Welcome to

Aakash + BYJU'S NOTES

Dual nature of radiation and matter

Nature of Light

Newton's Corpuscular Theory (1637) → Light is a particle

Huygens' Theory (1678) → Light is a wave

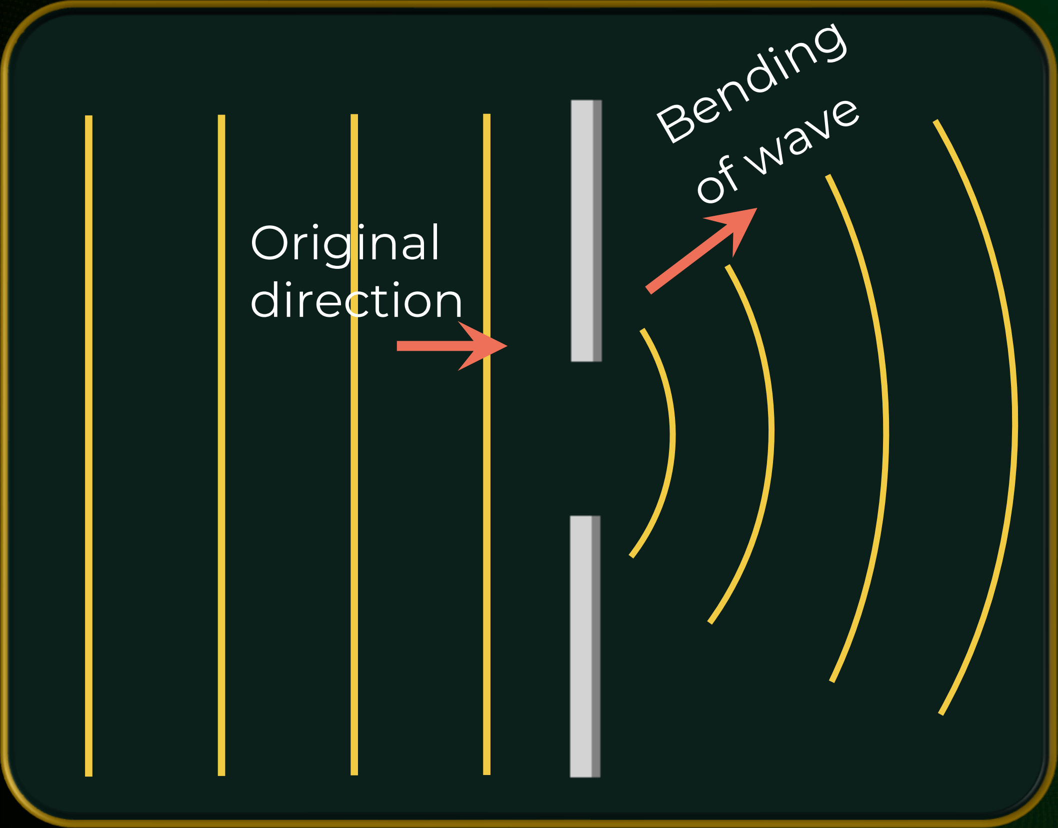
Maxwell's Electromagnetic Wave Theory (1861) → Light is a wave

Light is a particle ← Max Planck's Quantum Theory of Light (1900)

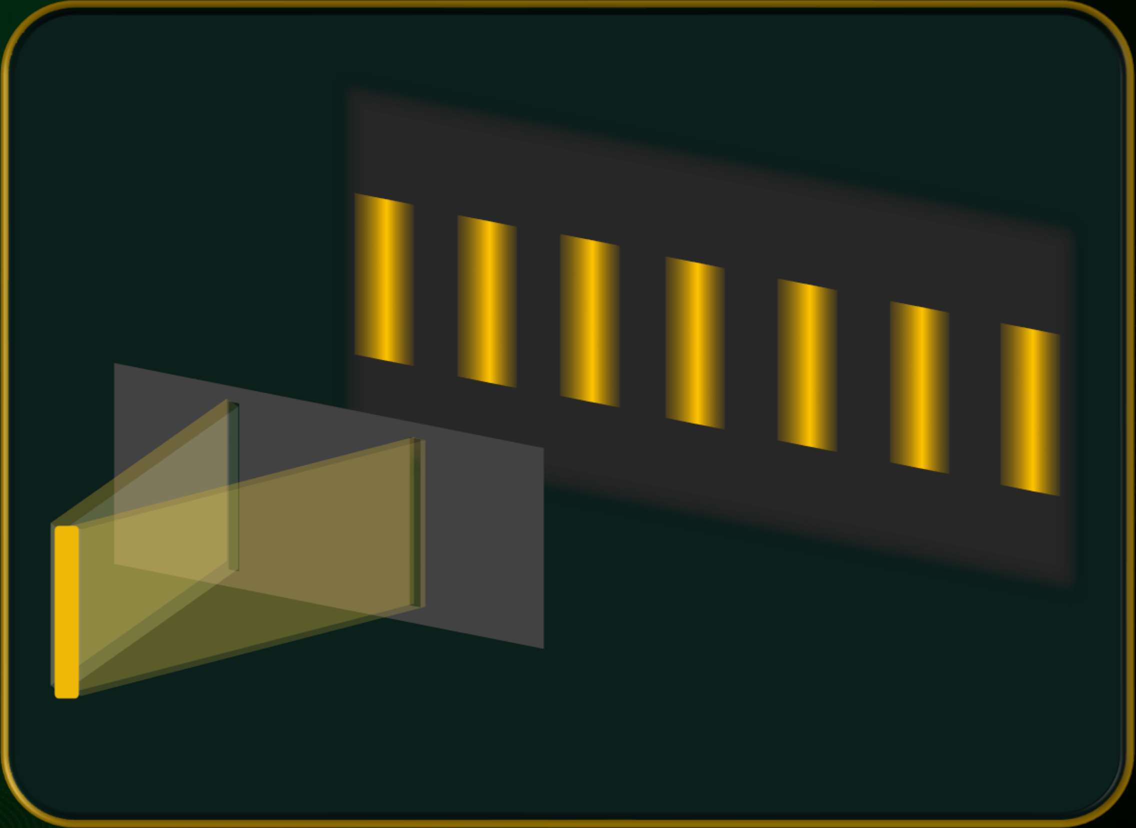
Dual nature of matter ← De Broglie Hypothesis (1924)

Wave Nature of Light

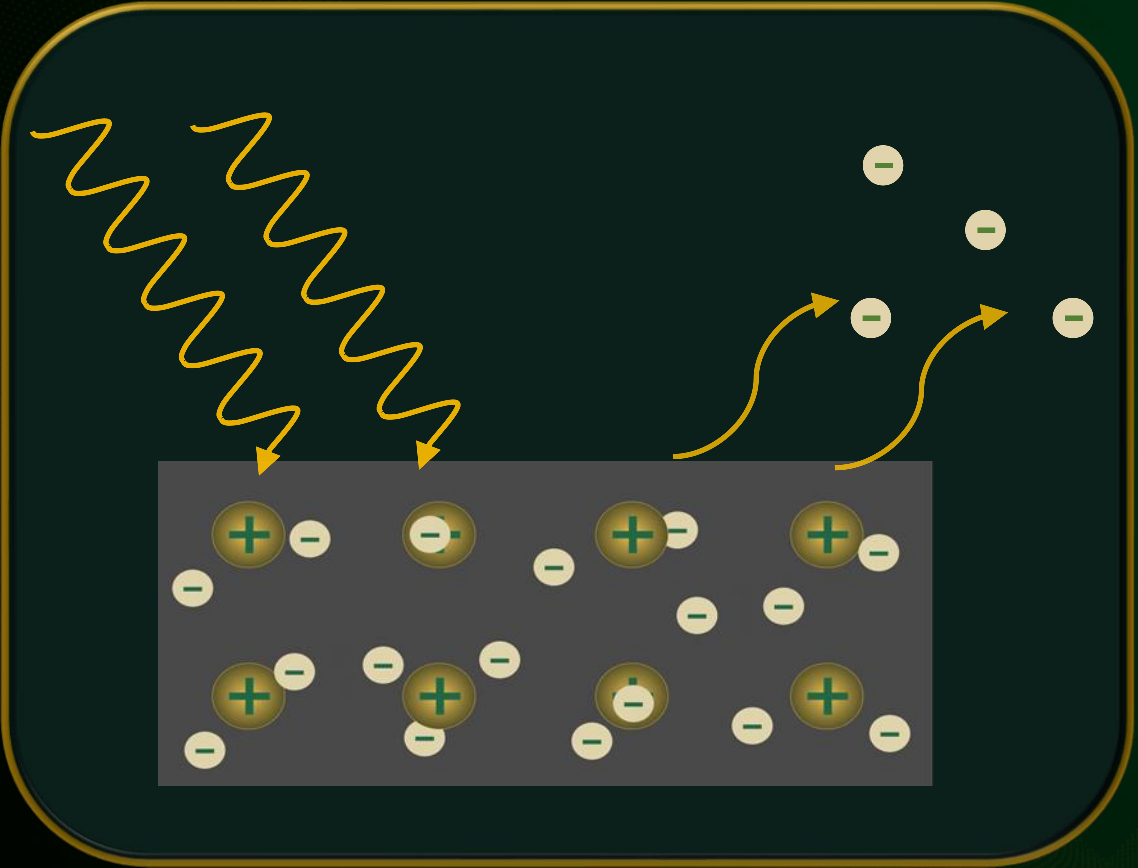
Diffraction of Light



Interference of Light

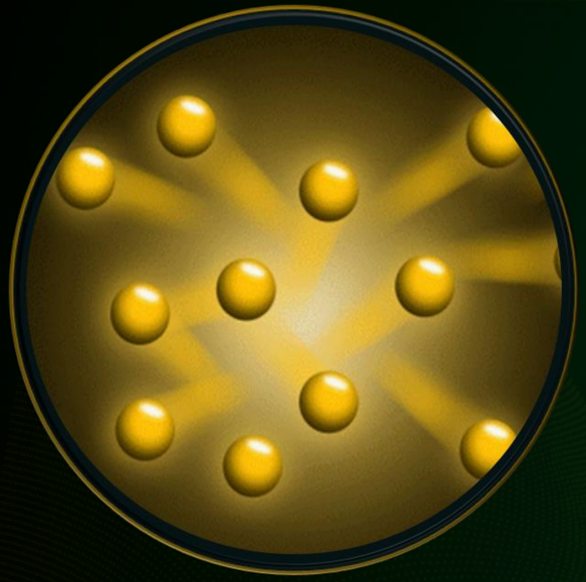


Particle Nature of Light – Photoelectric effect



- When light of suitable frequency illuminates a metal surface, free electrons are emitted from the metal surface.
- Photoelectric effect is explained by particle nature of light.

Photon Theory of Light



- Photons are smallest possible packets of electromagnetic energy.
- A photon has a definite energy and momentum.
- Photons always travel with the speed of light $(3 \times 10^8\ m/s)$ in vacuum.
- Energy of photon: $E = h\nu = \dfrac{hc}{\lambda}$

Planck constant $(h) = 6.626 \times 10^{-34}\ Js$

$h \approx 4.13 \times 10^{-15}\ eVs$

$hc \approx 2 \times 10^{-25}\ Jm \approx 12400\ eV\dot{A}$

eV

$$E = \frac{12400}{\lambda}$$

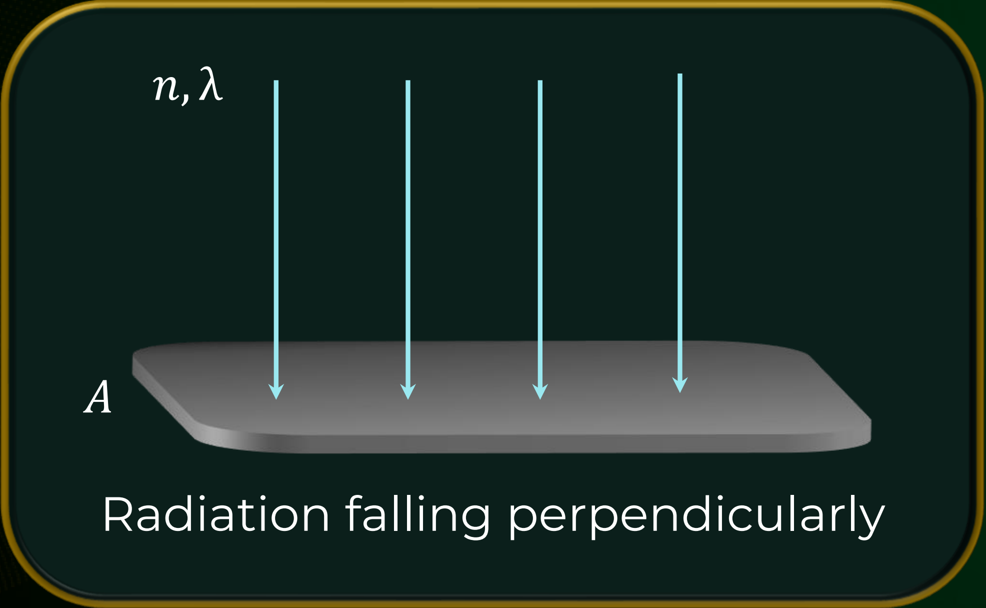
$eV\dot{A}$

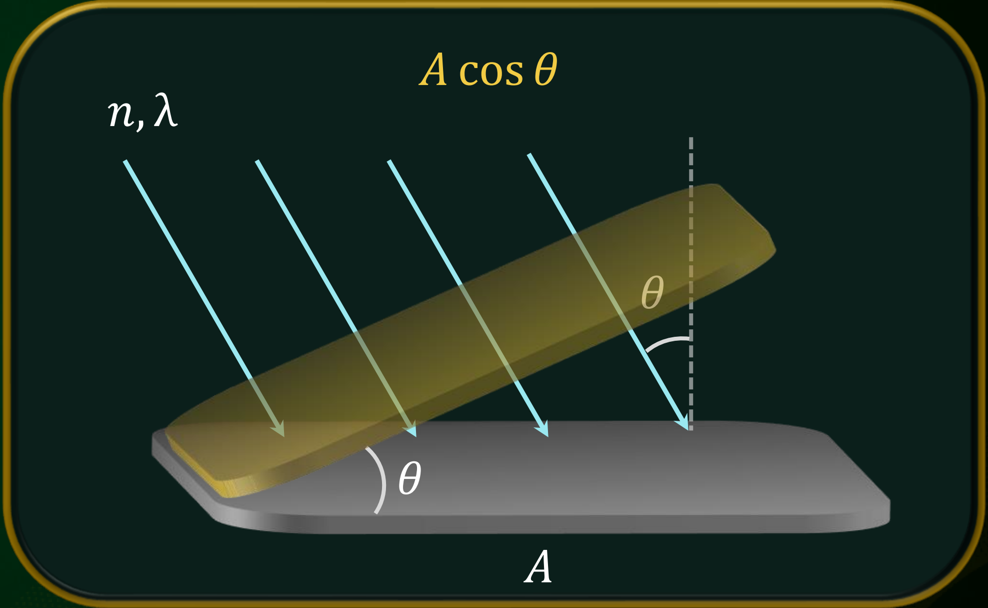
$\dot{A}$

- Momentum of a photon: $p = \dfrac{E}{c} = \dfrac{h}{\lambda}$
- Rest mass of photon is zero.
- Number of photons may not be conserved.
- Energy and momentum of a photon are always conserved.

Intensity of Light

Intensity of light(I): The energy crossing per unit area per unit time perpendicular to the direction of propagation.



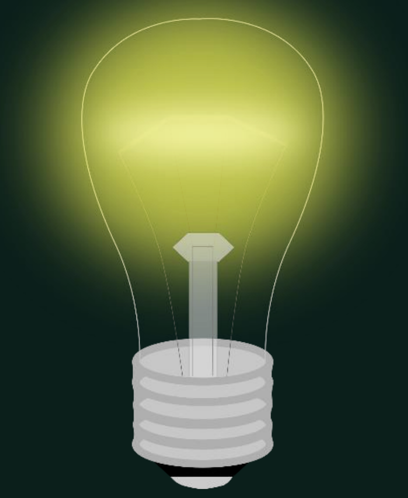


$$I = \frac{nhc}{\lambda A}$$

$$I = \frac{nhc}{\lambda A\cos\theta}$$

Note: n is the number of photons per second.

Photon Count



Power of the source: P

Monochromatic Light source

- Photon Count (n): Number of photons emitted per second by the source.

$$n = \frac{P\lambda}{hc}$$

- Photon Flux (ϕ): Number of photons incident normally on a surface per second per unit area.

$$I = \frac{nhc}{\lambda A} \Rightarrow \frac{n}{A} = \frac{I\lambda}{hc}$$

$$\phi = \frac{n}{A} = \frac{I\lambda}{hc}$$

When the sun is directly overhead, the surface of the earth receives $1.4 \times 10^3\ W/m^2$ of sunlight. Assume that the light is monochromatic with average wavelength $500\ nm$ and that no light is absorbed in between the sun and the earth's surface. The distance between the sun and the earth is $1.5 \times 10^{11}\ m$.

(a) Calculate the number of photons falling per second on each square meter of earth's surface directly below the sun.

A. 7×10^{21}

B. 3.5×10^{21}

C. 9×10^{20}

D. 1.4×10^{22}

Given: $I = 1.4 \times 10^3\ W/m^2,\ \lambda = 500\ nm,\ r = 1.5 \times 10^{11}\ m$

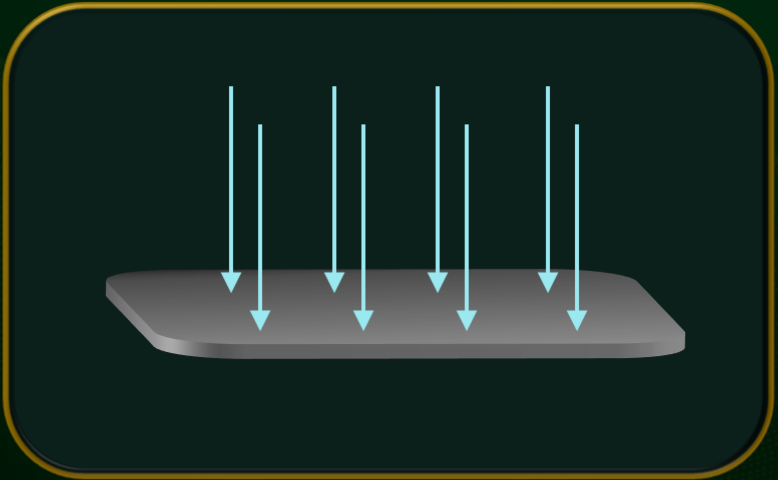
To find: n

Solution:

$$\boxed{\phi = \frac{n}{A} = \frac{I\lambda}{hc}} \Rightarrow n = \frac{I\lambda A}{hc}$$

$$n = \frac{1.4 \times 10^3 \times 500 \times 10^{-9} \times 1}{2 \times 10^{-25}}$$

$$n = 3.5 \times 10^{21}$$



When the sun is directly overhead, the surface of the earth receives $1.4 \times 10^3\ W/m^2$ of sunlight. Assume that the light is monochromatic with average wavelength $500\ nm$ and that no light is absorbed in between the sun and the earth's surface. The distance between the sun and the earth is $1.5 \times 10^{11}\ m$.

(b) How many photons are there in each cubic meter near the earth's surface at any instant?

Given: $I = 1.4 \times 10^3\ W/m^2, \lambda = 500\ nm,\ r = 1.5 \times 10^{11}\ m,\ n = 3.5 \times 10^{21}/m^2 s$

To find: N – number of photons in a cube of side $1\ m$.

