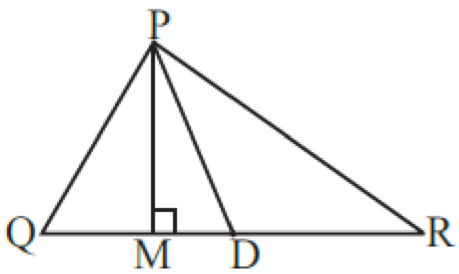
EXERCISE 6.1

PAGE: 116

1. In Δ PQR, D is the mid-point of $\overline{QR}$.



(i) $\overline{PM}$ is ___.

Solution:-

Altitude

An altitude has one endpoint at a vertex of the triangle and another on the line containing the opposite side.

(ii) PD is ___.

Solution:-

Median

A median connects a vertex of a triangle to the mid-point of the opposite side.

(iii) Is QM = MR?

Solution:-

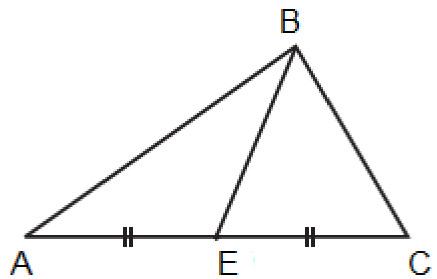
No, QM ≠ MR because D is the mid-point of QR.

2. Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

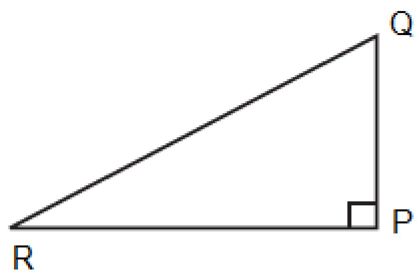
Solution:-

A median connects a vertex of a triangle to the mid-point of the opposite side.



(b) In ΔPQR, PQ and PR are altitudes of the triangle.

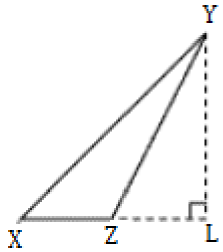
Solution:-



An altitude has one endpoint at a vertex of the triangle and another on the line containing the opposite side.

(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

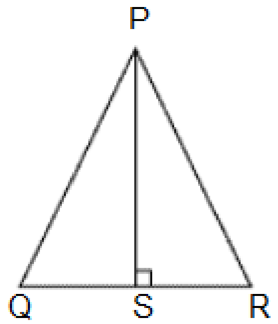
Solution:-



In the figure, we may observe that for ΔXYZ, YL is an altitude drawn exteriorly to side XZ which is extended up to point L.

3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be the same.

Solution:-

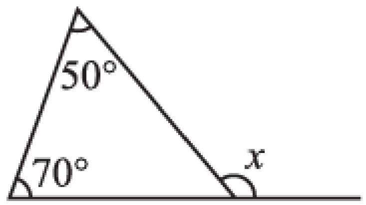


Draw a line segment PS ⊥ BC. It is an altitude for this triangle. Here, we observe that the length of QS and SR is also the same. So PS is also a median of this triangle.

EXERCISE 6.2 PAGE: 118

1. Find the value of the unknown exterior angle x in the following diagram:

(i)



Solution:-

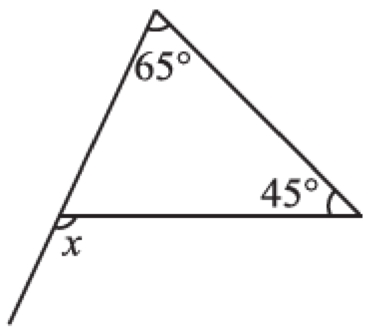
We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = 50° + 70°

= x = 120°

(ii)



Solution:-

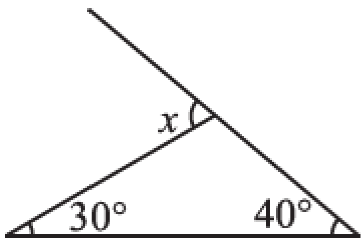
We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = 65° + 45°

= x = 110°

(iii)



Solution:-

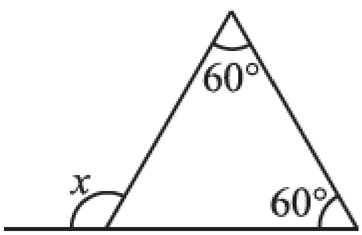
We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = 30° + 40°

= x = 70°

(iv)



Solution:-

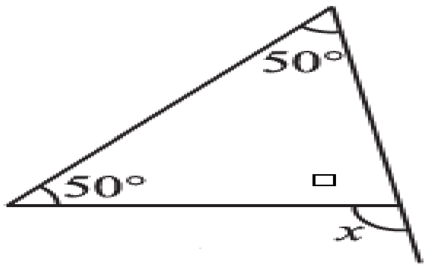
We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = 60° + 60°

= x = 120°

(v)



