Question 7.1:

A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the RMS value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Answer 7.1:

Given :

The resistance R of the resistor is 100 Ω

The source voltage V is 220 V

The frequency of the supply is 50 Hz.

a) To determine the value of the current in the connection, we can use the following relation:

$$I = \frac{V}{R}$$

Substituting values, we get

$$I = \frac{220}{100}$$

= 2.20 A

Therefore, the RMS value of the current in the connection is 2.20 A.

b) The total power consumed over an entire cycle can be calculated using the following formula:

P = V x I

Substituting values in the above equation, we get

= 220 x 2.2 = 484 W

Therefore, the total power consumed is 484 W.

Question 7.2:

a) The peak voltage of an ac supply is 300 V. What is the RMS voltage?

b) The RMS value of current in an ac circuit is 10 A. What is the peak current?

Answer 7.2:

a) The peak voltage of the AC supply is V_0 = 300 V.

We know that,

$$V_{RMS} = V = \frac{V_0}{\sqrt{2}}$$

Substituting the values, we get

$$V = \frac{300}{\sqrt{2}} = 212.2 \text{ V}$$

Hence, the RMS voltage of the AC supply is 212.2 V.

b) The RMS value of the current in the circuit is I = 10 A

We can calculate the peak current from the following equation,

$$I_0 = \sqrt{2}I$$

Substituting the values, we get

$$\sqrt{2} \times 10 = 14.1A$$

Therefore, the peak current is 14.1 A\end{array} \)

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Question 7.3:

A 44 mH inductor is connected to a 220 V, 50 Hz ac supply. Determine the RMS value of the current in the circuit.

Answer 7.3:

As given :

The inductor connected to the AC supply has an inductance of $L = 44 \text{ m H} = 44 \times 10^{-3}$ H

The magnitude of the source voltage V is 220 V

The frequency of the source is $\nu = 50$ Hz

The angular frequency of the source is given by $\omega = 2 \pi \nu$

The inductive reactance X_L can be calculated as follows:

$\omega_L = 2 \pi \nu L = 2\pi \times 50 \times 44 \times 10^{-3}\ \Omega$

To find the RMS value of the current, we use the following relation:

$$I = \frac{V}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}}$$

= 15.92 A

Therefore, the RMS value of the current in the network is 15.92 A.

Question 7.4:

A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the RMS value of the current in the circuit.

Answer 7.4:

Given:

The capacitance of the capacitor in the circuit is C = 60 μ F or 60×10^{-6} F

The source voltage is V = 110 V

The frequency of the source is ν = 60 Hz

The angular frequency can be calculated using the following relation,

ω = 2πν

The capacitive reactance in the circuit is calculated as follows:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}}$$

Ω

Now, the RMS value of the current is determined as follows:

$$I = \frac{V}{X_C} = \frac{110}{2\pi \times 60 \times 60 \times 10^{-6}} = 2.49 A$$

Therefore, the RMS current is 2.49 A.

Question 7.5:

In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Answer 7.5:

a) We know that in the case of the inductive network,

The RMS current value is I = 15.92 A

The RMS voltage value is V = 220 V

Therefore, the total power taken can be derived by the following equation:

P = VI cos Φ

Here,

Φ is the phase difference between V and I

In the case of a purely inductive circuit, the difference in the phase of an alternating voltage and an alternating current is 90°,

i.e. Φ = 90°.

Therefore, P = 0

Thus, the total power absorbed by the circuit is zero.

b) In the case of the capacitive network, we know that

The value of RMS current is given by, I = 2.49 A

The value of RMS voltage is given by, V = 110 V

Thus, the total power absorbed can be derived from the following equation:

P = VI Cos Φ

For a purely capacitive circuit, the phase difference between alternating voltage and alternating current is 90°

i.e. Φ = 90°.

Thus , P = 0

Therefore, the net power absorbed by the circuit is zero.

Question 7.6:

Obtain the resonant frequency ω_r of a series LCR circuit with L = 2.0H, C = 32 µF and R = 10 Ω. What is the Q-value of this circuit?

Answer 7.6:

Given:

The inductance of the inductor is L = 2.0 H

The capacitance of the capacitor, C = 32 µF = 32×10^{-6} F

The resistance of the resistor R = 10 Ω.

