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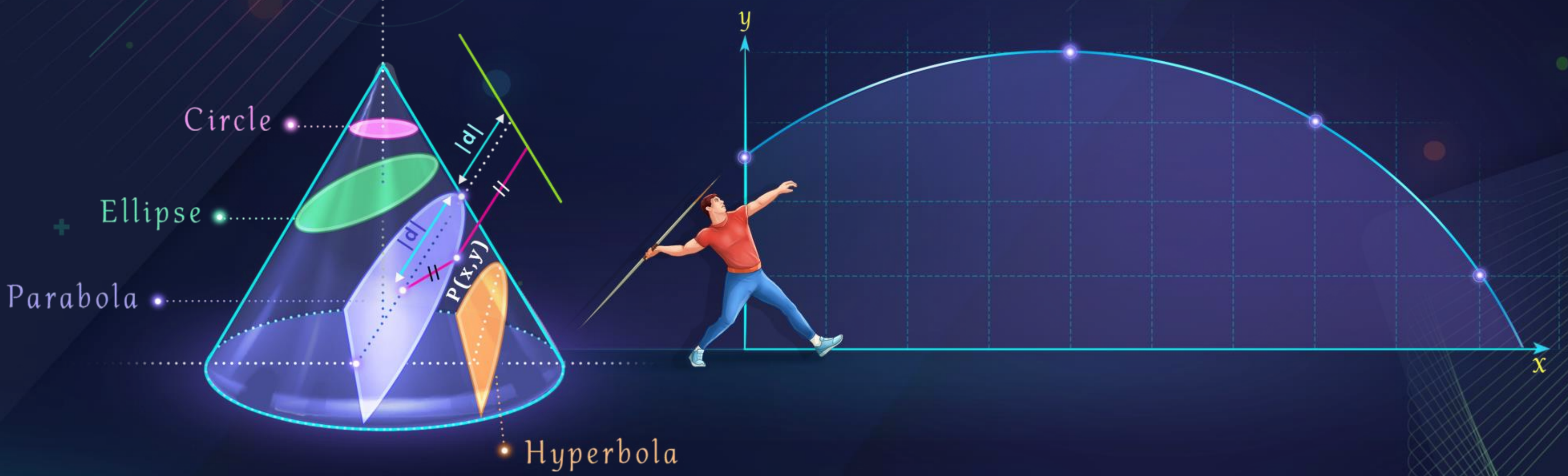
Aakash + BYJU'S LIVE

Parabola, Ellipse and Hyperbola



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Session 01

Introduction to Conic and Parabola

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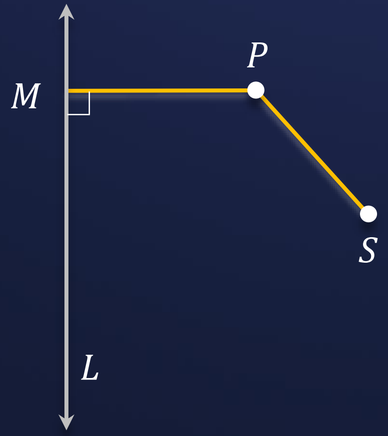
Definition of Conic:

It is the locus of a moving point such that ratio of its distance from a fixed point [focus (S)] to a fixed line [directrix (L)] is always constant. The constant ratio is known as eccentricity (e) of the conic.



$$\frac{SP}{PM} = e$$

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When focus lies on directrix then (Degenerated conics)

Condition	Nature of conic
$e > 1$	Pair of distinct straight lines
$e = 1$	Coincident lines
$0 < e < 1$	Point
$e \to \infty$	Parallel lines

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When focus doesn't lie on directrix then (Non - Degenerated conics)

Condition	Nature of conic
$e = 0$	Circle
$e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

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For , general 2 degree equation : $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Case I. $\Delta = 0$ (Degenerated conic)

S. no	Condition	Nature of conic
1	$h^2 > ab$	Pair of straight lines
2	$h^2 = ab$	Pair of coincident lines
3	$h^2 < ab$	Point

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Case II. $\Delta \neq 0$ (Non-degenerate conic)

S. no	Condition	Nature of conic
1	$h^2 > ab$	Hyperbola
2	$h = 0 \,\&\, a = b$	Circle
3	$h^2 = ab$	Parabola
4	$h^2 < ab$	Ellipse

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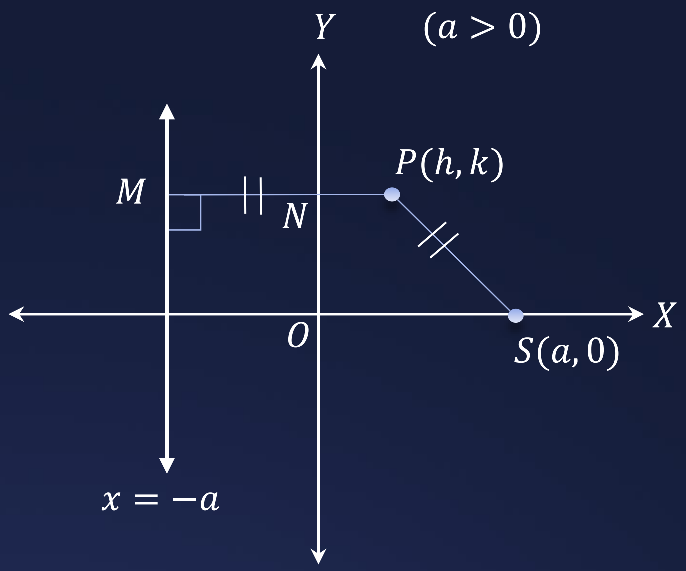
Key Takeaways

Standard Equation of Parabola:

$S(a, 0) \rightarrow$ focus
$l : x + a = 0 \rightarrow$ directrix

Let $P(h, k)$ be a point on the parabola. Then, we need to find the locus of $P(h, k)$.



By definition of parabola,

$$PS = PM$$

$$\Rightarrow \sqrt{(h-a)^2 + (k-0)^2} = PN + NM = h + a$$

$$\Rightarrow (h-a)^2 + k^2 = (h+a)^2$$

$$\Rightarrow k^2 = (h+a)^2 - (h-a)^2$$

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Key Takeaways

Standard Equation of Parabola:

$$\Rightarrow k^2 = (h+a)^2 - (h-a)^2$$

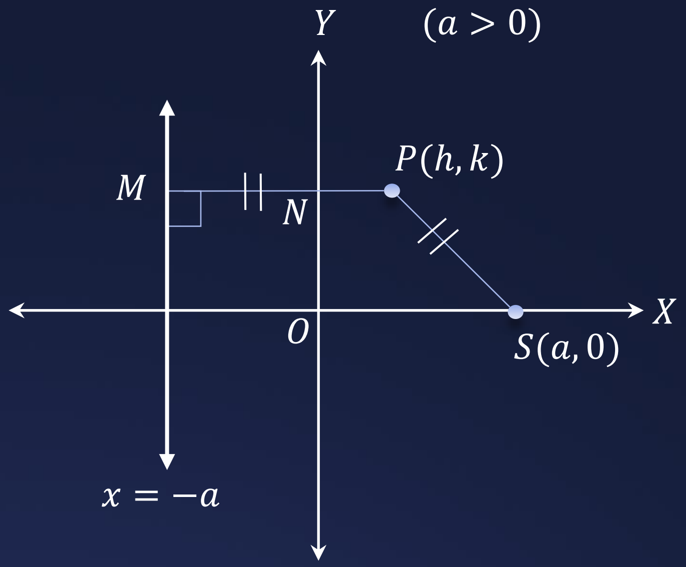
$$\Rightarrow k^2 = (h+a+h-a)(h+a-h+a)$$

$$\Rightarrow k^2 = 2h \times 2a = 4ah$$

$\therefore$ Required equation of parabola is

$$\boxed{y^2 = 4ax}$$

STANDARD EQUATION



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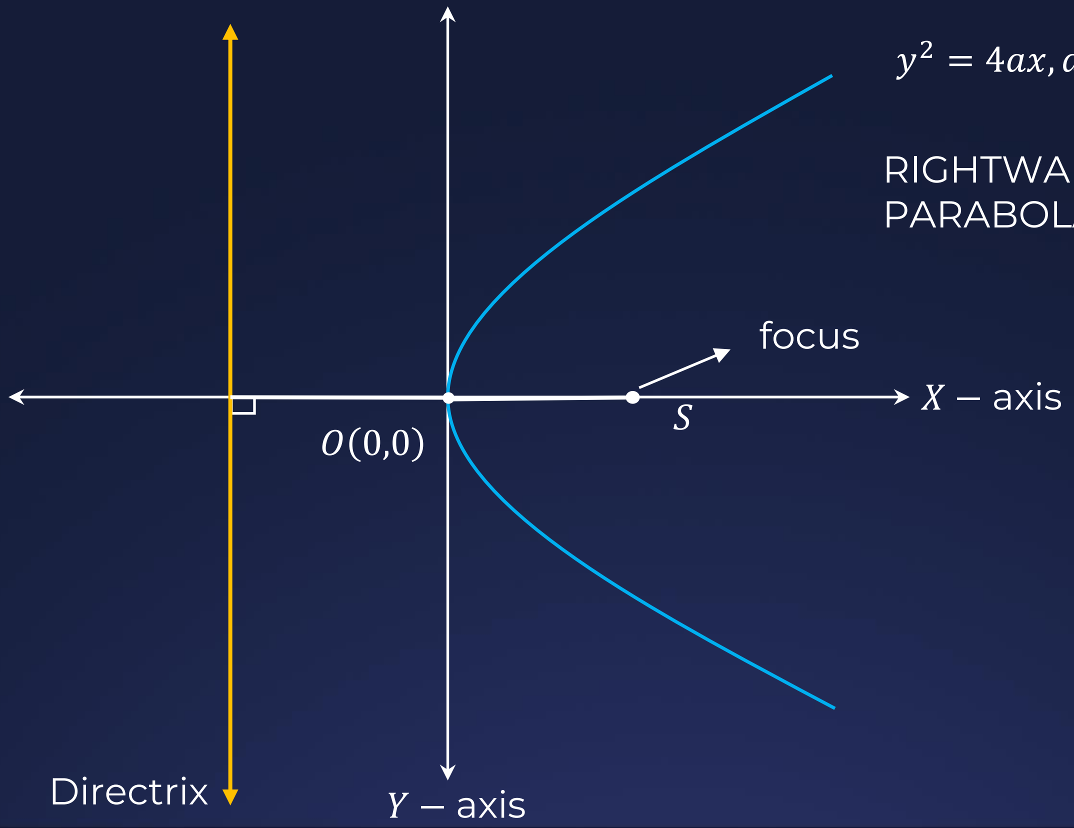
Key Takeaways



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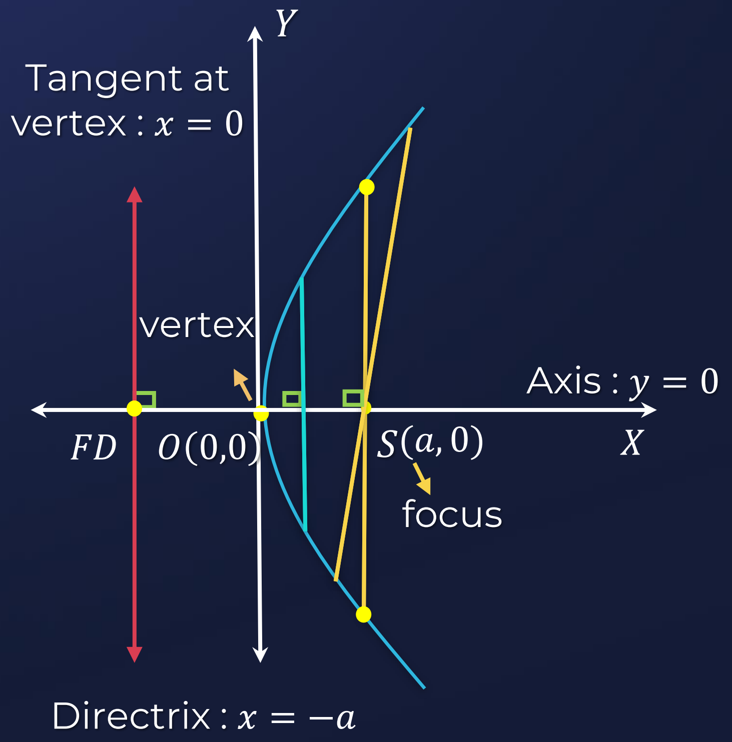
Terminologies

- Straight line passing through focus and perpendicular to directrix is called axis
- Vertex: A point at which parabola and axis intersect.
- Directrix: $\rightarrow x + a = 0$ or $x = -a$
- Foot at Directrix: $FD = (-a, 0)$ point where axis and directrix intersect.
- Focal chord: any chord passing through focus $(a, 0)$
- Double Ordinate: Any chord parallel to Directrix
- Tangent at Vertex: A line passing through vertex and parallel to Directrix
- Vertex $= \frac{FD+Focus}{2}$



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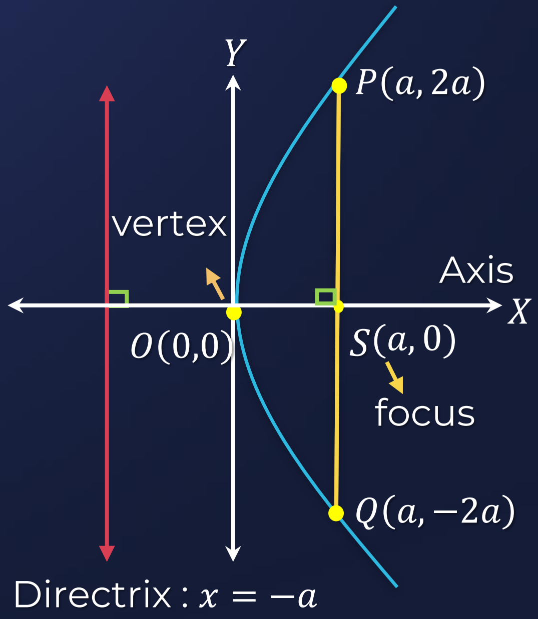
Important terms

- Latus Rectum: LR = Focal chord perpendicular to Axis

  L.R. : $x = a$

  Here P, Q are end points of L.R.

  $\therefore P \equiv (a, 2a)$ & $Q \equiv (a, -2a)$, $PQ = 4a$



Remark: Double ordinate passing through the focus is a latus rectum

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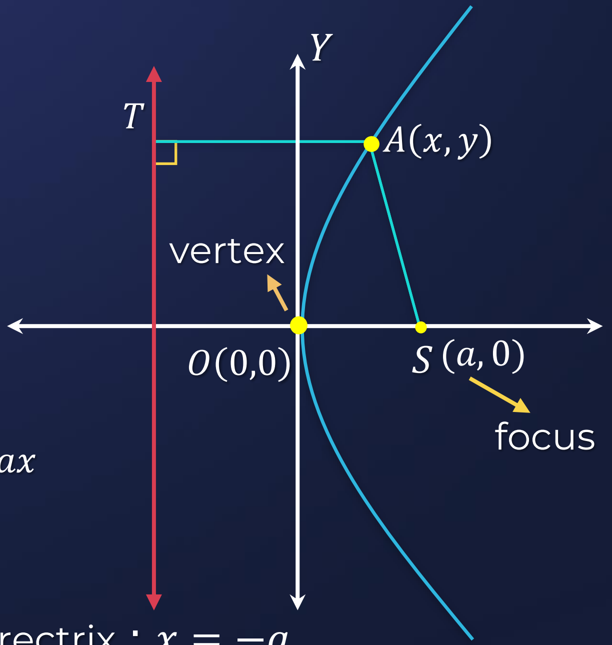
Important terms

Focal Distance :

Distance between any point on the curve and the focus

Let $A(x, y)$ be a point on the parabola $y^2 = 4ax$

$\Rightarrow AS$ = focal distance $= AT = x + a$



Directrix : $x = -a$

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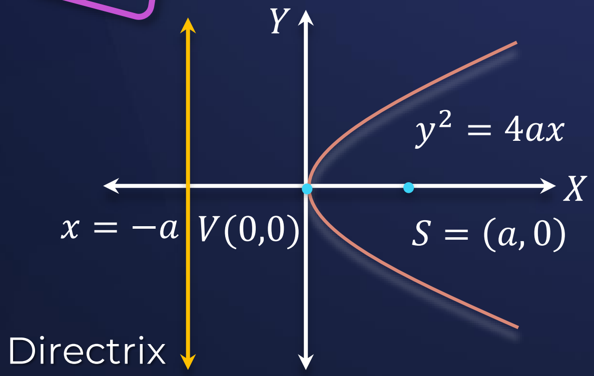
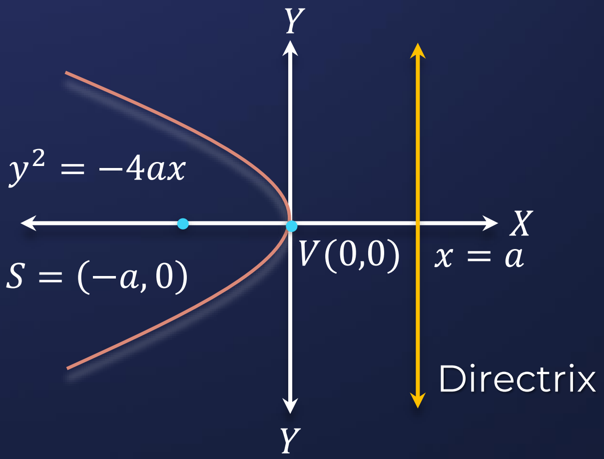
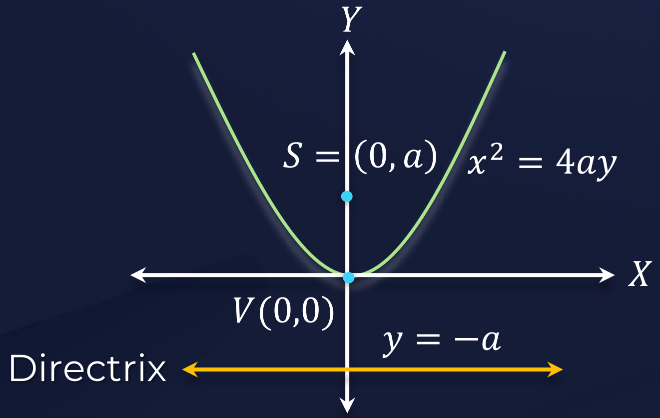
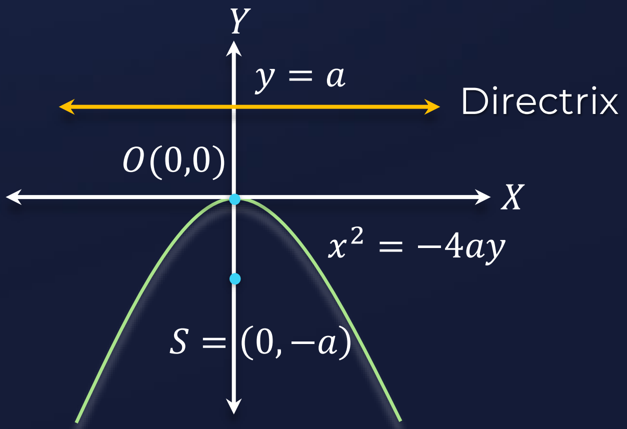
Standard parabolas :









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Standard parabola recap

Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Axis	X –axis	X –axis	Y –axis	Y –axis
Length of L.R	$4a$	$4a$	$4a$	$4a$
Focal distance	$x + a$	$-x + a$	$y + a$	$-y + a$
Tangent at vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$

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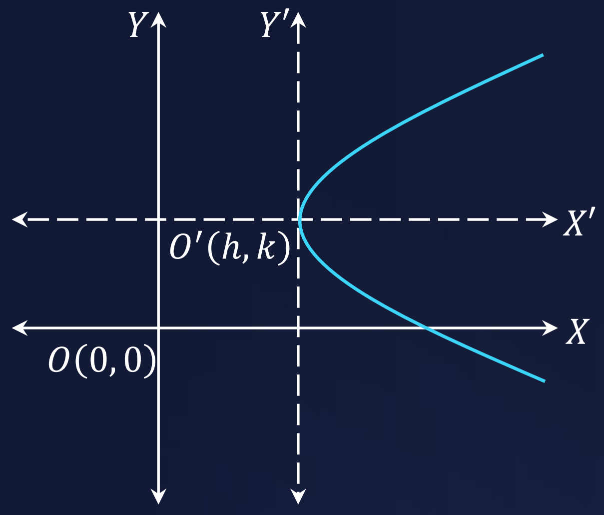
Key Takeaways

SHIFTING OF VERTEX:

Consider rightward opening parabola having Vertex is at (h, k) & axis of symmetry parallel to X – axis

Consider the new $X'Y'$ coordinate plane w.r.t the shifted origin $O'(h, k)$



Equation of parabola w.r.t the new coordinate system $X'Y'$ is :

$$(Y')^2 = 4AX'$$

$$\boxed{\Rightarrow\ (y-k)^2 = 4A(x-h)}$$

We know,
Under shifting of origin
$Y' = y - k$
$X' = x - h$

Required eqn. of the shifted parabola

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Key Takeaways

FOR $(y-k)^2 = 4A(x-h)$:

$(Y')^2 = 4AX'$

Vertex: $(x', y') = (h, k)$

Focus $= (A+h, k)$

Directrix: $x = h - A$

Axis of symmetry: $y = k$

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Axis of a parabola along x −axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x −axis then which of the following points does not lie on it?

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A $(4, -4)$

B $(6, 4\sqrt{2})$

C $(5, 2\sqrt{6})$

D $(8, 6)$

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Axis of a parabola along x –axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x –axis then which of the following points does not lie on it?

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A $(4, -4)$

B $(6, 4\sqrt{2})$

C $(5, 2\sqrt{6})$

D $(8, 6)$

Given:

Vertex is $(2, 0)$

Here $a = 2$

Equation of parabola will be:

$$(y - 0)^2 = 4 \cdot 2(x - 2)$$

$$\Rightarrow y^2 = 8(x - 2) \quad \cdots (1)$$

All the options satisfy equation (1) except option (D).

$\therefore$ $(8, 6)$ does not lie on the given parabola.

Hence, option (D) is the correct answer.

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Axis of a parabola along x −axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x −axis then which of the following points does not lie on it?

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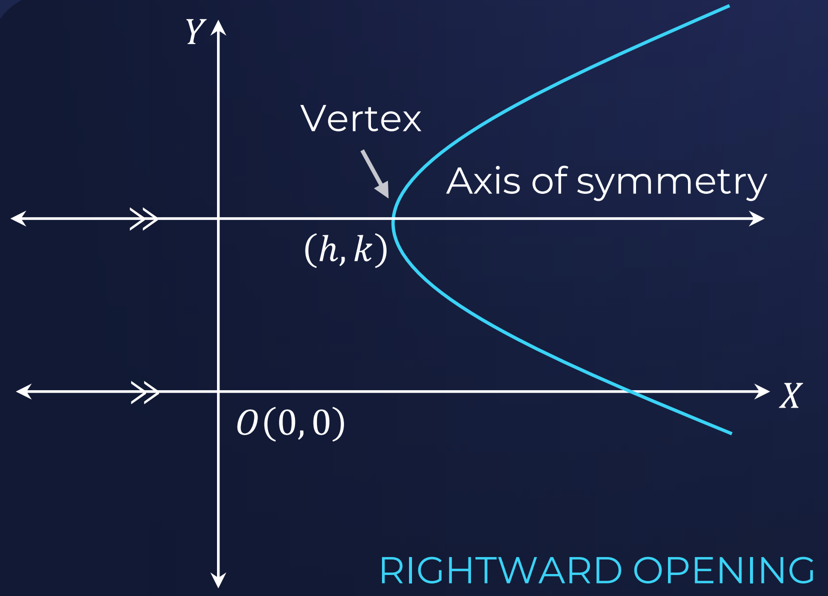
SHIFTING OF VERTEX:



$$(y-k)^2 = 4A(x-h)$$

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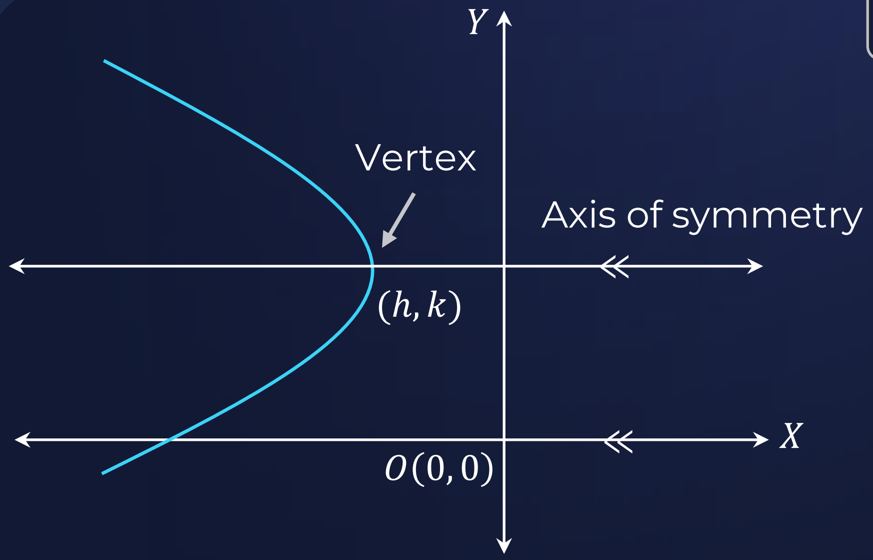
SHIFTING OF VERTEX:

$$(y-k)^2 = -4A(x-h)$$



LEFTWARD OPENING

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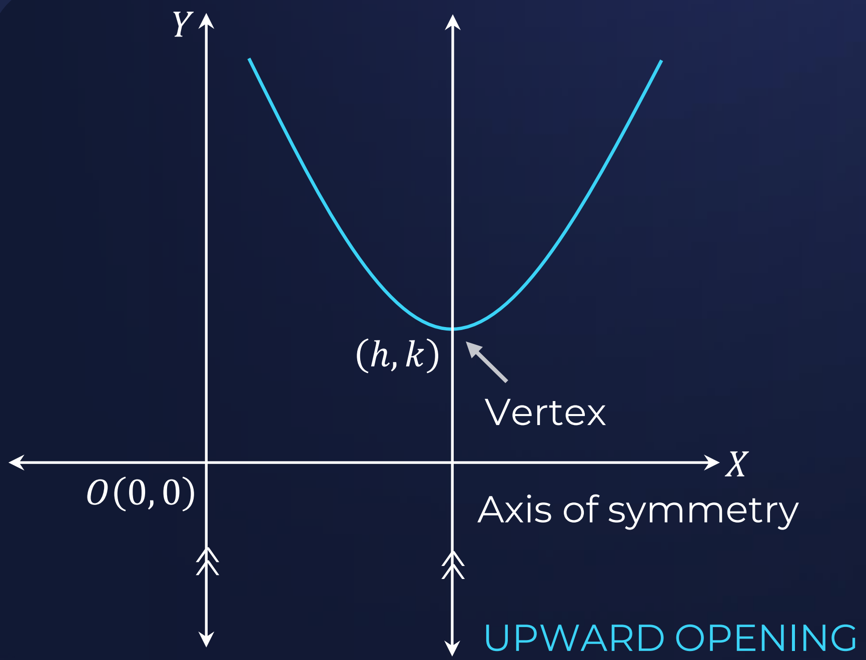
SHIFTING OF VERTEX:



$$(x-h)^2 = 4A(y-k)$$

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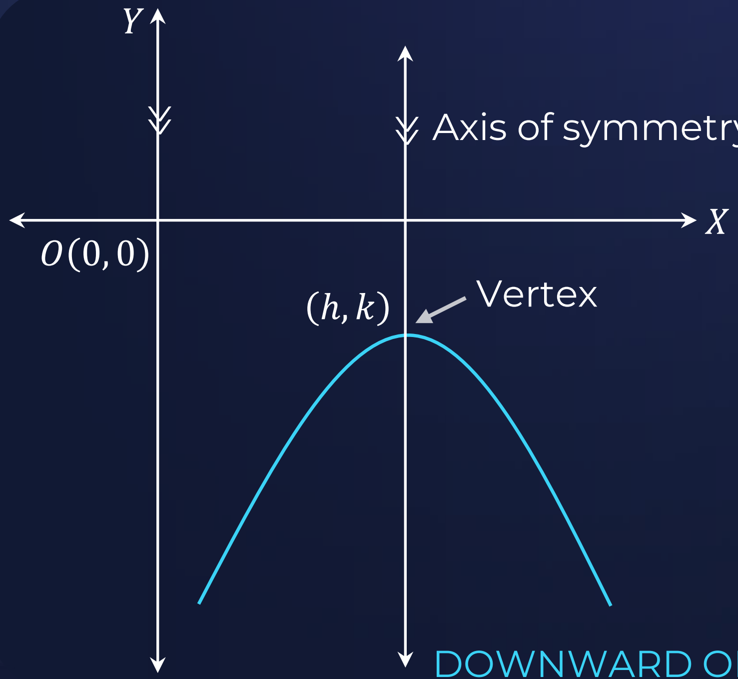
SHIFTING OF VERTEX:



DOWNWARD OPENING

$$(x-h)^2 = -4A(y-k)$$

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NOTE:

- On shifting the parabola, coordinates & equations changes but distances do not change.

Length of L.R.
Distance b/w vertex and focus
Distance b/w Directrix & Vertex

- The equation $y = Bx^2 + Cx + D$ or $(x-h)^2 = 4A(y-k)$ represents a vertical parabola.
- The equation $x = By^2 + Cy + D$ or $(y-k)^2 = 4A(x-h)$ represents a horizontal parabola.

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If $y^2 + 2y - x + 5 = 0$ represents a parabola. Find its vertex, axis of symmetry, focus, equation of directrix, equation of latus rectum, length of latus rectum & extremities of latus rectum.

We have, $y^2 + 2y - x + 5 = 0$ (Parabola)

$\Rightarrow y^2 + 2y + (1)^2 - (1)^2 - x + 5 = 0$

$\Rightarrow (y+1)^2 = x - 4$

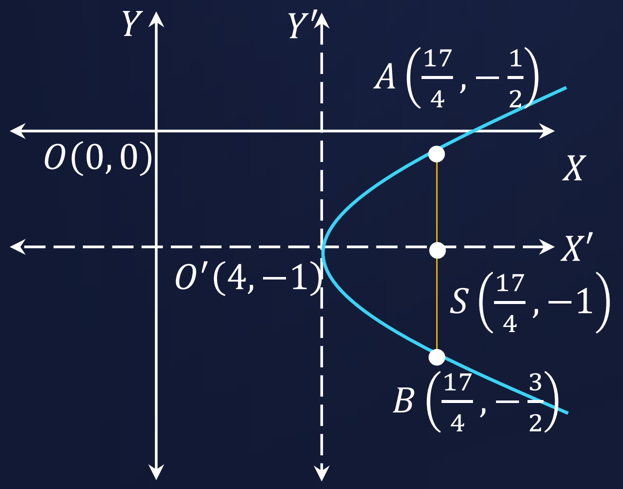
$$\Rightarrow \underbrace{(y+1)^2 = 1(x-4)}$$

Rightward opening parabola with axis parallel to X – axis & vertex $\equiv (4, -1)$

Comparing with $(y-k)^2 = 4A(x-h)$

Thus the transformed equation of the parabola is :

$(y - (-1))^2 = 1(x - 4)$



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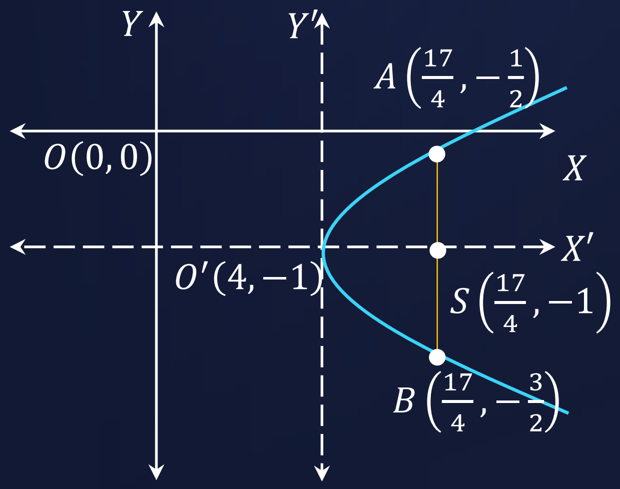
If $y^2 + 2y - x + 5 = 0$ represents a parabola. Find its vertex, axis of symmetry, focus, equation of directrix, equation of latus rectum, length of latus rectum & extremities of latus rectum.

i.e., $(y')^2 = 4A(x')$; $y' = y + 1$, $x' = x - 4$

$4A = 1 \Rightarrow A = \frac{1}{4}$



Axis of symmetry:

Equation : $y' = 0$

i.e., $y + 1 = 0 \Rightarrow y = -1$

Focus:

Coordinates : $(A, 0)$

$\Rightarrow x' = A \;\; \& \;\; y' = 0$

$\Rightarrow x - 4 = A \;\; \& \;\; y + 1 = 0$

$\Rightarrow x = A + 4 \;\; \& \;\; y = -1$

$\Rightarrow x = \frac{1}{4} + 4 \;\; \& \;\; y = -1$ i.e., $\left(\frac{17}{4}, -1\right)$

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If $y^2 + 2y - x + 5 = 0$ represents a parabola. Find its vertex, axis of symmetry, focus, equation of directrix, equation of latus rectum, length of latus rectum & extremities of latus rectum.

Directrix

Equation : $x' = -A$

$\Rightarrow x - 4 = -\frac{1}{4}$

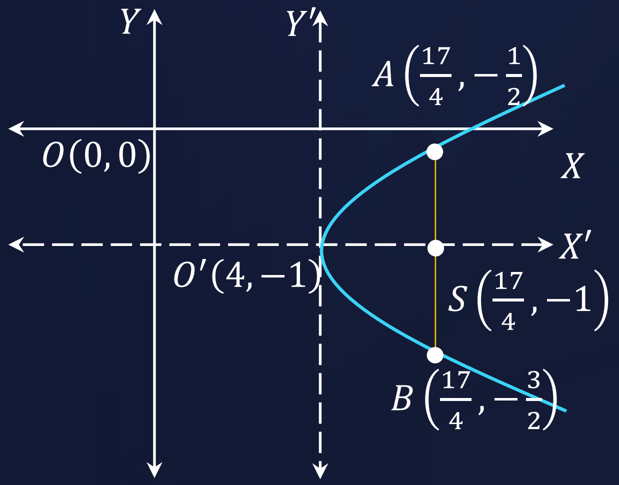
$\Rightarrow x = \frac{15}{4}$

Latus Rectum:

$= 4A$

$= 4 \times \frac{1}{4}$

$=$ 1 unit



Extremities of L.R.

Coordinates : $(A, \pm 2A)$

i.e., $X' = A$ & $Y' = \pm 2A$

i.e., $x - 4 = \frac{1}{4}$ & $y + 1 = \pm\frac{1}{2}$

i.e. $x = \frac{17}{4}$ & $y = -\frac{1}{2}, -\frac{3}{2}$ i.e. $\left(\frac{17}{4}, -\frac{1}{2}\right)$ & $\left(\frac{17}{4}, -\frac{3}{2}\right)$

Equation of L.R : $AB \Rightarrow x = \frac{17}{4}$

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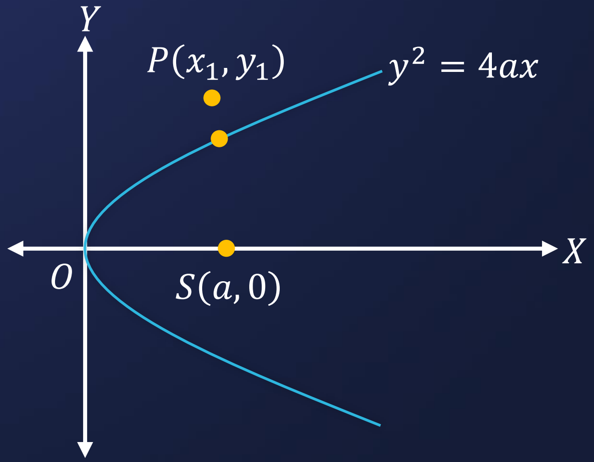
Position of a point w.r.t. Parabola:



Parabola $y^2 = 4ax$ and a point (x_1, y_1)

$y_1^2 - 4ax_1 > 0$

∴ Point is lying outside the parabola

$y_1^2 - 4ax_1 = 0$

∴ Point is lying on the parabola

$y_1^2 - 4ax_1 < 0$

∴ Point is lying inside the parabola

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Consider the parabola $y^2 = 4x$. $P(1,3)$ & $Q(1,1)$ are two points lying in the XY – plane. Then,

A. P & Q are exterior points.

B. P is interior & Q is exterior point.

C. P & Q are interior points.

D. P is exterior & Q is an interior point.

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Consider the parabola $y^2 = 4x$. $P(1,3)$ & $Q(1,1)$ are two points lying in the XY – plane. Then,

Given : $y^2 = 4x$

$$S \equiv y^2 - 4x$$

I. For $P(1,3)$

$$S_{P(1,3)} = 3^2 - 4(1) = 5 > 0$$

$\Rightarrow P(1,3)$ lies outside the parabola.

II. For $Q(1,1)$

$$S_{Q(1,1)} = 1^2 - 4(1) = -3 < 0$$

$\Rightarrow Q(1,1)$ lies inside the parabola.

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Consider the parabola $y^2 = 4x$. $P(1, 3)$ & $Q(1, 1)$ are two points lying in the XY – plane. Then,

A. P & Q are exterior points.

B. P is interior & Q is exterior point.

C. P & Q are interior points.

D. P is exterior & Q is an interior point.

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Session 02

Parametric Form and Tangent to Parabola

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Parametric Coordinates

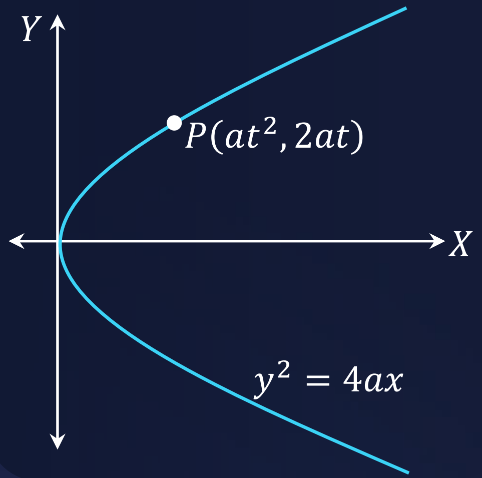
Let P be any point on the parabola $y^2 = 4ax$.

Then, $P \equiv (at^2, 2at)\ ;\ t \in \mathbb{R}$

$\because$ For all real values of t,

$x = at^2\ \&\ y = 2at$

satisfy the equation of the parabola $y^2 = 4ax$



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Parametric Coordinates

Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Parametric Coordinate	$(at^2, 2at)$	$(-at^2, 2at)$	$(2at, at^2)$	$(2at, -at^2)$
Parametric Equation	$x = at^2$ $y = 2at$	$x = -at^2$ $y = 2at$	$x = 2at$ $y = at^2$	$x = 2at$ $y = -at^2$

NOTE :

When we add h and k to x and y respectively we get translated parabola.

The parametric equations of the parabola $(y-k)^2 = 4a(x-h)$ are
$x = h + at^2$ & $y = k + 2at$.

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The locus of a point which divides the line segment joining the point $(0, -1)$ and a point on the parabola, $x^2 = 4y$, internally in the ratio $1:2$, is:

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A. $9x^2 - 12y = 8$

B. $4x^2 - 3y = 2$

C. $x^2 - 3y = 2$

D. $9x^2 - 3y = 2$

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The locus of a point which divides the line segment joining the point $(0,-1)$ and a point on the parabola, $x^2 = 4y$, internally in the ratio $1:2$, is:

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Let point P be $(2t, t^2)$ and Q be (h, k).

By section formula,

$$h = \frac{2t}{3}, \quad k = \frac{-2+t^2}{3}$$

Now, eliminating t from the above equations, we get

$$3k + 2 = \left(\frac{3h}{2}\right)^2$$

Replacing h and k by x and y respectively, we get the Locus of the curve as: $9x^2 - 12y = 8$.

Hence, The correct option is (A).

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The locus of a point which divides the line segment joining the point $(0, -1)$ and a point on the parabola, $x^2 = 4y$, internally in the ratio $1:2$, is:

JEE Main Jan 2020

A $9x^2 - 12y = 8$

B $4x^2 - 3y = 2$

C $x^2 - 3y = 2$

D $9x^2 - 3y = 2$

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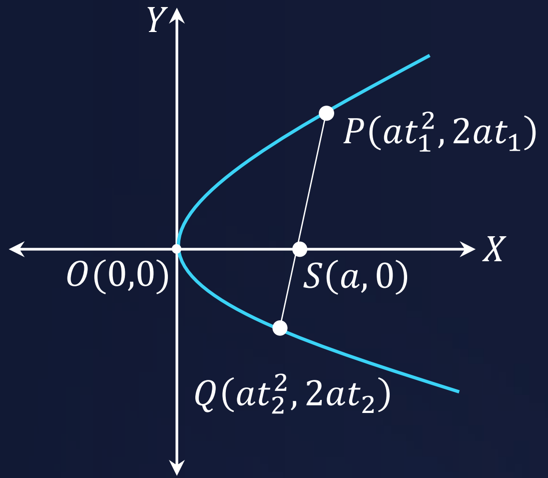
Key Takeaways

Focal Chord: Any chord passing through the focus of the parabola.



Properties of Focal Chord:

I. If $P(t_1)$ & $Q(t_2)$ are the ends point of a focal chord then $t_1 t_2 = -1$

Proof :

$P(t_1) \equiv (at_1^2, 2at_1)$ & $Q_2(at_2^2, 2at_2)$

Equation of PQ using Two Point form is :

$$y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1{}^2)$$

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Key Takeaways

$$\Rightarrow y - 2at_1 = \underbrace{\frac{2}{t_2+t_1}}_{\text{Slope of } PQ}(x - at_1^2) \ldots (i)$$

PQ is the focal chord

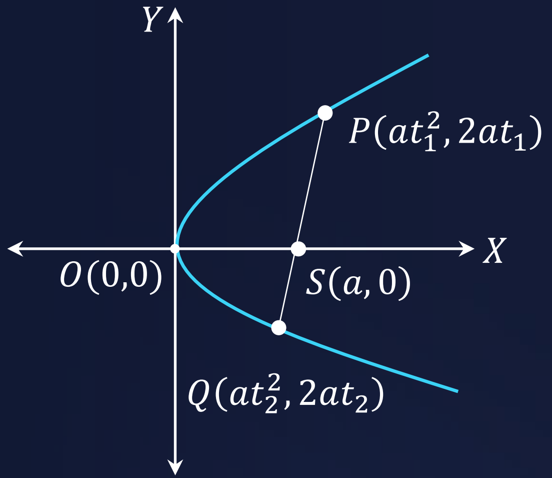
$\Rightarrow S(a, 0) \longrightarrow$ focus satisfies equation (i)

$$\Rightarrow 0 - 2at_1 = \frac{2}{t_2+t_1}(a - at_1^2)$$

$$\Rightarrow -2at_1^2 - 2at_1t_2 = 2a - 2at_1^2$$

$$\Rightarrow -2at_1t_2 = 2a$$

$$\Rightarrow t_1t_2 = -1$$



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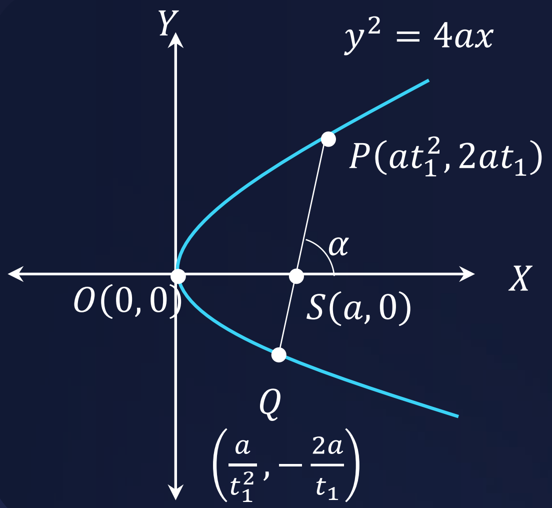
Properties of Focal Chord:

II. Length of the focal chord which makes an angle α with the positive direction of X – axis is

$\Rightarrow\ 4a\,\text{cosec}^2\,\alpha$



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NOTE :

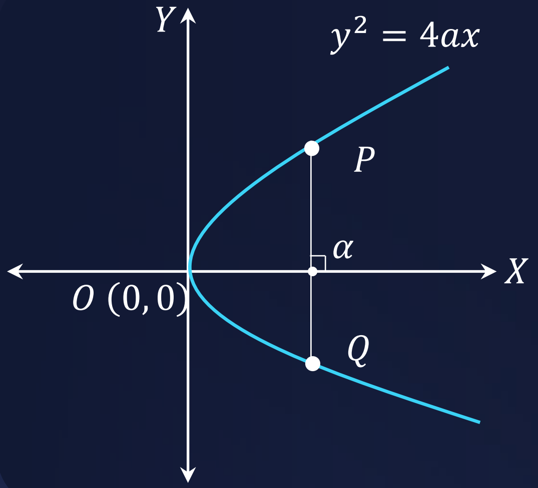
Smallest focal chord is the one perpendicular to the axis of symmetry i.e. Latus Rectum .

