

**Q1.** A steel wire of length 4.7 m and cross-sectional area  $3.0 \times 10^{-5} \text{ m}^2$  stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of  $4.0 \times 10^{-5} \text{ m}^2$  under a given load. What is the ratio of Young's modulus of steel to that of copper?

**Answer:**

Length of the steel wire,  $l_1 = 4.7 \text{ m}$

Cross-sectional area of the steel wire,  $a_1 = 3.0 \times 10^{-5} \text{ m}^2$

Length of the copper wire,  $l_2 = 3.5 \text{ m}$

Cross-section area of the copper wire,  $a_2 = 4.0 \times 10^{-5} \text{ m}^2$

Change in length =  $\Delta l_1 = \Delta l_2 = \Delta l$

Force applied in both cases =  $F$

Young's modulus of the steel wire

$$Y_1 = \frac{F}{a_1} \frac{l_1}{\Delta l}$$

$$Y_1 = \frac{F}{3.0 \times 10^{-5}} \frac{4.7}{\Delta l}$$

---(1)

Young's modulus of the copper wire

$$Y_2 = \frac{F}{a_2} \frac{l_2}{\Delta l}$$

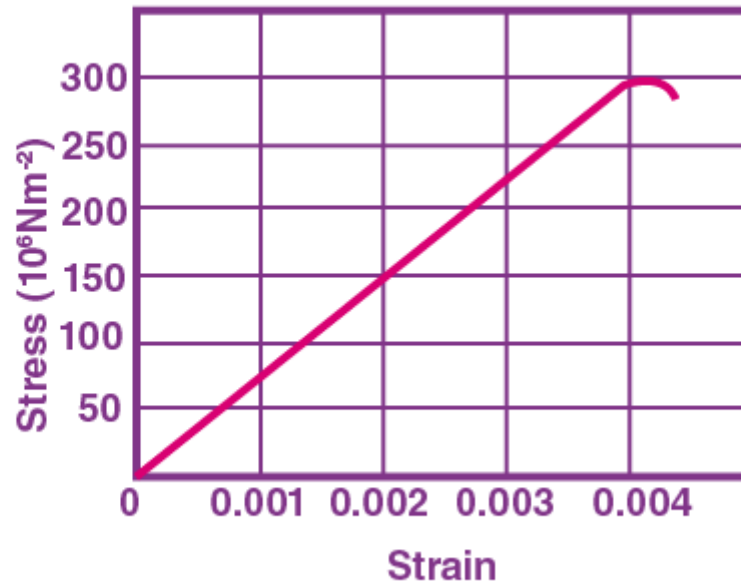
$$Y_2 = \frac{F}{4.0 \times 10^{-5}} \frac{3.5}{\Delta l}$$

---(2)

Dividing (1) by (2), we get

$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = 1.79$$

**Q2.** The figure below shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

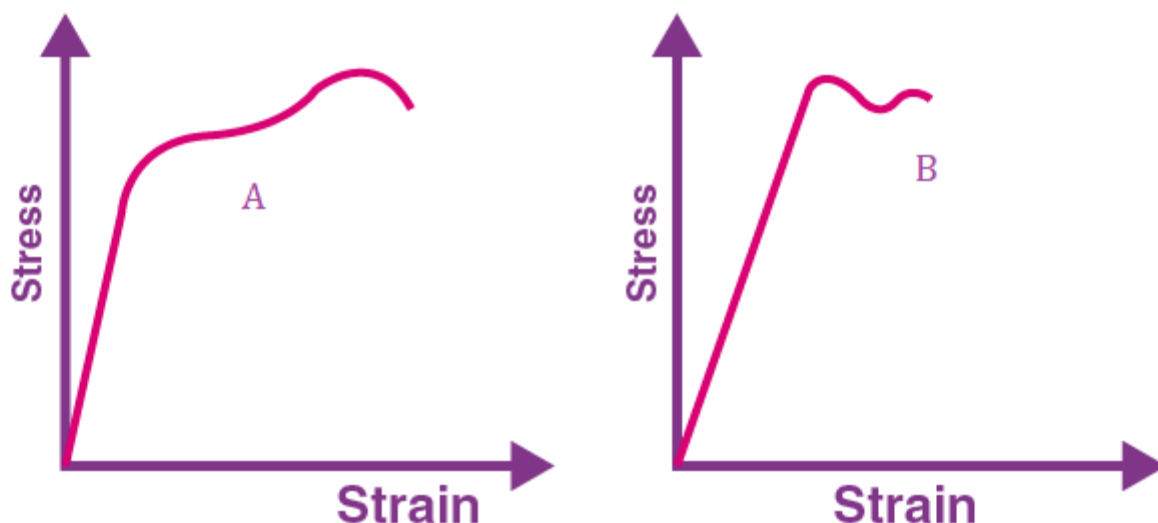


**Answer:**

$$\begin{aligned}
 \text{Young's modulus, } Y &= \text{Stress/Strain} \\
 &= 150 \times 10^6 / 0.002 \\
 &= 150 \times 10^6 / 2 \times 10^{-3} \\
 &= 75 \times 10^9 \text{ Nm}^{-2} \\
 &= 75 \times 10^{10} \text{ Nm}^{-2}
 \end{aligned}$$

(a) Yield strength of a material is the maximum stress that the material can sustain and retain its elastic property. From the graph, the approximate yield strength of the given material  
 $= 300 \times 10^6 \text{ Nm}^{-2}$   
 $= 3 \times 10^8 \text{ Nm}^{-2}$ .

