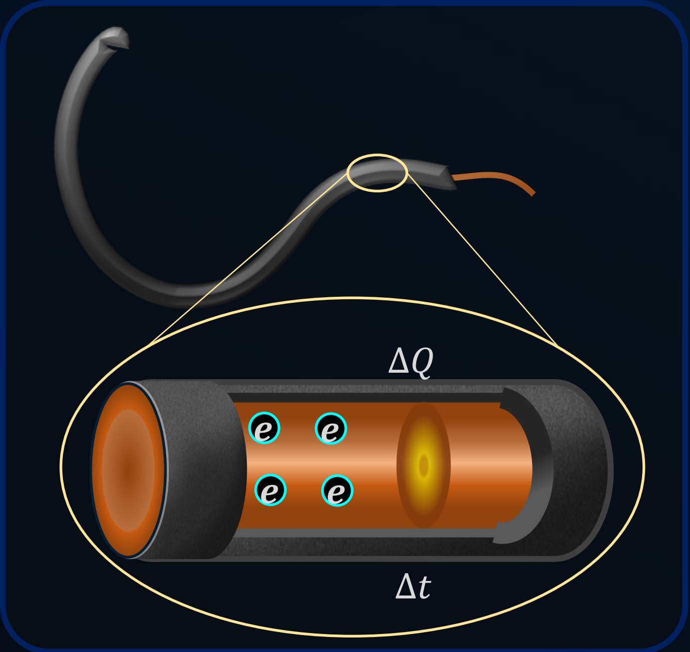
Aakash
BYJU'S NOTES

Current Electricity

Electric Current



- The net amount of charge flowing across the area in the time interval Δt, is defined to be the average current across the area.

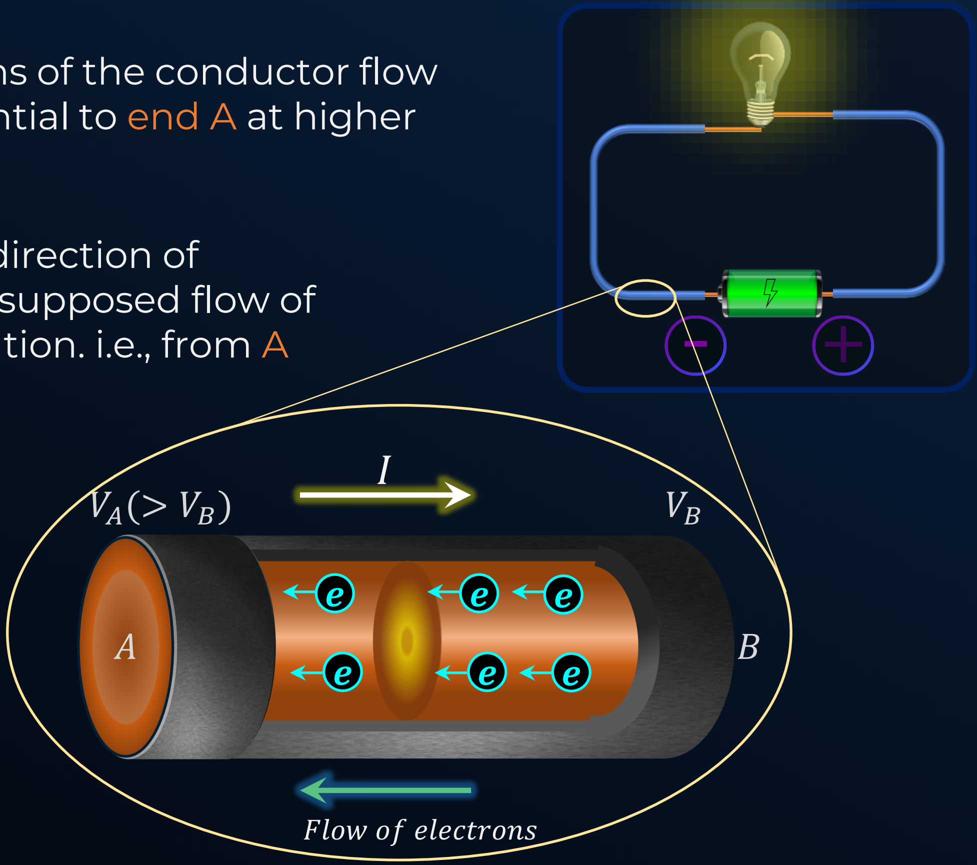
$$i_{avg} = \frac{\Delta Q}{\Delta t}$$

- SI unit of electric current is $ampere$ (A).
- Electric current has direction as well as magnitude but it is a scalar quantity.
- If charge dQ is flowing across the area in an infinitesimally small time interval dt, it is defined to be the instantaneous current across that area.

$$i_{inst} = \frac{dQ}{dt}$$

Direction of Conventional Electric Current

- In reality, the free electrons of the conductor flow from end B at lower potential to end A at higher potential.
- However, we choose the direction of electric current as that of supposed flow of positive charge as convention. i.e., from A to B.



The current in a conductor varies with time t as, $i = 2t + 3t^2\ A$. where i is in ampere and t is in second. Find the electric charge flowing through a section of the conductor during $t = 2\ s$ to $t = 3\ s$.

Given $i = (2t + 3t^2)\ A$

To find Q during $t = 2\ s$ to $t = 3\ s$

Charge flowing in a time interval is given by:

$$Q = \int_2^3 i\, dt$$

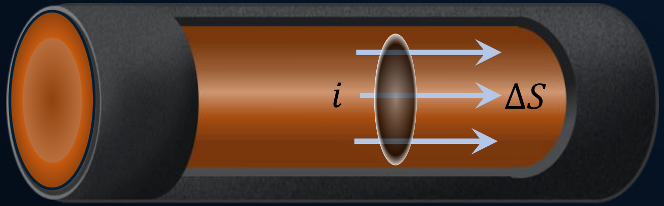
$$\Rightarrow Q = \int_2^3 (2t + 3t^2)\, dt$$

$$\Rightarrow Q = [t^2]_2^3 + [t^3]_2^3$$

$$\therefore Q = 24\ C$$

Electric Current Density

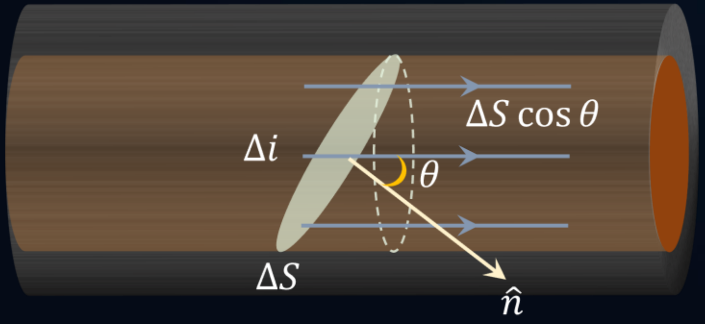
- A vector quantity whose magnitude is equal to electric current per unit normal cross-sectional area inside a conductor.



$$j_{avg} = \frac{\Delta i}{\Delta S}$$

$$j_{inst} = \frac{di}{dS}$$

- Under electrostatic conditions, $\vec{j} = 0$



- If the current Δi exists through an area ΔS which makes an angle θ with the current, as shown in the figure, then

$\Delta i = \vec{j}.\Delta\vec{S}$, $\vec{j} \rightarrow$ Uniform current density

The current density: $j = \dfrac{\Delta i}{\Delta S \cos\theta}$

- For non-uniform current density,

$$i = \int \vec{j}.d\vec{S}$$

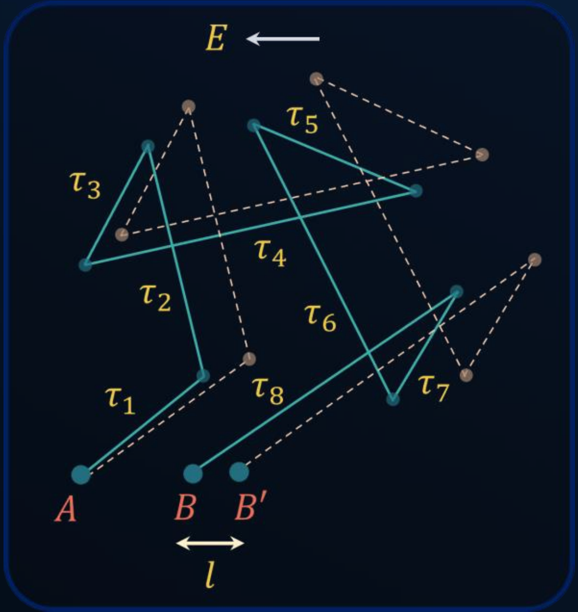
Drift Speed

- In a conductor, the free electrons exhibit a random motion (known as Brownian motion). During this motion, the electrons collide with the heavy fixed ions. But after collision, electrons emerge with same speed but in random directions.

- After application of an electric field, average velocity attained by electrons in a material is known as drift speed.

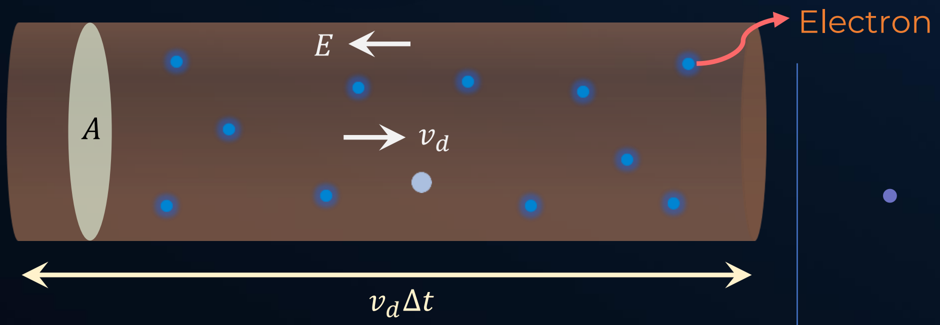
$$v_{avg} = v_d = \frac{eE}{m}\tau$$

- The average time interval between two successive electron collisions is called relaxation time(τ).



Mobility

- Physically, the mobility (μ) refers to the ease with which the charge carriers move in the medium.



n = No. of free electrons per unit volume

or

Mobile electron density

$$\boxed{\mu = \frac{v_d}{E} = \frac{e\tau}{m}}$$

- Electric current,

$$\boxed{i = neAv_d}$$

- Current density,

$$\boxed{j = nev_d}$$

By substituting the value of v_d in above equation, we get,

$$j = nev_d = \frac{ne^2\tau}{m}E = \sigma E$$

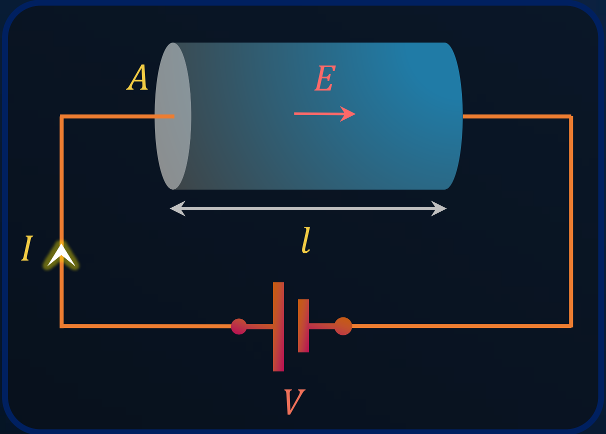
$\boxed{\vec{j} = \sigma\vec{E}}$, σ = electrical conductivity of material

Resistivity, $\boxed{\rho = \frac{1}{\sigma} = \frac{m}{ne\tau^2}}$

Ohm's Law

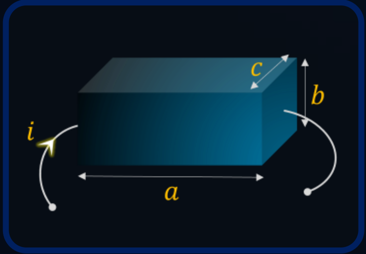
- The current through a conductor (I) is directly proportional to the potential difference (V) applied across its ends.

$$\boxed{V = RI}$$

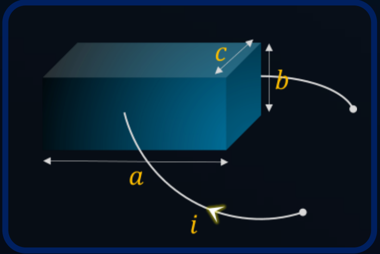


- Resistance of a given cylindrical conductor,

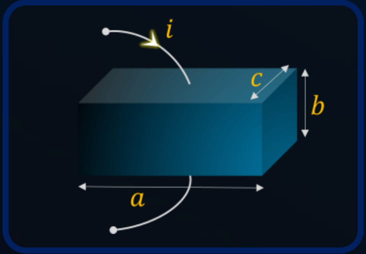
$$\boxed{R = \frac{\rho l}{A}}$$



$$R = \frac{\rho a}{bc}$$



$$R = \frac{\rho c}{ba}$$



$$R = \frac{\rho b}{ac}$$

Resistance of a conductor depends on:

- Resistivity of material
- Dimensions

Find the resistance of a copper coil of total wire-length $10\ m$ and area of cross-section $1.0\ mm^2$. What would be the resistance of a similar coil of aluminium? The resistivity of copper $= 1.7 \times 10^{-8}\ \Omega - m$ and that of aluminium $= 2.6 \times 10^{-8} \Omega - m$.

$$R_{Cu} = \frac{\rho_{Cu} \times l}{A} = \frac{1.7 \times 10^{-8} \times 10}{1 \times 10^{-6}} = 0.17\ \Omega$$

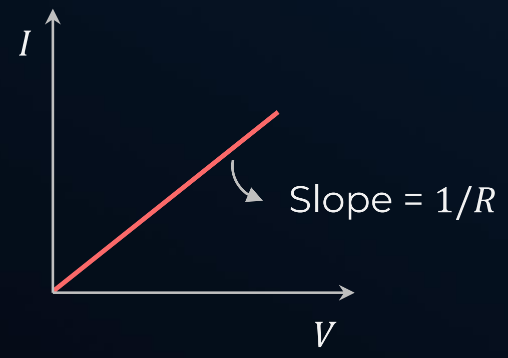
$$R = \frac{\rho l}{A} \Rightarrow R \propto \rho \qquad \text{[if } l \text{ \& } A \text{ are kept same]}$$

$$\frac{R_{Cu}}{R_{Al}} = \frac{\rho_{Cu}}{\rho_{Al}} \Rightarrow R_{Al} = \frac{\rho_{Al}}{\rho_{Cu}} \times R_{Cu} = \frac{2.6 \times 10^{-8}}{1.7 \times 10^{-8}} \times 0.17 = 0.26\ \Omega$$

Ohmic & Non-ohmic Materials

Validity of Ohm's law

Ohmic Materials



- Metals at low V and I (Temperature and other physical conditions must remain constant)
Ex. Nichrome

Non-ohmic Materials

- Semiconductors & alloys.

A conductor of length l has a circular cross section as shown. The radius of cross section varies from a to b. The resistivity of the material is ρ. Assume that $b - a \ll l$, find the resistance of the conductor across the ends P and Q.

Solution:

We know resistance of a cylindrical conductor, $\boxed{R = \frac{\rho l}{A}}$

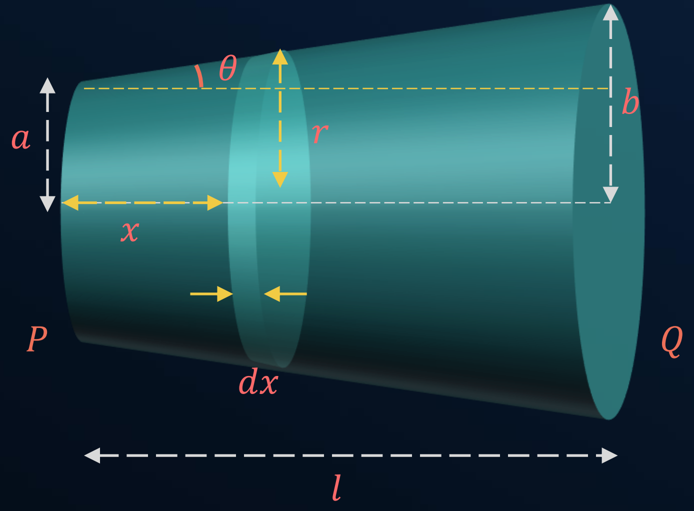
For elemental disc, $\Rightarrow dR = \frac{\rho dx}{\pi r^2}$

Radius of element at x distance from end P,

$$\Rightarrow r = a + \left(\frac{b-a}{l}\right)x$$

Resistance of the conductor,

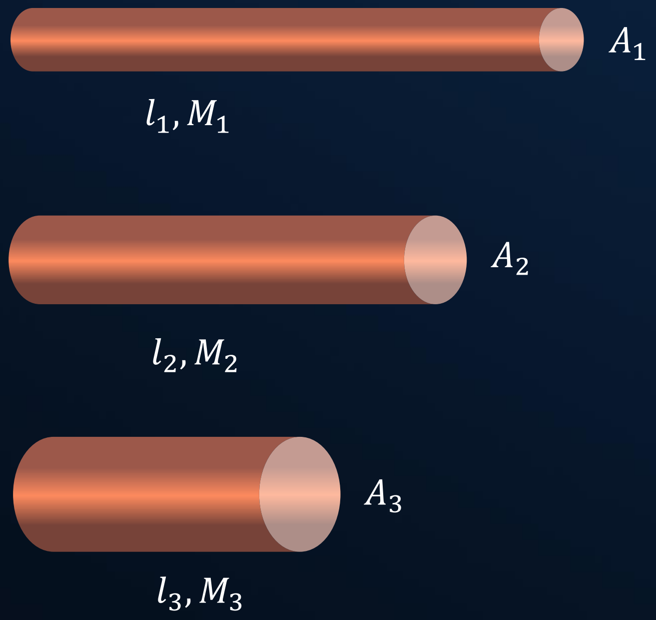
$$\Rightarrow R = \int_a^b \frac{\rho}{\pi r^2}\left(\frac{l}{b-a}\right)dr \quad \Rightarrow \quad \boxed{R = \frac{\rho l}{\pi ab}}$$



The masses of the wires of copper are in the ratio $1:3:5$ and their lengths are in the ratio $5:3:1$. The ratio of their electrical resistance is

Solution: $M_1:M_2:M_3 = 1:3:5, \quad l_1:l_2:l_3 = 5:3:1$

We know resistance of a cylindrical conductor, $\boxed{R = \dfrac{\rho l}{A}}$



$$M = Al \times d$$

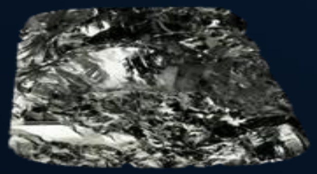
$$\Rightarrow R = \frac{\rho l^2 d}{M} \qquad \boxed{\Rightarrow R \propto \frac{l^2}{M}}$$

Ratio of R_1, R_2 & R_3,

$$\therefore R_1 : R_2 : R_3 = \frac{5^2}{1} : \frac{3^2}{3} : \frac{1^2}{5}$$

$$\boxed{R_1 : R_2 : R_3 = 125 : 15 : 1}$$

Resistivity Dependence Factors – Types of Material

Conductor	Semiconductor	Insulator
		
$\rho \approx 10^{-12} - 10^{-6}\ \Omega - m$	$\rho \approx 10^{-5} - 10^{5}\ \Omega - m$	$\rho \approx 10^{5} - 10^{16}\ \Omega - m$

$$\rho = \frac{m}{ne^2\tau}$$

Resistivity Dependence Factors – Temperature

Change in resistivity with temperature :

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

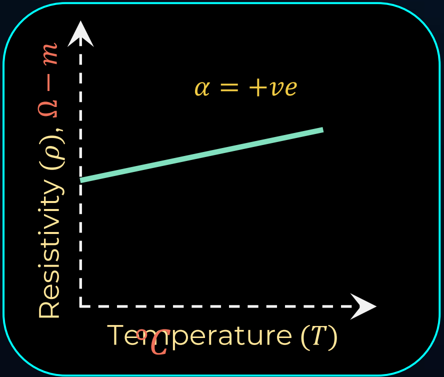
ρ = resistivity at temperature T
ρ_0 = resistivity at temperature T_0
α = temperature coefficient of resistivity

Resistance of a conductor at temperature T :

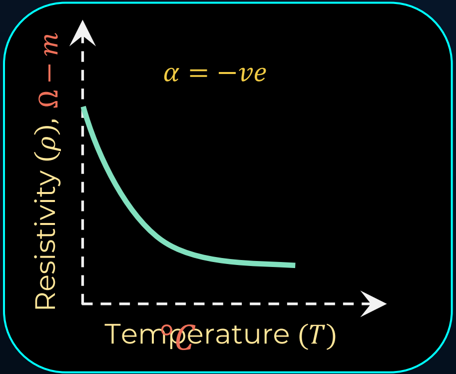
$$R = R_0[1 + \alpha(T - T_0)]$$

Effect of Temperature on Resistivity of Different Materials

For Conductors



For Semiconductors



The temperature coefficient of resistance of wire is $0.00125\ °C^{-1}$. At $300\ K$ its resistance is $1\ \Omega$. At what temperature will its resistance become 2Ω ?

Solution : $R_1 = 1\ \Omega, R_2 = 2\ \Omega, T_1 = 300\ K = 27\ °C, \alpha = 0.00125\ °C^{-1}$

Let the resistance of wire is R_0 at $T = 0\ °C$. Hence,

$$R_1 = R_0(1 + \alpha T_1) \quad \& \quad R_2 = R_0(1 + \alpha T_2)$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{(1 + \alpha T_1)}{(1 + \alpha T_2)}$$

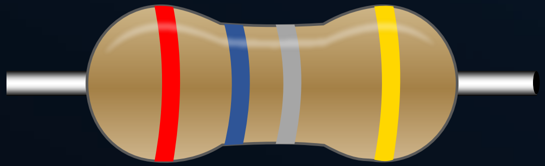
$$\Rightarrow \quad \alpha = \frac{R_1 - R_2}{R_2 T_1 - R_1 T_2}$$

$$1.25 \times 10^{-3} = \frac{1 - 2}{2(27) - 1(T_2)} \Rightarrow \quad T_2 = 854\ °C \quad \text{or} \quad \boxed{T_2 = 1127\ K}$$

Resistor and Colour Coding

- Resistor is an object with desired resistance.

Resistor



Symbol of resistor

1 2 3 4

4 → Colour 4 or no colour stands for tolerance or possible variation in percentage about the indicated values.

3 → Indicates the decimal multiplier.

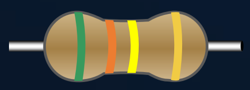
1, 2 → Indicate the first and second significant figures of the resistance in ohm.

