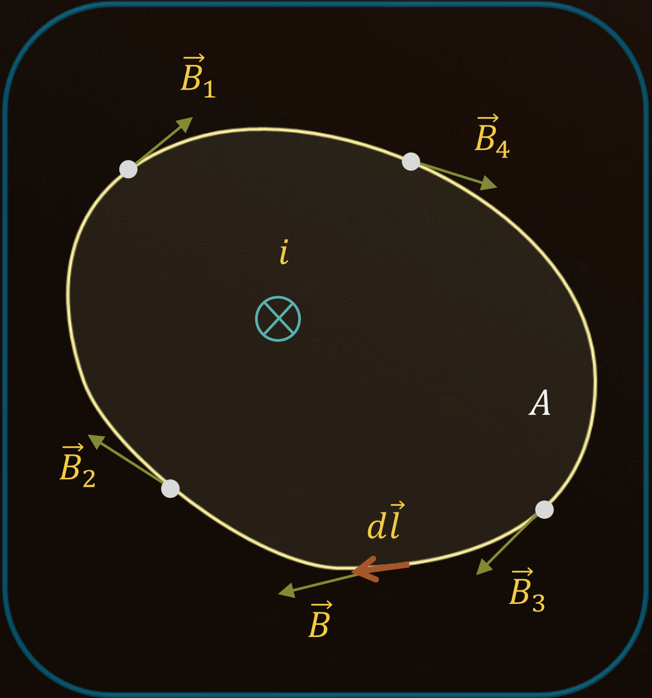
Welcome to

Aakash + BYJU'S NOTES

EM Waves

Ampere's Circuital Law



- Line integral or circulation of $\vec{B}$ along the curve

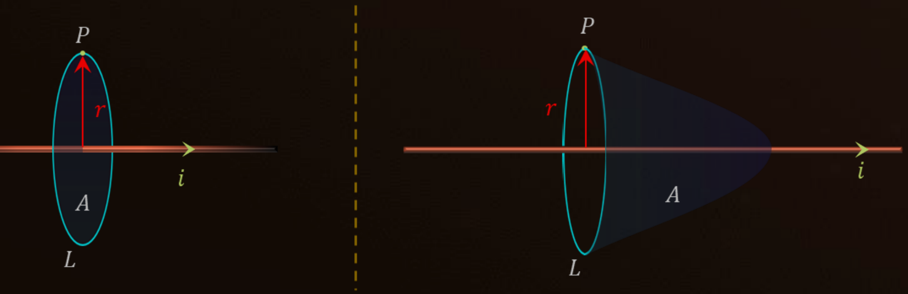
$$\oint \vec{B} . dl = \mu_0 i$$

μ_0 = Permeability of free space

A = Area bounded by the loop

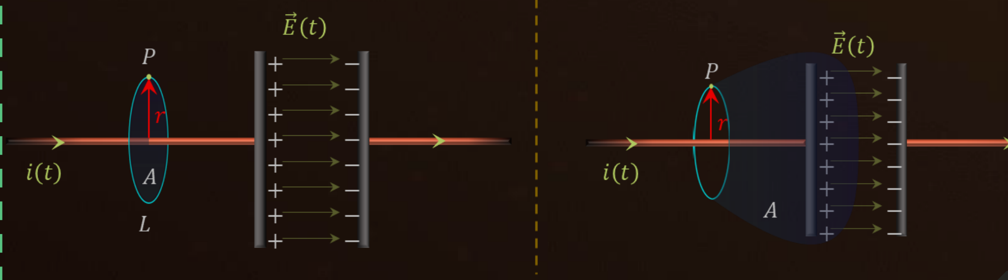
i = Net current intersecting the area

Ampere's Circuital Law



Ampere's law is consistent

$$B_P = \frac{\mu_0 i}{2\pi r}$$



$$B_P = \frac{\mu_0 i(t)}{2\pi r}$$

Inconsistency in Ampere's law

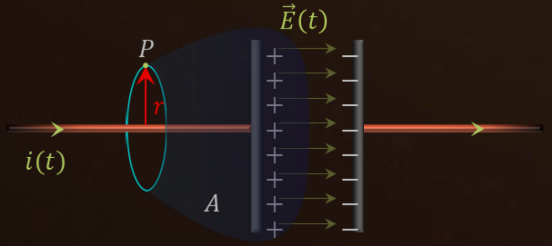
$$B_P = 0$$

Ampere's Circuital Law

- Displacement current :

A fictitious current which is produced due to the change of electric flux with respect to time.

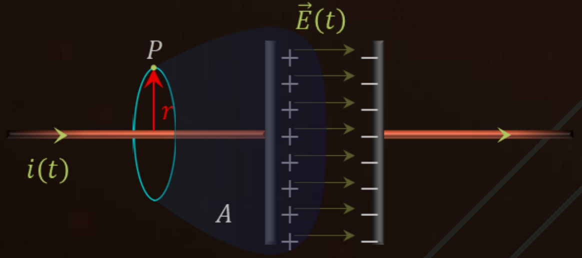
$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$



- Ampere-Maxwell Law :

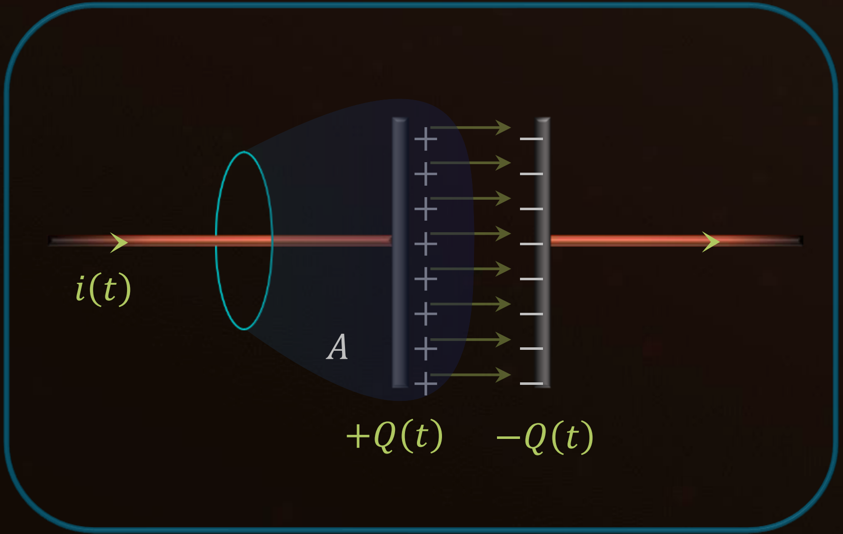
The line integral of magnetic field along a closed loop in free space is equal to μ_0 times the total current (sum of conduction current and displacement current) threading the loop.