Solution: n is the number of photons crossing the cube per second.

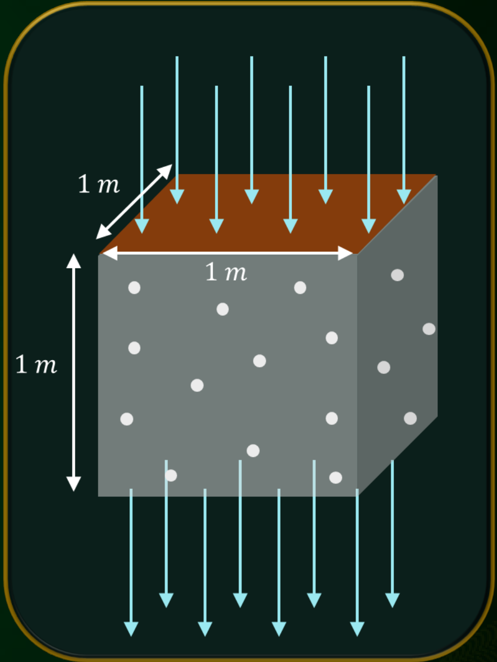
Let Δt be the time taken by the photon to travel $1\ m$ inside the cube.

$$\Delta t = \frac{1m}{c}$$

In Δt amount of time, N number photons enter the cube.

$$N = n \times \Delta t \times A$$

$$N = 3.5 \times 10^{21} \times \frac{1}{3 \times 10^8} \times (1 \times 1) \qquad N = 1.2 \times 10^{13}$$



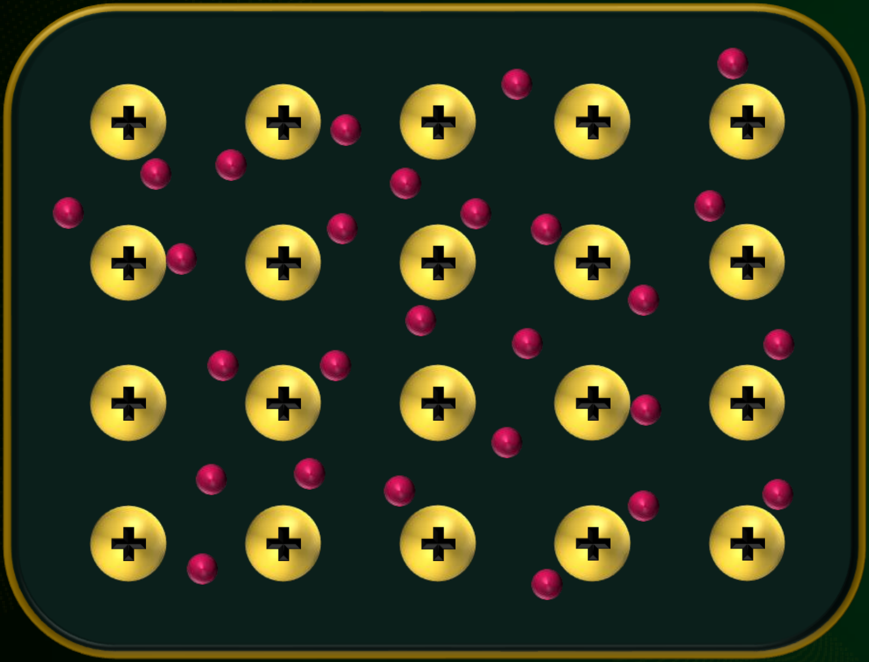
A 1.5×10^{13}

B 3.5×10^{21}

C 1.2×10^{13}

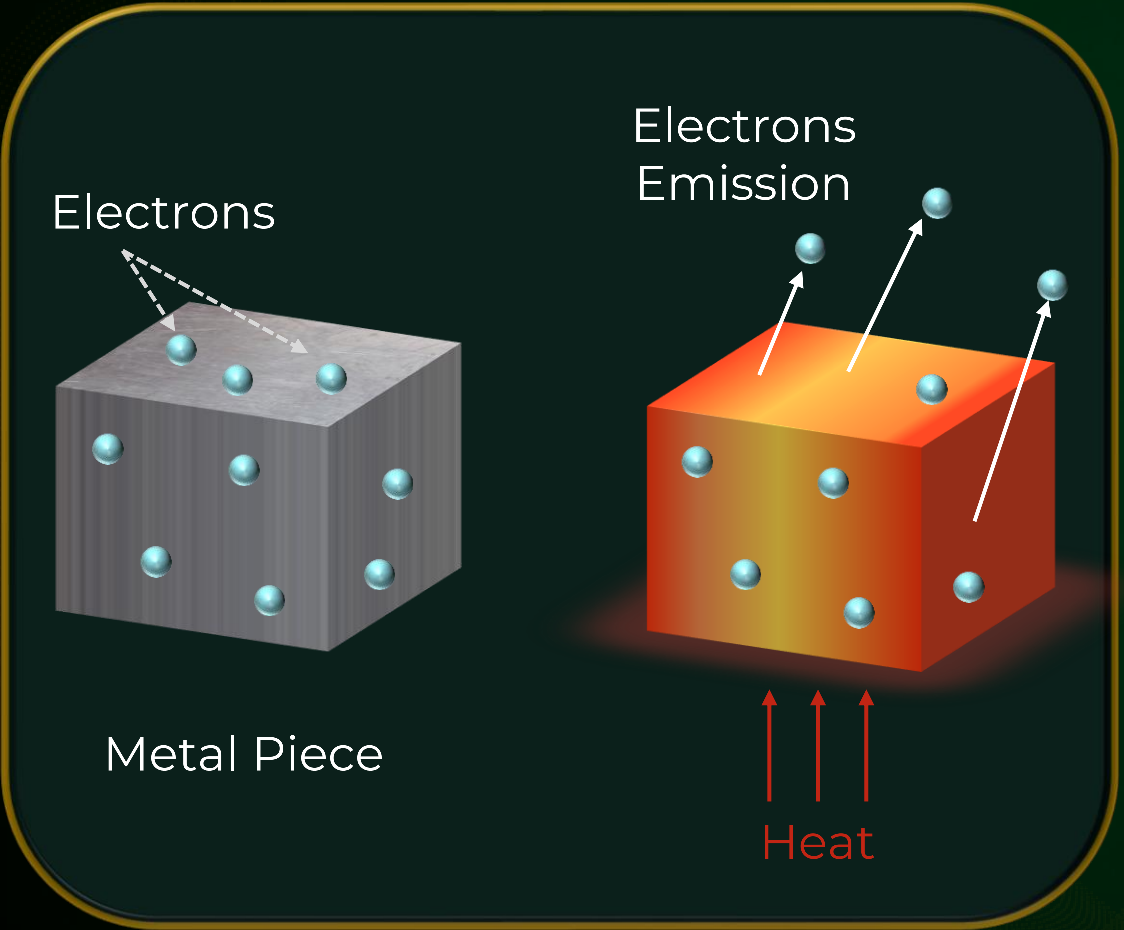
D 10.5×10^{29}

Free Electrons



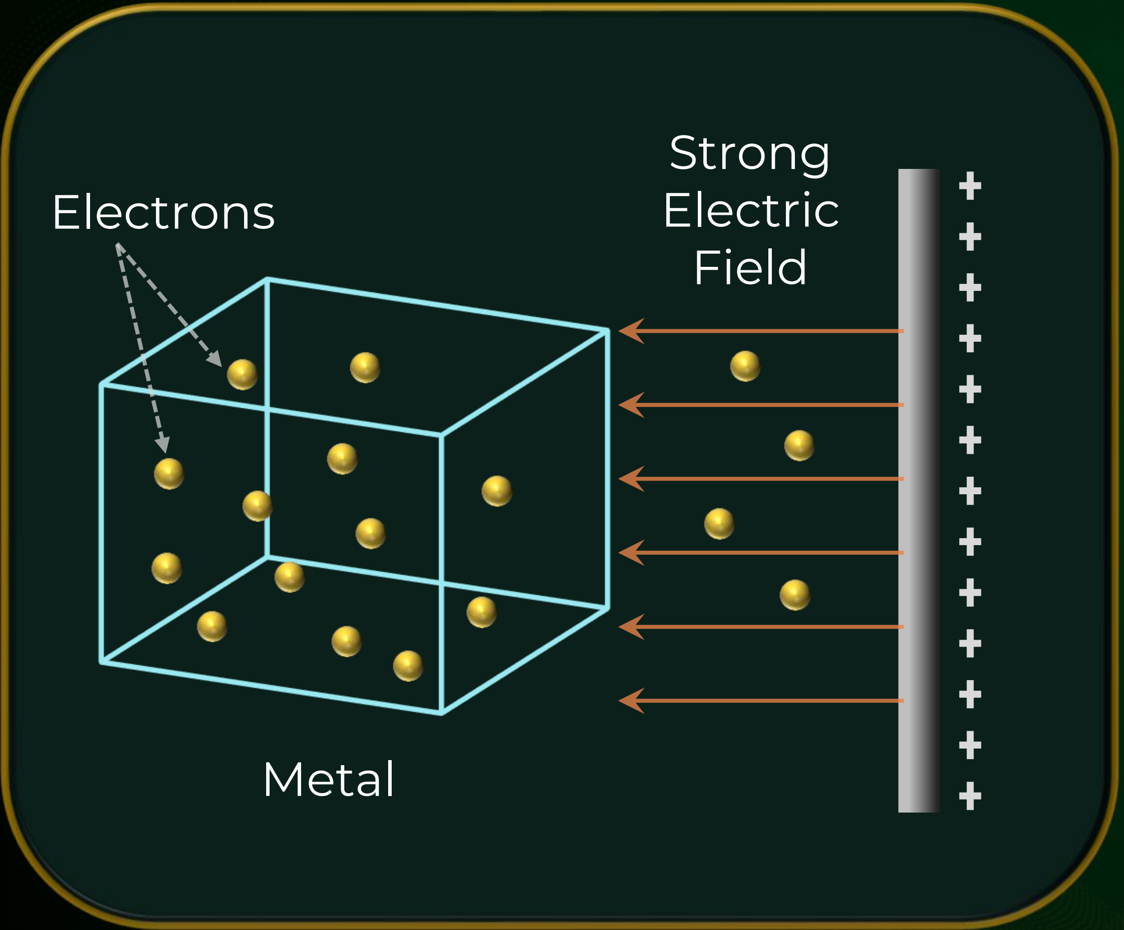
- In metals, the electrons in the outer shells of the atoms are loosely bound.
- They are quite free to move easily within the metal surface but can not leave the metal surface. Such loosely bound electrons are called free electrons.
- The minimum amount of energy required to remove an electron from the metal surface is called work function (ϕ).
- The work function depends on the nature of the metal surface.

Thermionic Emission



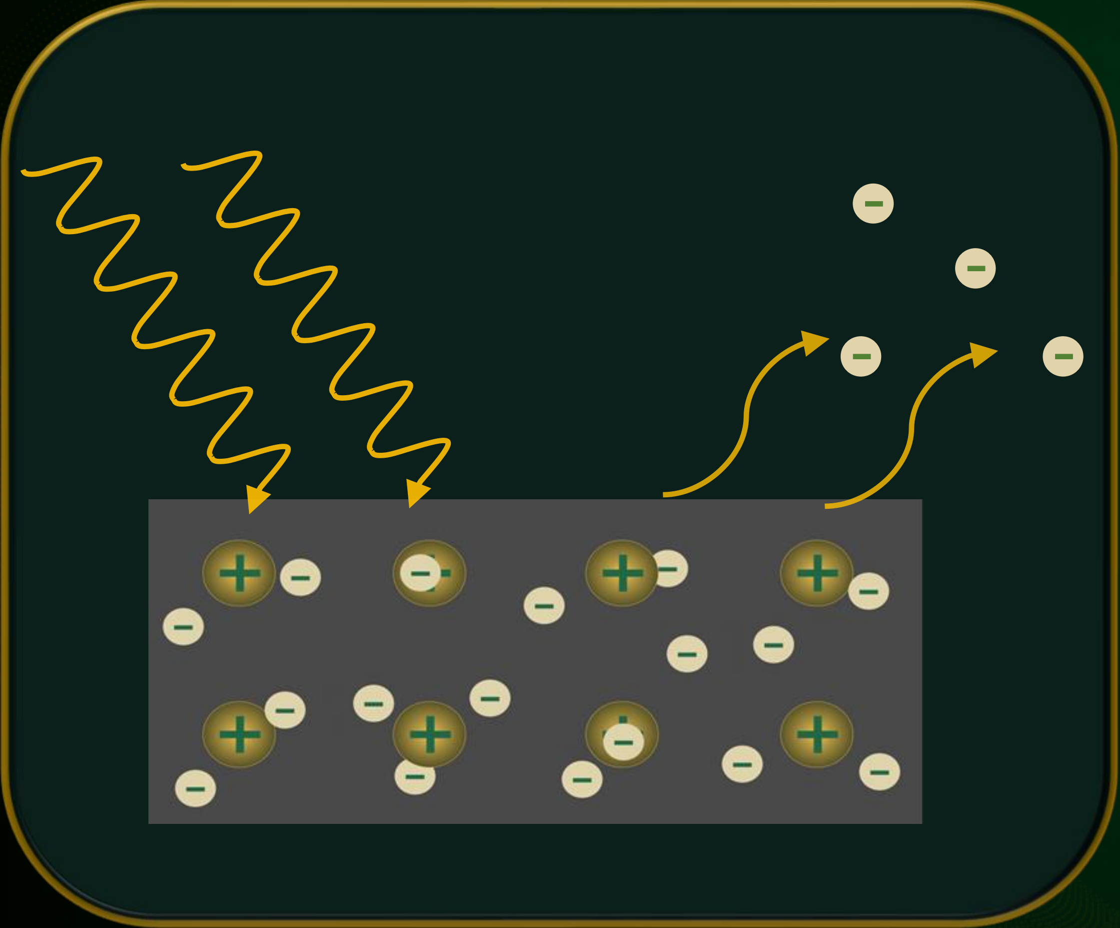
- By suitably heating, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal.

Field Emission



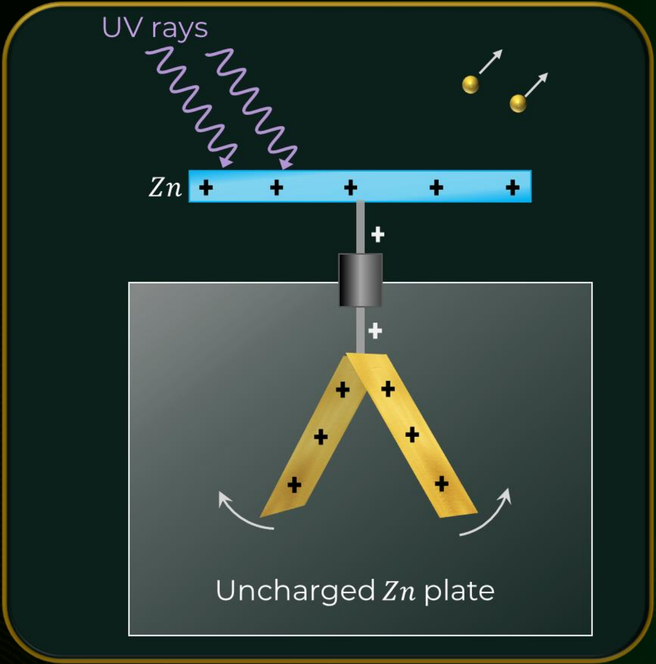
- By applying a very strong electric field of the order of $(10^8\ V/m)$ to a metal, electrons can be pulled out of the metal.

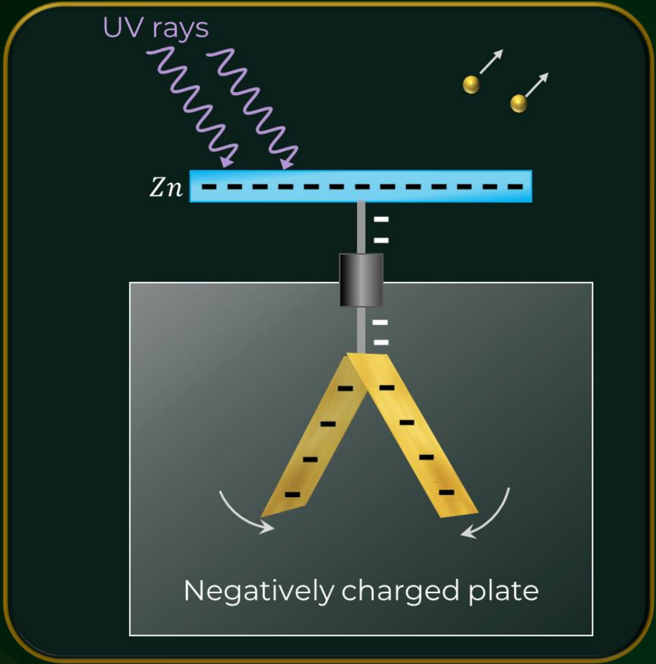
Photoelectric Emission

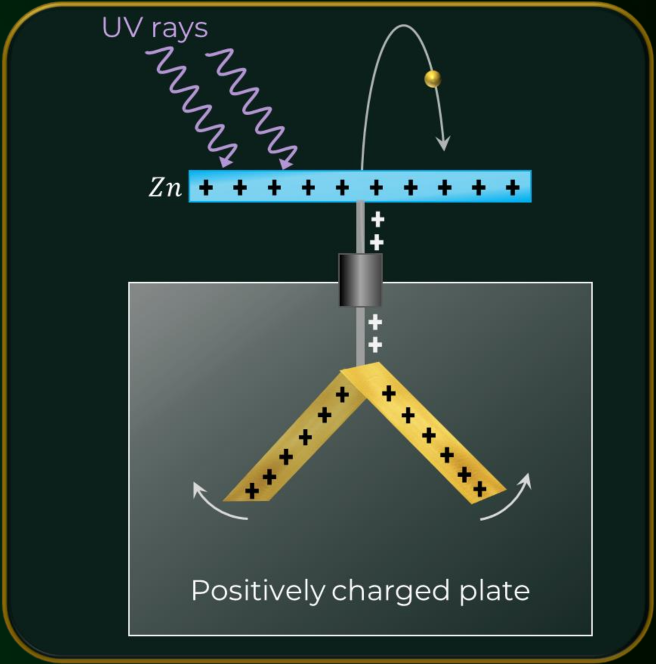


- When light of suitable frequency illuminates a metal surface, free electrons are emitted from the metal surface.
- These photo(light) generated electrons are called photoelectrons.