Colour Coding of Carbon Resistors

Trick
B
B
R
O
Y
Great
Britain has a
Very
Good
Wife
Gold
Silver
No colour

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	$\pm 5\%$
Silver		10^{-2}	$\pm 10\%$
No colour			$\pm 20\%$

Example:



Digit 1 $= 5$ (Green)

Digit 2 $= 3$ (Orange)

Multiplier $= 10^4$ (Yellow)

Tolerance $= \pm 5\,\%$ (Gold)

Resistance of Carbon resistor,

$$R = 53 \times 10^4 \pm 5\,\%$$

$$\boxed{R = 530\ k\Omega \pm 5\,\%}$$

Ohm's law & Sign Convention

- The current through a conductor (I) is directly proportional to the potential difference(V) applied across its ends.

$$\boxed{V = RI}$$

- Sign Convention :

Case 1: $V_A > V_B$

Along path $A \rightarrow B$

$$V_A - IR = V_B$$

$$\Rightarrow \boxed{V_A - V_B = IR}$$

Case 2: $V_A < V_B$

Along path $A \rightarrow B$

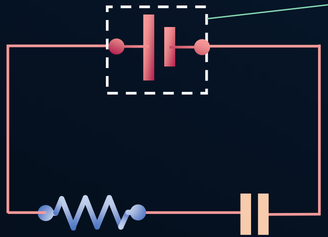
$$V_A + IR = V_B$$

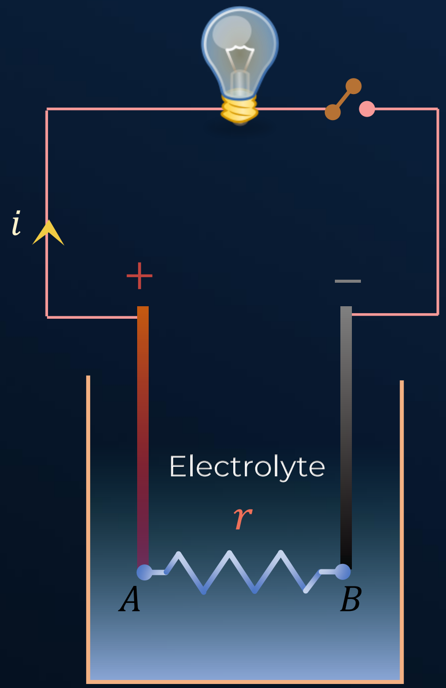
$$\Rightarrow \boxed{V_B - V_A = IR}$$

- Current, by convention is always from higher to lower potential. $(+)ve$ sign is taken for higher potential and $(-)ve$ sign for lower potential.

Electric Cell (Battery)

- Electric cell is a device which maintains potential difference between its terminals.
- Converts chemical energy into electrical energy.



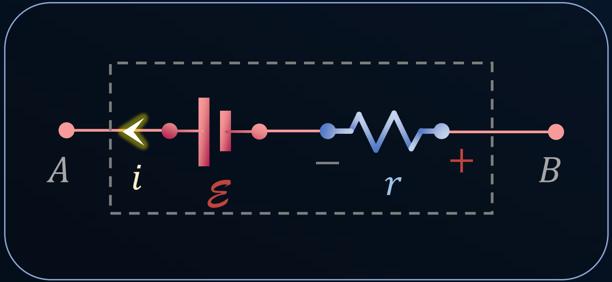


EMF of Electric Cell

- EMF of a battery is defined as the work done (W) in driving a unit charge (q) across the terminals of the cell.
- EMF is also written as the potential difference across the cell when it is not connected to the external circuit.
- r is internal resistance of the cell.
- Potential difference across an ideal cell is equal to its EMF.

Terminal Voltage of a Cell

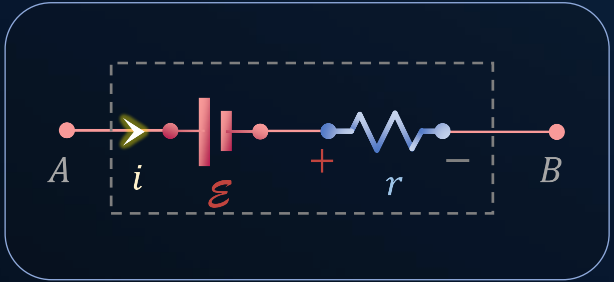
Discharging(Source)



$$V_{AB} = \mathcal{E} - ir$$

- Terminal Voltage is less than EMF of cell.

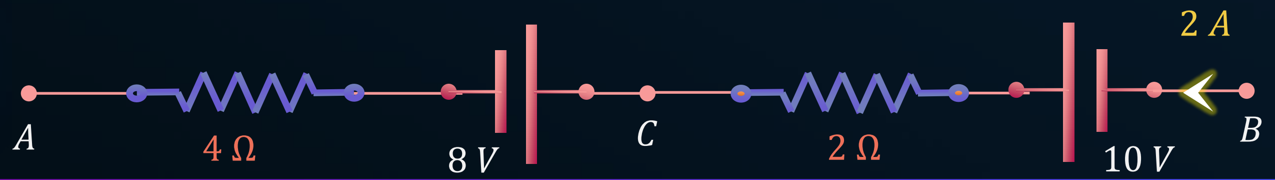
Charging (Load)



$$V_{AB} = \mathcal{E} + ir$$

- Terminal Voltage is Greater than EMF of cell.

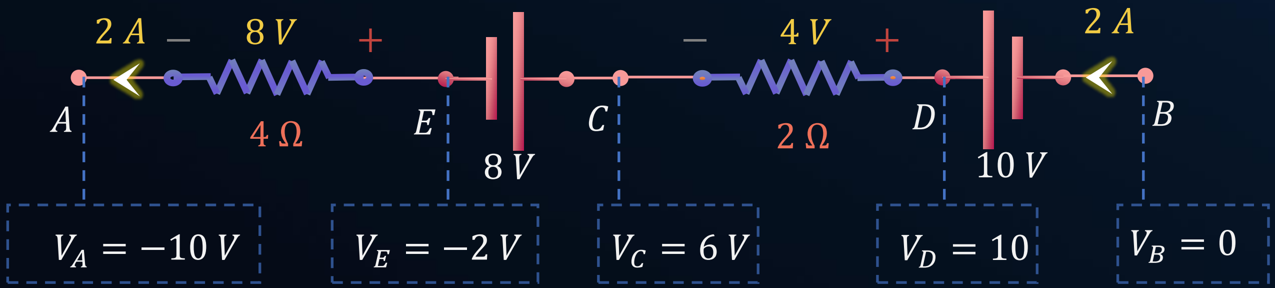
In the diagram shown below, find: $(a)\ V_{AB}$ $(b)\ V_{CA}$ in steady state.



- All elements are in a line. Hence current through all of them is $2\ A$.
- Potential drop across each resistor,

$$V_{2\Omega} = 2 \times 2 = 4\ V \quad \& \quad V_{4\Omega} = 2 \times 4 = 8\ V$$

Potential at every point.

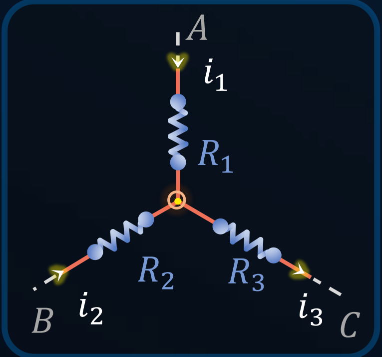


$$\boxed{V_{AB} = -10\ V}$$

$$\boxed{V_{CA} = 16\ V}$$

Kirchhoff's Laws

Kirchhoff's Current Law(KCL)



- Net current at a junction = 0

$i_1 + i_2 - i_3 = 0$

- Incoming current = Outgoing current

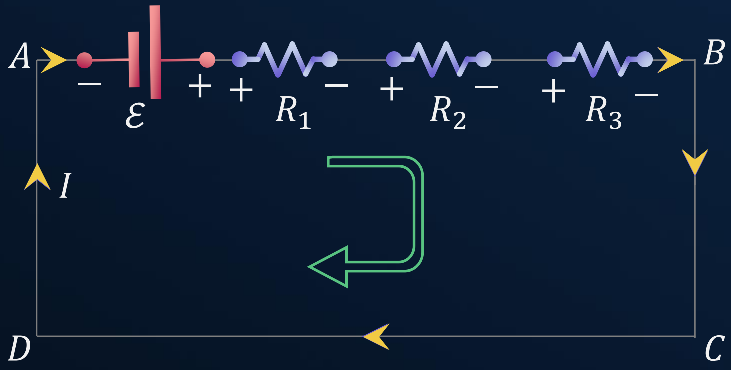
$$\boxed{i_1 + i_2 = i_3}$$

- At the junction $\Rightarrow Q_1 + Q_2 = Q_3$

Sum of charges entering the node = Sum of charges leaving the node

- KCL is based on the conservation of charge principle .

Kirchhoff's Voltage Law(KVL)



- In a closed loop, the algebraic sum of all the potential differences is zero.
- KVL is Based on energy conservation .
- Apply KVL from Point C,

$$+\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$\Rightarrow \boxed{\mathcal{E} = i\,(R_1 + R_2 + R_3)}$$

Sign convention for KVL

For Resistor

- If we choose to traverse along the direction of current , then

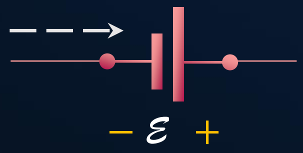
$\Delta V \rightarrow -ve$

- If we choose to traverse opposite to the direction of current then,

$\Delta V \rightarrow +ve$

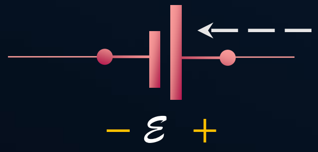
For Battery/Cell

- If you encounter $-ve$ terminal of the cell first, then,



$\Delta V \rightarrow +ve$

- If you encounter $+ve$ terminal of the cell first, then,



$\Delta V \rightarrow -ve$

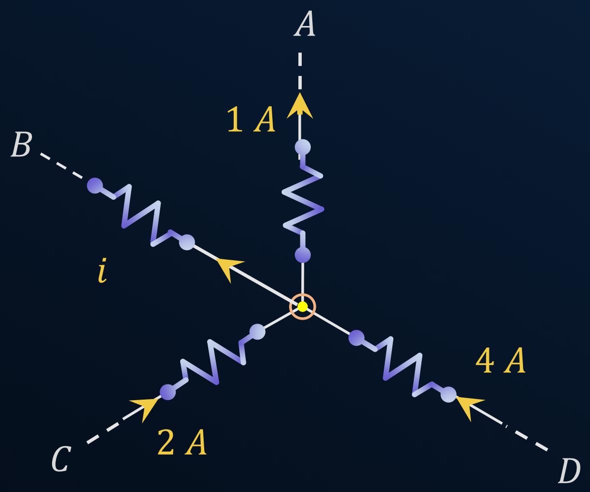
In the electrical circuit shown below, find the value of i.

Assuming direction of i as shown & using KCL ,

Incoming current = Outgoing current

$\Rightarrow 4 + 2 = 1 + i$

$\Rightarrow i = \boxed{5\ A}$



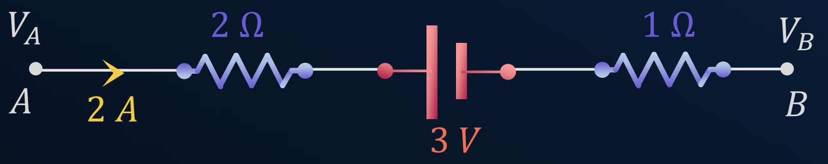
The potential difference $(V_A - V_B)$ across the points A and B in the given figure is ______.

A. $+3\ V$

B. $-3\ V$

C. $+9\ V$

D. $-9\ V$

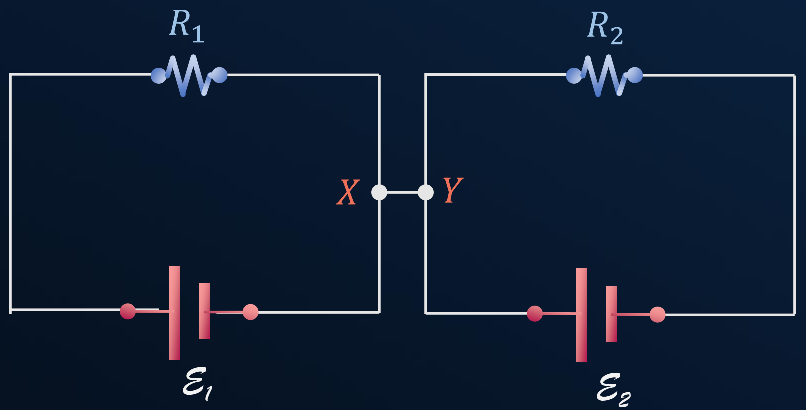


Applying KVL from A to B

$$V_A - 2(2) - 3 - 2(1) = V_B$$

$$V_A - V_B = \boxed{+9\ V}$$

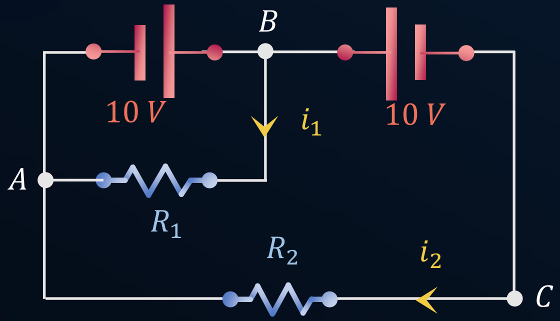
In the circuit shown below, the conductor XY is of negligible resistance. Then find the current through XY.



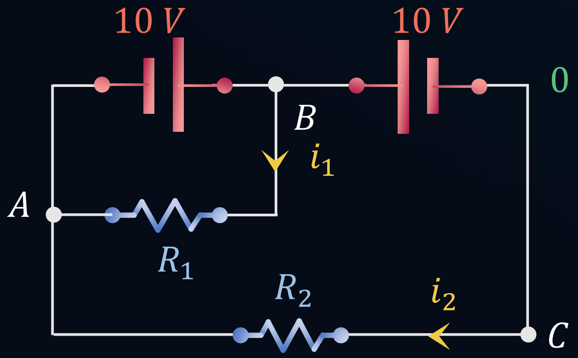
- In a loop entry and exit current of a cell always remains same.
- If some amount of current flows through XY, then enter and exit current will not be same for both the cells. Therefore, KCL will be violating.

No Current will flow through XY

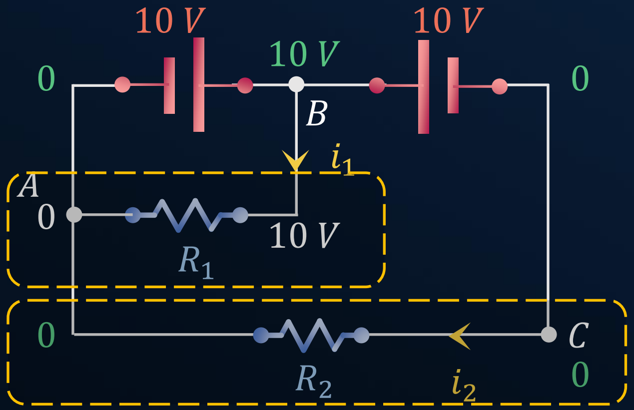
Each of the resistors shown in figure has a resistance of $10\ \Omega$ and each of the batteries has an emf of $10\ V$. Find the currents through the resistors R_1 and R_2 in following circuits:



- Step 1 : Choose zero potential at any point.



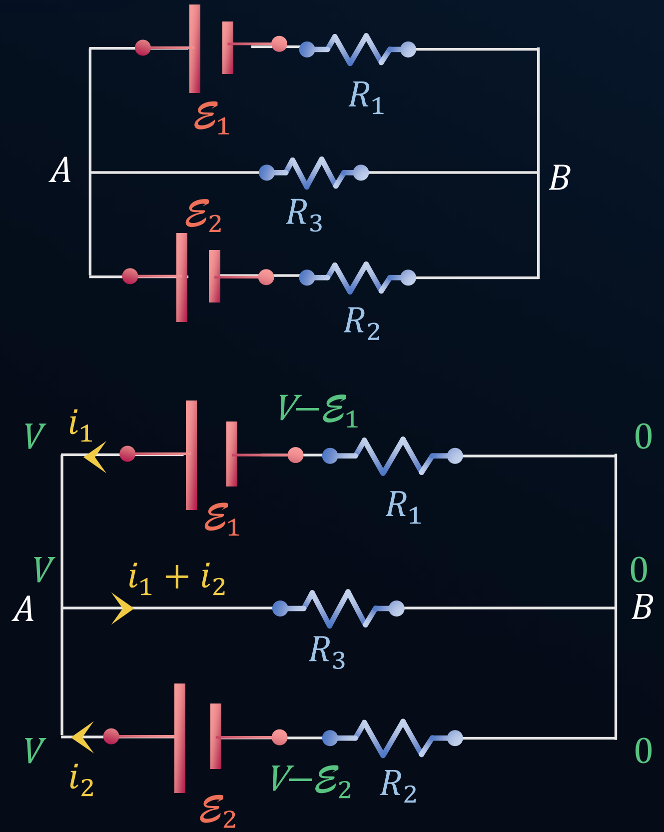
- Step 2 : Write potential of every point



- Current through resistor R_1 & R_2.

$$i_1 = 1\ A\ \&\ i_2 = 0\ A$$

Find the potential difference $V_A - V_B$. In the Following circuit.



- Since we have to find $V_A - V_B$, always try to take $V_B = 0$ for this type of the question, so that V will be equal to V_A

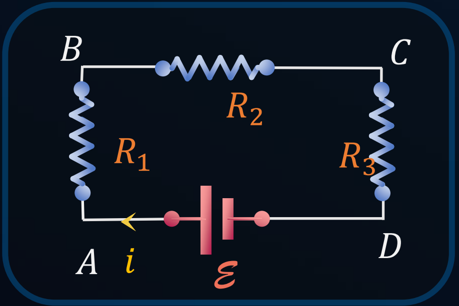
- Apply KCL at point B.

$$\frac{(V - \mathcal{E}_1) - 0}{R_1} + \frac{V - 0}{R_3} + \frac{(V - \mathcal{E}_2) - 0}{R_2} = 0$$

$$\Rightarrow \boxed{V = V_A - V_B = \frac{\frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}}$$

Combination of Resistances

Series Combination



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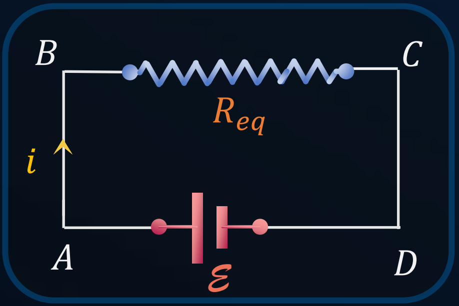
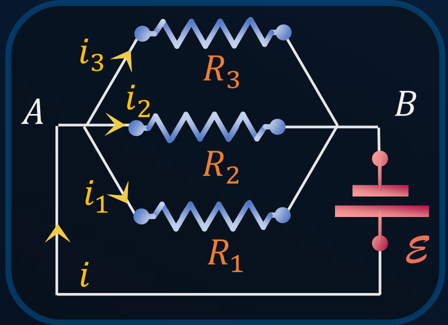


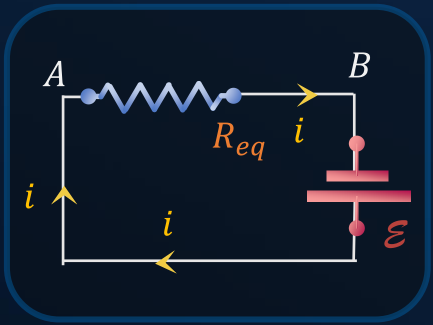
$$R_{eq} = R_1 + R_2 + R_3$$

- Current is same through all resistors.
- For n number of resistances in series,

$$R_{eq} = \sum R_i$$

Parallel Combination

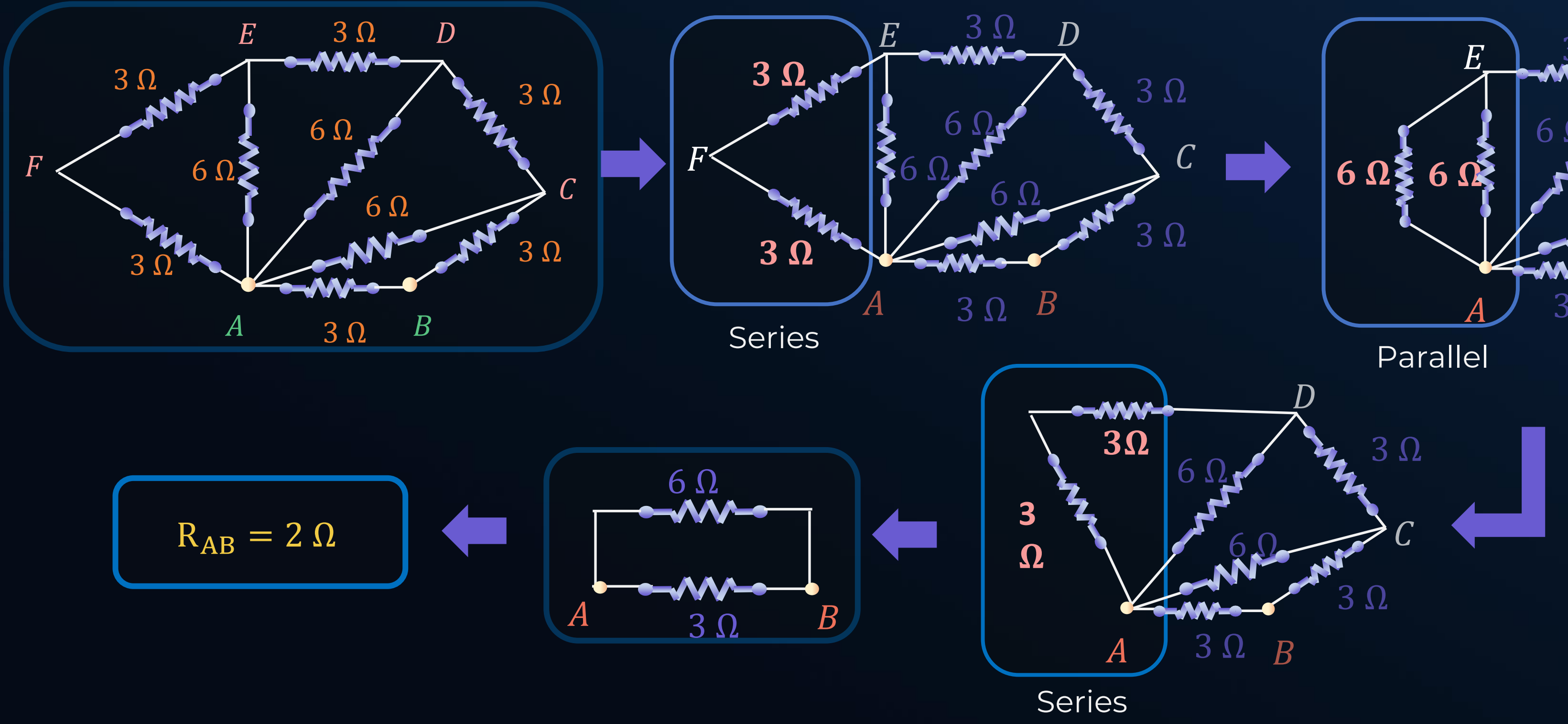




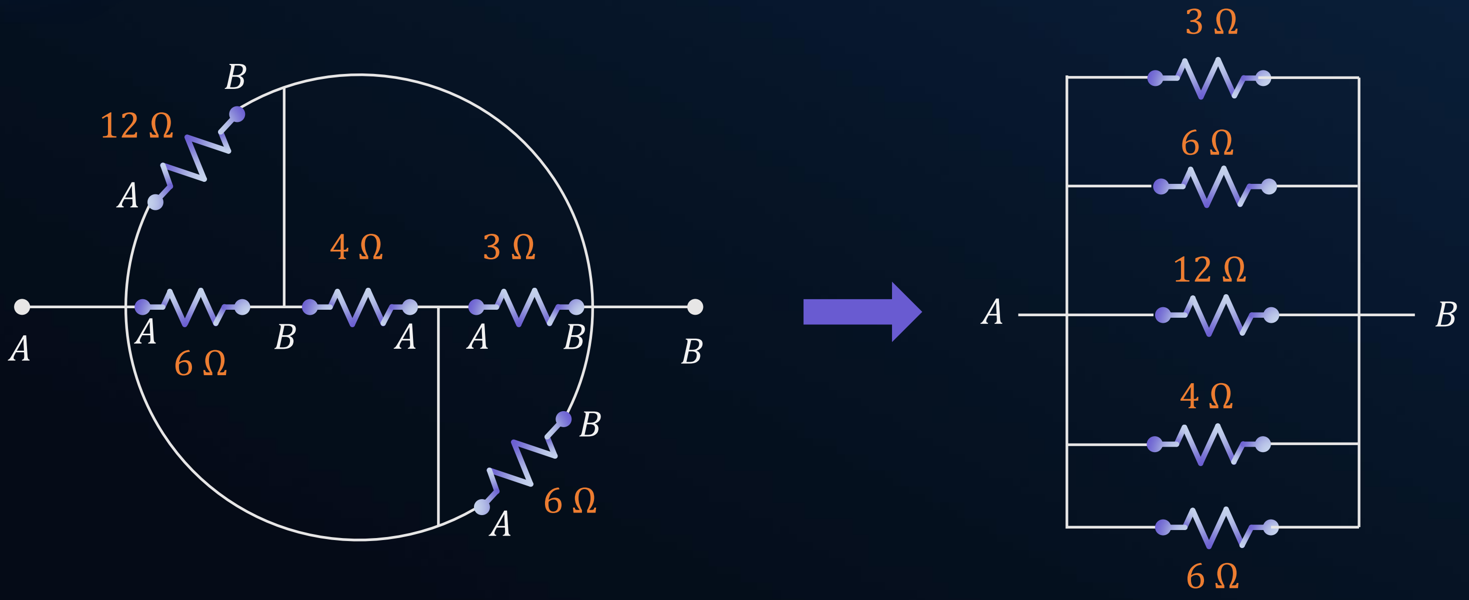
- Potential difference is same across all resistors.
- For n number of resistances in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Find the effective resistance between the points A and B of the network as shown in figure.