**Q3.** The stress-strain graphs for materials A and B are shown in the figure below.



The graphs are drawn to the same scale.

- (a) Which of the materials has the greater Young's modulus?  
(b) Which of the two is the stronger material?

**Answer:**

Young's modulus = Stress/Strain

- (a) From the graphs, we can see that for the given strain, stress for A is greater than that of B. Therefore, Young's modulus of A is greater than B.  
(b) Young's modulus is also a measure of the strength of the material. Young's modulus is greater for A; therefore, material A is stronger than B.

**Q4. Read the following two statements below carefully and state, with reasons, if it is true or false.**

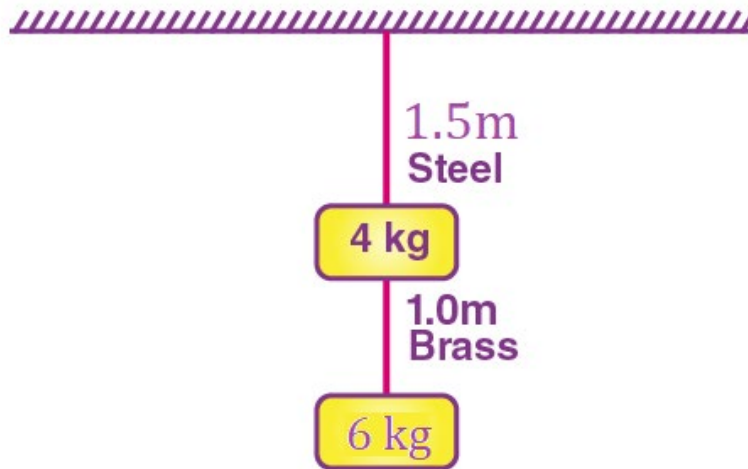
- (a) *The Young's modulus of rubber is greater than that of steel;*  
(b) *The stretching of a coil is determined by its shear modulus.*

**Answer.**

( a ) True. Stretching a coil does not change its length; only its shape is altered, and this involves shear modulus.

( b ) False. This is because, for the same value of stress, there is more strain in rubber than in steel. And as Young Modulus is an inverse of strain, it is greater in steel.

**Q5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass, are loaded, as shown in Fig. The unloaded length of steel wire is 1.5 m, and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires. [Young's modulus of steel is  $2.0 \times 10^{11}$  Pa. (1 Pa = 1 N m<sup>2</sup>)]**



**Answer:**

Diameter of the two wires,  $d = 0.25\text{m}$   
 Radius of the wires,  $r = d/2 = 0.125\text{cm}$   
 Unloaded length of the steel wire,  $l_1 = 1.5\text{m}$   
 Unloaded length of the brass wire,  $l_2 = 1.0\text{m}$

Force exerted on the steel wire  
 $F_1 = (4+6)g = 10 \times 9.8 = 98\text{N}$

Cross-section area of the steel wire,  $a_1 = \pi r_1^2$

Change in length of the steel wire =  $\Delta l_1$

Young's modulus for steel =  $2.0 \times 10^{11}\text{ Pa}$

$$Y_1 = \frac{F_1}{a_1} \frac{l_1}{\Delta l_1}$$

$$Y_1 = \frac{F_1}{\pi r_1^2} \frac{l_1}{\Delta l_1}$$

$$\Delta l_1 = \frac{F_1}{\pi r_1^2} \frac{l_1}{Y_1}$$

$$\Delta l_1 = \frac{9.8}{\pi(0.125 \times 10^{-2})^2} \frac{1.5}{2.0 \times 10^{11}}$$

