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Aakash + BYJU'S LIVE

Basics of Mathematics

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Inequalities

Inequalities are mathematical expressions involving $<, >, \leq, \geq$ symbols.

Examples:

- $7 > 4$
- $3x + 2 < 0$
- $\frac{2x+1}{4x-3} \geq 0$

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Interval

The set of all real numbers between any two real numbers is called an interval.

Types of Interval:

(i) Open interval

(ii) Closed interval

(iii) Semi open/closed interval

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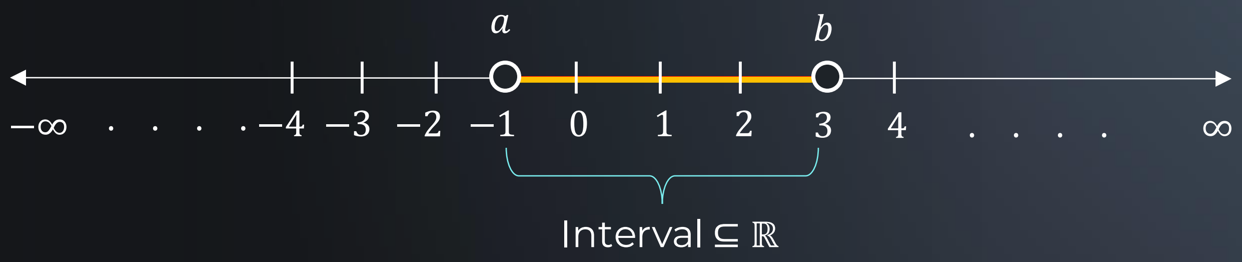
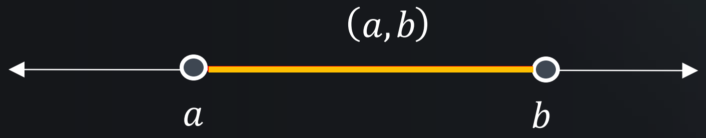
Open Interval



$\{x \in \mathbb{R} : -1 < x < 3\} \;= (-1,3)$ → Open Interval

$$(a,b) = \{x : a < x < b, x \in \mathbb{R}\}$$



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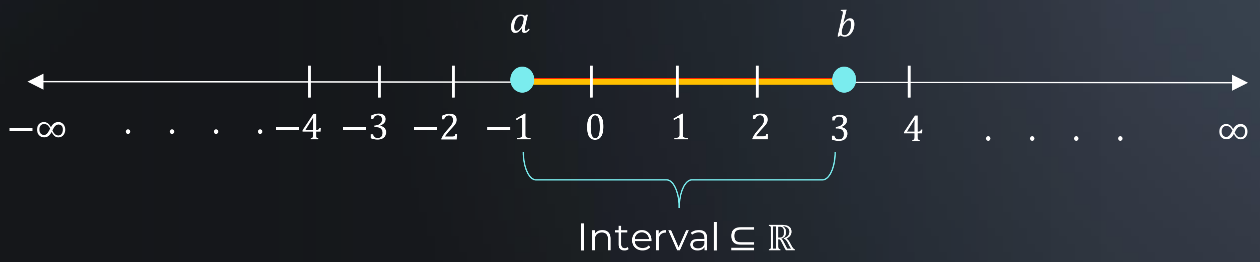
Closed Interval



$\{x \in \mathbb{R} : -1 \leq x \leq 3\} = [-1, 3]$ → Closed Interval

$$[a, b] = \{x : a \leq x \leq b\}$$

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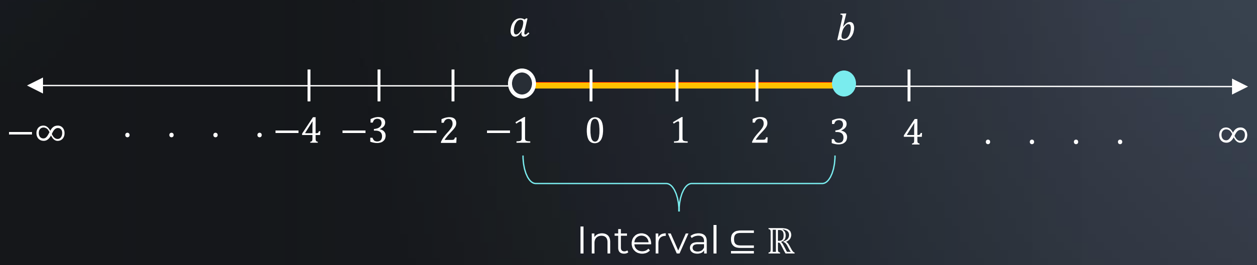
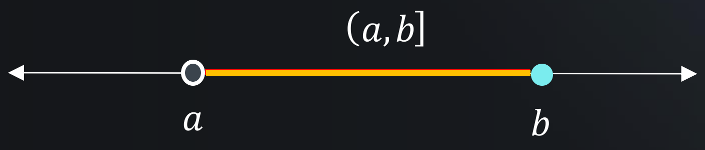
Semi Open/Closed Interval



