +∠RPS

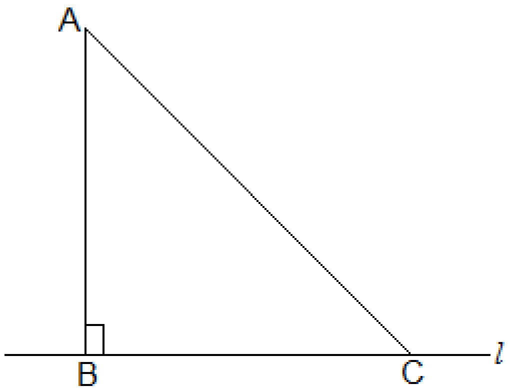
Thus, from (i), (ii), (iii) and (iv), we get

∠PSR > ∠PSQ

**6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.**

**Solution:**

First, let "*l*" be a line segment and "B" be a point lying on it. A line AB perpendicular to *l* is now drawn. Also, let C be any other point on *l*. The diagram will be as follows:



**To prove:**

$AB < AC$

**Proof:**

In $\Delta ABC$, $\angle B = 90°$

Now, we know that

$\angle A + \angle B + \angle C = 180°$

$\therefore \angle A + \angle C = 90°$

Hence, $\angle C$ must be an acute angle which implies $\angle C < \angle B$

So, $AB < AC$ (As the side opposite to the larger angle is always larger)