Rutherford's Model

Planck's Quantum Theory

SWE

J.J. Thomson's Model

Bohr's Model

de Broglie's hypothesis

Heisenberg's Uncertainty principle
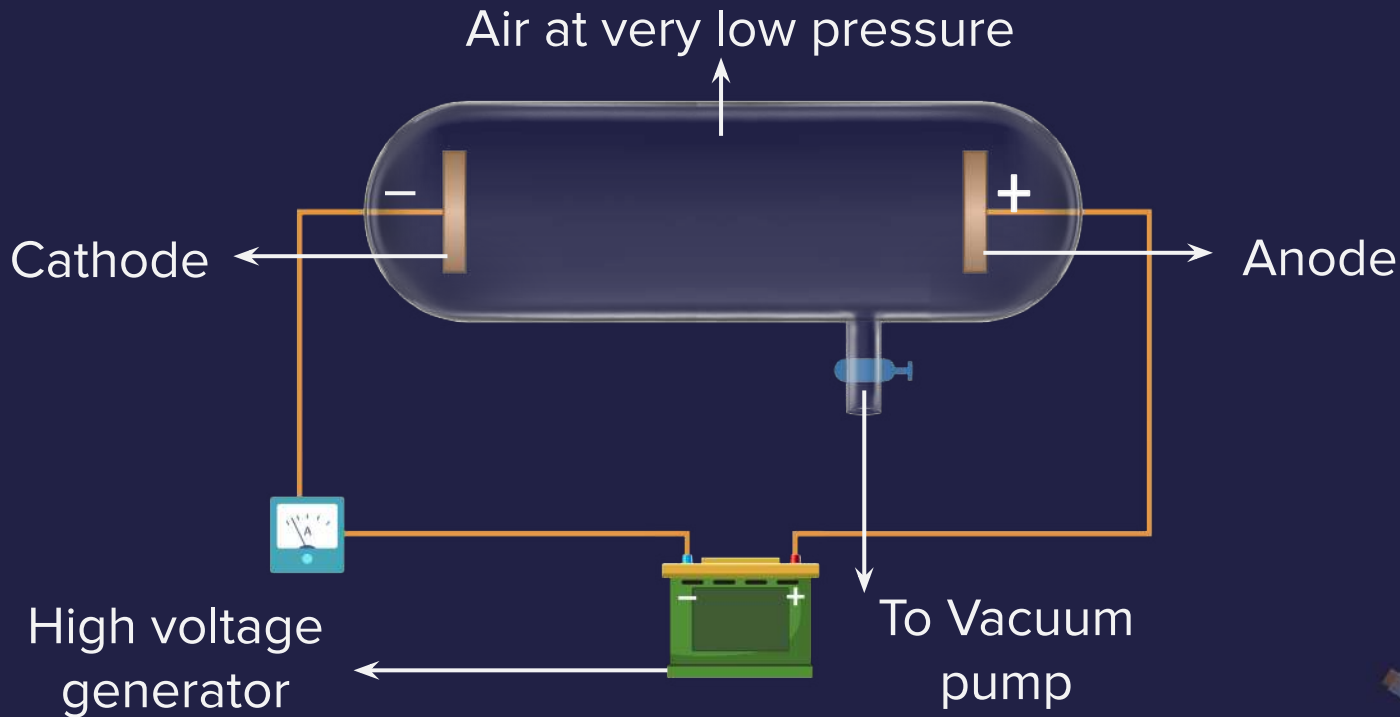
Discovery of electron

Cathode rays

In 1891, George Johnstone Stoney named the fundamental unit of electricity as 'electron'

J. J. Thomson and his team identified electron as a particle in 1897

Discovery of electron

Discharge tube: Cylindrical hard glass tube fitted with **two metallic electrodes** connected to **a battery.**
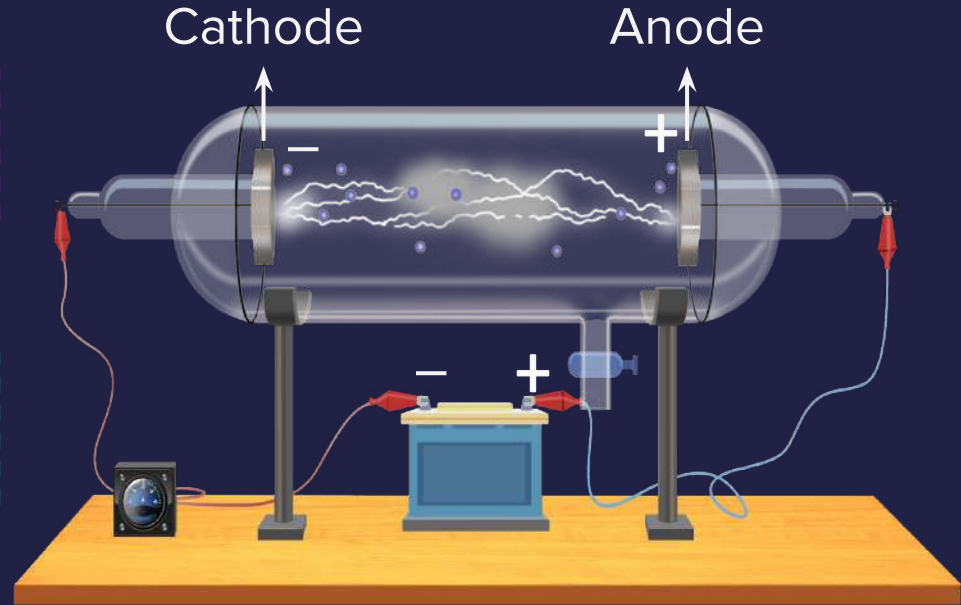
Air at very low pressure

−

+

Cathode

Anode

High voltage generator

To Vacuum pump

Discovery of electron

Observations

Readings of electric current was observed

Anode end of the tube showed a greenish glow on the ZnS screen

Cathode

Anode

−

+

−

+

Why are Gases used under LOW pressure?

