EXERCISE 15.3

1. From the data given below state which group is more variable, A or B?

Marks	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**
Group A	**9**	**17**	**32**	**33**	**40**	**10**	**9**
Group B	**10**	**20**	**30**	**25**	**43**	**15**	**7**

Solution:-

For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater C.V. is said to be more variable than the other. The series having lesser C.V. is said to be more consistent than the other.

Co-efficient of variation (C.V.) = $(\sigma/\bar{x}) \times 100$

Where, σ = standard deviation, $\bar{x}$ = mean

For Group A.

Marks	**Group A** f_i	**Midpoint** X_i	$Y_i = (x_i - A)/h$	$(Y_i)^2$	$f_i y_i$	$f_i(y_i)^2$
10 – 20	9	15	((15 – 45)/10) = -3	$(-3)^2 = 9$	– 27	81
20 – 30	17	25	((25 – 45)/10) = -2	$(-2)^2 = 4$	– 34	68
30 – 40	32	35	((35 – 45)/10) = – 1	$(-1)^2 = 1$	– 32	32
40 – 50	33	45	((45 – 45)/10) = 0	0^2	0	0
50 – 60	40	55	((55 – 45)/10) = 1	$1^2 = 1$	40	40
60 – 70	10	65	((65 – 45)/10) = 2	$2^2 = 4$	20	40

70 – 80	9	75	((75 – 45)/10) = 3	3^2 = 9	27	81
Total	150				-6	342

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A = 45,

and $y_i = (x_i – A)/h$

Here h = class size = 20 – 10

h = 10

So, $\bar{x}$ = 45 + ((-6/150) × 10)

= 45 – 0.4

= 44.6

Then, variance $\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (10^2/150^2)\,[150(342) – (-6)^2]$

= (100/22500) [51,300 – 36]

= (100/22500) × 51264

= 227.84

Hence, standard deviation = σ = √227.84

= 15.09

∴ C.V for group A = (σ/ $\bar{x}$) × 100

= (15.09/44.6) × 100

= 33.83

Now, for group B.

Marks	**Group B** f_i	**Midpoint** X_i	$Y_i = (x_i – A)/h$	$(Y_i)^2$	$f_i y_i$	$f_i(y_i)^2$

10 – 20	10	15	((15 – 45)/10) = -3	$(-3)^2 = 9$	– 30	90
20 – 30	20	25	((25 – 45)/10) = -2	$(-2)^2 = 4$	– 40	80
30 – 40	30	35	((35 – 45)/10) = – 1	$(-1)^2 = 1$	– 30	30
40 – 50	25	45	((45 – 45)/10) = 0	0^2	0	0
50 – 60	43	55	((55 – 45)/10) = 1	$1^2 = 1$	43	43
60 – 70	15	65	((65 – 45)/10) = 2	$2^2 = 4$	30	60
70 – 80	7	75	((75 – 45)/10) = 3	$3^2 = 9$	21	63
Total	150				-6	366

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A = 45,

h = 10

So, $\bar{x}$ = 45 + ((-6/150) × 10)

= 45 – 0.4

= 44.6

Then, variance $\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (10^2/150^2)\ [150(366) - (-6)^2]$

= (100/22500) [54,900 – 36]

= (100/22500) × 54,864

= 243.84

Hence, standard deviation = σ = √243.84

= 15.61

∴ C.V for group B = $(\sigma/\bar{x}) \times 100$

= (15.61/44.6) × 100

= 35

By comparing the C.V. of group A and group B.

C.V of Group B > C.V. of Group A

So, Group B is more variable.

2. From the prices of shares X and Y below, find out which is more stable in value.

X	**35**	**54**	**52**	**53**	**56**	**58**	**52**	**50**	**51**	**49**
Y	**108**	**107**	**105**	**105**	**106**	**107**	**104**	**103**	**104**	**101**

Solution:-

From the given data,

Let us make the table of the given data and append other columns after calculations.

X (x_i)	**Y (y_i)**	**X_i^2**	**Y_i^2**
35	108	1225	11664
54	107	2916	11449
52	105	2704	11025
53	105	2809	11025
56	106	8136	11236
58	107	3364	11449

52	104	2704	10816
50	103	2500	10609
51	104	2601	10816
49	101	2401	10201
Total = 510	1050	26360	110290

We have to calculate Mean for x,

Mean $\overline{x} = \sum x_i/n$

Where, n = number of terms

= 510/10

= 51

Then, Variance for x = $\frac{1}{n^2}\left[N\sum x_i^2 - (\sum x_i)^2\right]$

= $(1/10^2)[(10 \times 26360) - 510^2]$

= (1/100) (263600 – 260100)

