

Q1. In which of the following examples of motion can the body be considered approximately a point object?

- (a) A railway carriage moving without jerks between two stations.
- (b) A cap on top of a man cycling smoothly on a circular track.
- (c) A spinning cricket ball that turns sharply on hitting the ground.
- (d) A tumbling beaker that has slipped off the edge of a table.

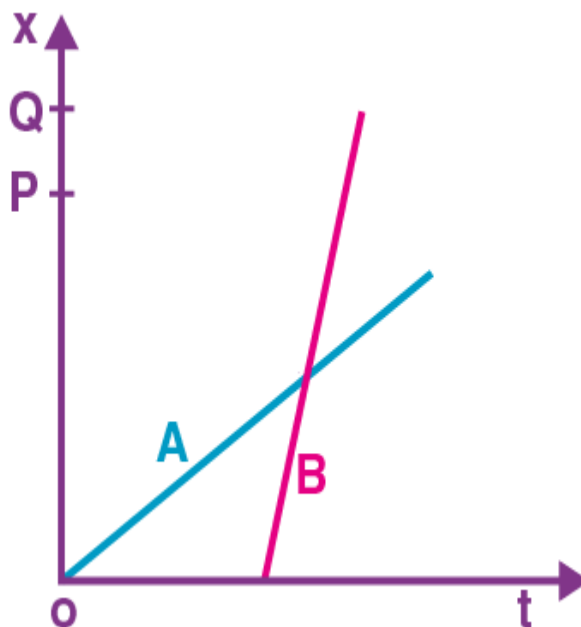
Ans.

(a), (b) The size of the railway carriage and the cap is very small as compared to the distance they've travelled, i.e. the distance between the two stations and the length of the race track, respectively. Therefore, the cap and the carriage can be considered as point objects.

The size of the cricket ball is comparable to the distance through which it bounces off after hitting the floor. Thus, the cricket ball cannot be treated as a point object. Likewise, the size of the beaker is comparable to the height of the table from which it drops. Thus, the beaker cannot be treated as a point object.

Q2. The position-time ($x-t$) graphs for two children, A and B, returning from their school O to their homes, P and Q, respectively, are shown in Fig. Choose the correct entries in the brackets below:

- (a) (A/B) lives closer to the school than (B/A)
- (b) (A/B) starts from the school earlier than (B/A)
- (c) (A/B) walks faster than (B/A)
- (d) A and B reach home at the (same/different) time
- (e) (A/B) overtakes (B/A) on the road (once/twice).

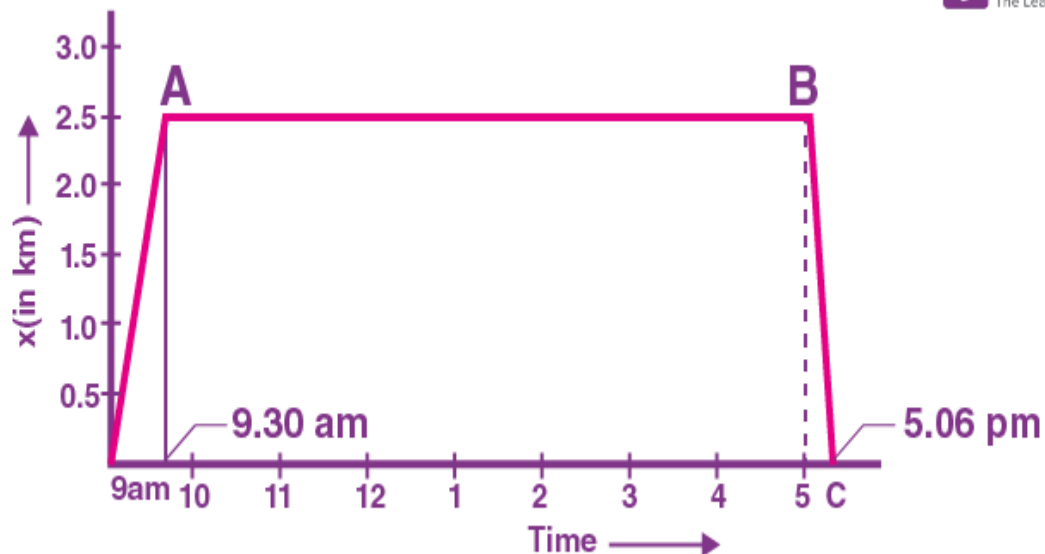


Ans.

- (a) A lives closer to the school than B, because A has to cover shorter distances [$OP < OQ$].
- (b) A starts from school earlier than B, because for $x=0$, $t=0$ for A but for B, t has some finite time.
- (c) The slope of B is greater than that of A; therefore, B walks faster than A.

- (d) Both A and B will reach their home at the same time.
(e) At the point of intersection, B overtakes A on the roads once.

Q3. A woman starts from her home at 9.00 am, walks at a speed of 5 km/h on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by auto at a speed of 25 km/h. Choose suitable scales and plot the x-t graph of her motion.



Ans.

Distance to her office = 2.5 km.

Walking speed the woman = 5 km/h

Time taken to reach office while walking = $(2.5/5) \text{ h} = (1/2) \text{ h} = 30 \text{ minutes}$

