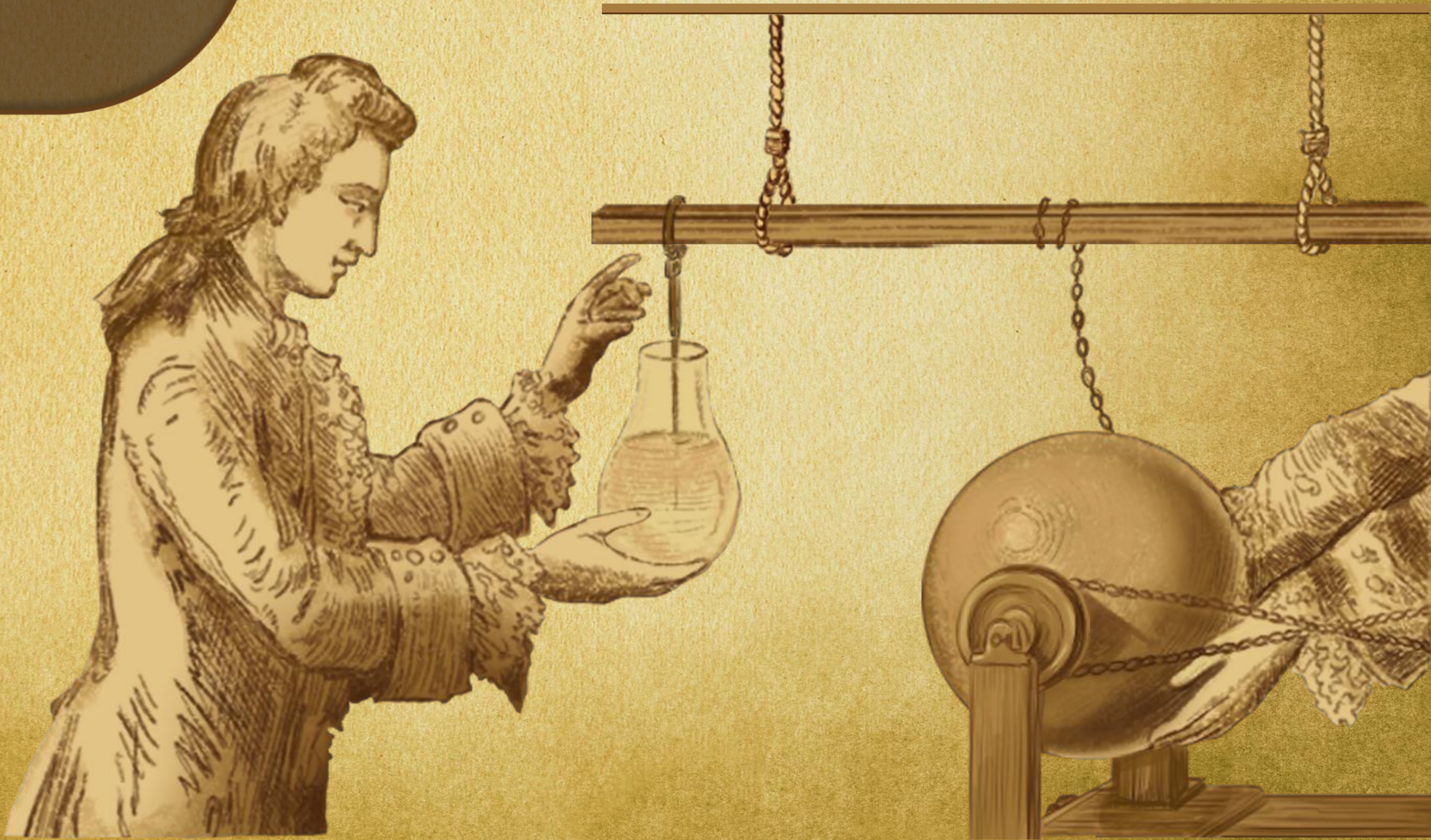
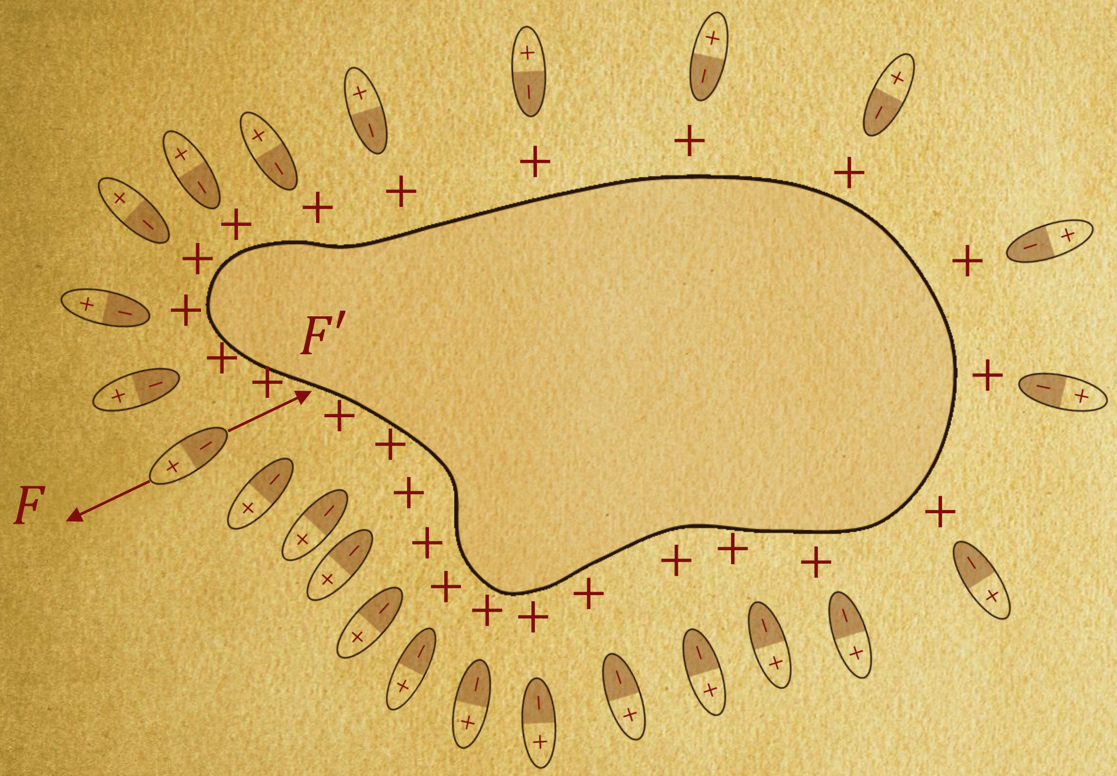
Aakash + BYJU'S NOTES

Capacitance



Charging Capacity of a Conductor

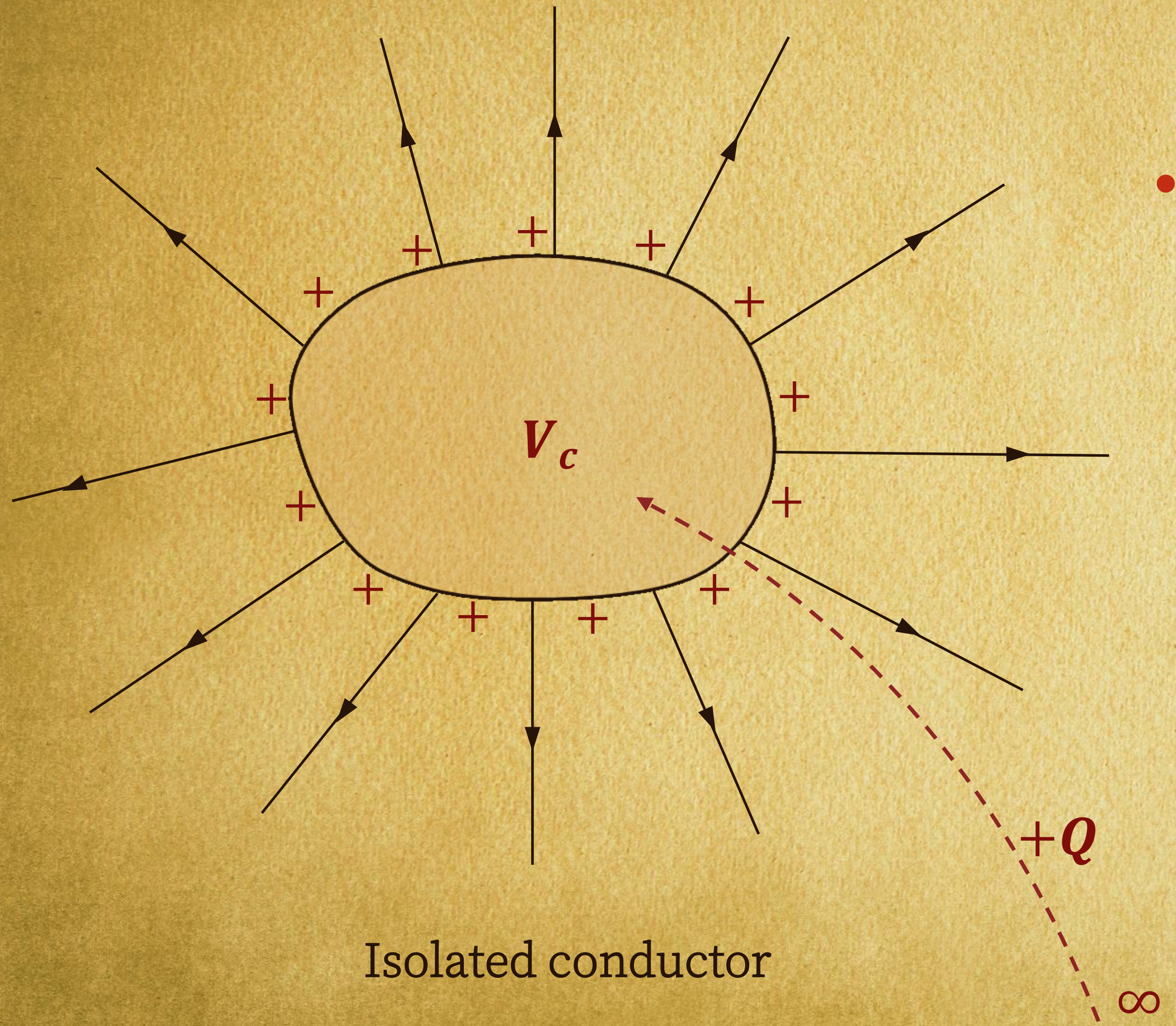


Isolated conductor placed in a non conducting medium

- As the charge on the conductor increases, potential increases thereby electric field strength increases.
- If a very high electric field is applied, phenomenon of dielectric breakdown takes place.
- Local surface charge density of a conductor is inversely proportional to local radius of curvature of the conductor, i.e., $\sigma \propto \frac{1}{r}$
- Maximum capacity of charging depends upon

1. Shape of conductor.
2. Breakdown strength of surrounding medium.

Capacitance of an Isolated Conductor



Isolated conductor

- The potential of the conductor w.r.t the reference point is given by,

$$V = V_C - V_\infty$$

- Potential difference is proportional to amount of charge present on an isolated conductor and vice-versa .

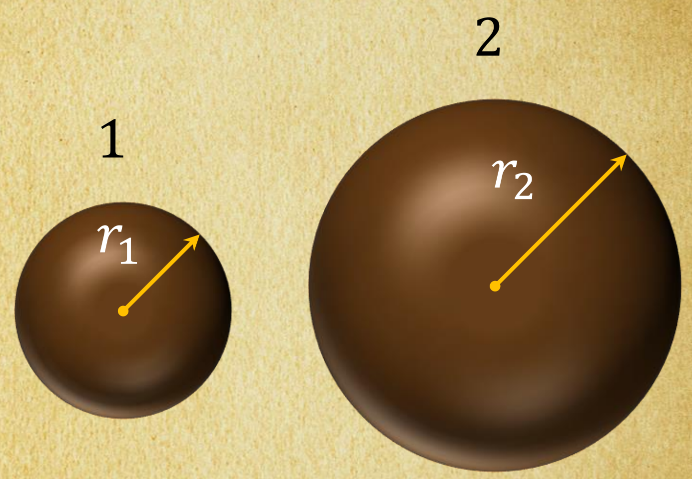
$$V \propto Q \Rightarrow \frac{Q}{V} = \text{Constant} = \text{Capacitance of the conductor}$$

- Capacitance of a conductor is defined as its ability to hold charge at a given potential

$$C = \frac{Q}{V}$$

- Capacitance is the ratio of the charge stored to the potential at the surface of the conductor.

Two spherical conductors having same potential are as shown in the figure. Find the relation between the capacitance of two conductors.



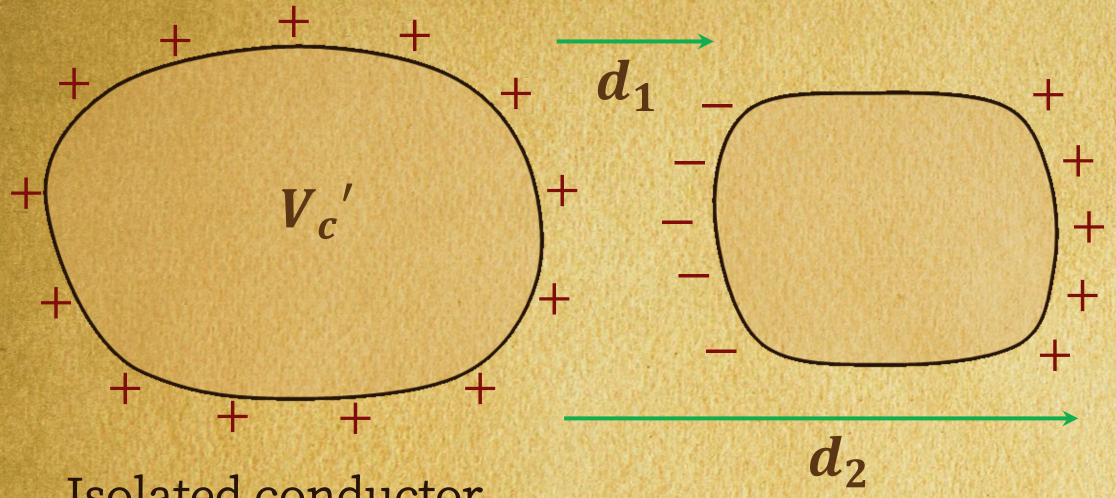
Two spherical conductors having same potential, but different surface area can store different amounts of charge.

$$C_1 = \frac{q_1}{V_1} \quad \& \quad C_2 = \frac{q_2}{V_2}$$

$$\frac{V_1}{V_2} = \frac{q_1/4\pi\epsilon_0 r_1}{q_2/4\pi\epsilon_0 r_2} \Rightarrow \frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 < q_2 \quad [\because V_1 = V_2]$$

$$\therefore C_1 < C_2$$

Effect of Placing a Conductor Near an Isolated Conductor



Isolated conductor

Neutral conductor

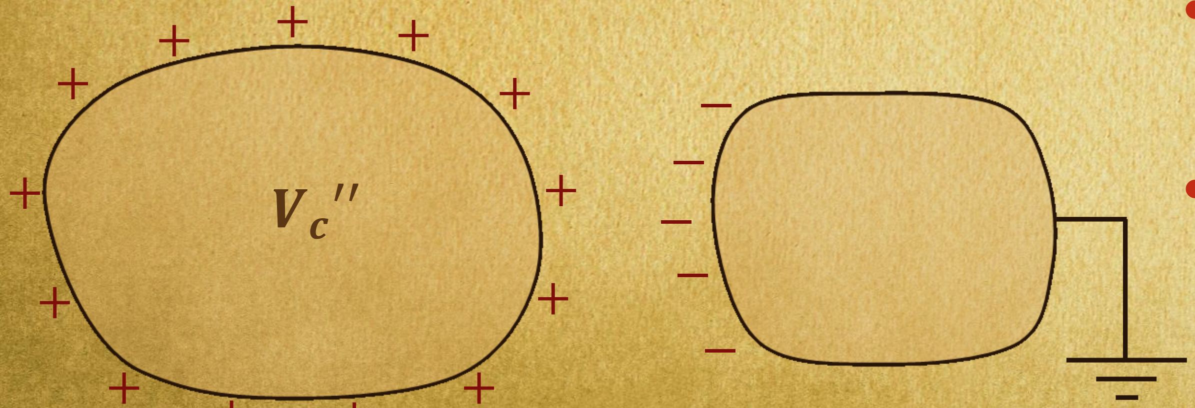
$$C = \frac{Q}{V_C}$$

- Potential of the charged conductor will be

$$V_C' = V_C + |V_+| - |V_-|$$

- $d_1 < d_2 \Rightarrow |V_-| > |V_+| \Rightarrow V_C' < V_C$
- Capacitance of the conductor will be,

$$C' = \frac{Q}{V_C'}$$



Isolated conductor

Neutral conductor

- Potential of the charged conductor will be,

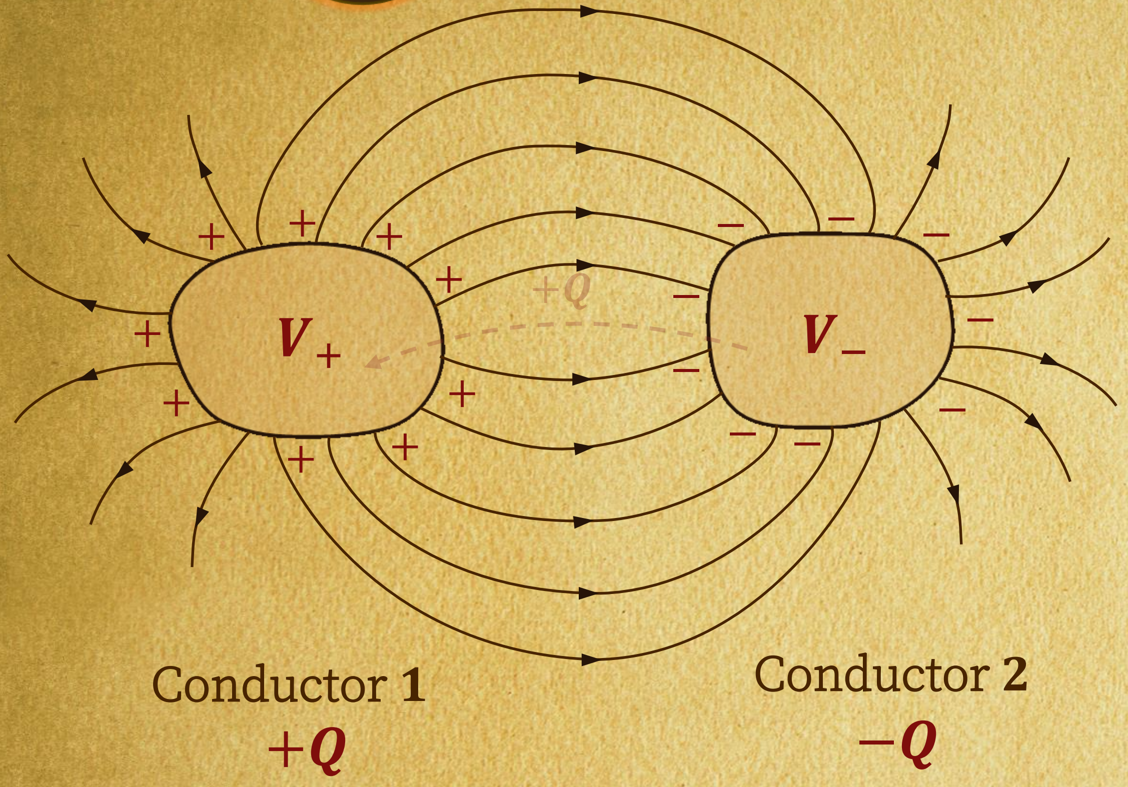
$$V_C'' = V_C - |V_-|$$

- Capacitance of the conductor will be,

$$C'' = \frac{Q}{V_C - |V_-|}$$

- Since $V_C'' < V_C' < V_C$, thus, $C'' > C' > C$

Capacitor



- A combination of two conductors placed close to each other, having equal and opposite charges is called a CAPACITOR.

Potential of conductor (1) w.r.t conductor (2) is given by, $V = V_+ - V_-$

- The charge on a capacitor having capacitance C is taken as, $Q = CV$.
- The net charge on a capacitor is always zero.

Capacitance of a capacitor depends on:

- Shape and size of the conductor.
- Geometrical placing of conductors.
- Medium between conductors.

Capacitance calculation

General 4 step plan :

(1) Assume a charge $+Q$ and $-Q$ on the conductors.

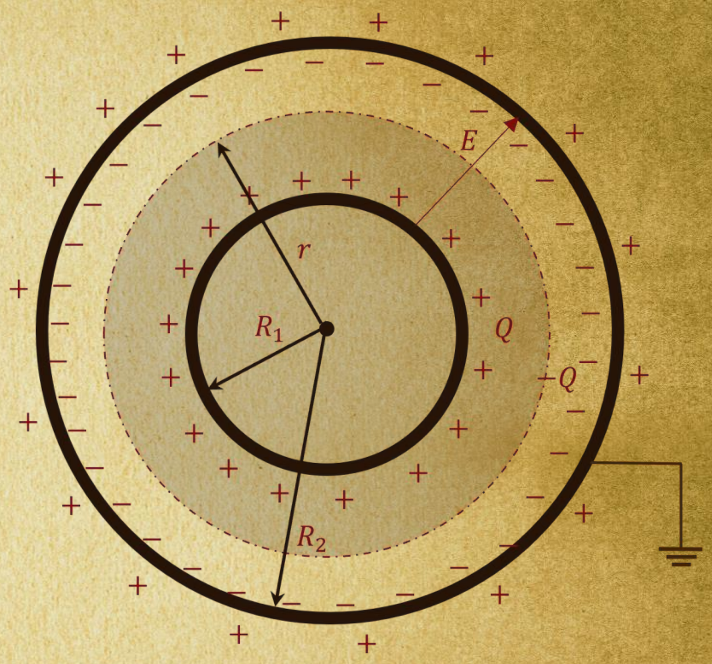
(2) Using Gauss's Law, calculate the electric field between the conductors in terms of charge.

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

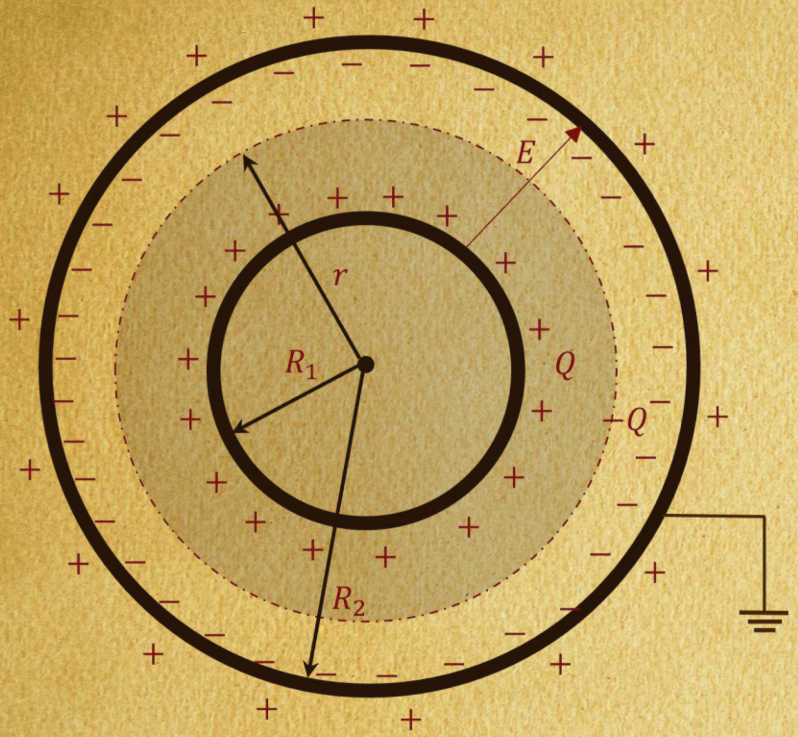
(3) Calculate the potential difference between the conductors using electric field.

$$V = -\int \vec{E} \cdot d\vec{r}$$

(4) Use equation $C = Q/V$



Spherical Capacitor



$$dV = -\vec{E}.d\vec{r} = -\frac{kQ}{r^2}dr \Rightarrow \int_{V_+}^{V_-} dV = kQ\int_{R_1}^{R_2} -\frac{1}{r^2}dr$$

$$V = V_+ - V_- = KQ\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$

$\vec{E}(r < R_1) = 0$ (Shell theorem)

$\vec{E}(r > R_2) = 0$ (Net charge of capacitor = 0)

$\vec{E}(R_1 < r < R_2) = \frac{kQ}{r^2}\hat{r}$ (Gauss's law)

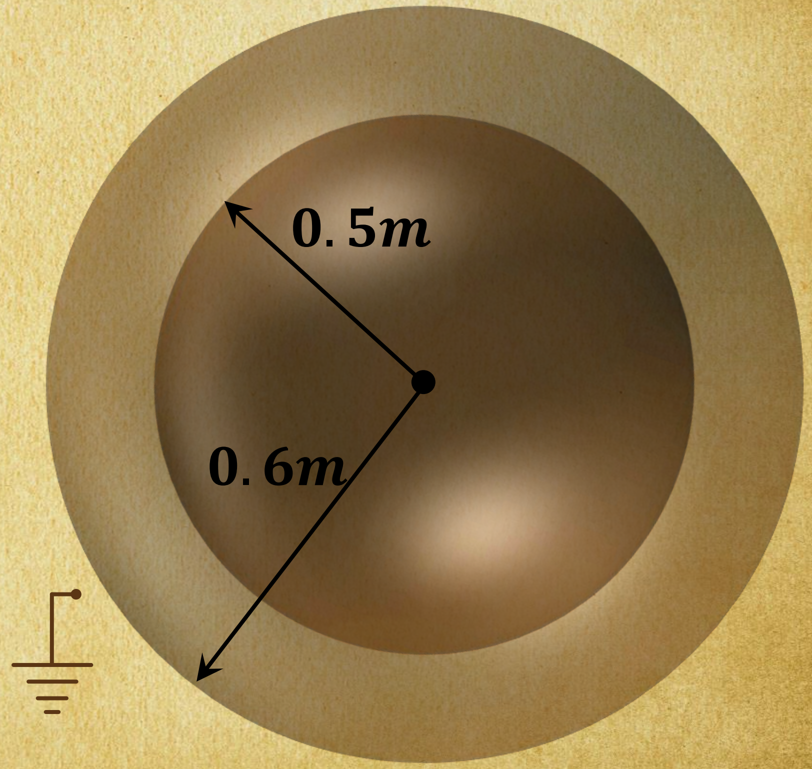
Calculate the capacitance of a spherical capacitor consisting of two concentric spherical shells of radius **$0.50\ m$** and **$0.60\ m$**. The material in the space between the two spheres is vacuum.

For a spherical capacitor, capacitance is given by:

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

$$C = \frac{0.5 \times 0.6}{9 \times 10^9 \times (0.6 - 0.5)}$$

$$\therefore C = 3.3 \times 10^{-10}\ F$$



Capacitance of Earth

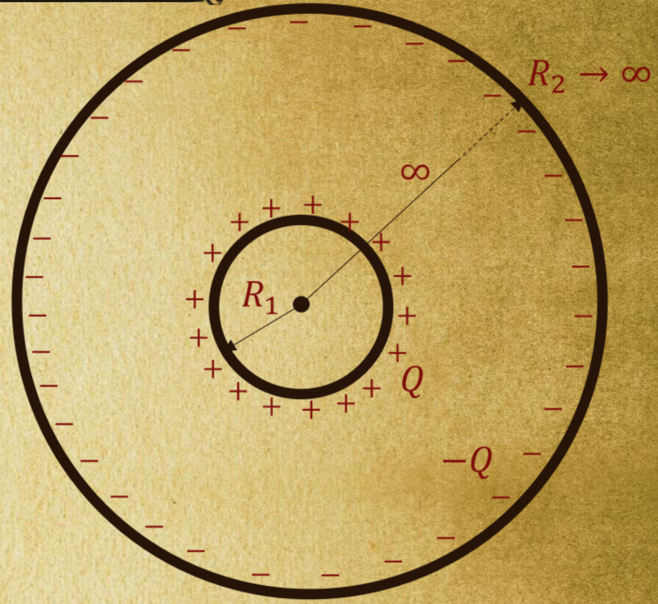
Capacitance of isolated sphere:

$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$R_1 = R$ (Radius of isolated sphere)

$R_2 \to \infty$ (For isolated sphere we can assume the outer sphere is at infinity)

$$C = 4\pi\varepsilon_0 R$$



- Earth is an infinite sink of charges.

$$C = 711.4\ \mu \frac{coulomb}{volt}$$

- SI unit of capacitance is Farad (F).
- 1 F capacitance is very large for practical purposes.
- For practical purposes we use :

$\mu F(10^{-6}\ F)$

$nF(10^{-9}\ F)$

$pF(10^{-12}\ F)$

An isolated sphere has a capacitance of $50\ pF$.

(a) Calculate its radius,

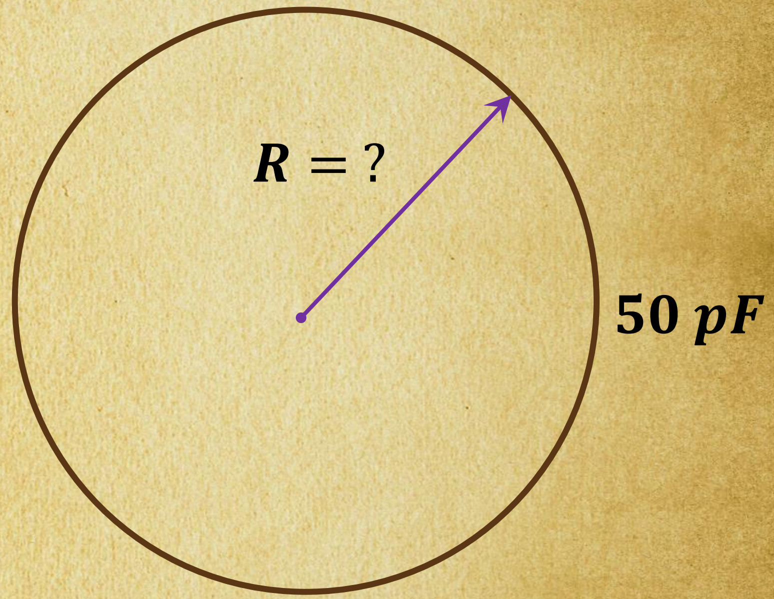
(b) How much charge should be placed on it to raise its potential to $10^4\ V$?