Hallwachs' Observation

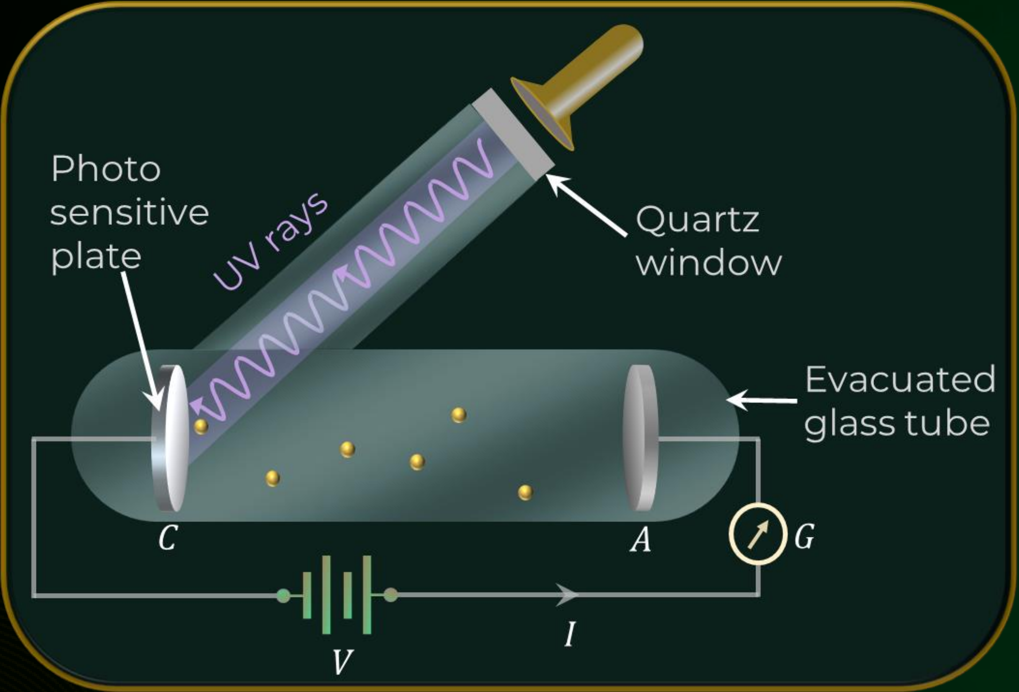






- When uncharged zinc plate is irradiated by ultraviolet light, it becomes positively charged and leaves diverge.
- If the negatively charged zinc plate is exposed to ultraviolet light, the leaves come closer as the charges leak away quickly.
- If the plate is positively charged, it becomes more positive upon irradiation of UV rays and the leaves diverge further.

Lenard's Observation



- When ultraviolet light is incident on the negative plate (C), an electric current flows in the circuit that is indicated by the deflection in the galvanometer.

- If the positive plate(A) is irradiated by the ultraviolet light, no current is observed in the circuit.

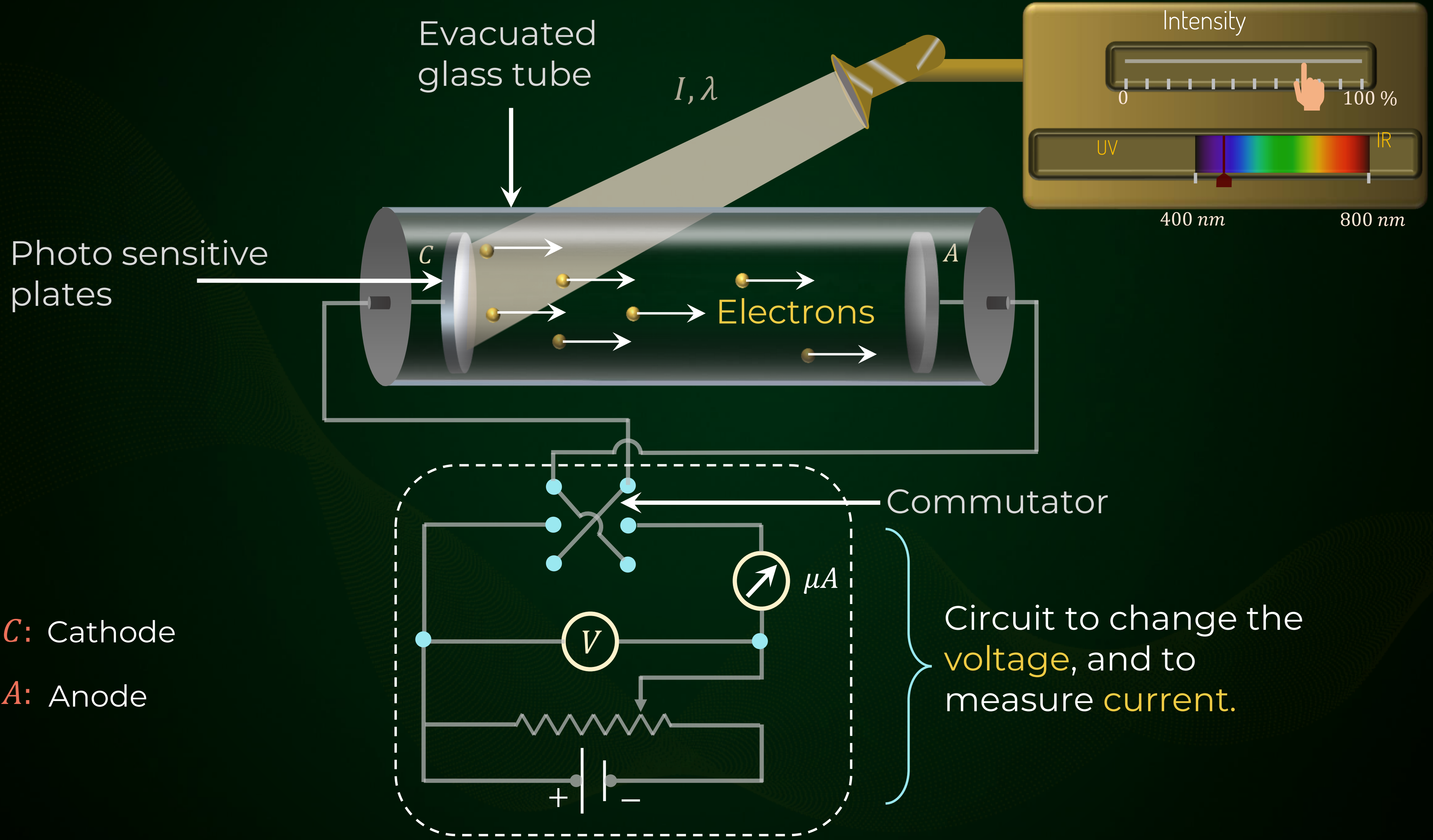
Conclusion:

- When ultraviolet light falls on the negative plate, electrons are ejected from it which are attracted by the positive plate A.

- On reaching the positive plate through the evacuated tube, the circuit is completed and the current flows in it.

- Thus, the UV light falling on the negative plate causes the electron emission from the surface of the plate.

Photoelectric Effect

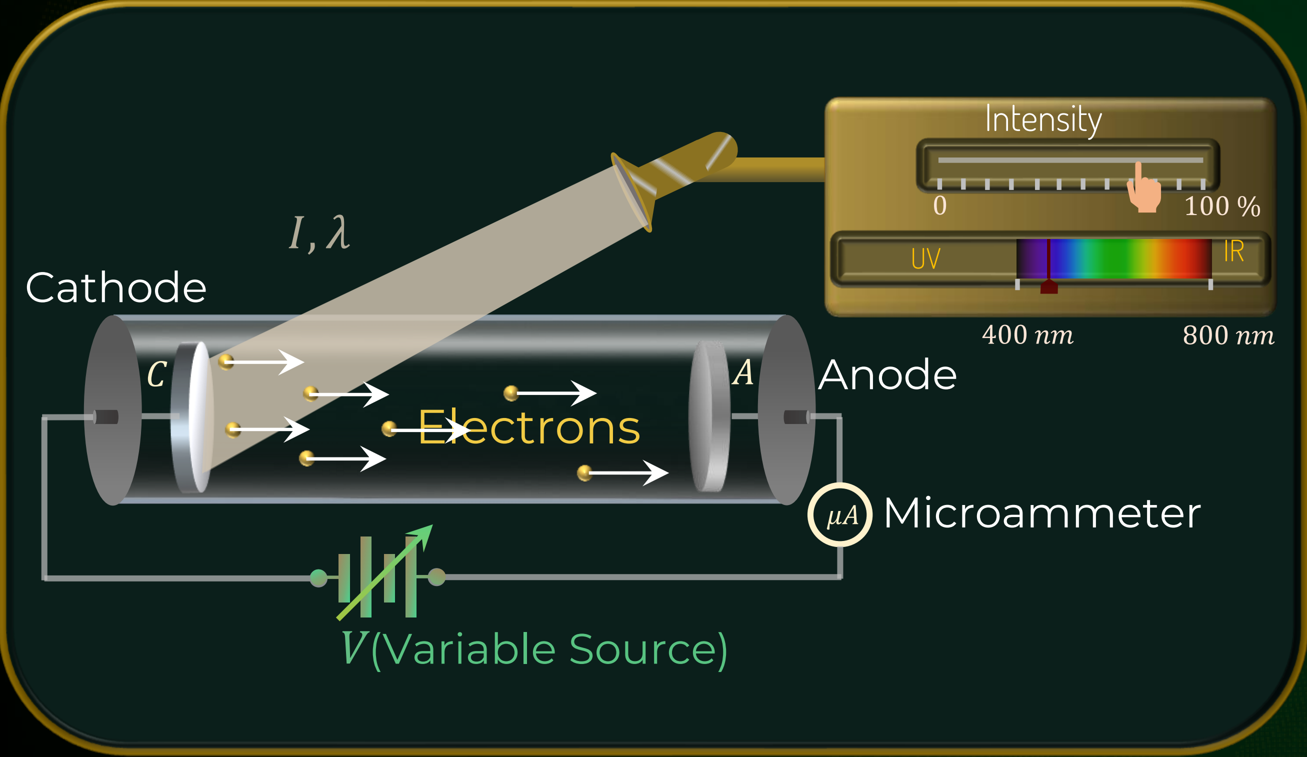


C: Cathode

A: Anode

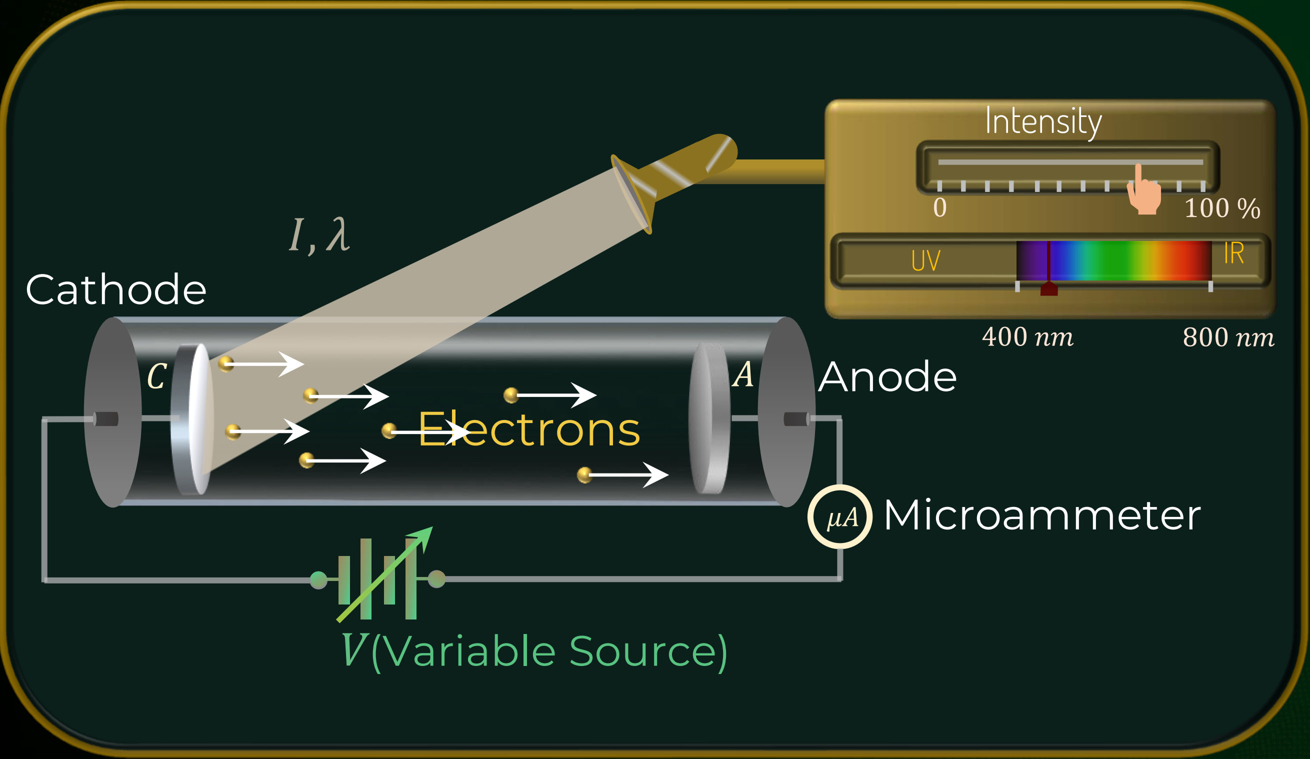
Photoelectric Effect

Ejection of electrons from material due to bombarding of photons on it is known as photoelectric effect.



- The required frequency or wavelength for ejection of electron depends on material.
- Below a particular wavelength, photocurrent is produced even at lower intensity.
- Above a particular wavelength, no photocurrent is produced even at very high intensity of light.

Photoelectric Effect – Einstein's explanation



- When a photon interacts with an electron, it imparts its entire energy to the electron. The electron can't absorb a fraction of the energy.
- Electron can only take energy equal to $h\nu$ and **not** multiples of $h\nu$.
- If an electron receives an energy E, it gets ejected from the metal surface with a kinetic energy of $E - \phi$, where ϕ is the work function of the metal.
- The maximum kinetic energy of an emitted electron:

$$KE_{max} = h\nu - \phi$$

Threshold Wavelength and Frequency

- The value of wavelength λ_0 above which no photocurrent is produced is known as threshold wavelength.

- The value of frequency ν_0 below which no photocurrent is produced is known as threshold frequency.

- Minimum energy of photon to eject electron is:

$$\boxed{E = \frac{hc}{\lambda_0}} \qquad \boxed{E = h\nu_0}$$

- This energy is also known as the Work Function (ϕ) of the metal.

- For Photoelectric effect to happen, the energy of incident photon must be greater than the work function of metal.

$$E \geq \phi \Rightarrow \frac{hc}{\lambda} \geq \phi \Rightarrow \boxed{\lambda \leq \frac{hc}{\phi}}$$

$$E \geq \phi \Rightarrow h\nu \geq \phi \Rightarrow \boxed{\nu \geq \frac{\phi}{h}}$$

- Maximum Kinetic energy of the emitted electron:

$$\boxed{KE_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}$$

$$\boxed{KE_{max} = h(\nu - \nu_0)}$$

The work function of a substance is $4\ eV$. The longest wavelength of light can cause photoelectron emission from this substance is approximately:

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Given: $\phi = 4\ eV$

Solution: Let λ_m = Longest wavelength of light

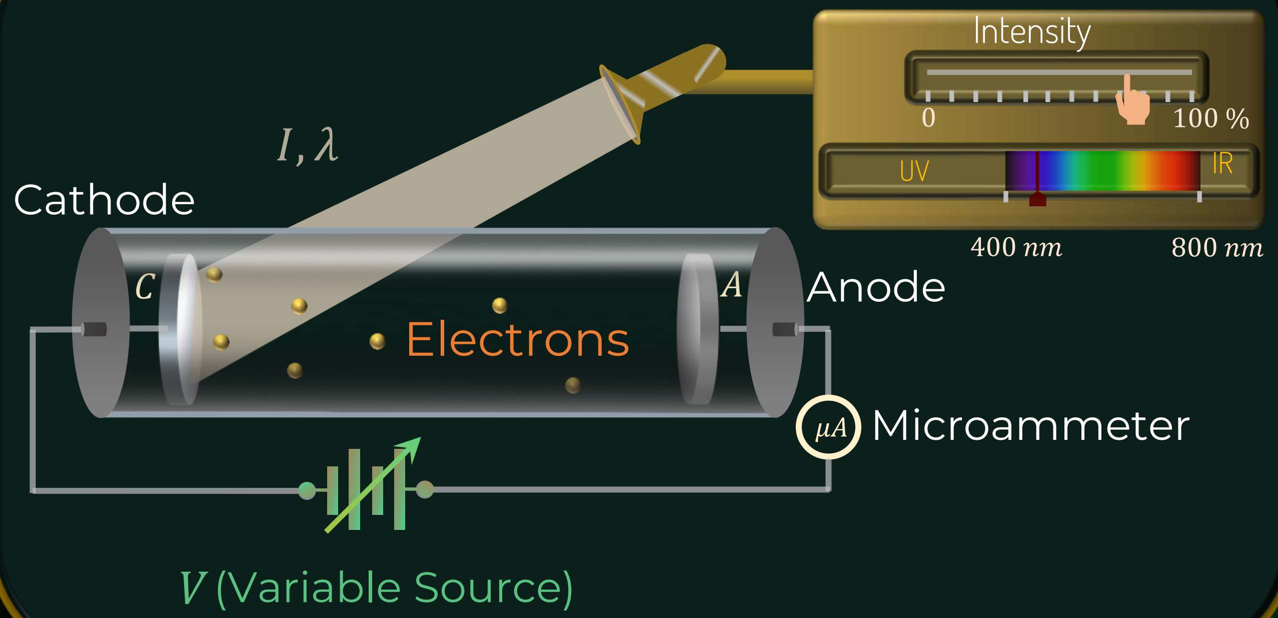
$$\frac{hc}{\lambda_m} = \phi$$

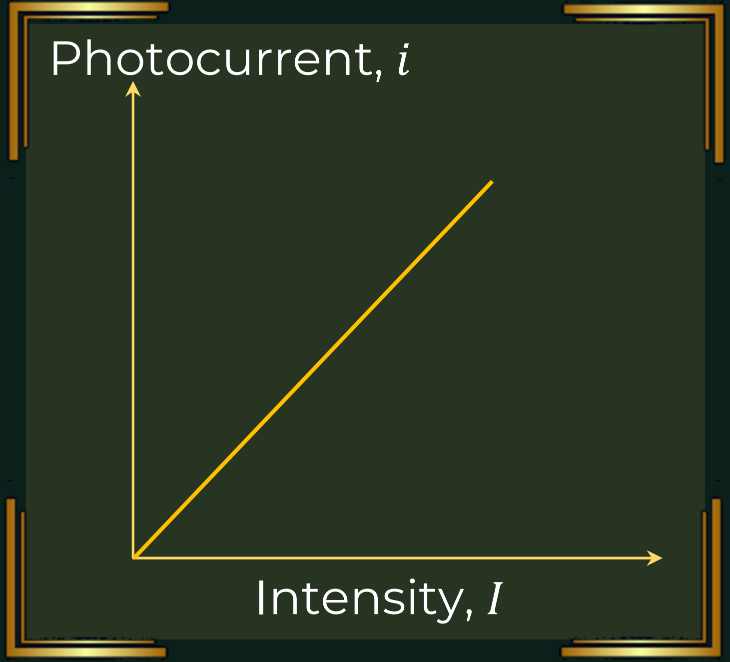
$$\Rightarrow \lambda_m = \frac{hc}{\phi} = \frac{12400\ eV\dot{A}}{4\ eV} = 3100\ \dot{A} = 310\ nm$$

$$\boxed{\lambda_m = 310\ nm}$$

Effect of Intensity of Light on Photocurrent

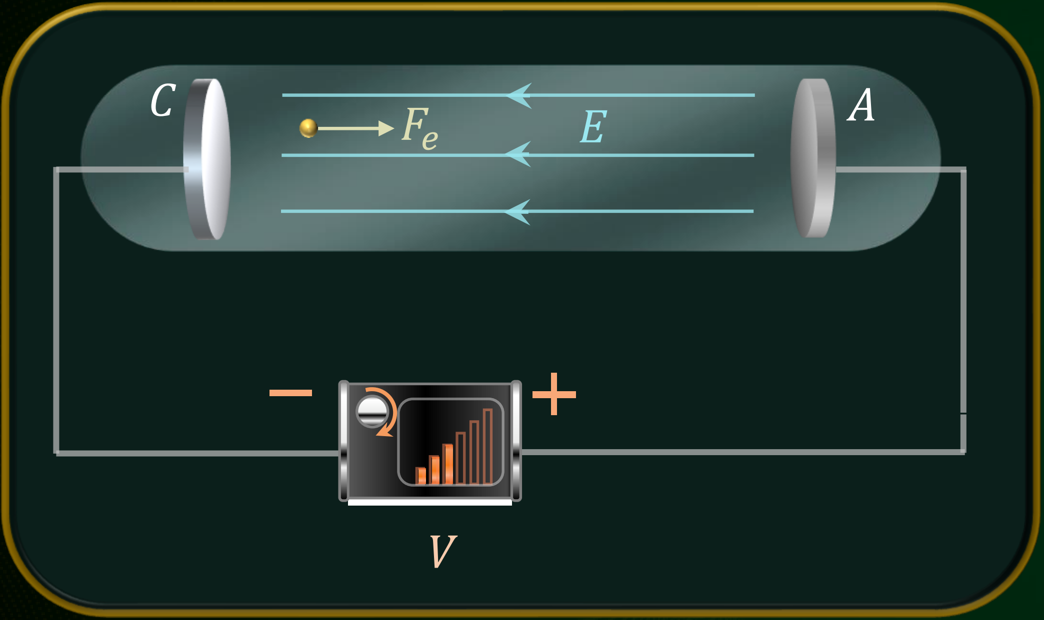
- Wavelength and Potential difference are kept constant, only intensity is increased.

