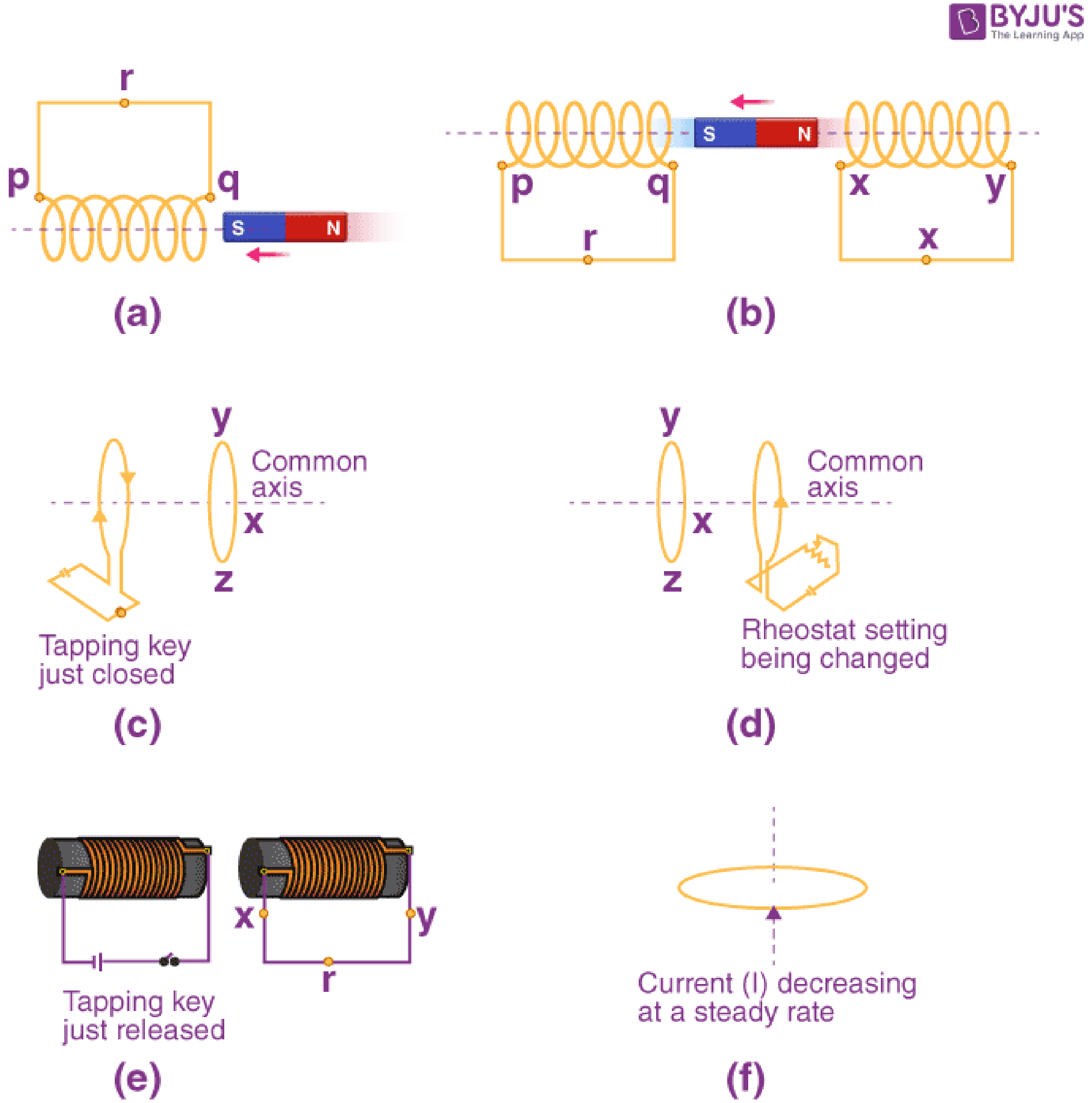
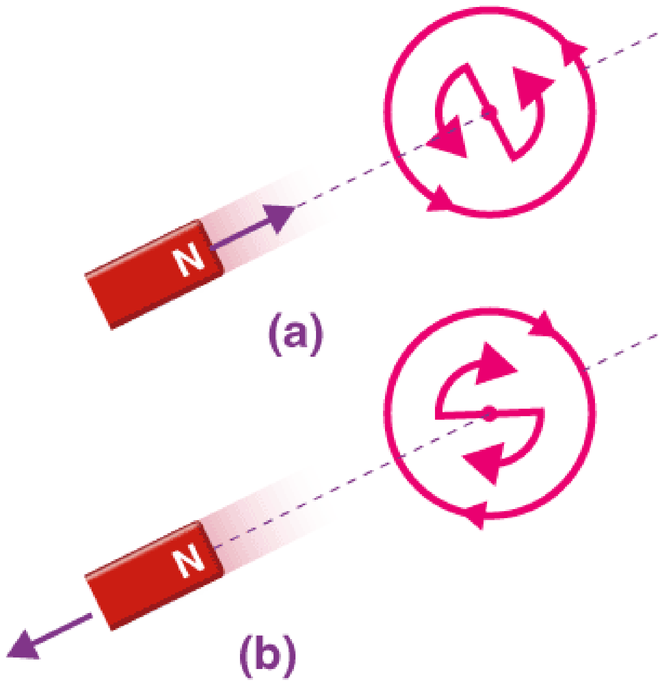
Q 1. Predict the direction of induced current in the situations described by the following figures.