At high pressure, more number of gas molecules are present and so there is more obstructions in the paths of electrons which prevents electrons from reaching the anode.
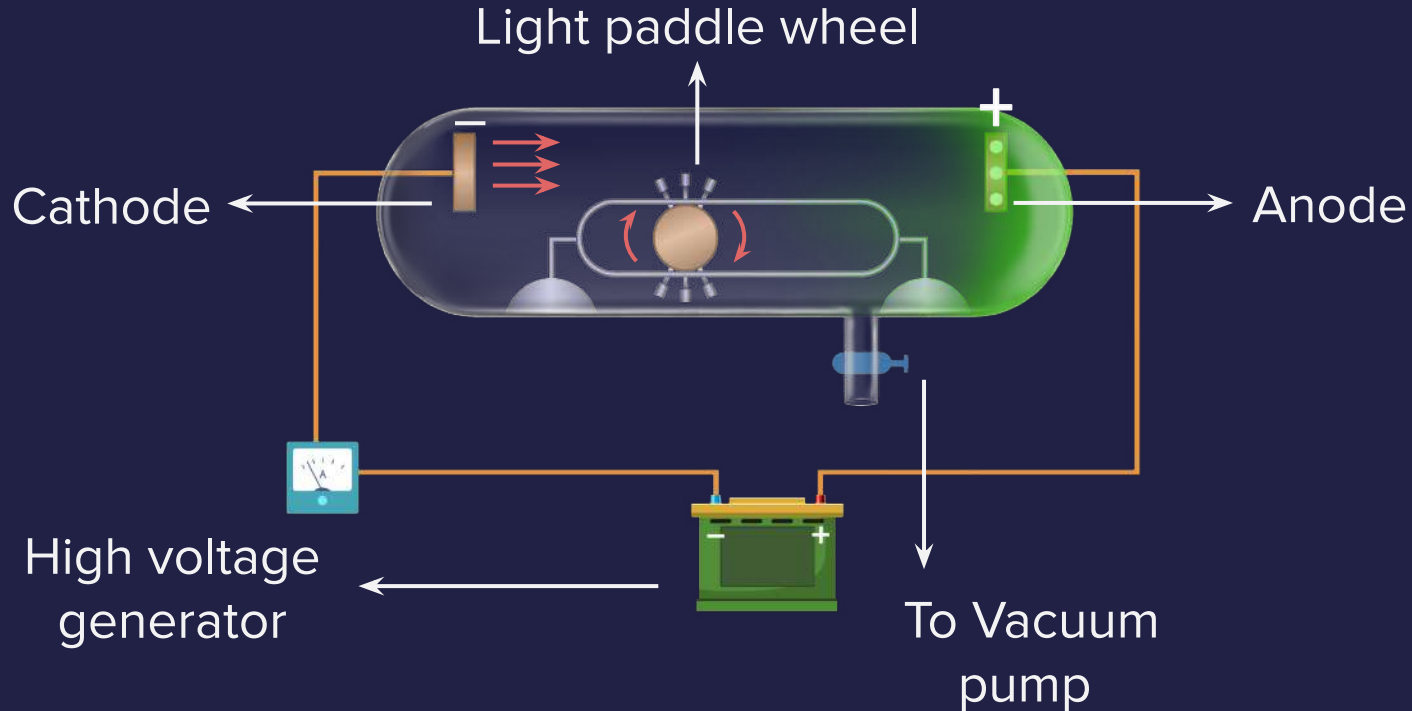
Discovery of electron

Observation and Characteristics

1. Cathode rays move from cathode to anode.

2. Cathode rays travel in a straight line with high velocity
in the absence of electric & magnetic fields.

3. Cathode rays are more efficiently observed with the help of a fluorescent or phosphorescent material like ZnS.

4. Cathode rays rotate the light paddle wheel placed in their path. It shows that the particles of cathode ray particles are **material particles** which have **mass** and **velocity.**
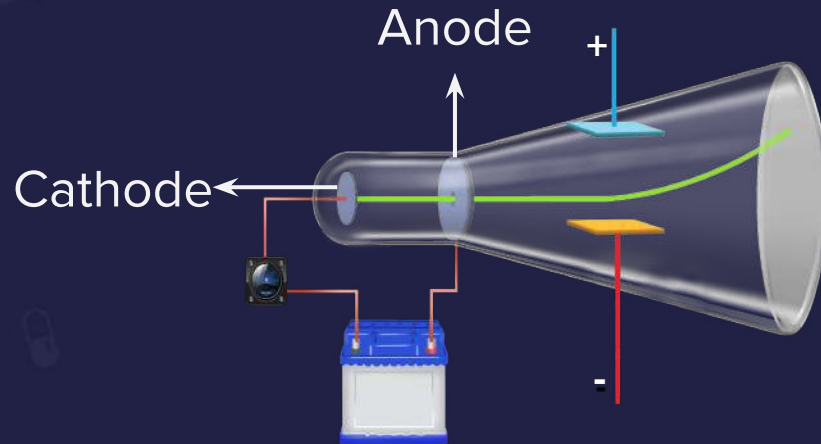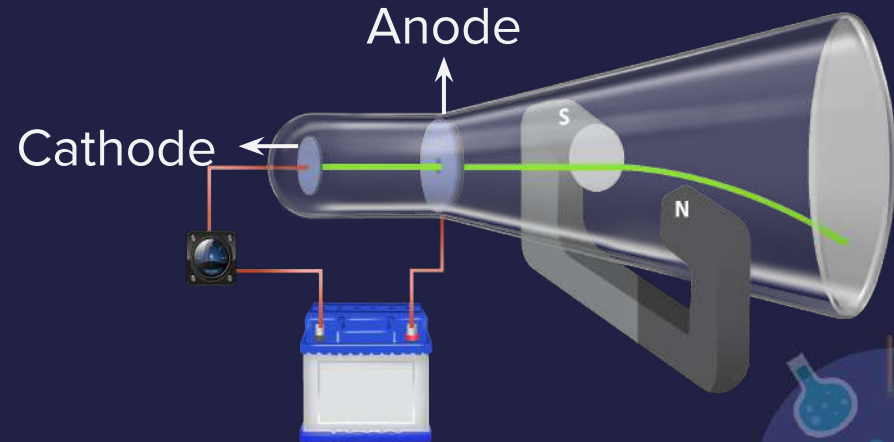
Discovery of electron

