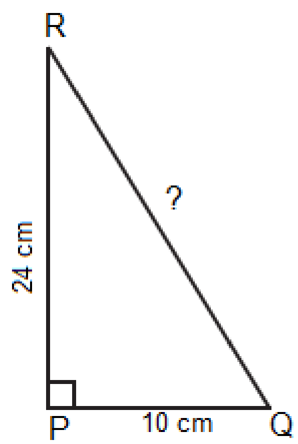
EXERCISE 6.5 PAGE: 130

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Solution:-

Let us draw a rough sketch of a right-angled triangle.



By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$QR^2 = PQ^2 + PR^2$

$QR^2 = 10^2 + 24^2$

$QR^2 = 100 + 576$

$QR^2 = 676$

$QR = \sqrt{676}$

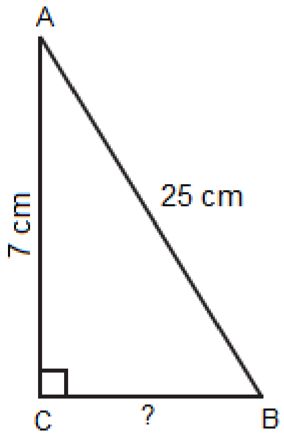
QR = 26 cm

Hence, the length of the hypotenuse QR = 26 cm

2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Solution:-

Let us draw a rough sketch of the right-angled triangle.



By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$AB^2 = AC^2 + BC^2$

$25^2 = 7^2 + BC^2$

$625 = 49 + BC^2$

By transposing 49 from RHS to LHS, it becomes – 49

$BC^2 = 625 – 49$

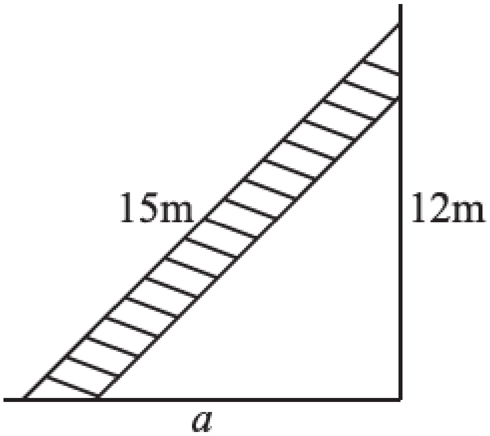
$BC^2 = 576$

$BC = \sqrt{576}$

BC = 24 cm

Hence, the length of the BC = 24 cm

3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Solution:-

By the rule of Pythagoras' Theorem,

Pythagoras' theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of squares on the legs.

In the above figure, RQ is the hypotenuse,

$15^2 = 12^2 + a^2$

$225 = 144 + a^2$

By transposing 144 from RHS to LHS, it becomes – 144

$a^2 = 225 – 144$

$a^2 = 81$

$a = \sqrt{81}$

a = 9 m

Hence, the length of a = 9 m

4. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2cm, 2.5 cm

In the case of right-angled triangles, identify the right angles.

Solution:-

(i) Let a = 2.5 cm, b = 6.5 cm, c = 6 cm

Let us assume the largest value is the hypotenuse side, i.e., b = 6.5 cm.

Then, by Pythagoras' theorem,

$b^2 = a^2 + c^2$

$6.5^2 = 2.5^2 + 6^2$

42.25 = 6.25 + 36

42.25 = 42.25

The sum of squares of two sides of the triangle is equal to the square of the third side,

∴ the given triangle is a right-angled triangle.

The right angle lies on the opposite of the greater side, 6.5 cm.

(ii) Let a = 2 cm, b = 2 cm, c = 5 cm

Let us assume the largest value is the hypotenuse side, i.e. c = 5 cm.

Then, by Pythagoras' theorem,

$c^2 = a^2 + b^2$

$5^2 = 2^2 + 2^2$

25 = 4 + 4

$25 \neq 8$

The sum of squares of two sides of the triangle is not equal to the square of the third side,

∴ the given triangle is not a right-angled triangle.

