Q.1: Answer the following.
(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
(b) An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?
(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises.) However, the tidal effect of the moon's pull is greater than the tidal effect of the sun. Why?

Solution:

(a). No, as of now, no method has been devised to shield a body from gravity because gravity is independent of the medium, and it is the virtue of each and every matter. So the shield would exert gravitational forces.

(b). Yes, if the spaceship is large enough, then the astronaut will definitely detect Mars's gravity.

(c). Gravitational force is inversely proportional to the square of the distance, whereas Tidal effects are inversely proportional to the cube of the distance. So as the distance between the earth and the moon is smaller than the distance between the earth and the sun, the moon will have a greater influence on the earth's tidal waves.

Q.2. Choose the correct alternative.
(a) Acceleration due to gravity increases/decreases with increasing altitude.
(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).
(c) Acceleration due to gravity is independent of the mass of the earth/mass of the body.
(d) The formula –G M m($1/r_2 – 1/r_1$) is more/less accurate than the formula mg($r_2 – r_1$) for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Solution:

(a) decreases
(b) decreases
(c) mass of the body
(d) more

Q.3: Suppose there existed a planet that went around the Sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Solution:

Time taken by the earth for one complete revolution, T_E = 1 Year

The radius of Earth's orbit, R_E = 1 AU

Thus, the time taken by the planet to complete one complete revolution

T_P =

$\frac{1}{2}$
T_E =
$\frac{1}{2}$
year

Let the orbital radius of this planet = R_P

Now, according to Kepler's third law of planetary motion,

$$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}}$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}} = \left(\frac{\frac{1}{2}T_E}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^{\frac{2}{3}} = 0.63$$

Therefore, the radius of the orbit of this planet is 0.63 times smaller than the radius of the orbit of the Earth.

Q.4: Io, one of the satellites of Jupiter, has an orbital period of 1.769 days, and the radius of the orbit is 4.22 × 10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Solution:

Given,

Orbital period of Io, T_{Io} = 1.769 days = 1.769 × 24 × 60 × 60 s

Orbital radius of Io, R_{Io} = 4.22 × 10^8 m

We know the mass of Jupiter

$M_J = 4\pi^2 R_{Io}^3 / G T_{Io}^2$ . . . . (1)

Where;

M_J = Mass of Jupiter

G = Universal gravitational constant

Also,

The orbital period of the earth

T_E = 365.25 days = 365.25 × 24 × 60 × 60 s

The orbital radius of the earth, R_E = 1 AU = 1.496 × 10^{11} m

We know that the mass of the sun is

M_s =

$$\frac{4\pi^2 R_E^3}{G T_E^2}$$
. . . . (2)

Therefore,

$$\frac{M_S}{M_J}$$
=
$$\frac{4\pi^2 R_E^3}{G T_E^2} \times \frac{G T_{10}^2}{4\pi^2 R_{10}^3}$$
=
$$\frac{R_E^3}{T_E^2} \times \frac{T_{10}^2}{R_{10}^3}$$

Now, on substituting the values, we will get

=

$$\left[\frac{1.769\times24\times60\times60}{365.25\times24\times60\times60}\right]^2 \times \left[\frac{1.496\times10^{11}}{4.22\times10^8}\right]^3$$
= 1045.04

Therefore,

$$\frac{M_S}{M_J}$$
~ 1000

$M_S \sim 1000 \times M_J$

[Which proves that the Sun's mass is 1000 times that of Jupiter's.]

Q.5. Let us assume that our galaxy consists of 2.5 × 10^{11} stars, each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5ly.

Solution:

Mass of our galaxy, $M=2.5\times10^{11}$ solar mass
1 Solar mass = Mass of Sun $=2\times10^{30}$ kg
Mass of our galaxy, $M=2.5\times10^{11}\times2\times10^{30}=5\times10^{41}$kg

Diameter of Milky Way, $d=10^5$ ly
Radius of Milky Way, $r=5\times10^4$ ly
1 ly$=9.46\times10^{15}$m

Therefore, $r=5\times104\times9.46\times10^{15}=4.73\times10^{20}$m

The time taken by the star to revolve around the galactic centre is given by the relation
$T=(4\pi^2r^3/GM)^{1/2}$
=

$$\sqrt{\frac{4\times(3.14)^2\times(4.73)^3\times10^{60}}{6.67\times10^{-11}\times5\times10^{41}}}$$

= 1.12×10^{16} s

=

$$\frac{1.12\times10^{16}}{365\times24\times60\times60}\ years$$

= 3.55×10^8 years

Q.6. Choose the correct alternative.
(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.
(b) The energy required to launch an orbiting satellite out of the earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of the earth's influence.