(a) Calculate its radius.

Capacitance of sphere, $C = 4\pi\epsilon_0 R$

Radius of sphere, $R = \dfrac{C}{4\pi\epsilon_0} = (50 \times 10^{-12}) \times (9 \times 10^9)\ m$

$R = 45 \times 10^{-2}\ m = 45\ cm$



(b) How much charge should be placed on it to raise its potential to $10^4\ V$.

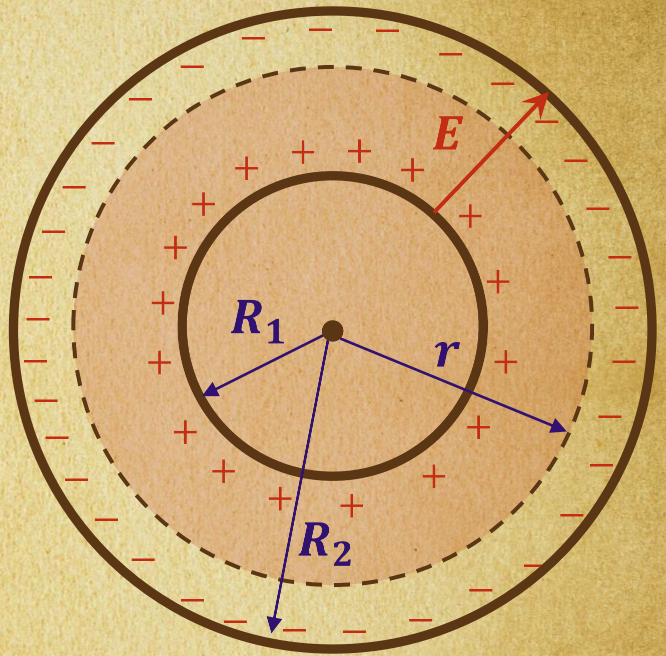
Charge to be placed,

$Q = CV = 50 \times 10^{-12} \times 10^4 = 5 \times 10^{-7} C = 0.5\ \mu C$

Energy Stored in a Charged Spherical Capacitor

- A capacitor stores energy in a region where electric field exists.
- For a spherical capacitor, electric field exists in region between two concentric spherical shells.
- The general expression of the density of energy stored in an electric field is,

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2$$



- Steps to follow to find the energy density

1. Find the expression for electric field
2. Substitute the expression for electric field in energy density
3. Do required mathematical operation to get energy stored

- The expression for energy stored is,

$$U = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

A spherical capacitor with charge Q is made up of two concentric spherical shells of radius $0.50\ m$ and $0.60\ m$. The outer shell is replaced with a shell of radius $0.80\ m$. Calculate the percentage change in the energy stored by the capacitor.

Case A:

$$R_1 = 0.50\ m$$

$$R_2 = 0.60\ m$$

$$U = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$U_A = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{0.5} - \frac{1}{0.6}\right] = \frac{Q^2}{24\pi\epsilon_0}$$

Case B:

$$R_1 = 0.50\ m$$

$$R_2 = 0.80\ m$$

$$U = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$U_B = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{0.5} - \frac{1}{0.8}\right] = \frac{3Q^2}{32\pi\epsilon_0}$$

$$\text{Percentage change} = \frac{U_B - U_A}{U_A} \times 100 = \frac{\frac{3}{32} - \frac{1}{24}}{\frac{1}{24}} \times 100 = 125\ \%$$

Capacitance of a Charged Cylindrical Capacitor

Find $\vec{E}$

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

Find V

$$V = -\int \vec{E}\cdot d\vec{r}$$

Get C

$$C = \frac{Q}{V}$$

- Electric field in the annular region:

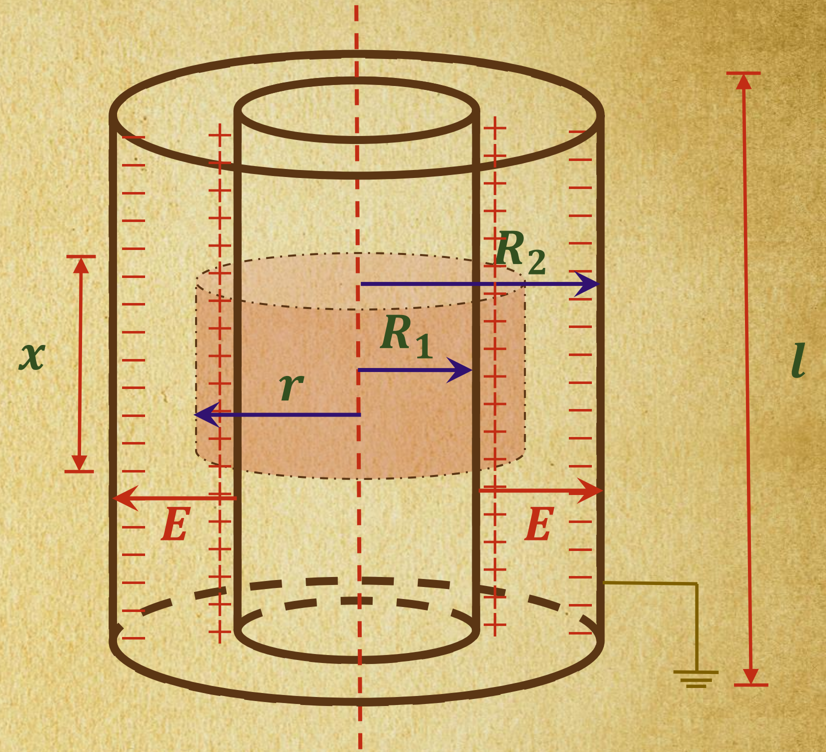
$$\vec{E}(R_1 < r < R_2) = \frac{Q}{2\pi\varepsilon_0 r l}\hat{r}$$

- Potential difference:

$$V = \frac{Q}{2\pi\varepsilon_0 l} ln\left(\frac{R_2}{R_1}\right)$$

- Capacitance of a charged cylindrical capacitor:

$$C = \frac{2\pi\varepsilon_0 l}{ln\left(\frac{R_2}{R_1}\right)}$$

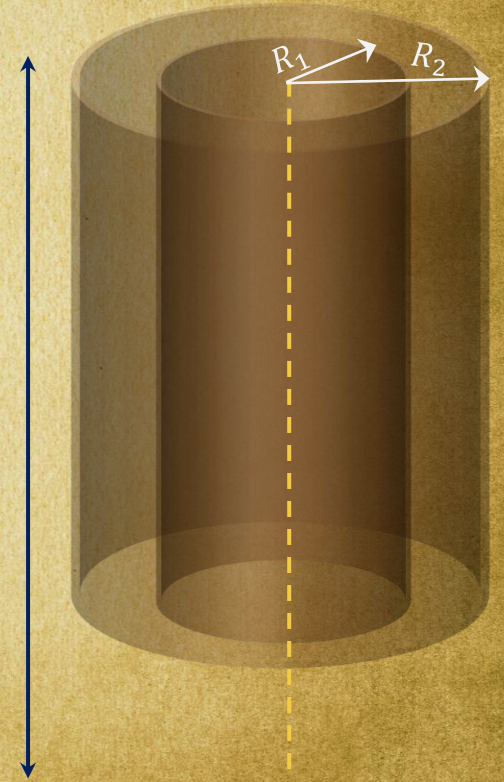


A cylindrical capacitor is constructed using two thin conducting coaxial cylinders of the same length **10 cm** and radii **5 mm** and **10 mm**, respectively.

a) Calculate the capacitance.

b) Another capacitor of same length is constructed with cylinders of radii **8 mm** and **16 mm**. Calculate the capacitance.

Capacitance of cylindrical capacitor : $C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}$



Case – a

$l = 10\ cm = 0.1\ m$ $\quad R_2 = 10\ mm$

$R_1 = 5\ mm$

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$C = \frac{2\pi \times 8.854 \times 10^{-12} \times 0.1}{\ln\left(\frac{10}{5}\right)}$$

$C \approx 8.03 \times 10^{-12}$
$\approx 8\ pF$

Case – b

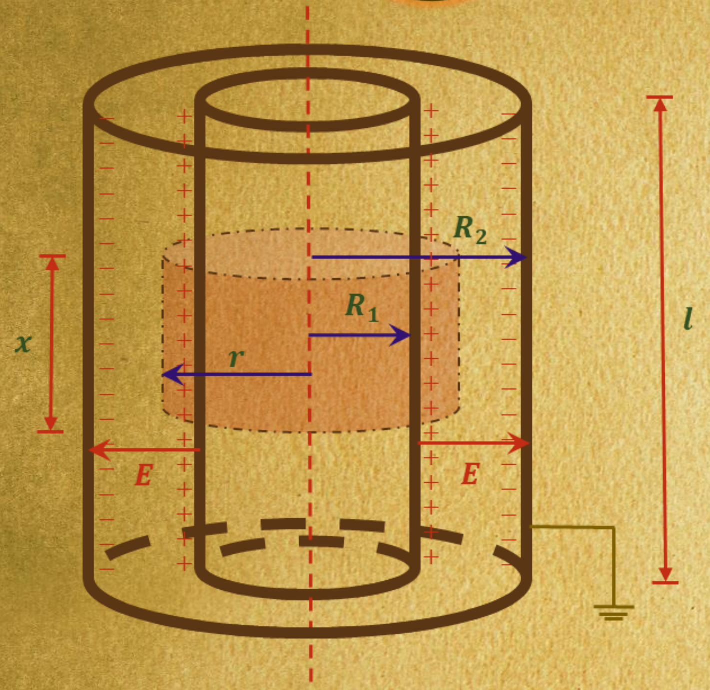
$l = 10\ cm = 0.1\ m R_2 = 16\ mm \quad R_1 = 8\ mm$

$$\ln\left(\frac{R_2}{R_1}\right) = \ln\left(\frac{16}{8}\right) = \ln 2 \equiv \ln\left(\frac{10}{5}\right)$$

The capacitance will remain same as case – a.

If length l and the ratio $\frac{R_2}{R_1}$ remains same for two cylindrical capacitors, the capacitance of both the capacitors will be same.

Energy Stored in a Charged Cylindrical Capacitor



1. Find the expression for electric field
2. Substitute the expression for electric field in energy density
3. Do required mathematical operation to get energy stored

- The expression for electric field,

$$\vec{E}(R_1 < r < R_2) = \frac{Q}{2\pi\varepsilon_0 r l}\hat{r}$$

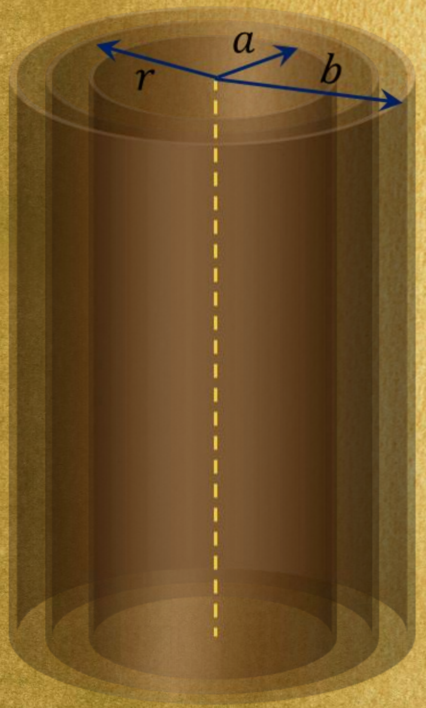
- The expression for energy density,

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2$$

- Energy stored in the capacitor

$$U = \frac{Q^2}{4\pi\varepsilon_0 l}\ln\left(\frac{R_2}{R_1}\right)$$

A cylindrical capacitor has radii a and b. Show that half the energy stored lies within a cylinder whose radius is $r = \sqrt{ab}$, where $a < r < b$.



Electric field at a distance r : $|\vec{E}| = \frac{Q}{2\pi\varepsilon_0 r l}$

Energy density : $\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2 = \frac{Q^2}{8\pi^2\varepsilon_0 r^2 l^2}$

Energy stored from $r = a$ to $r = r$:

$$U_r = \int_a^r \frac{Q^2}{8\pi^2\varepsilon_0 r^2 l^2}(2\pi r l)\, dr$$

$$U_r = \frac{Q^2}{4\pi\varepsilon_0 l}\ln\left(\frac{r}{a}\right)$$

Total energy stored : $$U_{total} = \frac{Q^2}{4\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

Given : $$\frac{U_r}{U_{total}} = \frac{1}{2}$$

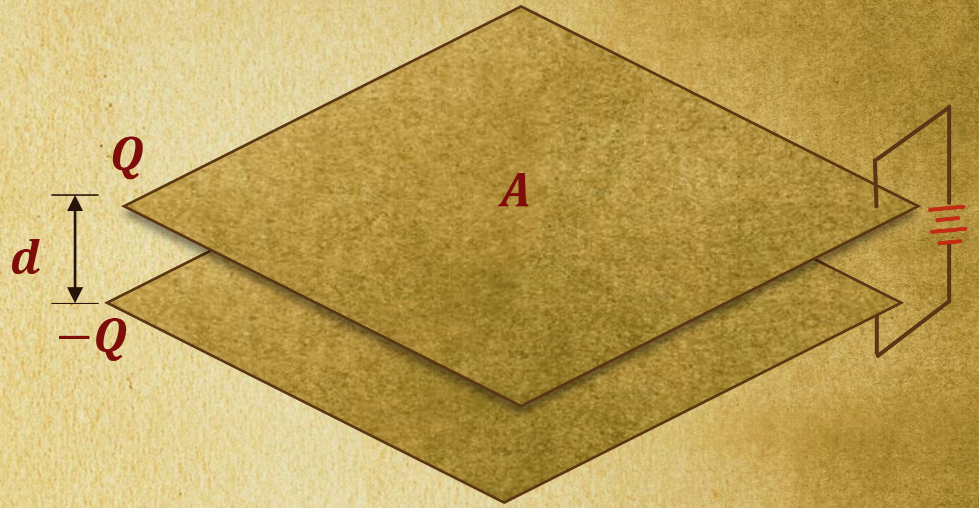
$$\ln\left(\frac{r}{a}\right) = \frac{1}{2}\ln\left(\frac{b}{a}\right)$$

$$\frac{r}{a} = \sqrt{\frac{b}{a}}$$

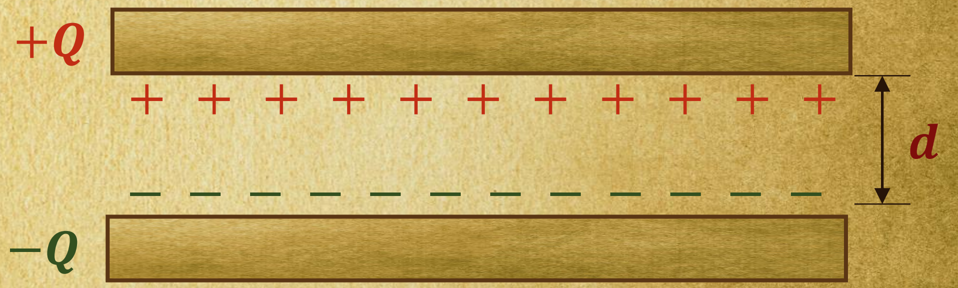
$$r = \sqrt{ab}$$

Parallel Plate Capacitor

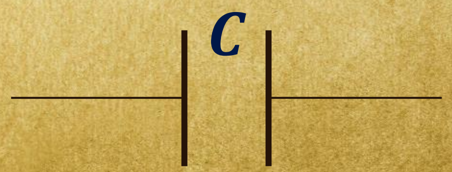
- When two parallel plates are connected across a battery, the plates are charged, and an electric field is established between them. This setup is known as the parallel plate capacitor.



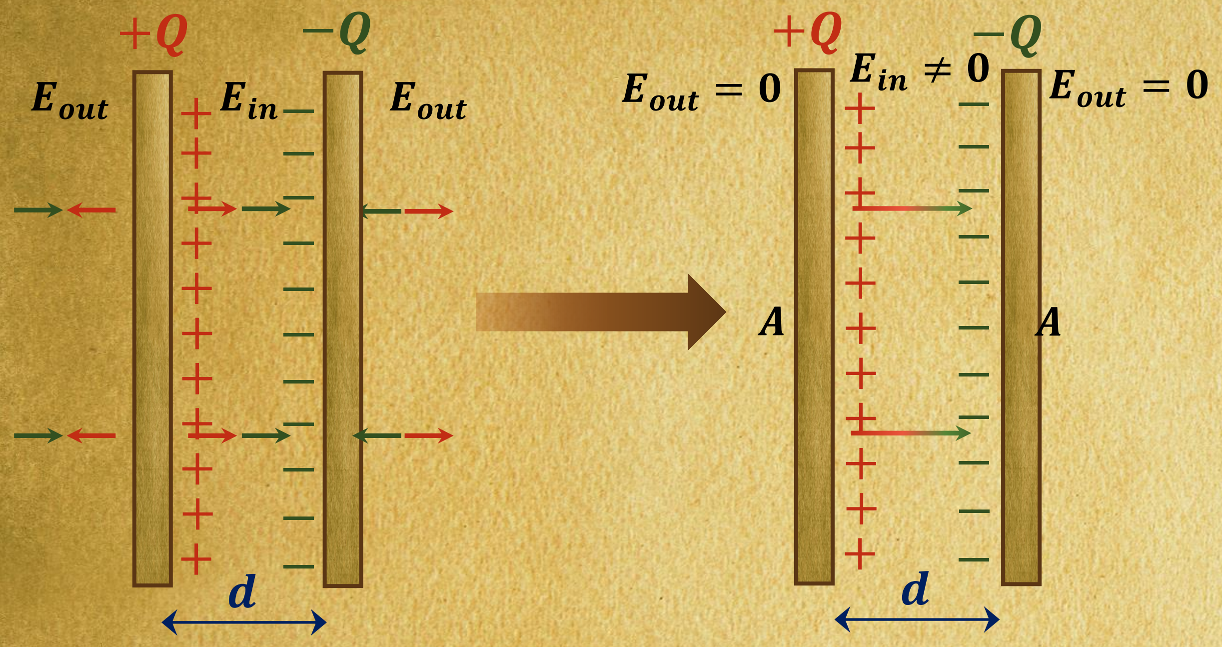
- For a parallel plate capacitor, the charge given on the plates redistributes itself in such a way that it exists only on the sides of the plates facing each other.
- Separation (***d***) between them is small.



- Equal and opposite charges appear on the facing surfaces.
- Charge on a capacitor means the magnitude of charge on either of the plates.
- Symbolic representation of capacitor in an electrical circuit:



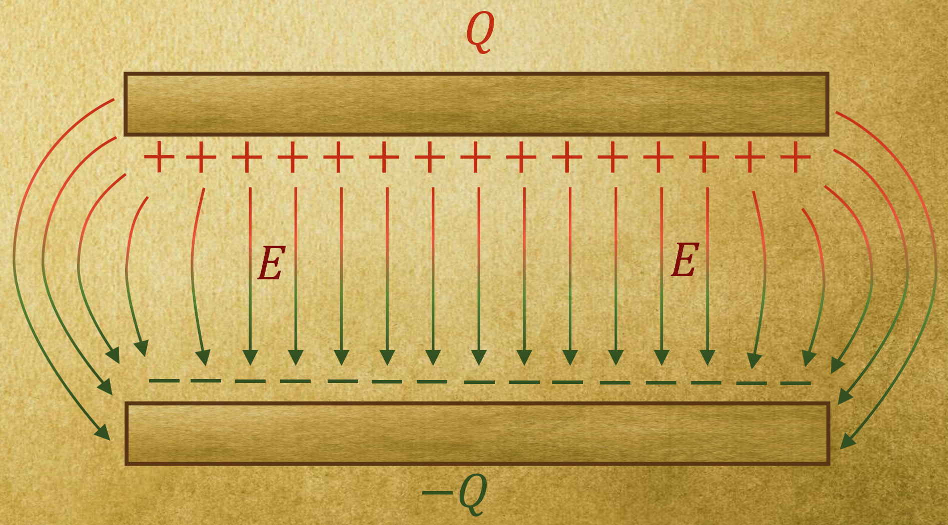
Electric Field in Parallel Plate Capacitor



Electric field exists only between the plates of the capacitor and the magnitude of such net electric field is,

$$|E_{in}| = \frac{Q}{A\varepsilon_0}$$

- FRINGING is the bending of the electric field lines near the edge of the parallel plates.
- In this chapter, fringing of the electric field at the edges of the plates is neglected.



Capacitance of a Parallel Plate Capacitor

Find $\vec{E}$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

Find V

$$V = -\int \vec{E} \cdot d\vec{r}$$

Get C

$$C = \frac{Q}{V}$$

- Electric field in between the plates :

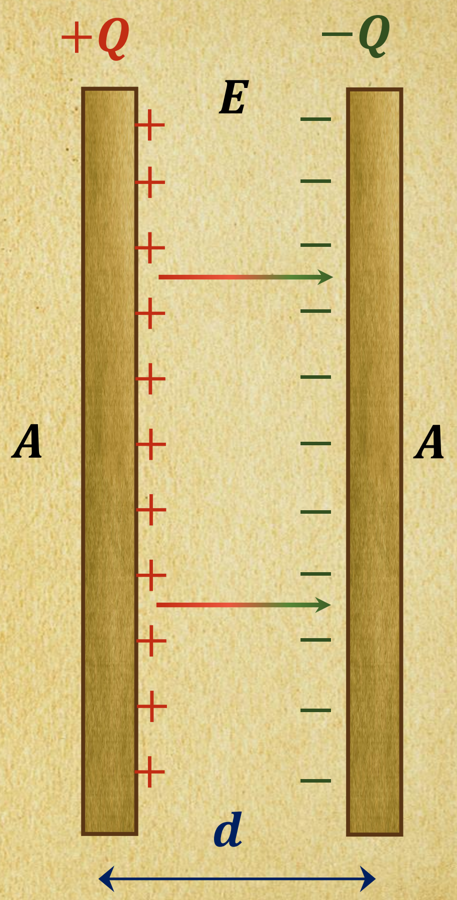
$$E = \frac{Q}{A\varepsilon_0}$$

- Potential difference between the plates :

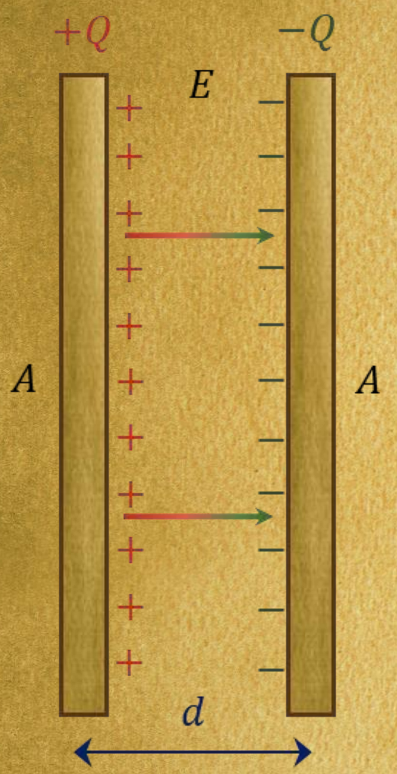
$$V = \frac{Q}{A\varepsilon_0} d$$

- Capacitance :

$$C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$



Energy Stored in a Charged Parallel Plate Capacitor



A capacitor can store energy in the form of electric field.

1. Find the expression for electric field
2. Substitute the expression for electric field in energy density
3. Do required mathematical operation to get energy stored

The expression for electric field,

$$E = \frac{Q}{A\varepsilon_0}$$

The expression for energy density,

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2$$

Energy stored in the capacitor

$$dU = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{A\varepsilon_0}\right)^2 dV$$

$$U = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{A\varepsilon_0}\right)^2 (Ad)$$

$$U = \frac{Q^2}{2C} \quad \because C = \frac{A\varepsilon_0}{d}$$

Expression of energy stored in a charged capacitor :

$$U = \frac{1}{2}CV^2 \xleftarrow{Q = CV} U = \frac{Q^2}{2C} \xrightarrow{C = Q/V} U = \frac{QV}{2}$$

Estimate the change in percentage of energy stored in a parallel plate capacitor, if the separation between its plates is reduced by **10 %** keeping the voltage of the source unchanged ?

Initial separation between plates = d

Final separation between plates :

$$d' = d - (10\%\ of\ d) = \frac{9d}{10}$$

Initial energy stored :

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$$

Final energy stored :

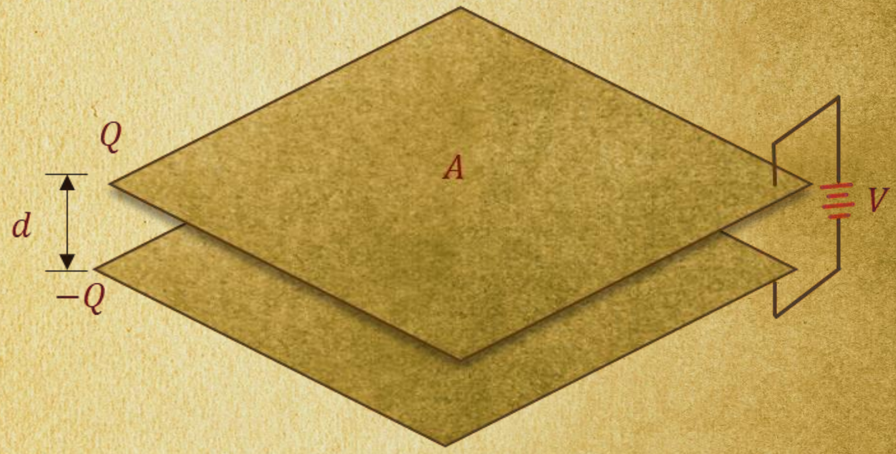
$$U_f = \frac{1}{2}C'V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d'}\right)V^2$$

Percentage change in energy stored:

$$\left[\frac{U_f}{U_i} - 1\right] \times 100$$

$$= \left[\frac{d}{d'} - 1\right] \times 100$$

$$= \left[\frac{10}{9} - 1\right] \times 100$$



+11.1 %

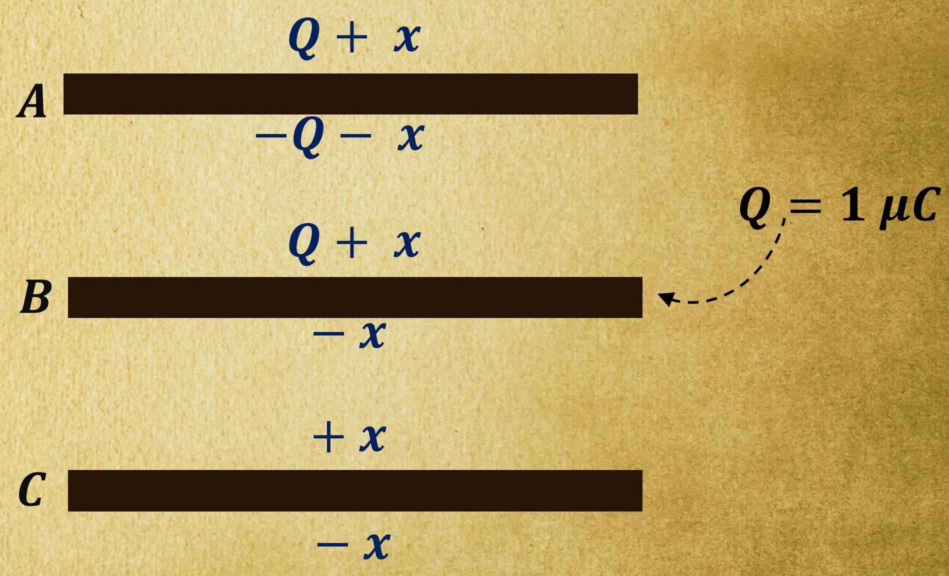
The capacitance of a capacitor formed by any two adjacent plates is **50 nF**. A charge of **$1.0\ \mu C$** is placed on the middle plate.

(a) What will be the charge on the outer surface of the upper plate ?

(b) Find the potential difference developed between the upper and the middle plates.

Rule :

Equate the charge on outer surface of top plate with the charge on outer surface of bottom plate



Calculation :

$$Q + x = -x$$
$$x = -\frac{Q}{2}$$

Given : $Q = 1\ \mu C$ → $x = -0.5\ \mu C$

The charge on outer surface of upper plate :

$Q + x = (1 - 0.5)\ \mu C$ → $0.5\ \mu C$

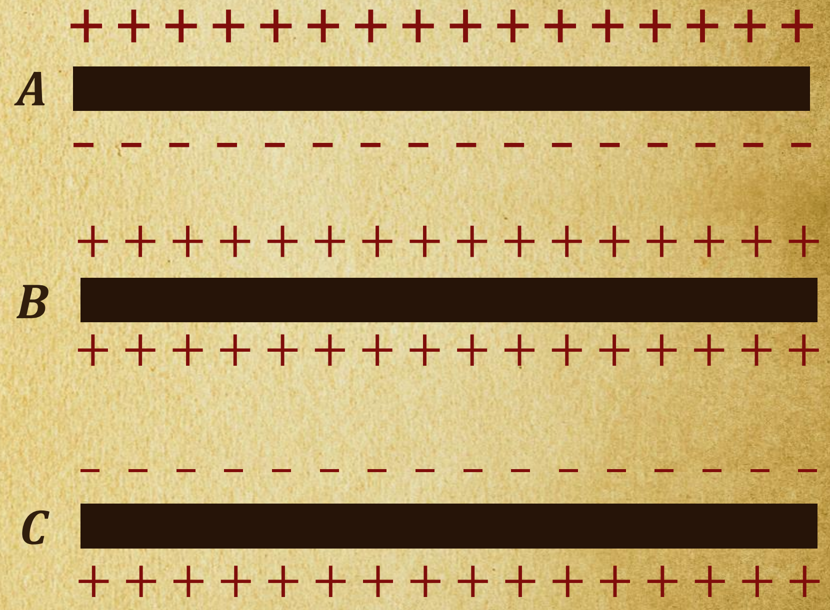
The capacitance of a capacitor formed by any two adjacent plates is **50 nF**. A charge of **1.0 μC** is placed on the middle plate.

(a) What will be the charge on the outer surface of the upper plate ?

(b) Find the potential difference developed between the upper and the middle plates.