$\Rightarrow \alpha = 90°$

$\therefore$ Its length $= 4a \operatorname{cosec}^2(90°) = 4a$

= Length of Latus Rectum

= Minimum length of a focal chord



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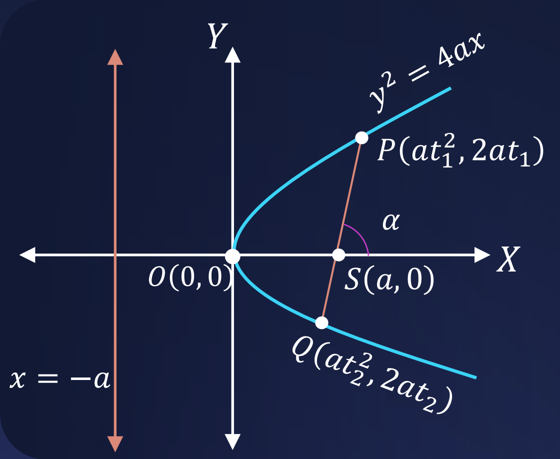
Properties of Focal Chord:

III. Semi latus rectum is the harmonic mean of SP and SQ where P and Q are extremities of the focal chord

$\Rightarrow |PS|, 2a, |QS|$ are in HP

$$2a = 2 \cdot \frac{|PS| \times |QS|}{|PS| + |QS|}$$

$\left(\begin{array}{c} a, b, c \text{ are in HP} \\ \text{if } b = \frac{2ac}{a+c} \end{array} \right)$



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If $(2, -8)$ is at an end of a focal chord of the parabola $y^2 = 32x$ and the other end of the chord is (α, β), then $\frac{\alpha+\beta}{8} =$ _____ .

A 3

C 8

B 18

D 32

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If $(2,-8)$ is at an end of a focal chord of the parabola $y^2 = 32x$ and the other end of the chord is (α, β), then $\frac{\alpha+\beta}{8} =$ _____ .

Given : $y^2 = 32x$

$P(t) \equiv (at^2, 2at)$

Here, $a = 8$

Coordinates of P are $(8t^2, 16t)$

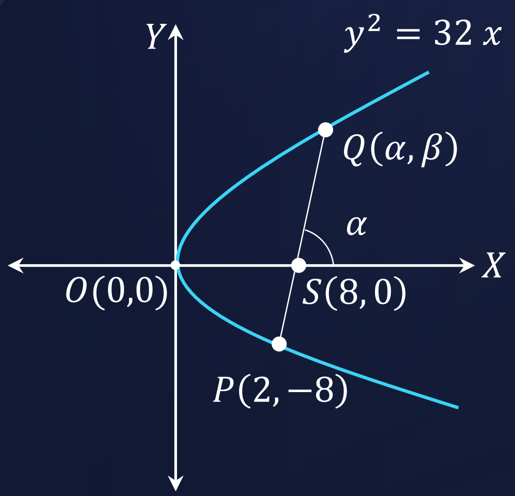
Given Coordinates of P are $(2, -8)$

On comparing, we get : $16\,t = -8$

$\Rightarrow t = -\frac{1}{2}$

Parameter of P is $t = -\frac{1}{2}$

$\Rightarrow$ Parameter of Q will be, $-\frac{1}{t} = 2$



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If $(2,-8)$ is at an end of a focal chord of the parabola $y^2 = 32x$ and the other end of the chord is (α, β), then $\frac{\alpha+\beta}{8} =$ ______ .

$(t_1 t_2 = -1)$

$Q\left(-\frac{1}{t}\right) \equiv \left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

$Q\left(-\frac{1}{t}\right) \equiv (32, 32)$

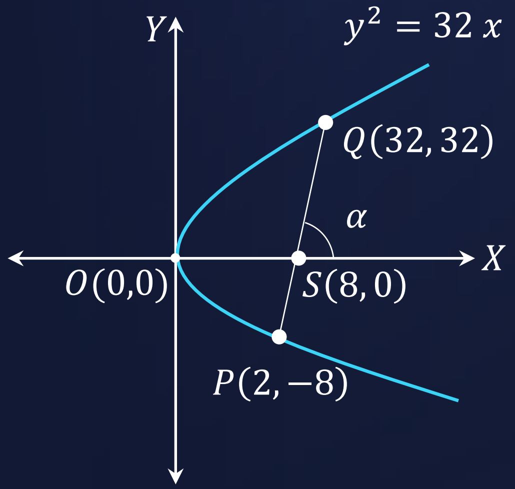
Coordinates of $Q = (\alpha, \beta)$

Comparing:

$(\alpha, \beta) = (32, 32)$

$\alpha = 32$ and $\beta = 32$

$\frac{\alpha+\beta}{8} = \frac{32+32}{8} = \frac{64}{8} = 8$



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If $(2, -8)$ is at an end of a focal chord of the parabola $y^2 = 32x$ and the other end of the chord is (α, β), then $\frac{\alpha+\beta}{8} =$ ______ .

A 3

C 8

B 18

D 32

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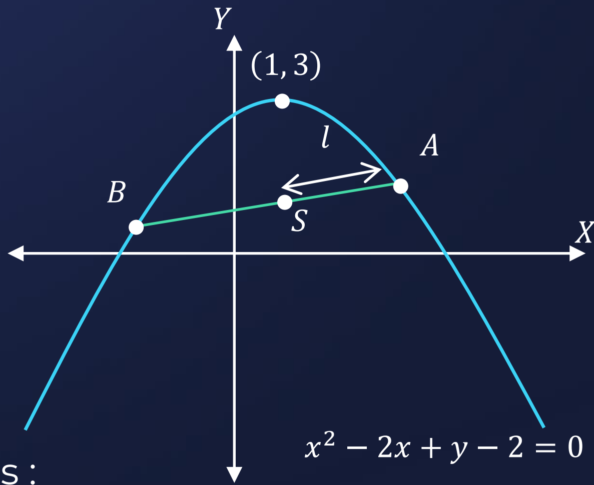
If AB is the focal chord of $x^2 - 2x + y - 2 = 0$ whose focus is S and $|AS| = l$, then $|BS| =?$

We have, $x^2 - 2x + y - 2 = 0$



Firstly, we will transform the given equation to a perfect square in x

$$\Rightarrow x^2 - 2x + (1)^2 - (1)^2 + y - 2 = 0$$

$$\Rightarrow (x-1)^2 = -(y-3)$$

Thus, The transformed equation of the parabola is :

$$(x-1)^2 = -(y-3)$$

$$\Rightarrow (x')^2 = -4A(y')\,;\; x' = x - 1;\, y' = y - 3$$

$$4\,A = 1 \Rightarrow A = \frac{1}{4}$$

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If AB is the focal chord of $x^2 - 2x + y - 2 = 0$ whose focus is S and $|AS| = l$, then $|BS| =?$

Length of Latus Rectum $= 4A = 4 \times \frac{1}{4} = 1$ units

Length of Semi Latus Rectum $= \frac{1}{2}$ units

$|AS|, \frac{1}{2}, |BS|$ are in HP

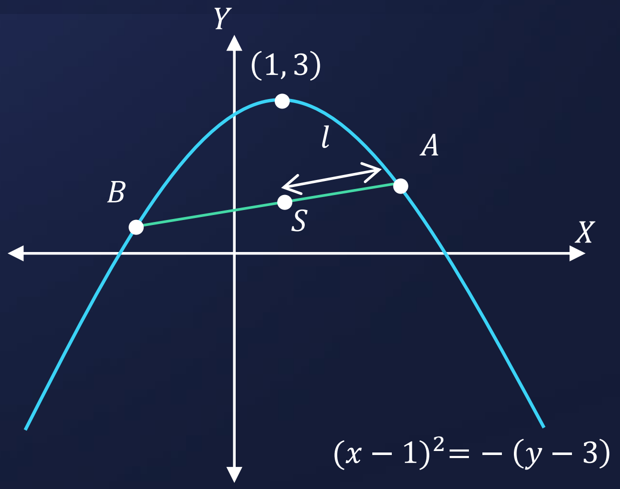
$\Rightarrow \frac{1}{2} = 2 \cdot \frac{l \times l'}{l+l'}$

$\Rightarrow l + l' = 4l \cdot l'$

$\Rightarrow l = 4l \cdot l' - l'$

$\Rightarrow l = l' \cdot (4l - 1) \Rightarrow l' = \frac{l}{4l-1}$

$\Rightarrow |BS| = \frac{l}{4l-1}$



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Key Takeaways

Position of a Line w.r.t. a Parabola:

Parabola : $y^2 = 4ax$ Line : $y = mx + c$

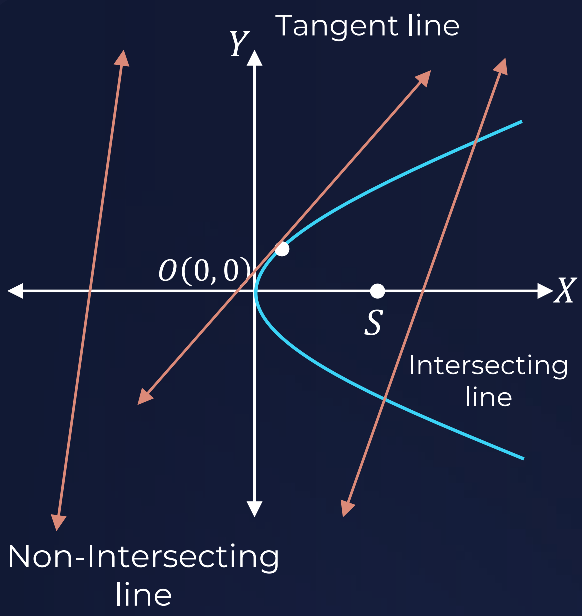
Solve simultaneously to get points of intersection

$(mx + c)^2 = 4ax$

$\Rightarrow m^2x^2 + c^2 + 2\,mcx = 4ax$

$$\boxed{\Rightarrow (m^2)x^2 + (2mc - 4a)x + c^2 = 0}$$

The above equation we get is quadratic in x



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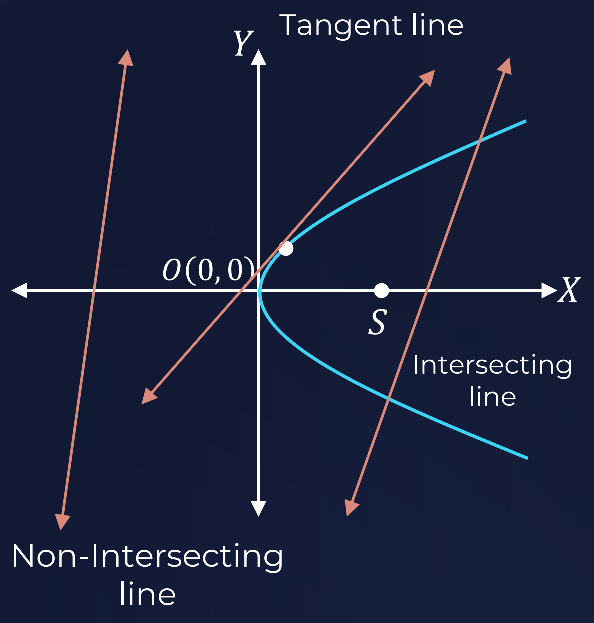
Key Takeaways

Position of a Line w.r.t. a Parabola:

$$(m^2)x^2 + (2mc - 4a)x + c^2 = 0$$

We have three cases:

$D > 0$	Two real & distinct roots Intersecting line
$D = 0$	Real & repeated roots Tangent line
$D < 0$	Imaginary roots Non-intersecting line



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Position of a Line w.r.t. a Parabola:

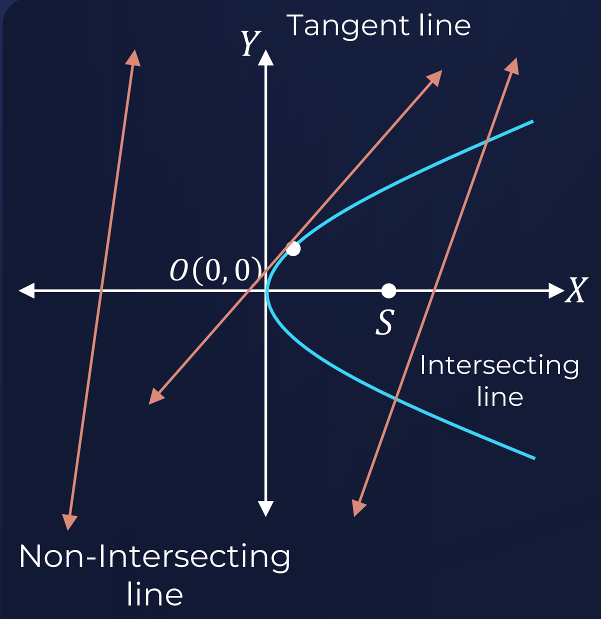
$$\Rightarrow (m^2)x^2 + (2mc - 4a)x + c^2 = 0$$

$$D = (2mc - 4a)^2 - 4(m^2)(c^2)$$

$$= 4m^2c^2 + 16a^2 - 16amc \ - 4m^2c^2$$

$$= 16a\,(a - mc)$$

$$\Rightarrow D = 16a\,(a - mc)$$



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Position of a Line w.r.t. a Parabola:

Case I: CONDITION FOR INTERSECTING LINE

$\Rightarrow$ Two points of intersection

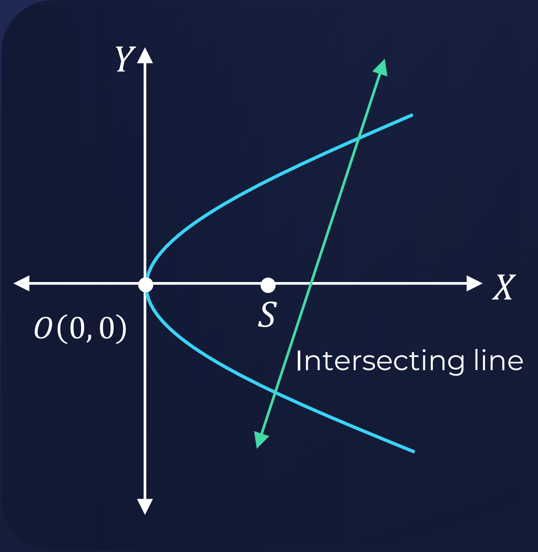
$\Rightarrow$ Two real and distinct roots

$\Rightarrow D > 0$

$$\boxed{\Rightarrow 16a\,(a - mc) > 0}$$

If $a > 0, \Rightarrow a - mc > 0$

$$\boxed{\Rightarrow a > mc}$$



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Position of a Line w.r.t. a Parabola:

Case II: CONDITION FOR TANGENT LINE

$\Rightarrow$ One points of intersection

$\Rightarrow$ One distinct root

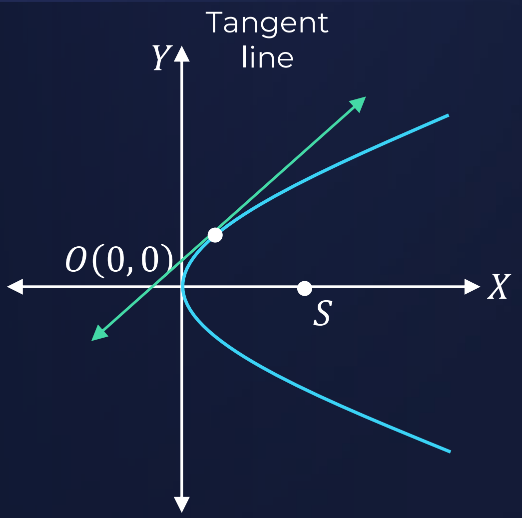
$\Rightarrow$ Two real and repeated roots

$\Rightarrow D = 0$

$\Rightarrow 16a\,(a - mc) = 0$

$\Rightarrow a - mc = 0$

$\boxed{\Rightarrow a = mc}$



Note: In $y = mx + c$, on replacing c with $\frac{a}{m}$, the line becomes tangent to the parabola $y^2 = 4ax$

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Position of a Line w.r.t. a Parabola:

Case III: CONDITION FOR NON-INTERSECTING LINE

$\Rightarrow$ No points of intersection

$\Rightarrow$ No real root

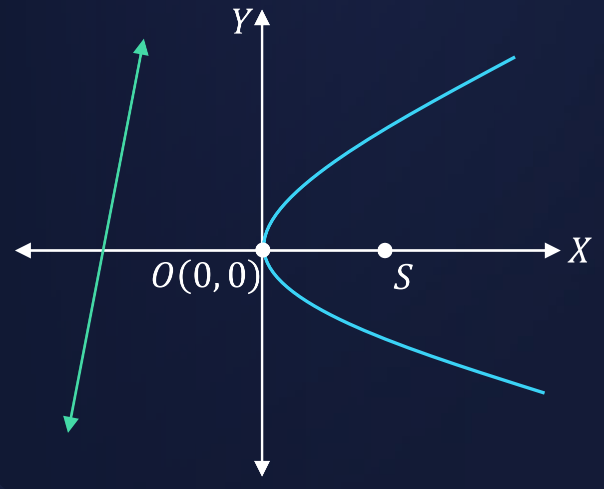
$\Rightarrow$ Two imaginary roots

$\Rightarrow D < 0$

$\boxed{\Rightarrow 16a\,(a - mc) < 0}$

If a $> 0, \Rightarrow\ a - mc < 0$

$\boxed{\Rightarrow a < mc}$



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Key Takeaways

Equation of Tangent:

An equation of Tangent divides into three forms:

POINT FORM

PARAMETRIC FORM

SLOPE FORM

Point Form:

The equation of tangent to the parabola

$y^2 = 4ax$ at $P(x_1, y_1)$ is $yy_1 = 2a\,(x + x_1)$.

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Key Takeaways

Equation of Tangent:

Proof:

For $y^2 = 4ax$

Equation of tangent at $P(x_1, y_1)$ becomes

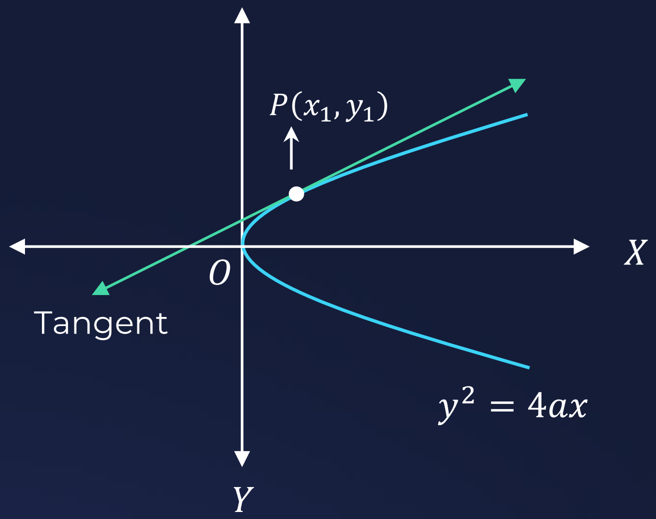
$$yy_1 = 4a\left(\frac{x+x_1}{2}\right)$$

Differentiating $y^2 = 4ax$ w.r.t x

$$\Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$$

Equation of tangent at $P(x_1, y_1)$



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Key Takeaways

Equation of Tangent:

Proof:

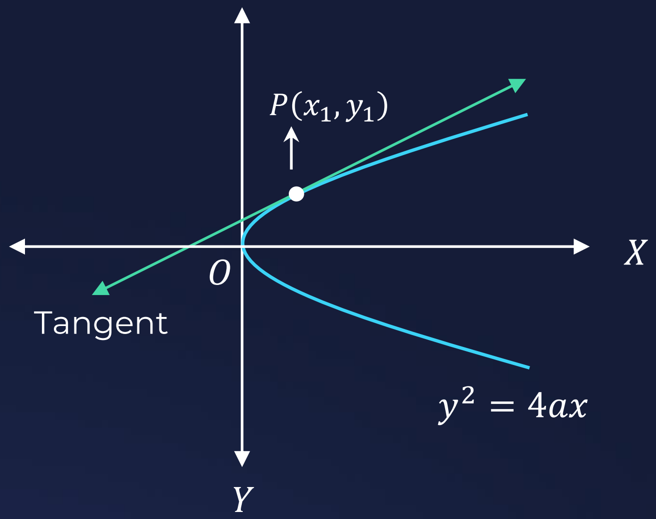
$$\Rightarrow y - y_1 = \frac{2a}{y_1}(x - x_1)$$

$$\Rightarrow yy_1 - 2ax = y_1^2 - 2ax_1$$

$$\Rightarrow yy_1 - 2ax - 2ax_1 = y_1^2 - 2ax_1 - 2ax_1$$

$$\Rightarrow yy_1 - 2a(x + x_1) = \underbrace{y_1^2 - 4ax_1}_{= 0}$$

$$\Rightarrow yy_1 = 2a(x + x_1)$$



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Key Takeaways

Equation of Tangent:

Note:

The equation of tangent is given by $T = 0$

where T expression is obtained by using the transformations:

$$\text{Replace} \begin{cases} x^2 \longrightarrow xx_1 \\ y^2 \longrightarrow yy_1 \\ x \longrightarrow \frac{x+x_1}{2} \\ y \longrightarrow \frac{y+y_1}{2} \\ xy \longrightarrow \frac{xy_1+yx_1}{2} \end{cases}$$

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Key Takeaways

Equation of Tangent:

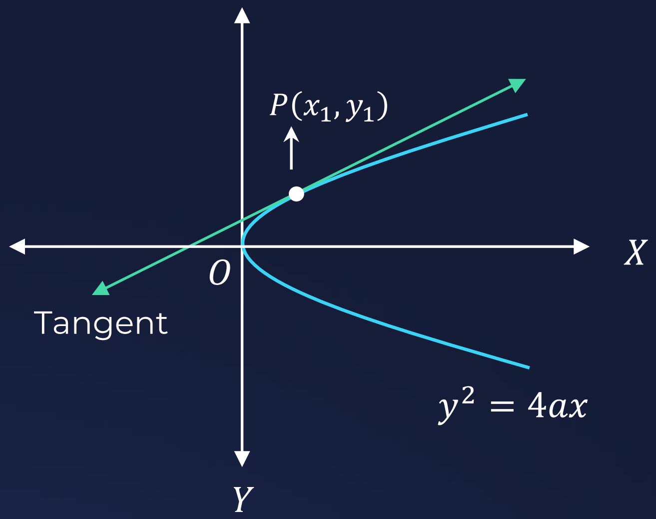
Note:

For $y^2 = 4ax$

Equation of tangent at $P(x_1, y_1)$ becomes

$$yy_1 = 4a\left(\frac{x+x_1}{2}\right)$$

$$\Rightarrow yy_1 = 2a\,(x + x_1)$$



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Key Takeaways

Equation of Tangent:

Equation of Parabola	Tangent at (x_1, y_1)
$y^2 = 4ax$	$yy_1 = 2a(x + x_1)$
$y^2 = -4ax$	$yy_1 = -2a(x + x_1)$
$x^2 = 4ay$	$xx_1 = 2a(y + y_1)$
$x^2 = -4ay$	$xx_1 = -2a(y + y_1)$

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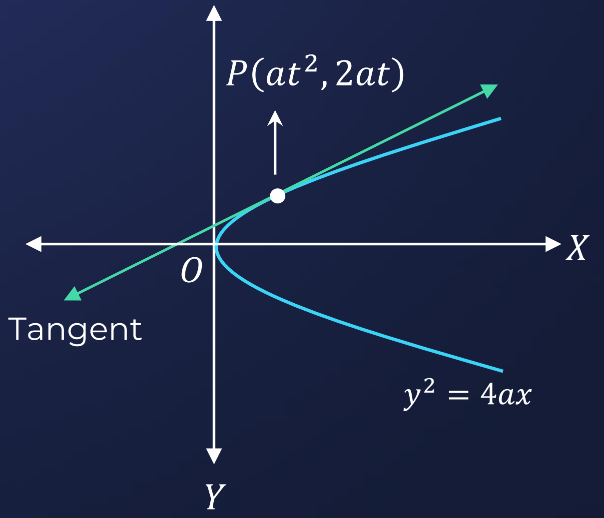
Equation of Tangent:

Parametric Form:

The equation of tangent to the parabola

$y^2 = 4ax$ at $P(t) \equiv P(at^2, 2at)$ is $ty = x + at^2$



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Equation of Tangent:

The equations of tangent of all standard parabolas at t:

Equation of Parabola	Parametric co-ordinates t	Tangent at t
$y^2 = 4ax$	$(at^2, 2at)$	$ty = x + at^2$
$y^2 = -4ax$	$(-at^2, 2at)$	$ty = -x + at^2$
$x^2 = 4ay$	$(2at, at^2)$	$tx = y + at^2$
$x^2 = -4ay$	$(2at, -at^2)$	$tx = -y + at^2$

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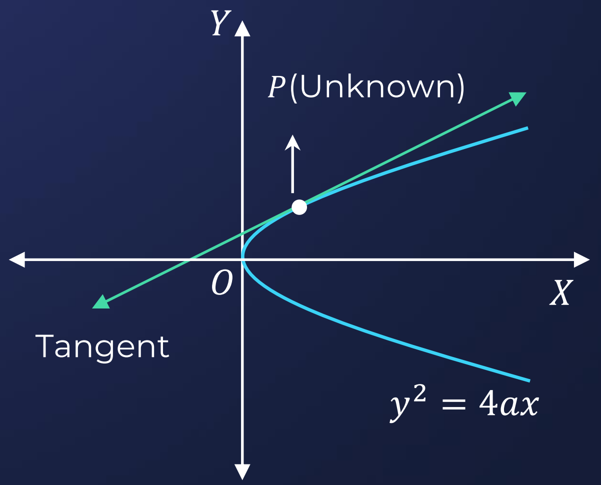
Equation of Tangent:

Slope Form:

The equation of tangent of slope m to the parabola $y^2 = 4ax$ is given by $y = mx + \frac{a}{m}$



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Equation of Tangent:

The equations of tangent of all standard parabolas in slope form:

Equation of Parabola	Point of contact in terms of slope (m)	Equation of tangent in terms of slope (m)
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$
$y^2 = -4ax$	$\left(-\frac{a}{m^2}, -\frac{2a}{m}\right)$	$y = mx - \frac{a}{m}$
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$
$x^2 = -4ay$	$(-2am, -am^2)$	$y = mx + am^2$

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A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

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A $(0, 3)$

B $(-6, 0)$

C $(4, 5)$

D $(5, 4)$

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A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

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For $y^2 = 6x,\ a = \frac{3}{2}$

Equation of tangent: $y = mx + c$

$\Rightarrow y = mx + \frac{3}{2m}$

$\because c = \frac{a}{m}$

And $\because m = \frac{1}{2}$

(Perpendicular to the line $2x + y = 1$)

Tangent is: $y = \frac{x}{2} + \frac{3}{1} \Rightarrow x - 2y + 6 = 0$

$(5,4)$ does not lie on the line $x - 2y + 6 = 0$

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A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

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A $(0, 3)$

B $(-6, 0)$

C $(4, 5)$

D $(5, 4)$

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Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the $x-$ axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then $2m$ is equal to _____.

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Given parabola $y^2 = x$

Let the coordinates of P be $\left(\frac{t^2}{4}, \frac{t}{2}\right)$

Slope of $OP = m \Rightarrow \frac{2}{t} = m \,..(i)$

Equation of tangent at P is:

$$yt = x + \frac{t^2}{4}$$

Now the coordinates of $Q \equiv \left(-\frac{t^2}{4}, 0\right)$

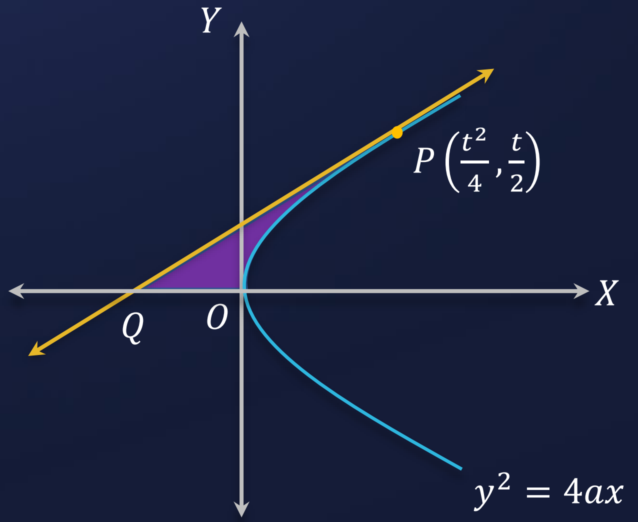
Area of $\Delta OPQ = \frac{1}{2} \times$ base $\times$ Height

$= \frac{1}{2} \times OQ \times$ Altitude from P

$$4 = \frac{1}{2} \times \frac{t^2}{4} \times \frac{t}{2}$$

$$\Rightarrow t = 4$$

$\therefore$ By $(i), m = \frac{1}{2} \quad \Rightarrow 2m = 1$



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Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $P(2,1)$ is :

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A $2x + y = 5$

B $x + 2y = 4$

C $x + 3y = 5$

D $x - y = 1$

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Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $P(2,1)$ is :

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Image of $y^2 = 4x$, about $y = x$ is $x^2 = 4y$

Equation of tangent at $P(2,1)$ is :

$$2x = 2(y + 1) \Rightarrow x - y = 1$$

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Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $P(2,1)$ is :

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A. $2x + y = 5$

B. $x + 2y = 4$

C. $x + 3y = 5$

D. $x - y = 1$

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Session 03

Equation of Normal in Different Forms

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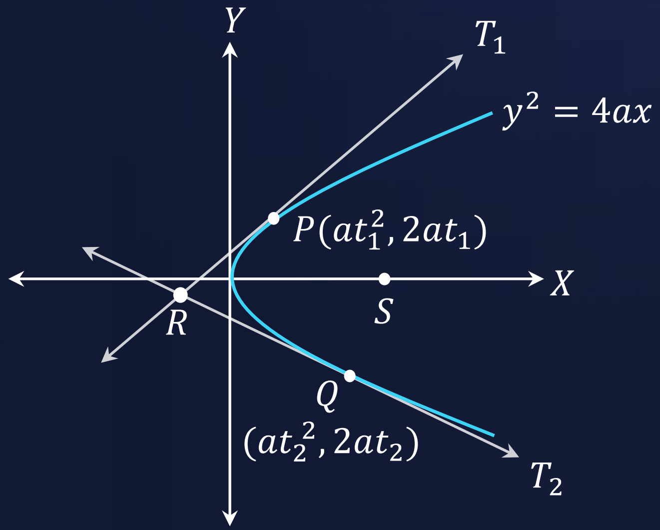
In each of the equations of tangent for the standard parabolas, replace $x \to (x-h)$ and $y \to (y-k)$ to get the equation of tangent for the respective translated parabolas.

Properties of Tangents :

I. Tangents at $P(t_1)$ and $Q(t_2)$ intersect at $R \equiv (at_1t_2, a(t_1+t_2))$.



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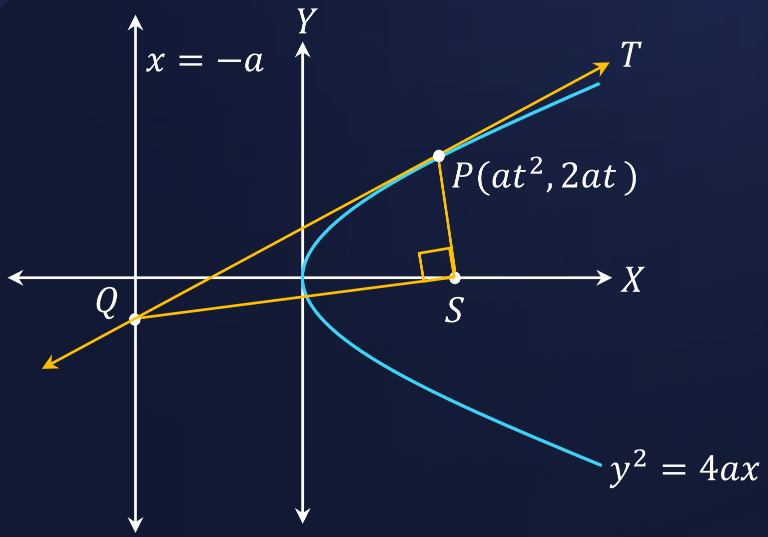
Properties of Tangents :

II. The portion of tangent between the point of contact $\left(P(t)\right)$ and the point where it meets the directrix $\left(Q(t)\right)$ subtends right angle at focus.



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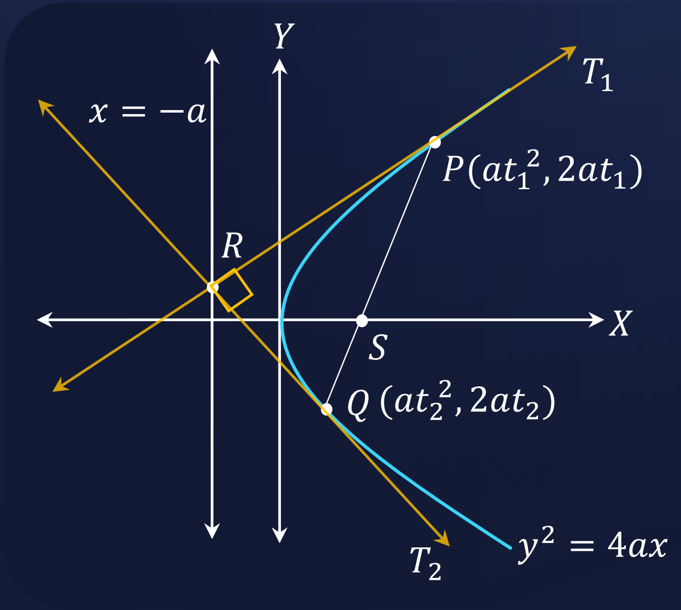
Properties of Tangents :

III. Tangent drawn at the extremities of focal chord are perpendicular and intersect on the directrix.



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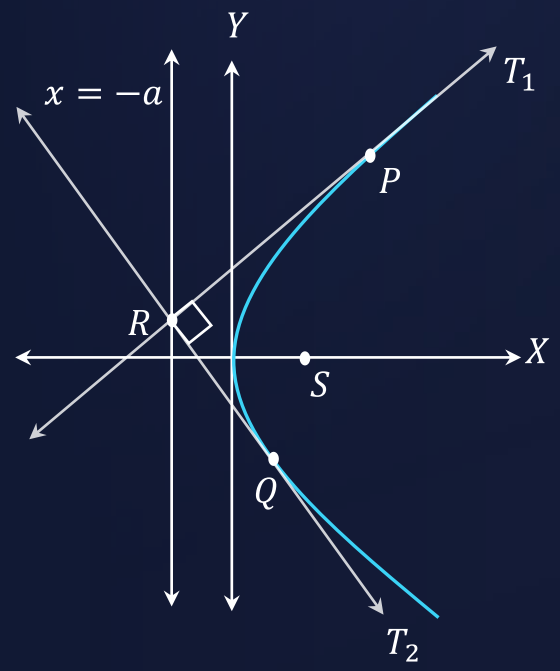
Properties of Tangents :

Note :

Each point on the directrix will give perpendicular tangents.

So, Equation of Director Circle to the parabola $y^2 = 4ax$ is

$x = -a$



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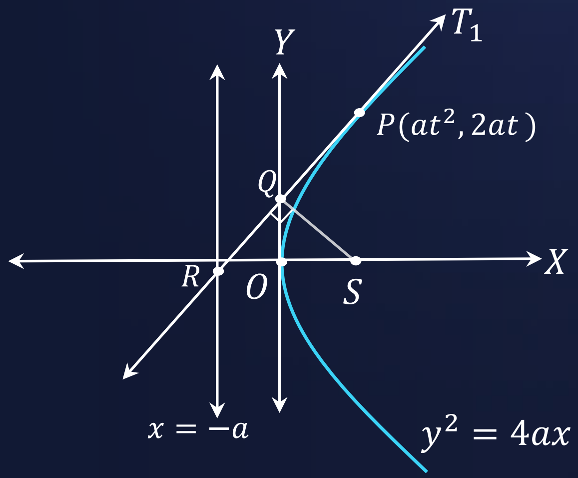
Properties of Tangents :

IV. The foot of the perpendicular drawn from focus upon any tangent lies on the tangent at vertex. Hence, circle described on any focal radii as diameter touches the tangent at vertex.



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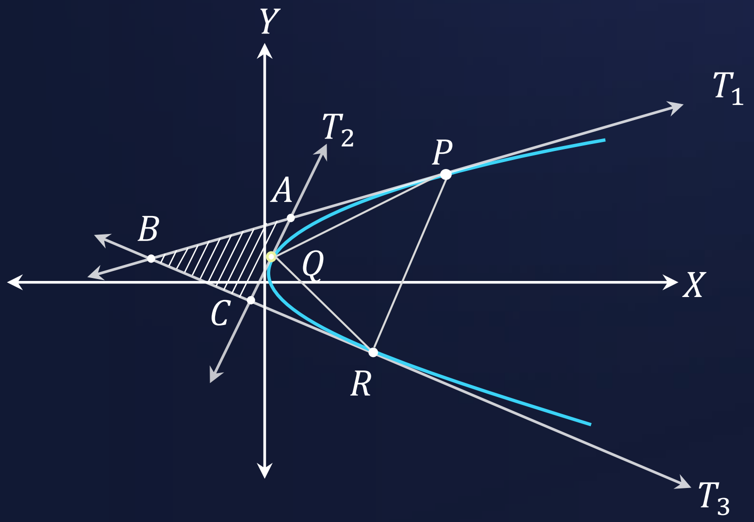
Properties of Tangents :

V. The area of the triangle formed by three points on the parabola is twice the area of the triangle formed by the tangents at these points.



$$Ar\ \triangle PQR = 2\ Ar\ \triangle ABC$$

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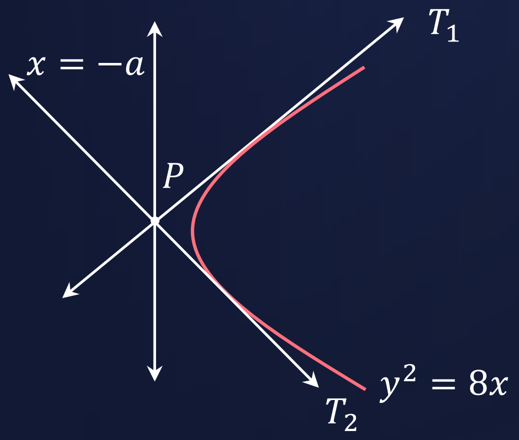
$y = x + 2$ is any tangent to the parabola $y^2 = 8x$. Find the point P on the tangent such that the other tangent from it is perpendicular to it.

$y^2 = 8x \quad \Rightarrow a = 2$

Tangent drawn at the extremities of focal chord are perpendicular and intersect on the directrix.

P lies on the directrix $x = -a = -2$

$\therefore P \equiv (-2,0)$



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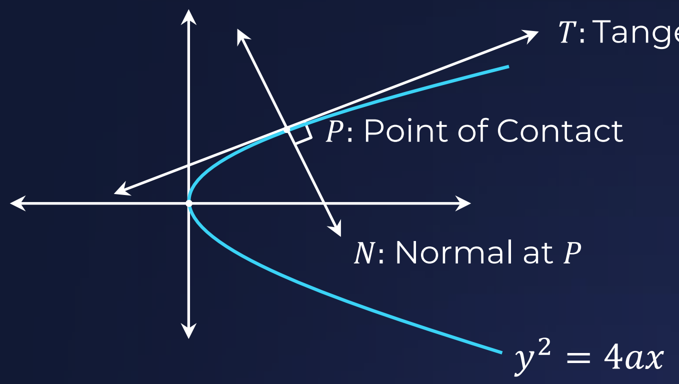
Key Takeaways

Normal :

The line perpendicular to the tangent at the point of contact is called the Normal to the parabola at that point.



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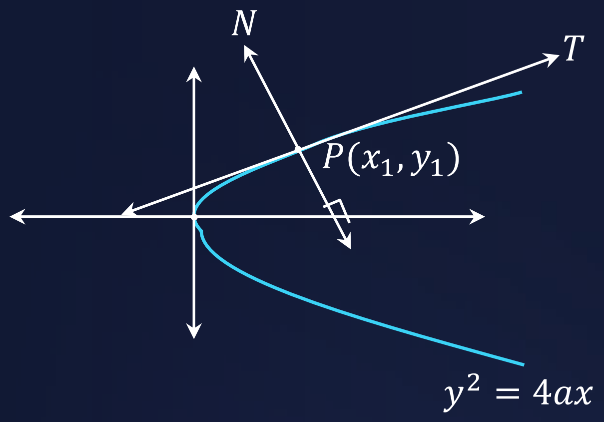
Key Takeaways

Equations of Normal : Point Form

The equation of normal to the parabola

$y^2 = 4ax$ at $P(x_1, y_1)$ is $y - y_1 = -\frac{y_1}{2a}(x - x_1)$.



Proof :

We have, $y^2 = 4ax$

Equation of tangent at $P(x_1, y_1)$ is $T = 0$.

$\Rightarrow yy_1 = 2a(x + x_1)$

$\Rightarrow m_{\text{tangent}} = \frac{2a}{y_1}$ = Slope of tangent at P

$\Rightarrow$ Slope of Normal $= -\frac{y_1}{2a}$ $\left(\because \text{Tangent} \perp \text{Normal}\right)$

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Key Takeaways

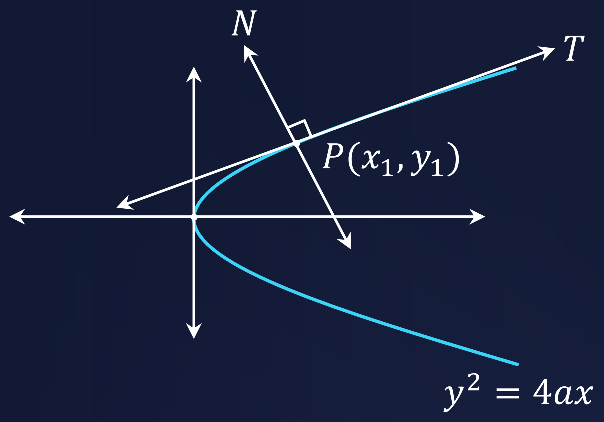
Equations of Normal : Point Form

$\Rightarrow$ Slope of Normal $= -\dfrac{y_1}{2a}$

$\left[\because \text{Tangent} \perp \text{Normal}\right]$

$\therefore$ By POINT SLOPE FORM,

Equation of normal is : $y - y_1 = -\dfrac{y_1}{2a}(x - x_1)$



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Key Takeaways

Equations of Normal : Point Form

Equation of normals of standard parabolas at (x_1, y_1) :

S. No.	Equation	Normal at (x_1, y_1)
1	$y^2 = 4ax$	$y - y_1 = -\frac{y_1}{2a}(x - x_1)$
2	$y^2 = -4ax$	$y - y_1 = \frac{y_1}{2a}(x - x_1)$
3	$x^2 = 4ay$	$y - y_1 = -\frac{2a}{x_1}(x - x_1)$
4	$x^2 = -4ay$	$y - y_1 = \frac{2a}{x_1}(x - x_1)$

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?

Let the tangent to the parabola $S: y^2 = 2x$ at the point $P(2,2)$ meet the x −axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

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A $\frac{25}{2}$

B $\frac{15}{2}$

C $\frac{35}{2}$

D 25

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Let the tangent to the parabola $S: y^2 = 2x$ at the point $P(2,2)$ meet the x –axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

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Solution:

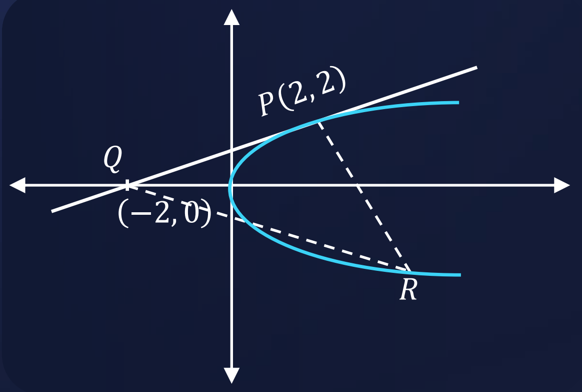
Tangent to $y^2 = 2x$ at $P(2,2)$ is $T = 0$

$\Rightarrow 2y = x + 2$

$\therefore Q(-2,0)$

Normal at $P(2,2)$ is $y + 2x = 6$

meets the curve at $R\left(\frac{9}{2}, -3\right)$



$$\Delta PQR = \left|\frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}\right|$$

Area of $\Delta PQR = \frac{25}{2}$ sq. units

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Let the tangent to the parabola $S: y^2 = 2x$ at the point $P(2, 2)$ meet the x –axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

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A $\frac{25}{2}$

B $\frac{15}{2}$

C $\frac{35}{2}$

D 25

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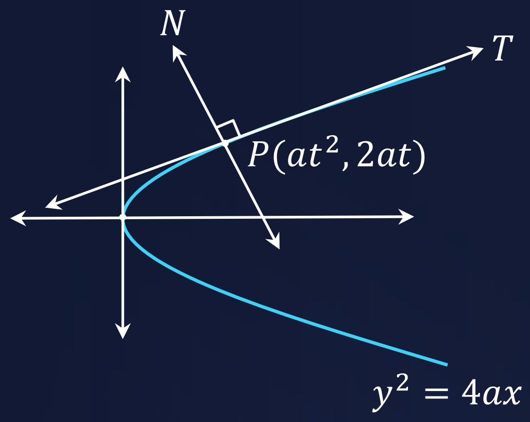
Key Takeaways

Equations of Normal : Parametric Form

The equation of normal to the parabola
$y^2 = 4ax$ at $P(t) \equiv P(at^2, 2at)$
is given by $y = -tx + 2at + at^3$.