Solution:-

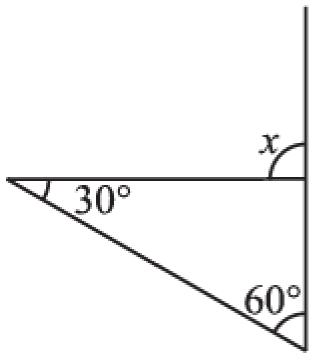
We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = $50^\circ + 50^\circ$

= x = 100°

(vi)



Solution:-

We know that,

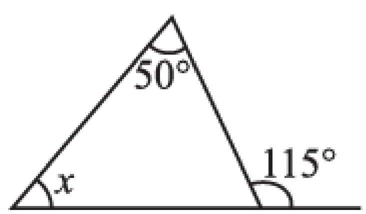
An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= x = $30^\circ + 60^\circ$

= x = 90°

2. Find the value of the unknown interior angle x in the following figures:

(i)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

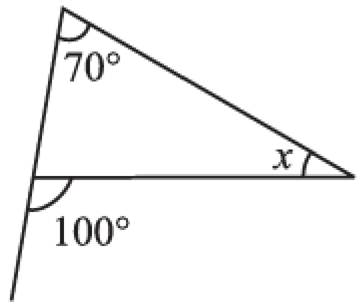
= x + 50° = 115°

By transposing 50° from LHS to RHS, it becomes – 50°

= x = 115° – 50°

= x = 65°

(ii)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

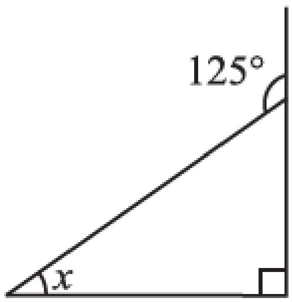
= 70° + x = 100°

By transposing 70° from LHS to RHS, it becomes – 70°

= x = 100° – 70°

= x = 30°

(iii)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right-angled triangle. So, the angle opposite to the x is 90°.

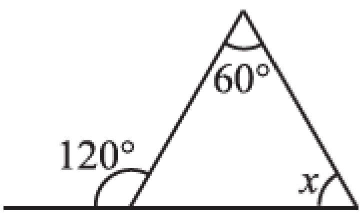
= x + 90° = 125°

By transposing 90° from LHS to RHS, it becomes – 90°

= x = 125° – 90°

= x = 35°

(iv)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

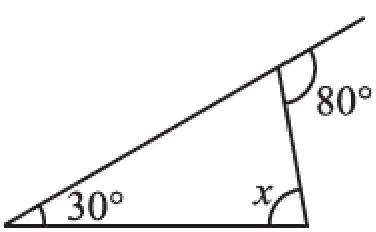
= x + 60° = 120°

By transposing 60° from LHS to RHS, it becomes – 60°

= x = 120° – 60°

= x = 60°

(v)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right-angled triangle. So, the angle opposite to the x is 90°.

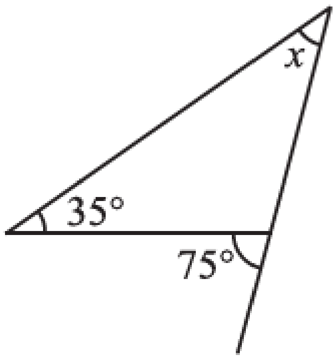
$= x + 30^\circ = 80^\circ$

By transposing 30° from LHS to RHS, it becomes $- 30^\circ$

$= x = 80^\circ - 30^\circ$

$= x = 50^\circ$

(vi)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right-angled triangle. So, the angle opposite to the x is 90°.

$= x + 35^\circ = 75^\circ$

By transposing 35° from LHS to RHS, it becomes $- 35^\circ$

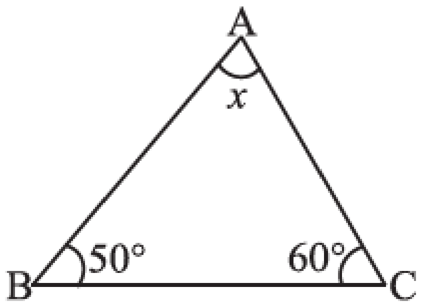
$= x = 75^\circ - 35^\circ$

$= x = 40^\circ$

EXERCISE 6.3 PAGE: 121

1. Find the value of the unknown x in the following diagrams:

(i)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= ∠BAC + ∠ABC + ∠BCA = 180°

= x + 50° + 60° = 180°

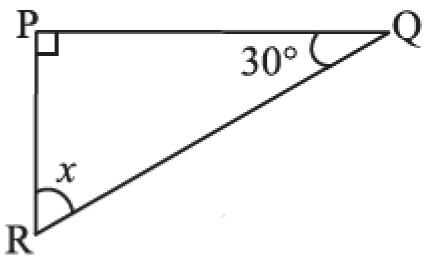
= x + 110° = 180°

By transposing 110° from LHS to RHS, it becomes – 110°

= x = 180° – 110°

= x = 70°

(ii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

The given triangle is a right-angled triangle. So, the ∠QPR is 90°.