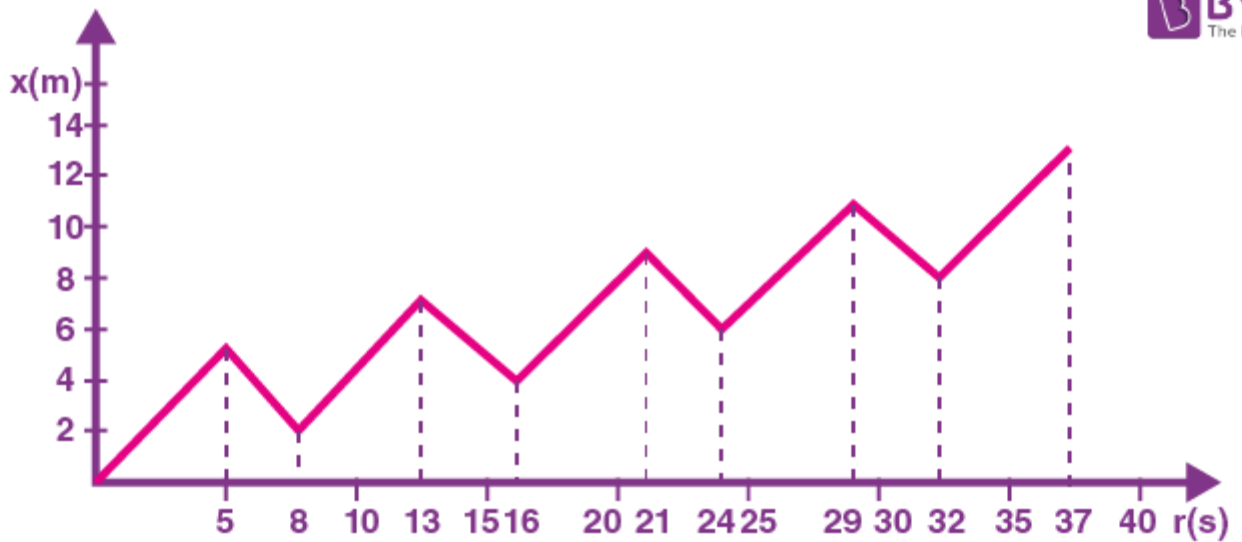
Speed of auto = 25 km/h

Time taken to reach home in auto = $2.5/25 = (1/10) \text{ h} = 0.1 \text{ h} = 6 \text{ minutes}$

In the graph, O is taken as the origin of the distance and the time, then at $t = 9.00 \text{ am}$, $x = 0$ and at $t = 9.30 \text{ am}$, $x = 2.5 \text{ km}$

OA is the portion on the x-t graph that represents her walk from home to the office. AB represents her time of stay in the office from 9.30 to 5. Her return journey is represented by BC.

Q4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backwards, followed again by 5 steps forward and 3 steps backwards, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.



Ans.

The time taken to go one step is 1 second. In 5 s, he moves forward through a distance of 5 m, and then in the next 3s, he comes back by 3 m. Therefore, in 8 s, he covers 2 m. So, to cover a distance of 8 m, he takes 32 s. He must take another 5 steps forward to fall into the pit. So, the total time taken is $32\text{ s} + 5\text{ s} = 37\text{ s}$ to fall into a pit 13 m away.

Q5. A jet aeroplane travelling at the speed of 500 km/h ejects its products of combustion at the speed of 1500 km/h relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?

Ans.

Speed of the jet aeroplane, $V_A = 500\text{ km/h}$

The speed at which the combustion products are ejected relative to the jet plane, $V_B - V_A$

$$= -1500\text{ km/h}$$

(The negative sign indicates that the combustion products move in a direction opposite to that of the jet)

Speed of combustion products w.r.t. observer on the ground, $V_B - 500 = -1500$

$$V_B = -1500 + 500 = -1000\text{ km/h}$$

Q6. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Ans.

The initial velocity of the car = u

The final velocity of the car = v

Distance covered by the car before coming to rest = 200 m

Using the equation,

$$v = u + at$$

$$t = (v - u)/a = 11.44\text{ sec.}$$

Therefore, it takes 11.44 sec for the car to stop.

Q7. Two trains, A and B, of length 400 m each, are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m s^{-2} . If, after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

Ans.

Length of trains A and B = 400 m

Speed of both the trains = $72 \text{ km/h} = 72 \times (5/18) = 20 \text{ m/s}$

Using the relation, $s = ut + (1/2)at^2$

Distance covered by the train B

$$S_B = u_B t + (1/2)at^2$$

Acceleration, $a = 1 \text{ m/s}^2$

Time = 50 s

$$S_B = (20 \times 50) + (1/2) \times 1 \times (50)^2$$

$$= 2250 \text{ m}$$

Distance covered by the train A

$$S_A = u_A t + (1/2)at^2$$

Acceleration, $a = 0$

$$S_A = u_A t = 20 \times 50 = 1000 \text{ m}$$

Therefore, the original distance between the two trains = $S_B - S_A = 2250 - 1000 = 1250 \text{ m}$

Q8. On a two-lane road, car A is travelling at a speed of 36 km/h . Two cars, B and C, approach car A in opposite directions with a speed of 54 km/h each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What is the minimum acceleration of car B required to avoid an accident?

Ans:

The speed of car A = $36 \text{ km/h} = 36 \times (5/18) = 10 \text{ m/s}$

Speed of car B = $54 \text{ km/h} = 54 \times (5/18) = 15 \text{ m/s}$

Speed of car C = $-54 \text{ km/h} = -54 \times (5/18) = -15 \text{ m/s}$ (negative sign shows B and C are in opposite directions)

Relative speed of A w.r.t. C, $V_{AC} = V_A - V_C = 10 - (-15) = 25 \text{ m/s}$

Relative speed of B w.r.t. A, $V_{BA} = V_B - V_A = 15 - 10 = 5 \text{ m/s}$

Distance between AB = Distance between AC = 1 km = 1000 m

Time taken by the car C to cover the distance AC, $t = 1000/V_{AC} = 1000/25 = 40 \text{ s}$

If a is the acceleration, then

$$s = ut + (1/2) at^2$$

$$1000 = (5 \times 40) + (1/2) a (40)^2$$

$$a = (1000 - 200) / 800 = 1 \text{ m/s}^2$$

Thus, the minimum acceleration of car B required to avoid an accident is 1 m/s^2

Q9. Two towns, A and B, are connected by regular bus service, with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min in the opposite direction. What is the period T of the bus service, and with what speed (assumed constant) do the buses ply on the road?

Ans.

Speed of each bus = V_b

Speed of the cyclist = $V_c = 20 \text{ km/h}$

The relative velocity of the buses plying in the direction of motion of the cyclist is $V_b - V_c$.

The buses playing in the direction of motion of the cyclist go past him after every 18 minutes, i.e. $(18/60) \text{ s}$.

$$\text{Distance covered} = (V_b - V_c) \times 18/60$$

Since the buses are leaving every T minutes. Therefore, the distance is equal to $V_b \times (T/60)$

$$(V_b - V_c) \times 18/60 = V_b \times (T/60) \text{ ---(1)}$$

The relative velocity of the buses plying in the direction opposite to the motion of the cyclist is $V_b + V_c$.