$$\oint \vec{B}.d\vec{l} = \mu_o\left[i_c + \frac{\varepsilon_o d\phi_E}{dt}\right] = \mu_o[i_c + i_d]$$



- Conduction current : Current in a conductor due to flow of charge carriers.

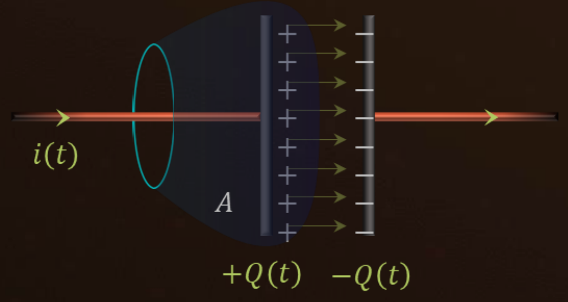
Show that the displacement current is equal to conduction current during charging of a capacitor.



Assumption :

Area of the plates of the capacitor = A

Instantaneous charge on the plates = $Q(t)$



Solution :

Electric field between plates of capacitor at any instant t :

$$E(t) = \frac{Q(t)}{A\varepsilon_0}$$

Electric flux of this field passing through the surface between the plates :

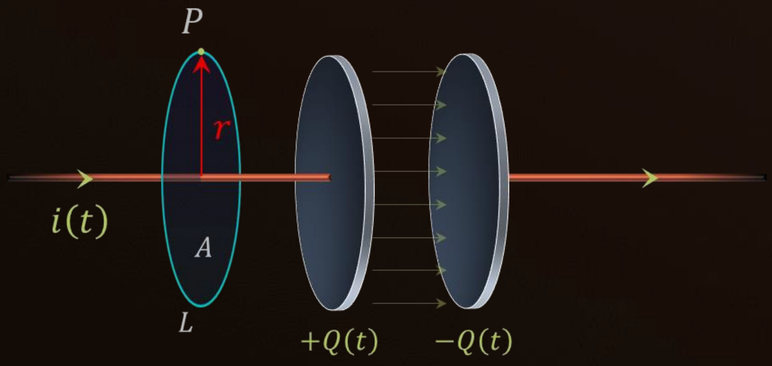
$$\phi_E = E(t) \times A = \frac{Q(t)}{\varepsilon_0}$$

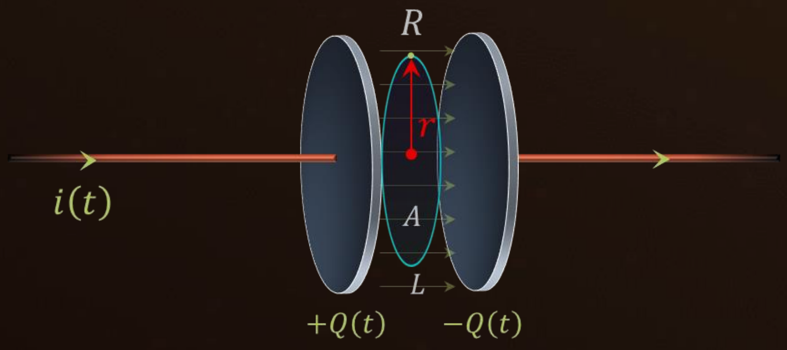
Displacement current : $$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \frac{dQ(t)}{dt}$$

$\frac{dQ(t)}{dt}$: It is also the rate at which charge is accumulating at positive plate of capacitor through conducting wire.

Therefore, conduction current is also equal to $\frac{dQ(t)}{dt}$. Hence, $$i_c = \frac{dQ(t)}{dt} = i_d$$

Finding The Induced Magnetic Field





$i_c = i(t)$ and $i_d = 0$

$$B_P = \frac{\mu_0 i(t)}{2\pi r}$$

$$\phi_E = E \times \pi r^2$$

$$\phi_E = \left(\frac{Q(t)}{\varepsilon_0 \pi R^2}\right) \times \pi r^2$$

$$\phi_E = \frac{Q(t) r^2}{\varepsilon_0 R^2}$$

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \frac{r^2}{R^2} i(t)$$

$$i_c = 0$$

$$B_R = \left(\frac{\mu_0 i(t)}{2\pi R^2}\right) r$$

Calculation of Induced Magnetic Field

- Inside the capacitor plates ($r > R$) :

Conduction current, $i_c = 0$

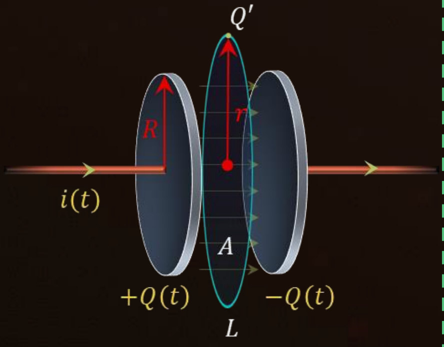
Electric field between the plates of capacitor :

$$E(t) = \frac{Q(t)}{A\varepsilon_0}$$

Electric flux of this field passing through the surface between the plates :

$$\phi_E = E(t) \times A = \frac{Q(t)}{\varepsilon_0}$$

Displacement current : $i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \frac{dQ(t)}{dt} = i(t)$