Then,

= ∠QPR + ∠PQR + ∠PRQ = 180°

= 90° + 30° + x = 180°

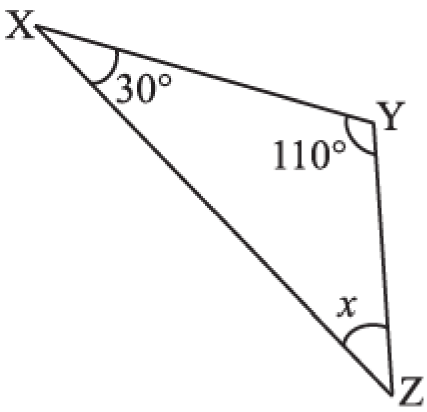
= 120° + x = 180°

By transposing 110° from LHS to RHS, it becomes – 110°

= x = 180° – 120°

= x = 60°

(iii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= ∠XYZ + ∠YXZ + ∠XZY = 180°

= 110° + 30° + x = 180°

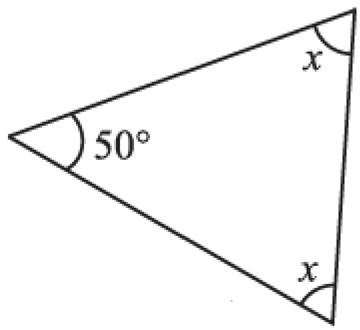
$= 140^\circ + x = 180^\circ$

By transposing 140° from LHS to RHS, it becomes $- 140^\circ$

$= x = 180^\circ - 140^\circ$

$= x = 40^\circ$

(iv)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= 50^\circ + x + x = 180^\circ$

$= 50^\circ + 2x = 180^\circ$

By transposing 50° from LHS to RHS, it becomes $- 50^\circ$

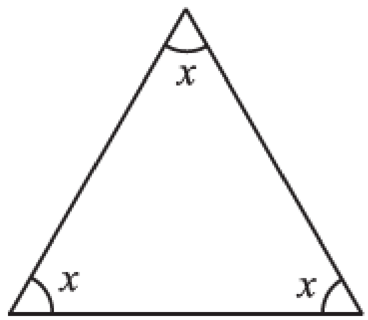
$= 2x = 180^\circ - 50^\circ$

$= 2x = 130^\circ$

$= x = 130^\circ/2$

$= x = 65^\circ$

(v)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= x + x + x = 180^\circ$

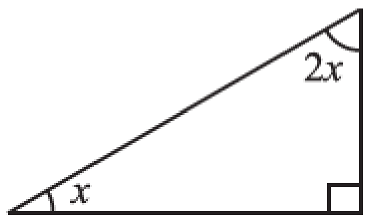
$= 3x = 180^\circ$

$= x = 180^\circ/3$

$= x = 60^\circ$

∴ the given triangle is an equiangular triangle.

(vi)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= 90^\circ + 2x + x = 180^\circ$

= $90^\circ + 3x = 180^\circ$

By transposing 90° from LHS to RHS, it becomes – 90°

= $3x = 180^\circ – 90^\circ$

= $3x = 90^\circ$

= $x = 90^\circ/3$

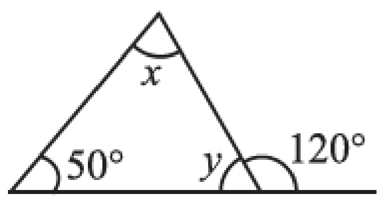
= $x = 30^\circ$

Then,

= $2x = 2 \times 30^\circ = 60^\circ$

2. Find the values of the unknowns x and y in the following diagrams:

(i)



Solution:-

We know that,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Then,

= $50^\circ + x = 120^\circ$

By transposing 50° from LHS to RHS, it becomes – 50°

= $x = 120^\circ – 50^\circ$

= $x = 70^\circ$

We also know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= $50^\circ + x + y = 180^\circ$

= $50^\circ + 70^\circ + y = 180^\circ$

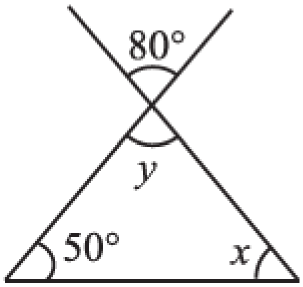
= $120^\circ + y = 180^\circ$

By transposing 120° from LHS to RHS, it becomes – 120°

= $y = 180^\circ - 120^\circ$

= $y = 60^\circ$

(ii)



Solution:-

From the rule of vertically opposite angles,

= $y = 80^\circ$

Then,

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= $50^\circ + 80^\circ + x = 180^\circ$