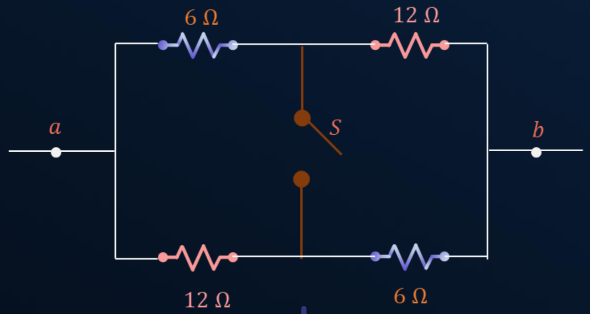
In the circuit shown below, find the equivalent resistance across A and B.

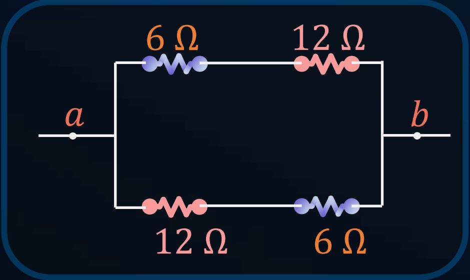


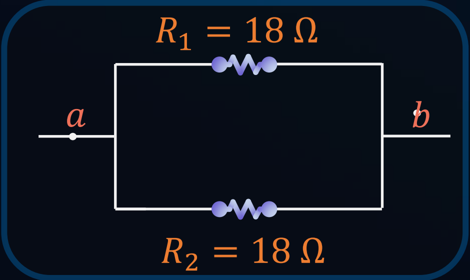
$$\frac{1}{R_{eq}} = \frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{4}+\frac{1}{6} = \frac{4+2+1+3+2}{12} \Rightarrow R_{eq} = \boxed{1\ \Omega}$$

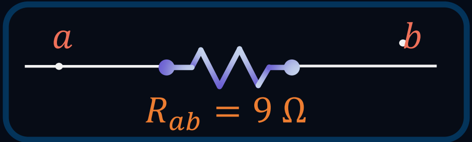
Find the equivalent resistance of the network between the points a and b when;

(a) The switch S is open and (b) The switch S is closed.









Step - 1

$$R_1 = 6 + 12 = 18\ \Omega$$

$$R_2 = 12 + 6 = 18\ \Omega$$

Step - 2

$$\frac{1}{R_{ab}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{R_{ab}} = \frac{2}{18}$$

$$\Rightarrow \boxed{R_{ab} = 9\ \Omega}$$

Step - 1

$$R_1 = \frac{6 \times 12}{6 + 12} = 4\ \Omega$$

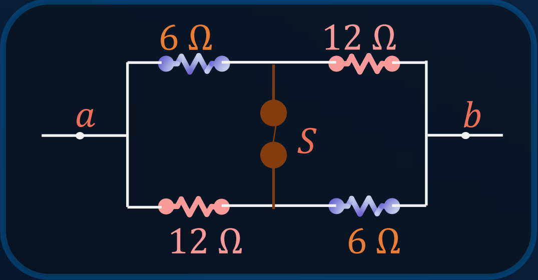
$$R_2 = \frac{6 \times 12}{6 + 12} = 4\ \Omega$$

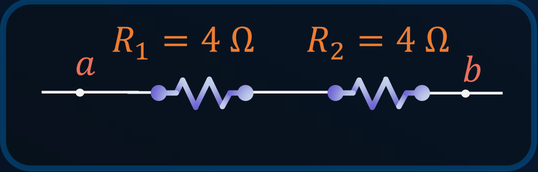
Step - 2

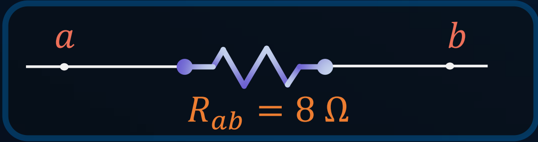
$$R_{ab} = R_1 + R_2$$

$$\Rightarrow R_{ab} = 4 + 4$$

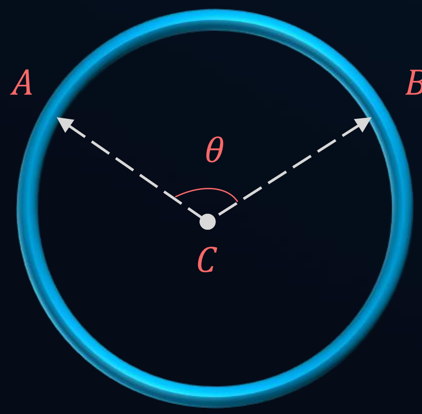
$$\boxed{R_{ab} = 8\ \Omega}$$



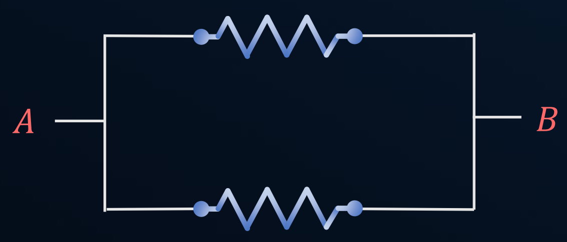




A and B are two points on a uniform ring of resistance R. The $\angle ACB = \theta$, where C is the centre of the ring. The equivalent resistance between A and B is



$\equiv$



$$R_1 = \frac{R}{2\pi}\theta$$

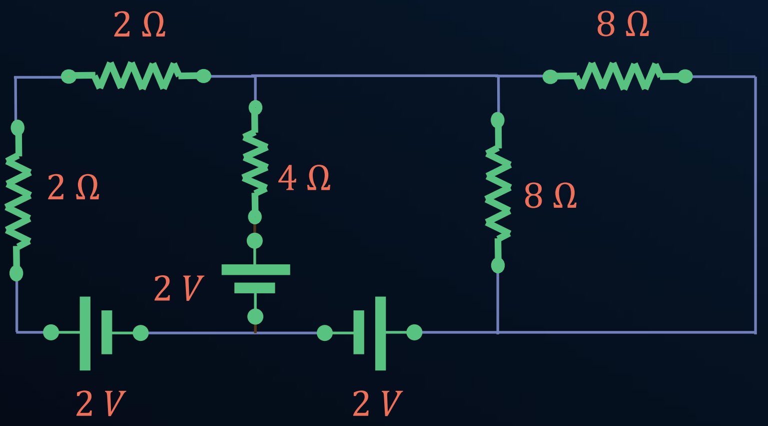
$$R_2 = \frac{R}{2\pi}(2\pi - \theta)$$

- Major & Minor arc are trapped between same points hence they exists in parallel combination.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{R\theta(2\pi - \theta)}{4\pi^2}$$

Find the currents in the different resistors shown in figure.



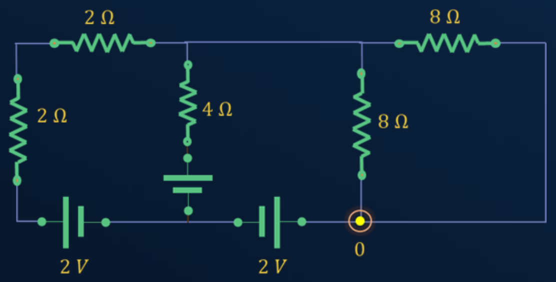
Applying KCL at junction A, we get,

$$\left(\frac{V-0}{4}\right)+\left(\frac{V-0}{4}\right)+\left[\left(\frac{V-0}{8}\right)+\left(\frac{V-0}{8}\right)\right]=0$$

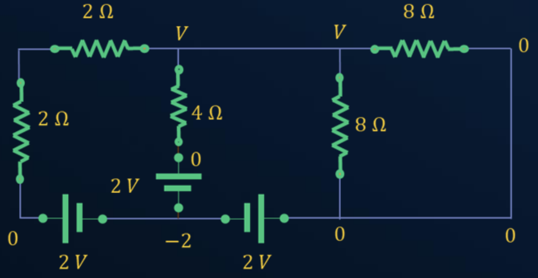
$$\boxed{V=0}$$

Current through all resistances is zero

1 Choose zero potential at a junction



2 Find potential at different junctions

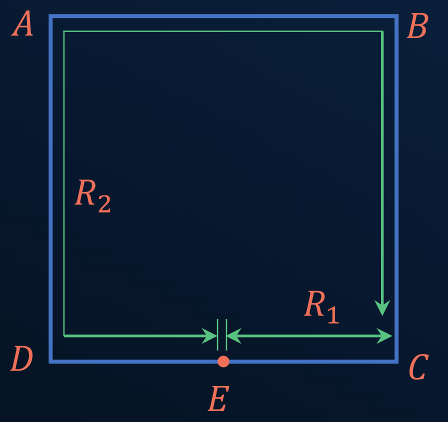


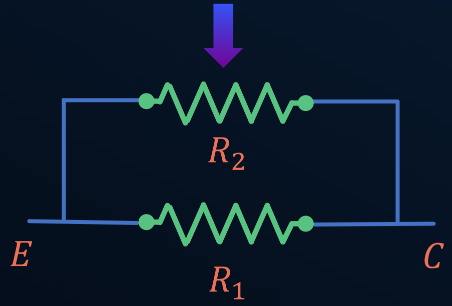
3 Apply KCL / KVL to find the voltage and current

A conducting wire of resistance R is bent to form a square $ABCD$ as shown in the figure. Find the effective resistance between E and C (E is mid-point of arm CD).

- Length of each side of the square = a
- Resistance per unit length of the square = $\frac{R}{4a}$
- Resistance, $R_1 = \frac{R}{4a} \times EC = \frac{R}{4a} \times \frac{a}{2} = \frac{R}{8}$
- Resistance, $R_2 = \frac{R}{4a} \times (EDABC) = \frac{R}{4a} \times \left[\frac{a}{2} + 3a\right] = \frac{7R}{8}$
- Effective resistance between E and C :

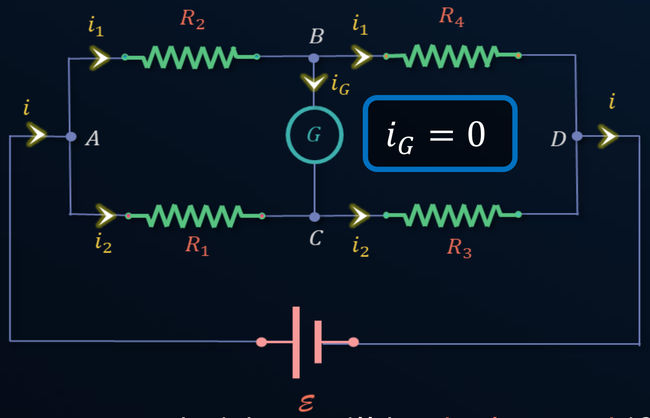
$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{\left(\frac{R}{8}\right) \times \left(\frac{7R}{8}\right)}{\left(\frac{R}{8} + \frac{7R}{8}\right)} = \frac{7R}{64}$$





Wheatstone Bridge

Balanced



$$i_G = 0$$

The wheatstone bridge will be balanced if,

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

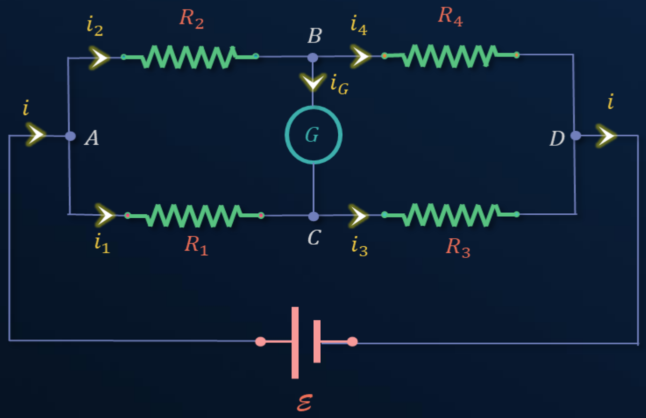
Or

$$R_1 R_4 = R_2 R_3$$

- Equivalent resistance :

$$R_{eq} = \frac{(R_2 + R_4)(R_1 + R_3)}{(R_1 + R_2 + R_3 + R_4)}$$

Unbalanced



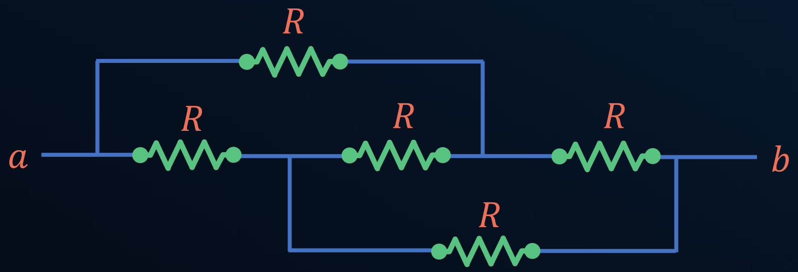
The wheatstone bridge will be unbalanced if,

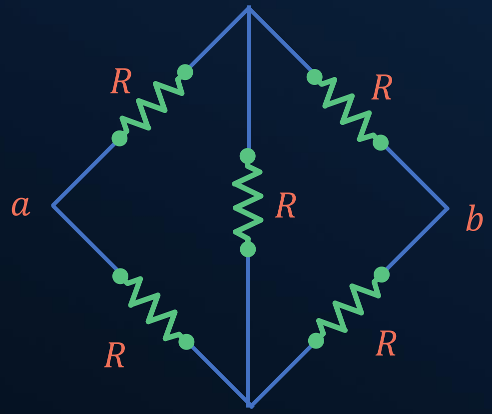
$$\frac{R_2}{R_1} \neq \frac{R_4}{R_3}$$

- Equivalent resistance :

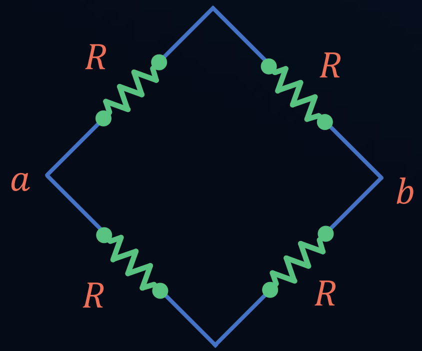
$$R_{eq} = \frac{\mathcal{E}}{i}$$

Find the equivalent resistance of the shown network between points a and b.



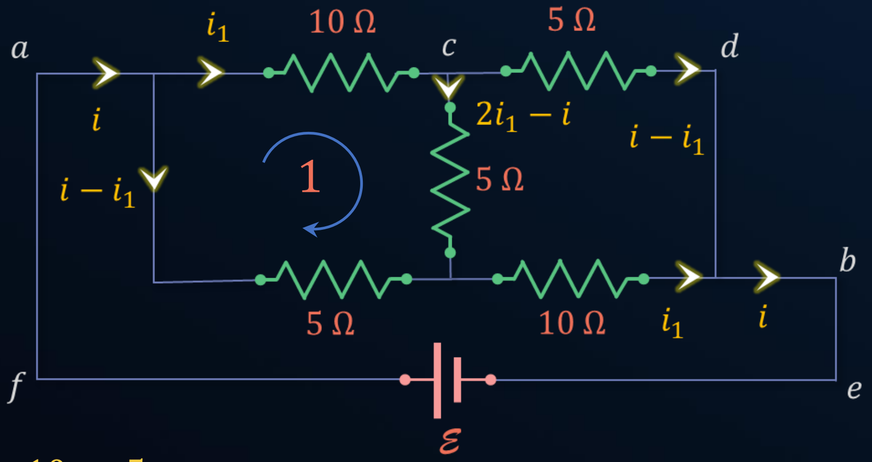


- The network is a balanced wheatstone bridge.



$$R_{eq} = \frac{2R \times 2R}{(2R + 2R)} = R$$

Find equivalent resistance between points a and b.



- $\frac{10}{5} \neq \frac{5}{10}$ ⟹ The wheatstone bridge is unbalanced.

- It possesses input/output symmetry.

⬇

- The current through R_1 and R_4 will be same, and the current through R_2 and R_3 will be same.

- Assuming an external battery of $\mathcal{E}$ supplies i current in the circuit :

$$\boxed{R_{eq} = \frac{E}{i}}$$

- Apply KVL for loop 1 :

$$-10i_1 - 5(2i_1 - i) + 5(i - i_1) = 0 \Rightarrow i_1 = \frac{2i}{5}$$

- Apply KVL for external loop $acdbefa$:

$$-10i_1 - 5(i - i_1) + \mathcal{E} = 0 \quad \Rightarrow \mathcal{E} = 5(i_1 + i)$$

$$\Rightarrow \mathcal{E} = 5\left(\frac{2i}{5} + i\right)$$

$$\boxed{R_{eq} = \frac{\mathcal{E}}{i} = 7\ \Omega}$$

In a Wheatstone bridge a battery of $2V$ is used as shown in the figure. Find the current through middle branch.

- The wheatstone bridge is unbalanced since $\frac{1}{2} \neq \frac{2}{3}$
- Apply KCL at node B :

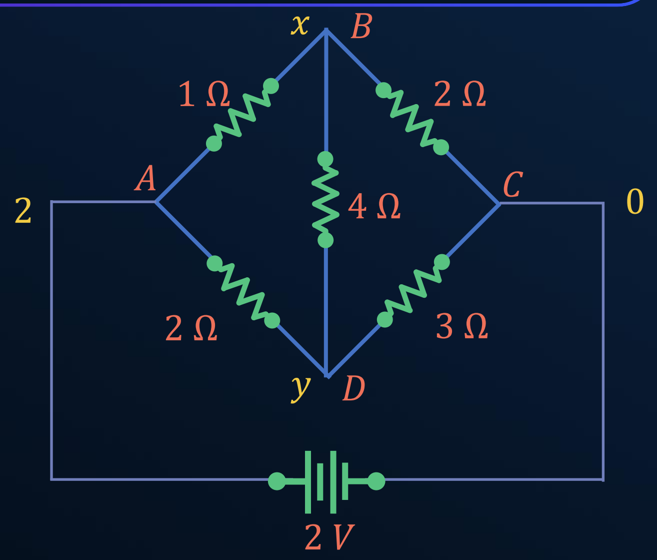
$$\frac{x-0}{2}+\frac{x-y}{4}+\frac{x-2}{1}=0 \qquad \Rightarrow 7x-y=8 \text{ ....(1)}$$

- Apply KCL at node D :

$$\frac{y-0}{3}+\frac{y-x}{4}+\frac{y-2}{2}=0 \qquad \Rightarrow 13y-3x=12 \text{ ....(2)}$$

Solving (1) and (2) we get:

$$\boxed{y=\frac{27}{22}\,V} \qquad \boxed{x=\frac{203}{154}\,V}$$

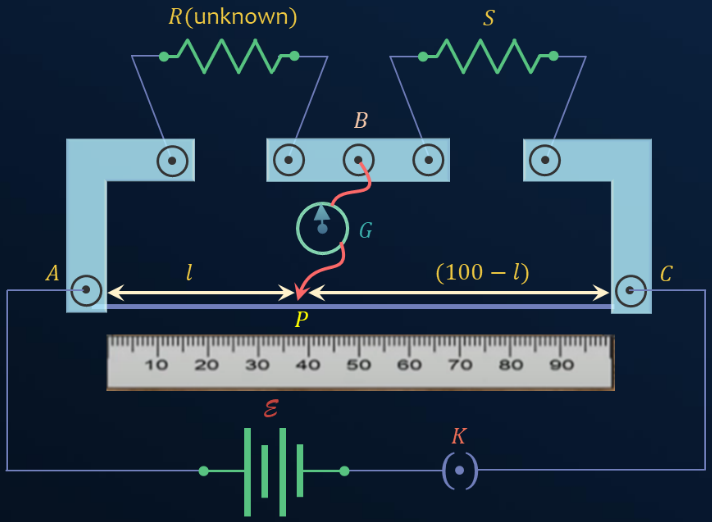


Current through middle branch :

$$I_{4\Omega}=\frac{x-y}{4} \rightarrow \boxed{I_{4\Omega}=\frac{7}{308}\,A}$$

Meter Bridge

- It is an electrical instrument used to find the value of unknown resistance. It consists of a $1\ m$ ($100\ cm$) wire with uniform cross-section.
- Null point : A point on the wire of the meter bridge for which no deflection will be shown by the needle of the galvanometer.
- It works based on the principle of “Balanced Wheatstone Bridge”.