- Inside the capacitor plates ($r > R$) :

Apply Ampere-Maxwell Law :

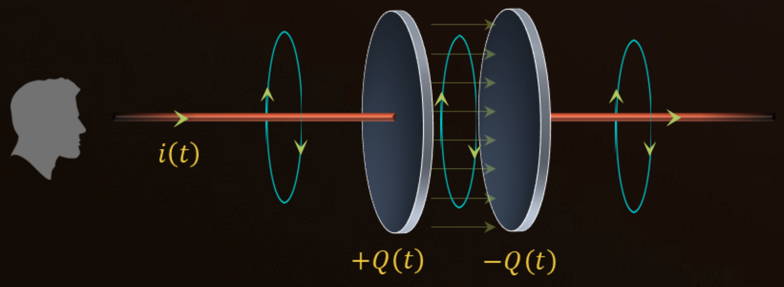
$$\oint_L \vec{B} . d\vec{l} = \mu_o[i_c + i_d]$$

$$B \times 2\pi r = \mu_o[0 + i(t)]$$

$$B = \frac{\mu_o i(t)}{2\pi r}$$

$$B_{Q'} = \frac{\mu_0 i(t)}{2\pi r}$$

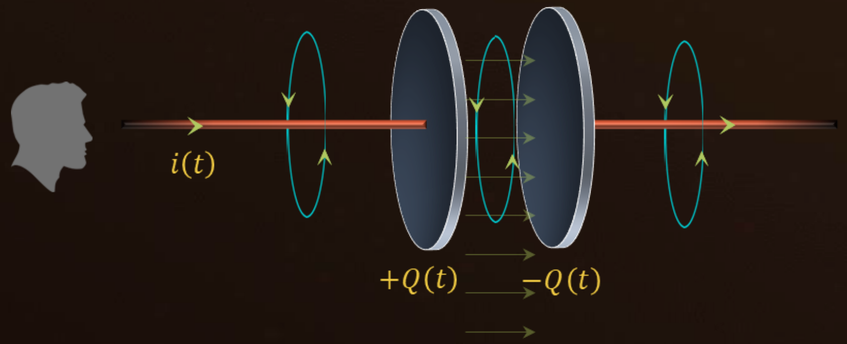
Direction of Induced Magnetic Field



Charging of capacitor

Observer : Induced magnetic field is in clockwise direction.

(Use "Right hand thumb rule")



Discharging of capacitor

Observer : Induced magnetic field is in anti-clockwise direction.

(Use "Right hand thumb rule")

A parallel plate capacitor with circular plates of radius $1\ m$ has a capacitance of $1\ nF$. At $t = 0$, it is connected for charging in series with a resistor $R = 1\ M\Omega$ across a $2\ V$ battery. Calculate the magnetic field at a point P, halfway between the centre and the periphery of the plates after $t = 10^{-3} s$. Take $q(t) = CV[1 - e^{-\frac{t}{\tau}}]$

Solution :

Instantaneous charge : $q(t) = CV[1 - e^{-\frac{t}{\tau}}]$

Conduction current at $t = 10^{-3}\ s$:

$$i_c = \frac{dq(t)}{dt} = \frac{V}{R}\left[e^{-\frac{t}{\tau}}\right] = \frac{V}{R}\left[e^{-\frac{t}{CR}}\right]$$

$$i_c = \frac{2}{10^6}\left[e^{-\frac{10^{-3}}{10^{-9}\times 10^6}}\right] = \frac{2}{10^6} \times \frac{1}{e} = 7.36 \times 10^{-7}\ A$$

Displacement current is assumed to flow uniformly throughout the volume.

Displacement current = Conduction current

Displacement current through the reddish patch :

$$i'_d = \frac{i_d}{\pi r^2} \times \pi\left(\frac{r}{2}\right)^2 = \frac{i_d}{4} = \frac{i_c}{4} = 1.84 \times 10^{-7}\ A$$

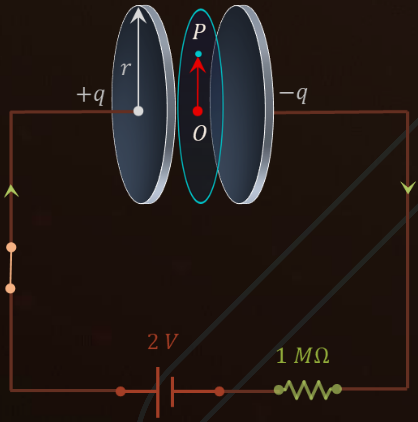
$$B.2\pi\frac{r}{2} = \mu_0(i_c + i'_d)$$

Conduction current between the plates is zero.

$$B.\pi r = \mu_0 i'_d$$

$$B = \frac{\mu_0 i'_d}{\pi r} = \frac{4\pi \times 10^{-7} \times 1.84 \times 10^{-7}}{\pi \times 1}$$

$$B = 0.74 \times 10^{-13}\ T$$

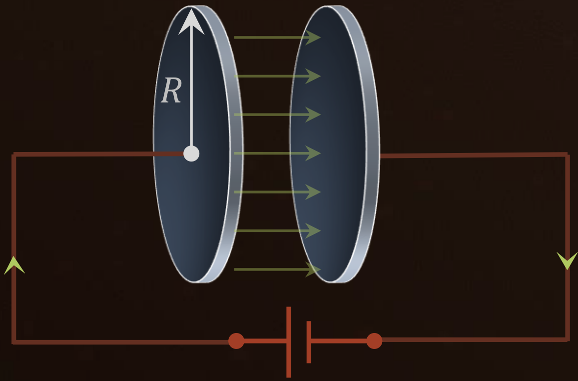


A parallel plate capacitor consists of two circular plates of radius $R = 0.1\ m$. They are separated by a distance $d = 0.5\ mm$. If electric field between the capacitor plates changes as $\frac{dE}{dt} = 5 \times 10^{13}\ \frac{V}{m \times s}$, find the displacement current between the plates.