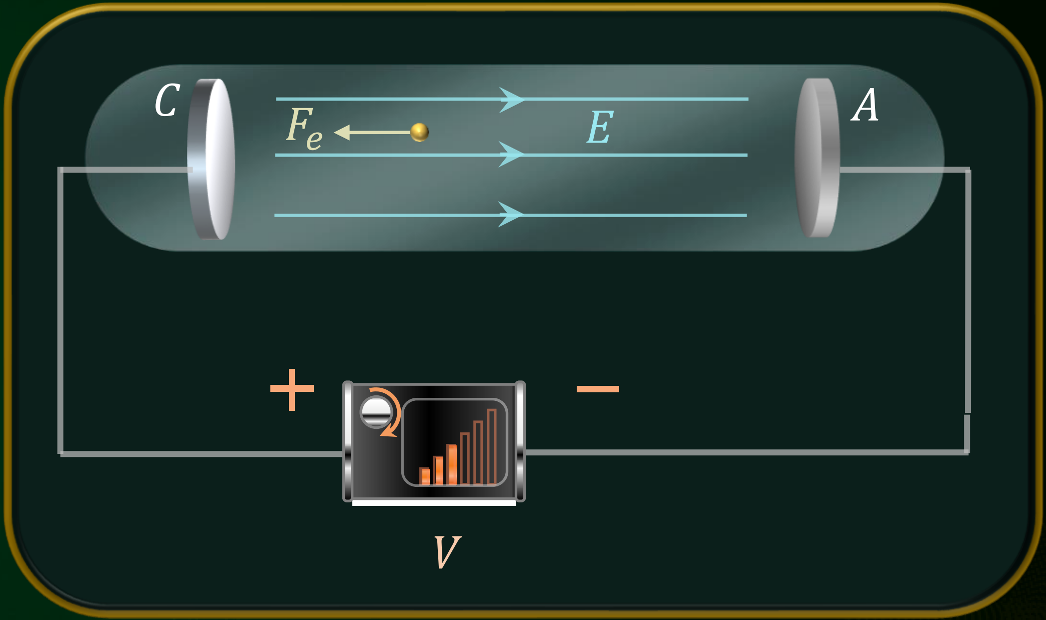




Photocurrent ∝ Intensity of light

Effect of Potential on Photocurrent

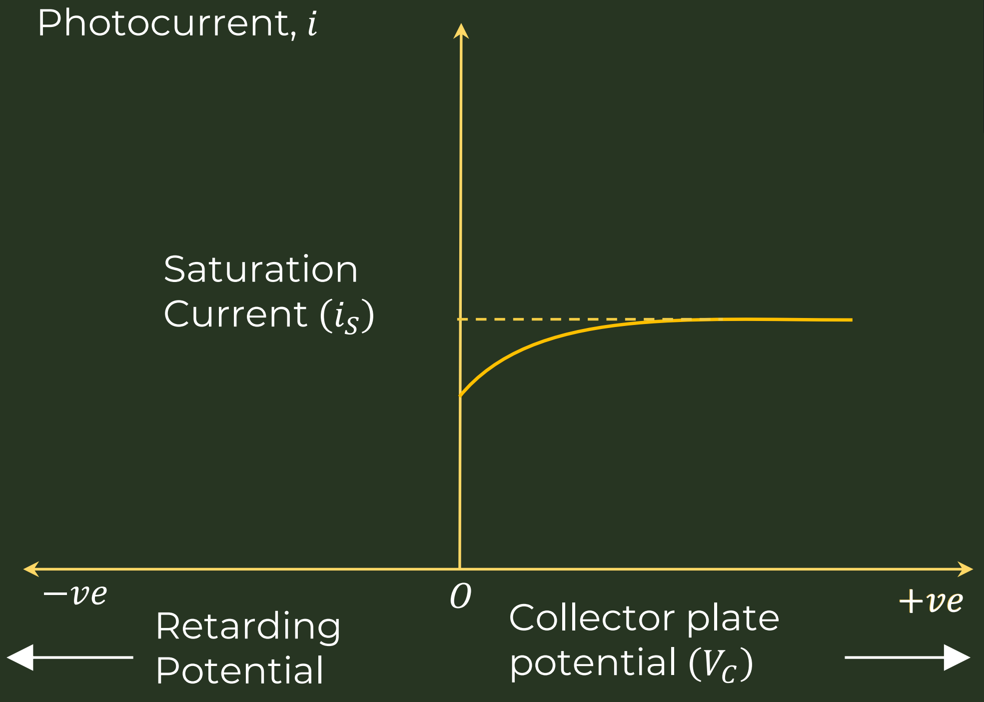




- When the collector plate is at higher potential, electric field accelerates the electrons.

- When the emitter plate is at higher potential, electric field decelerates the electrons.

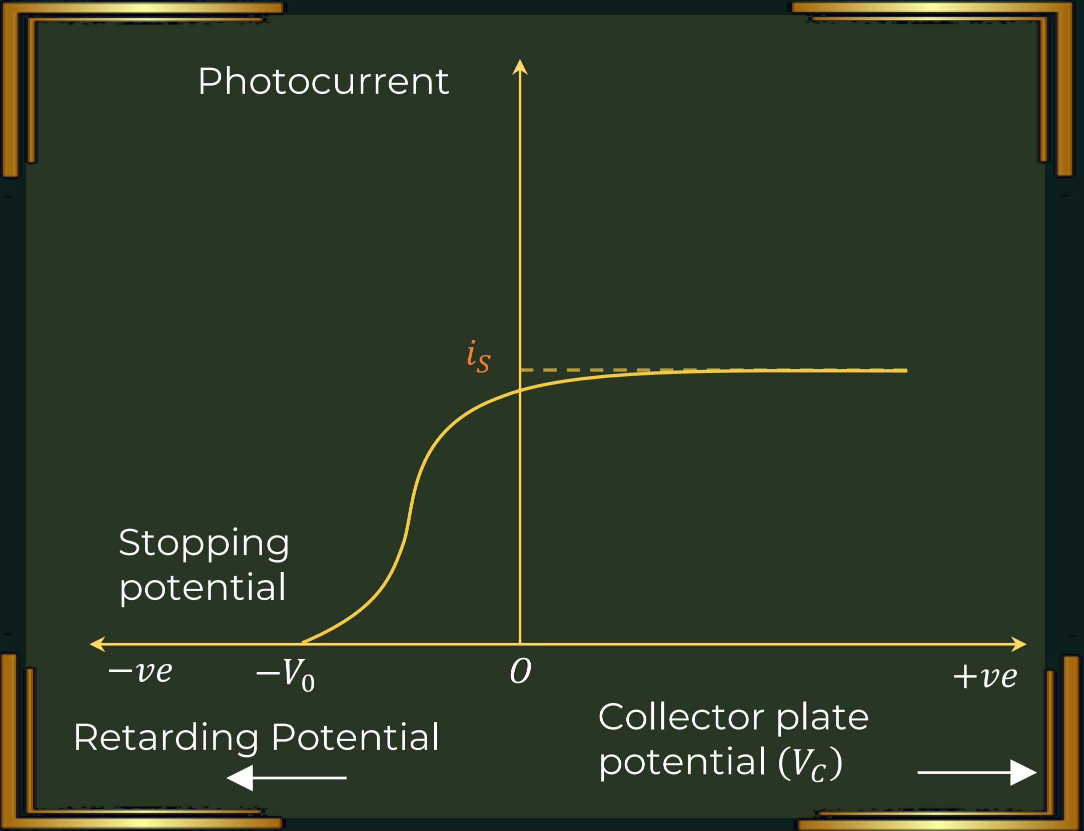
Effect of Potential on Photocurrent



- When $V_C = 0$, some of the emitted electrons reach collector because of their own kinetic energy.
- When V_C is increased, photocurrent increases.
- When all the photoelectrons emitted by the emitter reach the collector plate, the current reaches a maximum value which is known as Saturation Current (i_S).

Note: λ, I is kept constant.

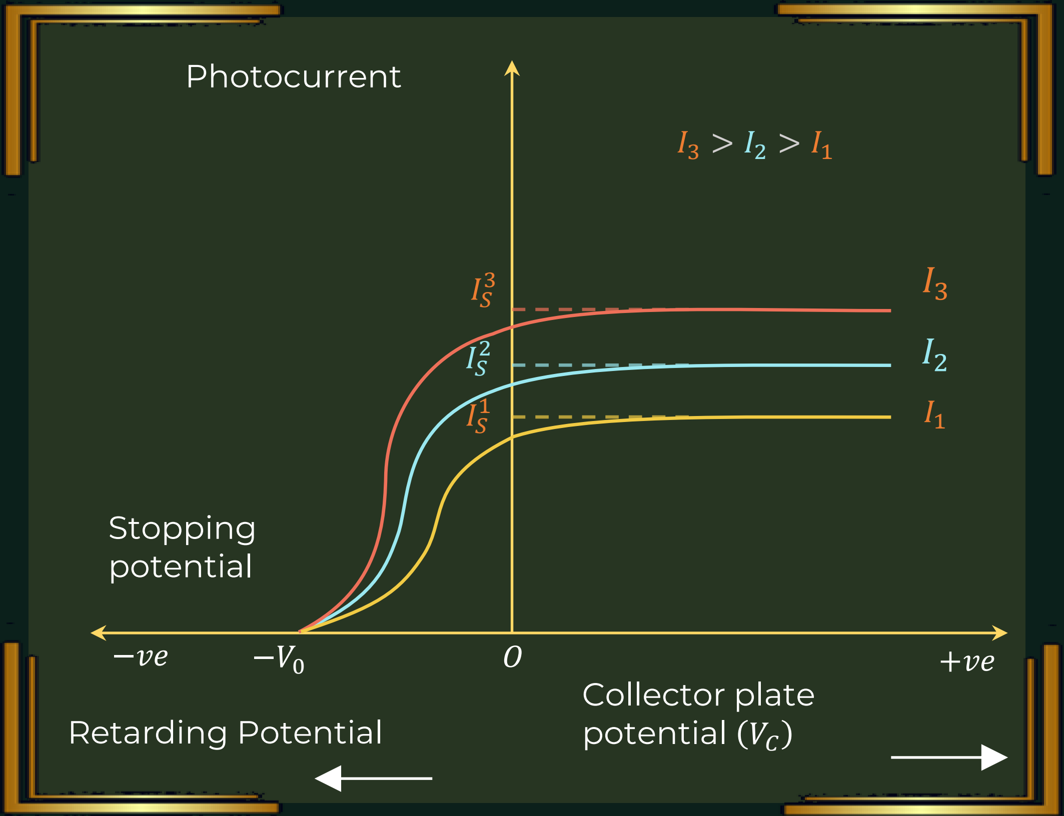
Effect of Potential on Photocurrent



- When Potential is reversed, electrons get repelled by the collector plates.
- As the potential increases, fewer photoelectrons reach the collector plate.
- The negative potential of emitter plate at which the photocurrent becomes zero is called Stopping Potential (V_0).

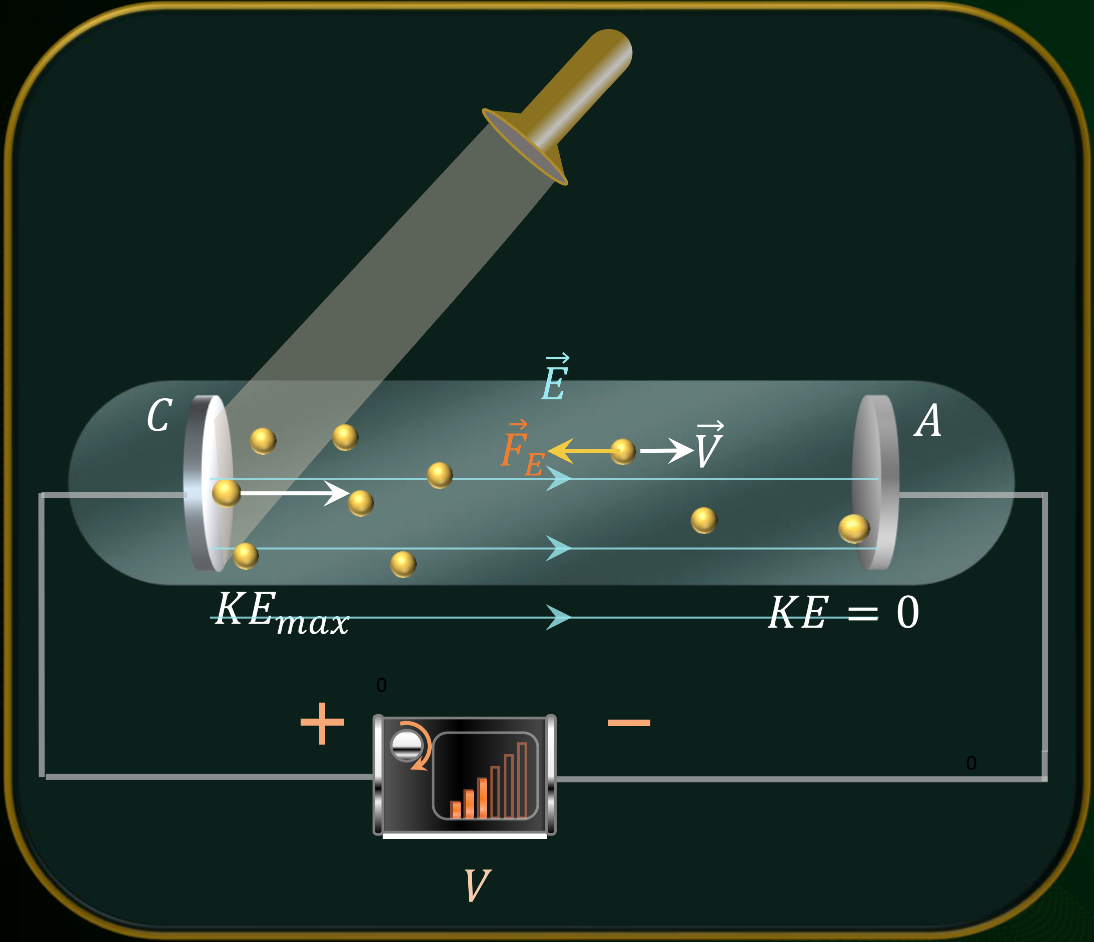
Note: λ, I are kept constant.

Effect of Intensity on Photocurrent



- Saturation Current increases with increase in intensity.
- No. of photoelectrons emitted ∝ Intensity
- At a given frequency of incident radiation, the stopping potential is independent of its intensity.

Stopping Potential



- Stopping potential is the voltage required to stop the most energetic photoelectron.

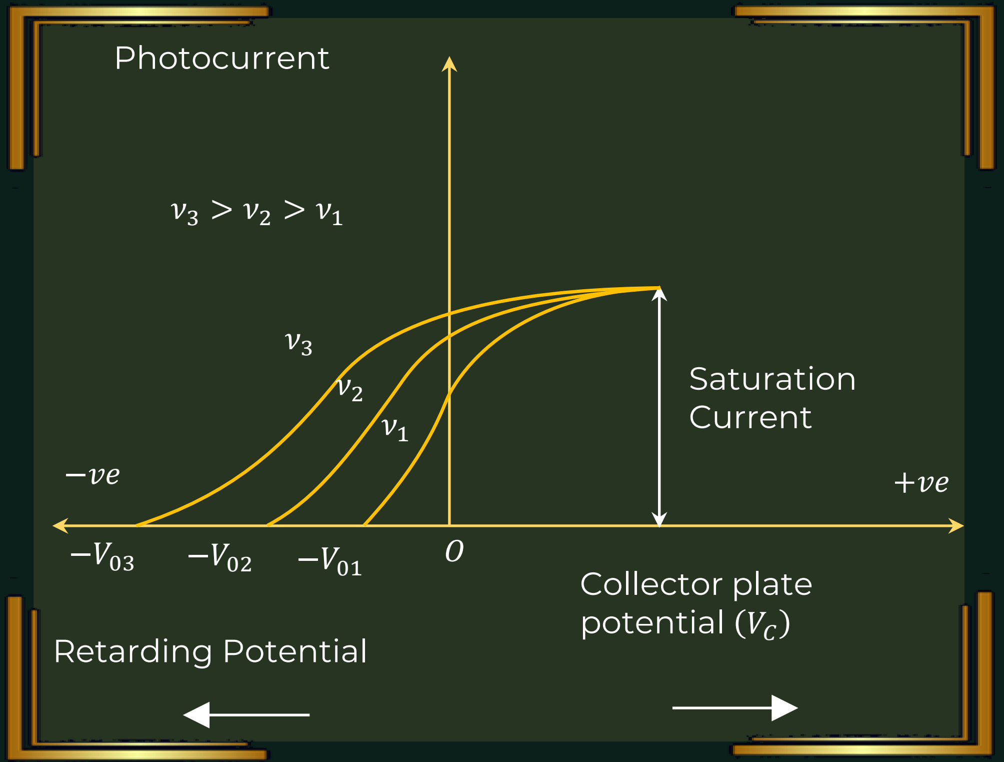
$$KE_{max} = eV_0$$

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

or

$$V_0 = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$

Effect of Frequency on Stopping Potential



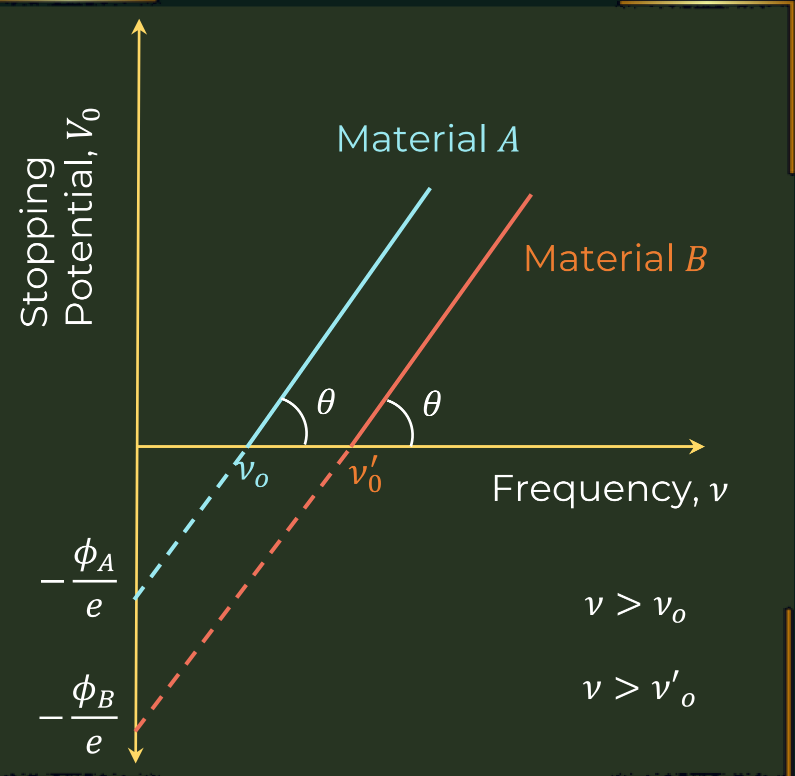
- Saturation current is same for all frequencies.
- Stopping Potential is higher in magnitude for higher frequencies.