- Condition of null-point :

$$\boxed{\frac{R}{S}=\frac{l}{(100-l)}}$$

- Unknown resistance : $R=S\dfrac{l}{(100-l)}$

A meter bridge is set-up, as shown, to determine an unknown resistance 'X' using a standard $10\ \Omega$ resistor. The galvanometer shows null point when tapping-key is at $52\ cm$ mark. The determined value of 'X' is

Distance of null point P from end A is,

$AP = 52\ cm$

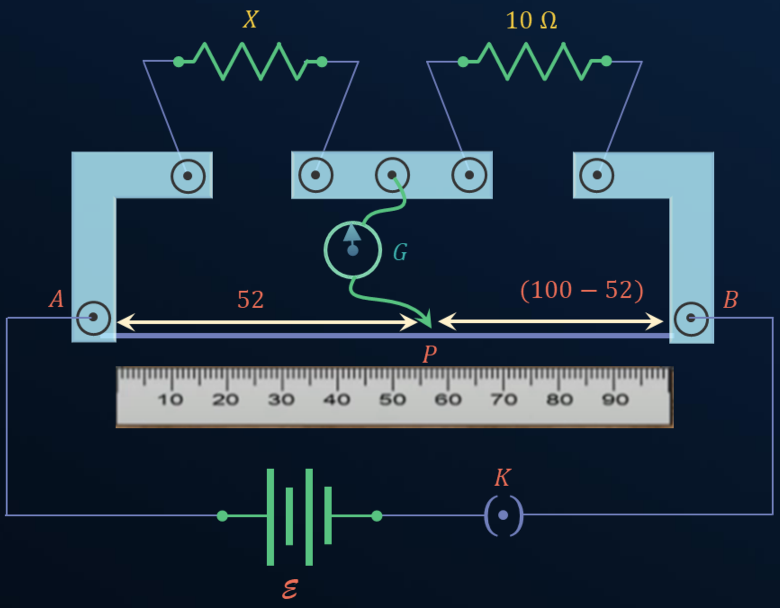
Distance of null point P from end B is,

$PB = (100 - 52) = 48\ cm$

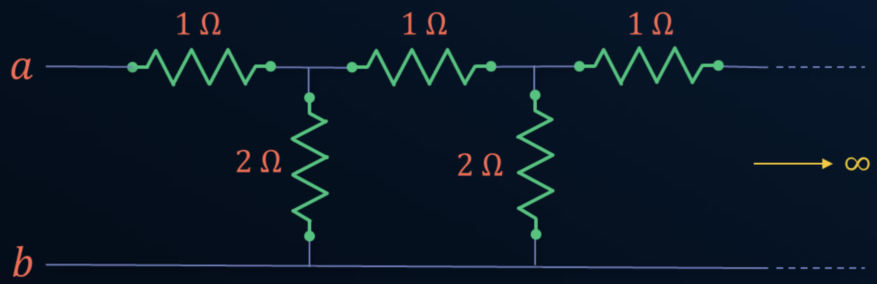
Unknown resistance:

$$X = 10 \times \frac{52}{48}$$

$$\boxed{X = 10.8\ \Omega}$$



Find equivalent resistance between points a and b of the infinite ladder network of resistances.



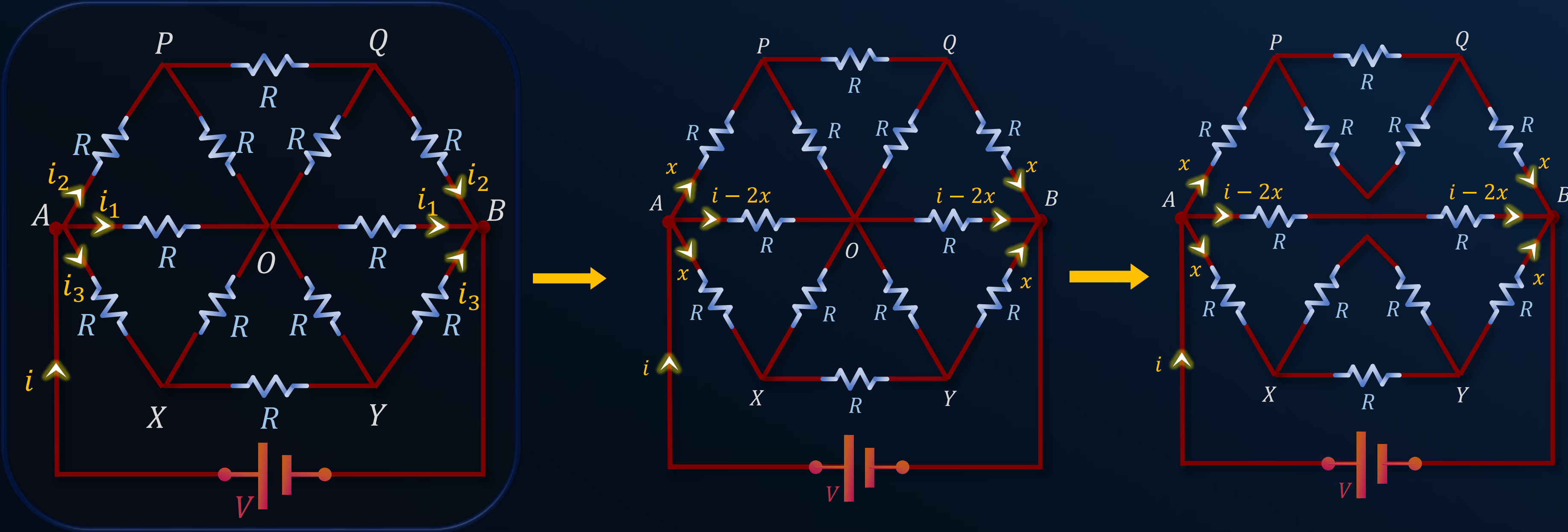
- Equivalent resistance between a and b is,

$$R_{ab} \equiv R_{eq} = 1 + \frac{2 \times R_{eq}}{(2 + R_{eq})}$$

$$R_{eq} = \frac{3R_{eq} + 2}{(2 + R_{eq})}$$

$$R_{eq}^2 - R_{eq} - 2 = 0 \qquad \boxed{R_{eq} = 2\ \Omega}$$

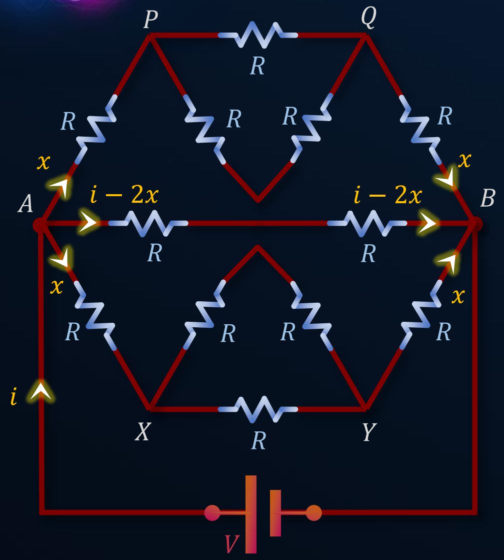
Symmetric Circuit

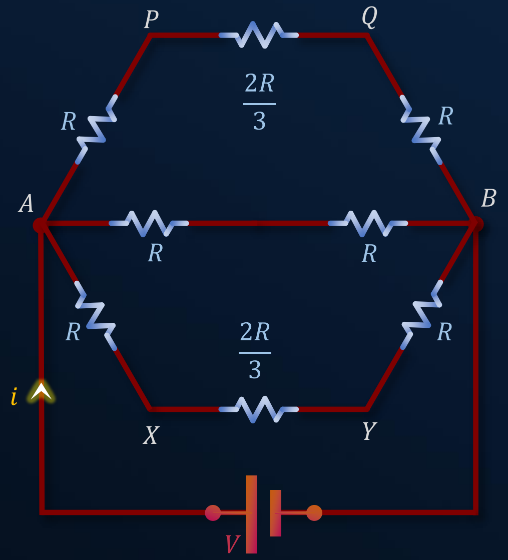


Find equivalent resistance of the circuit shown.

- The circuit has input & output symmetry & junction O is redundant.

Symmetric Circuit





$$R_{APQB} = R + \frac{2R}{3} + R = \frac{8R}{3}$$

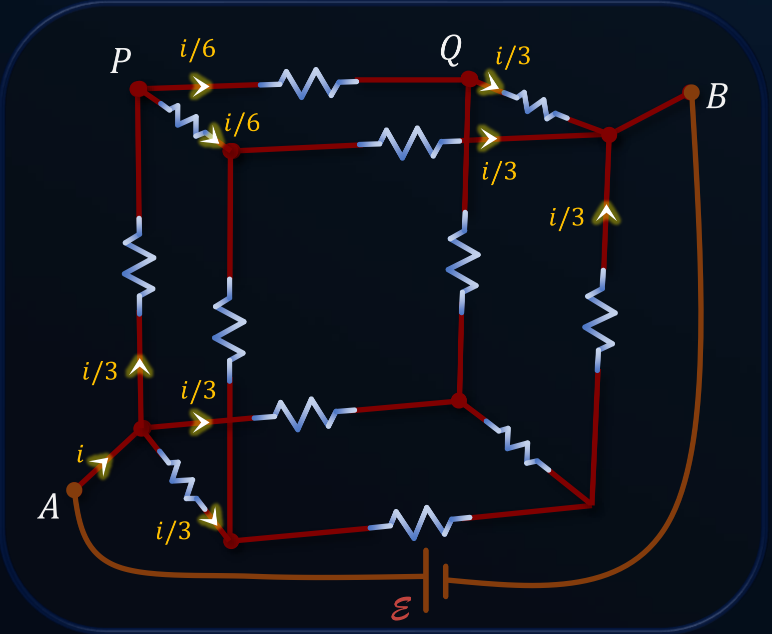
$$R_{AXYB} = R + \frac{2R}{3} + R = \frac{8R}{3}$$

$$R_{AB} = 2R$$

$$\frac{1}{R_{eq}} = \frac{3}{8R} + \frac{3}{8R} + \frac{1}{2R} = \frac{10}{8R} = \frac{5}{4R}$$

$$R_{eq} = \frac{4R}{5}$$

Find the equivalent resistance of following circuit between A and B , given that each resistance is R.



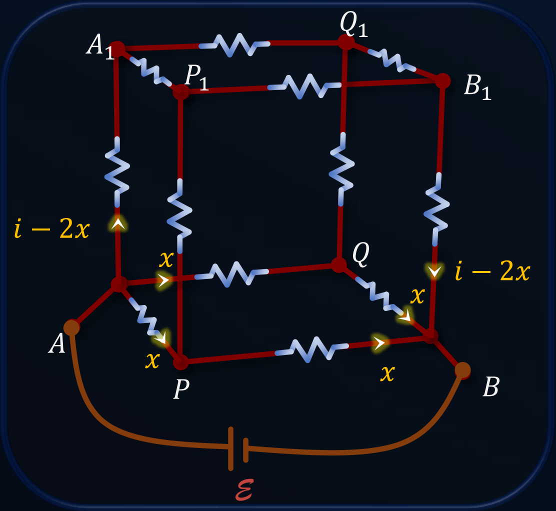
Applying KVL in the loop $APQBA$ on starting from point P, we get,

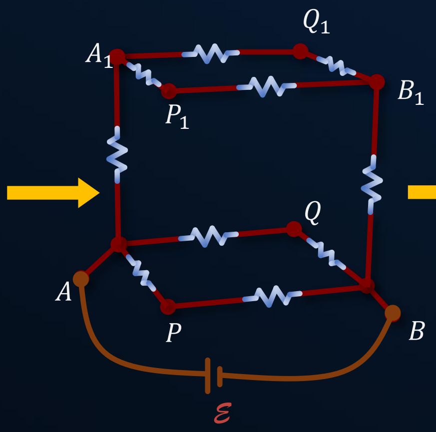
$$-\frac{iR}{6}-\frac{iR}{3}+\mathcal{E}-\frac{iR}{3}=0$$

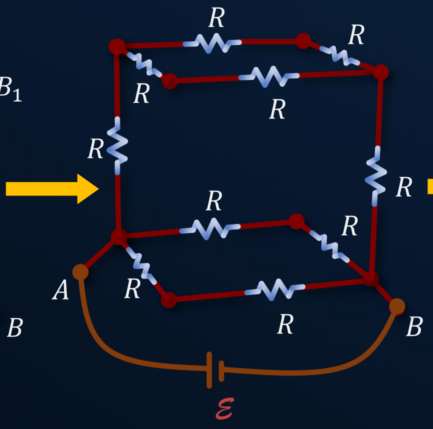
$$\Rightarrow \frac{\mathcal{E}}{i}=\frac{5R}{6}$$

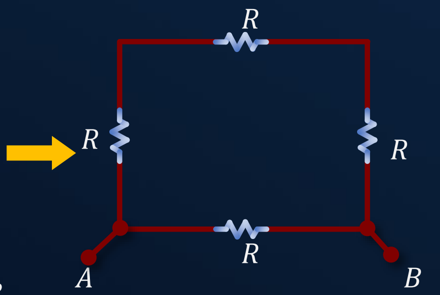
$$\Rightarrow R_{eq}=\frac{5R}{6}$$

Find the equivalent resistance of following circuit between A and B , given that each resistance is R.





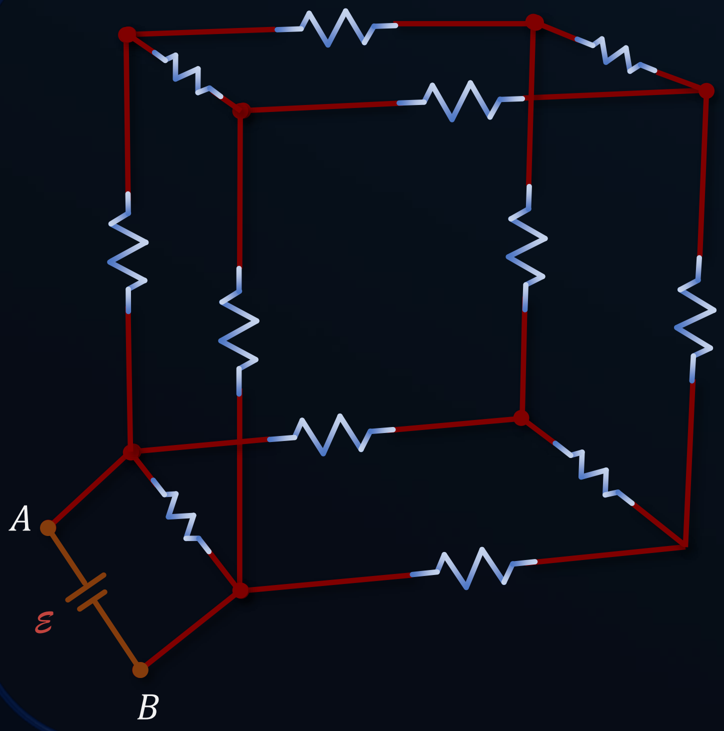


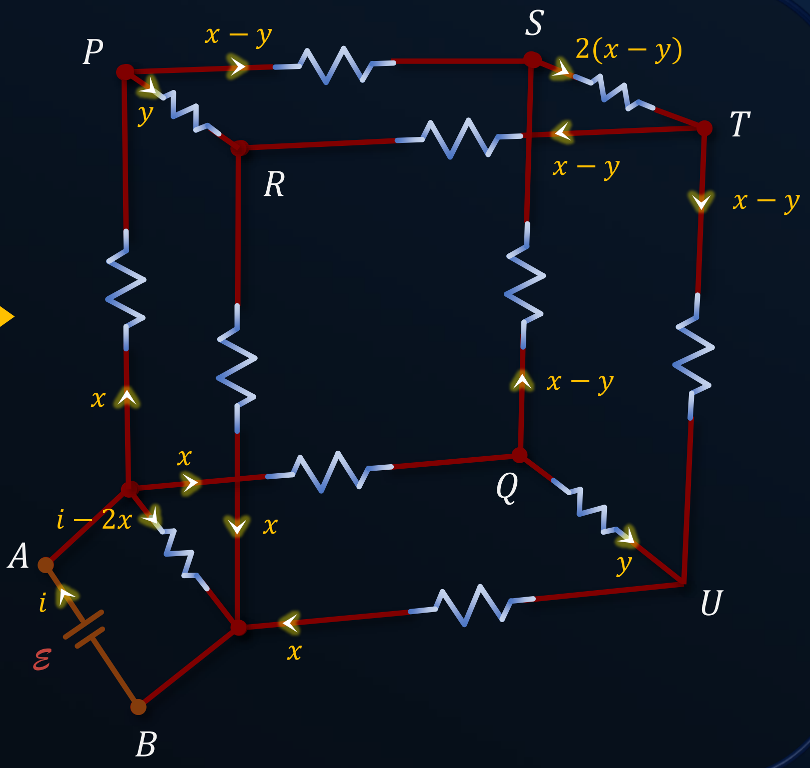


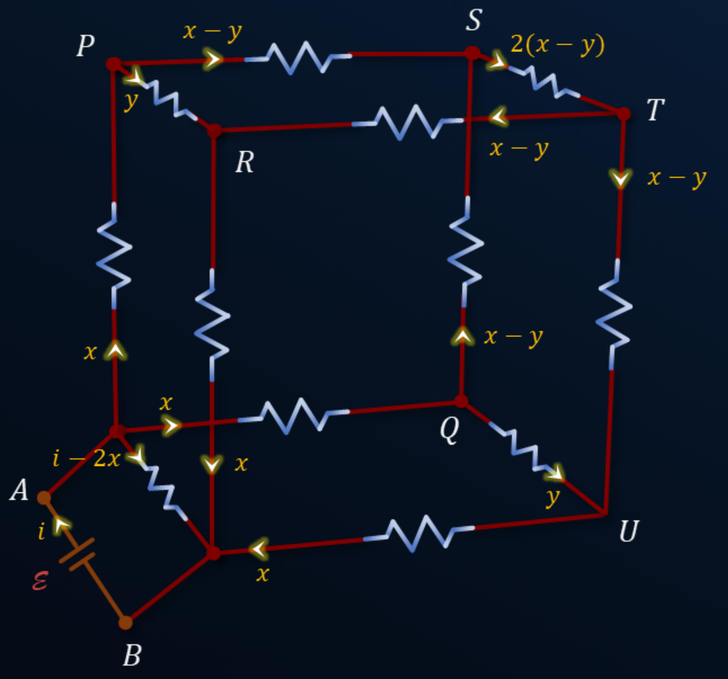
- The circuit has input & output symmetry & junction P & Q are redundant. Therefore current through branches PP_1 & QQ_1 are zero.

$$\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{R} \Rightarrow R_{eq} = \frac{3R}{4}$$

Find the equivalent resistance of following circuit between A and B, given that each resistance is R.







Applying KVL in loop $PSTRP$,

$$-(x-y)R-2(x-y)R-(x-y)R+yR=0$$

$$\Rightarrow 4x=5y \dots\dots (1)$$

Applying KVL in loop $AQUBA$,

$$-xR-yR-xR+(i-2x)R=0$$

$$\Rightarrow 4x+y=i \dots\dots (2)$$

Applying KVL in loop ABA,

$$(i-2x)R=\mathcal{E} \dots\dots (3)$$

From equation (1) & (2),

$$4x+\frac{4x}{5}=i \Rightarrow x=\frac{5i}{24}$$

Substituting value of x in equation (3),

$$\left(i-\frac{5i}{12}\right)R=\mathcal{E}$$

$$\Rightarrow R_{eq}=\frac{\mathcal{E}}{i}=\frac{7R}{12}$$

Combination of Cells

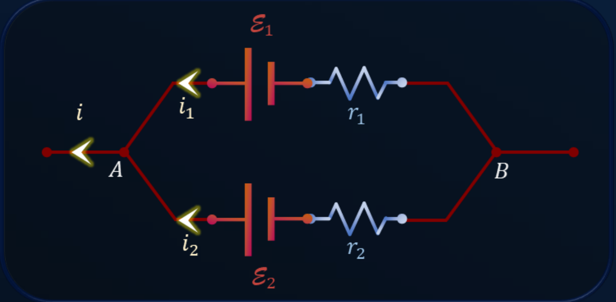
Series Combination

Net resistance, $\boxed{r_{eq} = \sum r_i}$

Net emf = $\boxed{\mathcal{E}_{eq} = \sum \mathcal{E}_i}$

Algebraic addition with sign

Parallel Combination



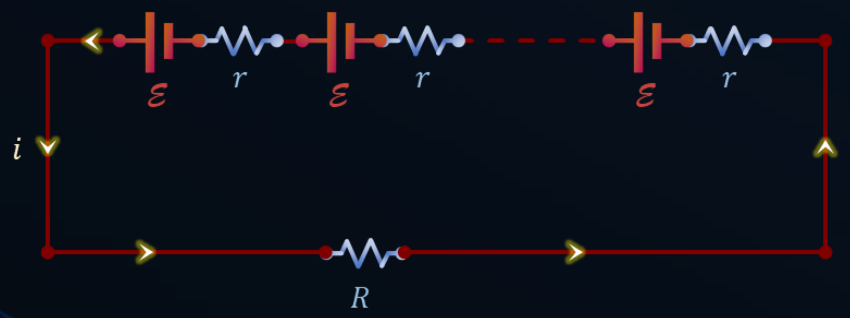
Net resistance, $\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$

Net emf = $\mathcal{E}_{eq} = \frac{\mathcal{E}_2 r_1 + \mathcal{E}_1 r_2}{r_1 + r_2}$

$$\boxed{\mathcal{E}_{eq} = r_{eq}\left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right)}$$

Combination of Cells

Series Combination

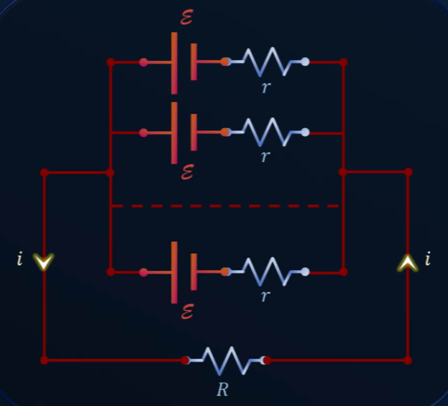


Net emf = $n\,\mathcal{E}$

Total resistance = $nr + R$

Current in circuit, $i = \dfrac{n\mathcal{E}}{nr + R}$

Parallel Combination

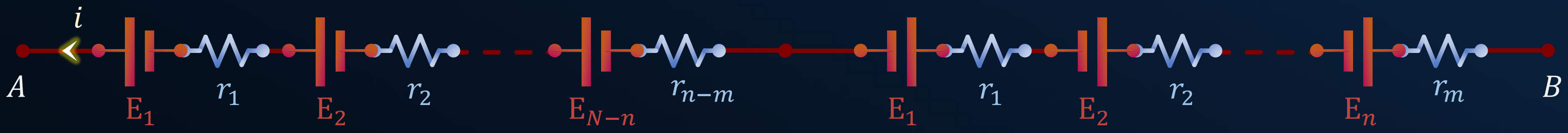


Net emf = $\mathcal{E}_{eq} = \mathcal{E}$

Net resistance, $R + \dfrac{r}{n}$

Current in circuit, $i = \dfrac{\mathcal{E}}{R + \dfrac{r}{n}}$

n identical cells, each of emf E and internal resistance r, are joined in series. Out of these, m cells are wrongly connected, i.e., their terminals are connected in reverse of that required for series connection $(m < n/2)$. Let E_0 be the emf of the resulting battery and r_o be its internal resistance, find E_0, r_o .



- Since n cells are connected in series, the internal resistances of the cells are also in series & since polarity doesn't affect resistance.

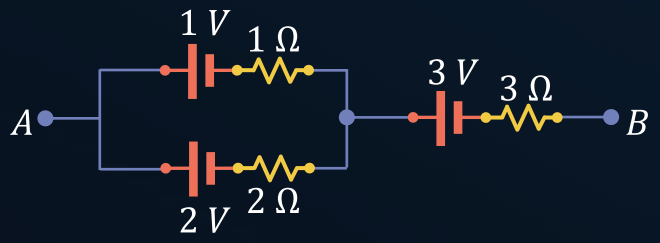
  $r_0 = nr$

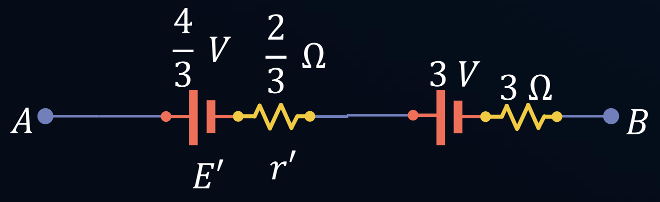
- Two oppositely connected cells connected in series nullify each other emfs. Therefore, this nullification always happens in pairs.