The buses go past the cyclist every 6 minutes, i.e. $(6/60) \text{ s}$.

$$\text{Distance covered} = (V_b + V_c) \times 6/60$$

$$\text{Therefore, } (V_b + V_c) \times 6/60 = V_b \times (T/60) \text{ ---(2)}$$

Dividing (2) by (1)

$$[(V_b - V_c) \times 18/60] / [(V_b + V_c) \times 6/60] = [V_b \times (T/60)] / [V_b \times (T/60)]$$

$$(V_b - V_c) 18 / (V_b + V_c) 6 = 1$$

$$(V_b - V_c) 3 = (V_b + V_c)$$

Substituting the value of V_c

$$(V_b - 20) 3 = (V_b + 20)$$

$$3V_b - 60 = V_b + 20$$

$$2V_b = 80$$

$$V_b = 80/2 = 40 \text{ km/h}$$

To find the value of T, substitute the values of V_b and V_c in equation (1)

$$(V_b - V_c) \times 18/60 = V_b \times (T/60)$$

$$(40 - 20) \times (18/60) = 40 \times (T/60)$$

$$T = (20 \times 18) / 40 = 9 \text{ minutes}$$

Q10. A player throws a ball upwards with an initial speed of 29.4 m/s .

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) Choose the $x = 0$ m and $t = 0$ s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of the x -axis, and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(d) To what height does the ball rise, and after how long does the ball return to the player's hands? (Take $g = 9.8 \text{ m s}^{-2}$ and neglect air resistance).

Ans.

(a) The acceleration due to gravity always acts downwards towards the centre of the Earth.

(b) At the highest point of its motion, the velocity of the ball will be zero, but the acceleration due to gravity will be 9.8 m s^{-2} acting vertically downward.

(c) If we consider the highest point of ball motion as $x = 0$, $t = 0$, and vertically downward direction to be +ve direction of the x -axis, then

(i) During the upward motion of the ball before reaching the highest point position, $x = +ve$, velocity, $v = -ve$ and acceleration, $a = +ve$.

(ii) During the downward motion of the ball after reaching the highest point position, velocity and acceleration, all three quantities are positive.

(d) Initial speed of the ball, $u = -29.4 \text{ m/s}$

The final velocity of the ball, $v = 0$

Acceleration = 9.8 m/s^2

Applying in the equation $v^2 - u^2 = 2gs$

$$0 - (-29.4)^2 = 2(9.8)s$$

$$s = -864.36/19.6 = -44.1$$

Height to which the ball rises = -44.1 m (negative sign represents upward direction)

Considering the equation of motion

$$v = u + at$$

$$0 = (-29.4) + 9.8t$$

$$t = 29.4/9.8 = 3 \text{ seconds}$$

Therefore, the total time taken for the ball to return to the player's hands is $3 + 3 = 6 \text{ s}$.

Q11. Read each statement below carefully and state with reasons and examples, if it is true or false; A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant

(b) with zero speed may have non-zero velocity

(c) with constant speed must have zero acceleration

(d) with positive value of acceleration must be speeding up

Ans.

(a) True

(b) False

(c) True (if the particle rebounds instantly with the same speed, it implies infinite acceleration, which is unphysical)

(d) False (true only when the chosen positive direction is along the direction of motion)

Q12. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s.

Ans.

Height from which the ball is dropped = 90 m

The initial velocity of the ball, $u = 0$

Let v be the final velocity of the ball

Using the equation

$$v^2 - u^2 = 2as \text{ ——(1)}$$

$$v_1^2 - 0 = 2 \times 10 \times 90$$

$$v_1 = 42.43 \text{ m/s}$$

Time taken for the first collision can be given by the equation

$$v = u + at$$

$$42.43 = 0 + (10) t$$

$$t_1 = 4.24 \text{ s}$$

The ball loses one-tenth of the velocity at collision. So, the rebound velocity of the ball is

$$v_2 = v - (1/10)v$$

$$v_2 = (9/10) v$$

$$v_2 = (9/10) (42.43)$$

$$= 38.19 \text{ m/s}$$

Time taken to reach maximum height after the first collision is

$$v = u + at$$

$$38.19 = 0 + (10)t_2$$

$$t_2 = 3.819 \text{ s}$$

The total time taken by the ball to reach the maximum height is

$$T = t_1 + t_2$$

$$T = 4.24 + 3.819 = 8.05 \text{ s}$$

Now the ball will travel back to the ground in the same time as it took to reach the maximum height = 3.819 s

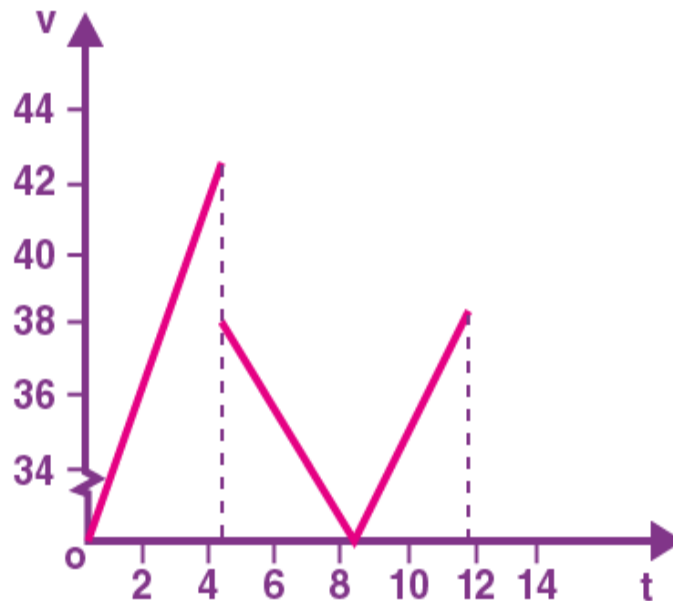
$$\text{Total time taken will be, } T = 4.24 + 3.819 + 3.819 = 11.86$$

Velocity after the second collision

$$v_3 = (9/10) (38.19)$$

$$v_3 = 34.37 \text{ m/s}$$

Using the above information, the speed time graph can be plotted



Q13. Provide clear explanations and examples to distinguish between:

(a) The total length of a path covered by a particle and the magnitude of displacement over the same interval of time.