$\{x \in \mathbb{R} : -1 < x \leq 3\} = (-1, 3]$ → Semi Open/Semi Closed Interval

$$(a, b] = \{x : a < x \leq b\}$$



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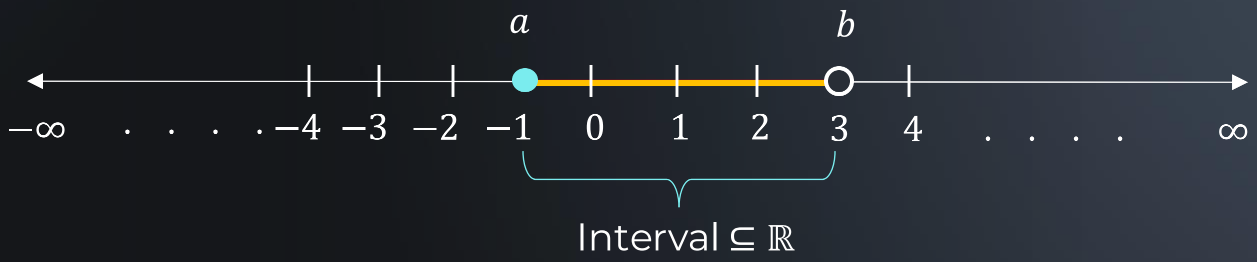
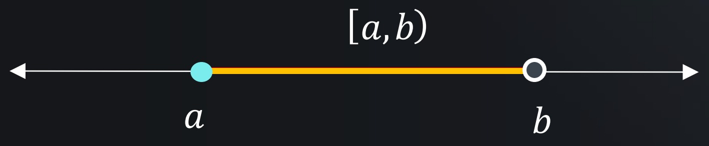
Semi Open/Closed Interval



$\{x \in \mathbb{R} : -1 \leq x < 3\} = [-1,3)$ → Semi Open/Semi Closed Interval

$$[a,b) = \{x: a \leq x < b\}$$



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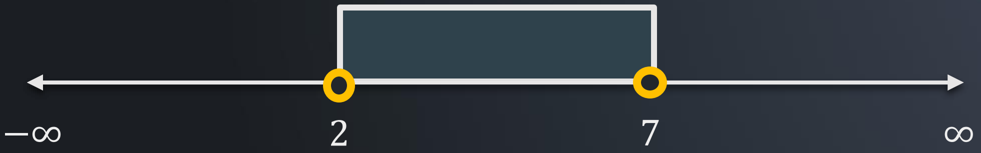
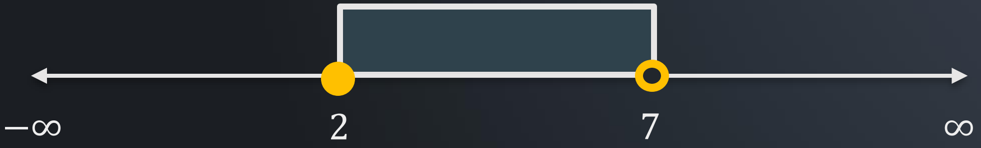
Which among the following represents the interval (2,7]