Capacitance between upper and middle plate: $C_{AB} = \dfrac{\left(Q/_2\right)}{V_{AB}}$

Capacitance between middle and lower plate: $C_{BC} = \dfrac{\left(Q/_2\right)}{V_{BC}}$



Given : $C_{AB} = C_{BC} = C = 50\ nF$

$Q = 1\ \mu C$

Potential difference between upper and middle plates:

$$V_{AB} = \frac{Q}{2C_{AB}} = \frac{Q}{2C} = \frac{10^{-6}}{2 \times 50 \times 10^{-9}} = 10\ Volts$$

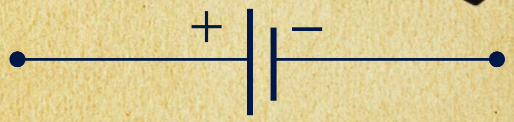
$$V_{AB} = 10\ volt$$

The battery and its properties

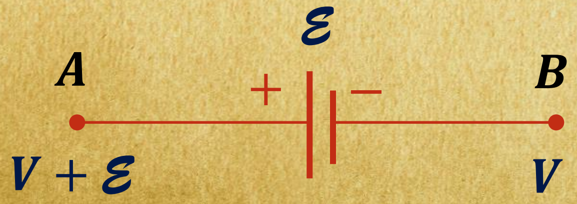
- To get charged and to store energy, a capacitor requires a potential difference to be applied and a battery produces the required potential difference.

- The battery is a device which creates potential difference externally between two points across which it is connected.

- Symbolic representation of a battery in an electrical circuit:



- The terminal at higher potential is called POSITIVE TERMINAL and the one at lower potential is called NEGATIVE TERMINAL.



Workdone by battery

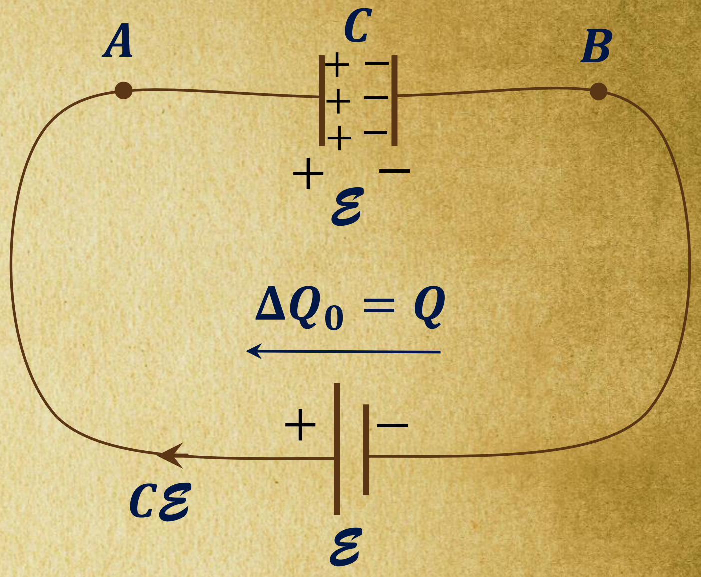
- Work done by a battery :

W_b = (Charge supplied or accepted by the positive terminal of the battery to the circuit)×(emf of the battery)

- Battery supplies charge Q to the circuit ➡ $W_b = +ve$

⬇

The battery is acting as a SOURCE.



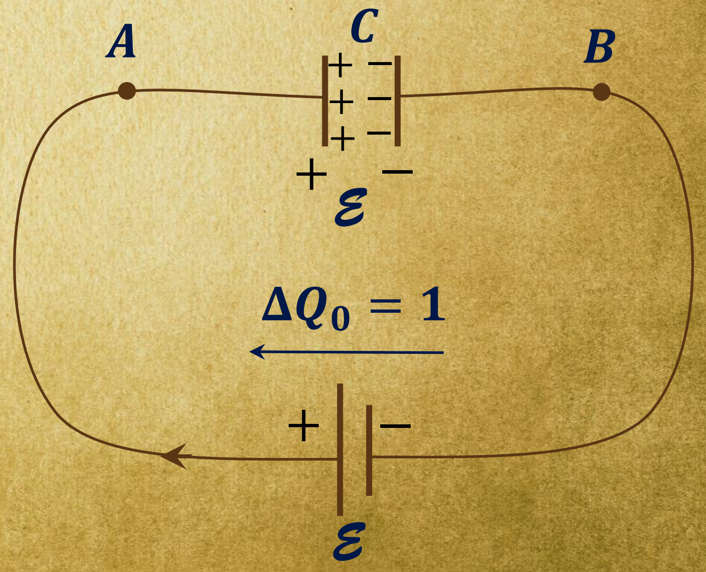
- Charge Q enters the positive plate of the battery ➡ $W_b = -ve$

⬇

The battery is acting as a LOAD.

- Electromotive Force (EMF) :

The EMF of a battery is defined as the work done by the battery in driving a unit charge around a complete circuit.



If at $t = 0$ switch S is closed, then for steady state, find
(a) Charge flow in the circuit (ΔQ_0).
(b) Final charge on the capacitor (Q).
(c) Heat generated (H).

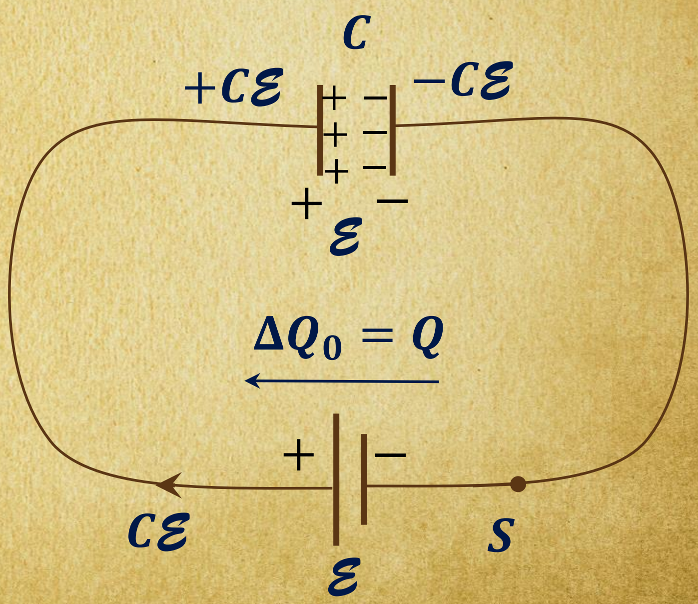
(a) Before achieving the steady state, the charge flown in the circuit, $\Delta Q_0 = C\mathcal{E}$

(b) In steady state, final charge on the capacitor, $Q = C\mathcal{E}$

$$U_{source} = U_{load} + Heat$$

- At $t = 0$, energy stored in capacitor, $U_i = 0$
- In steady state, energy stored in capacitor, $U_f = \frac{1}{2}C\mathcal{E}^2$
- Net energy stored in capacitor, $U_{load} = U_f - U_i = \frac{1}{2}C\mathcal{E}^2$
- Energy supplied by the battery, $U_{source} = C\mathcal{E}^2$

At steady state



(c) Heat generated, $H = U_{source} - U_{load} = \frac{1}{2}C\mathcal{E}^2$

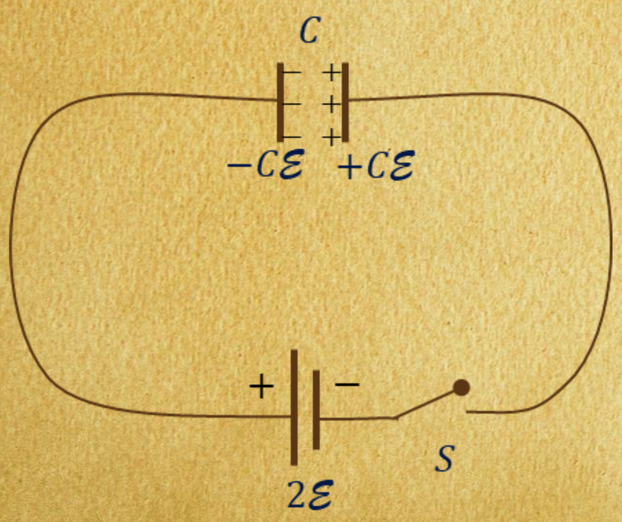
For the circuit given below, if at $t = 0$ switch S is closed, then for steady state find

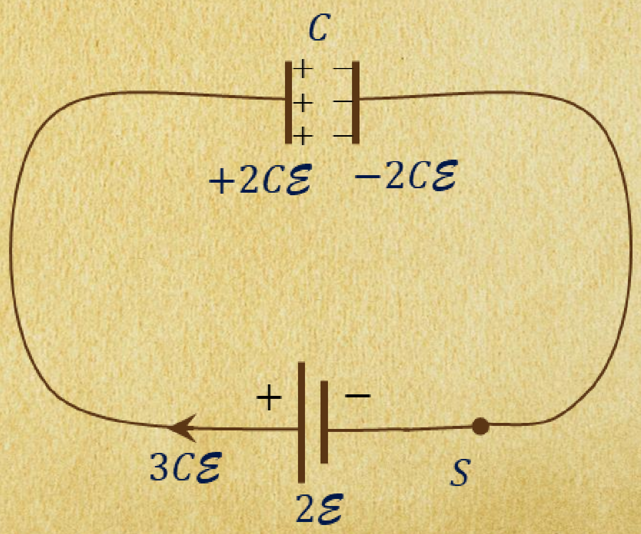
(a) Final charge on the capacitor (Q).

(b) Charge flow in the circuit (ΔQ_0).

(c) Heat generated (H).

- Before the connection is made with battery, the charge in the capacitor is, $Q' = C\mathcal{E}$
- The capacitor and the battery are connected with opposite polarity.





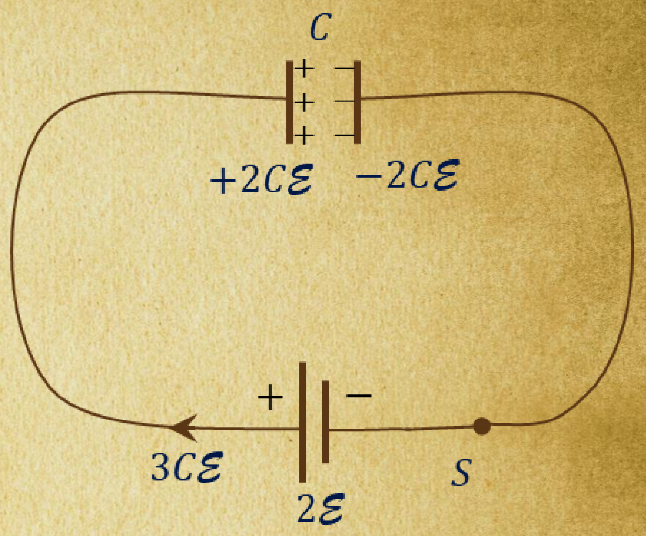
(a) Charge on the capacitor in steady state, $Q = 2C\mathcal{E}$

(b) Charge flown in the circuit, $\Delta Q_0 = 2C\mathcal{E} - (-C\mathcal{E}) = 3C\mathcal{E}$

- At $t = 0$, energy stored in capacitor, $U_i = \frac{1}{2}C\mathcal{E}^2$
- In steady state, energy stored in capacitor, $U_f = 2C\mathcal{E}^2$
- Work done by the battery, $W_b = 6C\mathcal{E}^2$

(c) Heat generated, $H = ?$

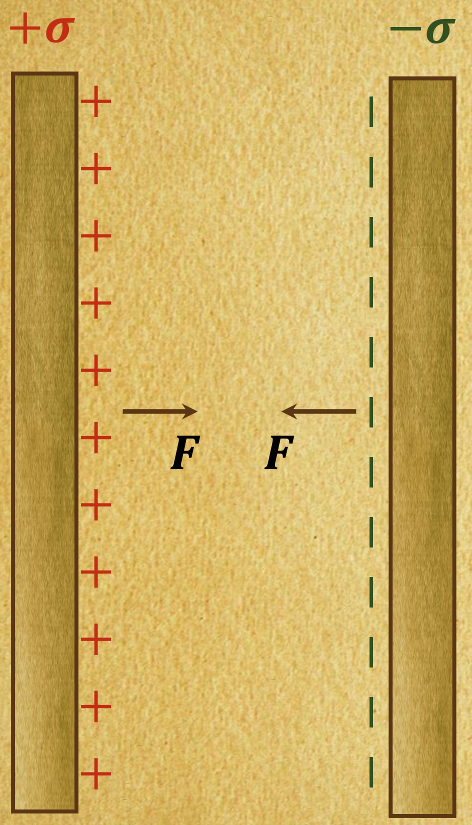
$$W_b = [U_f - U_i] + H \quad \Rightarrow H = 6C\mathcal{E}^2 - \frac{3}{2}C\mathcal{E}^2 = \frac{9}{2}C\mathcal{E}^2$$



Suppose Q_i and Q_f are the initial and final charge on the capacitor, respectively.

- When the capacitor and battery are connected with same polarity, charge flown in the circuit, $\Delta Q_0 = Q_f - Q_i$
- When the capacitor and battery are connected with opposite polarities, charge flown in the circuit, $\Delta Q_0 = Q_f - (-Q_i)$

Force between Plates of a Parallel Plate Capacitor



The oppositely charged capacitor plates attract one another.

The electric field created by the positively charged plate is,

$$E_+ = \frac{\sigma}{2\epsilon_0}$$

Force on the negative plate is,

$$F = QE_+ = \frac{Q^2}{2A\epsilon_0}$$

?

A capacitor of capacitance $10\ \mu F$ is connected to a $20\ V$ cell. The separation between the plates is reduced by half during a slow process.

a) Find the work done by the cell during the process.
b) Find the work done by the external agent during the process.
c) Find the ratio of energy density in electric field between the capacitor plates, before and after the plate separation is changed.

Given:

Initial capacitance $C_1 = 10\ \mu F$

Potential difference $V = 20\ V$

Initial charge $Q_1 = CV = 200\ \mu C$

Since distance between the plates is halved,

$C_2 = 2C = 20\ \mu F$

Final charge $Q_2 = C_2 V = 400\ \mu C$

(a) $W_{cell} = \Delta Q V = (Q_2 - Q_1)V$
$= 200 \times 20 = 4000\ \mu J$

(b) Change in energy stored in capacitor, $\Delta U = U_2 - U_1$

$$= \frac{1}{2}(2C)V^2 - \frac{1}{2}CV^2 \quad = \frac{1}{2}CV^2 = 2000\ \mu J$$

$$W_{ext} + W_{cell} = \Delta U \Rightarrow W_{ext} = \Delta U - W_{cell} = -2000\ \mu J$$

(c) Electric field between the plates, $E = \frac{V}{d}$

When d is halved, E doubles.

Energy density, $u = \frac{1}{2}\epsilon_0 E^2$

If E doubles, u becomes 4 times.

Kirchhoff's Law

Kirchhoff's Current Law (KCL)

- Also known as : (1) Junction Law
  (2) Nodal Analysis Principle
- The amount of charge flowing into a node is equal to the sum of charge flowing out of it.
- KCL is based on charge conservation principle.
- At a junction :

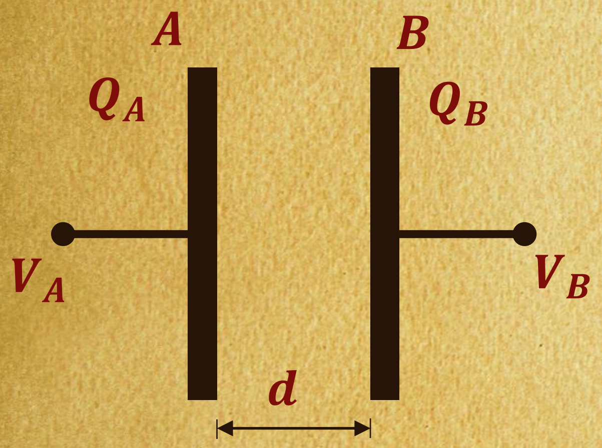
  Incoming charge = Outgoing charge

Kirchhoff's Voltage Law (KVL)

- Also known as : (1) Loop Law
  (2) Mesh Analysis Principle
- As one traverses a loop to reach the same point again, the algebraic sum of all the potential drops or gains and emfs encountered along the path is zero.
- KVL is based on the principle of energy conservation.

Convention of a Capacitor

Kirchhoff's Current Law (KCL)



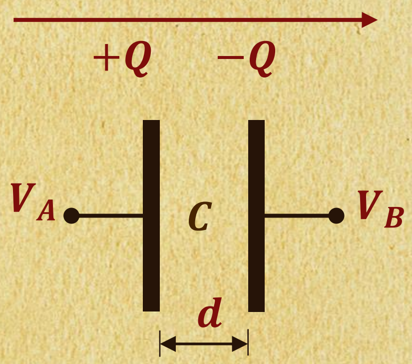
$$(V_B - V_A) = \frac{Q_B}{C}$$

Or

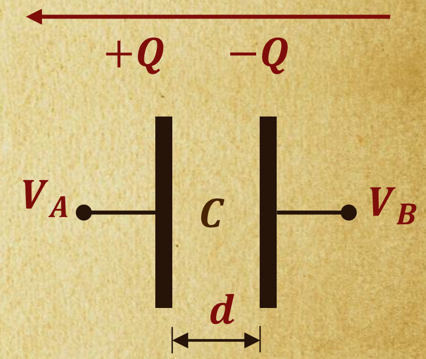
$$(V_A - V_B) = \frac{Q_A}{C}$$

Kirchhoff's Voltage Law (KVL)

- Sign Convention



$$V_A - \frac{Q}{C} = V_B$$

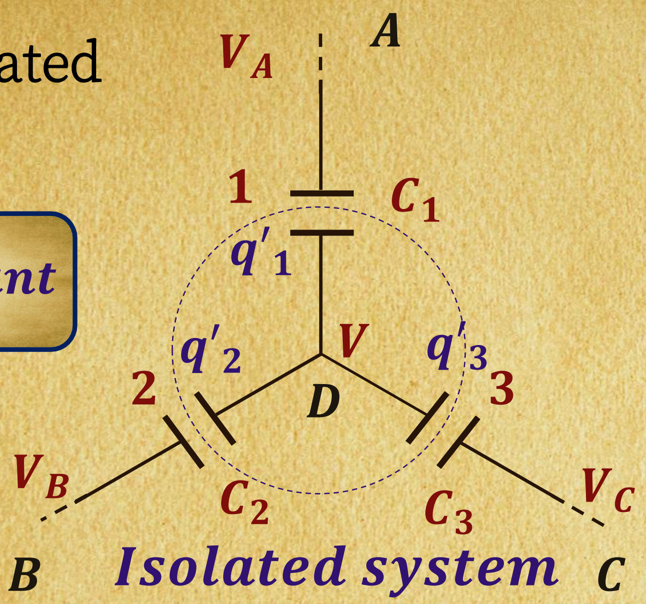


$$V_B + \frac{Q}{C} = V_A$$

Kirchhoff's Current Law (KCL)

- Net charge of an isolated system is constant.

$$q_1' + q_2' + q_3' = constant$$



- At junction ***D***,

$$[q_1' + q_2' + q_3']_{initial} = [q_1' + q_2' + q_3']_{final}$$

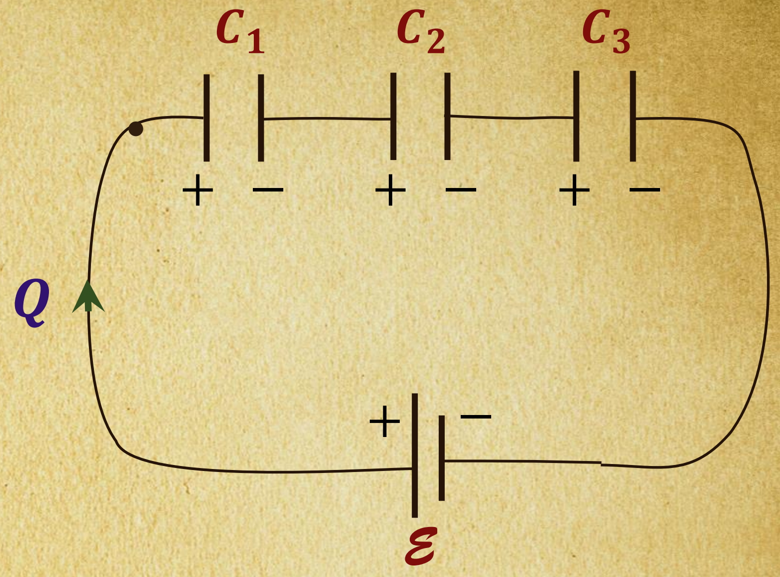
- Apply KCL at junction ***D***.

$$C_1(V - V_A) + C_2(V - V_B) + C_3(V - V_C) = 0$$

$$\Rightarrow \quad V = \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$$

Assume potential (V) of chosen junction (D) is maximum.

Kirchhoff's Voltage Law (KVL)



Applying KVL in Clockwise direction

$$-\frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} + \mathcal{E} = 0$$

Applying KVL in Anti Clockwise direction

$$-\mathcal{E} + \frac{Q}{C_3} + \frac{Q}{C_2} + \frac{Q}{C_1} = 0$$

$$\mathcal{E} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)Q$$

If at $t = 0$ switch S is closed, then for steady state find

(a) Charge flow in the circuit (ΔQ_0).

(b) Final charge on each capacitor.

(c) Heat generated (H).

(a) Charge flow in the circuit (ΔQ_0).

Applying KCL on the isolated system,

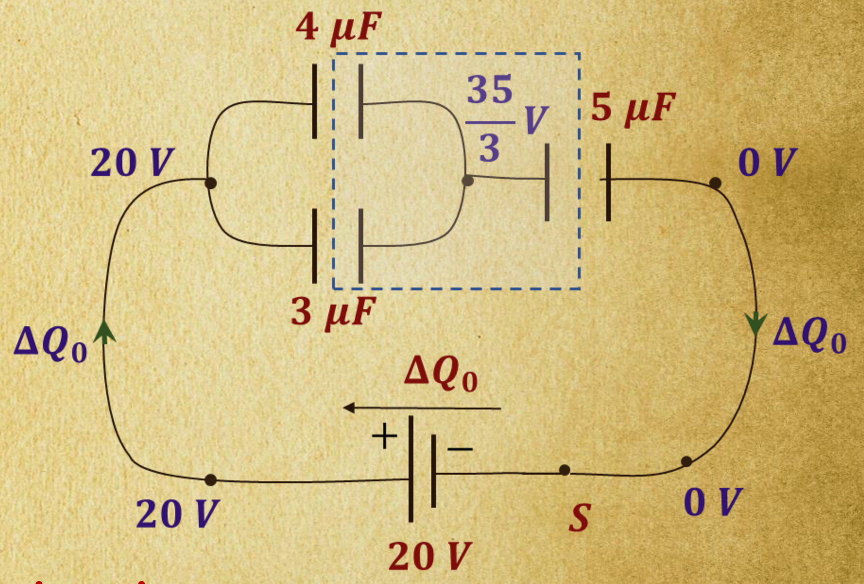
$$4[V-20]+3[V-20]+5[V-0]=0$$

$$\Rightarrow \quad V=\frac{35}{3}\ Volts$$

Charge flow through the capacitor $5\mu F$

$$\Rightarrow \quad Q_5=\left(\frac{35}{3}-0\right)5=\frac{175}{3}\mu F$$

Charge flow through the capacitor $5\mu F$ is same as in the circuit,



$$\Delta Q_0=\frac{175}{3}\mu F$$

(b) Final Charge on each capacitor.

Charge on capacitor of capacitance $4\ \mu F$

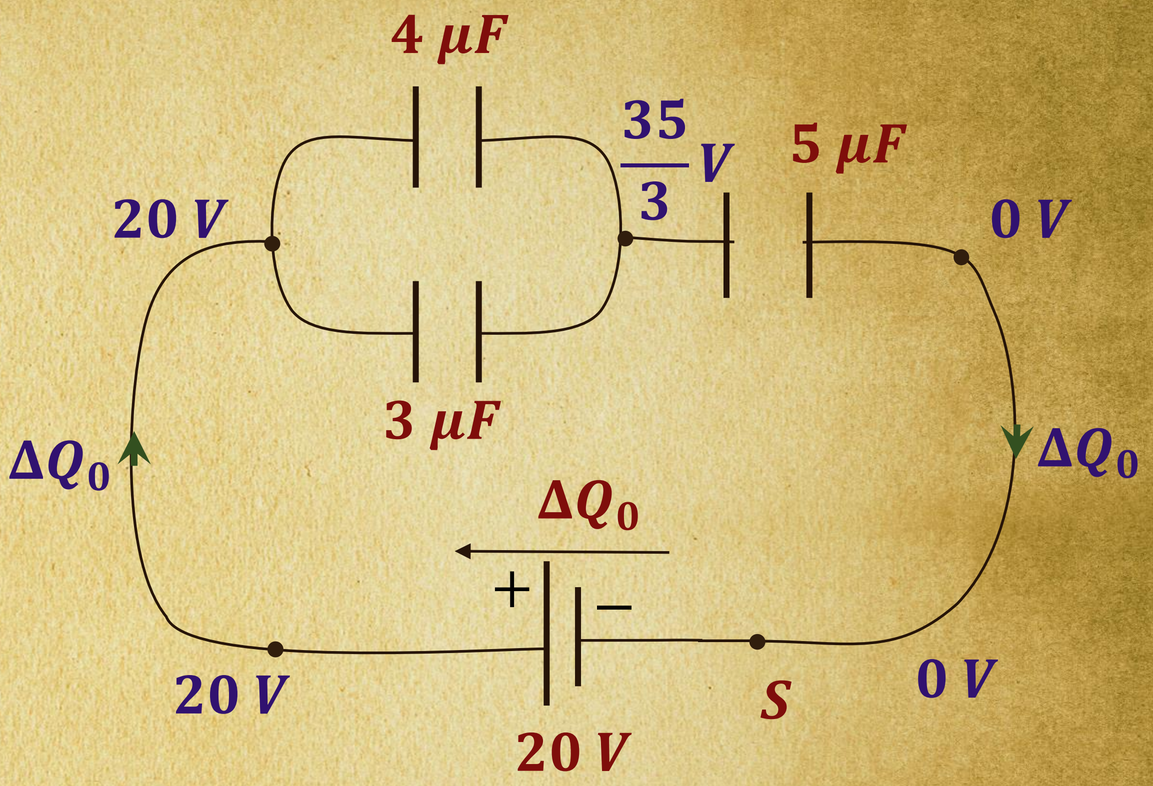
$$\Rightarrow \quad Q_4 = \left(20 - \frac{35}{3}\right)4 = \frac{100}{3}\ \mu C$$

Charge on capacitor of capacitance $3\ \mu F$

$$\Rightarrow \quad Q_3 = \left(20 - \frac{35}{3}\right)3 = 25\ \mu C$$

Charge on capacitor of capacitance $5\ \mu F$

$$\Rightarrow \quad Q_5 = \left(\frac{35}{3} - 0\right)5 = \frac{175}{3}\ \mu C$$



(c) Heat generated (H).

Energy stored in each capacitor,

$$\Rightarrow \quad U_4 = \frac{1}{2}(4 \times 10^{-6})\left(20 - \frac{35}{3}\right)^2 = \frac{1250}{9}\ \mu J$$

$$\Rightarrow \quad U_3 = \frac{1}{2}(3 \times 10^{-6})\left(20 - \frac{35}{3}\right)^2 = \frac{1875}{18}\ \mu J$$

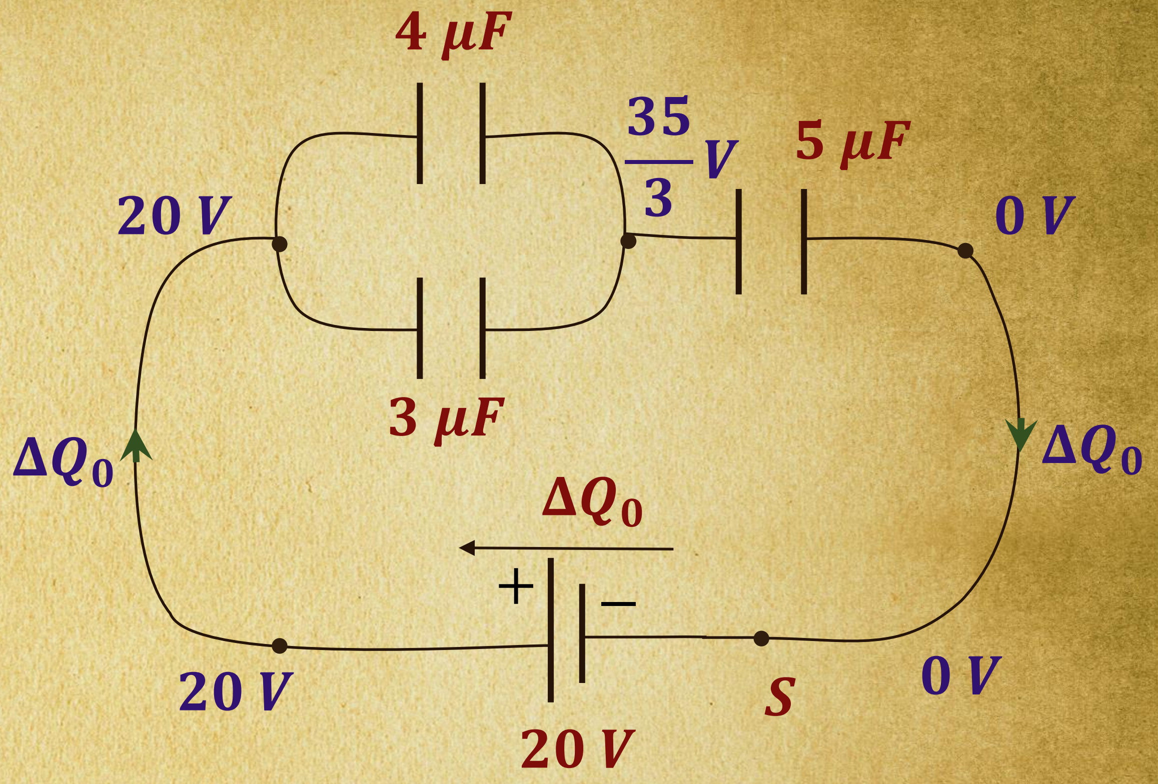
$$\Rightarrow \quad U_5 = \frac{1}{2}(5 \times 10^{-6})\left(\frac{35}{3} - 0\right)^2 = \frac{6125}{18}\ \mu J$$

Work done by battery,

$$\Rightarrow \quad W_b = \left(\frac{175}{3} \times 10^{-6}\right)(20) = \frac{3500}{3}\ \mu J$$

By energy conservation

$$W_b = U_3 + U_4 + U_5 + H$$



Heat generated in the Circuit

$$H = \frac{1750}{3}\ \mu J$$

Find the charges on the three capacitors shown below.

