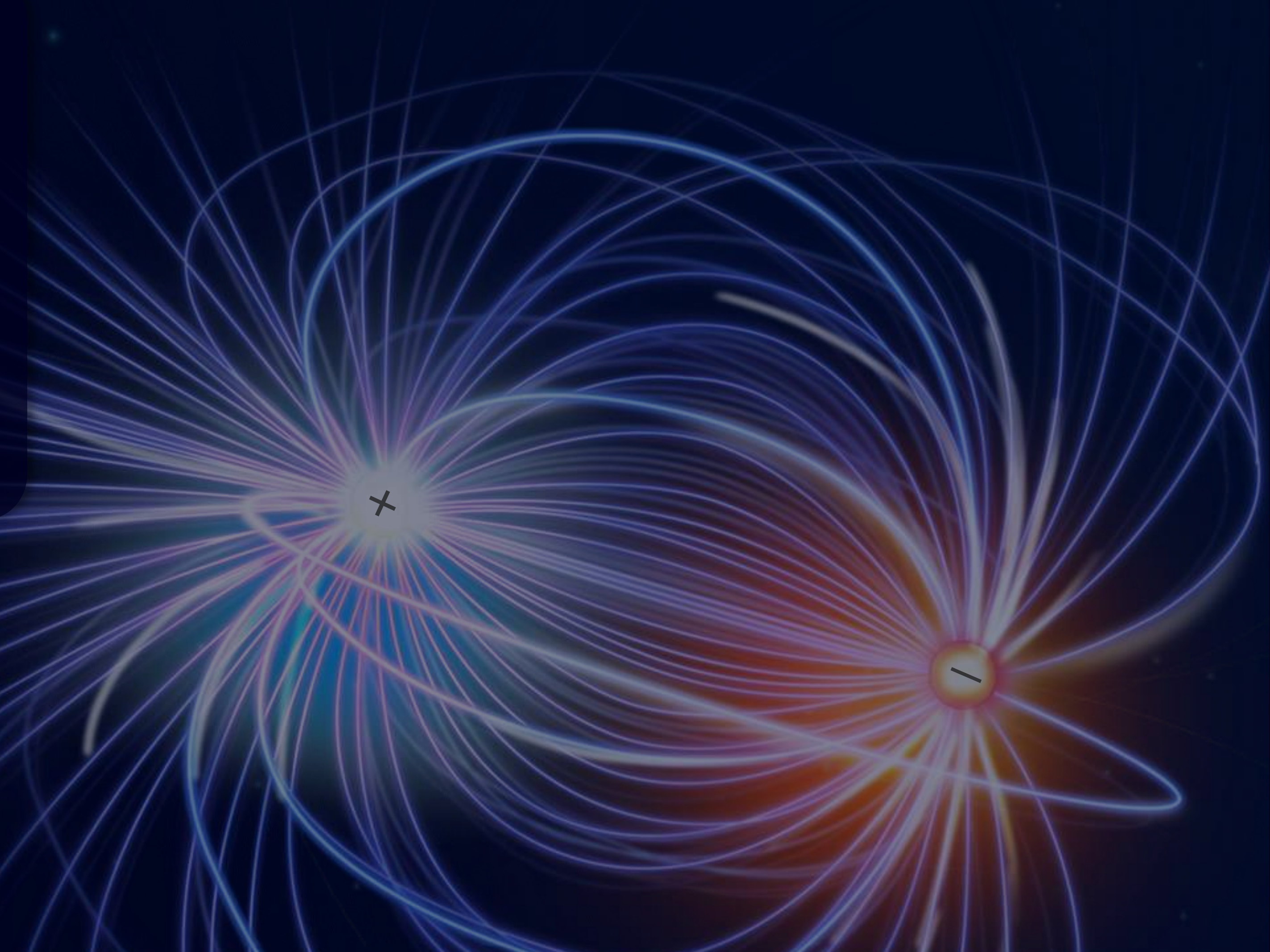
Welcome to

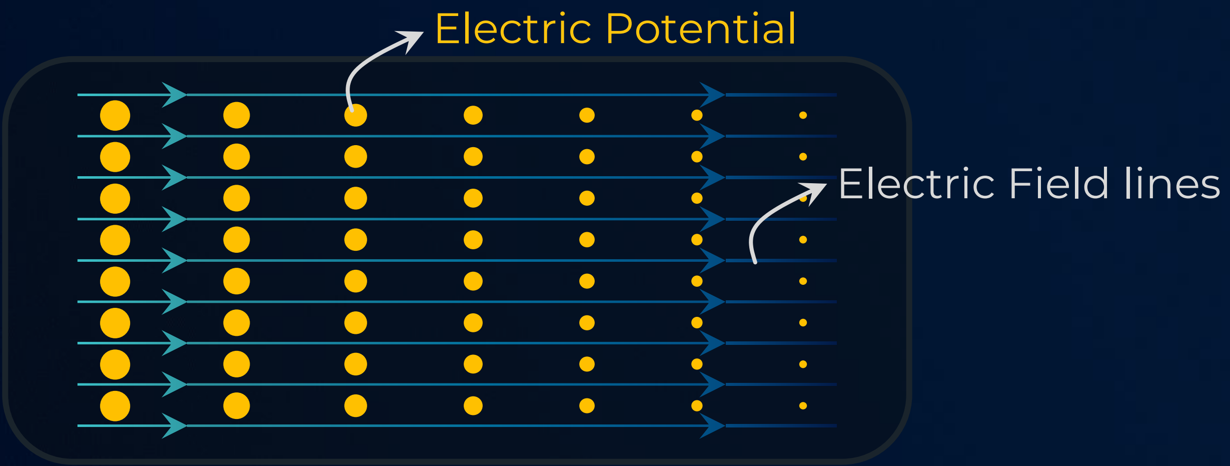
Aakash + BYJU'S NOTES

Electric Potential



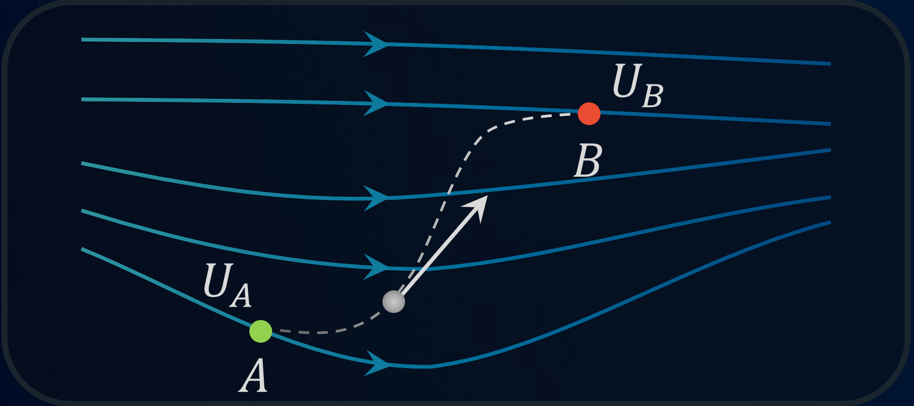
Electric Potential

- Electrical Potential is a scalar quantity capable of describing the nature of the electric field in the 3D space.



- When a charge $+q$ is moved from point A to B in a region of electric field under the action of an external force, the potential energy change is given by,

  $U_B - U_A = W_{ext}$ provided $\Delta KE = 0$.



- The change in electric potential is defined as the change in potential energy per unit charge.

$$V_B - V_A = \frac{U_B - U_A}{q}$$

The kinetic energy of a charged particle decreases by $10\,J$ as it moves from a point at potential $100\,V$ to a point at potential $200\,V$. Find the charge on the particle.

Solution :

$$\Delta KE = -10\,J$$

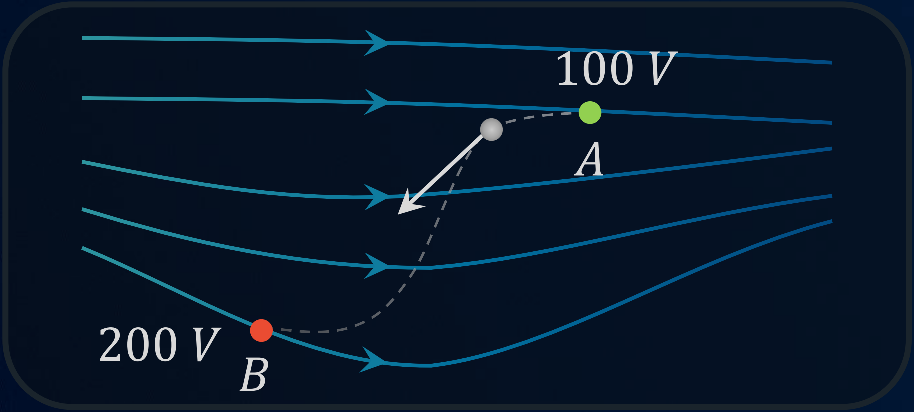
From the conservation of mechanical energy principle -

$$W_{ext} + W_{el} = \Delta KE$$

$$W_{el} = \Delta KE = -\Delta U \quad (\text{Considering } W_{ext} = 0)$$

$$-(U_f - U_i) = \Delta KE = -q(V_f - V_i)$$

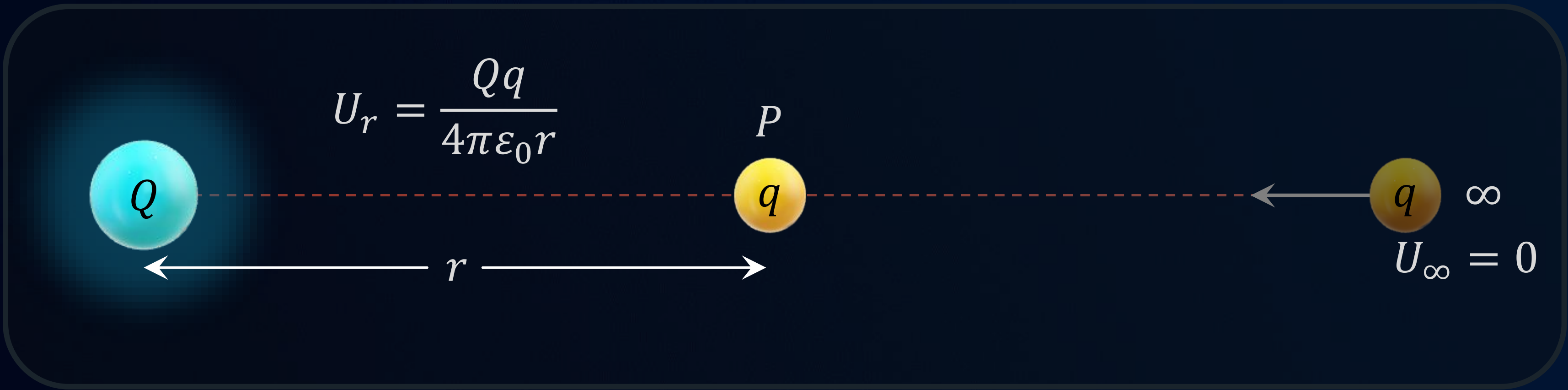
$$-q(V_f - V_i) = -10\,J$$



$$-q(200 - 100) = -10\,J$$

$$\boxed{q = 0.10\,C}$$

Electric Potential of a Point Charge



Change in potential energy of the system in moving charge q from $r = \infty$ to $r = r$: $U_P - U_\infty = \frac{Qq}{4\pi\varepsilon_0 r}$

$\Rightarrow V_P(r) = \frac{Q}{4\pi\varepsilon_0 r}$ = Electric potential at point P due to charge Q

Point Charge	$\vec{E}(\hat{r})$	$V_P(r)$
$+Q$	$\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$	$\frac{Q}{4\pi\varepsilon_0 r}$
$-Q$	$\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$	$\frac{-Q}{4\pi\varepsilon_0 r}$

Four electric charges $+q, +q, -q$ and $-q$ are placed at the corners of a square of side $2L$. The electric potential at point A, midway between the two charges $+q$ and $+q$, is:

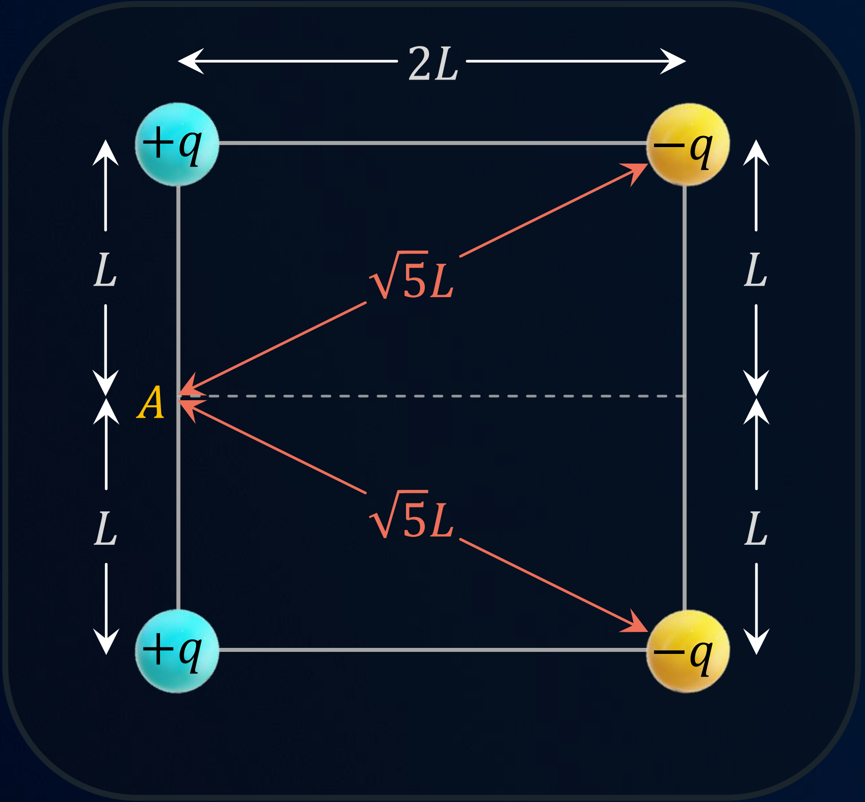
Solution :

$$V_{net} = V_1 + V_2 + V_3 + V_4$$

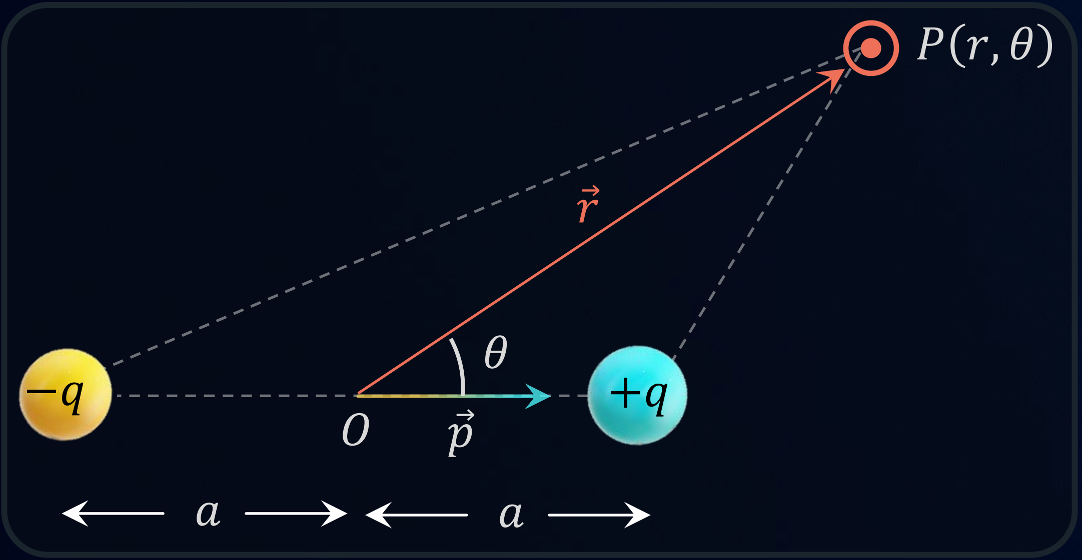
$$= \frac{kq}{L} + \frac{kq}{L} + \left(-\frac{kq}{\sqrt{5}L}\right) + \left(-\frac{kq}{\sqrt{5}L}\right)$$

$$= \frac{2kq}{L} - \frac{2kq}{\sqrt{5}L}$$

$$\boxed{V_{net} = \frac{2kq}{L}\left(1 - \frac{1}{\sqrt{5}}\right)}$$



Electric Dipole



Location of P	$\vec{E}(r)$	$V(r)$
At the axis	$\dfrac{2\vec{p}}{4\pi\varepsilon_0 r^3}$	$\dfrac{p}{4\pi\varepsilon_0 r^2}$
At the equatorial plane	$-\dfrac{\vec{p}}{4\pi\varepsilon_0 r^3}$	0
At any general point	$\dfrac{p(1+3\cos^2\theta)^{\frac{1}{2}}}{4\pi\varepsilon_0 r^3}$ (Magnitude)	$\dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$

