EXERCISE 12.2 PAGE NO. 273

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4)

(iv) (2, –1, 3) and (–2, 1, 3)

Solution:

(i) (2, 3, 5) and (4, 3, 1)

Let P be (2, 3, 5) and Q be (4, 3, 1)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 2, y_1 = 3, z_1 = 5$

$x_2 = 4, y_2 = 3, z_2 = 1$

Distance PQ = $\sqrt{[(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2]}$

$= \sqrt{[(2)^2 + 0^2 + (-4)^2]}$

$= \sqrt{[4 + 0 + 16]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

∴ The required distance is $2\sqrt{5}$ units.

(ii) (–3, 7, 2) and (2, 4, –1)

Let P be (– 3, 7, 2) and Q be (2, 4, –1)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -3, y_1 = 7, z_1 = 2$

$x_2 = 2$, $y_2 = 4$, $z_2 = -1$

Distance PQ = $\sqrt{[(2 - (-3))^2 + (4 - 7)^2 + (-1 - 2)^2]}$

= $\sqrt{[(5)^2 + (-3)^2 + (-3)^2]}$

= $\sqrt{[25 + 9 + 9]}$

= $\sqrt{43}$

∴ The required distance is $\sqrt{43}$ units.

(iii) (–1, 3, –4) and (1, –3, 4)

Let P be (– 1, 3, – 4) and Q be (1, – 3, 4)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = -1$, $y_1 = 3$, $z_1 = -4$

$x_2 = 1$, $y_2 = -3$, $z_2 = 4$

Distance PQ = $\sqrt{[(1 - (-1))^2 + (-3 - 3)^2 + (4 - (-4))^2]}$

= $\sqrt{[(2)^2 + (-6)^2 + (8)^2]}$

= $\sqrt{[4 + 36 + 64]}$

= $\sqrt{104}$

= $2\sqrt{26}$

∴ The required distance is $2\sqrt{26}$ units.

(iv) (2, –1, 3) and (–2, 1, 3)

Let P be (2, – 1, 3) and Q be (– 2, 1, 3)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 2$, $y_1 = -1$, $z_1 = 3$

$x_2 = -2$, $y_2 = 1$, $z_2 = 3$

Distance PQ = $\sqrt{[(-2 - 2)^2 + (1 - (-1))^2 + (3 - 3)^2]}$

$= \sqrt{[(-4)^2 + (2)^2 + (0)^2]}$

$= \sqrt{[16 + 4 + 0]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

∴ The required distance is $2\sqrt{5}$ units.

2. Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Solution:

If three points are collinear, then they lie on the same line.

First, let us calculate the distance between the 3 points

i.e., PQ, QR and PR

Calculating PQ

$P \equiv (-2, 3, 5)$ and $Q \equiv (1, 2, 3)$

By using the formula,

Distance PQ $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -2, y_1 = 3, z_1 = 5$

$x_2 = 1, y_2 = 2, z_2 = 3$

Distance PQ $= \sqrt{[(1 - (-2))^2 + (2 - 3)^2 + (3 - 5)^2]}$

$= \sqrt{[(3)^2 + (-1)^2 + (-2)^2]}$

$= \sqrt{[9 + 1 + 4]}$

$= \sqrt{14}$

Calculating QR

$Q \equiv (1, 2, 3)$ and $R \equiv (7, 0, -1)$

By using the formula,

Distance QR $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1, y_1 = 2, z_1 = 3$

$x_2 = 7$, $y_2 = 0$, $z_2 = -1$

Distance QR = $\sqrt{[(7 - 1)^2 + (0 - 2)^2 + (-1 - 3)^2]}$

= $\sqrt{[(6)^2 + (-2)^2 + (-4)^2]}$

= $\sqrt{[36 + 4 + 16]}$

= $\sqrt{56}$

= $2\sqrt{14}$

Calculating PR

P ≡ (–2, 3, 5) and R ≡ (7, 0, – 1)

By using the formula,

Distance PR = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = -2$, $y_1 = 3$, $z_1 = 5$

$x_2 = 7$, $y_2 = 0$, $z_2 = -1$

Distance PR = $\sqrt{[(7 - (-2))^2 + (0 - 3)^2 + (-1 - 5)^2]}$

= $\sqrt{[(9)^2 + (-3)^2 + (-6)^2]}$

= $\sqrt{[81 + 9 + 36]}$

= $\sqrt{126}$

= $3\sqrt{14}$

Thus, PQ = $\sqrt{14}$, QR = $2\sqrt{14}$ and PR = $3\sqrt{14}$

So, PQ + QR = $\sqrt{14} + 2\sqrt{14}$

= $3\sqrt{14}$

= PR

∴ The points P, Q and R are collinear.

3. Verify the following:
(i) (0, 7, –10), (1, 6, –6), and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) are the vertices of a right-angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) are the vertices of a parallelogram.