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

- $\nu_3 > \nu_2 > \nu_1 \quad \Rightarrow \quad |V_{03}| > |V_{02}| > |V_{01}|$

Note: I is kept constant.

Effect of Frequency on Stopping Potential



$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

- Stopping potential varies linearly with the frequency for a given photosensitive material.
- Slope of graph $= \tan\theta = \frac{h}{e} \rightarrow$ constant for all materials
- y-intercept $= -\frac{\phi}{e}$
- x-intercept $= \nu_0 = \frac{\phi}{h}$
- Where ν_0 is the threshold frequency of the material, below which no photo electrons are released.

Light of wavelength $\lambda = 400\ nm$ is incident on a metal surface of work function $\phi = 1.6\ eV$. Find the magnitude of stopping potential.

Given: $\lambda = 400\ nm,\ \phi = 1.6\ eV$

To find: Stopping potential (V_0)

Solution: $KE_{max} = \dfrac{hc}{\lambda} - \phi$

$$\Rightarrow eV_0 = \frac{1240\ \ eV}{\lambda\ (in\ nm)} - \phi$$

$$\Rightarrow eV_0 = \frac{1240}{400}\ eV - 1.6\ eV$$

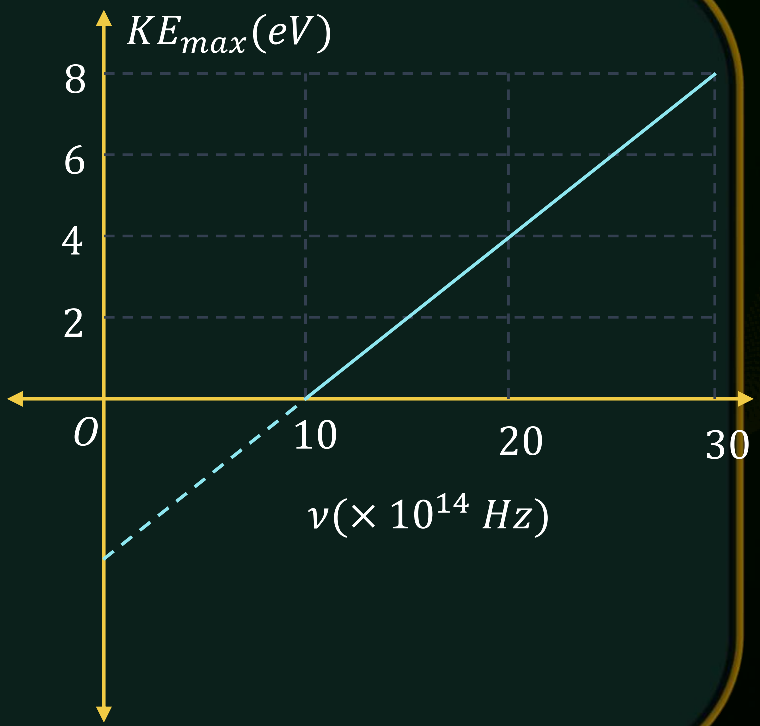
$$\Rightarrow eV_0 = 3.1\ eV - 1.6\ eV = 1.5\ eV$$

$$\boxed{V_0 = 1.5\ V}$$

A graph regarding photoelectric effect is drawn between the maximum kinetic energy of emitted electrons and the frequency of the incident light. Based on this graph, calculate:
a) $h \rightarrow$ Planck's constant b) Work function

To find: h and ϕ

Solution:



a) From the graph,
threshold frequency$(\nu_0) = 10 \times 10^{14}\ Hz$

Slope $= h = \dfrac{\Delta KE_{max}}{\Delta \nu} = \dfrac{8 \times 1.6 \times 10^{-19}}{20 \times 10^{14}}$

$\Rightarrow h = 6.4 \times 10^{-34}\ J - s$

b) Work function is given by,

$\phi = h\nu_0$

$\Rightarrow \phi = 6.4 \times 10^{-34} \times 10 \times 10^{14} = 6.4 \times 10^{-19} J$

$\boxed{\Rightarrow \phi = 4\ eV}$

The Intensity Problem

Wave Theory

INTENSITY → Energy of wave → Energy transferred to e^- → K.E

The intensity of light falling on metal is increased.

K.E. of emitted photoelectrons should also increase.

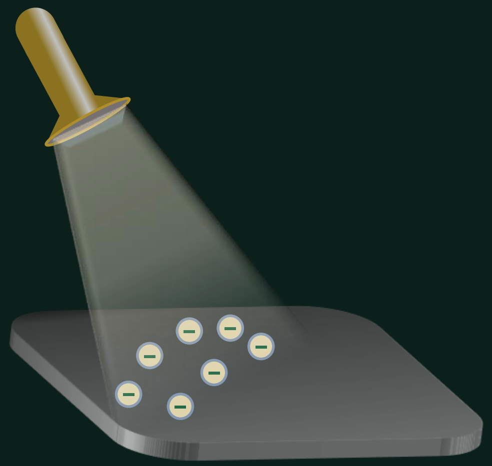
Particle Theory

$$KE_{max} = h\nu - \phi$$

The maximum kinetic energy of a photoelectron doesn't depend on the intensity of the incident light.

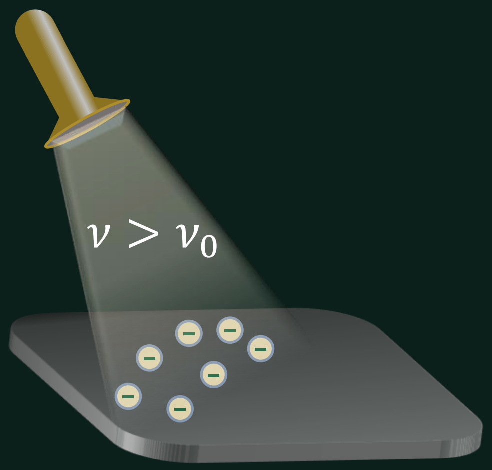
The Frequency Problem

Wave Theory



The photoelectric effect should occur for any frequency of the light, provided that the light is intense enough to eject the photoelectrons.

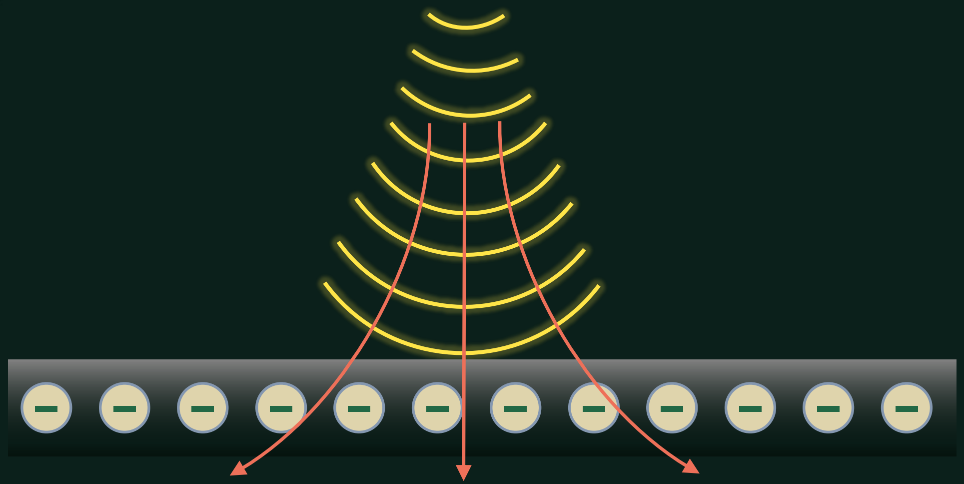
Particle Theory



Photoelectrons will get ejected only when incident light frequency is more than the threshold value for any intensity.

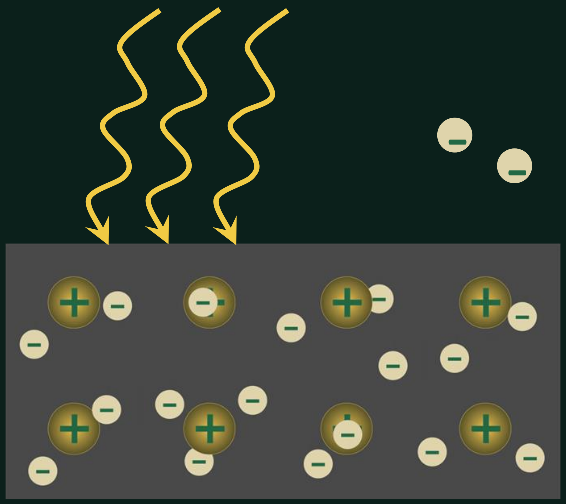
The Time Delay Problem

Wave Theory



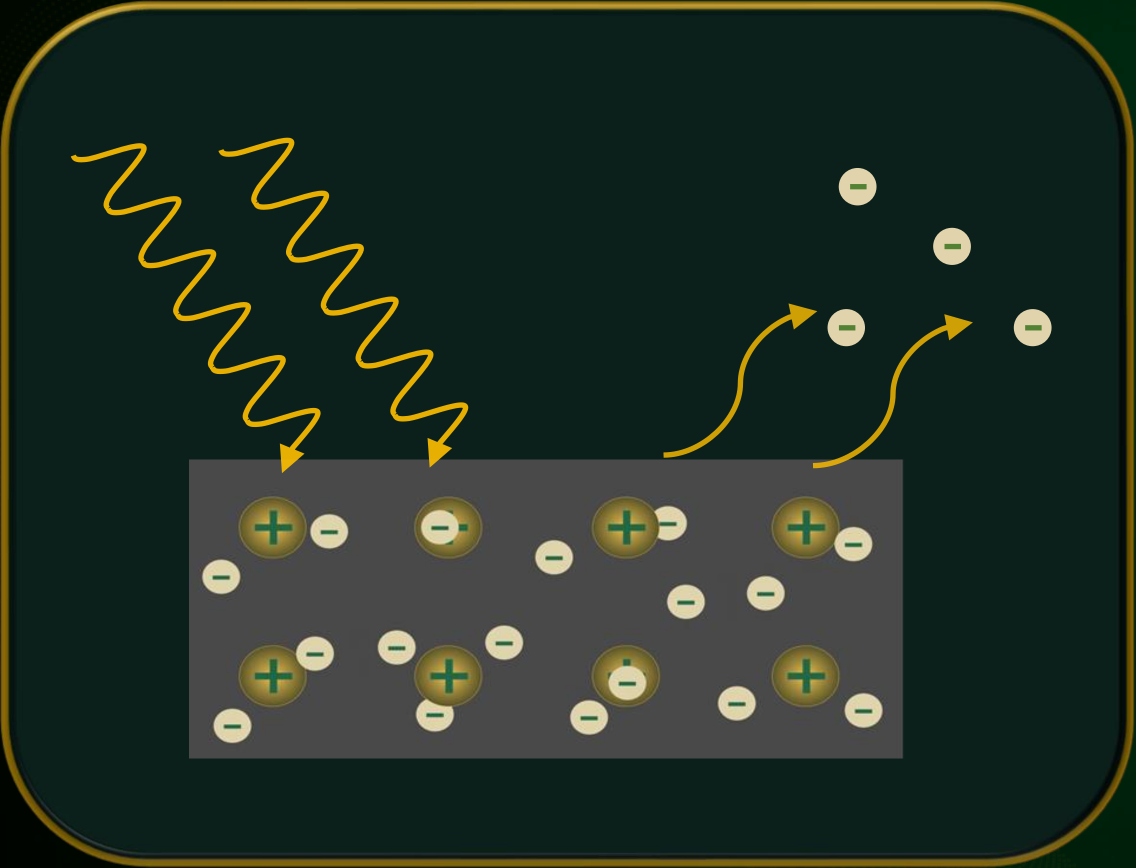
Light energy is uniformly distributed among the electrons. The electron will take some time to accumulate enough energy to escape from the metal surface. Hence there should be a time lag.

Particle Theory



Whole of the energy associated with a photon is absorbed by a free electron. Hence emission is instantaneous.

Explanation by Photon Theory of Light



- An electron can receive only one photon. So, it doesn't matter how many photons are being incident. It only matters how much energy each electron has $(h\nu)$.
- An electron needs to overcome ϕ amount of energy to get out of the metal.
- Electron emission is instantaneous. Photons immediately impart energy, whereas if it was a wave, it would slowly eject electrons.

Wave - Particle Duality of Light

Phenomenon	Wave nature	Particle nature
Reflection	✓	✓
Refraction	✓	
Interference	✓	
Diffraction	✓	
Polarization	✓	
Photoelectric effect		✓

The stopping potential of a metal is $3\ V$, when it is illuminated by light of wavelength $500\ nm$. What will be the stopping potential of the metal when the wavelength is $600\ nm$? (Photoelectric emission takes place in both the cases)

Given: $\lambda_1 = 500\ nm; V_{0_1} = 3\ V; \lambda_2 = 600\ nm$

To find: New stopping potential $\left(V_{0_2}\right)$

Solution:

According to Einstein's photoelectric equation,

$$eV_0 = \frac{hc}{\lambda} - \phi$$

Case 1: $eV_{0_1} = \frac{hc}{\lambda_1} - \phi \quad \ldots(1)$

Case 2: $eV_{0_2} = \frac{hc}{\lambda_2} - \phi \quad \ldots(2)$

$(2) - (1) \Rightarrow eV_{0_2} - eV_{0_1} = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$

$$eV_{0_2} - e(3\ V) = \frac{12400}{6000} - \frac{12400}{5000}$$

$$eV_{0_2} - 3\ eV = 2.06\ eV - 2.48\ eV$$

$$\Rightarrow eV_{0_2} = 2.58\ eV$$

$$\Rightarrow \boxed{V_{0_2} = 2.58\ V}$$

When a certain photosensitive surface is illuminated by a monochromatic light of frequency ν, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by a monochromatic light of frequency $\frac{\nu}{2}$, the stopping potential is $\frac{V_0}{3}$. The threshold frequency for photoelectric emission is:

Given: $\nu_1 = \nu; V_{0_1} = V_0; \nu_2 = \frac{\nu}{2}; V_{0_2} = \frac{V_0}{3}$

To find: Threshold frequency (ν_0)

Solution:

According to Einstein's photoelectric equation,

$$KE_{max} = E - \phi \implies eV_0 = h\nu - h\nu_0$$

Case 1: $eV_0 = h(\nu - \nu_0) \quad ...(1)$

Case 2: $\frac{eV_0}{3} = h\left(\frac{\nu}{2} - \nu_0\right) \quad ...(2)$

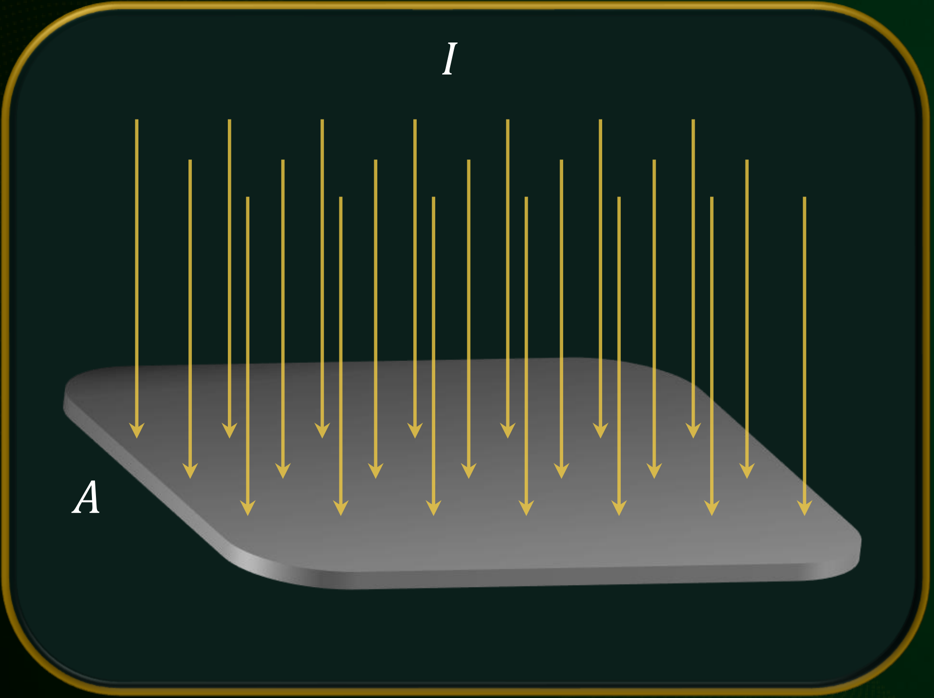
$$\frac{(1)}{(2)} \implies 3 = \frac{\nu - \nu_0}{\left(\frac{\nu - 2\nu_0}{2}\right)}$$

$$\implies 3\nu - 6\nu_0 = 2\nu - 2\nu_0$$

$$\implies \boxed{\nu_0 = \frac{\nu}{4}}$$

Radiation Pressure

Radiation pressure : The pressure experienced by the surface exposed to the radiation.



$$P = \frac{F}{A}$$

P = Radiation pressure

F = Normal force on plate due to photons

A = Area of plate

- The magnitude of radiation pressure is so minute that it has no effect on our normal life.