Two electric dipoles, A, B with respective dipole moments $d_A = -4aq\,\hat{\imath}$ and $d_B = -2aq\,\hat{\imath}$ are placed on the x-axis with a separation R, as shown in the figure. The distance from A at which both produce the same potential is

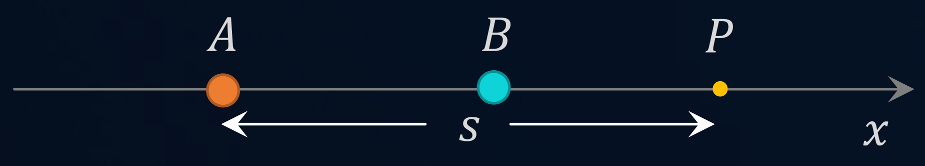
Solution :

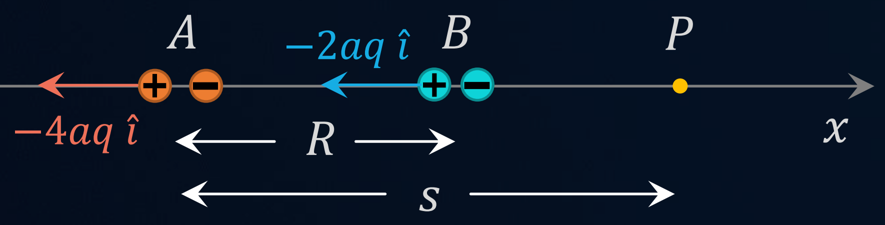
Electric potential produced at point P by both electric dipoles -

$$-\frac{4aq}{4\pi\varepsilon_0 s^2} = -\frac{2aq}{4\pi\varepsilon_0 (s-R)^2}$$

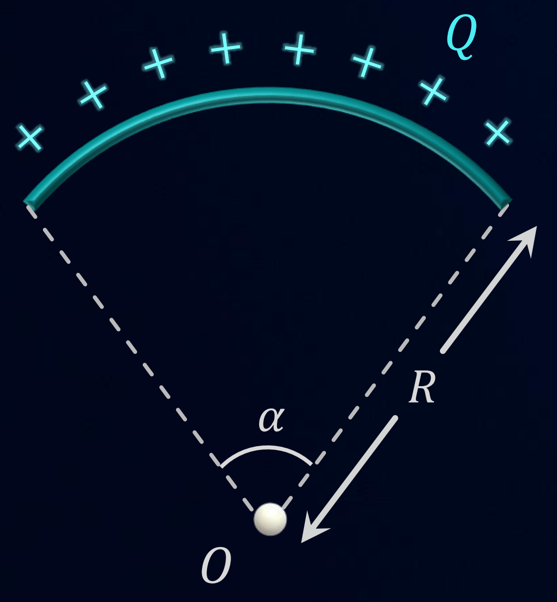
$$\Rightarrow (s-R)^2 = \frac{s^2}{2}$$

$$s = \frac{\sqrt{2}R}{\sqrt{2}-1} \quad (\because s > R)$$





Electric Potential due to a Circular Arc



$$V_o = \frac{KQ}{R}$$

$$V_o = \frac{KQ}{R}$$

Relation between Electric Field and Potential

Work done by the electric field

$$dW_{el} = \vec{F}_{el} \cdot d\vec{r} = q\vec{E} \cdot d\vec{r}$$

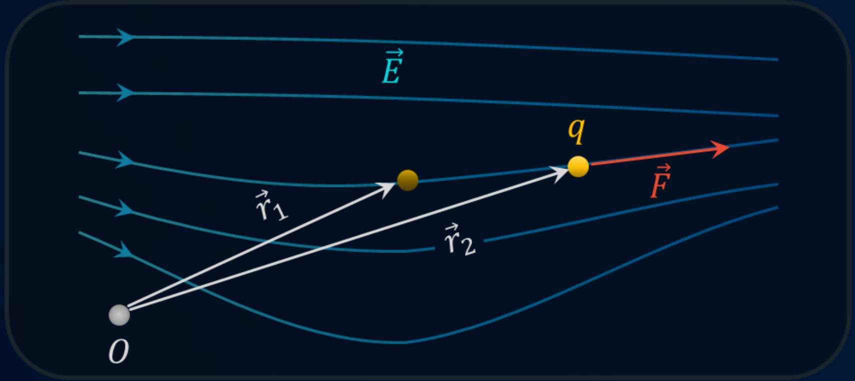
$$dU = -dW_{el}$$

$$dU = -q\vec{E} \cdot d\vec{r}$$

Change in electric potential

$$dV = \frac{dU}{q} = -\vec{E} \cdot d\vec{r}$$

$$V_2 - V_1 = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

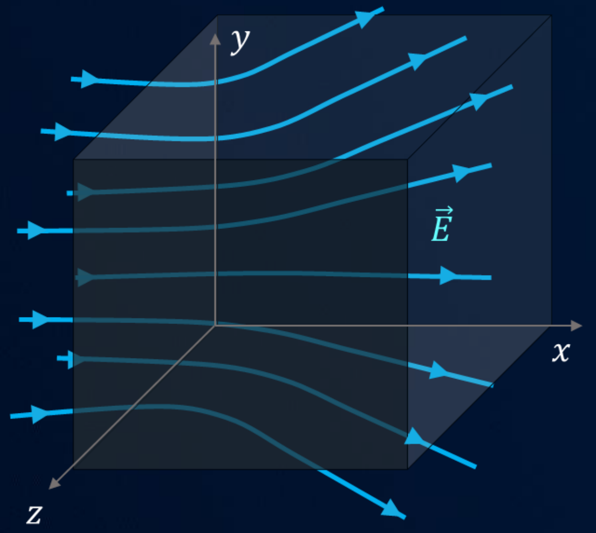


In Cartesian coordinate system

For $V = f(x, y, z)$ corresponding to the electric field $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}$

$$dV = -\vec{E} \cdot d\vec{r}$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$



The electric potential existing in space is $V(x, y, z) = A(xy + yz + zx)$

a. Find the expression for the electric field.

b. If A is $10\ V\ m^{-2}$, then find the magnitude of the electric field at $(1\ m, 1\ m, 1\ m)$

Solution :

$$E_x = -\frac{\partial V}{\partial x} = -A[y + 0 + z] = -A(y + z)$$

$$E_y = -\frac{\partial V}{\partial y} = -A[x + z + 0] = -A(x + z)$$

$$E_z = -\frac{\partial V}{\partial z} = -A[0 + y + x] = -A(x + y)$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

$$\boxed{\vec{E} = -A\left[(y + z)\hat{\imath} + (x + z)\hat{\jmath} + (x + y)\,\hat{k}\right]}$$

$$\boxed{\left|\vec{E}(1, 1, 1)\right| = 20\sqrt{3}\ \ Vm^{-1}}$$

Find out the potential difference between points A and B.

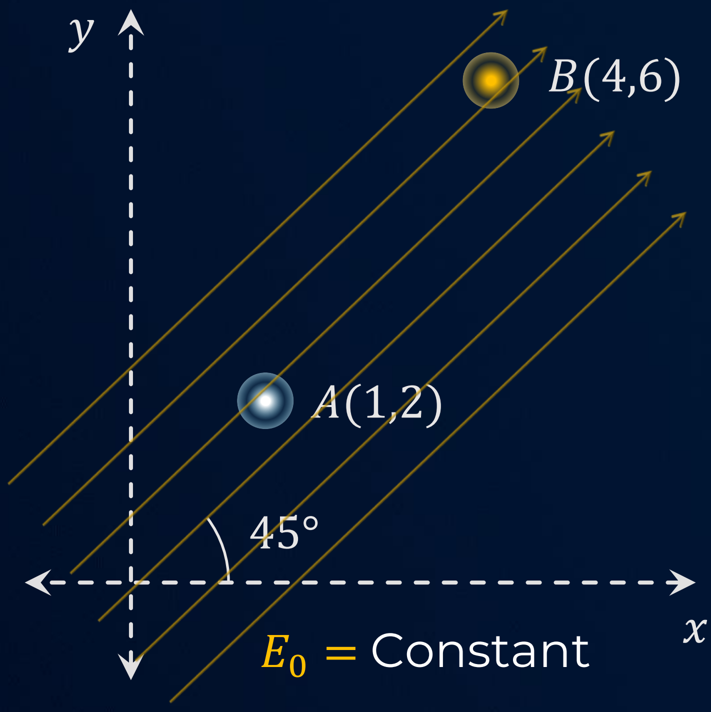
Solution :

We know : $V_i - V_f = \int \vec{E}.d\vec{r}$

Displacement : $\vec{r} = (4-1)\hat{\imath} + (6-2)\hat{\jmath} = 3\hat{\imath} + 4\hat{\jmath}$

Electric field : $\vec{E} = E_0 \cos 45° \, \hat{\imath} + E_0 \sin 45° \, \hat{\jmath}$

$$\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$$



As the electric field is uniform,

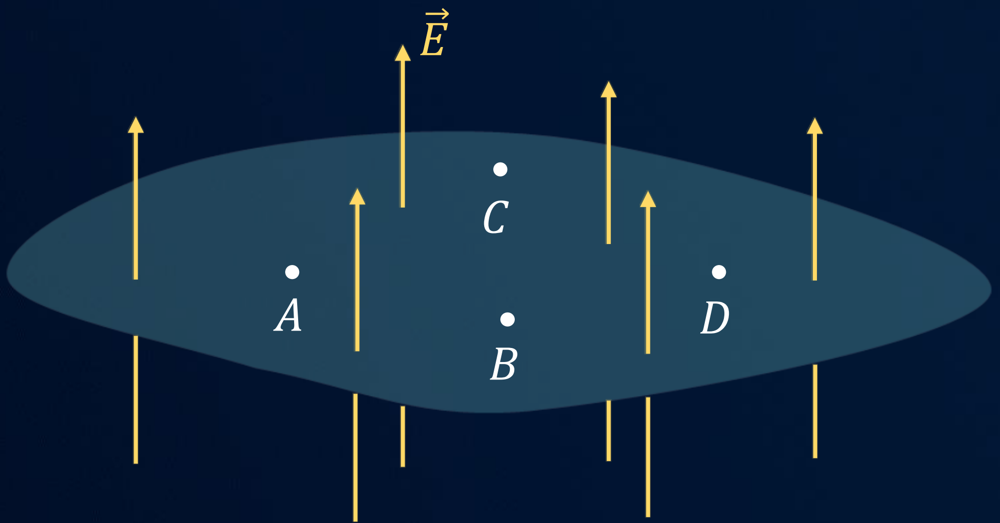
$$V_A - V_B = \vec{E}.\vec{r}$$

$$\Rightarrow V_A - V_B = \frac{E_0}{\sqrt{2}}(\hat{\imath} + \hat{\jmath}).(3\hat{\imath} + 4\hat{\jmath})$$

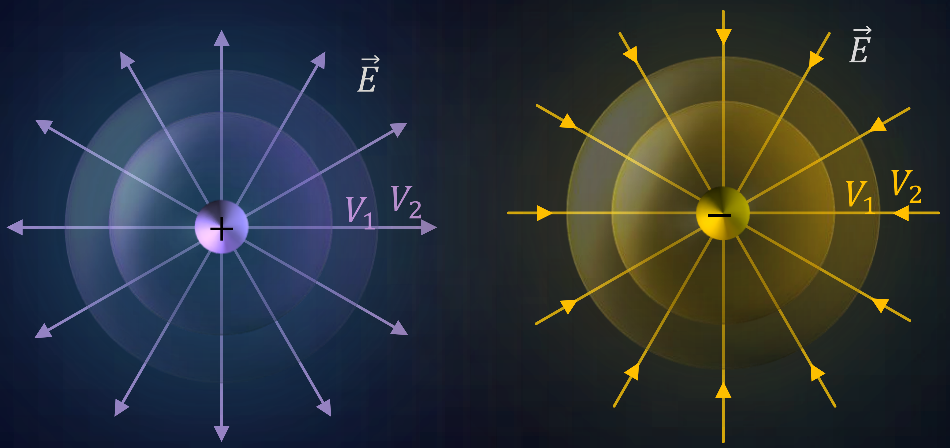
$$\boxed{V_A - V_B = \frac{7E_0}{\sqrt{2}}}$$

Equipotential Surface

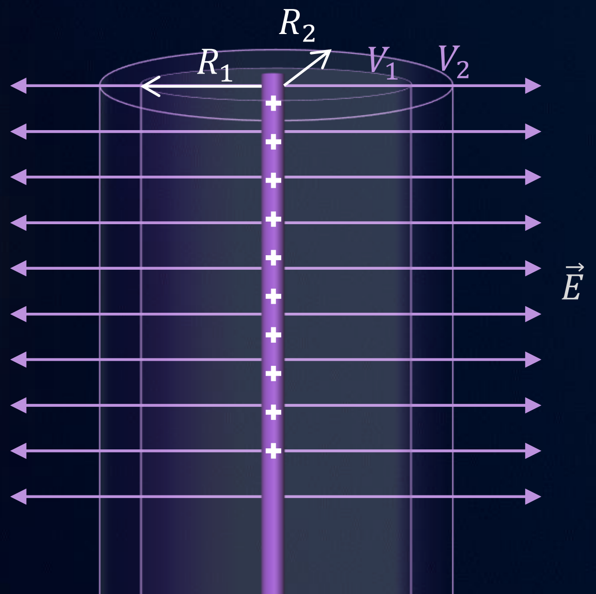
- A surface on which potential is same at every point.
- Electric field at any point on an equipotential surface is perpendicular to it. ($\because \vec{E} \cdot d\vec{r} = 0$)
- Work done by electrostatic force in moving a charge from one point to the other on such a surface is zero.



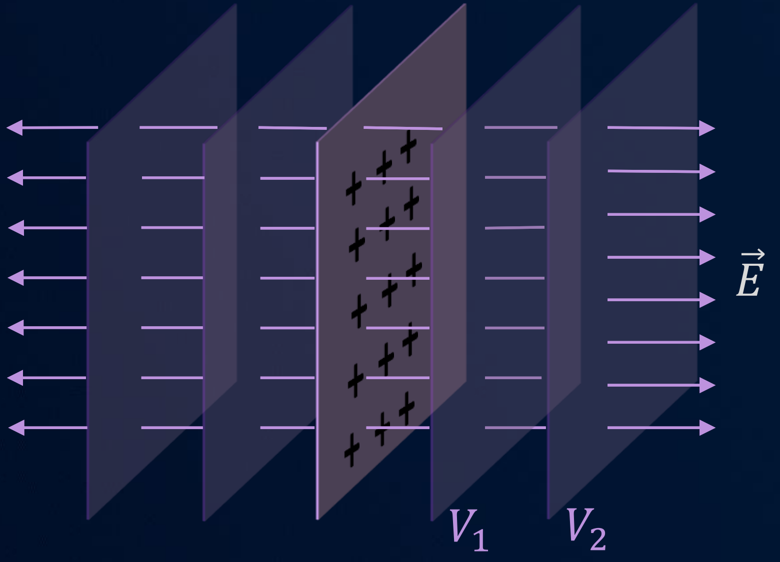
Point Charge



Uniform Line Charge



Plane Sheet of Charge



Some equipotential surfaces are shown. Find the magnitude of the electric field and the angle it makes with the x-axis.