Solution:

(i) (0, 7, –10), (1, 6, –6), and (4, 9, – 6) are the vertices of an isosceles triangle.

Let us consider the points,

P(0, 7, –10), Q(1, 6, –6) and R(4, 9, –6)

If any 2 sides are equal, it will be an isosceles triangle

So, first, let us calculate the distance of PQ, QR

Calculating PQ

P ≡ (0, 7, – 10) and Q ≡ (1, 6, – 6)

By using the formula,

Distance PQ = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here,

$x_1 = 0, y_1 = 7, z_1 = – 10$

$x_2 = 1, y_2 = 6, z_2 = – 6$

Distance PQ = $\sqrt{[(1 – 0)^2 + (6 – 7)^2 + (-6 – (-10))^2]}$

$= \sqrt{[(1)^2 + (-1)^2 + (4)^2]}$

$= \sqrt{[1 + 1 + 16]}$

$= \sqrt{18}$

Calculating QR

Q ≡ (1, 6, – 6) and R ≡ (4, 9, – 6)

By using the formula,

Distance QR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = 1, y_1 = 6, z_1 = – 6$

$x_2 = 4, y_2 = 9, z_2 = – 6$

Distance QR = $\sqrt{[(4 – 1)^2 + (9 – 6)^2 + (-6 – (-6))^2]}$

$= \sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$

$= \sqrt{[9 + 9 + 0]}$

$= \sqrt{18}$

Hence, PQ = QR

18 = 18

2 sides are equal

∴ PQR is an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) are the vertices of a right-angled triangle.

Let the points be

P(0, 7, 10), Q(– 1, 6, 6) & R(– 4, 9, 6)

First, let us calculate the distance of PQ, OR and PR

Calculating PQ

P ≡ (0, 7, 10) and Q ≡ (– 1, 6, 6)

By using the formula,

Distance PQ = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = –1, y_2 = 6, z_2 = 6$

Distance PQ = $\sqrt{[(-1 – 0)^2 + (6 – 7)^2 + (6 – 10)^2]}$

$= \sqrt{[(-1)^2 + (-1)^2 + (-4)^2]}$

$= \sqrt{[1 + 1 + 16]}$

$= \sqrt{18}$

Calculating QR

Q ≡ (1, 6, –6) and R ≡ (4, 9, –6)

By using the formula,

Distance QR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = 1, y_1 = 6, z_1 = –6$

$x_2 = 4, y_2 = 9, z_2 = –6$

Distance QR = $\sqrt{[(4 – 1)^2 + (9 – 6)^2 + (-6 – (-6))^2]}$

= $\sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$

= $\sqrt{[9 + 9 + 0]}$

= $\sqrt{18}$

Calculating PR

P ≡ (0, 7, 10) and R ≡ (– 4, 9, 6)

By using the formula,

Distance PR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = – 4, y_2 = 9, z_2 = 6$

Distance PR = $\sqrt{[(-4 – 0)^2 + (9 – 7)^2 + (6 – 10)^2]}$

= $\sqrt{[(-4)^2 + (2)^2 + (-4)^2]}$

= $\sqrt{[16 + 4 + 16]}$

= $\sqrt{36}$

Now,

$PQ^2 + QR^2 = 18 + 18$

= 36

= PR^2

By using the converse of Pythagoras theorem,

∴ The given vertices P, Q & R are the vertices of a right–angled triangle at Q.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) are the vertices of a parallelogram.

Let the points: A(–1, 2, 1), B(1, –2, 5), C(4, –7, 8) & D(2, –3, 4)

ABCD can be vertices of parallelogram only if opposite sides are equal.

i.e., AB = CD and BC = AD

First, let us calculate the distance

Calculating AB

A ≡ (–1, 2, 1) and B ≡ (1, –2, 5)

By using the formula,

Distance AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -1$, $y_1 = 2$, $z_1 = 1$

$x_2 = 1$, $y_2 = -2$, $z_2 = 5$

Distance AB = $\sqrt{[(1 - (-1))^2 + (-2 - 2)^2 + (5 - 1)^2]}$

$= \sqrt{[(2)^2 + (-4)^2 + (4)^2]}$

$= \sqrt{[4 + 16 + 16]}$

$= \sqrt{36}$

$= 6$

Calculating BC

B ≡ (1, –2, 5) and C ≡ (4, –7, 8)

By using the formula,

Distance BC = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1$, $y_1 = -2$, $z_1 = 5$

$x_2 = 4$, $y_2 = -7$, $z_2 = 8$

Distance BC = $\sqrt{[(4 - 1)^2 + (-7 - (-2))^2 + (8 - 5)^2]}$

$= \sqrt{[(3)^2 + (-5)^2 + (3)^2]}$

$= \sqrt{[9 + 25 + 9]}$

$= \sqrt{43}$

Calculating CD

C ≡ (4, –7, 8) and D ≡ (2, –3, 4)