(b) The magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].

In (a) and (b), compare and find which of the two quantities is greater.

When can the given quantities be equal? [For simplicity, consider one-dimensional motion only].

Ans.

(a) Let us consider an example of a football; it is passed to player B by player A and then instantly kicked back to player A along the same path. Now, the magnitude of displacement of the ball is 0 because it has returned to its initial position. However, the total length of the path covered by the ball = AB +BA = 2AB. Hence, it is clear that the first quantity is greater than the second.

(b) Taking the above example, let us assume that football takes t seconds to cover the total distance. Then,

The magnitude of the average velocity of the ball over time interval t = Magnitude of displacement/time interval

$$= 0 / t = 0.$$

The average speed of the ball over the same interval = total length of the path/time interval

$$= 2AB/t$$

Thus, the second quantity is greater than the first.

The above quantities are equal if the ball moves only in one direction from one player to another (considering one-dimensional motion).

Q14. A man walks on a straight road from his home to a market 2.5 km away at a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h⁻¹. What is the

(a) Magnitude of average velocity, and

(b) Average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min? [Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time and not as the magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero !]

Ans.

Distance to the market = 2.5 km = 2500 m

Speed of the man walking to the market = 5 km/h = $5 \times (5/18) = 1.388$ m/s

Speed of the man walking when he returns back home = 7.5 km/h = $7.5 \times (5/18) = 2.08$ m/s

(a) Magnitude of the average speed is zero since the displacement is zero

(b)

(i) Time taken to reach the market = Distance/Speed = $2500/1.388 = 1800$ seconds = 30 minutes

So, the average speed over 0 to 30 minutes is 5 km/h or 1.388 m/s

(ii) Time taken to reach back home = Distance/Speed = $2500/2.08 = 1200$ seconds = 20 minutes

So, the average speed is

Average speed over a interval of 50 minutes = distance covered/time taken = $(2500 + 2500)/3000 = 5000/3000 = 5/3 = 1.66$ m/s

= 6 km/h

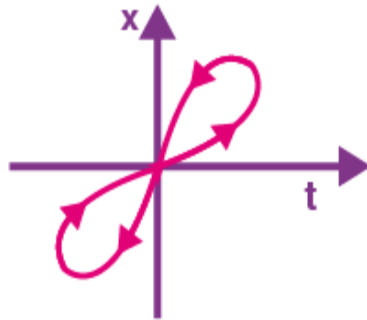
(ii) Average speed over an interval of 0-40 minutes = distance covered/ time taken = $(2500+ 1250)/2400 = 1.5625$ seconds = 5.6 km/h

Q15. In Exercises 3.13 and 3.14, we have carefully distinguished between average speed and the magnitude of average velocity. No such distinction is necessary when we consider the instantaneous speed and the magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

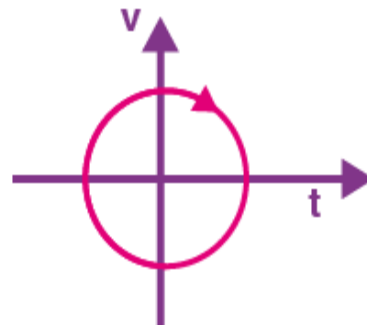
Ans.

Instantaneous velocity and instantaneous speed are equal for a small interval of time because the magnitude of the displacement is effectively equal to the distance travelled by the particle.

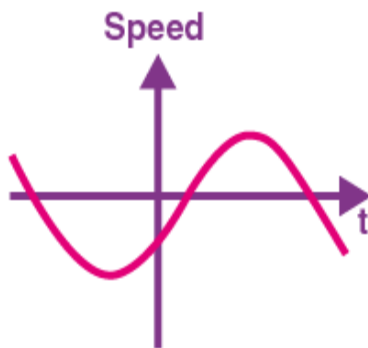
Q16. Look at the graphs (a) to (d) carefully and state, with reasons, which of these cannot possibly represent the one-dimensional motion of a particle?



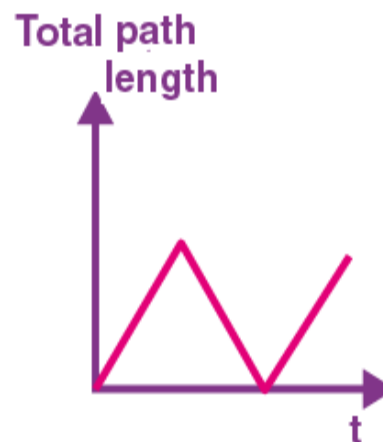
(a)



(b)



(c)



(d)

Ans.

None of the four graphs shows a one-dimensional motion.

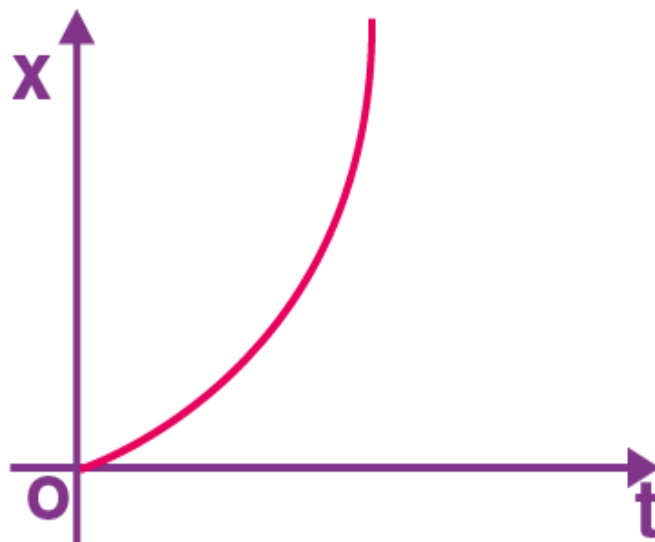
(a) Shows two positions at the same time, which is not possible.

(b) A particle cannot have velocity in two directions at the same time.

(c) Graph shows negative speed, which is impossible. Speed is always positive.

(d) Path length decreases in the graph; this is also not possible.

Q17. The figure shows the x - t plot of the one-dimensional motion of a particle. Is it correct to say from the graph that shows that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.



Ans.