Given :

Radius of the circular plates of capacitor : $R = 0.1\ m$

Rate of change of electric field : $\frac{dE}{dt} = 5 \times 10^{13}\ \frac{V}{m \times s}$



Solution :

Displacement current : $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

$$i_d = \varepsilon_0 \frac{d}{dt}(E \times \pi R^2)$$

$$i_d = \varepsilon_0 \pi R^2 \frac{dE}{dt}$$

$$i_d = 8.854 \times 10^{-12} \times \pi \times (0.1)^2 \times 5 \times 10^{13}$$

$$i_d = 13.9\ A$$

A. $10\ A$

B. $15\ A$

C. $13.9\ A$

D. $21\ A$

In charging a parallel plate capacitor of capacity $10\ \mu F$, it takes $0.5\ s$ to reach potential difference of $50\ V$. If plate area of the capacitor is $10 \times 10^{-12}\ m^2$, then find:
(a) Average conduction current during that time.
(b) Average displacement current during that time.
(c) Rate of change of electric field during that time.

Given: $\frac{\Delta V}{\Delta t} = 100\ V/s$ Area, $S = 10 \times 10^{-2}\ m^2$

Solution :

(a) Charge on a capacitor,

$Q = CV$

Average conduction current,

$$(i_c)_{av} = \frac{\Delta Q}{\Delta t} = C\left(\frac{\Delta V}{\Delta t}\right)$$

$$(i_c)_{av} = 10 \times 10^{-6} \times 100\ \ A$$

$$(i_c)_{av} = 10^{-3}\ A$$

(b) Displacement current,

$$i_d = \varepsilon_o \frac{d\phi_E}{dt}$$

Electric Flux : $\phi_E = \frac{Q}{S\varepsilon_o} . S = \frac{Q}{\varepsilon_o}$

Average displacement current,

$$(i_d)_{av} = \varepsilon_o \frac{\Delta \phi_E}{\Delta t} = \frac{\Delta Q}{\Delta t} = (i_c)_{av}$$

$$(i_d)_{av} = 10^{-3}\ A$$

(c) Electric field : $E = \frac{Q}{S\varepsilon_o}$

Rate of change of electric field :

$$\frac{\Delta E}{\Delta t} = \frac{1}{S\varepsilon_o}\frac{\Delta Q}{\Delta t} = \frac{(i_c)_{av}}{S\varepsilon_o}$$

$$\frac{\Delta E}{\Delta t} = \frac{10^{-3}}{10 \times 10^{-12} \times 8.854 \times 10^{-12}}$$

$$\frac{dE}{dt} = 1.1 \times 10^{19} \frac{V}{m \times s}$$

Maxwell's Equations

Unification of theories of electricity & magnetism when fields are varying with time.

Gauss's Law for Electrostatics

$$\oint \vec{E}\,.\,d\vec{S} = \frac{Q_{in}}{\varepsilon_0}$$

Gauss's law for Magnetism

$$\oint \vec{B}\,.\,d\vec{S} = 0$$

JAMES CLERK MAXWELL
(1831-1879)

Faraday's law of EMF

$$\oint \vec{E}\,.\,d\vec{l} = -\frac{d\phi_B}{dt}$$

Ampere – Maxwell law

$$\oint \vec{B}\,.\,d\vec{l} = \mu_0 i_c + \varepsilon_0\mu_0\frac{d\phi_E}{dt}$$

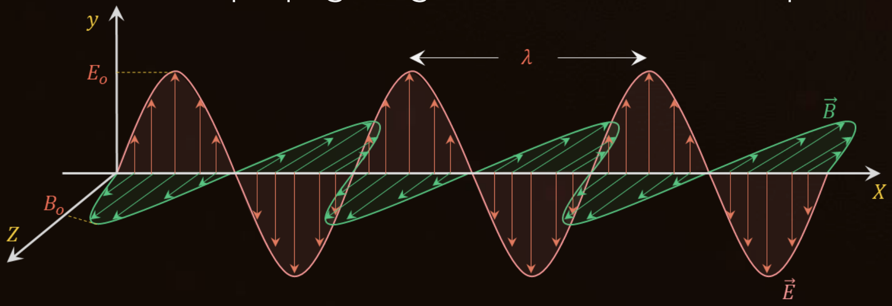
Maxwell's Equations

- Accelerated motion of a charged particle generates EM Waves.

- Accelerated charged particle produces time varying electric field and it produces time varying magnetic field.

- The oscillating E.F. and M.F. are perpendicular to each other.

- The energy associated with the propagating wave comes at the expense of the energy of the source.



Some Other Sources of Electromagnetic Waves

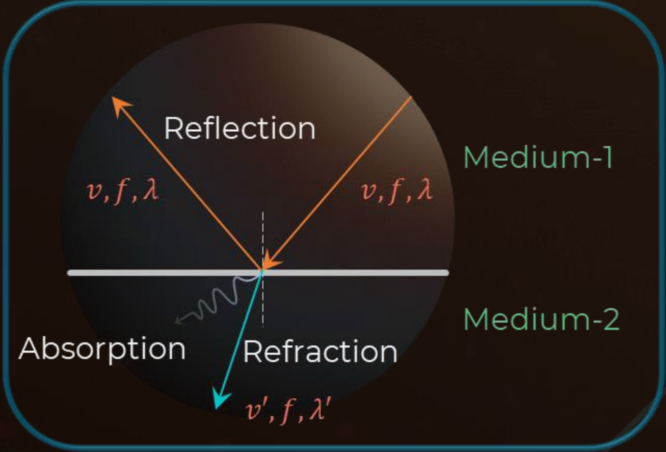
- An electron orbiting around its nucleus in a stationary orbit does not emit electromagnetic wave. It will emit only during transition from higher energy orbit to lower energy orbit.

- Electromagnetic wave (X-ray) is produced when high speed electron enters into a target of high atomic weight.

