

Welcome to



Aakash



BYJU'S

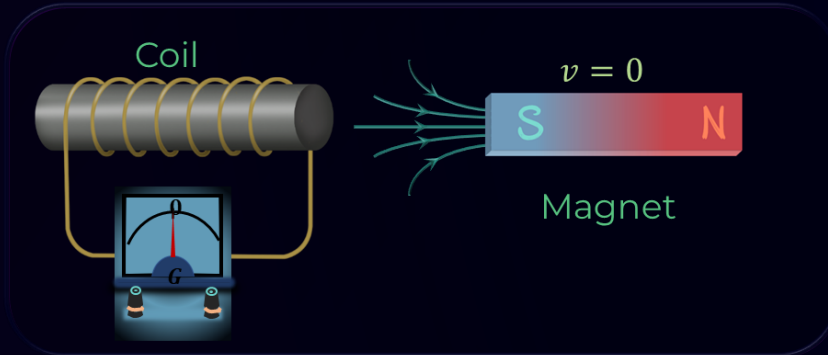
NOTES

Electromagnetic induction

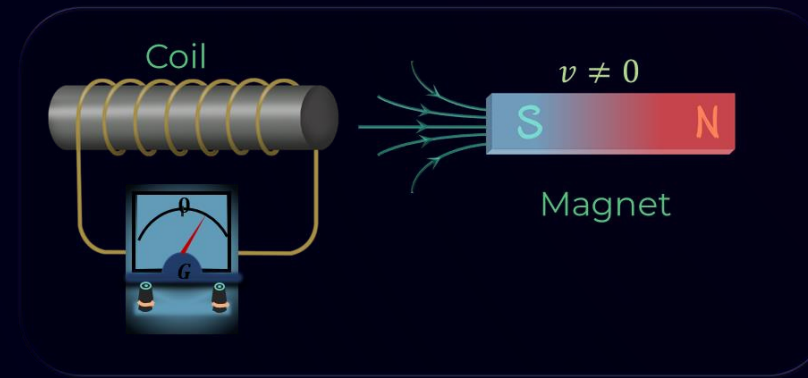




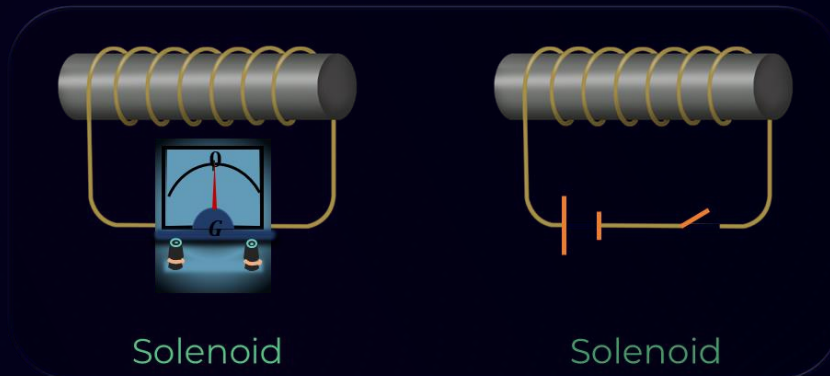
Faraday's Experiment



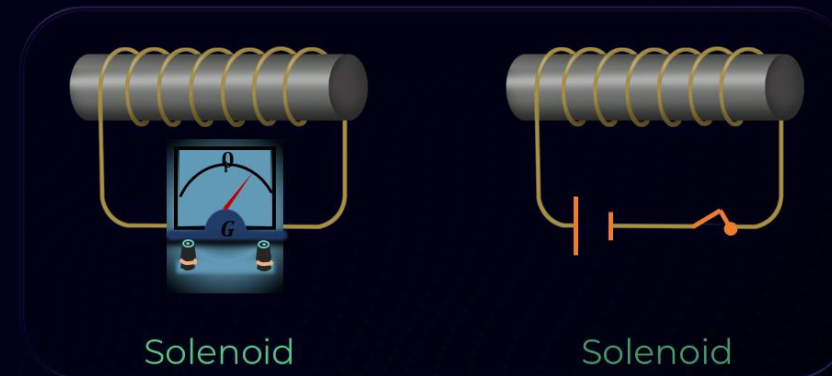
- If there is **no relative** motion in the coil and magnet, then there is **no deflection** in galvanometer.



- If a **relative** motion is caused between the coil and magnet, then there is a **deflection**.



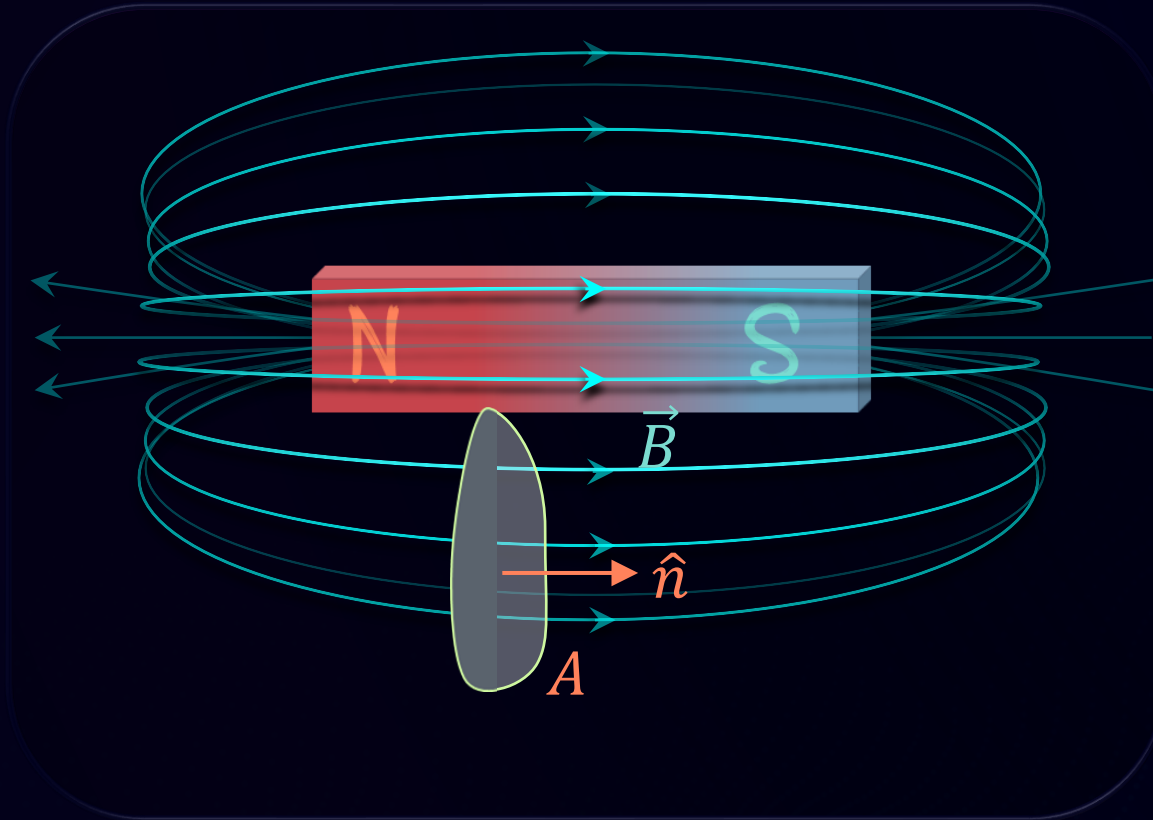
- When the switch is closed, a deflection is observed in the galvanometer for very small time.



- When the key is opened, a deflection is observed in galvanometer (opposite direction).



Magnetic Flux



- Measure of number of magnetic field lines crossing normally through a given area.

\hat{n} : area vector

$$\phi = \int \vec{B} \cdot d\vec{A}$$

$$\phi = BA \cos\theta$$



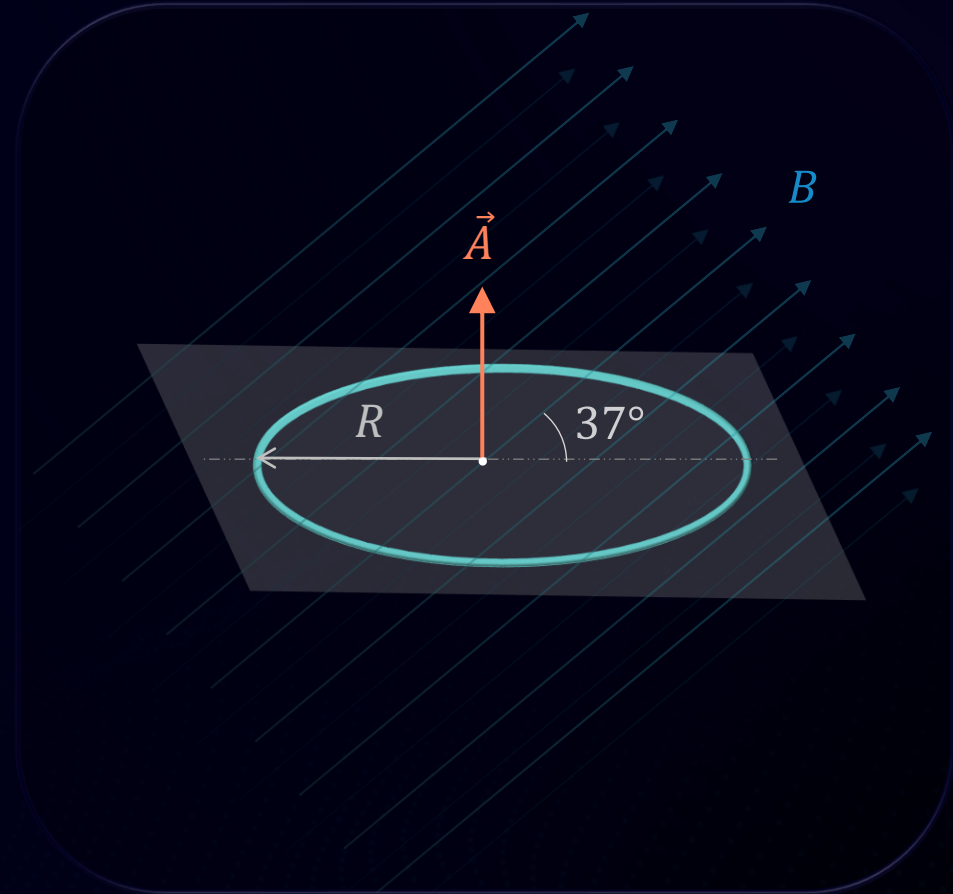
Find the **magnetic flux** passing through the closed loop as shown in the figure.

Given: $\theta = 90^\circ - 37^\circ = 53^\circ$

Formula: $\phi = \int \vec{B} \cdot d\vec{A}$

Solution: $\phi = B\pi R^2 \cos 53^\circ$

$$\phi = \frac{3}{5} B\pi R^2 \quad (\cos 53^\circ = \frac{3}{5})$$



?_T

A square conducting loop of side a is placed near a long straight wire carrying a current i as shown. Find the magnetic flux passing through the square loop.

Formula: $\phi = \int \vec{B} \cdot d\vec{A}$

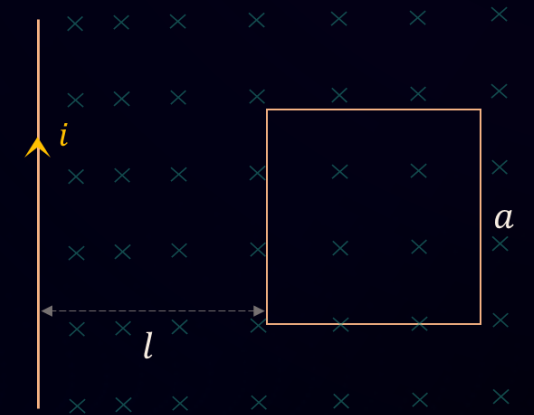
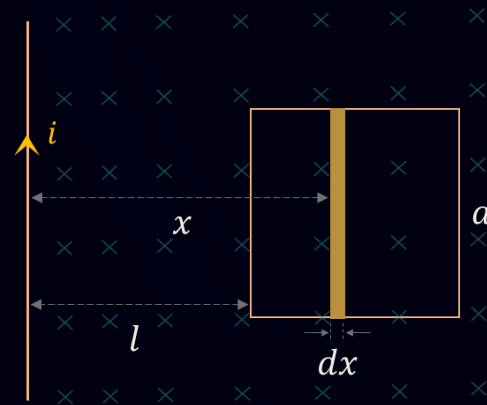
Solution: $B = \frac{\mu_0 i}{2\pi x}$

$$dA = a \, dx$$

$$d\phi = \frac{\mu_0 i}{2\pi x} (a \, dx)$$

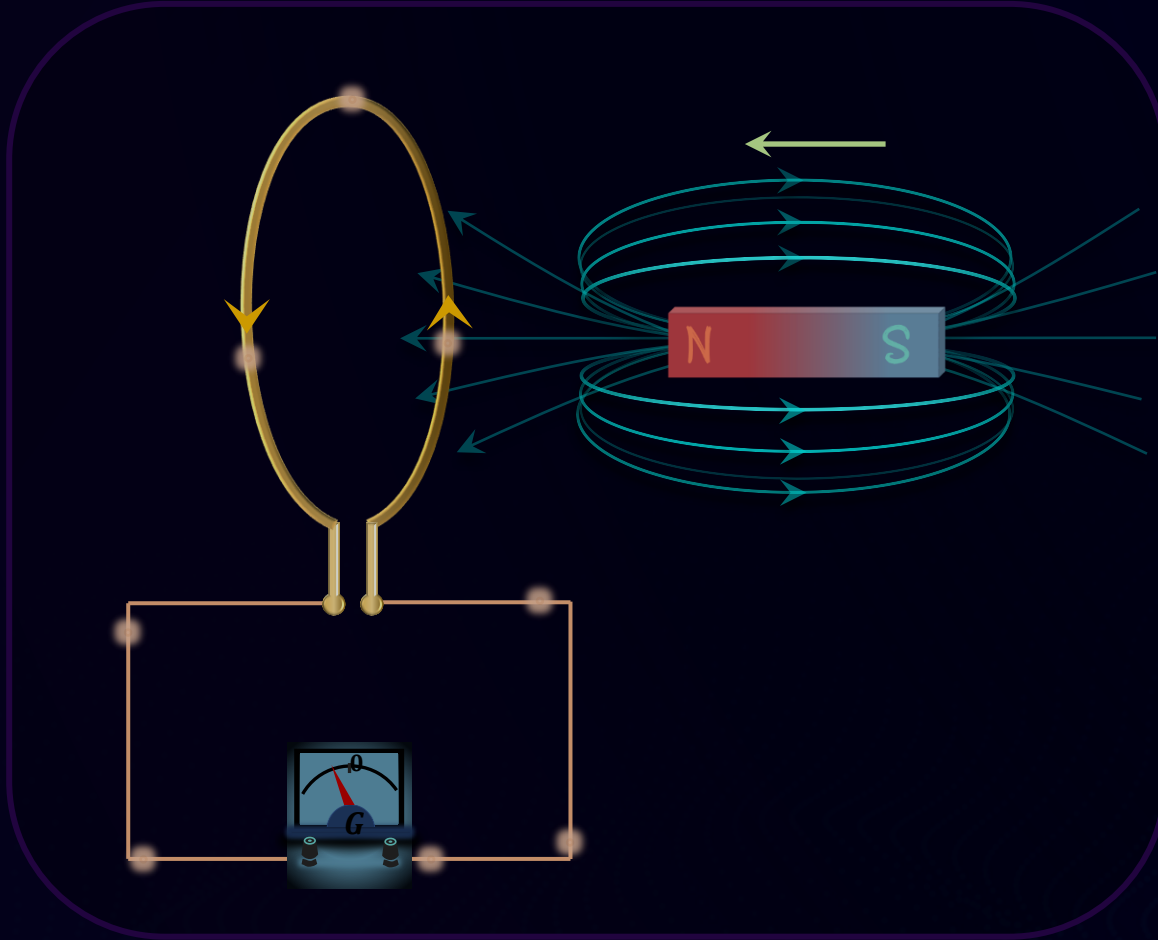
$$\phi = \int_l^{l+a} \frac{\mu_0 i}{2\pi x} a \, dx$$

$$\phi = \frac{\mu_0 i a}{2\pi} \ln \left(\frac{l+a}{l} \right)$$





Faraday's Law of Electromagnetic Induction



- The magnitude of the **emf induced** in a conducting loop is equal to the **rate** with which the **magnetic flux** through that loop changes with time.

$$\varepsilon = - \frac{d\phi}{dt}$$

?

A conducting circular loop is placed in a uniform magnetic field $B = 0.020 \text{ T}$ with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of 1.0 mm s^{-1} . Find the induced emf in the loop at an instant when the radius is 2 cm .

Solution: Flux through the loop

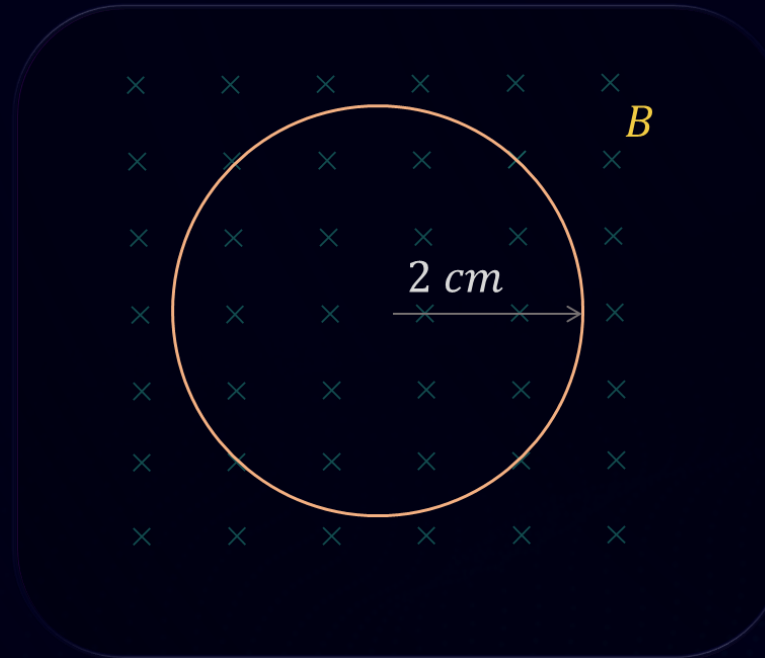
$$\phi = BA \cos \theta$$

$$\phi = B(\pi r^2) \cos 180^\circ = -\pi B r^2$$

Induced emf-

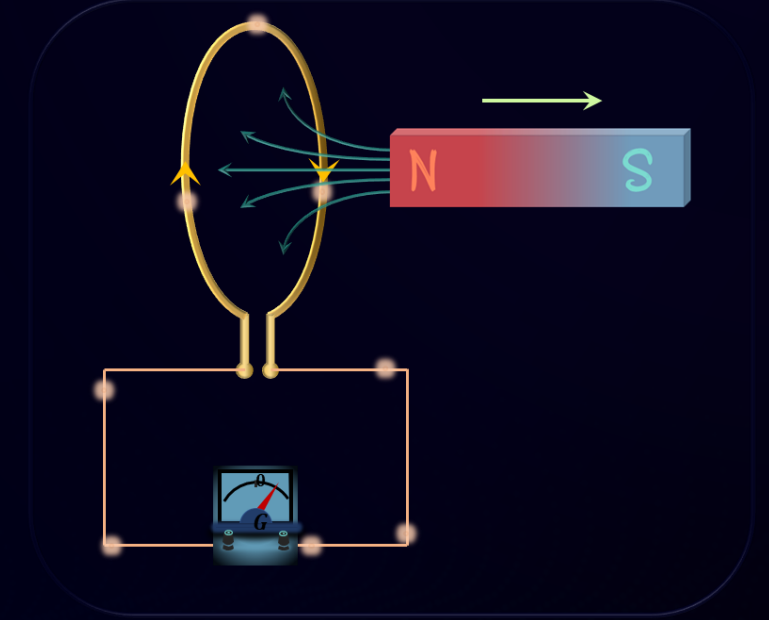
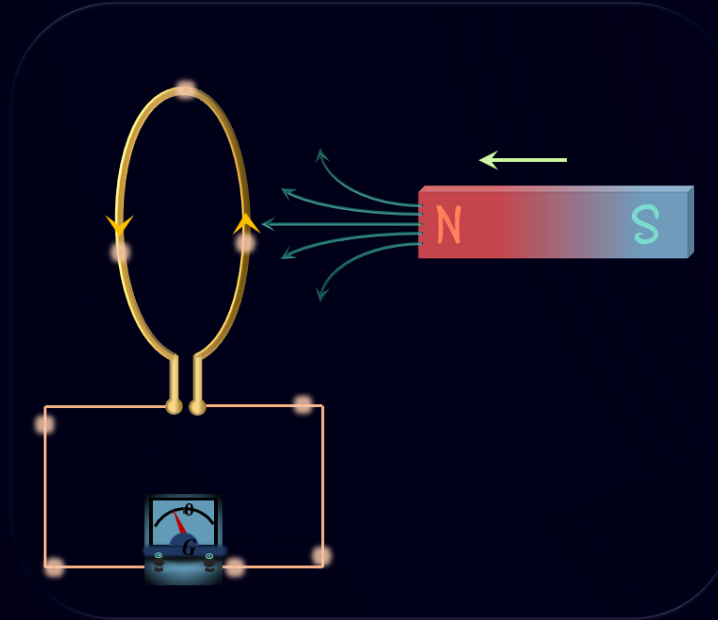
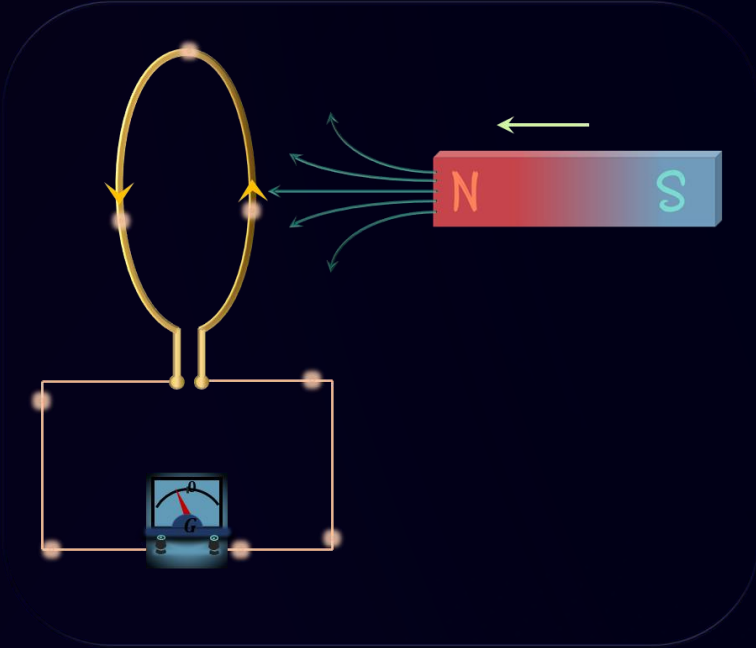
$$\varepsilon = -\frac{d\phi}{dt} = 2\pi r B \frac{dr}{dt}$$

$$\varepsilon = 2.5 \mu\text{V}$$





Lenz's Law

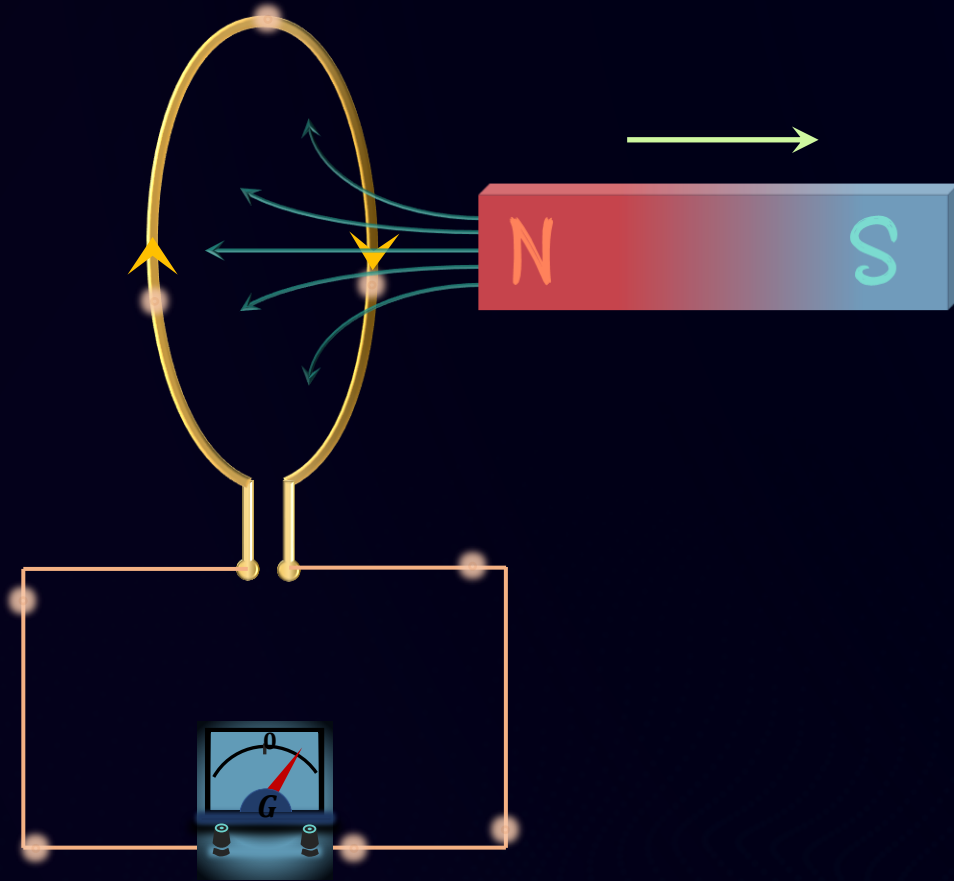


- The direction of induced current in a conducting loop is such that it opposes the change that has induced it.