We know that resonant frequency can be calculated by the following relation,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times32\times10^{-6}}} = \frac{1}{8\times10^{-3}} = 125\,\frac{rad}{sec}$$

Now, the Q-value of the circuit can be calculated as follows:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}} = \frac{1}{10\times4\times10^{-3}} = 25$$

Thus, the Q-value of the circuit is 25.

Question 7.7:

A charged 30 µF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Answer 7.7:

Given capacitance value of the capacitor, C = 30 µF = 30 × 10 $^{-6}$ F

Given inductance value of the charged inductor, L = 27 mH = 27 × 10 $^{-3}$ H

Angular frequency is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$
$$\omega_r = \frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}} = \frac{1}{9\times10^{-4}} = 1.11\times10^{3}\, rad\, sec^{-1}$$

Therefore, the calculated angular frequency of free oscillation of the connection is 1.11 x 10 ^{3} s^{-1}.

Question 7.8:

Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at a later time?

Answer 7.8:

Given capacitance value of the capacitor, C = 30 µF = 30 × 10 $^{-6}$ F

Inductance of the inductor, L = 27 mH = 27 × 10 $^{-3}$ H

Charge on the capacitor, Q = 6 mC = 6 × 10 $^{-3}$ C

The total energy stored in the capacitor can be calculated as follows:

$$E = \frac{1}{2}\times\frac{Q^2}{C} = \frac{1}{2}\frac{(6\times10^{-3})^2}{30\times10^{-6}} = \frac{6}{10} = 0.6J$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

Question 7.9:

A series LCR circuit with R = 20 Ω, L = 1.5 H, and C = 35 µF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Answer 7.9:

The supply frequency and the natural frequency are equal at resonance conditions in the circuit.

The given resistance of the resistor, R = 20 Ω

The given inductance of the inductor, L = 1.5 H

The given capacitance of the capacitor, C = 35 µF = 30 × 10 $^{-6}$ F

An AC source with a voltage of V = 200 V is connected to the LCR circuit

We know that the impedance of the above combination can be calculated by the following relation:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonant condition in the circuit, $X_L = X_C$

Therefore , Z = R = 20 Ω

We know that the current in the network is given by the relation:

$$I = \frac{V}{Z} = \frac{200}{20} = 10A$$

Therefore, the average power that is being transferred to the circuit in one full cycle:

V I = 200 x 10 = 2000 W

Question 7.10:

A radio can tune over the frequency range of a portion of the MW broadcast band: 800 kHz to 1200 kHz. If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[Hint: The condition for tuning is that the natural frequency, which is the frequency of free oscillations of the LC network, must be of the same value as the radio wave frequency]

Answer 7.10:

Given the frequency range of (ν) of radio is 800 kHz to 1200 kHz.

Given that the lower tuning frequency of the circuit, $\nu_1 = 800$ kHz $= 800 \times 10^3$ Hz

Given that the upper tuning frequency of the circuit, $\nu_2 = 1200$ kHz $= 1200 \times 10^3$ Hz

Given that the effective inductance of the inductor in the circuit is L = 200 μH = 200×10^{-6} H

We know that the capacitance of variable capacitor for ν_1 can be calculated as follows :

$$C_1 = \frac{1}{\omega_1^2 L}$$

Here, the variables are,

ω_1 = Angular frequency for capacitor C_1

$\omega_1 = 2\pi\nu_1$

$\omega_1 = 2\pi \times 800 \times 10^3$ rad/s

Therefore,

$$C_1 = \frac{1}{\left(2\pi \times 800 \times 10^3\right)^2 \times 200 \times 10^{-6}} = 1.9809 \times 10^{-10} F = 198.1 pF$$

The variable capacitor for v_2 has a capacitance of

$$C_2 = \frac{1}{\omega_2^2 L}$$

Here, the variables are,

ω_2 = Angular frequency for capacitor C_2

$\omega_2 = 2\pi\nu_2$

$\omega_2 = 2\pi \times 1200 \times 10^3$ rad/s

Therefore,

$$C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}} = 0.8804 \times 10^{-10} F = 88.04 pF$$

Thus, the variable capacitor has a range from 88.04 pF to 198.1 pF.