Proof :

Here, $(x_1, y_1) \equiv (at^2, 2at)$

Equation of N : $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ (POINT FORM)

$\Rightarrow y - 2at = \frac{-2at}{2a}(x - at^2)$

$\Rightarrow y = -tx + 2at + at^3$

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Key Takeaways

Equations of Normal : Parametric Form

Equation of normals of standard parabolas at t :

S. No.	Equation	Parametric Co-ordinates	Normal at t
1	$y^2 = 4ax$	$(at^2, 2at)$	$y + tx = 2at + at^3$
2	$y^2 = -4ax$	$(-at^2, 2at)$	$y - tx = 2at + at^3$
3	$x^2 = 4ay$	$(2at, at^2)$	$x + ty = 2at + at^3$
4	$x^2 = -4ay$	$(2at, -at^2)$	$x - ty = 2at + at^3$

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Equation of parabola	Point of contact	Equation of Normal
$y^2 = 4ax$	$(am^2, -2am)$	$y = mx - 2am - am^3$
$y^2 = -4ax$	$(-am^2, 2am)$	$y = mx + 2am + am^3$
$x^2 = 4ay$	$\left(-\frac{2a}{m}, \frac{a}{m^2}\right)$	$y = mx + 2a + \frac{a}{m^2}$
$x^2 = -4ay$	$\left(\frac{2a}{m}, -\frac{a}{m^2}\right)$	$y = mx - 2a - \frac{a}{m^2}$

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?

If the parabolas $y^2 = 4b(x-c)$ and $y^2 = 8ax$ have a common normal, then which one of following is a valid choice for the ordered triplets (a, b, c)?

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A $\left(\frac{1}{2}, 2, 0\right)$

B $(1, 1, 3)$

C $(1, 1, 0)$

D $\left(\frac{1}{2}, 2, 3\right)$

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If the parabolas $y^2 = 4b(x-c)$ and $y^2 = 8ax$ have a common normal, then which one of following is a valid choice for the ordered triplets (a, b, c)?

JEE Main JAN 2019

A $\left(\frac{1}{2}, 2, 0\right)$

B $(1, 1, 3)$

C $(1, 1, 0)$

D $\left(\frac{1}{2}, 2, 3\right)$

If m is the slope of the common normal, then the equation of normal to parabola $y^2 = 4b(x-c)$ is given by

$$y = m(x-c) - 2bm - bm^3 \quad \cdots (1)$$

Equation of normal to parabola $y^2 = 8ax$ is

$$y = mx - 4am - 2am^3 \quad \cdots (2)$$

From equation (1) and (2), we get

$$m(x-c) - 2bm - bm^3 = mx - 4am - 2am^3$$

$$\Rightarrow m^2(2a-b) = c - 2(2a-b)$$

$$\Rightarrow m^2 = \frac{c}{2a-b} - 2$$

$$\because m^2 \geq 0 \Rightarrow \frac{c}{2a-b} - 2 \geq 0$$

$$\therefore \frac{c}{2a-b} \geq 2 \quad \cdots (3)$$

Hence $(1, 1, 3)$ satisfies equation (3)

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If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of following is a valid choice for the ordered triplets (a, b, c)?

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A $\left(\frac{1}{2}, 2, 0\right)$

B $(1, 1, 3)$

C $(1, 1, 0)$

D $\left(\frac{1}{2}, 2, 3\right)$

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Find the minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$

Given: Curve $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$

Shortest distance between curves occur along common normal.

Equation of normal to $y^2 = 4x : tx + y = 2t + t^3$

$\therefore$ It should pass through centre of circle $(6, 0)$

$\Rightarrow 6t + 0 = 2t + t^3$

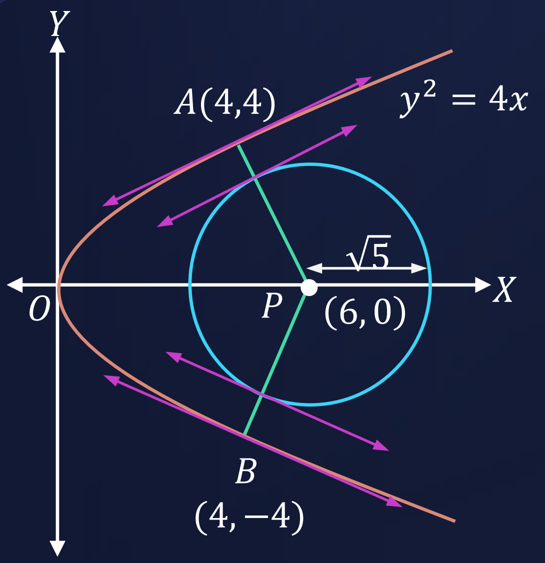
$\Rightarrow t^3 - 4t = 0$

$\Rightarrow t(t-2)(t+2) = 0$

$t = 0$ (origin)

$t = 2$ (A)

$t = -2$ (B)



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Find the minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$

$A = (t^2, 2t) = (4,4)$ & $B = (t^2, 2t) = (4,-4)$

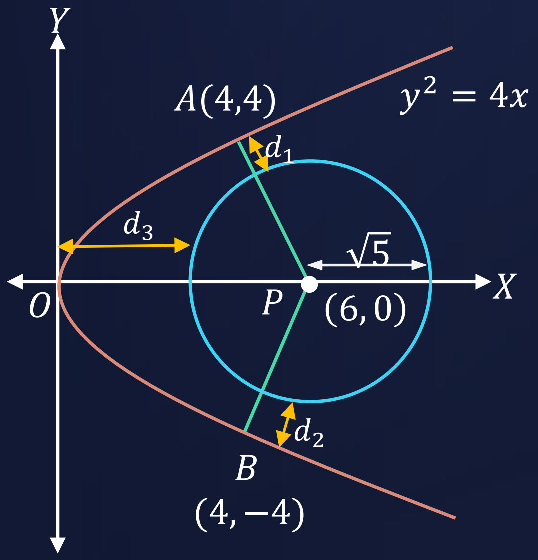
AP = distance between $(4,4)$ & $(6,0) = \sqrt{20}$

$CC' = d_3 = 6 - \sqrt{5}$

$\Rightarrow$ Shortest distance $= AP - \sqrt{5}$

$= \sqrt{20} - \sqrt{5}$

$\therefore$ Shortest distance $= \sqrt{20} - \sqrt{5}$



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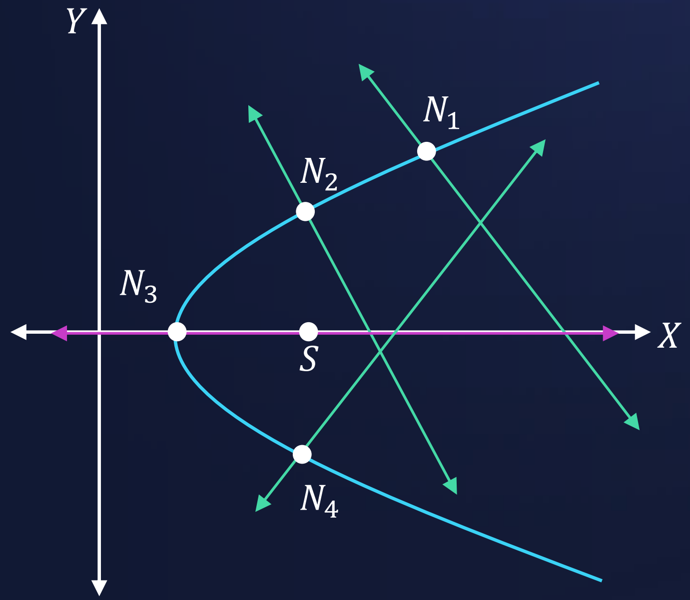
Properties of Normal

I. Normal other than axis of parabola never passes through the focus.



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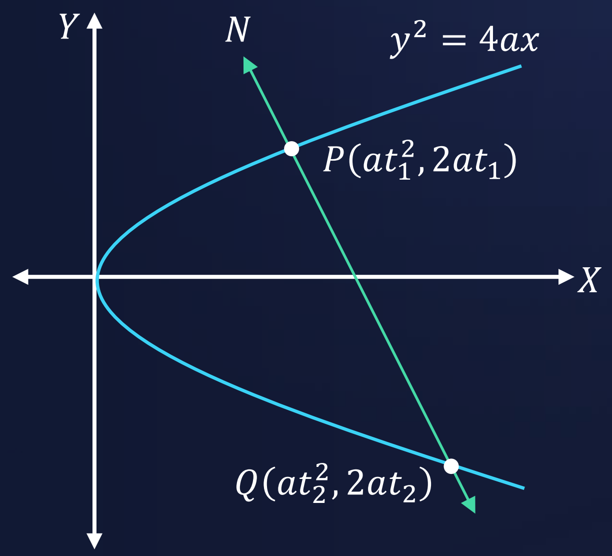
Properties of Normal

II. Normal at $P(t_1)$ meets the curve again at $Q(t_2)$, then

$$\boxed{t_2 = -t_1 - \frac{2}{t_1}}$$



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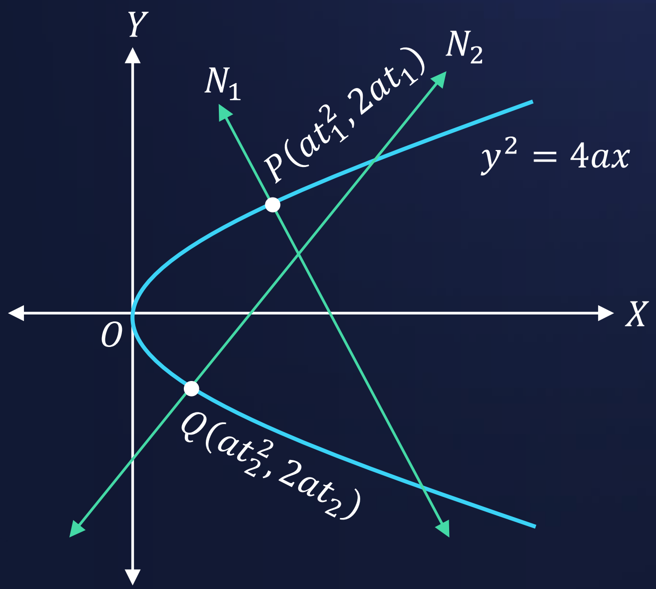
Properties of Normal

III. The point of intersection of normals at $P(t_1)$ and $Q(t_2)$ is $\left(2a + a(t_1^2 + t_1 t_2 + t_2^2), -at_1 t_2(t_1 + t_2)\right)$



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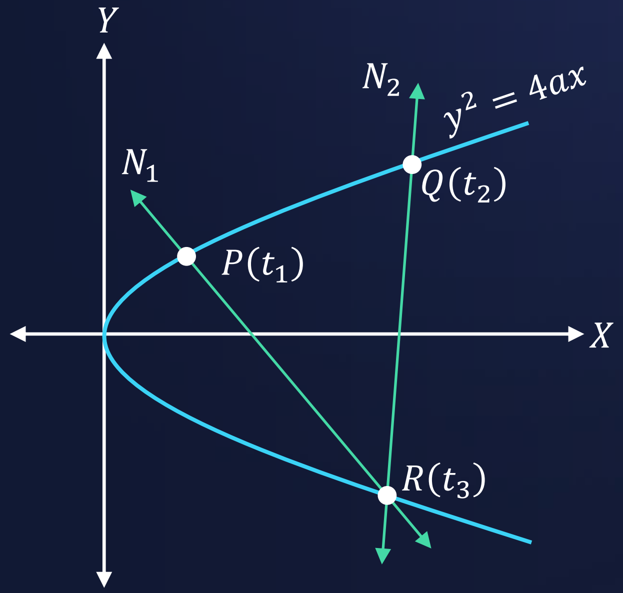
Properties of Normal

IV. If the normals to the parabola $y^2 = 4ax$ at points $P(t_1)$ and $Q(t_2)$ intersect again on the parabola at the point $R(t_3)$ then, $t_1 t_2 = 2$ and $t_1 + t_2 + t_3 = 0$.



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A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then find the slope of Normal.

Given: $y^2 = 4ax$

Slope of normal $N = m = -\mathrm{t}_1$

$m_{OA} \times m_{OB} = -1$

$\Rightarrow \left(\frac{2at_1 - 0}{at_1^2 - 0}\right) \times \left(\frac{2at_2 - 0}{at_2^2 - 0}\right) = -1$

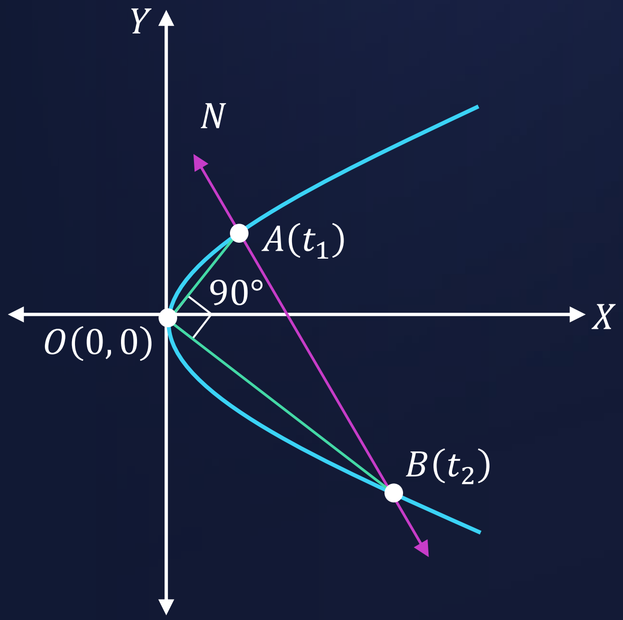
$\Rightarrow \frac{4a^2 t_1 t_2}{a^2 t_1^2 t_2^2} = -1$

$\Rightarrow t_1 t_2 = -4 \cdots (i)$

Using property II: $t_2 = -t_1 - \frac{2}{t_1} \cdots (ii)$

Solving (i) and (ii):

$t_2 = -4/t_1$



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A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then find the slope of Normal.

$$\therefore -\frac{4}{t_1} = -t_1 - \frac{2}{t_1} \Rightarrow t_1 = \frac{2}{t_1}$$

$$\Rightarrow t_1^2 = 2$$

$$\Rightarrow t_1 = \pm\sqrt{2}$$

$$\boxed{\therefore m = \pm\sqrt{2}}$$

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Key Takeaways

Conormal Points

Points on the parabola through which normals drawn are concurrent.

Here, $(A, B$ and $C) \rightarrow$ Conormal points

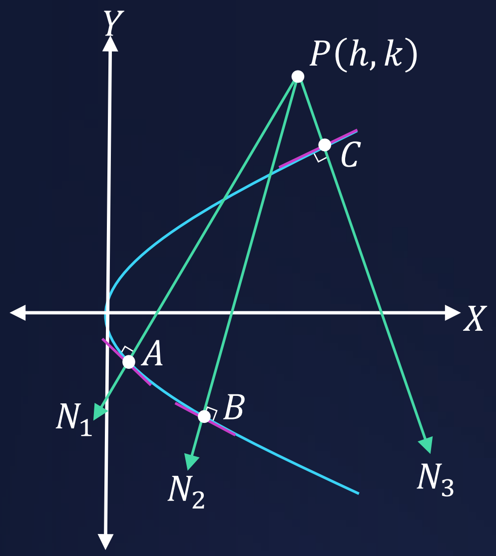
Feet of normal drawn from $P(h, k)$

Equation of Normal: $y = mx - 2am - am^3$

$\therefore$ Normal passes through $P(h, k)$

$\Rightarrow k = mh - 2am - am^3$

$\Rightarrow am^3 + 2am - mh + k = 0$



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Key Takeaways

Conormal Points

$$\underbrace{\boxed{\Rightarrow am^3 + (2a - h)m + k = 0}}_{\text{Cubic equation in } m} \to \left.\begin{matrix} m_1 \\ m_2 \\ m_3 \end{matrix}\right\} \in \mathbb{R}$$

Cubic equation in m

At most 3 real roots and at least one real root

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Key Takeaways

Conormal Points

Note:

From any given point $P(h, k)$; maximum three normals can be drawn to the parabola.

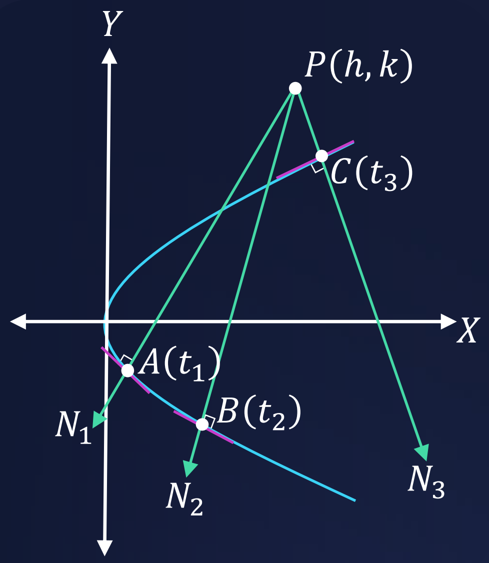
Also, If m_1, m_2, m_3 represent their slopes, then,

$$\boxed{m_1 + m_2 + m_3 = 0}$$

$$\boxed{m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{2a-h}{a}}$$

$$\boxed{m_1 m_2 m_3 = -\frac{k}{a}}$$

$$\boxed{t_1 + t_2 + t_3 = 0}$$



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If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0), a \neq 0$ then $'a'$ must be greater than:

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A 1

B $\frac{1}{2}$

C $-\frac{1}{2}$

D -1

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If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0), a \neq 0$ then $'a'$ must be greater than:

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Let the equation of the normal is :

$$y = mx - 2am - am^3$$

Here,

$$4a = 2 \quad \Rightarrow a = \frac{1}{2}$$

$$y = mx - m - \frac{1}{2}m^3$$

It passes through $A(a, 0)$ then

$$0 = ma - m - \frac{1}{2}m^3$$

$$m = 0, \quad m^2 - 2(a-1) = 0$$

For real values of m

$$2(a-1) > 0 \quad \therefore a > 1$$

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If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0), a \neq 0$ then $'a'$ must be greater than:

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A 1

B $\frac{1}{2}$

C $-\frac{1}{2}$

D -1

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Session 04

Chord of Contact and Introduction to Ellipse

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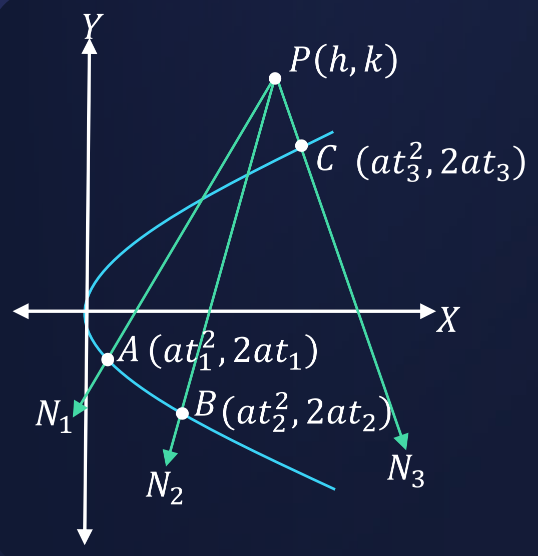
Properties of Conormal Points:

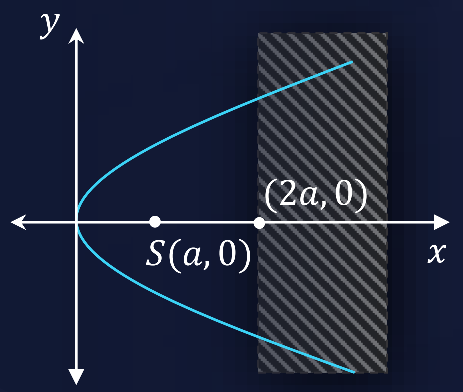
I. The algebraic sum of ordinates of the feet of 3 normal (conormal points) drawn to a parabola from a given point is 0.

NOTE: $(m_1 + m_2 + m_3) = 0 = (t_1 + t_2 + t_3)$



II. Centroid of the Δ formed by conormal points as vertices lie on the axis of the parabola.

III. If three normals drawn to any parabola $y^2 = 4ax$ from a given point (h, k) be real, then $h > 2a$.



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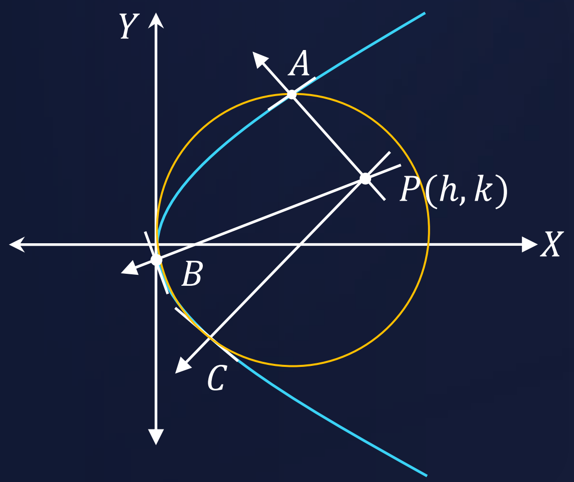
Properties of Conormal Points:

IV. A circle circumscribing the triangle formed by three conormal points passes through the vertex of the parabola and its equation is $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$



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The number of distinct normals that can be drawn from $(-2, 1)$ to the parabola $y^2 - 4x - 2y - 3 = 0$

A. 1

B. 2

C. 3

D. 0

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The number of distinct normals that can be drawn from $(-2, 1)$ to the parabola $y^2 - 4x - 2y - 3 = 0$

Given, $y^2 - 4x - 2y - 3 = 0$

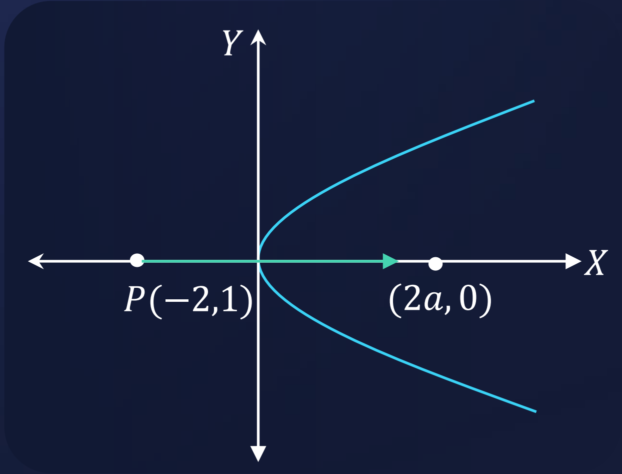
$\Rightarrow (y-1)^2 = 4(x+1)$

So, the axis is $y - 1 = 0$

Also, $(-2, 1)$ lies on the axis of parabola and it is exterior to the parabola because

$1^2 - 4(-2) - 2(1) - 3 = 4 > 0$

Hence, only one normal is possible.



Given diagram of $Y^2 = 4X$

$Y = y - 1$
$X = x + 1$

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The number of distinct normals that can be drawn from $(-2, 1)$ to the parabola $y^2 - 4x - 2y - 3 = 0$

A 1

B 2

C 3

D 0

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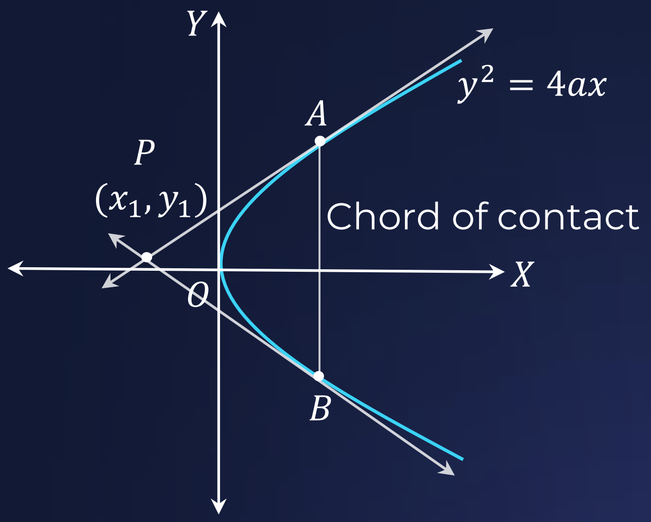
Key Takeaways

Chord of Contact :

A chord joining two points of contact of a pair of a tangents drawn from an external point.



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Key Takeaways

Chord of Contact :

Equation of chord of contact AB is $T = 0$.

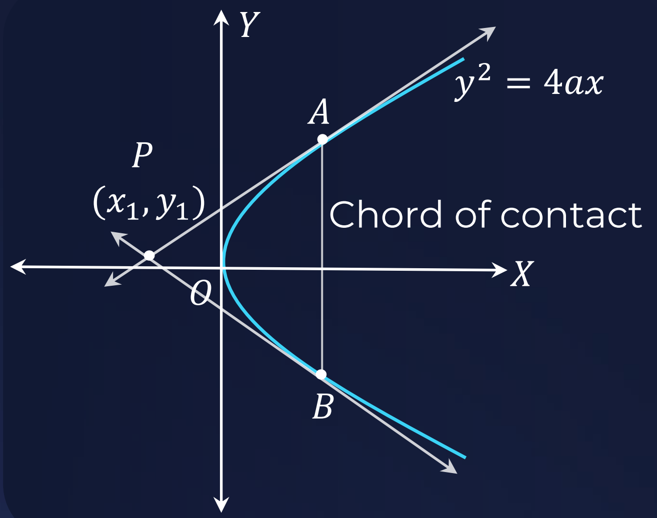
For $y^2 = 4ax$, equation of tangent is :

$$T = 0 \Rightarrow yy_1 - \frac{4a(x+x_1)}{2} = 0$$

$$\Rightarrow yy_1 - 2a(x + x_1) = 0$$

$\therefore$ Equation of chord of contact is

$$yy_1 - 2a(x + x_1) = 0$$



Note :

I. If $P(x_1, y_1)$ is an external point to the parabola, then $T = 0$ represents the equation of chord of contact w.r.t P.

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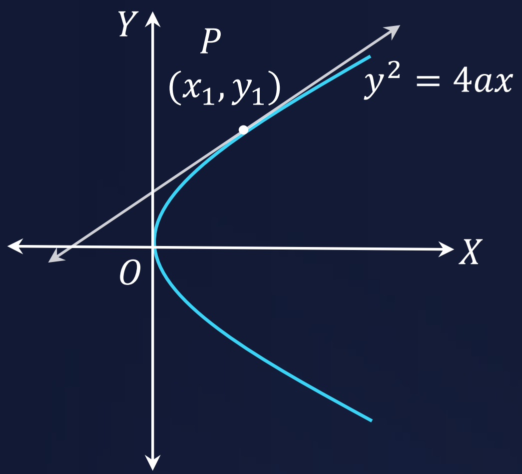
Key Takeaways

Note :

II. If $P(x_1, y_1)$ lies on the parabola,

then $T = 0$ represents the equation of

tangent through P (point of contact).



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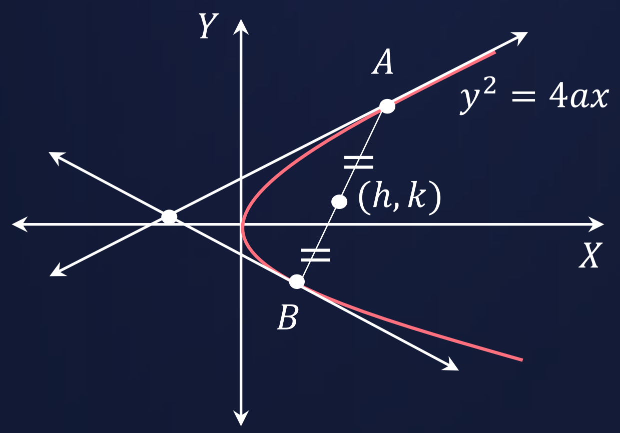
Chord Bisected at point :

For $y^2 = 4ax$ i.e. $S \equiv y^2 - 4ax = 0$

Equation of AB is $T = S_1$

$\Rightarrow yy_1 = 2a(x + x_1) = y_1^2 - 4ax_1$

Represents equation of the chord AB bisected at point $P(x_1, y_1)$



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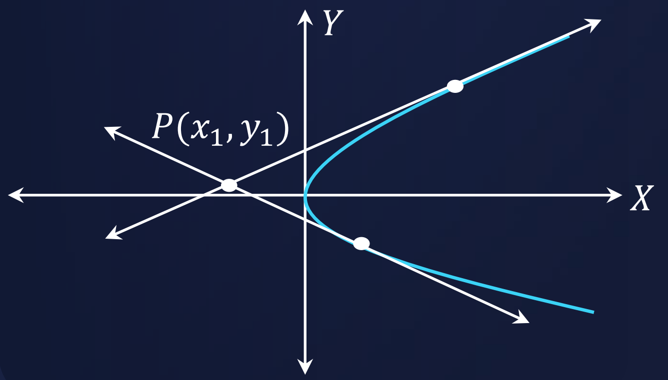
Note :

T_1 and T_2 are pair of tangents drawn from an external point $P(x_1, y_1)$.



Then, combined equation of T_1 and T_2 is:

$SS_1 = T^2$

For $y^2 = 4ax$ i.e. $S \equiv y^2 - 4ax$

We know, $S_1 \equiv y_1^2 - 4ax_1$ and $T \equiv yy_1 - 2a(x + x_1)$

Then, $SS_1 = T^2$ i.e. $(y^2 - 4ax)(y_1^2 - 4ax_1) = \left(yy_1 - 2a(x + x_1)\right)^2$

Represents the combined equation of T_1 and T_2

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If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the the following is (are) possible value(s) of p, h and k?

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A $p = -1, h = 1, k = -3$

B $p = 2, h = 3, k = -4$

C $p = -2, h = 2, k = -4$

D $p = 5, h = 4, k = -3$

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If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is (are) possible value(s) of p, h and k?

JEE Adv. 2017

Equation of chord: $T = S_1$

$$ky - 8(x + h) = k^2 - 16h$$

$$\Rightarrow -\frac{8}{k}x + y = k - \frac{8h}{k}$$

Comparing with given equation of chord, $2x + y = p$

$$-\frac{8}{k} = 2 \quad \Rightarrow k = -4$$

$$k - \frac{8h}{k} = p \quad \Rightarrow p = 2h - 4$$

$(\because p \neq -2, \text{line is not tangent})$

$\therefore p = 2, h = 3, k = -4$ is possible value

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If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is (are) possible value(s) of p, h and k?

JEE Adv. 2017

A $p = -1, h = 1, k = -3$

B $p = 2, h = 3, k = -4$

C $p = -2, h = 2, k = -4$

D $p = 5, h = 4, k = -3$

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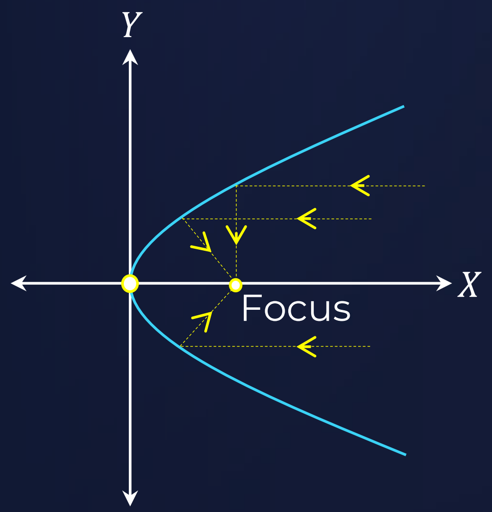
Reflection Property of Parabola :

- Any ray parallel to the axis of the parabola will bounce off the parabola and pass through the Focus.

- Conversely, any ray (light ray) emanating from the focus will reflect off the parabola in a straight line parallel to the axis.



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A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $(y-4)^2 = 8(x+1)$. After reflection, the ray always passes through the point (α, β), find the value of $\alpha + \beta + 10$.

As we know that all rays of light parallel to axis of the parabola are reflected through the focus of the parabola.

The equation of the given parabola is

$(y-4)^2 = 8(x+1) \Rightarrow Y^2 = 8X$

where $Y = y - 4$ and $X = x + 1$

Now the focus of the parabola is $(a, 0)$

Therefore, $X = a, Y = 0$

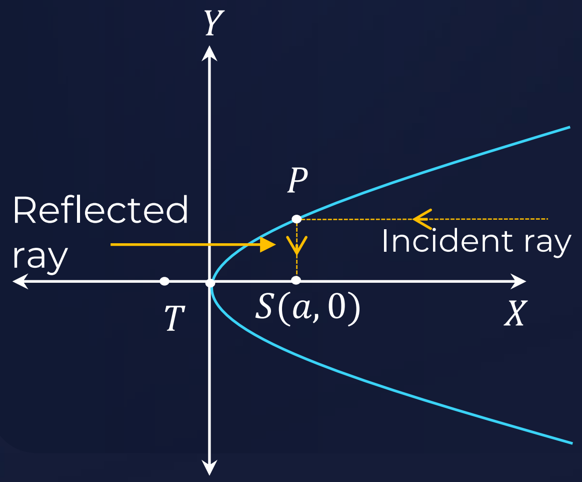
$\Rightarrow x + 1 = 2$ and $y - 4 = 0$

$\Rightarrow x = 1$ and $y = 4$

Hence, the focus is $(1,4)$

Thus $\alpha = 1$ and $\beta = 4$

Now, $\alpha + \beta + 10 = 1 + 4 + 10 = 15$



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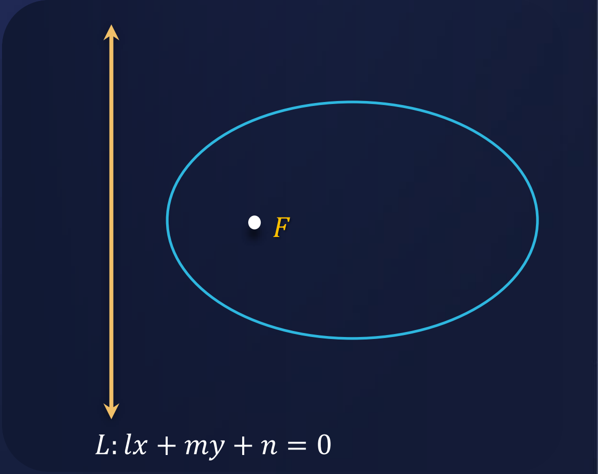
ELLIPSE:

- An ellipse is the locus of a moving point such that the ratio of its distance from a fixed point (focus) and a fixed line (directrix) is a positive constant (eccentricity) which is always less than 1 ($0 < e < 1$).
- Let the fixed point be F (focus) and the fixed line (directrix) be $L: lx + my + n = 0$ then $\frac{FP}{PM} = e < 1$.
- For 2^{nd} degree Non-Homogenous equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$.
  If $\Delta \neq 0, h^2 < ab$ then it represents ellipse



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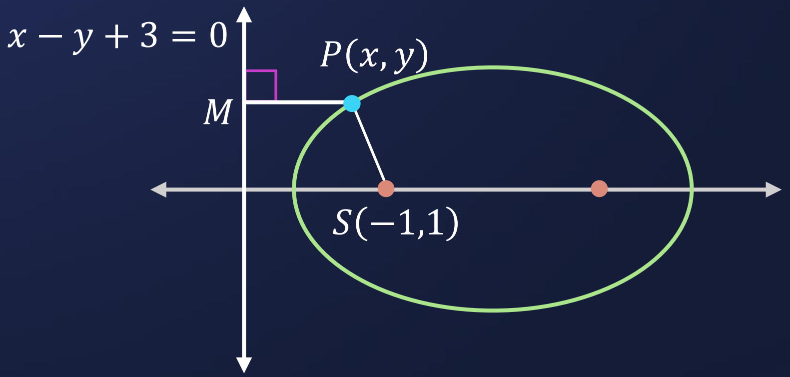
Find equation of ellipse with focus as $(-1,1)$, eccentricity= $\frac{1}{2}$ and equation of directrix as $x-y+3=0$



We know $SP = ePM$

Let $P = (x,y)$

Now, the equation of ellipse will be:

$$\sqrt{(x+1)^2+(y-1)^2} = \frac{1}{2}\cdot\frac{|x-y+3|}{\sqrt{1^2+1^2}}$$

Squaring both sides

$$8(x+1)^2+8(y-1)^2 = (x-y+3)^2$$

$$\Rightarrow 7x^2+7y^2+10x-10y+2xy+7=0$$

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Standard Equation of an Ellipse

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ This is the standard equation of the ellipse.

  where $b^2 = a^2(1 - e^2)$

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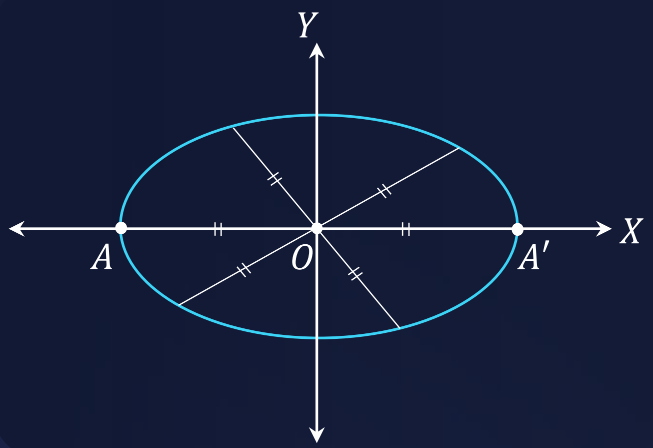
Key Takeaways

Terms associated with an Ellipse

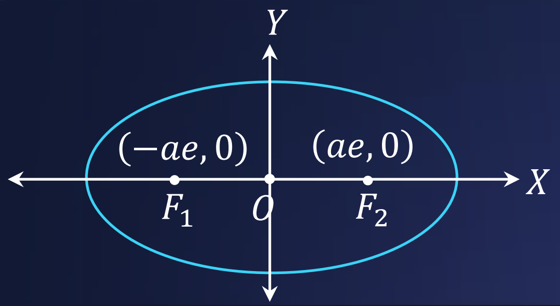
Centre :

- The point which bisects every chord of the ellipse drawn through it is called center of the ellipse.



- Here center is $O \equiv (0, 0)$.

- Foci : The foci are $F_1 \equiv (-ae, 0)$ and $F_2 \equiv (ae, 0)$.



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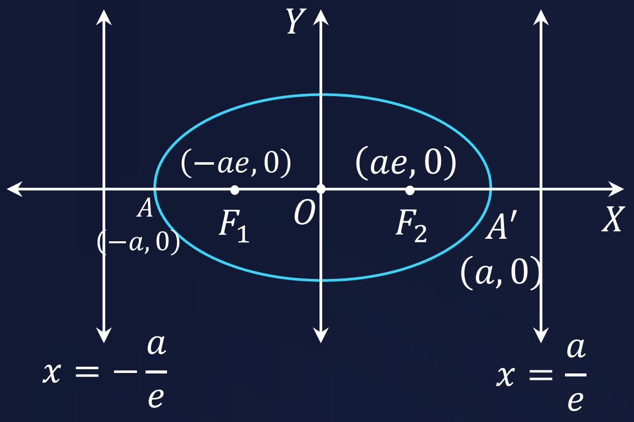
Key Takeaways

Terms associated with an Ellipse



Directrices :

- The equation of directrices are $x = -\frac{a}{e}$ and $x = \frac{a}{e}$.

Vertices :

- The points of intersection f the ellipse with the line passing through the foci are called vertices.

  Here Vertices are $A \equiv (-a, 0)$ and $A' \equiv (a, 0)$

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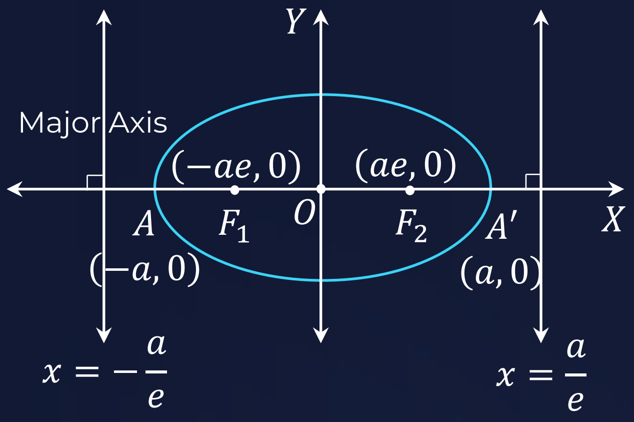
Key Takeaways

Terms associated with an Ellipse

Major Axis :

- The line which passes through the foci and is perpendicular to the directrices is called major axis.
  Here the major axis intersects the ellipse at the vertices.



$\Rightarrow$ The length of major axis is the distance between its vertices $= AA' \Rightarrow AA' = 2a$

$\Rightarrow$ The major axis is the longest chord of an ellipse.

$\Rightarrow$ Here the major axis is along the x – axis.

$\therefore$ Equation of major axis is $y = 0$.

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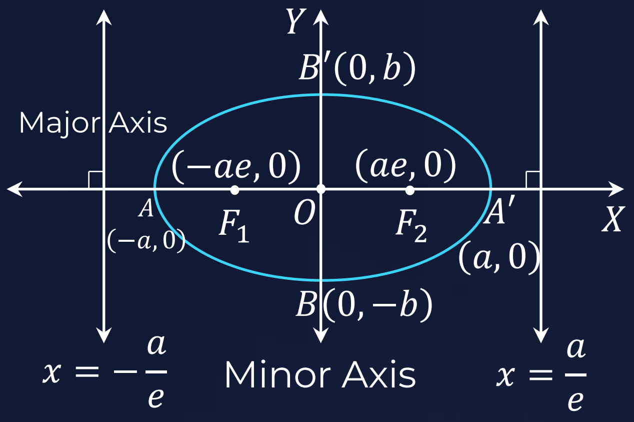
Key Takeaways

Terms associated with an Ellipse

Minor Axis :

- The axis which is perpendicular bisector of the major axis is called minor axis.

  Here the major axis intersects the ellipse at the vertices.



Length of minor axis :

$\Rightarrow$ Let the ellipse intersect the minor axis at B and B'

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- At B and $B', x = 0$

$\Rightarrow y^2 = b^2 \Rightarrow y = \pm b \qquad \therefore B \equiv (0, -b)$ and $B' \equiv (0, b)$

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Key Takeaways

Terms associated with an Ellipse

Length of minor axis :

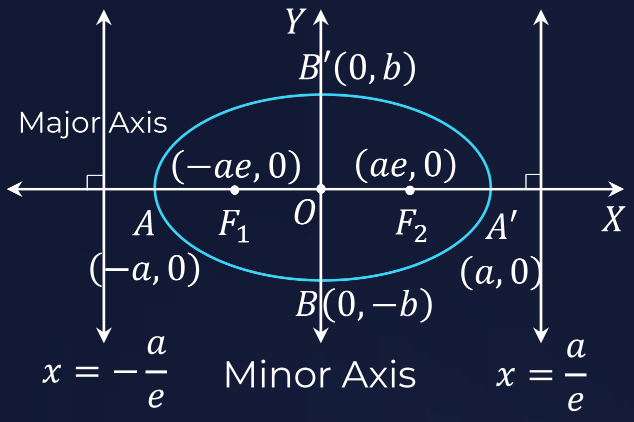
At B and B', $x = 0$

$\Rightarrow y^2 = b^2 \Rightarrow y = \pm b$

- $\therefore B \equiv (0, -b)$ and $B' \equiv (0, b)$

$\boxed{\therefore \text{ Length of minor axis is } 2b.}$

Where $b^2 = a^2(1 - e^2)$



Equation of minor axis :

- Here the minor axis is along the $y -$ axis.

$\therefore$ Equation of minor axis is $x = 0$

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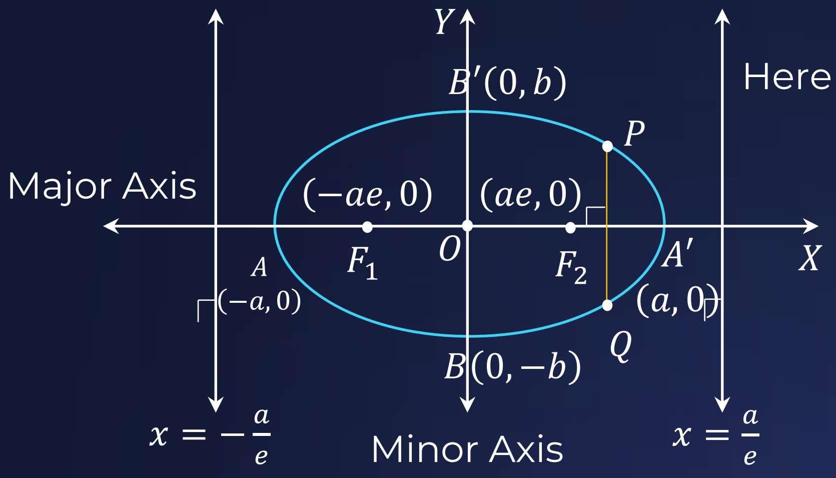
Key Takeaways

Terms associated with an Ellipse

Double Ordinate :

A chord perpendicular to major axis is called double ordinate.