The **direction of induced current** in a conducting loop is such that it **opposes the change that has induced it**.



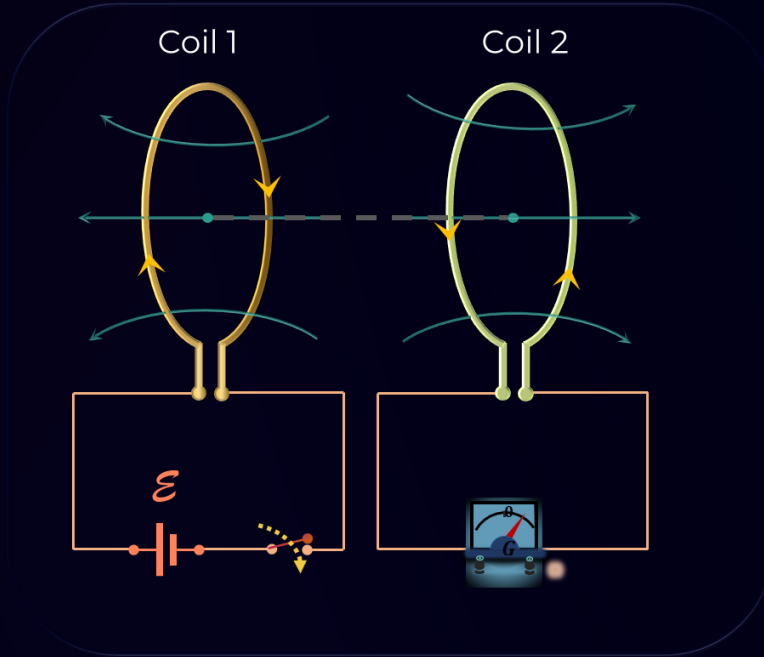
Lenz's Law and energy conservation



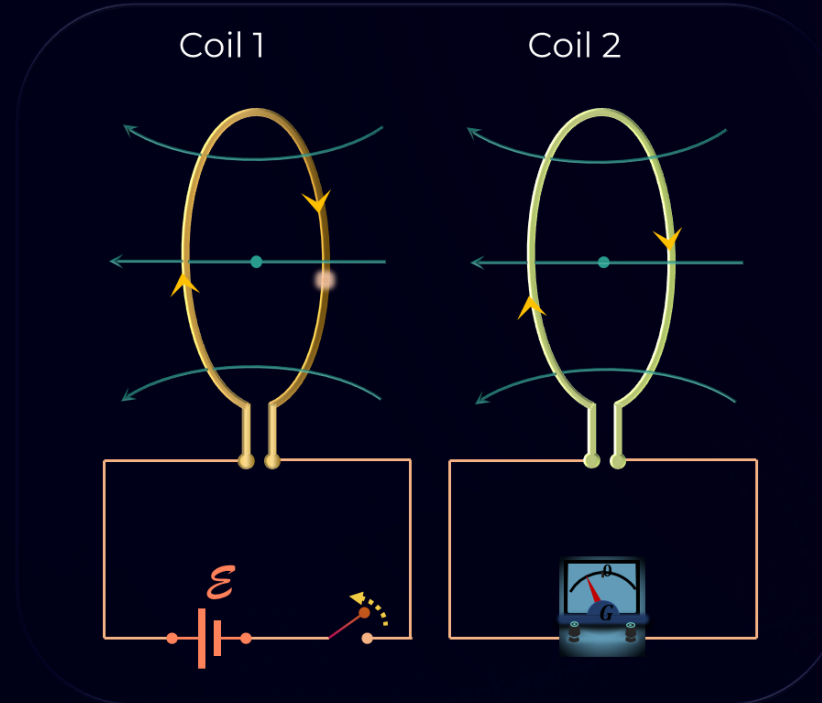
- The **current** which is induced in the coil produces **heat**.
- From the principle of conservation of energy, since heat is produced, there must be an equal **decrease of energy** in some other form.
- The production of heat is compensated by the **decrease in kinetic energy** of the magnet. Thus, Lenz's law justifies the conservation of energy principle.



Lenz's Law with coils



- As the switch is **closed** current **increases** in coil 1 so **flux** through coil 2 also **increases** and induced current flows in coil 2.
- After some time current in coil 1 becomes **constant** so flux becomes constant and induced current becomes **zero** in coil 2



- When switch is **opened** from closed state then current through the first coil **decreases** hence flux is also decreased in coil 2, it opposes the change of flux and **current is induced**.

?_T

A square loop ACDE of area 20 cm^2 and resistance 5Ω is rotated in a magnetic field $B = 2 \text{ T}$ through 180° , (a) in 0.01 s and (b) in 0.02 s . Find the magnitudes of average values of ε and i in both the cases.

Solution: (a) in 0.01 s

$$\varepsilon_{avg} = \frac{\Delta\phi}{\Delta t}$$

$$\phi_1 = BA \cos 180^\circ = 2 \times 20 \times 10^{-4}(-1)$$

$$\phi_1 = -4 \times 10^{-3} \text{ Wb}$$

$$\phi_2 = 4 \times 10^{-3} \text{ Wb}$$

$$\Delta\phi = 8 \times 10^{-3} \text{ Wb}$$

$$\Rightarrow |\varepsilon| = 8 \times \frac{10^{-3}}{0.01} = 0.8 \text{ V}$$

$$\Rightarrow i = \frac{\varepsilon}{R} = \frac{0.8}{5} = 0.16 \text{ A}$$

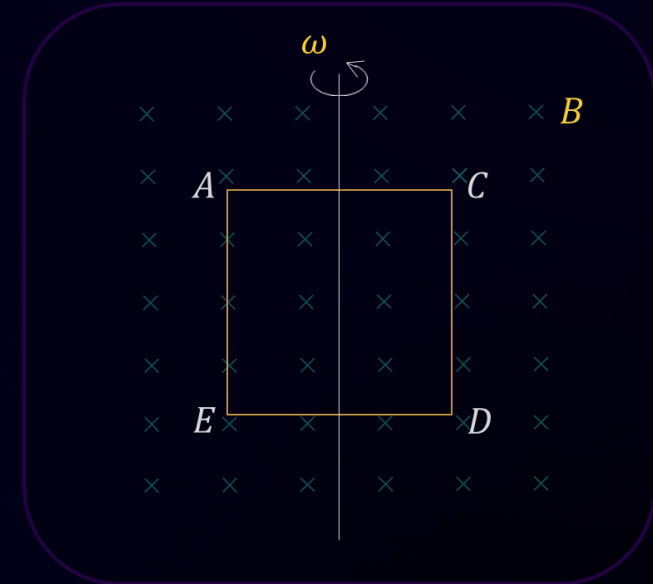
(b) in 0.02 s

$$|\varepsilon_{avg}| = \frac{\Delta\phi}{\Delta t}$$

$$\Delta\phi = 8 \times 10^{-3} \text{ Wb}$$

$$\Rightarrow |\varepsilon| = 8 \times \frac{10^{-3}}{2 \times 0.01} = 0.4 \text{ V}$$

$$\Rightarrow i = \frac{\varepsilon}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$



?

A conducting loop of radius R having resistance $r \Omega$ is placed in the varying magnetic field $B = B_0 t^3$. Find the charge flown in the loop in time $t = t_0$.

Solution: Given: $B = B_0 t^3$

To find : Charge flown in the loop

Solution : EMF generated

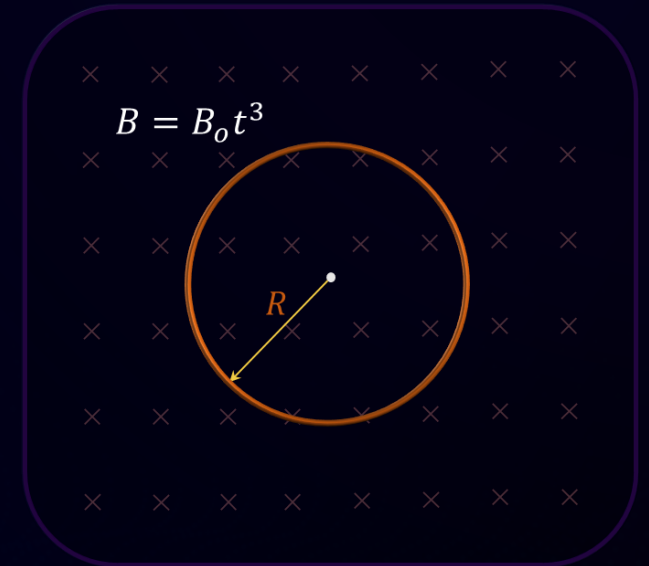
$$\varepsilon = \left| \frac{d\phi}{dt} \right|$$

$$= 3B_0 \pi R^2 t^2 \quad (\phi = B \cdot A)$$

$$i(t) = \frac{\varepsilon}{r} = \frac{3B_0 \pi R^2 t^2}{r}$$

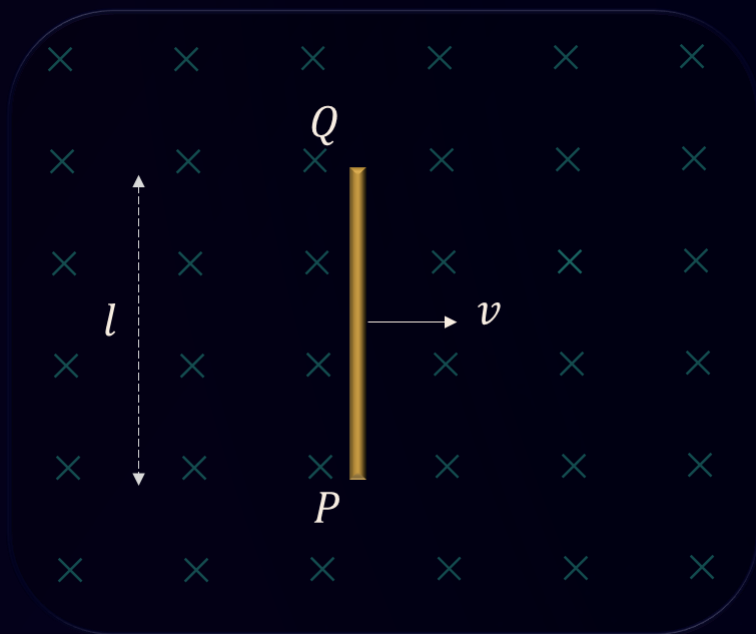
$$q = \int i(t) dt$$

$$q = \frac{B_0 \pi R^2 t_0^3}{r}$$





Origin of Induced EMF



Motional EMF

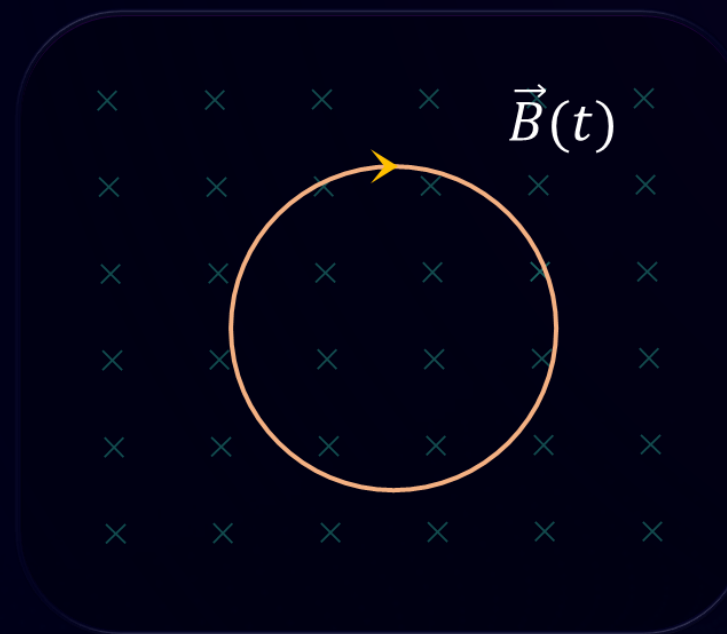
- Moving whole or a part of conducting loop or change in area of the loop keeping \vec{B} constant

$$\phi = BA \cos\theta$$

B : constant

A : variable

$$\mathcal{E} = -\frac{d\phi}{dt}$$



Induced Electric field

- When area of the loop is constant and time varying magnetic field is present.

$\vec{B}(t)$: time varying magnetic field

$$\phi = BA \cos\theta$$

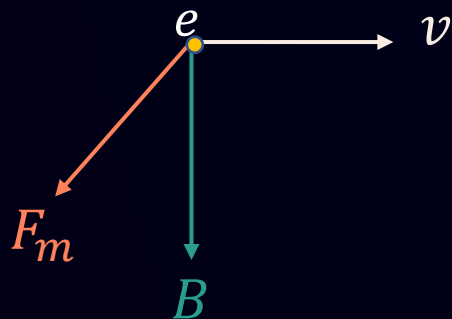
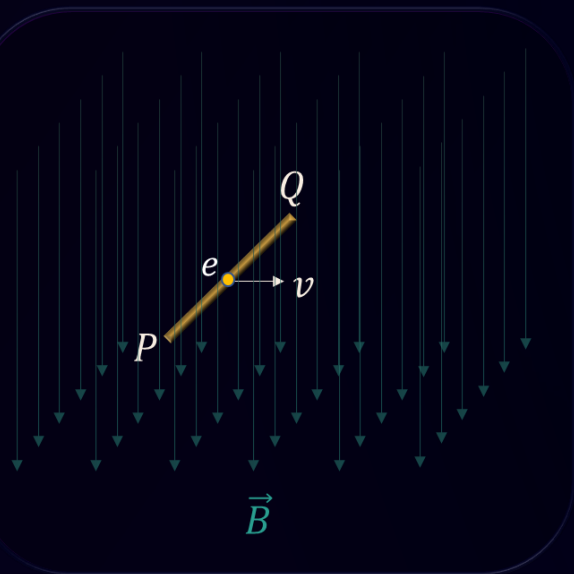
A : constant

B : variable

$$\mathcal{E} = -\frac{d\phi}{dt}$$



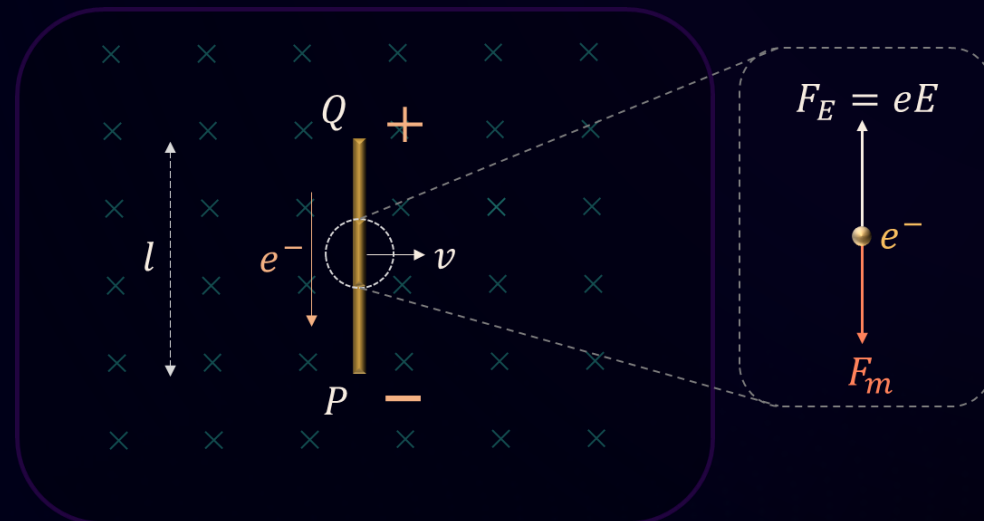
Motional EMF



- As free electron moving in magnetic field, magnetic field apply force (\vec{F}_m) on electron (e^-).

$$\vec{F}_m = -e(\vec{v} \times \vec{B})$$

- Negative charge accumulating at P and simultaneously positive charge accumulating at point Q.
- Electric field produced from Q to P.



F_E : Electric force on charge

F_m : Magnetic force on charge

For equilibrium

$$F_E = F_m$$

$$\Rightarrow eE = evB$$

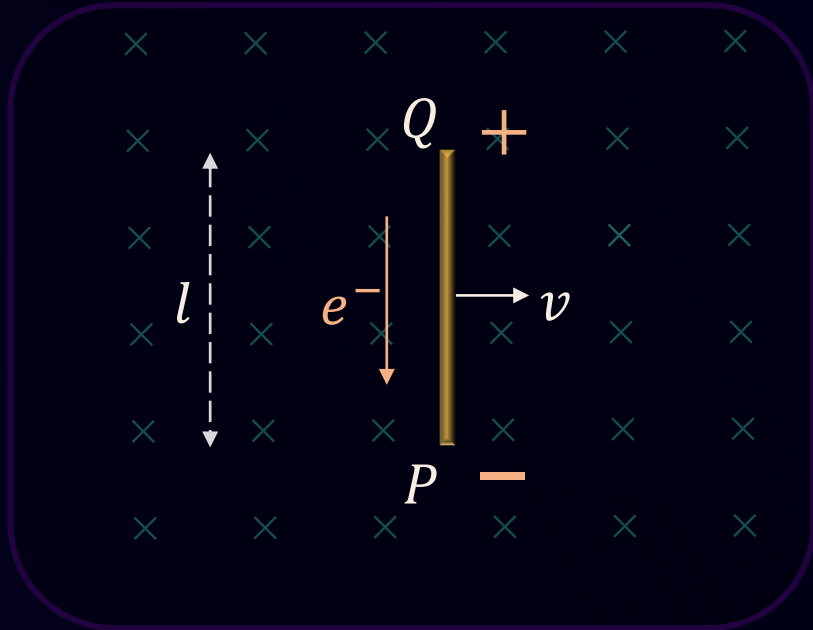
$$\Rightarrow E = vB$$

$$\Rightarrow \frac{\varepsilon}{l} = vB$$

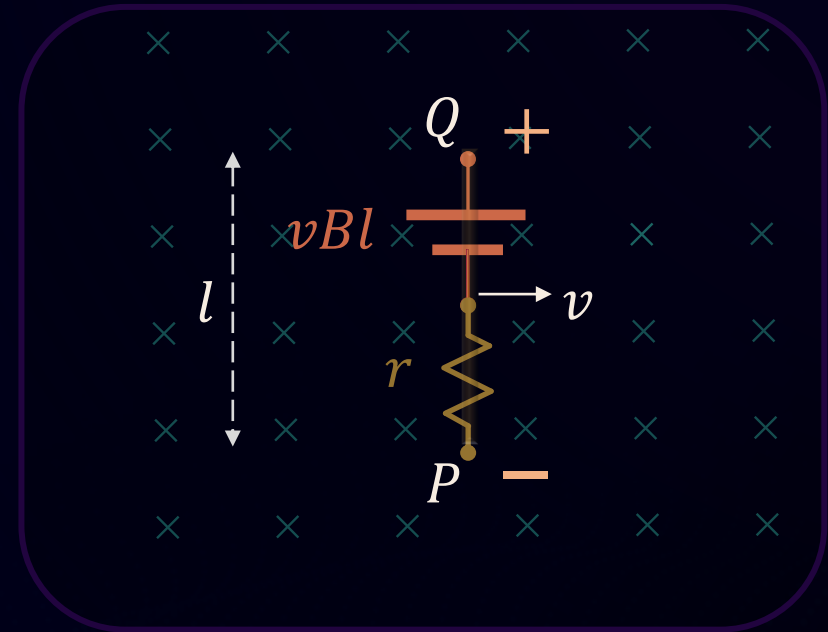
$$\varepsilon = Bvl$$



Motional EMF Equivalent to Battery



\equiv



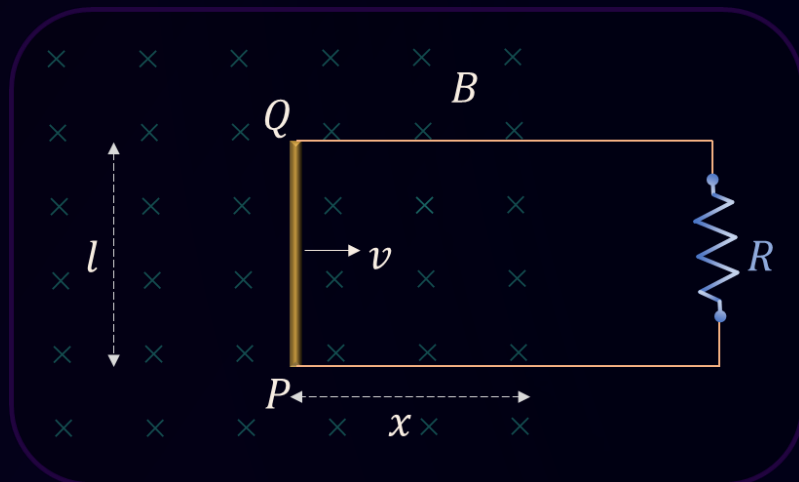
EMF of battery(ε) = Bvl

Resistance of rod = (r)

- Consider a moving rod in magnetic field as a battery with emf = Bvl .