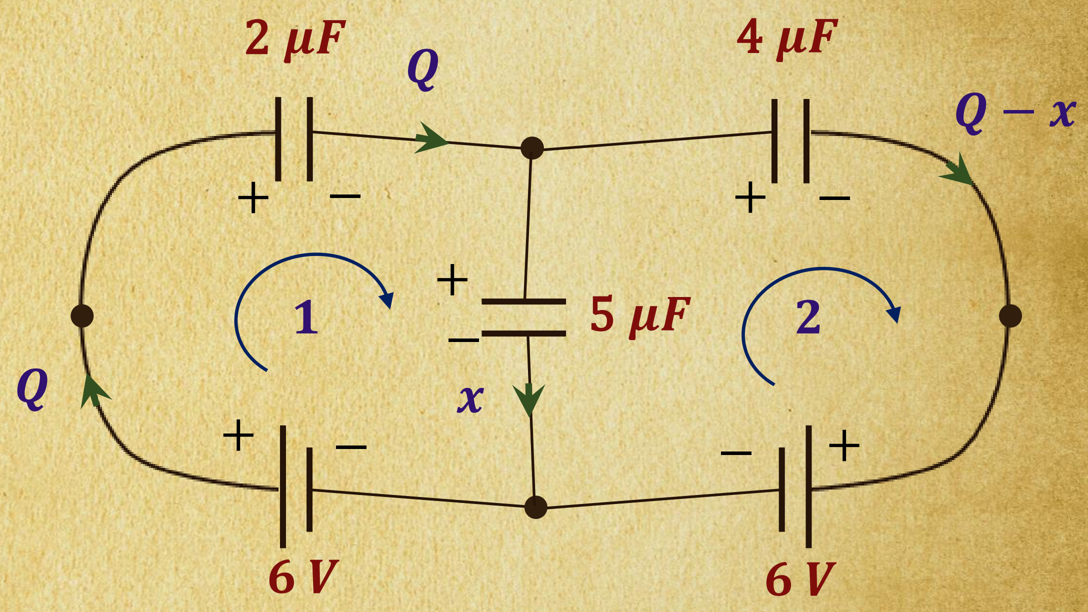
Applying KVL in loop 1

$$\Rightarrow -\frac{Q}{2} - \frac{x}{5} + 6 = 0 \quad \Rightarrow 5Q + 2x = 60$$

Applying KVL in loop 2

$$\Rightarrow -\frac{Q-x}{4} - 6 + \frac{x}{5} = 0 \quad \Rightarrow 5Q - 9x = -120$$

Solving above equations



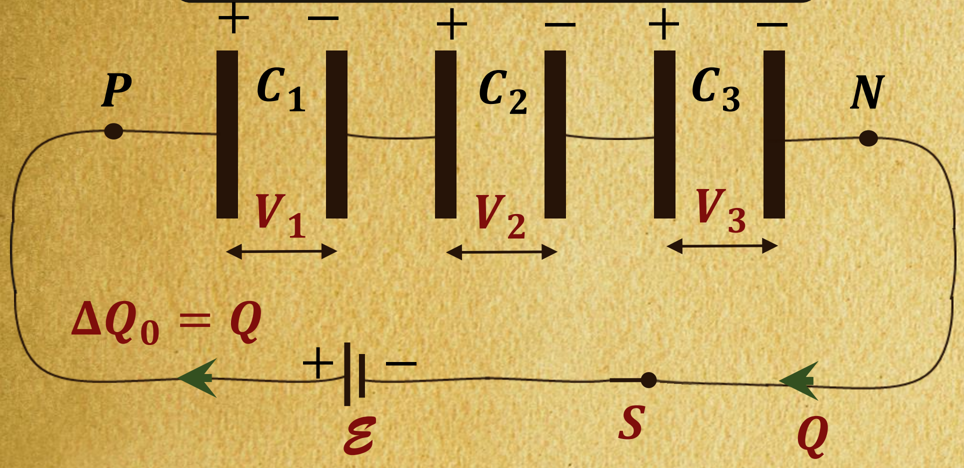
$Q_{2\mu F} = 5.45\ \mu C$

$Q_{4\mu F} = 10.9\ \mu C$

$Q_{5\mu F} = 16.35\ \mu C$

Combination of Capacitors

Series Combination



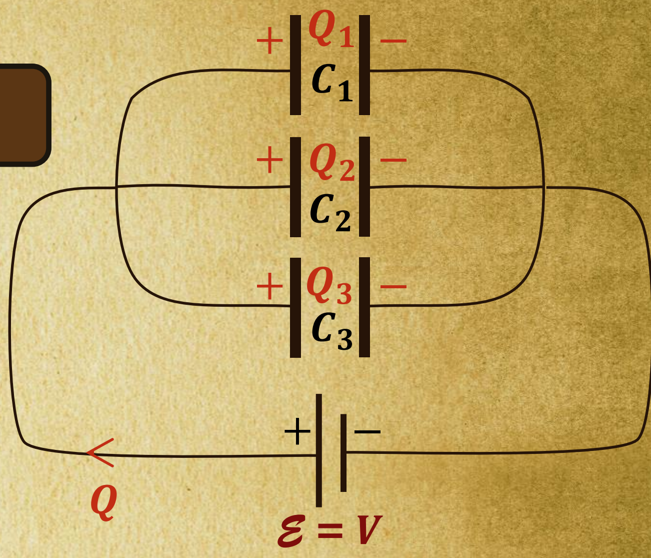
- Charge on each capacitor is same, $Q = C_1V_1 = C_2V_2 = C_3V_3$

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

- Equivalent Capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Parallel Combination



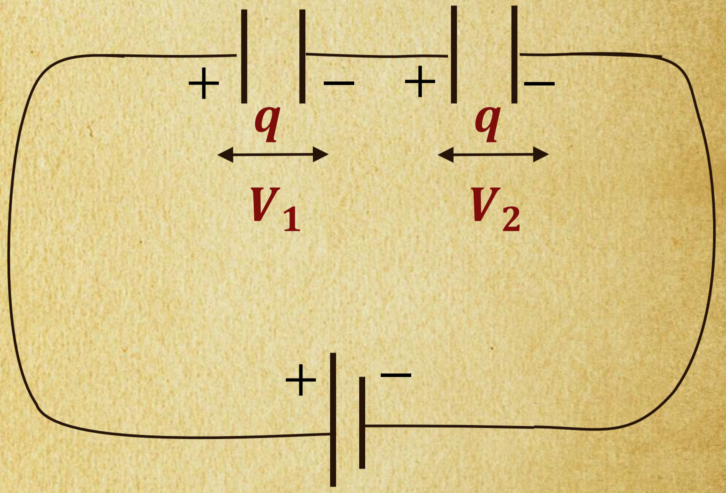
- Potential difference across each capacitor is same. $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

- Equivalent Capacitance is given by

$$C_{eq} = C_1 + C_2 + C_3$$

In the circuit shown below, Find
(a) The equivalent capacitance.
(b) The charge stored in each capacitor.
(c) The potential difference across each capacitor.



(a) The equivalent capacitance.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1 \times 4}{1 + 4} \Rightarrow \boxed{C_{eq} = 0.80\ \mu F}$$

(b) The charge q, stored in each capacitor is,

$$\Rightarrow q = C_{eq} V = (0.80 \times 10^{-6})(100) C \Rightarrow \boxed{q = 80\ \mu C}$$

(c) The potential difference across each capacitor

$$V \propto \frac{1}{C} \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right) V = \left(\frac{4}{1+4}\right)(100) \Rightarrow \boxed{V_1 = 80\ V}$$

$$V_2 = V - V_1 = 100 - 80 \Rightarrow \boxed{V_2 = 20\ V}$$

In the figure shown below, the charge on the left plate of the $10\ \mu F$ capacitor is $-30\ \mu C$. The charge on the right plate of the $6\ \mu F$ capacitor is

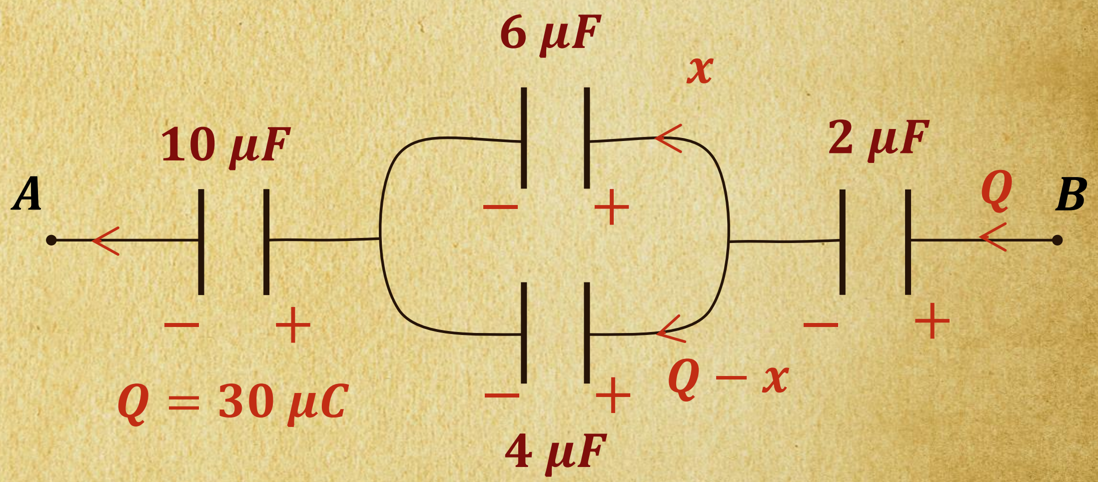
$V_{upper} = V_{lower}$ ($\because$ Parallel connection)

$\Rightarrow \frac{x}{6} = \frac{Q - x}{4}$

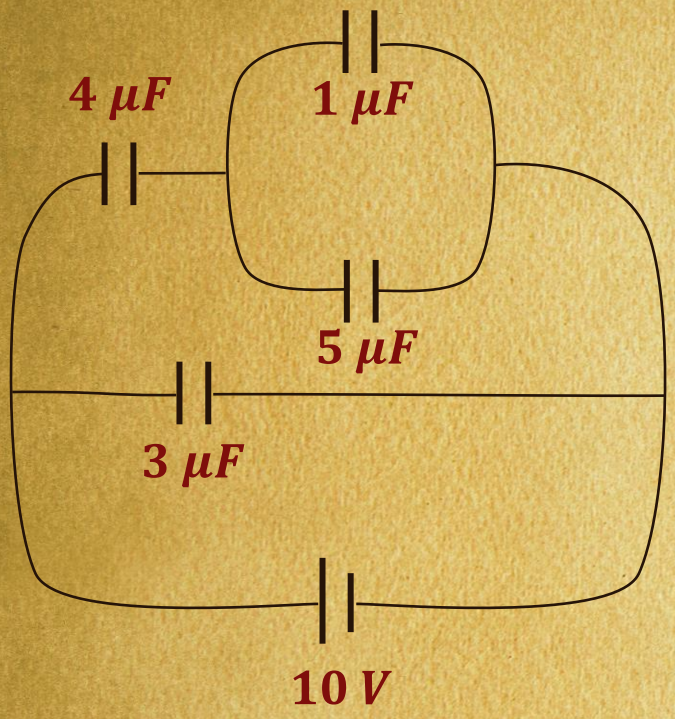
$\Rightarrow \frac{x}{6} = \frac{30 - x}{4}$

$\Rightarrow 4x = 180 - 6x$

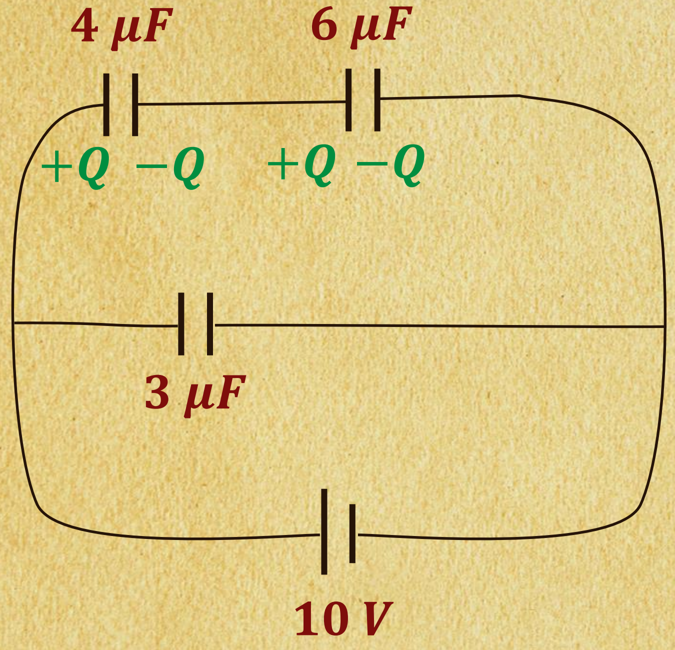
$x = 18\ \mu C$



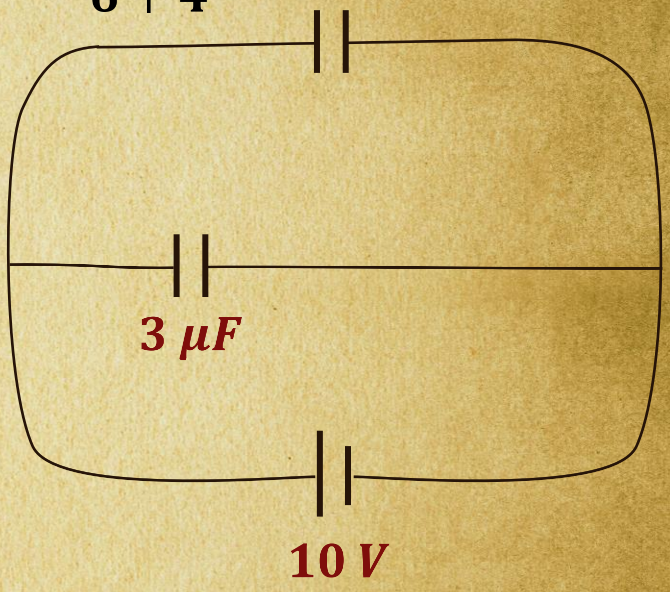
In the given circuit, find the charge on $4\ \mu F$ capacitor.



$1 + 5 = 6\ \mu F$



$$C_1' = \frac{6 \times 4}{6 + 4} = 2.4\ \mu F$$



Charge supplied to the upper branch -

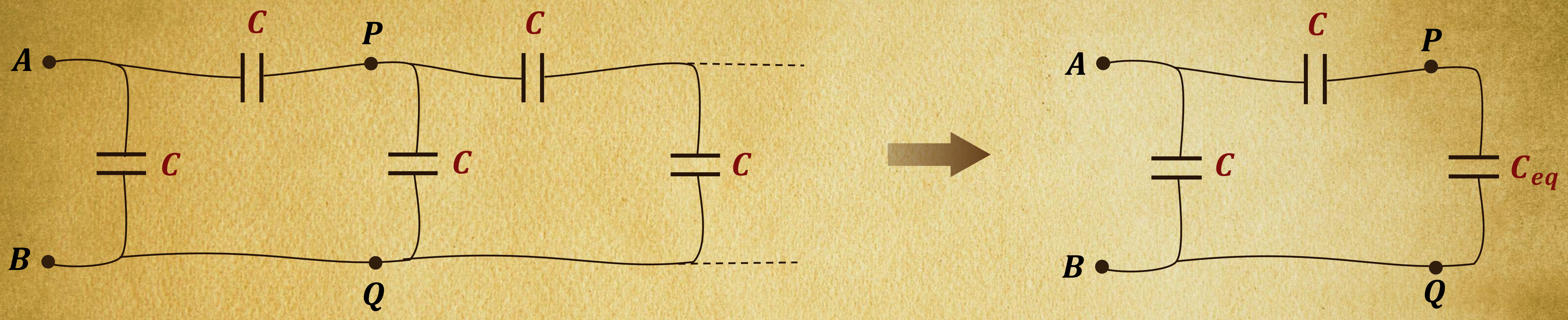
$$Q = 2.4 \times 10\ \mu C$$

$$= 24\ \mu C$$

Charge on $4\ \mu F$ capacitor -

$$Q = 24\ \mu C$$

Find the equivalent capacitance of the infinite ladder network shown below.



By substituting the infinite ladder with C_{eq},

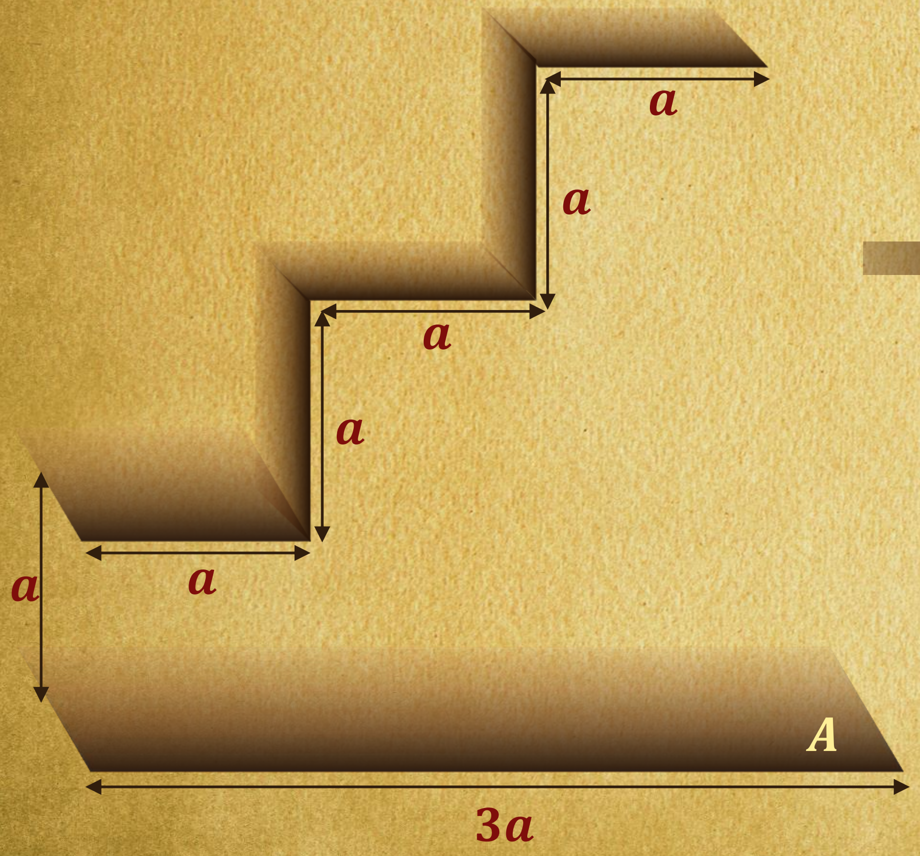
$$C_{AB} = C + \frac{C \times C_{eq}}{C + C_{eq}} = C_{eq}$$

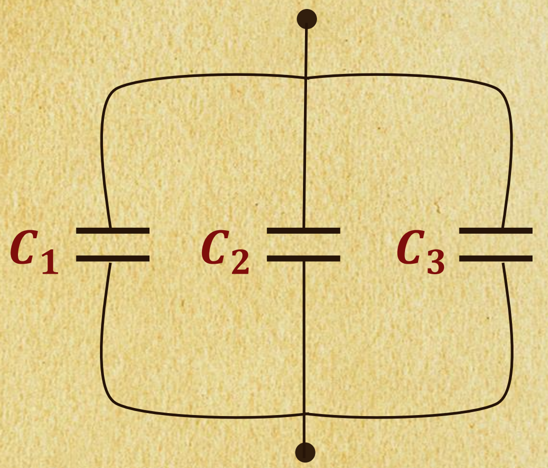
$$C_{eq}^2 - C(C_{eq}) - C^2 = 0$$

$$C_{eq} = \frac{C + \sqrt{C^2 + 4C^2}}{2} \quad (\because C \nless 0)$$

$$C_{eq} = \frac{1 + \sqrt{5}}{2} C$$

A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown. The width and the height of each stair is a. Find the capacitance of the assembly.





Equivalent capacitance of parallel combination,

$$C_{eq} = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3a} + \frac{\varepsilon_0 A}{6a} + \frac{\varepsilon_0 A}{9a}$$

$$C_{eq} = \frac{11\varepsilon_0 A}{18a}$$

A capacitance of $2\ \mu F$ is required in an electrical circuit across a potential difference of $1\ kV$. A large number of $1\ \mu F$ capacitors are available which can withstand a potential difference of not more than $300\ V$. The minimum number of capacitors required to achieve this is

As voltage across a capacitor should not exceed $300\ V$,

$$n \times 300 > 1000$$

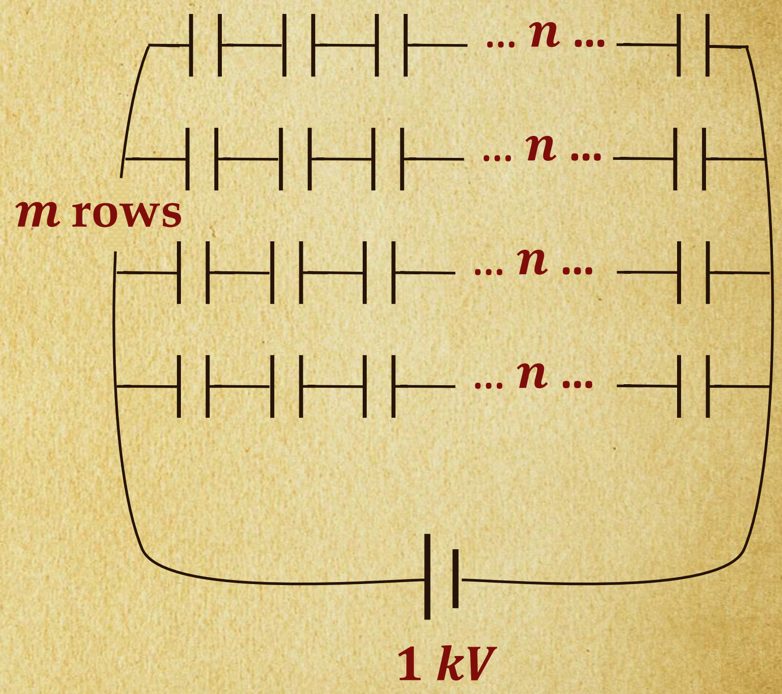
$$n_{min} = 4 \quad \text{(in a branch)}$$

Equivalent capacitance of the network -

$$C_{eq} = m \times \frac{C}{n} = 2$$

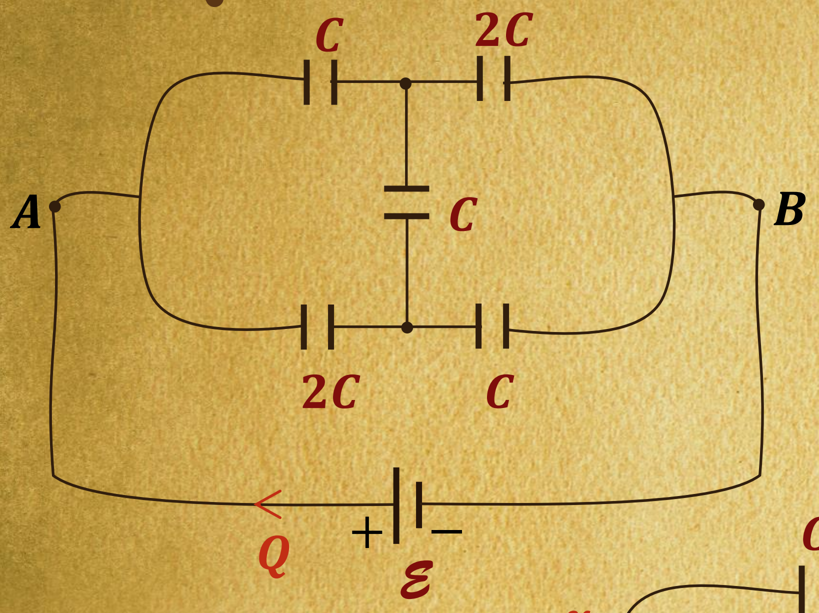
$$= m \times \frac{1}{4} = 2$$

$$m = 8$$

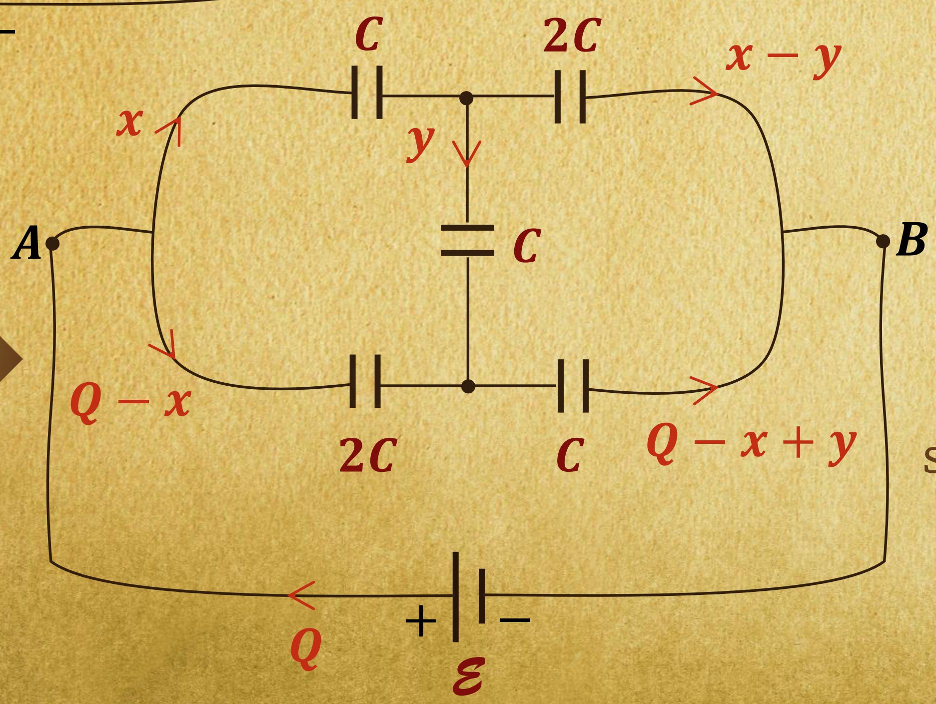


No. of capacitors required $= m \times n = 8 \times 4 = 32$

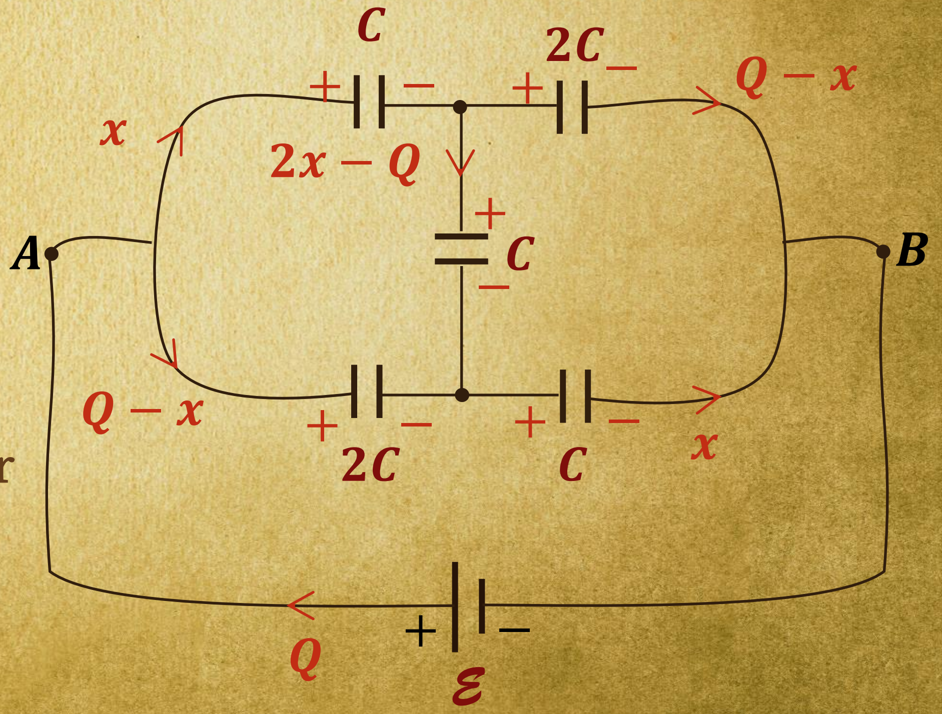
Find the equivalent capacitance of network shown below.