Solution:

(a) If the zero potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.
(b) The energy required to launch an orbiting satellite out of Earth's gravitational influence is less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

Q. 7. Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

Solution:

The escape speed is given by the expression

$$v=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}$$

(a) The escape speed of a body from the Earth does not depend on the mass of the body.
(b) The escape speed of a body from the Earth does not depend on the location from where a body is projected.
(c) The escape speed does not depend on the direction of the projection of a body.
(d) The escape speed of a body depends upon the height of the location from where the body is launched since the escape velocity depends on the gravitational potential at the point from which it is launched. This potential, in turn, depends on the height.

Q.8. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed (b) angular speed (c) angular momentum (d) kinetic energy (e) potential energy (f) total energy

throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.
Solution:

(a) According to Kepler's second law, the linear speed of the comet keeps changing. When the comet is near the sun, its speed will be the fastest, and when it is far away from the sun, its speed will be the least.
(b) Angular speed also varies slightly.
(c) Comet has constant angular momentum.
(d) Kinetic energy changes.
(e) Potential energy changes along the path.
(f) Total energy will remain constant throughout the orbit

Q9. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

Solution:

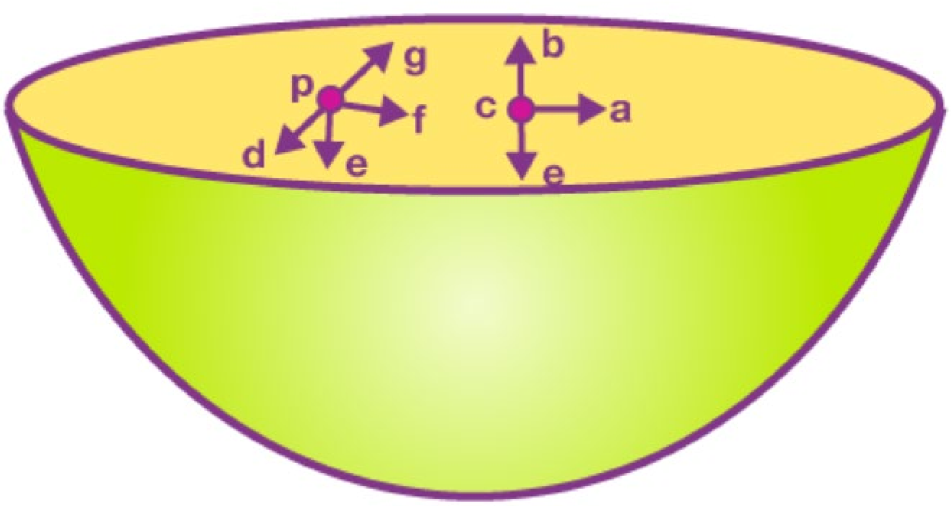
(a). In zero gravity, the blood flow to the feet isn't increased, so the astronaut does not get **swollen feet.**

(b). There is more supply of blood to the face of the astronaut. Therefore, the astronaut will have a **swollen face.**

(c). Due to increased blood supply to their faces, astronauts can be affected by **headaches.**

(d). Space has different orientations, so **orientational problems** can affect an astronaut.

Q.10: In the following two exercises, choose the correct answer from among the given ones.
The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig) (i) a, (ii) b, (iii) c, (iv) 0



Solution: (iii) c

Reason:

Inside a hollow sphere, gravitational forces on any particle at any point are symmetrically placed. However, in this case, the upper half of the sphere is removed. Since gravitational intensity is gravitational force per unit mass, it will act in a direction point downwards along ‘c’.

Q.11: For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Solution: (ii) e

Reason:

Making use of the logic/explanation from the above answer, we can conclude that the gravitational intensity at P is directed downwards along e.

Q.12. A rocket is fired from the earth towards the sun. At what distance from the earth’s centre is the gravitational force on the rocket zero? Mass of the sun = 2 x 10^{30} kg, and the mass of the earth = 6 x 10^{24} kg. Neglect the effect of other planets, etc. (Orbital radius = 1.5 x 10^{11} m)

Solution:

Mass of Sun, M_{sun}= 2 x 10^{30} kg

Mass of Earth, M_{earth} = 6 x 10^{24} kg

Orbital radius, r = 1.5 x 10^{11} m

Mass of the rocket = m

Let P be the point at a distance x at which the gravitational force on the rocket is due to Earth.