Motional EMF by Faraday's law



At time t ,

x : length of the loop inside the magnetic field region

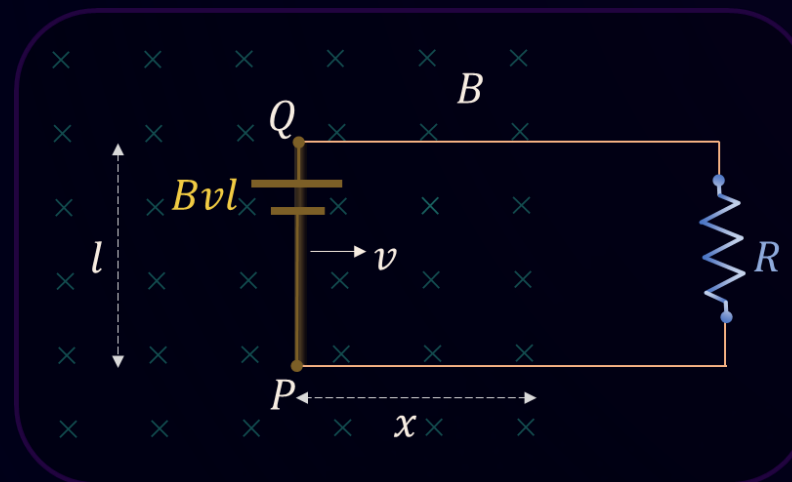
$$\phi = BA \cos\theta$$

$$\Rightarrow \phi = B(lx) \quad (1)$$

$$\mathcal{E} = \frac{d\phi}{dt}$$

$$\mathcal{E} = Bl \frac{dx}{dt} = Blv$$

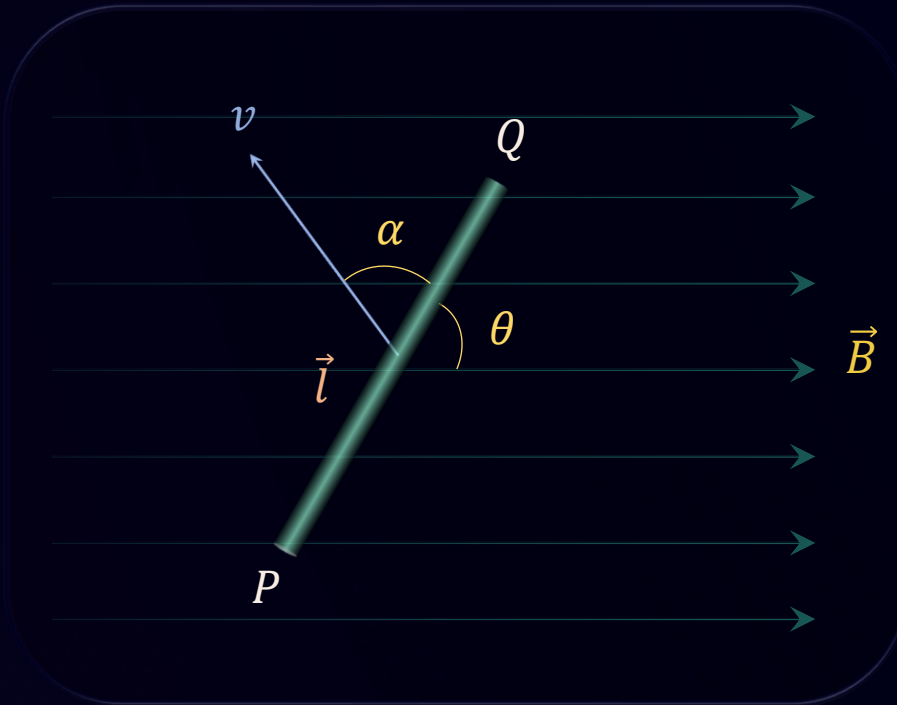
$$\mathcal{E} = Blv$$



- For direction: Stretch your right hand fingers in the direction of velocity then curl your fingers along the magnetic field, your thumb will point to positive polarity of battery.



Motional EMF (Vector Form)



At steady state,

$$\vec{F}_E = -\vec{F}_m$$

$$P.D. = - \int \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

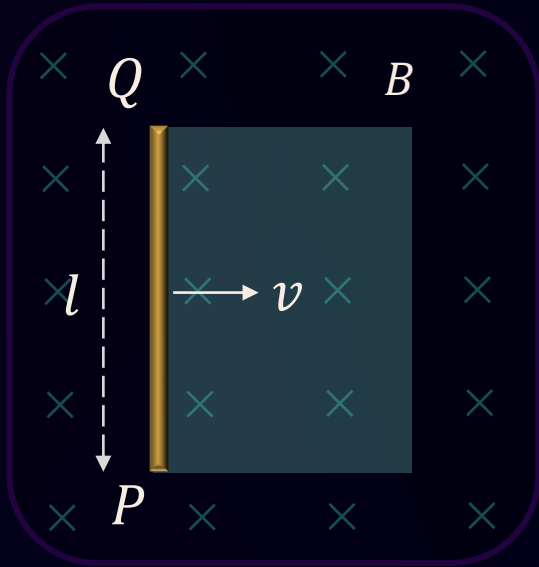
$$\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

If magnetic field is uniform , then,

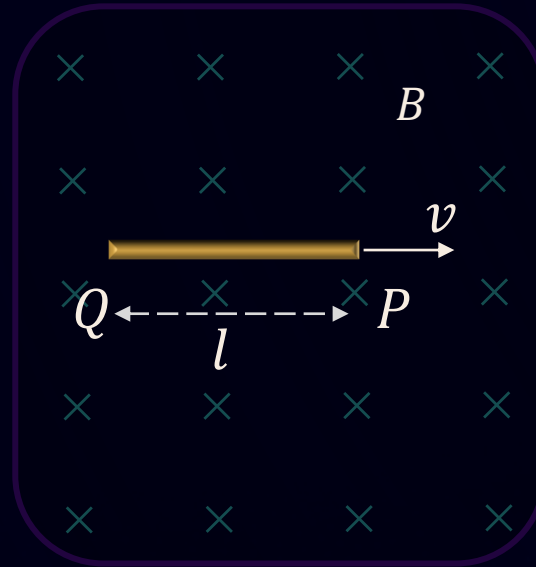
$$\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$



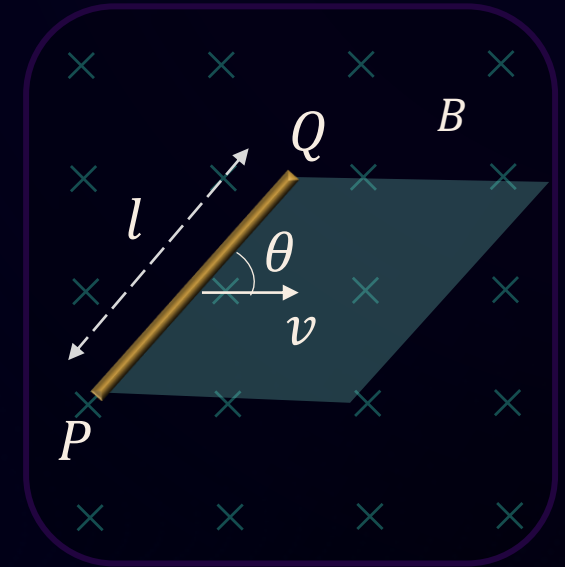
Motional EMF Different Cases



$$\mathcal{E} = Bvl$$



$$\mathcal{E} = 0$$

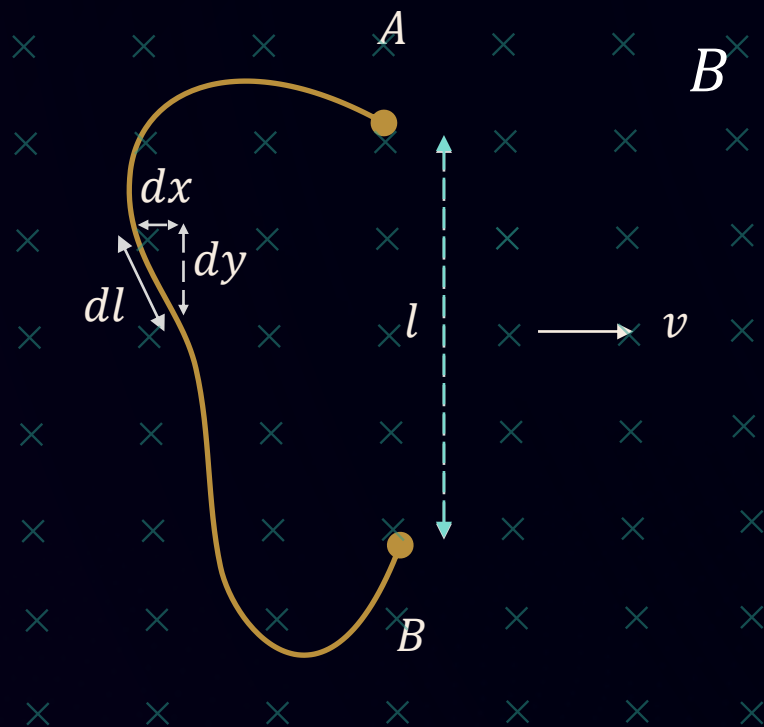


$$\mathcal{E} = Bvl \sin\theta$$

- length(l), velocity (v) and magnetic field (B) are **perpendicular to each other**.
- length(l), velocity (v) are parallel to each other.
- If any two quantities are parallel to each other then induced emf will be zero.
- length(l), velocity (v) are at θ angle.



Motional EMF for Random Shape Wire



Induced emf for small elementary

$$d\mathcal{E} = Bv \sin\theta \, dl$$

$$\Rightarrow d\mathcal{E} = Bv (dl \sin\theta)$$

$$\Rightarrow d\mathcal{E} = Bv (dy)$$

$$\Rightarrow \int d\mathcal{E} = Bv \int dy$$

$$\Rightarrow \mathcal{E} = Bvl$$

l is effective length between ends of wire

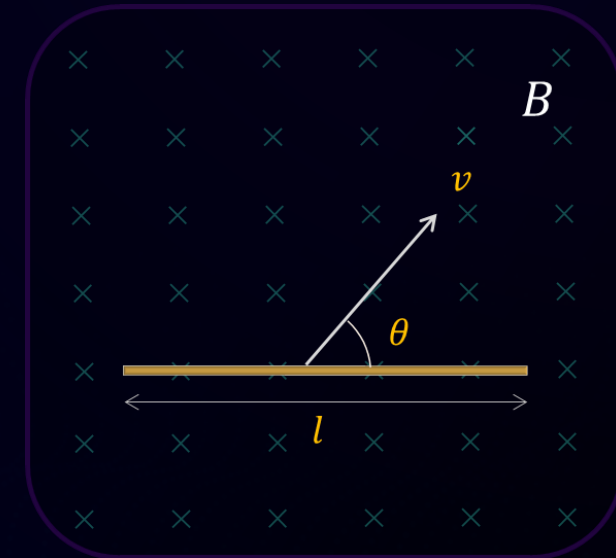
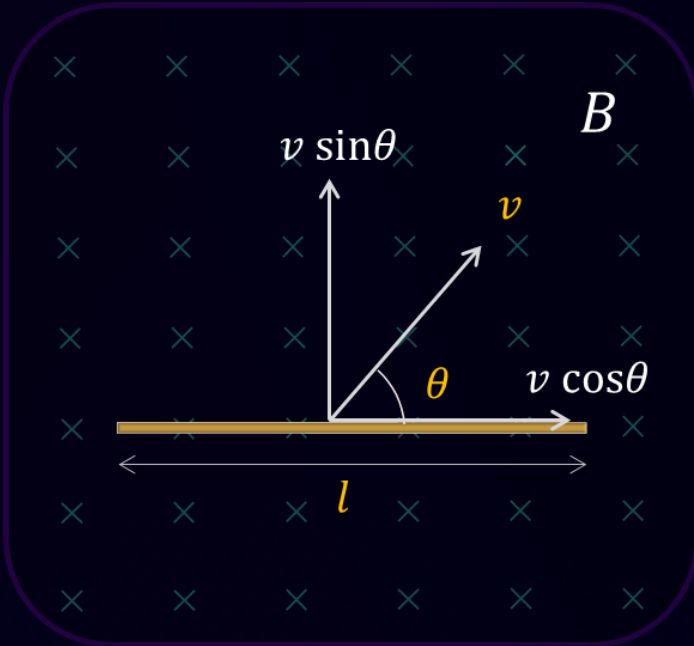
? T

A rod of length l is translating at a velocity v making an angle θ with its length. A uniform magnetic field B exists in a direction perpendicular to the plane of motion. Calculate the **emf induced** in the rod. Draw a figure representing the induced emf by an equivalent battery.

Solution:

- Only $v \sin \theta$ is considered because this component is perpendicular to length and magnetic field.

$$\varepsilon = Bl v \sin \theta$$



?

An angle $\angle aob$ made of a conducting wire moves along its bisector through a magnetic field B . Find the **emf induced** between the two free ends if the magnetic field is perpendicular to the plane of the angle.

Solution: Induced emf-

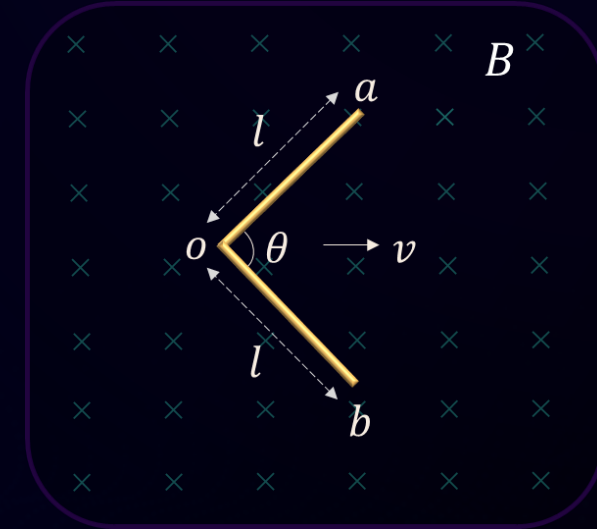
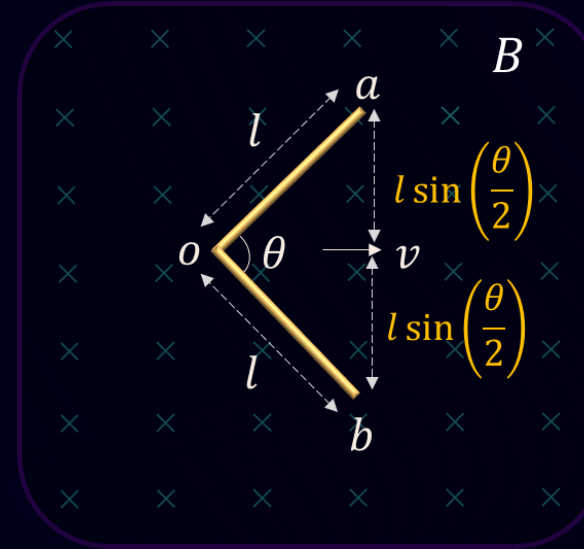
$$\varepsilon = V_a - V_b$$

$$\Rightarrow \varepsilon = (V_a - V_o) + (V_o - V_b)$$

$$\Rightarrow \varepsilon = \left(Bvl \sin\left(\frac{\theta}{2}\right) \right) + \left(Bvl \sin\left(\frac{\theta}{2}\right) \right)$$

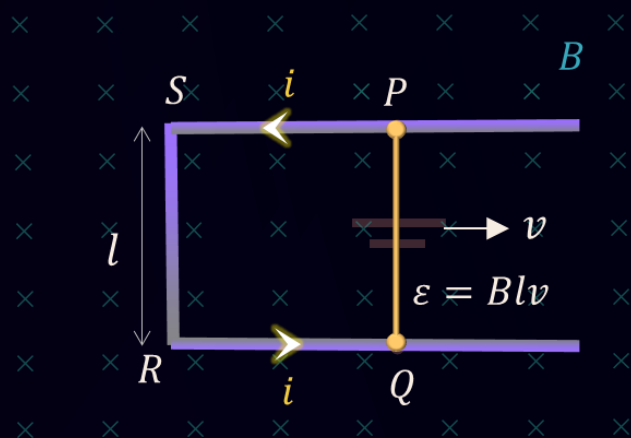
$$\Rightarrow \varepsilon = 2Bvl \sin\left(\frac{\theta}{2}\right)$$

For direct emf, effective length could be calculated by joining its ends.





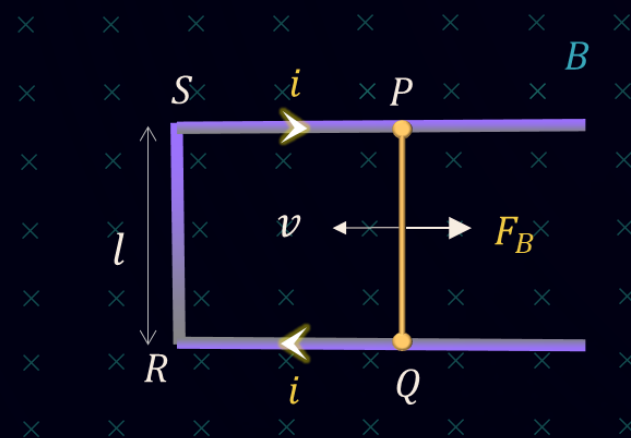
Concept of Energy Consideration



- The rod moving on two rails in magnetic field acts like a battery and the potential difference across the rod is,

$$\mathcal{E} = Blv$$

- Motional EMF can be converted into mechanical/electrical energy.



- In a constant magnetic field, if we change the flux, an emf is induced in the $PQRS$. PQ is the movable arm.

$$\mathcal{E} = \left| -\frac{d\phi}{dt} \right|$$

$$\text{Current} = i = \frac{Blv}{r}$$

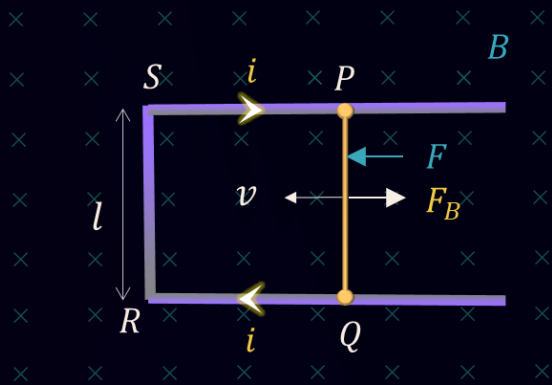
r is the resistance of movable arm PQ

- As the magnetic field is present, there will also be a force F_B acting as,

$$F_B = ilB$$



Concept of Energy Consideration



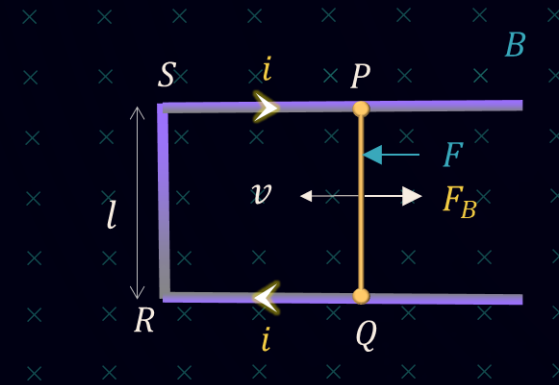
- This force is directed towards right in the direction opposite to the velocity of the rod.

$$F_B = \frac{B^2 l^2 v}{r}$$

- The arm PQ is pushed towards left with force F to make it move with constant velocity. So, the mechanical power required is:

Power = Force x Velocity

$$P = F \cdot v = \frac{B^2 l^2 v^2}{r}$$



- The Power dissipated in form of Electrical Power is:

$$P = i^2 r$$

$$\text{Current}(i) = \frac{Blv}{r}$$

$$P = \frac{B^2 l^2 v^2}{r^2} \times r$$

$$P = \frac{B^2 l^2 v^2}{r}$$

Electrical Power = Mechanical Power

- The concept of Energy consideration here shows that the Lenz's Law is consistent with the conservation of Energy.



A horizontal magnetic field which is uniform above the dotted line and is zero below it is shown. A long, rectangular, conducting loop of width L , mass m and resistance R is placed partly above and partly below the dotted line with the lower edge parallel to it. With what velocity should it be pushed downwards so that it may continue to fall **without any acceleration**?