$$= 1.49 \times 10^{-4}\text{ m}$$

Force of the brass wire,  $F_2 = 6 \times 9.8 = 58.8\text{ N}$

Cross-section area of the brass wire,  $a_2 = \pi r_2^2$

Change in length of the brass wire =  $\Delta l_2$

Young's modulus of the brass wire =  $0.91 \times 10^{11}$  Pa

$$Y_2 = \frac{F_2}{a_2} \frac{l_2}{\Delta l_2}$$

$$\Delta l_2 = \frac{F_2}{\pi r_2^2} \frac{l_2}{Y_2}$$

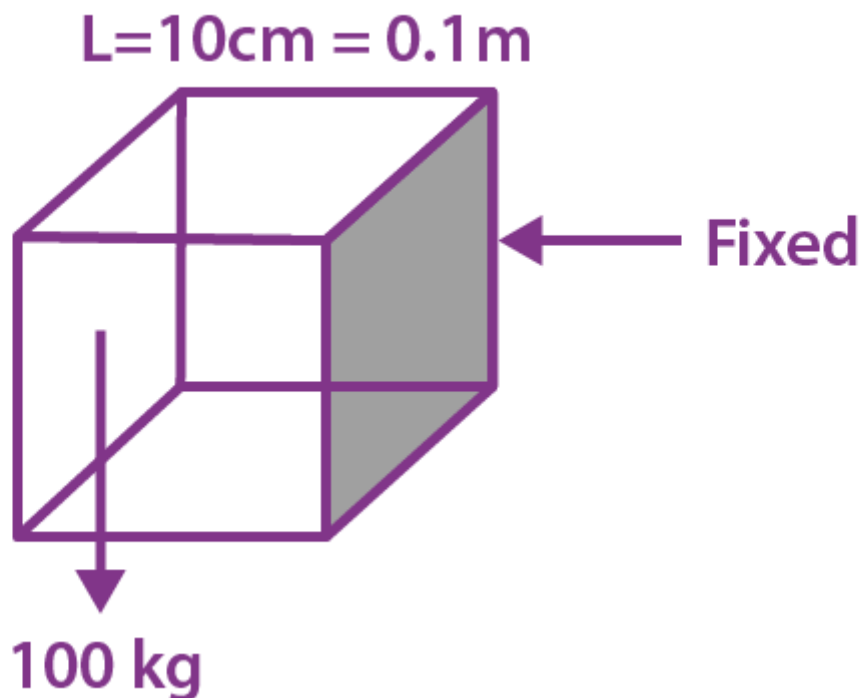
$$\Delta l_2 = \frac{58.8}{\pi(0.125 \times 10^{-2})^2} \frac{1}{0.91 \times 10^{11}}$$

$$= 1.3 \times 10^{-4} \text{ m}$$

The elongation of the steel wire is  $1.49 \times 10^{-4}$  m, and that of brass is  $1.3 \times 10^{-4}$  m.

**Q6.** The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

**Answer:**



Edge of the aluminium cube,  $L = 10 \text{ cm} = 10/100 = 0.1 \text{ m}$   
Area of each face,  $A = (0.1)^2 = 0.01 \text{ m}^2$

Mass attached to the opposite face of the cube = 100 kg

Tangential force acting on the face,  $F = 100 \text{ kg} = 100 \times 9.8 = 980 \text{ N}$

Shear modulus,  $\eta = \text{Tangential stress/Shearing strain}$

Shearing strain = Tangential stress/ Shear modulus

$$= F/A\eta = 980/(0.01 \times 25 \times 10^9) = 3.92 \times 10^{-6}$$

Shearing strain = Lateral strain/Side of the cube

Lateral strain = Shearing strain  $\times$  Side of the cube =  $3.92 \times 10^{-6} \times 0.1$

$$= 3.92 \times 10^{-7} \text{ m} \approx 4 \times 10^{-7} \text{ m}$$

**Q7. Four identical hollow cylindrical columns of mild steel support a big structure with a mass of 50,000 kg. The inner and outer radii of each column are 30 and 60 cm, respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.**

**Answer:**

Mass of the big structure,  $M = 50,000 \text{ kg}$

Total force exerted on the four columns = Total weight of the structure =  $50000 \times 9.8 \text{ N}$

The compressional force on each column =  $Mg/4 = (50000 \times 9.8)/4 \text{ N} = 122500 \text{ N}$

Therefore, Stress = 122500 N

Young's modulus of steel,  $Y = 2 \times 10^{11} \text{ Pa}$

Young's modulus,  $Y = \text{Stress/Strain}$

Strain = Young's modulus/Stress

$$\text{Strain} = (F/A)/Y$$

Inner radius of the column,  $r = 30 \text{ cm} = 0.3 \text{ m}$

Outer radius of the column,  $R = 60 \text{ cm} = 0.6 \text{ m}$

Where,

$$\text{Area, } A = \pi(R^2 - r^2) = \pi((0.6)^2 - (0.3)^2) = 0.27 \pi \text{ m}^2$$

$$\text{Strain} = 122500/[0.27 \times 3.14 \times 2 \times 10^{11}] = 7.22 \times 10^{-7}$$

Hence, the compressional strain of each column is  $7.22 \times 10^{-7}$ .

**Q8. A piece of copper having a rectangular cross-section of 15.2 mm  $\times$  19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.**

**Answer:**

Area of the copper piece,  $A = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4} \text{ m}^2$

Tension force applied on the piece of copper,  $F = 44,500 \text{ N}$

Modulus of elasticity of copper,  $Y=42 \times 10^9 \text{ Nm}^{-2}$

Modulus of elasticity ( $Y$ ) = Stress / Strain

$$\begin{aligned} &= (F/A) / \text{Strain} \\ \text{Strain} &= F/(YA) \\ &= 44500 / (2.9 \times 10^{-4} \times 42 \times 10^9) \\ &= 3.65 \times 10^{-3} \end{aligned}$$

**Q9. A steel cable with a radius of 1.5 cm supports a chairlift in a ski area. If the maximum stress is not to exceed  $10^8 \text{ Nm}^{-2}$ . What is the maximum load the cable can support?**

**Answer:**

Radius of the steel cable,  $r=1.5 \text{ cm} = 0.015 \text{ m}$

$$\begin{aligned} \text{Cross-sectional area of the cable} &= \pi r^2 = 3.14 \times (0.015)^2 \\ &= 7.06 \times 10^{-4} \text{ m}^2 \end{aligned}$$

Maximum stress allowed on the steel cable =  $10^8 \text{ N/m}^2$

Maximum load the cable can support = Maximum stress  $\times$  Area of cross-section

$$\begin{aligned} &= 10^8 \times 7.06 \times 10^{-4} \\ &= 7.065 \times 10^4 \text{ N} \end{aligned}$$

Hence, the cable can support the maximum load of  $7.065 \times 10^4 \text{ N}$ .