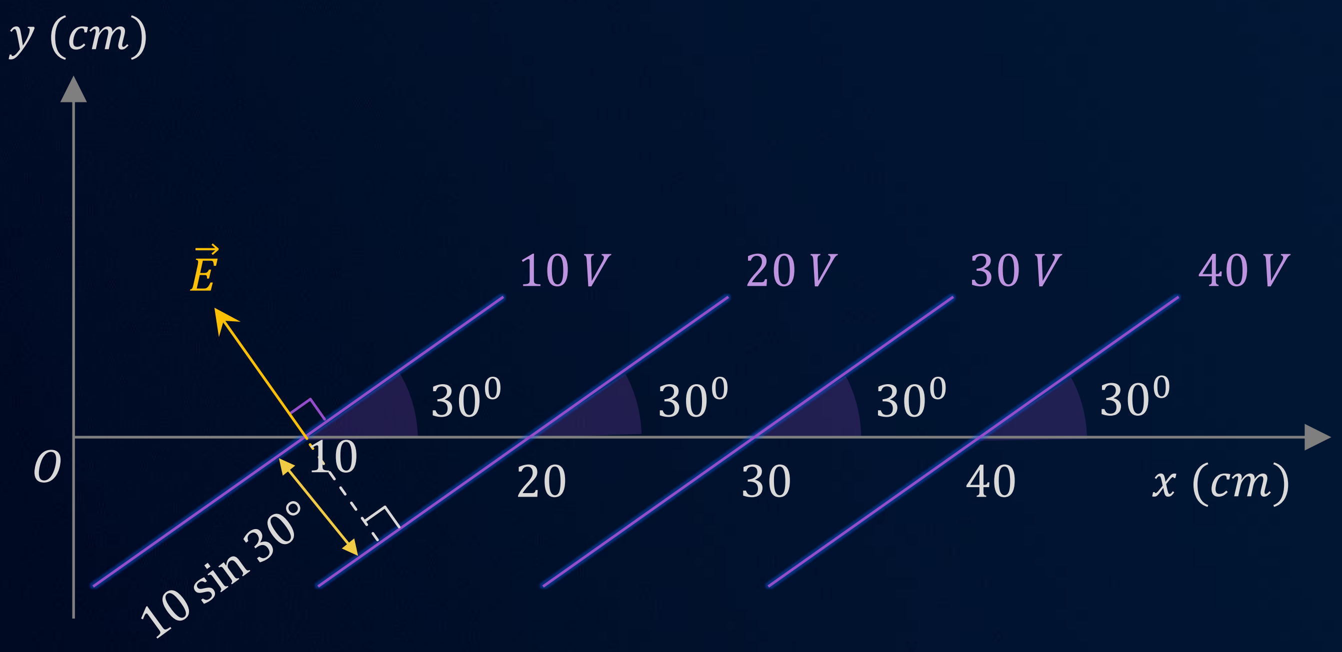
Solution :

$$E = \frac{V_2 - V_1}{d}$$

$$= \frac{20 - 10}{10 \sin 30° \times 10^{-2}} \frac{V}{m}$$

$$= \frac{10}{5} \times 100 \frac{V}{m}$$

$$\boxed{E = 200\ V/m \ \ (120° \text{ w.r.t. } x\text{–axis})}$$



- Direction of electric field is from high potential to low potential.
- Electric field vector is perpendicular to the equipotential surface.

Figure shows lines of constant potential in a region in which an electric field is present. At which point among P, Q, R and S the magnitude of the electric field will be greatest?

Solution :

Relation between electric field and potential :

$$E = -\frac{dV}{dr}$$

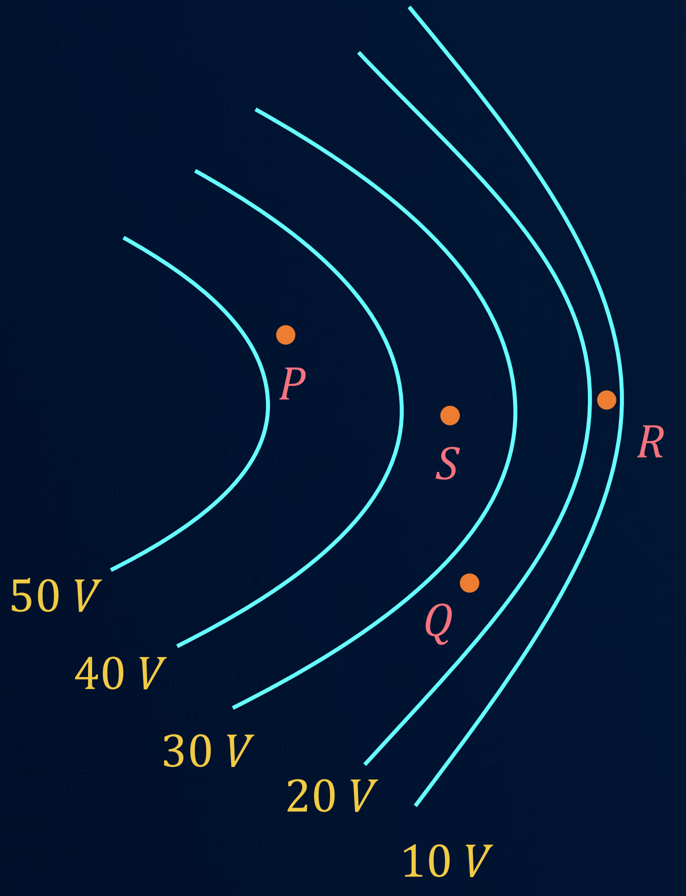
Here, difference in potential between equipotential surfaces is constant ($10\ V$).

For equipotential surfaces, this relation can be written as :

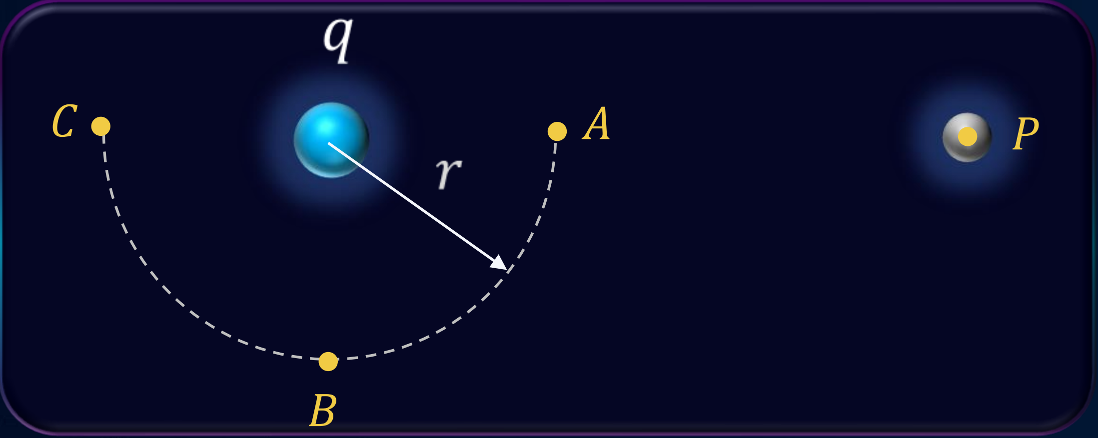
$$E \propto \frac{1}{dr}$$

So, magnitude of the electric field will be greatest where spacing between equipotential surfaces will be less.

The magnitude of electric field will be greatest at point R.



Consider the situation given in the figure. The work done in taking a point charge from P to A is W_A, from P to B is W_B, and from P to C is W_C. Find the relation between $W_A,\ W_B,\ W_C$.

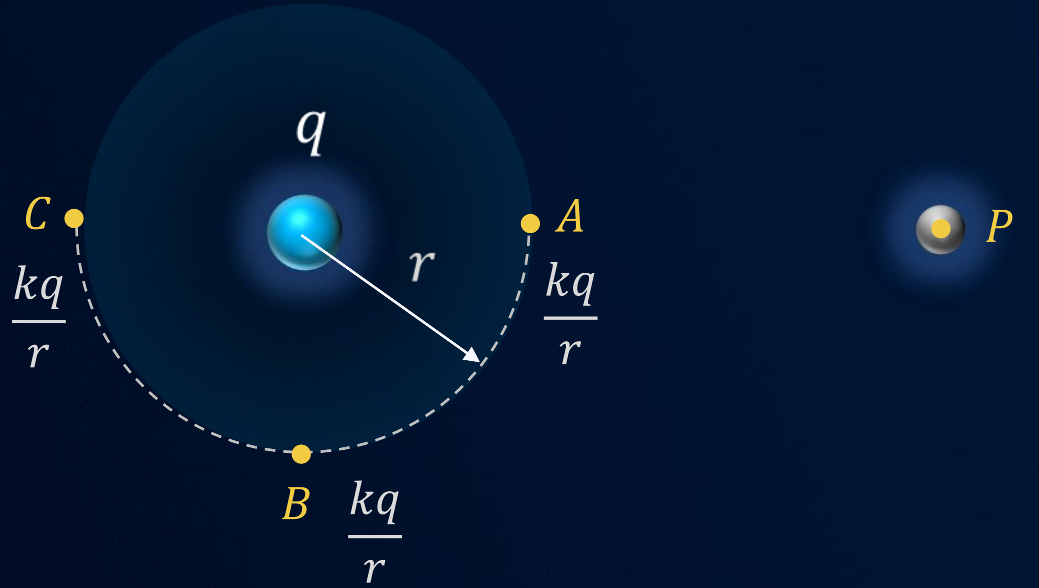


Solution :

- Equipotential surface of a point charge is a sphere.
- Potential of each point A, B and C is same.

$$W_{12} = q\int \vec{E}.d\vec{r} = q(V_1 - V_2)$$

- Work done to go from point P to point A, B and C through individual paths will also be same.



Electric Potential due to Uniformly Charged Ring

- Electric potential at point P due to a charge element dq :

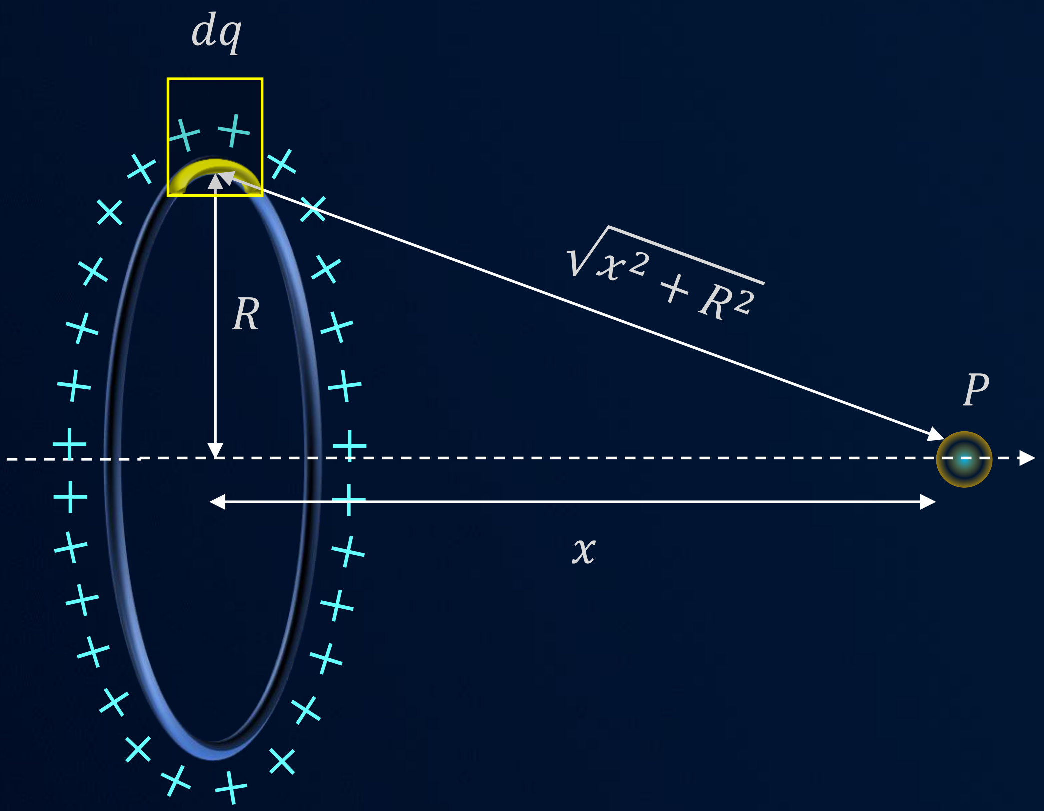
$$dV = \frac{k\,dq}{\sqrt{x^2 + R^2}}$$

- Net electric potential at point P due to whole ring :

$$V = \int dV$$

$$V = \int \frac{k\,dq}{\sqrt{x^2 + R^2}}$$

$$V = \frac{kQ}{\sqrt{x^2 + R^2}}$$



$$\boxed{V = \frac{kQ}{\sqrt{x^2 + R^2}}}$$

Electric Field due to Uniformly Charged Ring

- Electric potential at point P on the axis of uniformly charged ring:

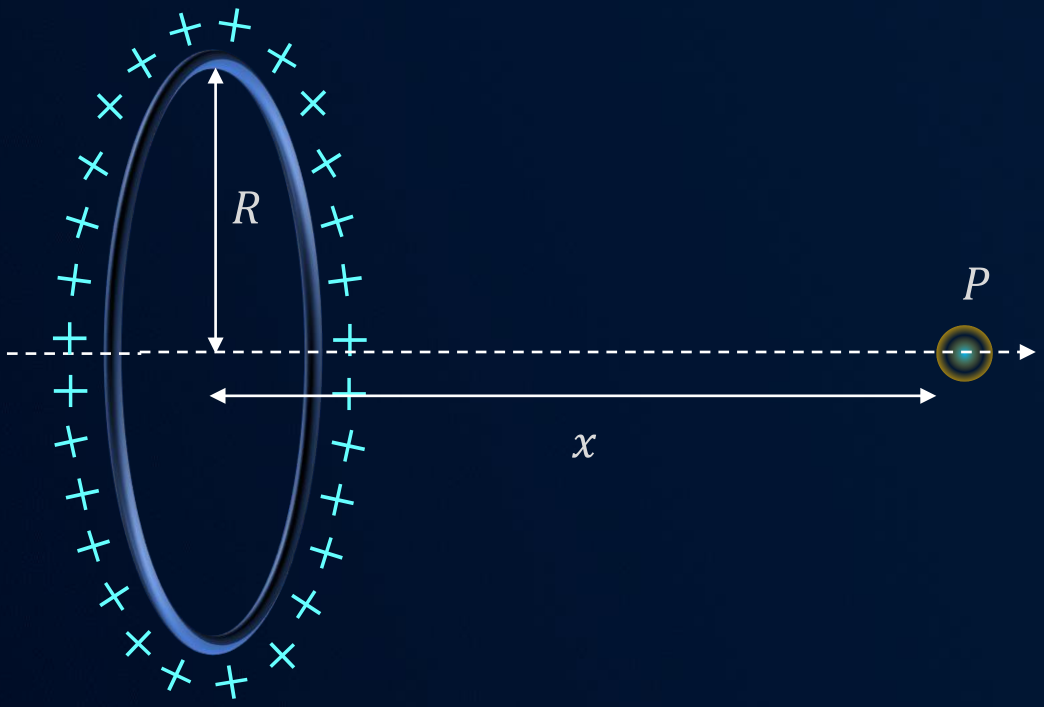
$$V = \frac{kQ}{\sqrt{x^2 + R^2}}$$

- V varies with x only. So, the electric field at P :

$$\vec{E} = -\frac{dV}{dx}\hat{x}$$

$$\vec{E} = -\frac{d}{dx}\left(\frac{kQ}{\sqrt{x^2 + R^2}}\right)\hat{x}$$

$$\vec{E} = (-kQ)\left(-\frac{1}{2}\right)\left((x^2 + R^2)^{-\frac{3}{2}}\right)(2x)\hat{x}$$



$$\boxed{\vec{E} = \frac{kQx}{(x^2 + R^2)^{\frac{3}{2}}}\hat{x}}$$

A non-conducting ring of radius $0.5\ m$ carries a total charge of $1.11 \times 10^{-10}\ C$ distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l}$ ($l = 0$ being the centre of the ring) in volt is ____

Solution :

$$\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} = \int_{l=\infty}^{l=0} dV \qquad \Rightarrow \int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} = V(centre) - V(infinity)$$

If the point at infinity is chosen as reference point, then, $V(infinity) = 0$

$$\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} = V(centre) = \text{Potential at centre of ring}$$

$$\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} = V(centre) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} = \frac{(9 \times 10^9)(1.11 \times 10^{-10})}{0.5} \qquad \Rightarrow \boxed{\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l} \approx 2\ V}$$

A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is

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Solution :

Potential at any on the axis of the ring at distance z from its centre is,

$$V_P = \frac{kq}{\sqrt{R^2 + z^2}} = \frac{kq}{\sqrt{(3a)^2 + (4a)^2}} = \frac{kq}{5a}$$