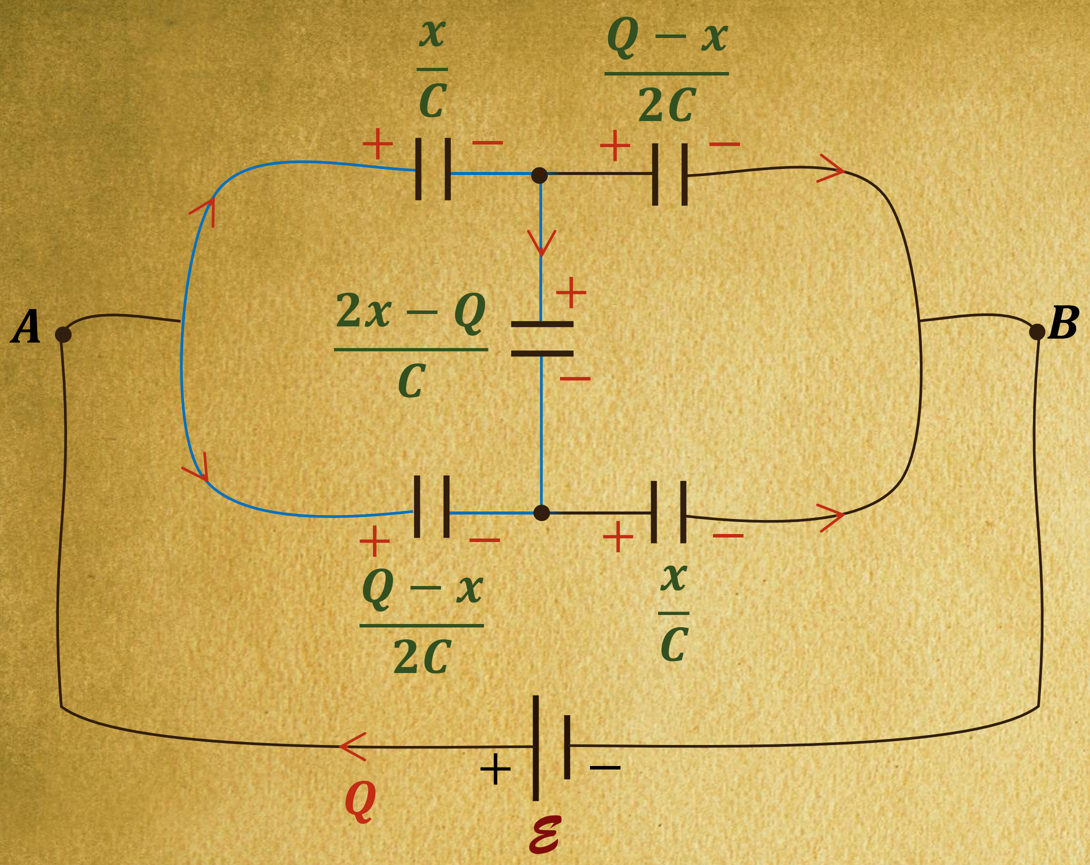


Equivalent capacitance of the network, $C_{eq} = \frac{Q}{\mathcal{E}}$



I/O symmetry

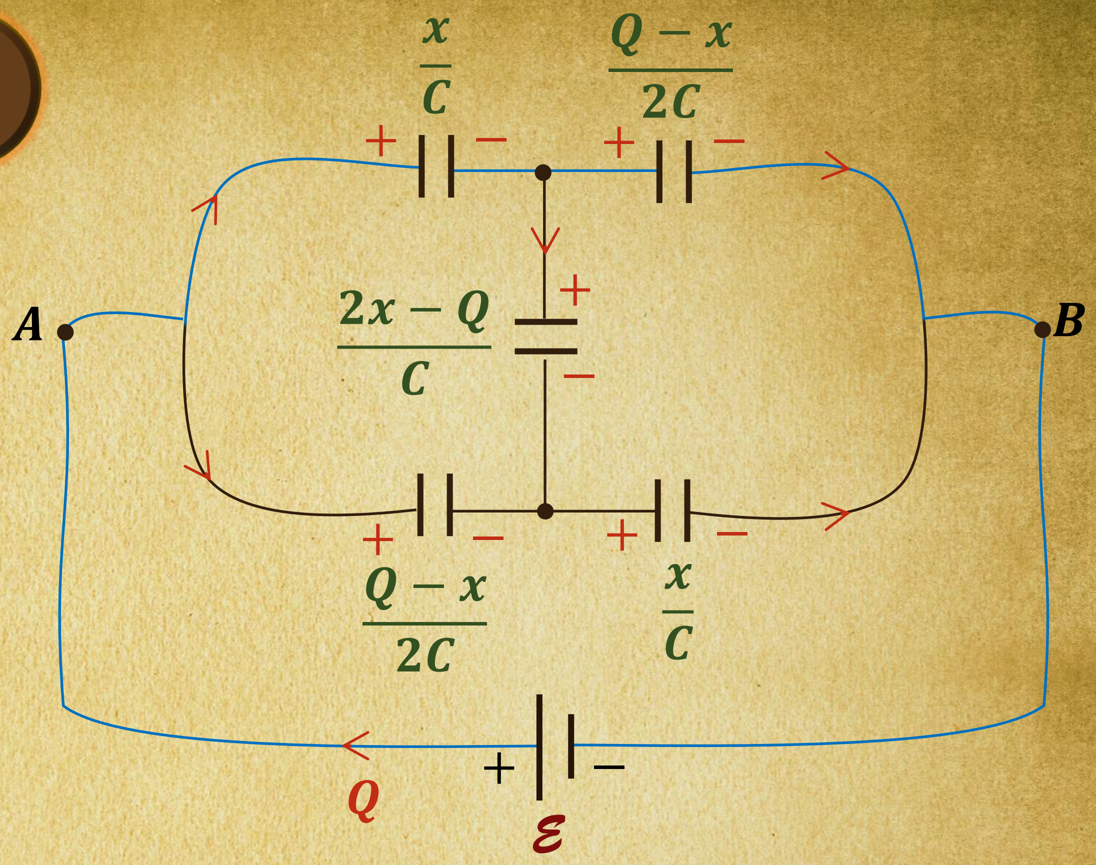




Applying KVL for the inner loop,

$$-\frac{x}{C}-\frac{(2x-Q)}{C}+\frac{Q-x}{2C}=0$$

$$x=\frac{3Q}{7}$$

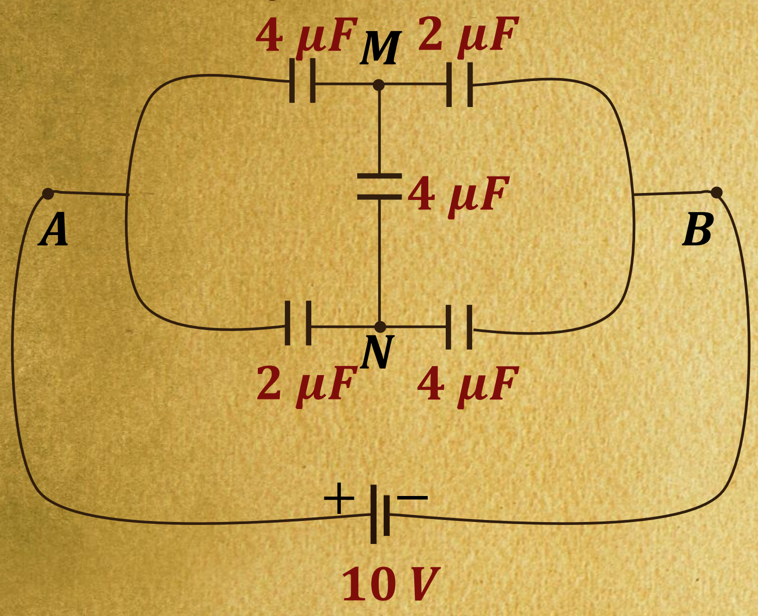


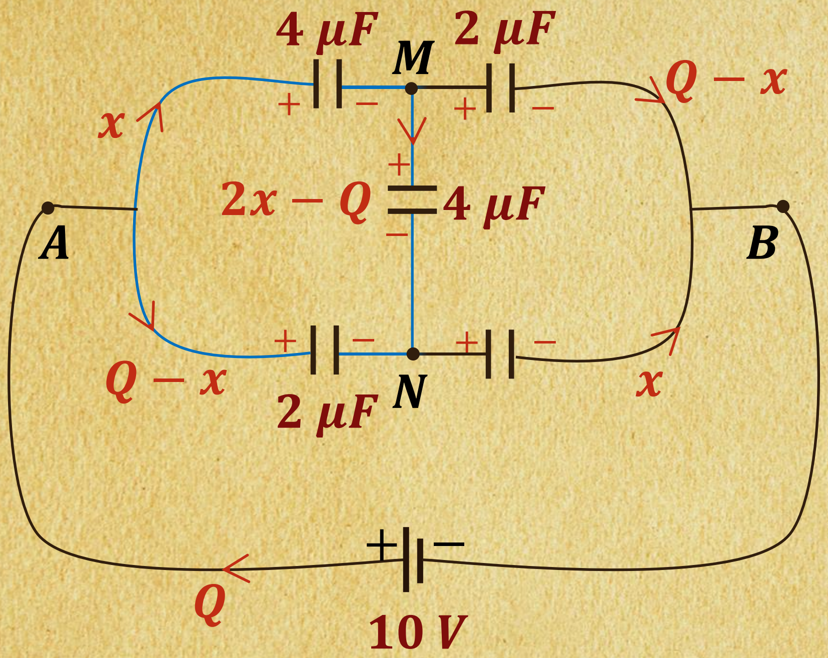
Applying KVL for the outer loop,

$$-\frac{x}{C}-\frac{(Q-2x)}{2C}+\mathcal{E}=0$$

$$C_{eq}=\frac{Q}{\mathcal{E}}=\frac{7C}{5}$$

Find the charge flowing through the middle capacitor ($4\ \mu F$) placed between the points M and N.

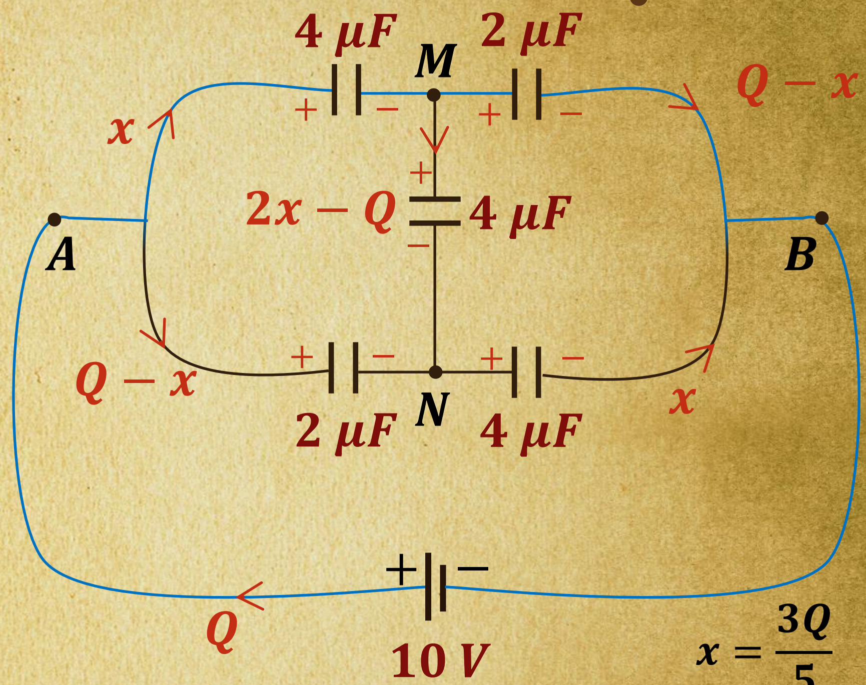




Applying KVL for the inner loop,

$$-\left(\frac{x}{4}\right)-\left(\frac{2x-Q}{4}\right)+\left(\frac{Q-x}{2}\right)=0$$

$$x=\frac{3Q}{5}$$



$$x=\frac{3Q}{5}$$

Applying KVL for the outer loop,

$$-\left(\frac{x}{4}\right)-\left(\frac{Q-x}{2}\right)+10=0$$

$$2Q-x=40 \quad \Rightarrow Q=\frac{200}{7}\mu C$$

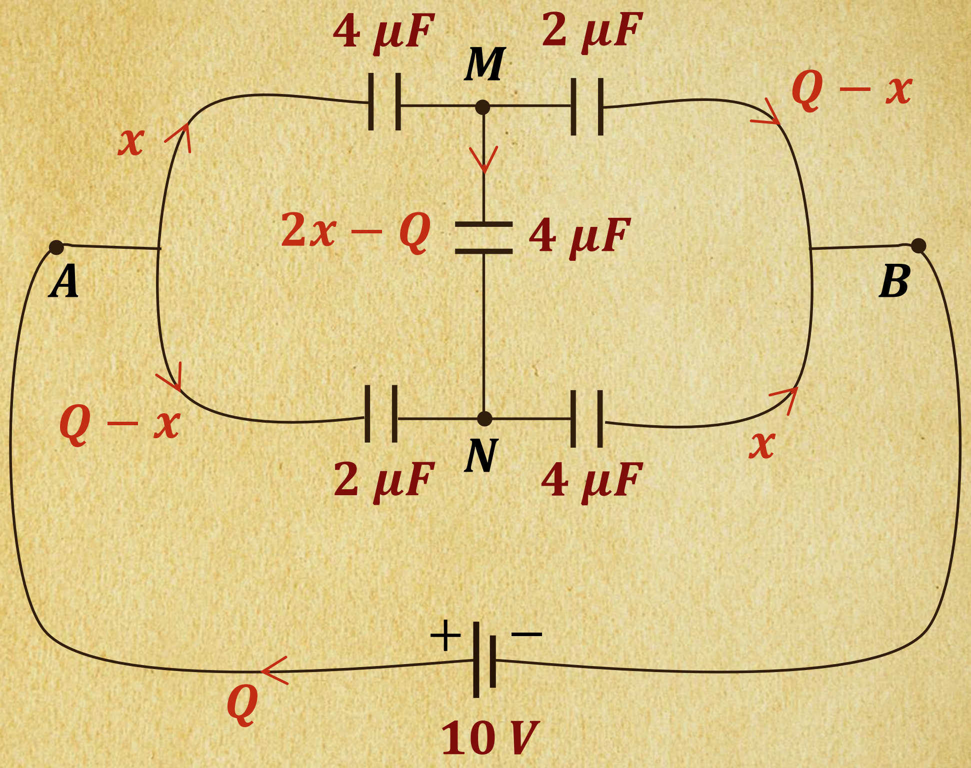
$$x = \frac{3Q}{5} \qquad Q = \frac{200}{7}\,\mu C$$

Charge flowing through the middle capacitor

$$Q_{mid} = 2x - Q = \frac{Q}{5} \quad \left(\because x = \frac{3Q}{5}\right)$$

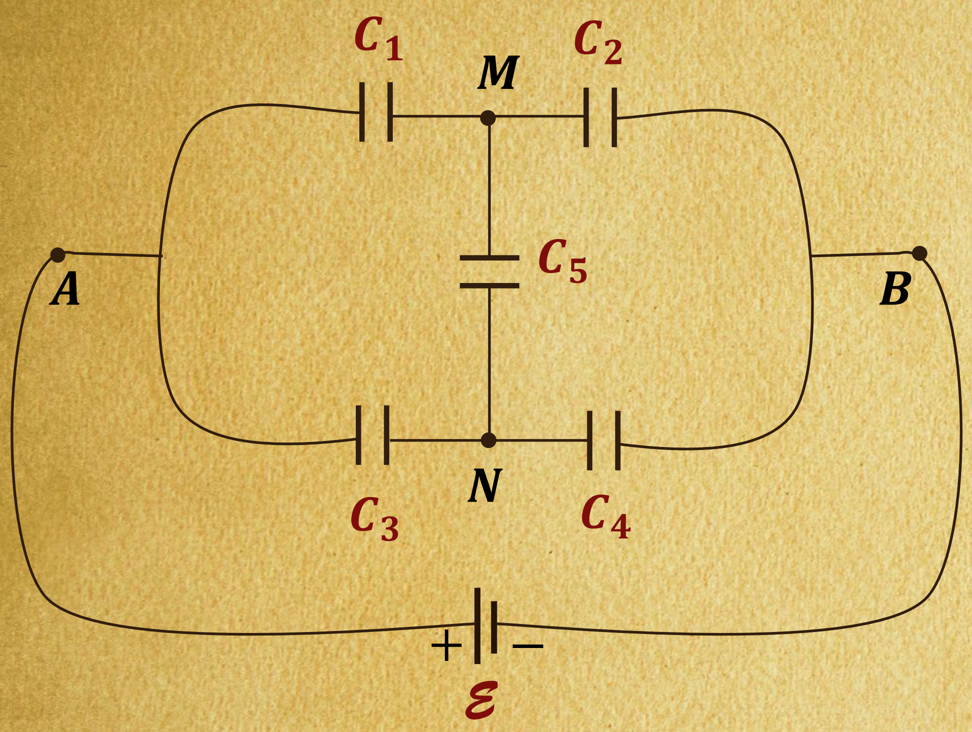
$$= \frac{200}{7} \times \frac{1}{5}$$

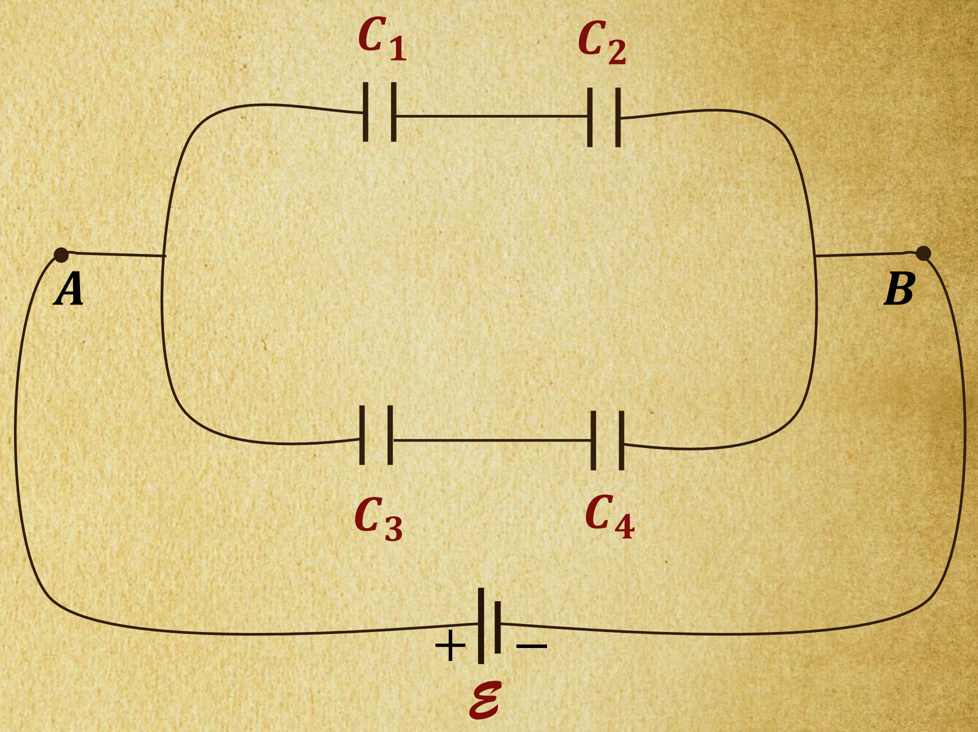
$$Q_{mid} = \frac{40}{7}\,\mu C$$



$$Q = \frac{200}{7}\,\mu C$$

Balanced Wheatstone Bridge





Condition for a balanced Wheatstone bridge

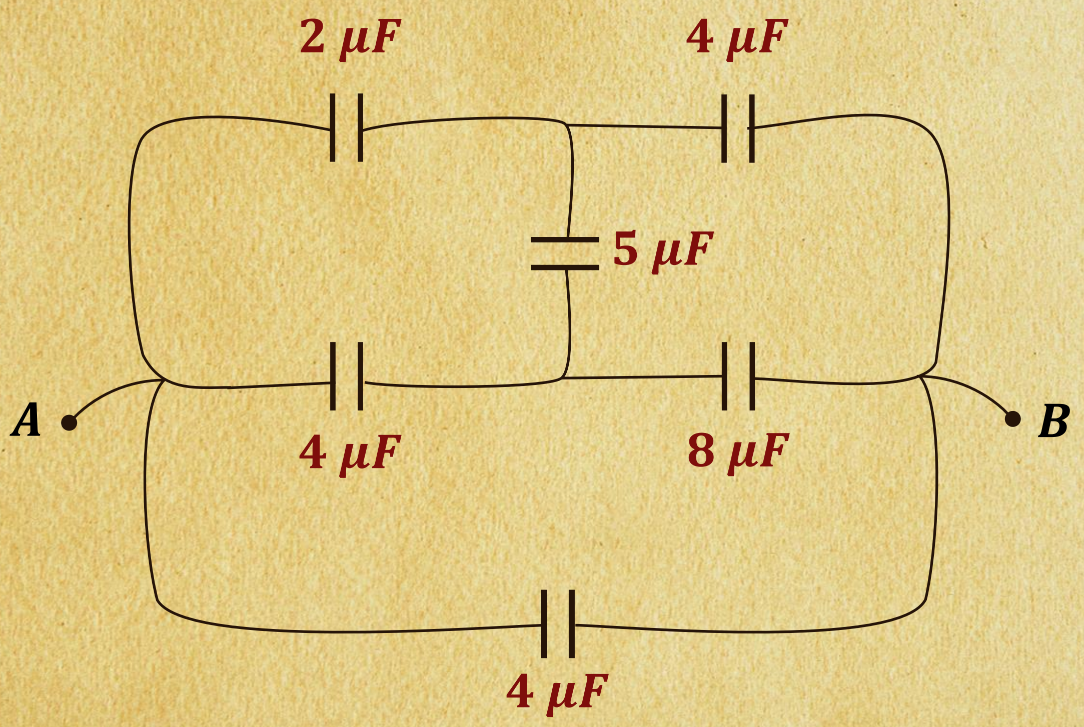
$$\frac{C_1}{C_3} = \frac{C_2}{C_4}$$

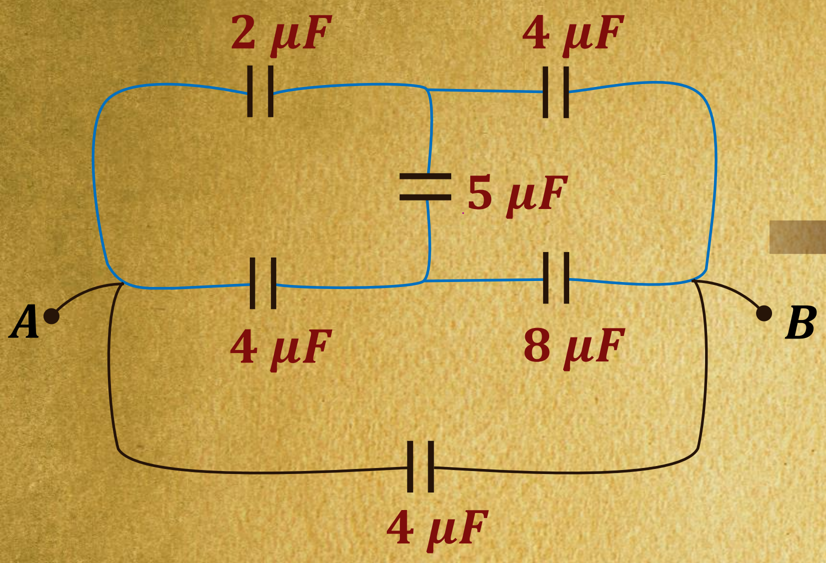
$$\Rightarrow V_M = V_N$$

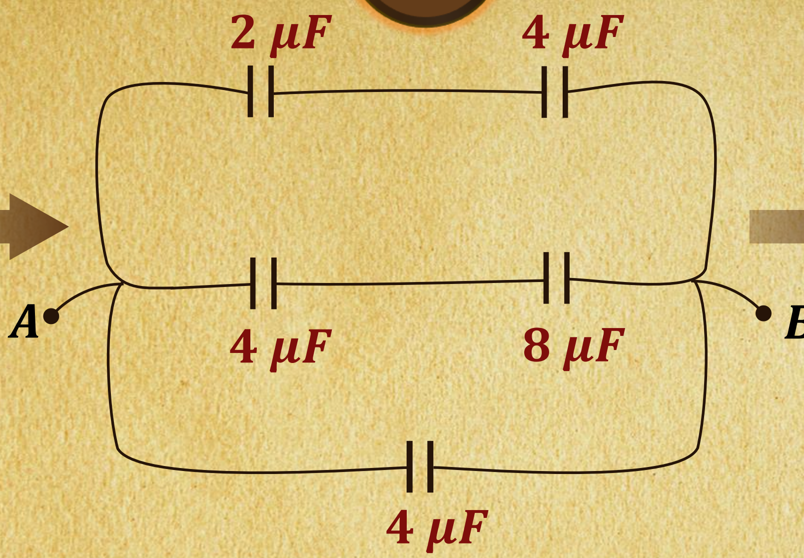
Equivalent Capacitance of the circuit -

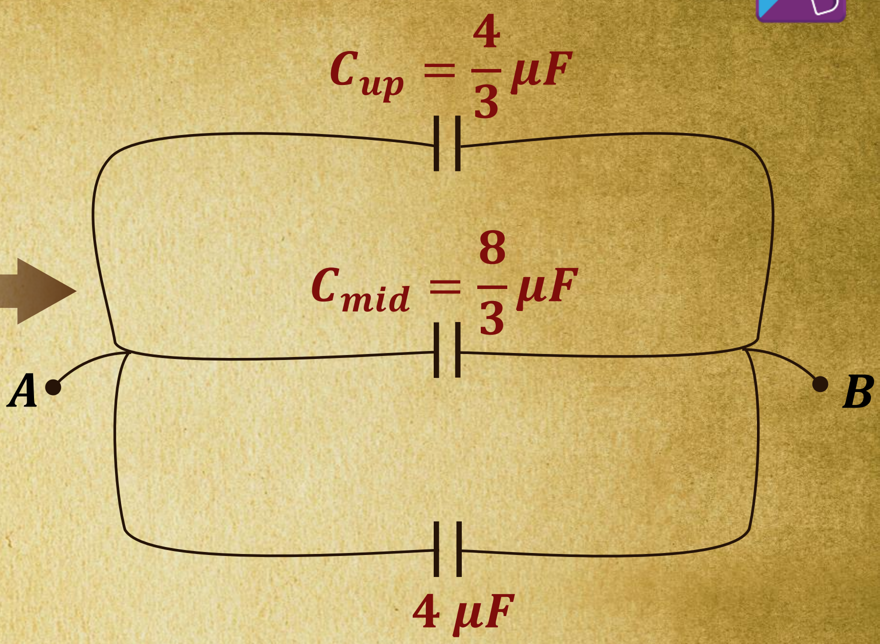
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

What will be the equivalent capacitance of the network between the points A and B ?









$$\frac{2\ \mu F}{4\ \mu F} = \frac{4\ \mu F}{8\ \mu F} = \frac{1}{2}$$

Balanced Wheatstone bridge

$$C_{up} = \frac{2 \times 4}{2 + 4} = \frac{4}{3} \mu F$$

$$C_{mid} = \frac{4 \times 8}{4 + 8} = \frac{8}{3} \mu F$$

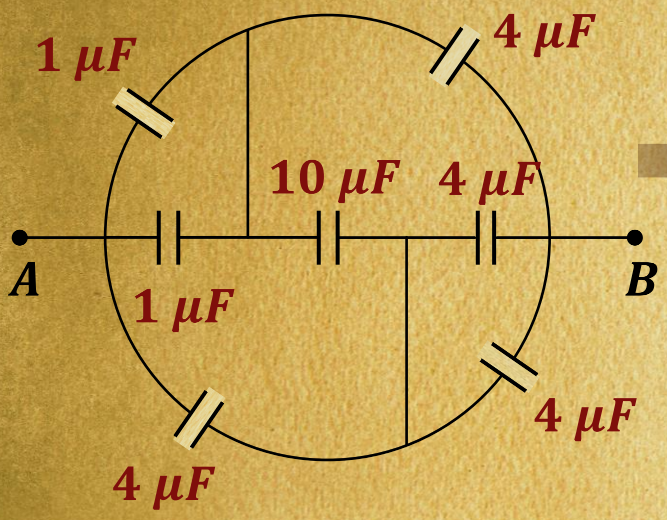
Equivalent Capacitance of the circuit between *A* and *B* -

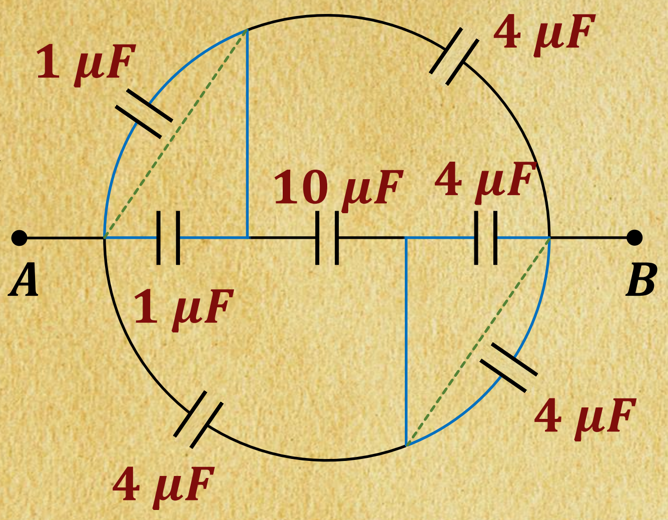
$$C_{eq} = C_{up} + C_{mid} + 4\ \mu F$$

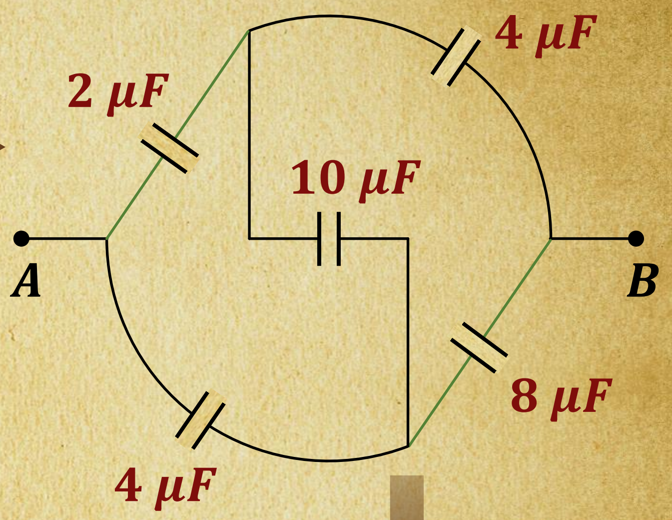
$$= \frac{4}{3} + \frac{8}{3} + 4$$

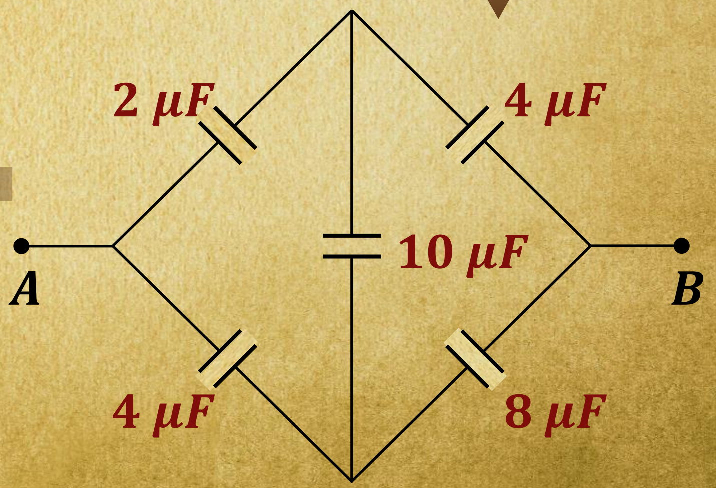
$$C_{eq} = 8\ \mu F$$

What will be the equivalent capacitance of the network between the points *A* and *B* ?