It is not correct to say that the particle moves in a straight line for $t < 0$ (i.e., -ve) and on a parabolic path for $t > 0$ (i.e., + ve) because the x-t graph does not represent the path of the particle.

A suitable physical context for the graph can be the particle is dropped from the top of a tower at $t = 0$.

Q18. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1} . If the muzzle speed of the bullet is 150 m s^{-1} , with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car).

Ans.

Speed of the police van = $30 \text{ km/h} = 30 \times (5/18) = 25/3 \text{ m/s}$

Speed of a thief's car = $192 \text{ km/h} = 192 \times (5/18) = 160/3 \text{ m/s}$

Muzzle Speed of the bullet = 150 m/s

Speed of the bullet = speed of the police van + muzzle speed of the bullet

= $(25/3) + 150 = 475/3 \text{ m/s}$

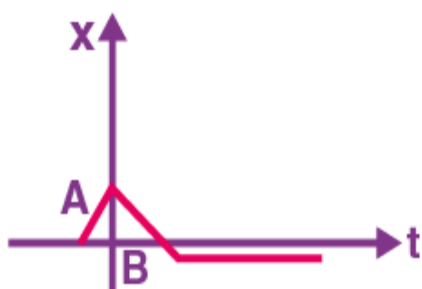
The relative velocity of the bullet w.r.t. the thief's car is

$v = \text{Speed of the bullet} - \text{Speed of a thief's car}$

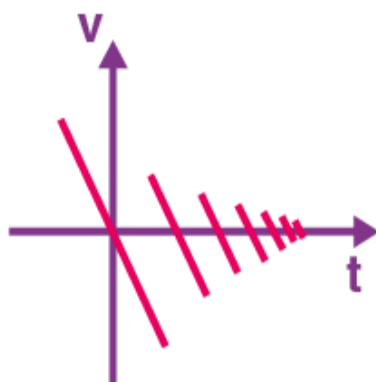
= $(475/3) - (160/3) = 105 \text{ m/s}$

The bullet hits the thief's car at a speed of 105 m/s

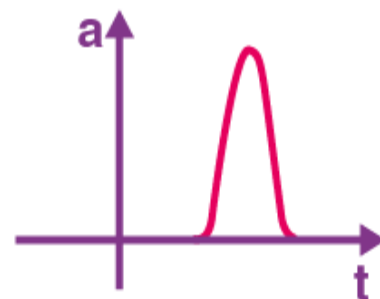
Q19. Suggest a suitable physical situation for each of the following graphs.



(a)



(b)



(c)

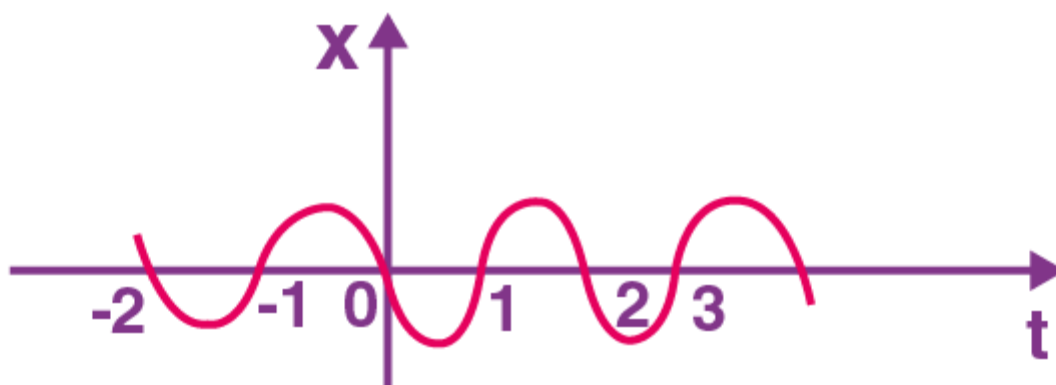
Ans.

(a) The graph is similar to kicking a ball, and then it hits the wall and rebounds with a reduced speed. The ball then moves in the opposite direction and hits the opposite wall, which stops the ball.

(b) The graph shows a continuous change in the velocity of the object, and at some instant, it loses some velocity. Therefore, it may represent a situation where a ball falls on the ground from a certain height and rebounds with a reduced speed.

(c) A cricket ball moving with a uniform speed is hit by a bat for a very short time interval.

Q20. The following figure gives the x - t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2 s, -1.2 s.



Ans.

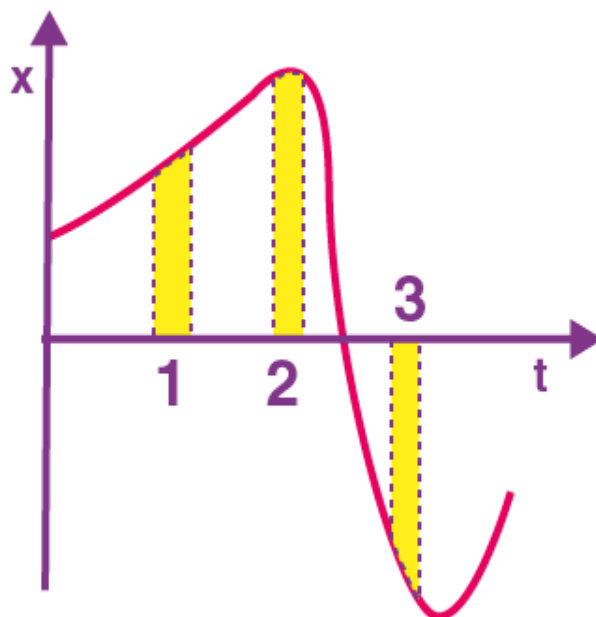
In S.H.M., acceleration, $a = -\omega^2 x$, ω is the angular frequency —(1)

(i) At $t = 0.3$ s, $x < 0$, i.e. position is negative. Moreover, as x becomes more negative with time, it shows that velocity is negative (i.e., $v < 0$). However, using equation (1), acceleration will be positive.

(ii) At $t = 1.2$ s, Positions and velocity will be positive. Acceleration will be negative.

(iii) At $t = -1.2$ s, Position, x is negative. Velocity and acceleration will be positive.