- Electromagnetic wave (γ-ray) is produced during de-excitation of nucleus in radioactivity.

Characteristics and Nature of EM Waves

- EM waves do not require medium for their propagation.
- These waves travel with speed of light in vacuum.
- These waves are transverse in nature.
- They carry both energy and momentum.
- They can be reflected, refracted or absorbed.
- Frequency is inherent characteristic of EM waves. When it travels from one medium to another only its speed and wavelength changes.



Summary

$$E = E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right)$$

$$B = B_Z = B_0 \sin\omega\left(t - \frac{x}{c}\right)$$

$$\oint \vec{E}\,.\,d\vec{l} = -\frac{d\phi_B}{dt}$$

Faraday's Law
(for vacuum)

$$E_0 = cB_0$$

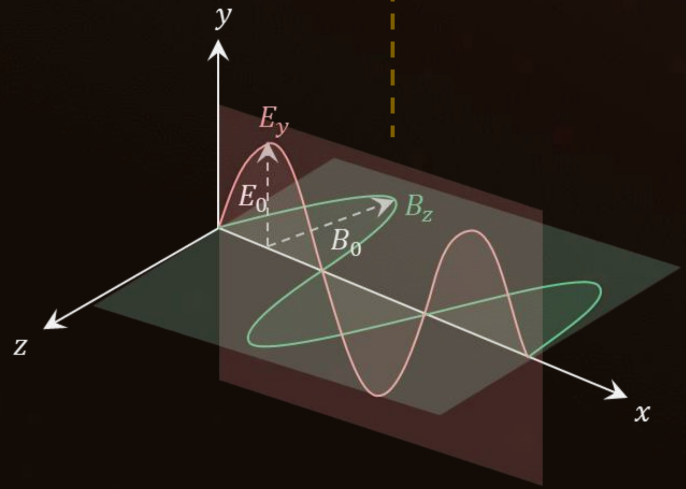
$$E = E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right)$$

$$B = B_Z = B_0 \sin\omega\left(t - \frac{x}{c}\right)$$

$$\oint \vec{B}\,.\,d\vec{l} = \varepsilon_0\mu_0\frac{d\phi_E}{dt}$$

Ampere's Law
(for vacuum)

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$



Speed of EM Wave in a medium

- Speed of EM Wave in vacuum : $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
- Speed of EM Wave in medium

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$v = \frac{1}{\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}}$$

$$v = \frac{1}{\sqrt{(\mu_0\varepsilon_0)(\mu_r\varepsilon_r)}}$$

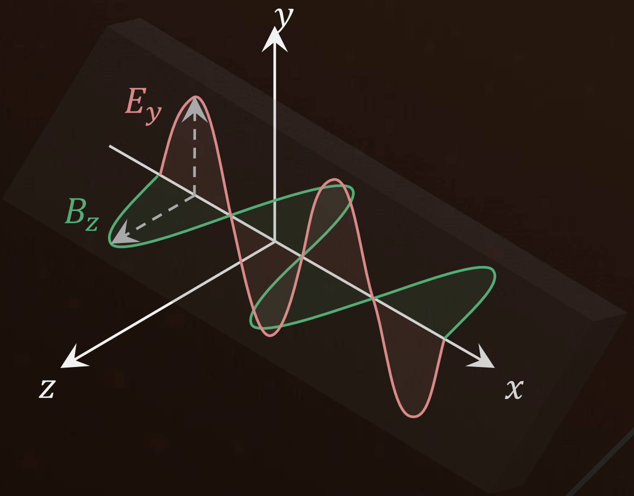
$$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

μ : Permeability of the medium

ε : Permittivity of the medium

μ_r : Relative permeability of the medium

ε_r : Relative permittivity of the medium



The magnetic field in a plane electromagnetic wave is given by
$B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\ T$.
(a) What is the wavelength and frequency of the wave?
(b) Write an expression for the electric field.

(a) Given : $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\ T$

General equation magnetic field of EM wave :

$$B_y = B_0 \sin(kx + \omega t)$$

Therefore,

$B_0 = 2 \times 10^{-7}\ T$ $\quad \omega = 1.5 \times 10^{11}\ rad/s$ $\quad k = 0.5 \times 10^3\ m^{-1}$

Wavelength of the wave :

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{0.5 \times 10^3}$$

$$\lambda = 1.26\ cm$$

Frequency of the wave :

$$f = \frac{\omega}{2\pi} = \frac{1.5 \times 10^{11}}{2 \times 3.14}$$

$$f = 23.9\ GHz$$

(b) EM Waves are transverse wave. So, electric field of the given EM Wave will be along z-axis.

$$E_z = E_0 \sin(kx + \omega t)$$

We know : $E_0 = cB_0$

$$E_0 = (3 \times 10^8\ ms^{-1}) \times (2 \times 10^{-7}\ T)$$

$$E_0 = 60\ V/m$$

Therefore, the expression of electric field :

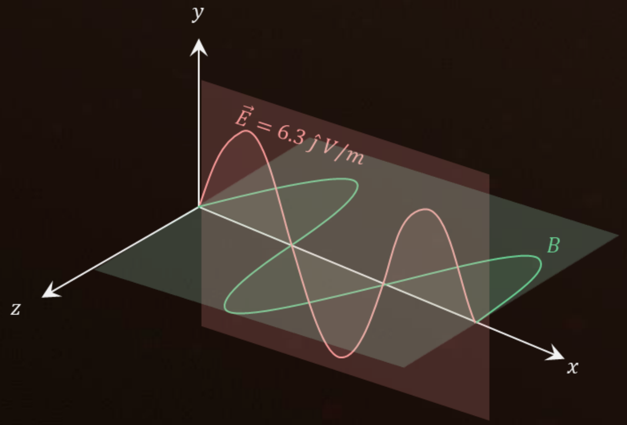
$$E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\ V/m$$

A plane electromagnetic wave of frequency $25\ MHz$ travels in free space along the x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}\ V/m$. What is $\vec{B}$ at this point?