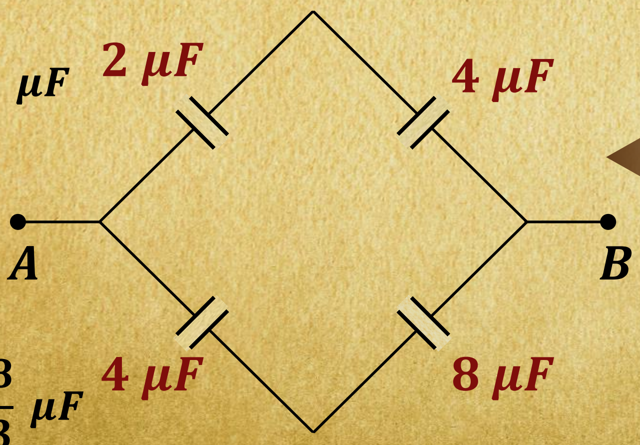








$$C_{up} = \frac{2 \times 4}{2+4} = \frac{4}{3}\ \mu F$$



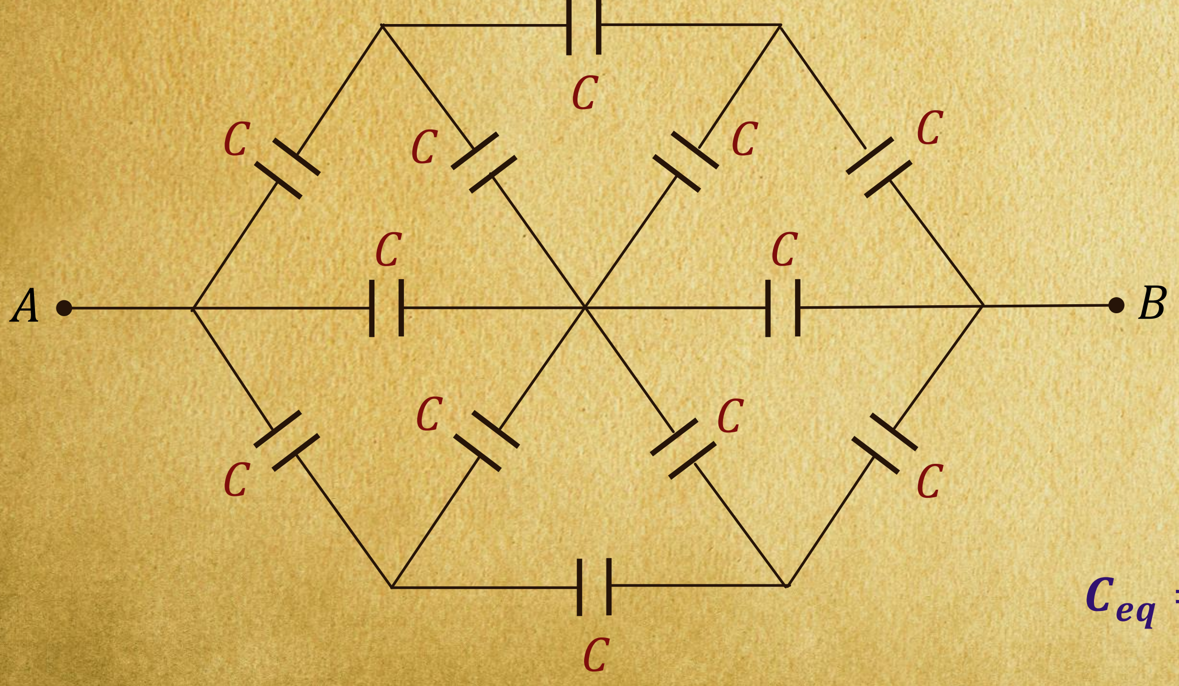
$$C_{low} = \frac{4 \times 8}{4+8} = \frac{8}{3}\ \mu F$$

$$C_{eq} = \frac{4}{3} + \frac{8}{3}\ \mu F$$

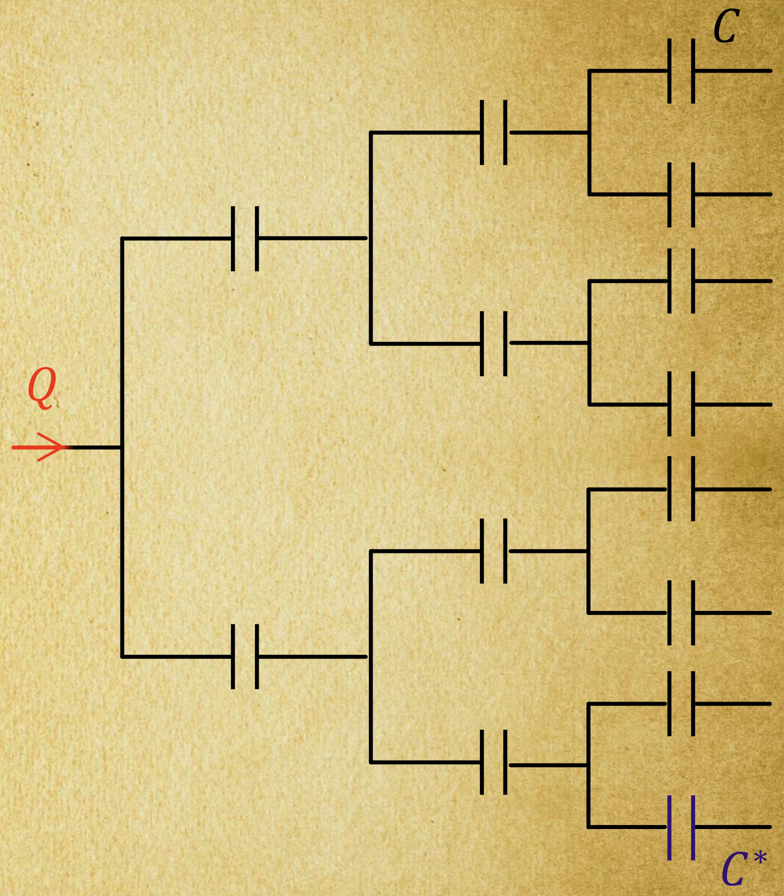
$$C_{eq} = 4\ \mu F$$

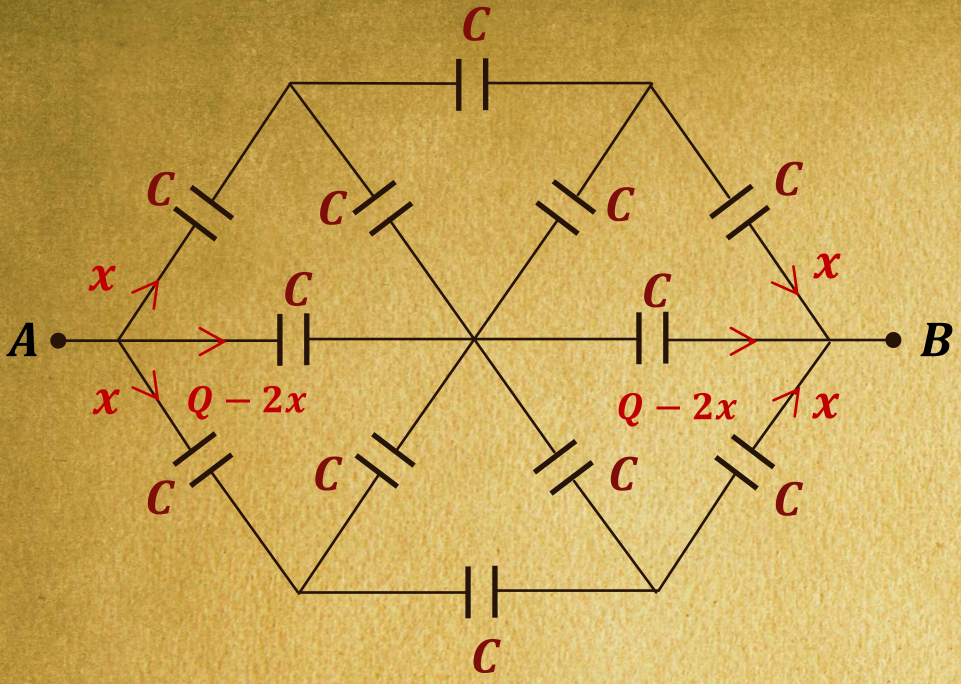
Symmetric Circuits

At a junction, current/charge will divide itself equally into all branches if and only if the future distribution of branches is also exactly identical till the very end.

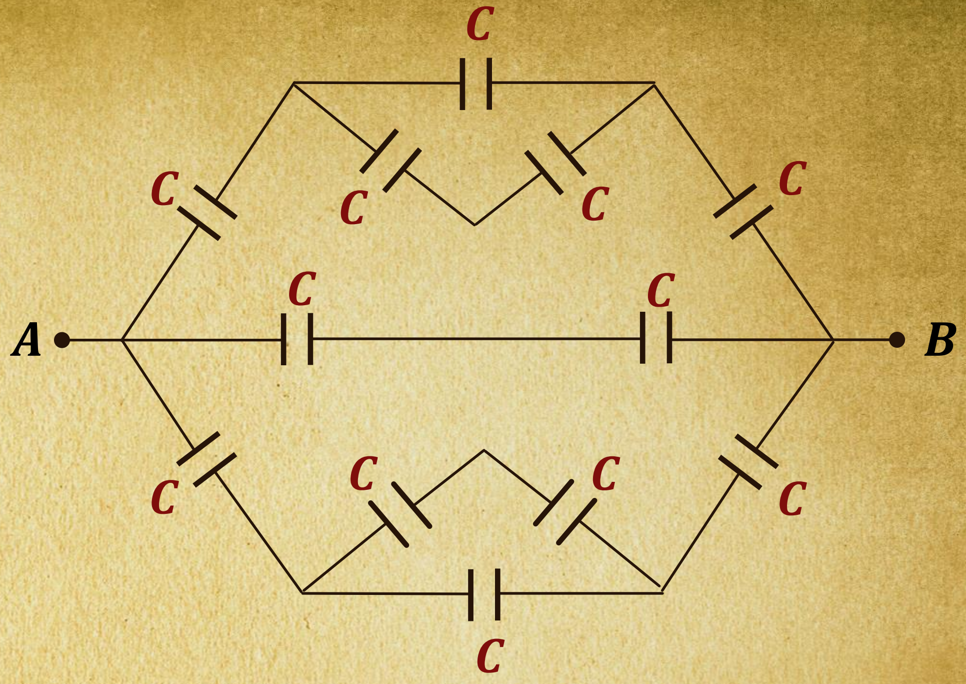


$C_{eq} = ?$

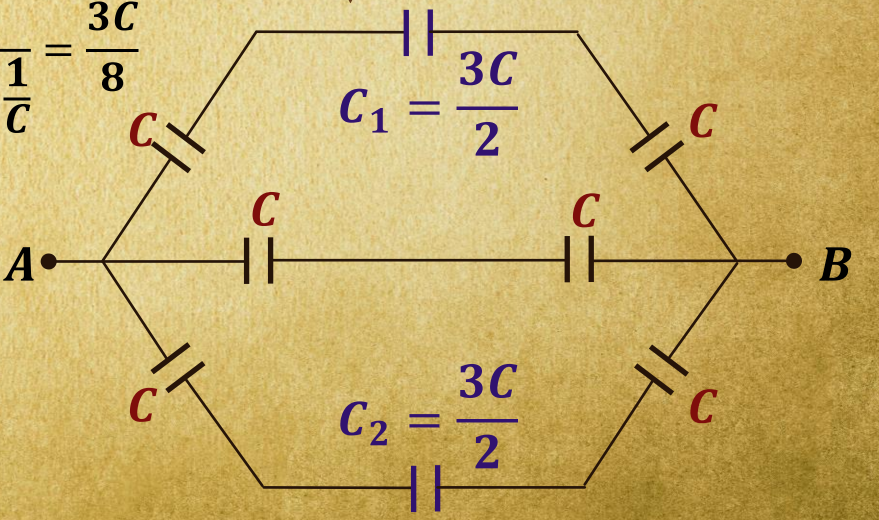




Folding + I/O symmetry

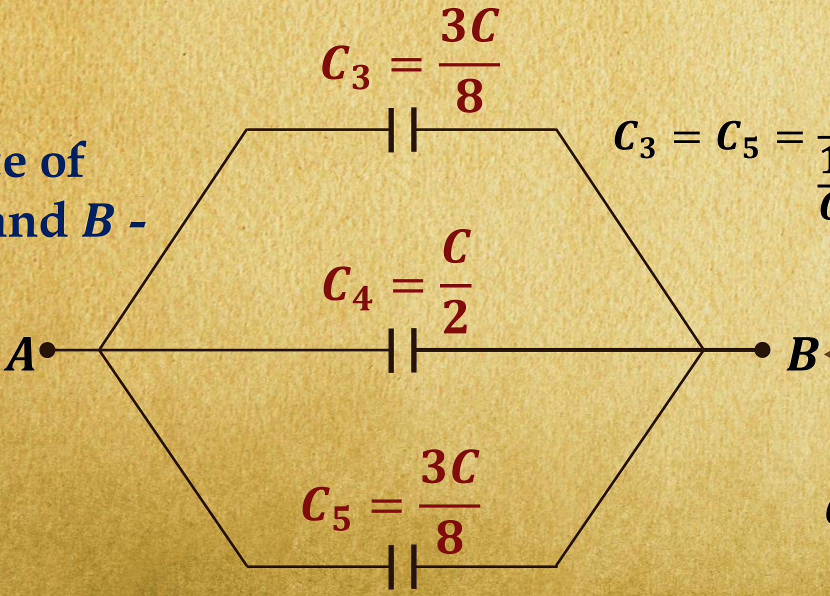


$$C_1 = C_2 = C + \frac{C \times C}{C + C} = \frac{3}{2}C$$



$$C_3 = C_5 = \frac{1}{\frac{1}{C} + \frac{2}{3C} + \frac{1}{C}} = \frac{3C}{8}$$

$$C_4 = \frac{C}{2}$$



Equivalent Capacitance of the circuit between *A* and *B* -

$$C_{eq} = \frac{3C}{8} + \frac{C}{2} + \frac{3C}{8}$$

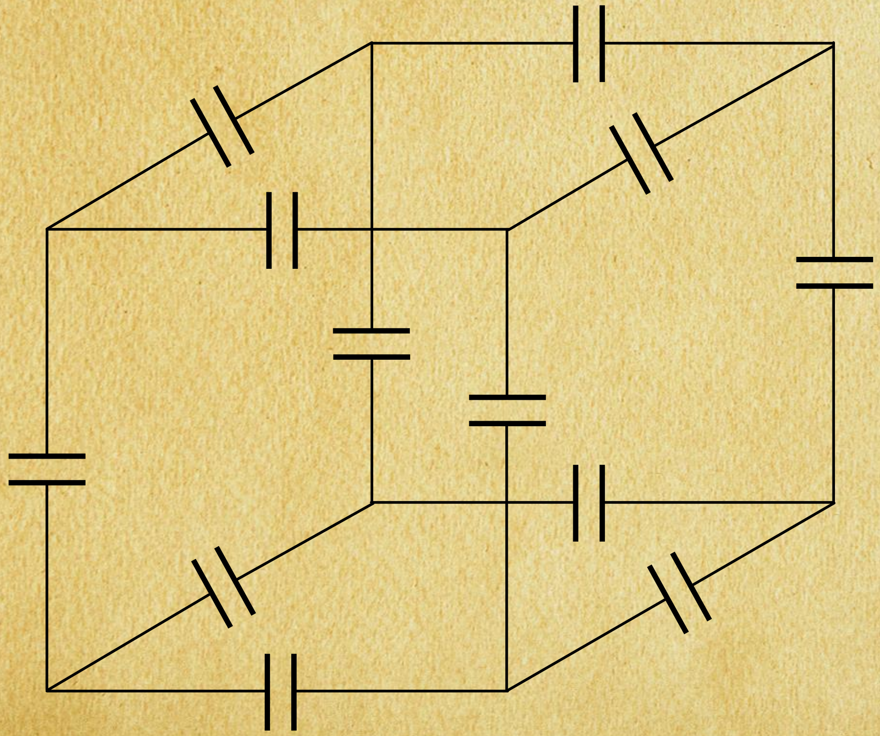
$$= \frac{5}{4}C$$

Twelve capacitors, each having a capacitance C, are connected to form a cube. Find the equivalent capacitance

(a) Along body diagonal

(b) Along an edge

(c) Along face diagonal



(a) Along body diagonal

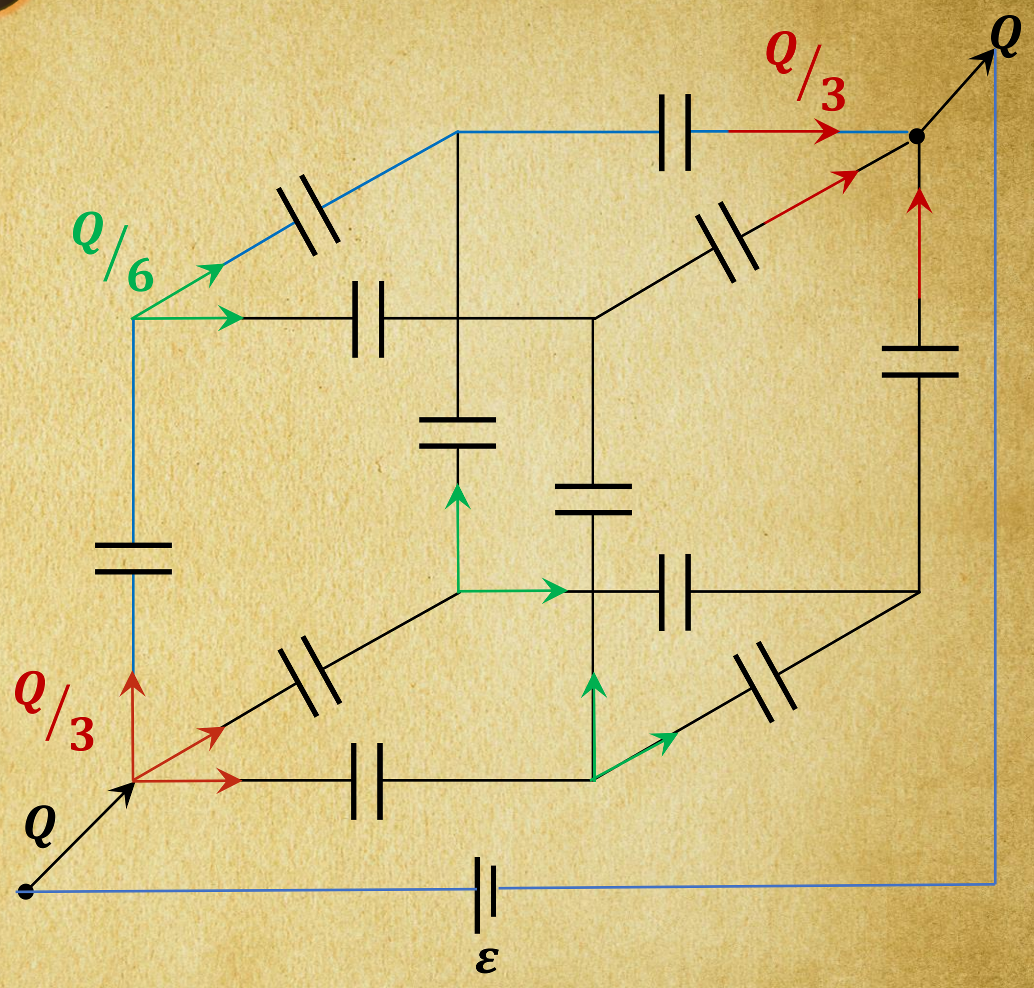
Applying KVL in the blue loop,

$$-\frac{\frac{Q}{3}}{C}-\frac{\frac{Q}{6}}{C}-\frac{\frac{Q}{3}}{C}+\varepsilon=0$$

$$\frac{Q}{C}\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right]=\varepsilon$$

Simplifying, we get

$$C_{body}=\frac{Q}{\varepsilon}=\frac{6C}{5}$$



(b) Along an edge

Using KVL on the roof (clockwise),

$$-\frac{2y}{C}-\frac{y}{C}+\frac{(x-y)}{C}-\frac{y}{C}=0$$

$$x=5y$$

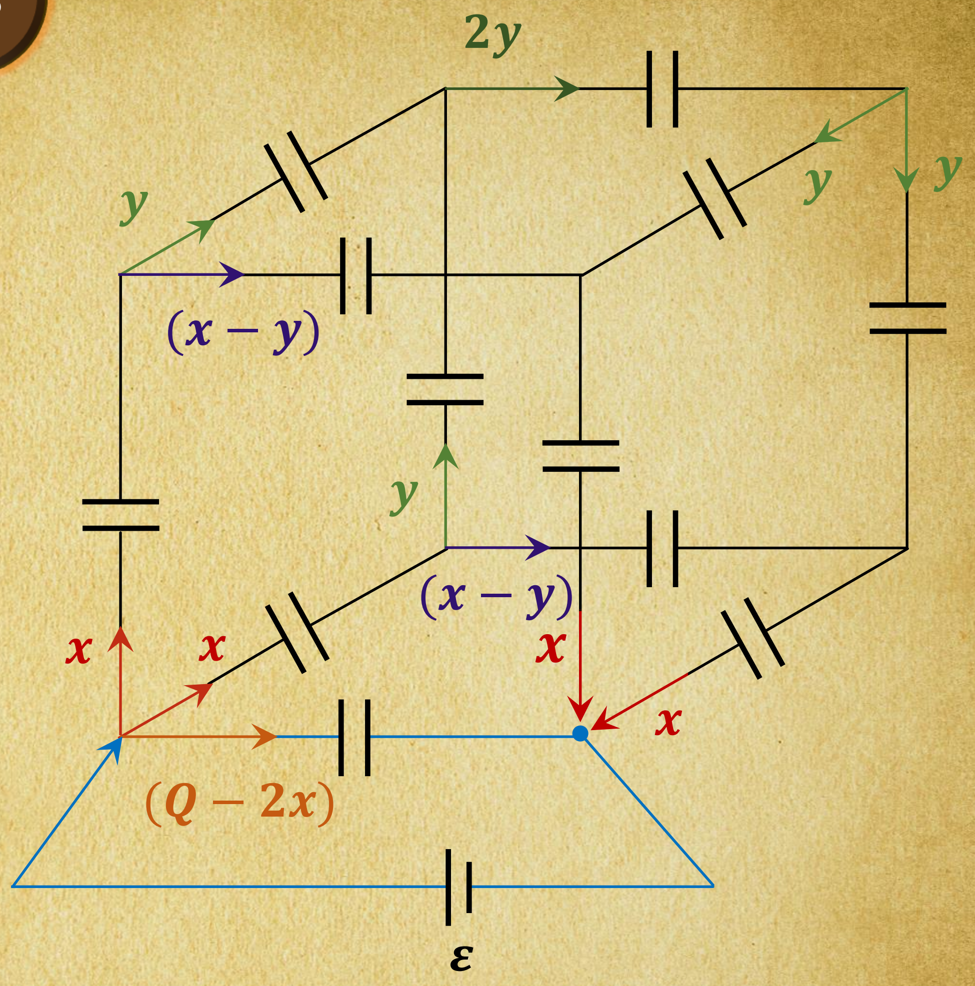
Using KVL on the front wall (clockwise),

$$-\frac{(x-y)}{C}-\frac{x}{C}+\frac{(Q-2x)}{C}-\frac{x}{C}=0$$

$$5x-y=Q$$

Using KVL on the outside loop (clockwise),

$$\frac{Q-2x}{C}+\varepsilon=0 \qquad \Rightarrow Q-2x=C\varepsilon$$



(b) Along an edge

The three derived relations are,

$x = 5y \qquad 5x - y = Q \qquad Q - 2x = C\varepsilon$

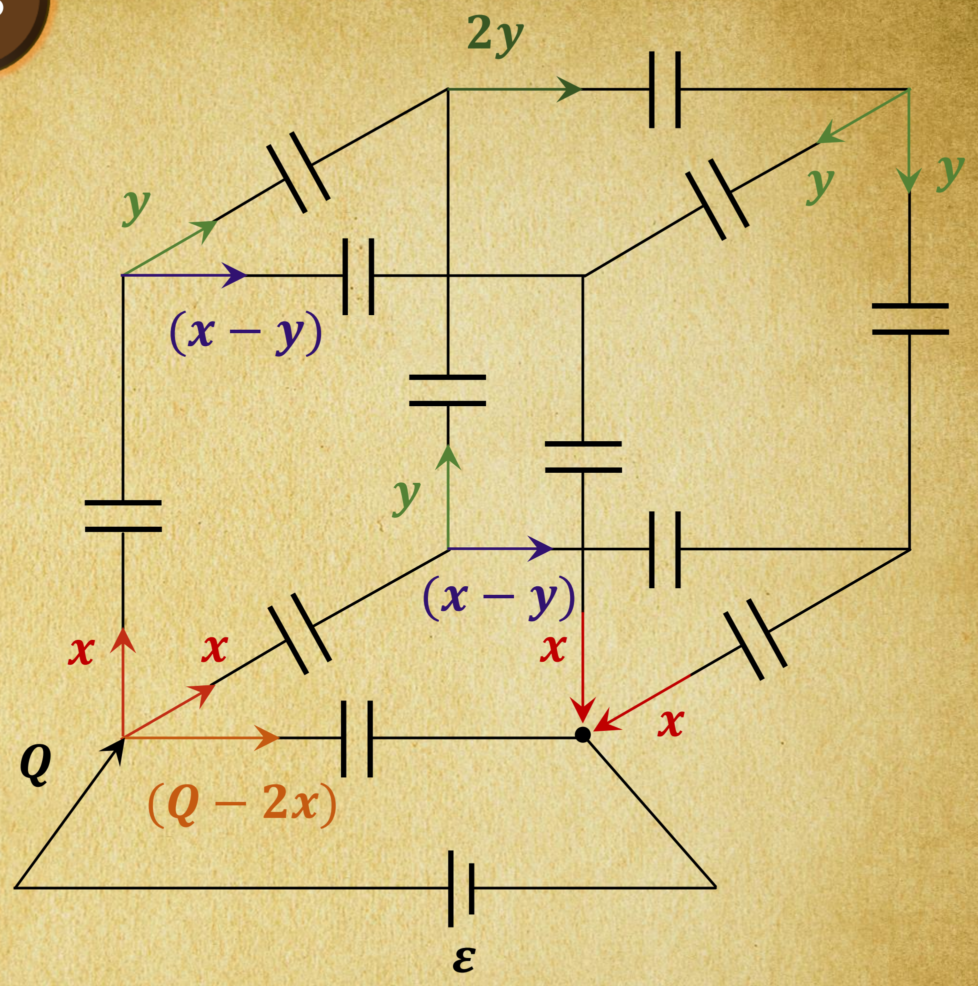
Putting value of x in terms of Q in equation,

$$Q - 2x = C\varepsilon \quad \Rightarrow Q - 2\left[\frac{5Q}{24}\right] = C\varepsilon$$