By using the formula,

Distance CD = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 4, y_1 = –7, z_1 = 8$

$x_2 = 2, y_2 = –3, z_2 = 4$

Distance CD = $\sqrt{[(2 – 4)^2 + (-3 – (-7))^2 + (4 – 8)^2]}$

$= \sqrt{[(-2)^2 + (4)^2 + (-4)^2]}$

$= \sqrt{[4 + 16 + 16]}$

$= \sqrt{36}$

$= 6$

Calculating DA

D ≡ (2, –3, 4) and A ≡ (–1, 2, 1)

By using the formula,

Distance DA = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = 2, y_1 = – 3, z_1 = 4$

$x_2 = –1, y_2 = 2, z_2 = 1$

Distance DA = $\sqrt{[(-1 – 2)^2 + (2 – (-3))^2 + (1 – 4)^2]}$

$= \sqrt{[(-3)^2 + (5)^2 + (-3)^2]}$

$= \sqrt{[9 + 25 + 9]}$

$= \sqrt{43}$

Since AB = CD and BC = DA (given),

In ABCD, both pairs of opposite sides are equal.

∴ ABCD is a parallelogram.

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Solution:

Let A (1, 2, 3) & B (3, 2, –1)

Let point P be (x, y, z)

Since it is given that point P(x, y, z) is equal distance from point A(1, 2, 3) & B(3, 2, –1)

i.e. PA = PB

First, let us calculate

Calculating PA

$P \equiv (x, y, z)$ and $A \equiv (1, 2, 3)$

By using the formula,

Distance PA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = x, y_1 = y, z_1 = z$

$x_2 = 1, y_2 = 2, z_2 = 3$

Distance PA = $\sqrt{[(1 - x)^2 + (2 - y)^2 + (3 - z)^2]}$

Calculating PB

$P \equiv (x, y, z)$ and $B \equiv (3, 2, -1)$

By using the formula,

Distance PB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = x, y_1 = y, z_1 = z$

$x_2 = 3, y_2 = 2, z_2 = -1$

Distance PB = $\sqrt{[(3 - x)^2 + (2 - y)^2 + (-1 - z)^2]}$

Since PA = PB

Square on both sides, we get

$PA^2 = PB^2$

$(1 - x)^2 + (2 - y)^2 + (3 - z)^2 = (3 - x)^2 + (2 - y)^2 + (-1 - z)^2$

$(1 + x^2 - 2x) + (4 + y^2 - 4y) + (9 + z^2 - 6z)$

$(9 + x^2 - 6x) + (4 + y^2 - 4y) + (1 + z^2 + 2z)$

$-2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$4x - 8z = 0$

$x - 2z = 0$

∴ The required equation is $x - 2z = 0$.

5. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(–4, 0, 0) is equal to 10.

Solution:

Let A(4, 0, 0) & B(– 4, 0, 0)

Let the coordinates of point P be (x, y, z)

Calculating PA

P ≡ (x, y, z) and A ≡ (4, 0, 0)

By using the formula,

Distance PA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = x$, $y_1 = y$, $z_1 = z$

$x_2 = 4$, $y_2 = 0$, $z_2 = 0$

Distance PA = $\sqrt{[(4 - x)^2 + (0 - y)^2 + (0 - z)^2]}$

Calculating PB,

P ≡ (x, y, z) and B ≡ (– 4, 0, 0)

By using the formula,

Distance PB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = x$, $y_1 = y$, $z_1 = z$

$x_2 = -4$, $y_2 = 0$, $z_2 = 0$

Distance PB = $\sqrt{[(-4 - x)^2 + (0 - y)^2 + (0 - z)^2]}$

It is given that,

PA + PB = 10

PA = 10 – PB

Square on both sides, we get

$PA^2 = (10 - PB)^2$

$PA^2 = 100 + PB^2 - 20\ PB$

$(4 - x)^2 + (0 - y)^2 + (0 - z)^2$

$100 + (-4 - x)^2 + (0 - y)^2 + (0 - z)^2 - 20\ PB$

$(16 + x^2 - 8x) + (y^2) + (z^2)$

$100 + (16 + x^2 + 8x) + (y^2) + (z^2) - 20\ PB$

$20\ PB = 16x + 100$

$5\ PB = (4x + 25)$

Square on both sides again, we get

$25\ PB^2 = 16x^2 + 200x + 625$

$25\ [(-4 - x)^2 + (0 - y)^2 + (0 - z)^2] = 16x^2 + 200x + 625$

$25\ [x^2 + y^2 + z^2 + 8x + 16] = 16x^2 + 200x + 625$

$25x^2 + 25y^2 + 25z^2 + 200x + 400 = 16x^2 + 200x + 625$

$9x^2 + 25y^2 + 25z^2 - 225 = 0$

∴ The required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.