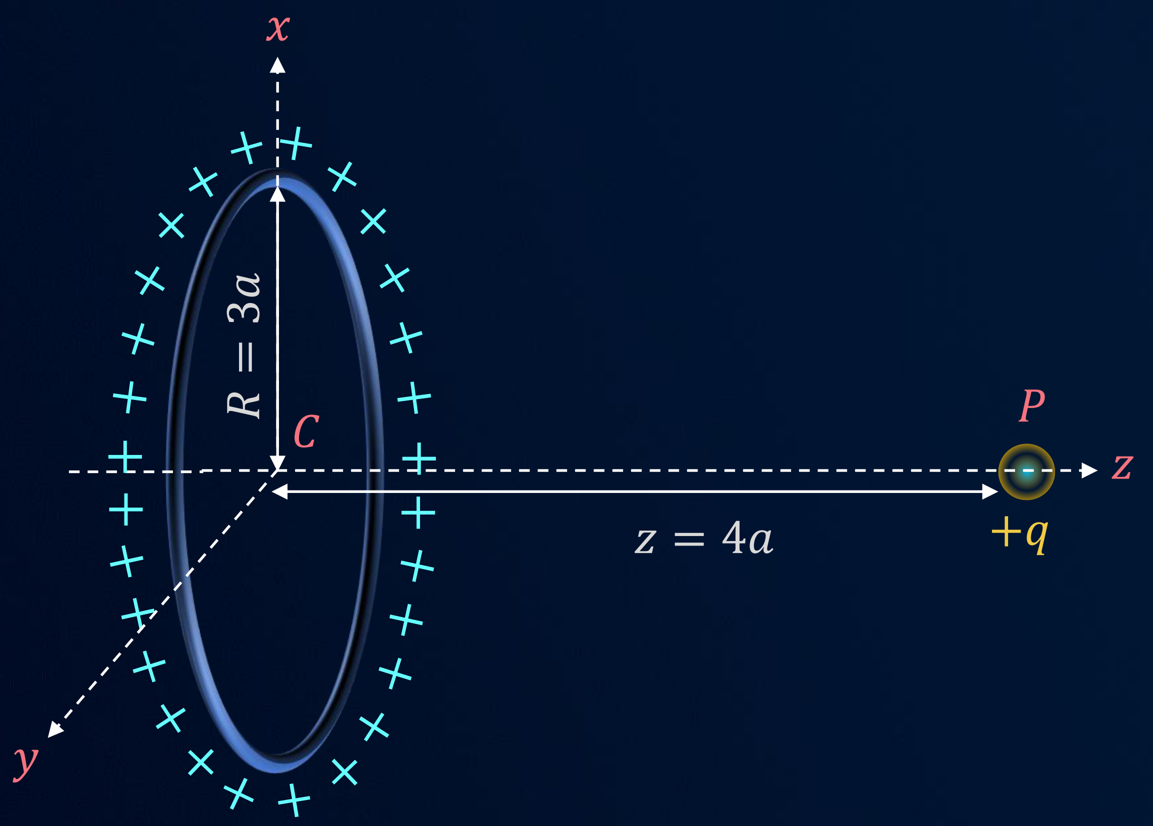
Potential at the centre of the ring is,

$$V_C = \frac{kq}{\sqrt{R^2 + z^2}} = \frac{kq}{\sqrt{(3a)^2 + (0)^2}} = \frac{kq}{3a}$$

Energy required to move the point charge from $x = 4a$ to the centre is,

$$\Delta U = q\Delta V = q(V_C - V_P)$$

$$\Delta U = \frac{kq^2}{a}\left[\frac{1}{3} - \frac{1}{5}\right] \Rightarrow \Delta U = \frac{2}{15}\frac{kq^2}{a}$$



The particle will able to cross the centre of the ring, if,

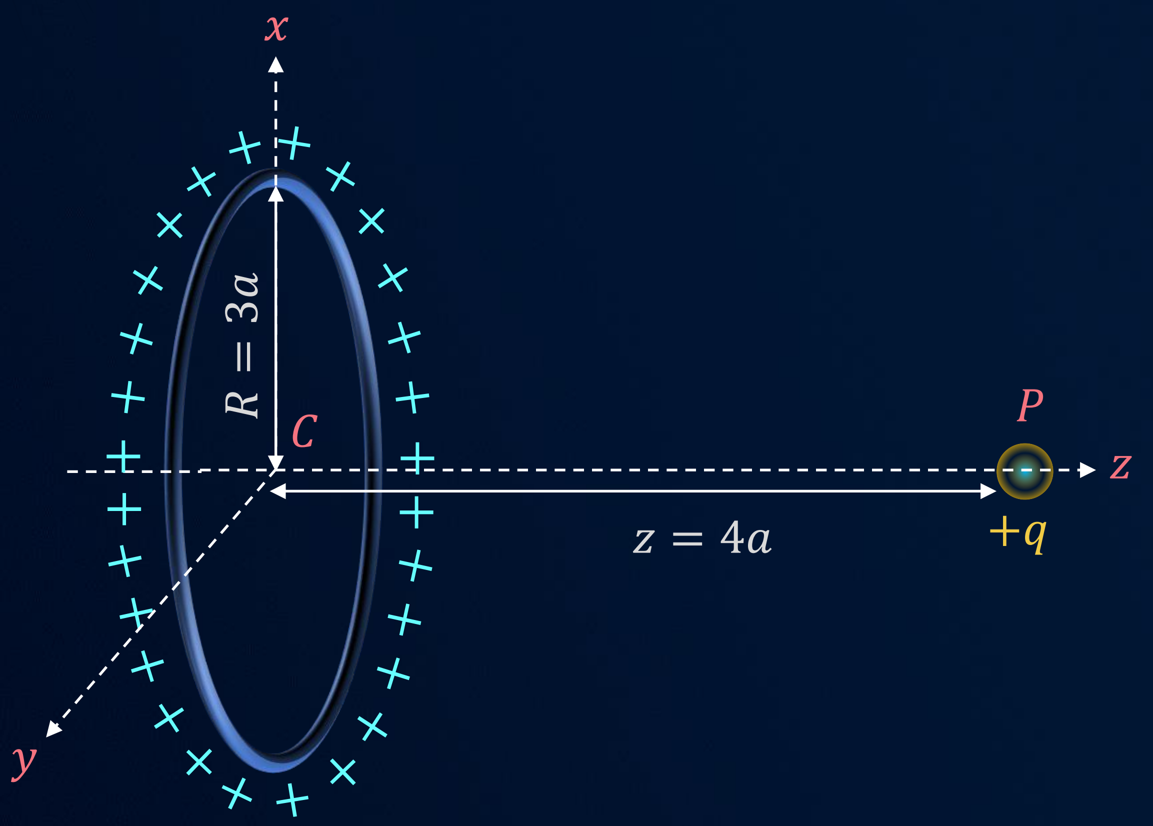
$$\Delta U \leq \frac{1}{2}mv^2$$

So, minimum energy required is,

$$\frac{1}{2}mv^2 = \frac{2}{15}\frac{kq^2}{a}$$

$$v^2 = \frac{2}{m}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)$$

$$\boxed{v = \sqrt{\frac{2}{m}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)}^{1/2}}$$



Electric Potential due to Uniformly Charged Disc

- Due to symmetry, a disc can be thought of as a collection of concentric rings.

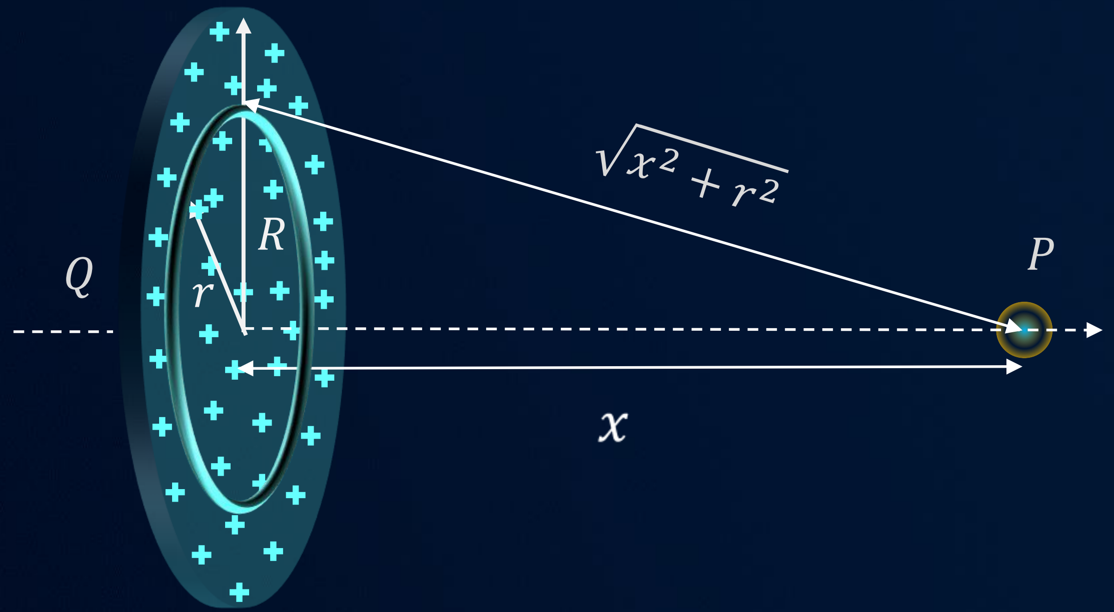
- Surface charge density : $\sigma = \dfrac{Q}{\pi R^2}$

- Electric potential at point P due to a elementary charged ring of charge dq, radius r and thickness dr :

$$dV = \frac{k\,(\sigma \times 2\pi r\,dr)}{\sqrt{x^2 + r^2}}$$

- Electric potential at point P due to whole disc :

$$V = \int_0^R \frac{k\,(\sigma \times 2\pi r\,dr)}{\sqrt{x^2 + r^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{x^2 + r^2}}$$



$$\boxed{V(x) = \frac{\sigma}{2\varepsilon_0}\left[(x^2 + R^2)^{\frac{1}{2}} - x\right]}$$

Electric Field due to Uniformly Charged Disc

- Electric potential at point P on the axis of uniformly charged disc :

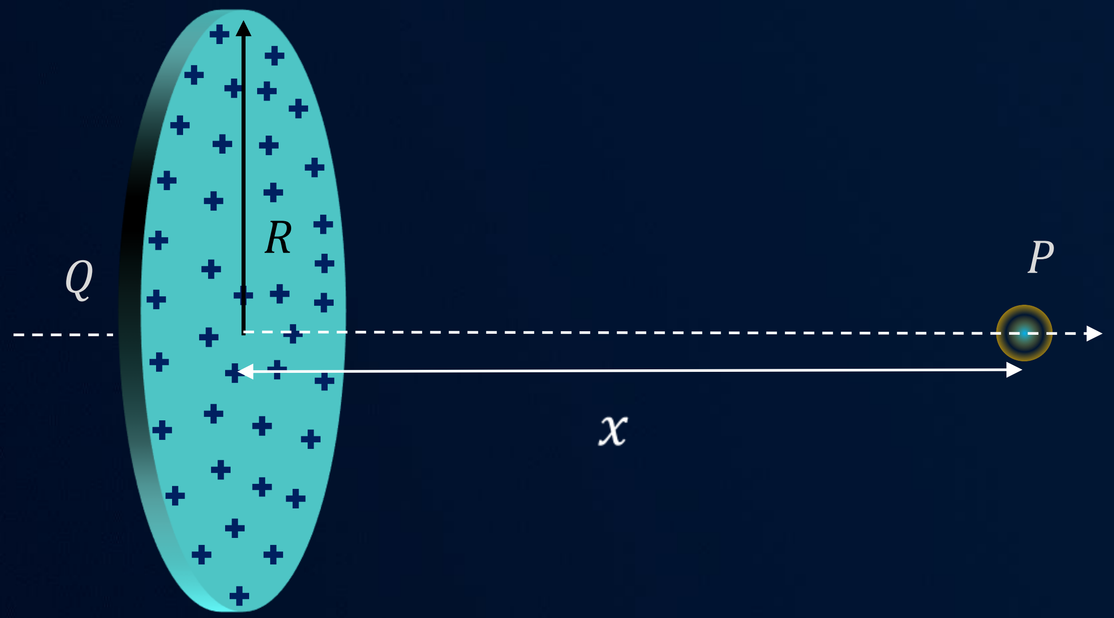
$$V(x) = \frac{\sigma}{2\varepsilon_0}\left[(x^2 + R^2)^{\frac{1}{2}} - x\right]$$

- V varies with x only. So, the electric field at P :

$$\vec{E} = -\frac{dV}{dx}\hat{x}$$

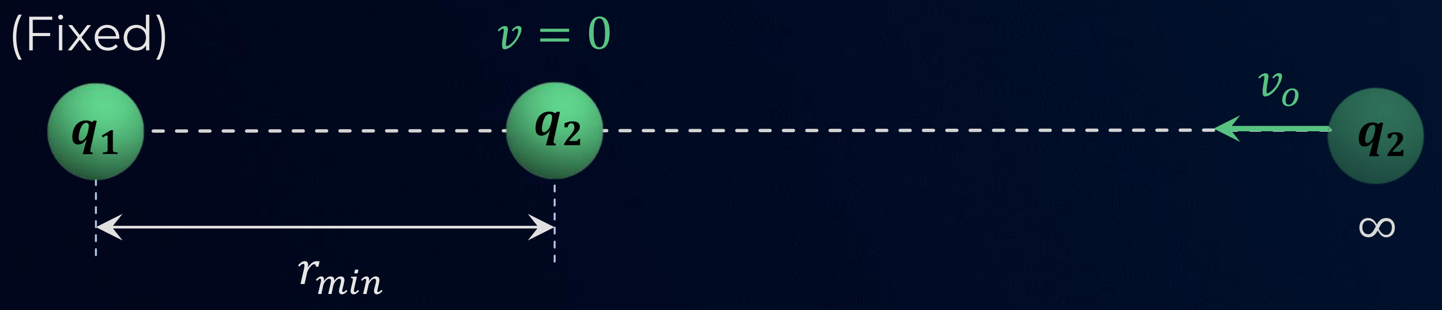
$$\vec{E} = \left(-\frac{\sigma}{2\varepsilon_0}\right)\left[\left\{\left(\frac{1}{2}\right)\left((x^2 + R^2)^{-\frac{1}{2}}\right)(2x)\right\} - 1\right]\hat{x}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}}\right]\hat{x}}$$



A point charge q_1 is fixed and a point charge q_2 of mass m at infinity starts moving towards q_1 with velocity v_o as shown in figure. Find the minimum separation distance between the two charges.

Solution :



Using Principle of Conservation of Mechanical Energy,

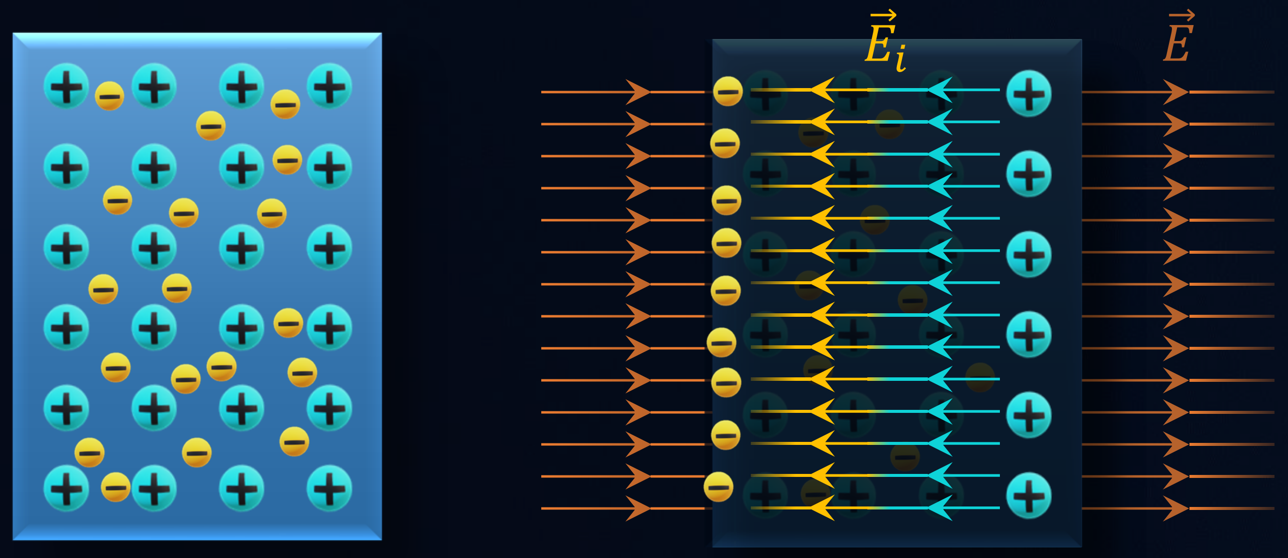
$$U_i + K_i = U_f + K_f$$

$$\Rightarrow 0 + \frac{1}{2}mv_0^2 = \frac{kq_1q_2}{r_{min}} + 0$$

$$\boxed{r_{min} = \frac{2kq_1q_2}{mv_0^2}}$$

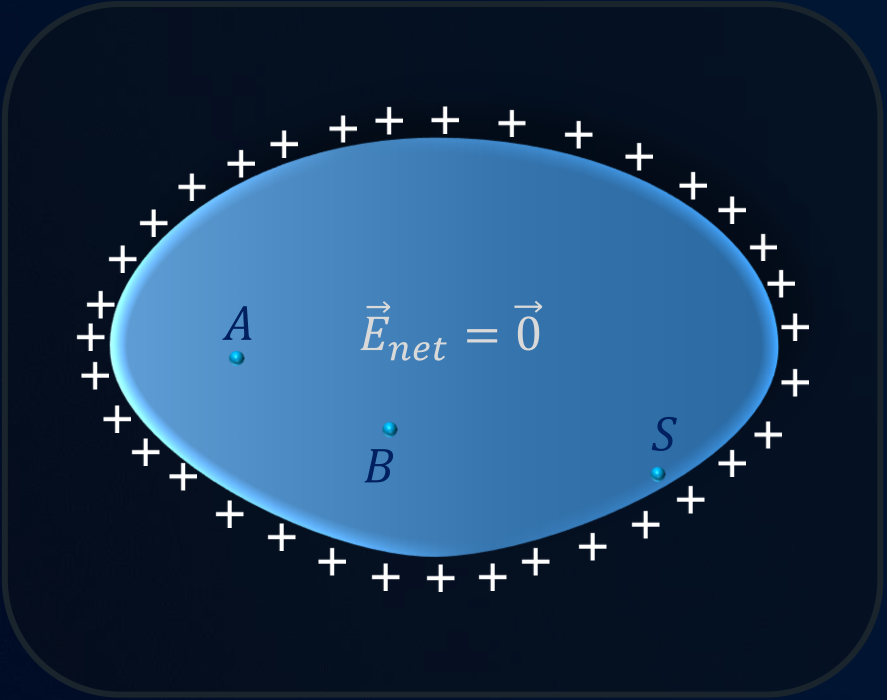
Properties of a Conductor

Excess charge given to an isolated conductor redistributes itself on the conductor's surface in order to minimize the potential energy and maximize stability.