Here $PQ \rightarrow$ Double ordinate

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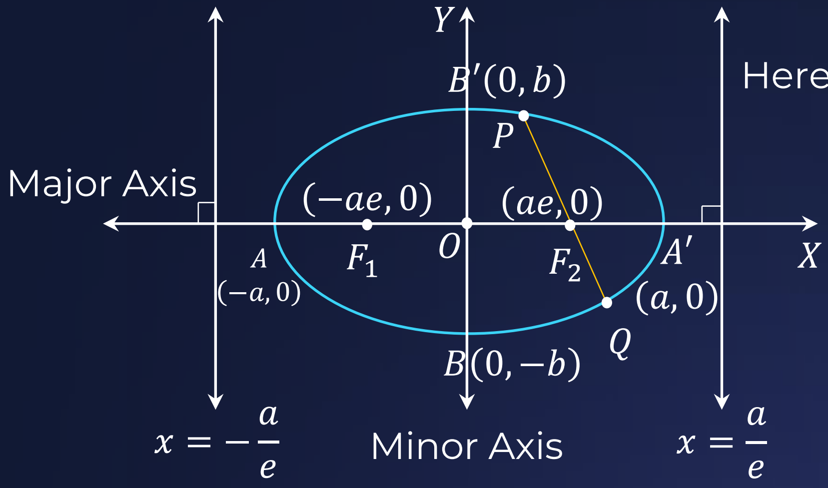
Key Takeaways

Terms associated with an Ellipse

Focal Chord :

A chord passing through focus is called focal chord.



Here $PQ \rightarrow$ Focal chord

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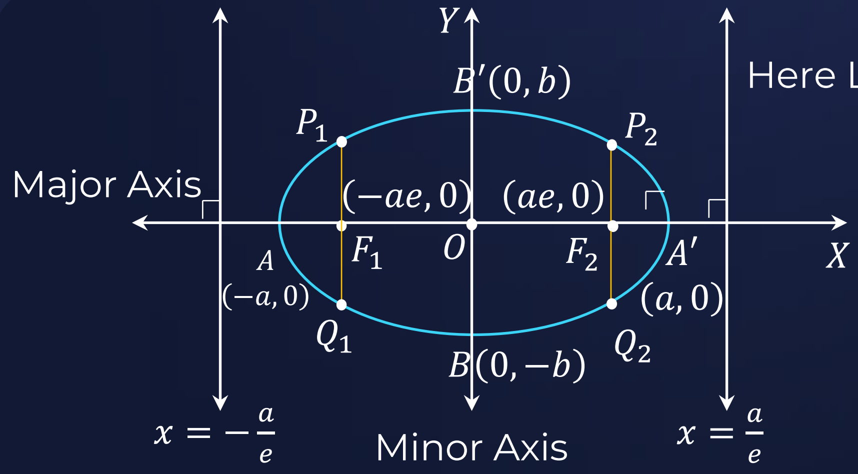
Terms associated with an Ellipse

Latus Rectum :

A focal chord perpendicular to the major axis is called Latus Rectum.



Here Latus rectum $\rightarrow P_1Q_1 = P_2Q_2$

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Terms associated with an Ellipse

Length of Latus Rectum :

The length of latus rectum is given by $\frac{2b^2}{a}$.

Here Latus rectum $\rightarrow P_1Q_1 = P_2Q_2$

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If the distance between the focii of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

JEE Main 2020

A $2\sqrt{3}$

B $\sqrt{3}$

C $\frac{3}{\sqrt{2}}$

D $3\sqrt{2}$

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If the distance between the focii of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

JEE Main 2020

Let the equation of the ellipse be: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ \ (a>b)$

Now, $2ae=6$ and $\frac{2a}{e}=12$

$\Rightarrow ae=3$ and $\frac{a}{e}=6$

$\Rightarrow a^2=18$

$\Rightarrow (ae)^2=a^2-b^2=9$

$\Rightarrow b^2=9$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{\sqrt{18}}=3\sqrt{2}$

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If the distance between the focii of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

JEE Main 2020

A $2\sqrt{3}$

B $\sqrt{3}$

C $\frac{3}{\sqrt{2}}$

D $3\sqrt{2}$

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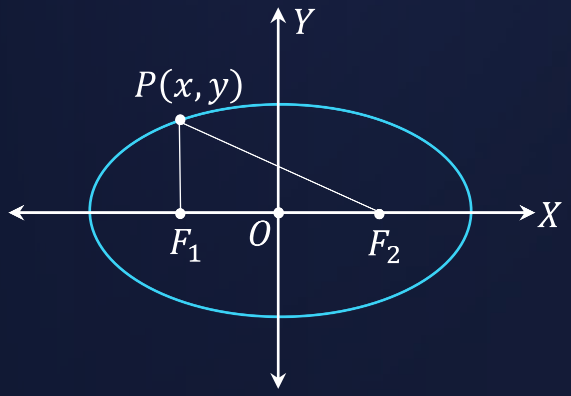
Terms associated with an Ellipse

Focal Distance :

- The distance between the focus to any point on the ellipse is called focal distance or focal radii.



Let $P \equiv (x, y)$ be any point on the ellipse.

- Then focal distance is PF_1 or PF_2.

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Focal Distance :

Let $P(x, y)$ be a point on the ellipse.

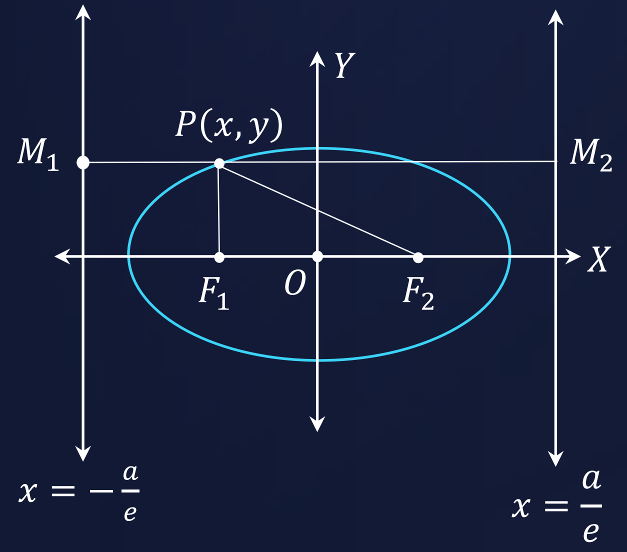
$$\frac{PF_1}{PM_1} = e \quad \text{and} \quad \frac{PF_2}{PM_2} = e$$

$$\Rightarrow PF_1 = e(PM_1) \text{ and } PF_2 = e(PM_2) \cdots (i)$$

$$PM_1 = \frac{a}{e} - x \quad \text{and} \quad PM_2 = \frac{a}{e} + x$$

$$(i) \Rightarrow PF_1 = e\left(\frac{a}{e} - x\right) \text{ and } PF_2 = e\left(\frac{a}{e} + x\right)$$

$\therefore$ Focal distances of $P(x, y) = a \pm ex$



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Let PQ be a focal chord of the parabola $y^2 = 4x$, such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3,0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is :

JEE Main 2022

A. $1 + \sqrt{2}$

B. $3 + 2\sqrt{2}$

C. $4 + 5\sqrt{3}$

D. $1 + 2\sqrt{3}$

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Let PQ be a focal chord of the parabola $y^2 = 4x$, such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3,0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \, , a^2 > b^2$. If e is the eccentricity of the ellipse E , then the value of $\frac{1}{e^2}$ is :

JEE Main 2022

$$P \equiv (t^2 , 2t) \, , Q \equiv \left(\frac{1}{t^2} , -\frac{2}{t}\right)$$

$$m_{PR}.m_{QR} = -1 \Rightarrow \frac{\left(\frac{2}{t}\right)}{\left(3 - \frac{1}{t^2}\right)} \cdot \frac{2t}{t^2 - 3} = -1$$

$$\Rightarrow t = \pm 1$$

$$P \equiv (1, 2) \, , Q \equiv (1 , -2)$$

For the ellipse :

$$\frac{1}{a^2} + \frac{4}{b^2} = 1 \ \& \ ae = 1$$

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Let PQ be a focal chord of the parabola $y^2 = 4x$, such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3,0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is :

JEE Main 2022

$$\frac{1}{a^2} + \frac{4}{b^2} = 1 \;\&\; ae = 1$$

$$\frac{1}{a^2} + \frac{4}{a^2(1-e^2)} = 1$$

$$e^2 + \frac{4e^2}{(1-e^2)} = 1$$

$$\Rightarrow e^2 = 3 - 2\sqrt{2}$$

$$\Rightarrow \frac{1}{e^2} = 3 + 2\sqrt{2}$$

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Let PQ be a focal chord of the parabola $y^2 = 4x$, such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3,0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is :

JEE Main 2022

A. $1 + \sqrt{2}$

B. $3 + 2\sqrt{2}$

C. $4 + 5\sqrt{3}$

D. $1 + 2\sqrt{3}$

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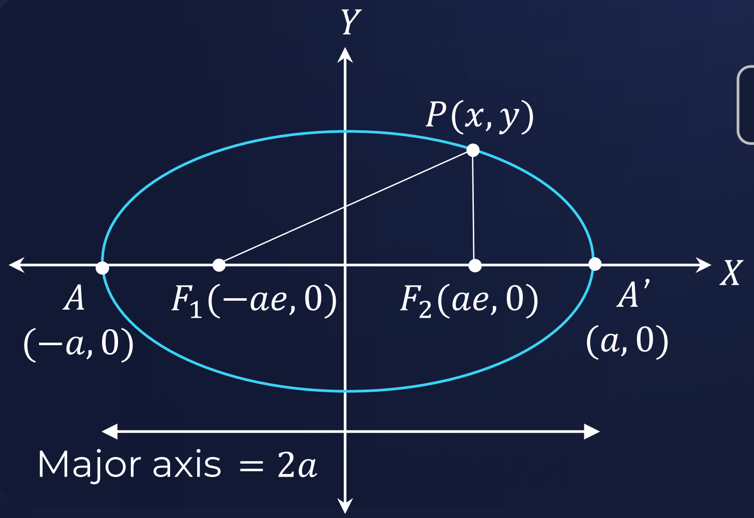
Alternate definition of an Ellipse :

An ellipse is the set of all points in a plane; the sum of whose distances from two fixed points (Foci) in the plane is a constant (length of major axis= $2a$).



$$PF_1 + PF_2 = 2a > F_1F_2 = 2ae$$

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What is the eccentricity of ellipse if length of minor axis is equal to focal length.

A $\frac{1}{2}$

B $\frac{1}{\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$

D $\frac{3}{4}$

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What is the eccentricity of ellipse if length of minor axis is equal to focal length.

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $a > b$

According to the given condition, $2b = 2ae$

We know, $c^2 = a^2 - b^2$

$\Rightarrow c^2 = a^2 - a^2e^2$

$\Rightarrow 2a^2e^2 = a^2 \qquad (\because c = ae)$

$\Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

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What is the eccentricity of ellipse if length of minor axis is equal to focal length.

A $\frac{1}{2}$

B $\frac{1}{\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$

D $\frac{3}{4}$

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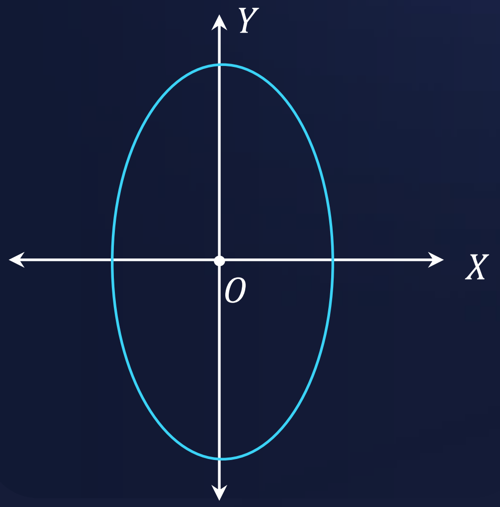
Vertical Ellipse :

- Major axis is along y-axis and Minor axis is along x-axis
- Equation of a vertical ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where, $0<a<b$ and $a^2=b^2(1-e^2)$



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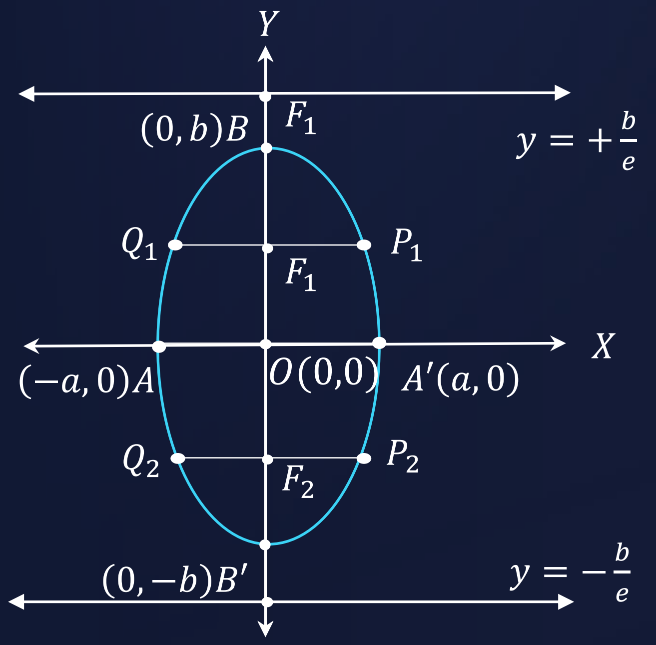
Vertical Ellipse :

Center :	$O(0,0)$
Foci:	$F_1(0,be), F_2(0,-be)$
Vertices:	$B(0,b), B'(0,-b)$
Major Axis:	BB'
Length of Major Axis :	$2b$
Minor Axis:	AA'
Length of Minor Axis:	$2a$
Directrices:	$y = \pm\frac{b}{e}$
Length of L.R. :	$\frac{2a^2}{b}$
Focal distances of $P(x,y)$:	$b \pm ey$



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Vertical Ellipse :

Note :

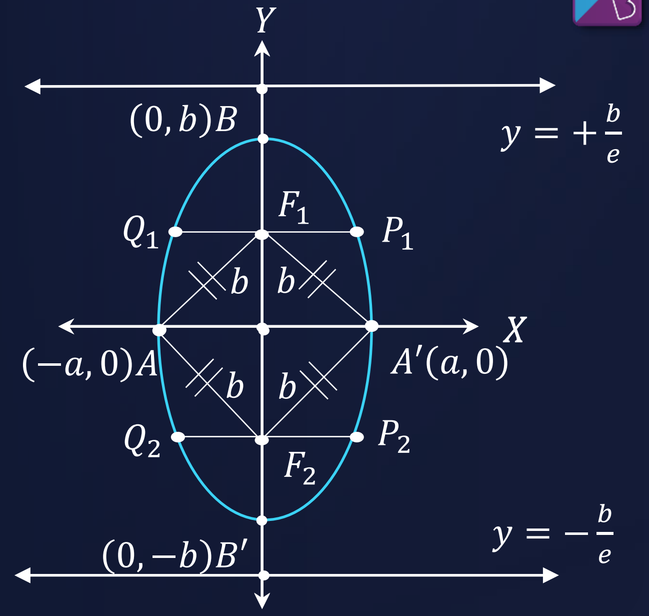
- $PF_1 + PF_2 = 2b$ (length of the major axis)

  $AF_1 = A'F_1 = AF_2 = A'F_2 = b$

  Where, $AF_1 = \sqrt{a^2 + b^2e^2} = \sqrt{b^2} = b$

  The ellipse is symmetric about both X and Y-axis.

  Eccentricity, $e = \sqrt{1 - \dfrac{a^2}{b^2}}$



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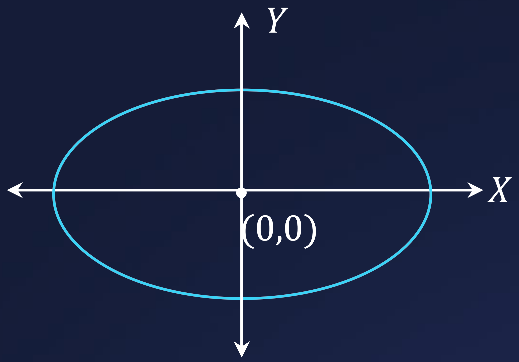
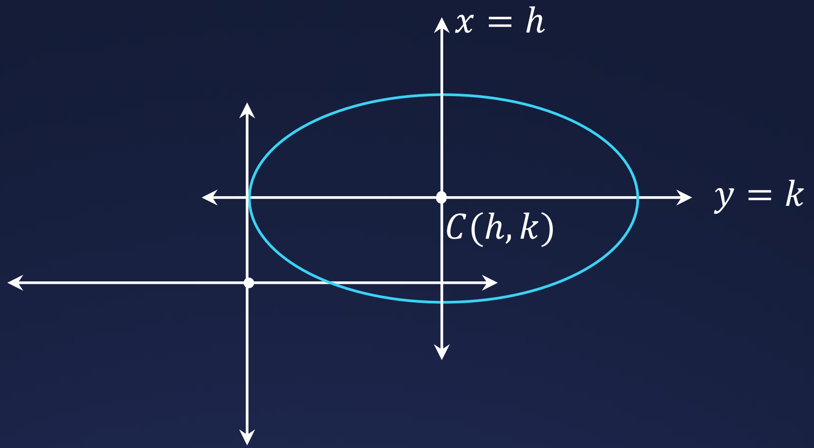
Key Takeaways

Axes of ellipse parallel to co-ordinate axes :



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

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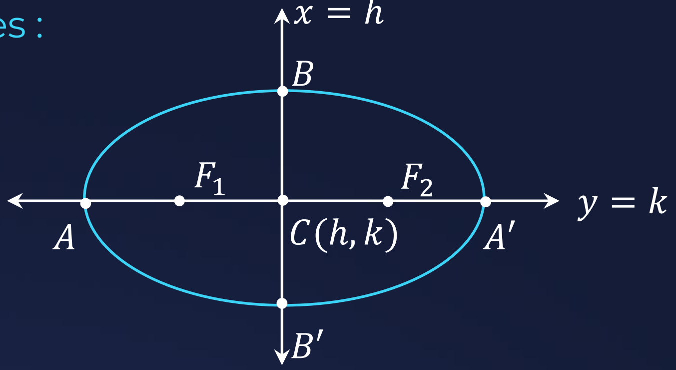
Key Takeaways

Axes of ellipse parallel to co-ordinate axes :

Case (i): Major axis parallel to x-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Where, $a>b, b^2=a^2(1-e^2)$



Center :	$C(h,k)$
Foci :	$F_1(h-ae,k), F_2(h+ae,k)$
Vertices:	$A(h-a,k), A'(h+a,k)$

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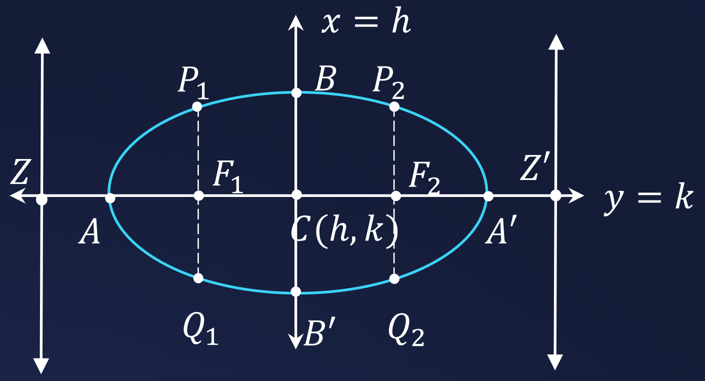
Key Takeaways

Axes of ellipse parallel to co-ordinate axes :

Case (i): Major axis parallel to x-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$



Length of Major Axis :	$AA'=2a$
Length of Minor Axis :	$BB'=2b$
Equation of Directrix:	$x=h\pm\frac{a}{e}$
Eccentricity :	$e=\sqrt{1-\frac{b^2}{a^2}}$

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Session 05

Auxiliary Circle and Parametric Equation of an Ellipse

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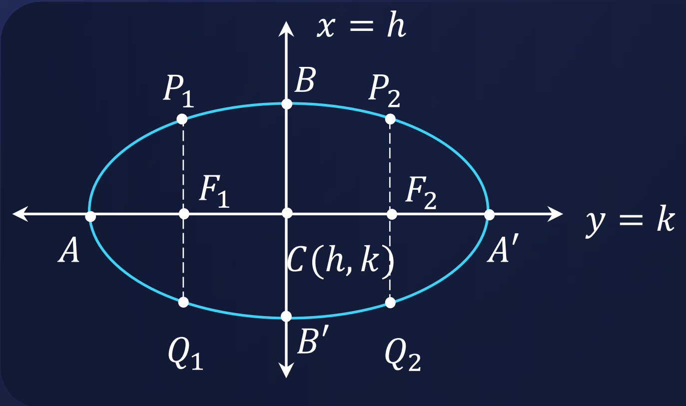
Axes of ellipse parallel to co-ordinate axes :

Case (i): Major axis parallel to x-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$



Ends of L.R. :	$P_1\left(h-ae,k+\frac{b^2}{a}\right), Q_1\left(h-ae,k-\frac{b^2}{a}\right)$ $P_2\left(h+ae,k+\frac{b^2}{a}\right), Q_2\left(h+ae,k-\frac{b^2}{a}\right)$
Length of L.R.:	$\frac{2b^2}{a}$

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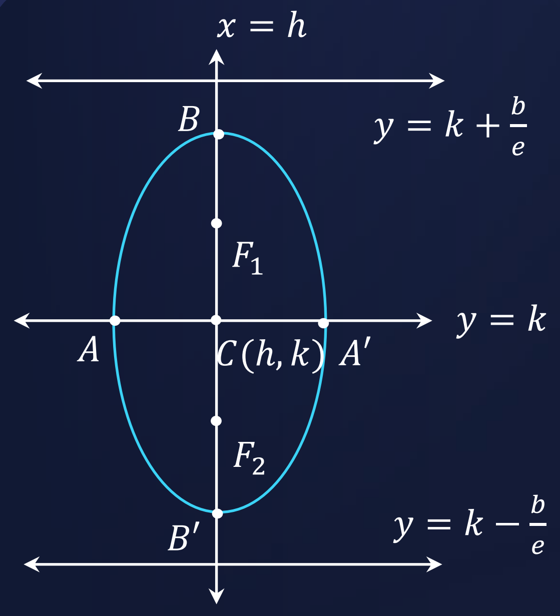
Axes of ellipse parallel to co-ordinate axes :

Case (ii): Major axis parallel to y-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Where $a<b, a^2=b^2(1-e^2)$

Center :	$C(h,k)$
Foci :	$F_1(h,k+be), F_2(h,k-be)$
Vertices:	$B(h,k+b), B'(h,k-b)$



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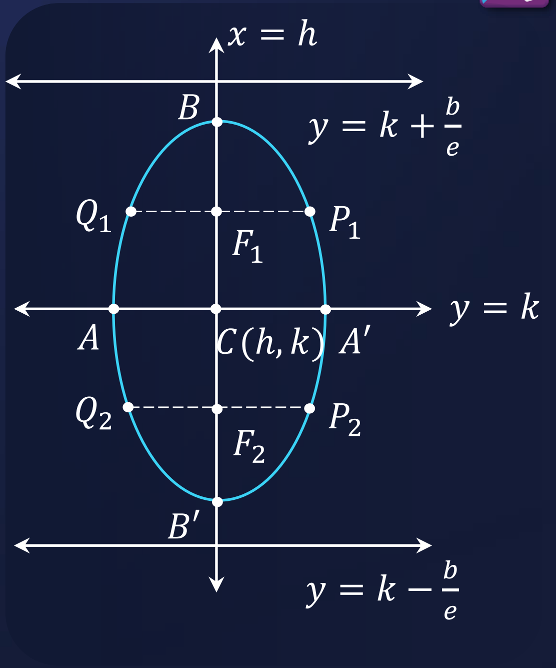
Axes of ellipse parallel to co-ordinate axes :

Case (ii): Major axis parallel to y-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Ends of L.R. :	$P_1\left(h + \frac{a^2}{b}, k + be\right), Q_1\left(h - \frac{a^2}{b}, k + be\right)$ $P_2\left(h + \frac{a^2}{b}, k - be\right), Q_2\left(h - \frac{a^2}{b}, k - be\right)$
Length of L.R.:	$\frac{2a^2}{b}$



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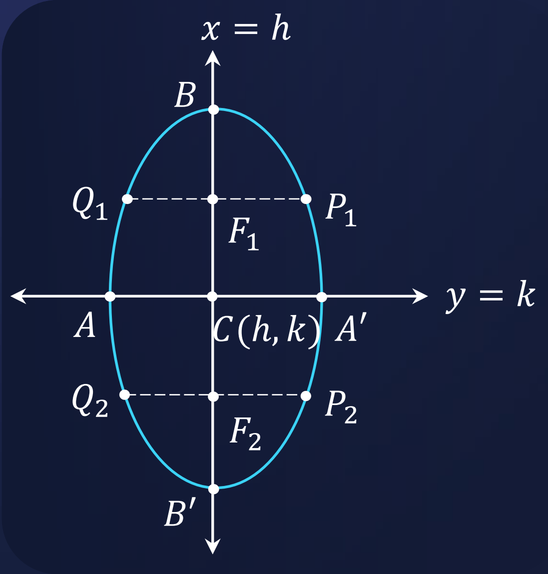
Axes of ellipse parallel to co-ordinate axes :

Case (ii): Major axis parallel to y-axis

Equation of ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



Length of Major Axis:	$BB' = 2b$
Length of Minor Axis:	$AA' = 2a$
Equation of Directrix:	$y = k \pm \frac{b}{e}$
Eccentricity :	$e = \sqrt{1 - \frac{a^2}{b^2}}$

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In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is ______.

JEE MAIN 2019

A 5

B 10

C 8

D 6

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In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is ______.

JEE MAIN 2019

Solution:

Given : An ellipse with Centre $\equiv (0,0)$

Focus $\equiv (0, 5\sqrt{3})$, lies on Y- axis

$\therefore$ Major axis is along Y-axis. $\therefore$ $be = 5\sqrt{3} \cdots (i)$

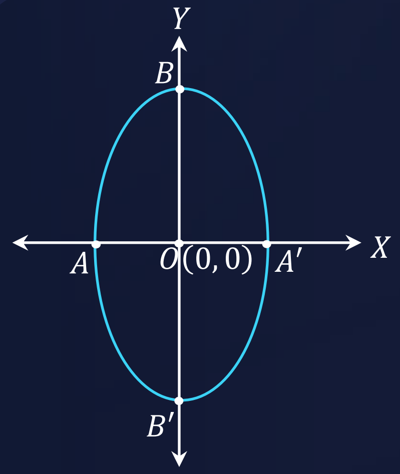
Also $2b - 2a = 10 \Rightarrow b - a = 5 \cdots (ii)$

$a^2 = b^2(1 - e^2)$

$\Rightarrow b^2e^2 = b^2 - a^2 \Rightarrow (5\sqrt{3})^2 = (b+a)(b-a)$

$\Rightarrow 75 = (b+a)(5)$

$\Rightarrow b + a = 15 \cdots (iii)$



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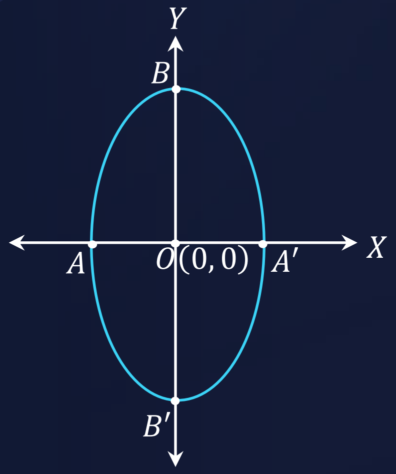
In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is ______.

JEE MAIN 2019

Solution:



$\Rightarrow b - a = 5 \cdots (ii)$

$\Rightarrow b + a = 15 \cdots (iii)$

Solving (ii) and (iii), we get;

$b = 10$ and $a = 5$

Length of latus rectum $= \frac{2a^2}{b} = 5$

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In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is ______.

JEE MAIN 2019

A. 5

B. 10

C. 8

D. 6

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An ellipse, with foci at $(0,2)$ and $(0,-2)$ and minor axis of length 4, passes through which of the following points ?

JEE MAIN APR 2019

A $(2, \sqrt{2})$

B $(1, 2\sqrt{2})$

C $(\sqrt{2}, 2)$

D $(2, 2\sqrt{2})$

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An ellipse, with foci at $(0,2)$ and $(0,-2)$ and minor axis of length 4, passes through which of the following points ?

JEE MAIN APR 2019

A $(2,\sqrt{2})$

B $(1,2\sqrt{2})$

C $(\sqrt{2},2)$

D $(2,2\sqrt{2})$

Solution:

Foci are $(0,2)$ and $(0,-2)$

So, major axis is y –axis.

Let the equation of ellipse be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Given: $2b=4$ and $2c=4$

$\Rightarrow b=2, c=2$

$\Rightarrow c^2=4 \Rightarrow a^2-b^2=4$

$\Rightarrow a^2=8$

Therefore, equation of the ellipse is $\frac{x^2}{4}+\frac{y^2}{8}=1$

By putting all the points, we get that

$(\sqrt{2},2)$ lies on the ellipse.

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An ellipse, with foci at $(0, 2)$ and $(0, -2)$ and minor axis of length 4, passes through which of the following points ?

JEE MAIN APR 2019

A $(2, \sqrt{2})$

B $(1, 2\sqrt{2})$

C $(\sqrt{2}, 2)$

D $(2, 2\sqrt{2})$

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Let S and S' be foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right-angled triangle with right angle at B and area of $\Delta S'BS = 8$ sq. units, then the length of a latus rectum of the ellipse (in units) is:

JEE MAIN JAN 2019

A. 2

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. 4

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Let S and S' be foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right-angled triangle with right angle at B and area of $\Delta S'BS = 8$ sq. units, then the length of a latus rectum of the ellipse (in units) is:

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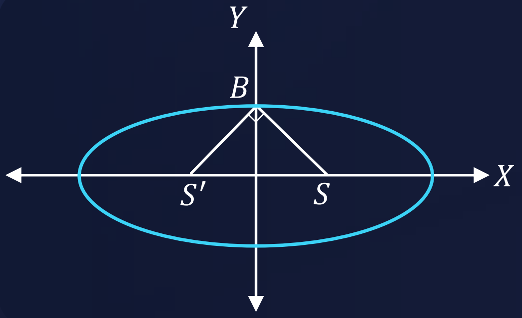
Solution:

Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$



Where $S \equiv (ae, 0), S' \equiv (-ae, 0)$ and $B \equiv (0, b)$

$\because$ $\Delta S'BS$ is a right-angled triangle with right angle at B

$\therefore (S'B)^2 + (SB)^2 = (SS')^2$

$\Rightarrow a^2e^2 + b^2 + a^2e^2 + b^2 = 4a^2e^2$

$\Rightarrow b^2 = a^2e^2$

$\Rightarrow e^2 = \frac{b^2}{a^2}$

$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{b^2}{a^2}$

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Let S and S' be foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right-angled triangle with right angle at B and area of $\Delta S'BS = 8$ sq. units, then the length of a latus rectum of the ellipse (in units) is:

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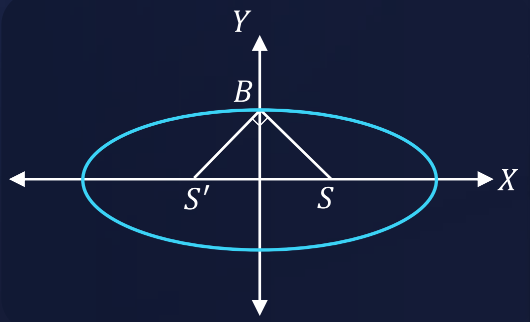
Solution:



$\Rightarrow a^2 = 2b^2 \Rightarrow a = \sqrt{2}b$

Given, area of $\Delta(S'BS) = 8$ sq. units

$\Rightarrow \frac{1}{2} \times 2ae \times b = 8$

$\Rightarrow \sqrt{2}b \times \frac{b}{\sqrt{2}b} \times b = 8$

$\Rightarrow b^2 = 8$

$\therefore b = 2\sqrt{2} \Rightarrow a = 4$

So, length of latus rectum

$= \frac{2b^2}{a} = \frac{2(8)}{4} = 4$ units

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Let S and S' be foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right-angled triangle with right angle at B and area of $\Delta S'BS = 8$ sq. units, then the length of a latus rectum of the ellipse (in units) is:

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A) 2

B) $2\sqrt{2}$

C) $4\sqrt{2}$

D) 4

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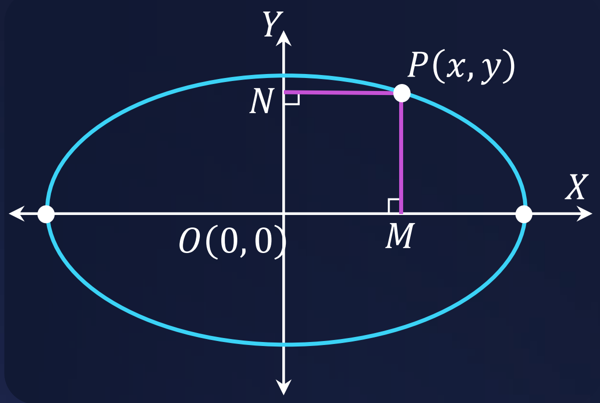
Key Takeaways

Equation of an Ellipse Referred to Two Perpendicular Lines as Axes:

Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $(a > b)$



Let $P(x, y)$ be any point on the ellipse

Then, $PM = y, PN = x$

$\therefore$ Equation of ellipse: $\frac{PN^2}{a^2} + \frac{PM^2}{b^2} = 1$

Suppose, if minor axis is $a_1x + b_1y + c_1 = 0 \longrightarrow L_1$

and major axis is $b_1x - a_1y + c_2 = 0 \longrightarrow L_2$

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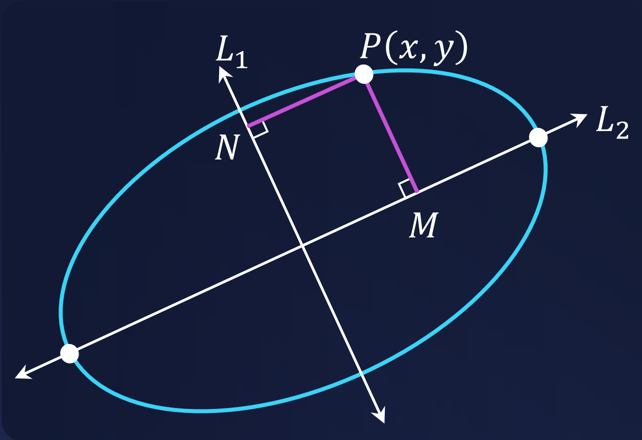
Key Takeaways

Equation of an Ellipse referred to Two Perpendicular Lines as Axes:

$$PN = \frac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}} \text{ and } PM = \frac{|b_1x-a_1y+c_2|}{\sqrt{b_1^2+a_1^2}}$$

$$\frac{(PN)^2}{a^2} + \frac{(PM)^2}{b^2} = 1; a > b$$



$$\text{Equation of ellipse: } \frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2} + \frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{b_1^2+a_1^2}}\right)^2}{b^2} = 1$$

Where $a > b$

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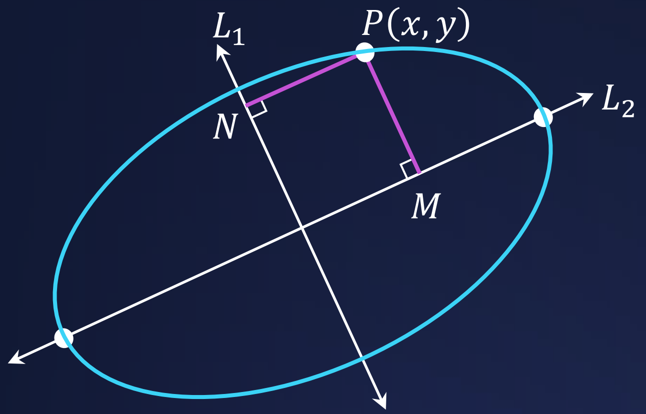
Key Takeaways

Note:

- Centre is the point of intersection of L_1 and L_2
- If $a > b$, then major axis lies along L_2 and minor axis along L_1
- If $a > b,\ b^2 = a^2(1 - e^2)$
- If $a < b,\ a^2 = b^2(1 - e^2)$



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Position of a Point W.R.T an Ellipse:

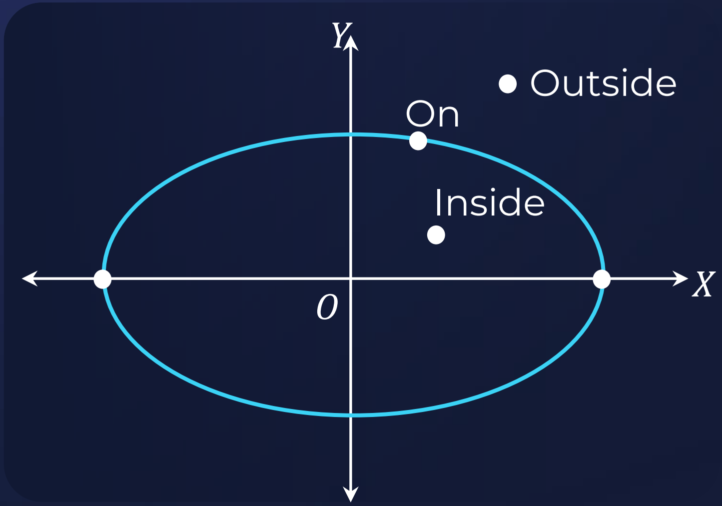
Let $P(x_1, y_1)$ be any point in the plane of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

i. If $S_1 > 0$ $\longrightarrow$ P lies outside the ellipse

ii. If $S_1 = 0$ $\longrightarrow$ P lies on the ellipse

iii. If $S_1 < 0$ $\longrightarrow$ P lies inside the ellipse



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If $x^2 + 9y^2 - 4x + 3 = 0, x, y \in \mathbb{R}$, then x and y respectively lie in the intervals

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A. $[1, 3]$ and $[1, 3]$

B. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

C. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$

D. $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

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If $x^2 + 9y^2 - 4x + 3 = 0, x, y \in \mathbb{R}$, then x and y respectively lie in the intervals

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C. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$

D. $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

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If $x^2 + 9y^2 - 4x + 3 = 0, x, y \in \mathbb{R}$, then x and y respectively lie in the intervals

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Solution:

Given: $x^2 + 9y^2 - 4x + 3 = 0$

$\Rightarrow (x^2 - 4x + 4) + 9y^2 - 1 = 0$

$\Rightarrow (x - 2)^2 + 9y^2 = 1$

$\Rightarrow \frac{(x-2)^2}{1} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

Since it is the equation of an ellipse,

Therefore, x and y will be on circumference of the ellipse.

$x - 2 \in [-1, 1]$

$\Rightarrow x \in [1, 3]$

And $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

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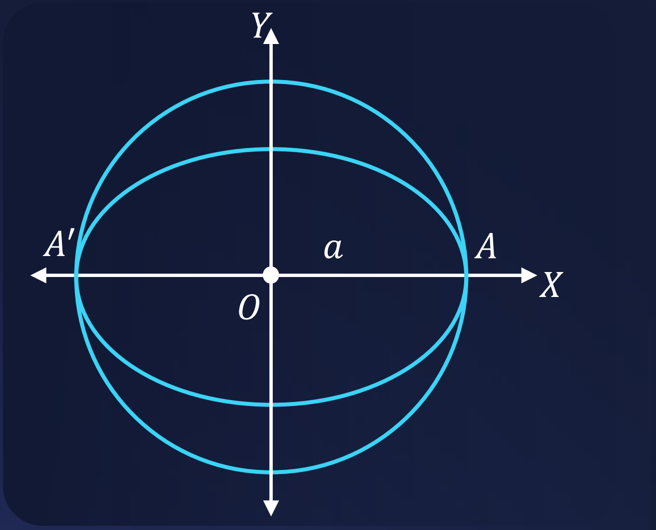
Key Takeaways

Auxiliary Circle:

The circle described on the major axis as diameter is called the Auxiliary Circle of the given ellipse.