Light paddle wheel

Cathode

Anode

High voltage generator

To Vacuum pump

Discovery of electron

Observation and Characteristics

5. Cathode rays are deflected in the presence of an electric field.

6. Cathode rays are deflected in the presence of a magnetic field.

Anode

Cathode

\+

\-

Anode

Cathode

S

N

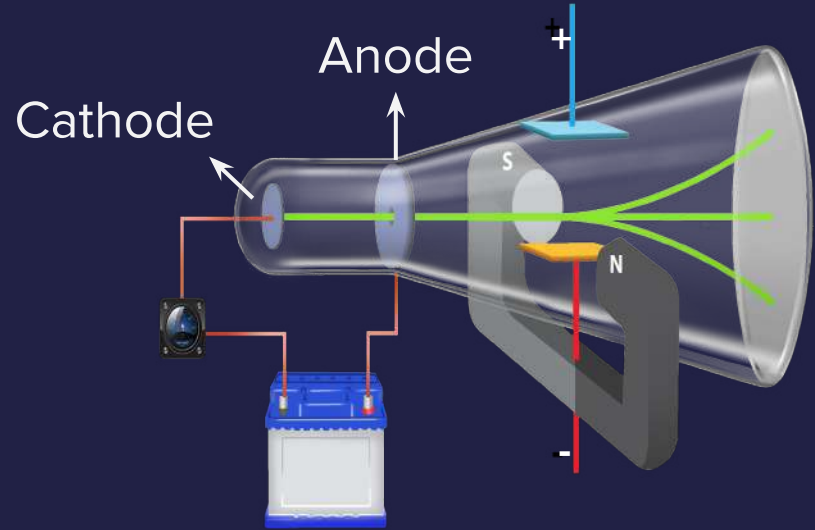# Discovery of electron

Conclusions

Cathode rays consist of negatively charged particles and identified as **electrons.**

Charge to mass ratio

In 1897, **J.J. Thomson**

- Measured the charge (e) to mass (m) ratio of an electron.
- Electric & magnetic fields were applied perpendicular to each other & to the path of electrons

$$\frac{e}{m} = 1.758820 \times 10^{11} \text{ C/kg}$$
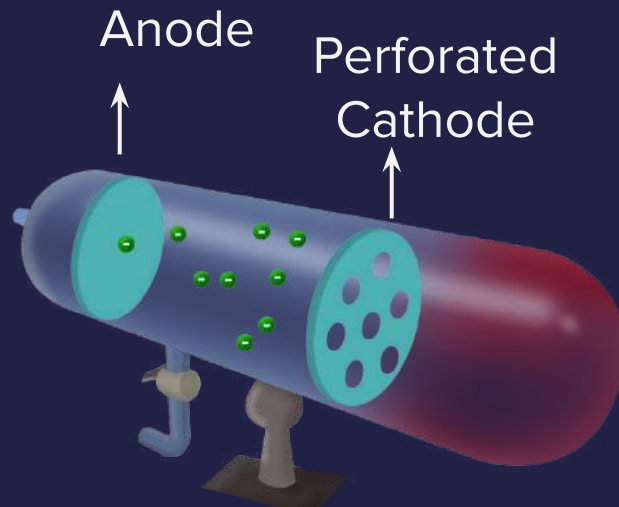
Anode

Cathode

Discovery of Anode Rays

Discovered by Goldstein

He repeated experiment with a discharge tube by using a perforated cathode.

Existence of positively charged particles was shown using anode rays.