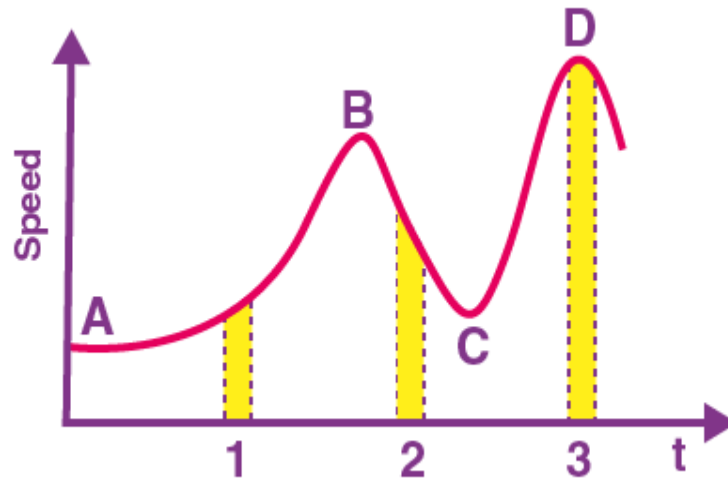
Q21. The figure gives the x - t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



Ans.

Interval 3 is the greatest, and 2 is the least. The average velocity is positive for intervals 1 and 2, and it is negative for interval 3.

Q22. The following figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion gives the signs of v and a in the three intervals. What are the accelerations at points A, B, C and D?



Ans.

The change in the speed with time is maximum in interval 2. Therefore, the average acceleration is greatest in magnitude in interval 2.

The average speed is maximum in interval 3.

The sign of velocity is positive in intervals 1, 2 and 3. The acceleration depends on the slope. The acceleration is positive in intervals 1 and interval 3, as the slope is positive. The acceleration is negative in interval 2, as the slope is negative.

Acceleration at A, B, C and D is zero since the slope is parallel to the time axis at these instants.

Q23. A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the n th second ($n = 1, 2, 3, \dots$) versus n . What do you expect this plot to be during accelerated motion: a straight line or a parabola?

Ans.

For a straight line, the distance covered by a body in n th second is:

$$S_n = u + a(2n - 1)/2 \quad \dots \dots \dots (1)$$

Where,

a = Acceleration

u = Initial velocity

n = Time = 1, 2, 3, \dots , n

In the above case,

$a = 1 \text{ m/s}^2$ and $u = 0$.

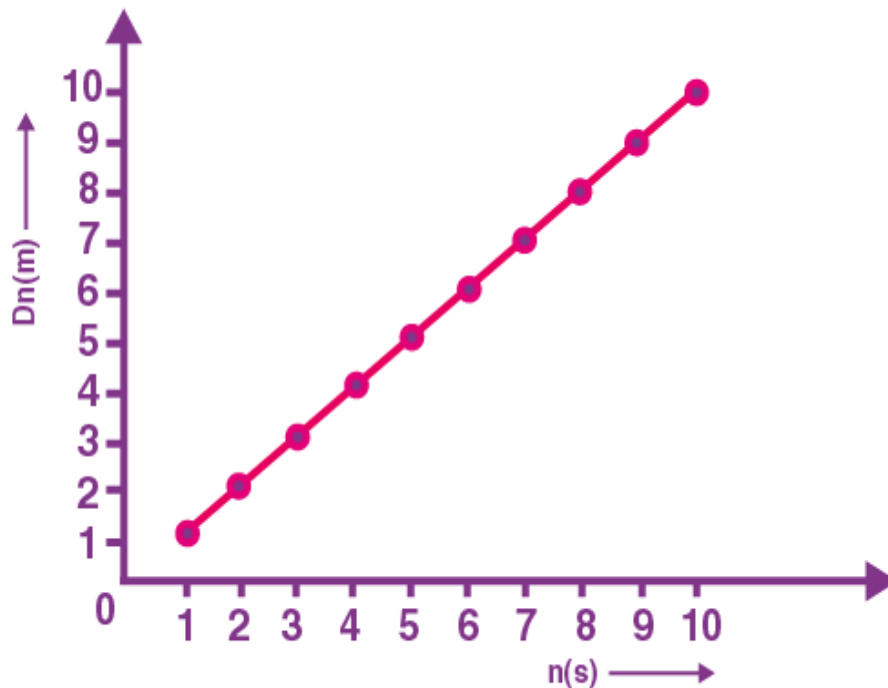
$$\therefore S_n = (2n - 1) / 2 \quad \dots \dots \dots (2)$$

This relation shows that:

$$S_n \propto n \quad \dots \dots \dots (3)$$

Now substituting different values of n in equation (2) we get:

n	1	2	3	4	5	6	7	8	9
S_n	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5



This plot is expected to be a straight line.

Q24. A boy, standing on a stationary lift (open from above), throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1} . How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Ans.

The initial velocity of the ball, $u = 49 \text{ m/s}$

Case: I

The boy throws the ball upwards when the lift is stationary. The vertically upward direction is taken as the positive direction. The displacement of the ball is zero.

Considering the equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$0 = (49)t + \frac{1}{2}(-9.8)t^2$$

$$t = (49 \times 2)/9.8 = 98/9.8 = 10 \text{ sec}$$

Case: II

As the lift starts moving with a speed of 5 m/s, the initial speed of the ball will be 49 m/s + 5 m/s = 54 m/s

The displacement of the ball will be $s = 5t'$

Therefore, the time taken can be calculated using the formula

$$s = ut + (1/2) at^2$$

$$5t' = (54) t' + (1/2)(-9.8) t'^2$$

$$t' = 2(54 - 5)/9.8 = 10 \text{ sec}$$

The time taken will remain the same in both cases.

