EXERCISE 14.5 PAGE NO. 342

1. Show that the statement

***p*: "If *x* is a real number such that $x^3 + 4x = 0$, then *x* is 0" is true by**

(i) direct method

(ii) method of contradiction

(iii) method of contrapositive

Solution:

Let p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0'

q: x is a real number such that $x^3 + 4x = 0$

r: x is 0

(i) We assume that q is true to show that statement p is true and then show that r is true.

Therefore, let statement q be true

Hence, $x^3 + 4x = 0$

$x (x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

Since x is real, it is 0.

So, statement r is true.

Hence, the given statement is true.

(ii) By contradiction, to show statement p to be true, we assume that p is not true.

Let x be a real number such that $x^3 + 4x = 0$ and let $x \neq 0$

Hence, $x^3 + 4x = 0$

$x (x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

$x = 0$ or $x^2 = -4$

However, x is real. Hence, $x = 0$, which is a contradiction since we have assumed that $x \neq 0$.

Therefore, the given statement p is true.

(iii) By the contrapositive method, to prove statement p to be true, we assume that r is false and prove that q must be false.

$\sim r$: $x \neq 0$

Clearly, it can be seen that

$(x^2 + 4)$ will always be positive

$x \neq 0$ implies that the product of any positive real number with x is not zero.

Now, consider the product of x with $(x^2 + 4)$

$\therefore x (x^2 + 4) \neq 0$

$x^3 + 4x \neq 0$

This shows that statement q is not true.

Hence, it proved that

$\sim r \Rightarrow \sim q$

Hence, the given statement p is true.

2. Show that the statement "For any real numbers *a* and *b*, $a^2 = b^2$ implies that $a = b$" is not true by giving a counter-example.

Solution:

The given statement can be written in the form of 'if then' is given below:

If a and b are real numbers such that $a^2 = b^2$, then a = b

Let p: a and b are real numbers such that $a^2 = b^2$

q: a = b

The given statement has to be proved false. To show this, two real numbers, a and b, with $a^2 = b^2$, are required such that $a \neq b$.

Let us consider a = 1 and b = – 1

$a^2 = (1)^2$

= 1 and

$b^2 = (-1)^2$

= 1

Hence, $a^2 = b^2$

However, $a \neq b$

Therefore, it can be concluded that the given statement is false.

3. Show that the following statement is true by the method of contrapositive.

p*: *If x is an integer and x^2 is even, then x is also even.

Solution:

Let p: If x is an integer and x^2 is even, then x is also even

Let q: x be an integer and x^2 be even

r: x is even

By the contrapositive method, to prove that p is true, we assume that r is false and prove that q is also false

Let x is not even

To prove that q is false, it has to be proved that x is not an integer or x^2 is not even.

x is not even indicates that x^2 is also not even.

Hence, statement q is false.

Therefore, the given statement p is true.

4. By giving a counter example, show that the following statements are not true.

(i) *p*: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

(ii) *q*: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Solution:

(i) Let q: All the angles of a triangle are equal

r: The triangle is an obtuse angled triangle

The given statement p has to be proved false.

To show this, the required angles of a triangle should not be an obtuse angle.

We know that sum of all the angles of a triangle is 180^0. Therefore, if all three angles are equal, then each angle measures 60^0, which is not obtuse.

In an equilateral triangle, all angles are equal. However, the triangle is not an obtuse-angled triangle.

Hence, it can be concluded that the given statement p is false.

(ii) The given statement is

q: The equation $x^2 – 1 = 0$ does not have a root lying between 0 and 2.

This statement has to be proven false

To show this, let us consider

$x^2 – 1 = 0$

$x^2 = 1$

$x = \pm 1$

One root of the equation $x^2 – 1 = 0$, i.e. the root $x = 1$, lies between 0 and 2

Therefore, the given statement is false.

5. Which of the following statements are true and which are false? In each case, give a valid reason for saying so.

(i) *p*: Each radius of a circle is a chord of the circle.

(ii) *q*: The centre of a circle bisects each chord of the circle.

(iii) *r*: Circle is a particular case of an ellipse.

(iv) *s*: If *x* and *y* are integers such that $x > y$, then $–x < –y$.

(v) *t*: $\sqrt{11}$ is a rational number.

Solution:

(i) The given statement p is false.

As per the definition of a chord, it should intersect the circle at two distinct points.

(ii) The given statement q is false.

The centre will not bisect that chord which is not the diameter of the circle.

In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

If we put a = b = 1, then, we get

$x^2 + y^2 = 1$, which is an equation of a circle

Hence, a circle is a particular case of an ellipse.

Therefore, statement r is true.

(iv) $x > y$

By the rule of inequality

$-x < -y$

Hence, the given statement s is true.

(v) 11 is a prime number

We know that the square root of any prime number is an irrational number.

Therefore $\sqrt{11}$ is an irrational number.

Hence, the given statement t is false.