Anode

Perforated Cathode

Red glow is due to anode particles which passes through perforated cathode and strikes the wall of the tube at the cathode side.
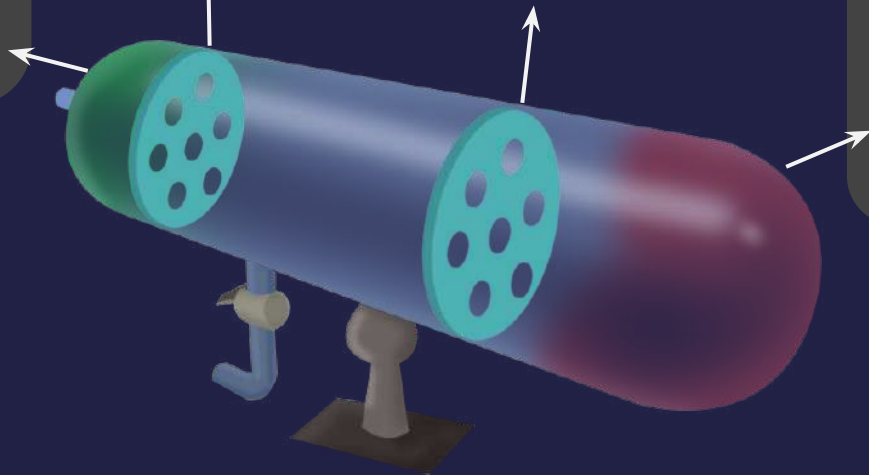
Discovery of Anode Rays

Green colour fluorescence observed due to Cathode rays

Perforated Anode

Perforated Cathode

Red colour fluorescence observed due to Anode rays

Observations and Characteristics

1. Anode rays possess positive charge

Concluded by their directions of deflections in the presence of electric & magnetic fields

2. Anode rays travel in straight lines in the absence of both electric and magnetic fields.

Observations and Characteristics

3. e/m ratio of the canal rays is different for different gases

Properties of anode rays depends on nature of the gas taken in the discharge tube

In 1919, Rutherford discovered that the smallest and the lightest positive ions are obtained from hydrogen and called them protons
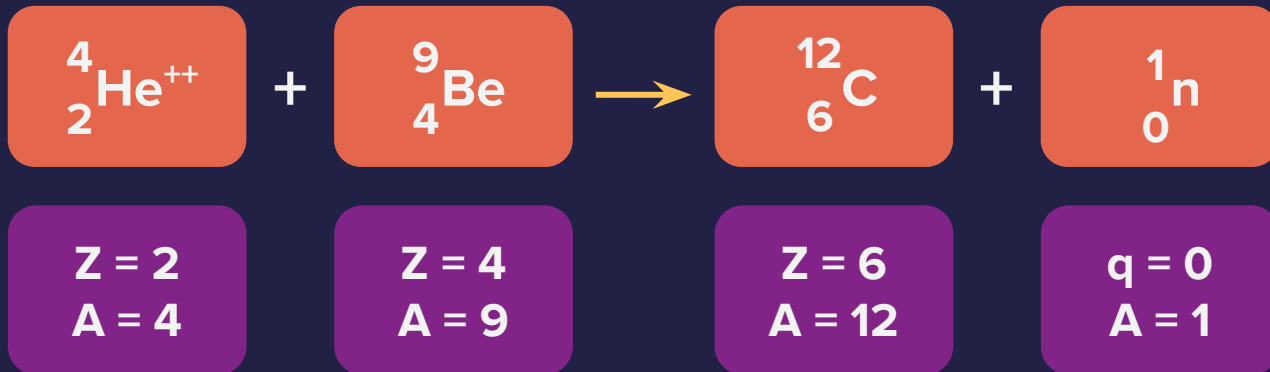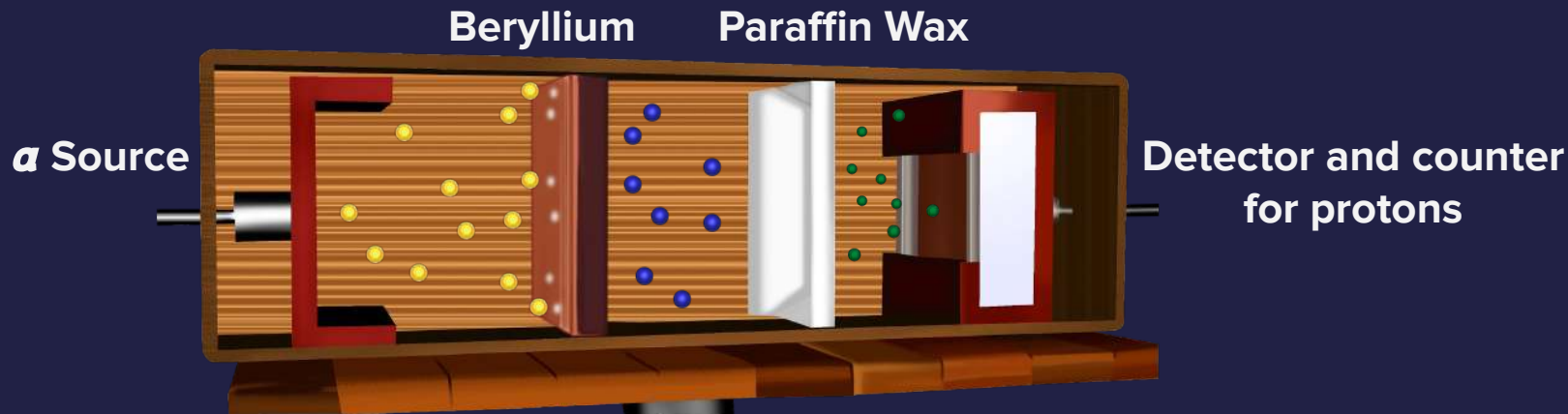
Discovery of Neutrons