Solution:

Induced EMF-

$$\varepsilon = BvL$$

$$i = \frac{BvL}{R} \quad (\text{induced current})$$

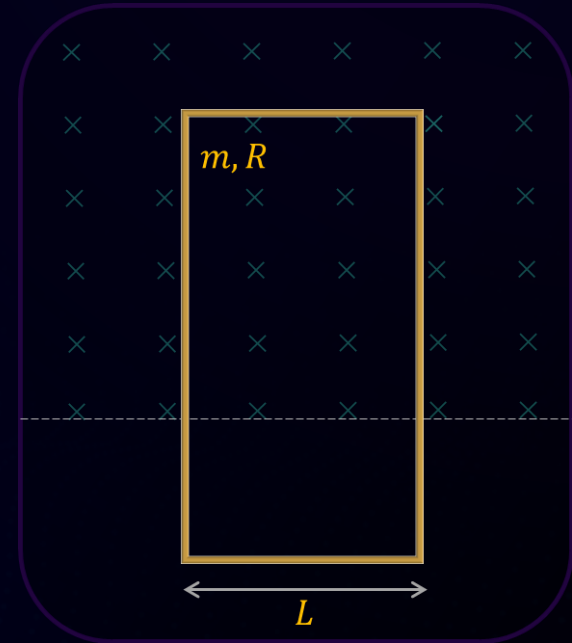
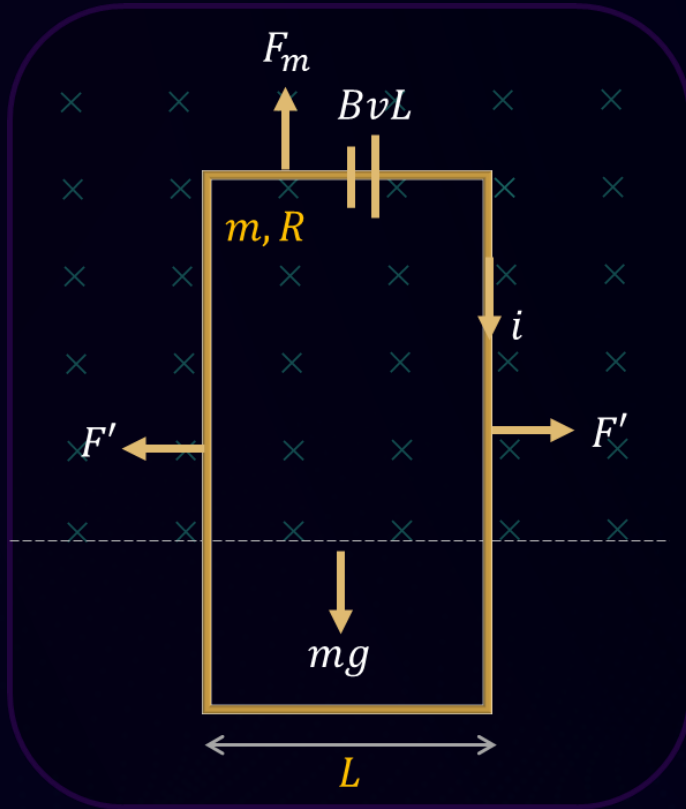
For zero acceleration, net force should be zero-

$$F_m = mg \quad (F_m: \text{Magnetic Force on loop})$$

$$\Rightarrow BiL = mg$$

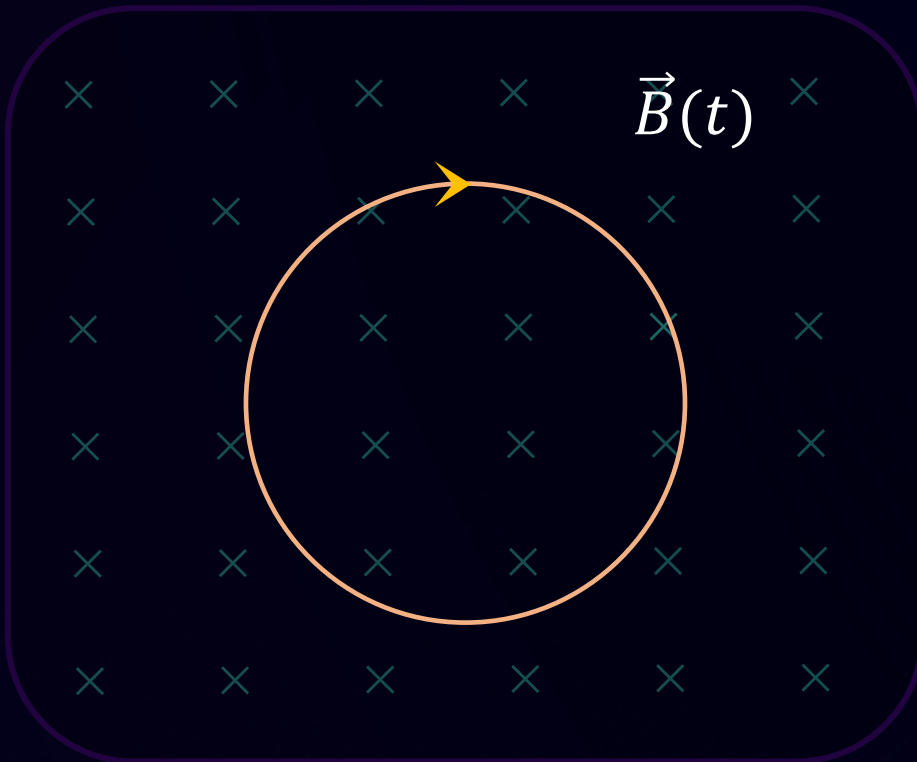
$$\Rightarrow B \left(\frac{BvL}{R} \right) L = mg$$

$$\Rightarrow v = \frac{mgR}{L^2 B^2}$$





Induced Electric field



Induced electric field:

- When a time varying magnetic field is present in space then simultaneously a field is produced known as induced electric field.
- $\vec{B}(t)$: time varying magnetic field

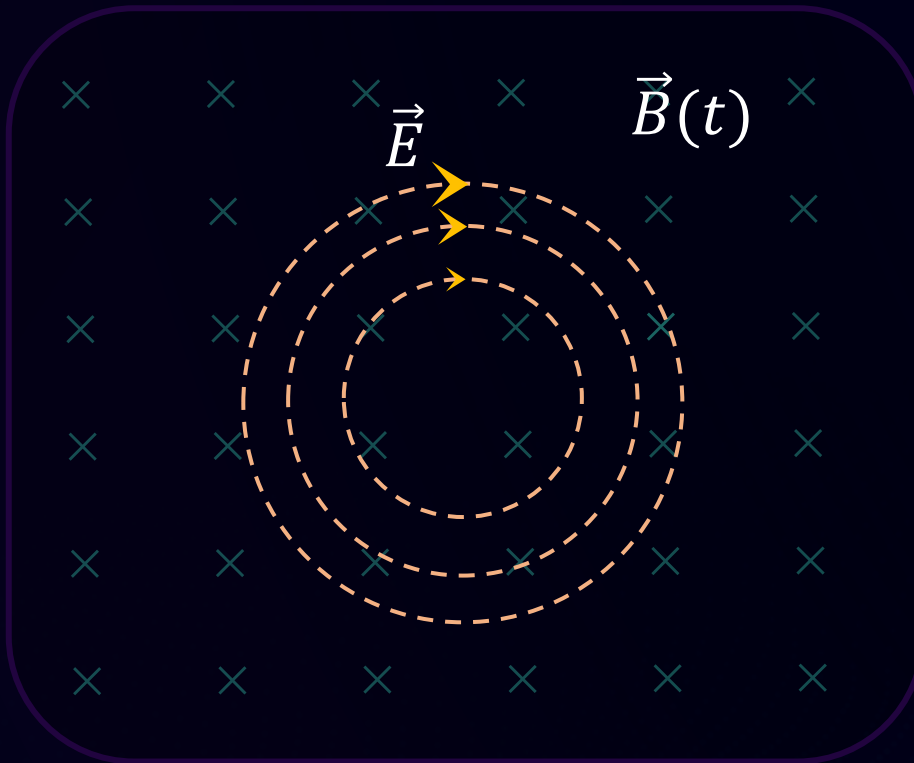


Properties of Induced Electric field



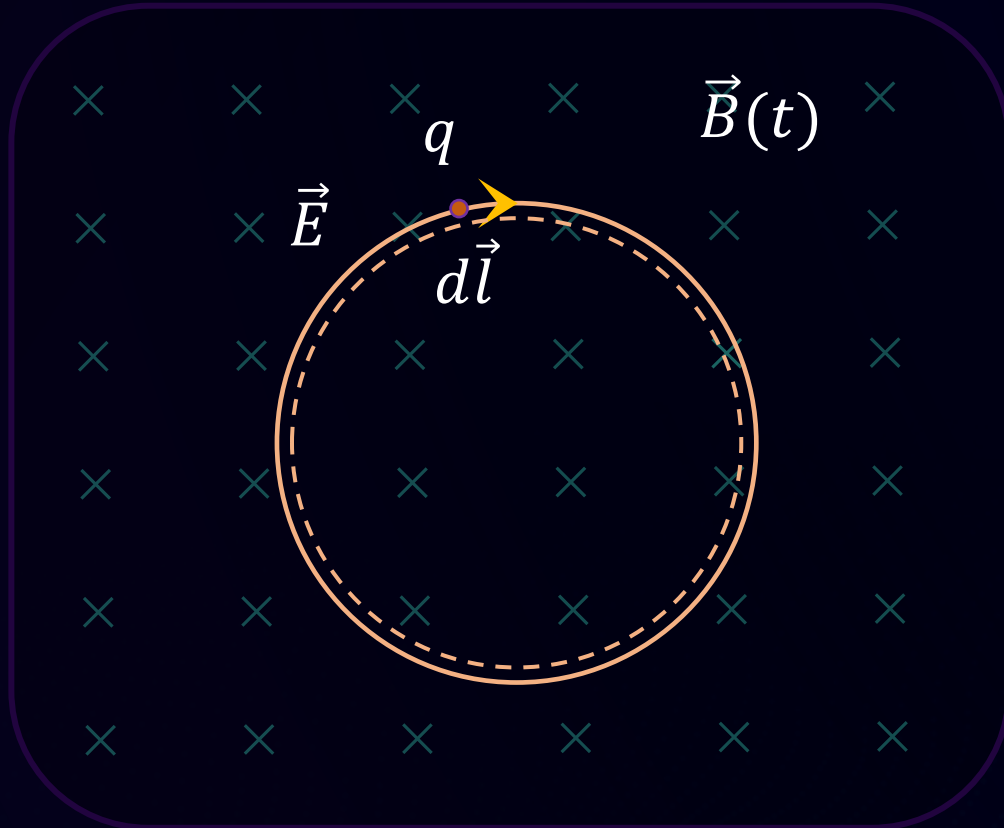
Induced electric field is –

- **Non – electrostatic**
This electric field is produced by time varying magnetic field not by static charge.
- **Non – conservative**
Work done on the charge in a close loop is nonzero.
- **Electric field lines can be closed loop**
Electric field due to Static charges never form closed loops but this electric field forms closed loops.





Relation between Magnetic Flux and Induced Electric Field



Work done on charge by induced electric field

$$dW = \vec{F} \cdot d\vec{l} = q\vec{E} \cdot d\vec{l}$$

Net work done in moving it along the closed loop

$$\Rightarrow W = \oint q\vec{E} \cdot d\vec{l}$$

$$\Rightarrow q\mathcal{E} = \oint q\vec{E} \cdot d\vec{l}$$

$$\Rightarrow \mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

The work done on charge is non zero in a close loop so this proving that field is non conservative.

?

A metal rod of length l rotates about an end with a uniform angular velocity ω . A uniform magnetic field B exists in the direction of the axis of rotation. Calculate the emf induced between the ends of the rod. Neglect the centripetal force acting on the free electrons as they move in circular paths.

Solution:

Each point of rod has different velocity

Velocity of elementary length

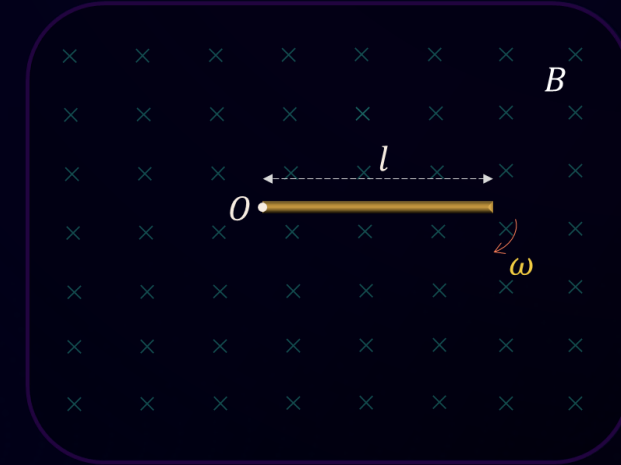
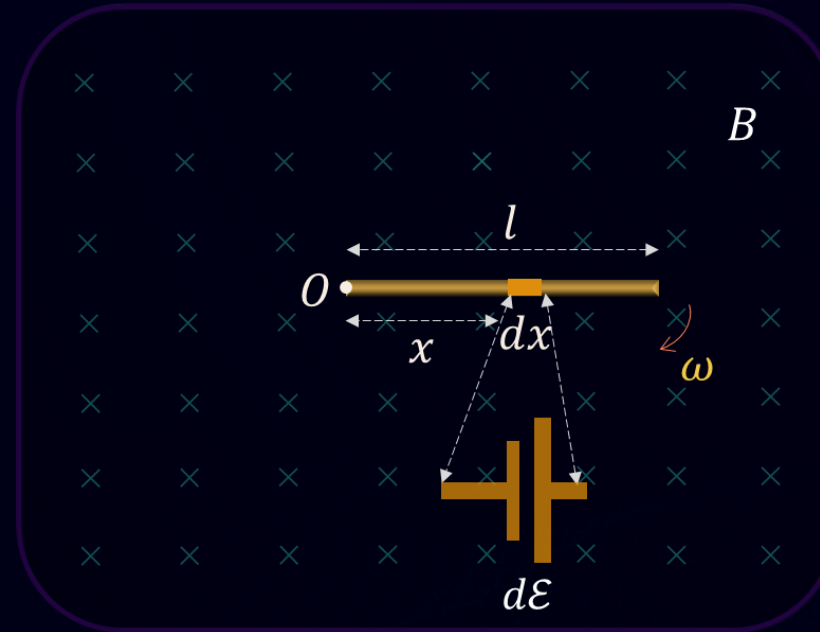
$$v = \omega x$$

Induced emf across elementary length ' dx '

$$d\mathcal{E} = B(dx)\omega x$$

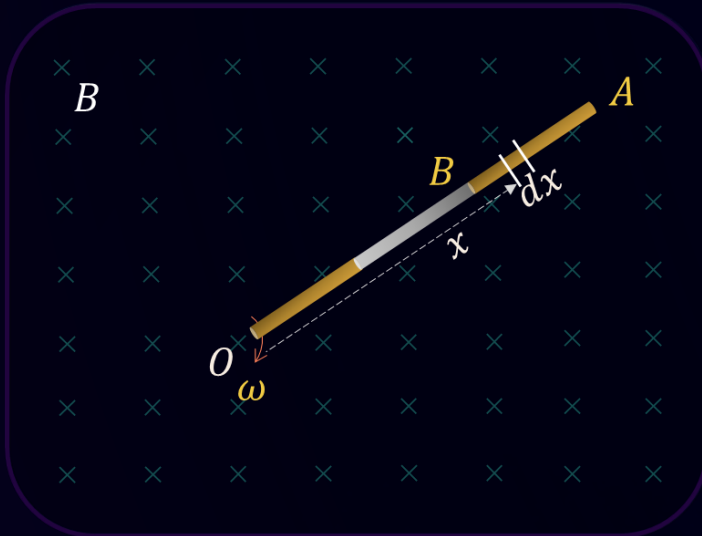
$$\int d\mathcal{E} = \int_0^l B\omega x \, dx$$

$$\mathcal{E} = \frac{1}{2} B\omega l^2$$



? A rod of length **10 cm** is made up of conducting and non-conducting materials (middle part is non-conducting). The rod is rotated with constant angular velocity **10 rad/s** about point **O** in uniform magnetic field of magnitude **2 T**. Find the induced emf between point **A** and point **B**.

Solution:



Induced emf across elementary dx

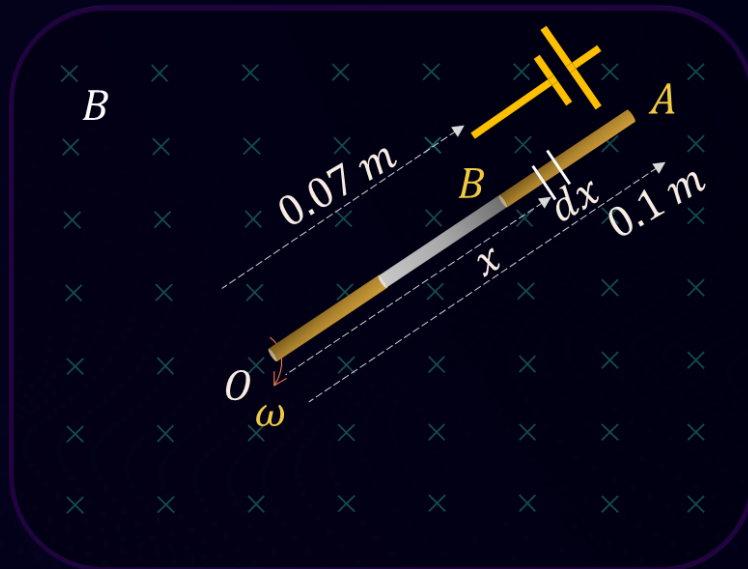
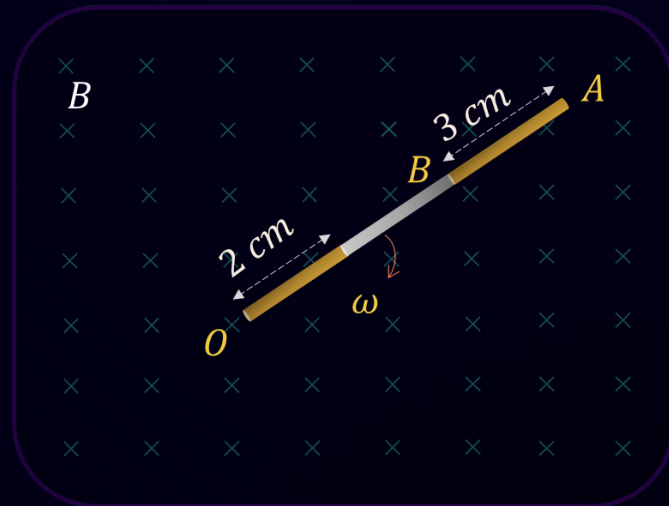
$$d\mathcal{E} = B(\omega x)dx$$

$$\mathcal{E} = \int_{0.07}^{0.1} B\omega x dx$$

$$\mathcal{E} = \frac{1}{2} B\omega [x^2]_{0.07}^{0.1}$$

$$\mathcal{E} = \frac{1}{2} \times 2 \times 10 \times (0.1^2 - 0.07^2)$$

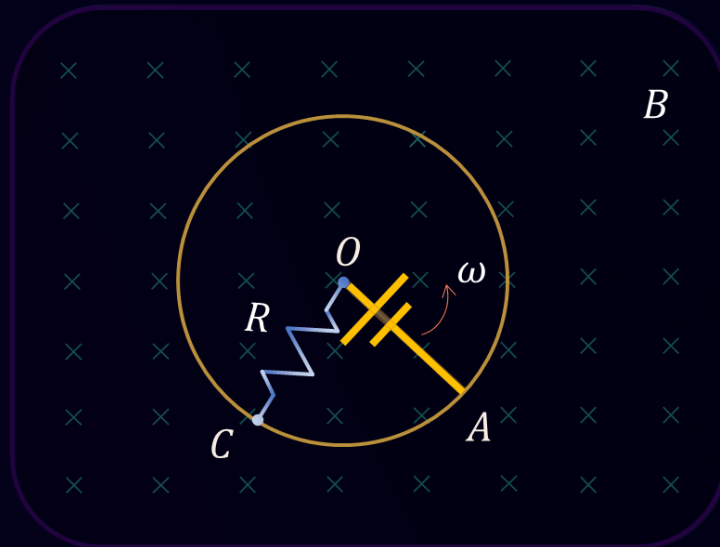
$$\mathcal{E} = 0.051 \text{ V}$$



?

A conducting circular loop of radius a is placed in a uniform, perpendicular magnetic field B . A metal rod OA is pivoted at the centre O of the loop. The other end A of the rod touches the loop. The rod OA and the loop are **resistanceless** but a resistor having a resistance R is connected between O and a fixed point C on the loop. The rod OA is made to rotate anticlockwise at a small but uniform angular speed ω by an external force. Find (a) the current in the resistance R .

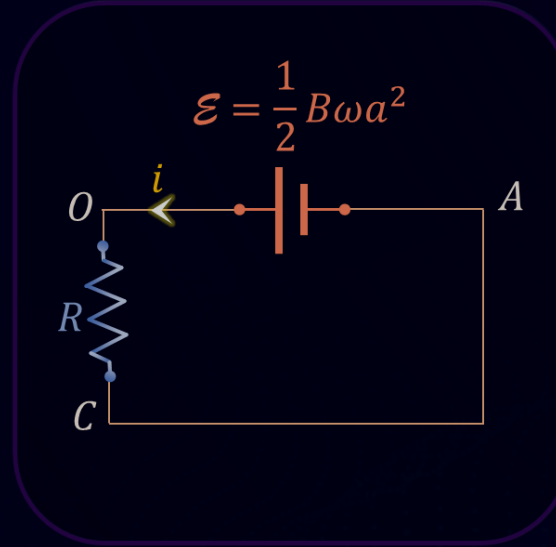
Solution:



Induced emf

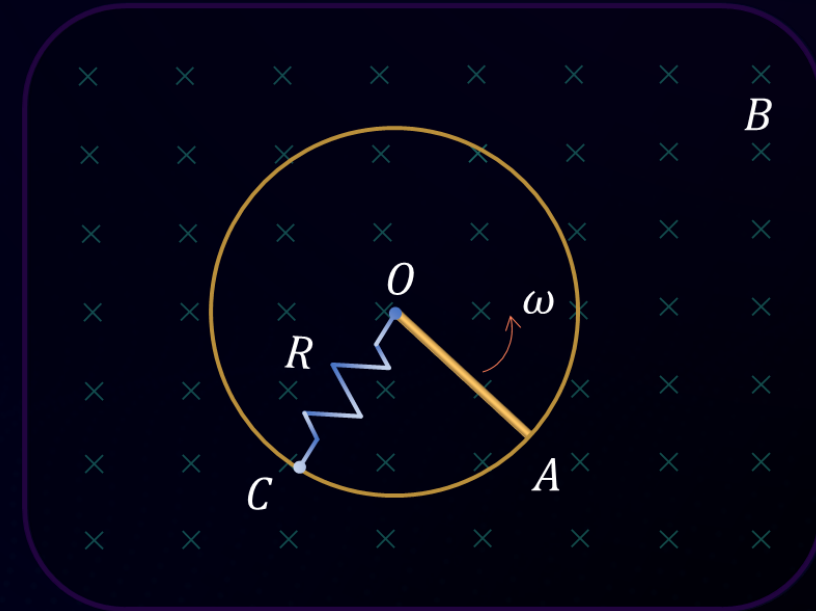
$$\mathcal{E} = \frac{1}{2} B \omega a^2$$

\equiv



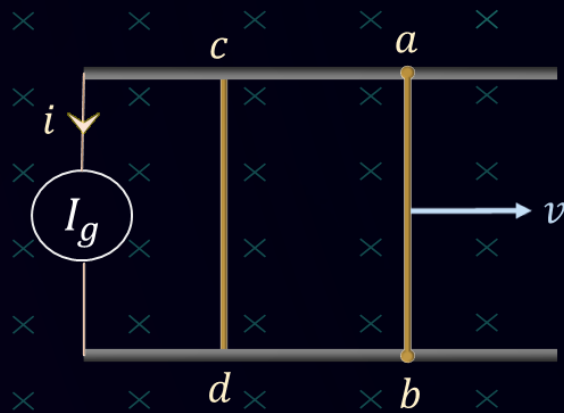
Induced current

$$i = \frac{\mathcal{E}}{R} = \frac{B \omega a^2}{2R}$$



?

The current generator I_g sends a constant current i through the circuit. The wire cd is fixed and ab is made to slide on the smooth, thick rails with a constant velocity v towards right. Each of these wires has resistance r . Find the current through the wire cd .