  So, if m cells are connected wrongly among n cells,

  Net emf, $E_0 = (n - 2m)E$

Calculate E_{eq} and r_{eq} for the given combination of cells between A & B.





$$E' = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\Rightarrow E' = \frac{4}{3} V$$

$$r' = \frac{r_1 r_2}{r_1 + r_2}$$

$$\Rightarrow r' = \frac{2}{3} \Omega$$

$$E_{eq} = E' + 3$$

$$= \frac{4}{3} + 3$$

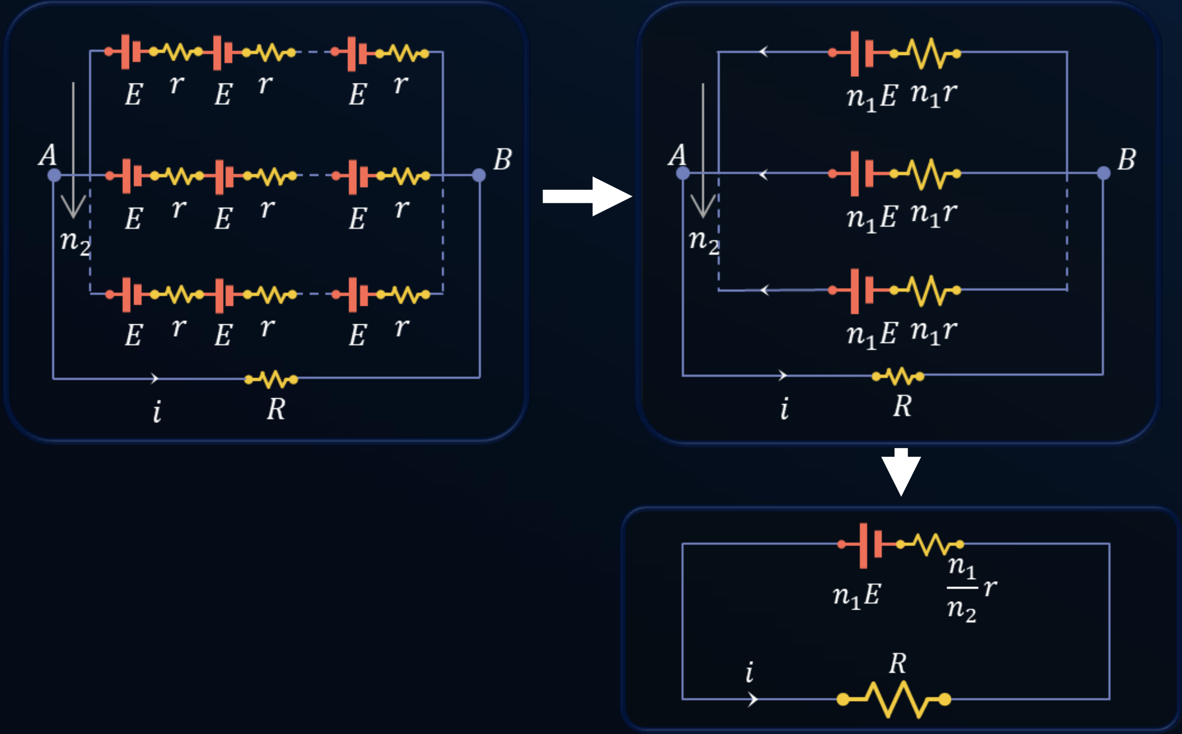
$$\Rightarrow \boxed{E_{eq} = \frac{13}{3} V}$$

$$r_{eq} = r' + 3$$

$$= \frac{2}{3} + 3$$

$$\Rightarrow \boxed{r_{eq} = \frac{11}{3} \Omega}$$

Consider $N = n_1 n_2$ identical cells, each of emf E and internal resistance r. Suppose n_1 cells are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance R.
(a) Find the current through the external resistance.



$$\frac{1}{r_{eq}} = \frac{1}{n_1 r} + \frac{1}{n_1 r} + \cdots \frac{1}{n_1 r} = \frac{n_2}{n_1 r}$$

$$\Rightarrow r_{eq} = \frac{n_1 r}{n_2}$$

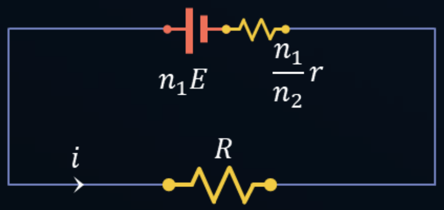
$$E_{eq} = \frac{\frac{n_1 E}{n_1 r} + \frac{n_1 E}{n_1 r} + \cdots \frac{n_1 E}{n_1 r}}{\frac{1}{n_1 r} + \frac{1}{n_1 r} + \cdots \frac{1}{n_1 r}} = \frac{\frac{n_2 E}{r}}{\frac{n_2}{n_1 r}}$$

$$\Rightarrow E_{eq} = n_1 E$$

$$i = \frac{n_1 E}{\frac{n_1 r}{n_2} + R}$$

$$\boxed{i = \frac{NE}{n_1 r + n_2 R}}$$

Consider $N = n_1 n_2$ identical cells, each of emf E and internal resistance r. Suppose n_1 cells are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance R.
(b) Assuming that n_1 and n_2 can be continuously varied ($N = n_1 n_2$ still holds), find the relation between n_1, n_2, R and r for which the current in R is maximum.



Current in the circuit,

$$i = \frac{NE}{n_1 r + n_2 R}$$

$$= \frac{NE}{n_1 r + \frac{NR}{n_1}}$$

$$i = \frac{n_1 NE}{n_1{}^2 r + NR}$$

When i is maximum,

$$\frac{di}{dn_1} = 0$$

$$\Rightarrow \frac{di}{dn_1} = \frac{(NE)[n_1{}^2 r + NR] - n_1(NE)(2n_1 r)}{[n_1{}^2 r + NR]^2} = 0$$

$$\Rightarrow n_1{}^2 r + NR - 2n_1{}^2 r = 0$$

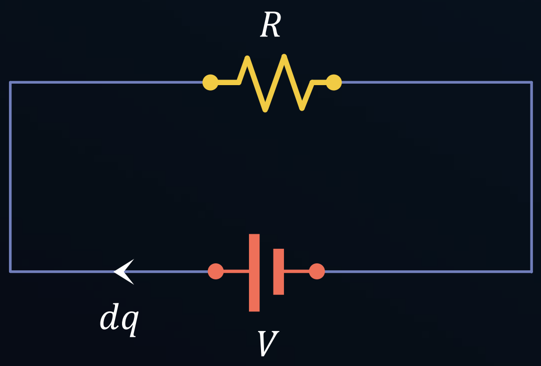
$$\Rightarrow n_1 n_2 R = n_1{}^2 r \qquad (\because N = n_1 n_2)$$

$$\Rightarrow \boxed{n_1 r = n_2 R}$$

$\therefore$ Condition for maximum current in R is, $n_1 r = n_2 R$

Heating Effect of Electric Current

When a charge moves through a potential difference, electrical work is done. This work represents the loss of potential energy of charges.



dq- Charge passing through a potential difference, V

- Work done, $dW = Vdq$
- Taking V in $volt$, I in $ampere$ and R in ohm,

$$W = VIt = I^2Rt = \frac{V^2}{R}t \; joule$$

- The work done appears as increased thermal energy of conductor and conductor gets heated.

$$\therefore H = VIt = I^2Rt = \frac{V^2}{R}t \; joule$$

Power

Power – rate at which energy is transferred

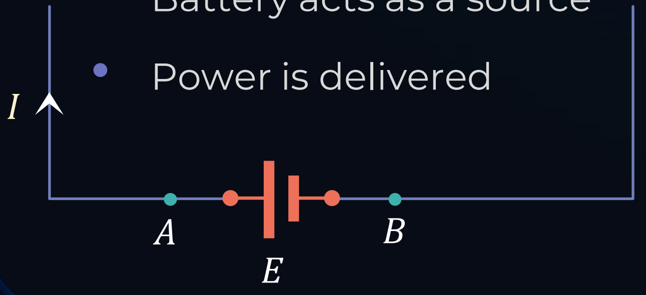
For any electrical element, $P = VI$

P - Instantaneous Power

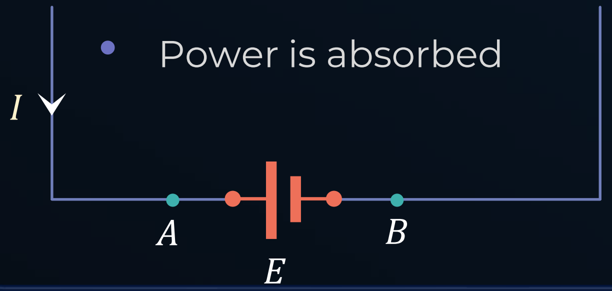
V - Potential difference across the element

I - Instantaneous current through the element

- Battery acts as a source
- Power is delivered

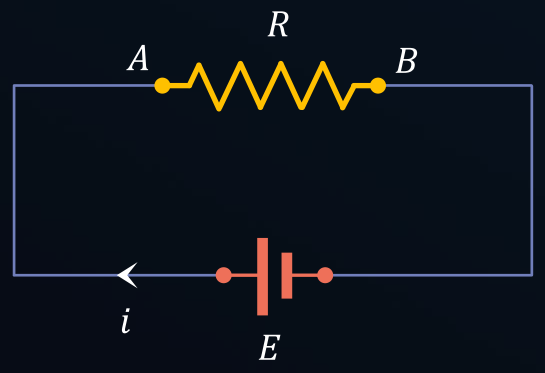


- Battery acts as a load
- Power is absorbed



Power Dissipation Across a Resistor

It is the rate of generation of thermal energy across the resistor.



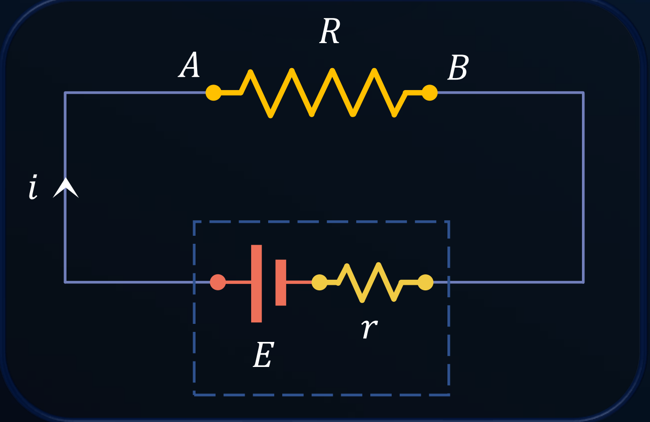
- A resistance always acts as a load

$$P_R = Vi = (iR)i = i^2R$$

$$= V\left[\frac{V}{R}\right] = \frac{V^2}{R}$$

$$\boxed{P = i^2R = Vi = \frac{V^2}{R}}$$

Maximum Power Dissipation Across a Resistor



- $i = \dfrac{E}{R + r}$
- $P_E = Ei = \dfrac{E^2}{R + r}$
- $P_R = i^2 R = \dfrac{E^2 R}{(R + r)^2}$

$P_R + P_r = \dfrac{E^2}{R + r}$

- $P_r = i^2 r = \dfrac{E^2 r}{(R + r)^2}$

$$\boxed{\Rightarrow P_R + P_r = P_E}$$

$$P_R = \frac{E^2 R}{(R + r)^2}$$

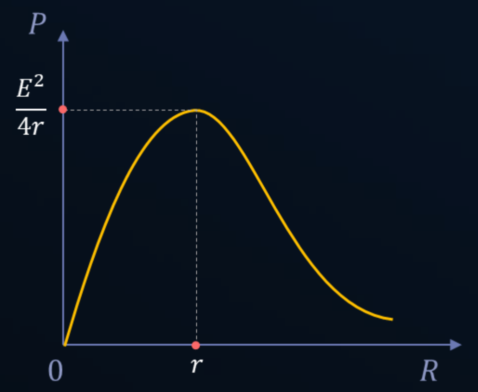
When P_R is maximum, $\dfrac{dP_R}{dR} = 0$

$$\Rightarrow \frac{(E^2)(R + r)^2 - 2(R + r).(E^2 R)}{(R + r)^4} = 0$$

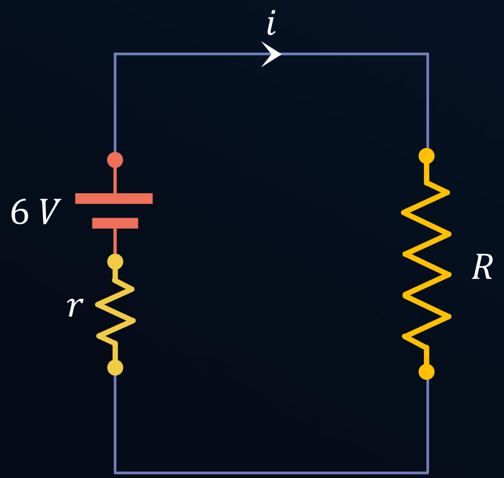
$$\Rightarrow E^2 (R + r)(R + r - 2R) = 0$$

$$\Rightarrow R = r$$

$$\boxed{(P_{max})_R = \frac{E^2}{4r} \quad (\text{for } R = r)}$$



A battery has an open circuit potential difference of $6\,V$ between its terminals. When a load resistance of $60\,\Omega$ is connected across the battery, the total power supplied by the battery is $0.4\,W$. What should be the load resistance R, so that maximum power will be dissipated in R. Calculate this power. What is the total power supplied by the battery when such a load is connected?



$$i = \frac{6}{60 + r}$$

$$P_{battery} = 6i = \frac{36}{60 + r}$$

$$\Rightarrow \frac{36}{60 + r} = 0.4 = \frac{2}{5}$$

$$\Rightarrow r = 30\,\Omega$$

For maximum P_R, $R = r$

$$\boxed{R = 30\,\Omega}$$

$$(P_{max})_R = \frac{E^2}{4r} = \frac{6^2}{4 \times 30}$$

$$\boxed{\Rightarrow (P_{max})_R = 0.3\,W}$$

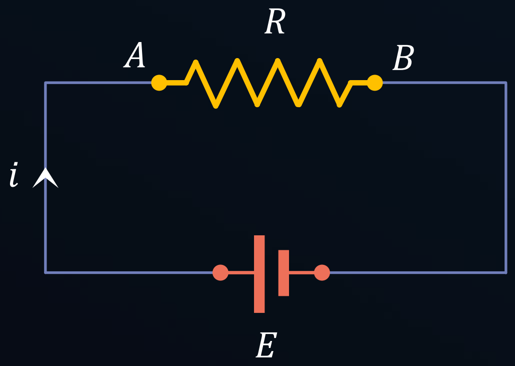
$$P_r = (P_{max})_R \qquad (\because R = r)$$

$$\Rightarrow P_r = 0.3\,W$$

$$P_E = (P_{max})_R + P_r$$

$$\boxed{P_E = 0.6\,W}$$

Measuring Electric Energy



- SI unit of energy is joule(J)
- kWh is a larger unit

$$1\ kWh = P(kilowatt) \times t(hour)$$

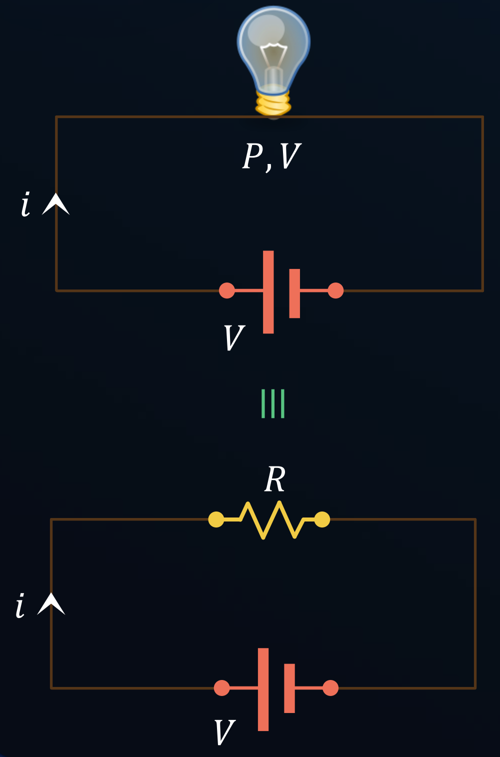
$$1\ kWh = 1000 \times 3600\ joule$$

$$1\ kWh = 3.6 \times 10^6\ joule$$

- No. of units $(N) = \dfrac{watt \times hour}{1000}$ OR

$$N = \frac{\text{(Power consumed in watts)(Time of consumption in hours)}}{1000}$$

Resistance of an Electric Bulb



Bulb Rating:

P - Power
V - Potential difference

Bulb consumes P power for an applied potential difference of V.

Resistance of the bulb,

$$\boxed{R = \frac{V^2}{P}}$$

How will the Bulbs Glow?

$$R = \frac{V^2}{P}$$

$$R = \frac{220^2}{60}$$

$$P_1 = \frac{{V_1}^2}{R}$$

$$= 150^2 \times \frac{60}{220^2}$$

$$\Rightarrow P_1 < 60\ W$$

- Bulb will glow dimmer

$$P_2 = \frac{{V_2}^2}{R}$$

$$= 220^2 \times \frac{60}{220^2}$$

$$\Rightarrow P_2 = 60\ W$$

- Bulb will glow with its rated power

$$P_3 = \frac{{V_3}^2}{R}$$

$$= 400^2 \times \frac{60}{220^2}$$

$$\Rightarrow P_3 > 60\ W$$

- Bulb will fuse

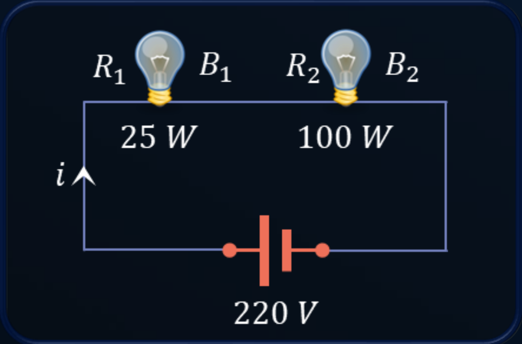
Two electric bulbs B_1 and B_2 rated at $25\ W$, $220\ V$ and $100\ W$, $220\ V$ are given. Which bulb glows more brightly and what is the value of output powers P_1 and P_2 , if

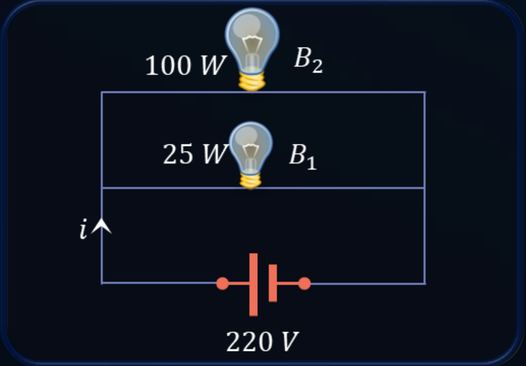
(a) they are connected in series across a $220\ V$ battery

(b) connected in parallel across a $220\ V$ battery

$$R_1 = \frac{{V_1}^2}{P} = \frac{220^2}{25}$$

$$R_2 = \frac{{V_2}^2}{P} = \frac{220^2}{100}$$





(a) $R_1 = 4R_2 \Longrightarrow V_1 = 4V_2$

$$\therefore V_1 = \frac{4}{5} \times 220 \qquad V_2 = \frac{1}{5} \times 220$$

$$P_1 = \frac{{V_1}^2}{R_1} = \frac{16 \times 220^2}{25} \times \frac{25}{220^2} \qquad P_2 = \frac{{V_2}^2}{R_2} = \frac{220^2}{25} \times \frac{100}{220^2}$$

$$\boxed{P_1 = 16\ W} \qquad \boxed{P_2 = 4\ W}$$

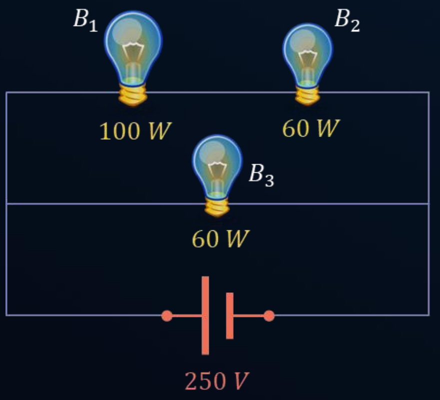
Bulb B_1 glows more brightly

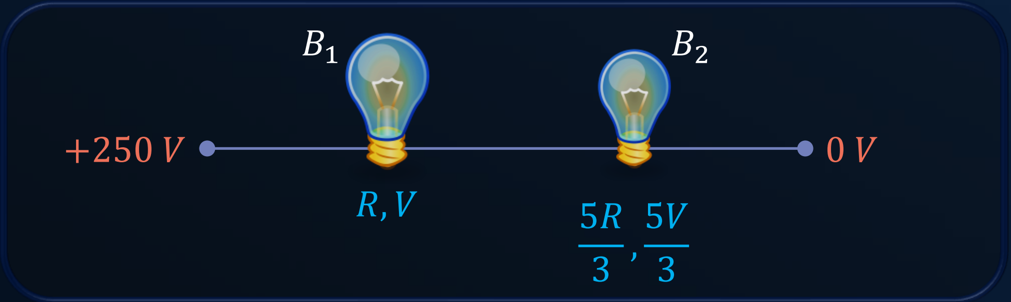
(b) Applied voltage = Rated voltage

$$\boxed{P_1 = 25\ W,\ P_2 = 100\ W}$$

Bulb B_2 glows more brightly

A $100\ W$, $250\ V$ bulb B_1, and two $60\ W$, $250\ V$ bulbs B_2 and B_3, are connected to a $250\ V$ source as shown. Now W_1, W_2 and W_3 are the output powers of bulbs B_1, B_2 and B_3. Then find relation between W_1, W_2 and W_3 .