Complete Absorption and Normal Incidence

Initial momentum of each photon, $p_i = \frac{h}{\lambda}$

Final momentum of each photon, $p_f = 0$

Magnitude of Change in momentum of each photon,

$$\Delta p = p_f - p_i = \frac{h}{\lambda}$$

Total Change in momentum per sec, $\frac{\Delta p}{\Delta t} = \frac{nh}{\lambda}$

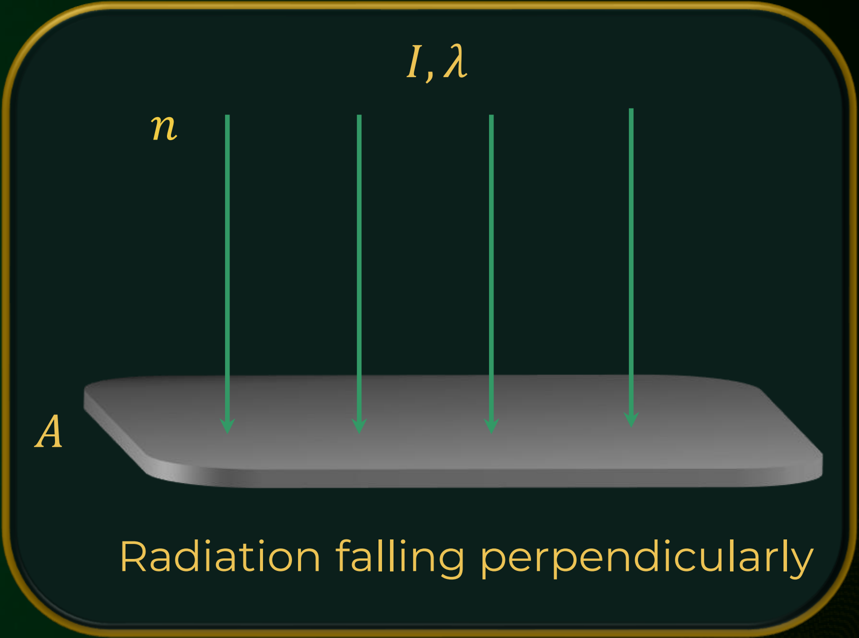
Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{nh}{\lambda}$

Intensity, $I = \frac{nhc}{\lambda A} \Rightarrow \frac{nh}{\lambda} = \frac{IA}{c}$

∴ Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{IA}{c}$

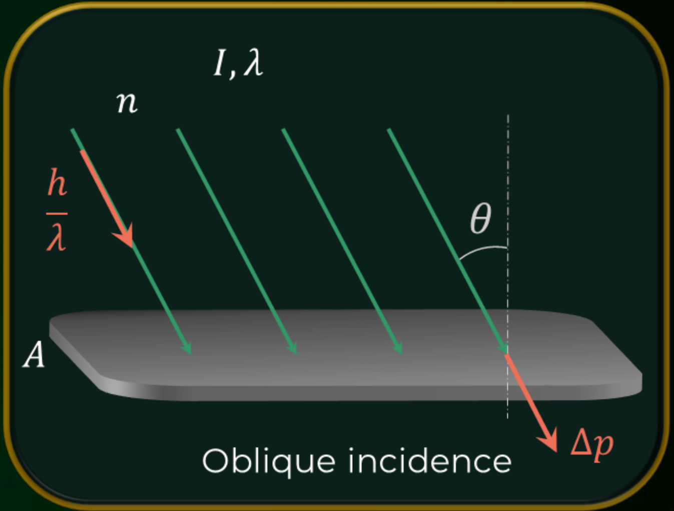
Radiation pressure, $P = \frac{F}{A} = \frac{\frac{IA}{c}}{A}$

$$\boxed{P = \frac{I}{c}}$$



n = No. of photons incident per second

Complete Absorption and Oblique Incidence



n = No. of photons incident per second

Initial momentum of each photon, $p_i = \frac{h}{\lambda}$

Final momentum of each photon, $p_f = 0$

Magnitude of Change in momentum of each photon, $\Delta p = p_f - p_i = \frac{h}{\lambda}$

Total Change in momentum per sec, $\frac{\Delta p}{\Delta t} = \frac{nh}{\lambda}$

Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{nh}{\lambda}$

Intensity, $I = \frac{nhc}{\lambda A \cos\theta} \quad \Rightarrow \frac{nh}{\lambda} = \frac{IA\cos\theta}{c}$

$\therefore$ Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{IA\cos\theta}{c}$

Force normal to the surface, $F_\perp = F\cos\theta$

Radiation pressure, $P = \frac{F_\perp}{A} = \frac{\frac{IA\cos^2\theta}{c}}{A}$

$$\boxed{P = \frac{I}{c}\cos^2\theta}$$

Complete Reflection and Normal Incidence

Initial momentum of each photon, $p_i = \frac{h}{\lambda}$

Final momentum of each photon, $p_f = -\frac{h}{\lambda}$

Magnitude of Change in momentum of each photon, $\Delta p = p_f - p_i = \frac{2h}{\lambda}$

Total Change in momentum per sec, $\frac{\Delta p}{\Delta t} = \frac{n2h}{\lambda}$

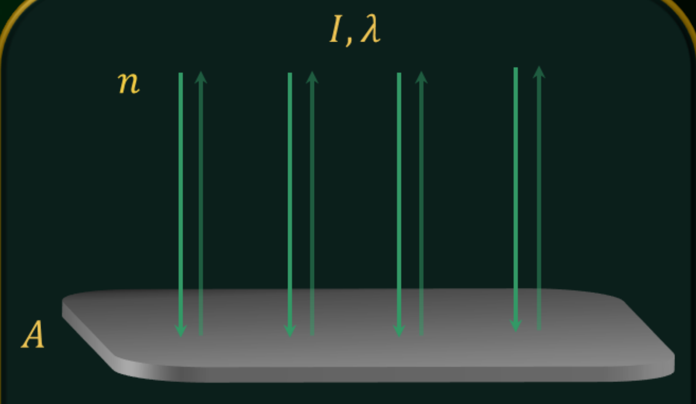
Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{n2h}{\lambda}$

Intensity, $I = \frac{nhc}{\lambda A} \Rightarrow \frac{nh}{\lambda} = \frac{IA}{c}$

$\therefore$ Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{2IA}{c}$

Radiation pressure, $P = \frac{F}{A} = \frac{\frac{2IA}{c}}{A}$

$$\boxed{P = \frac{2I}{c}}$$



Radiation falling perpendicularly

n = No. of photons incident per second

Assumption:- Elastic collision

Complete Reflection and Oblique Incidence

Initial momentum of each photon, $p_i = \frac{h}{\lambda}$

Final momentum of each photon, $p_f = -\frac{h}{\lambda}$

Magnitude of Change in momentum of each photon, $\Delta p = p_f - p_i = \frac{2h}{\lambda}\cos\theta$

Total Change in momentum per sec, $\frac{\Delta p}{\Delta t} = n\frac{2h}{\lambda}\cos\theta$

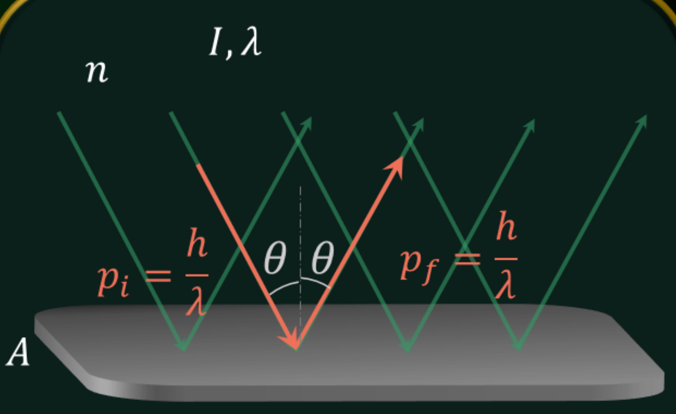
Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = n\frac{2h}{\lambda}\cos\theta$

Intensity, $I = \frac{nhc}{\lambda A\cos\theta} \Rightarrow \frac{nh}{\lambda} = \frac{IA\cos\theta}{c}$

$\therefore$ Net force acting on the surface, $F = \frac{\Delta p}{\Delta t} = \frac{2IA\cos^2\theta}{c}$

Radiation pressure, $P = \frac{F}{A} = \frac{\frac{2IA\cos^2\theta}{c}}{A}$

$$\boxed{P = \frac{2I}{c}\cos^2\theta}$$



Oblique incidence

n = No. of photons incident per second

Assumption:- Elastic collision

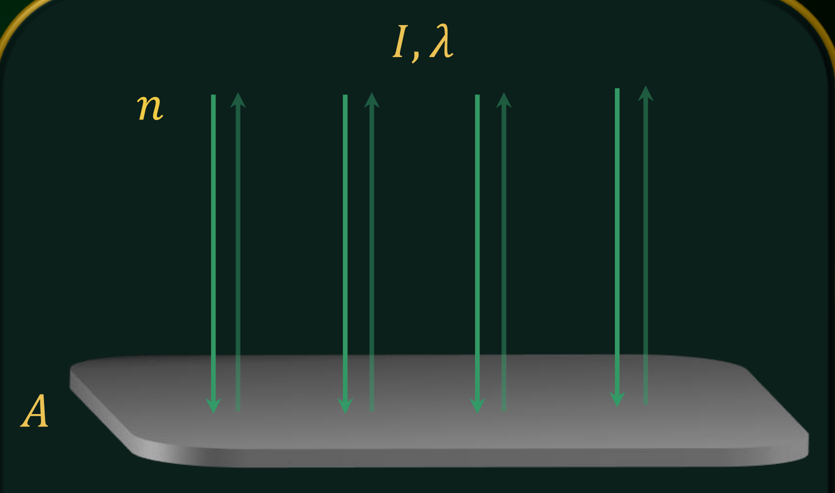
Partial Reflection and Normal Incidence

$0 < a < 1, \quad 0 < r < 1, \quad a + r = 1$

a = Absorption coefficient

r = Reflection coefficient

n = No. of photons incident per second



No. of photons absorbed per second, $n_a = a \times n$

No. of photons reflected per second, $n_r = r \times n$

Radiation pressure due to absorption, $P_a = a\frac{I}{c} = (1 - r)\frac{I}{c}$

Radiation pressure due to reflection, $P_r = r\frac{2I}{c}$

Net Radiation pressure, $P = P_a + P_r$

$\Rightarrow P = (1 - r)\frac{I}{c} + r\frac{2I}{c}$

$$\boxed{P = (1 + r)\frac{I}{c}}$$

Partial Reflection and Oblique Incidence

$$0 < a < 1, \quad 0 < r < 1, \quad a + r = 1$$

$a =$ Absorption coefficient

$r =$ Reflection coefficient

$n =$ No. of photons incident per second

No. of photons absorbed per second, $n_a = a \times n$

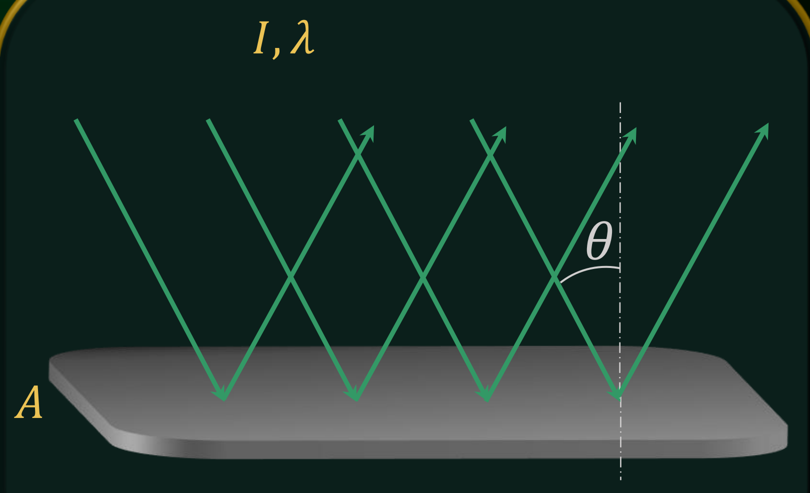
No. of photons reflected per second, $n_r = r \times n$

Radiation pressure due to absorption, $P_a = a\frac{I}{c}\cos^2\theta = (1-r)\frac{I}{c}\cos^2\theta$

Radiation pressure due to reflection, $P_r = r\frac{2I}{c}\cos^2\theta$

Net Radiation pressure, $P = P_a + P_r$

$$\Rightarrow P = (1-r)\frac{I}{c}\cos^2\theta + r\frac{2I}{c}\cos^2\theta \qquad \boxed{P = (1+r)\frac{I}{c}\cos^2\theta}$$

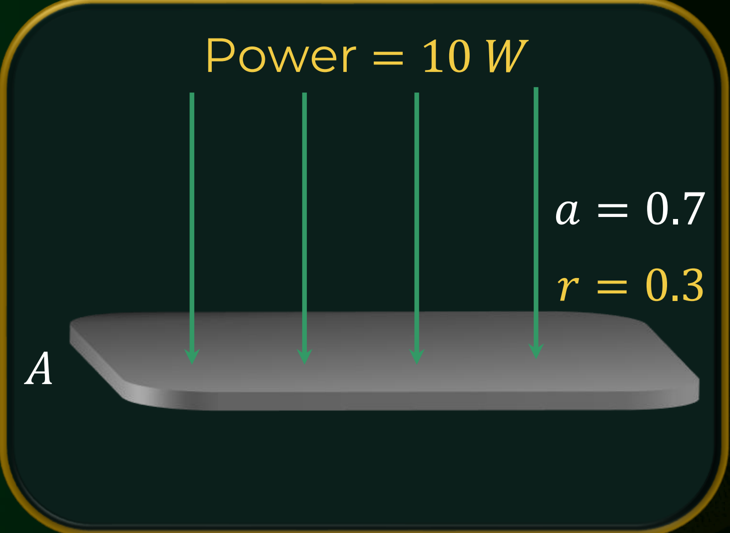


Oblique incidence

A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries $10\ W$ of power, find the force exerted by it on the surface.

Given: Power $= 10\ W,\ a = 0.7 \Rightarrow r = 0.3$

To find: Force exerted on the surface, F



Solution:

Intensity, $I = \dfrac{E}{At} = \dfrac{\text{Power}}{A} = \dfrac{10}{A}$

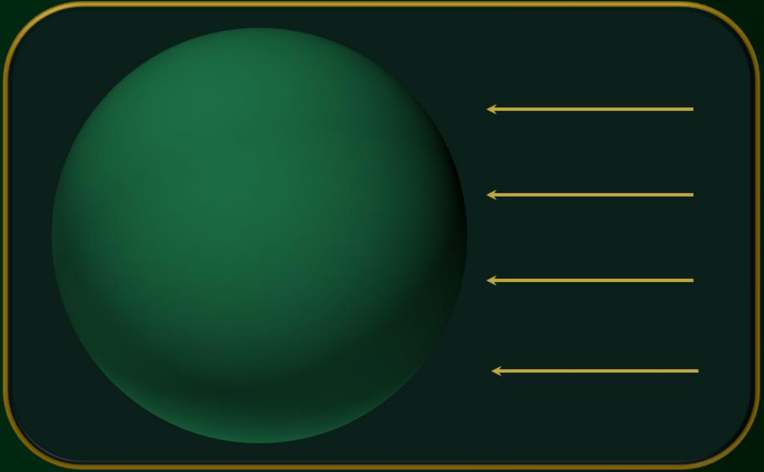
Force exerted on the surface, $F = PA = \dfrac{I}{c}(1+r)A = \dfrac{10}{Ac}(1+r)A$

$$\Rightarrow F = \frac{10}{3\times 10^8}(1+0.3) \Rightarrow \boxed{F = 4.3\times 10^{-8}\ N}$$

A perfectly reflecting solid sphere of radius R is placed in the path of a parallel beam of light of large aperture. If the beam carries an intensity I, the force exerted by the beam on the sphere is

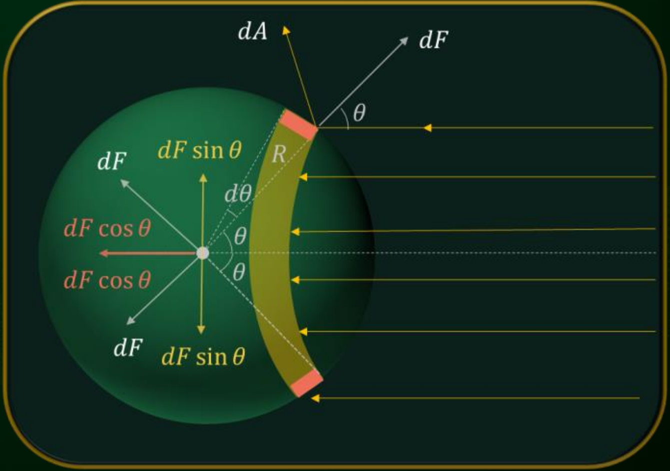
Given: Radius $= R$, Intensity $= I$

To find: Force



Solution:

dF = Force exerted by the light on the small elemental area dA of the thin ring.



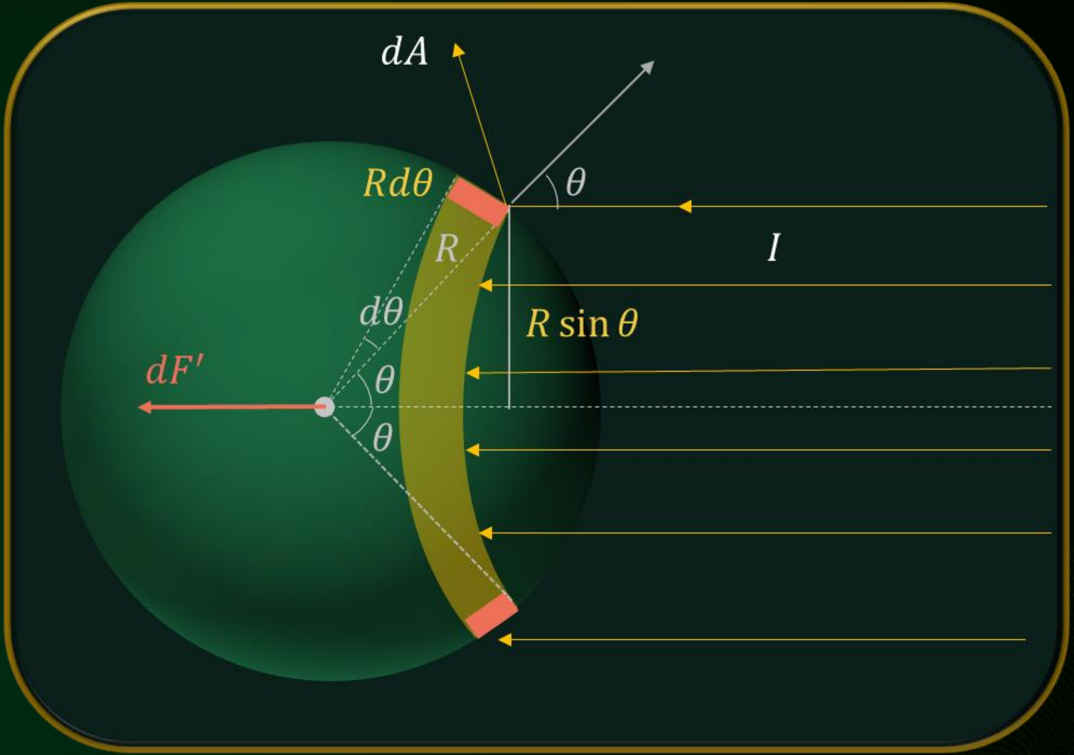
The vertical components $dF \sin\theta$ of forces on two symmetrical elemental areas of the ring cancel each other and the net force is due to the horizontal component $dF \cos\theta$.

dA' = Total area of thin ring $= \int dA$

dF' = Net force on this thin ring $= \int dF \cos\theta$

Radiation pressure, $P = \dfrac{dF}{dA} = \dfrac{2I}{c}\cos^2\theta$

Area of thin ring, $dA' = \int dA = 2\pi R\sin\theta \times R d\theta$



Net force on thin ring,

$$dF' = \int dF\cos\theta$$

$$= \int \frac{2I}{c}\cos^2\theta\, dA\cos\theta$$

$$= \frac{2I}{c}\cos^3\theta \int dA$$

$$dF' = \frac{2I}{c}\cos^3\theta\,(2\pi R\sin\theta \times R d\theta)$$

Force on entire sphere, $F = \displaystyle\int_0^{\pi/2} dF' = \int_0^{\pi/2} \frac{2I}{c}\cos^3\theta\,(2\pi R\sin\theta \times R d\theta)$

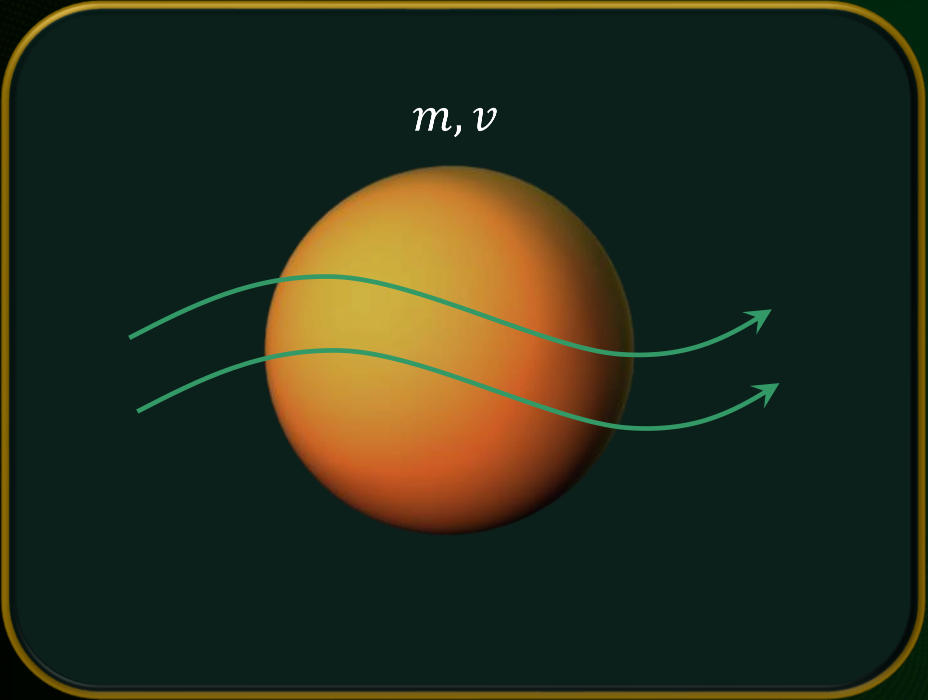
$$= \frac{4\pi r^2 I}{c}\int_0^{\pi/2} \cos^3\theta\sin\theta\, d\theta$$

$$= -\frac{4\pi r^2 I}{c}\left[\frac{\cos^4\theta}{4}\right]_0^{\frac{\pi}{2}}$$

$$\boxed{F = \frac{\pi r^2 I}{c}}$$

Matter Waves

The waves associated with moving particles are called Matter waves or de Broglie waves.