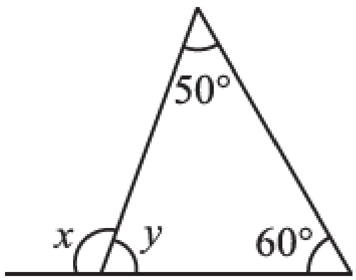
= $130^\circ + x = 180^\circ$

By transposing 130° from LHS to RHS, it becomes – 130°

= $x = 180^\circ - 130^\circ$

= $x = 50^\circ$

(iii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= 50° + 60° + y = 180°

= 110° + y = 180°

By transposing 110° from LHS to RHS, it becomes – 110°

= y = 180° – 110°

= y = 70°

Now,

From the rule of linear pair,

= x + y = 180°

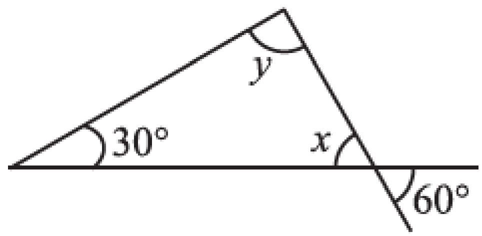
= x + 70° = 180°

By transposing 70° from LHS to RHS, it becomes – 70°

= x = 180° – 70

= x = 110°

(iv)



Solution:-

From the rule of vertically opposite angles,

= x = 60°

Then,

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= 30° + x + y = 180°

= 30° + 60° + y = 180°

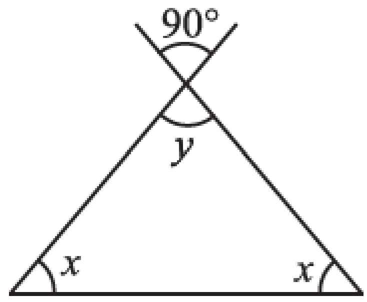
= 90° + y = 180°

By transposing 90° from LHS to RHS, it becomes – 90°

= y = 180° – 90°

= y = 90°

(v)



Solution:-

From the rule of vertically opposite angles,

= $y = 90^{\circ}$

Then,

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= $x + x + y = 180^{\circ}$

= $2x + 90^{\circ} = 180^{\circ}$

By transposing 90° from LHS to RHS, it becomes – 90°

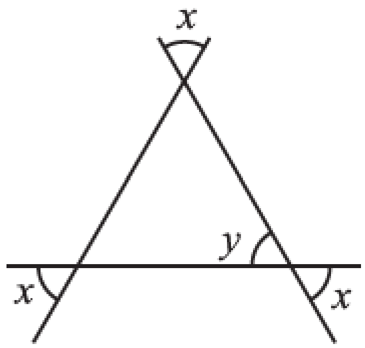
= $2x = 180^{\circ} – 90^{\circ}$

= $2x = 90^{\circ}$

= $x = 90^{\circ}/2$

= $x = 45^{\circ}$

(vi)



Solution:-

From the rule of vertically opposite angles,

= $x = y$

Then,

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= x + x + x = 180^\circ$

$= 3x = 180^\circ$

$= x = 180^\circ/3$

$= x = 60^\circ$

EXERCISE 6.4 PAGE: 126

1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm

Solution:-

Clearly, we have

(2 + 3) = 5

5 = 5

Thus, the sum of any two of these numbers is not greater than the third.

Hence, it is not possible to draw a triangle whose sides are 2 cm, 3 cm and 5 cm.

(ii) 3 cm, 6 cm, 7 cm

Solution:-

Clearly, we have

(3 + 6) = 9 > 7

(6 + 7) = 13 > 3

(7 + 3) = 10 > 6

Thus, the sum of any two of these numbers is greater than the third.

Hence, it is possible to draw a triangle whose sides are 3 cm, 6 cm and 7 cm.

(iii) 6 cm, 3 cm, 2 cm

Solution:-

Clearly, we have

(3 + 2) = 5 < 6

Thus, the sum of any two of these numbers is less than the third.

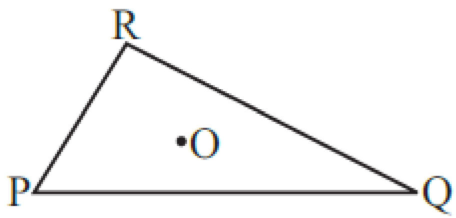
Hence, it is not possible to draw a triangle whose sides are 6 cm, 3 cm and 2 cm.

2. Take any point O in the interior of a triangle PQR. Is

(i) OP + OQ > PQ?

(ii) OQ + OR > QR?

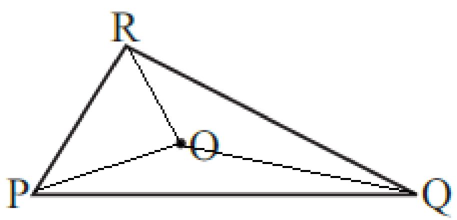
(iii) OR + OP > RP?



Solution:-

If we take any point O in the interior of a triangle PQR and join OR, OP, OQ.

Then, we get three triangles ΔOPQ, ΔOQR and ΔORP are shown in the figure below.



We know that,

The sum of the length of any two sides is always greater than the third side.

(i) Yes, ΔOPQ has sides OP, OQ and PQ.