(iii) Let a = 1.5 cm, b = 2 cm, c = 2.5 cm

Let us assume the largest value is the hypotenuse side, i.e., b = 2.5 cm.

Then, by Pythagoras' theorem,

$b^2 = a^2 + c^2$

$2.5^2 = 1.5^2 + 2^2$

6.25 = 2.25 + 4

6.25 = 6.25

The sum of squares of two sides of the triangle is equal to the square of the third side,

∴ the given triangle is a right-angled triangle.

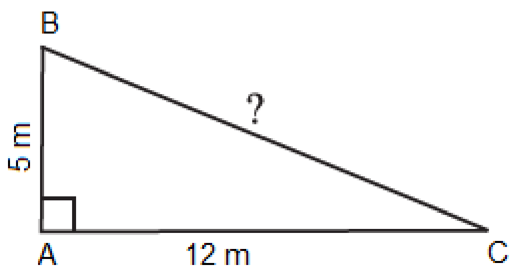
The right angle lies on the opposite of the greater side 2.5 cm.

5. A tree is broken at a height of 5 m from the ground, and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Solution:-

Let ABC is the triangle and B be the point where the tree is broken at the height of 5 m from the ground.

Treetop touches the ground at a distance of AC = 12 m from the base of the tree,



By observing the figure, we came to conclude that a right-angle triangle is formed at A.

From the rule of Pythagoras' theorem,

$BC^2 = AB^2 + AC^2$

$BC^2 = 5^2 + 12^2$

$BC^2 = 25 + 144$

$BC^2 = 169$

$BC = \sqrt{169}$

BC = 13 m

Then, the original height of the tree = AB + BC

= 5 + 13

= 18 m

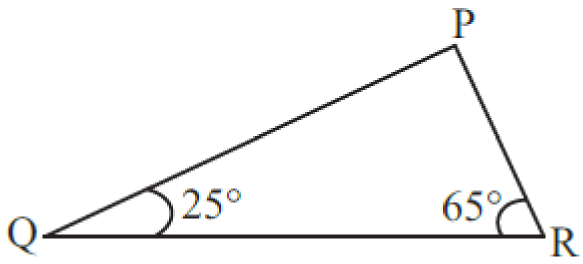
6. Angles Q and R of a ΔPQR are 25° and 65°.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Solution:-

Given that $\angle Q = 25^\circ$, $\angle R = 65^\circ$

Then, $\angle P = ?$

We know that sum of the three interior angles of a triangle is equal to 180°.