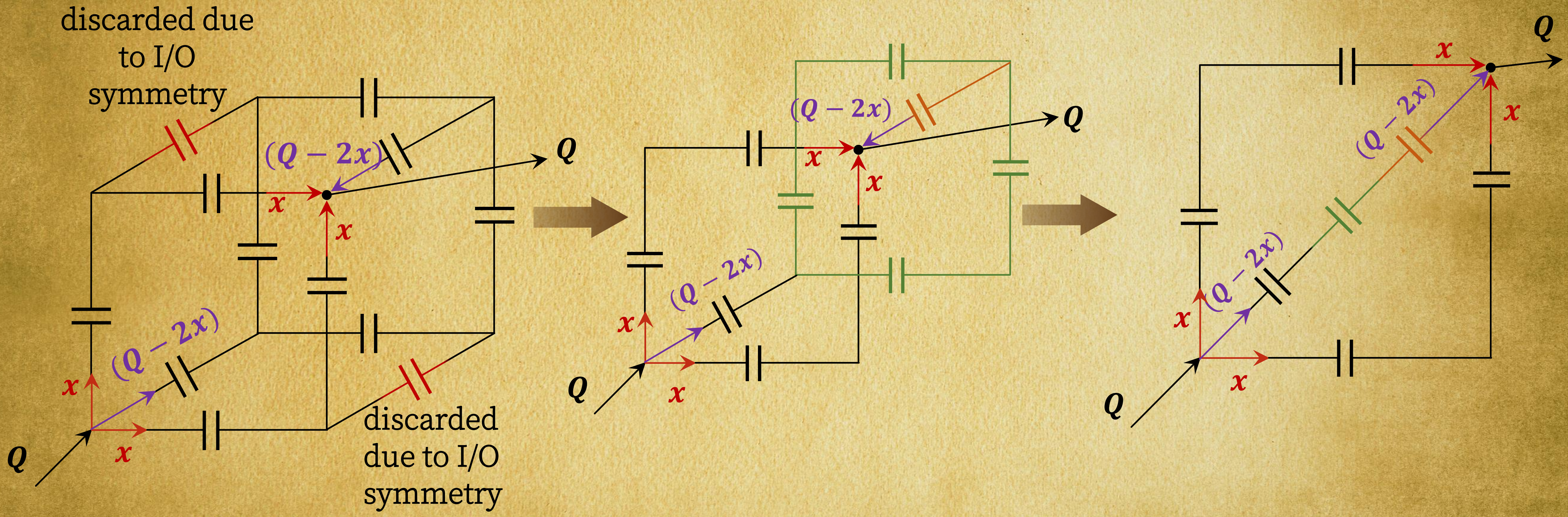
$$\Rightarrow \frac{7Q}{12} = C\varepsilon$$

$$\Rightarrow \frac{Q}{\varepsilon} = \frac{12}{7}C$$

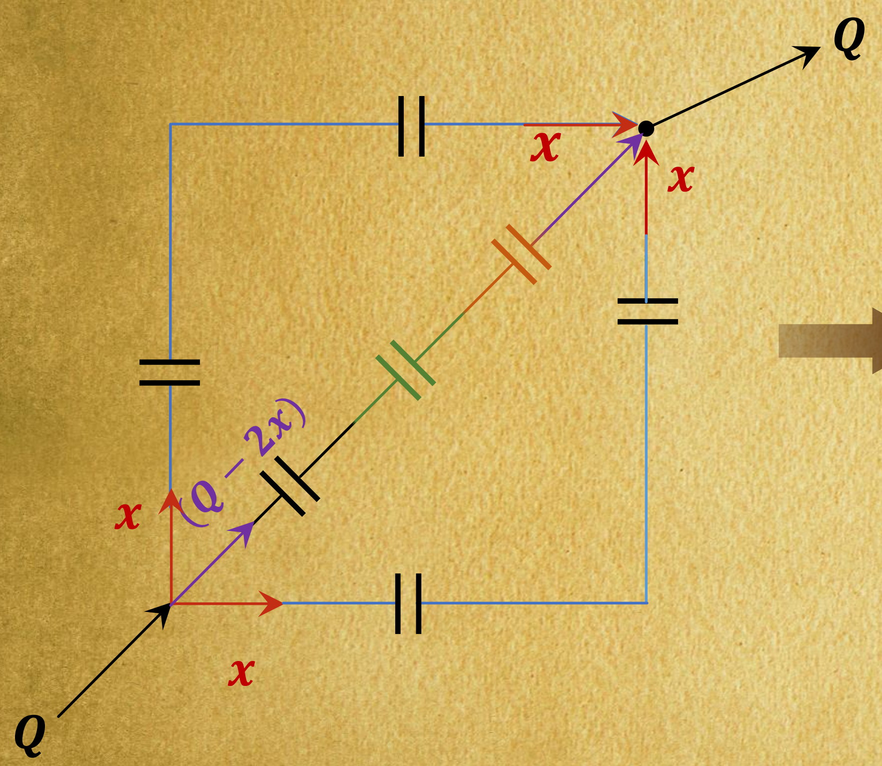
$$C_{edge} = \frac{12}{7}C$$

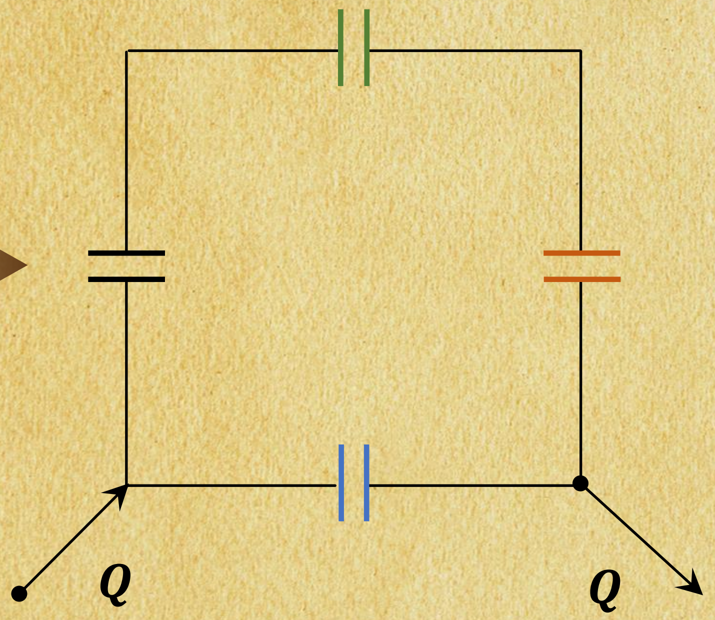


(c) Along face diagonal



(c) Along face diagonal



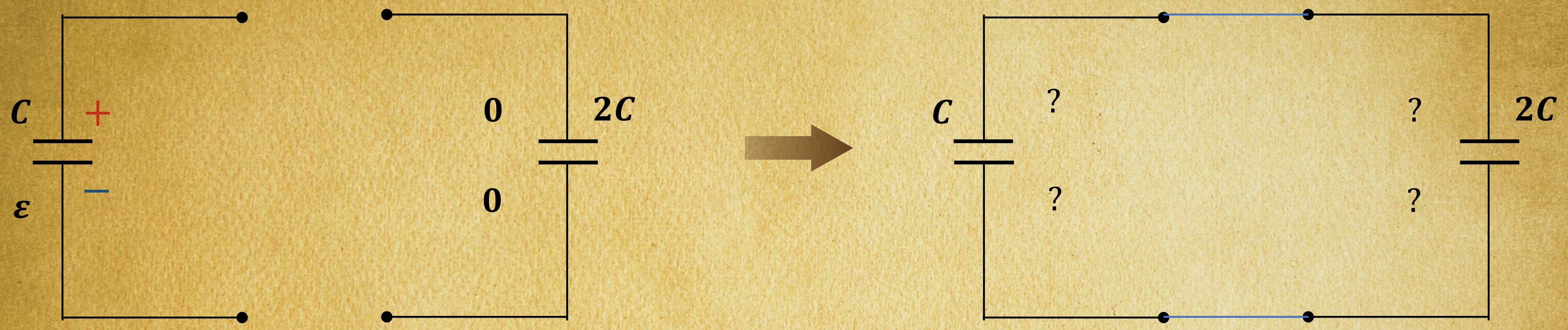


The equivalent capacitance of the network is,

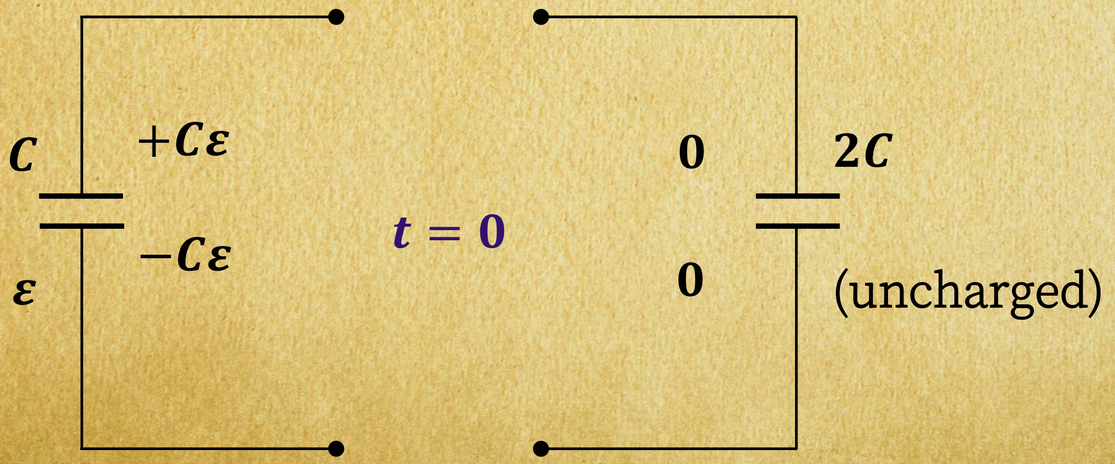
$$C_{eq} = \frac{C}{3} + C$$

Simplifying, we get

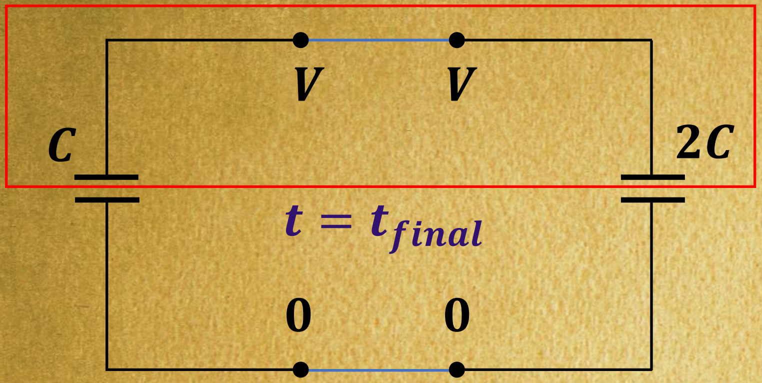
$$C_{face} = \frac{4C}{3}$$



What happens when a charged capacitor is connected to an uncharged capacitor?



Using KCL

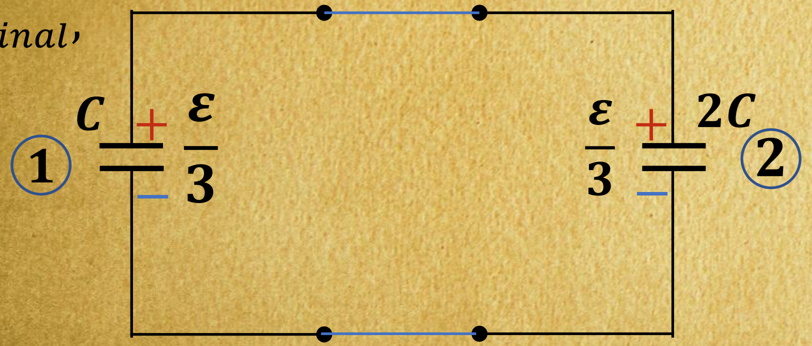


Applying KCL in the isolated region,

$$C[V-0] + 2C[V-0] = [C\varepsilon + 0]$$

$$CV + 2CV = C\varepsilon \quad \Rightarrow V = \frac{\varepsilon}{3}$$

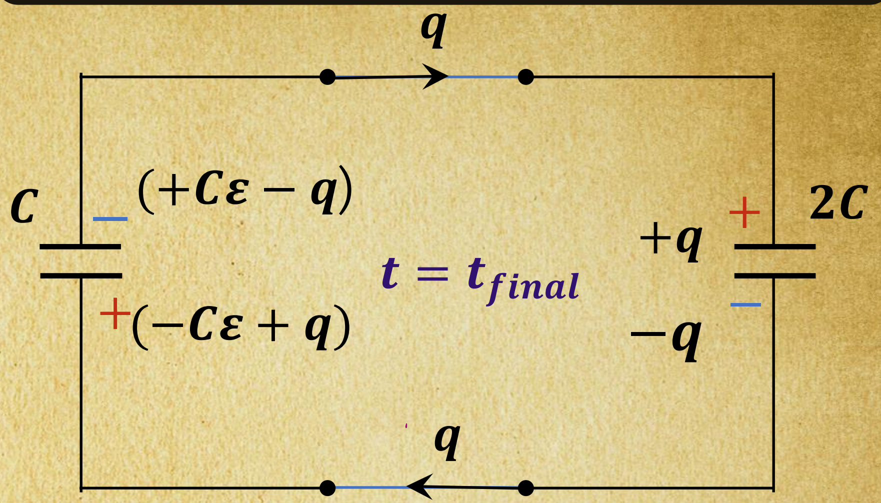
At t_{final},



Thus, the charges in capacitor 1 and 2 will be,

$$Q_1 = \frac{C\varepsilon}{3} \qquad Q_2 = \frac{2C\varepsilon}{3}$$

Using KVL



Applying KVL on this loop,

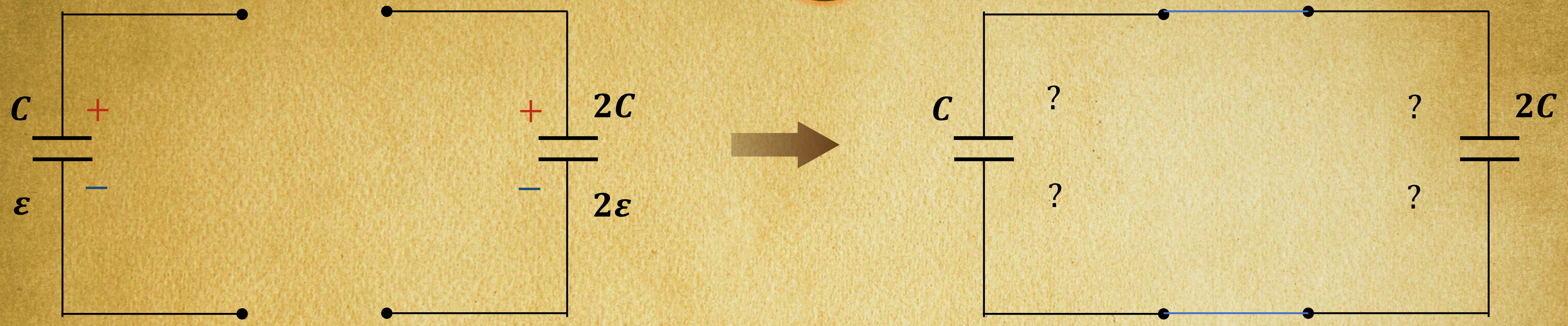
$$-\frac{q}{2C} - \left[\frac{-C\varepsilon + q}{C}\right] = 0$$

$$-q - 2(-C\varepsilon + q) = 0$$

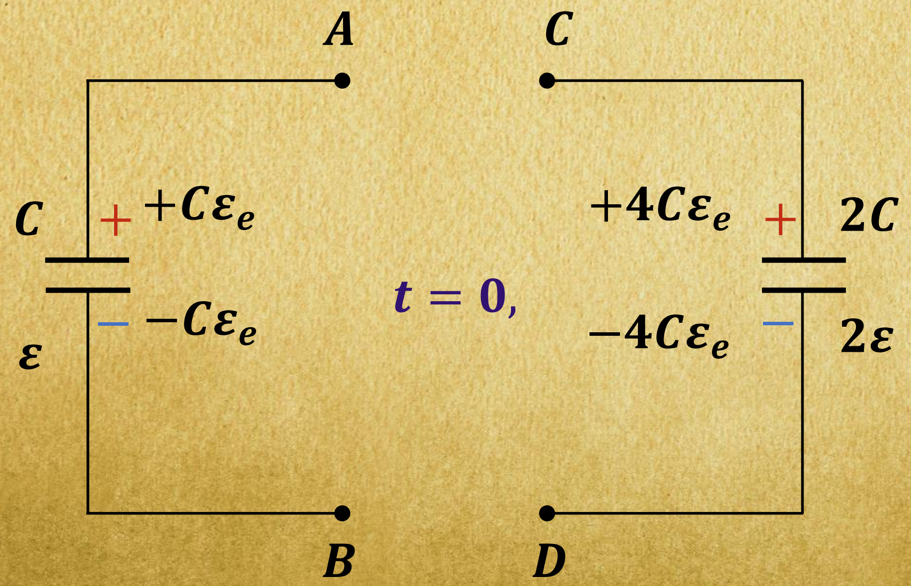
$$q = \frac{2C\varepsilon}{3}$$

Thus, the charges on either capacitor are,

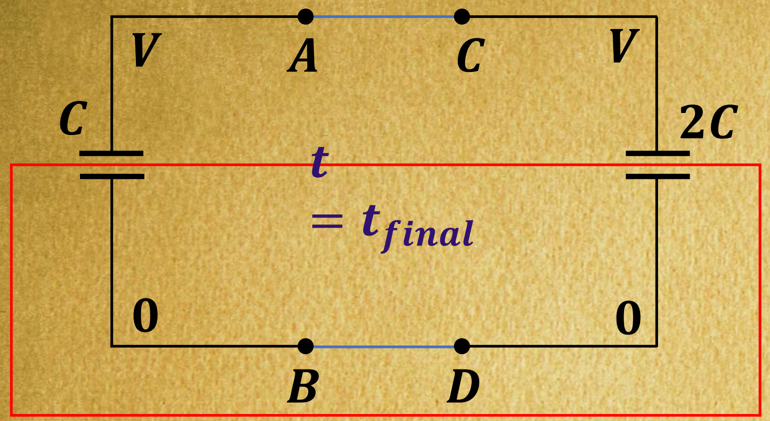
$$Q_1 = \frac{C\varepsilon}{3} \qquad Q_2 = \frac{2C\varepsilon}{3}$$



What happens when two charged capacitors of the same polarities are connected?



Using KCL

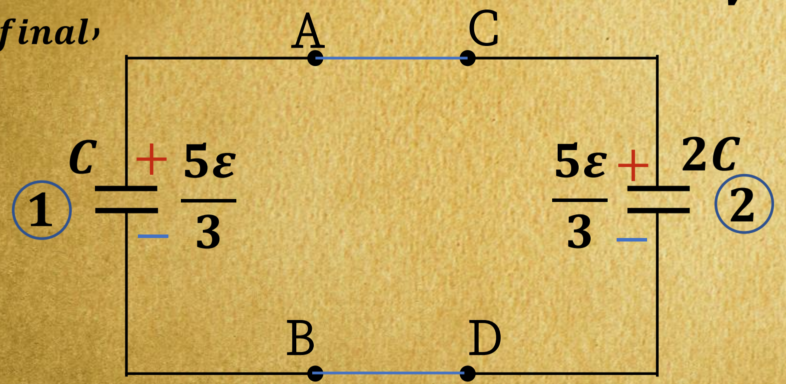


Applying KCL in the isolated portion,

$$C[0 - V] + 2C[0 - V] = [-C\varepsilon - 4C\varepsilon]$$

$$V = \frac{5\varepsilon}{3}$$

At t_{final},

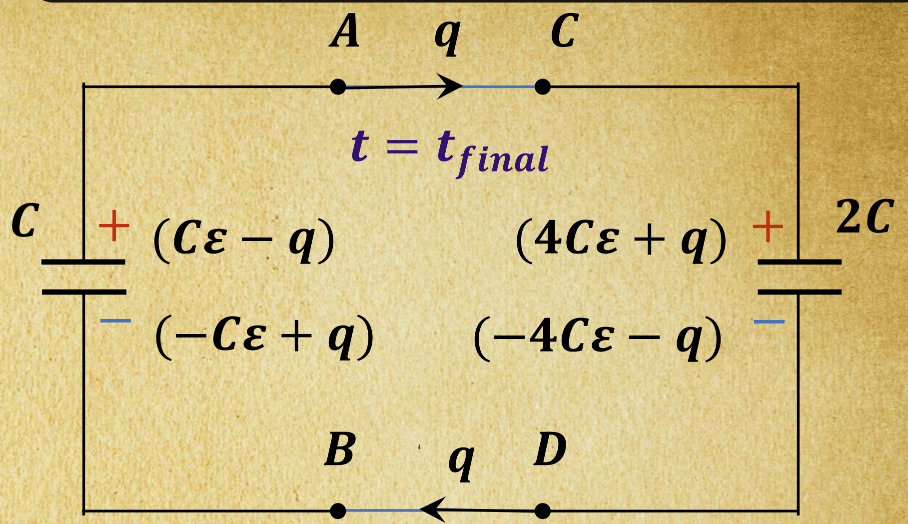


Thus, the charges in capacitor 1 and 2 will be,

$$Q_1 = \frac{5C\varepsilon}{3}$$

$$Q_2 = \frac{10C\varepsilon}{3}$$

Using KVL



Applying KVL in the loop, $-\left[\frac{4C\varepsilon + q}{2C}\right] + \left[\frac{C\varepsilon - q}{C}\right] = 0$

$$-4C\varepsilon - q + 2C\varepsilon - 2q = 0$$

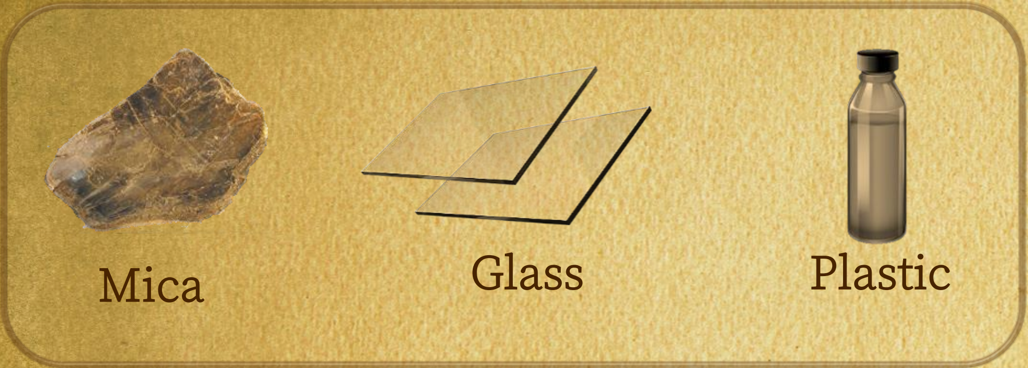
$$q = -\frac{2C\varepsilon}{3}$$

Thus, the charges on either capacitor are,

$$Q_1 = \frac{5C\varepsilon}{3}$$

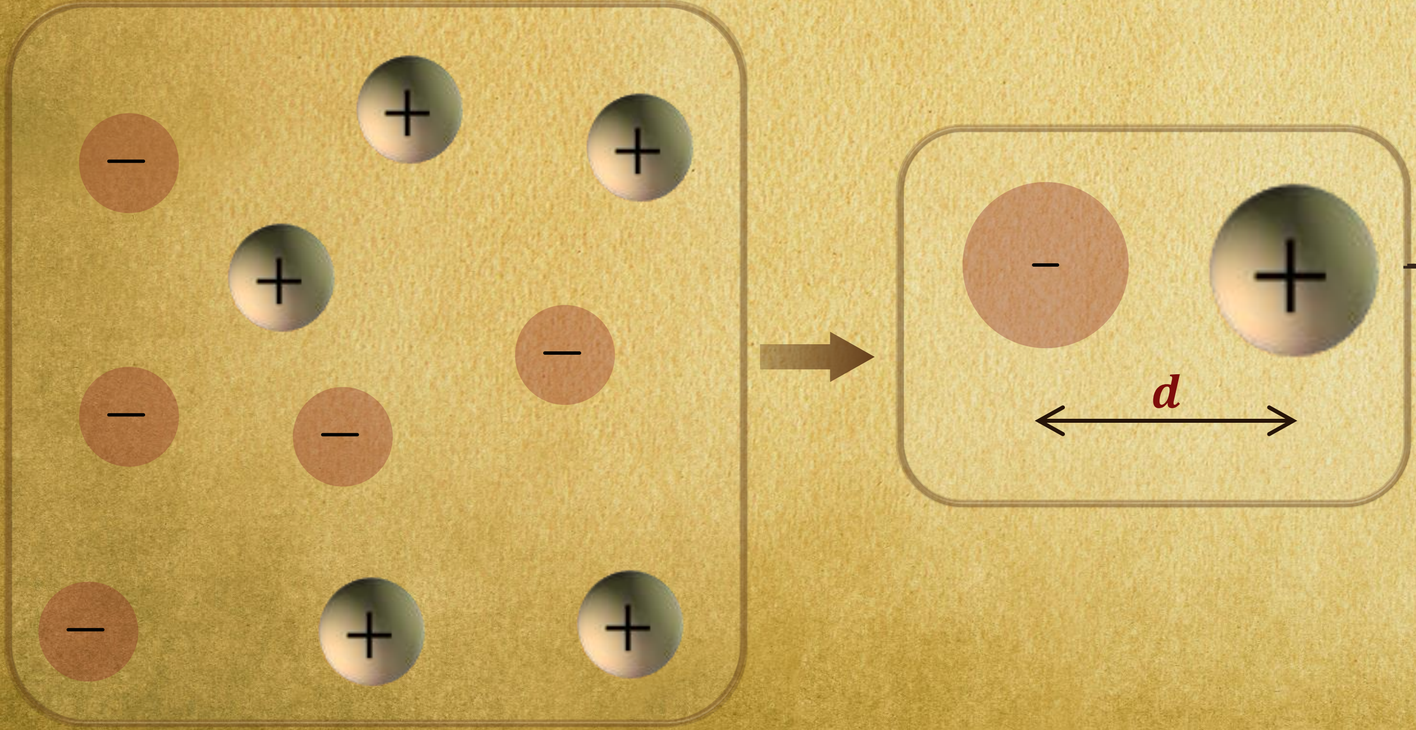
$$Q_2 = \frac{10C\varepsilon}{3}$$

Dielectrics



Dielectrics are:

- Non-conducting substances
- Have no free electrons
- Can hold an electrostatic charge while dissipating minimal energy in the form of heat
- Can be polar or non-polar



Polar

$d \neq 0$ $\vec{p} \neq 0$

Non-Polar

$d = 0$ $\vec{p} = 0$

Turns into dipoles in the presence of external electric field

Electric Polarization ($\vec{P}$)

$\vec{E}_{in} = \vec{E}_o + \vec{E}_{int} \Rightarrow E_{in} = E_o - E_{int}$

Experimentally, $E_{in} = \dfrac{E_o}{K}$

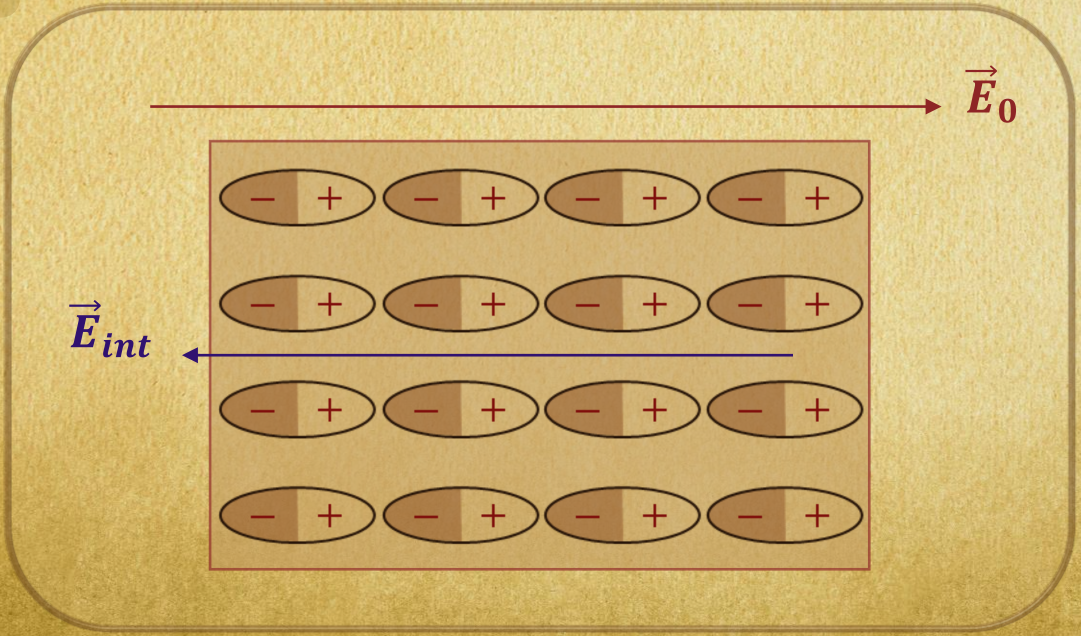
K is the Dielectric Constant of the medium

But, $E_{in} < E_o$ $\Rightarrow K > 1$

When a dielectric material is placed in an electric field, dipole moment appears in its volume

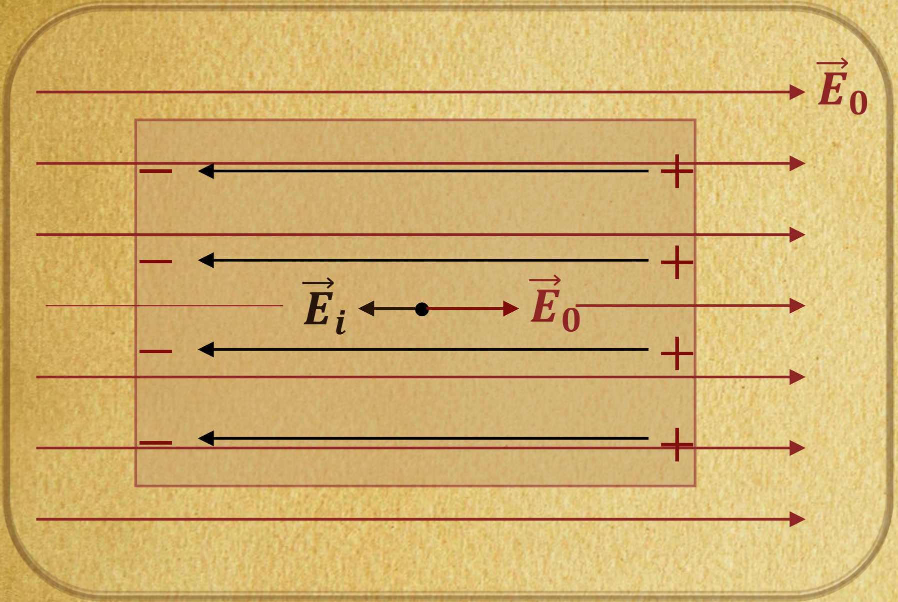
Dipole moment per unit volume of a dielectric material

$$\vec{P} = \frac{\vec{p}}{V}$$



Induced or Bound Charge

The charge appearing on the surface of a dielectric when placed in an electric field are not free but bound to an atom/ molecule lying in or near the surface.



$$|\vec{E}| = |\vec{E}_0| - |\vec{E}_i|, \ \vec{E} = \frac{\vec{E}_0}{K}$$

Where,

$$K = \varepsilon_r = \frac{|\vec{E}_0|}{|\vec{E}|}$$

K- Dielectric constant of the medium

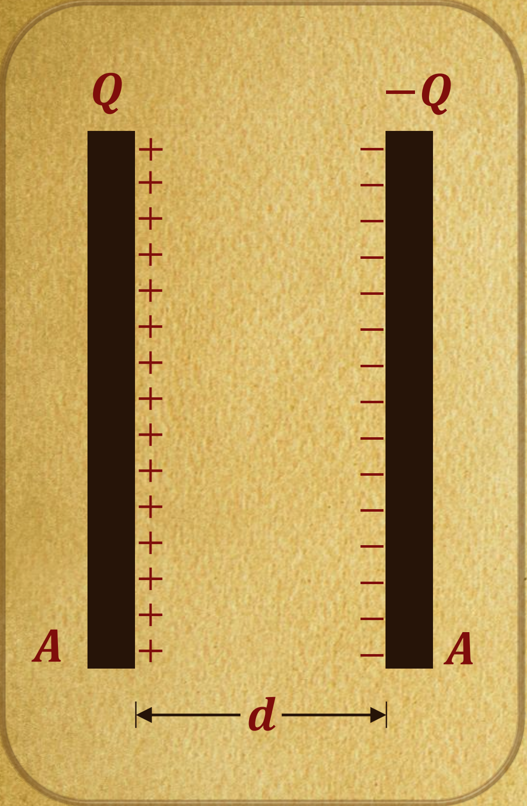
ε_r - Relative permittivity of the medium

Dielectric constant of conductor:

Electric field inside a conductor is zero. $\vec{E} = 0$

$$K = \varepsilon_r = \frac{|\vec{E}_0|}{|\vec{E}|} \Rightarrow K = \infty$$

Parallel Plate Capacitor with Dielectric



Before dielectric is inserted,

$$C_o = \frac{\varepsilon_o A}{d} \qquad E_o = \frac{Q}{\varepsilon_o A}$$

After dielectric is inserted,

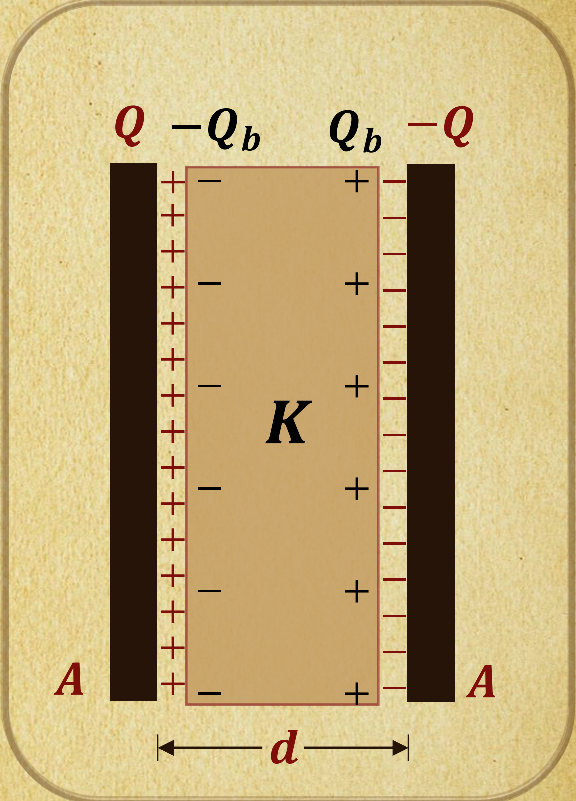
$$E = \frac{E_o}{K} = \frac{Q}{K\varepsilon_o A}$$

$$V = Ed = \frac{Qd}{K\varepsilon_o A}$$

$$\Rightarrow \frac{Q}{V} = \frac{K\varepsilon_o A}{d}$$

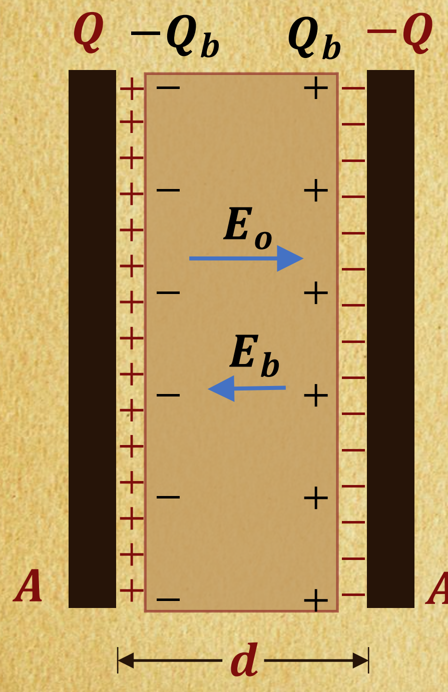
$$\Rightarrow C = K\left[\frac{\varepsilon_o A}{d}\right]$$

$$\Rightarrow C = KC_o$$



If the space between plates of parallel plate capacitor is completely filled with dielectric, its Capacitance increases by K times.