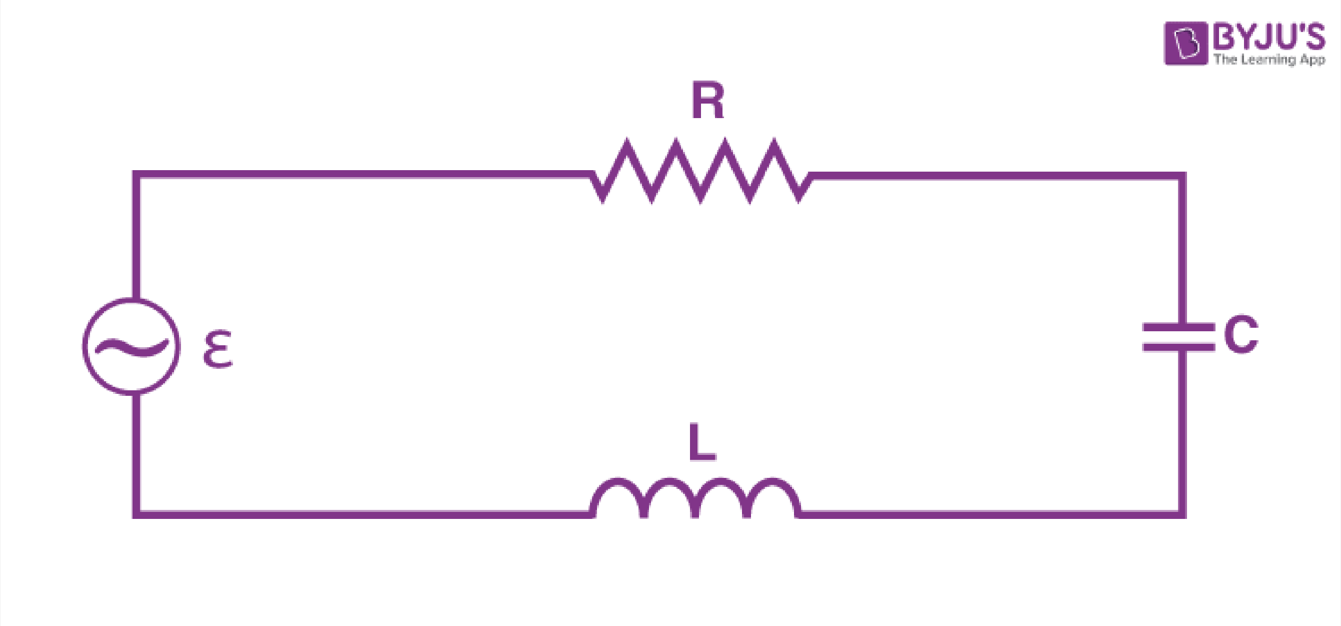
Question 7.11:

The figure below shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω.

L = 5.0 H,

C = 80 μF,

R = 40 Ω



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the RMS potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Answer 7.11:

Given that the inductance of the inductor in the circuit, L = 5.0 H

Given that the capacitance of the capacitor in the circuit, C = 80 μF = 80 × 10 $^{-6}$ F

Given that the resistance of the resistor in the circuit, R = 40 Ω

Value of potential of the variable voltage supply, V = 230 V

(a) We know that the resonance angular frequency can be obtained by the following relation:

$$\omega_r = \frac{1}{\sqrt{LC}}$$
$$\omega_r = \frac{1}{\sqrt{5\times 80\times 10^6}}$$
$$\omega_r = \frac{10^3}{20} = 50 rad/sec$$

Thus, the circuit encounters resonance at a frequency of 50 rad/s.

(b) We know that the Impedance of the circuit can be calculated by the following relation:

$$Z = \sqrt{R^2 + (X_L – X_C)^2}$$

At resonant condition,

$X_L = X_C$

Z = R = 40 Ω

At resonating frequency, the amplitude of the current can be given by the following relation:

$$I_0 = \frac{V_0}{Z}$$

Where,

V_0 = peak voltage =

$$\sqrt{2}V_{rms}$$

Therefore,

$$I_0 = \frac{\sqrt{2}V_{rms}}{Z} = \frac{\sqrt{2}\times 230}{40} = 8.13A$$

Thus, at resonant condition, the impedance of the circuit is calculated to be 40 Ω, and the amplitude of the current is found to be 8.13 A

c) The rms potential drop across the inductor in the circuit,

$(V_L)_{rms} = I \times \omega_r L$

Where,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} = \frac{23}{4}A$$

Therefore, $(V_L)_{rms}$

$$\frac{23}{4} \times 50 \times 5 = 1437.5V$$

We know that the potential drop across the capacitor can be calculated with the following relation:

$$(V_c)_{rms} = I \times \frac{1}{\omega_r C} = \frac{23}{4} \times \frac{1}{50\times80\times10^{-6}} = 1437.5V$$