Resistance provided by the bulbs,

$$R_1 = \frac{(250)^2}{100}\ \Omega = R \text{ (say)}$$

$$R_3 = R_2 = \frac{(250)^2}{60}\ \Omega = \frac{5R}{3}$$

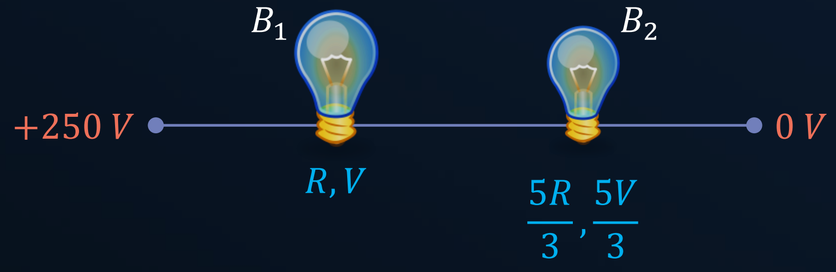
For the given series connection,

$$V + \frac{5V}{3} = 250 \quad \Rightarrow V = \frac{3}{8}(250)\ V$$

$$\Rightarrow V_1 = V = \frac{3}{8}(250)\ V$$

$$\Rightarrow V_2 = \frac{5V}{3} = \frac{5}{8}(250)\ V$$

$$\Rightarrow V_1 = V = \frac{3}{8}(250)\ V \qquad \Rightarrow V_2 = \frac{5V}{3} = \frac{5}{8}(250)\ V$$



Output powers of bulbs B_1 and B_2,

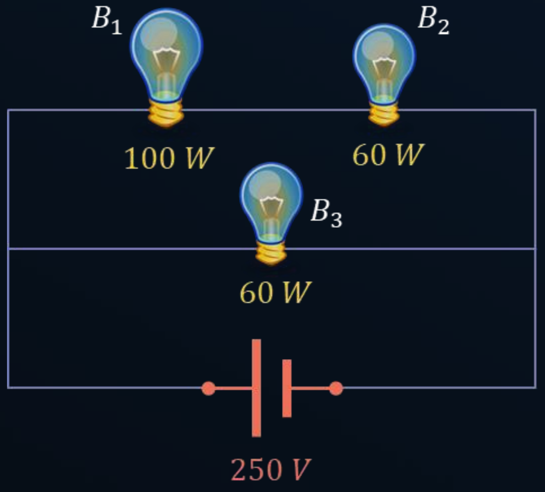
$$W_1 = \frac{V_1^2}{R_1} = \frac{9}{64}(250)^2 \times \frac{100}{(250)^2} = 14.06\ W$$

$$W_2 = \frac{V_2^2}{R_2} = \frac{25}{64}(250)^2 \times \frac{60}{(250)^2} = 23.44\ W$$

Since B_3 is at its rated voltage,

$$W_3 = 60\ W$$

$$\boxed{W_1 < W_2 < W_3}$$



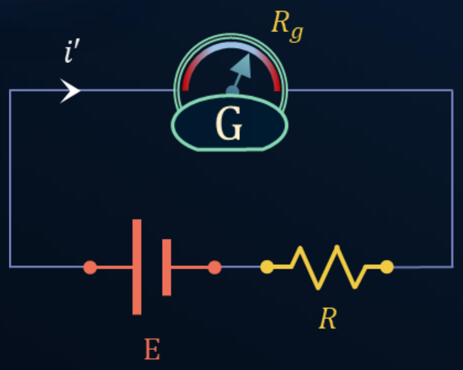
Electrical Measuring Instruments

Galvanometer

Ammeter

Voltmeter

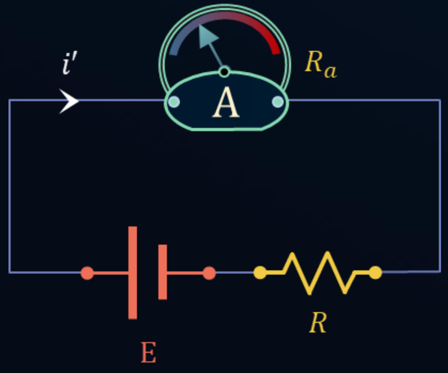
Potentiometer



- Detects the presence of electric current in a circuit.
- Reading of Galvanometer $\propto i$
- Current measured by the galvanometer is,

$$i' = \frac{E}{R + R_g}$$

$\left(R_g \approx 0 \text{ for accurate reading} \right)$



- Measures current passing through it
- Minimal effect on existing current in the circuit (negligible resistance)
- Connected in series in the circuit

$$i' = \frac{E}{R + R_a}$$

$\left(R_a \approx 0 \text{ for an ideal ammeter} \right)$

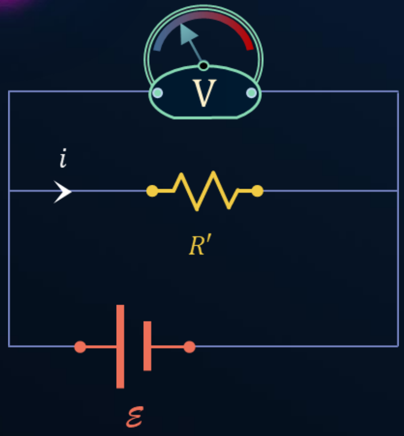
Electrical Measuring Instruments

Galvanometer

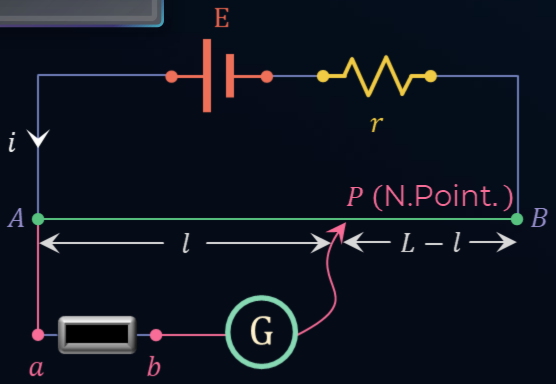
Ammeter

Voltmeter

Potentiometer



- Measures potential difference across it
- Connected in parallel with the circuit
- Resistance of an ideal voltmeter is infinite so that no current passes through it



- Measures potential difference without drawing any current from the given circuit.
- Jockey is moved from one end to the other end of the wire AB to find the null point (zero deflection)

$$V_a - V_b = V_A - V_P = Zl$$

Where, $\boxed{Z = \text{Potential gradient} = \frac{V_A - V_B}{L} l}$

Conversion of Galvanometer into Ammeter

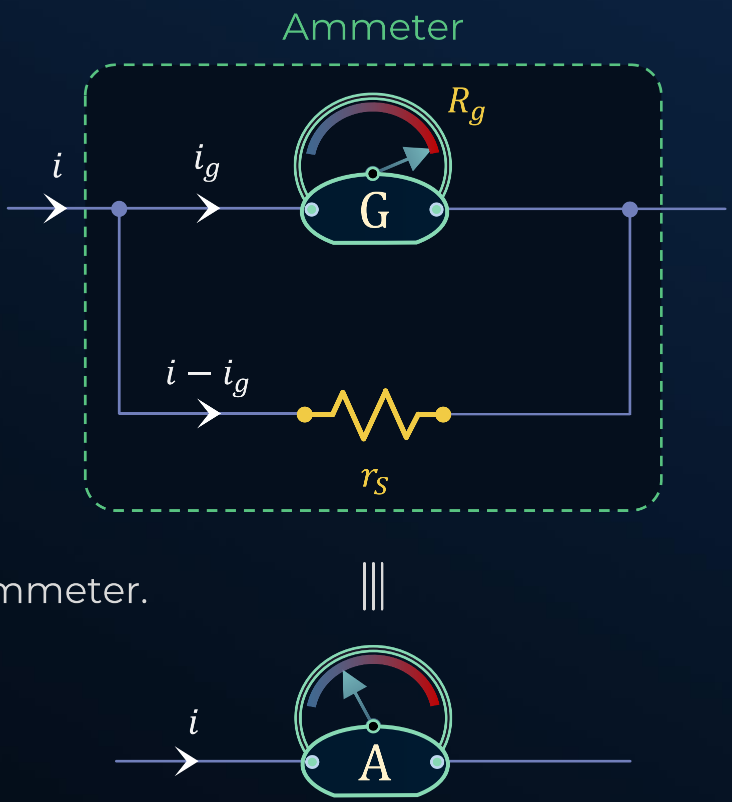
- Full-scale deflection current (i_g) is the maximum current that is allowed to pass through the galvanometer.

- A very small resistance r_S (shunt) is connected in parallel to minimize the resistance of the ammeter.

$$V_g = V_S$$

$i_g R_g = (i - i_g) r_S$, where i is the range of the ammeter.

$$\boxed{r_S = \frac{i_g R_g}{i - i_g}}$$



A galvanometer has resistance $2\ \Omega$ with maximum current measuring capacity $0.1\ A$. Find the shunt resistance S of an ammeter which can measure current up to $10\ A$.

Maximum current allowed inside galvanometer is, $i_g = 0.1\ A$

Shunt S is such that galvanometer can measure up to $10\ A$ current flowing through the circuit.

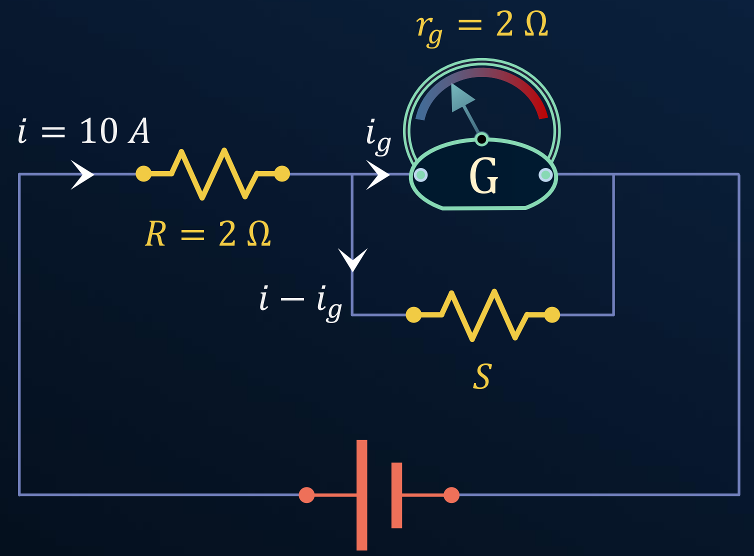
Applying KVL inside the loop containing G and S,

$$r_g i_g = (i - i_g)S$$

$$2 \times 0.1 = (10 - 0.1) \times S$$

$$0.2 = 9.9S$$

$$\boxed{S = \frac{2}{99}\ \Omega}$$



Conversion of Galvanometer into Voltmeter

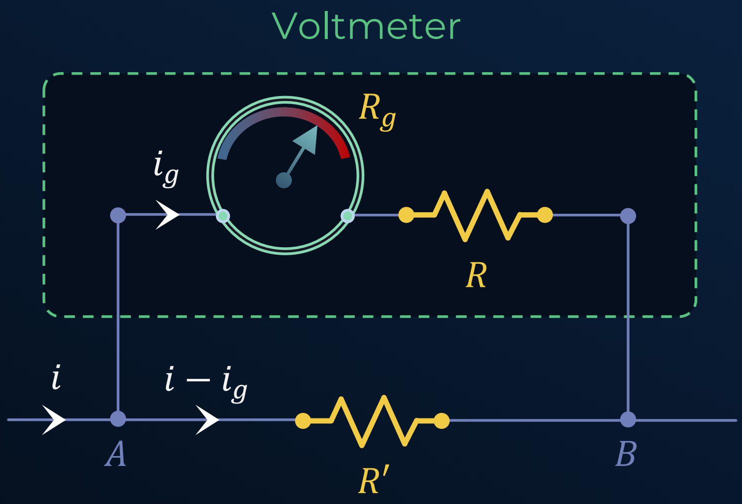
- Max potential difference that can be measured with voltmeter is,

$$V_{AB} = i_g(R_g + R) \approx i_g R \quad (\because R_g \ll R)$$

- If the reading of the newly calibrated voltmeter is V, then,

$$V = i_g(R_g + R)$$

$$\boxed{R = \frac{V}{i_g} - R_g}$$



A galvanometer has a coil of resistance $100\ \Omega$ showing a full-scale deflection at $50\ \mu A$. What resistance should be added to use it as a voltmeter of range $50\ V$?

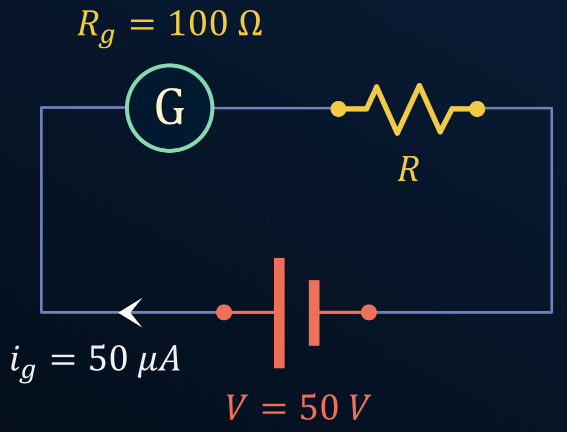
Applying KVL across the circuit,

$$i_g(R_g + R) = V$$

$$\Rightarrow R = \frac{V}{i_g} - R_g$$

$$= \frac{50}{50 \times 10^{-6}} - 100$$

$$\boxed{R \approx 10^6\ \Omega}$$

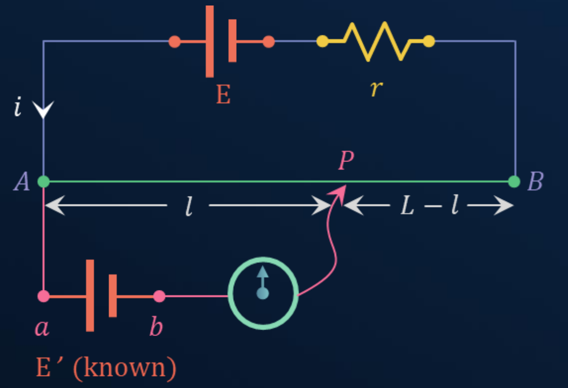


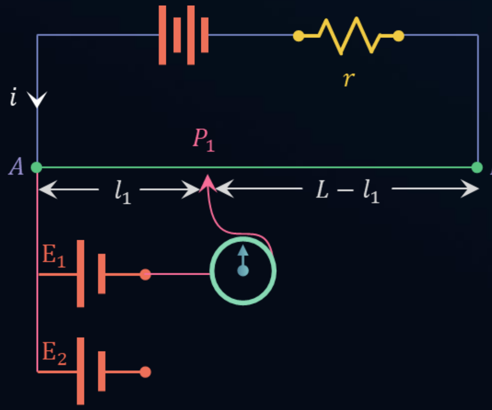
Calibration of Potentiometer

- If the null point is found at point P which is at a distance l from point A, then,

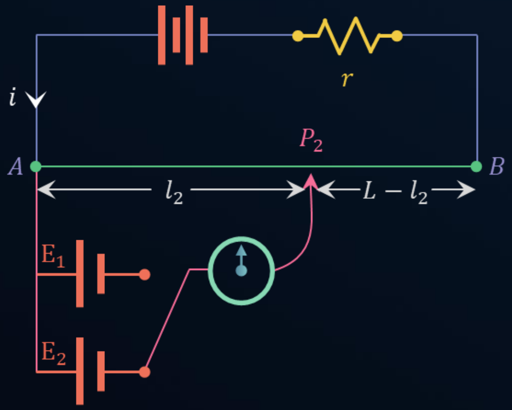
$$V_A - V_P = \mathrm{E}' = \frac{V_O}{L} l, \quad \text{where,} \quad V_O = V_A - V_B = \frac{\mathrm{E}R}{(R+r)}$$

R = Resistance of the potentiometer wire





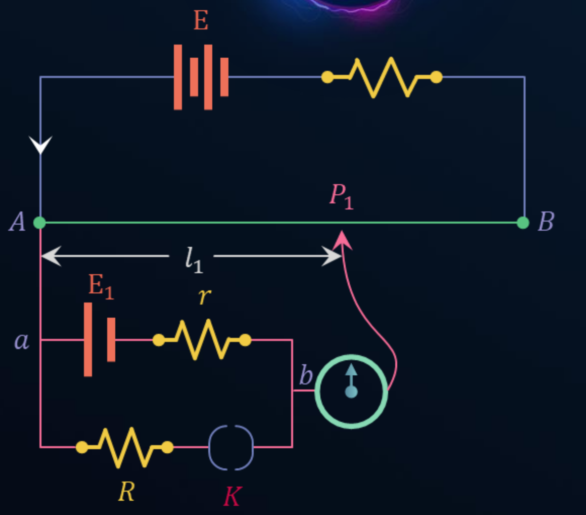
$$\mathrm{E}_1 = \frac{V_{AB}}{L} l_1$$



$$\mathrm{E}_2 = \frac{V_{AB}}{L} l_2$$

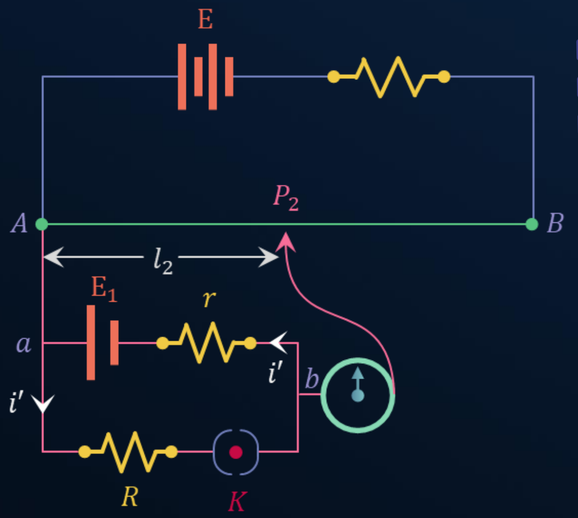
$$\boxed{\frac{\mathrm{E}_1}{\mathrm{E}_2} = \frac{l_1}{l_2}}$$

Measurement of Internal Resistance



- When the key is open, no current flows through the secondary circuit. If the null point is found at point P (length l_1), then,

$$E_1 = V_{AB}\frac{l_1}{L}$$



- At null deflection (P_2),

$$V_A - V_{P_2} = V_a - V_b$$

$$\Rightarrow V_{AB}\frac{l_2}{L} = i'R$$

$$\Rightarrow V_{AB}\frac{l_2}{L} = \frac{E_1 R}{R+r}$$

$$E_1 = V_{AB}\frac{l_1}{L}; \quad \frac{E_1 R}{R+r} = V_{AB}\frac{l_2}{L}$$

By dividing these two,

$$\frac{l_1}{l_2} = \frac{R+r}{R}$$

$$= 1 + \frac{r}{R}$$

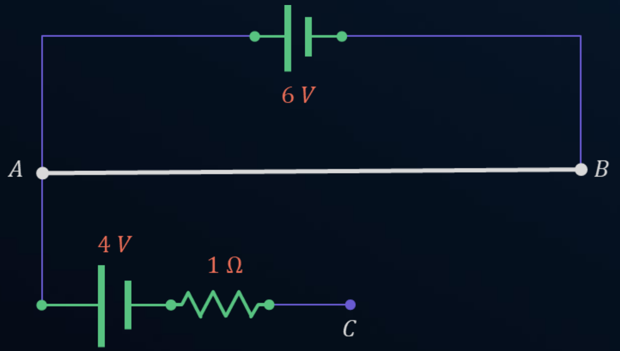
$$\boxed{r = \frac{R(l_1 - l_2)}{l_2}}$$

A $6\ V$ battery of negligible internal resistance is connected across a uniform wire AB of length $100\ cm$. The positive terminal of another battery of emf $4\ V$ and internal resistance $1\ \Omega$ is joined to the point A as shown. Take the potential at B to be zero.

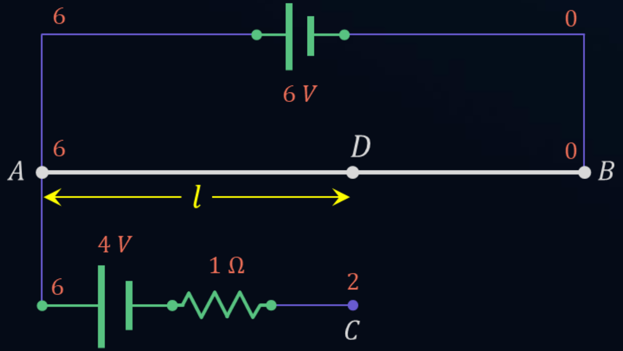
(a) What are the potentials at A and C?

(b) At what point D of the wire AB, the potential is equal to that at C?

(c) If points C and D are connected by a wire, what will be the current through it?



Extrapolating potential at various points,



Thus, the voltages at A and C are, $\boxed{V_A = 6\ V, \qquad V_C = 2\ V}$

Given, $V_D = V_C$

Also, $V_A = 6\ V \qquad V_C = 2\ V \qquad AB = L = 100\ cm$

Taking the potential gradient, $$\frac{V_A - V_B}{L} = \frac{V_A - V_D}{l}$$

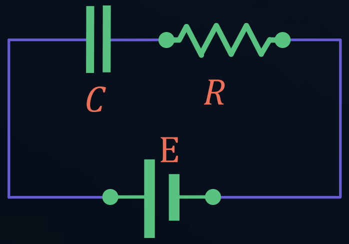
Since, $V_A - V_B = 6\ V$ and $V_A - V_D = 4\ V$

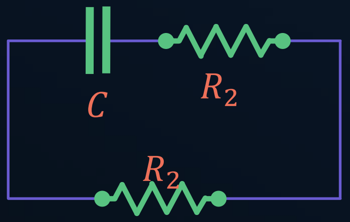
We get, $l = \dfrac{2}{3}L$ $\qquad \boxed{AD = 66.67\ cm}$

Thus, current across DC, $i = \dfrac{(V_D - V_c)}{R}$ $\qquad \boxed{i = 0}$

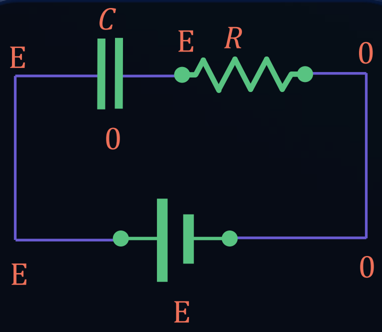
Charging a RC Circuit

An RC circuit is a circuit defined by the combination of a resistor and a capacitor

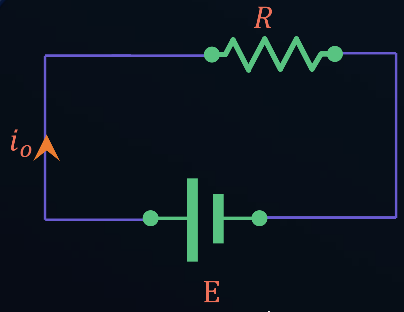




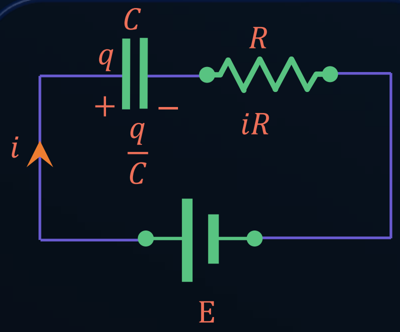
- At time t, the charge on the capacitor is q and current i
- At time $t = 0$, the charge on the capacitor is 0 and current $i = i_0 = \frac{E}{R}$



At $t = 0$

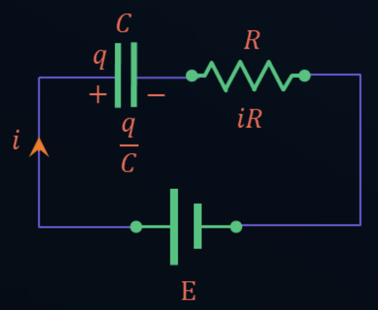


At $t = 0^+$



At t

Derivation of Charging RC Circuit



At t

Applying KVL At time t,

$$-\frac{q}{C} - iR + \mathrm{E} = 0$$

Thus, we get,

$$i = \frac{dq}{dt} = \frac{C\mathrm{E} - q}{RC}$$

Integrating using variable separable method,

$$\int_0^q \frac{dq}{C\mathrm{E} - q} = \frac{1}{RC}\int_0^t dt$$

$$\ln\left[\frac{C\mathrm{E} - q}{-1}\right]_o^q = \frac{1}{RC}[t]_o^t$$

Applying the limits,

$$\ln\left[\frac{C\mathrm{E} - q}{C\mathrm{E}}\right] = -\frac{t}{RC}$$

Thus, we get,

$$\boxed{q = C\mathrm{E}\left(1 - e^{-\frac{t}{RC}}\right)}$$

Differentiating w.r.t time, we get current as,

$$\boxed{i = \frac{\mathrm{E}}{R}e^{-\frac{t}{RC}}}$$

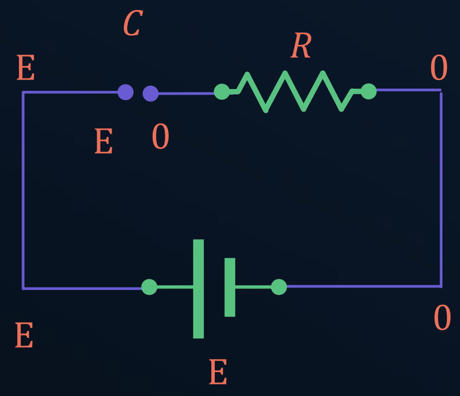
Derivation of Charging RC Circuit

$$q = CE\left(1 - e^{-\frac{t}{RC}}\right)$$

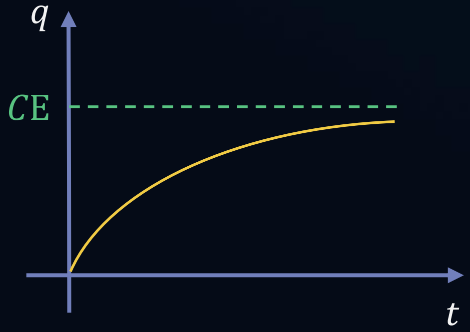
$$i = \frac{E}{R}e^{-\frac{t}{RC}}$$

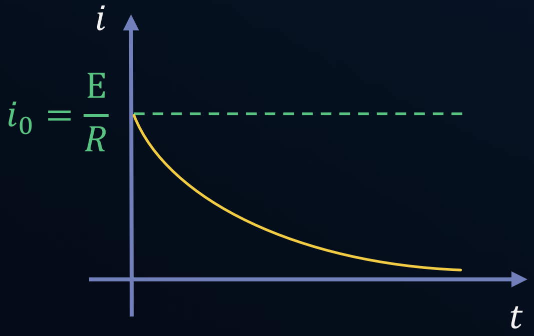
- As time goes on, the charge on the capacitor increases.
- As time goes on, current across the circuit decreases.