Given: $R_{cd} = R_{ab} = r$

To find: i_1

Solution:

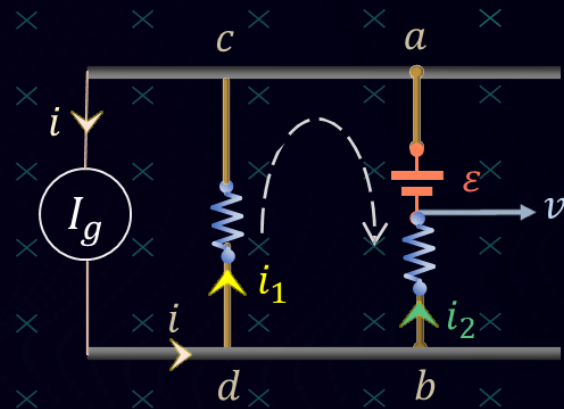
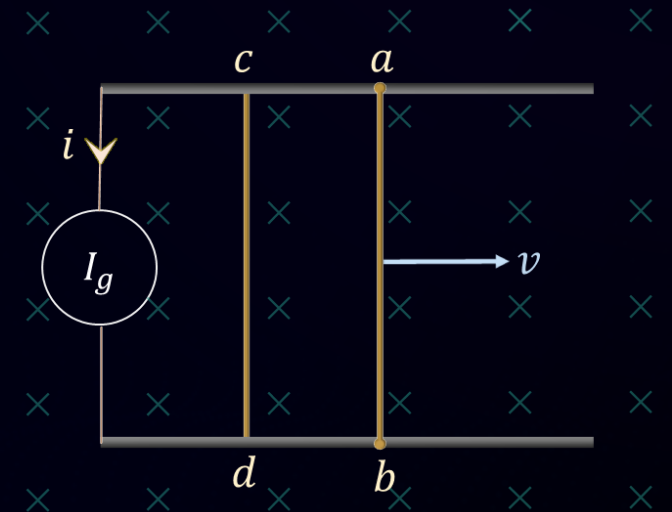
The Emf induced in wire = Blv

Apply KVL in loop $dcabd$ clockwise:

$$-i_1 r - Blv + (i - i_1)r = 0$$

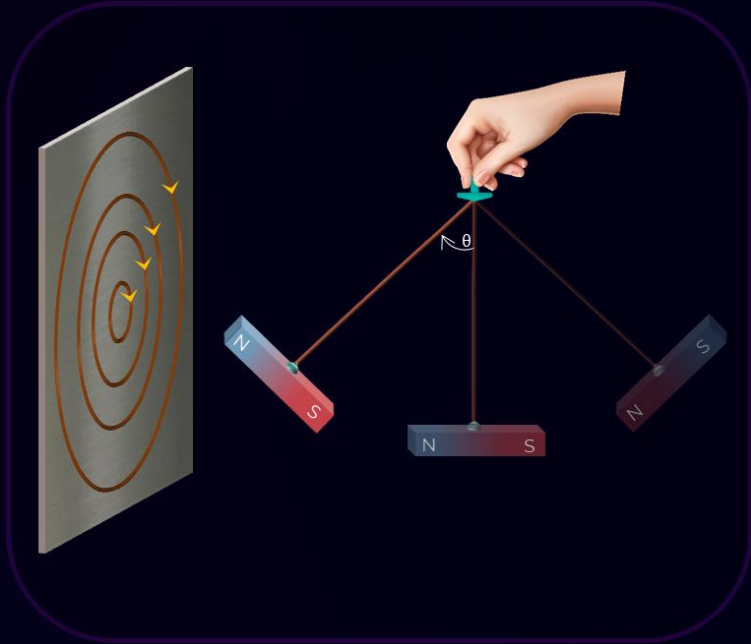
$$-2i_1 r - Blv + ir = 0$$

$$i_1 = \frac{ir - Blv}{2r}$$



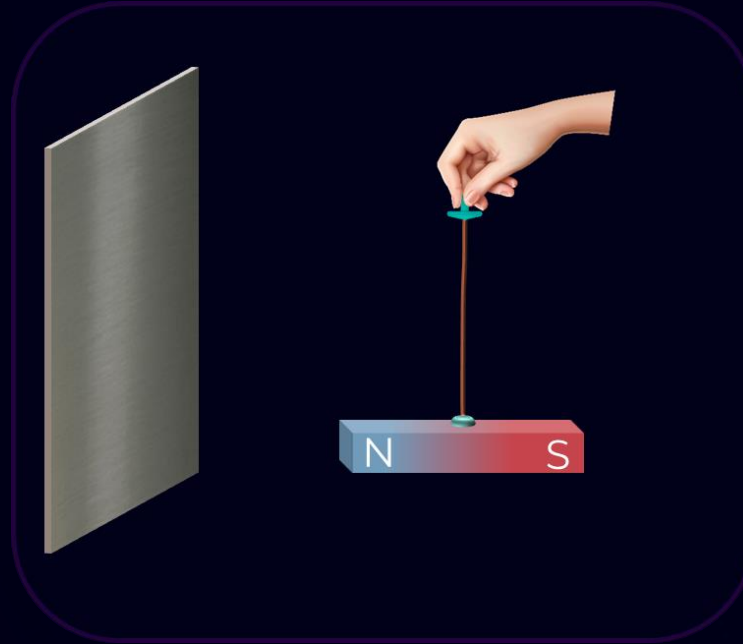


Eddy Current



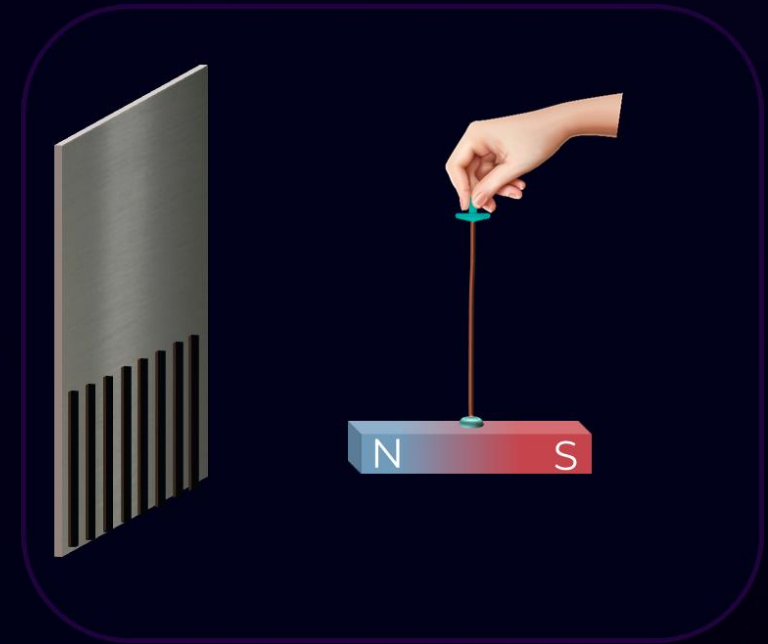
Eddy currents are the loops of electric current induced within the body of conductors.

- They are in a pattern like eddies in water.
- The direction of this induced current is given by Lenz's law.



Eddy currents produce heat in the metal which comes at the cost of kinetic energy of bar magnet.

The bar magnet slows down and eventually comes to rest. This phenomenon is called as "Electromagnetic Damping".



Metal sheet are cut into slots to reduce the possible path of eddy current.

It also reduces the surface area through which the magnetic field passes.

"Slotting of conductor reduces eddy current formation".

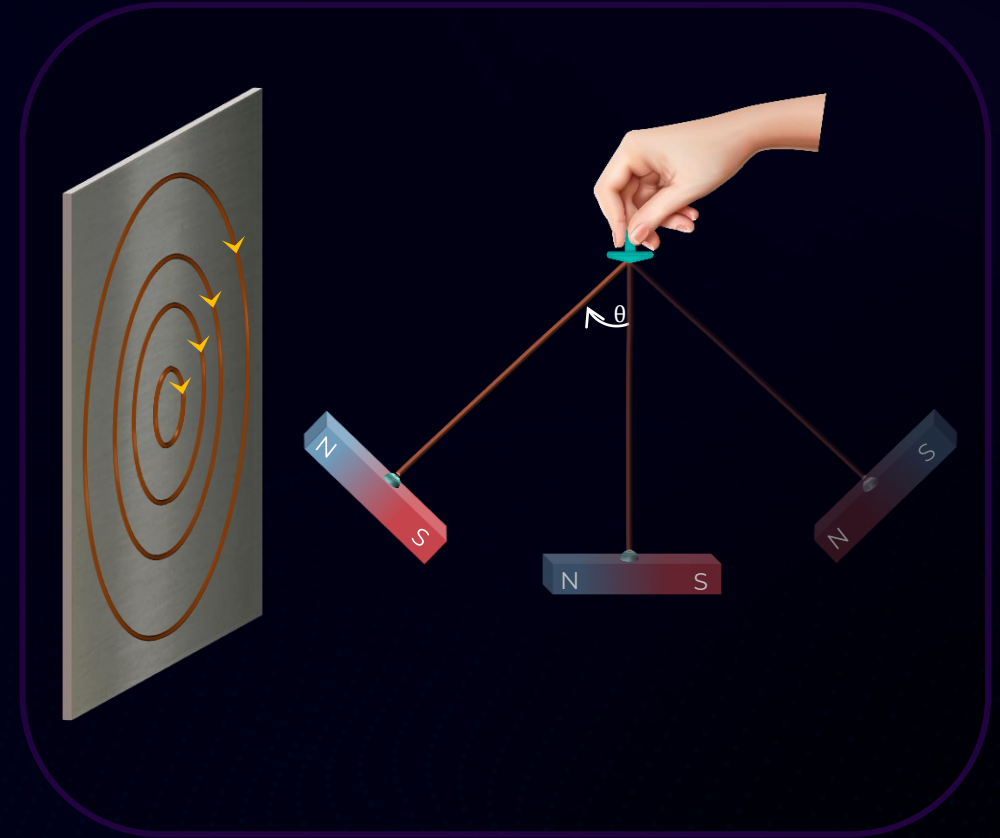


Applications of Eddy Current



Eddy currents are used to advantage in certain applications like:

- Magnetic braking in trains
- Electromagnetic damping
- Induction furnace
- Electric power meters





Classification of Electrical Components



Active Electrical Components

- Active element generates energy for any device. It is the core component to operate the device.
- Active components control the **flow of charges** in electrical or electronic circuits.

- Ex:



Cell

Passive Electrical Components

- A passive element is an electrical component that does not generate power but dissipates, stores and / or releases it.
- Ex:



Resistor



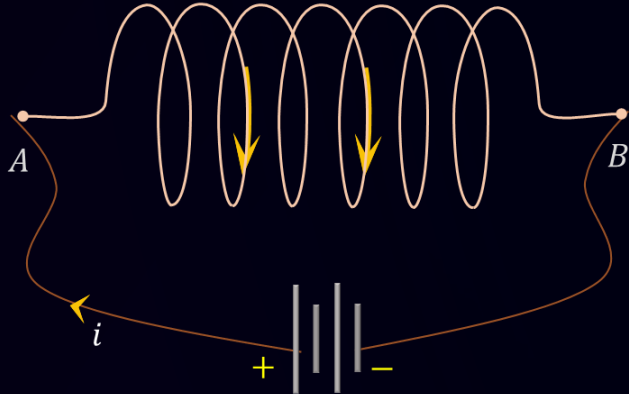
Capacitor



Inductor



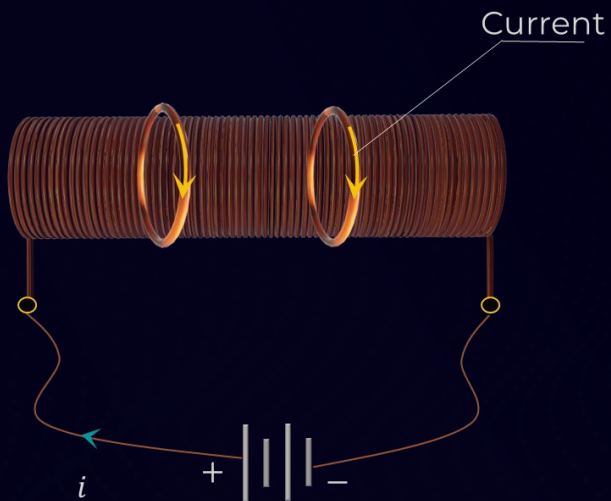
Self Inductance



The property of a coil, by which an **induced emf** is developed due to **change** in strength of **current** flowing through the coil itself.

- The **induced emf** will **resist** the rise of current when the current increases and vice-versa.
- **Applicable** for **time-varying current** and not for steady current.

Measurement of Self Inductance



$$\phi \propto i$$

$$\phi_T = \text{Total flux} = N\phi$$

$$\phi_T = Li = N\phi$$

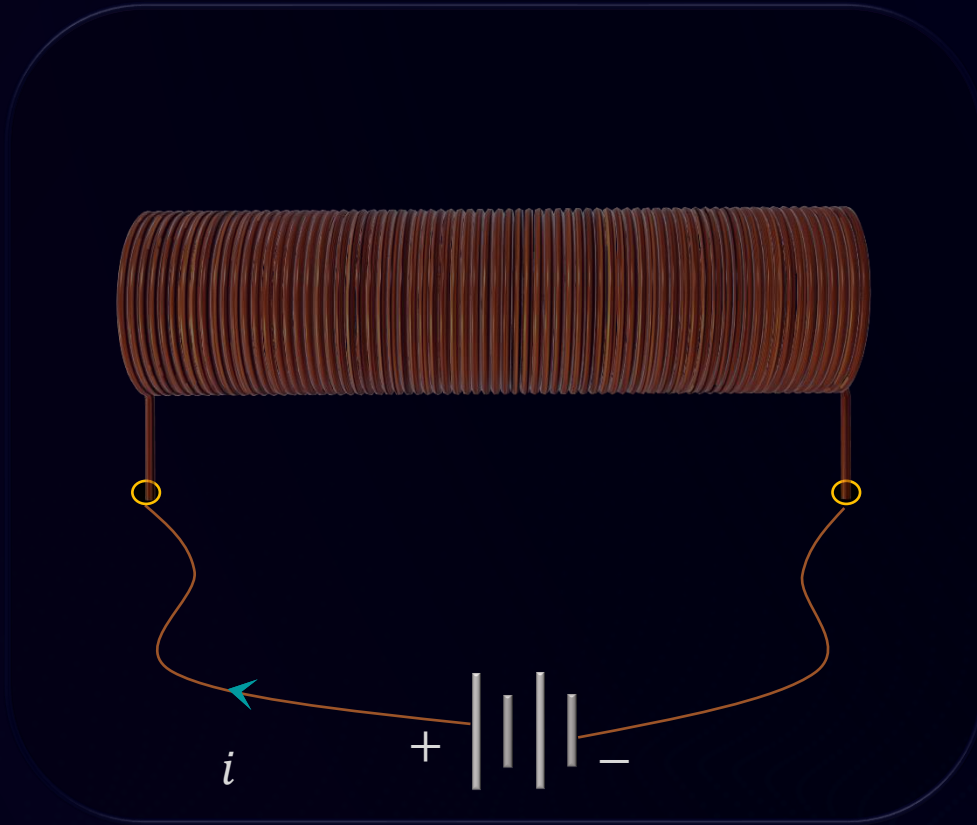
$$L = \frac{N\phi}{i}$$



Coefficient of Self Inductance



Coefficient of self inductance is the ratio of magnitudes of emf produced in a circuit by self induction to the rate of change of current producing it.



$$\varepsilon = - \frac{d\phi_T}{dt}$$

$$\phi_T = Li$$

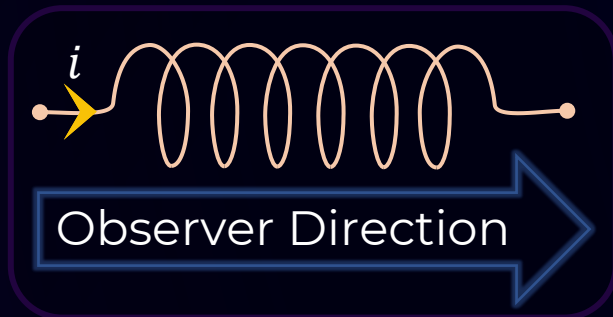
$$\varepsilon = -L \frac{di}{dt}$$

$$L = \frac{|\varepsilon|}{\left| \frac{di}{dt} \right|}$$

- **SI Unit:** $\frac{\text{weber}}{\text{ampere}} = \text{henry (H)}$

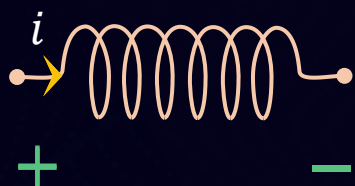


Direction of Induced Emf

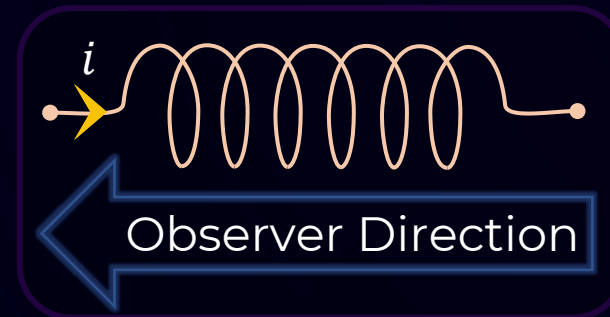
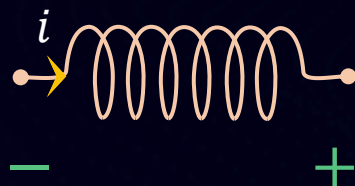


$$\varepsilon = -L \frac{di}{dt}$$

$$\frac{di}{dt} > 0$$

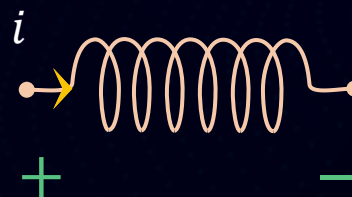


$$\frac{di}{dt} < 0$$

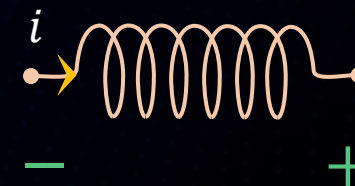


$$\varepsilon = L \frac{di}{dt}$$

$$\frac{di}{dt} > 0$$

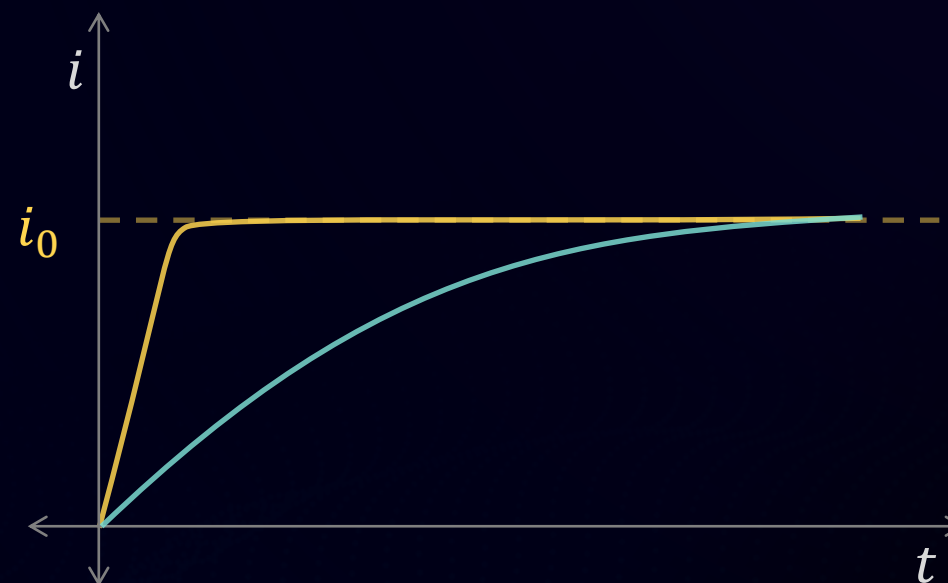


$$\frac{di}{dt} < 0$$



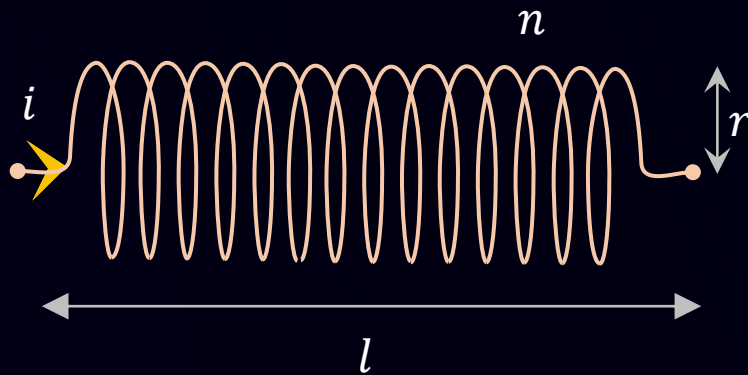


Understanding Self Induction





Self Inductance of Long Solenoid



l = Length of the solenoid

n = Number of turns per unit length

i = Current through the solenoid

$N = nl$ = Total number of turns

r = Radius of the coil

$$B = \mu_0 ni$$

$$\phi = \vec{B} \cdot \vec{A} = BA$$

$$\phi = \mu_0 ni \pi r^2$$
 (Flux through one coil)

$$\phi_T = N\phi$$
 (Total Flux)

$$Li = (nl)(\mu_0 ni \pi r^2)$$

$$L = \mu_0 n^2 \pi r^2 l$$

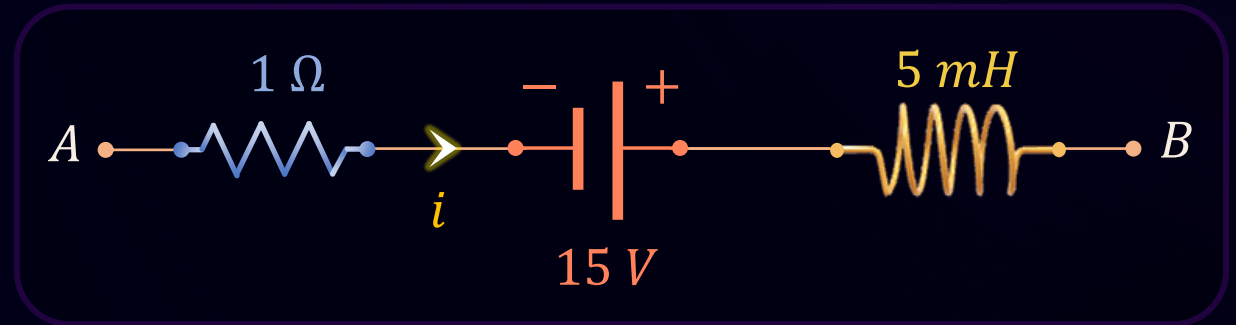
?

The network shown in the figure is part of a complete circuit. If at a certain instant, the current is $5A$, and is decreasing at a rate $10^3 A/s$, then $(V_B - V_A)$ is

Given: $i = 5A$, $\frac{di}{dt} = -10^3 A/s$

To Find: $(V_B - V_A)$

Solution:



Apply **KVL** between point A and B in the direction of current

The potential difference between point A and B is:

$$(V_B - V_A) = -ir + E + \left(-L \frac{di}{dt} \right) \longrightarrow \text{(Because we are going in direction of current)}$$

$$(V_B - V_A) = -5 + 15 - 5 \times 10^{-3}(-10^3) = 15V$$



The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate $\frac{di}{dt} = 0.03 \text{ A/s}$. Find the self-induced EMF.