$\angle PQR + \angle QRP + \angle RPQ = 180^\circ$

$25^\circ + 65^\circ + \angle RPQ = 180^\circ$

$90^\circ + \angle RPQ = 180^\circ$

$\angle RPQ = 180 - 90$

$\angle RPQ = 90^\circ$

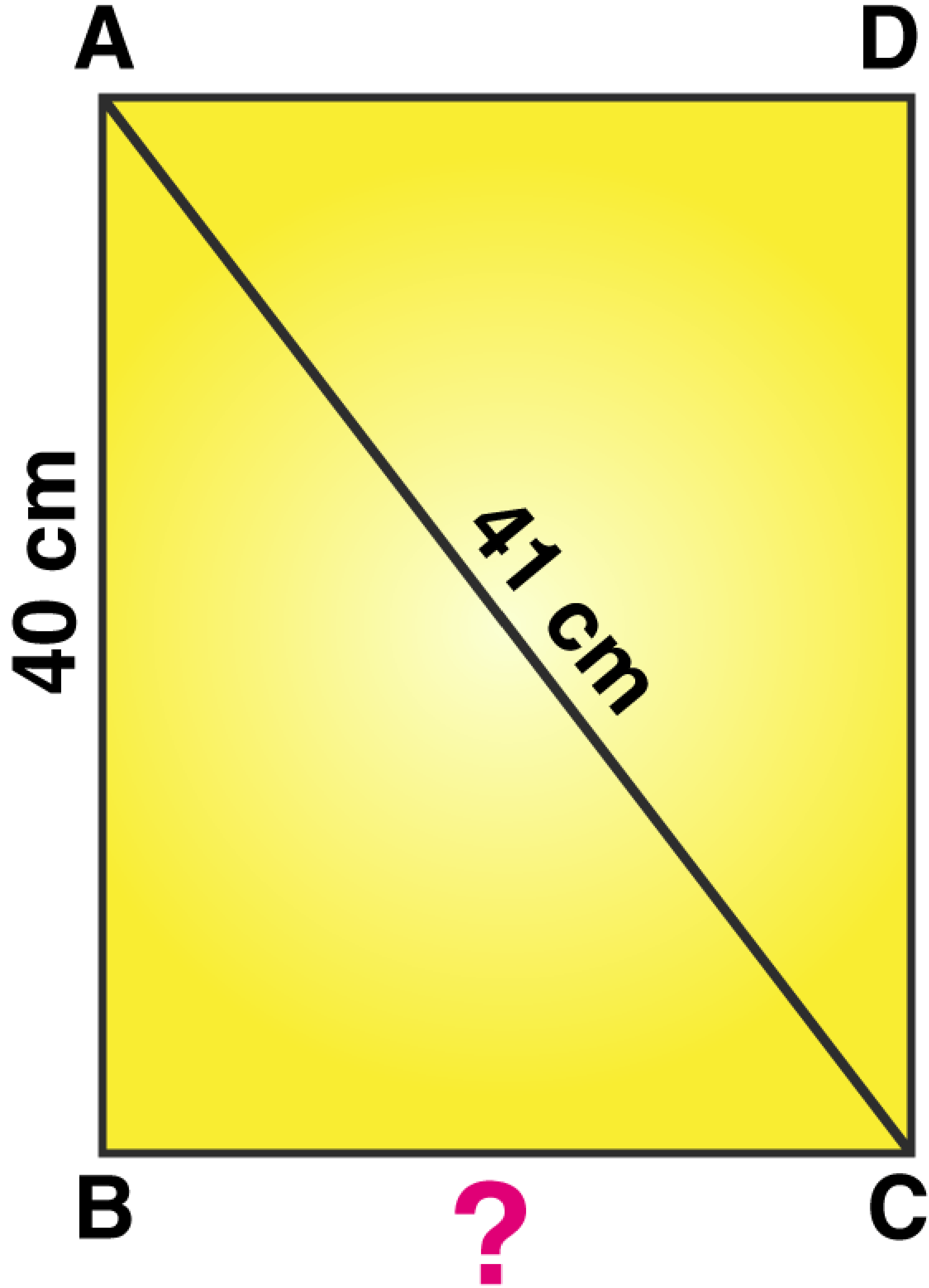
Also, we know that the side opposite to the right angle is the hypotenuse.

$\therefore QR^2 = PQ^2 + PR^2$

Hence, (ii) is true.

7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Solution:-


A
D
40 cm
41 cm
B
?
C

Let ABCD be the rectangular plot.

Then, AB = 40 cm and AC = 41 cm

BC =?

According to Pythagoras' theorem,

From the right angle triangle ABC, we have

= $AC^2 = AB^2 + BC^2$

= $41^2 = 40^2 + BC^2$

= $BC^2 = 41^2 - 40^2$

= $BC^2 = 1681 - 1600$

= $BC^2 = 81$

= $BC = \sqrt{81}$

= BC = 9 cm

Hence, the perimeter of the rectangle plot = 2 (length + breadth)