We know that the potential drop across the resistor can be calculated with the following relation:

$$(V_R)_{rms} = IR = \frac{23}{4} \times 40 = 230V$$

Now, the potential drop across the LC connection can be obtained by the following relation:

$V_{LC} = I(X_L - X_C)$

At resonant condition,

$X_L = X_C$

$V_{LC} = 0$

Therefore, it has been proved from the above equation that the potential drop across the LC connection is equal to zero at a frequency at which resonance occurs.

Question 7.12:

An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible.
Let the instant the circuit is closed by t = 0.
(a) What is the total energy stored initially? Is it conserved during LC oscillations?
(b) What is the natural frequency of the circuit?
(c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?
(d) At what times is the total energy shared equally between the inductor and the capacitor?
(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Answer 7.12:

L = 20 mH

C = 50 μF

q = 10 mC

(a) The total energy stored initially is

E = $q^2/2C$

= $(10 \times 10^{-3})^2/(2\times 50 \times 10^{-6})$

= 1 J

The energy is conserved during LC oscillation.

(b) Natural frequency of the circuit

$$f = \frac{1}{2\pi\sqrt{LC}}$$
$$f = \frac{1}{2\times 3.14\sqrt{20\times10^{-3}\times50\times10^{-6}}}$$
$$f = \frac{1}{6.28\sqrt{1000\times10^{-3}\times10^{-6}}}$$
$$f = \frac{1}{0.00628}$$

f = 159.24 Hz

(c) At any instant t, the charge on the capacitor, $q = q_0 \cos\omega t$

$q = q_0 \text{Cos}(2\pi/T)t$

(i) Energy stored is completely electrical at t = 0, T/2, T,3T/2 ……….
(ii) Energy stored is completely magnetic (i.e., electrical energy is zero) at t = T/4, 3T/4,5T/4 ——-here T = 1/f = 1 /159.24 = 6.3 mS

(d) Equal sharing of energy means energy of the capacitor = (1/2) maximum energy

$q^2/2C = (1/2)\, q_0^2/2C$

$q = q_0/\sqrt{2}$

But $q = q_0 \cos\omega t$

$q = q_0\cos(2\pi/T)t$

Therefore, $q_0/\sqrt{2} = q_0\cos(2\pi/T)t$

$1/\sqrt{2} = \cos(2\pi/T)t \Rightarrow t = (2n+1)\, T/8$

The energy will be half on the capacitor and inductor at t = T/8, 3T/8, 5T/8…..

(e) Resistor damps out the LC oscillations eventually. The whole of the initial energy (= 1.0 J) is eventually dissipated as heat.

Question 7.13:

A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.
(a) What is the maximum current in the coil?
(b) What is the time lag between the voltage maximum and the current maximum?

Answer 7.13:

The inductance of the coil, L = 0.50 H

Resistance of the coil, R= 100 Ω

Supply voltage, = 240 V

Frequency of the supply,ν= 50 Hz

(a) Peak voltage $V_0 = \sqrt{2}V$

$V_0 = \sqrt{2} \times 240 = 1.414 \times 240 = 339.36$

Angular frequency of the supply, ω = 2πν

= 2π x 50 = 100 π rad/sec

The maximum current in the coil is,

$$I_0 = \frac{V_0}{\sqrt{R^2+\omega^2 L^2}}$$
$$I_0 = \frac{339.36}{\sqrt{100^2+(100\pi)^2(0.50)^2}}$$
$$I_0 = \frac{339.36}{\sqrt{34649}}$$
$$I_0 = \frac{339.36}{186.14}$$
$$I_0 = 1.82$$

(b) The time lag between the maximum voltage and maximum current is Φ/ω

Phase angle Φ is given by the relation

tan Φ = ωL/R

= 2πνL/R

= (2π x 50 x 0.50)/100

= 1.57

$\Phi = \tan^{-1}(1.57) = 57.5^0 = (57.5 \times \pi)/180$

ωt = (57.5 x π)/180

t = (57.5 x π)/(180 x 100π) = 57.5/18000

= 3.19×10^{-3} s

Question 7.14:

Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high-frequency supply (240 V, 10 kHz). Hence, explain the statement that at a very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a DC circuit after the steady state?