Answer:

Lenz's law shows the direction of the induced current in a closed loop. The given two figures show the direction of induced current when the North pole of a bar magnet is moved towards and away from a closed loop, respectively.



We can predict the direction of induced current in different situations by using Lenz's rule:

(i) The direction of the induced current is along qrpq.

(ii) The direction of the induced current is along prq along yzx.

(iii) The direction of the induced current is along yzxy.

(iv) The direction of the induced current is along zyxz.

(v) The direction of the induced current is along xryx.

(vi) No current is induced since the field lines are lying in the plane of the closed loop.

Q 2. We are rotating a 1 m long metallic rod with an angular frequency of 400 red s^{-1} with an axis normal to the rod passing through its one end. And on to the other end of the rod, it is connected with a circular metallic ring. There exist a uniform magnetic field of 0.5 T, which is parallel to the axis everywhere. Find out the emf induced between the centre and the ring.
Ans:

Length of the rod = 1m

Angular frequency,

$$\omega = 400\ rad/s$$

Magnetic field strength, B = 0.5 T

At one of the ends of the rod, it has zero liner velocity, while on its other end, it has a linear velocity of

$$I\omega$$

Average linear velocity of the rod,

$$v = \frac{I\omega + 0}{2} = \frac{I\omega}{2}$$

Emf developed between the centre and ring.

$$e = Blv = Bl\left(\frac{I\omega}{2}\right) = \frac{Bl^2\omega}{2}$$

$$= \frac{0.5 \times (1)^2 \times 400}{2} = 100V$$

Hence, the emf developed between the centre and the ring is 100 V.

Q 3. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans:

Number of turns on the solenoid – 15 turns/cm = 1500 turns / m

Number of turns per unit length, n = 1500 turns

The solenoid has a small loop of area, A = 2.0 cm^2 = 2×10^{-4} m^2

The current carried by the solenoid changes from 2 A to 4 A.

Therefore, Change in current in the solenoid, di = 4 – 2 = 2 A

Change in time, dt = 0.1 s

According to Faraday's law, induced emf in the solenoid is given by:

$e = \frac{d\phi}{dt} \qquad ...(1)$

Where,

ϕ
= Induced flux through the small loop
= BA . . . (2)

B = Magnetic field

=

$\mu_0 ni$

μ_0
= Permeability of free space

=

$4\pi \times 10^{-7}\ H/m$

Hence, equation (1) can be reduced to:

$e = \frac{d}{dt}(BA)$

$e = A\,\mu_0\,n \times \left(\frac{di}{dt}\right)$

$= 2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$

$= 7.54 \times 10^{-6}\ V$

Hence, the induced voltage in the loop is

$$7.54 \times 10^{-6}\ V$$

Q 4. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of the uniform magnetic field of magnitude 0.3 T directed, normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side and (b) shorter side of the loop? For how long does the induced voltage last in each case?