We know : $E_0 = cB_0$

$$B_0 = \frac{E_0}{c}$$

$$B_0 = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}\ T$$



Since the wave is propagating along $+ve\ x$-direction, the magnetic field should be along $+ve\ z$-direction.

This is because $(\vec{E} \times \vec{B})$ should be parallel to the direction of propagation of wave.

$\vec{B} = 2.1 \times 10^{-8}\hat{k}\ T$

A. $\vec{B} = 2.1 \times 10^{-8}\hat{k}\ T$

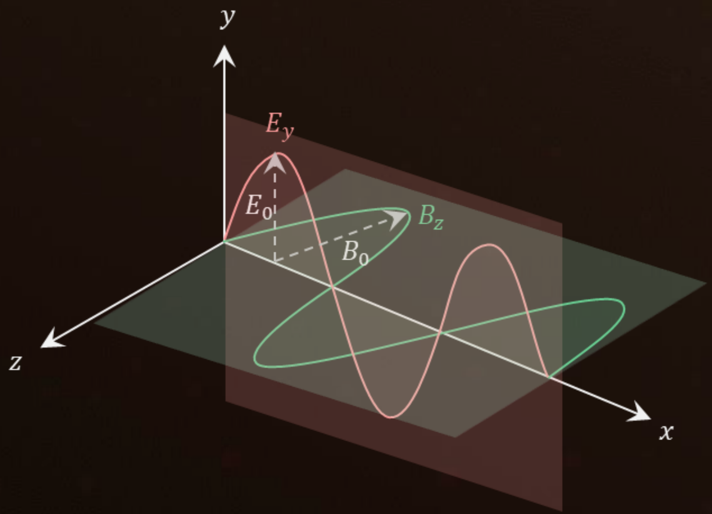
B. $\vec{B} = 2.1 \times 10^{-8}\hat{j}\ T$

C. $\vec{B} = 2.1 \times 10^{-10}\hat{j}\ T$

D. $\vec{B} = 6 \times 10^{-8}\hat{k}\ T$

Summary

- Energy stored in a unit volume (dV) in any electric/magnetic field is known as Energy density.
- Energy density in electric field : $\frac{1}{2}\varepsilon_o E^2$
- Energy density in magnetic field : $\frac{B^2}{2\mu_o}$



- Average energy density in electric field :

$$(U_E)_{avg} = \frac{1}{T}\int_0^T \frac{1}{2}\varepsilon_o E^2\, dt = \frac{\varepsilon_o}{2T}\int_0^T E_0^2 \sin^2(\omega t - kx)\, dt$$

$$(U_E)_{avg} = \frac{\varepsilon_o}{2T} \times E_0^2 \times \frac{T}{2}$$

$$(U_E)_{avg} = \frac{1}{4}\varepsilon_o E_0^2$$

- Average energy density in magnetic field :

$$(U_B)_{avg} = \frac{1}{T}\int_0^T \frac{1}{2\mu_0} B^2\, dt = \frac{1}{2\mu_0 T}\int_0^T B_0^2 \sin^2(\omega t - kx)\, dt$$

$$(U_B)_{avg} = \frac{1}{4\mu_o} B_0^2$$

- Average energy density of EM Wave :

$$U_{avg} = \frac{1}{4}\varepsilon_o E_0^2 + \frac{1}{4\mu_o} B_0^2$$

Summary

$$E_0 = cB_0$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$(U_E)_{avg} = \frac{1}{4}\varepsilon_o E_0^2 = \frac{1}{4} \times \frac{1}{c^2 \mu_0} \times c^2 B_0^2$$

$$(U_E)_{avg} = \frac{1}{4\mu_o} B_0^2 = (U_B)_{avg}$$

- Average energy density of EM Wave :

$$U_{avg} = \frac{1}{4}\varepsilon_o E_0^2 + \frac{1}{4\mu_o} B_0^2$$

Or

$$U_{avg} = \frac{1}{2}\varepsilon_o E_0^2 = \frac{1}{2\mu_o} B_0^2$$

A plane electromagnetic wave has frequency $2 \times 10^{10}\ Hz$ and its energy density is $1.02 \times 10^{-8}\ J/m^3$ in vacuum. The amplitude of the magnetic field of the wave is close to :

Given :

Frequency of EM wave, $f = 2 \times 10^{10}\ Hz$

Energy density, $U_{avg} = 1.02 \times 10^{-8}\ j/m^3$

Solution :

Energy density,

$$U_{avg} = \frac{B_0^2}{2\mu_0}$$

$$B_0 = \sqrt{2 \times \mu_0 \times U_{avg}}$$

$$B_0 = \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}}\ T$$

$$B_0 = 160 \times 10^{-9}\ T$$

$$B_0 = 160\ nT$$

A. $150\ nT$

B. $160\ nT$

C. $180\ nT$

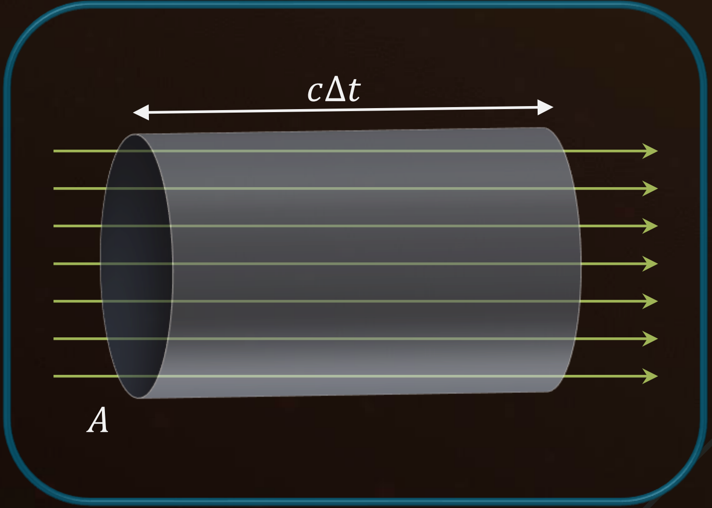
D. $190\ nT$

Intensity of EM Waves

- The energy crossing per unit area per unit time perpendicular to the direction of propagation of wave is called the intensity of wave.
- Energy passing through cylindrical area A per unit time:

$$\frac{U}{\Delta t} = \frac{1}{2}\varepsilon_o E_o^2 \times A \times c = \frac{B_o^2}{2\mu_o} \times A \times c$$

$$I = \frac{U}{A\Delta t} = \frac{1}{2}\varepsilon_o E_o^2 c = \frac{B_o^2}{2\mu_o} c$$