**Q10. A rigid bar of mass 15 kg is supported symmetrically by three wires, each 2.0 m long. Those at each end are of copper, and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.**

**Answer.**

As the tension on the wires is the same, the extension of each wire will also be the same. Now, as the length of the wires is the same, the strain on them will also be equal.

Now, we know

$$Y = \text{Stress} / \text{Strain} \\ = (F/A) / \text{Strain} = (4F/\pi d^2) / \text{Strain} \dots\dots\dots (1)$$

Where,

A = Area of cross-section

F = Tension force

d = Diameter of the wire

We can conclude from equation (1) that  $Y \propto (1/d^2)$ .

We know that Young's modulus for iron,  $Y_1 = 190 \times 10^9 \text{ Pa}$

Let the diameter of the iron wire =  $d_1$

Also, Young's modulus for copper,  $Y_2 = 120 \times 10^9 \text{ Pa}$

Let the diameter of the copper wire =  $d_2$

Thus, the ratio of their diameters can be given as

$$\frac{d_1}{d_2} = \sqrt{\frac{Y_1}{Y_2}}$$

=

$$\sqrt{\frac{190 \times 10^9}{120 \times 10^9}}$$

= 1 : 25 : 1

**Q11.** A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm<sup>2</sup>. Calculate the elongation of the wire when the mass is at the lowest point of its path.

**Answer:**

Mass, m = 14.5 kg

Length of the steel wire, l = 1 m

Angular velocity, v = 2 rev/s

Cross-sectional area of the wire, A = 0.065 × 10<sup>-4</sup> m<sup>2</sup>

Total pulling force on the steel wire when the mass is at the lowest point of the vertical circle, F = mg + mr ω<sup>2</sup>

$$= 14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2$$

$$= 2429.53 \text{ N}$$

Young's modulus = Stress / Strain

$$Y = \frac{F l}{A \Delta l}$$

$$\Delta l = \frac{F l}{A Y}$$

$$\Delta l = (2429.53 \times 1) / (0.065 \times 10^{-4}) \times (2 \times 10^{11}) = 1.87 \times 10^{-3} \text{ m}$$

Hence, the elongation of the wire when the mass is at the lowest is 1.87 × 10<sup>-3</sup> m.

**Q12.** Compute the bulk modulus of water from the following data: Initial volume = 100.0 litres, Pressure increase = 100.0 atm (1 atm = 1.013 × 10<sup>5</sup> Pa), Final volume = 100.5 litres. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

**Answer:**

Initial volume, V<sub>1</sub> = 100.0 litre = 100.0 × 10<sup>-3</sup> m<sup>3</sup>

Final volume, V<sub>2</sub> = 100.5 litre = 100.5 × 10<sup>-3</sup> m<sup>3</sup>

Change in the volume, ΔV = V<sub>2</sub> - V<sub>1</sub> = 0.5 × 10<sup>-3</sup> m<sup>3</sup>



Pressure increase,  $p = 100.0 \text{ atm} = 100 \times 1.013 \times 10^5 \text{ Pa}$

$$= 101.3 \times 10^5 \text{ Pa}$$

Bulk modulus of water =  $p / (\Delta V / V_1) = p V_1 / \Delta V$

$$= 101.3 \times 10^5 \times 100 \times 10^{-3} / (0.5 \times 10^{-3})$$

$$= 2.026 \times 10^9 \text{ Pa}$$

Bulk modulus of air =  $1 \times 10^5 \text{ Pa}$

$$\text{Bulk modulus of water} / \text{Bulk modulus of air} = 2.026 \times 10^9 / (1 \times 10^5)$$

$$= 2.026 \times 10^4$$

The intermolecular force in liquids is much larger than in air as the distance between the molecules is much lesser in liquid than in air. Therefore, at the same temperature, the strain for water is much more than for air.

**Q13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is  $1.03 \times 10^3 \text{ kg m}^{-3}$ ?**

Answer.

Let the depth be the alphabet 'd'.

Given,

Pressure at the given depth,  $p = 80.0 \text{ atm} = 80 \times 1.01 \times 10^5 \text{ Pa}$

Density of water at the surface,  $\rho_1 = 1.03 \times 10^3 \text{ kg m}^{-3}$

Let  $\rho_2$  be the density of water at the depth d.

$V_1$  be the volume of water of mass m at the surface.

Then, let  $V_2$  be the volume of water of mass m at the depth h.

And  $\Delta V$  is the change in volume.

$$\Delta V = V_1 - V_2$$

$$= m \left[ \frac{1}{\rho_1} - \frac{1}{\rho_2} \right] \therefore \text{Volumetric strain} = \Delta V / V_1$$

$$= m \left[ \frac{1}{\rho_1} - \frac{1}{\rho_2} \right] \times (\rho_1 / m)$$

$$\Delta V / V_1 = 1 - (\rho_1 / \rho_2) \quad \dots \dots \dots (1)$$

We know, Bulk modulus,  $B = p V_1 / \Delta V$

$$\Rightarrow \Delta V / V_1 = p / B$$

Compressibility of water =  $(1/B) = 45.8 \times 10^{-11} \text{ Pa}^{-1}$

$$\therefore \Delta V / V_1 = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 2.78 \times 10^{-3} \quad \dots \dots \dots (2)$$

Using equation (1) and equation (2), we get

$$1 - (\rho_1 / \rho_2) = 2.78 \times 10^{-3}$$

$$\rho_2 = 1.03 \times 10^3 / [1 - (2.78 \times 10^{-3})] = 1.032 \times 10^3 \text{ kg m}^{-3}$$

Therefore, at depth d, water has a density of  $1.034 \times 10^3 \text{ kg m}^{-3}$ .