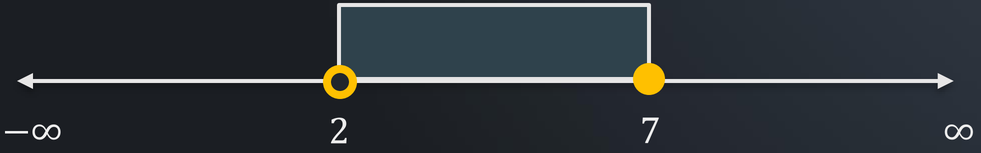
A



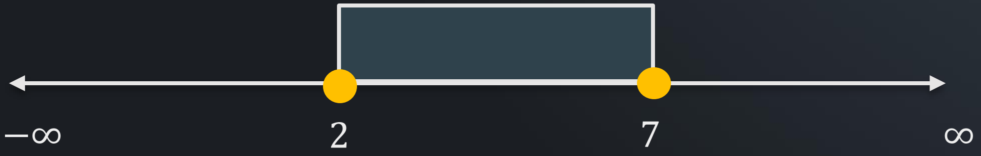
B



C



D



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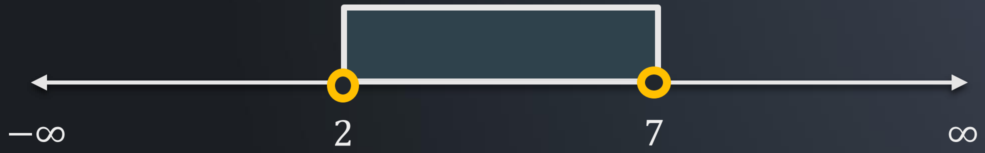
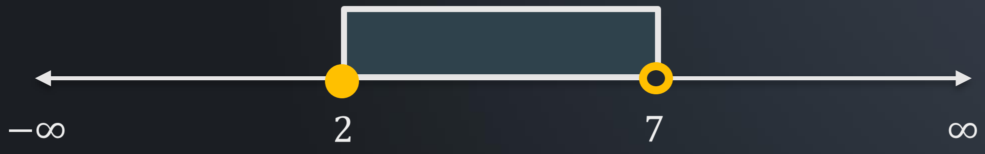
Which among the following represents the interval (2,7]

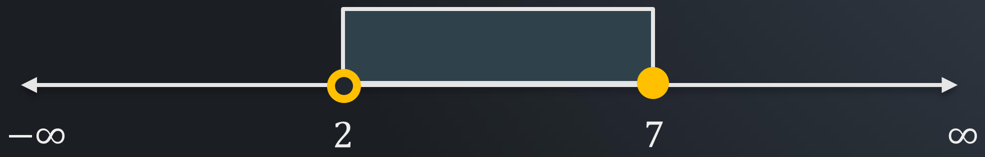
A



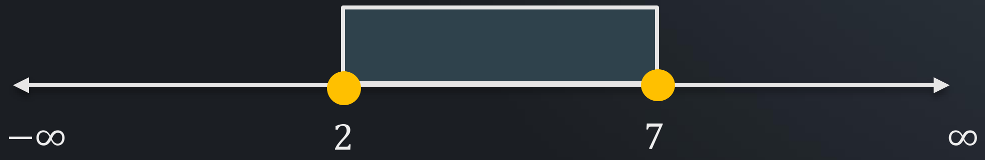
B



C



D



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Key Takeaways

Rules for Solving an Inequality

Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sides of inequality.

Example

- $x - 5 > 8$

  Adding 5 on both sides,

  $\Rightarrow x - 5 + 5 > 8 + 5$

  $\Rightarrow x > 13$ i.e., $x \in (13, \infty)$

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Key Takeaways

Rules for Solving an Inequality

Both sides of an inequality can be multiplied (or divided) by the same positive number, but when both sides are multiplied (or divided) by a negative number, then the sign of inequality is reversed.

Examples

- $5 > 2$

  Multiply 3 on both sides,

  $\Rightarrow 15 > 6$

- $2 > 1$

  Multiply -1 on both sides,

  $\Rightarrow -2 < -1$

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Key Takeaways

Rules for Solving an Inequality

Note:

- For an inequality, squaring the terms on both the sides is allowed only if both the quantities are non-negative.

Example $-5 < 2$ but $25 \nless 4$

- If an inequality consists of fractions or rational terms, cross multiplication of only positive quantity is allowed.