James Chadwick

Named the electrically neutral particles emitted as neutrons
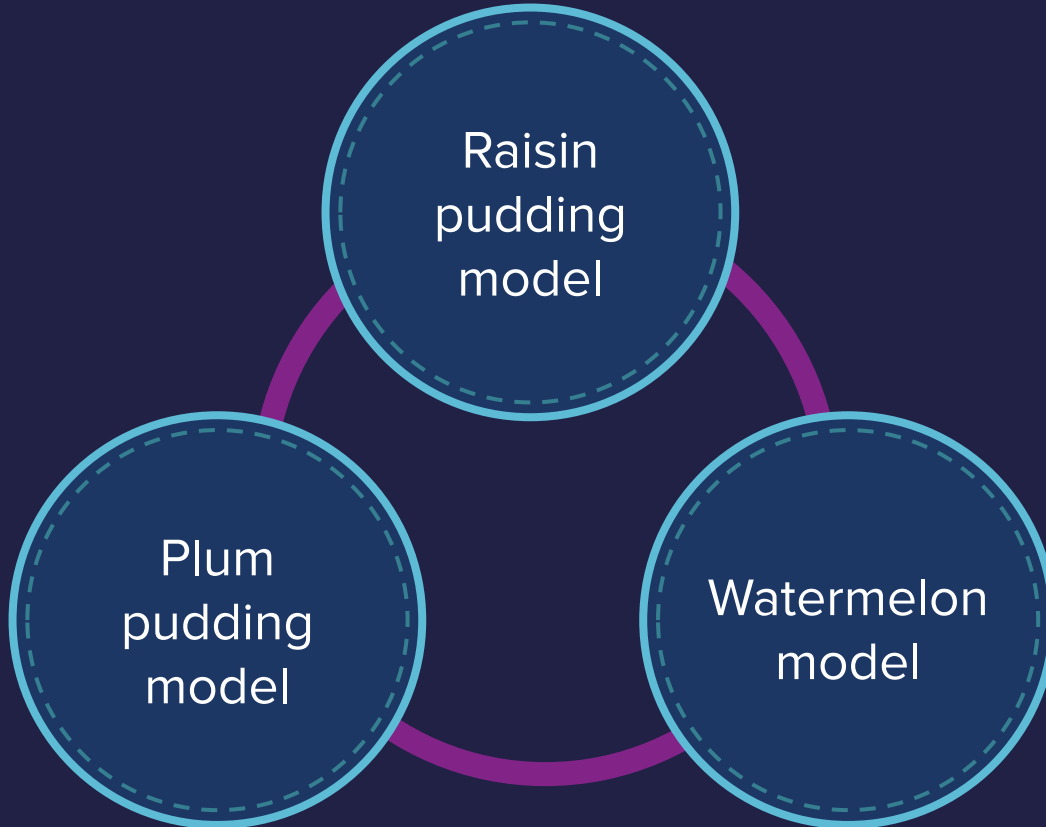
Discovered neutrons in 1932

Mass of neutrons is slightly greater than that of protons

Bombarded a thin sheet of beryllium ($^{9}_{4}$Be) with alpha particles ($^{4}_{2}$He^{2+})

Discovery of Neutrons

Beryllium

Paraffin Wax

α Source

Detector and counter for protons

$$^{4}_{2}He^{++} + ^{9}_{4}Be \rightarrow ^{12}_{6}C + ^{1}_{0}n$$

Z = 2
A = 4

Z = 4
A = 9

Z = 6
A = 12

q = 0
A = 1

Thomson's Model

Raisin pudding model

Plum pudding model

Watermelon model

Thomson's Model

1. An atom has a spherical shape (radius ~10^{-10} m)

2. Positive charge is uniformly distributed throughout the sphere

3. Negatively charged electrons are embedded in it like raisins in a pudding

4. Mass of the atom is assumed to be uniformly distributed all over it
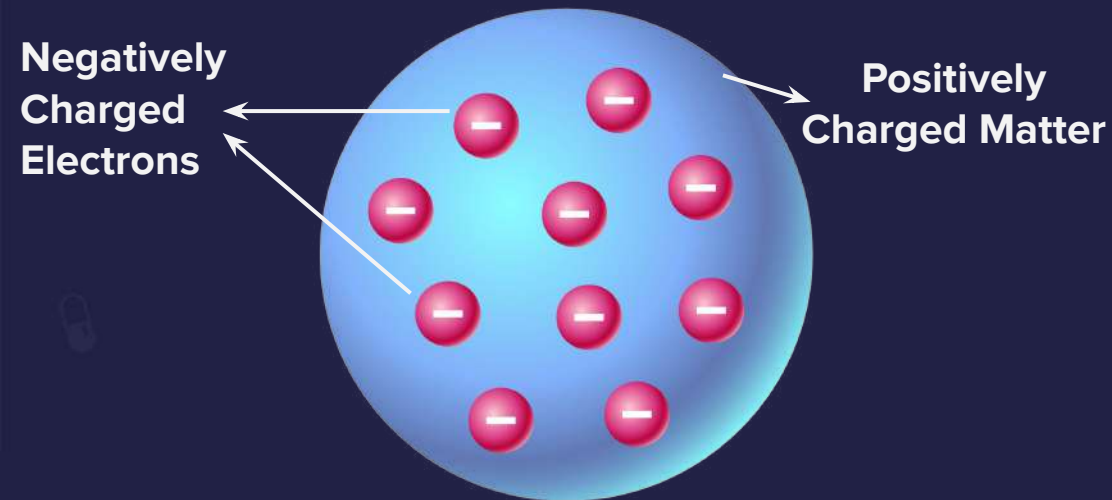
Plum Pudding