**Q14. Compute the fractional change in volume of a glass slab when subjected to a hydraulic pressure of 10 atm.**

Answer.

Given,

Pressure acting on the glass plate,  $p = 10 \text{ atm} = 10 \times 1.013 \times 10^5 \text{ Pa}$

We know,

Bulk modulus of glass,  $B = 37 \times 10^9 \text{ Nm}^{-2}$

$\Rightarrow$  Bulk modulus,  $B = p / (\Delta V/V)$

Where,

$\Delta V/V =$  Fractional change in volume

$\therefore \Delta V/V = p / B$

$= [10 \times 1.013 \times 10^6] / (37 \times 10^9)$

$= 2.73 \times 10^{-4}$

Therefore, the fractional change in the volume of the glass plate is  $2.73 \times 10^{-4}$ .

**Q15. Determine the volume contraction of a solid copper cube, 10 cm on edge, when subjected to a hydraulic pressure of  $7.0 \times 10^6 \text{ Pa}$ .**

**Answer:**

Side of the copper cube,  $a = 10 \text{ cm}$

Therefore, the volume of the copper cube,  $V = a^3 = 10^{-3} \text{ m}^3$

Hydraulic pressure,  $p = 7.0 \times 10^6 \text{ Pa}$

Bulk modulus of copper  $B = 140 \text{ G Pa} = 140 \times 10^9 \text{ Pa}$ .

Bulk modulus,  $K = -P/(\Delta V/V)$

We get the value of volume contraction as  $\Delta V = -PV/K$   
 $= -(7 \times 10^6 \times 0.001) / (140 \times 10^9)$

$= -0.05 \times 10^{-6} \text{ m}^3$

**Q16. How much should the pressure on a litre of water be changed to compress it by 0.10%?**

**Answer:**

Volume of water,  $V = 1 \text{ litre}$

Water should be compressed by 0.10%

The fractional change in volume,  $\Delta V/V = (0.1/100) \times 1 = 10^{-3}$

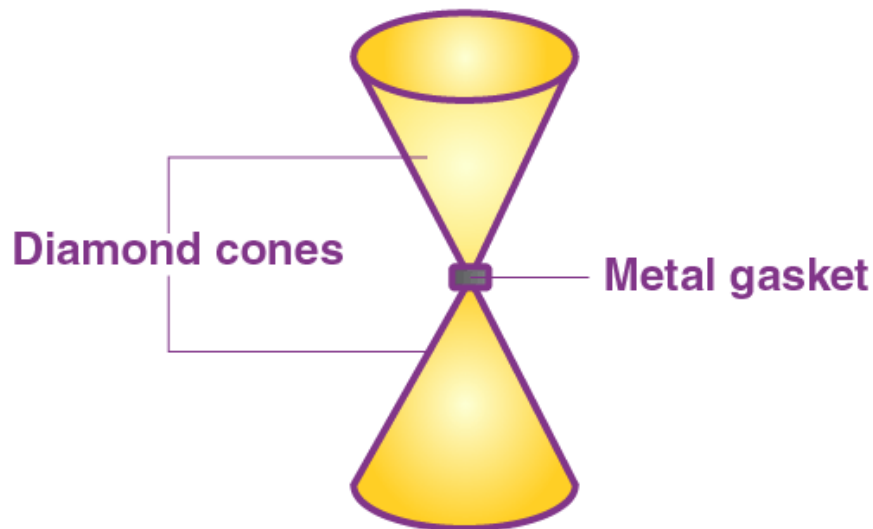
Bulk modulus,  $B = P/(\Delta V/V) = PV/\Delta V$

$P = B \times (\Delta V/V)$

Bulk modulus of water,  $B = 2.2 \times 10^9 \text{ Nm}^{-2}$

Pressure on water,  $P = 2.2 \times 10^9 \times 10^{-3} = 2.2 \times 10^6 \text{ Pa}$

**Q17. Anvils made of single crystals of diamond, with the shape as shown in the figure, are used to investigate the behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?**



**Answer:**

Flat faces at the narrow end of the anvil have a diameter,  $d=0.50\text{mm}=0.5\times 10^{-3}\text{ m}$