When an isolated conductor is placed in an external electric field, internal division of charges occurs, resulting an internal Electric Field ($\vec{E}_i$) in the direction opposite to external Field ($\vec{E}$). This phenomenon is call Polarisation of charges.

Conductor under Electrostatic Steady State Conditions



Net electric field inside a conductor is zero.

$$\left(\vec{E}_{net} = \vec{E}_i + \vec{E} = \vec{0}\right)$$

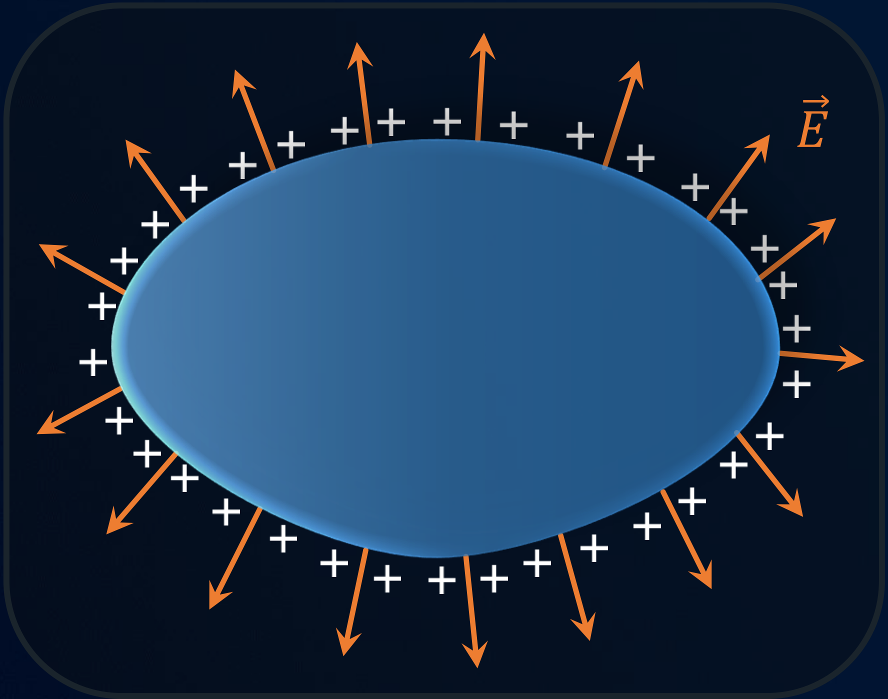
Conductor is an equipotential body under electrostatic conditions.

$$\left(dV = \vec{E} \cdot d\vec{r} = 0\right)$$

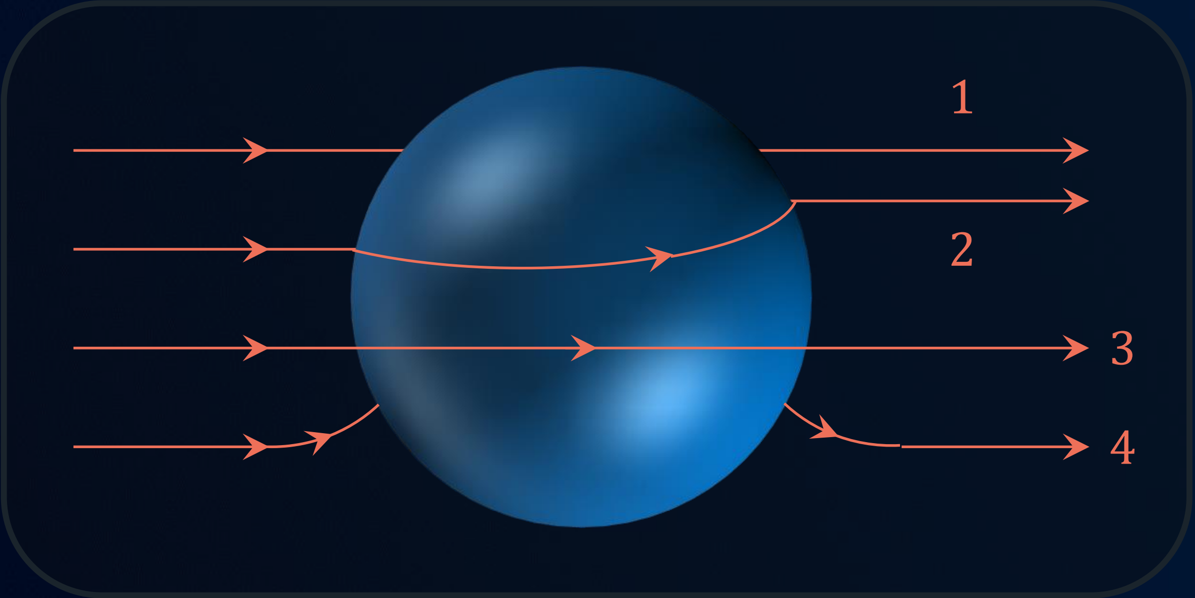
Electrostatics Steady State

- For $\vec{E}_t(tangetial\ component\ of\ E) \neq 0$, the charge will start moving along the boundary of conductor, which means the conditions of electrostatic steady state are no longer satisfied.

- Hence, the net electric field at the surface of the conductor is always perpendicular to the surface.



A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as



A 1

B 2

C 3

D 4

Conductors with Cavity

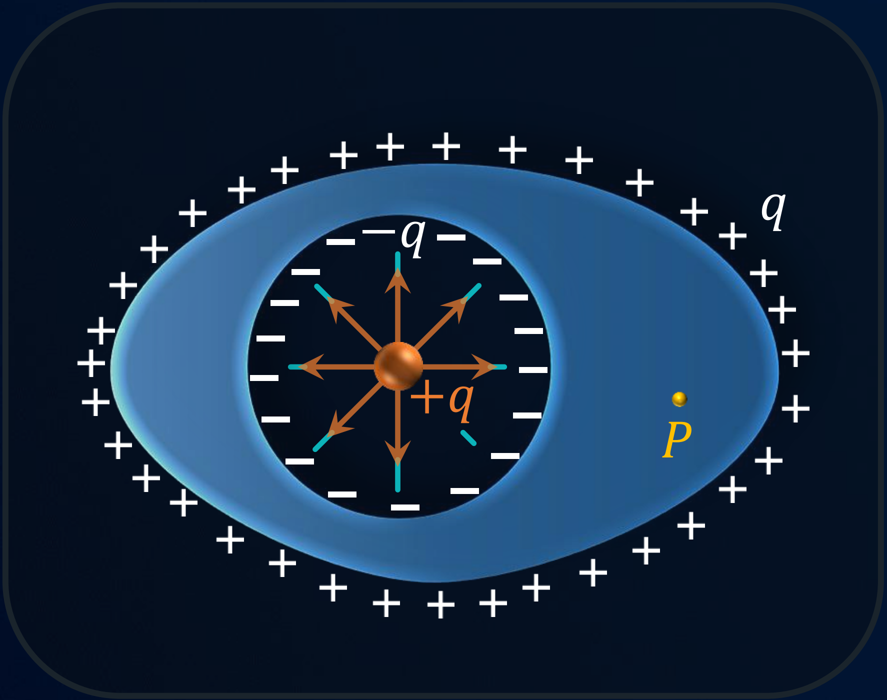
- Under electrostatic steady state condition, net electric field in the material of the conductor is zero.

- In accordance with the Uniqueness theorem,

  Inside the conductor material,

$$\vec{E}_q + \vec{E}_{induced} = 0$$

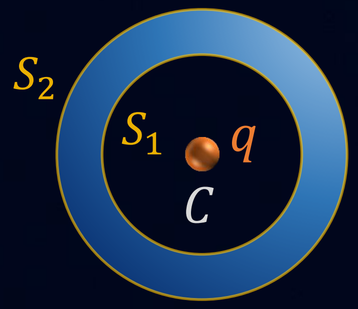
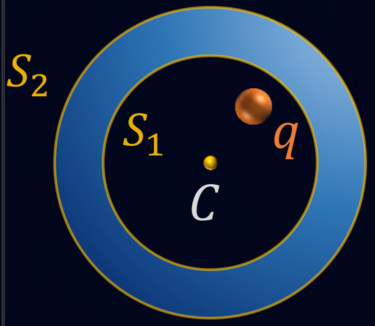
$$\vec{E}_{surface} = 0$$

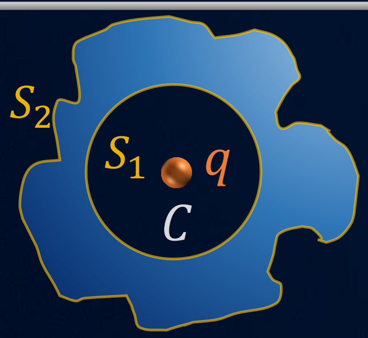
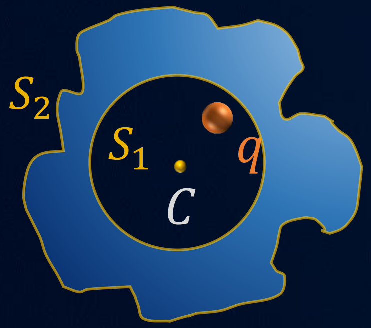


- The charge distribution on the outer surface is unaffected by the movement of the point charge inside the cavity.

- External charge brought near the conductor will only affects the outer surface charge distribution of the conductor.

Conductors with Cavity (Geometry)

Shape of Conductor	Charge distribution(S_1)	Charge distribution(S_2)
	Uniform	Uniform
	Non–uniform	Uniform

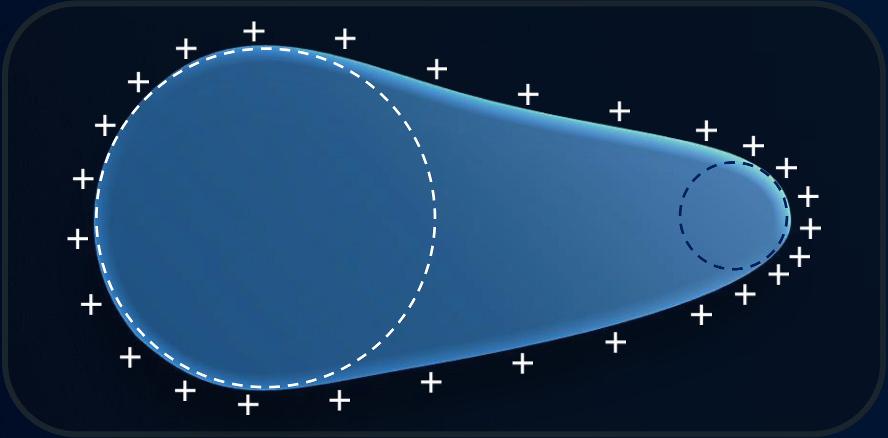
Shape of Conductor	Charge distribution(S_1)	Charge distribution(S_2)
	Uniform	Non–uniform
	Non–uniform	Non–uniform

Conductors with Cavity (Geometry)

Shape of Conductor	Charge distribution (S_1)	Charge distribution (S_2)
	Non-uniform	Uniform
	Non-uniform	Non-uniform

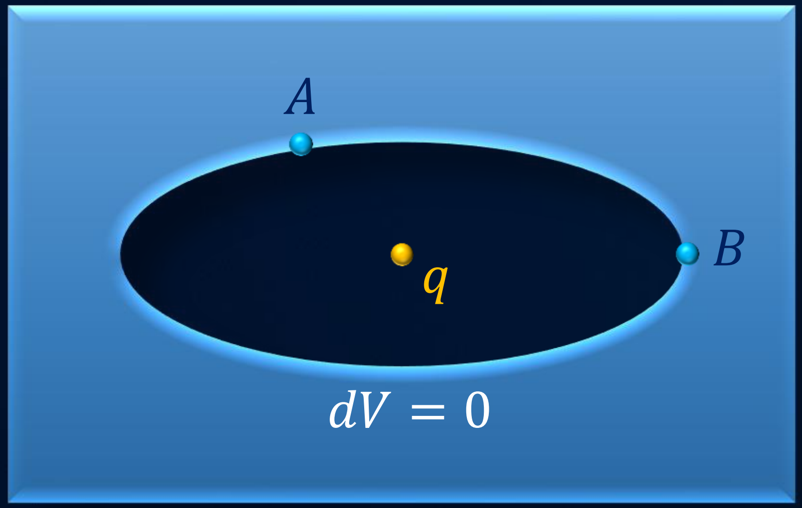
Under electrostatic condition, the surface charge density at any point is inversely proportional to its radius of curvature.

$$\sigma \propto \frac{1}{r}$$



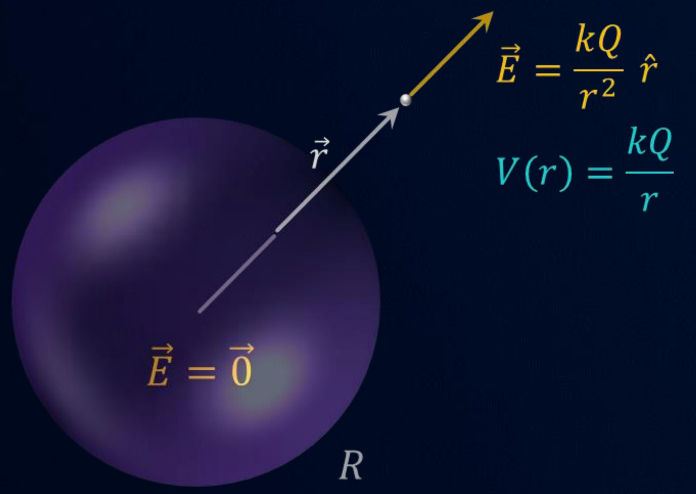
An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then

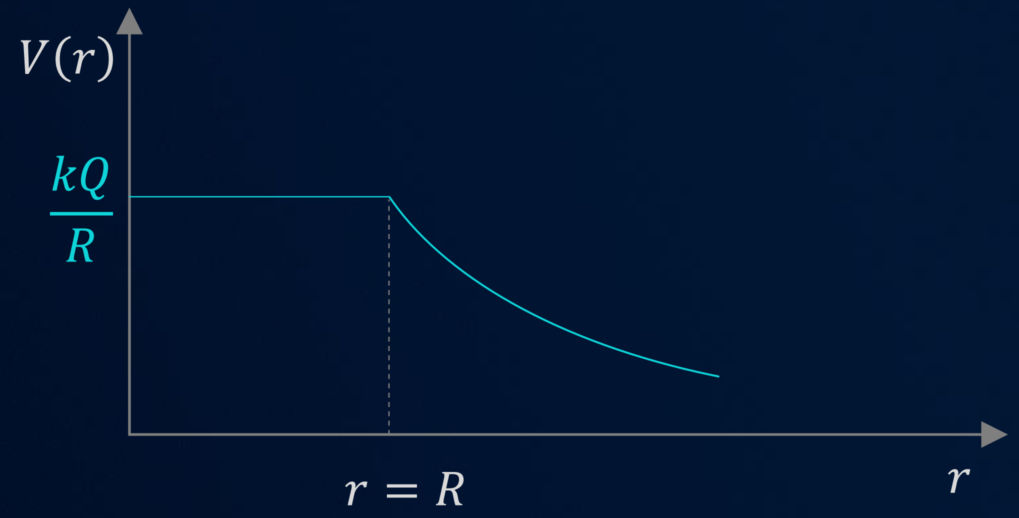
- **A** electric field near A in the cavity = electric field near B in the cavity.
- **B** charge density at A = charge density at B
- **C** potential at A = potential at B
- **D** None of these