(i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1;\ (a > b)$



Equation of auxiliary circle $x^2 + y^2 = a^2$

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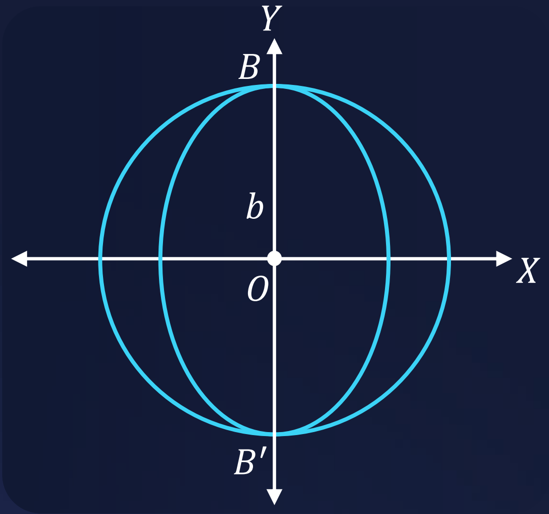
Key Takeaways

Auxiliary Circle:

$$(ii)\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1;\ (a<b)$$

Equation of auxiliary circle $x^2+y^2=b^2$



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Eccentric Angle and Parametric Equation:

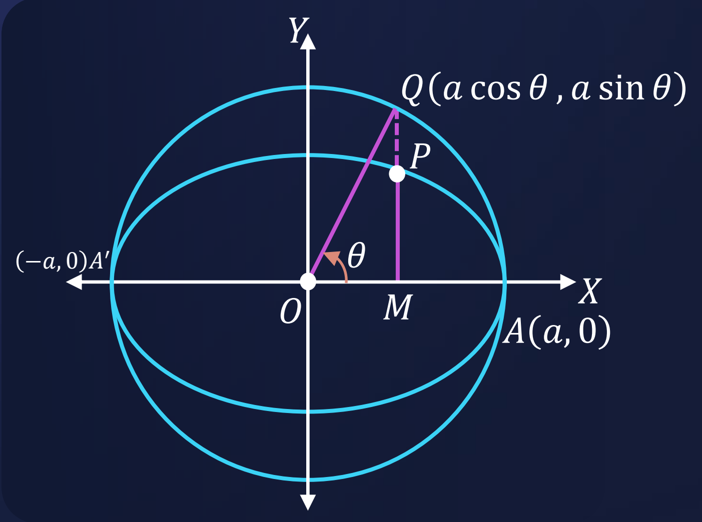
Consider the ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

Auxiliary circle: $x^2+y^2=a^2$

Let Q be a point on Auxiliary circle

$QM \perp$ Major-axis $\longrightarrow$ Meets the ellipse at P

P is called corresponding point of Q



The angle, $\theta \equiv \angle QOA$ is called Eccentric angle of the point P on the ellipse $(0 \le \theta < 2\pi)$

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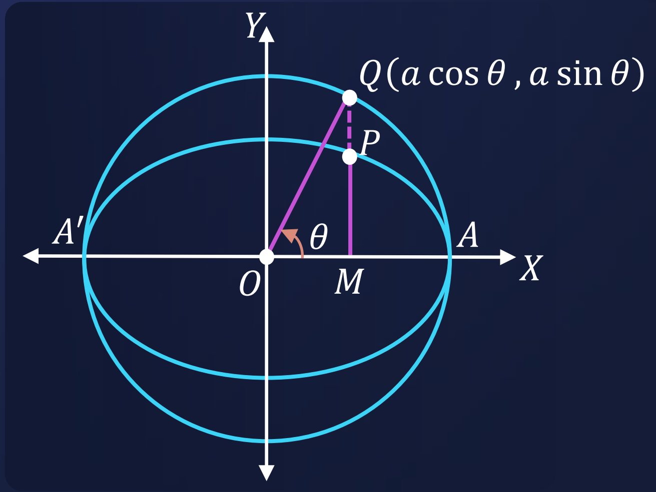
Eccentric Angle and Parametric Equation:

- Also, $x = a\cos\theta$, $y = b\sin\theta$ are the Parametric equation of the ellipse

  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$



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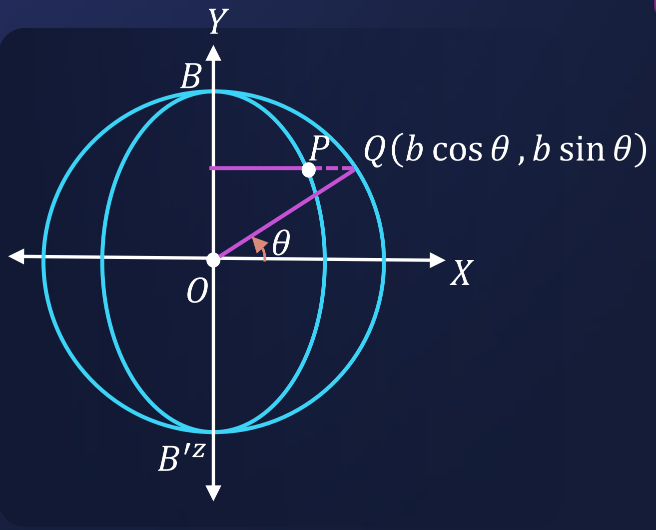
Eccentric Angle and Parametric Equation:



- For Ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a<b)$

  Equation of Auxiliary circle: $x^2+y^2=b^2$

  and $Q \equiv (b\cos\theta, b\sin\theta)$

  $P \equiv (a\cos\theta, b\sin\theta) \longrightarrow$ Parametric form

  $x=a\cos\theta, y=b\sin\theta \longrightarrow$ Parametric equation

Parametric equation:

$$\boxed{x=h+a\cos\theta, y=k+b\sin\theta}$$

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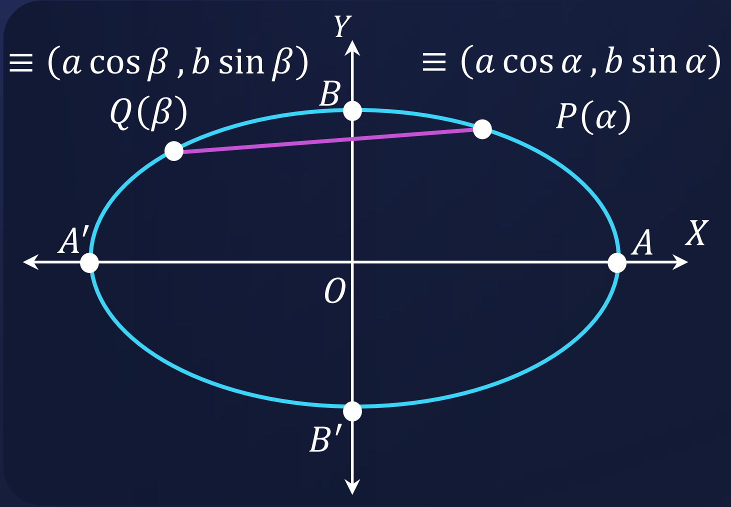
Chord Joining Two Points:

Equation of a chord joining the two points $P(\alpha)$ and $Q(\beta)$ on the ellipse $S = 0$

$$\boxed{\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)}$$



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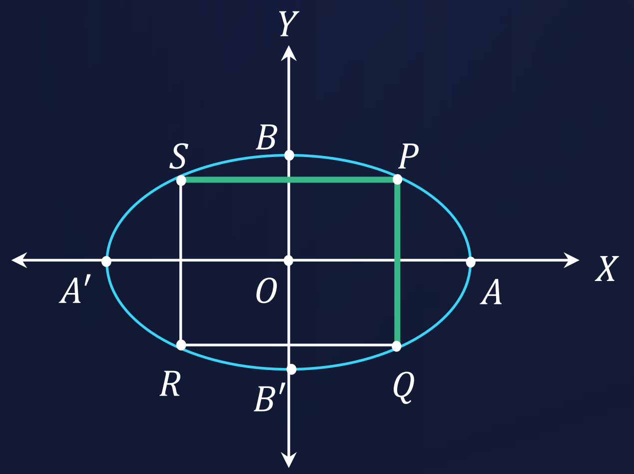
Find the maximum area of the rectangle that can be inscribed in the ellipse $S = 0$

Solution:

Let equation of ellipse: $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Rectangle $PQRS$ inscribed with its sides ∥ axes



Let $P \equiv (a\cos\theta\,, b\sin\theta)$

Then, $PQ = 2b\sin\theta$ and, $PS = 2a\cos\theta$

∴ Area of rectangle $= PQ \times PS$

$= 2b\ \sin\theta \times 2a\ \cos\theta \ = 2ab\sin 2\theta$

$= 2ab\sin 2\theta \longrightarrow$ Maximum

Maximum area of the rectangle $= 2ab$

<u>Note:</u> Area of the ellipse $S = 0 \longrightarrow \pi ab$

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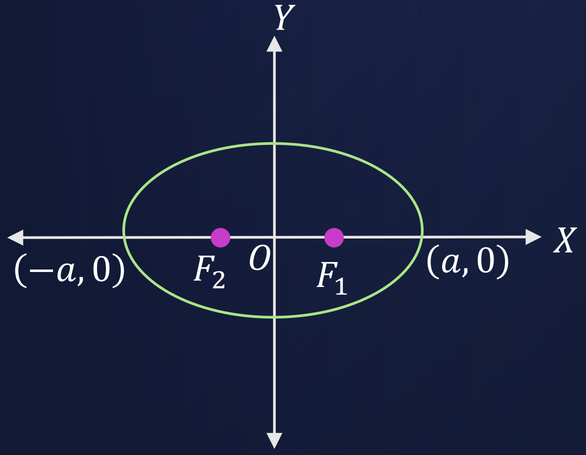
Position of point w.r.t ellipse:

$(i) E(P) > = < 0$

$$P \text{ lies } \begin{cases} \text{Outside } S_1 > 0 \\ \text{On } S_1 = 0 \\ \text{Inside } S_1 < 0 \end{cases}$$

$(ii). PF_1 + PF_2 > = < 2a$

$$P \text{ lies } \begin{cases} \text{Outside } PF_1 + PF_2 > 2a \\ \text{On } PF_1 + PF_2 = 2a \\ \text{Inside } PF_1 + PF_2 < 2a \end{cases}$$



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Line and ellipse

Line : $y = mx + c$

Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

- Solving equation of line with ellipse , we get
  $x^2(a^2m^2 + b^2) + 2a^2cmx + a^2(c^2 - b^2) = 0$
  $$D = a^2b^2(a^2m^2 + b^2 - c^2)$$

$D > 0$ $a^2m^2 + b^2 > c^2$ Chord	$D = 0$ $a^2m^2 + b^2 = c^2$ Tangent	$D < 0$ $a^2m^2 + b^2 < c^2$ Neither chord nor tangent

$\boxed{c = \pm\sqrt{a^2m^2 + b^2}}$ ➡ Condition of tangency

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Key Takeaways

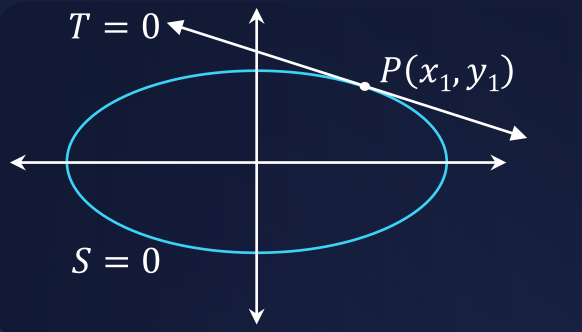
Point Form :

Equation of tangent to an ellipse $S : \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ at a point $P(x_1, y_1)$ is $T = 0$

$x^2 \rightarrow xx_1$

$y^2 \rightarrow yy_1$

$\therefore$ Equation $\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$



For $S : \frac{(x')^2}{a^2} + \frac{(y')}{b^2} - 1 \;\Rightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - 1 = 0$

$T : \frac{x' x_1}{a^2} + \frac{y' y_1}{b^2} - 1 = 0 \;\Rightarrow \frac{(x-h)x_1}{a^2} + \frac{(y-k)y_1}{b^2} - 1 = 0$

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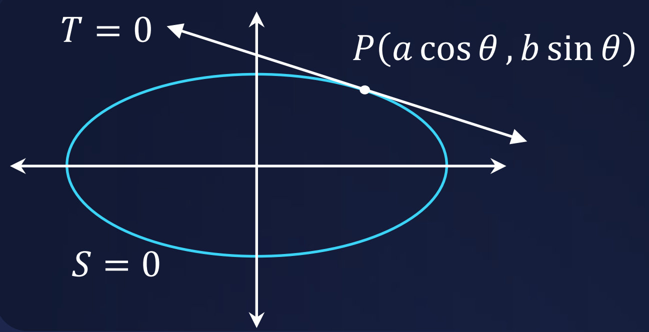
Key Takeaways

Parametric Form :

Equation of tangent to an ellipse $S : \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

at a point $P(x_1, y_1)$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$



Let $P(x_1, y_1) \equiv P(a\cos\theta, b\sin\theta)$

$$\Rightarrow \frac{x(a\cos\theta)}{a^2} + \frac{y(b\sin\theta)}{b^2} - 1 = 0$$

$$\Rightarrow \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta - 1 = 0$$

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Slope Form :

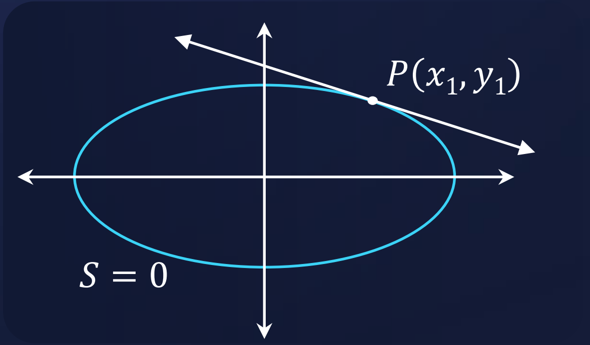
The line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

If $a^2m^2 + b^2 = c^2$

$\Rightarrow c = \pm\sqrt{a^2m^2 + b^2}$

So, equation of tangent to an ellipse $S = 0$ having slope $'m'$ is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$



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Let L be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$ then the value of b is equal to :

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A. 20

B. 14

C. 16

D. 11

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Let L be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$ then the value of b is equal to :

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Solution:

Equation of parabola is: $y^2 = 4x - 20$

Tangent at $P(6, 2)$ will be:

$$2y = 4\left(\frac{x+6}{2}\right) - 20$$

$$\Rightarrow 2y = 2x + 12 - 20$$

$$\Rightarrow y = x - 4 \ldots (1)$$

This is tangent to the ellipse, $\frac{x^2}{2} + \frac{y^2}{b} = 1$ also

$\therefore$ Applying condition of tangency, $c^2 = a^2m^2 + b^2$

$$(-4)^2 = (2)(1) + b$$

$$\Rightarrow b = 14$$

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Let L be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$ then the value of b is equal to :

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A. 20

B. 14

C. 16

D. 11

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If the tangents to the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (p, q) are perpendicular to each other, then the value p^2 is equal to

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A $\frac{128}{7}$

B $\frac{67}{7}$

C $\frac{4}{17}$

D $\frac{2}{17}$

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If the tangents to the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (p, q) are perpendicular to each other, then the value p^2 is equal to

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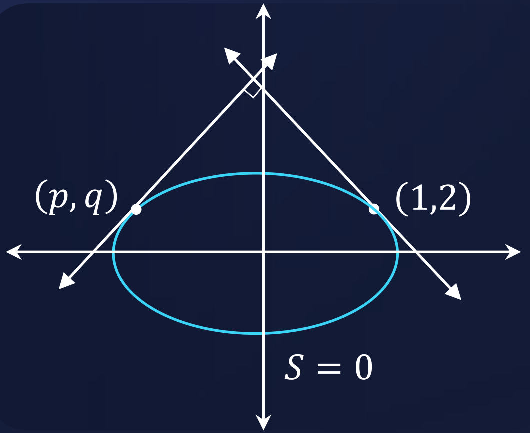
Solution:

Given : Equation of the ellipse $4x^2 + y^2 = 8$



Equation of tangent at (x_1, y_1) is $T = 0$

$\therefore$ Equation of the tangent at $(1,2)$ is
$4x\,(1) + y\,(2) = 8$

$\Rightarrow 2x + y = 4 \cdots (i)$

Equation of the tangent at (p, q) is $4x\,(p) + y\,(q) = 8$

$\Rightarrow 4px + qy = 8 \cdots (ii)$

Given, (i) is perpendicular to (ii)

$\Rightarrow$ Product of their slopes $= -1$

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If the tangents to the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (p, q) are perpendicular to each other, then the value p^2 is equal to

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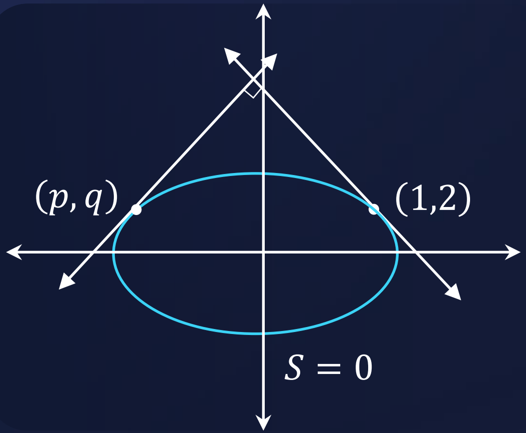
Solution:



$\Rightarrow 2x + y = 4 \cdots (i)$

$\Rightarrow 4px + qy = 8 \cdots (ii)$

$\Rightarrow$ Product of their slopes $= -1$

$\Rightarrow \left(-\frac{2}{1}\right) \times \left(-\frac{4p}{q}\right) = -1$

$\Rightarrow q = -8p \cdots (iii)$

Also, the point (p, q) lies on the ellipse.

So, $4p^2 + q^2 = 8$

From (iii) and (iv) by eliminating 'q' we get

$4p^2 + (-8p)^2 = 8$

$p^2 = \frac{8}{68} \Rightarrow p^2 = \frac{2}{17}$

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If the tangents to the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (p, q) are perpendicular to each other, then the value p^2 is equal to

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A $\frac{128}{7}$

B $\frac{67}{7}$

C $\frac{4}{17}$

D $\frac{2}{17}$

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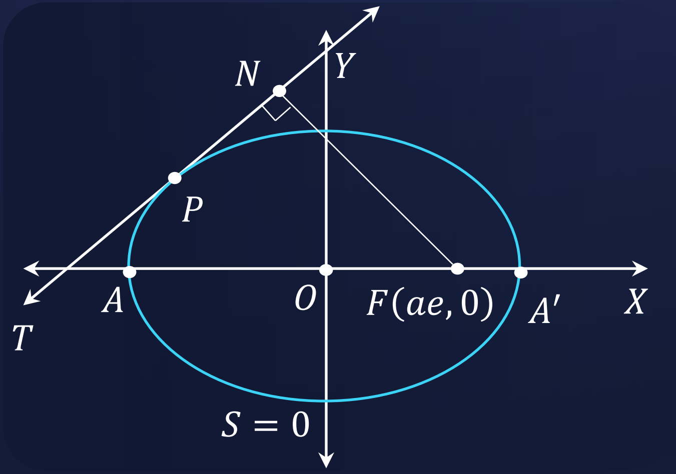
Note :

Locus of feet of perpendicular from foci upon any tangent is an auxiliary circle.



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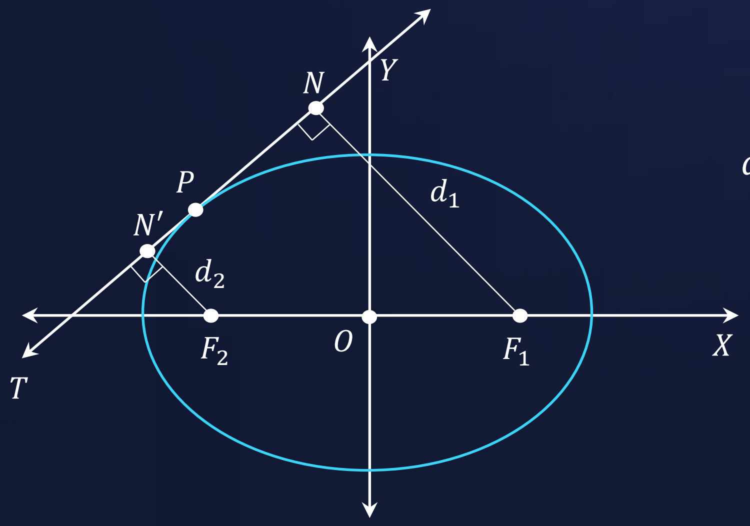
Note :

Product of the lengths of perpendiculars from foci upon any tangent of an ellipse is equal to the square of the semi minor axis.



$$d_1 . d_2 = b^2$$

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Session 06

Director Circle and Normal to Ellipse

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If the tangents to the parabola $y^2 = x$ at a point (α, β) where $\beta > 0$, is also tangent to the ellipse $x^2 + 2y^2 = 1$, then α is equal to :

a) $\sqrt{2} + 1$ b) $\sqrt{2} - 1$ c) $2\sqrt{2} + 1$ d) $2\sqrt{2} - 1$

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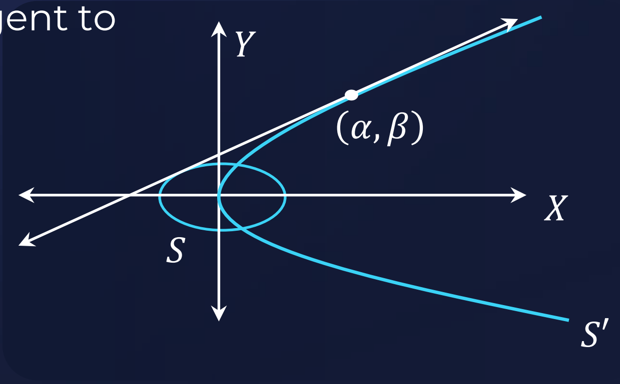
Solution:

Tangent at (α, β),$(\beta > 0)$, to $S' : y^2 = x$ is also tangent to

$S : x^2 + 2y^2 = 1 \Rightarrow \frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1.$



(α, β) lies on $y^2 = x \Rightarrow \beta^2 = \alpha \cdots (i)$

Equation of the tangent at (α, β) to the parabola $y^2 = x$ is $T = 0$.

$$\Rightarrow y(\beta) = \frac{1}{2}(x + \alpha)$$

$$y = \frac{x}{2\beta} + \frac{\alpha}{2\beta} \cdots (ii)$$

(ii) is also a tangent to $S : \frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$

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If the tangents to the parabola $y^2 = x$ at a point (α, β) where $\beta > 0$, is also tangent to the ellipse $x^2 + 2y^2 = 1$, then α is equal to :

a) $\sqrt{2}+1$ b) $\sqrt{2}-1$ c) $2\sqrt{2}+1$ d) $2\sqrt{2}-1$

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Solution:

Condition of tangency : $c^2 = a^2m^2 + b^2$

Here $c = \frac{\alpha}{2\beta}, m = \frac{1}{2\beta}, a^2 = 1, b^2 = \frac{1}{2}$

$\therefore \frac{\alpha^2}{4\beta^2} = 1.\left(\frac{1}{4\beta^2}\right) + \frac{1}{2}$

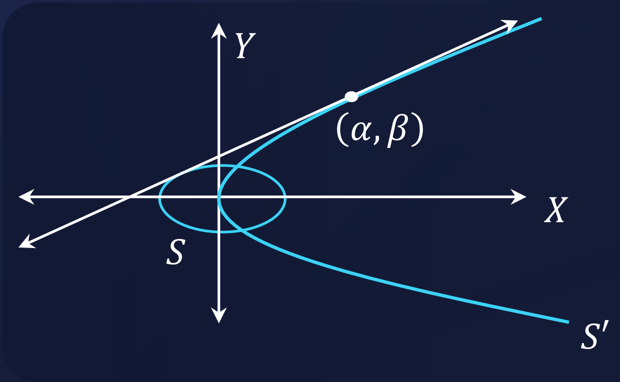
$\Rightarrow 1 + 2\beta^2 = \alpha^2$

$\Rightarrow 1 + 2\alpha = \alpha^2 \quad (\because \beta^2 = \alpha)$

$\Rightarrow \alpha^2 - 2\alpha - 1 = 0$

$\Rightarrow \alpha = 1 \pm \sqrt{2}$ ($\because \alpha = \beta^2$, it cannot be negative)

$\Rightarrow \alpha = 1 + \sqrt{2}$



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If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals:

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Solution:

Since (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ ... (1)

Tangent to the parabola, $y = mx + \frac{1}{m}$ passes through (α, β).

So, $\alpha m^2 - \beta m + 1 = 0$ has roots m_1 and $4m_1$.

$\therefore m_1 + 4m_1 = \frac{\beta}{\alpha}$ and $m_1 \cdot 4m_1 = \frac{1}{\alpha}$

Gives that $4\beta^2 = 25\alpha$ ... (2)

From (1) and (2), we get:

$25(\alpha^2 + \alpha) = 1$

$\Rightarrow (\alpha^2 + \alpha) = \frac{1}{25}$

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If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals:

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Solution:

Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$

$$= 25(2\alpha + 1)^2 + (100\alpha + 50)^2 \quad (\because 4\beta^2 = 25\alpha)$$

$$= 25(2\alpha + 1)^2 + 2500(2\alpha + 1)^2$$

$$= 25 \times 101(4\alpha^2 + 4\alpha + 1)$$

$$= 25 \times 101\left(\frac{4}{25} + 1\right) = 2929$$

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If m is the slope of a common tangent to the curves $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2+y^2=12$, then $12m^2$ is equal to:

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A 6

B 9

C 10

D 12

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If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

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Solution:

$$C_1: \frac{x^2}{16} + \frac{y^2}{9} = 1 \quad \text{and} \quad C_2: x^2 + y^2 = 12$$

Let $y = mx \pm \sqrt{16m^2 + 9}$ be any tangent to C_1 and if this is also tangent to C_2 then

$$\left|\frac{\sqrt{16m^2+9}}{\sqrt{m^2+1}}\right| = \sqrt{12}$$

$$\Rightarrow 16m^2 + 9 = 12m^2 + 12$$

$$\Rightarrow 4m^2 = 3$$

$$\Rightarrow 12m^2 = 9$$

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If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

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A. 6

B. 9

C. 10

D. 12

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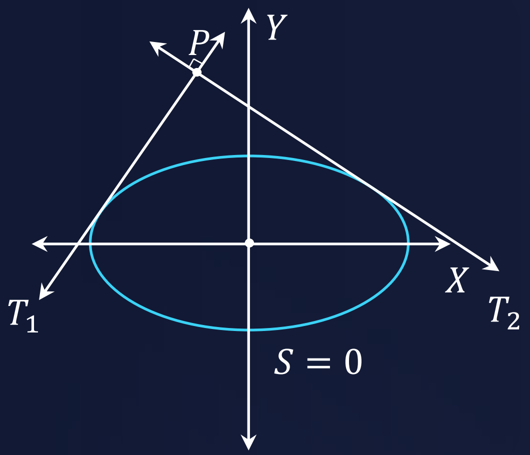
Key Takeaways

Director Circle :

The locus of the points of intersection of perpendicular tangents to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a concentric circle called director circle, given by $x^2 + y^2 = a^2 + b^2$.



Proof :

Let $P \equiv (h, k)$ be a point outside the ellipse $S = 0$

Equation of tangents to the ellipse is $y = mx \pm \sqrt{a^2m^2 + b^2}$

Tangents pass through $P \equiv (h, k)$

$\Rightarrow k = mh \pm \sqrt{a^2m^2 + b^2}$ $\Rightarrow k - mh = \pm\sqrt{a^2m^2 + b^2}$

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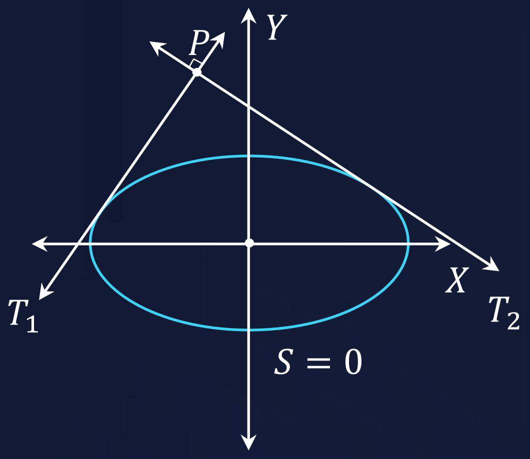
Key Takeaways



$$\Rightarrow k - mh = \pm\sqrt{a^2m^2 + b^2}$$

Squaring both sides,

$$\Rightarrow k^2 + m^2h^2 - 2mhk = a^2m^2 + b^2$$

$$\Rightarrow m^2(h^2 - a^2) - 2mhk + (k^2 - b^2) = 0 \cdots (i)$$

(i) is a quadratic equation in m whose roots are m_1 and m_2 (slopes of tangents T_1 and T_2)

$$\Rightarrow m^2(h^2 - a^2) - 2mhk + (k^2 - b^2) = 0 \cdots (i)$$

$T_1 \perp^r T_2 \Rightarrow m_1.m_2 = -1 \Rightarrow$ Product of roots of $(i) = -1$

$$\Rightarrow \frac{k^2-b^2}{h^2-a^2} = -1 \quad \Rightarrow h^2 + k^2 = a^2 + b^2$$

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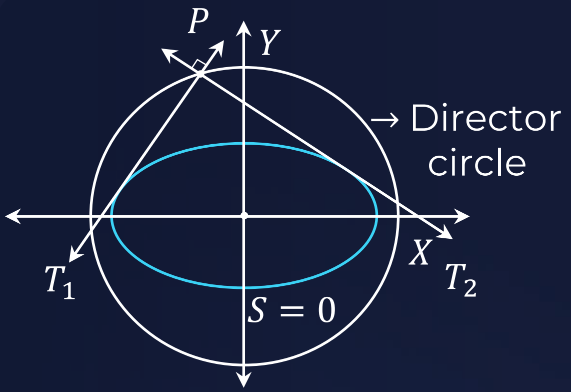
Key Takeaways

$$\Rightarrow h^2 + k^2 = a^2 + b^2$$

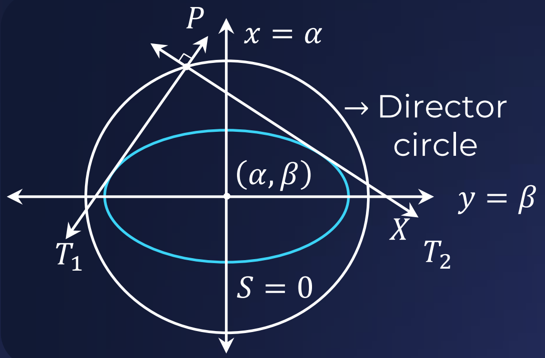
Replace $h \to x$ and $k \to y$

$$\Rightarrow x^2 + y^2 = a^2 + b^2 \to \text{Director circle}$$



Director circle of $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$

is $(x-\alpha)^2 + (y-\beta)^2 = a^2 + b^2$



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Find the angle between the tangents drawn from any point on the circle $x^2 + y^2 = 41$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

A $\frac{\pi}{2}$

B $\frac{\pi}{4}$

C π

D $\frac{3\pi}{2}$

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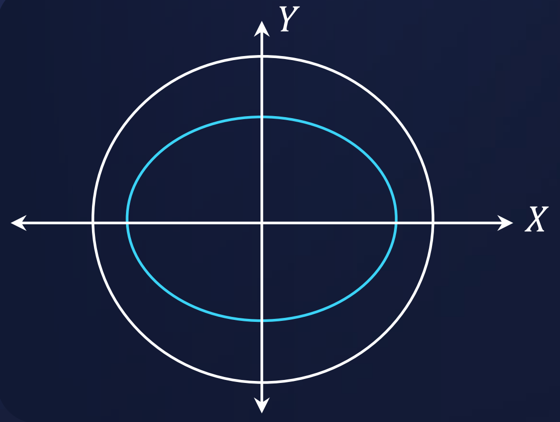
Find the angle between the tangents drawn from any point on the circle $x^2 + y^2 = 41$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Solution:



Given : $\frac{x^2}{25} + \frac{y^2}{16} = 1$ → Ellipse

$x^2 + y^2 = 41$ → Circle

$x^2 + y^2 = 41 = 25 + 16$

$\Rightarrow x^2 + y^2 = a^2 + b^2$ → Director circle

Director circle is the locus of points of intersection of perpendicular tangents to the ellipse.

$\therefore$ Angle between the tangents is $\frac{\pi}{2}$

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Find the angle between the tangents drawn from any point on the circle $x^2 + y^2 = 41$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

A $\frac{\pi}{2}$

B $\frac{\pi}{4}$

C π

D $\frac{3\pi}{2}$

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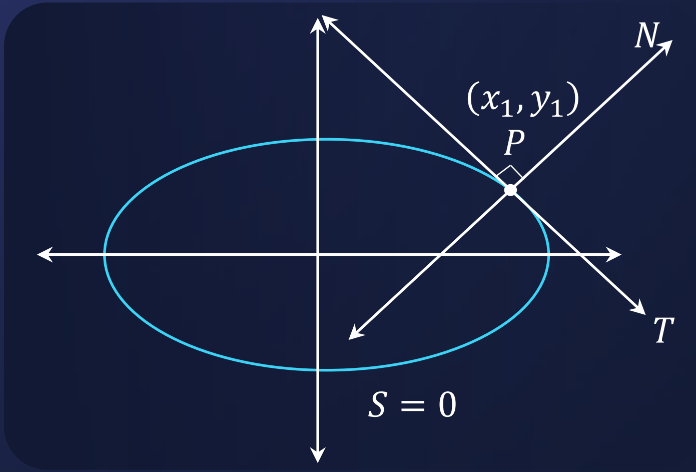
Equation of Normal : Point Form

The equation of normal at $P \equiv (x_1, y_1)$ on the ellipse $S = 0$ is given by

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

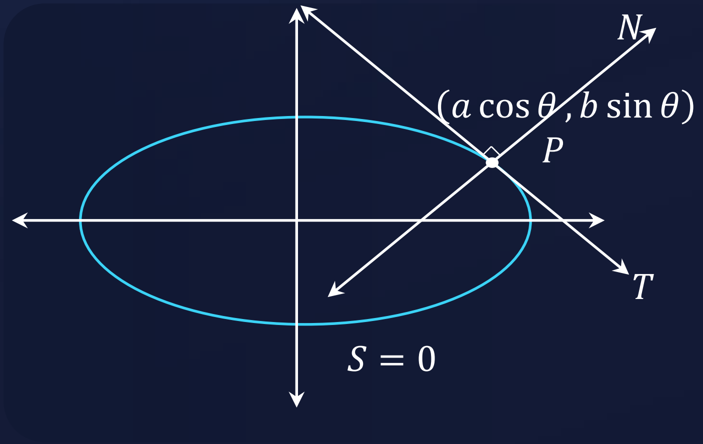


Equation of Normal : Parametric Form

The equation of normal at $P(\theta)$ on the ellipse $S = 0$ is given by

$$\frac{a\,x}{\cos\theta} - \frac{b\,y}{\sin\theta} = a^2 - b^2 = a^2 e^2$$

where $P(\theta) \equiv (a\cos\theta, b\sin\theta)$



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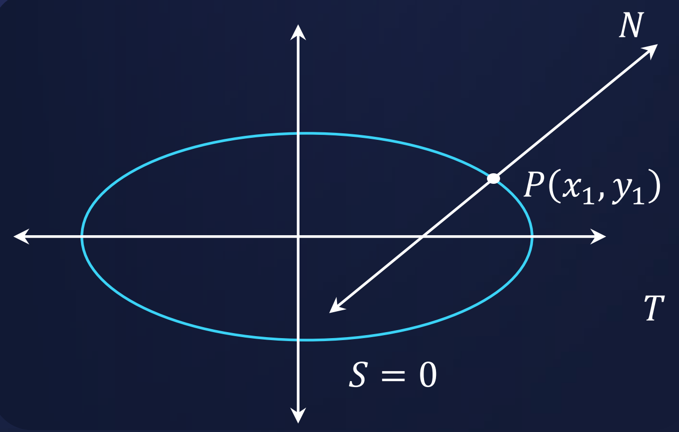
Equation of Normal : Slope Form

The equation of normal to the ellipse $S = 0$ whose slope is m is given by

$$y = mx \mp \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m}}$$

where $P \equiv \left(\pm\frac{a^2}{\sqrt{a^2+b^2m^2}}, \pm\frac{mb^2}{\sqrt{a^2+b^2m^2}}\right)$



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Let $x = 4$ be a directrix to an ellipse whose centre is at origin and its eccentricity is 0.5. If $P(1, \beta)$, $\beta > 0$, is a point on this ellipse, then the equation of the normal to it at P is

a) $8x - 2y = 5$ b) $4x - 2y = 1$ c) $7x - 4y = 1$ d) $4x - 3y = 2$

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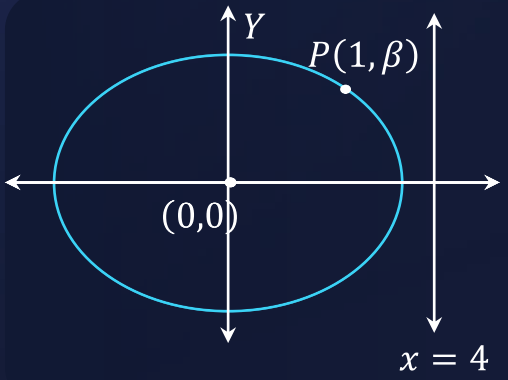
Solution:

Ellipse : Center $\equiv (0,0)$, Directrix $x = 4$ and $e = 0.5$



Directrix : $x = \frac{a}{e} = 4 \Rightarrow \frac{a}{0.5} = 4 \Rightarrow a = 2$

$$b^2 = a^2(1 - e^2) = 4\left(1 - \frac{1}{4}\right) = 3$$

$\therefore$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$P(1, \beta)$ lies on ellipse

$$\Rightarrow \frac{1}{4} + \frac{\beta^2}{3} = 1 \Rightarrow \beta = \frac{3}{2} \; (\because \beta > 0)$$

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Let $x = 4$ be a directrix to an ellipse whose centre is at origin and its eccentricity is 0.5. If $P(1, \beta)$, $\beta > 0$, is a point on this ellipse, then the equation of the normal to it at P is

a) $8x - 2y = 5$ b) $4x - 2y = 1$ c) $7x - 4y = 1$ d) $4x - 3y = 2$

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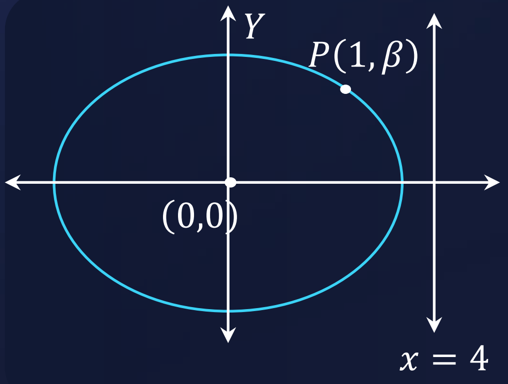
Solution:

$\therefore$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$



$$\Rightarrow \beta = \frac{3}{2} \; (\because \beta > 0)$$

$$\therefore P \equiv (1, \beta) \equiv \left(1, \frac{3}{2}\right)$$

Equation of normal : $\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$

$$\Rightarrow \frac{4x}{1} - \frac{3y}{\left(\frac{3}{2}\right)} = 4 - 3$$

$$\Rightarrow 4x - 2y = 1$$

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?

If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

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A $e^4 + 2e^2 - 1 = 0$

B $e^2 + 2e - 1 = 0$

C $e^4 + e^2 - 1 = 0$

D $e^4 + e - 1 = 0$

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If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

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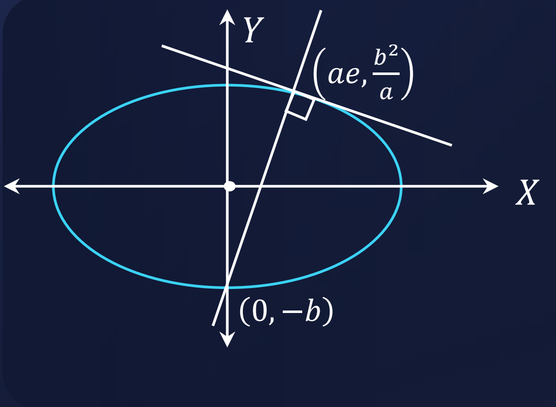
Solution:

Let the ellipse be: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Equation of normal at $\left(ae, \frac{b^2}{a}\right)$



$$\frac{a^2x}{ae} - \frac{b^2y}{\frac{b^2}{a}} = a^2e^2$$

$$\Rightarrow \frac{ax}{e} - ay = a^2e^2$$

$$\Rightarrow \frac{x}{e} - y = ae^2$$

Since, it passes through $(0, -b)$

$$\therefore b = ae^2$$

$$\Rightarrow b^2 = a^2e^4$$

$$\Rightarrow a^2(1 - e^2) = a^2e^4$$

$$\Rightarrow e^4 + e^2 - 1 = 0$$

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If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

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A $e^4 + 2e^2 - 1 = 0$

B $e^2 + 2e - 1 = 0$

C $e^4 + e^2 - 1 = 0$

D $e^4 + e - 1 = 0$

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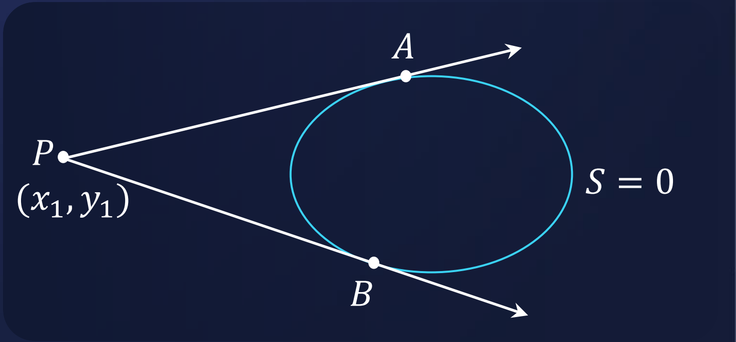
Pair of Tangents :

The combined equation of the pair of tangents from an external point $P(x_1, y_1)$ to the ellipse $S = 0$ is $T^2 = S \cdot S_1$

Where $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

$$T: \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$$

$$S_1: \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$$



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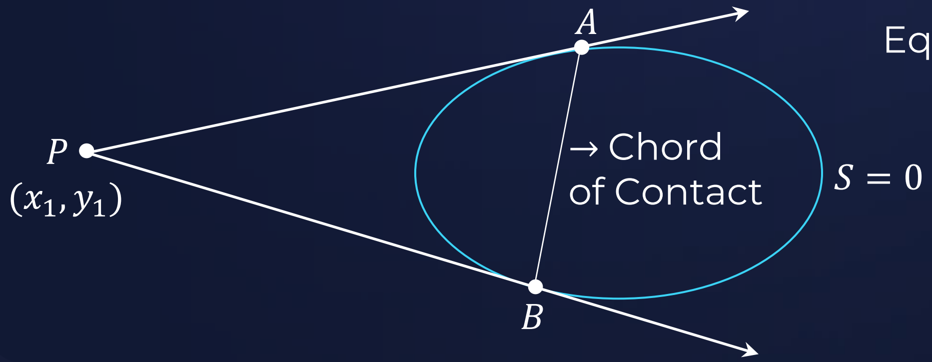
Chord of Contact :

If the pair of tangents drawn from an external point $P(x_1, y_1)$ to the ellipse $S=0$ touch it at points A and B, then AB is called chord of contact.



Equaton of AB is $T=0$

If equation of AB is $lx+my+n=0$, then

$$P \equiv \left(\frac{-a^2 l}{n}, \frac{-b^2 m}{n}\right)$$

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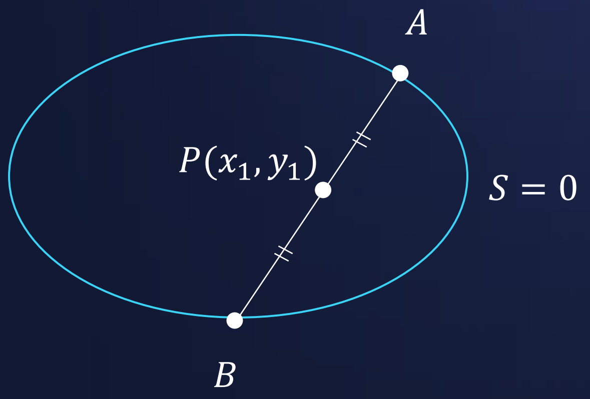
Chord with given midpoint :

The equation of a chord of the ellipse $S = 0$ whose midpoint is $P(x_1, y_1)$ is $T = S_1$.



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The chords of contact of all the points on the line $x - y - 5 = 0$ with respect to the ellipse $x^2 + 4y^2 = 4$ always passes through a fixed point ______.

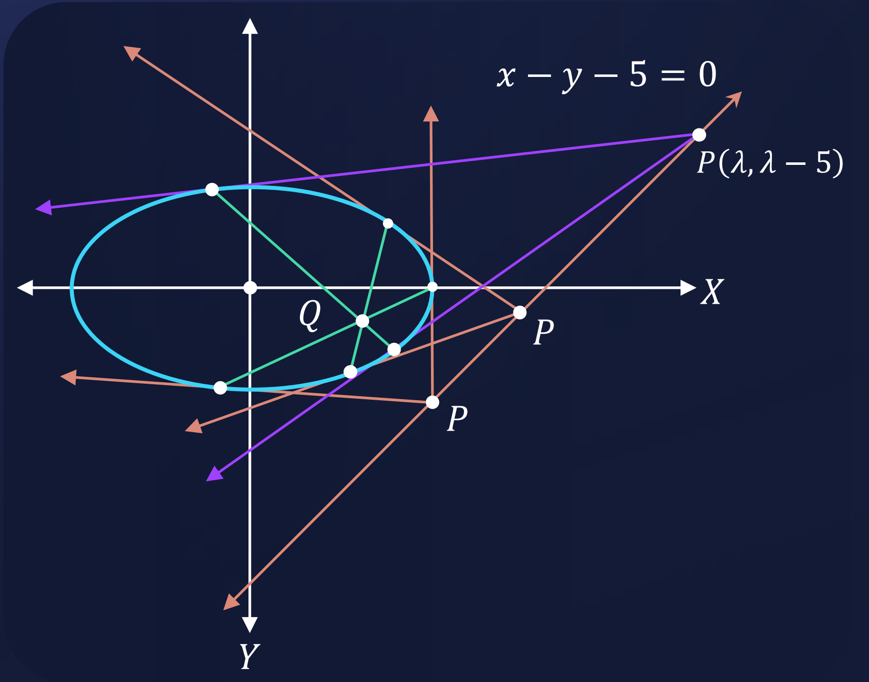
Solution:

Given: Ellipse: $x^2 + 4y^2 = 4$

Line: $x - y - 5 = 0$

$\Rightarrow S \equiv \frac{x^2}{4} + \frac{y^2}{1} - 1 = 0$

Let $P(\lambda, \lambda - 5)$ be any point on $x - y - 5 = 0,\ \lambda \in R$



Equation of chord of contact of P is $T = 0$

$\Rightarrow \frac{x(\lambda)}{4} + \frac{y(\lambda-5)}{1} = 1$

$\Rightarrow \lambda x + 4\lambda y - 20y = 4$

$\Rightarrow (-20y - 4) + \lambda(x + 4y) = 0$

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The chords of contact of all the points on the line $x - y - 5 = 0$ with respect to the ellipse $x^2 + 4y^2 = 4$ always passes through a fixed point ______.