Radius,  $r=d/2=0.25\times 10^{-3}\text{ m}$

Compressional force,  $F=50000\text{N}$

Pressure at the tip of the anvil

$P=\text{Force}/\text{Area}$

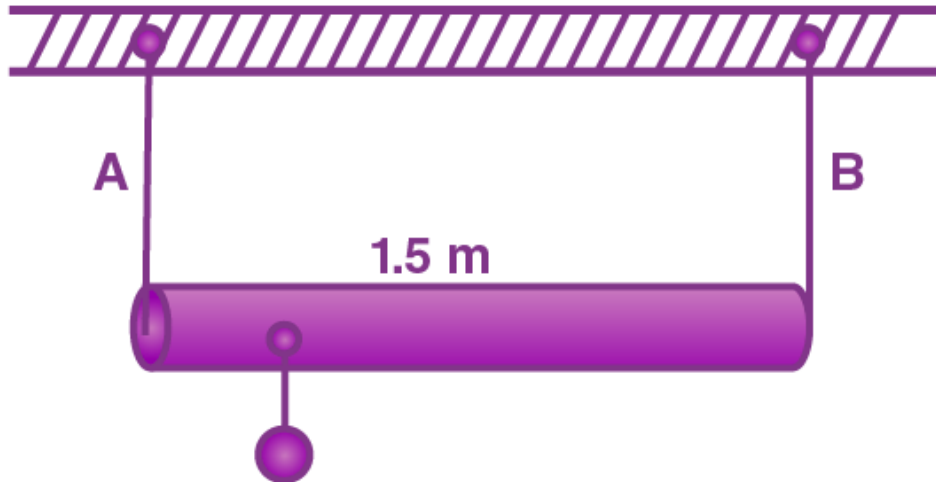
$$\text{Area} = \pi r^2 = 3.14 \times (0.25 \times 10^{-3})^2 = 0.1925 \times 10^{-6} \text{m}^2$$

Pressure at the tip of the anvil =  $F/A$

$$= 50000/0.1925 \times 10^{-6}$$

$$= 2.59 \times 10^{11} \text{Pa}$$

**Q18.** A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths, as shown in the figure. The cross-sectional areas of wires A and B are  $1.0\text{ mm}^2$  and  $2.0\text{ mm}^2$ , respectively. At what point along the rod should a mass  $m$  be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?



Answer.

Given,

Cross-sectional area of wire A,  $a_1 = 1.0 \text{ mm}^2 = 1.0 \times 10^{-6} \text{ m}^2$

Cross-sectional area of wire B,  $a_2 = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$

We know Young's modulus for steel,  $Y_1 = 2 \times 10^{11} \text{ Nm}^{-2}$

Young's modulus for aluminium,  $Y_2 = 7.0 \times 10^{10} \text{ Nm}^{-2}$

( i ) Let a mass  $m$  be hung on the stick at a distance  $y$  from the end where wire A is attached.

Stress in the wire = Force / Area =  $F / a$

Now, it is given that the two wires have equal stresses

$$F_1 / a_1 = F_2 / a_2$$

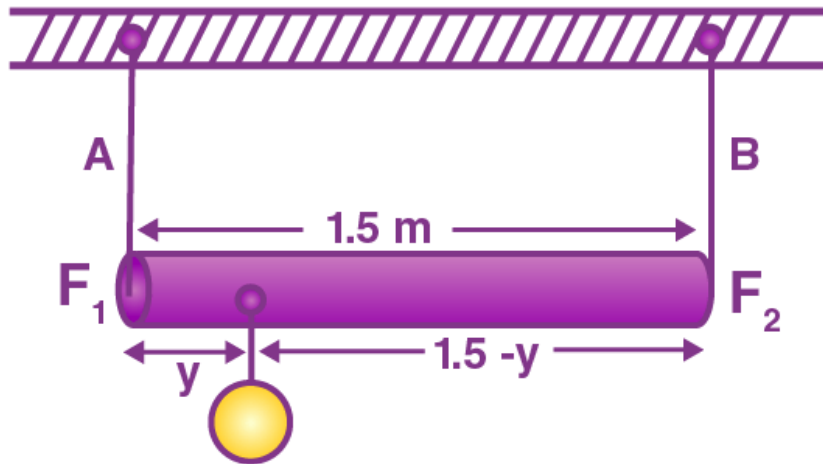
Where,

$F_1$  = Force acting on wire A

and  $F_2$  = Force acting on wire B

$$F_1 / F_2 = a_1 / a_2 = 1 / 2 \dots\dots\dots ( 1 )$$

The above situation can be represented as



Moment of forces about the point of suspension, we have

$$F_1 y = F_2 (1.5 - y)$$

$$F_1 / F_2 = (1.5 - y) / y \quad \dots \dots \dots (2)$$

Using equation (1) and equation (2), we can write

$$(1.5 - y) / y = 1 / 2$$

$$2(1.5 - y) = y$$

$$y = 1 \text{ m}$$

Therefore, the mass needs to be hung at a distance of 1 m from the end where wire A is attached in order to produce equal stress in the two wires.

(ii) We know,

Young's modulus = Stress / Strain

$$\Rightarrow \text{Strain} = \text{Stress} / \text{Young's modulus} = (F/a) / Y$$

It is given that the strain in the two wires is equal

$$(F_1/a_1) / Y_1 = (F_2/a_2) / Y_2$$

$$F_1 / F_2 = a_1 Y_1 / a_2 Y_2$$

$$a_1 / a_2 = 1 / 2$$

$$F_1 / F_2 = (1 / 2) (2 \times 10^{11} / 7 \times 10^{10}) = 10 / 7 \quad \dots \dots \dots (3)$$

Let the mass m be hung on the stick at a distance  $y_1$  from the end where the steel wire is attached in order to produce equal strain.