- The wavelength associated with a moving particle is known as de Broglie wavelength.

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

h = Planck's constant

p = Momentum

Matter Waves

- In ordinary situation, de Broglie wavelength is very small and wave nature of matter can be ignored.

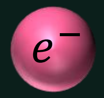
Baseball

Mass: 146 g

Speed: 30 m/s

De Broglie wavelength, $\boxed{\lambda = 1.51 \times 10^{-34}\ m}$

Electron

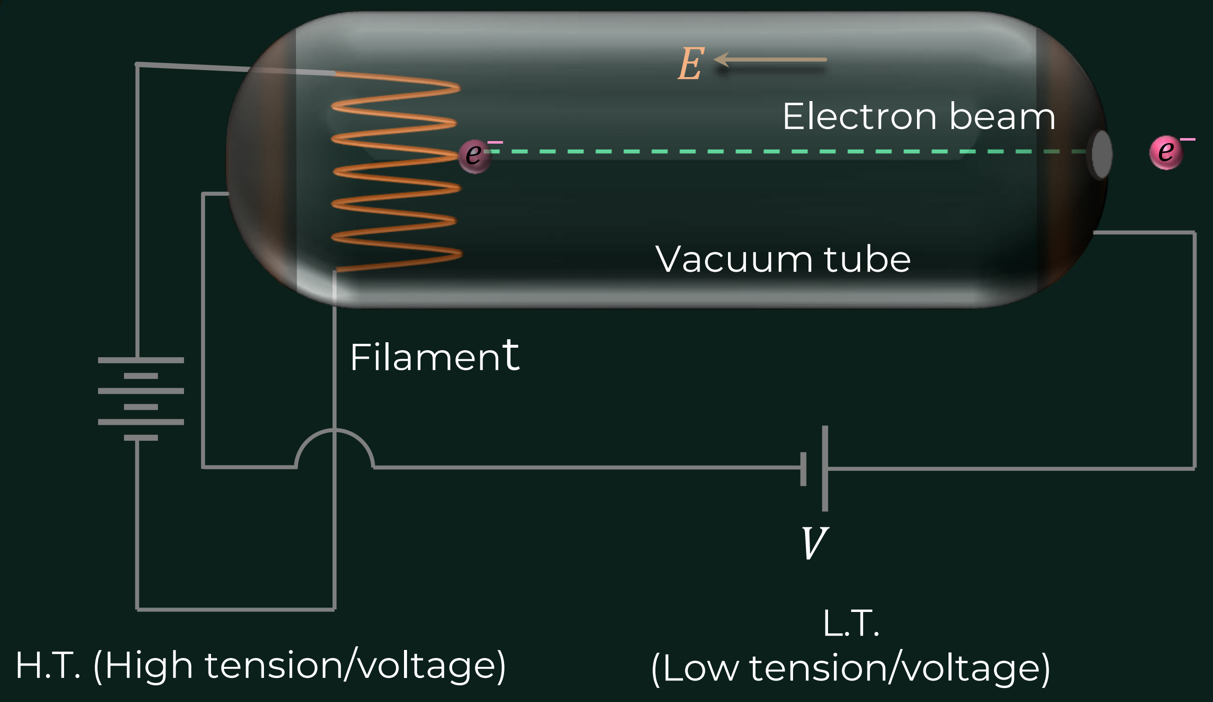


Mass: 9.1 x $10^{-31}\ kg$

Speed: $3 \times 10^{7}\ m/s$

De Broglie wavelength, $\boxed{\lambda = 2.42 \times 10^{-11}\ m}$

De Broglie Wavelength of an Electron



Electron Gun

- If an electrons is accelerated with the voltage V, then its kinetic energy becomes:

$$\boxed{K = eV}$$

- De Broglie wavelength becomes,

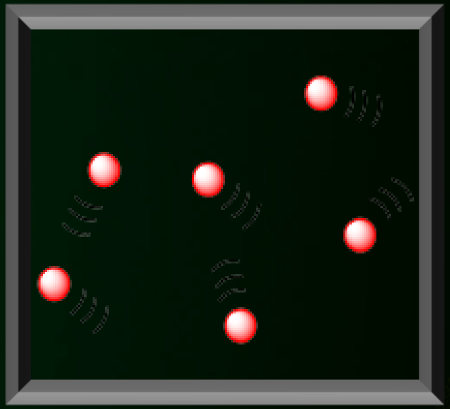
$$\lambda = \frac{h}{\sqrt{2m\,K}} = \frac{h}{\sqrt{2m\,eV}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}}$$

Find the de Broglie wavelength of a monoatomic gas particle at temperature T.

Given: Monoatomic gas at temperature T

To find: De Broglie wavelength (λ)



Solution: K.E of each particle of monoatomic gas, $E_1 = \frac{3}{2}kT$

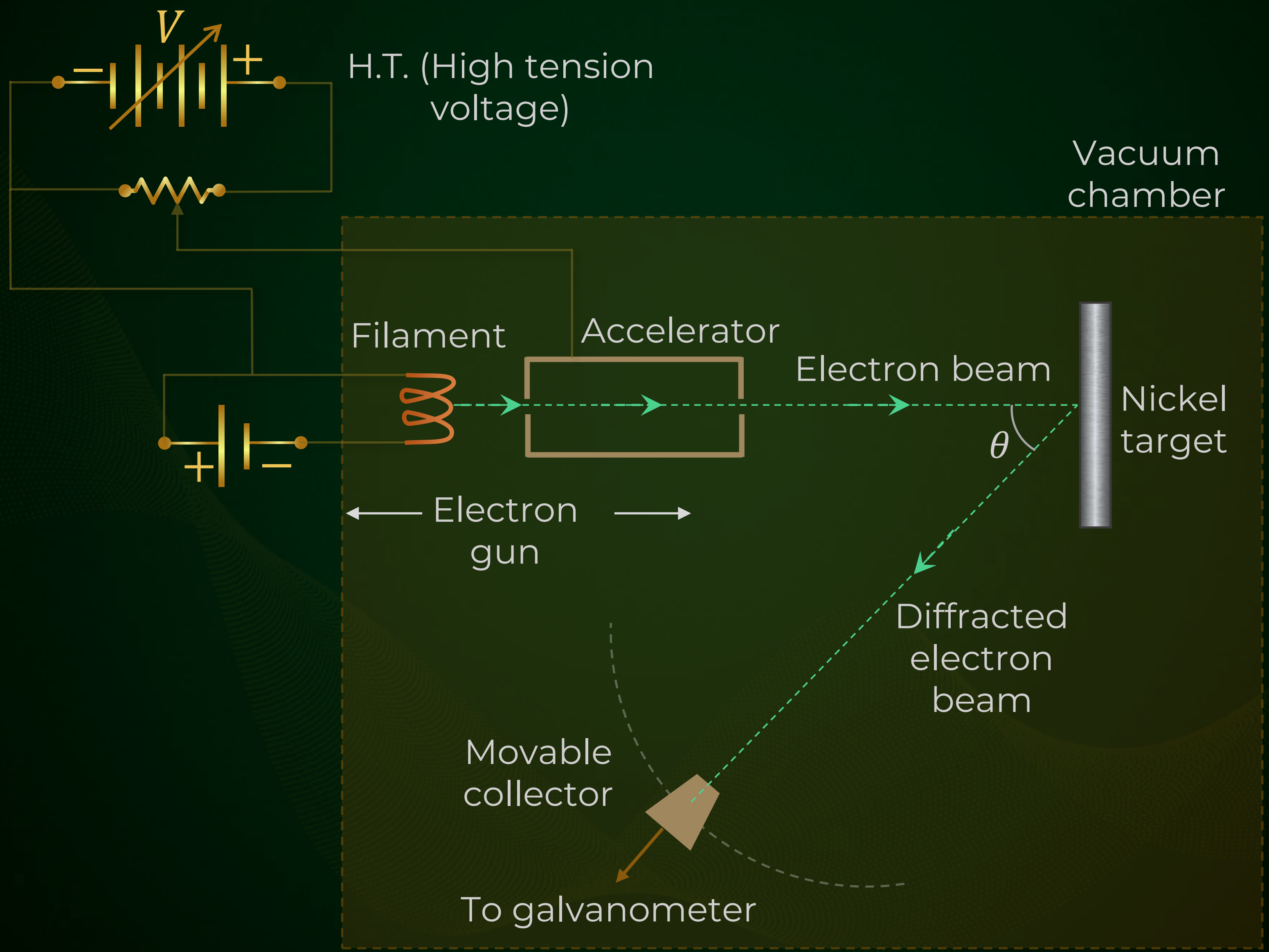
De Broglie wavelength, $\lambda = \frac{h}{\sqrt{2m\,K}} = \frac{h}{\sqrt{2mE_1}}$

$$\lambda = \frac{h}{\sqrt{2m \times \left(\frac{3}{2}kT\right)}}$$

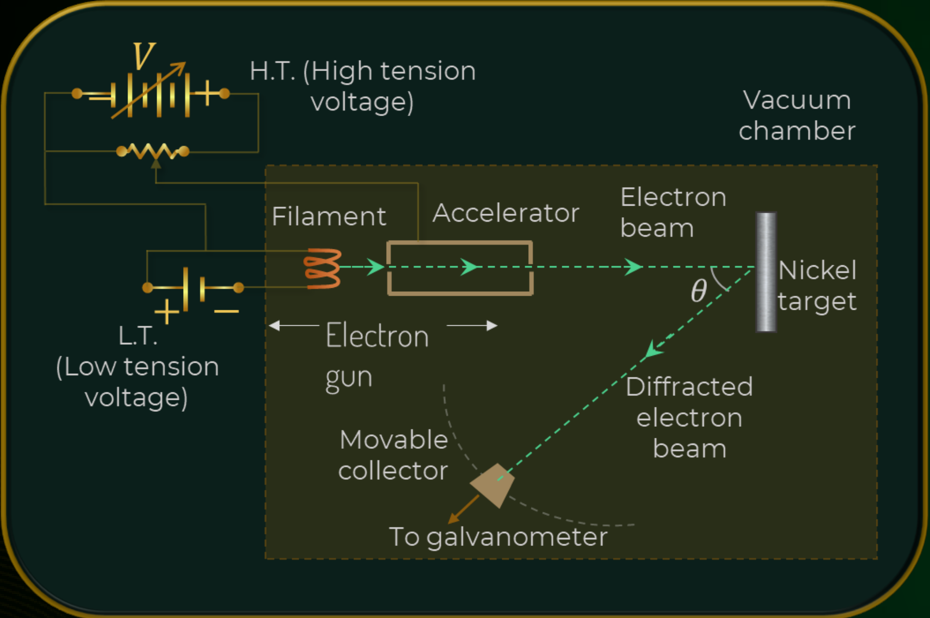
(k = Boltzmann constant)

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

Davisson and Germer Experiment



Davisson and Germer Experiment



- The experiment was performed by varying the accelerating voltage from $44\ V$ to $68\ V$.

- The intensity (I) of the scattered electrons is measured at different angles of scattering (θ).

- A strong peak is appeared in intensity (I) of the scattered electron for an accelerating voltage of $54\ V$ at a scattering angle $\theta = 50°$.

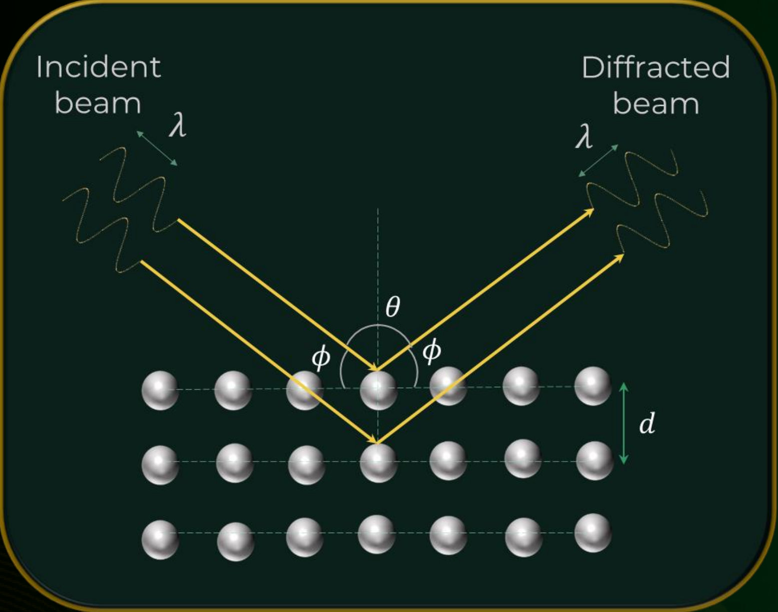
- This peak was the result of constructive interference of electrons scattered from the nickel target

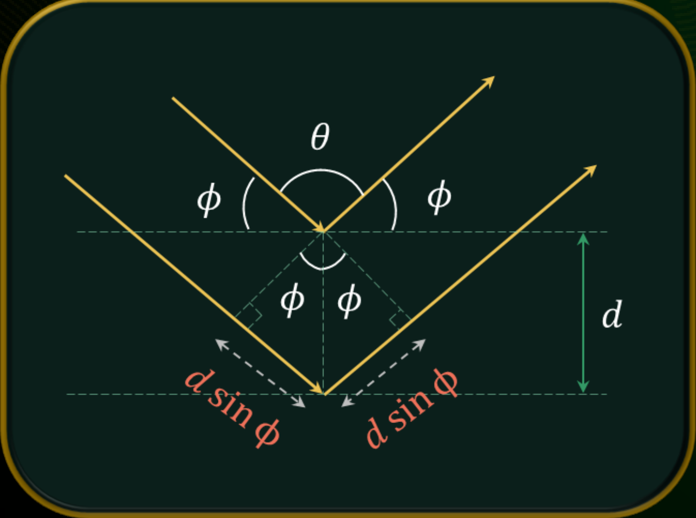
For accelerating voltage $V = 54\ V$, the de Broglie wavelength of electron is

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å} \qquad \lambda = \frac{12.27}{\sqrt{54}}\ \text{Å}$$

$$\boxed{\lambda = 1.67\ \text{Å}}$$

Bragg's Law





θ = Scattering angle

ϕ = Angle between incident beam and crystallographic plane

d = Distance between atomic layer (also known as interplanar spacing)

For constructive interference,

Path difference $= n\lambda$

$$\boxed{2d\sin\phi = n\lambda}$$

For $\theta = 50°$ $\Rightarrow \phi = 65°$

Putting $n = 1, d = 0.91\text{Å}$, and $\phi = 65°$

$$\boxed{\lambda = 1.66\ \text{Å}}$$

Electrons accelerated by a potential V are diffracted from a crystal. If $d = 1Å$ and $i = 30°$, V should be about

Solution:

For 1^{st} order maxima, $2d \sin\theta = \lambda$

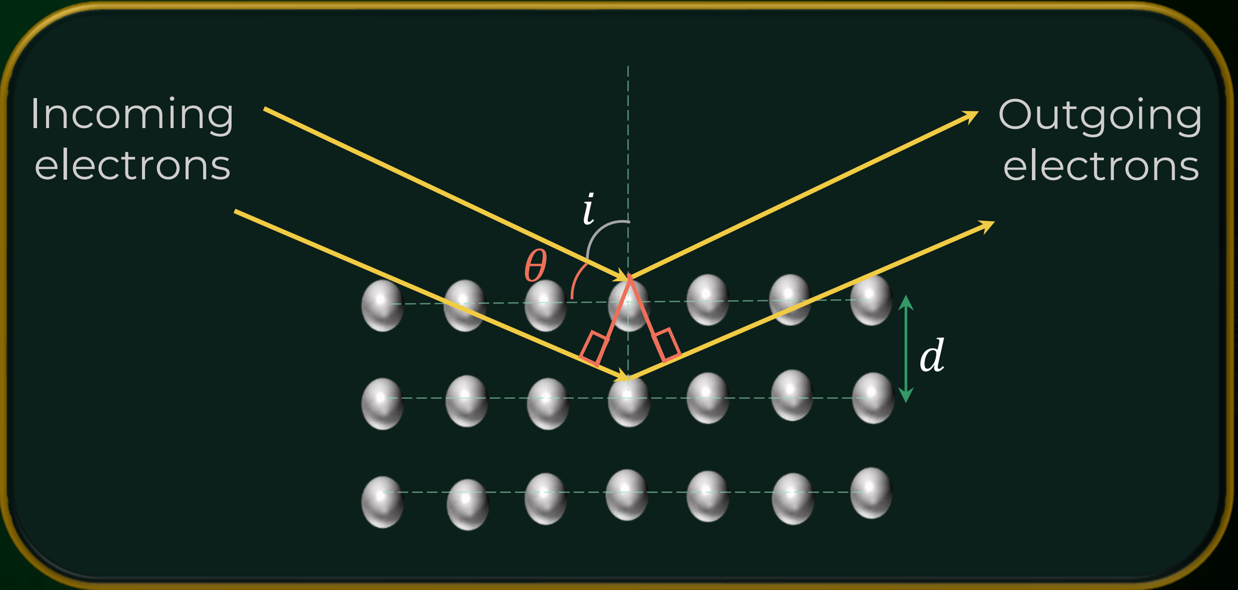
$$\Rightarrow 2d \sin(90° - i) = \lambda$$

$$\Rightarrow 2d \cos i = \lambda$$

$$\Rightarrow 2 \times 1 \times \cos 30° = \frac{12.27}{\sqrt{V}}$$

$$\Rightarrow \sqrt{V} = \frac{12.27}{\sqrt{3}}$$

$$\boxed{V = 50.18\ V \approx 50\ V}$$



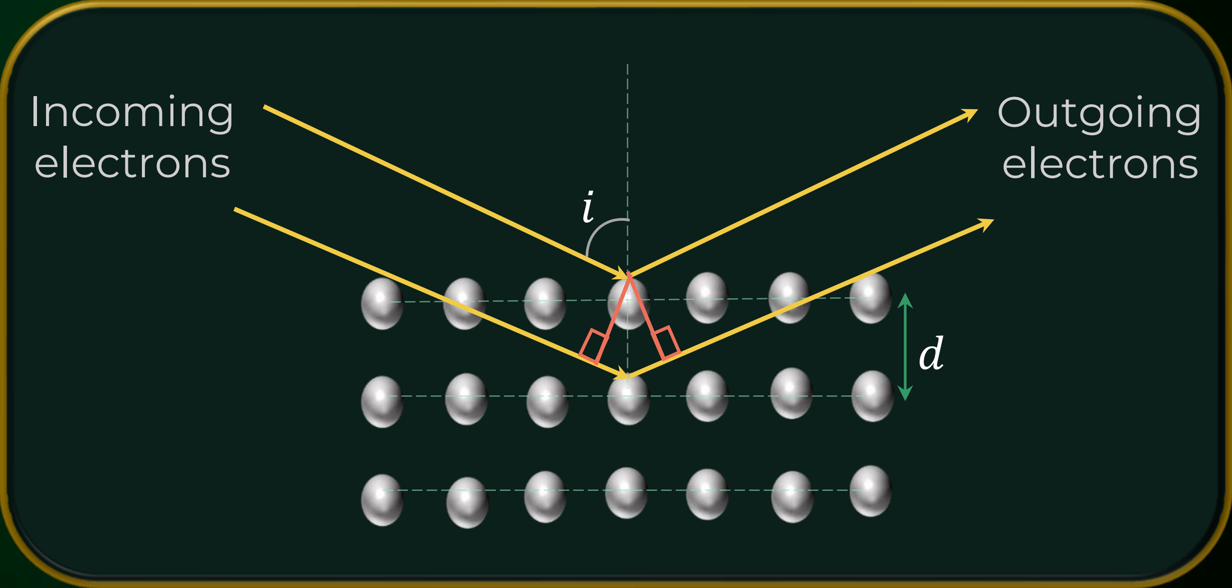
Electrons accelerated by a potential V are diffracted from a crystal. If $d = 1\text{Å}$ and $i = 30°$, V should be about

A $1000\ V$

B $2000\ V$

C $50\ V$

D $500\ V$



Physical Interpretation of Matter Waves

- Matter wave represents the probability of finding a particle in space.