Answer 7.14:

The inductance of the coil, L = 0.50 H

Resistance of the coil, R= 100 Ω

Supply voltage, = 240 V

Frequency of the supply,ν= 10 kHz = 10^4 Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 10^4$ rad/sec

(a) Peak voltage $V_0 = \sqrt{2}V$

$V_0 = \sqrt{2} \times 240 = 1.414 \times 240 = 339.36$

The maximum current in the circuit is,

$$I_0 = \frac{V_0}{\sqrt{R^2+\omega^2L^2}}$$
$$I_0 = \frac{339.36}{\sqrt{100^2+(2\pi\times10^4)^2(0.50)^2}}$$
$$I_0 = 1.1 \times 10^{-2}\ A$$

(b) The time lag between the maximum voltage and maximum current is Φ/ω

Phase angle Φ is given by the relation

$\tan\Phi = \omega L/R$

$= 2\pi\nu L/R$

$= (2\pi \times 10^4 \times 0.50)/100$

$= 100\pi = 314$

$\Phi = \tan^{-1}(314) = 89.82^0$

$\omega t = (89.82 \times \pi)/180$

$t = (89.82 \times \pi)/(180 \times 2\pi \times 10^4) = 25\mu s$

I_0 is much smaller than the low-frequency case (Exercise 7.13), showing thereby that at high frequencies, L nearly amounts to an open circuit. In a DC circuit (after steady state), $\omega = 0$, so, here, L acts like a pure conductor.

Question 7.15:

A 100 µF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

Answer 7.15:

Capacitance of the capacitor, $C = 100\ \mu F = 100 \times 10^{-6}$ F

The resistance of the resistor = 40 Ω

Supply voltage, V = 110 V

Frequency, $\upsilon = 60$ Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 = 120\pi$ rad/s

(a) Maximum current in the circuit

$I_0 = V_0/Z$

Here,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$
$$Z = \sqrt{40^2 + \frac{1}{(120\pi)^2(10^{-4})^2}}$$
$$Z = \sqrt{1600 + \frac{10^8}{(120\pi)^2}}$$
$$Z = \sqrt{1600 + \frac{10^8}{141978}}$$
$$Z = \sqrt{1600 + 704}$$
$$Z = \sqrt{2304}$$

Z = 48

$I_0 = 110\sqrt{2}/48$

$I_0 = 155.6/48 = 3.24$

(b) In the capacitor circuit, the voltage will lag behind the current by a phase angle Φ.

Therefore, $\tan\Phi = 1/\omega CR$

$= 1/120\pi \times 10^{-4} \times 40$

= 0. 6635

$\Phi = \tan^{-1}(0.6635) = 33.56^0$

$= 33.56\pi/180$ rad

Time lag, $t = \Phi/\omega = 33.56\pi/(180 \times 120\pi) = 1.55 \times 10^{-3}$ S = 1.55 ms

Question 7.16:

Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply. Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a DC circuit after the steady state.

Answer 7.16:

Capacitance of the capacitor, C = 100 μF = 100×10^{-6} F

The resistance of the resistor, R= 40 Ω

Supply voltage, V = 110 V

Frequency, υ = 12×10^3 Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 12 \times 10^3 = 24\pi \times 10^3$ rad/s

The maximum current in the circuit

$I_0 = V_0/Z$

$V_0 = V\sqrt{2} = 110\sqrt{2} = 155.6$ V

Here,

$$Z=\sqrt{R^2+\frac{1}{\omega^2C^2}}$$
$$Z=\sqrt{40^2+\frac{1}{(24\pi\times10^3)^2(10^{-4})^2}}$$
$$Z=\sqrt{1600+\frac{1}{(24\pi\times10^3\times10^{-4})^2}}$$
$$Z=\sqrt{1600+0.178}$$
$$Z=\sqrt{1600.178}$$

Z = 40

$I_0 = 110\sqrt{2}/40$

I_0 = 155.6/40= 3.89 A

(b) In the capacitor circuit, the voltage will lag behind the current by a phase angle Φ.