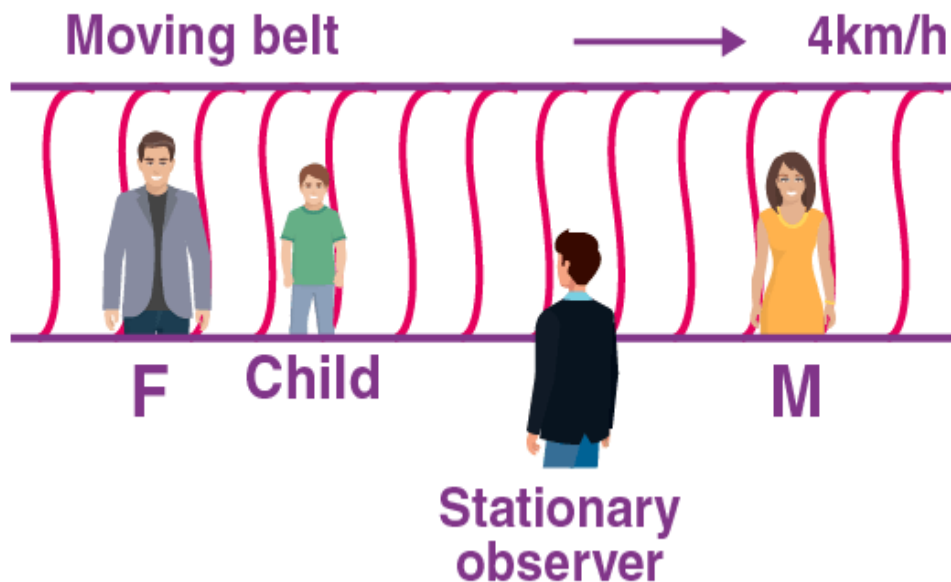
Q25. On a long, horizontally moving belt figure, a child runs to and fro with a speed 9 km h⁻¹ (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves at a speed of 4 km h⁻¹. For an observer on a stationary platform outside, what is the

(a) speed of the child running in the direction of motion of the belt?

(b) speed of the child running opposite to the direction of motion of the belt?

(c) time taken by the child in (a) and (b)?

Which of the answers alter if motion is viewed by one of the parents?



Ans.

Speed of child = 9 km h⁻¹

Speed of belt = 4 km h⁻¹

(a) When the boy runs in the direction of motion of the belt, then his speed as observed by the stationary observer = (9 + 4) km h⁻¹ = 13 km h⁻¹.

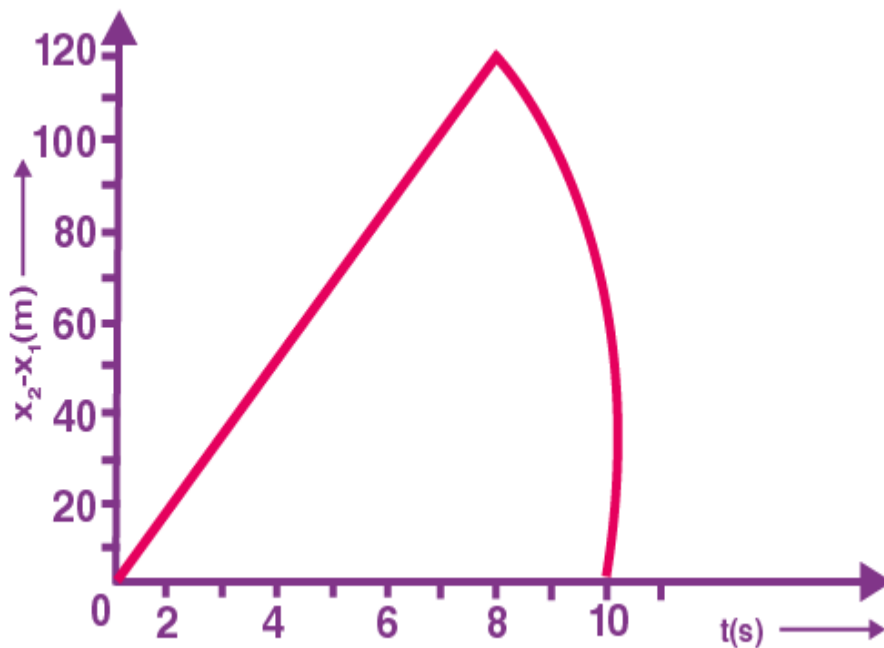
(b) When the boy runs opposite to the direction of motion of the belt, then speed of the child as observed by the stationary observer = $(9 - 4) \text{ km h}^{-1} = 5 \text{ km h}^{-1}$

(c) Distance between the two parents = 50 m = 0.05 km

The speed of the boy, as observed by both the parents, is 9 km h^{-1}

Time taken by the boy to move towards one of the parents = $0.05 \text{ km} / 9 \text{ km h}^{-1} = 0.0056 \text{ h} = 20 \text{ S}$

Q26. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1} . Verify that the graph shown in Fig. 3.27 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10 \text{ m s}^{-2}$. Give the equations for the linear and curved parts of the plot.



Ans.

For the first stone:

Given,

Acceleration, $a = -g = -10 \text{ m/s}^2$

Initial velocity, $u_1 = 15 \text{ m/s}$

Now, we know

$$s_1 = s_0 + u_1 t + (1/2) a t^2$$

Given, the height of the tree, $s_0 = 200 \text{ m}$

$$s_1 = 200 + 15t - 5t^2 \dots\dots\dots (1)$$

When this stone hits the jungle floor, $s_1 = 0$

$$\begin{aligned} \therefore -5t^2 + 15t + 200 &= 0 \\ t^2 - 3t - 40 &= 0 \\ t^2 - 8t + 5t - 40 &= 0 \\ t(t - 8) + 5(t - 8) &= 0 \\ t = 8 \text{ s or } t = -5 \text{ s} \end{aligned}$$

Since the stone was thrown at time $t = 0$, the negative sign is not possible

$$\therefore t = 8 \text{ s}$$

For the second stone:

Given,

Acceleration, $a = -g = -10 \text{ m/s}^2$

Initial velocity, $u_{11} = 30 \text{ m/s}$

We know,

$$\begin{aligned} s_2 &= s_0 + u_{11}t + (1/2)at^2 \\ &= 200 + 30t - 5t^2 \dots \dots \dots (2) \end{aligned}$$

when this stone hits the jungle floor; $s_2 = 0$

$$\begin{aligned} -5t^2 + 30t + 200 &= 0 \\ t^2 - 6t - 40 &= 0 \\ t^2 - 10t + 4t + 40 &= 0 \\ t(t - 10) + 4(t - 10) &= 0 \\ (t - 10)(t + 4) &= 0 \\ t = 10 \text{ s or } t = -4 \text{ s} \end{aligned}$$

Here again, the negative sign is not possible

$$\therefore t = 10 \text{ s}$$

Subtracting equation (1) from equation (2), we get

$$\begin{aligned} s_2 - s_1 &= (200 + 30t - 5t^2) - (200 + 15t - 5t^2) \\ s_2 - s_1 &= 15t \dots \dots \dots (3) \end{aligned}$$

Equation (3) represents the linear trajectory of the two stones because to this linear relation between $(s_2 - s_1)$ and t , the projection is a straight line till 8 s.