Ans:

Length of the wired loop, l = 8 cm = 0.08 m

Width of the wired loop, b = 2 cm = 0.02 m

Since the loop is a rectangle, the area of the wired loop,

A = lb

= 0.08 × 0.02

=

16×10−4�2

Strength of magnetic field, B = 0.3 T

Velocity of the loop, v = 1 cm / s = 0.01 m / s

(i) Emf developed in the loop is given as:

e = Blv

= 0.3 × 0.08 × 0.01 =

$$16 \times 10^{-4}\, m^2$$

Time taken to travel along the width ,

$$t = \frac{Distance\ travelled}{Velocity} = \frac{b}{v}$$

$$= \frac{0.02}{0.01} = 2s$$

Hence, the induced voltage is

$$2.4 \times 10^{-4}\ V$$,

which lasts for 2 s.

(ii) Emf developed, e = Bbv

= 0.3 × 0.02 × 0.01 =

0.6×10−4�
The time taken to travel along the length,

$$t = \frac{Distance\ travelled}{Velocity} = \frac{l}{v}$$

$$= \frac{0.08}{0.01} = 8s$$

Hence, the induced voltage is

$$0.6 \times 10^{-4}\,V$$,
which lasts for 8 s.

Q 6. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0 × 10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans:

Maximum emf induced = 0.603 V

Average emf induced = 0 V

Maximum current in the coil = 0.0603 A

Power loss (average) = 0.018 W

(Power which is coming from external rotor)

Circular coil radius, r = 8 cm = 0.08 m

Area of the coil,

$$A = \pi r^2 = \pi \times (0.08)^2 m^2$$

Number of turns on the coil, N = 20

Angular speed,

$$\omega = 50\ rad/s$$

Strength of magnetic,

$$B = 3 \times 10^{-2}\ T$$

Total resistance produced by the loop,

$$R = 10\Omega$$

Maximum emf induced is given as:

$$\begin{aligned} e &= N\omega\ AB \\ &= 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} \\ &= 0.603\ V \end{aligned}$$

The maximum emf induced in the coil is 0.603 V.

Over a full cycle, the average emf induced in the coil is zero.

The maximum current is given as:

$$\begin{aligned} I &= \frac{e}{R} \\ &= \frac{0.603}{10} = 0.0603\ A \end{aligned}$$

Average power because of the Joule heating:

$$\begin{aligned} P &= \frac{el}{2} \\ &= \frac{0.603 \times 0.0603}{2} = 0.018\ W \end{aligned}$$

The torque produced by the current induced in the coil opposes the normal rotation of the coil. To keep the rotation of the coil continuous, we must find a source of torque which opposes the torque by the emf, so here the rotor works as an external agent. Hence, dissipated power comes from the external rotor.

Q 7. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30 × 10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?
(b) What is the direction of the emf?
(c) Which end of the wire is at the higher electrical potential?

Ans:

Wire's length, l = 10 m

Speed of the wire with which it is falling, v = 5.0 m/s

Strength of magnetic field, B =

$$0.3 \times 10^{-4} Wb\, m^{-2}$$

(a) EMF induced in the wire, e = Blv

$$= 0.3 \times 10^{-4} \times 5 \times 10$$
$$= 1.5 \times 10^{-3} V$$

(b) We can determine the direction of the induced current by using Fleming's right-hand thumb rule; here, the current is flowing in the direction from West to East.

(c) In this case, the eastern end of the wire will have a higher potential.

Q 8. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit.

Ans:

Current at initial point,

I_1
: 5.0 A

Current at final pint,

I_2
: 0.0 A

Therefore, change in current is, dI =

$I_1 – I_2$
= 5 A

Total time taken, t = 0.1 s

Average EMF, e = 200 V

We have the relation for self–inductance (L) and average emf of the coil:

$$e = L\frac{di}{dt}$$

$$L = \frac{e}{\left(\frac{di}{dt}\right)}$$

$$= \frac{200}{\frac{5}{0.1}} = 4H$$

Hence, the self-induction of the coil is 4 H.