Taking the moment of the force about the point where mass m is suspended

$$F_1 y_1 = F_2 (1.5 - y_1)$$

$$F_1 / F_2 = (1.5 - y_1) / y_1 \quad \dots \dots \dots (4)$$

From equations (3) and (4), we get

$$(1.05 - y_1) / y_1 = 10 / 7$$

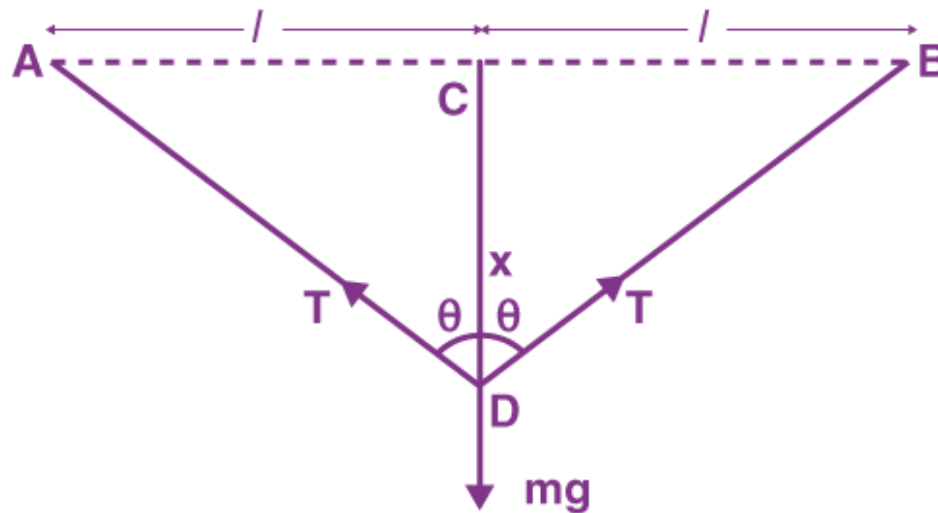
$$7(1.05 - y_1) = 10y_1$$

$$y_1 = 0.432 \text{ m}$$

Therefore, the mass needs to be hung at a distance of 0.432 m from the end where wire A is attached in order to produce equal strain in the two wires.

**Q19. 9** A mild steel wire of length 1.0 m and cross-sectional area  $0.50 \times 10^{-2} \text{ cm}^2$  is stretched well within its elastic limit horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

**Answer.**



Steel wire length = 1.0 m

Area of cross-section,  $A = 0.50$

$$\times 10^{-2} \text{ cm}^2$$

Mass of the given object,  $m = 100 \text{ g} = 0.1 \text{ kg}$

From the given diagram,  
We can deduce that

$x$   
is the depression at the midpoint, that is,  
 $CD =$   
 $x$

It is given that,  
AD = BD =

$$(I^2 + x^2)^{\frac{1}{2}}$$

AC = CB = l = 0.5m  
m = 100g = 0.100kg  
Gain in length,

$$\begin{aligned} \Delta I &= AD + DB - AB = 2AD - AB \\ &= \\ 2(I^2 + x^2)^{\frac{1}{2}} - 2I &= \left(1 + \frac{x^2}{I^2}\right)2I - (2I) \\ &= \\ 2I \left(1 + \frac{x^2}{2I^2}\right) - 2I &= \frac{x^2}{I} \end{aligned}$$

Thus, strain =

$$\frac{\Delta I}{2I} = \frac{x^2}{2I^2}$$

If the tension of the wire is

$$\begin{aligned} T & \\ , \text{ then} & \\ 2T \cos \theta &= mg \end{aligned}$$

$$\therefore T = \frac{mg}{2 \cos \theta}$$

Now,

$$\begin{aligned} \cos \theta & \\ = & \\ \frac{x}{(I^2 + x^2)^{\frac{1}{2}}} &= \frac{x}{I \left(1 + \frac{x^2}{I^2}\right)^{\frac{1}{2}}} = \frac{x}{\left(1 + \frac{x^2}{I^2}\right)} \end{aligned}$$

Since,

$x$

$\ll 1$ , then  $1 \gg$

$$\frac{1x^2}{2I^2}$$

&

$$1 + \frac{1}{2} \frac{x^2}{I^2} \approx 1$$

Therefore,

$$T = \frac{mg}{2\left(\frac{x}{l}\right)} = \frac{mgI}{2x}$$

Stress =

$$\frac{T}{A} = \frac{mgI}{2Ax}$$

Young's modulus

$$Y = \frac{\text{stress}}{\text{strain}}$$

=

$$\frac{mgI}{2Ax} \times \frac{2I^2}{x^2} = \frac{mgI^3}{Ax^3}$$

Therefore,

$$x = I \left( \frac{mg}{YA} \right)^{\frac{1}{3}} = 0.5 \left( \frac{0.1 \times 10}{20 \times 10^{11} \times 10^{-6}} \right)$$

=

$$1.074 \times 10^{-2} \text{ m}$$

**Q20.** Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed  $6.9 \times 10^7$  Pa? Assume that each rivet is to carry one-quarter of the load.

**Answer:**

Diameter of the metal strips = 6 mm =  $6 \times 10^{-3}$  m

Radius,  $r = 3 \times 10^{-3}$  m;

Shearing stress on the rivet =  $6.9 \times 10^7$  Pa

Maximum load or force on a rivet

= Maximum stress  $\times$  cross-sectional area

=  $6.9 \times 10^7 \times 3.14 \times (3 \times 10^{-3})^2$  N = 1950 N

Maximum tension =  $4 \times 1950$  N = 7800 N

**Q21.** The Mariana trench is located in the Pacific Ocean, and in one place, it is nearly eleven km beneath the surface of the water. The water pressure at the bottom of the trench is about  $1.1 \times 10^8$  Pa. A steel ball of initial volume  $0.32$  m<sup>3</sup> is dropped into the ocean and falls to the bottom of the



trench. What is the change in the volume of the ball when it reaches the bottom?