Example $\dfrac{10}{-3} < 1 \Rightarrow 10 \nless -3$

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Solve the following for x

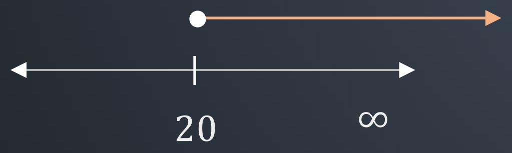
$(i)\ 3x \geq 60 \quad (ii)\ -4x > 15$

Solution:

$(i)\ 3x \geq 60$

Divide by 3 on both sides ,

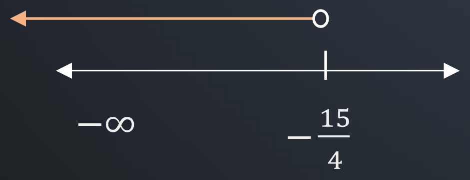
$\Rightarrow x \geq 20, x \in [20, \infty)$



$(ii)\ -4x > 15$

Divide by -4 on both sides ,

$$\Rightarrow x < -\frac{15}{4}, x \in \left(-\infty, -\frac{15}{4}\right)$$



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Solve $(x-5)(x-7) \geq 0$

Solution:

$(x-5)(x-7) \geq 0$

$ab > 0 \Rightarrow a$ & b are of same sign.

$(x-5) \geq 0$ & $(x-7) \geq 0$

or

$(x-5) \leq 0$ & $(x-7) \leq 0$

$(x-5) \geq 0$ & $(x-7) \geq 0$

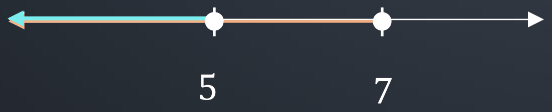
$\boxed{x \geq 5}$ $\boxed{x \geq 7}$

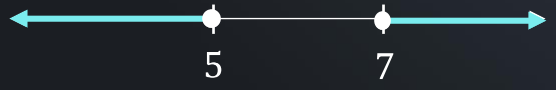
$\Rightarrow x \geq 7$

$(x-5) \leq 0$ & $(x-7) \leq 0$

$\boxed{x \leq 5}$ $\boxed{x \leq 7}$

$\Rightarrow x \leq 5$





$\therefore x \in (-\infty, 5] \cup [7, \infty)$

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Key Takeaways

Important Results

If $a, b \in \mathbb{R}\ (b \neq 0)$, then

- $ab > 0$ or $\frac{a}{b} > 0 \Rightarrow a$ & b are of same sign.
- $ab < 0$ or $\frac{a}{b} < 0 \Rightarrow a$ & b are of opposite sign.

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Wavy Curve Method

The wavy Curve method or method of intervals is a strategy used to solve inequalities of the form $\frac{f(x)}{g(x)} > 0$, $(> , < , \geq , \leq)$

Where $f(x)$ and $g(x)$ are functions of x.

Examples

$(i)\ x^2 - 12x + 35 \geq 0$

$(ii)\ (x-2)^2 \cdot (x^2 + x - 42) < 0$

$(iii) \frac{(x-3)^3(x+2)^9(x+5)^5}{(x+1)(x-7)^7} < 0$

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Solve the inequality $x^2 + 35 > 12x$

Solution:

Step 1 : Bring everything to the L.H.S and make R.H.S $= 0$

$$\underbrace{x^2 - 12x + 35}_{\text{Expression}} > 0$$

Step 2 : Factorize the expression into as many linear factors as possible.

$$\underbrace{x^2 - 12x + 35}_{\text{Expression}} > 0$$

$$\Rightarrow (x-5)(x-7) > 0$$

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Solve the inequality $x^2 + 35 > 12x$

Solution:

Step 3 : Identify the critical points. (The value of x at which each individual factor is equal to zero)

$$\underbrace{\underset{(i)}{(x-5)}\underset{(ii)}{(x-7)}}_{\text{Expression}} > 0$$

Critical Points

$(i)\ x - 5 = 0$

$(ii)\ x - 7 = 0$

Critical Points are 5, 7

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Solve the inequality $x^2 + 35 > 12x$

Solution:

$$\underbrace{(x-5)(x-7)}_{\text{Expression}} > 0$$

Critical Points are 5, 7

Step 4 : Plot the critical points on the real line and determine the regions.

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Solve the inequality $x^2 + 35 > 12x$

Solution:

$$\underbrace{(x-5)(x-7)}_{\text{Expression}} > 0$$

Step 5: Determine the sign of the expression in each region.

- If critical point satisfies the given expression, then mention solid circle on number line, else mention hollow circle on the number line.
- Take any real value of x greater than the extreme right most critical point and put it in the expression to get the sign in I.
- Repeat the same to get the sign of the expression in II and III.