Q 9. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Ans:

Given,

Mutual inductance,

$$\mu = 1.5\ H$$

Current at initial point,

I_1
: 0 A

Current at final point,

I_2
: 20 A

Therefore, change in current is, dI =

$$I_2 – I_1$$
$$= 20 - 0 = 20\text{ A}$$

The time taken for the change, t = 0.5s

Emf induced,

$$e = \frac{d\phi}{dt} \quad \ldots (1)$$

$d\phi$
= change in the flux linkage with the coil.
The relation of emf and inductance is:

$$e = \mu \frac{dI}{dt} \quad \ldots (2)$$

On equating both the equation, we get

$$\frac{d\phi}{dt} = \mu \frac{dI}{dt}$$
$$\frac{d\phi}{dt} = 1.5 \times (20)$$

$$= 30\ Wb$$

Therefore, the change in flux linkage is 30 Wb.

Q 10. A jet plane is travelling towards the west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5 × 10^{-4} T and the dip angle is 30°?

Ans:

Speed of the plane with which it is moving, v = 1800 km/h = 500 m/s

Wing span of the jet, l = 25 m

Magnetic field strength by earth, B =

$$5 \times 10^{-4} \, T$$

Dip angle,

$$\delta = 30^{\circ}$$

The vertical component of Earth's magnetic field,

$$B_v = B \sin \delta$$
$$= 5 \times 10^{-4} \sin 30^{\circ}$$
$$= 2.5 \times 10^{-4} T$$

The difference in voltage between both ends can be calculated as:

$$e = (B_v) \times l \times v$$

$$= 2.5 \times 10^{-4} \times 25 \times 500$$

$$= 3.125 V$$

Hence, the voltage difference developed between the ends of the wings is 3.125 V.

Additional Exercises

Q 11. Let us assume that the loop in question number 4 is stationary or constant, but the current source, which is feeding the electromagnet which is producing the magnetic field, is slowly decreased. It had an initial value of 0.3 T, and the rate of reducing the field was 0.02 T / sec. If the cut is joined to form the loop having a resistance of

1.6Ω
, calculate how much power is lost in the form of heat. What is the source of this power?

Ans:

The rectangular loop has sides of 8 cm and 2 cm.

Therefore, the area of the loop will be, A = L × B

= 8 cm × 2 cm

=

$$16cm^2$$

=

$$16 \times 10^{-4}cm^2$$

Value of magnetic field at the initial phase, B' = 0.3 T

Magnetic fields decreasing rate,

$$\frac{dB}{dt} = 0.02T/s$$

Emf induced in the loop is:

$$e = \frac{d\phi}{dt}$$
$$d\phi$$

= change in flux in the loop area
= AB

$$\therefore e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$= 16 \times 10^{-4} \times 0.02 = 0.32 \times 10^{-4}V$$

Resistance in the loop will be, R =

1.6Ω
The current developed in the loop will be:

$$i = \frac{e}{R}$$

$$= \frac{032 \times 10^{-4}}{1.6} = 2 \times 10^{-5}A$$

Power loss in the loop in the form of heat is:

$$P = i^2 R$$

$$= \left(2 \times 10^{-5}\right)^2 \times 1.6$$

$$= 6.4 \times 10^{-10} W$$

An external agent is a source of this heat loss, which is responsible for the change in the magnetic field with time.

Q 12. We have a square loop having a side of 12 cm, and its sides parallel to the x and the y-axis is moved with a velocity of 8 cm/s in the positive x-direction in a region which have a magnetic field in the direction of the positive z-axis. The field is not uniform, whether in the case of its space or in the case of time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction(i.e. its value increases by $10^{-3}\ T\ cm^{-1}$ as we move from positive to negative direction), and it is reducing in the case of time with the rate of $10^{-3}\ T\ s^{-1}$. Calculate the magnitude and direction of induced current in the loop (Given: Resistance = $4.50\ m\Omega$).

Ans:

Side of the Square loop, s = 12cm = 0.12m

Area of the loop, A = s × s = 0.12 × 0.12 = 0.0144 m^2

Velocity of the loop, v = 8 $cm\ s^{-1}$ = 0.08 $m\ s^{-1}$

The gradient of the magnetic field along the negative x-direction,

$$\frac{dB}{dx} = 10^{-3}\ T\,cm^{-1} = 10^{-1}\ m^{-1}$$

And the rate of decrease of the magnetic field,

$$\frac{dB}{dt} = 10^{-3}\, T\, s^{-1}$$

Resistance, R =

$$4.50\ m\Omega = 4.5 \times 10^{-3}\ \Omega$$

The rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as**:**

$$\frac{dB}{dt} = A \times \frac{dB}{dx} \times v$$

$$= 144 \times 10^{-4} m^2 \times 10^{-1} \times 0.08$$

$$= 11.52 \times 10^{-5}\ Tm^2 s^{-1}$$

The rate of change of the flux due to explicit time variation in field B is given as:

$$\frac{d\phi}{dt} = A \times \frac{dB}{dt}$$

$$= 144 \times 10^{-4} \times 10^{-3}$$

$$= 1.44 \times 10^{-5} T\, m^2 s^{-1}$$

Since the rate of change of the flux is the induced emf, the total induced emf in the loop can be calculated as:

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$= 12.96 \times 10^{-5} V$$

$$\therefore Induced\ current,\ i = \frac{e}{R}$$

$$= \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$

$$i = 2.88 \times 10^{-2}\ A$$

Hence, the direction of the induced current is such that there is an increase in the flux through the loop along the positive z-direction.

Q 13. We have a powerful loudspeaker magnet and have to measure the magnitude of the field between the poles of the speaker. And a small search coil is placed, normal to the field direction and then quickly removed out of the field region; the coil is of

2��2
area and has 25 closely wound turns. Similarly, we can give the coil a quick
90∘
turn to bring its plane parallel to the field direction. We measure the total charge flown in the coil by using a ballistic galvanometer, and it comes to 7.5 mC. Total resistance after combining the coil and the galvanometer is 0.50Ω
. Estimate the field strength of the magnet.
Ans:

Given,

Coil's Area, A =

$$2\,cm^2 = 2 \times 10^{-4} m^2$$

Number of turns on the coil, N = 25

Total Charge in the coil, Q = 7.5 mC =

$$7.5 \times 10^{-3}\ C$$

Total resistance produced by the combo of coil and galvanometer,

$$R = 0.50\ \Omega$$

Current generated in the coil,

$$I = \frac{Induced\ emf\ (e)}{R} \qquad \ldots(1)$$

The EMF induced is shown as:

$$e = -N\frac{d\Phi}{dt} \qquad \ldots(2)$$

Where,

$d\phi$
= Change in flux
From equations (1) and (2), we have

$$I = -\frac{N\frac{d\Phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \qquad \ldots(3)$$

Flux through the coil at the initial phase,

$$\phi_i = BA$$

Where B = Strength of the magnetic field

Flux through the coil at the final phase,

$$\phi_f = 0$$

After integrating equation (3) on both sides, we get

$\int Idt = \frac{-N}{R} \int_{\phi_i}^{\phi_f} d\phi$

Total Charge,

$Q = \int Idt$

$\therefore Q = \frac{-N}{R}(\phi_f - \phi_i) = \frac{-N}{R}(-\phi_i) = +\frac{N\phi_i}{R}$

$Q = \frac{NBA}{R}$

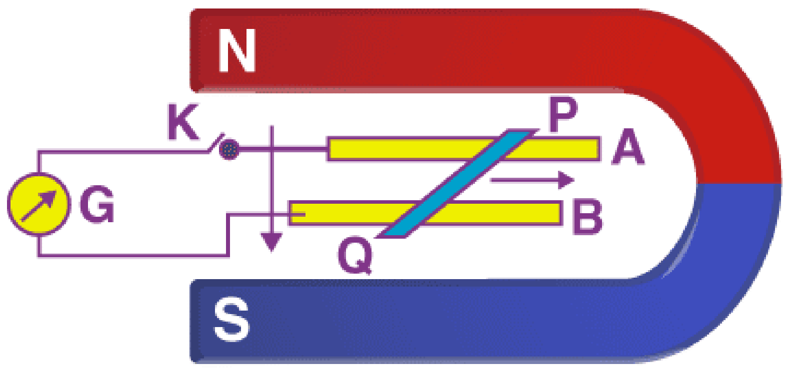
$\therefore B = \frac{QR}{NA}$

$= \frac{7.5\times10^{-3}\times0.5}{25\times2\times10^{-4}} = 0.75\ T$

Hence, the field strength is 0.75 T.