Watermelon

Thomson's Model

Electrons are embedded in an atom in such a way that the most stable electrostatic arrangement is achieved.
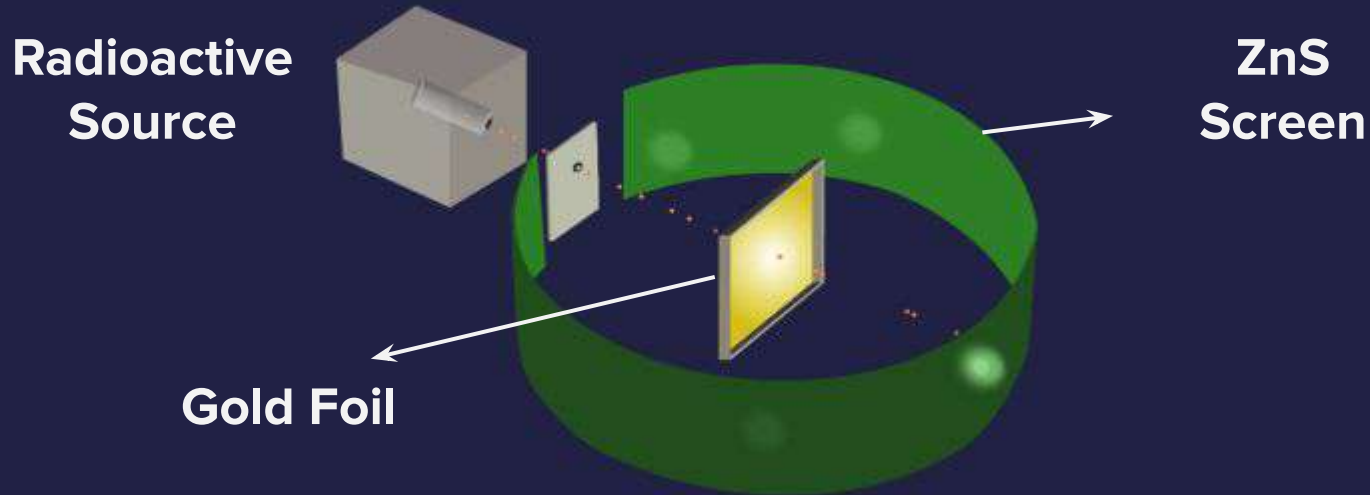
Explains the overall neutrality of an atom

**Drawback:
Not consistent with the results of later experiments**

Negatively Charged Electrons

Positively Charged Matter

Rutherford's Experiment

A stream of high energy α–particles was directed at a thin gold foil (thickness ~ 100 nm)
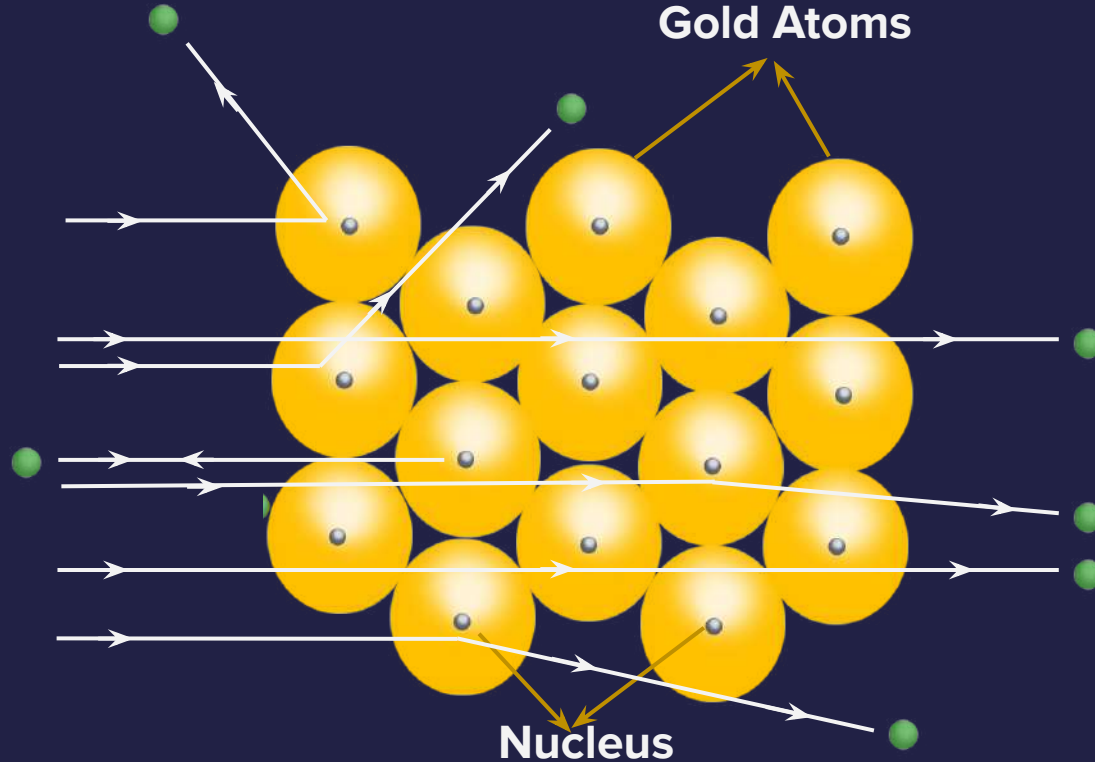
Radioactive Source