So, OP + OQ > PQ

(ii) Yes, ΔOQR has sides OR, OQ and QR.

So, OQ + OR > QR

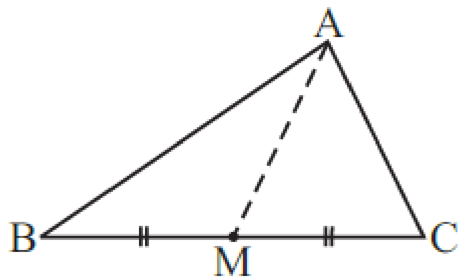
(iii) Yes, ΔORP has sides OR, OP and PR.

So, OR + OP > RP

3. AM is a median of a triangle ABC.

Is AB + BC + CA > 2 AM?

(Consider the sides of triangles ΔABM and ΔAMC.)



Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔABM,

Here, AB + BM > AM … [equation i]

Then, consider the ΔACM

Here, AC + CM > AM … [equation ii]

By adding equations [i] and [ii], we get,

AB + BM + AC + CM > AM + AM

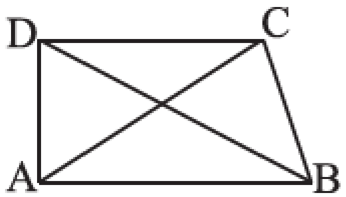
From the figure we have, BC = BM + CM

AB + BC + AC > 2 AM

Hence, the given expression is true.

4. ABCD is a quadrilateral.

Is AB + BC + CD + DA > AC + BD?



Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔABC,

Here, AB + BC > CA … [equation i]

Then, consider the ΔBCD

Here, BC + CD > DB … [equation ii]

Consider the ΔCDA

Here, CD + DA > AC … [equation iii]

Consider the ΔDAB

Here, DA + AB > DB … [equation iv]

By adding equations [i], [ii], [iii] and [iv], we get,

AB + BC + BC + CD + CD + DA + DA + AB > CA + DB + AC + DB

2AB + 2BC + 2CD + 2DA > 2CA + 2DB

Take out 2 on both the side,

2(AB + BC + CA + DA) > 2(CA + DB)

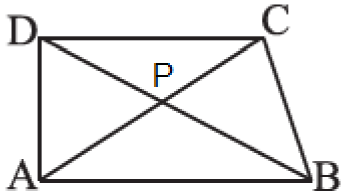
AB + BC + CA + DA > CA + DB

Hence, the given expression is true.

5. ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)

Solution:-

Let us consider ABCD as a quadrilateral, and P is the point where the diagonals intersect. As shown in the figure below.



We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔPAB,

Here, PA + PB < AB … [equation i]

Then, consider the ΔPBC

Here, PB + PC < BC … [equation ii]

Consider the ΔPCD

Here, PC + PD < CD … [equation iii]

Consider the ΔPDA

Here, PD + PA < DA … [equation iv]

By adding equations [i], [ii], [iii] and [iv], we get,

PA + PB + PB + PC + PC + PD + PD + PA < AB + BC + CD + DA

2PA + 2PB + 2PC + 2PD < AB + BC + CD + DA

2PA + 2PC + 2PB + 2PD < AB + BC + CD + DA

2(PA + PC) + 2(PB + PD) < AB + BC + CD + DA

From the figure, we have, AC = PA + PC and BD = PB + PD

Then,

2AC + 2BD < AB + BC + CD + DA

2(AC + BD) < AB + BC + CD + DA

Hence, the given expression is true.

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

From the question, it is given that two sides of the triangle are 12 cm and 15 cm.

So, the third side length should be less than the sum of the other two sides,

12 + 15 = 27 cm

Then, it is given that the third side can not be less than the difference of the two sides, 15 – 12 = 3 cm

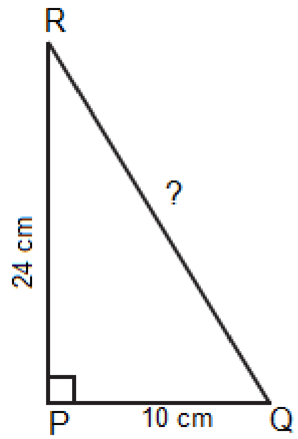
So, the length of the third side falls between 3 cm and 27 cm.

EXERCISE 6.5 PAGE: 130

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Solution:-

Let us draw a rough sketch of a right-angled triangle.



By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$QR^2 = PQ^2 + PR^2$

$QR^2 = 10^2 + 24^2$

$QR^2 = 100 + 576$

$QR^2 = 676$

$QR = \sqrt{676}$

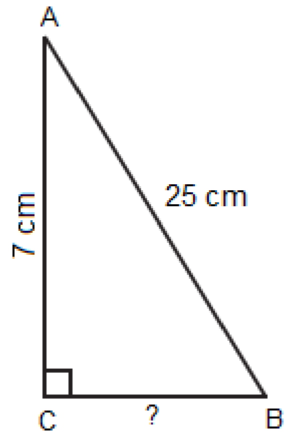
QR = 26 cm

Hence, the length of the hypotenuse QR = 26 cm

2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Solution:-

Let us draw a rough sketch of the right-angled triangle.