Q 14. In the given figure, we have a metal rod PQ which is put on the smooth rails AB, and they are kept in between the two poles of permanent magnets. All three (rod, rails and the magnetic field) are in a mutually perpendicular direction. There is a galvanometer 'G' connected through the rails by using a switch 'K'.Given, Rod's length = 15 cm , Magnetic field strength, B = 0.50 T, Resistance produced by the closed-loop =

$9.0\ m\Omega$ **. Let's consider the field is uniform.**



(i) Determine the polarity and the magnitude of the induced emf if we will keep the K open, and the rod will be moved with the speed of 12 cm/s in the direction shown in the figure.

(ii) When the K was open, was there any excess charge built up? Assuming that K is closed, then what will happen after it?

(iii) When the rod was moving uniformly, and the K was open, then on the electron in the rod PQ, there was no net force even though they did not experience any magnetic field because of the motion of the rod. Explain.

(iv) After closing the K, calculate the retarding force.

(v) When the K will be closed, calculate the total external power which will be required to keep moving the rod at the same speed (12 cm/s), and also calculate the power required when K will be closed.

(vi) What would be the power loss (in the form of heat) when the circuit is closed? What would be the source of this power?

(vii) Calculate the emf induced in the moving rod if the direction of the magnetic field is changed from perpendicular to parallel to the rails?

Ans:

Length of the rod, l = 15 cm = 0.15 m

Strength of the magnetic field, B = 0.50 T

Resistance produced by the closed-loop, R =

$$9.0\ m\Omega = 9 \times 10^{-3}\Omega$$

(i)

The polarity of the emf induced is in such a way that its P end is showing positive, while the other end, i.e. Q, is showing negative.

Since, speed, v = 12cm/s = 0.12 m/s

Emf induced is: e = Bvl

= 0.5 × 0.12 × 0.15

=

$$9 \times 10^{-3}\ V$$

= 9 mV

Here, the polarity of the emf is induced in a way that the P end shows +ve and the Q end shows -ve.

(ii) Yes, when the key K was opened, then at both ends, there was excess charge built up.

An excess charge was also built up when the key K was closed, and that charge was maintained by the continuous flow of current.

(iii) Because of the electric charge set-up, there was an excess charge of the opposite nature at both ends of the rod. Because of that, the Magnetic force was cancelled up.

When the key K was opened, then there was no net force on the electrons in the rod PQ, and the rod was moving uniformly. It is because of the cancelled magnetic field on the rod.

(iv) Regarding force exerted on the rod, F = IBl

Where,

I = current flowing through the rod

$$= \frac{e}{R} = \frac{9\times10^{-3}}{9\times10^{-3}} = 1\ A$$

$$\therefore F = 1 \times 0.5 \times 0.15$$

$$= 75 \times 10^{-3}\ N$$

(v)

No power will be expended when the key K is opened.

Speed of the rod, v = 12 cm/s = 0.12 m /s

Hence,

Power, P = Fv

$$= 75 \times 10^{-3} \times 0.12$$

$$= 9 \times 10^{-3}\ W$$

$$= 9\ mW$$

When the key K is opened, no power is expended.

(vi) 9mW,

Power is provided by an external agent.

Power loss in the form of heat is given as follows:

=

$$I^2R$$

=

$$1^2 \times 9 \times 10^{-3}$$

= 9 mW

(vii) Zero (0)

There would be no emf induced in the coil. As the emf induces if the motion of the rod cuts the field lines. But in this case, the movement of the rod does not cut across the field lines.

Q 15. We have an air-cored solenoid having a length of 30 cm, whose area is

$25\ cm^2$ **and the number of turns is 500. And the solenoid has carried a current of 2.5 A. Suddenly the current is turned off, and the time taken for it is**

10^{-3} **s. What would be the average value of the induced back-emf by the ends of the open switch in the circuit? (Neglect the variation in the magnetic fields near the ends of the solenoid.)**

Ans:

Given,

Length of the solenoid, l = 30 cm = 0.3 m

Area of the solenoid, A =

$$25\ cm^2 = 25 \times 10^{-4}\ m^2$$

Number of turns on the solenoid, N = 500

Current in the solenoid, I = 2.5 A

Time duration for the current flow, t =

$$10^{-3}$$

Average back emf,

$$e = \frac{d\phi}{dt} \qquad \ldots(1)$$

change in flux
= NAB . . . . (2)