Given: $L = 0.54 \text{ H}$ $\frac{di}{dt} = -0.03 \text{ A/sec}$

To Find: induced emf

Solution: Self induced EMF is given as:

$$\varepsilon = -L \frac{di}{dt}$$

$$\Rightarrow \varepsilon = (-0.54)(-0.03)$$

$$\varepsilon = 1.62 \times 10^{-2} \text{ V}$$





Growth of Current in L-R circuit



At $t = 0, i = 0$

Apply KVL clockwise in the circuit,

$$-L \frac{di}{dt} - iR + \varepsilon = 0$$

$$\varepsilon - iR = L \frac{di}{dt}$$

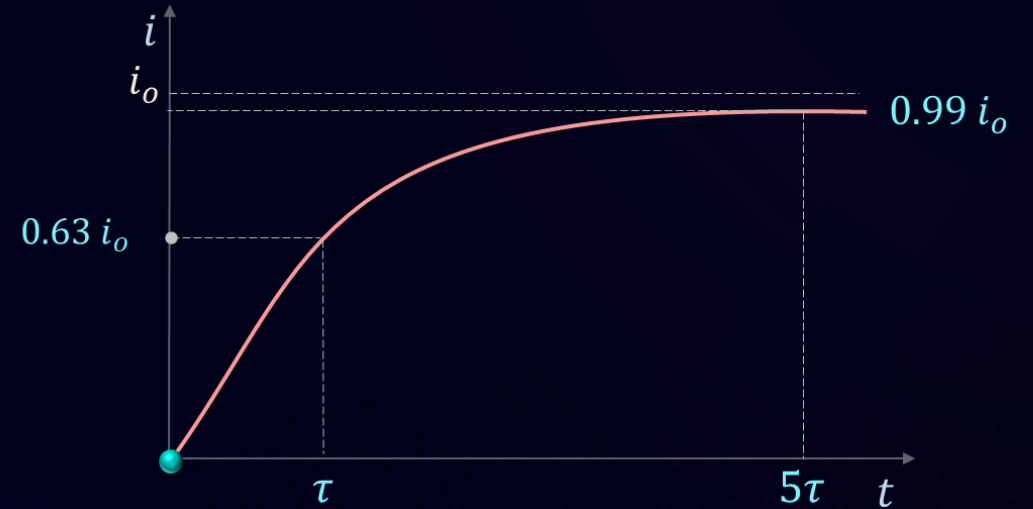
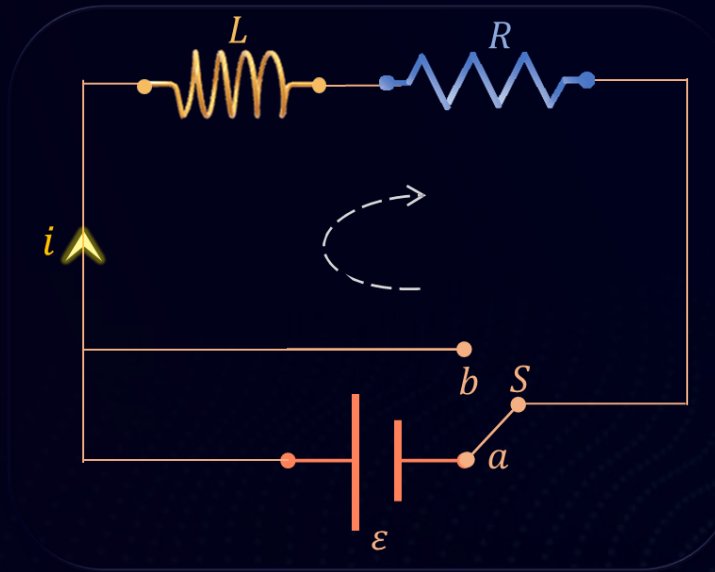
$$\int_0^i \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

$$i(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau})$$

$$i(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau})$$

$$\frac{L}{R} = \tau = \text{time constant}; \frac{\varepsilon}{R} = i_o = \text{steady current}$$

$$i(t) = i_o (1 - e^{-t/\tau})$$



- In one time constant, i is increased to 63% of its steady state value.
- It takes around 5 – time constants to reach steady state.

t	i
t	$\frac{\varepsilon}{R} (1 - e^{-t/\tau})$
0	0
τ	$0.63 i_o$
∞	i_o

(Steady current)

?

An $L - R$ circuit with $L = 12.6\text{ H}$, $R = 6.3\ \Omega$ and $\varepsilon = 20\text{ V}$ is switched on at $t = 0$. Find the power dissipated by the resistor at $t = 2\text{ s}$. ($\frac{1}{e} = 0.37$)

Given: $L = 12.6\text{ H}$, $R = 6.3\ \Omega$, $\varepsilon = 100\text{ V}$

To find: P (When $t = 2\text{ s}$)

Solution: The time constant of the circuit is:

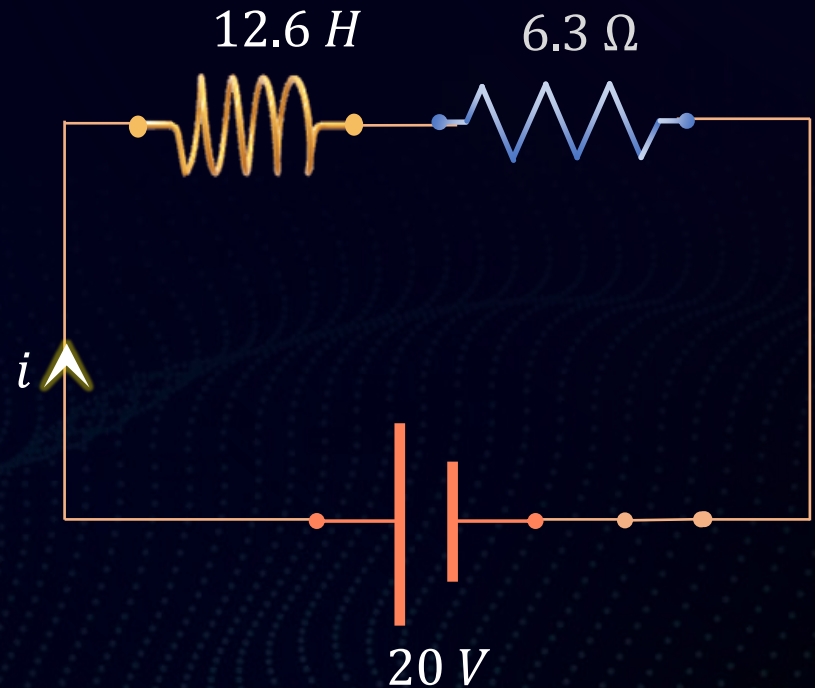
$$\tau = \frac{L}{R} = \frac{12.6}{6.3} = 2\text{ s}$$

The current at $t = 2\text{ s}$ is given by,

$$i = \frac{\varepsilon}{R} (1 - e^{-t/\tau}) = \frac{20}{6.3} \times 0.63 = 2\text{ A}$$

The power dissipated is given by:

$$P = i^2 R = 4 \times 6.3 = 25.2\text{ W}$$





Instantaneous behaviour of Inductor



at $t = 0$

$$i(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau})$$

$$i(t = 0) = \frac{\varepsilon}{R} (1 - e^{-0}) = \frac{\varepsilon}{L} (1 - 1)$$

$\therefore i = 0$ at $t = 0$



at $t \rightarrow \infty$

$$i(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau})$$

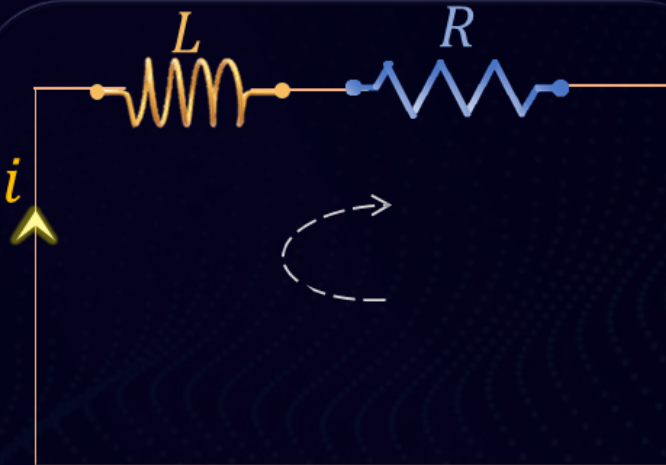
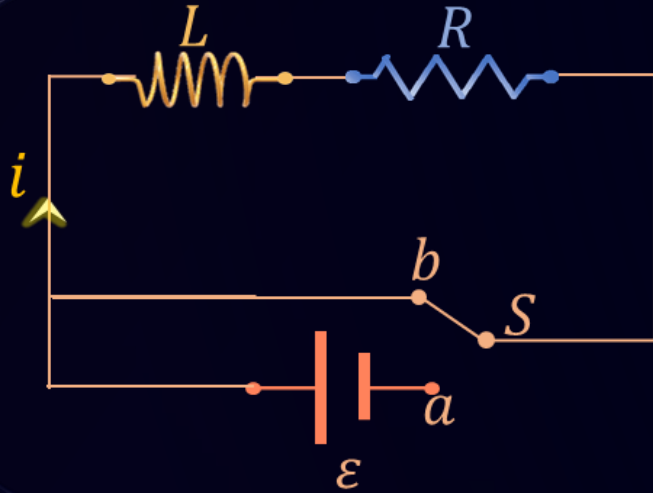
$$i(t \rightarrow \infty) = \frac{\varepsilon}{R} (1 - e^{-\infty}) = \frac{\varepsilon}{R} (1 - 0)$$

$\therefore i = \frac{\varepsilon}{R} = i_o$ at $t \rightarrow \infty$





Decay of current in L-R circuit



At $t = 0$, $i = i_o = \frac{\varepsilon}{R}$

Apply KVL clockwise in the circuit,

$$-L \frac{di}{dt} - iR = 0$$

$$-iR = L \frac{di}{dt}$$

$$\int_{i_o}^i \frac{di}{i} = \int_0^t \frac{-Rdt}{L}$$

$$i(t) = \frac{\varepsilon}{R} e^{-\frac{Rt}{L}} \quad \frac{L}{R} = \tau = \text{time constant}$$

$$i(t) = i_o e^{-\frac{t}{\tau}}$$

$$i(t) = i_0 e^{-\frac{t}{\tau}}$$

at $t = 0$

$$i(t = 0) = i_0 e^{-0} = i_0 \times 1 = i_0$$

at $t = \tau = \frac{L}{R}$

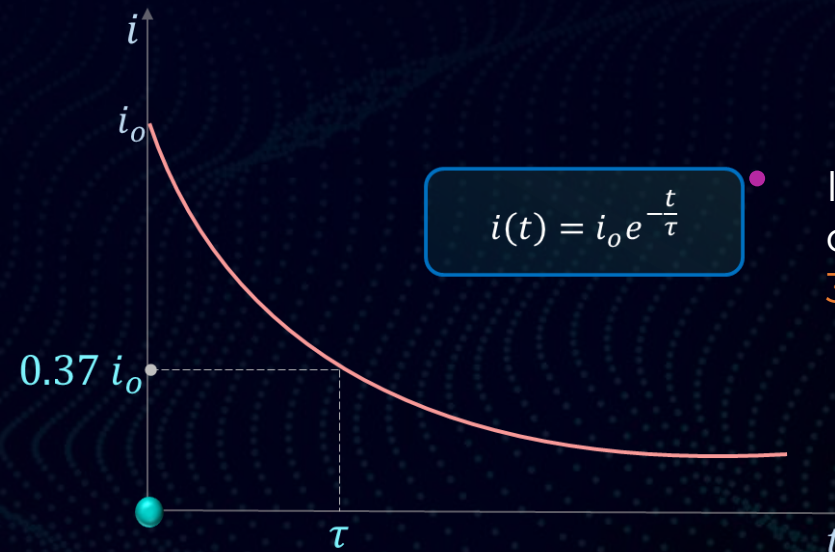
$$i\left(t = \frac{L}{R}\right) = i_0 e^{-\frac{R}{L} \times \frac{L}{R}} = i_0 \times e^{-1} = \frac{i_0}{e^1} = 0.37 i_0$$

at $t \rightarrow \infty$

$$i(t \rightarrow \infty) = i_0 e^{-\frac{1}{\tau} \times \infty} = i_0 \times e^{-\infty} = \frac{i_0}{e^{\infty}} = 0$$

t	i
t	$i_0 e^{-\frac{t}{\tau}}$
0	i_0
τ	$0.37 i_0$
∞	0

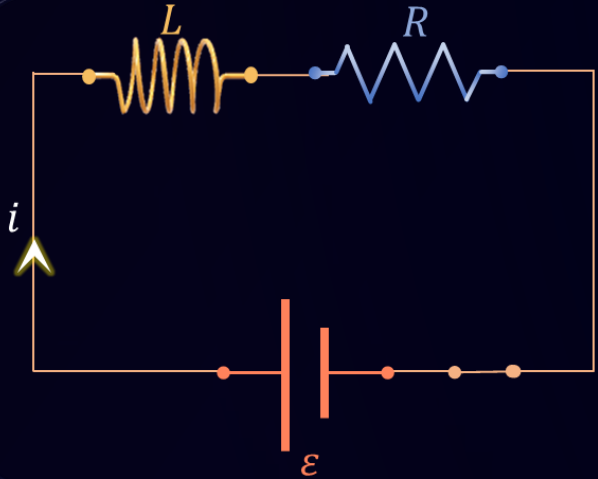
(Steady state)



In one time constant, current reduces to 37% of its initial value



Energy stored in an Inductor



Applying KVL in clockwise direction

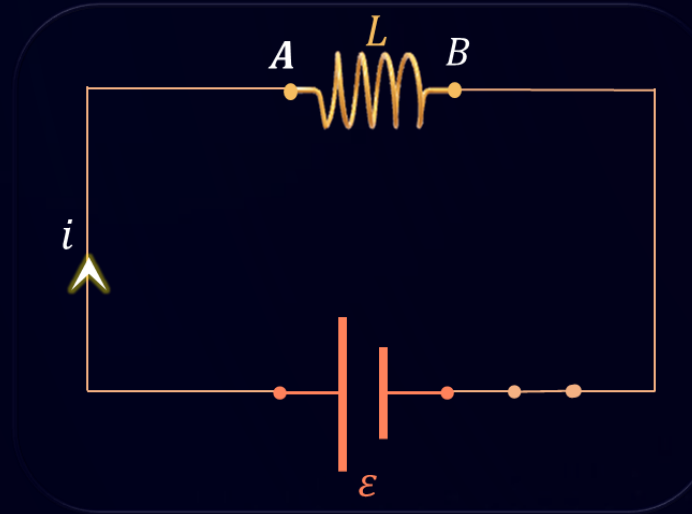
$$-L \frac{di}{dt} - iR + \varepsilon = 0$$

Multiply both sides by " i " and integrate w.r.t " t "

$$\int_0^t \varepsilon i dt = \int_0^i L i dt + \int_0^t i^2 R dt$$

Work done by battery Energy stored in inductor Heat loss at resistor

$$U = \frac{1}{2} Li^2$$



Let a current i be flowing through an inductor of inductance L .

$$\text{Power} = \varepsilon i$$

$$\therefore \frac{dW}{dt} = \left(L \frac{di}{dt} \right) i$$

$$\Rightarrow dW = L i di$$

Integrating both sides, we get;

$$\Rightarrow W = \frac{1}{2} Li^2 \quad (\text{This work done by battery is stored in inductor as magnetic energy.})$$

$$U = \frac{1}{2} Li^2$$



?

The magnetic potential energy stored in a certain inductor is 25 mJ when the current in the inductor is 60 mA . The **value of inductance** is:

Given: $W = 25 \text{ mJ}$ $i = 60 \text{ mA}$

To find: Inductance (L)

Solution: The potential energy stored is equal to the work done which is given by:

$$W = \frac{1}{2} Li^2$$

$$\Rightarrow 25 \times 10^{-3} \text{ J} = \frac{1}{2} \times L \times (60 \times 10^{-3})^2$$

$$\therefore L = 13.89 \text{ H}$$

A

1.389 H

B

0.138 H

C

138.88 H

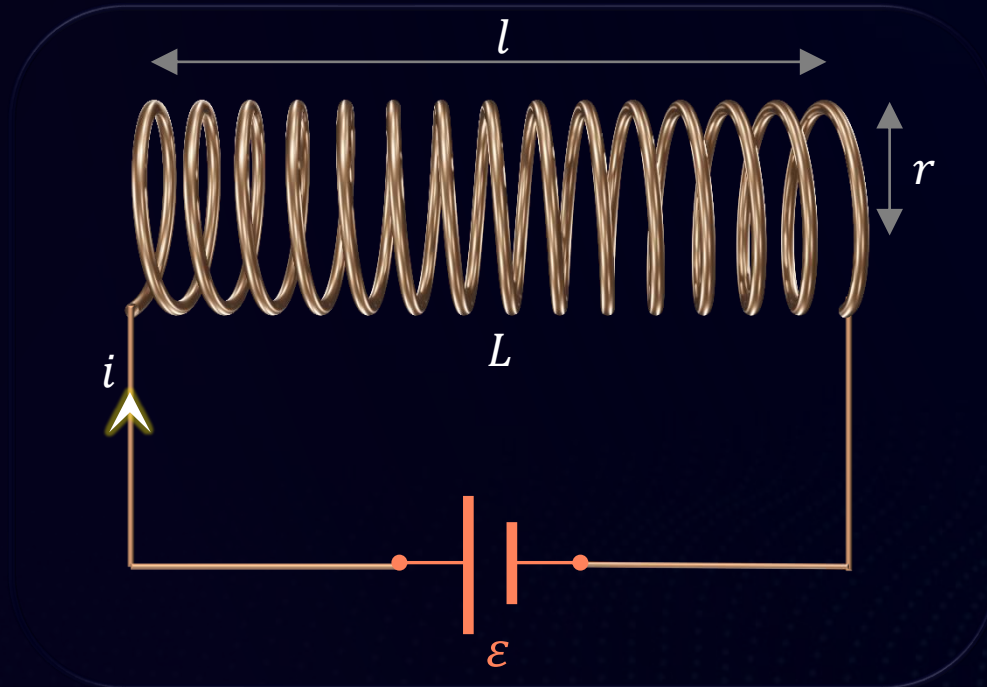
D

13.89 H



Energy Density of Magnetic field

+



The Energy stored in the inductor is

$$U = \frac{1}{2} Li^2$$

Energy Density in the field in an inductor

$$u = \frac{U}{V} = \frac{\frac{1}{2} Li^2}{\pi r^2 l}$$

$$L = \mu_0 n^2 \pi r^2 l$$

$$u = \frac{1}{2\mu_0} B^2$$

?

Consider a small cube of volume 1 mm^3 at the centre of a circular loop of radius 10 cm carrying a current of 4 A . Find the magnetic energy stored inside the cube.

Given: $V = 1 \text{ mm}^3 = 10^{-9} \text{ m}^3$, $R = 10 \text{ cm} = 0.1 \text{ m}$, $i = 4 \text{ A}$

To Find: Magnetic energy(U)

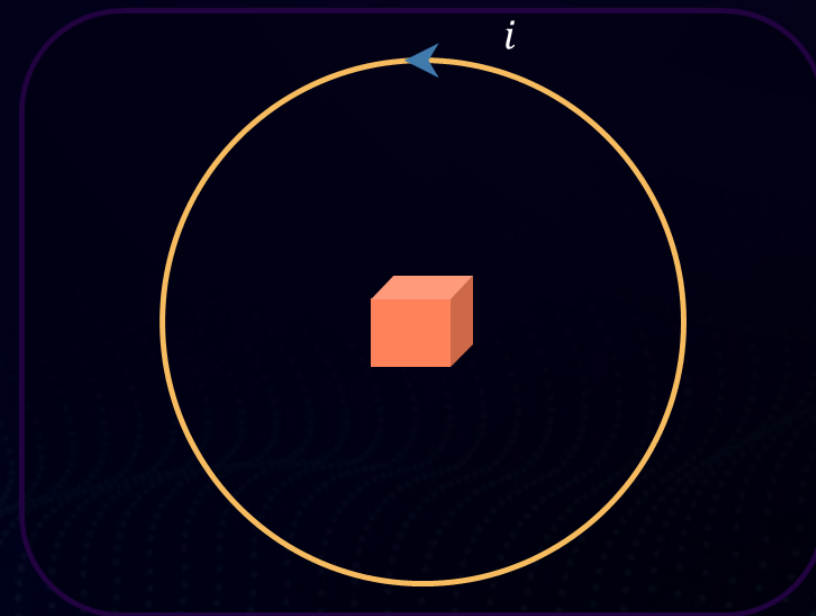
Solution: The magnetic field due to a **circular** loop at the **center**

$$B = \frac{\mu_0 i}{2r}$$

The **magnetic energy** stored inside the cube:

$$U = uV = \frac{1}{2\mu_0} B^2 V = \frac{1}{(2 \times \mu_0)} \left(\frac{\mu_0 \times 4}{2 \times 0.1} \right)^2 (10^{-9})$$

$$U = 8\pi \times 10^{-14} \text{ J}$$





A cell of EMF 15 V is connected across an inductor of 10 H and a resistance of $10\text{ }\Omega$. The ratio of magnitude of current at $t \rightarrow \infty$ to that of at $t = 1$ is,

Given: $R = 10\text{ }\Omega, L = 10\text{ H}, V = 15\text{ V}$

To Find: $\frac{i_{t \rightarrow \infty}}{i_{t=1}}$

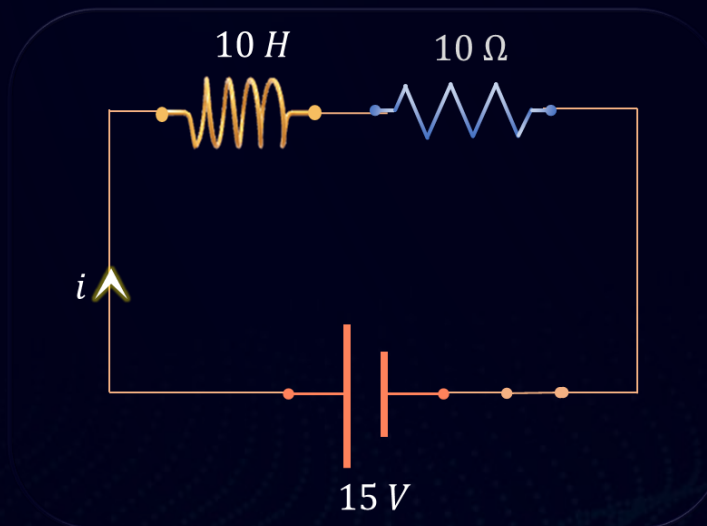
Solution:

$$\tau = \frac{L}{R} = \frac{10}{10} = 1\text{ sec.}$$

At $t \rightarrow \infty$, $i = \frac{V}{R} = \frac{15}{10} = 1.5\text{ Amp.}$

$$\begin{aligned} \text{At } t = \tau, i &= i_0(1 - e^{-\frac{t}{\tau}}) \\ &= 1.5(1 - e^{-1}) = 1.5\left(\frac{e - 1}{e}\right)\text{ Amp.} \end{aligned}$$

$$\frac{i_{t \rightarrow \infty}}{i_{t=1}} = \left(\frac{e}{e - 1}\right)$$



A

$$\frac{e}{e - 1}$$

B

$$\frac{e^{1/2}}{e^{1/2} - 1}$$

C

$$1 - e^{-1}$$

D

$$e^{-1}$$

?

A coil having an inductance L and resistance R is connected to a battery of emf \mathcal{E} . Find the time taken for the magnetic energy stored in the circuit to change from one fourth of the steady-state value to half of the steady-state value.