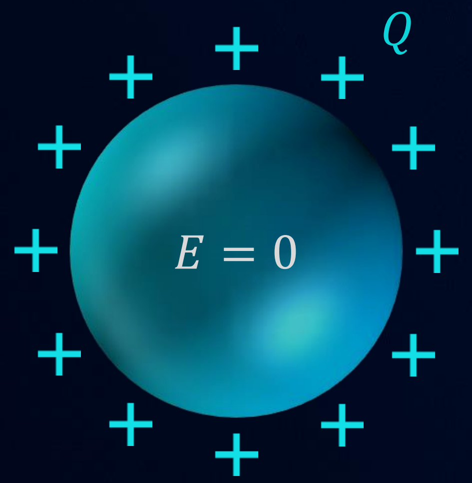
Electric Potential

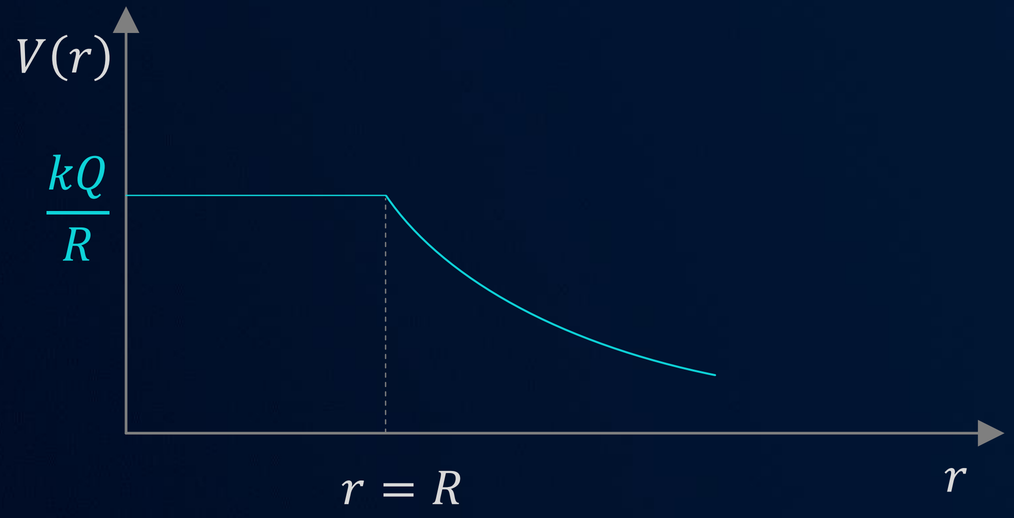
Uniformly Charged Thin Spherical Shell





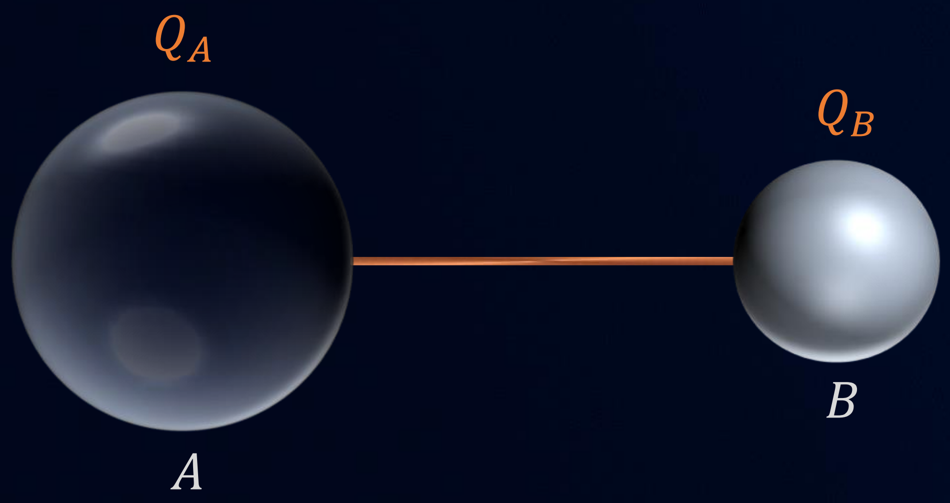
Solid Conducting Sphere





A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then find out;

1. Electric field inside shell A



The electric field inside a conducting shell is always zero.

Electric field inside shell A $= 0$

2. *relation between* Q_A & Q_B

When both the shell and sphere are connected by a metal wire, their potentials become equal.

$$\Rightarrow V_A = V_B$$

$$\Rightarrow \frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$$\Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} > 1$$

$$\Rightarrow Q_A > Q_B$$

3. $\frac{\sigma_A}{\sigma_B}$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B}$$

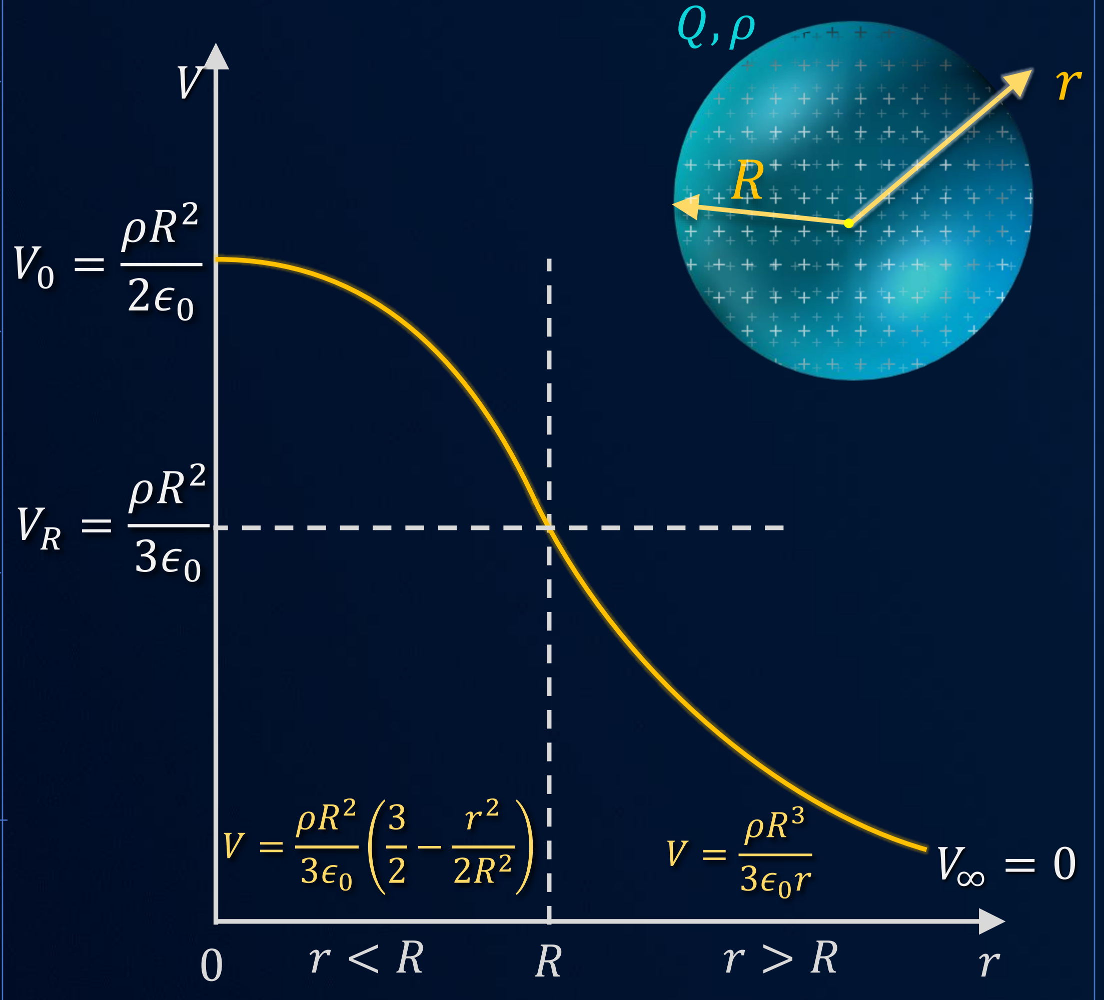
$$\Rightarrow \frac{Q_A}{4\pi R_A^2} \times \frac{4\pi R_B^2}{Q_B} = \frac{R_A}{R_B} \times \frac{4\pi R_B^2}{4\pi R_A^2}$$

$$\Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$$

Electric Potential

Uniformly Charged Non-Conducting Sphere

		In terms of ρ:
Outside the sphere $(r > R)$	$V(r) = \frac{kQ}{r}$	$V(r) = \frac{\rho R^3}{3\varepsilon_0 r}$
At the surface of the sphere $(r = R)$:	$V(r) = \frac{kQ}{R}$	$V(r) = \frac{\rho R^2}{3\varepsilon_0}$
Inside the sphere $(r < R)$:	$V(r) = \frac{kQ}{2R^3}(3R^2 - r^2)$	$V(r) = \frac{\rho R^2}{3\varepsilon_0}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$
At the centre $(r = 0)$:	$V(r) = \frac{3kQ}{2R}$	$V(r) = \frac{\rho R^2}{2\varepsilon_0}$



Consider a uniformly charged non-conducting sphere of radius R. Electric potential at distance $2R$ from its centre was found to be $20\ V$. What would be the potential at the centre of the sphere?

Solution :

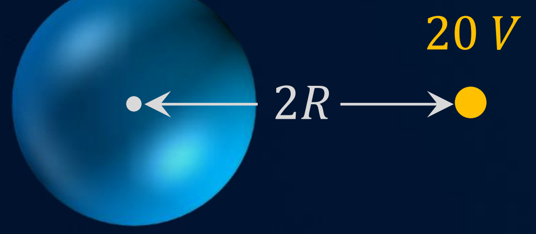
Electric potential outside the uniformly charged non-conducting sphere is given by,

$$V(r) = \frac{\rho R^3}{3\epsilon_0 r} \quad \text{for } r > R$$

$$\Rightarrow V(r = 2R) = \frac{\rho R^3}{3\epsilon_0 \times 2R} = \frac{\rho R^2}{6\epsilon_0}$$

Electric potential at the center of the uniformly charged non-conducting sphere is given by,

$$V(r = 0) = \frac{\rho R^2}{2\epsilon_0}$$



$$\Rightarrow 3 \times V(r = 2R) = V(r = 0)$$

$$\Rightarrow 3 \times 20 = V(r = 0)$$

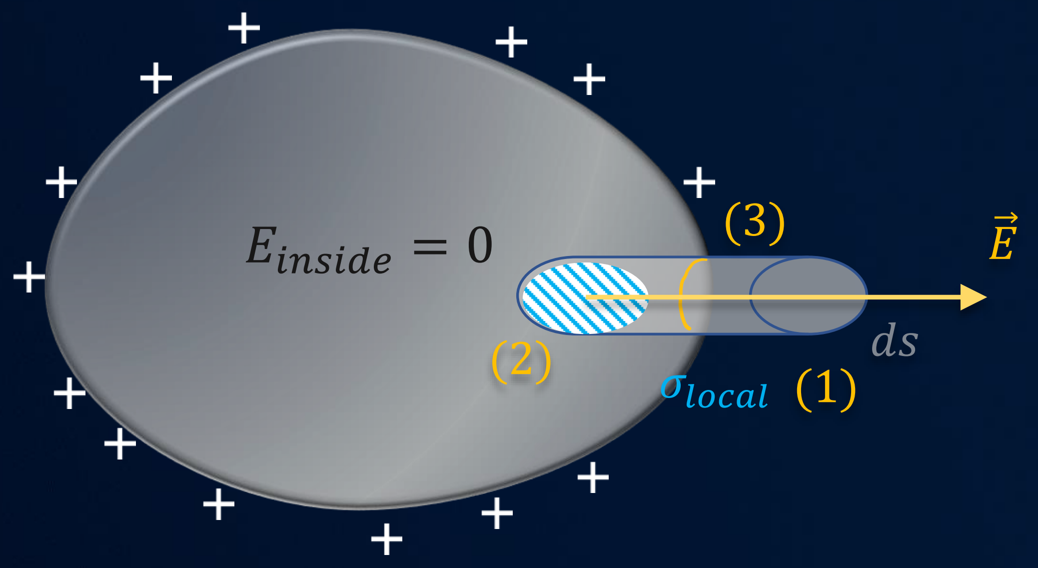
$$\boxed{V(r = 0) = 60\ V}$$

Electric Field outside Charged Conductor's surface

We take a small area ds of the conductor of charge density σ_{local},

Take a Gaussian cylindrical surface as shown,

Applying Gauss's Theorem,



$$\phi_{net} = \phi_1 + \phi_2 + \phi_3$$

$$\phi_1 = E \cdot ds = \frac{\sigma_{local} \cdot ds}{\varepsilon_0} \Rightarrow E = \frac{\sigma_{local}}{\varepsilon_0}$$

$\phi_2 = 0$ (E is zero inside the conductor)

$\phi_3 = 0$ (E is parallel to the curved surface)

Thus, Electric field just outside the charged conductor's surface is,

$$\boxed{E_{net} = \frac{\sigma_{local}}{\varepsilon_0}}$$

Electric Pressure on a Charged Metal Surface

Let us consider the electric field at points X and Y,

At X, $\vec{E}_X = \vec{E}_{dS} + \vec{E}_{rest} = \dfrac{\sigma_{local}}{\varepsilon_0}\hat{n}$

At Y, $\vec{E}_Y = \vec{E}_{dS} + \vec{E}_{rest} = \vec{0}$

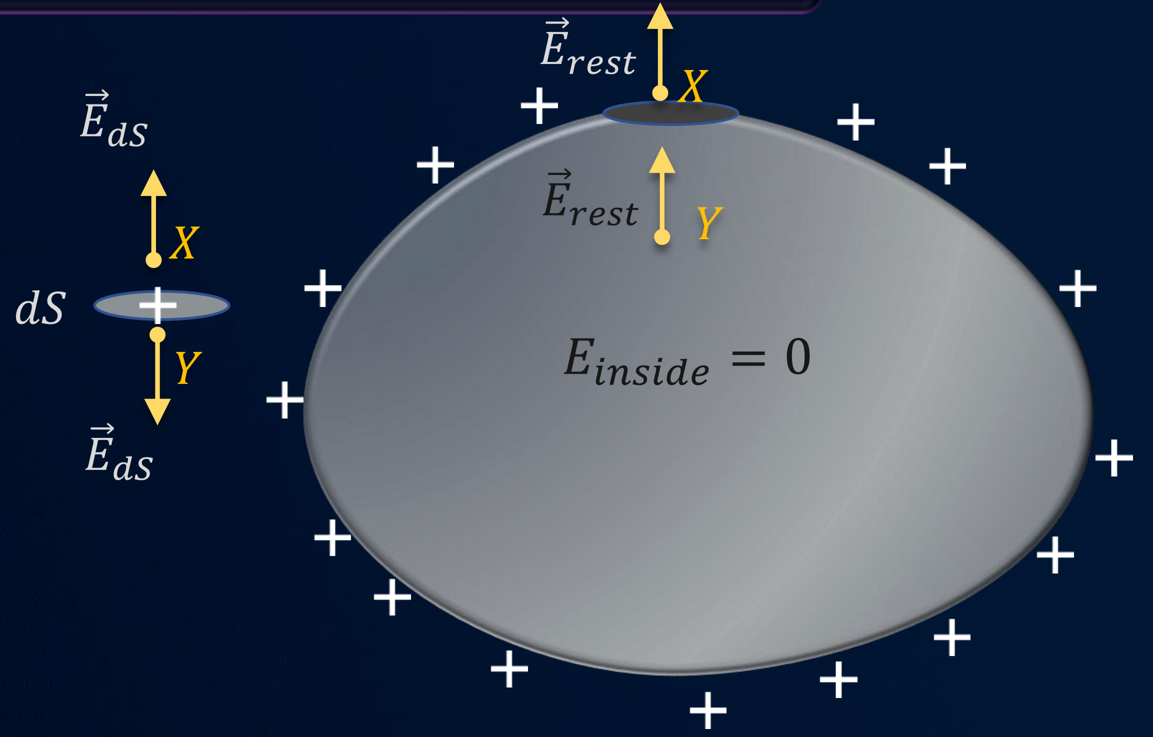
From the two equations,

$$\vec{E}_{dS} = \frac{\sigma_{local}}{2\varepsilon_0}\hat{n} \Rightarrow |\vec{E}_{rest}| = |\vec{E}_{dS}| = \frac{\sigma_{local}}{2\varepsilon_0}$$