Where,

B = Strength of the magnetic field

$$= \mu_0 \frac{NI}{l} \qquad \ldots(3)$$

Where,

μ_0
= Permeability of free space =
$4\pi \times 10^{-7}\ T\ m\ A^{-1}$

Using equations (2) and (3) in equation (1), we get

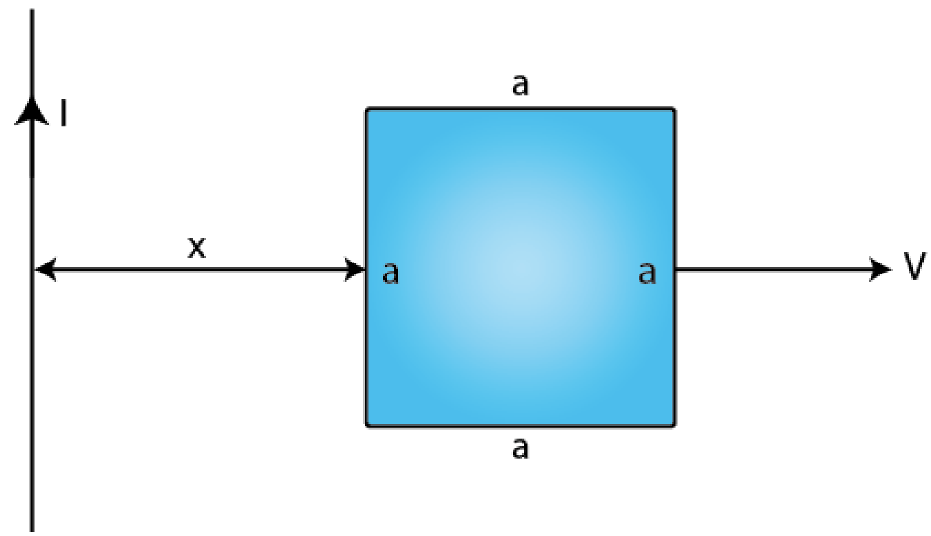
$$e = \frac{\mu_0 N^2 I A}{lt}$$

$$= \frac{4\pi\times10^{-7}\times(500)^2\times2.5\times25\times10^{-4}}{0.3\times10^{-3}} = 6.5V$$

Hence, the average back emf induced in the solenoid is 6.5 V.

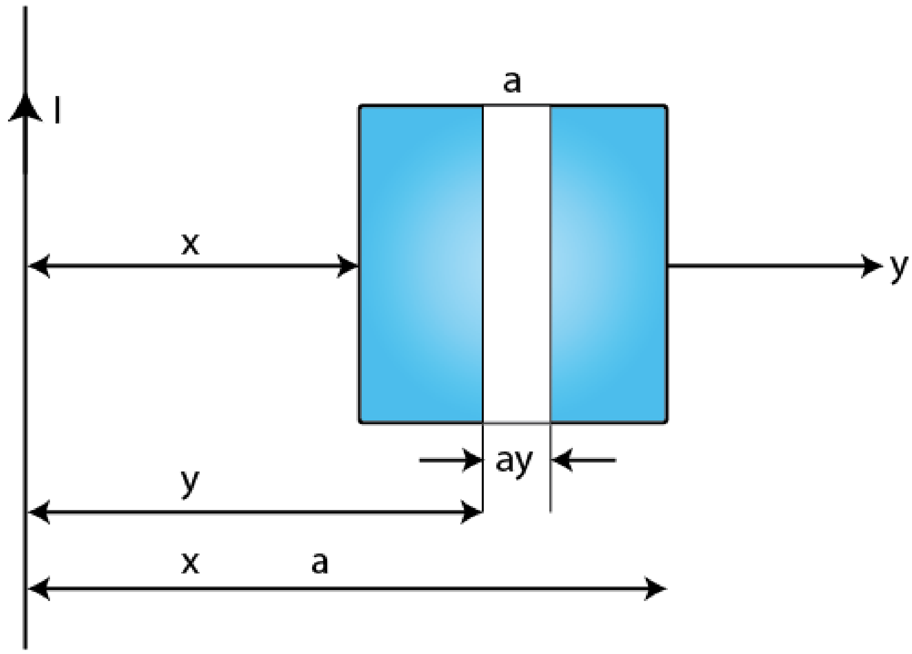
Q 16. (i) We are given a long straight wire and a square loop of a given size (refer to figure). Find out an expression for the mutual inductance between both.