Solution:

In steady-state,

$$i_o = \frac{\mathcal{E}}{R} \quad U_o = \frac{1}{2} L (i_o)^2$$

Let

t_1 = time when $U = U_1 = \frac{1}{4} \times U_o$ and $i = i_1$

t_2 = time when $U = U_2 = \frac{1}{2} \times U_o$ and $i = i_2$

On comparison,

$$U_1 = \frac{1}{4} \times U_o = \frac{1}{8} L (i_o)^2 = \frac{1}{2} L (i_1)^2$$

$$U_2 = \frac{1}{2} \times U_o = \frac{1}{4} L (i_o)^2 = \frac{1}{2} L (i_2)^2$$

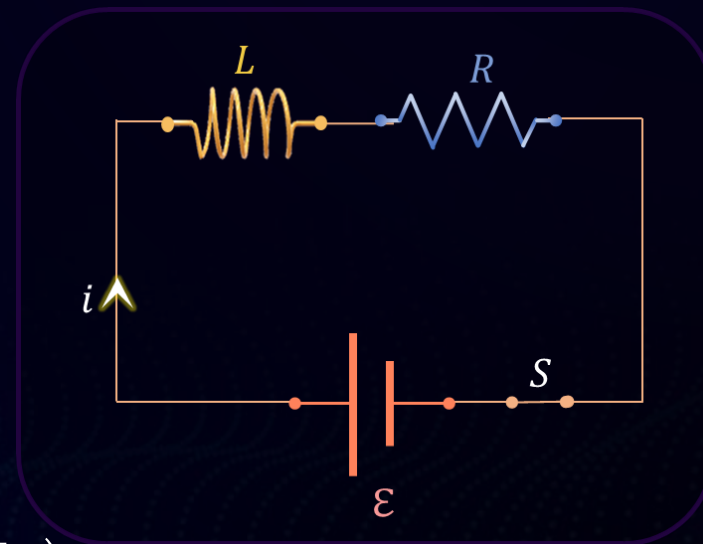
$$i_1 = \frac{i_o}{2} \quad i_2 = \frac{i_o}{\sqrt{2}}$$

$$i(t) = \frac{\mathcal{E}}{R} (1 - e^{-\frac{t}{\tau}})$$

$$\text{when } i = i_1 = \frac{i_o}{2} \Rightarrow t_1 = \tau \ln(2)$$

$$\text{when } i = i_2 = \frac{i_o}{\sqrt{2}} \Rightarrow t_2 = \tau \ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$$

$$t_2 - t_1 = t = \tau \ln\left(\frac{1}{2 - \sqrt{2}}\right)$$



?

A metal bar AB can slide on two parallel thick metallic rails separated by a distance l . A resistance R and an inductance L are connected to rails. A long wire carrying a constant current I_0 is placed in plane of the rails and perpendicular to them. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Find

a) Relation among i , $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where ϕ is the flux

Given: At $t = 0$, wire starts sliding on rails towards right

To find: Relation among i , $\frac{di}{dt}$ and $\frac{d\phi}{dt}$

Solution: Magnitude of EMF induced in the bar:

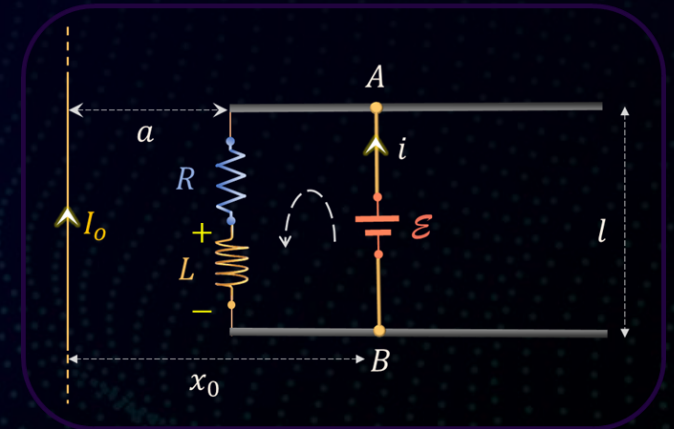
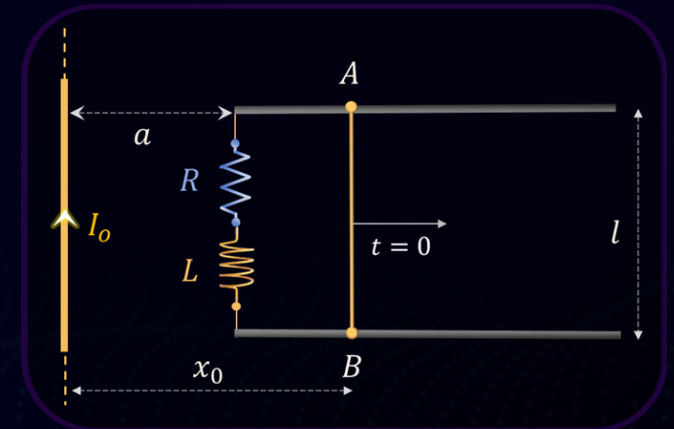
$$\mathcal{E} = \frac{d\phi}{dt}$$

Apply KVL in the anti-clockwise direction:

$$-iR - L \frac{di}{dt} + \mathcal{E} = 0$$

$$\Rightarrow -iR - L \frac{di}{dt} + \frac{d\phi}{dt} = 0$$

$$\frac{d\phi}{dt} = iR + L \frac{di}{dt}$$



?
T

A coil of inductance $L = 2.0 \mu H$ and resistance $R = 1 \Omega$ is connected to an ideal source of constant emf $\mathcal{E} = 3 V$ as shown in the figure. The total amount of **heat** produced in the coil after the switch S is disconnected is

Given: $L = 2.0 \mu H$ $R = 1 \Omega$ $\mathcal{E} = 3 V$

To find: Total heat produced in coil after S is disconnected

Solution: $(i_2)_o = \frac{\mathcal{E}}{R} = \frac{3}{1} = 3 \text{ Amp. (steady state current)}$

$$E = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \mu H \times (3)^2$$

$$E = 9 \mu J \text{ (in steady state)}$$

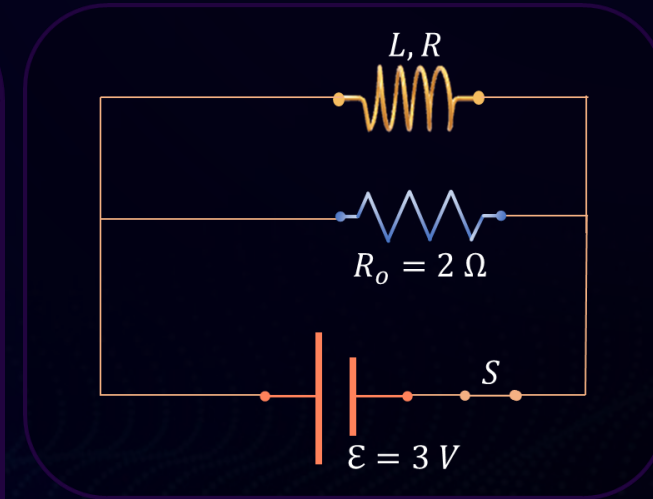
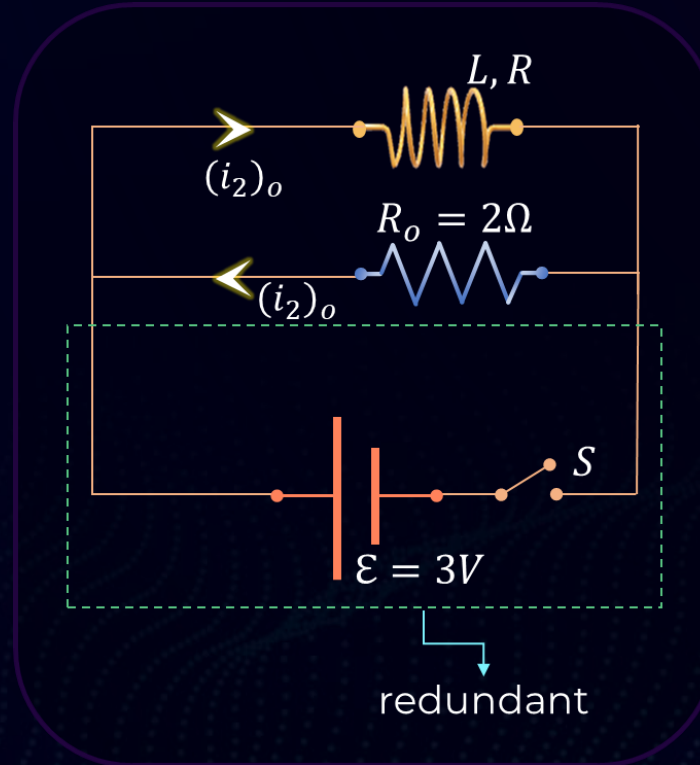
After switch is disconnected:

$$\frac{E_R}{E_{R_o}} = \frac{\text{energy dissipated across } R}{\text{energy dissipated across } R_o} = \frac{R}{R_o} = \frac{1}{2}$$

$$E = E_R + E_{R_o} = 9 \mu J$$

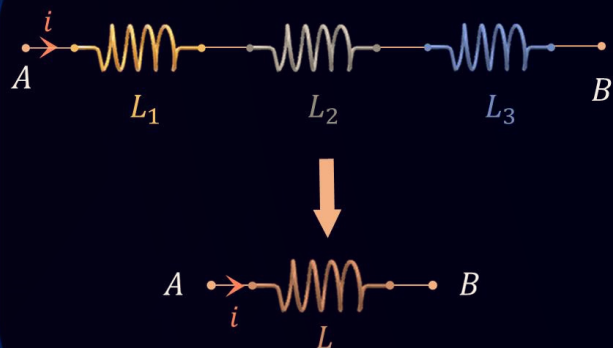
$$\Rightarrow E_R = 3 \mu J$$

$$\Rightarrow E_{R_o} = 6 \mu J$$





Equivalent Inductance in Series combination



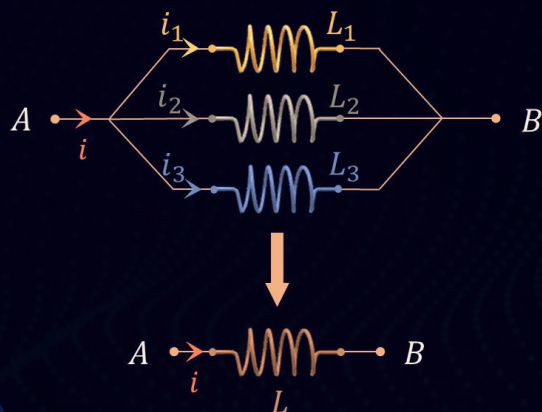
The **equivalent inductance** between A and B is:

$$L_{eq.} = L_1 + L_2 + L_3$$

- Inductors in series are added just like **resistors** are added in series.



Equivalent Inductance in Parallel combination



The **equivalent inductance** between A and B is:

$$\frac{1}{L_{eq.}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

- Inductors in parallel are added just like **resistors** are added in parallel.

?

Three inductors of inductances 60 mH , 120 mH and 75 mH respectively, are connected together in parallel combination. Calculate the **total inductance** of the parallel combination.

To find: Equivalent **inductance** of the given configuration

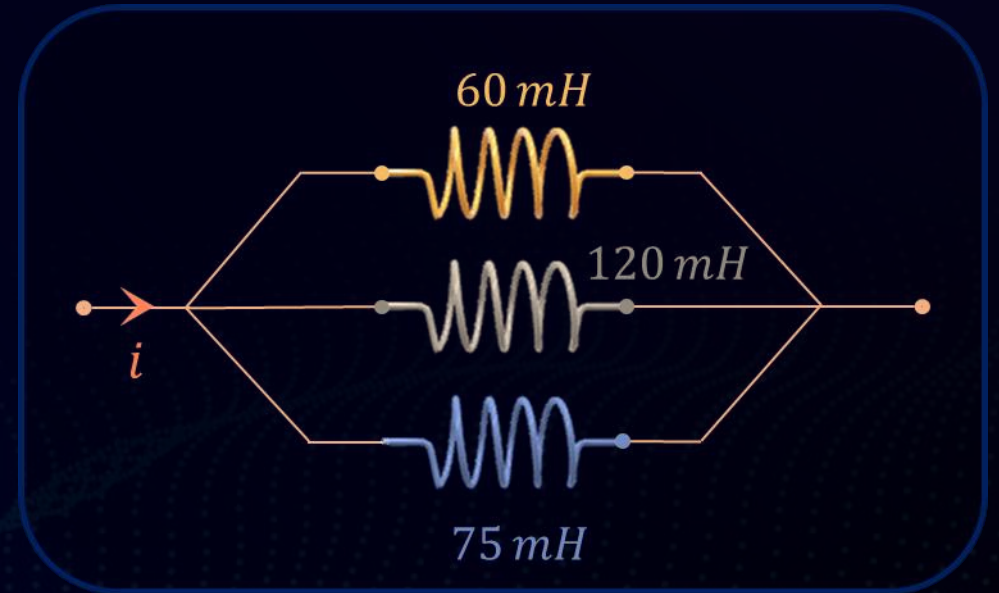
Solution: The **equivalent inductance** in parallel combination is:

$$\frac{1}{L_{eq.}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\frac{1}{L_{eq.}} = \frac{1}{60} + \frac{1}{120} + \frac{1}{75}$$

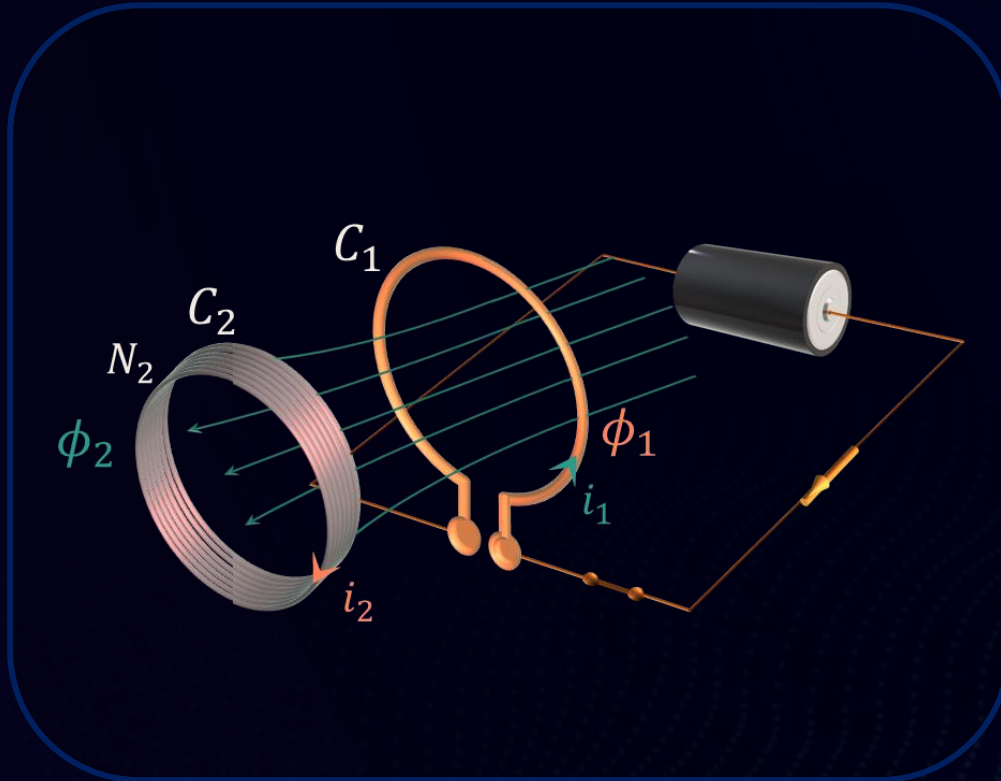
$$\frac{1}{L_{eq.}} = \frac{10 + 5 + 8}{600} = \frac{23}{600}$$

$$L_{eq.} \approx 26\text{ mH}$$





Mutual Induction



Mutual Induction:

The phenomenon of induction of emf in a coil due to **changing current** in a neighboring coil.

$$N_2\phi_2 \propto i_1$$

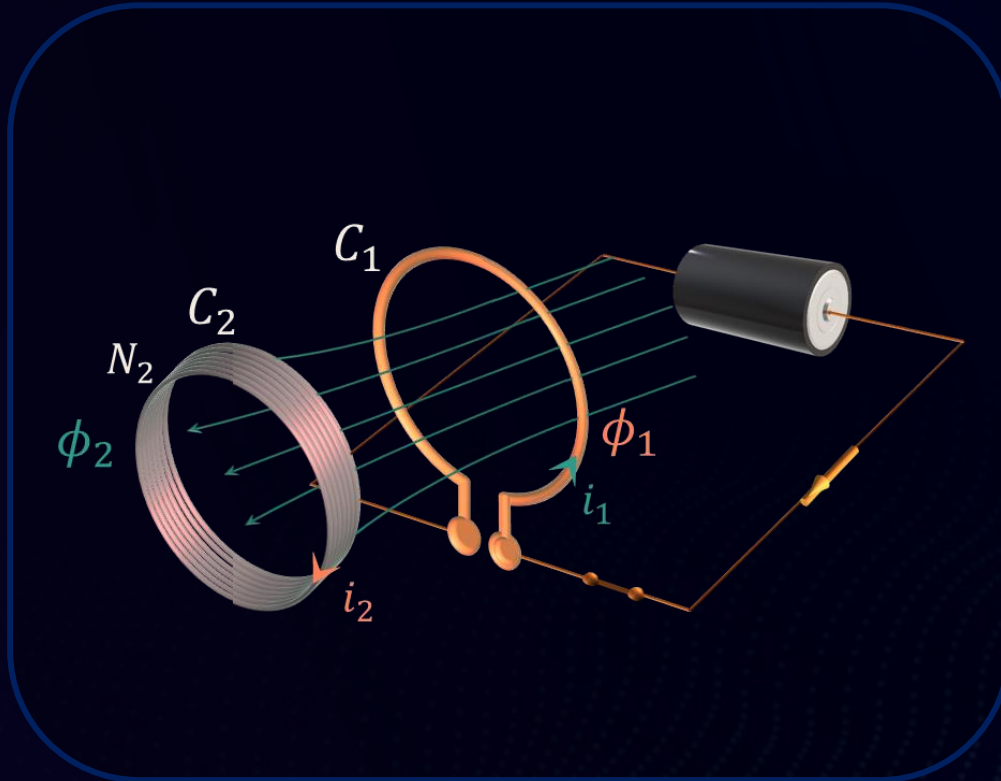
$$N_2\phi_2 = M i_1$$

$$M = \frac{N_2\phi_2}{i_1}$$

M = Coefficient of mutual induction for coils C_1 and C_2



Factors on which Mutual Inductance depends

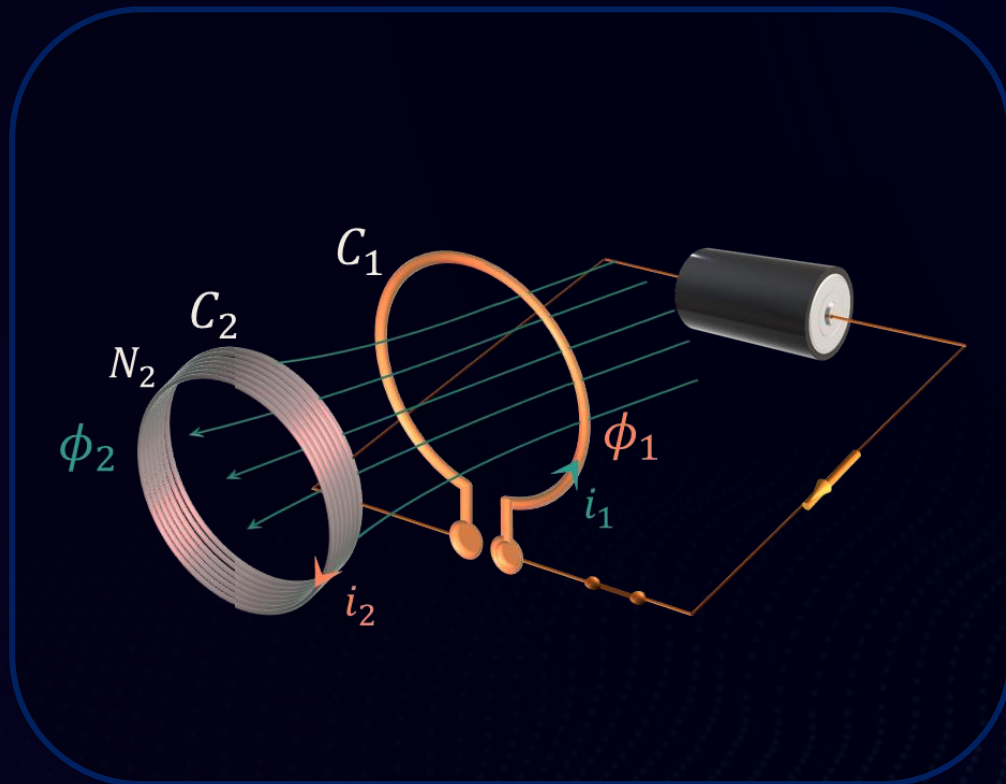


Mutual Inductance of a pair of coils depends upon:

- Geometry of two coils
- Relative orientation of two coils
- Number of turns in two coils
- Separation and medium in between two coils



Emf Induced in Mutual Induction



Total flux associated with coil C_2 :

$$N_2\phi_2 = Mi_1$$

Emf induced in coil C_2 :

$$\mathcal{E}_2 = -N_2 \frac{d\phi_2}{dt}$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \left(\phi_2 = \frac{Mi_1}{N_2} \right)$$

$$M = \left| \frac{-\mathcal{E}_2}{di_1/dt} \right|$$



Current in the secondary coil changes from 10 A to 0 A in 0.5 s . If the EMF induced in the primary coil is 220 V , find the **mutual inductance** between the two coils.