Calculating the force experienced by dS,

$$(dF)_{dS} = (dq)_{dS} \times E_{rest}$$

$$\Rightarrow (dF)_{dS} = \frac{(\sigma_{local})^2}{2\varepsilon_0} dS$$



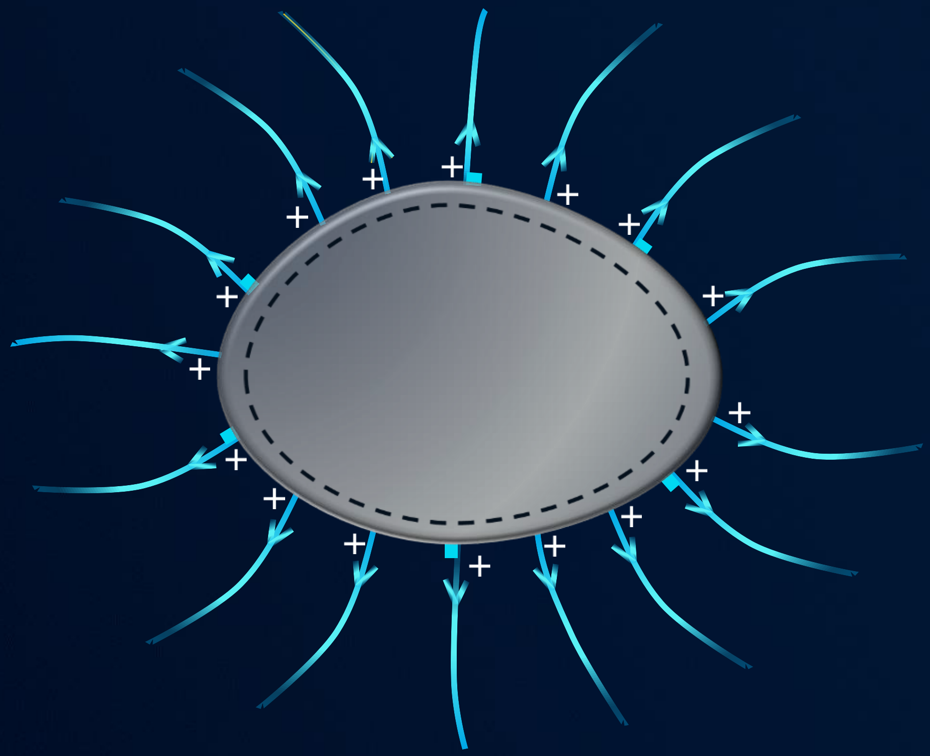
The pressure experienced by the small section of area dS is,

$$\boxed{P_e = \frac{(\sigma_{local})^2}{2\varepsilon_0}}$$

Field Energy of Electric Field

We know that the electrostatic pressure on the surface of the conductor due to mutual repulsion of the charges is $P_e = \frac{(\sigma_{local})^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E_{net}^2$

Let dV be the change in volume due to this pressure.



Thus, work done dW, $|dW| = dU = P_e dV$

$$dU = \left(\frac{1}{2}\varepsilon_0 E_{net}^2\right) dV$$

Thus, energy density is, $\boxed{\frac{dU}{dV} = \left(\frac{1}{2}\varepsilon_0 E_{net}^2\right)}$

The energy associated with a uniform electric field is,

$$\boxed{U = \int dU = \left(\frac{1}{2}\varepsilon_0 E_{net}^2\right) V}$$

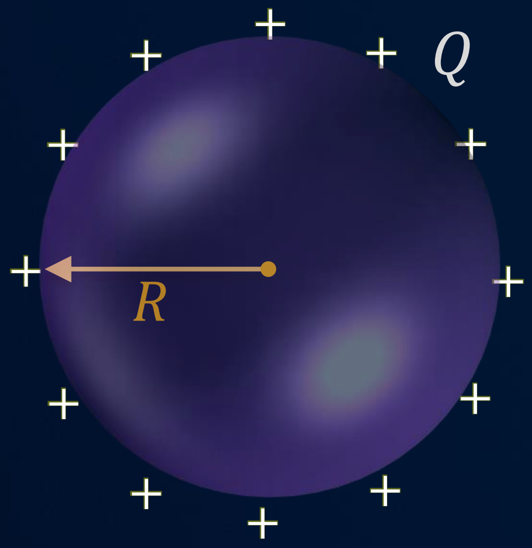
The energy associated with a non-uniform electric field is,

$$\boxed{U = \int dU = \int \left[\frac{1}{2}\varepsilon_0 E^2\right] dV}$$

Self Energy of a Uniform Spherical Shell

The total work required to assemble the charges on the shell will be stored as the potential energy of the shell. This stored energy is known as its self energy

Let's take an initially uncharged thin spherical shell and charge it by bringing charges externally from infinity to the shell.



Electric potential due to the charge q on surface is,

$$V = \frac{kq}{R}$$

Total work to assemble Q charge on the shell is,

$$W_{net} = \int dW_{net} = \int V dq = \int_0^Q \frac{kq}{R} dq$$

Thus, self energy of the uniformly charged spherical shell is,

$$\boxed{U_{net} = \frac{kQ^2}{2R}}$$

Self Energy

Consider two regions of storing the self energy of the shell: space inside the boundary of the shell, and space outside the boundary

$$U_{self} = U_{inside} + U_{outside}$$

Since the electric field inside is zero,

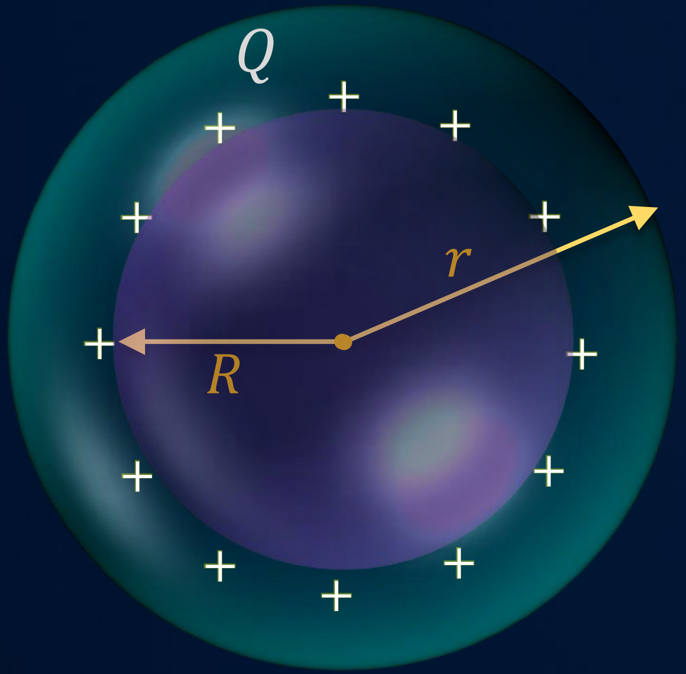
$$U_{self} = 0 + \int \frac{1}{2}\varepsilon_0 E_{outside}{}^2\, dV = \frac{\varepsilon_0}{2} \int_{outside} E^2 dV$$

Using a hypothetical shell of radius $r (r > R)$, and thickness dr, we get $dV = 4\pi r^2 dr$

$$U_{self} = \frac{\varepsilon_0}{2} \int_R^{\infty} \left[\frac{Q}{4\pi\varepsilon_0 r^2}\right]^2 4\pi r^2 dr$$

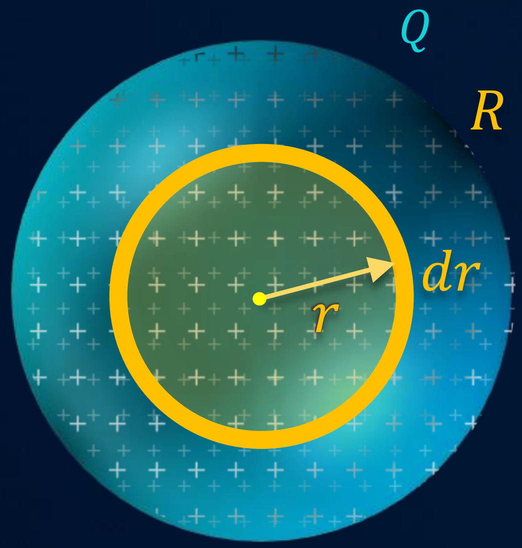
Solving, we get

$$\boxed{U_{self} = \frac{kQ^2}{2R}}$$



Self Energy of a Uniformly Charged Sphere

Volume Charge density of the sphere is, $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3} = \dfrac{3Q}{4\pi R^3}$



Charge contained inside the sphere of radius r is,

$$q = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi r^3 \times \frac{3Q}{4\pi R^3} = \frac{Qr^3}{R^3}$$

Potential at the surface of the considered shell is,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{Qr^3}{R^3} \times \frac{1}{4\pi\varepsilon_0 r} = \frac{Qr^2}{4\pi\varepsilon_0 R^3}$$

Charge required to increase the radius from r to $r + dr$ is,

$$dq = \rho \cdot 4\pi r^2 dr = \frac{3Q}{4\pi R^3} \times 4\pi r^2 dr = \frac{3Q}{R^3} r^2 dr$$

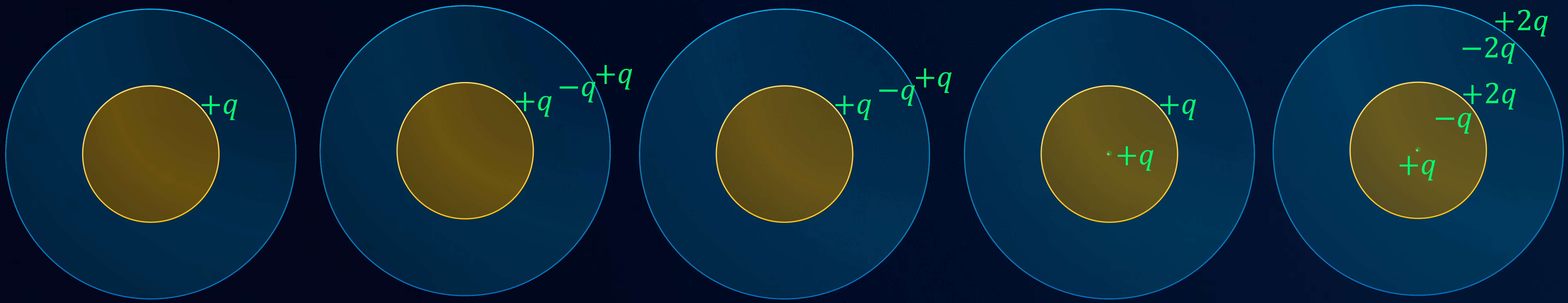
The total work done in assembling the charged sphere of radius R is,

$$W_{net} = \int_0^Q V\,dq = \int_0^R \frac{Qr^2}{4\pi\varepsilon_0 R^3} \times \frac{3Q}{R^3} r^2 dr$$

$$= \frac{3Q^2}{4\pi\varepsilon_0 R^6} \int_0^R r^4\, dr = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

$$\boxed{U_{self} = \frac{3Q^2}{20\pi\varepsilon_0 R}}$$

Induction and Redistribution of Charge



Earthing of Conductors

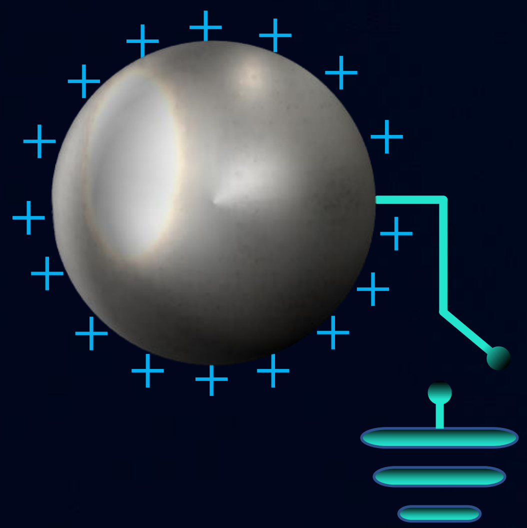
Electrostatically,

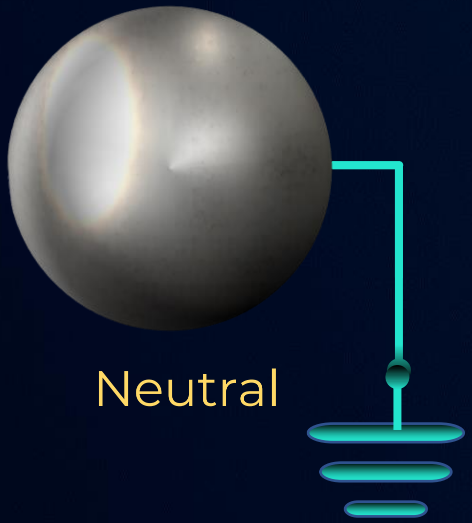
- Earth is an infinite resource and a sink of charges
- Earth's potential is assumed to be zero

i.e. $$V_e = \frac{kQ}{R_e} = 0$$

Whenever a conductor is grounded, it results in the charge transfer to/from the Earth till its potential becomes zero.

It is not necessary that the charge on the body becomes zero. There are certain cases where the potential of the body becomes zero but the net charge remains non-zero





Thus, after earthing,

$$\boxed{V_{conductor} = V_{earth} = 0}$$

There are two uncharged identical metallic spheres of radius a, separated by a distance d. A charged metallic sphere (charge q) of same radius is brought and it touches sphere 1. After some time, it is moved to a far off distance. After this, the sphere 2 is earthed. Find the charge on sphere 2.

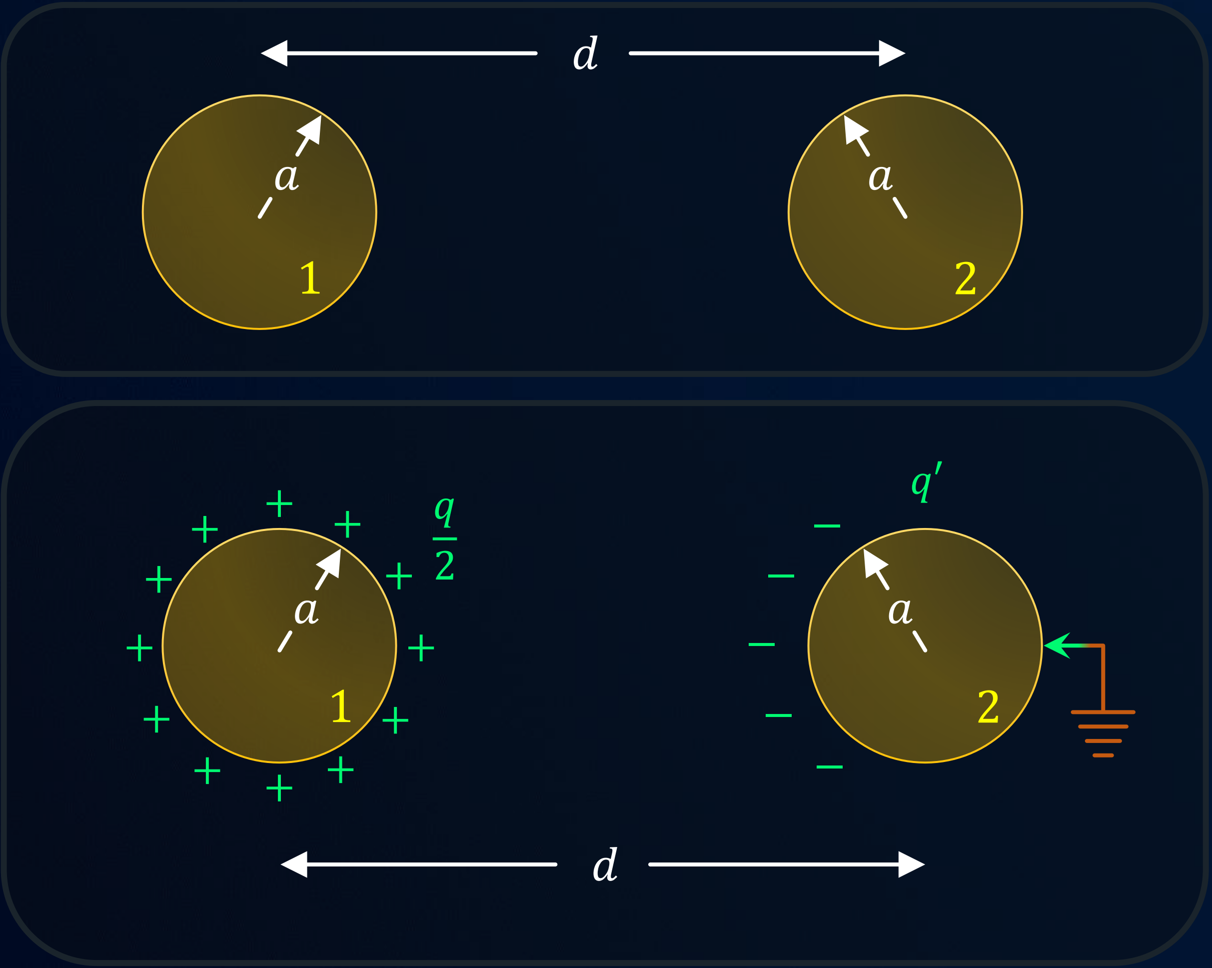
Solution :

Since the sphere 2 is grounded,

$$V_2 = \frac{k\left(\frac{q}{2}\right)}{d} + \frac{kq'}{a} = 0$$

$$\Rightarrow \frac{q}{2d} + \frac{q'}{a} = 0$$

$$\boxed{q' = -\frac{qa}{2d}}$$



A charge q is distributed uniformly on the surface of a solid sphere of radius R. It is covered by a concentric hollow conducting sphere of radius $2R$. Find the charges on the inner and outer surfaces of the hollow sphere if it is earthed.