(ii) Now, consider that we passed an electric current through the straight wire of 50 A, and the loop is then moved to the right with constant velocity, v = 10 m/s. Find the emf induced in the loop at an instant where x = 0.2 m. Take a = 0.01 m and assume that the loop has a large resistance.



Ans:

(i) Take a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).



Magnetic flux associated with element

$$dy,\ d\phi = a\,dy$$

3

B = Magnetic field at distance

$$y = \frac{\mu_0 I}{2\pi y}$$

I = Current in the wire

μ_0

= Permeability of free space =

$$4\pi \times 10^{-7}$$

$$\therefore d\phi = \frac{\mu_0 I a}{2\pi} \frac{dy}{y}$$

$$\phi = \frac{\mu_0 I a}{2\pi} \int \frac{dy}{y}$$

$$y \; tends \; from \; x \; to \; a + x$$

$$\therefore \phi = \frac{\mu_0 I a}{2\pi} \int_x^{a+x} \frac{dy}{y}$$

$$= \frac{\mu_0 I a}{2\pi} [\log_e y]_x^{a+x}$$

$$= \frac{\mu_0 I a}{2\pi} \log_e \frac{a+x}{x}$$

For mutual inductance M, the flux is given as:

$$\phi = MI$$

$$\therefore MI = \frac{\mu_0 I a}{2\pi} \log_e \left(\frac{a}{x} + 1\right)$$

$$M = \frac{\mu_0 a}{2\pi} \log_e \left(\frac{a}{x} + 1\right)$$

(ii) EMF induced in the loop, e = B’av

$$= \frac{\mu_0 I}{2\pi x} av$$

Given, I = 50 A

x = 0.2 m

a = 0.1 m

v = 10 m/s

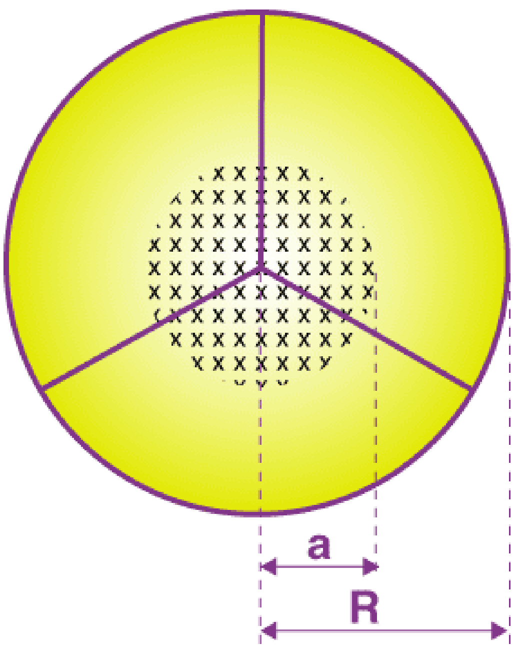
$$e = \frac{4\pi\times10^{-7}\times50\times0.1\times10}{2\pi\times0.2}$$

$$e = 5\times10^{-5}\ V$$

Q 17.A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,

$$B = -B_0\, k\, (r \leq a; a < R)$$ **= 0 (otherwise)**

What is the angular velocity of the wheel after the field is suddenly switched off?



Ans:

Line charge per unit length =

$$\lambda = \frac{Total\ charge}{Length} = \frac{Q}{2\pi r}$$

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

The radius of the wheel = R

Magnetic field,

$$\vec{B} = -B_0\dot{k}$$

At distance r, the magnetic force is balanced by the centripetal force, i.e.

$$BQv = \frac{Mv^2}{r}$$

Where,

v = linear velocity of the wheel

$$\therefore B2\pi r\lambda = \frac{Mv}{r}$$

$$v = \frac{B2\pi\lambda r^2}{M}$$

$$\therefore Angular\ Velocity, \omega = \frac{v}{R}$$

$$= \frac{B2\pi\lambda r^2}{MR}$$

$$For\ r \le a\ and\ a < R, we\ get$$

$$\omega = -\frac{2\pi B_0 a^2 \lambda}{MR}\hat{k}$$