Where, length = 40 cm, breadth = 9 cm

Then,

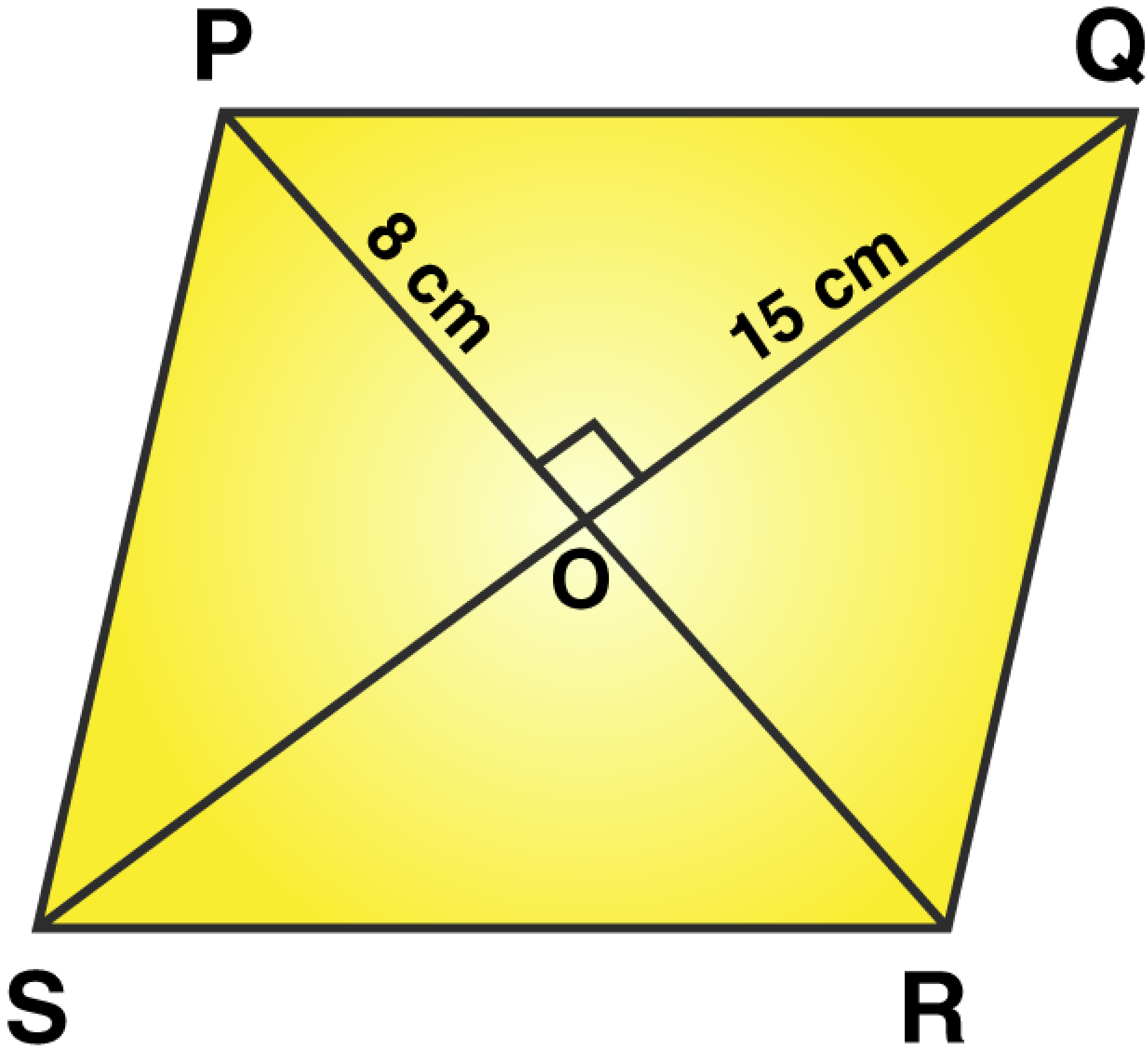
= 2(40 + 9)

= 2×49

= 98 cm

8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Solution:-



Let PQRS be a rhombus, all sides of the rhombus have equal length, and its diagonal PR and SQ are intersecting each other at point O. Diagonals in the rhombus bisect each other at 90°.

So, PO = (PR/2)

= 16/2

= 8 cm

And, SO = (SQ/2)

= 30/2

= 15 cm

Then, consider the triangle POS and apply the Pythagoras theorem,

$PS^2 = PO^2 + SO^2$

$PS^2 = 8^2 + 15^2$

$PS^2 = 64 + 225$

$PS^2 = 289$

$PS = \sqrt{289}$

PS = 17 cm

Hence, the length of the side of the rhombus is 17 cm

Now,

The perimeter of the rhombus = 4 × side of the rhombus

= 4 × 17

= 68 cm

∴ the perimeter of the rhombus is 68 cm.