Therefore, tanΦ = 1/ωCR

= 1/24π x10^3 x 10^{-4} x 40

tan Φ = 1/96 π

Φ = tan-1(0.0033) = 0.2^0

= 0.2 π/180 rad

Time lag, t =Φ /ω = 0.2 π/(180 x 24π x 10^3)

= 0.2/ (4320 x 10^3)

= 4.6 x 10^{-8} s = 0.04 μs

Thus, at a high frequency, the capacitor acts like a conductor. For a DC circuit, after a steady state, ω = 0, the capacitor amounts to an open circuit.

Question 7.17:

Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C and R, are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current RMS value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Answer 7.17:

Inductance of the inductor in the circuit, L = 5.0 H

Capacitance of the capacitor in the circuit, C = 80 μF = 80 × 10^{-6} F

Resistance of the resistor in the circuit, R = 40 Ω

Potential of the variable voltage supply, V = 230 V

The impedance of the given parallel LCR circuit is,

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Here, ω = angular frequency

At resonance,

$$\frac{1}{\omega L} - \omega C = 0$$
$$\omega = \frac{1}{\sqrt{LC}}$$
$$\omega = \frac{1}{\sqrt{5 \times 80 \times 10^6}} = 50rad/s$$

Therefore, Z is maximum at 50rad/s.

The RMS current flowing through the inductor is given as

I_L= V/ωL

= 230/(50 x 5) = 0.92 A

RMS current flowing through the resistor R is

I_R = V/R

= 230/40 = 5.75 A

RMS current flowing through the capacitor C

I_c = V/(1/ωC) = ωVC

= 50 x 80 x 10^{-6} x 230 = 0.92 A

Question 7.18

A circuit containing an 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.
(a) Obtain the current amplitude and RMS values.
(b) Obtain the RMS values of potential drops across each element.
(c) What is the average power transferred to the inductor?
(d) What is the average power transferred to the capacitor?
(e) What is the total average power absorbed by the circuit? [‘Average’ implies ‘averaged over one cycle]

Answer 7.18:

Inductance, L = 80mH = 80 x 10^{-3} H

Capacitance, C = 60μF = 60 x 10^{-6} F

Supply voltage, V = 230 V

Frequency, ν = 50 Hz

Angular frequency, $\omega = 2\pi\nu = 100\pi$ rad/s

Peak voltage, $V_0 = V\sqrt{2} = 230\sqrt{2}$

(a) RMS value of the current

$$I_0 = \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)}$$

$$I_0 = \frac{230\sqrt{3}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)}$$

$$I_0 = -11.63 \text{ A}$$

The negative sign shows $\omega L < (1/\omega C)$

Amplitude of the maximum current,

$$|I_0| = 11.63A$$

$I = I_0/\sqrt{2}$

$= -11.63/\sqrt{2} = -8.22$ A

(b) Potential difference across the inductor

$V_L = I \times \omega L$

$= 8.22 \times 100\pi \times 80 \times 10^{-3}$

= 206.61 V

The potential difference across the capacitor,

$V_c = I \times (1/\omega C)$

$= 8.22 \times (1/100\pi \times 60 \times 10^{-6})$

= 437 V

(c) Whatever may be the current in the inductor, the actual voltage leads to current by $\pi/2$. Therefore, the average power consumed by the inductor is zero.
(d) For the capacitor, voltage lags by $\pi/2$. So, the average power consumed by the capacitor is zero.
(e) Total average power absorbed by the circuit is zero.

Question 7.19:

Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit and the total power absorbed.

Answer 7.19:

Inductance, $L = 80\text{mH} = 80 \times 10^{-3}$ H

Capacitance, $C = 60\mu\text{F} = 60 \times 10^{-6}$ F

Supply voltage, V = 230 V

Frequency, $\nu = 50$ Hz

Resistance of the resistor, R = 15 Ω

Angular frequency of the signal, ω = 2πv = 2π x 50 = 100π rad/s

The impedance of the circuit,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
$$Z = \sqrt{15^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)^2}$$
$$Z = \sqrt{15^2 + (25.12 - 53.08)^2} = 31.728\Omega$$