ZnS Screen

Gold Foil

Observations of Rutherford's Experiment

When an α–particle strikes the screen, a glow was produced at that point on the screen
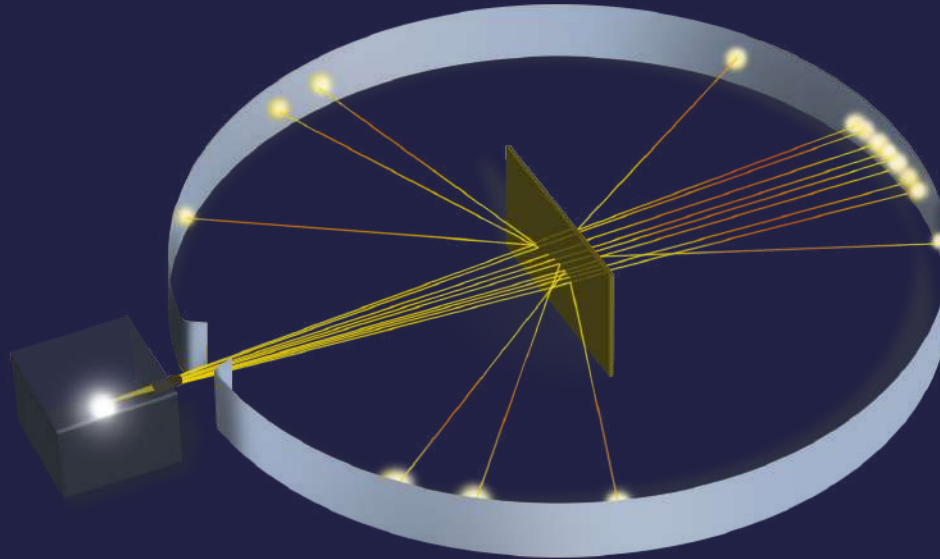
Observations of Rutherford's Experiment

a–particles

1. Most of them passed undeflected

2. Very small fraction was deflected by large angles

3. Very few were deflected by 180° (~1 in 20,000)

4. Small fraction was deflected by small angles

Observation	Conclusion
Most **α**-particles passed through the foil without deflection.	Presence of large empty space in the atom.
Few **α**-particles were deflected by small angles.	Positive charge is concentrated in a very small region.
Very few **α**-particles (~1 of 20,000) deflected at 180°.	Small positively charged core at the centre.

Nucleus

Atom consists of a **small positively charged core** at the center which carries almost the entire mass of the atom

It has **negligible volume** compared to the volume of the atom.

Nucleus

Both, protons and neutrons present in the nucleus are collectively called nucleons.

$$R = R_0 A^{\frac{1}{3}}$$

Radius of the atom ← $\sim 10^{-10}$ m

$\sim 10^{-15}$ m → Radius of the nucleus

R = Radius of nucleus of an element

A = Mass number of element

R_0 = 1.11 x 10^{-15} m to 1.44 x 10^{-15} m

Extranuclear part
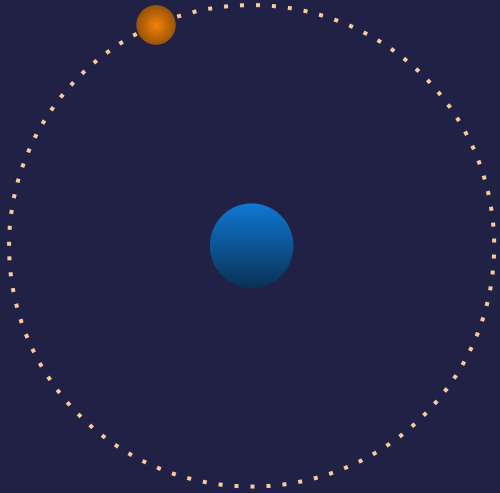
Electrons and nucleus are held together by electrostatic forces of attraction.
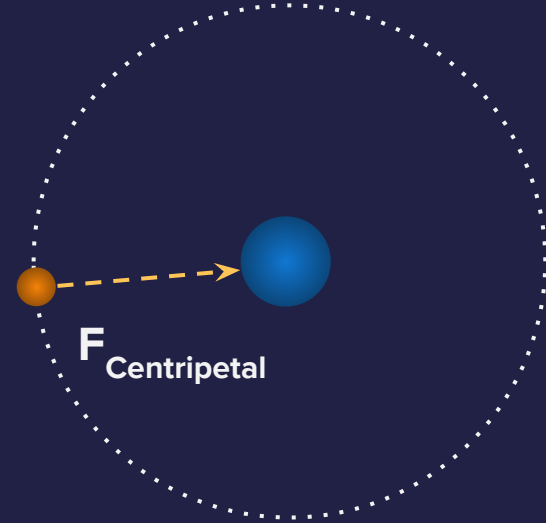
$$F_{Centripetal}$$

Nucleus is surrounded by revolving electrons.

$$F_{Electrostatic} = F_{Centripetal}$$

Drawbacks of Rutherford's Model

It could not explain stability of the atom.

It could not explain line spectrum of the H atom.