Applying Newton’s law of gravitation, the gravitational force acting on the rocket at point P is equated.

$$\frac{GmM_{sun}}{(r-x)^2} = \frac{GmM_{earth}}{x^2}$$
$$\frac{(r-x)^2}{x^2} = \frac{M_{sun}}{M_{earth}} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{1/2} = 577.35$$

1.5 x 10^{11} – x = 577.35x
x = 1.5 x 10^{11}/578.35 = 2.59 x 10^{8} m

Q.13: How will you ‘weigh the sun’, that is, estimate its mass? The mean orbital radius of the earth around the sun is 1.5 × 10^{8} km.

Solution:

Given:
Earth’s orbit, r = 1.5 × 10^{11} m
Time taken by the Earth for one complete revolution
T = 1 year = 365.25 days
i.e. T = (365.25 × 24 × 60 × 60) seconds

Since, Universal gravitational constant G = 6.67 × 10^{-11} Nm^2 kg^{-2}
Therefore, mass of the Sun M =

$$\frac{4\ \pi^2 r^3}{G\ T^2}$$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow$$

$$M = \frac{1.331\times10^{35}}{66.425\times10^{3}} = 2.004 \times 10^{30}\ kg$$

Therefore, the estimated mass of the Sun is 2.004 × 10^{30} Kg.

Q. 14. A Saturn year is 29.5 times the Earth year. How far is Saturn from the Sun if the Earth is 1.50 x 10^8 km away from the Sun?

Solution:

According to Kepler's third law of planetary motion,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

From the above equation, we get $T^2 \propto r^3$

T_s and r_s are the orbital periods and the mean distance of Saturn from the Sun, respectively.

T_e and r_e are the orbital periods and the mean distance of Earth from the Sun, respectively.

The time period of Saturn,T_s = 29.5 T_e

$\Rightarrow r_s^3/r_e^3 = T_s^2/T_e^2$

$r_s = r_e(T_s/T_e)^{2/3}$

= 1.5 x 10^8 x $29.5^{2/3}$

= 14.3 x 10^8 km

Q.15: A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Solution:

Given:

Weight of the man, W = 63 N

We know that acceleration due to gravity at height 'h' from the Earth's surface is

g' =

$$\frac{g}{\left[1+\left(\frac{h}{R_e}\right)\right]^2}$$

Where g = Acceleration due to gravity on the Earth's surface
And, R_e = Radius of the Earth
For h =

$\frac{R_e}{2}$

g' =

$$\frac{g}{\left[1+\left(\frac{R_e}{2R_e}\right)\right]^2}$$

$\Rightarrow$

g' =

$$\frac{g}{\left[1+\left(\frac{1}{2}\right)\right]^2} = \frac{4}{9}g$$

Also, the weight of a body of mass 'm' kg at the height of 'h' meters can be represented as
W' = mg
= m ×

$\frac{4}{9}$ g = $\frac{4}{9}$ mg = $\frac{4}{9}$ W = $\frac{4}{9}$ × 63 = 28 N

Q. 16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh halfway down to the centre of the earth if it weighed 250 N on the surface?

Solution:

Weight of a body on the Earth's surface, W=mg=250 N

Radius of the Earth = R_e
Let d be at a distance halfway to the centre of the earth, $d=R_e/2$

Acceleration due to gravity at d is given by the relation
$g_d=(1-d/R_e)g$
$g_d=(1-R_e/2R_e)g$
$=g/2$

Weight of the body at depth d
$W'=mg_d$
$=mg/2 = W/2 = 250/2$
$=125$ N

Q.17: A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 × 10^{6} m; G = 6.67 × 10^{-11} N m^{2} kg^{-2}

Solution:

Given:
Velocity of the missile, v = 5 km/s = 5 × 10^3 m/s
Mass of the Earth, M_E = 6 × 10^{24} kg
Radius of the Earth, R_E = 6.4 × 10^6 m

Let the height reached by the missile be 'h' and the mass of the missile be 'm'.
Now, at the surface of the Earth
The total energy of the rocket at the surface of the Earth = Kinetic energy + Potential energy
T_{E1} =

$$\frac{1}{2}mv^2 + \frac{-GM_E\,m}{R_E}$$

Now, at the highest point, 'h'