I = V/Z = 230/31.728 = 7.25 A

The average power transferred to resistance is given as

$P_R = I^2 R$

$= (7.25)^2$ x 15 = 788.44 W

Average power transferred to the capacitor, $P_C = 0$

Average power transferred to the capacitor, $P_L= 0$

Therefore, the total power absorbed by the circuit = $P_R + P_C + P_L$ = 788.44 + 0 + 0 = 788. 44 W

Question 7.20:

A series LCR circuit with L = 0.12 H, C = 480 nF, and R = 23 Ω is connected to a 230 V variable frequency supply.
(a) What is the source frequency for which the current amplitude is maximum? Obtain this maximum value.
(b) What is the source frequency for which the average power absorbed by the circuit is maximum? Obtain the value of this maximum power.
(c) For which frequencies of the source are the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?
(d) What is the Q-factor of the given circuit?

Answer 7.20:

Inductance, L = 0.12 H

Capacitance, C = 480 nF = 480 x 10^{-9} F

Resistance, R = 23 Ω

Supply voltage, V = 230 V

Peak voltage is given as

$V_o = V\sqrt{2} = 230\sqrt{2} = 325.22$ V

(a) Current flowing the circuit is given by the relation

$$I_0 = \frac{V_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

At resonance, we have

$$\left(\omega_R L - \frac{1}{\omega_R C}\right) = 0$$

ω_R is the resonance angular frequency

$\omega_R = 1/\sqrt{LC}$

$= 1/\sqrt{0.12 \times 480 \times 10^{-9}} = 4166.67$ rad/s

Resonant frequency, $\nu_R = \omega R/2\pi = 4166.67/2 \times 3.14 = 663.48$ Hz

Maximum current, $(I_0)_{max} = V_0/R = 325.22/23 = 14.14$ A

(b) Maximum average power absorbed by the circuit is

$P_{Max} = (1/2)\,(I_0)^2_{max}\,R$

$= (1/2) \times (14.14)^2 \times 23 = 2300$ W

(c) The power transferred to the circuit is half the power at the resonant frequency

$P = V_{rms}\, I_{rms} \cos \Phi$

$= [(V_{rms})^2/Z\,] \times (R/Z)$

$= (V_{rms}^2 \times R)/Z^2$

$P_{max} = (V_{rms})^2/Z$

Given $P = (1/2)\, P_{max}$

Therefore, we get

$(V_{rms}^2 \times R)/Z^2 = (V_{rms})^2/2Z$

$2R^2 = Z^2$

$2R^2 = R^2 + (X_C - X_L)^2$

$X_C - X_L = \pm R$

Let us take $X_C - X_L = + R$

$(1/C\omega) - L\omega = R$

$1 - LC\omega^2 = RC\omega$

$LC\omega^2 + RC\omega - 1 = 0$

$$\omega = \frac{-RC+\sqrt{R^2C^2-4LC}}{2LC}$$

$$\omega = \frac{-23\times480\times10^{-9}+\sqrt{(23\times480\times10^{-9})^2-4\times0.12\times480\times10^{-9}}}{2\times0.12\times480\times10^{-9}}$$

ω = 4263.63 rad/s

Let us consider

$X_C - X_L = -R$

(1/Cω) – Lω = – R

1 – LCω² = RCω

LCω² – RCω -1 = 0

= (CR)² – 4 (LC)

Substituting the values, we get a negative value. Therefore, it will not be considered.

So, ω = 4263.63 rad/s

Therefore, 2πf = ω

⇒ f = 4263.63/(2 x 3.14) =4263.63/6.28 = 678.57 Hz

At this frequency, the current frequency is given as,

I' = (1/√2) x $(I_0)_{max}$

= 14.14/√2 = 10 A

(d) Q factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{23}\sqrt{\frac{0.12}{480\times10^{-9}}} = 21.74$$

Question 7.21:

Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Answer 7.21:

Inductance, L = 3.0 H

Capacitance, C = 27μF = 27 x 10^{-6} F

Resistance, R = 7.4 Ω

For the LCR series circuit, the resonant frequency of the source is

$$\omega = \frac{1}{\sqrt{LC}}$$
$$\omega = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = 111.11 rad/s$$

Q-factor of the series

Q = ωL/R

= (111.11 x 3)/7.4 = 45.04

To improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2, we should reduce the resistance to half.