= 3500/100

= 35

WKT Standard deviation = √variance

= √35

= 5.91

So, co-efficient of variation = $(\sigma/\overline{x}) \times 100$

= (5.91/51) × 100

= 11.58

Now, we have to calculate Mean for y,

Mean $\bar{y} = \sum y_i/n$

Where, n = number of terms

= 1050/10

= 105

Then, Variance for y = $\frac{1}{n^2}\left[N\sum y_i^2 - (\sum y_i)^2\right]$

= $(1/10^2)[(10 \times 110290) - 1050^2]$

= (1/100) (1102900 – 1102500)

= 400/100

= 4

WKT Standard deviation = √variance

= √4

= 2

So, co-efficient of variation = $(\sigma/\bar{x}) \times 100$

= (2/105) × 100

= 1.904

By comparing C.V. of X and Y,

C.V of X > C.V. of Y

So, Y is more stable than X.

3. An analysis of monthly wages paid to workers in two firms, A and B, belonging to the same industry, gives the following results:

	Firm A	**Firm B**
No. of wages earners	**586**	**648**
Mean of monthly wages	**Rs 5253**	**Rs 5253**

Variance of the distribution of wages	**100**	**121**

(i) Which firm, A or B, pays a larger amount as monthly wages?

(ii) Which firm, A or B, shows greater variability in individual wages?

Solution:-

(i) From the given table,

Mean monthly wages of firm A = Rs 5253

and Number of wage earners = 586

Then,

Total amount paid = 586 × 5253

= Rs 3078258

Mean monthly wages of firm B = Rs 5253

Number of wage earners = 648

Then,

Total amount paid = 648 × 5253

= Rs 34,03,944

So, firm B pays larger amount as monthly wages.

(ii) Variance of firm A = 100

We know that, standard deviation (σ)= $\sqrt{100}$

=10

Variance of firm B = 121

Then,

Standard deviation (σ)=$\sqrt{121}$

=11

Hence, the standard deviation is more in case of Firm B. That means, in firm B, there is greater variability in individual wages.

4. The following is the record of goals scored by team A in a football session:

No. of goals scored	**0**	**1**	**2**	**3**	**4**
No. of matches	**1**	**9**	**7**	**5**	**3**

For team B, the mean number of goals scored per match was 2, with a standard deviation 1.25 goals. Find which team may be considered more consistent?

Solution:-

From the given data,

Let us make the table of the given data and append other columns after calculations.

Number of goals scored x_i	**Number of matches f_i**	**f_ix_i**	**X_i^2**	**$f_ix_i^2$**
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
Total	25	50		130

First we have to calculate Mean for Team A,

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{50}{25} = 2$

Then,

$$\text{Variance } = \frac{1}{N^2}\left[N\sum f_i x_i^2 - (\sum f_i x_i)^2\right]$$

$$= \frac{1}{25^2}[25 \times 130 - 2500] = \frac{750}{625} = 1.2$$

We know that, Standard deviation σ = √variance = √1.2 = 1.09

Hence co-efficient of variation of team A,

$$C.V._A = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.09}{2} \times 100 = 54.5$$

For team B

Given, $\bar{x}$ = 2

Standard deviation σ = 1.25

So, co-efficient of variation of team B,

$$\Rightarrow C.V._B = \frac{1.25}{2} \times 100 = 62.5$$

C.V. of firm B is greater.

∴ Team A is more consistent.

5. The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8, \sum_{i=1}^{50} y_i = 261, \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying, the length or weight?

Solution:-

First, we have to calculate Mean for Length x.

Mean = $\bar{x} = \frac{\sum x_i}{n} = \frac{212}{50} = 4.24$

Then,

$$\text{Variance} = \frac{1}{N^2}\left[N\sum f_i x_i^2 - (\sum f_i x_i)^2\right]$$

$= (1/50^2)\ [(50 \times 902.8) - 212^2]$

$= (1/2500)\ (45140 - 44944)$

$= 196/2500$

$= 0.0784$

We know that, Standard deviation σ = √variance

$= \sqrt{0.0784}$

$= 0.28$

Hence co-efficient of variation of team A,

$$C.V._x = \frac{\sigma}{\bar{x}} \times 100 = \frac{0.28}{4.24} \times 100 = 6.603$$

Now we have to calculate mean of Weight y

$\bar{y} = \sum y_i/n$

= 261/50

= 5.22

Then,

Variance = $(1/N^2)\ [(N\sum f_i y_i^2) - (\sum f_i y_i)^2]$

= $(1/50^2)\ [(50 \times 1457.6) - 261^2]$

= (1/2500) (72880 - 68121)

= 4759/2500

= 1.9036

We know that, Standard deviation σ = √variance

= √1.9036

= 1.37

So, co-efficient of variation of team B,

$$C.V._Y = \frac{\sigma}{\overline{x}} \times 100 = \frac{1.37}{5.22} \times 100 = 26.24$$

Since C.V. of firm weight y is greater

∴ Weight is more varying.