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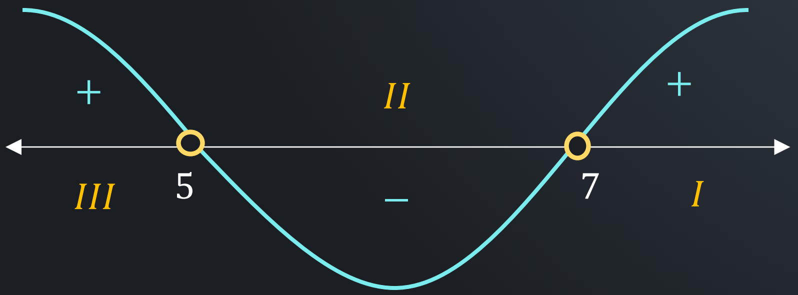
Solve the inequality $x^2 + 35 > 12x$

Solution:

$$\underbrace{(x-5)(x-7)}_{\text{Expression}} > 0$$

Step 6 : Generate the wave diagram starting from the rightmost region moving towards the leftmost region and hence write the solution as per the given question.



$\therefore x \in (-\infty, 5) \cup (7, \infty)$

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The number of distinct critical points of $\frac{x(1-x)^3(x-3)^5}{(x-2)(x^2-2)}$ is

A 3

B 4

C 5

D 6

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The number of distinct critical points of $\frac{x(1-x)^3(x-3)^5}{(x-2)(x^2-2)}$ is

$$\frac{x(1-x)^3(x-3)^5}{(x-2)(x^2-2)}$$

Critical Points → The value of x at which each individual factor is equal to zero

(i) $x = 0$

(ii) $(1 - x) = 0$

(iii) $(x - 3) = 0$

(iv) $(x - 2) = 0$

(v) $(x^2 - 2) = 0$

$$x = 0, 1, 3, 2, \pm\sqrt{2}$$

∴ Number of distinct critical points is 6

A 3

B 4

C 5

D 6

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Solve the inequality $\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2} \leq 0$

Solution:

Step 1 : Bring everything to the L.H.S and make R.H.S $= 0$

Step 2 : Factorize the expression into as many linear factors as possible.

$$\underbrace{\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2}}_{\text{Expression}} \leq 0$$

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Solve the inequality $\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2} \leq 0$

Solution:

Step 3 : Identify the critical points. (The value of x at which each individual factor is equal to zero)

$$\underbrace{\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2}}_{\text{Expression}} \leq 0$$

Critical Points

$(i)\ x+9=0 \quad (ii)\ x-6=0 \quad (iii)\ x+7=0 \quad (iv)\ x-2=0$

Critical Points are $-9,\ -7, 2, 6$

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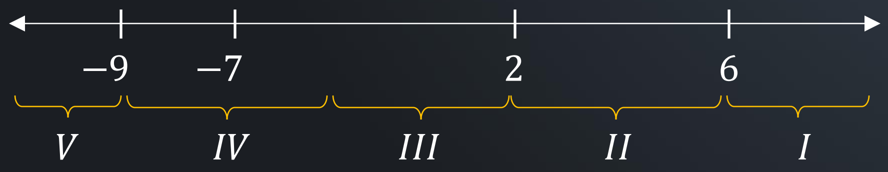
Solve the inequality $\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2} \leq 0$

Solution:

$$\underbrace{\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2}}_{\text{Expression}} \leq 0$$

Critical Points are $-9,\ -7, 2, 6$

Step 4 : Plot the critical points on the real line and determine the regions.



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Solve the inequality $\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2} \leq 0$