Solution:

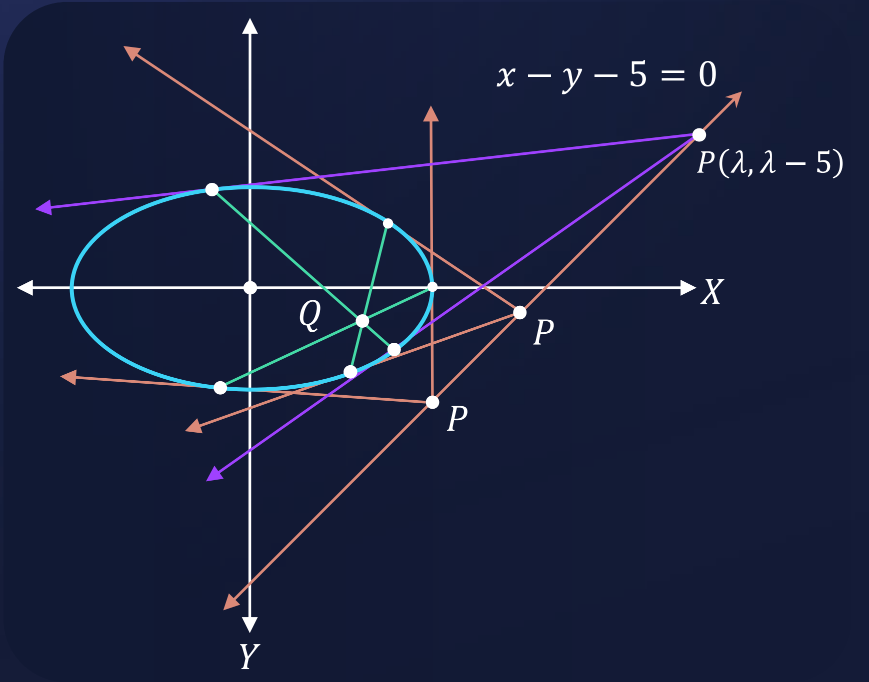
$$\Rightarrow \underbrace{(-20y - 4)}_{L_1} + \lambda\underbrace{(x + 4y)}_{L_2} = 0$$

$$L_1 + \lambda L_2 = 0$$

Family of concurrent lines, passing through the point of intersection of the two lines.

$$\left.\begin{aligned} -20y - 4 &= 0 \\ x + 4y &= 0 \end{aligned}\right\} \text{Point of intersection}$$

$$\boxed{\text{Point of intersection} \equiv \left(\frac{4}{5}, \frac{-1}{5}\right)}$$



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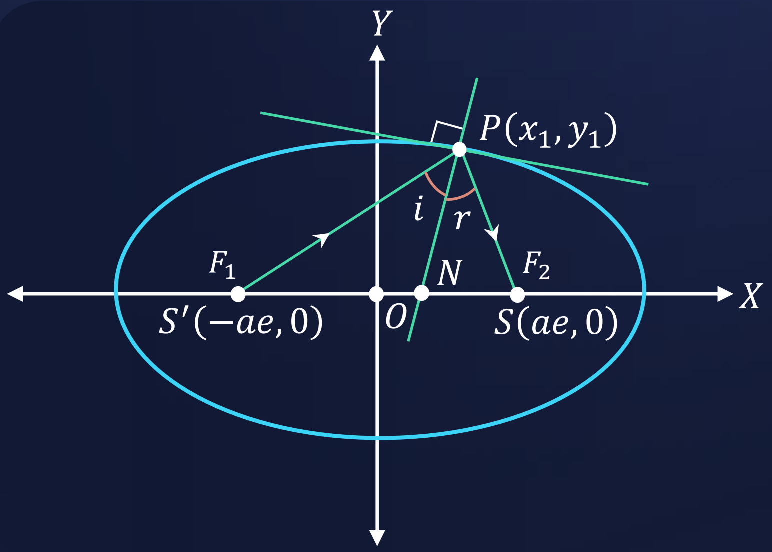
Geometrical Properties Of Ellipse:

i. The incident ray from focus S' $/F_1$ after reflection by ellipse at point P passes through other focus S/F_2.



Reflection Property

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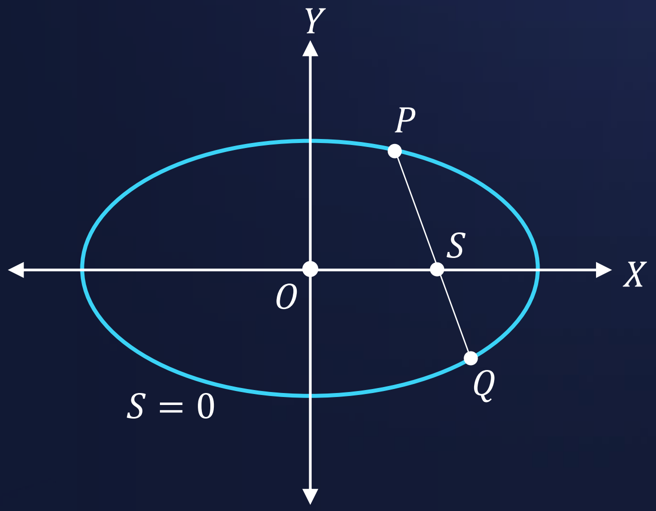
Geometrical Properties Of Ellipse:

ii. If PSQ is a focal chord of the ellipse $S = 0$, then $\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$ or Semi latus rectum is the H.M. of SP and SQ.



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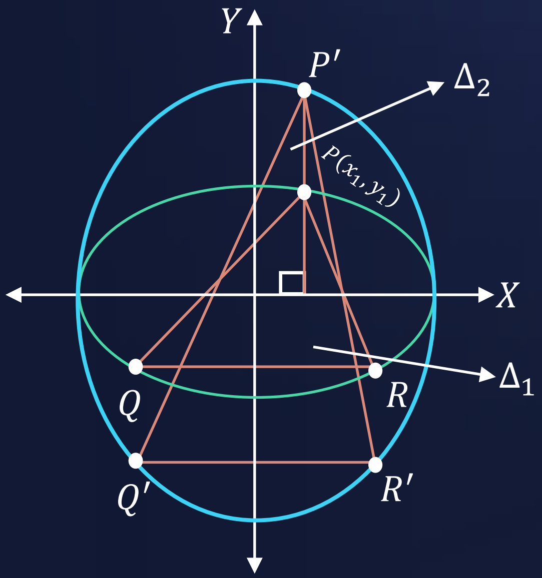
Geometrical Properties Of Ellipse:

iii. The ratio of area of any triangle inscribed in an ellipse to the area of triangle formed by corresponding points on the auxiliary circle is equal to the ratio of semi-minor axis to semi major axis.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; a > b$$

$$\frac{Ar.\Delta_1}{Ar.\Delta_2} = \frac{b}{a}$$

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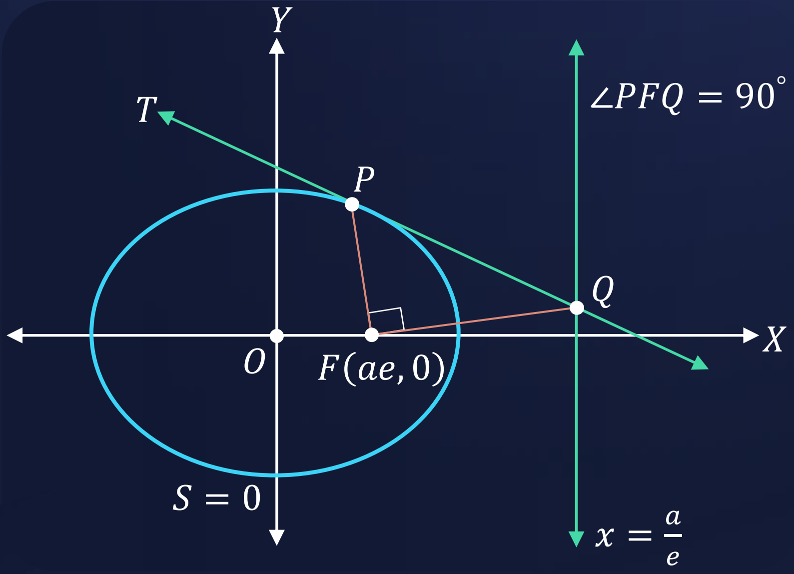
Geometrical Properties Of Ellipse:

iv. The portion of the tangent to an ellipse between the point of contact and the directrix subtends a right angle at the corresponding focus.



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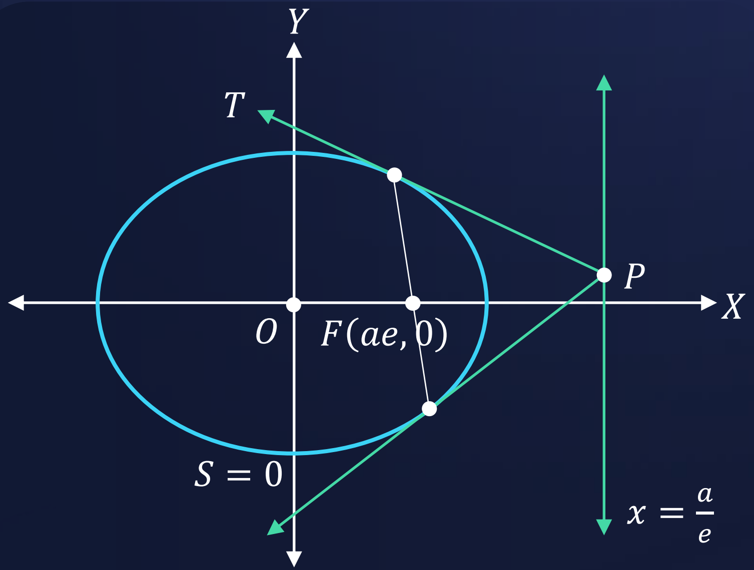
Geometrical Properties Of Ellipse:

v. Chord of contact of any point on the directrix passes through the corresponding focus.



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Session 07

Introduction to Hyperbola

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If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to:

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A $\frac{5\sqrt{5}}{2}$ units

B $\frac{\sqrt{157}}{2}$ units

C $\frac{\sqrt{61}}{2}$ units

D $\frac{\sqrt{221}}{2}$ units

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If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to:

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Solution:

Equation of ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Let point which is normal to the ellipse be $P\left(2\cos\theta, \sqrt{3}\sin\theta\right)$

$\therefore$ Equation of normal: $2x\,sec\,\theta - \sqrt{3}y\,cosec\,\theta = 1$

Equation of normal parallel to the line, $2x + y = 4$

Therefore $\frac{2\,sec\,\theta}{\sqrt{3}\,cosec\,\theta} = -2$

$\Rightarrow \tan\theta = -\sqrt{3} \ldots (1)$

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If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4,4)$ then PQ is equal to:

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Solution:

Similarly, equation of tangent at point P:

$$\frac{x \cdot 2\cos\theta}{4} + \frac{y \cdot \sqrt{3}\sin\theta}{3} = 1$$

$\because$ tangent passes through point $Q(4,4)$

$\therefore 2\sqrt{3}\cos\theta + 4\sin\theta = \sqrt{3} \ldots (2)$

Solving equation (1) and (2), we get

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$\therefore P\left(-1, \frac{3}{2}\right)$ & $Q(4,4)$

Hence $PQ = \frac{5\sqrt{5}}{2}$ units

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If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to:

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A. $\frac{5\sqrt{5}}{2}$ units

B. $\frac{\sqrt{157}}{2}$ units

C. $\frac{\sqrt{61}}{2}$ units

D. $\frac{\sqrt{221}}{2}$ units

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

i. Then the product of the lengths of the perpendicular segments from the foci on tangent at point P is ____.

A. 20

B. 45

C. 40

D. Can not be Determined

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

ii. Then if the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e')____.

A. $\frac{3}{\sqrt{10}}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{3}{\sqrt{5}}$

D. $\frac{\sqrt{10}}{3}$

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

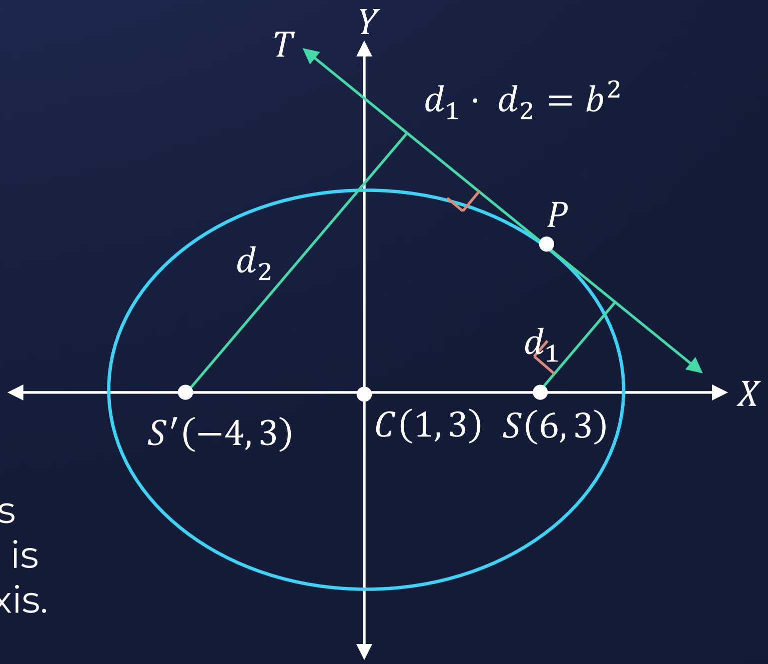
i. Then the product of the lengths of the perpendicular segments from the foci on tangent at point P is _____.

Given: Centre $C \equiv (1,3)$

Focus $S \equiv (6,3), S' \equiv (-4,3)$

{Mid-point of foci is centre}

Point on Ellipse, $P \equiv (4,7)$



Product of the lengths of perpendiculars from foci upon any tangent of an ellipse is equal to the square of the semi minor axis.

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

i. Then the product of the lengths of the perpendicular segments from the foci on tangent at point P is _____.

We know, $PS + PS' = 2a$

$$b^2 = a^2(1 - e^2)$$

$$SS' = 2ae$$

$$PS = \sqrt{(4+4)^2 + (7-3)^2} = 4\sqrt{5}$$

$$PS' = \sqrt{(6-4)^2 + (7-3)^2} = 2\sqrt{5}$$

$$\Rightarrow PS + PS' = 6\sqrt{5} = 2a$$

$$\Rightarrow a = 3\sqrt{5}$$

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COMPREHENSION

An ellipse E has its centre $C(1, 3)$ focus at $S(6, 3)$ and passing through the point $P(4, 7)$.

i. Then the product of the lengths of the perpendicular segments from the foci on tangent at point P is ____.

Also, $SS' = 10 = 2ae$

$\Rightarrow ae = 5$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 45\left(1 - \frac{5}{9}\right)$

$\boxed{\Rightarrow b^2 = 20}$

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COMPREHENSION

An ellipse E has its centre $C(1, 3)$ focus at $S(6, 3)$ and passing through the point $P(4, 7)$.

i. Then the product of the lengths of the perpendicular segments from the foci on tangent at point P is _____.

A. 20

B. 45

C. 40

D. Can not be Determined

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

ii. Then if the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e')____.

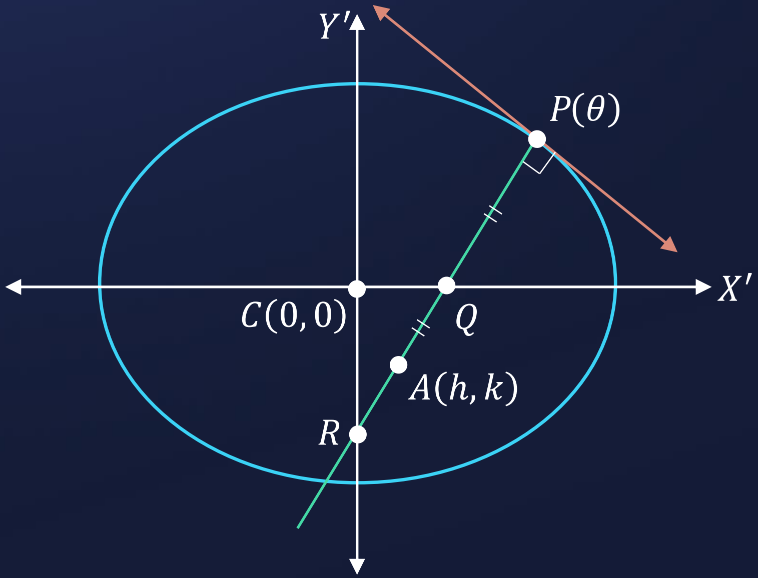
Given: Centre $C \equiv (1,3)$

Focus $S \equiv (6,3), S' \equiv (-4,3)$

Equation of ellipse: $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$

$\Rightarrow \frac{(x-1)^2}{45} + \frac{(y-3)^2}{20} = 1$

or $\frac{x'^2}{45} + \frac{y'^2}{20} = 1$



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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

ii. Then if the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e')____.

Equation of normal at P : $\frac{ax'}{\cos\theta} - \frac{by'}{\sin\theta} = a^2 - b^2$

$$\Rightarrow \frac{x'}{\frac{(a^2-b^2)\cos\theta}{a}} - \frac{y'}{\frac{(a^2-b^2)\sin\theta}{b}} = 1$$

Let mid-point of $QR \equiv (h,k)$

$$h = \frac{(a^2-b^2)\cos\theta}{2a}, k = \frac{(a^2-b^2)\sin\theta}{2b}$$

$$\Rightarrow \cos^2\theta + \sin^2\theta = \frac{4a^2h^2}{(a^2-b^2)^2} + \frac{4b^2k^2}{(a^2-b^2)^2} = 1$$

Replacing (h,k) with (x',y')

$$\Rightarrow \frac{x'^2}{\frac{(a^2-b^2)^2}{4a^2}} + \frac{y'^2}{\frac{(a^2-b^2)^2}{4b^2}} = 1$$

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

ii. Then if the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e')____.

Comparing with: $\frac{x'^2}{A^2} + \frac{y'^2}{B^2} = 1, (B > A)$

$\Rightarrow A^2 = B^2(1 - e'^2)$

$\Rightarrow \frac{(a^2-b^2)^2}{16a^2} = \frac{(a^2-b^2)^2}{16b^2}(1 - e'^2)$

$\Rightarrow b^2 = a^2(1 - e'^2)$

We will get same eccentricity as the eccentricity of given ellipse $\therefore e = \frac{\sqrt{5}}{3}$.

We have calculated : $a^2 = 45, b^2 = 20$

$\Rightarrow a^2 > b^2$

$\Rightarrow \frac{1}{a^2} < \frac{1}{b^2}$

$\Rightarrow \frac{1}{4a^2} < \frac{1}{4b^2}$

$\Rightarrow \frac{(a^2-b^2)^2}{4a^2} < \frac{(a^2-b^2)^2}{4b^2}$

$\Rightarrow B > A$

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COMPREHENSION

An ellipse E has its centre $C(1,3)$ focus at $S(6,3)$ and passing through the point $P(4,7)$.

ii. Then if the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e')____.

A $\frac{3}{\sqrt{10}}$

B $\frac{\sqrt{5}}{3}$

C $\frac{3}{\sqrt{5}}$

D $\frac{\sqrt{10}}{3}$

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Key Takeaways

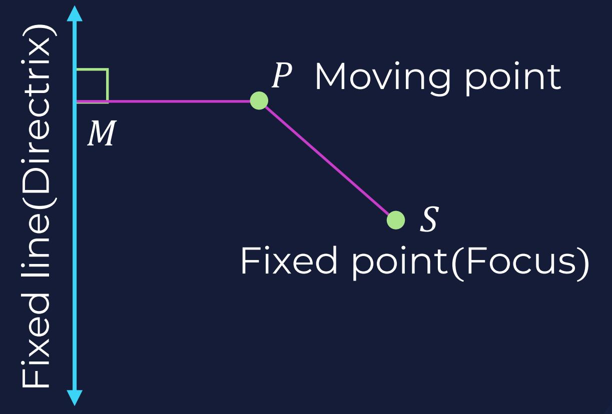
Analytical Interpretation

- A conic is the locus of a moving point such that ratio of its distance from a fixed point (focus) to a fixed straight line (directrix) is always a constant.

$$\frac{Distance\ from\ a\ fixed\ point}{Distance\ from\ a\ fixed\ line} = CONSTANT$$

Conics → The collection of all points P such that

$$\frac{PS}{PM} = \text{Constant} = \text{Eccentricity } (e) \geq 0$$

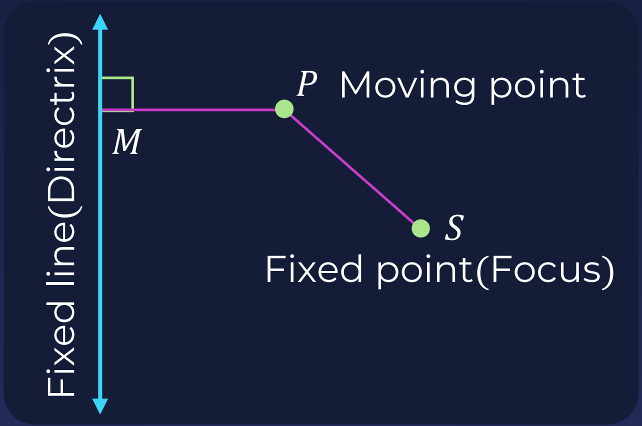


Key Takeaways

Hyperbola:

Hyperbola is the locus of a moving point such that the ratio of its distance from a fixed point (focus) and a fixed line (directrix) is a constant which is always greater than 1 $(e > 1)$.

Let the fixed point be F (focus) and the fixed line (directrix) be $L: lx + my + n = 0$.



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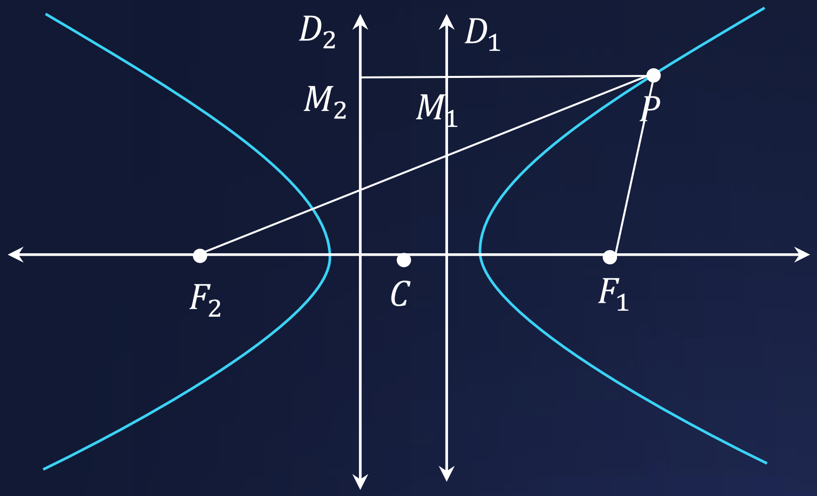
Key Takeaways

Hyperbola:



Hence for hyperbola, there are two foci (F_1 and F_2) and two directrices (D_1 and D_2) and C is the centre.

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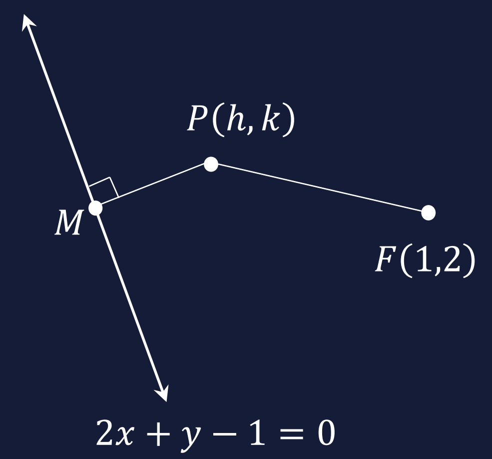
Find the equation of the hyperbola whose focus is (1,2), directrix is $2x + y - 1 = 0$ and eccentricity is $\sqrt{3}$.

Solution:



Given :

Focus, $F \equiv (1,2)$, $e = \sqrt{3}$

Directrix: $L \equiv 2x + y - 1 = 0$

Let $P(h, k)$ be a moving point.

$$\frac{PF}{PM} = e \Rightarrow PF = e \cdot PM$$

$$\Rightarrow \sqrt{(h-1)^2 + (k-2)^2} = \sqrt{3} \cdot \frac{|2h+k-1|}{\sqrt{5}}$$

Squaring both sides

$$5(h^2 - 2h + 1 + k^2 - 4k + 4) = 3(4h^2 + k^2 + 1 + 4hk - 2k - 4h)$$

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Find the equation of the hyperbola whose focus is (1,2), directrix is $2x + y - 1 = 0$ and eccentricity is $\sqrt{3}$.

Solution:

$$5(h^2 - 2h + 1 + k^2 - 4k + 4) = 3(4h^2 + k^2 + 1 + 4hk - 2k - 4h)$$

$$\Rightarrow 7h^2 + 12hk - 2k^2 - 2h + 14k - 22 = 0 \rightarrow \text{Locus of } P(h,k)$$

Replace $h \rightarrow x$ and $k \rightarrow y$

$$\boxed{\Delta \neq 0, h^2 > ab}$$

$$\Rightarrow \underbrace{7x^2 + 12xy - 2y^2 - 2x + 14y - 22 = 0}_{\text{Equation of the hyperbola}}$$

$h^2 = (6)^2 = 36$ and $ab = -14$

$$\Rightarrow h^2 > ab$$

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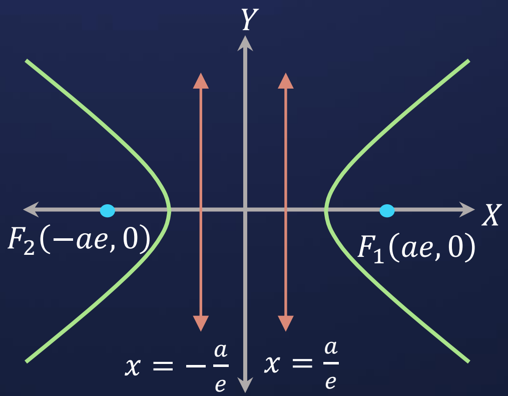
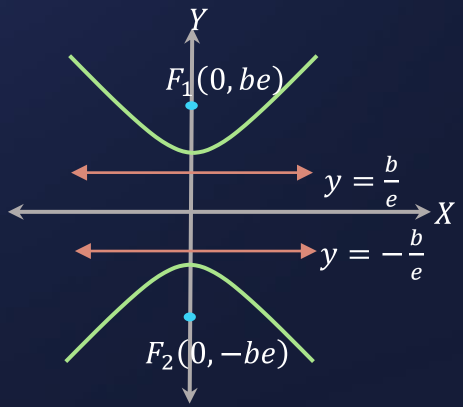
Standard vs Conjugate Hyperbola:

Equation of hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Figure		
Centre	$(0, 0)$	$(0, 0)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$

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Standard vs Conjugate Hyperbola:

Equation of hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Eccentricity	$e = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \sqrt{1 + \frac{a^2}{b^2}}$
Equation of directrix	$x = \pm\frac{a}{e}$	$y = \pm\frac{b}{e}$
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm be)$
Length of LR	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Length of Transverse Axis	$2a$	$2b$
Length of Conjugate Axis	$2b$	$2a$

Note: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ is said to be conjugate hyperbola for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and if e_H and e_{CH} are eccentricities of hyperbola and its conjugate , then $\frac{1}{{e_H}^2} + \frac{1}{{e_{CH}}^2} = 1$

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Equation of a Hyperbola referred to two Perpendicular Lines as axes:

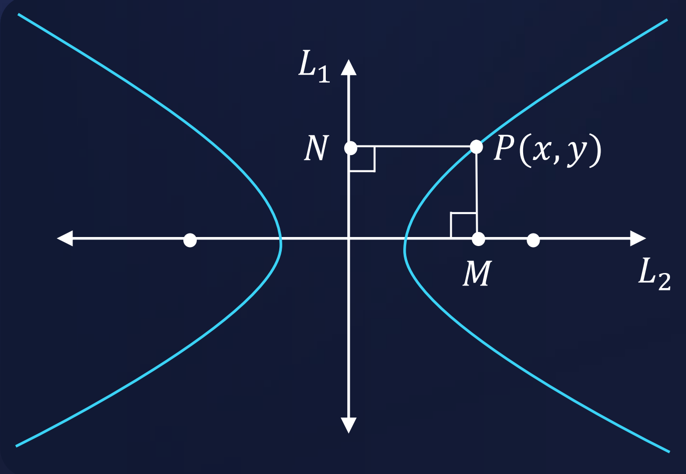
Equation of hyperbola:

$$\frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2}-\frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{b_1^2+a_1^2}}\right)^2}{b^2}=1$$

Suppose, if conjugate axis is $L_1: a_1x+b_1y+c_1=0$
And transverse axis is $L_2: b_1x-a_1y+c_2=0$

$$\boxed{e=\sqrt{\frac{a^2+b^2}{a^2}}}$$

Centre of the hyperbola is the point of intersection of the lines $L_1=0$ and $L_2=0$



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Let $S = \left\{(x, y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$ where $r \neq \pm 1$. Then S represents:

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A. An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

B. An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$

C. A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$

D. A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$

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Let $S = \left\{(x, y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$ where $r \neq \pm 1$. Then S represents:

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Solution:

$$\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$$

Since, $r \neq \pm 1$, we have two cases:

(i) When $r > 1$, S represents ellipse

$$\therefore \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$$

$$\Rightarrow \frac{y^2}{1+r} + \frac{x^2}{r-1} = 1$$

So, $a^2 = 1 + r,\ \ b^2 = r - 1$

$$\Rightarrow e = \sqrt{1 - \frac{r-1}{r+1}} \qquad \left(\because e = \sqrt{1 - \frac{b^2}{a^2}}\right)$$

$$\Rightarrow e = \sqrt{\frac{2}{r+1}}$$

(ii) When $0 < r < 1$, S represents hyperbola

$$\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1 \quad \text{(Conjugate hyperbola)}$$

So, $b^2 = 1 + r,\ \ a^2 = 1 - r$

$$\Rightarrow e = \sqrt{1 + \frac{1-r}{r+1}} \qquad \left(\because e = \sqrt{1 + \frac{a^2}{b^2}}\right)$$

$$\Rightarrow e = \sqrt{\frac{2}{r+1}}$$

Hence, only option (A) is correct

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Let $S = \left\{(x, y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$ where $r \neq \pm 1$. Then S represents:

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A. An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

B. An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$

C. A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$

D. A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$

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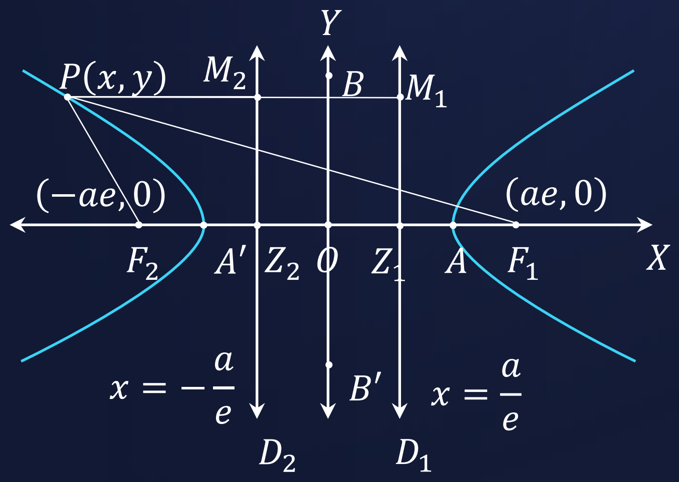
Alternative Definition of Hyperbola:

Hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points (foci) in the plane is a constant $(2a)$.

$$|PF_1 - PF_2| = 2a$$



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The locus of centre of a circle which touches externally two given circles is:

A. Pair of straight lines

B. Circle

C. Parabola

D. Hyperbola

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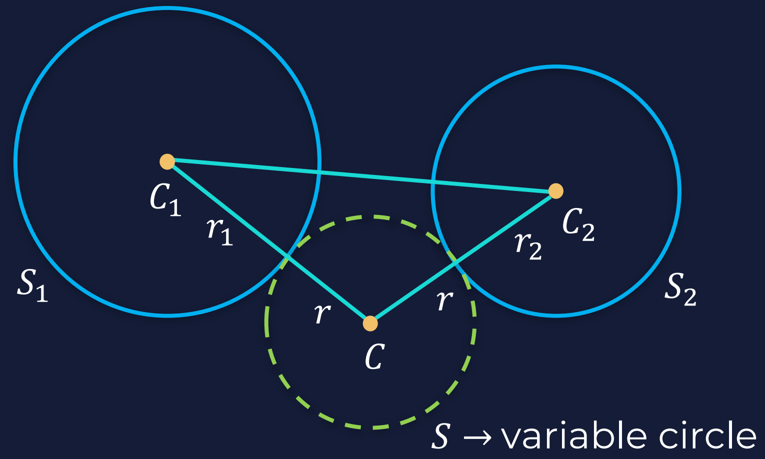
The locus of centre of a circle which touches externally two given circles is:

Solution:

Let there be two circles with centers C_1 & C_2 and radii r_1 & r_2 and let there be a third circle with centre C and radius r touch both the circles externally.



Now,

$CC_1 = r + r_1$

$CC_2 = r + r_2$

$\Rightarrow |CC_2 - CC_1| = |r_2 - r_1|$

i.e. The locus of C is a hyperbola

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The locus of centre of a circle which touches externally two given circles is:

A Pair of straight lines

B Circle

C Parabola

D Hyperbola

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Key Takeaways

Rectangular Hyperbola:

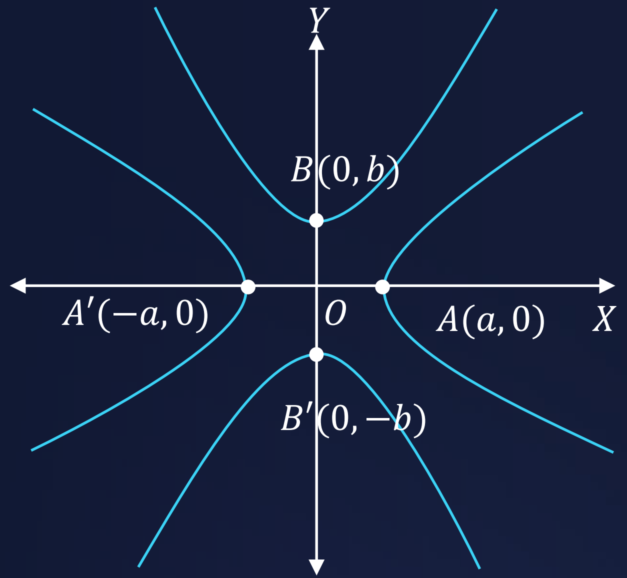
If the length of transverse axis and conjugate axis of a hyperbola are equal, then it is called Rectangular/ Equilateral hyperbola.

$2a = 2b \Rightarrow a = b$

Equation of a rectangular hyperbola in standard form is:

Horizontal: $x^2 - y^2 = a^2$

Vertical: $x^2 - y^2 = -a^2$



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Key Takeaways

Rectangular Hyperbola:

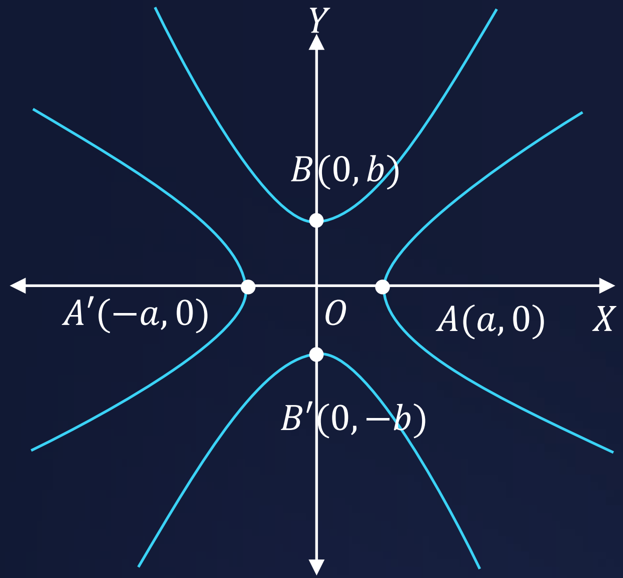
Equation of a rectangular hyperbola in standard form is:

Case 1:

$$\frac{x^2}{a^2}-\frac{y^2}{a^2}=1, e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{a^2}{a^2}}=\sqrt{2}$$

Case 2:

$$\frac{x^2}{a^2}-\frac{y^2}{a^2}=-1, e=\sqrt{1+\frac{a^2}{b}}=\sqrt{1+\frac{a^2}{a^2}}=\sqrt{2}$$



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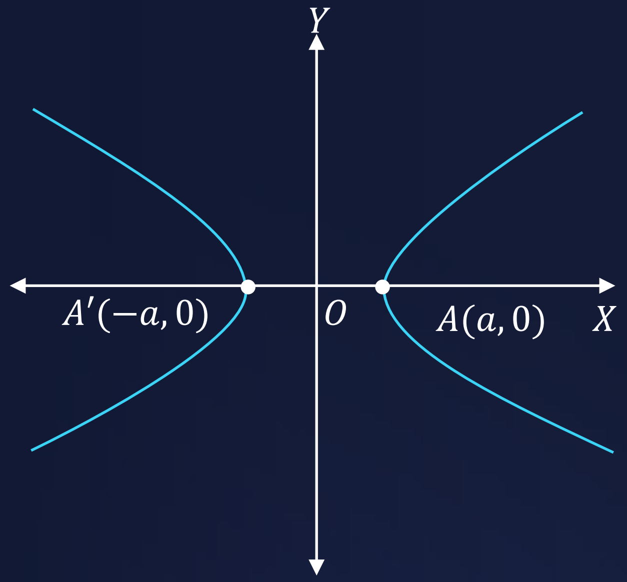
Key Takeaways

Rectangular Hyperbola:

Eccentricity of a rectangular hyperbola is $\sqrt{2}$.

Proof:

$$e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{a^2+a^2}{a^2}} = \sqrt{2} \qquad \boxed{\therefore a = b}$$



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Note:

Foci of the hyperbola and its conjugate hyperbola are concyclic and form a square.

$$ae_1 = a\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{a^2 + b^2}$$

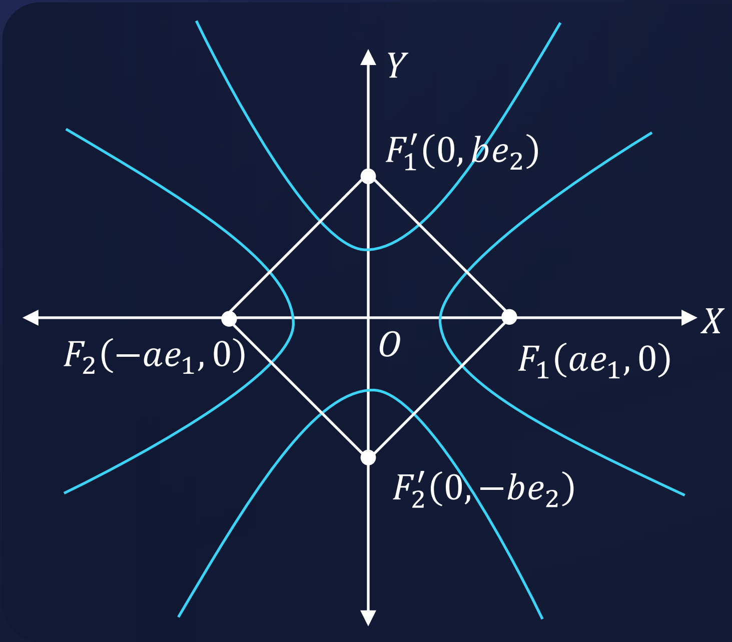
Similarly,

$$be_2 = b\sqrt{1 + \frac{a^2}{b^2}} = \sqrt{a^2 + b^2}$$

Diagonals of a square bisect each other at 90^0

F_1F_2 and $F_1''F_2''$ are the diagonals.

Taking O as a center we can draw a circle with radius $OF_1 = OF_1'$ and passing through four points which is the focus.



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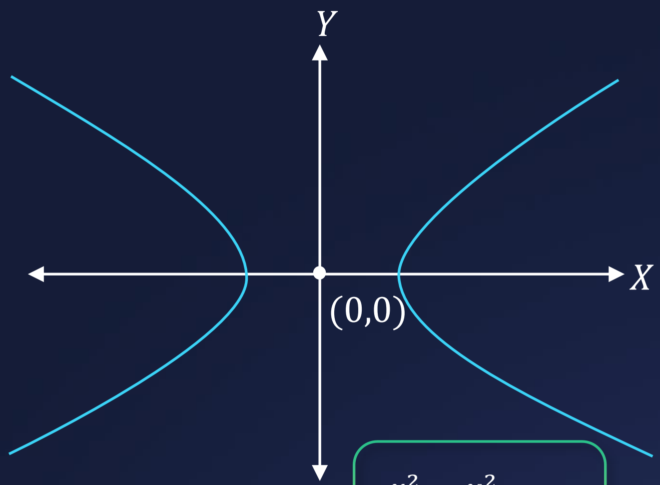
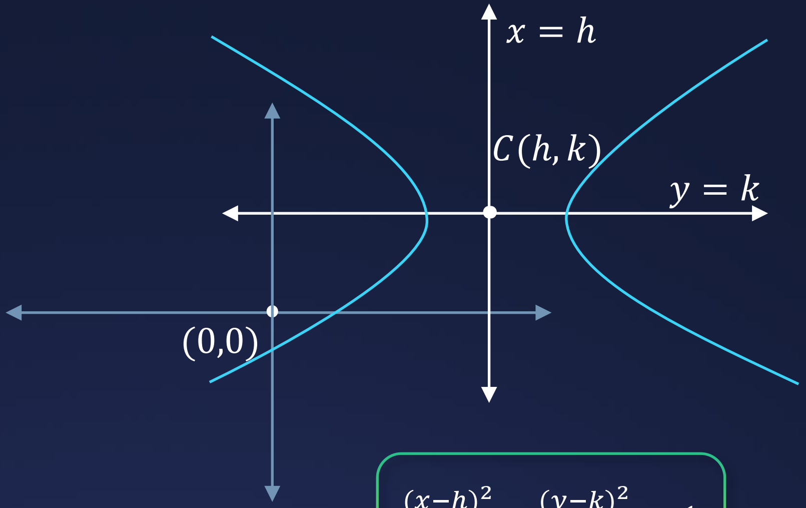
Key Takeaways

Axes of Hyperbola Parallel to co-ordinate axes



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Eccentricity remains same for both these hyperbola.

Similar procedure can be followed for vertical hyperbola.

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Find the center of the hyperbola $4x^2 - 9y^2 + 32x + 54y - 53 = 0$:

A $(-4,4)$

C $(-4, 3)$

B $(1, 2)$

D $(-3, 4)$

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Find the center of the hyperbola $4x^2 - 9y^2 + 32x + 54y - 53 = 0$:

Solution:

The standard form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Thus, $4x^2 - 9y^2 + 32x + 54y - 53 = 0$:

$$4(x^2 + 8x) - 9(y^2 - 6y) - 53 = 0$$

$$4[(x+4)^2 - 16] - 9[(y-3)^2 - 9] - 53 = 0$$

$$4(x+4)^2 - 9(y-3)^2 = 36$$

$$\frac{(x+4)^2}{3^2} - \frac{(y-3)^2}{2^2} = 1$$ $\therefore$ Centre $= (-4, 3)$

A $(-4,4)$

C $(-4, 3)$

B $(1, 2)$

D $(-3, 4)$

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Key Takeaways

Equation of a Hyperbola referred to two Perpendicular Lines as axes:

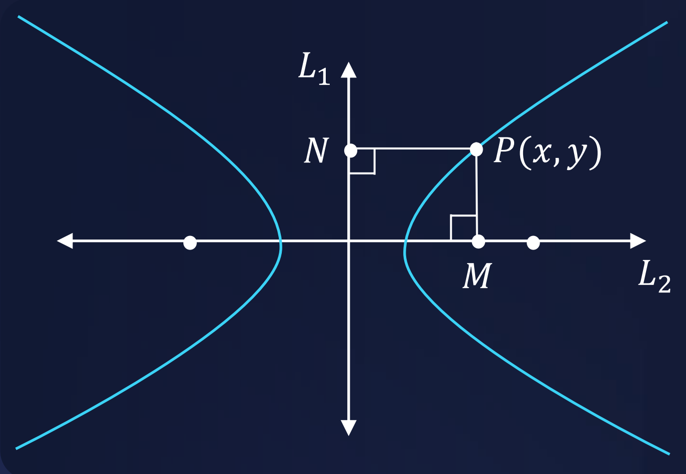
Equation of hyperbola:

$$\frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2}-\frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{b_1^2+a_1^2}}\right)^2}{b^2}=1$$

Suppose, if conjugate axis is $a_1x+b_1y+c_1=0 : L_1$ and transverse axis is $b_1x-a_1y+c_2=0 : L_2$

$$\boxed{e=\sqrt{\frac{a^2+b^2}{a^2}}}$$

Centre of the hyperbola is the point of intersection of the lines $L_1=0$ and $L_2=0$



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Session 08

Auxiliary Circle and Director Circle of Hyperbola

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Let $a > 0, b > 0$. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14} l$ and $(e')^2 = \frac{11}{8} l'$, then the value of $77a + 44b$ is equal to :

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A. 100

B. 110

C. 120

D. 130

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Let $a > 0, b > 0$. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14} l$ and $(e')^2 = \frac{11}{8} l'$, then the value of $77a + 44b$ is equal to :

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Solution:

$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then

$e^2 = \frac{11}{14} l$, (l be the length of LR)

$\Rightarrow a^2 + b^2 = \left(\frac{11}{14}\right)\left(\frac{2b^2}{a}\right) a^2$

$\Rightarrow a^2 + b^2 = \frac{11}{7} b^2 a \ldots (i)$

And $(e')^2 = \frac{11}{8} l'$

(l' be the length of LR of conjugate hyperbola)

$\Rightarrow a^2 + b^2 = \frac{11}{4} a^2 b \ldots (ii)$

By (i) and (ii)

$7a = 4b$

Then by (i)

$\frac{16}{49} b^2 + b^2 = \frac{11}{7} b^2 \cdot \frac{4b}{7}$

$\Rightarrow 44b = 65$ and $77a = 65$

$\Rightarrow 77a + 44b = 130$

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Let $a > 0, b > 0$. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14} l$ and $(e')^2 = \frac{11}{8} l'$, then the value of $77a + 44b$ is equal to :

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A. 100

B. 110

C. 120

D. 130

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Key Takeaways

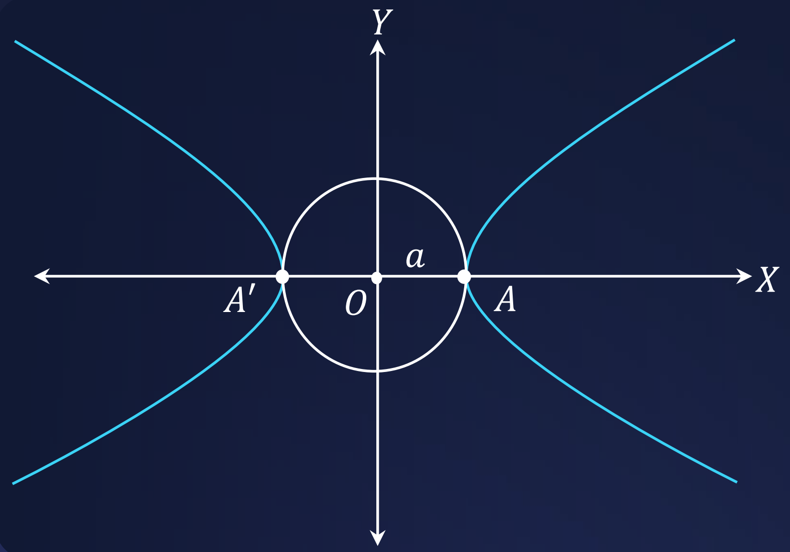
Auxiliary Circle:

The circle described on the transverse axis as diameter is called the Auxiliary Circle of the given hyperbola.

1) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Equation of auxiliary circle

$$\boxed{x^2 + y^2 = a^2}$$



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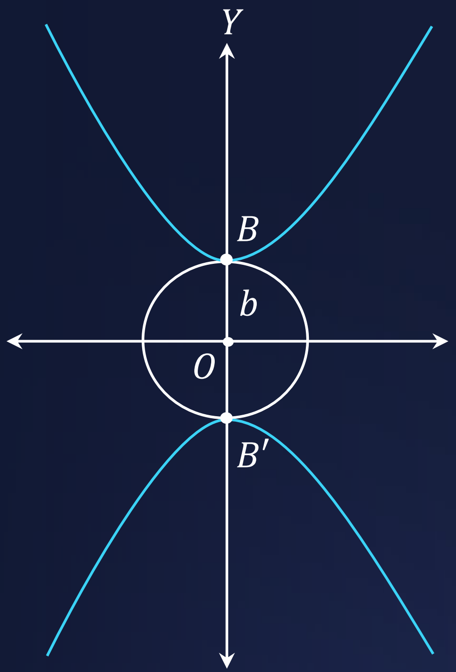
Key Takeaways

Auxiliary Circle:

2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Equation of auxiliary circle

$$\boxed{x^2 + y^2 = b^2}$$



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Key Takeaways

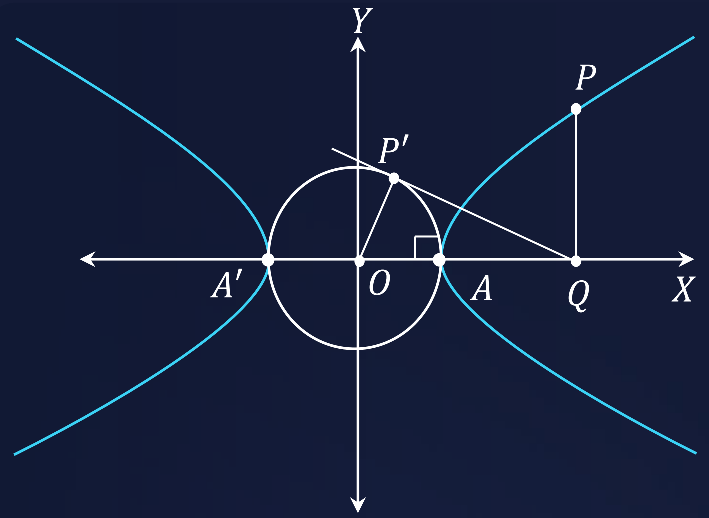
Parametric Equations :

Consider the hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Auxiliary circle: $x^2 + y^2 = a^2$

Let $P(x, y)$ be a point on Hyperbola

$PQ \perp x$-axis



QP' is called tangent from Q to Auxiliary circle.

Auxiliary circle: $x^2 + y^2 = a^2$

So, Equation of tangent at P' is $xx_1 + yy_1 = a^2$

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Key Takeaways

Parametric Equations :

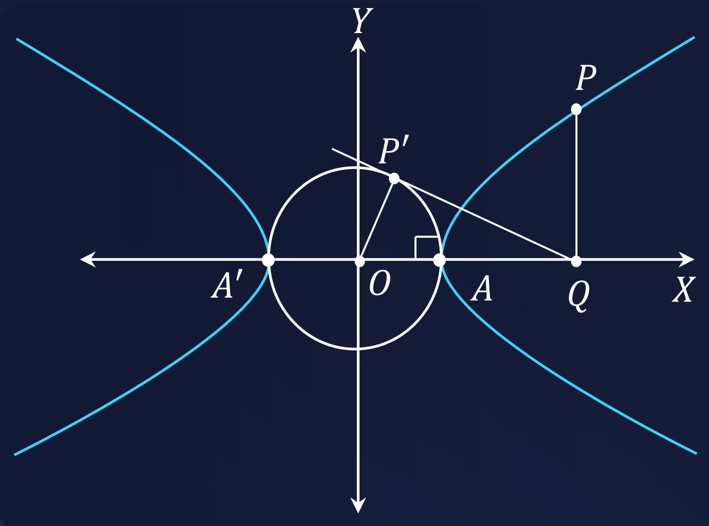
$= x(a\cos\theta) + y(\mathrm{b}\sin\theta) = a^2$

Tangent is cutting X –axis at Q

Let $\theta \equiv \angle P'OQ$ where $\theta \in [0,2\pi) - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

In $\Delta P'OQ, \cos\theta = \frac{a}{OQ} \Rightarrow OQ = \mathrm{a}\sec\theta$

$\therefore Q \equiv (a\sec\theta\,, 0)$



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Key Takeaways

Parametric Equations :

$\therefore Q \equiv (a \sec\theta, 0)$

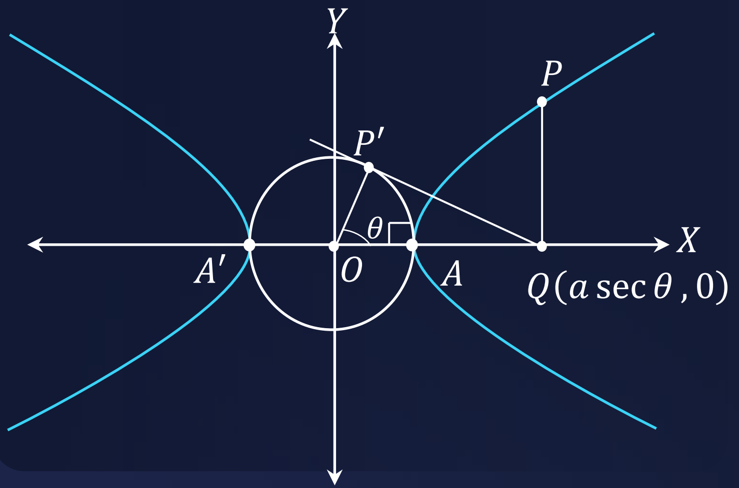
Hence, Substituting the value in the hyperbola

$P \equiv (a \sec\theta, b \tan\theta)$ Parametric point

$P' \equiv (a \cos\theta, a \sin\theta)$

P and P' are called corresponding points.

$x = a \sec\theta, y = b \tan\theta$ are the parametric equations of the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



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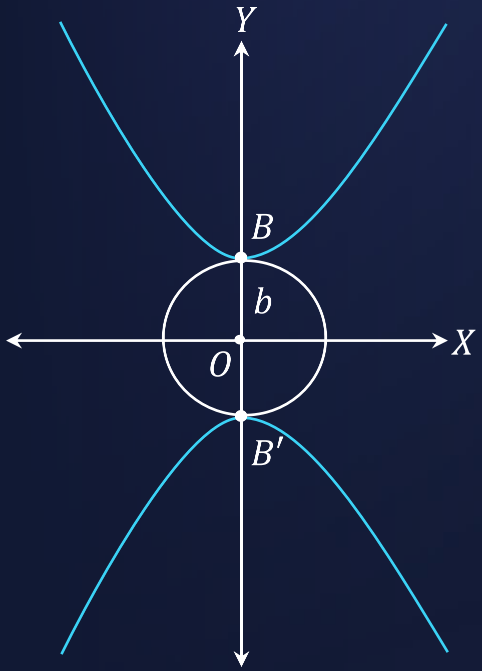
Parametric Equations :

For Hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$,

Parametric equation:

$$x = a\tan\theta\,, y = b\sec\theta$$



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The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is

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A $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

B $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

C $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

D $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

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The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is

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Solution:

$H \equiv 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) = 144$

$\equiv \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

$e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

Foci $(\pm ae - 1,\ 2) = (\pm 5 - 1, 2)$

Foci are $(4, 2)$ and $(-6, 2)$

Let a general point be $P(-1 + 3\sec\theta, 2 + 4\tan\theta)$

Let centroid be (h, k)

$h = \frac{4-6-1+3\sec\theta}{3},\ k = \frac{2+2+2+4\tan\theta}{3}$

$\Rightarrow h = -1 + \sec\theta$ and

$3k = 6 + 4\tan\theta$

$\Rightarrow \sec\theta = (h + 1)$ and $\frac{3}{4}(k - 2) = \tan\theta$

Now, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow\ 16(h + 1)^2 - 9(k - 2)^2 = 16$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

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The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is

JEE Main 2021

A $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

B $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

C $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

D $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

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Position of a Point:

Let $P \equiv (x_1, y_1)$ be any point in the plane of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

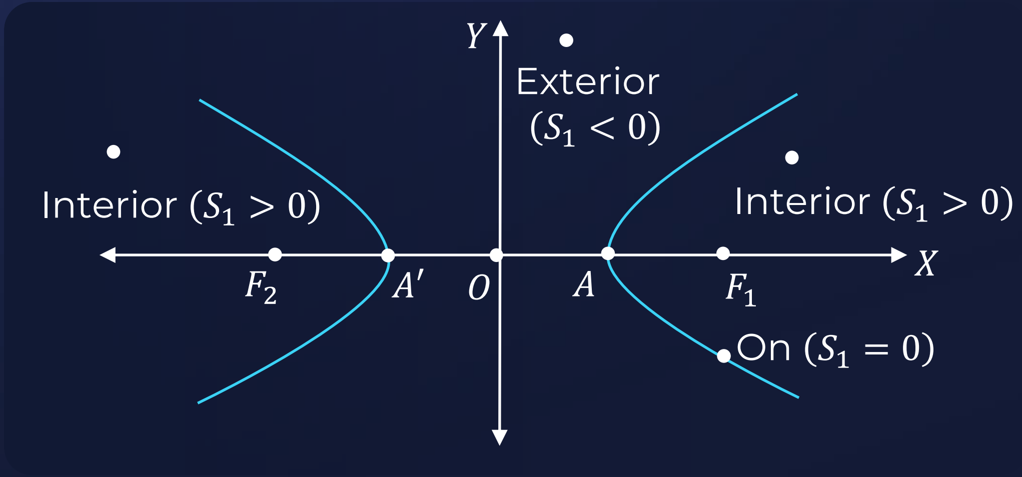
$S : \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$

$S_1 : \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$

If $S_1 > 0 \rightarrow$ Point lies inside

If $S_1 < 0 \rightarrow$ Point lies outside

If $S_1 = 0 \rightarrow$ Point lies on the Hyperbola



To justify above conditions, we can substitute the foci and center in S_1.

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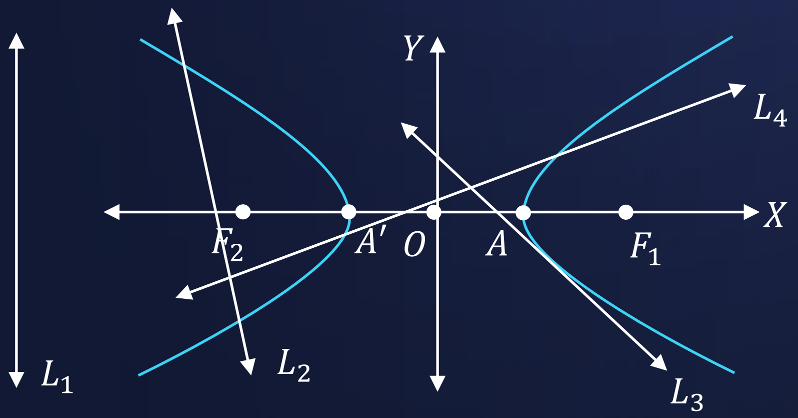
A Line and A Hyperbola:



A line can intersect Hyperbola at not more than 2 points as when we solve equation of line (linear) with the equation of hyperbola (2^{nd} degree), we get a Quadratic Equation, which has at most 2 distinct real roots.

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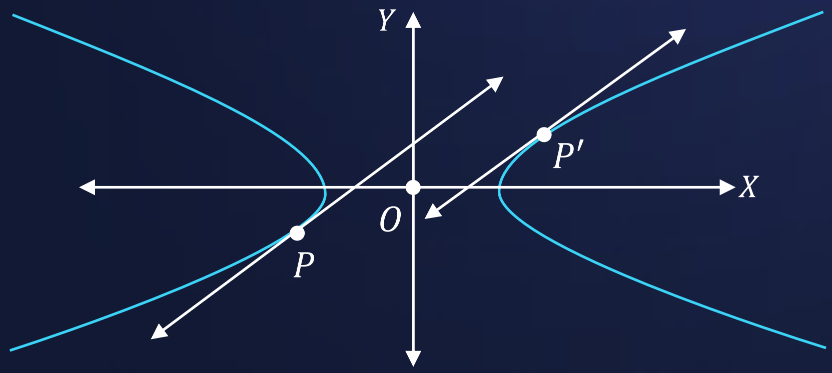
A Line and A Hyperbola:



A line cannot be tangent to both the branches of the hyperbola simultaneously as either point P or P' corresponds to two real and repeated roots of the quadratic equation..

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Key Takeaways

Position of a Line:

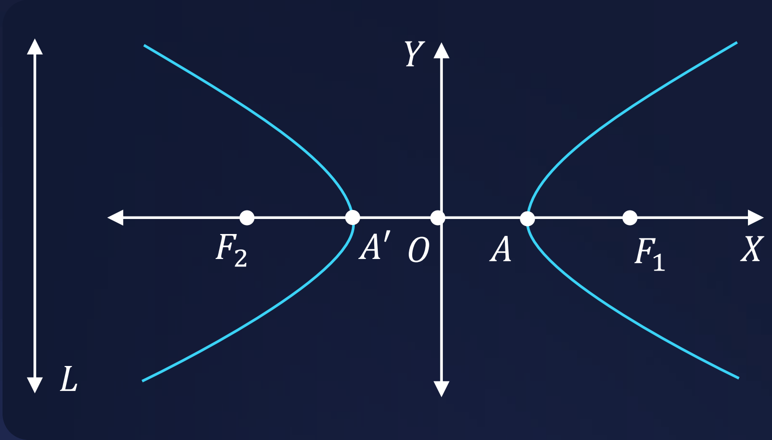
Consider a line $L:\ y = mx + c$ and a hyperbola : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Solving line and hyperbola we get :

$$(b^2 - a^2m^2)x^2 - 2a^2cmx - (a^2c^2 + a^2b^2) = 0$$

Discriminant of the above equation is :

$$\boxed{D = 4a^2b^2(c^2 + b^2 - a^2m^2)}$$



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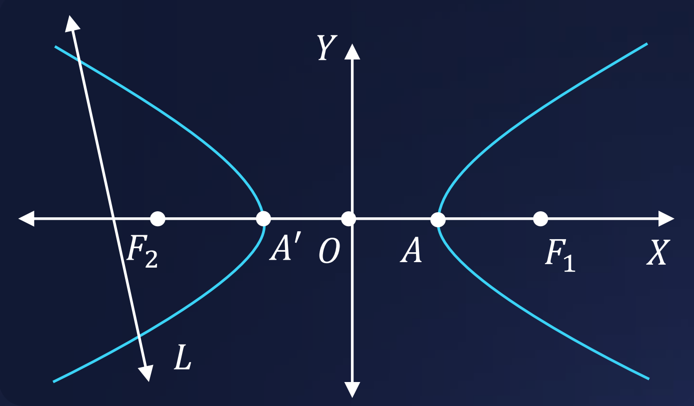
Key Takeaways

Position of a Line:

Case (i) : L meets the hyperbola at two distinct points



If line L meets the hyperbola at two distinct points, then equation has two distinct real roots.

$$\therefore D > 0 \text{ or } c^2 > a^2m^2 - b^2$$

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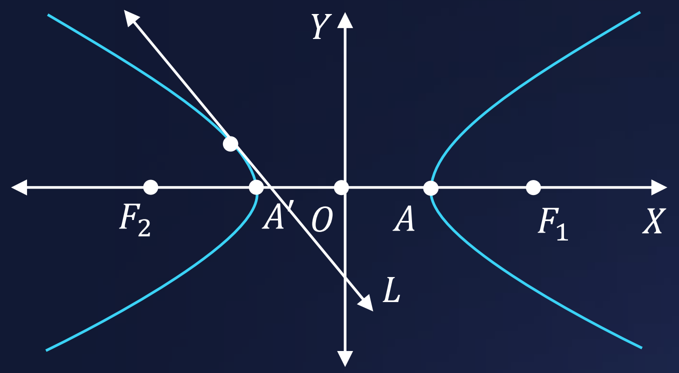
Key Takeaways

Position of a Line:

Case (ii) : L touches the hyperbola at one point



Hence, equation of the Line: $\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$

We are obtaining 2 equations with same slope as tangents to the hyperbola.

If line L touches the hyperbola at one point, then equation has two equal roots.

$\boxed{\therefore D = 0 \text{ or } c^2 = a^2m^2 - b^2}$

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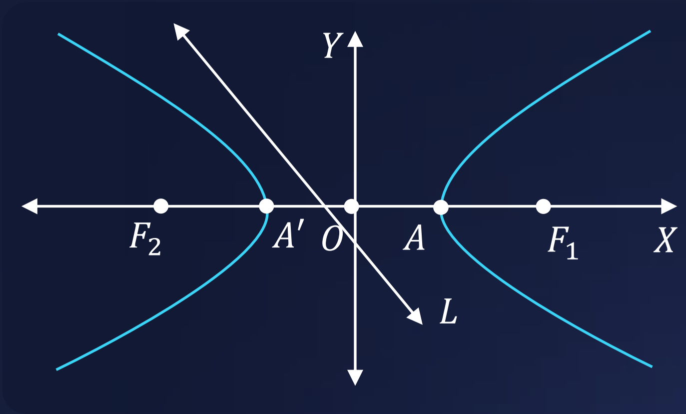
Key Takeaways

Position of a Line:

Case (iii) : L doesn't meet the hyperbola



If line L doesn't meet the hyperbola, then equation has imaginary roots.

$$\boxed{\therefore\ D < 0 \text{ or } c^2 < a^2m^2 - b^2}$$

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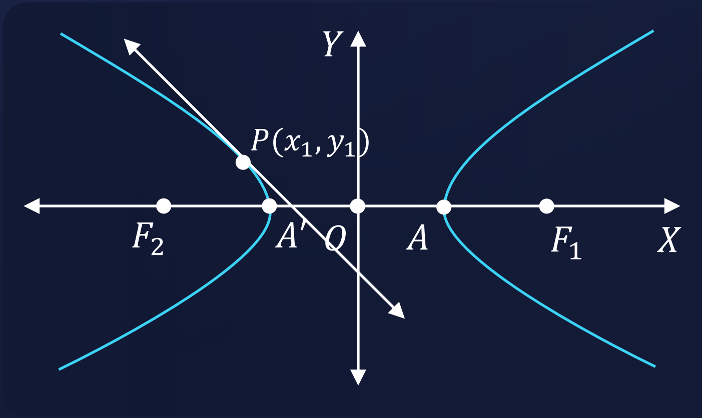
Equation of Tangent:

Point Form:

Equation of tangent to a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

at a point $P(x_1, y_1)$ is $T = 0$

$$\Rightarrow \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = 0$$



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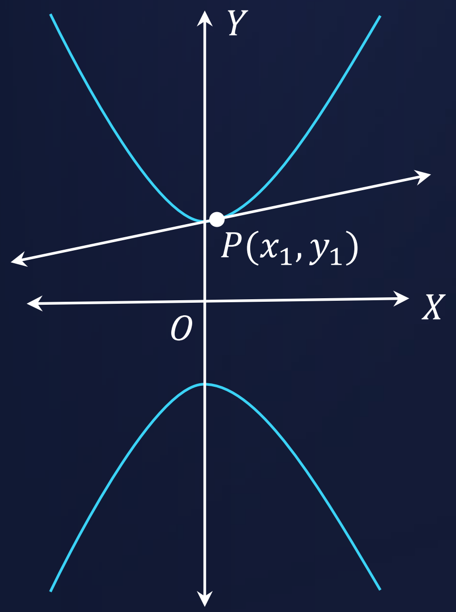
Equation of Tangent:

Point Form:

Equation of tangent to a hyperbola $S : \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

at a point $P(x_1, y_1)$ is given by

$$\boxed{\Rightarrow \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = -1}$$



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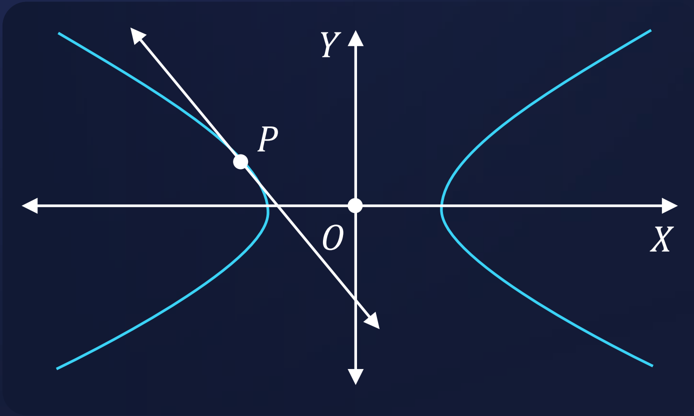
Equation of Tangent:

Parametric Form:

Equation of tangent to a hyperbola $S : \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

at a point $P(\theta) \equiv (a \sec\theta, b \tan\theta)$ is

$$\boxed{\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta - 1 = 0}$$



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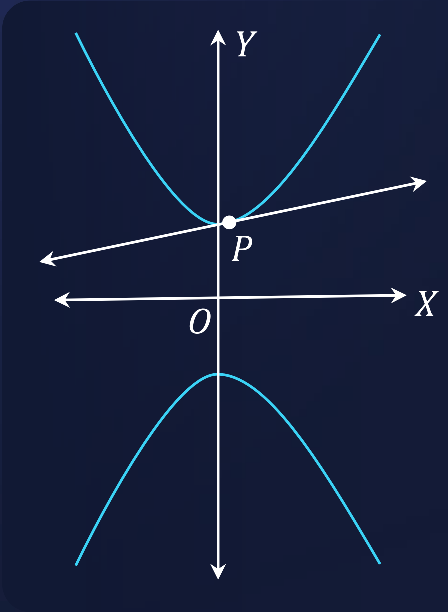
Equation of Tangent:

Parametric Form:

Equation of tangent to a hyperbola $S : \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$

at a point $P(\theta) \equiv (a\tan\theta, b\sec\theta)$ is

$$\boxed{\frac{x}{a}\tan\theta - \frac{y}{b}\sec\theta + 1 = 0}$$



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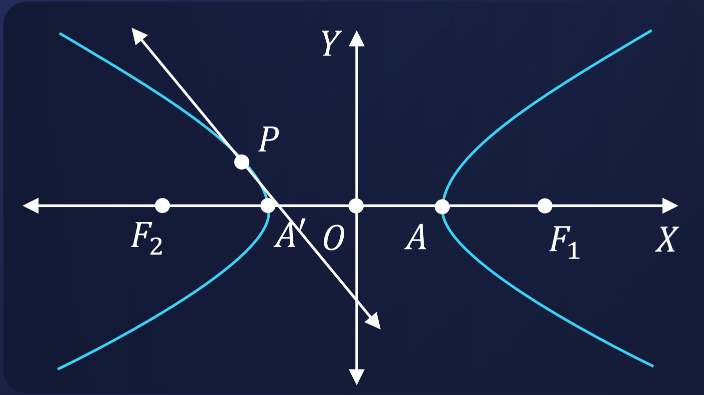
Equation of Tangent:

Slope Form:

Equation of tangent to a hyperbola

$S : \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ having slope m is

$$\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$$

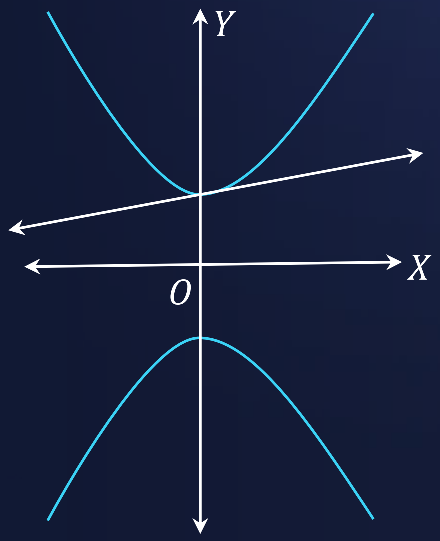


Point of contact of the tangent is $\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$ where $c = \pm\sqrt{a^2m^2 - b^2}$

NOTE:

$y = mx + c$ will be tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

if $\boxed{c^2 = b^2 - a^2m^2.}$



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A pair of tangents are drawn from the point $(4,3)$ to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$. Let θ be the acute angle between them. Which of the following is/are true ?

A. $\tan\theta=\frac{4}{3}$

B. One of the tangent is horizontal

C. One of the tangent is $3x-4y=0$

D. One of the tangent is vertical

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A pair of tangents are drawn from the point (4,3) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Let θ be the acute angle between them. Which of the following is/are true ?

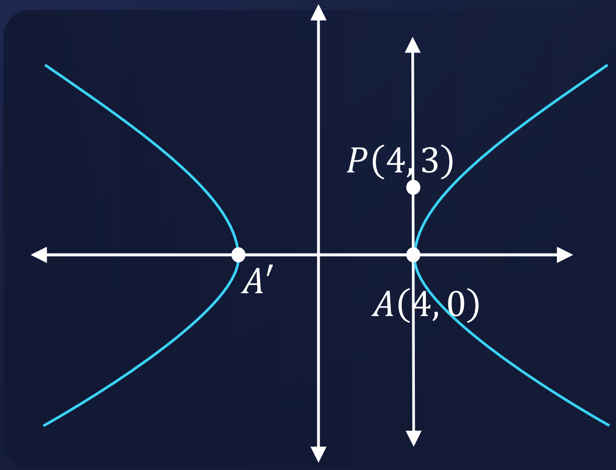
Solution:

Let $P \equiv (4,3)$

$$S: \frac{x^2}{16} - \frac{y^2}{9} - 1 \quad \Rightarrow a^2 = 16, b^2 = 9$$

$$S_P = \frac{4^2}{16} - \frac{3^2}{9} - 1 = -1 < 0$$

Hence P is the exterior point.



By observation, tangent through $P(4,3)$ passes through the vertex $A(4,0)$.

$\therefore$ One tangent is vertical.

Equation of tangent : $y = mx \pm \sqrt{a^2m^2 - b^2}$

Equation of tangent at P: $3 = 4m \pm \sqrt{16m^2 - 9}$

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A pair of tangents are drawn from the point (4,3) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Let θ be the acute angle between them. Which of the following is/are true ?

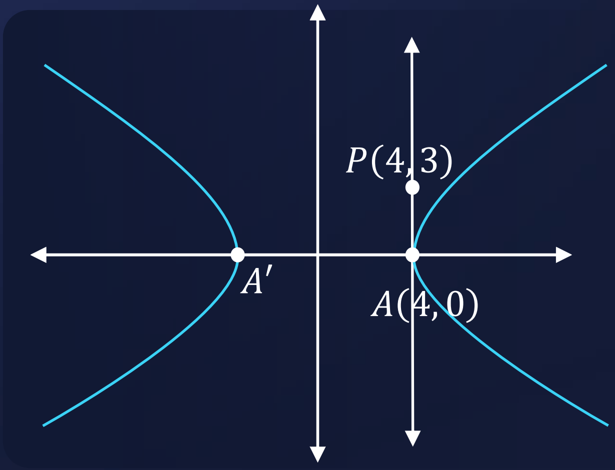
Solution:

$$\Rightarrow (3-4m)^2 = 16m^2 - 9$$

$$\Rightarrow 9 + 16m^2 - 24m = 16m^2 - 9$$

$$\Rightarrow m = \frac{18}{24} = \frac{3}{4}$$

$\therefore$ Equation of other tangent is $3x - 4y = 0$



Let α be the inclination of the tangent with x –axis.

$$\Rightarrow \tan\alpha = \frac{3}{4}$$

$\therefore$ The acute angle between tangents,

$$\theta = 180 - (90 + \alpha)$$

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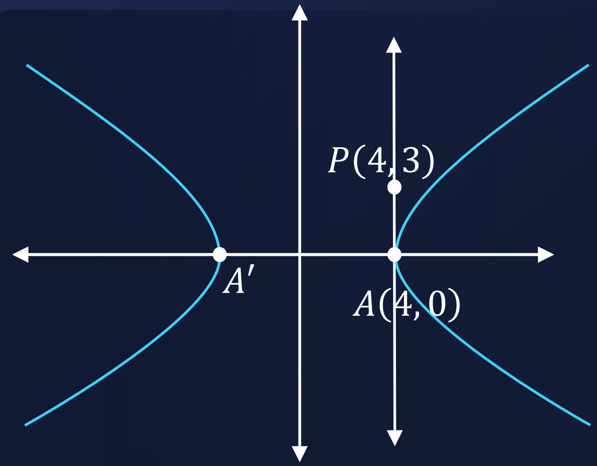
A pair of tangents are drawn from the point (4,3) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Let θ be the acute angle between them. Which of the following is/are true ?

Solution:

$$\tan\theta = \tan(90 - \alpha)$$

$$\tan\theta = \cot\alpha$$

$$\tan\theta = \frac{4}{3}$$



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Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to

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Solution:

Equation of L_1 is: $\frac{x \sec\theta}{4} - \frac{y \tan\theta}{2} = 1 \ldots (i)$

Equation of L_2 is: $\frac{x \tan\theta}{2} + \frac{y \sec\theta}{4} = 0 \ldots (ii)$

Required point of intersection of L_1 and L_2 is (x_1, y_1) then,

$\frac{x_1 \sec\theta}{4} - \frac{y_1 \tan\theta}{2} - 1 = 0 \ldots (iii)$

And $\frac{x_1 \tan\theta}{2} + \frac{y_1 \sec\theta}{4} = 0 \ldots (iv)$

From equation (iii) and (iv)

$\sec\theta = \frac{4x_1}{x_1^2+y_1^2}$ and $\tan\theta = -\frac{2y_1}{x_1^2+y_1^2}$

Required locus of (x_1, y_1) is:

$(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$

$\therefore \alpha = 16$ and $\beta = -4$

Thus, $\alpha + \beta = 12$

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?

The locus of the midpoint of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is:

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A $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

B $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

C $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

D $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

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The locus of the midpoint of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is:

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Solution:

Tangent to the hyperbola: $y = mx + \sqrt{9m^2 - 16} \dots (i)$

Which is a chord of the circle with mid-point (h, k)

So, equation of chord, $T = S_1$

$$hx + ky = h^2 + k^2$$

$$\Rightarrow y = -\frac{hx}{k} + \frac{h^2+k^2}{k} \dots (ii)$$

By (i) and (ii)

$m = -\frac{h}{k}$ and $\sqrt{9m^2 - 16} = \frac{h^2+k^2}{k}$

$$\Rightarrow 9\frac{h^2}{k^2} - 16 = \frac{\left(h^2+k^2\right)^2}{k^2}$$

$\therefore$ Locus is: $9x^2 - 16y^2 = (x^2 + y^2)^2$

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The locus of the midpoint of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is:

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A. $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

B. $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

C. $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

D. $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

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A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

JEE Main 2020

A 6

B 10

C 8

D 5

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A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

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Solution:

$T: \frac{xx_1}{4} - \frac{yy_1}{2} = 1 \ldots (i)$

$t: 2x - y = 0$ is parallel to T

$\Rightarrow T: 2x - y = \lambda \ldots (ii)$

Now, comparing the slopes, $x_1 = 4y_1$

(x_1, y_1) lies on the hyperbola

$\Rightarrow 4y_1^2 - \frac{y_1^2}{2} = 1$

$\Rightarrow 7y_1^2 = 2$

Now, $x_1^2 + 5y_1^2 = 21y_1^2 = 6$

A 6

B 10

C 8

D 5

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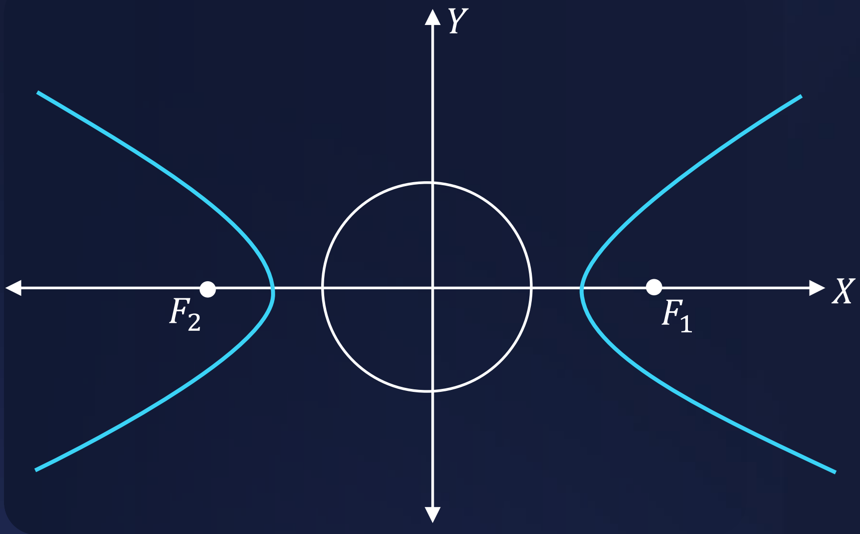
Key Takeaways

Director Circle:

Locus of the point of intersection of perpendicular tangents to a hyperbola is a circle concentric with the hyperbola which is called Director Circle.



For hyperbola S: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Director Circle: $x^2 + y^2 = a^2 - b^2$

$$(x-0)^2 + (y-0)^2 = \left(\sqrt{a^2 - b^2}\right)^2$$

Director Circle of the hyperbola is concentric with the hyperbola.

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Director Circle:

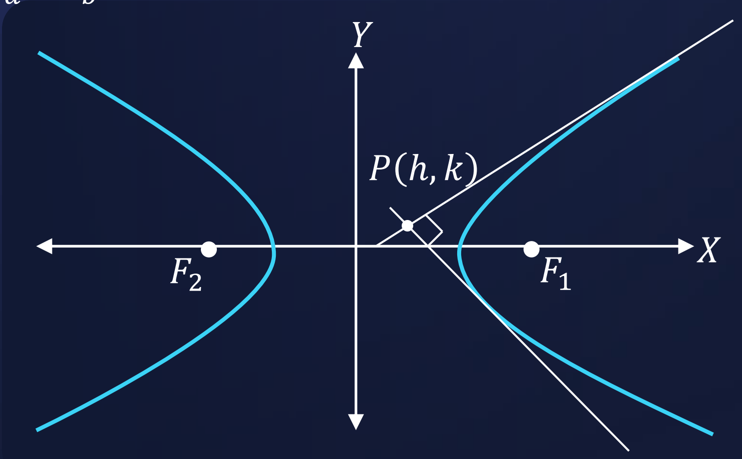
Equation of tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$y^2 + m^2x^2 - 2mxy = a^2m^2 - b^2$$

passing through $P(h, k)$

$$\underbrace{m^2(h^2 - a^2) - 2mhk + k^2 + b^2 = 0}_{\boxed{(m_1, m_2)}}$$



For director circle $m_1 \cdot m_2 = -1$

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Director Circle:

For director circle $m_1 \cdot m_2 = -1$

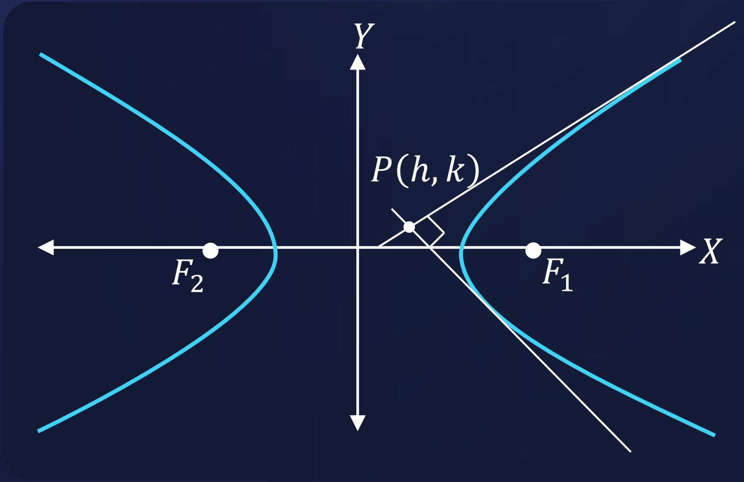
$$\Rightarrow \frac{k^2+b^2}{h^2-a^2} = -1$$

$$(h, k) \rightarrow (x, y)$$

$$\Rightarrow y^2 + b^2 = a^2 - x^2$$

$$\Rightarrow x^2 + y^2 = a^2 - b^2$$

CENTER $(0,0)$ & $r = \sqrt{a^2 - b^2}$



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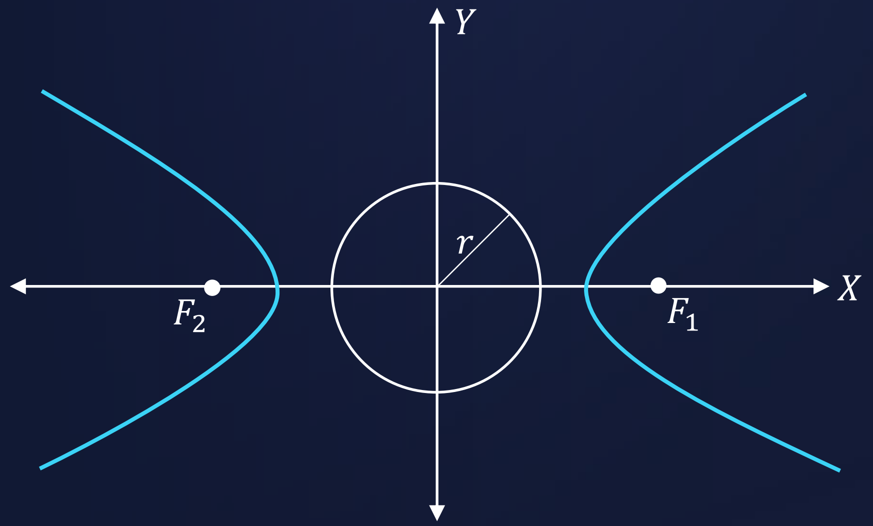
Note:

For Director Circle $r = \sqrt{a^2 - b^2}$

If $a^2 > b^2$ real circle

$a^2 = b^2$ point circle

$a^2 < b^2$ imaginary circle



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Note:

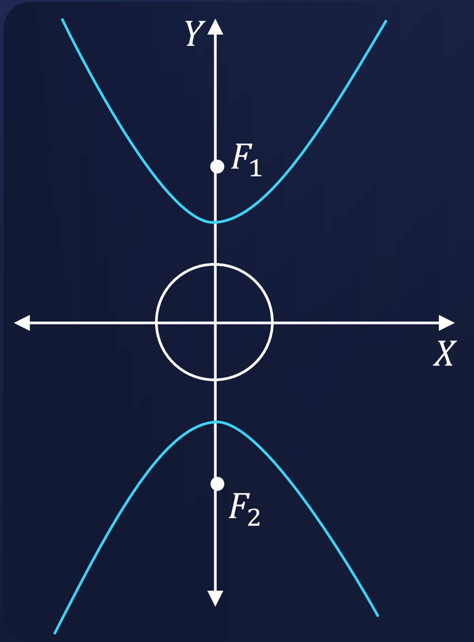
Equation of Director Circle for the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is $\boxed{x^2 + y^2 = b^2 - a^2}$

$b > a \rightarrow$ Real Circle

$b = a \rightarrow$ Point Circle

$b < a \rightarrow$ Imaginary Circle



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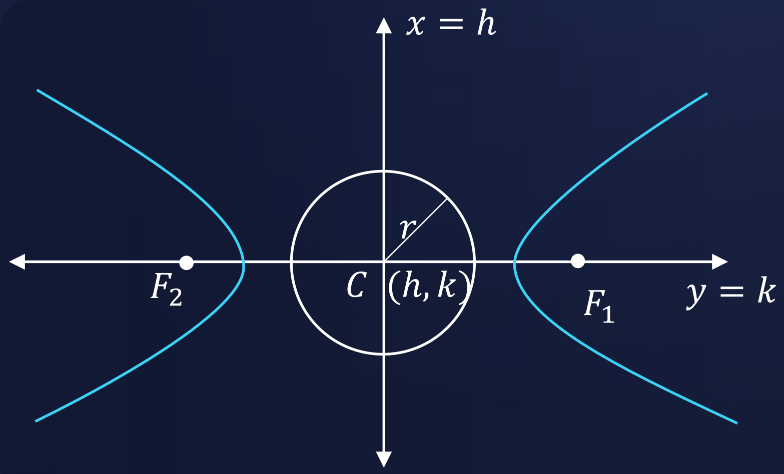
Note:

Equation of Director Circle of $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ is $\boxed{(x-h)^2 + (y-k)^2 = a^2 - b^2}$



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Session 09

Asymptotes and Geometrical Properties of Hyperbola

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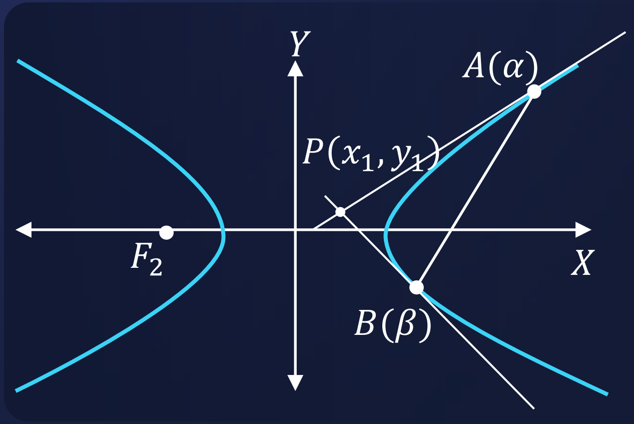
Chord of Contact:

If the pair of tangents drawn from an external point $P(x_1, y_1)$ to the hyperbola $S = 0$ touch it at the points A and B, then AB is called Chord Of Contact.

Equation of AB is $T = 0$

Point of intersection of the tangents $A(\alpha), B(\beta)$ on the hyperbola $S = 0$ is

$$\boxed{\left(\frac{a\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}\right)}$$



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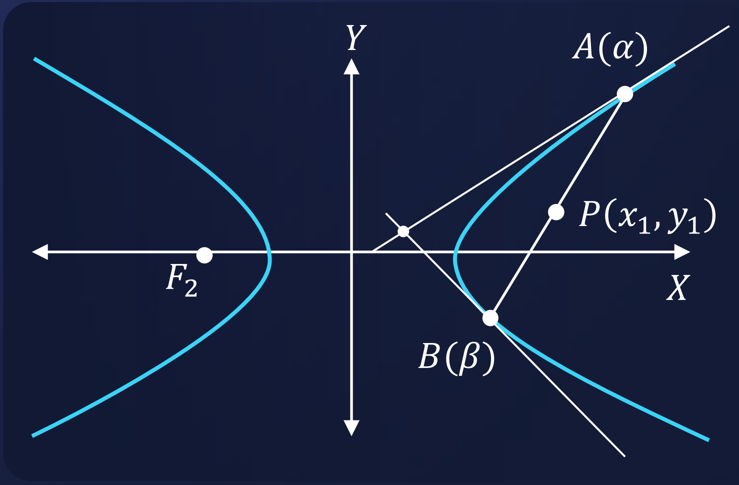
Chord with given mid point:

The equation of a chord of the hyperbola

$S = 0$ whose mid point is $P(x_1, y_1)$ is $\boxed{T = S_1}$



Where:

$$S = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$$

$$S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$$

$$T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$$

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?