Kinetic Energy = 0 [Since, v = 0] And, Potential energy =

$$\frac{-GM_E\,m}{R_E+h}$$

Therefore, the total energy of the missile at the highest point 'h'

T_{E2} = 0 +

$$\frac{-GM_E\,m}{R_E+h}$$
$\Rightarrow$
T_{E2} =
$$\frac{-GM_E\,m}{R_E+h}$$

According to the law of conservation of energy, we have
Total energy of the rocket at the Earth's surface T_{E1} = Total energy at height 'h' T_{E2}

$$\Rightarrow \frac{1}{2}mv^2 + \frac{-GM_E\,m}{R_E} = \frac{-GM_E\,m}{R_E+h}$$
$$\Rightarrow \frac{1}{2}v^2 + \frac{-GM_E}{R_E} = \frac{-GM_E}{R_E+h}$$
$$\Rightarrow v^2 = 2GM_E \times \left[\frac{1}{R_E} - \frac{1}{(R_E+h)}\right] = 2GM_E\left[\frac{h}{(R_E+h)R_E}\right]$$

$v^2 = GM_Eh/R_E^2 + R_Eh$

Where,

$$g = \frac{GM}{R_E^2}$$

= 9.8 ms^{-2}
$V^2 = 2gR_E^2h/R_E^2+R_Eh$

$$\Rightarrow v^2 = \frac{2g\,R_E\,h}{R_E+h}$$

Therefore, $v^2 (R_E + h) = 2gR_Eh$

$\Rightarrow$
$v^2R_E = h\,(2gR_E - v^2)$

$\Rightarrow$
$$h = \frac{R_E\,v^2}{2gR_E - v^2} = \frac{(6.4\times10^6)\times(5\times10^3)^2}{2\times9.8\times6.4\times10^6-(5\times10^3)^2}$$

$\Rightarrow$

$h = 1.6 \times 10^6$ m
Therefore, the missile reaches a height of 1.6×10^6 m from the surface.
Now, the distance from the center of the earth = h + R_E = 1.6×10^6 + 6.4×10^6 = 8.0×10^6 m

Therefore, the greatest height the missile can attain before falling back to the earth is 1.6×10^6 m, and the final distance of the missile from the centre of the earth is 8.0×10^6 m.

Q.18. The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Solution:

Given,
Escape speed of the projectile on the Earth's surface, V_e = 11.2 km/s
Speed of projection of the body, v = 3Ve = 3×11.2 = 33.6 km/s
Let v and v' be the speed of the body at the time of projection and at a point far from the earth.
At the time of projection
Initial kinetic energy of the body = $1/2mv^2$
Initial gravitational potential energy of the body = $-GM_em/R_e$
M_e is the mass of the Earth.
R_e is the radius of the Earth.

At a larger distance from the earth's surface
KE of the body = $1/2mv'^2$
The gravitational potential energy of the body = 0

According to the law of conservation of energy
Total energy at the point of projection = total energy very far from the earth's surface
$1/2mv^2 + (-GM_em/Re) = 1/2mv'^2$
$1/2\ mv'^2 = 1/2mv^2 – GM_em/R_e$———(1)
If V_e is the escape velocity, then,
$1/2\ mV_e^2 = GM_em/R_e$———(2)

Substituting (2) in (1),
$1/2mv'^2 = 1/2mv^2 – 1/2mV_e^2$
$v'^2 = v – V_e^2$
$= (3V_e)^2 – V_e^2$
$= 8V_e^2$
$v'^2 = 8V_e^2$
$v' = \sqrt{8} \times V_e$
v = 2 × 1.414 × 11.2 km/s
= 31.68 km/s
Speed of the body far away from the earth = 31.68 km/s
Q. 19. A satellite orbits the earth at the height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m^2 kg^{-2}
Solution:
Height of the satellite, h = 400 km = 4×10^5 m
Mass of the Earth, M = 6.0×10^{24} kg
Mass of the satellite, m = 200 kg
Radius of the Earth, $R_e = 6.4 \times 10^6$ m
Total energy of the satellite at height h = Kinetic energy + Potential energy
$= (1/2)mv^2 + [-GmM_e / (R_e + h)]$
Orbital velocity of the satellite,
$$v = \sqrt{\frac{GM_e}{R_e+h}}$$
Total energy of height, h =
$$= \frac{1}{2}\frac{GM_em}{R_e+h} - \frac{GM_em}{R_e+h}$$
Total Energy=
$$-\frac{1}{2}\frac{GM_em}{R_e+h}$$

The negative sign indicates that the satellite is bound to the earth. This is known as the binding energy of a satellite.