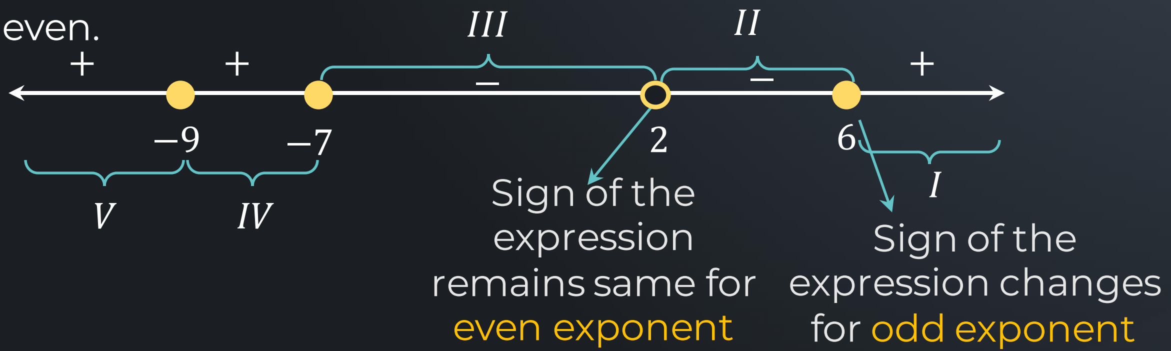
Solution:

$$\underbrace{\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2}}_{\text{Expression}} \leq 0$$

Critical Points are $-9, \ -7, 2, 6$

Step 5: Short cut for determining the sign of the expression in each region.

- If the factor corresponding to the critical point has odd power, then the sign will change, whereas the sign will remain the same incase the power is even.



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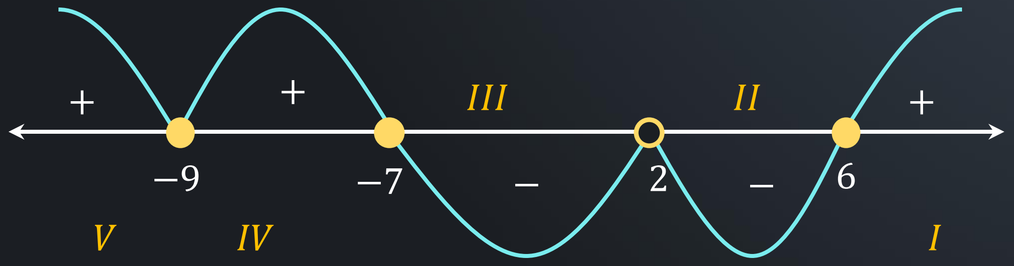
Solve the inequality $\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2} \le 0$

Solution:

$$\underbrace{\frac{(x+9)^2(x-6)(x+7)}{(x-2)^2}}_{\text{Expression}} \le 0$$

Critical Points are $-9,\ -7, 2, 6$

Step 6 : Generate the wave diagram starting from the rightmost region moving towards the leftmost region and hence write the solution as per the given question.



$\therefore x \in [-7, 2) \cup (2, 6]$

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Solve $(7-x)(x^3-8)>0$

Solution:

Step 1 : Bring everything to the L.H.S and make R.H.S $=0$

$$\underbrace{(7-x)(x^3-8)}_{\text{Expression}}>0$$

$$\Rightarrow (7-x)(x-2)(x^2+2x+4)>0$$

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Solve $(7-x)(x^3-8)>0$

Solution:

Step 2 : Factorize the expression into as many linear factors as possible and make the coefficient of x positive in each and every bracket

$$\underbrace{(7-x)(x^3-8)}_{\text{Expression}} > 0$$

$$\Rightarrow (7-x)(x-2)(x^2+2x+4)>0$$

Note:

- Here for $x^2+2x+4=0$, $D=2^2-4(1)(4)<0$

So, the quadratic equation will have non-real roots and hence it will not have critical points

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Solve $(7-x)(x^3-8)>0$

Solution:

Step 2 : Factorize the expression into as many linear factors as possible and make the coefficient of x positive in each and every bracket

$$\underbrace{(7-x)(x^3-8)}_{\text{Expression}}>0$$

Note:

- Ensure each linear factor to have positive coefficient of x in this step.

$$\Rightarrow (7-x)(x-2)(x^2+2x+4)>0$$

$$\Rightarrow -(x-7)(x-2)(x^2+2x+4)>0$$

$$\Rightarrow (x-7)(x-2)(x^2+2x+4)<0$$

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Solve $(7-x)(x^3-8) > 0$

Solution:

Step 3 : Identify the critical points.

$$\underbrace{\underset{(i)}{(x-7)}\underset{(ii)}{(x-2)}(x^2+2x+4)}_{\text{Expression}} < 0$$

Critical Points

$$\underbrace{(i)\ x-7=0 \qquad (ii)\ x-2=0}_{\text{Critical Points are } 2,\,7}$$

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Solve $(7-x)(x^3-8)>0$

Solution:

$$\underbrace{(x-7)(x-2)(x^2+2x+4)}_{\text{Expression}} < 0$$

Critical Points are $2, 7$

Step 4: Plot the critical points on the real line and determine the regions.