= R/2 = 7.4/2 = 3. 7 Ω

Question 7.22:

Answer the following questions:
(a) In any AC circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for RMS voltage?
(b) Why a capacitor is used in the primary circuit of an induction coil?
(c) An applied voltage signal consists of a superposition of a DC voltage and an AC voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the DC signal will appear across C and the AC signal across L.
(d) A choke coil in series with a lamp is connected to a DC line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an AC line.
(e) Why is choke coils needed in the use of fluorescent tubes with AC mains? Why can we not use an ordinary resistor instead of the choke coil?

Answer 7.22:

(a) Yes, the applied instantaneous voltage is equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit. The same is not true for RMS voltage because voltages across different elements may not be in phase.
(b) When the circuit is broken, the high induced voltage is used to charge the capacitor, thus avoiding sparks, etc.
(c) For a DC signal, the impedance across the inductor is negligible, and the impedance of the capacitor is very high (infinite). So the DC signal appears across the capacitor. For high-frequency AC, the impedance across the inductor is high, and that of a capacitor is low. So, the AC signal appears across the inductor.
(d) For a steady state DC, L has no effect, even if it is increased by an iron core. For AC, the lamp will shine dimly because of the additional impedance of the choke. It will dim further when the iron core is inserted, which increases the choke's impedance.
(e) A choke coil reduces the voltage across the fluorescent tube without wasting power. A resistor would waste power as heat.

Question 7.23:

A power transmission line feeds input power at 2300 V to a step-down transformer, with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Answer 7.23:

The input voltage to the transformer, V_1 = 2300 V

Primary windings in the step-down transformer, n_1= 4000 turns

Output voltage, V_2 = 230 V

Let n_2 be the number of turns in the secondary

$n_2 = (n_1 \times V_2)/V_1$

= (4000 x 230)/2300 = 400 turns

So, the number of turns in the secondary windings is 400.

Question 7.24:

At a hydroelectric power plant, the water pressure head is at the height of 300 m, and the water flow available is 100 m^3s^{-1}. If the turbine generator efficiency is 60%, estimate the electric power available from the plant (g = 9.8 ms^{-2}).

Answer 7.24:

h = 300 m

Volume of water flowing = 100 m^3s^{-1}

Density of water, ρ = 1000 kg/m^3

Electric power, P= hρgAv = hρgβ

Here, β = Av (Volume of water flowing per second, β= 100 m^3s^{-1})

⇒ P = hρgβ = 300 x 1000 x 9.8 x 100 = 29.4 x 10^7W

The efficiency of the turbine generator = 60%

Electric power available for the plant = (29.4 x 10^7 x 60)/100

= 176.4 x 10^6 W

Question 7.25:

A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V.
The resistance of the two wirelines carrying power is 0.5 Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a substation in the town.
(a) Estimate the line power loss in the form of heat.
(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?
(c) Characterise the step-up transformer at the plant.

Answer 7.25:

Power required , P = 800 kW = 800 x 10^3 W

Power = Voltage x Current

⇒ 800 x 10^3 = 4000 x I

Therefore, RMS current in the line, I = (800 x 10^3)/4000

I = 200 A

Total resistance of the two wire line, R = 2 x 15 x 0.5 = 15 Ω

(a) Line power loss = I^2 R= $(200)^2$ x 15 = 60 x 10^4 W = 600 kW

(b) Power supply to the plant= 800 kW + 600 kW= 1400 kW

(c) Voltage drop on the line = I R = 200 x 15 = 3000 V

Transmission voltage = 4000 V + 3000 V= 7000 V

The step-up transformer at the plant is 440 V – 7000 V

Question 7.26:

Do the same exercise as above with the replacement of the earlier transformer by a 40,000-220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high-voltage transmission is preferred.

Answer 7.26:

The RMS current in the line, I = $(800 \times 10^3)/40{,}000$

I = 20 A

Total resistance of the two-wire line, R = 2 x 15 x 0.5 = 15 Ω

(a) Line power loss = $I^2 R = (20)^2 \times 15 = 60 \times 10^2$ W = 6 kW

(b) Power supply to the plant= 800 kW + 6 kW= 806 kW

(c) Voltage drop on the line = I R = 20 x 15 = 300 V

The step-up transformer is 440 V – 40 V=300 V. It is clear that percentage power loss is greatly reduced by high voltage transmission. In Question 7.25, this power loss is (600/1400) × 100 = 43%. Here, it is only (6/806) × 100 = 0.74%.