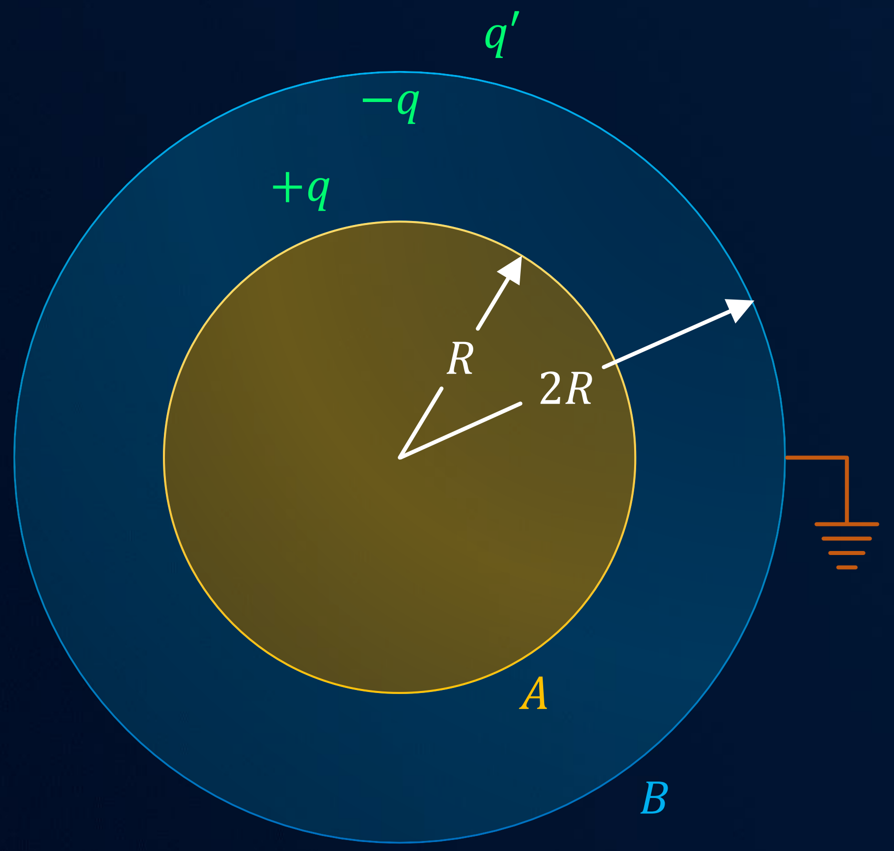
Solution :

Since the shell B is grounded outside,

$$V_B = \frac{k(+q)}{2R} + \frac{k(-q)}{2R} + \frac{kq'}{2R} = 0$$

$$\Rightarrow \frac{kq'}{2R} = 0$$

$$\boxed{q' = 0}$$



Two concentric shells A and B have radii a and b as shown in the figure. The shell B is given charge q and the shell A is earthed. Find the charge appearing on the outer surface of shell A.

Solution :

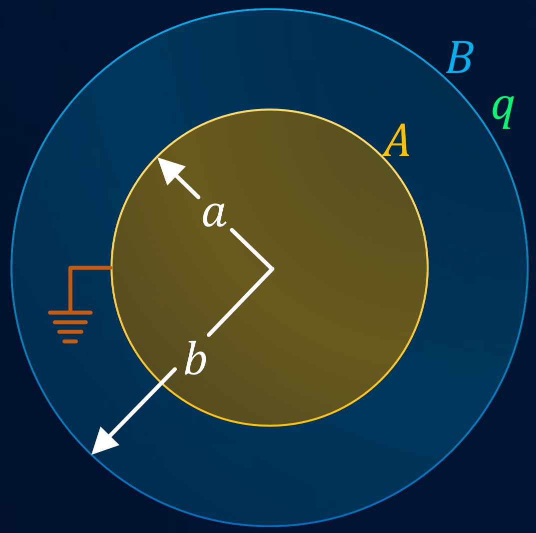
Since the shell A is grounded outside,

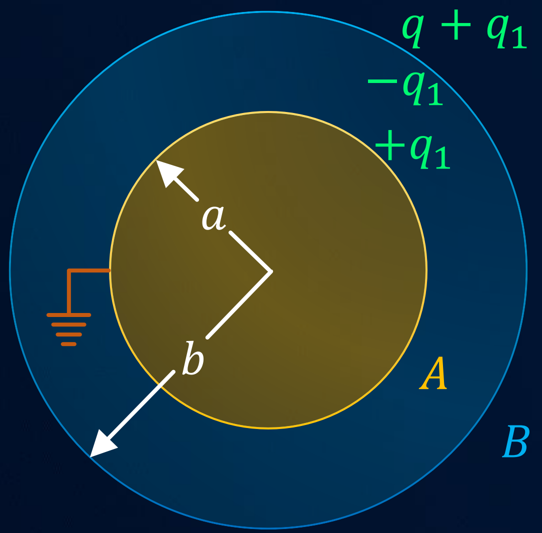
$$V_A = \frac{k(q_1)}{a} + \frac{k(-q_1)}{b} + \frac{k(q+q_1)}{b} = 0$$

$$\Rightarrow \frac{q_1}{a} - \frac{q_1}{b} + \frac{q}{b} + \frac{q_1}{b} = 0$$

$$\Rightarrow q_1 = -\frac{a}{b}q$$

$$\boxed{(q_A)_{outer} = -\frac{qa}{b}}$$





Three concentric shells A, B and C have radii a, b and c. The shells A and C are given charges q and $-q$ respectively and shell B is earthed. Find the charges appearing on the outer surface of B and C.

Solution :

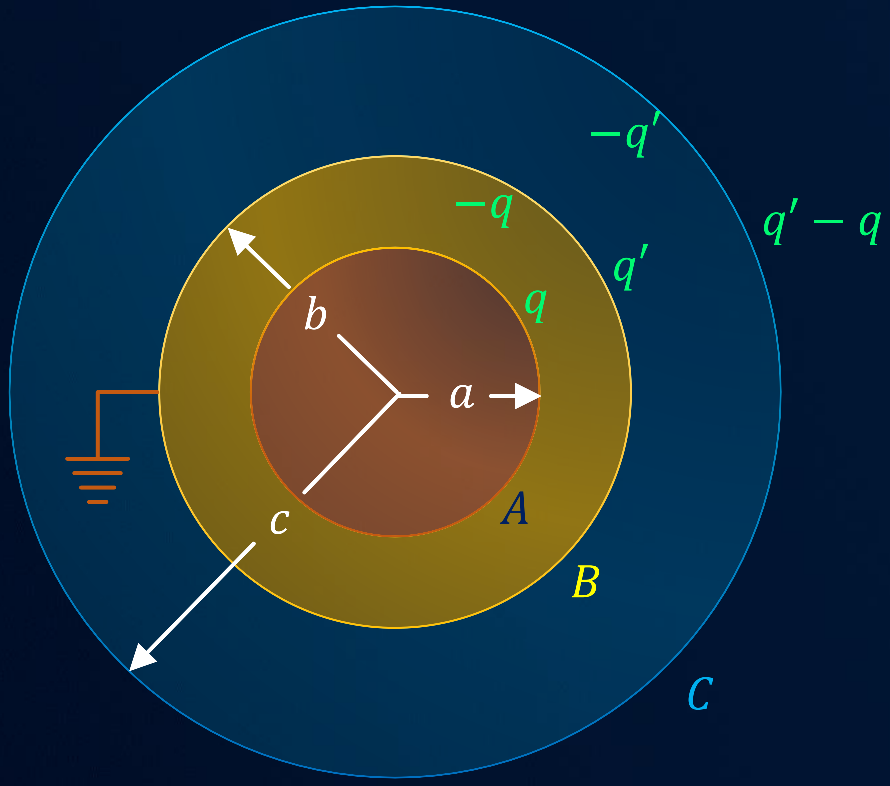
Since the shell B is grounded outside,

$$(V_{net})_B = \frac{kq}{b} + \frac{k(-q)}{b} + \frac{kq'}{b} + \frac{k(-q')}{c} + \frac{k(q'-q)}{c} = 0$$

$$\Rightarrow \frac{q}{b} - \frac{q}{b} + \frac{q'}{b} - \frac{q'}{c} + \frac{q'}{c} - \frac{q}{c} = 0$$

$$\boxed{q' = \frac{b}{c}q}$$

$$(q_C)_{outer} = q' - q = q\left(\frac{b}{c} - 1\right)$$



Three concentric shells A, B and C have radii a, b and c respectively. The shells A and C are connected via a thin conducting wire and shell B is earthed. If the shell A was given charge q at the beginning, find the charge appearing on the outer surface of C.

Solution :

The potentials of shells A and C are the same.

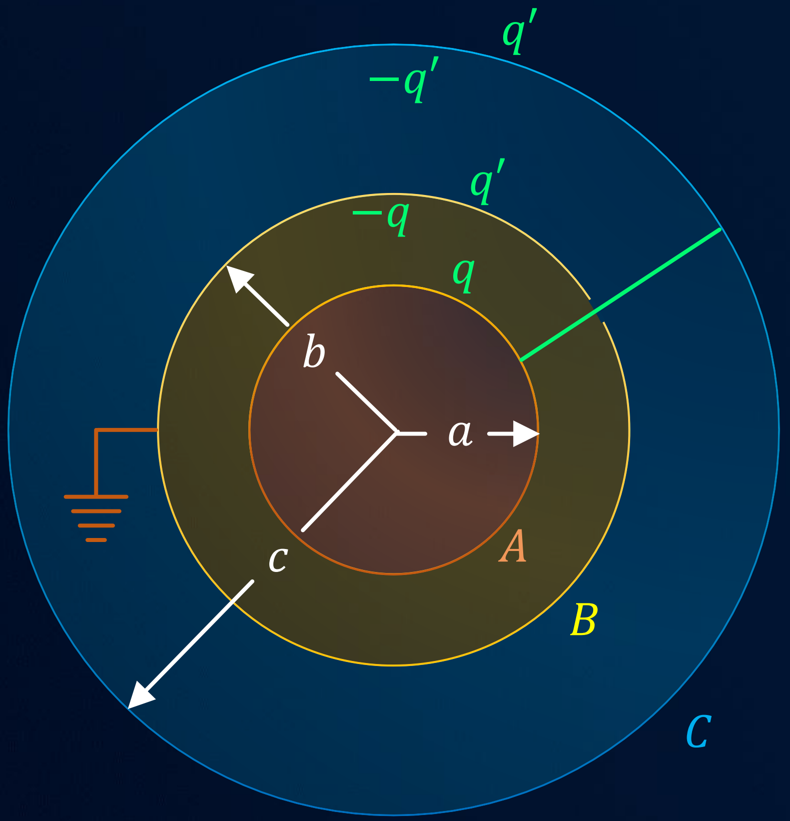
$$V_A = V_C$$

$$\frac{kq}{a} - \frac{kq}{b} + \frac{kq'}{b} - \frac{kq'}{c} + \frac{kq'}{c} = \frac{kq}{c} - \frac{kq}{c} + \frac{kq'}{c}$$

$$q\left(\frac{1}{a} - \frac{1}{b}\right) = q'\left(\frac{1}{c} - \frac{1}{b}\right)$$

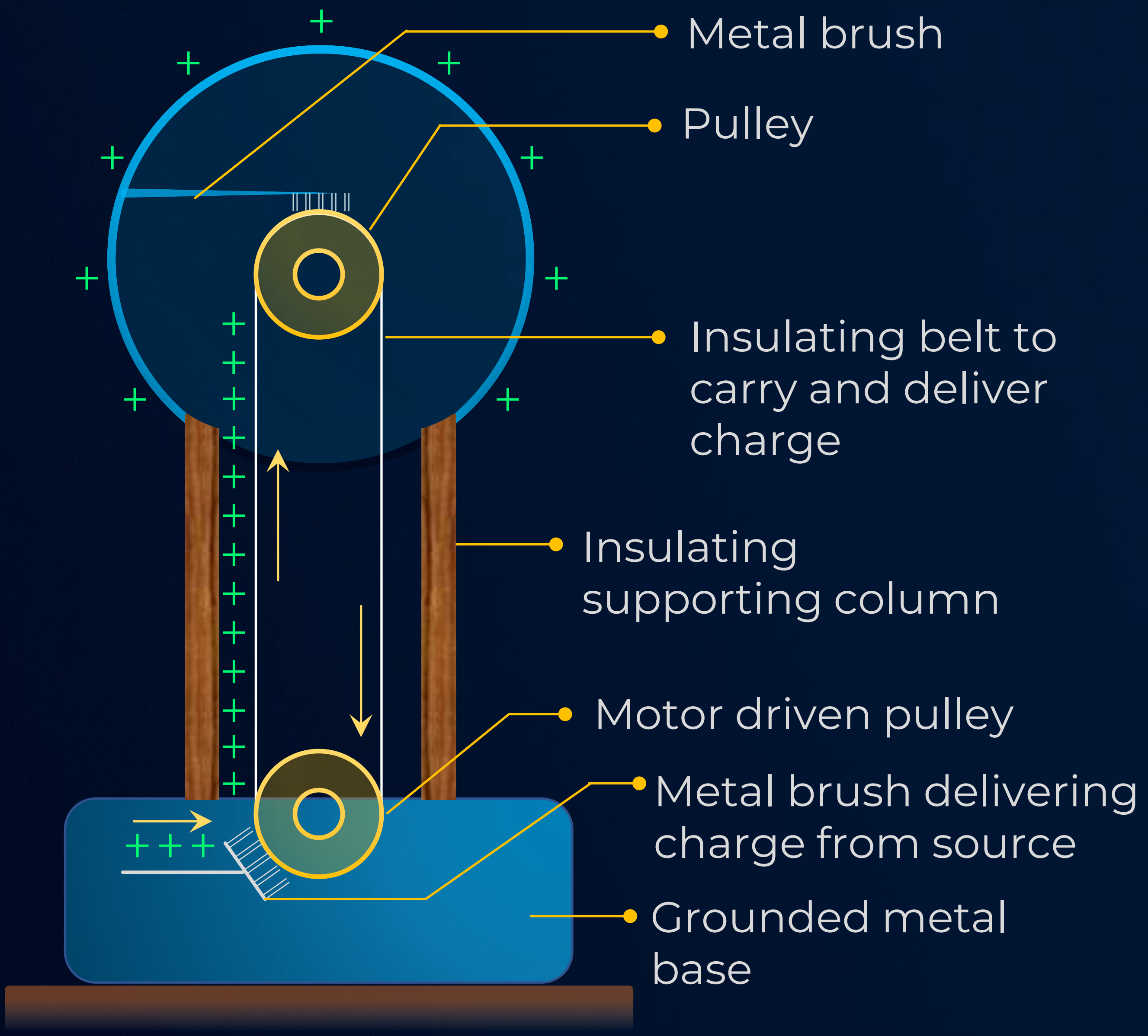
$$q' = \frac{qc(b-a)}{a(b-c)}$$

$$\boxed{(Q_C)_{out} = \frac{qc(b-a)}{a(b-c)}}$$



Van de Graaff Generator

- An arrangement developed to create high voltage (typically few Megavolts).
- Based on the accumulation of charge on the outer sphere due to its lower potential compared to the inner sphere.
- A large spherical shell is held at several meters high above the ground. (Insulated)
- Charge is continuously transferred to the smaller shell by means of a moving belt.
- Over time, larger shell acquires enormous amount of charge until the electrical breakdown.



The dielectric strength of air is $3 \times 10^6\ V\ m^{-1}$. With a sphere of radius $1.5\ m$, how much electrical potential can be achieved with the Van De Graaff generator?

Solution :

Electric field just outside the sphere is -

$$E = \frac{kQ}{R^2}$$

Electric potential just outside the sphere is -

$$V = \frac{kQ}{R} = ER$$

$$\Rightarrow E = \frac{V}{R}$$

The electric field generated by the generator cannot exceed the dielectric strength of air.

$$\Rightarrow E = \frac{V}{R} < E_{max}$$

$$\Rightarrow V < RE_{max}$$

$$\Rightarrow V_{max} = 1.5 \times 3 \times 10^6$$

$$\boxed{V_{max} = 4.5 \times 10^6\ V}$$