The maximum distance between the two stones is at $t = 8 \text{ s}$.

$$(s_2 - s_1)_{\text{max}} = 15 \times 8 = 120 \text{ m}$$

This value has been depicted correctly in the above graph.

After 8 s, only the second stone is in motion, whose variation with time is given by the quadratic equation:

$$s_2 - s_1 = 200 + 30t - 5t^2$$

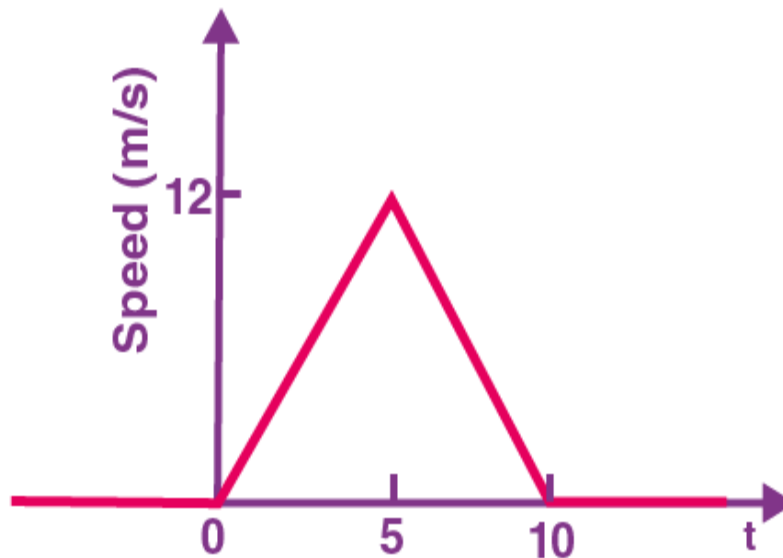
Therefore, the equation of linear and curved path is given by:

$$s_2 - s_1 = 15t \text{ (Linear path)}$$

$$s_2 - s_1 = 200 + 30t - 5t^2 \text{ (Curved path)}$$

Q27. The speed-time graph of a particle moving along a fixed direction is shown in the figure. Obtain the distance traversed by the particle between

- (a) $t = 0 \text{ s to } 10 \text{ s}$,
- (b) $t = 2 \text{ s to } 6 \text{ s}$.



Ans.

(a) Distance traversed by the particle between $t = 0$ s and $t = 10$ s

= area of the triangle = $(1/2) \times \text{base} \times \text{height}$

= $(1/2) \times 10 \times 12 = 60$ m

The average speed of the particle is $60 \text{ m} / 10 \text{ s} = 6 \text{ m/s}$

(b) The distance travelled by the particle between $t = 2$ s and $t = 6$ s

= Let S_1 be the distance travelled by the particle in time 2 to 5 s and S_2 be the distance travelled by the particle in time 5 to 6 s.

For the motion from 0 sec to 5 sec

Now, $u = 0$, $t = 5$, $v = 12 \text{ m/s}$

From the equation $v = u + at$, we get

$a = (v - u)/t = 12/5 = 2.4 \text{ m/s}^2$

Distance covered from 2 to 5 s, $S_1 = \text{distance covered in 5 sec} - \text{distance covered in 2 sec}$

= $(1/2) a (5)^2 - (1/2) a (2)^2 = (1/2) \times 2.4 \times (25 - 4) = 1.2 \times 21 = 25.2$ m

For the motion from 5 sec to 10 sec, $u = 12 \text{ m/s}$ and $a = -2.4 \text{ m/s}^2$

and $t = 5$ sec to $t = 6$ sec means $n = 1$ for this motion

Distance covered in the 6th sec is $S_2 = u + (1/2) a (2n - 1)$

= $12 - (2.4/2) (2 \times 1 - 1) = 10.8$ m

Therefore, the total distance covered from $t = 2$ s to $t = 6$ s = $S_1 + S_2$

= $25.2 + 10.8 = 36$ m

Q28. The velocity-time graph of a particle in one-dimensional motion is shown in the figure.

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (1/2)a(t_2 - t_1)^2$

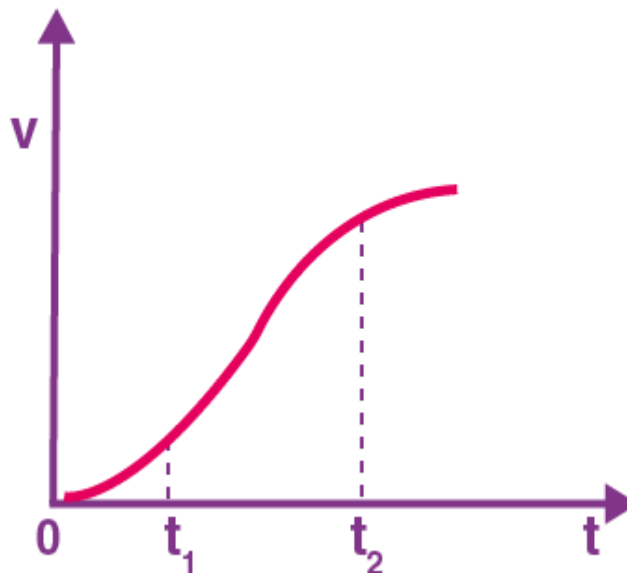
(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(c) $V_{\text{average}} = [x(t_2) - x(t_1)] / (t_2 - t_1)$

(d) $a_{\text{average}} = [v(t_2) - v(t_1)] / (t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{\text{av}}(t_2 - t_1) + (1/2)a_{\text{av}}(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1) = \text{Area under the } v\text{-}t \text{ curve bounded by } t\text{-axis and the dotted lines.}$



Ans.

The graph has a non-uniform slope between the intervals t_1 and t_2 (since the graph is not a straight line). The equations (a), (b) and (e) does not describe the motion of the particle. Only the relations (c), (d) and (f) are correct.