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Solve $(7-x)(x^3-8) > 0$

Solution:

$$\underbrace{(x-7)(x-2)(x^2+2x+4)}_{\text{Expression}} < 0$$

Critical Points are 2, 7

Step 5: Determine the sign of the expression in each region.

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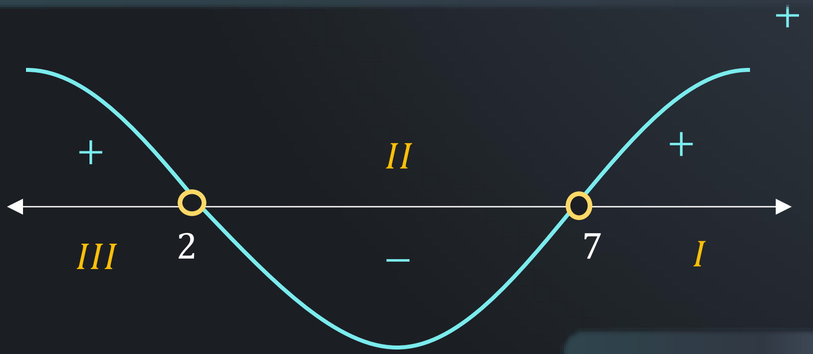
Solve $(7-x)(x^3-8)>0$

Solution:

$$\underbrace{(x-7)(x-2)(x^2+2x+4)}_{\text{Expression}}<0$$

Critical Points are 2, 7

Step 6 : Generate the wave diagram starting from the rightmost region moving towards the leftmost region and hence write the solution as per the given question.



$\therefore x \in (2,7)$

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Solve: $\dfrac{x^{2020}(x-17)^{2019}(x-13)^{2018}}{(x+7)^{2021}(x-31)^{2022}} \geq 0$

$$\frac{x^{2020}(x-17)^{2019}(x-13)^{2018}}{(x+7)^{2021}(x-31)^{2022}} \geq 0$$

Critical point at $x = -7, 0, 17, 13, 31$

Steps

Step 1 : Bring everything to the L.H.S and make R.H.S $= 0$

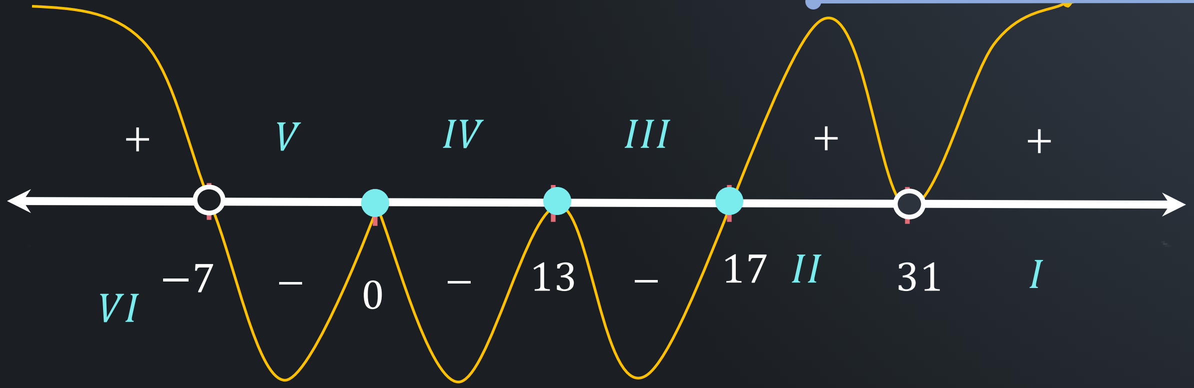
Step 2 : Factorize the expression into as many linear factors as possible.

Step 3 : Identify the critical points. (The value of x at which each individual factor is equal to zero)

Step 4 : Plot the critical points on the real line and determine the regions.

Step 5 : Determine the sign of the expression.

Step 6 : Write the solution set as per the question.



$\therefore x \in \{0, 13\} \cup (-\infty, -7) \cup [17, 31) \cup (31, \infty)$

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THANK
YOU