Intensity due to a Point Source

- The power of the source : P
- Intensity at a distance r due to the source :

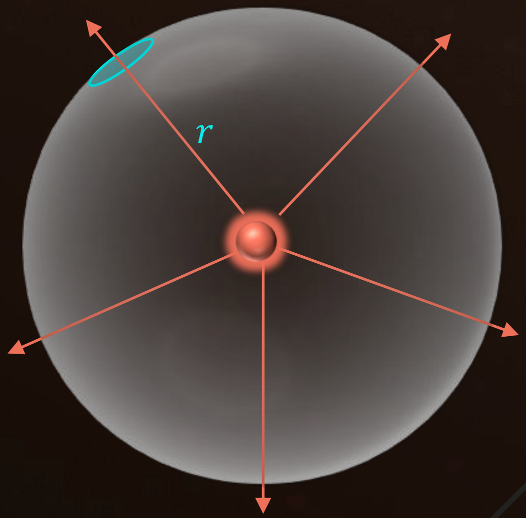
$$I = \frac{E}{A \times t} = \frac{P}{A}$$

$$I = \frac{P}{4\pi r^2}$$

Intensity is inversely proportional to square of r.

So, the intensity due to a point source obeys the inverse square law.

- Unit: $Watt/m^2$ or $\frac{J}{s\ m^2}$



A $27\ mW$ laser beam has a cross-sectional area of $10\ mm^2$. The magnitude of the maximum electric field in this electromagnetic wave is given by :

Given :

Power of laser beam, $P = 27\ mW = 27 \times 10^{-3}\ W$

Cross-sectional area, $A = 10\ mm^2 = 10 \times 10^{-6}\ m^2$

Solution :

Intensity of EM wave,

$$I = \frac{Power}{Area} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$$

$$E_0 = \sqrt{2 \times 10^6}\ V/m$$

$$E_0 = 1.414 \times 10^3\ V/m$$

$$E_0 = 1.4\ kV/m$$

A. $2\ kV/m$

B. $1\ kV/m$

C. $0.7\ kV/m$

D. $1.4\ kV/m$

Momentum and Radiation Pressure

- Electromagnetic wave carries energy and momentum with it.
- Momentum of EM waves :

$$p = \frac{U}{c}$$

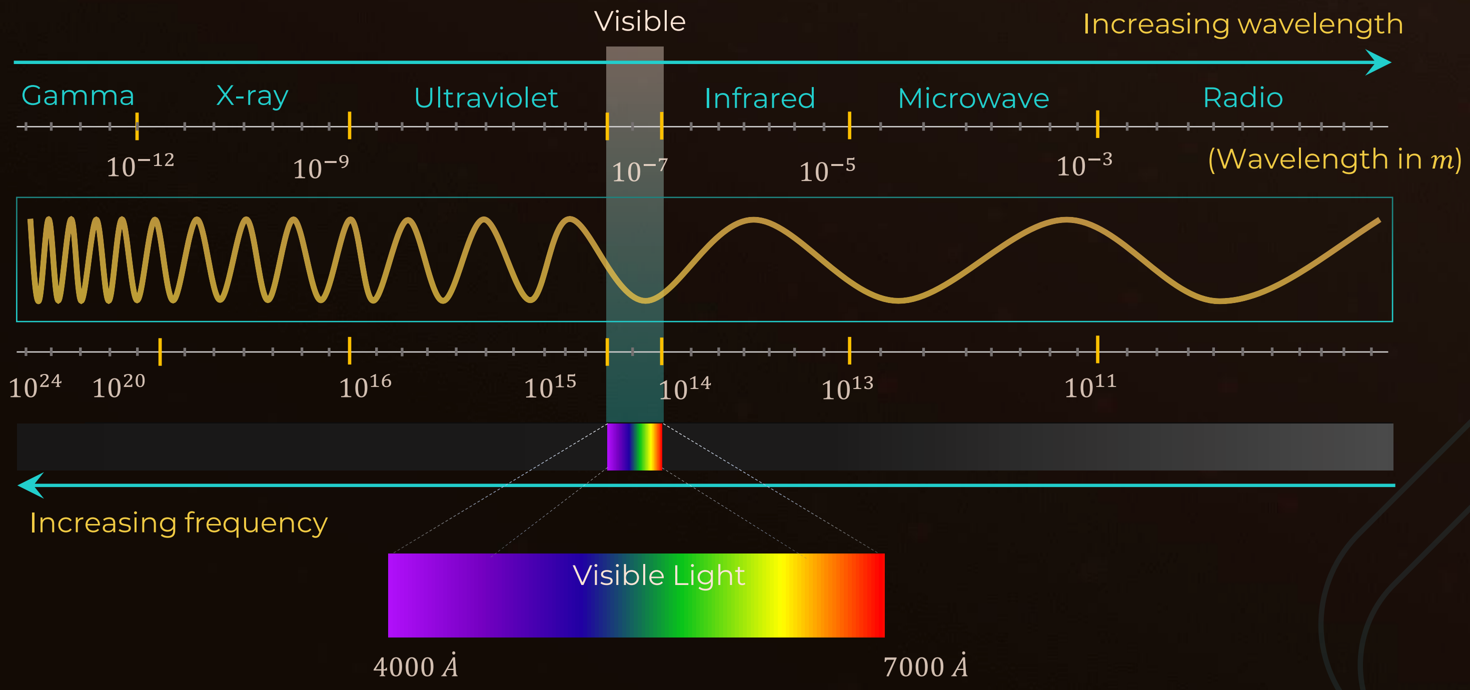
U: Energy carried by EM wave in free space

- Radiation pressure is defined as "Force exerted by EM Wave on unit surface area".