Given:

$$i_{\text{initial}} = 10\text{ A}$$

$$i_{\text{final}} = 0\text{ A}$$

$$\Delta t = 0.5\text{ s}$$

$$\mathcal{E}_1 = 220\text{ V}$$

To find:

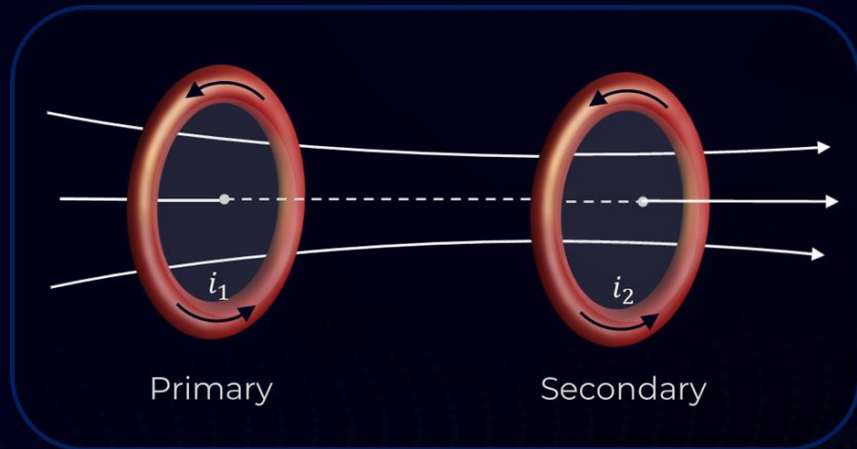
$$M$$

Solution:

$$\mathcal{E}_1 = M \left| \frac{di_2}{dt} \right|$$

$$220 = M \left| \frac{(0 - 10)}{0.5} \right|$$

$$M = 11\text{ H}$$



A

22 H

B

0.09 H

C

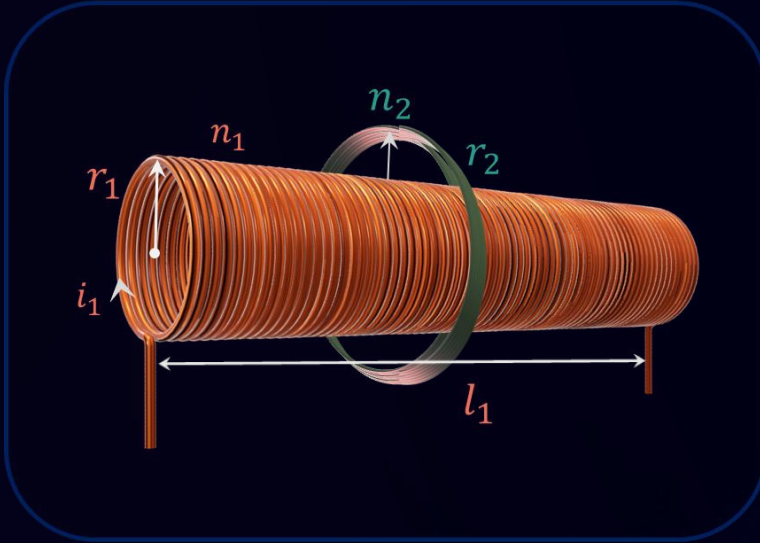
11 H

D

0



Mutual Inductance of pair of Solenoids



Magnetic field **inside** the solenoid:

$$B_1 = \mu_0 n_1 i_1$$

Magnetic field **outside** the solenoid is **zero**

Flux through **one turn** of outer coil

$$(\phi_2)_{\text{one turn}} = \vec{B}_1 \cdot \vec{A}_1$$

$$\Rightarrow \phi_2 = B_1 A_1 \cos \theta$$

$$\Rightarrow \phi_2 = \mu_0 n_1 i_1 \pi (r_1)^2$$

Total turns of outer coil = $N_2 = n_2 l_2$

Total flux through **outer** coil:

$$(\phi_2)_{\text{total}} = N_2 (\phi_2)_{\text{one turn}}$$

$$\Rightarrow (\phi_2)_{\text{total}} = \mu_0 n_1 n_2 \pi (r_1)^2 l_2 i_1$$

Also,

$$(\phi_2)_{\text{total}} = M i_1$$

On comparison,

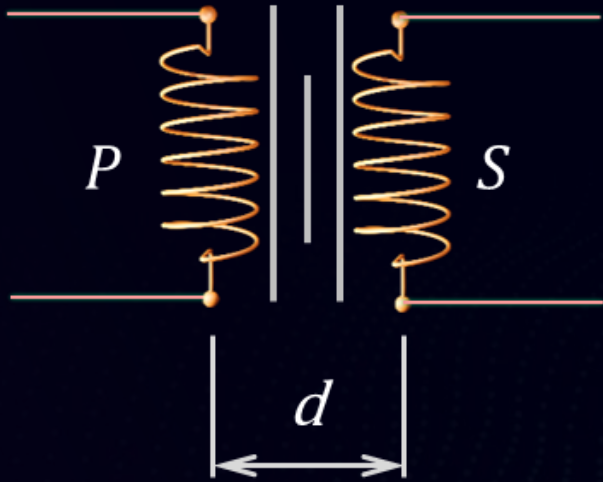
$$M = \mu_0 n_1 n_2 \pi (r_1)^2 l_2$$



Coefficient of Coupling



The coefficient of coupling of two coils gives a **measure** of how the two coils are coupled together i.e., the **interaction** between two coils in terms of **mutual induction**.



$$K = \frac{\text{Magnetic flux linked in secondary coil}}{\text{Magnetic flux linked in primary coil}} = \frac{\phi_2}{\phi_1}$$

Also,

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{\phi_2}{\phi_1} \quad (0 \leq K \leq 1)$$

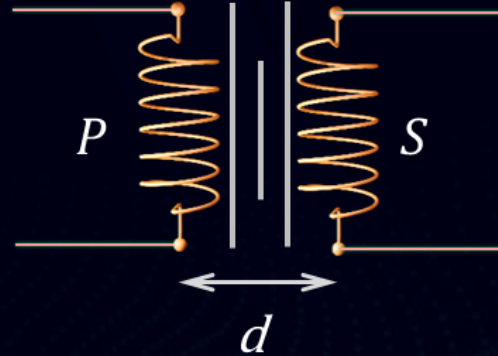


Coefficient of Coupling



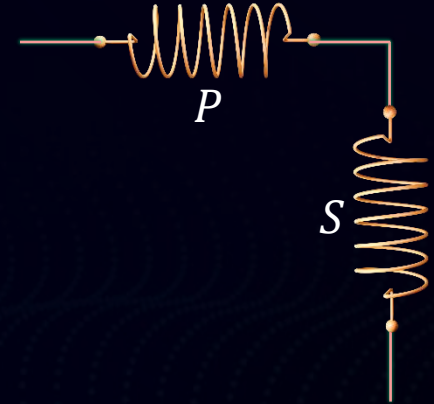
Co-axial coupling (Ideal)

$$K = 1; M = \sqrt{L_1 L_2}$$



Normal coupling

$$0 < K < 1; M = K\sqrt{L_1 L_2}$$



No coupling

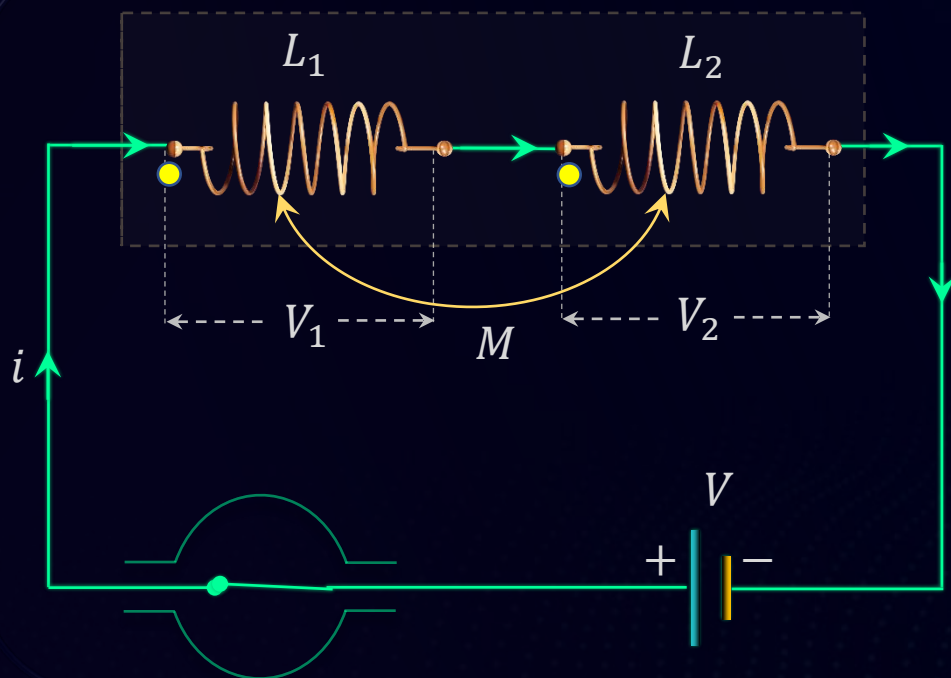
$$K = 0; M = 0$$



Series Combination Of Inductors

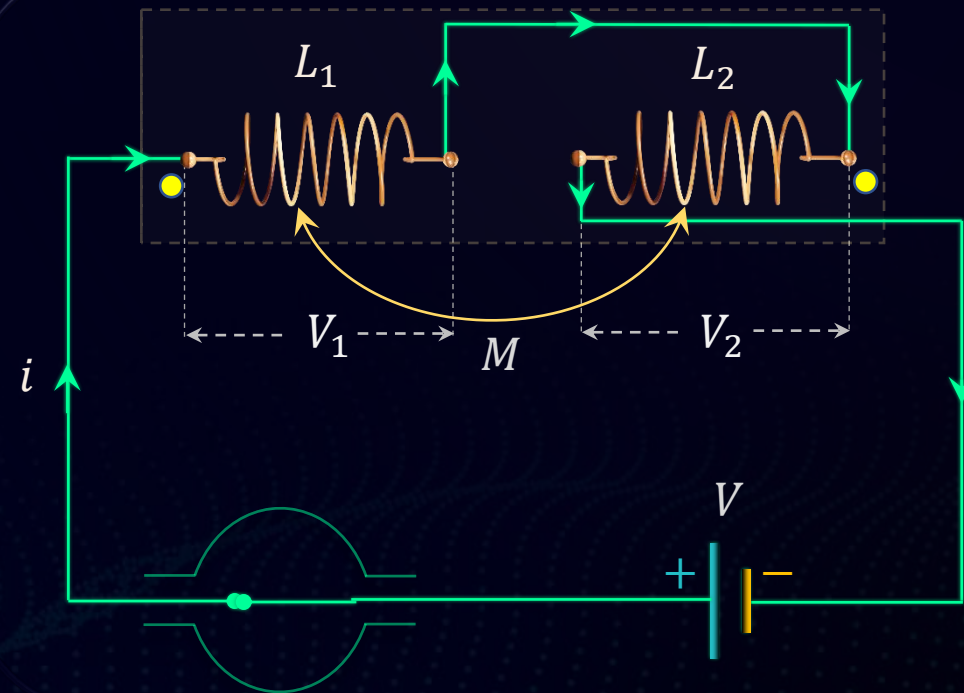


Series Combination (Aiding)



$$L_{eq} = L_1 + L_2 + 2M$$

Series Combination (Opposing)



$$L_{eq} = L_1 + L_2 - 2M$$

?

A solenoid of length 20 cm , area of cross-section 4.0 cm^2 and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area 8.0 cm^2 and length 10 cm . Find the **mutual inductance** between the solenoids.

Given: Cross-sectional area of $S_1 = 4.0\text{ cm}^2$
 Number of turns in $S_1 = N_1 = 4000$
 Cross-sectional area of $S_2 = 8.0\text{ cm}^2$
 Number of turns in $S_2 = N_2 = 2000$

To find: Mutual inductance

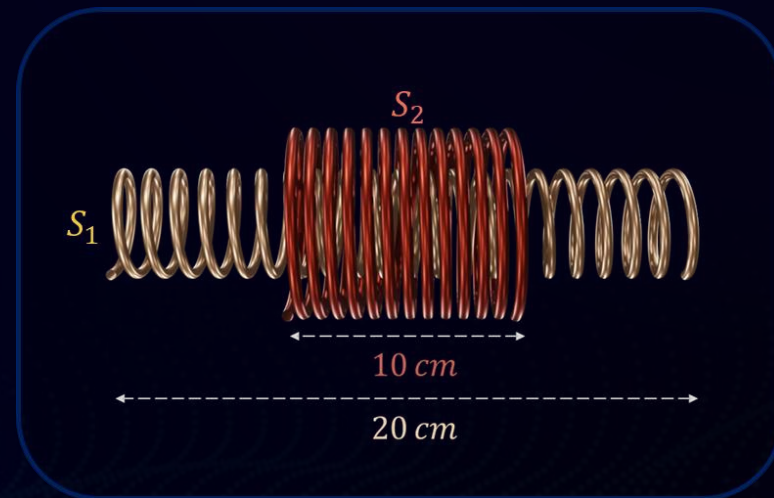
Solution: $n_1 = \frac{N_1}{l_1} = \frac{4000}{20 \times 10^{-2}} = 20000$

$$n_2 = \frac{N_2}{l_2} = \frac{2000}{10 \times 10^{-2}} = 20000$$

$$M = \mu_0 n_1 n_2 A_1 l_2$$

$$M = 4\pi \times 10^{-7} \times 20000 \times 20000 \times 4 \times 10^{-4} \times 0.1\text{ H}$$

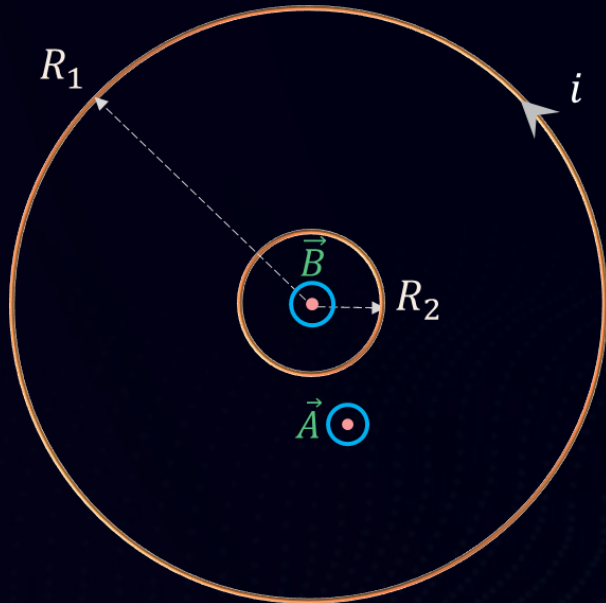
$$M \approx 2 \times 10^{-2}\text{ H}$$



?

Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centers coinciding. Find the **mutual inductance** between them assuming $R_2 \ll R_1$.

Solution:



Magnetic field at **centre** is:

$$B = \frac{\mu_0 i}{2R_1}$$

= Field is out of plane

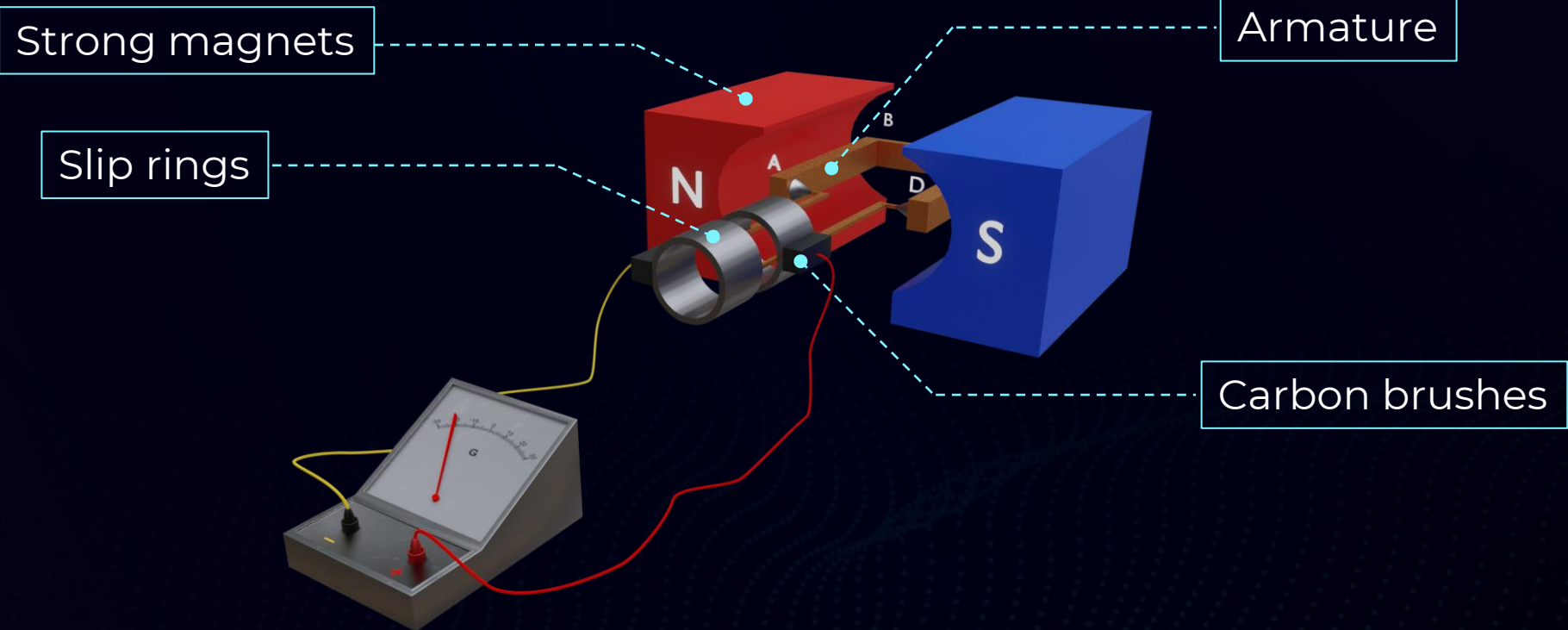
$$\phi = \vec{B} \cdot \vec{A} = BA \cos \theta$$

$$\Rightarrow \phi = \frac{\mu_0 i}{2R_1} \times \pi(R_2)^2 = Mi$$

$$M = \frac{\mu_0 \pi (R_2)^2}{2R_1}$$

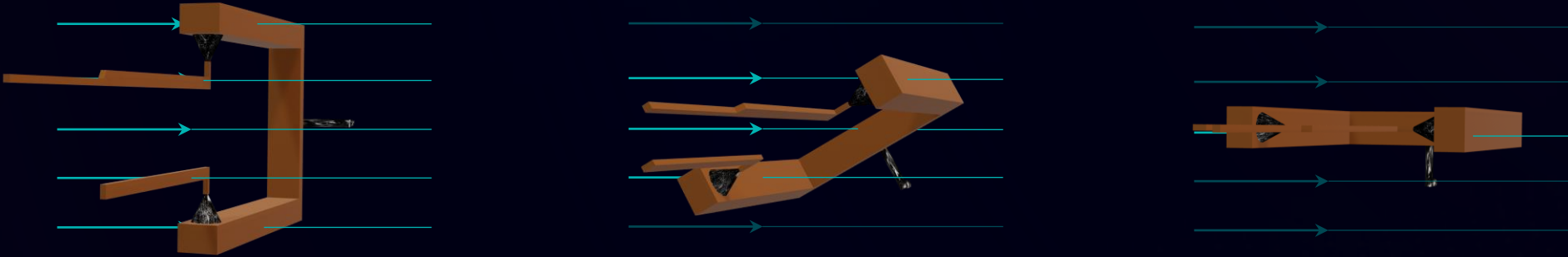


AC Generator - Construction





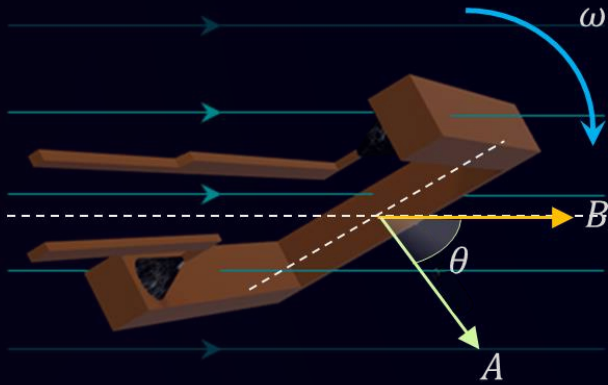
AC Generator – Working Principle



- Magnetic flux changes as the armature rotates in the magnetic field.
- Faraday's Law tells us that the change in magnetic flux generates emf in the coil.



AC Generator – Induced Current and EMF Derivation



For an armature moving at a constant angular speed ω ,

$$\theta = \omega t$$

The flux (ϕ) passing through the armature at any instant is:

$$\phi = NBA \cos(\omega t)$$

For N turns.

Where,

B = Magnetic field due to external magnets

A = Area of the armature coil

N = Number of turns

Magnitude of induced EMF:

$$\mathcal{E} = -\frac{d\phi}{dt} = BA\omega \sin(\omega t)$$

For N turns:

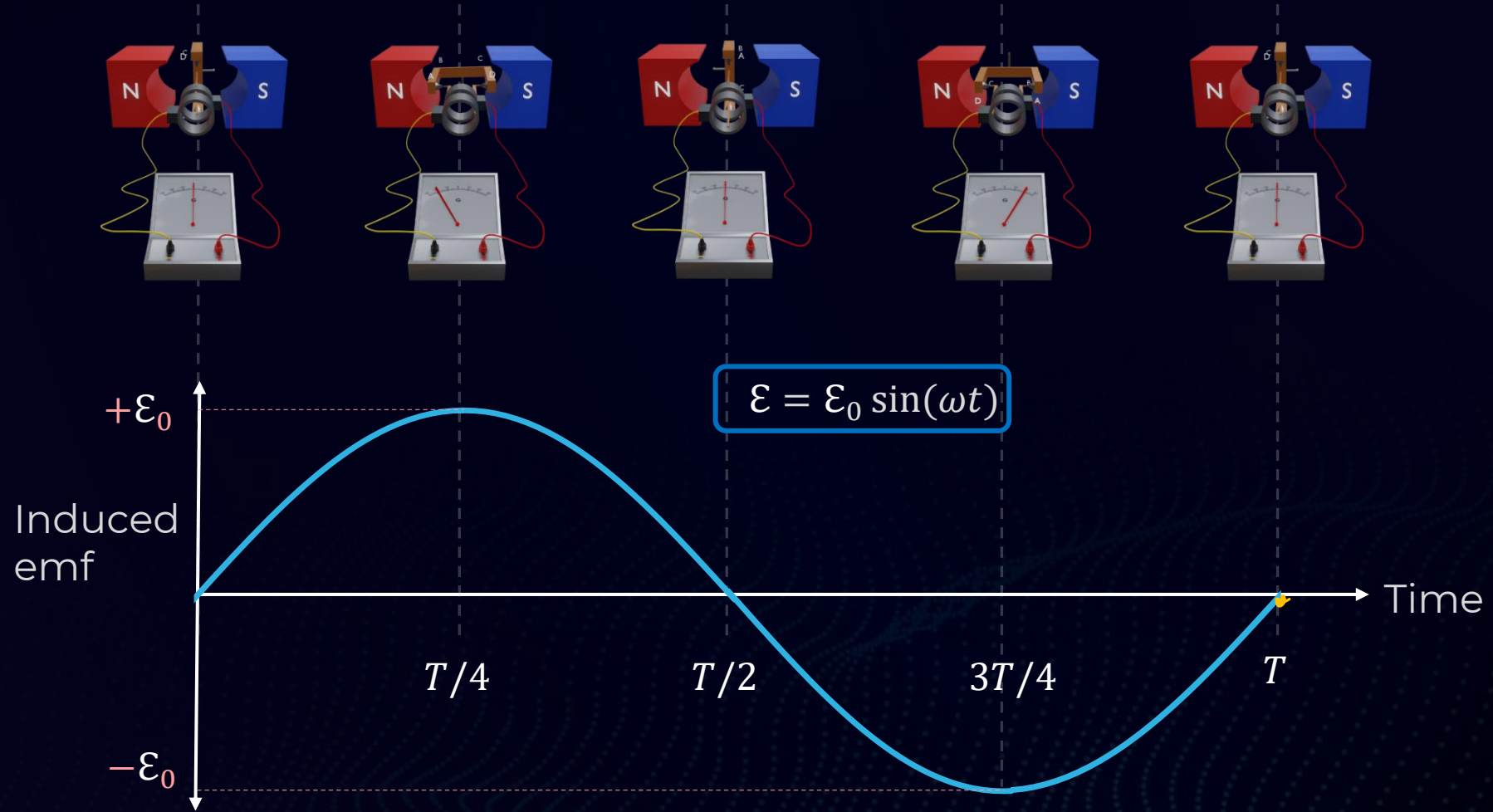
$$\mathcal{E} = NBA\omega \sin(\omega t)$$

$$\therefore \mathcal{E} = \mathcal{E}_0 \sin(\omega t)$$

Where, $\mathcal{E}_0 = NBA\omega$



AC Generator – Graphical Analysis



- The current produced by an AC generator is **sinusoidal** in nature.



Migration of Birds



- Birds migrating from Siberia to India rely on the **distribution of the Earth's magnetic field** for navigation.
- One possible explanation is that they might be using **motional EMF** to sense the magnitude of the magnetic field.
- By sensing the **magnitude of motional EMF**, the **magnitude and direction** of the magnetic field can be determined
- Fishes like sharks have been known to use the **magnetic field of the Earth** for the purpose of navigation.