- The intensity (square of the amplitude) of the matter wave at a point determines the probability density (probability per unit volume) of the particle at that point.

Probability of finding a particle in space:

$$\boxed{P = I \times \Delta V}$$

I = Intensity of matter wave

ΔV = Small volume

Heisenberg Uncertainty Principle

Werner Karl Heisenberg
1901 – 1976

According to Heisenberg's uncertainty principle, "It is not possible to measure both the position and momentum of a particle at the same time exactly".

The product of error in measurement of position and momentum is in the order of $\hbar$.

$$\boxed{\Delta x \cdot \Delta p \approx \hbar}$$

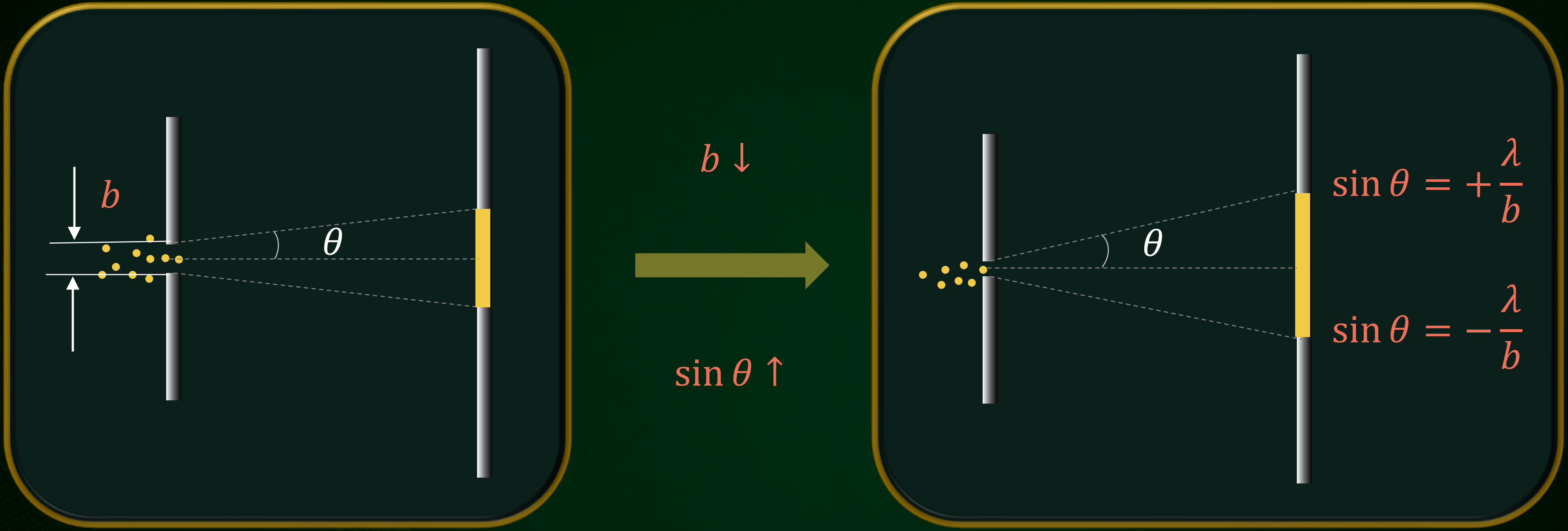
Δx = Uncertainty in position

Δp = Uncertainty in momentum

$$\hbar = \frac{h}{2\pi}$$

$$\boxed{\Delta x \cdot \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}}$$

Heisenberg Uncertainty Principle

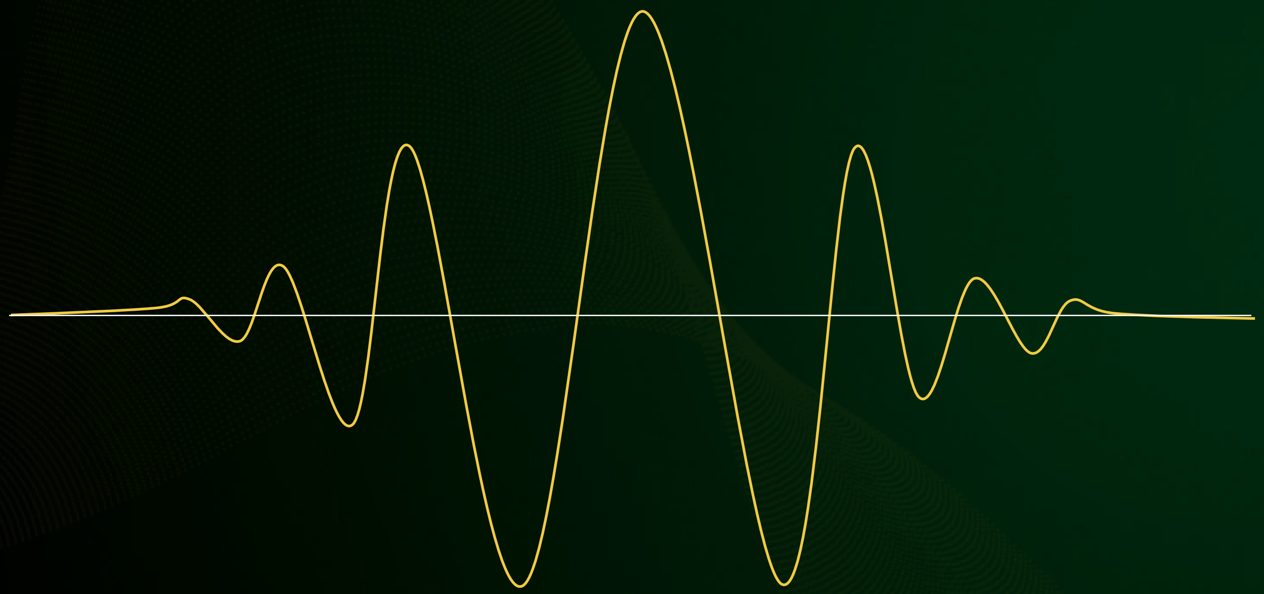


- When the slit is made narrower there is a lesser space for the photons to go through which means the uncertainty in the position has become smaller and the uncertainty in momentum has become larger.

- Therefore the uncertainty in the velocity of photons has also become larger that's why now the photons can go at higher angles.

Wave Packets

The wave packet corresponds to a spread of wavelength around some central wavelength.

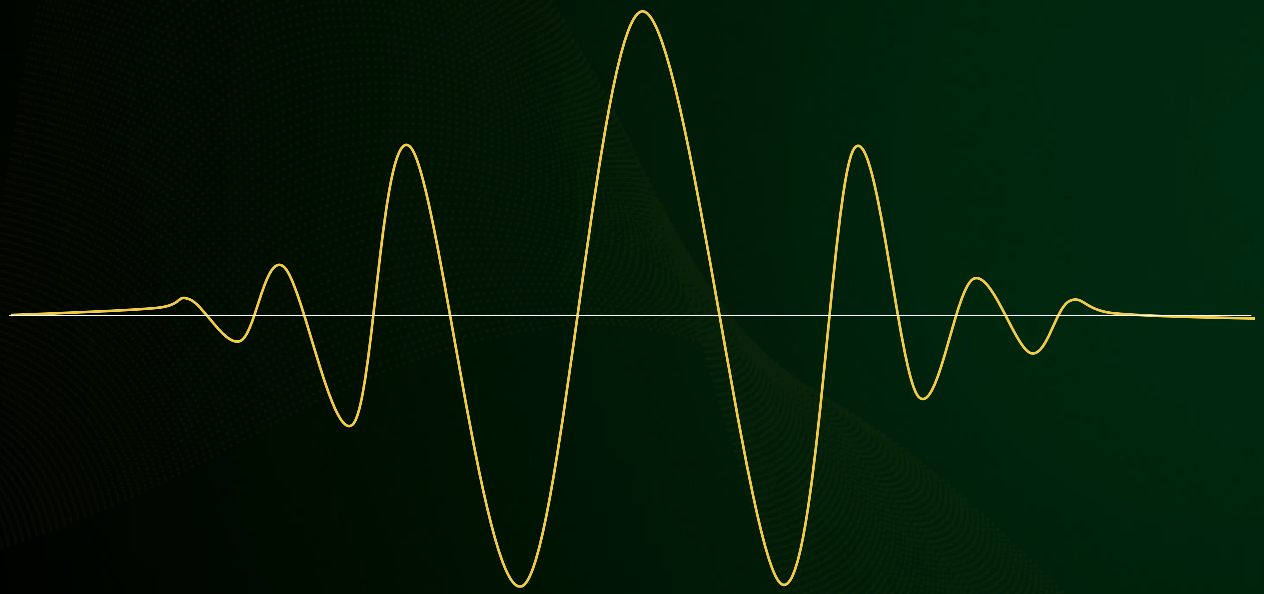


- The matter wave associated with the electron is not extended all over space (because a wave of definite wavelength extends all over space). It is a wave packet extending over some finite region of space.

- In that case Δx is not infinite but has some finite value depending on the extension of the wave packet.

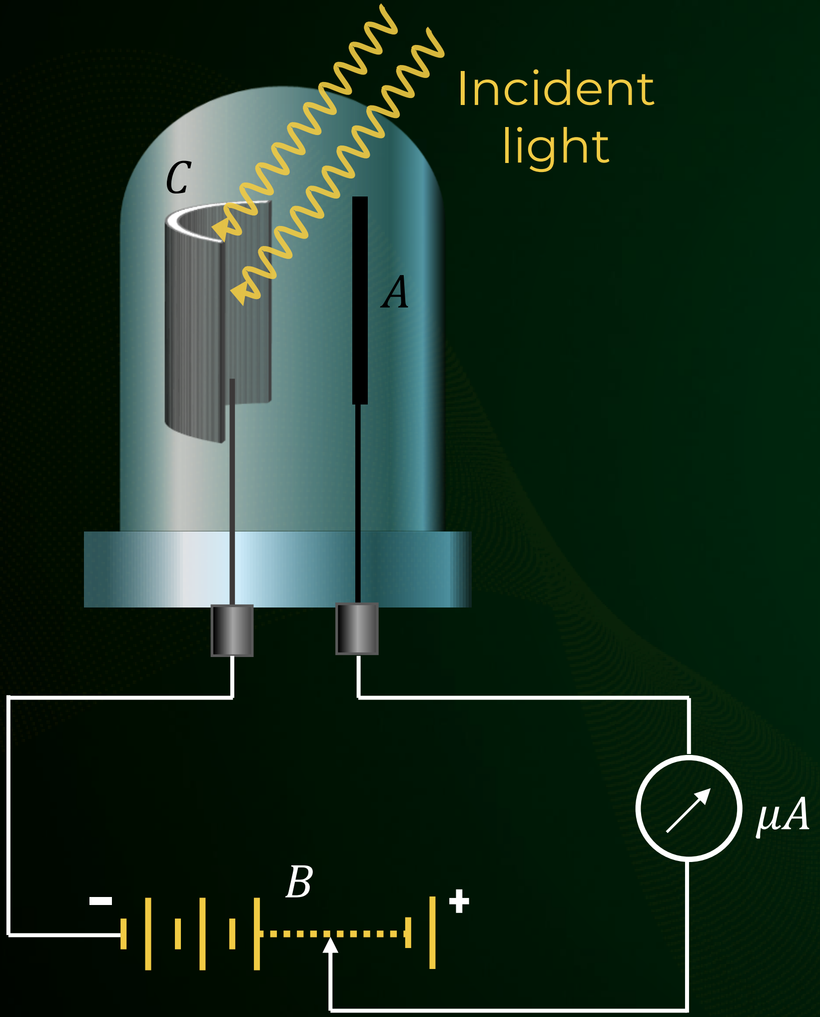
Wave Packets

The wave packet corresponds to a spread of wavelength around some central wavelength.



- By de Broglie's relation, then, the momentum of the electron will also have a spread – an uncertainty Δp. This is as expected from the uncertainty principle.

Photocell

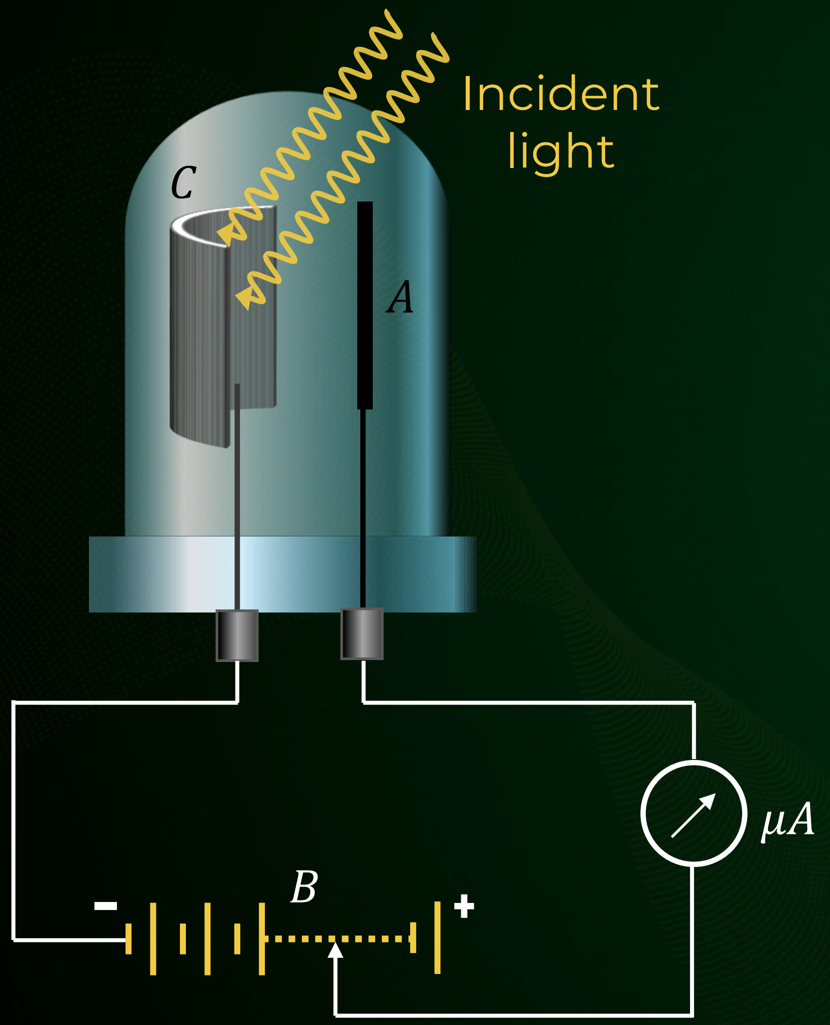


A photocell works on the principle of photoelectric effect.

Working:

- When light of suitable wavelength falls on the emitter C, photoelectrons are emitted.
- These photoelectrons are drawn to the collector A. Photocurrent of the order of a few microampere can be normally obtained from a photocell.
- A photocell converts a change in intensity of illumination into a change in photocurrent. This current can be used to operate control systems and in light measuring devices.

Photocell



Applications:

- The activation/deactivation process of streetlights is mainly controlled by photocells.
- These are used as timers in a running race to calculate the runner's speed.
- Photocells are used to count the vehicles on the road.
- These are used in burglar alarms to protect from a thief.

An electron, a doubly ionized helium ion (He^{2+}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths $\lambda_e, \lambda_{He^{2+}}$ and λ_p is

Solution:

De Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK.E.}}$

Also, $K.E._e = K.E._{He^{2+}} = K.E._p$

$m_{He^{2+}} > m_p > m_e$

$\therefore \lambda_{He^{2+}} < \lambda_p < \lambda_e$

A $\lambda_e > \lambda_{He^{2+}} = \lambda_p$

B $\lambda_e < \lambda_{He^{2+}} = \lambda_p$

C $\lambda_e > \lambda_p > \lambda_{He^{2+}}$

D $\lambda_e < \lambda_p < \lambda_{He^{2+}}$

An electron (mass m) with initial velocity $\vec{v} = v_0\hat{\imath} + v_0\hat{\jmath}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by

Given: $\vec{v} = v_0\hat{\imath} + v_0\hat{\jmath}$ $\Rightarrow |\vec{v}| = \sqrt{v_0^2 + v_0^2} = v_0\sqrt{2}$

$\vec{E} = -E_0\hat{k}$

To find: de-Broglie wavelength at time t (λ')

Solution:

Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv} = \frac{h}{mv_0\sqrt{2}} \quad \text{..................}(1)$$

Force on electron, $\vec{F} = -e\vec{E} = eE_0\hat{k}$

$$\Rightarrow m\vec{a} = eE_0\hat{k}$$

$$\Rightarrow \vec{a} = \frac{eE_0}{m}\hat{k}$$

$$\vec{v'} = \vec{v} + \vec{a}t = v_0\hat{\imath} + v_0\hat{\jmath} + \frac{eE_0}{m}t\,\hat{k}$$

$$\Rightarrow |\vec{v'}| = \sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$$

de-Broglie wavelength at time t,

$$\lambda' = \frac{h}{mv'} = \frac{h}{m\sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}} \quad \text{..................}(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{\lambda'}{\lambda_0} = \frac{\sqrt{2}v_0}{\sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}} \Rightarrow \boxed{\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}}$$