By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$AB^2 = AC^2 + BC^2$

$25^2 = 7^2 + BC^2$

$625 = 49 + BC^2$

By transposing 49 from RHS to LHS, it becomes – 49

$BC^2 = 625 – 49$

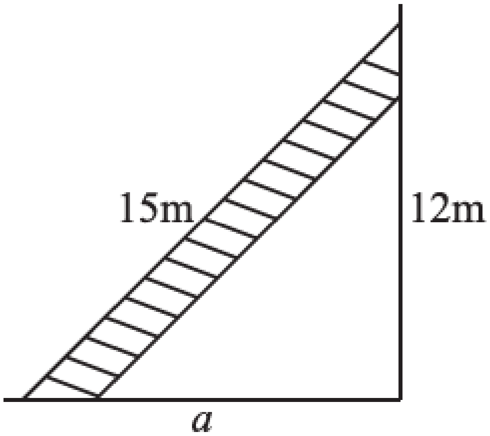
$BC^2 = 576$

$BC = \sqrt{576}$

BC = 24 cm

Hence, the length of the BC = 24 cm

3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Solution:-

By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$15^2 = 12^2 + a^2$

$225 = 144 + a^2$

By transposing 144 from RHS to LHS, it becomes – 144

$a^2 = 225 – 144$

$a^2 = 81$

$a = \sqrt{81}$

a = 9 m

Hence, the length of a = 9 m

4. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2cm, 2.5 cm

In the case of right-angled triangles, identify the right angles.

Solution:-

(i) Let a = 2.5 cm, b = 6.5 cm, c = 6 cm

Let us assume the largest value is the hypotenuse side, i.e., b = 6.5 cm.

Then, by Pythagoras' theorem,

$b^2 = a^2 + c^2$

$6.5^2 = 2.5^2 + 6^2$

$42.25 = 6.25 + 36$

$42.25 = 42.25$

The sum of squares of two sides of the triangle is equal to the square of the third side,

∴ the given triangle is a right-angled triangle.

The right angle lies on the opposite of the greater side, 6.5 cm.

(ii) Let a = 2 cm, b = 2 cm, c = 5 cm

Let us assume the largest value is the hypotenuse side, i.e. c = 5 cm.

Then, by Pythagoras' theorem,

$c^2 = a^2 + b^2$

$5^2 = 2^2 + 2^2$

$25 = 4 + 4$

$25 \neq 8$

The sum of squares of two sides of the triangle is not equal to the square of the third side,

∴ the given triangle is not a right-angled triangle.

(iii) Let a = 1.5 cm, b = 2 cm, c = 2.5 cm

Let us assume the largest value is the hypotenuse side, i.e., b = 2.5 cm.

Then, by Pythagoras' theorem,

$b^2 = a^2 + c^2$

$2.5^2 = 1.5^2 + 2^2$

$6.25 = 2.25 + 4$

$6.25 = 6.25$

The sum of squares of two sides of the triangle is equal to the square of the third side,

∴ the given triangle is a right-angled triangle.

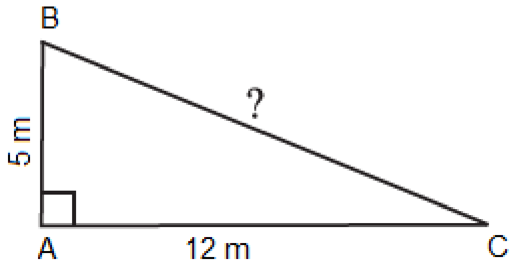
The right angle lies on the opposite of the greater side 2.5 cm.

5. A tree is broken at a height of 5 m from the ground, and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Solution:-

Let ABC is the triangle and B be the point where the tree is broken at the height of 5 m from the ground.

Treetop touches the ground at a distance of AC = 12 m from the base of the tree,



By observing the figure, we came to conclude that a right-angle triangle is formed at A.

From the rule of Pythagoras' theorem,

$BC^2 = AB^2 + AC^2$

$BC^2 = 5^2 + 12^2$

$BC^2 = 25 + 144$

$BC^2 = 169$

$BC = \sqrt{169}$

BC = 13 m

Then, the original height of the tree = AB + BC

= 5 + 13

= 18 m

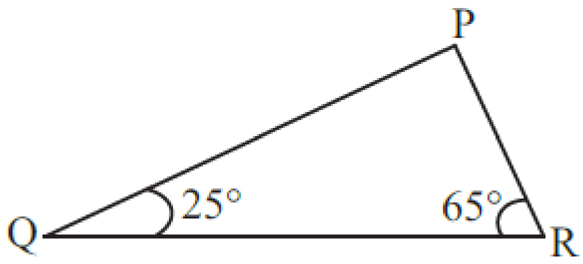
6. Angles Q and R of a ΔPQR are 25° and 65°.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Solution:-

Given that $\angle Q = 25^\circ$, $\angle R = 65^\circ$

Then, $\angle P$ =?