At $t = \infty$, $q = CE\,;\, i = 0$ (Steady State)



At $t = \infty$





Time Constant of a RC Circuit

The product RC is called the time constant (τ) and its SI unit is seconds.

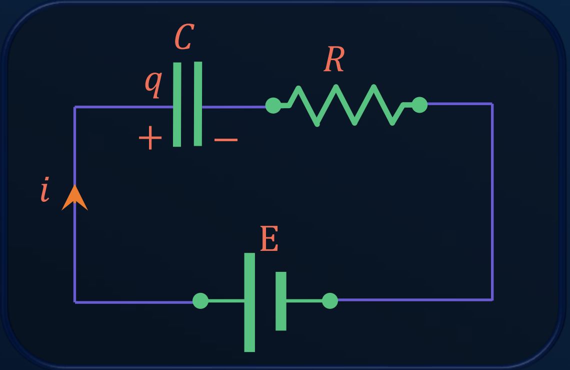
$$RC = \tau$$

Thus, at $t = \tau$, we get,

$$q = CE\,(1 - e^{-1}) \approx 0.63CE$$

Similarly, at $t = \tau$, we get current as,

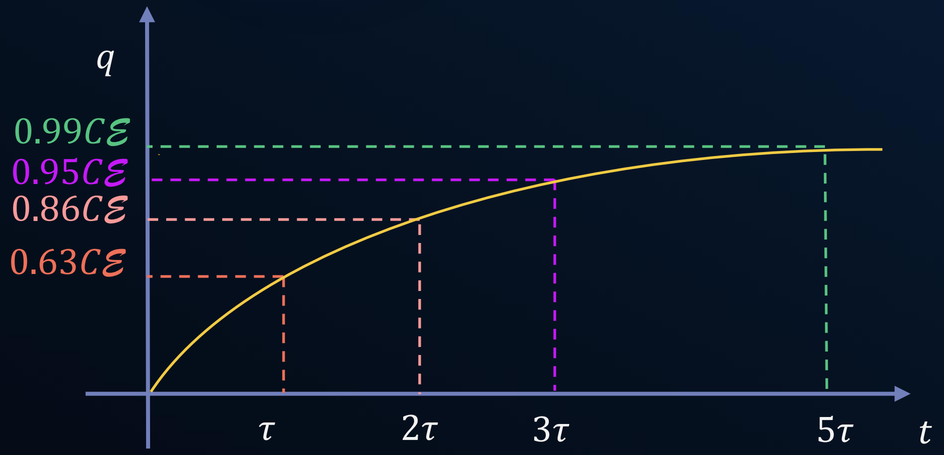
$$i = \frac{E}{R} e^{-1} = 0.37 \frac{E}{R}$$

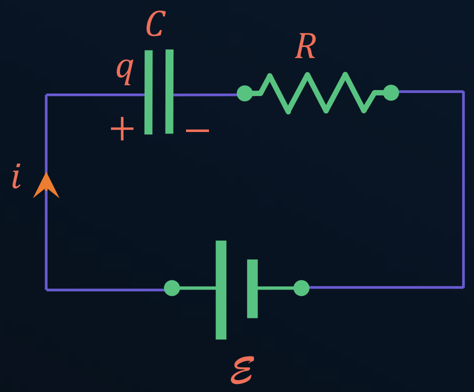


Values of charge and current at different time instances are as follows,

t	q	i
t	$CE\left(1 - e^{-\frac{t}{RC}}\right)$	$\frac{E}{R} e^{-\frac{t}{RC}}$
0	0	$\frac{E}{R}$
τ	$0.63CE$	$0.37\frac{E}{R}$
∞	CE	0

Time Constant of an RC Circuit

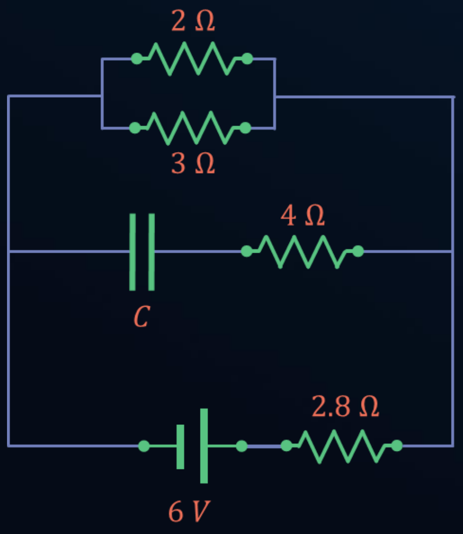


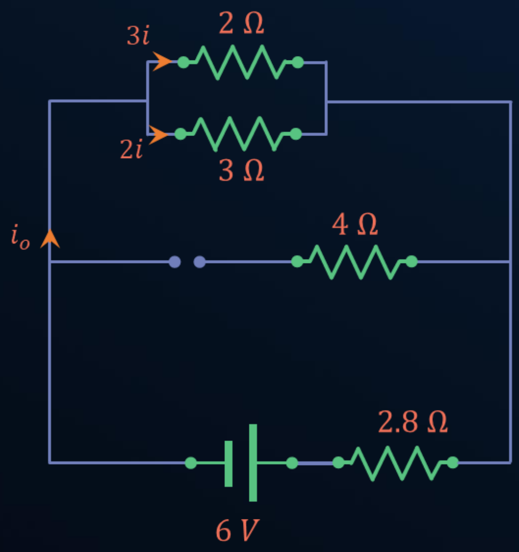


$$q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

- Nearly 95% of the charging occurs within three time constants.
- It takes five time constants to reach almost (99%) steady state.

What is the steady state current in the $2\,\Omega$ resistor shown in the circuit? The internal resistance of battery is negligible and the capacitance of condenser C is $0.2\ \mu F$.





At steady state, current across the condenser is zero and behaves as an open circuit (branch)

Thus, at $t = \infty$, we get,

$$i_o = \frac{6}{R_{eq}} = \frac{6}{2.8 + \frac{1}{\frac{1}{2} + \frac{1}{3}}} = \frac{6}{2.8 + 1.2} = \frac{6}{4} = 1.5\ A$$

By ratio of currents across parallel resistors,

$$3i + 2i = 1.5\ A \quad \Rightarrow i = 0.3\ A$$

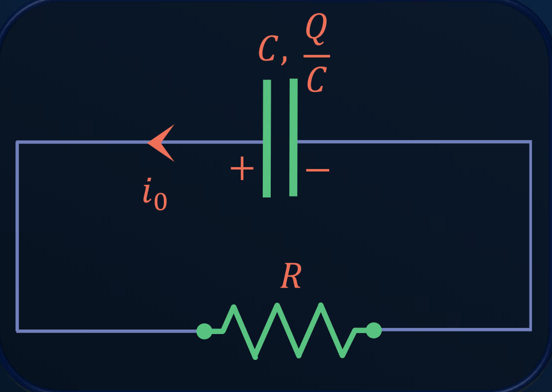
Thus, current across $2\,\Omega$ resistor = $3i$

$$\boxed{i_{2\Omega} = 0.9\ A}$$

Discharging RC Circuit

- At time $t = 0$, the p.d. across the capacitor is, $V = \frac{Q}{C}$

- Just after the circuit is completed, an initially charged capacitor behaves as a battery having EMF $\frac{Q}{C}$ and the current through the resistor is, $i_0 = \frac{Q}{RC}$

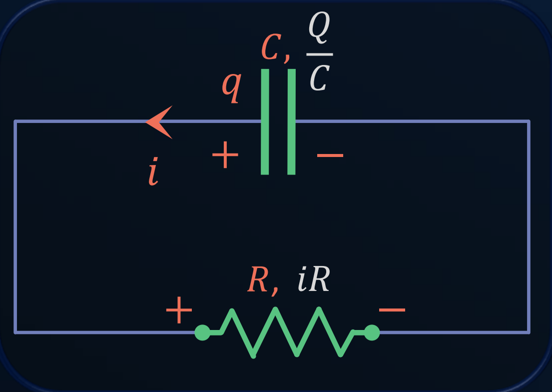


- The relation between i and q is, $\boxed{i = -\frac{dq}{dt}}$

  The $-ve$ sign indicates that the charge in the capacitor is decreasing as time progresses.

- Apply KVL in the circuit at time t:

$$-\frac{q}{C} + iR = 0 \qquad i = \frac{q}{RC} = -\frac{dq}{dt} \qquad \frac{dq}{q} = -\frac{dt}{RC}$$

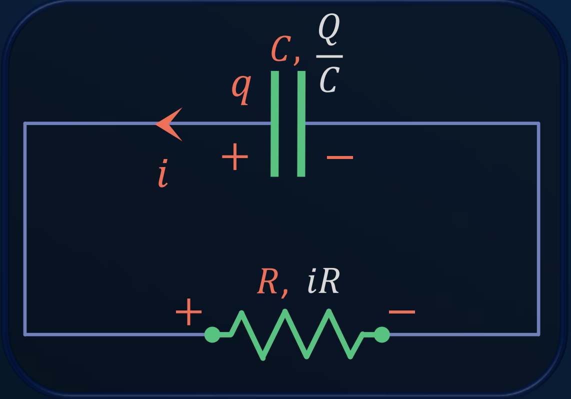


Derivation of Discharging RC Circuit

Integrating, we get, $$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

After solving we get, $$\boxed{q = Qe^{-\frac{t}{RC}}}$$

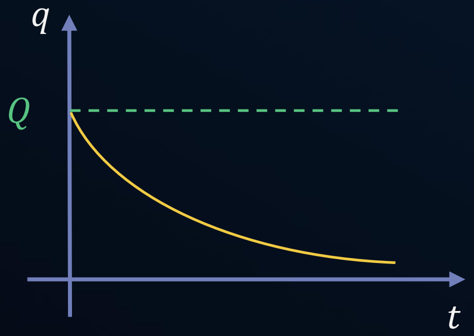
Differentiating w.r.t time, we get current as, $$\boxed{i = \frac{Q}{RC}e^{-\frac{t}{RC}}}$$

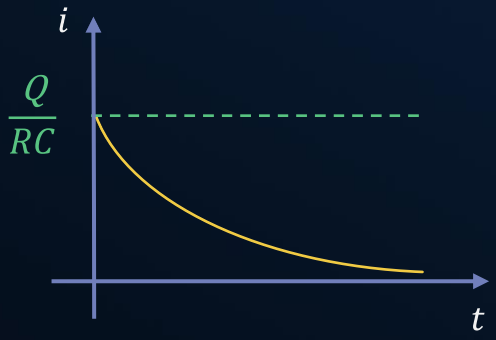


- As time goes on, the charge on the capacitor decreases.
- As time goes on, current across the circuit decreases.

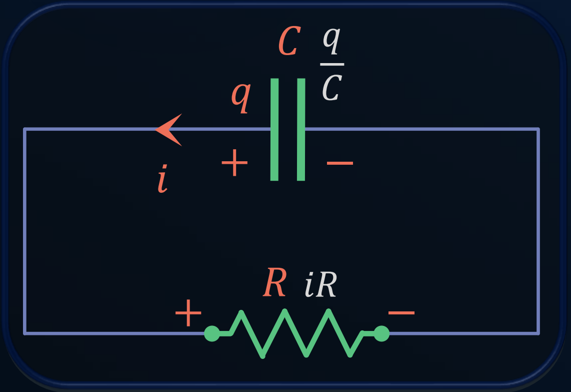
At $t = \infty$,

$q = CE\,;\, i = 0$ (Steady State)



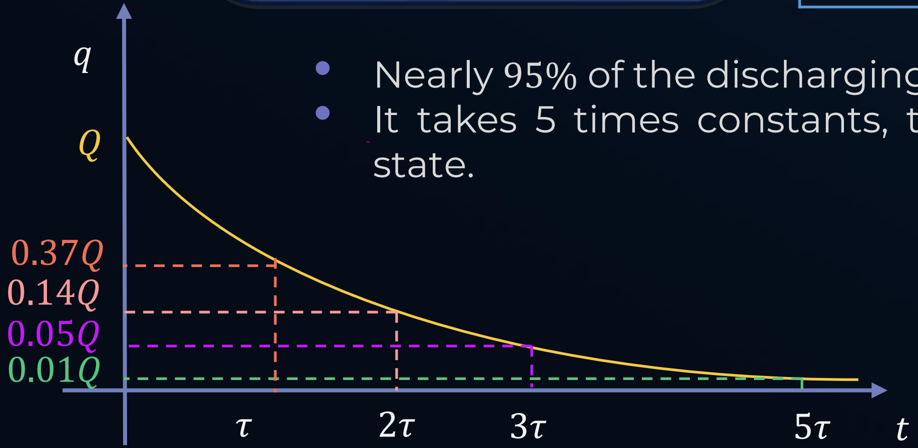


Discharging RC Circuit



t	q	i
t	$Qe^{-\frac{t}{RC}}$	$\frac{Q}{RC}e^{-\frac{t}{RC}}$
0	Q	$\frac{Q}{RC}$
τ	$0.37Q$	$0.37\frac{Q}{RC}$
∞	0	0

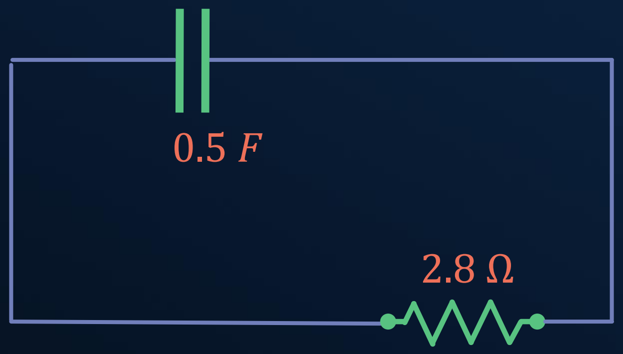
- Nearly 95% of the discharging occurs within 3 times constants.
- It takes 5 times constants, to reach almost (99% discharging) steady state.



How many time constants will elapse before the charge on a capacitor falls to 0.1% of its maximum value in a discharging RC circuit?

We know,

$$q = Qe^{-\frac{t}{\tau}} \qquad i = \frac{Q}{RC}e^{-\frac{t}{\tau}} = i_0 e^{-\frac{t}{\tau}}$$



Let $t = n\tau$. Thus, at t,

$$q = \frac{0.1}{100}Q = \frac{Q}{1000}$$

$$\frac{Q}{1000} = Qe^{-\frac{n\tau}{\tau}} \qquad \frac{1}{1000} = e^{-n} \qquad \Rightarrow e^{n} = 1000$$

To find n,

$$n = 3\ln 10$$

$\therefore$ $\boxed{n \approx 6.9}$,i.e., Time taken is $6.9\,\tau$

Find the time constant for the given RC circuit.
[Given: $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 4\ \mu F$, $C_1 = 2\ \mu F$]

- Time constant for RC circuit : $\boxed{\tau = R_{eq} C_{eq}}$

- Equivalent resistance :

  $R_{eq} = R_1 + R_2$ (Resistors are in series)

  $R_{eq} = 1\ \Omega + 2\ \Omega = 3\ \Omega$

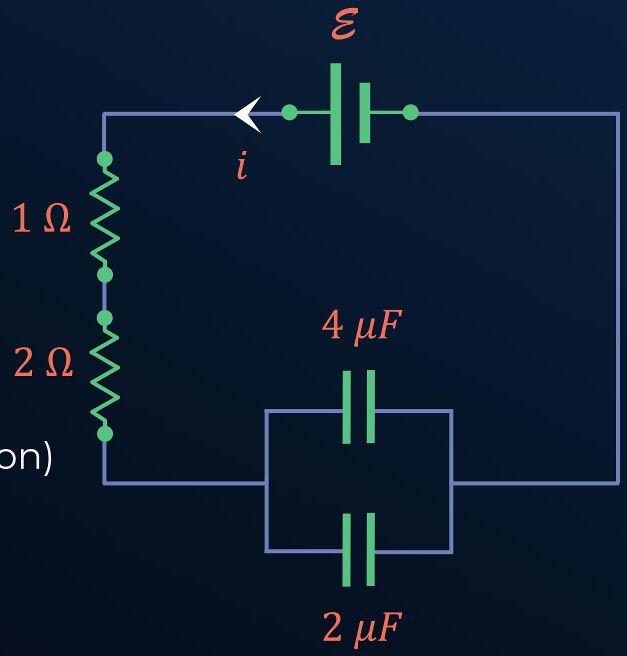
- Equivalent capacitance :

  $C_{eq} = C_1 + C_2$ (Capacitors are in parallel connection)

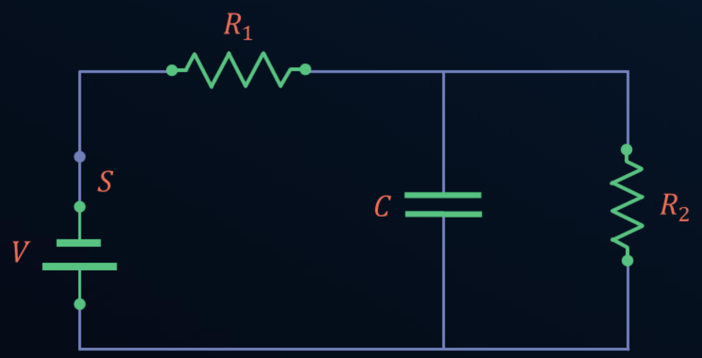
  $C_{eq} = 4\ \mu F + 2\ \mu F = 6\ \mu F$

- Time constant:

  $\tau = R_{eq} C_{eq} = 3 \times 6 = 18\ \mu s$ $\boxed{\tau = 18\ \mu s}$



At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_0(1 - e^{-\alpha t})$. Obtain the value of Q_0 and α in terms of given circuit parameters.



- For charging RC circuit : $Q = Q_0\left(1 - e^{-\frac{t}{\tau}}\right)$

Q_0 = Charge stored in the capacitor at steady state

$= C \times$ [P.D. across the capacitor at steady state]

$= C \times$ [P.D. across the resistance R_2]

$= C \times [R_2 \times$ (Current through the resistance R_2)]

$$= CR_2 \times \frac{V}{(R_1 + R_2)}$$

$$\boxed{Q_0 = \frac{CVR_2}{(R_1 + R_2)}}$$

- Given : $Q = Q_0(1 - e^{-\alpha t})$
- We have : $Q = Q_0\left(1 - e^{-t/\tau}\right)$

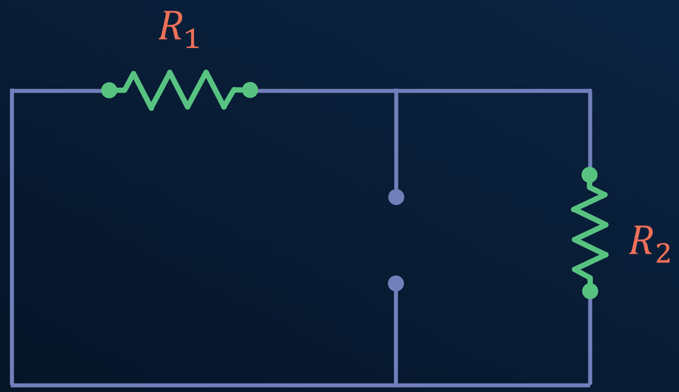
$$\boxed{\alpha = \frac{1}{\tau}}$$

- Time constant of the given circuit :

$\tau = CR_{eq}$

R_{eq} = Equivalent resistance across the capacitor after short circuiting the battery.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



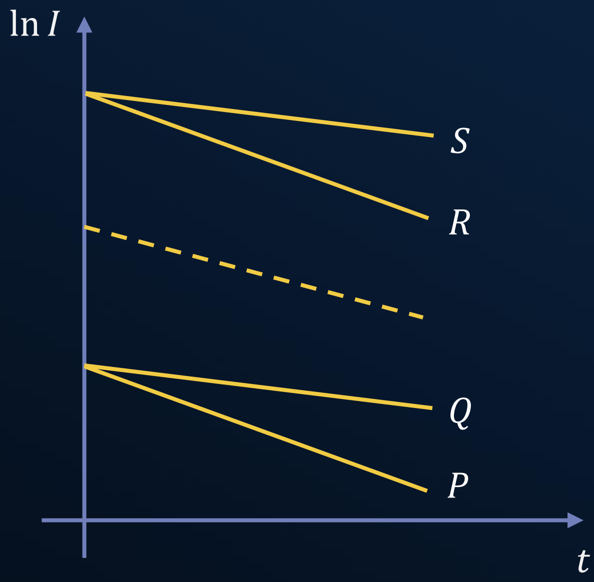
- Finally,

$$\tau = CR_{eq} = \frac{CR_1R_2}{R_1 + R_2} \longrightarrow \alpha = \frac{1}{\tau} \longrightarrow \boxed{\alpha = \frac{R_1 + R_2}{CR_1R_2}}$$

A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of $\ln I$ with respect to time. If the resistance is changed to $2x$, the new graph will be

- For charging RC circuit : $I = \frac{\mathcal{E}}{R} e^{-t/RC}$
- Taking $\ln$ on both sides: $\ln I = \ln\left(\frac{\mathcal{E}}{R}\right) - \frac{t}{RC}$ → Slope = $-\frac{1}{RC}$
- Resistance is increased from x to $2x$ → Slope will become less steeper
- At $t = 0$, we have : $\ln I = \ln\left(\frac{\mathcal{E}}{R}\right)$

As resistance is increased, $\ln I$ will be decreased.



Graph Q represents the correct behaviour

?

How many time constants will elapse before the energy stored in a capacitor reaches half of its equilibrium value in a charging RC circuit.