Magnitude of Induced or Bound Charge



$$\vec{E} = \frac{\vec{E}_o}{K}$$

But,

$$\vec{E} = \vec{E}_o + \vec{E}_b$$

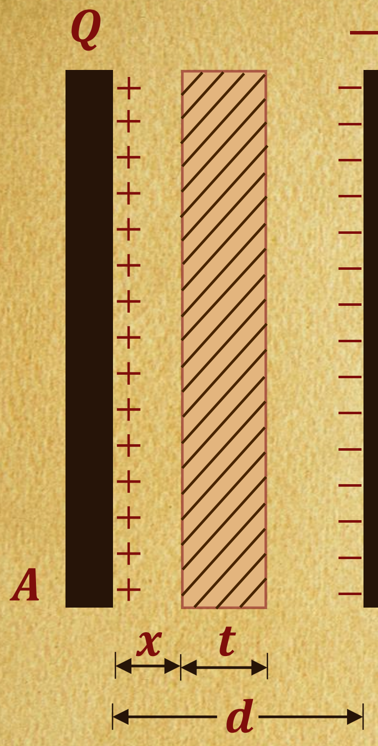
$$\Rightarrow \vec{E}_o + \vec{E}_b = \frac{\vec{E}_o}{K}$$

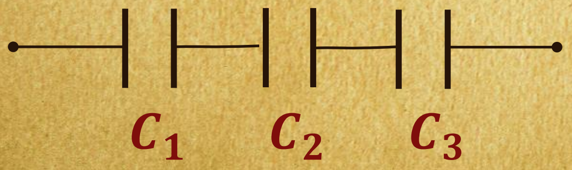
$$\Rightarrow \frac{Q}{A\varepsilon_o}\hat{\imath} + \frac{Q_b}{A\varepsilon_o}(-\hat{\imath}) = \frac{Q}{KA\varepsilon_o}\hat{\imath}$$

$$\Rightarrow Q - Q_b = \frac{Q}{K}$$

$$Q_b = Q\left[1 - \frac{1}{K}\right]$$

Capacitor Partially Filled with a Dielectric





$$C_1 = \frac{\varepsilon_o A}{x}$$

$$C_2 = \frac{K\varepsilon_o A}{t}$$

$$C_3 = \frac{\varepsilon_o A}{(d - x - t)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{x}{\varepsilon_o A} + \frac{t}{K\varepsilon_o A} + \frac{(d - x - t)}{\varepsilon_o A}$$

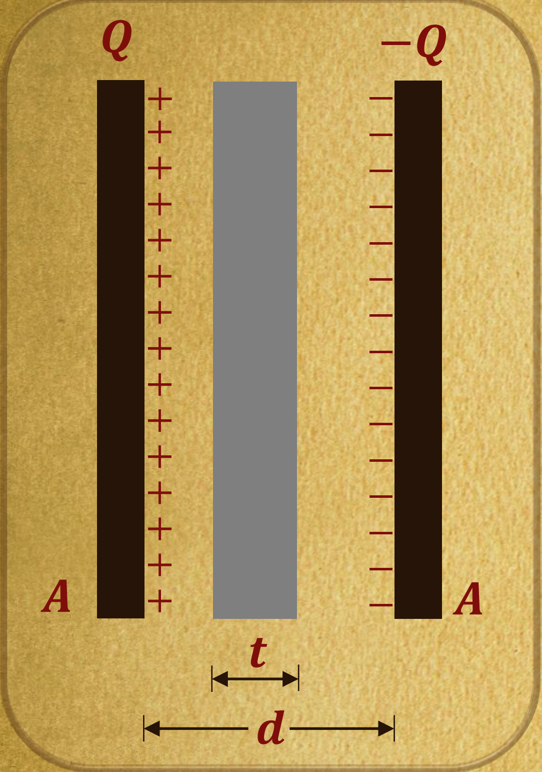
$$= \frac{t + K(d - t)}{K\varepsilon_o A}$$

$$\frac{1}{C_{eq}} = \frac{\frac{t}{K} + (d - t)}{\varepsilon_o A}$$

$$C_{eq} = \frac{\varepsilon_o A}{(d - t) + \frac{t}{K}}$$

The equivalent capacitance (C_{eq}) is independent of the position (x) at which the dielectric is placed.

Capacitor Partially Filled with a Conductor



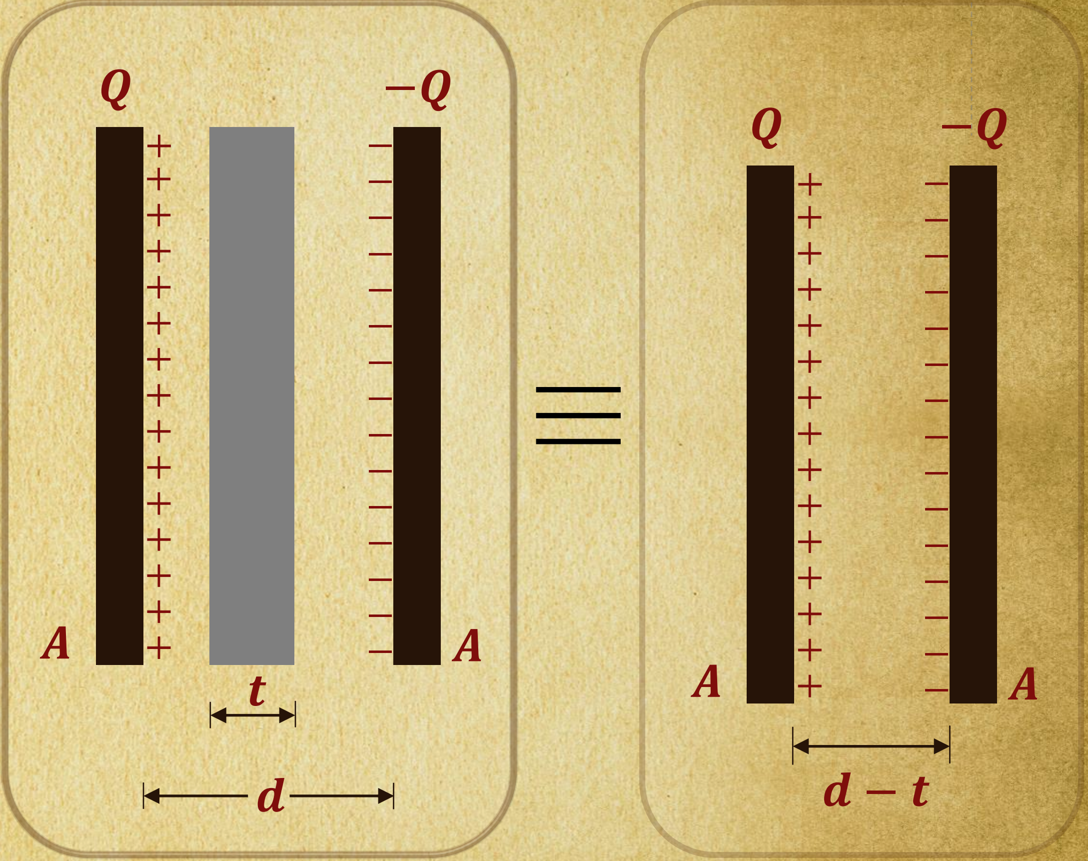
$$C_{eq} = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{K}}$$

$$K = \infty$$

$$C = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{K}}$$

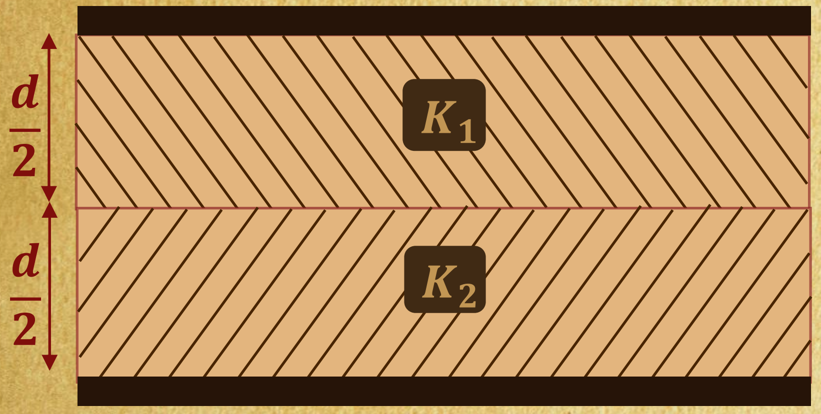
$$C = \frac{\varepsilon_0 A}{(d-t)+0}$$

$$C = \frac{\varepsilon_0 A}{(d-t)}$$



$$C = \frac{\varepsilon_0 A}{(d-t)}$$

A parallel plate capacitor of plate area A & plate separation d is filled with two different dielectric materials having dielectric constants K_1 and K_2 as shown. The capacitance of this capacitor is



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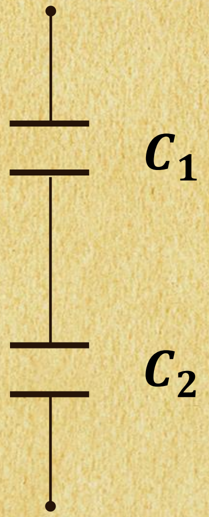


$$C_1 = \frac{2K_1\varepsilon_o A}{d}, \quad C_2 = \frac{2K_2\varepsilon_o A}{d}$$

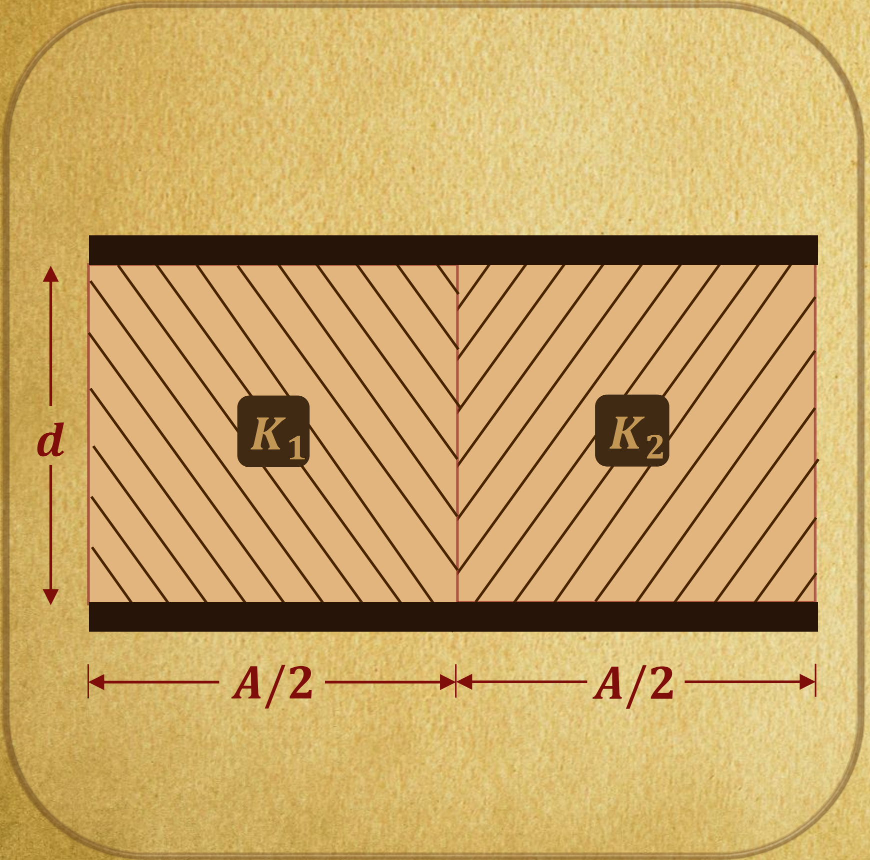
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

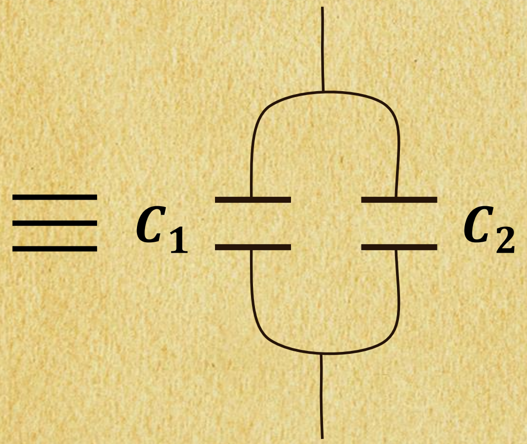
$$= \frac{d}{2K_1\varepsilon_o A} + \frac{d}{2K_2\varepsilon_o A}$$

$$= \frac{d(K_1 + K_2)}{2K_1K_2\varepsilon_o A}$$

$$\Rightarrow \boxed{C_{eq} = \frac{2K_1K_2\varepsilon_o A}{d(K_1 + K_2)}}$$

A parallel plate capacitor of plate area A & plate separation d is filled with two different dielectric materials having dielectric constants K_1 and K_2 as shown. The capacitance of this capacitor is





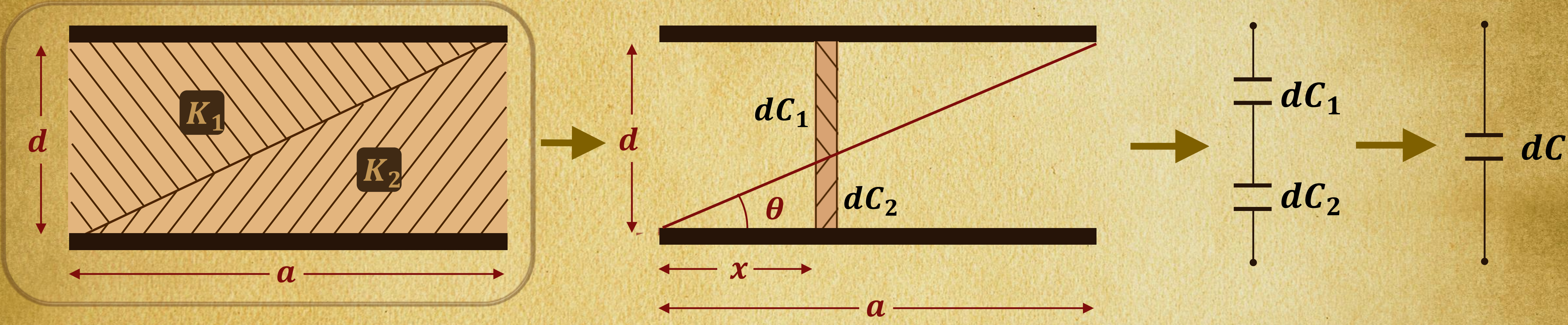
$$C_1 = \frac{K_1 \epsilon_o A}{2d}, \quad C_2 = \frac{K_2 \epsilon_o A}{2d}$$

$$C_{eq} = C_1 + C_2$$

$$= \frac{K_1 \epsilon_o A}{2d} + \frac{K_2 \epsilon_o A}{2d}$$

$$C_{eq} = \frac{\epsilon_o A}{2d}[K_1 + K_2]$$

Capacitor is formed by two square metal plates of edge a, separated by a distance d. Dielectrics of dielectric constant K_1 and K_2 are filled in the gap as shown. Find the capacitance of this capacitor.



$$\tan\theta = \frac{d}{a}$$

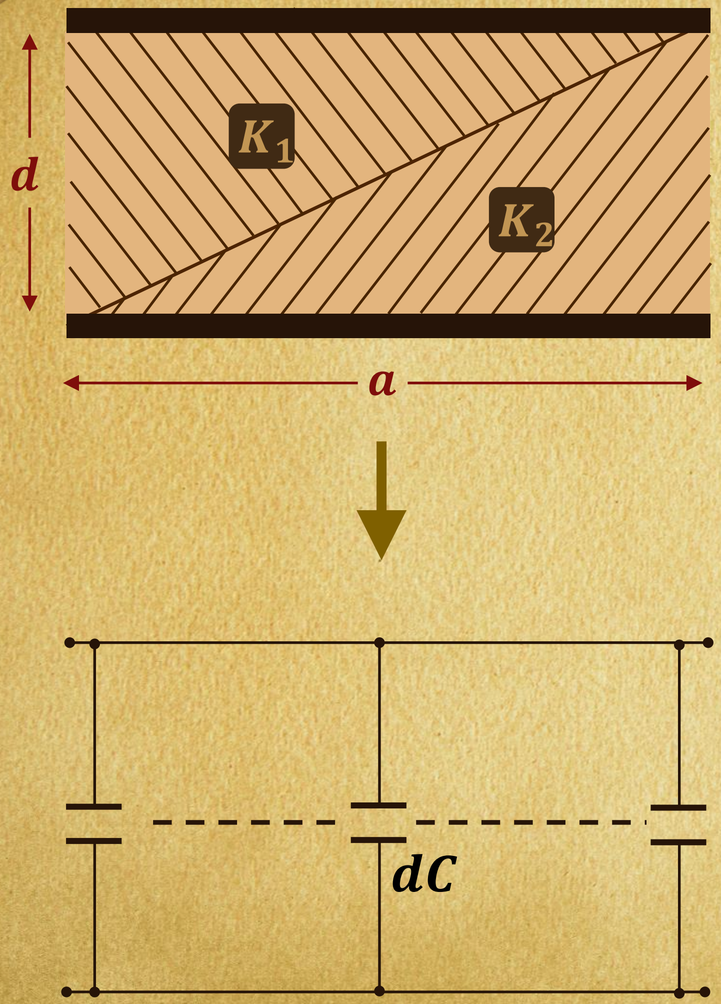
$$dC_1 = \frac{K_1\varepsilon_o(adx)}{(d - x\tan\theta)}$$

$$dC_2 = \frac{K_2\varepsilon_o(adx)}{(x\tan\theta)}$$

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

$$\frac{1}{dC} = \frac{(d - x\tan\theta)}{K_1\varepsilon_o(adx)} + \frac{(x\tan\theta)}{K_2\varepsilon_o(adx)}$$

$$\Rightarrow dC = \frac{K_1K_2\varepsilon_o(adx)}{K_2d + (K_1 - K_2)x\tan\theta}$$



$$\Rightarrow dC = \frac{K_1 K_2 \varepsilon_o (a\,dx)}{K_2 d + (K_1 - K_2) x \tan\theta}$$

$$C = \int dC$$

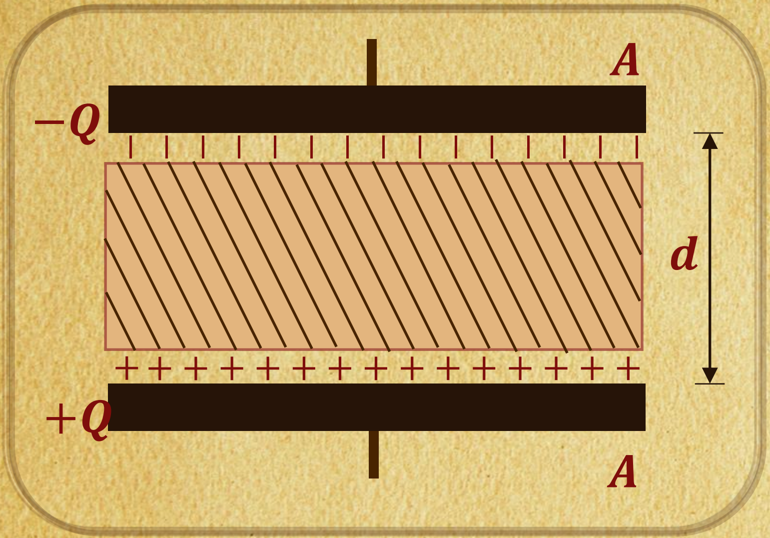
$$= \frac{K_1 K_2 \varepsilon_o a^2}{d} \int_0^a \frac{dx}{K_2 a + (K_1 - K_2)x}$$

$$= \frac{K_1 K_2 \varepsilon_o a^2}{d} \left[\frac{\ln[K_2 a + (K_1 - K_2)x]}{(K_1 - K_2)} \right]_0^a$$

$$C = \frac{\epsilon_o a^2 K_1 K_2}{d(K_1 - K_2)} \ln\left[\frac{K_1}{K_2}\right]$$

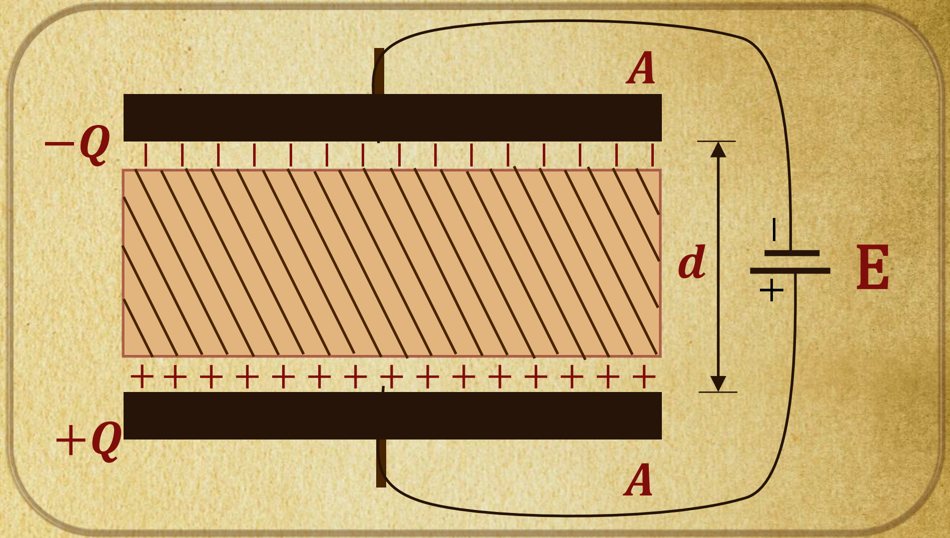
Inserting Dielectric in Capacitor

CASE 1: Battery is disconnected



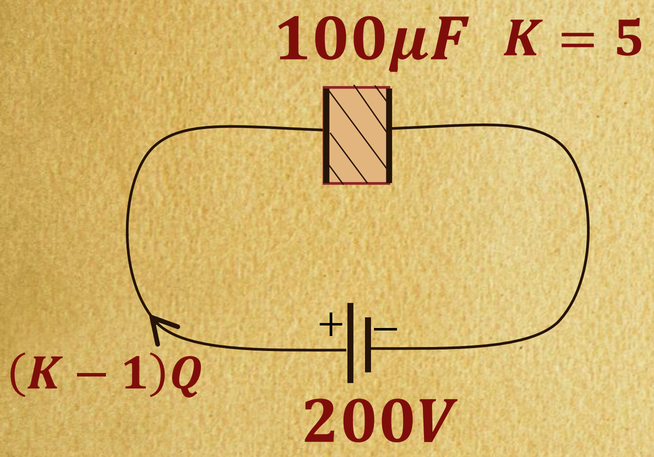
	$C = \frac{\varepsilon_0 A}{d}$	$V = \frac{Q}{C}$	$E = \frac{V}{d}$	$\boldsymbol{Q = CV}$	$U = \frac{Q^2}{2C}$
Before insertion	$\boldsymbol{C}$	$\boldsymbol{V}$	$\boldsymbol{E}$	$\boldsymbol{Q}$	$\boldsymbol{U}$
After insertion	KC	$\boldsymbol{\frac{V}{K}}$	$\boldsymbol{\frac{E}{K}}$	$\boldsymbol{Q}$	$\boldsymbol{\frac{U}{K}}$

CASE 2: Battery remains connected



	$C = \frac{\varepsilon_0 A}{d}$	$V = \frac{Q}{C}$	$E = \frac{V}{d}$	$\boldsymbol{Q = CV}$	$U = \frac{CV^2}{2}$
Before insertion	$\boldsymbol{C}$	$\boldsymbol{V}$	$\boldsymbol{E}$	$\boldsymbol{Q}$	$\boldsymbol{U}$
After insertion	KC	$\boldsymbol{V}$	$\boldsymbol{E}$	$\boldsymbol{KQ}$	$\boldsymbol{KU}$

A parallel-plate capacitor of capacitance $100\ \mu F$ is connected to a power supply of $200\ V$. A dielectric slab of dielectric constant 5 is now inserted in the gap between the plates.
(a) Find the extra charge flown through the power supply and the work done by the supply.
(b) Find the change in the electrostatic energy of the electric field in the capacitor.



a) $Q_i = Q,\ \ Q_f = KQ$

$\therefore Q_{extra} = (K-1)Q$

$= (K-1)CV$

$= 4 \times 100 \times 10^{-6} \times 200$

$= 80\ mC$

$W_{battery} = Q_{extra} \times V$

$= 80 \times 10^{-3} \times 200$

$= 16\ J$

b) $U_i = U = \frac{1}{2}CV^2$

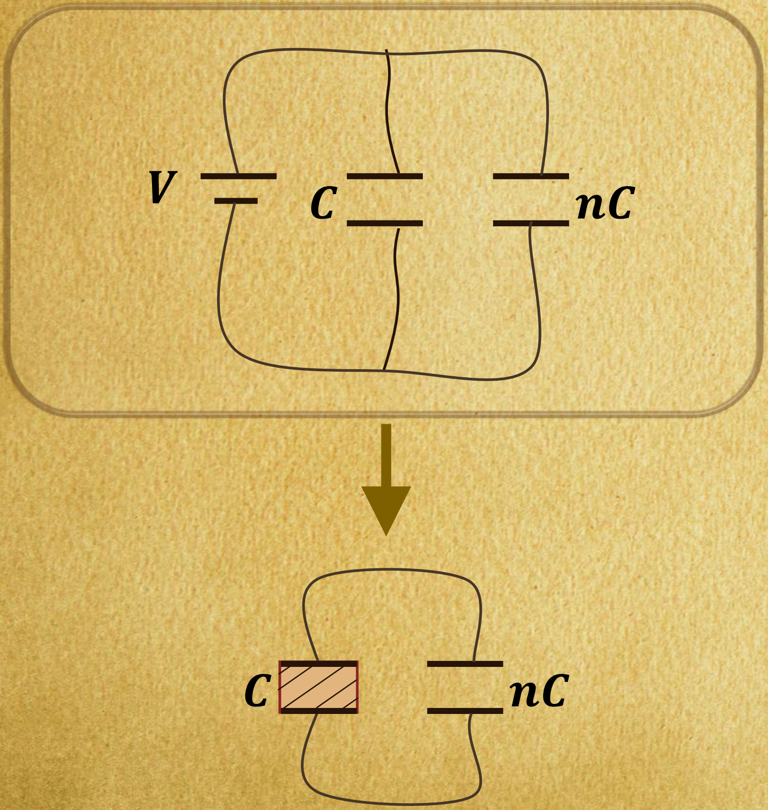
$U_f = KU = K \times \left[\frac{1}{2}CV^2\right]$

$\Rightarrow \Delta U = (k-1)\left[\frac{1}{2}CV^2\right]$

$= 4 \times \frac{1}{2} \times 10^{-4} \times 4 \times 10^4$

$\Delta U = 8J$

The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is



Before inserting dielectric,

$$C_{eq} = C + nC = C(n+1)$$

$$Q = C_{eq}V = CV(n+1)$$

After inserting dielectric,

$$\therefore V' = \frac{q}{C_{eq}} = \frac{CV(n+1)}{C(K+n)}$$

$$\Rightarrow V' = \frac{(n+1)V}{(K+n)}$$