We know that sum of the three interior angles of a triangle is equal to 180°.

$\angle PQR + \angle QRP + \angle RPQ = 180^\circ$

$25^\circ + 65^\circ + \angle RPQ = 180^\circ$

$90^\circ + \angle RPQ = 180^\circ$

$\angle RPQ = 180 - 90$

$\angle RPQ = 90^\circ$

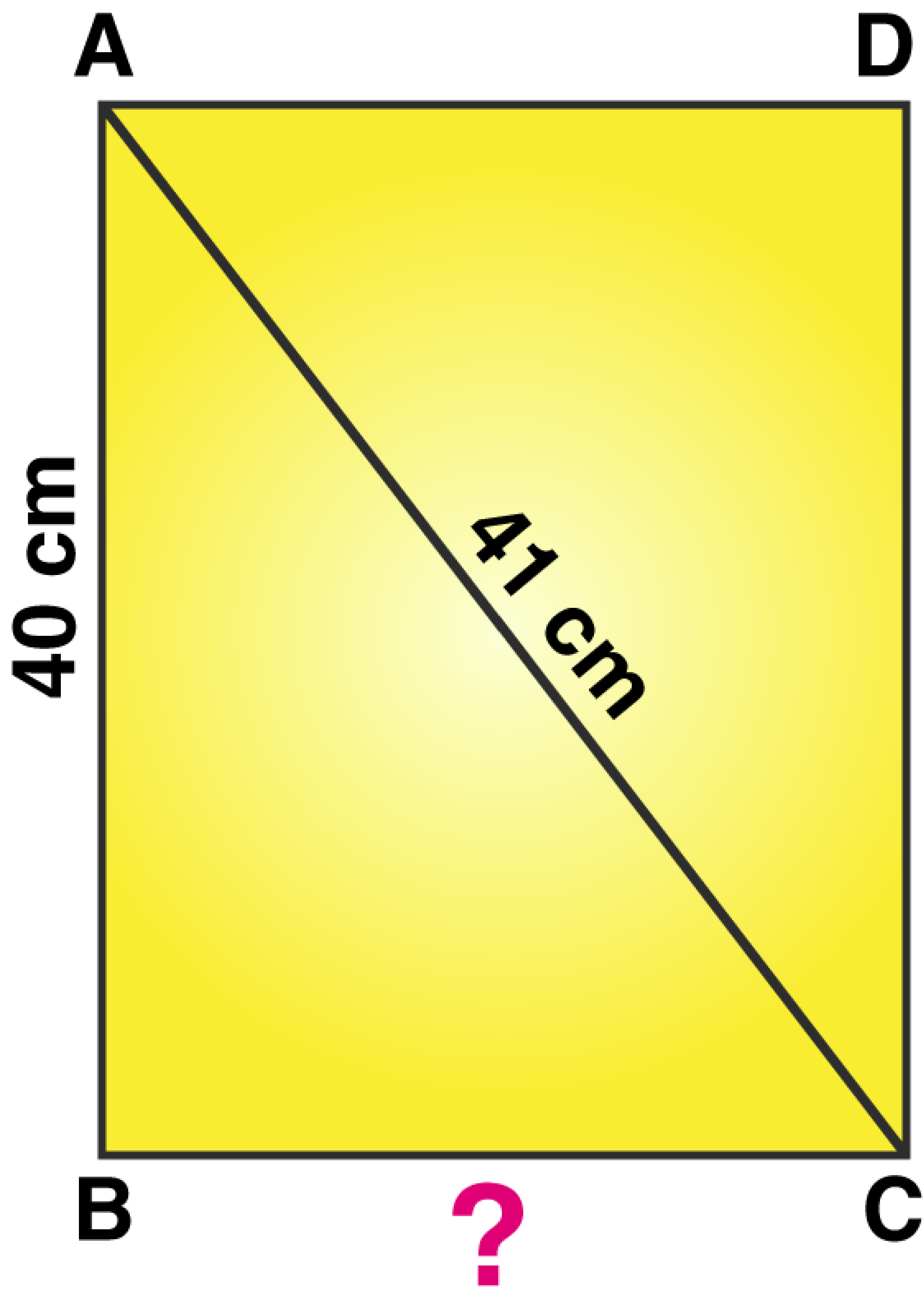
Also, we know that the side opposite to the right angle is the hypotenuse.

$\therefore QR^2 = PQ^2 + PR^2$

Hence, (ii) is true.

7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Solution:-


A
D
40 cm
41 cm
B
?
C

Let ABCD be the rectangular plot.

Then, AB = 40 cm and AC = 41 cm

BC =?