Condition	Radiation Pressure
Surface completely absorbs the radiation falling on it	$\frac{I}{c}$
Surface completely reflects the radiation falling on it	$\frac{2I}{c}$

Electromagnetic Spectrum

- The orderly distribution of electromagnetic radiations according to their frequency (or wavelength) is called "Electromagnetic spectrum".



Different Region of Electromagnetic Spectrum

Gamma Rays

- Produced by the disintegration of radioactive atomic nuclei.
- Used in medicine to destroy cancer cells.

Frequency: $10^{19}\ Hz - 10^{24}\ Hz$

Wavelength: $< 10^{-11}\ m$

X - Rays

- Produced due to sudden deceleration of fast-moving electrons when they collide and interact with the target anode.
- Used as a diagnostic tool in medicine to detect fracture of bones and foreign bodies.

Frequency: $10^{16}\ Hz - 10^{20}\ Hz$

Wavelength: $10^{-8}\ m - 10^{-13} m$

Different Region of Electromagnetic Spectrum

Ultraviolet Rays

- Produced by special lamps and very hot bodies (E.g., sun).
- Useful for eye surgery, detect skin disease, water purification etc.

Frequency: $10^{15}\ Hz - 10^{17}\ Hz$

Wavelength: $400\ nm - 1\ nm$

Visible Rays

- Produced by atomic excitation.
- It is detected by the eyes, photocells and photographic films.

Frequency: $4 \times 10^{14}\ Hz - 7 \times 10^{14}\ Hz$

Wavelength: $700\ nm - 400\ nm$

Different Region of Electromagnetic Spectrum

Infrared Rays

- Produced by hot bodies i.e., vibration of atoms and molecules.
- Used in the remote switches of household electronic system.

Frequency: $10^{12}\ Hz - 10^{14}\ Hz$

Wavelength: $25\ \mu m - 2.5 \mu m$

Microwaves

- Produced by special vacuum tubes. (E.g., Klystrons, Gunn diodes etc.)
- Used in radar system for aircraft navigation and microwave ovens.

Frequency: $3 \times 10^{11}\ Hz - \ 10^{13}\ Hz$

Wavelength: $1\ mm - 25\ mm$

Different Region of Electromagnetic Spectrum

Radio waves

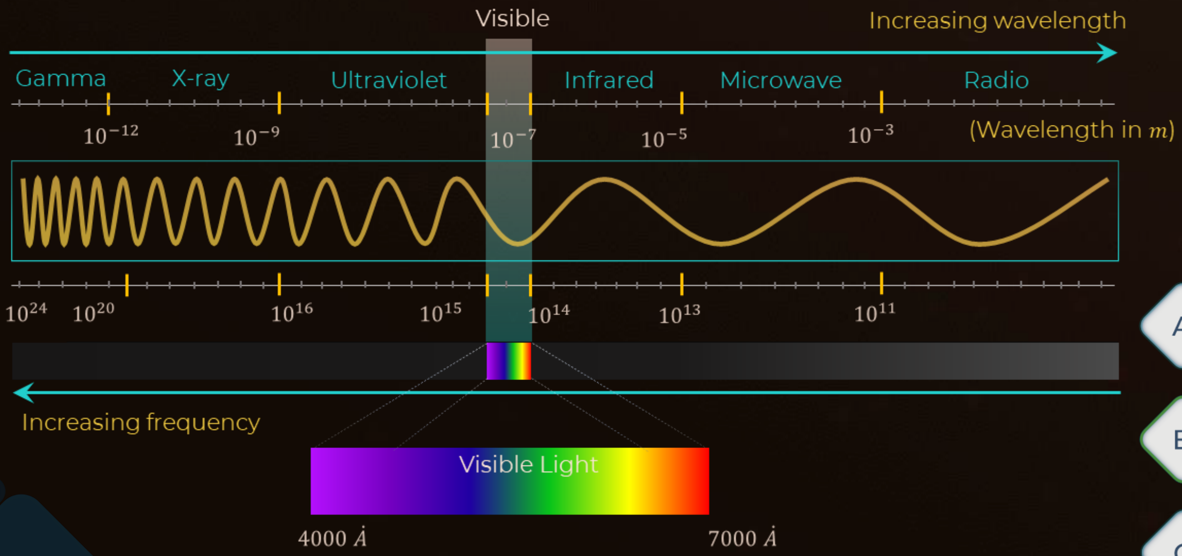
Frequency: $< 3 \times 10^{11}$ Hz | Wavelength: $> 1\ mm$

- **Amplitude modulated Radio Wave** ($530\ \text{kHz} - 1710\ \text{kHz}$)
- **Television Radio Wave** ($54\ \text{MHz} - 890\ \text{MHz}$)
- **Frequency modulated Radio Wave** ($88\ \text{MHz} - 108\ \text{MHz}$)

- They are produced by the accelerated motion of charges in conducting wires.
- They are used in radio and television communication system.

Choose the correct option relating wavelength of different parts of electromagnetic wave spectrum :

Solution :



A. $\lambda_{visible} < \lambda_{microwaves} < \lambda_{radiowaves} < \lambda_{X-rays}$

B. $\lambda_{radiowaves} > \lambda_{microwaves} > \lambda_{visible} > \lambda_{X-rays}$

C. $\lambda_{X-rays} < \lambda_{microwaves} < \lambda_{radiowaves} < \lambda_{visible}$

D. $\lambda_{visible} > \lambda_{X-rays} > \lambda_{radiowaves} < \lambda_{microwaves}$