The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is:

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A. $y^2(x-2) = x^3$

B. $y^3(x-2) = x^2$

C. $x^3(x-2) = y^2$

D. $x^2(x-2) = y^3$

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The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is:

Solution:

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Let the mid point of chord of hyperbola, $x^2 - y^2 = 4$ be (x_1, y_1)

$\therefore$ Equation of chord is : $xx_1 - yy_1 - 4 = x_1^2 - y_1^2 - 4$

$$\therefore y = \frac{x_1}{y_1}x + \frac{y_1^2 - x_1^2}{y_1} \quad \ldots (i)$$

$\because$ Equation (i) is tangent to parabola $y^2 = 8x$, then

$$\frac{y_1^2 - x_1^2}{y_1} = \frac{2}{\frac{x_1}{y_1}}$$

$$\Rightarrow (y_1^2 - x_1^2)x_1 = 2y_1^2$$

$$\Rightarrow y_1^2(x_1 - 2) = x_1^3$$

Thus, the required locus is: $y^2(x - 2) = x^3$

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The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is:

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A $y^2(x-2) = x^3$

B $y^3(x-2) = x^2$

C $x^3(x-2) = y^2$

D $x^2(x-2) = y^3$

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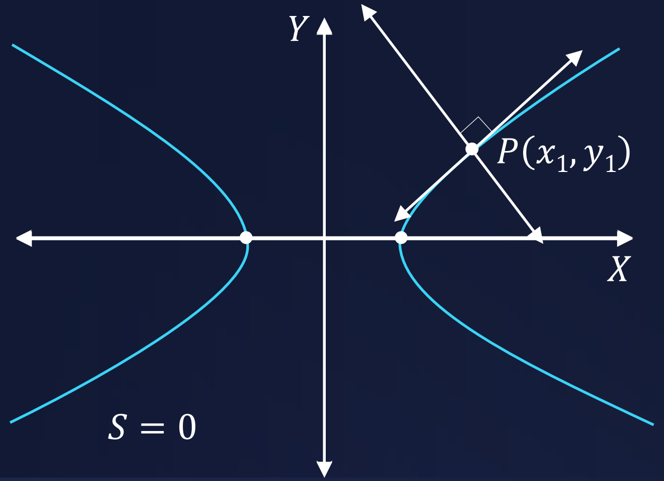
Key Takeaways

Point Form:

Equation of normal to the hyperbola $S = 0$ at $P(x_1, y_1)$ is

$$\boxed{\Rightarrow \frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2}$$



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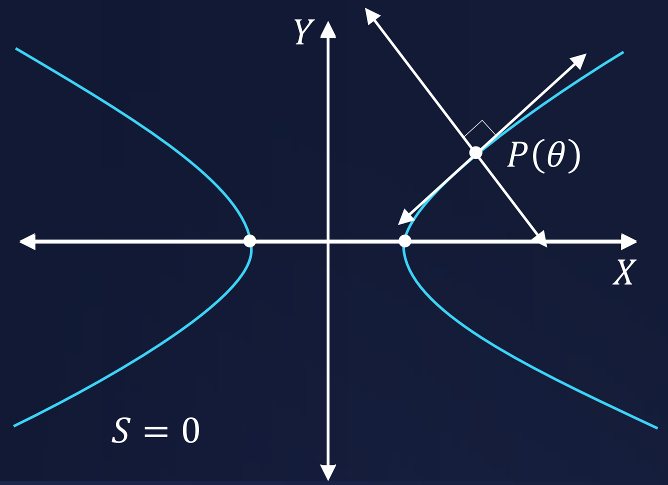
Key Takeaways

Parametric Form:

Equation of normal to the hyperbola $S = 0$ at $P(\theta)$ is

$$\boxed{\Rightarrow \frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2}$$



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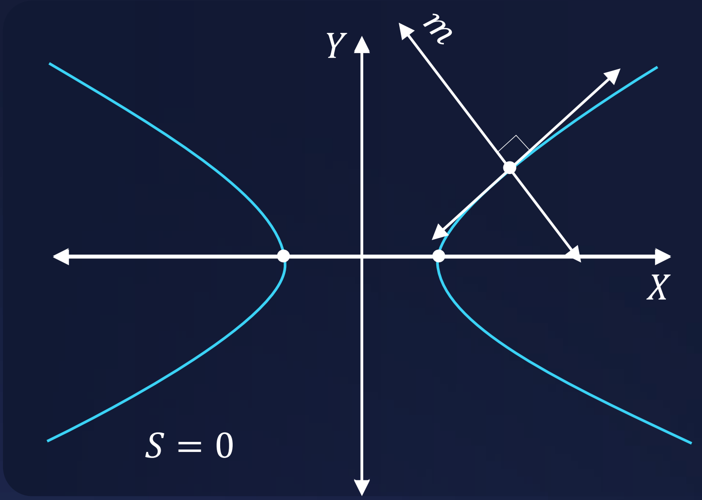
Key Takeaways

Slope Form:

Equation of normal to the hyperbola $S = 0$ whose slope is m is:

$$\boxed{\Rightarrow y = mx \pm \frac{(a^2+b^2)m}{\sqrt{a^2-b^2m^2}}, \text{ where } m \in \left[\frac{-a}{b}, \frac{a}{b}\right]}$$



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Let $P(3,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to

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A $(9,3)$

B $\left(\frac{9}{2}, 2\right)$

C $\left(\frac{9}{2}, 3\right)$

D $\left(\frac{3}{2}, 2\right)$

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Let $P(3,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to

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Solution:

Given : Hyperbola : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

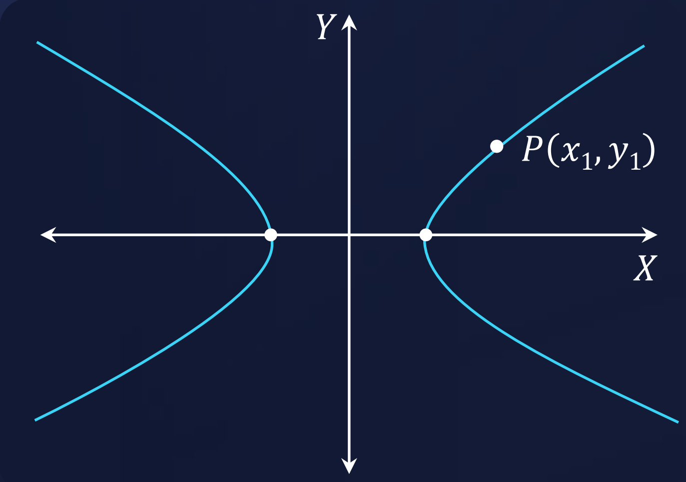
Point $P \equiv (3,3)$ lies on it

$$\Rightarrow \frac{9}{a^2} - \frac{9}{b^2} = 1 \cdots (i)$$

Equation of normal at P :

$$\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$$

$$\Rightarrow \frac{a^2 x}{3} + \frac{b^2 y}{3} = a^2 e^2 \quad \left(\because e^2 = \frac{a^2+b^2}{a^2}\right)$$



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Let $P(3,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to

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Solution:

Above normal intersects the x-axis at $(9,0)$

$$\therefore \frac{a^2(9)}{3} + 0 = a^2e^2$$

$$\Rightarrow e^2 = 3$$

Now, $e^2 = \frac{a^2+b^2}{a^2} \Rightarrow 3 = 1 + \frac{b^2}{a^2}$

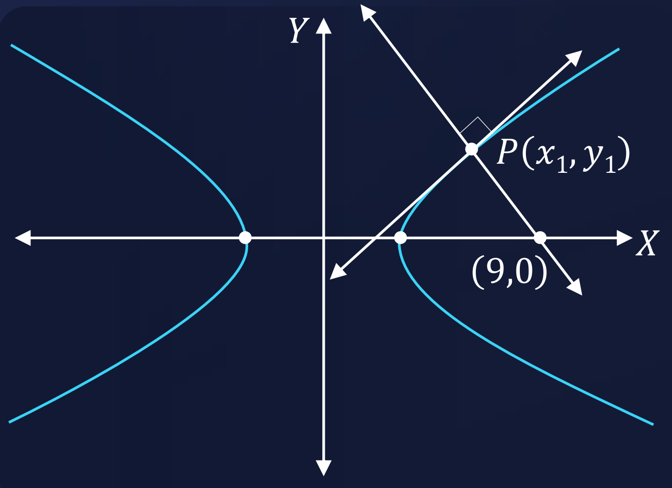
$$\Rightarrow b^2 = 2a^2 \cdots (ii)$$

Substitute in (i),

$$\frac{9}{a^2} - \frac{9}{2a^2} = 1$$

$$\Rightarrow a^2 = \frac{9}{2}$$

$$\boxed{\therefore (a^2, e^2) \equiv \left(\frac{9}{2}, 3\right)}$$



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Let $P(3,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to

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A $(9,3)$

B $\left(\frac{9}{2}, 2\right)$

C $\left(\frac{9}{2}, 3\right)$

D $\left(\frac{3}{2}, 2\right)$

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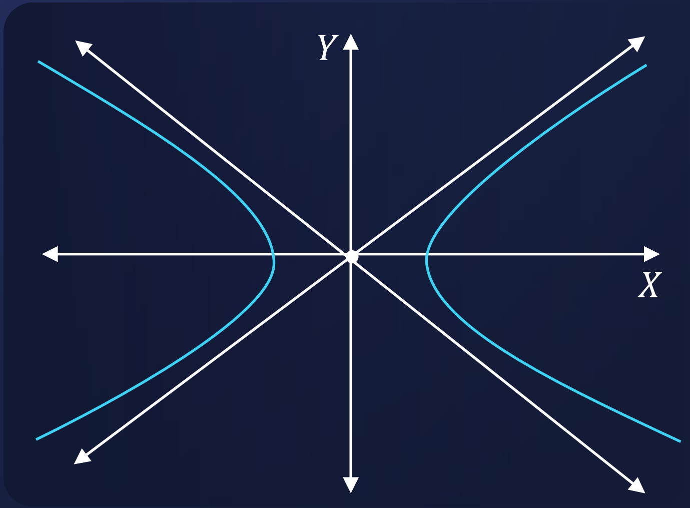
Asymptotes:

If the distance between hyperbola and a line tends to zero when a one or both of x and y coordinates approach to infinity, then that line is called 'Asymptote' to the hyperbola.

Asymptote is tangent to hyperbola at infinity.



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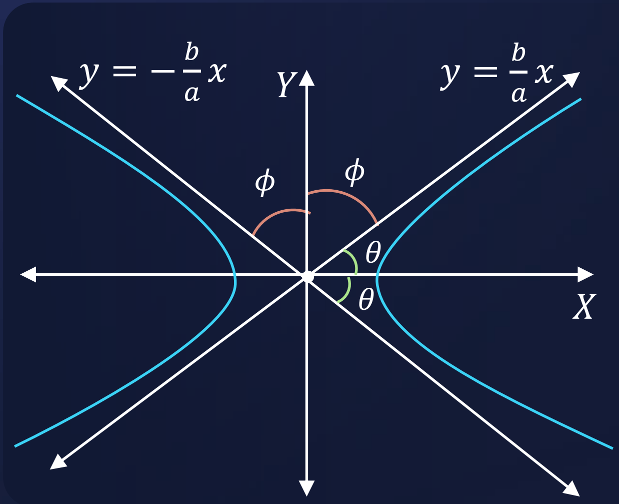
Note:

Asymptotes always pass through the center of hyperbola.

Transverse and the conjugate axes act as angle bisectors of the angle between the asymptotes.



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Note:

$i)$ If θ is the acute angle between asymptote.

$$m_1 = \frac{b}{a}, m_2 = -\frac{b}{a}$$

$$\tan\theta = \left|\frac{\frac{b}{a}+\frac{b}{a}}{1-\frac{b^2}{a^2}}\right| = \left|\frac{2ab}{a^2-b^2}\right|$$

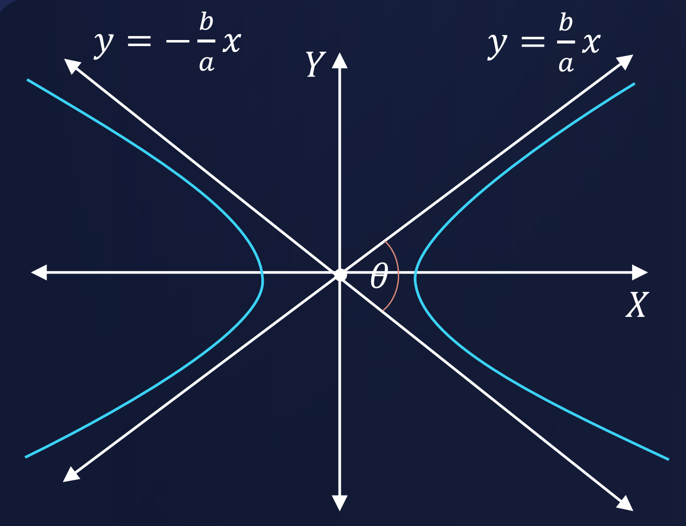
$$\boxed{\tan\theta = \left|\frac{m_1-m_2}{1+m_1 m_2}\right|}$$

$$\tan\frac{\theta}{2} = \frac{b}{a}$$

$$\Rightarrow \theta = 2\tan^{-1}\frac{b}{a}$$

Also, $\frac{b}{a} = \sqrt{e^2-1} = \tan\frac{\theta}{2}$

$$e^2 = 1+\tan^2\frac{\theta}{2} \;\Rightarrow e = \sec\frac{\theta}{2}$$



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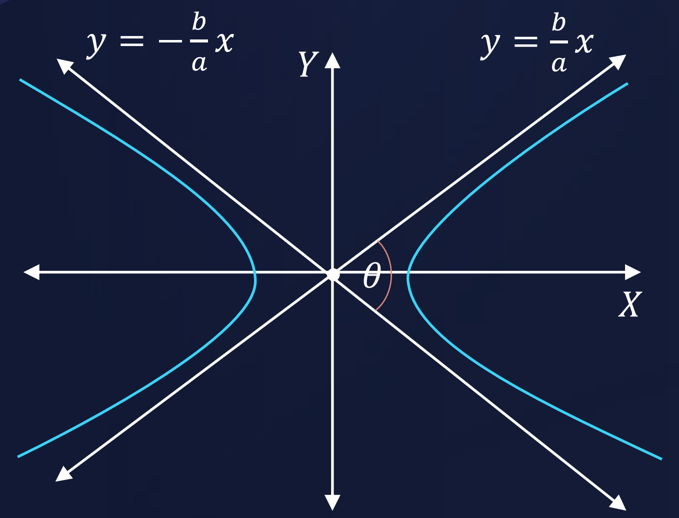
Note:

ii) A hyperbola and its conjugate have the same asymptotes.

$$\text{H}: \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$$

$$\text{A}: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$\text{C}: \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$$



NOTE:

Hyperbola, pair of asymptotes and conjugate hyperbola differ by a constant with $\mathrm{H} + \mathrm{C} = 2\mathrm{A}$

If $H: ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

then $A: ax^2 + by^2 + 2hxy + 2gx + 2fy + c + k = 0$

($\Delta = 0$ for straight line)

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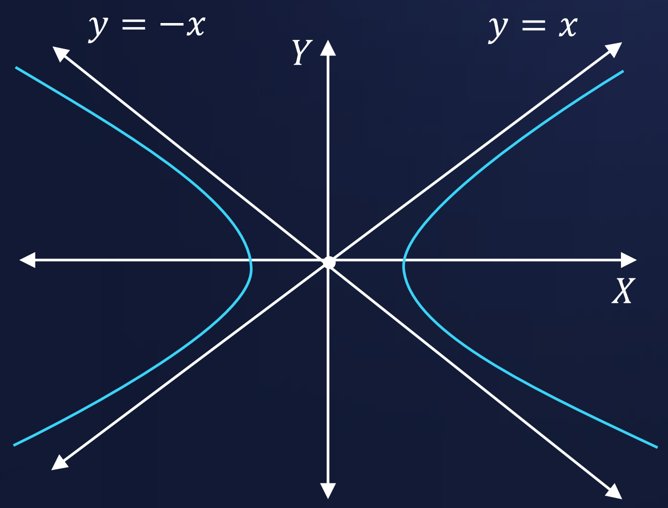
Note:

$iii)$ For a rectangular hyperbola $x^2 - y^2 = a^2$, equation of asymptotes is $y = \pm x$ which are at right angles.



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Find the equation of the asymptotes of the hyperbola $xy - 3y - 2x = 0$.

Solution:

Given hyperbola : $xy - 3y - 2x = 0$

Combined equation of pair of asymptotes is

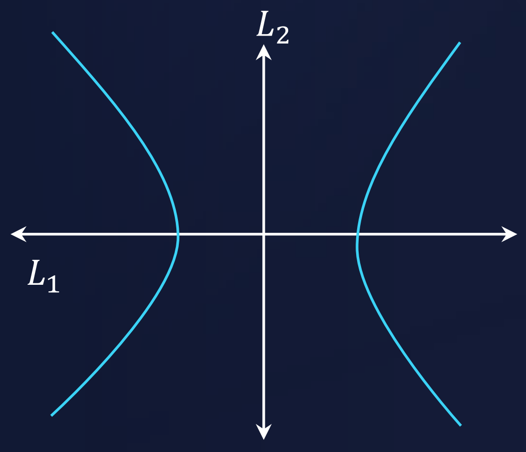
$xy - 3y - 2x + \lambda = 0$ (represents a pair of lines)

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 0 + \frac{3}{2} - 0 - 0 - \frac{\lambda}{4} = 0$

$\Rightarrow \lambda = 6$

$$\left(\begin{array}{l} a = 0, b = 0, c = \lambda \\ h = \frac{1}{2}, g = -1, f = -\frac{3}{2} \end{array} \right)$$



Equation of pair of asymptotes is $xy - 2x - 3y + 6 = 0$

$\Rightarrow x(y-2) - 3(y-2) = 0$

$\Rightarrow (x-3)(y-2) = 0$

$\therefore$ Equation of the asymptotes are $x - 3 = 0$ and $y - 2 = 0$

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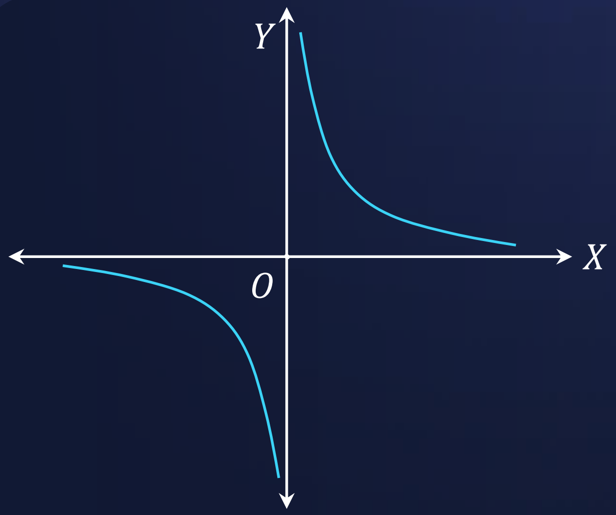
Rectangular Hyperbola: $xy = c^2$

When the centre of any rectangular hyperbola is at the origin and its asymptotes coincide with the co-ordinate axes, its equation is $xy = c^2$.



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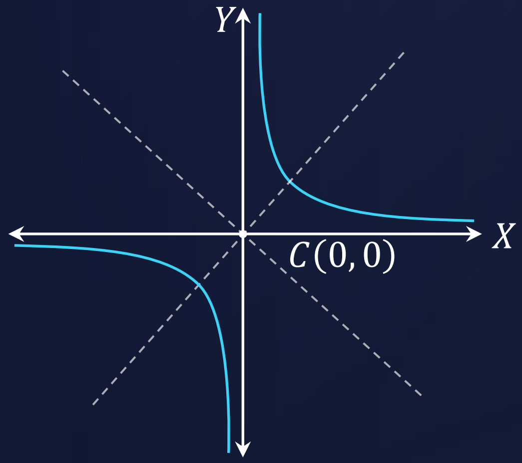
Terms related to Hyperbola : $xy = c^2$

Asymptotes:	$x = 0, y = 0$
Eccentricity:	$\sqrt{2}$
Center:	$(0, 0)$
Equation of T.A.:	$y = x$
Equation of C.A..	$y = -x$



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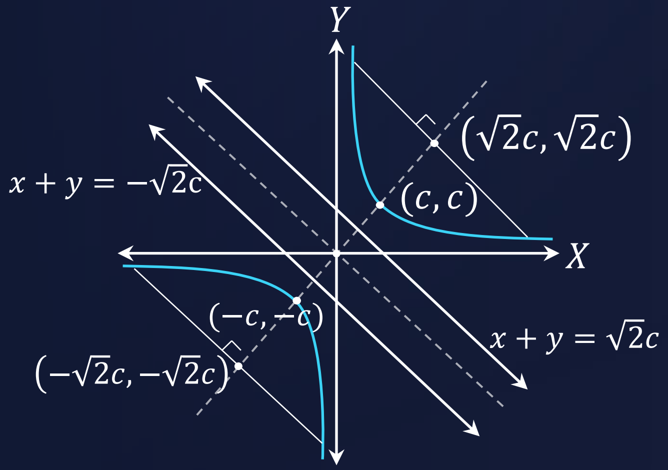
Terms related to Hyperbola : $xy = c^2$

Vertices:	$(c, c), (-c, -c)$
Length of $T.A. (= C.A.)$:	$2\sqrt{2}c$
Foci:	$(\sqrt{2}c, \sqrt{2}c), (-\sqrt{2}c, -\sqrt{2}c)$
Equation of L.R.:	$x + y = \pm 2\sqrt{2}c$
Length of L.R.:	$2\sqrt{2}c$
Equation of directrices:	$x + y = \pm\sqrt{2}c$



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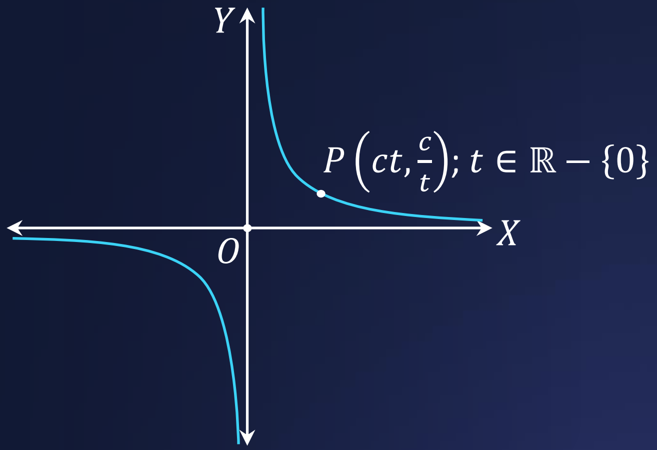
Key Takeaways

Parametric Equation

For $xy = c^2$

$$x = ct\ , y = \frac{c}{t} \quad (t \neq 0)$$

is the parametric form of equation.



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Key Takeaways

Equation of Tangent : $xy = c^2$

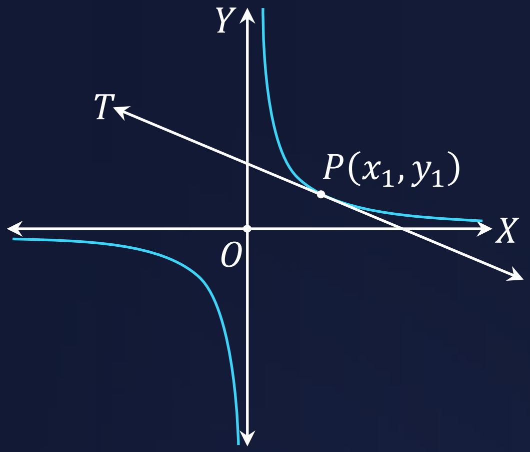
For $xy = c^2$

Equation of tangent in point form is $T = 0$.

Replace $xy \rightarrow \frac{xy_1 + yx_1}{2}$

$\therefore$ Equation of tangent becomes $xy_1 + x_1 y = 2c^2$

Slope of tangent is $-\frac{y_1}{x_1}$



Note :

- Slope of tangent is always negative.

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Key Takeaways

For $xy = c^2$

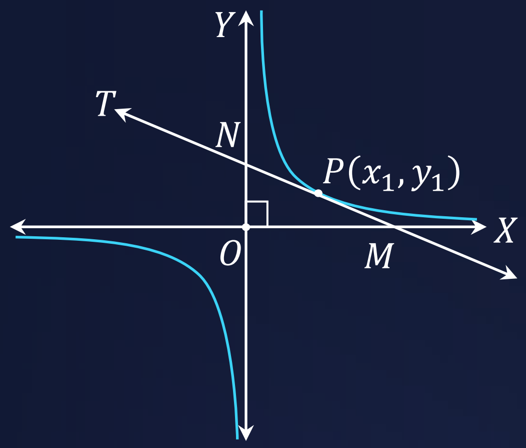
Equation of tangent $xy_1 + x_1y = 2c^2$

x – intercept i.e., $OM = \frac{2c^2}{y_1}$ and

y – intercept i.e., $ON = \frac{2c^2}{x_1}$

$\therefore$ Area formed by the tangent w.r.t coordinate axes i.e., $Ar(\Delta \mathrm{OMN})$

$$= \frac{1}{2} \times \frac{2c^2}{y_1} \times \frac{2c^2}{x_1} = \frac{2c^2 \times 2c^2}{2c^2} = 2c^2 sq.\text{ units.}$$



Note :

- Area formed by the tangent w.r.t coordinate axes is always constant.

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Let a line $L: 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

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A. -24

B. 24

C. 12

D. -12

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Let a line $L: 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

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Solution:

$x^2 - y^2 = 3$ is a rectangular hyperbola.

Given line $L: 2x + y = k, k > 0$

$\Rightarrow L: y = -2x + k$ is tangent to $x^2 - y^2 = 3$

$\therefore k = \sqrt{12 - 3} = 3$

$\Rightarrow L: y = -2x + 3$

$\because L = 0$ is also tangent to $y^2 = \alpha x$

$\therefore 3 = \frac{\frac{\alpha}{4}}{-2} \Rightarrow \alpha = -24$

A -24

B 24

C 12

D -12

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Key Takeaways

Equation of Normal : $xy = c^2$

$xy = c^2$

$T : xy_1 + yx_1 = c^2; \quad m_T = -\frac{y_1}{x_1}$

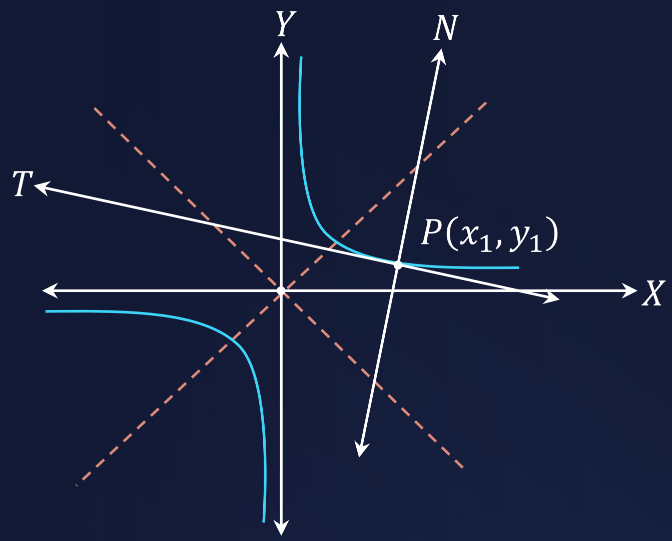
$\therefore m_N = \frac{x_1}{y_1}$

$N : y - y_1 = \frac{x_1}{y_1}(x - x_1)$

$yy_1 - y_1^2 = xx_1 - x_1^2$

$\therefore$ Equation of normal in the point form is

$xx_1 - yy_1 = x_1^2 - y_1^2$



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Key Takeaways

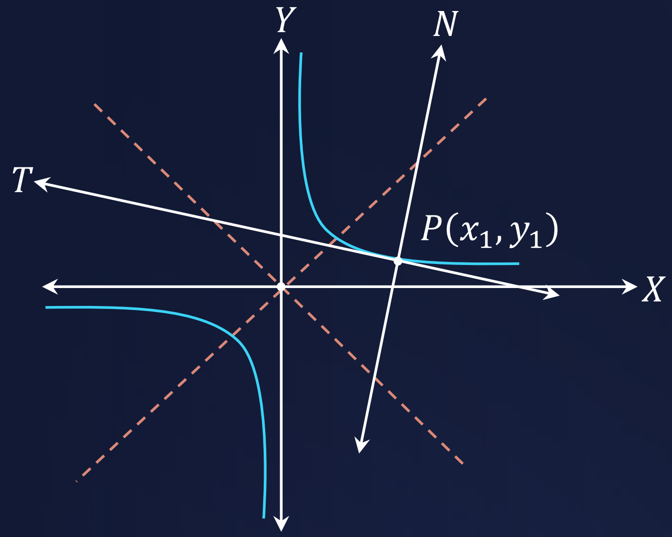
Note :

$$xy = c^2$$

$$N : xx_1 - yy_1 = x_1^2 - y_1^2$$

$$\therefore m_N = \frac{x_1}{y_1} \begin{cases} x_1 > 0, y_1 > 0 \rightarrow +ve \\ x_1 < 0, y_1 < 0 \rightarrow +ve \end{cases}$$

- Slope of normal is always positive.



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Key Takeaways

Parametric Form :

$$xy = c^2$$

$$T : xy_1 + yx_1 = c^2; \quad m_T = -\frac{y_1}{x_1}$$

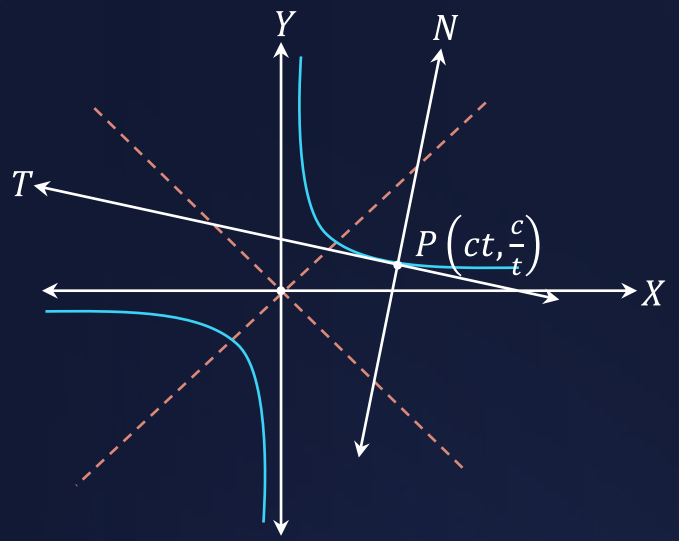
$$x_1 = ct, y_1 = \frac{c}{t}$$

Differentiating $xy = c^2$ w.r.t x.

$$y + x\frac{dy}{dx} = 0$$

$$\therefore m_T = \left.\frac{dy}{dx}\right|_P = -\left.\frac{y}{x}\right|_P = -\frac{\frac{c}{t}}{ct} = -\frac{c}{t} \times \frac{1}{ct} = -\frac{1}{t^2}$$

$$\therefore m_N = t^2$$



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For the hyperbola $xy = 16$, which of the following is/are true ?

A. Length of Latus Rectum is $8\sqrt{2}$

B. Tangent at $(2, 8)$ is $2x + y - 12 = 0$

C. Chord with midpoint $(5, 6)$ is $6x + 5y = 60$

D. Normal at $(4,3)$ is $4x - 3y + 7 = 0$

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For the hyperbola $xy = 16$, which of the following is/are true ?

Solution:

Given: Rectangular Hyperbola $xy = 16$

$$\Rightarrow c^2 = 16$$

(i) L.L.R. $= \dfrac{2b^2}{a}$

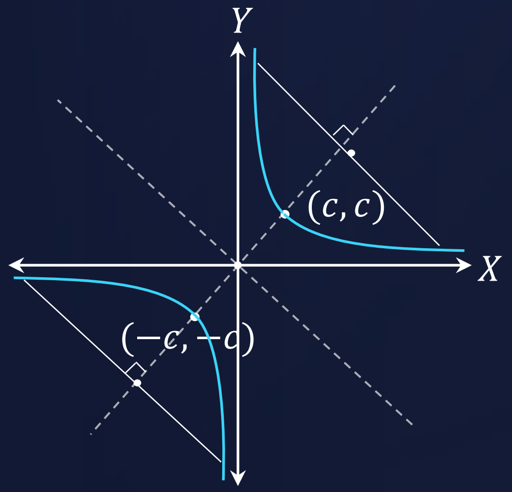
$= 2a \quad (\because a = b)$

$= 2\sqrt{2}c = 8\sqrt{2} \quad (\because a = \sqrt{2}c)$

(ii) Tangent at (2,8): $xy_1 + yx_1 = 2c^2$

$$\Rightarrow 8x + 2y = 32$$

$$\Rightarrow 4x + y = 16$$



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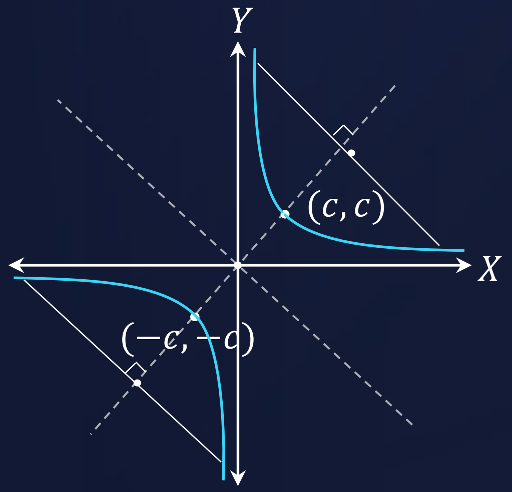
For the hyperbola $xy = 16$, which of the following is/are true ?

Solution:

Given: Rectangular Hyperbola $xy = 16$

$$\Rightarrow c^2 = 16$$



(iii) Chord with mid-point (5,6): $T = S_1$

$$\Rightarrow \frac{xy_1 + yx_1}{2} - c^2 = x_1 y_1 - c^2$$

$$\Rightarrow \frac{6x + 5y}{2} - 16 = 30 - 16$$

$$\Rightarrow 6x + 5y = 60$$

(iv) Normal at (4,3): $xx_1 - yy_1 = x_1^2 - y_1^2$

$$\Rightarrow 4x - 3y = 7$$

$$\Rightarrow 4x - 3y - 7 = 0$$

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For the hyperbola $xy = 16$, which of the following is/are true ?

A. $L.L.R$ is $8\sqrt{2}$

B. Tangent at $(2,8)$ is $2x + y - 12 = 0$

C. Chord with midpoint $(5,6)$ is $6x + 5y = 60$

D. Normal at $(4,3)$ is $4x - 3y + 7 = 0$

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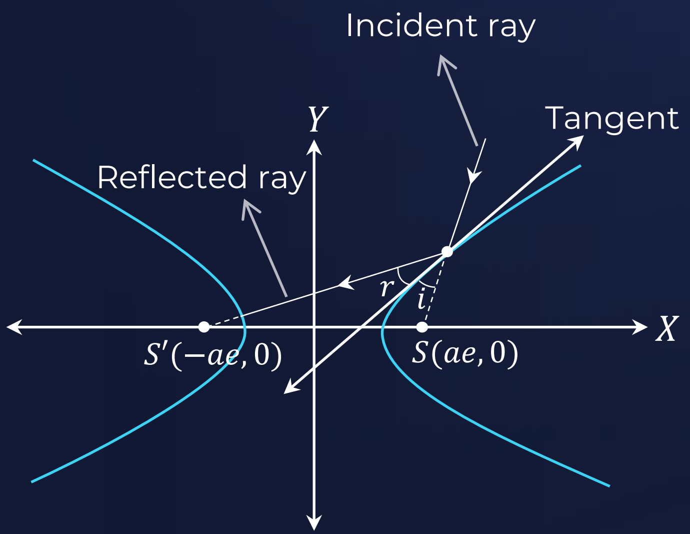
Geometrical properties of hyperbola :

(i) If an incoming light ray passing through one focus S strike convex side of the hyperbola then it will get reflected towards other focus S'.



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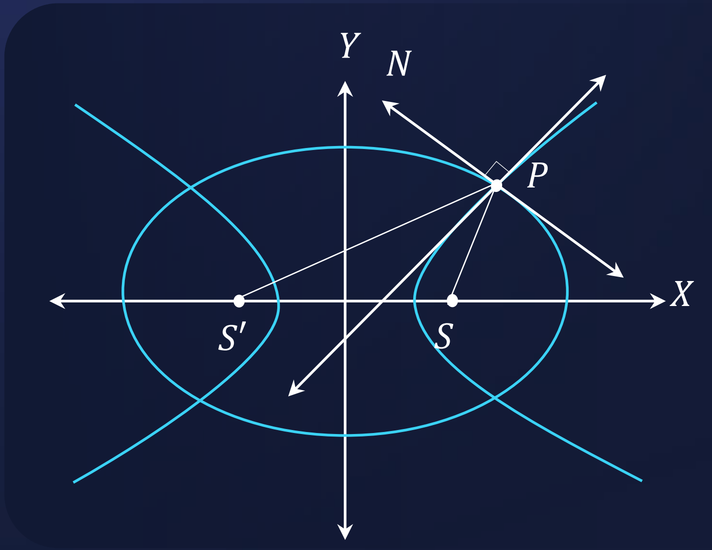
Geometrical properties of hyperbola :

(ii) If an ellipse and a hyperbola are confocal, then they intersect orthogonally.



Note:

Conversely, if ellipse and hyperbola intersect orthogonally, then they are confocal.

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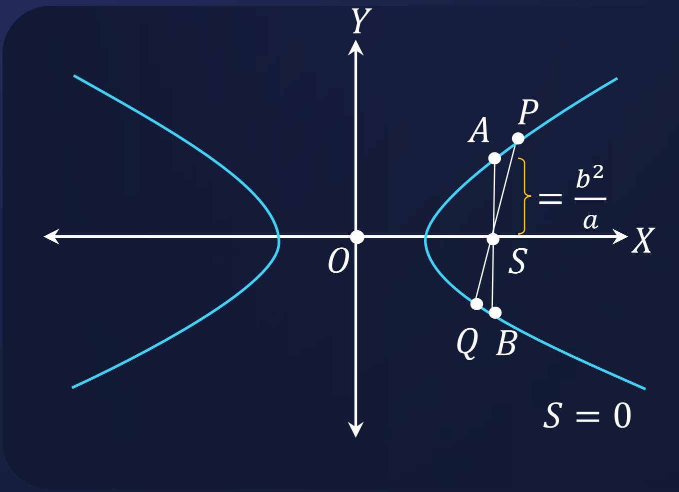
Geometrical properties of hyperbola :

(iii) If PSQ is a focal chord of the hyperbola $S = 0$, then $\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$ or Semi latus rectum is the H.M. of SP and SQ.



$\frac{b^2}{a}$ = Harmonic mean $(PS \,\&\, QS)$

$|PS|, \frac{b^2}{a}, |QS| \rightarrow$ H.P.

$\Rightarrow \frac{1}{|PS|}, \frac{a}{b^2}, \frac{1}{|QS|} \rightarrow$ A.P.

$$\Rightarrow 2\left(\frac{a}{b^2}\right) = \frac{1}{|PS|} + \frac{1}{|QS|}$$

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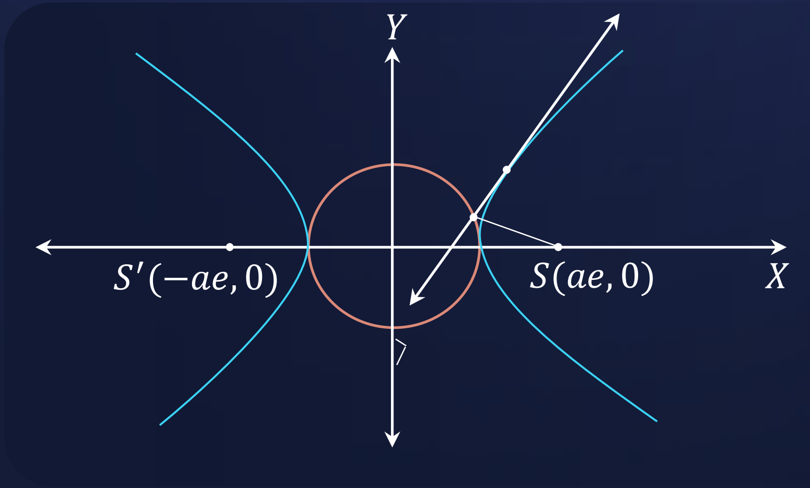
Geometrical properties of hyperbola :

(iv) Feet of perpendiculars from the foci upon any tangent to a hyperbola, lie on the auxiliary circle.



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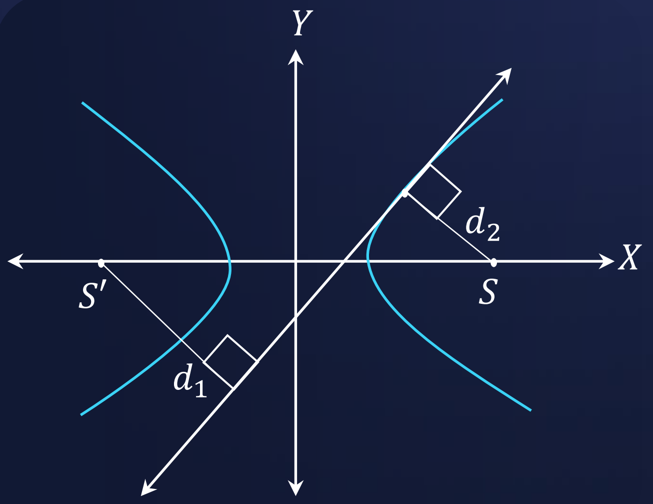
Geometrical properties of hyperbola :

(v) Product of the lengths of perpendiculars from foci upon any tangent of a hyperbola, is equal to the square of the semi-conjugate axis.



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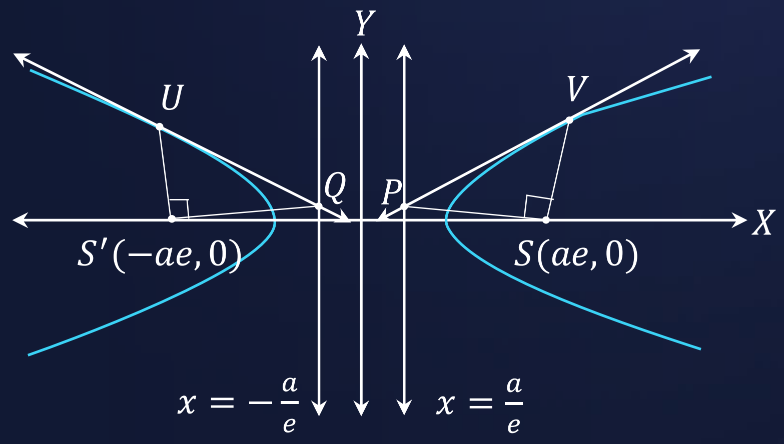
Geometrical properties of hyperbola :

(vi) The portion of the tangent to a hyperbola between the point of contact and the directrix subtends a right angle at the corresponding focus.



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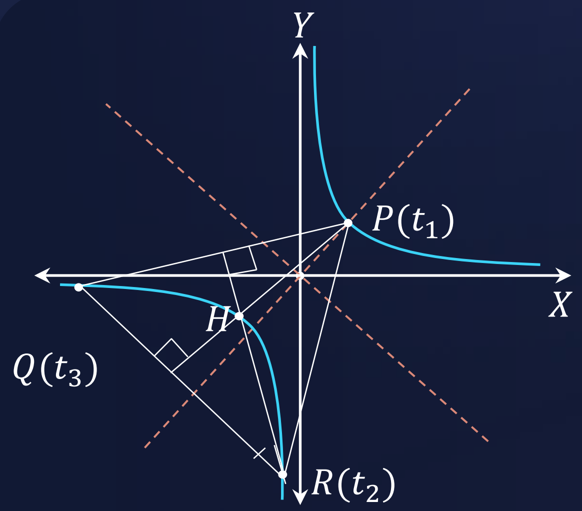
Geometrical properties of hyperbola :

(vii) If a triangle has its vertices on a rectangular hyperbola, then its orthocentre also lies on the same hyperbola.

$$H \equiv \left(-\frac{c}{t_1t_2t_3}, -ct_1t_2t_3\right)$$



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Sum of length of perpendiculars drawn from foci to any real tangent to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is always greater than or equal to a, then find the value of $'a'$.

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Solution:

Property: Product of the lengths of perpendiculars from foci upon any tangent of a hyperbola, is equal to the square of the semi-conjugate axis.

$$P_1 \cdot P_2 = b^2$$

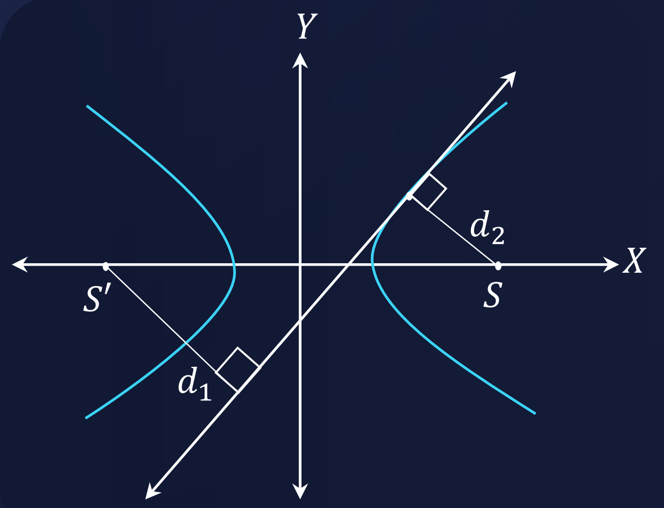
$$P_1 \cdot P_2 = 9$$

$$A.M. \geq G.M.$$

$$\frac{P_1+P_2}{2} \geq \sqrt{9}$$

$$P_1 + P_2 \geq 6$$

$$\therefore a = 6$$



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Thank You

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