- For charging RC circuit : $q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$

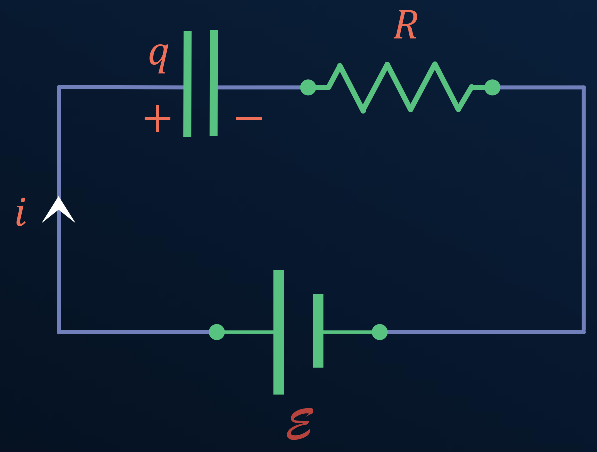
- The energy stored in the capacitor at equilibrium,

$$U_0 = \frac{Q^2}{2C}$$

- Assume that energy stored in the capacitor goes from U_0 to $\frac{U_0}{2}$ in time $t = n\tau$.

- At $t = n\tau$, energy stored in the capacitor,

$$U = \frac{q^2}{2C} = \frac{U_0}{2} \rightarrow \frac{q^2}{2C} = \frac{Q^2}{4C} \rightarrow q = \frac{Q}{\sqrt{2}}$$



- Finally,

$$q = Q\left(1 - e^{-\frac{t}{\tau}}\right)$$

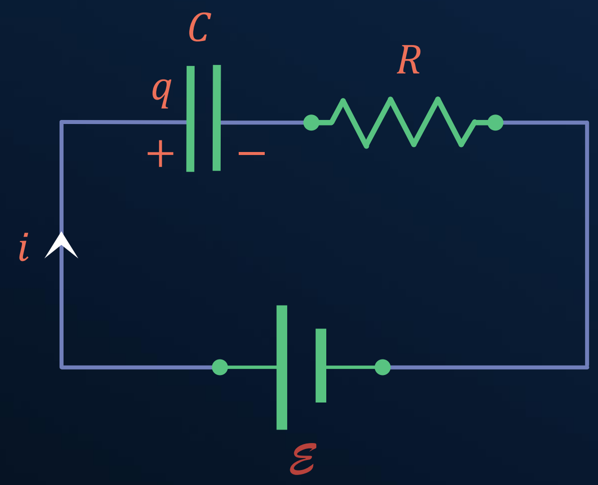
$$\frac{Q}{\sqrt{2}} = Q\left(1 - e^{-\frac{n\tau}{\tau}}\right)$$

$$e^{-n} = 1 - \frac{1}{\sqrt{2}}$$

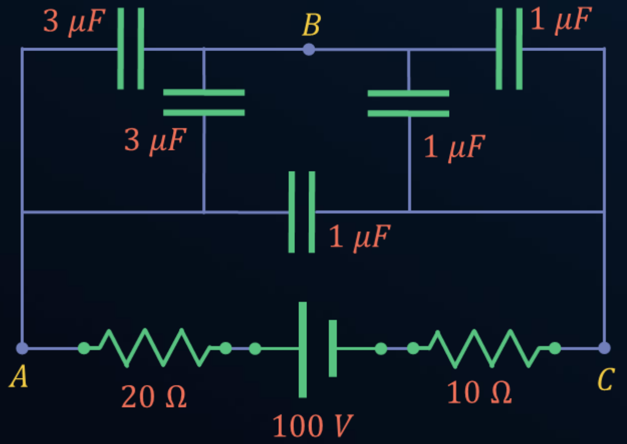
$$e^{n} = \frac{\sqrt{2}}{\sqrt{2} - 1} = \sqrt{2}(\sqrt{2} + 1)$$

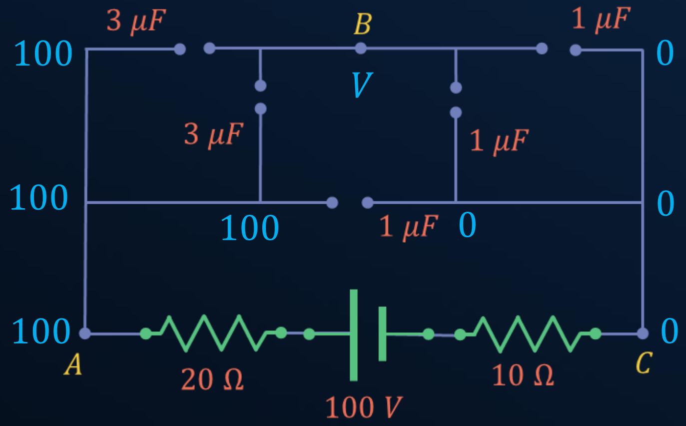
$$n = \ln\left(2 + \sqrt{2}\right) = 1.23$$

$$\boxed{t = 1.23\tau}$$



Find the potential difference between the points A and B and between the points B and C at the steady state.





- At steady state, capacitors behave as an open circuit.
- Choose zero potential at any specific junction and find potential of other junctions with respect to that.
- Now, if any particular junction is left out, choose its potential to be V.

- Apply KCL (Conservation of charge) in the annotated island:

$$3(V-100)+3(V-100)+1(V-0)+1(V-0)=0$$

$$V = 75\,Volts$$

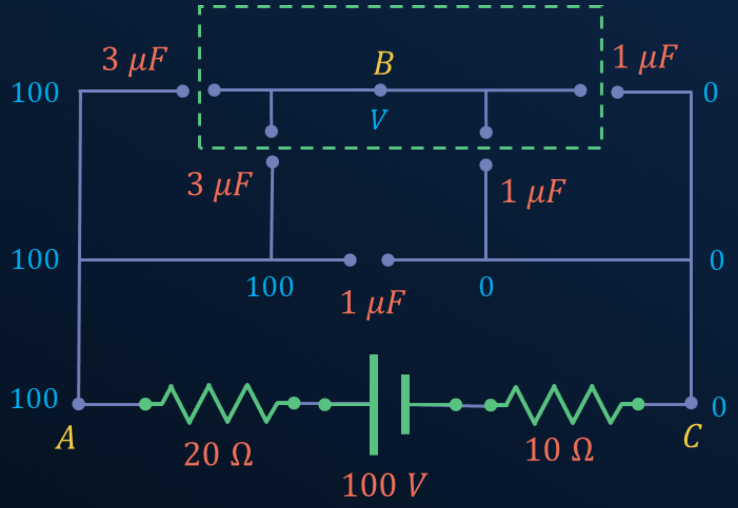
- Potential difference between any two junctions of a circuit should be positive just like difference between any two natural numbers is positive.

- Potential difference between A and B :

$$\boxed{V_{AB} = |V_A - V_B| = 25\,Volts}$$

- Potential difference between B and C :

$$\boxed{V_{BC} = |V_B - V_C| = 75\,Volts}$$



A capacitor of capacitance C charged to a potential difference V is discharged by connecting its plates through a resistance R. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i^2R\,dt$ and also by finding the decrease in the energy stored in capacitor.

- For discharging RC circuit :

$$i = \frac{Q}{RC}e^{-\frac{t}{RC}} = i_0 e^{-\frac{t}{\tau}}$$

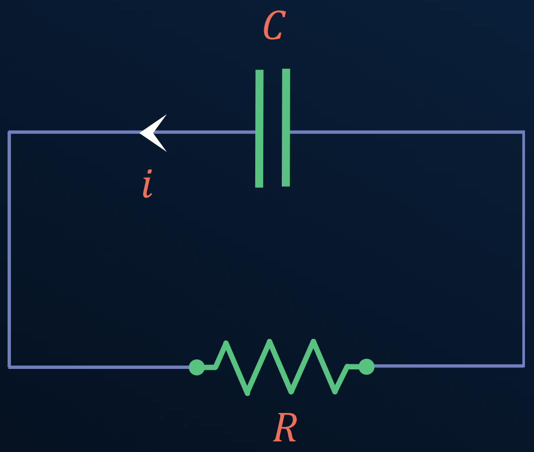
- Heat dissipated in one time constant :

$$H = \int_0^{\tau} i^2R\,dt = \int_0^{\tau} i_0^2 R\, e^{-\frac{2t}{\tau}}\,dt$$

$$\boxed{H = \frac{Q^2}{2C}\left(1 - \frac{1}{e^2}\right)}$$

Or

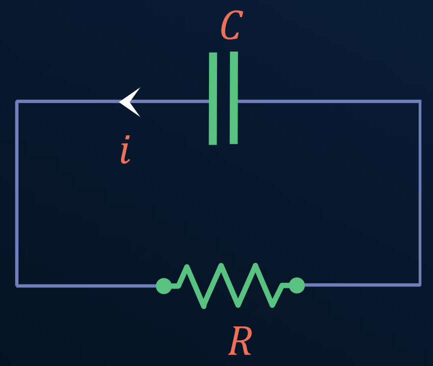
$$\boxed{H = \frac{1}{2}CV^2\left(1 - \frac{1}{e^2}\right)}$$



A capacitor of capacitance C charged to a potential difference V is discharged by connecting its plates through a resistance R. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i^2R\,dt$ and also by finding the decrease in the energy stored in capacitor.

- Initial charge on the capacitor = Q

  Initially, energy stored in the capacitor, $U_0 = \dfrac{Q^2}{2C}$



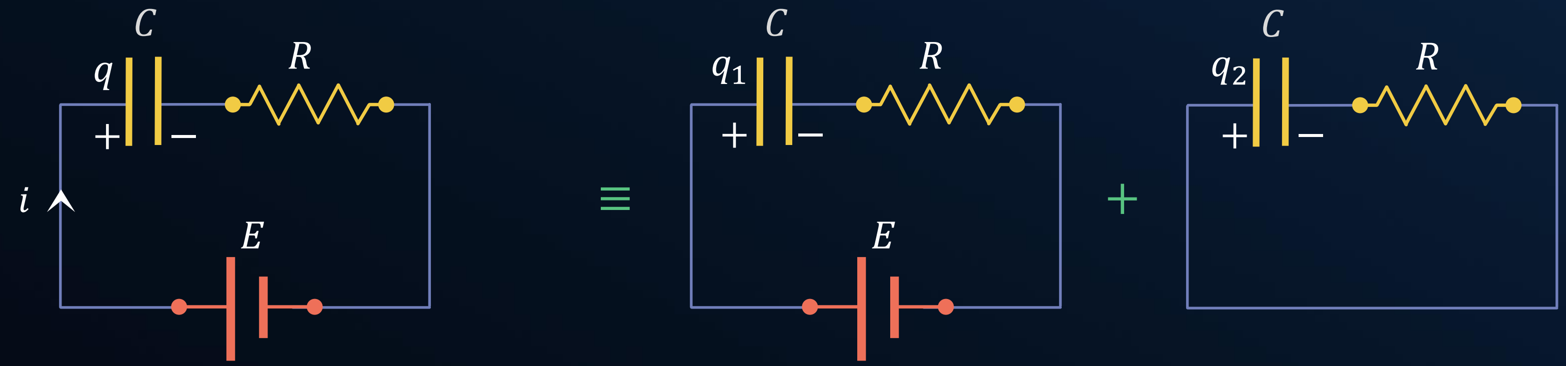
- After one time constant,

  Charge remaining on the capacitor, $q = 0.37Q = \dfrac{Q}{e}$

  Energy stored in the capacitor, $U_\tau = \dfrac{q^2}{2C} = \dfrac{\left(Q^2/_{2C}\right)}{e^2}$

- Energy lost, $U_{lost} = U_0 - U_\tau = \dfrac{Q^2}{2C}\left(1 - \dfrac{1}{e^2}\right) = H$

A capacitor of capacitance C is given a charge Q. At $t = 0$, it is connected to an ideal battery of emf E through a resistance R. Find the charge on the capacitor at time t.



At $t = 0, \; q = Q$

At $t = 0, \; q_1 = 0$

At $t = 0, \; q_2 = Q$

$$q_1(t) = CE(1 - e^{-t/RC})$$

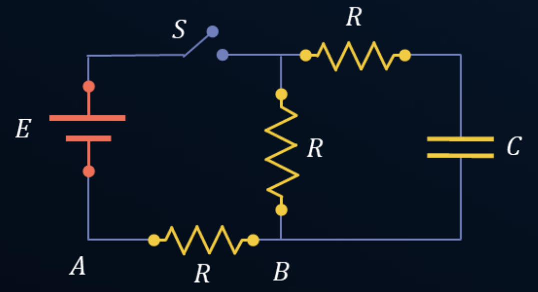
$$q_2(t) = Q(e^{-t/RC})$$

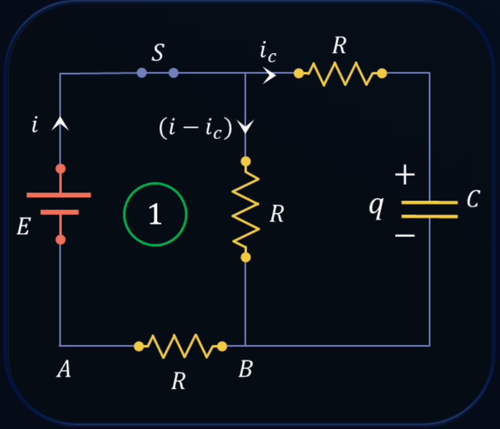
$$q(t) = q_1(t) + q_2(t)$$

$$\Rightarrow \boxed{q(t) = CE\left[1 - e^{-\frac{t}{RC}}\right] + Q\left[e^{-\frac{t}{RC}}\right]}$$

In the circuit shown in the figure, the battery is an ideal one, with emf E, the capacitor is initially uncharged. The switch is closed at $t = 0$.

(a) Find the charge on the capacitor at time t.





Outer loop:

$$-i_cR - \frac{q}{C} - iR + E = 0$$

$$\Rightarrow E = iR + i_cR + \frac{q}{C} \quad \text{——— (1)}$$

Loop 1:

$$-(i - i_c)R - iR + E = 0$$

$$\Rightarrow E = 2iR - i_cR \quad \text{——— (2)}$$

Putting value of iR from (2) in (1),

$$E = \left(\frac{E + i_cR}{2}\right) + i_cR + \frac{q}{C}$$

$$\Rightarrow i_c = \frac{CE - 2q}{3RC} \quad \Rightarrow \frac{dq}{dt} = \frac{CE - 2q}{3RC}$$

$$\Rightarrow \frac{dq}{CE - 2q} = \frac{dt}{3RC}$$

Integrating,

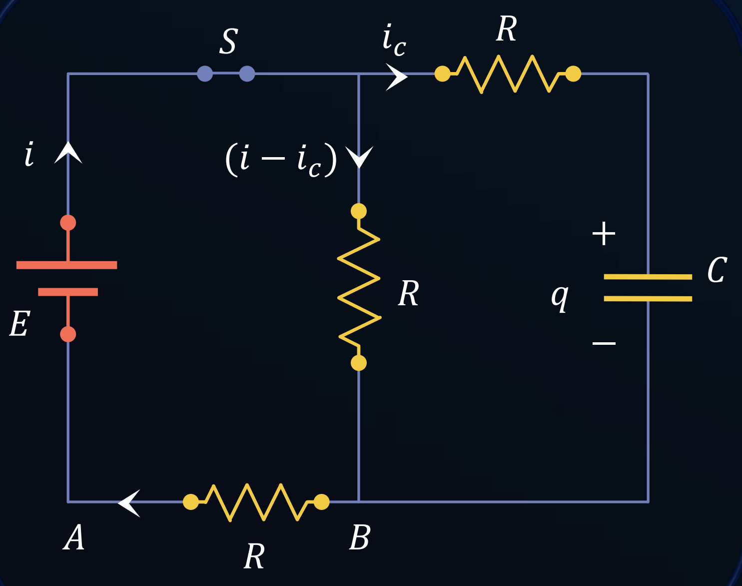
$$\Rightarrow \int_0^q \frac{dq}{CE - 2q} = \frac{1}{3RC}\int_0^t dt$$

$$\Rightarrow \left[\frac{\ln(CE - 2q)}{-2}\right]_0^q = \frac{1}{3RC}[t]_0^t$$

$$\Rightarrow \boxed{q = \frac{CE}{2}\left[1 - e^{-\frac{2t}{3RC}}\right]}$$

In the circuit shown in the figure, the battery is an ideal one, with emf E, the capacitor is initially uncharged. The switch is closed at $t = 0$.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$.



$$q = \frac{CE}{2}\left[1 - e^{-\frac{2t}{3RC}}\right] \qquad i_c = \frac{dq}{dt} = \frac{E}{3R}e^{-\frac{2t}{3RC}}$$

Apply KVL in the outer loop:

$$-i_c R - \frac{q}{C} - iR + E = 0$$

$$\Rightarrow i = \frac{E}{R} - i_c - \frac{q}{RC}$$

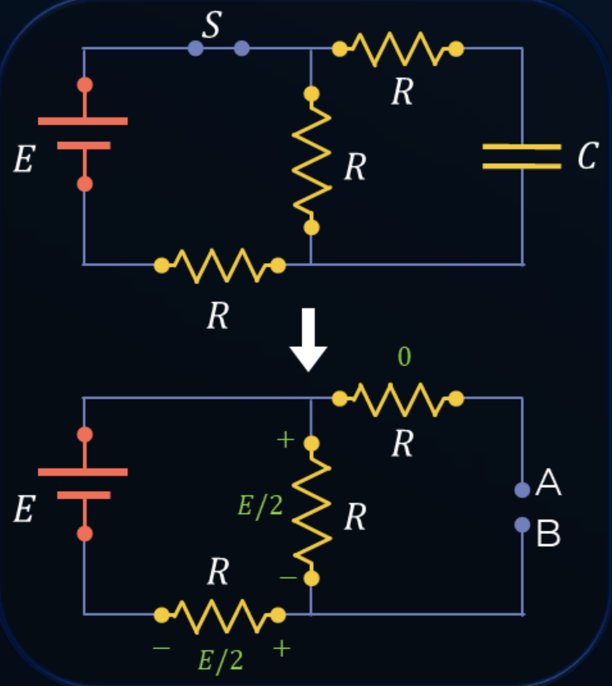
$$\Rightarrow i = \frac{E}{R} - \frac{E}{3R}e^{-\frac{2t}{3RC}} - \frac{1}{RC}\left[\frac{CE}{2}\left[1 - e^{-\frac{2t}{3RC}}\right]\right]$$

$$\Rightarrow i = \frac{E}{2R} + \frac{E}{6R}e^{-\frac{2t}{3RC}}$$

At $t = \infty$: $\boxed{i = \frac{E}{2R}}$

In the circuit shown in the figure, the battery is an ideal one, with emf E, the capacitor is initially uncharged. The switch is closed at $t = 0$.
(a) Find the charge on the capacitor at time t.

Alternate Solution :



Calculate τ_{eq}:

$$\tau_{eff} = CR_{eq}$$

$$R_{eq} = R_{AB}$$

$$= \frac{R \times R}{R + R} + R = \frac{3R}{2}$$

$$\Rightarrow R_{eq} = \frac{3R}{2}$$

$$\therefore \tau_{eff} = \frac{3RC}{2}$$

Calculate V_∞:

V_∞ = Steady state voltage across C

$$= V_{AB} = \frac{E}{2}$$

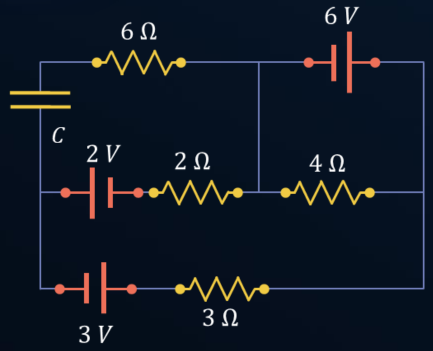
$$\Rightarrow V_\infty = \frac{E}{2}$$

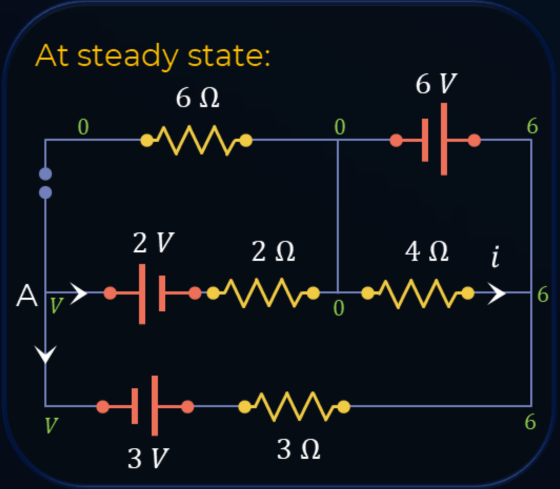
Charge on the capacitor:

$$q = CV_\infty\left(1 - e^{-\frac{t}{\tau_{eff}}}\right)$$

$$\Rightarrow \boxed{q = \frac{CE}{2}\left(1 - e^{-\frac{2t}{3RC}}\right)}$$

In the given circuit, $C = 5\ \mu F$. Find the current in R_3 and the energy stored in the capacitor.





Current in the 4Ω,

$$i = \frac{6-0}{4} \Rightarrow i = 1.5\ A$$

Applying junction rule at A,

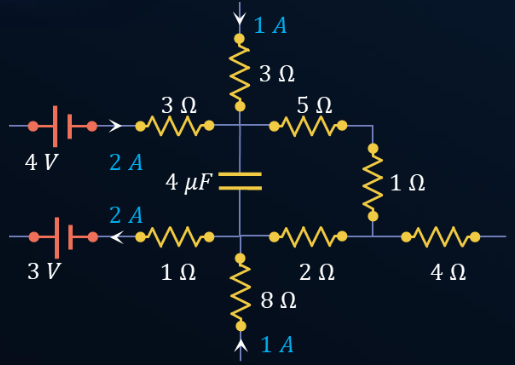
$$\frac{V-2-0}{2} + \frac{V+3-6}{3} = 0$$

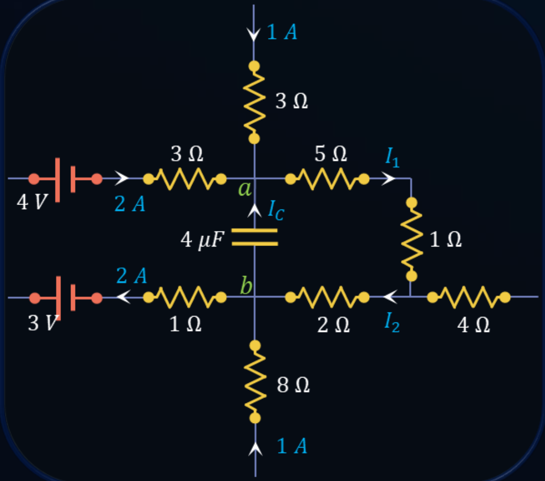
$$\Rightarrow V = \frac{12}{5}\ V$$

Energy in stored in the capacitor,

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times \left(\frac{12}{5}\right)^2 \Rightarrow \boxed{E = 1.44 \times 10^{-5}\ J}$$

Part of a circuit in a steady state along with the currents flowing in the branches, is shown in the figure. Calculate the energy stored in the capacitor.





Current across the capacitor in steady state,

$$I_C = 0\ A$$

Apply KCL at a:

$$2 + 1 + 0 = I_1$$

$$\Rightarrow I_1 = 3\ A$$

Apply KCL at b:

$$-2 + 1 + I_2 = 0$$

$$\Rightarrow I_2 = 1\ A$$

Apply KVL in the loop 1:

$$V_a - 5I_1 - I_1 - 2I_2 - V_b = 0$$

$$V_a - V_b = 6I_1 + 2I_2$$

$$= 6 \times 3 + 2 \times 1$$

$$= 20\ V$$

Energy stored in the capacitor:

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times 20^2$$

$$\Rightarrow \boxed{E = 8 \times 10^{-4}\ J}$$