The energy required to send the satellite out of its orbit = – (Bound energy)

=

$$\frac{1}{2}\frac{GM_e m}{R_e + h}$$

Putting values of all terms,

=

$$\frac{6.67\times10^{-11}\times6\times10^{24}\times200}{2(6.4\times10^{6}+4\times10^{5})}$$

= 5.9×10^9 J

Q. 20. Two stars, each of one solar mass (= 2×10^{30} kg), are approaching each other for a head-on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars remain undistorted until they collide. (Use the known value of G.)

Solution:

Mass of each star, M = 2×10^{30} kg

Radius of each star, R = 10^4 km = 10^7 m

Distance between the stars, r = 10^9 km = 10^{12}m

Initial potential energy of the system = -GMm/r ———(1)

Total K.E. of the stars = $1/2Mv^2 + 1/2Mv^2 = Mv^2$

Where v is the speed of stars with which they collide. The distance between the centres of the stars at collision r = 2 R.

Therefore, the final potential energy of two stars when they are close to each other = -GMm/2R

Thus, the total energy of the two stars just before the collision

E = Mv^2 -GMm/2R

Since the energy is conserved, the initial energy is equal to the final energy

Mv^2 -GMm/2R = -GMm/r

v^2 =(Gm/2R) – (Gm/r)

= Gm (1/2R – 1/r)

= $6.67 \times 10^{-11} \times 2 \times 10^{30}$ [(1/2 x 10^7) – (1/10^{12})]

$v^2 = 6.67 \times 10^{12}$

v = 2.58×10^6 m/s

Q.21: Two heavy spheres, each of mass 100 kg and radius 0.10 m, are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the midpoint of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

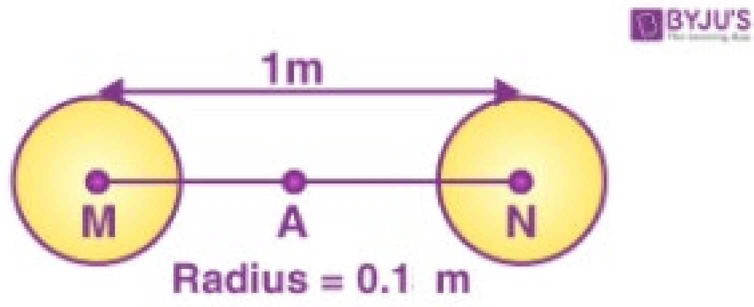
Solution:

Given:

Radius of spheres, R = 0.10 m

Distance between two spheres, r = 1.0 m

Mass of each sphere, M = 100 kg



From the above figure, 'A' is the mid-point since each sphere will exert the gravitational force in the opposite direction. Therefore, the gravitational force at this point will be zero.

Gravitational potential at the midpoint (A) is

U=

$$\left[\frac{-GM}{\frac{r}{2}} + \frac{-GM}{\frac{r}{2}}\right]$$

U=

$$\left[\frac{-4GM}{r}\right]$$

U=

$$\left[\frac{-4\times(6.67\times10^{-11})\times(1000)}{1.0}\right]$$
$\Rightarrow$

U= -2.668 x 10^{-7} J /kg
Therefore, the gravitational potential and force at the mid-point of the line connecting the centres of the two spheres is = -2.668 x 10^{-7} J /kg

The net force on an object placed at the mid-point is zero. However, if the object is displaced even a little towards any of the two bodies, it will not return to its equilibrium position. Thus, the body is in an unstable equilibrium.

Q.22: As you have learnt in the text, a geostationary satellite orbits the earth at the height of nearly 36,000 km from the surface of the earth. What is the potential due to the earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero.) Mass of the earth = 6.0×10^{24} kg, radius = 6400 km

Solution:

Given:

Radius of the Earth, R = 6400 km = 0.64 × 10^{7} m

Mass of Earth, M = 6 x 10^{24} kg

Height of the geostationary satellite from the earth's surface, h = 36000 km = 3.6 x 10^{7} m

Therefore, gravitational potential at height 'h' on the geostationary satellite due to the earth's gravity

G_P =

$$\frac{-GM}{R+h}$$

$\Rightarrow$

G_P =

$$\frac{-(6.67\times10^{-11})\times(6\times10^{24})}{(0.64\times10^{7})+(3.6\times10^{7})}$$

$\Rightarrow$

G_P =

$$\frac{-40.02\times10^{13}}{4.24\times10^{7}}$$

= -9.439 × 10^{6} J/Kg

Therefore, the gravitational potential due to Earth's gravity on a geostationary satellite orbiting earth is -9.439 × 10^{6} J/Kg

Q.23: A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category.) Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2×10^{30} kg)

Solution:

Any matter/object will remain stuck to the surface, if the outward centrifugal force is lesser than the inward gravitational pull.