**Answer:**

Water pressure at the bottom of the trench,  $p=1.1 \times 10^8 \text{ Pa}$

Initial volume of the steel ball,  $V=0.32 \text{ m}^3$

Bulk modulus of steel,  $B=1.6 \times 10^{11} \text{ Nm}^{-2}$

The ball falls at the bottom of the trench, which is nearly 11 km beneath the surface of the water.

The volume change of the ball after reaching the bottom of the trench is  $\Delta V$ .

Bulk modulus,  $B=p/(\Delta V/V) = pV/\Delta V$

$\Delta V= pV/B$

$= (1.1 \times 10^8 \times 0.32) / (1.6 \times 10^{11})$

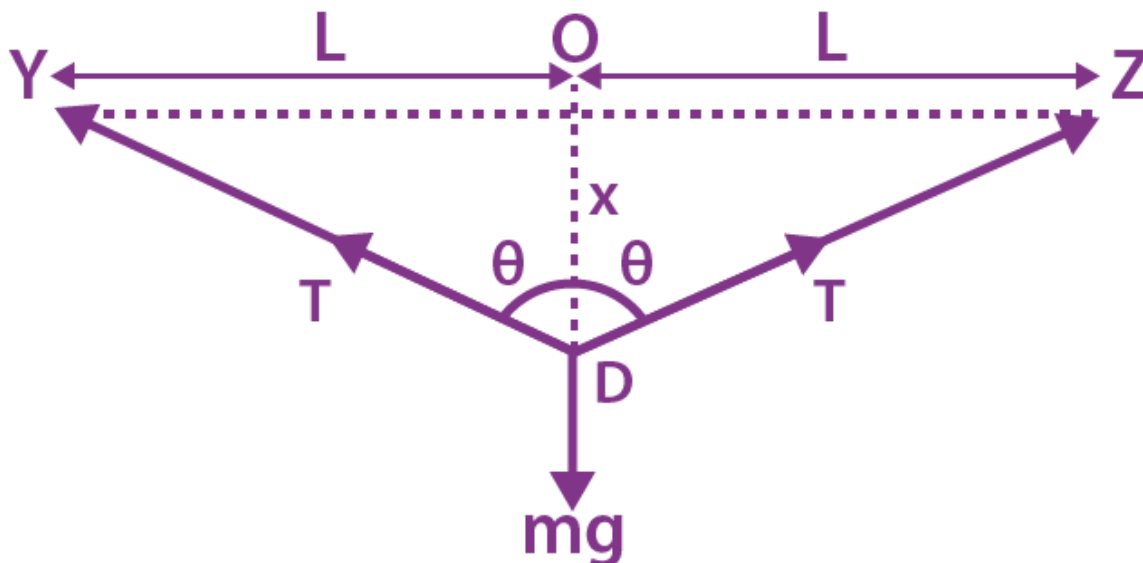
$= 0.352 \times 10^8 / 1.6 \times 10^{11}$

$= 0.22 \times 10^{-3} \text{ m}^3$

The change in volume of the ball on reaching the bottom of the trench is  $0.22 \times 10^{-3} \text{ m}^3$

**Q22.** A mild steel wire of cross-sectional area  $0.60 \times 10^{-2} \text{ cm}^2$  and length 2 m is stretched (not beyond its elastic limit) horizontally between two columns. If a 100g mass is hung at the midpoint of the wire, find the depression at the midpoint.

Answer.



Let YZ be the mild steel wire of length  $2l = 2\text{ m}$  and cross-sectional area  $A = 0.60 \times 10^{-2} \text{ cm}^2$ . Let the mass of  $m = 100 \text{ g} = 0.1 \text{ kg}$  be hung from the midpoint O, as shown in the figure. And let  $x$  be the depression at the midpoint, i.e., OD

From the figure,

$$ZO = YO = l = 1 \text{ m}$$

$$M = 0.1 \text{ KG}$$

$$ZD = YD = (l^2 + x^2)^{1/2}$$

$$\text{Increase in length, } \Delta l = YD + DZ - ZY$$

$$= 2YD - YZ \quad (\text{As } DZ = YD)$$

$$= 2(l^2 + x^2)^{1/2} - 2l$$

$$\Delta l = 2l(x^2/2l^2) = x^2 / l$$

$$\text{Therefore, longitudinal strain} = \Delta l / 2l = x^2/2l^2 \dots\dots\dots (i)$$

If T is the tension in the wires, then in equilibrium  $2T \cos \theta = 2mg$

$$\text{Or, } T = mg / 2 \cos \theta$$

$$= [mg (l^2 + x^2)^{1/2}] / 2x = mgl / 2x$$

$$\text{Therefore, Stress} = T / A = mgl / 2Ax \dots\dots\dots (ii)$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{mgl}{2Ax} \times \frac{2l^2}{x^2}$$

$$= \frac{mgl^3}{2Ax^3}$$

$$x =$$

$$l \left[ \frac{mg}{YA} \right]^{1/3}$$

$$= 1 \left[ \frac{0.1 \times 10}{20 \times 10^{11} \times 0.6 \times 10^{-6}} \right]^{1/3}$$

$$= 9.41 \times 10^{-3} \text{ m.}$$