According to Pythagoras' theorem,

From the right angle triangle ABC, we have

= $AC^2 = AB^2 + BC^2$

= $41^2 = 40^2 + BC^2$

= $BC^2 = 41^2 – 40^2$

= $BC^2 = 1681 – 1600$

= $BC^2 = 81$

= BC = $\sqrt{81}$

= BC = 9 cm

Hence, the perimeter of the rectangle plot = 2 (length + breadth)

Where, length = 40 cm, breadth = 9 cm

Then,

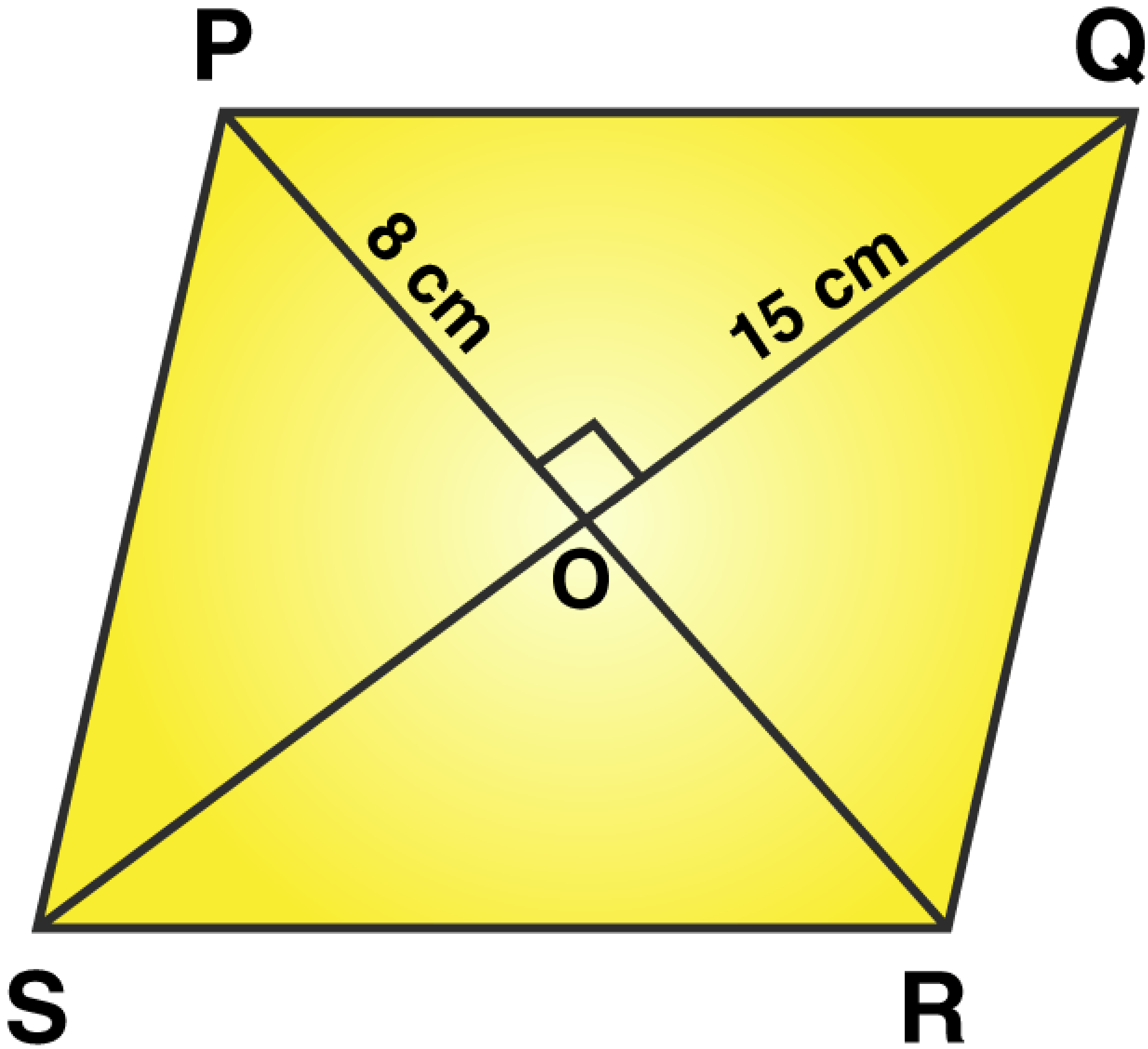
= 2(40 + 9)

= 2 × 49

= 98 cm

8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Solution:-



Let PQRS be a rhombus, all sides of the rhombus have equal length, and its diagonal PR and SQ are intersecting each other at point O. Diagonals in the rhombus bisect each other at 90°.

So, PO = (PR/2)

= 16/2

= 8 cm

And, SO = (SQ/2)

= 30/2

= 15 cm

Then, consider the triangle POS and apply the Pythagoras theorem,

$PS^2 = PO^2 + SO^2$

$PS^2 = 8^2 + 15^2$

$PS^2 = 64 + 225$

$PS^2 = 289$

$PS = \sqrt{289}$

PS = 17 cm

Hence, the length of the side of the rhombus is 17 cm

Now,

The perimeter of the rhombus = 4 × side of the rhombus

= 4 × 17

= 68 cm

∴ the perimeter of the rhombus is 68 cm.