Gravitational force, f_G =

$$\frac{GM\,m}{R^2}$$

[Neglecting negative sign]
Here,
M = Mass of the star = 2.5× 2 × 10^{30} = 5 × 10^{31} kg
m = Mass of the object
R = Radius of the star = 12 km = 1.2 ×10^{4} m
Therefore, f_G =

$$\frac{(6.67\times10^{-11})\times(5\times10^{31})\ m}{(1.2\times10^{4})^2} = 1.334\times10^{13} m\ N$$

Now, Centrifugal force, f_c= m r ω^2
Here, ω = Angular speed = 2πv
v = Angular frequency = 10 rev s^{-1}
f_c = m R $(2\pi v)^2$

$f_c = m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = (6.82 \times 10^5 m)$ N

As $f_G > f_C$, the object will remain stuck to the surface of the black hole.

Q. 24. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? The mass of the spaceship = 1000 kg; the mass of the sun = 2×10^{30} kg; the mass of Mars = 6.4×10^{23} kg; the radius of Mars = 3395 km; the radius of the orbit of Mars = 2.28×10^8 km; G = 6.67×10^{-11} N m^2kg^{-2}.

Solution:

The mass of the spaceship, m_s =1000 kg
the mass of the Sun, $M=2\times10^{30}$kg
the mass of Mars, $m_m=6.4\times10^{23}$kg
The radius of the orbit of Mars, $R=2.28\times10^{11}$ m
The radius of Mars, $r=3395\times10^3$m
Universal gravitational constant, $G=6.67\times10^{-11}Nm^2kg^{-2}$

The potential energy of the spaceship due to the gravitational attraction of the Sun, $U_s=-GMm_s/R$
The potential energy of the spaceship due to the gravitational attraction of Mars, $U_m=-Gm_mm_s/r$

Total energy of the spaceship, $E= U_m+U_s =[-GMm_s/R]+[-Gm_mm_s/r]$

The negative sign indicates that the satellite is bound to the system.
The energy required to launch the spaceship out of the solar system =−(total energy of the spaceship)

$-E = [GMm_s/R]+[Gm_mm_s/r]$

$= Gm_s(M/R + m_m/r)$

=

$$6.67\times10^{-11}\times10^3\times\left(\frac{2\times10^{30}}{2.28\times10^{11}}+\frac{6.4\times10^{23}}{3.395\times10^6}\right)$$

$= 5.91 \times 10^{11}$ J

Q.25. A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? The mass of mars = 6.4×10^{23} kg; the radius of mars = 3395 km; G = 6.67×10^{-11} N m^2 kg^{-2}.

Solution:

Speed of the rocket fired from the surface of mars (v) = 2 km/s

The mass of the rocket = m
The mass of Mars (M) = 6.4×10^{23} Kg
The radius of Mars (R) = 3395 km = 3.395×10^{6} m

The initial Potential energy = – GMm/R
The initial kinetic energy of the rocket = 1/2 mv^2

Total initial energy = 1/2 mv^2 – GMm/R

Due to atmospheric resistance, 20% of the kinetic energy is lost by the rocket.
The remaining Kinetic energy= 80% of 1/2mv^2
= 2/5 mv^2= 0.4 mv^2 ———(1)

When the rocket reaches the highest point, at the height h above the surface, the kinetic energy will be zero, and the potential energy is equal to – GMm/(R+h)

Applying the Law of conservation of energy,
– GMm/(R+h) = – GMm/R + (0.4) mv^2

GM/(R+h) = 1/R [GM – 0.4Rv^2] (R+h/R) = GM/[GM – 0.4Rv^2]

h/R = {GM/[GM – 0.4Rv^2]} – 1

h/R = (0.4Rv^2/GM – 0.4Rv^2)

h = (0.4R^2v^2/GM – 0.4Rv^2)

$$h = \frac{0.4\times(2\times10^3)\times(3.395\times10^6)^2}{6.67\times10^{-11}\times6.4\times10^{23}-0.4\times(2\times10^3)^2\times3.395\times10^6}$$
h = 495 km