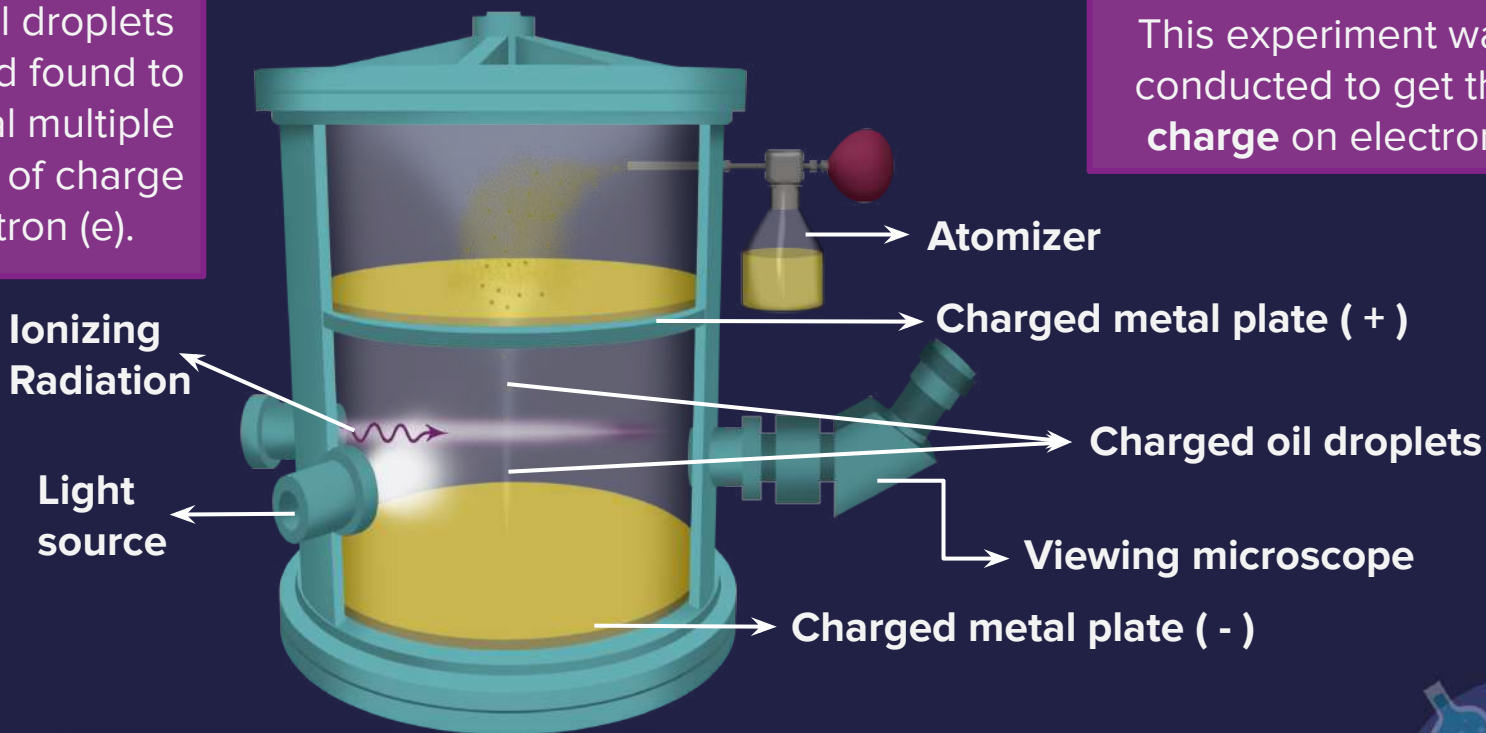It could not explain the electronic structure of the atom.

R.A. Millikan's Oil drop experiment

Charge on oil droplets measured and found to be an integral multiple of magnitude of charge on an electron (e).

This experiment was conducted to get the **charge** on electron.

Atomizer

Charged metal plate (+)

Ionizing Radiation

Charged oil droplets

Light source

Viewing microscope

Charged metal plate (-)

Charge on electron — 1.602176×10^{-19} C

Mass of the electron

Thomson's e/m ratio

Charge from Millikan's experiment

1.75882×10^{11} C kg^{-1}

$- 1.602176 \times 10^{-19}$ C

From **Thomson's experiment,** e/m ratio calculated and from **Oil drop experiment, charge** of electron calculated. Using the data from these two experiments, **mass of the electron** was determined.

Subatomic Particles

Subatomic Particles	Mass (u)	Absolute Mass (kg)
Electron	0.0005	9.1×10^{-31}
Proton	1.007	1.6722×10^{-27}
Neutron	1.008	1.6749×10^{-27}

Subatomic Particles	Relative Charge	Absolute Charge (C)
Electron	-1	-1.602×10^{-19}
Proton	+1	1.602×10^{-19}
Neutron	0	0

Quantization of Charge
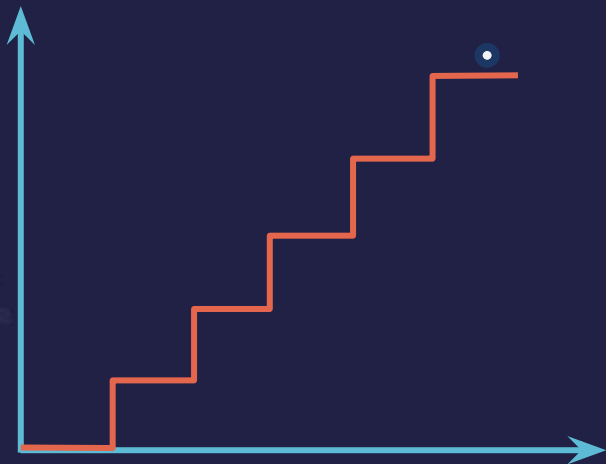
$$q = n \, (1.6 \times 10^{-19} \text{ C})$$

$$q = n \, (4.8 \times 10^{-10} \text{ esu})$$

$$n = 1, 2, 3 \ldots$$

$$q = n \, (e)$$

The charge can't have continious range of values but only take values in multiple of charge on one electron. The magnitude of charge on an electron is the smallest unit and denoted as "e". Thus charge on an electron is -e and on a proton, it is +e.

Electrostatic Force

$$F_{12} = F_{21} = K\frac{q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$$

F_{12} ← +q_1 +q_2 F_{21} →

+q_1 F_{12} →   ← F_{21} -q_2

← - - - r - - - →
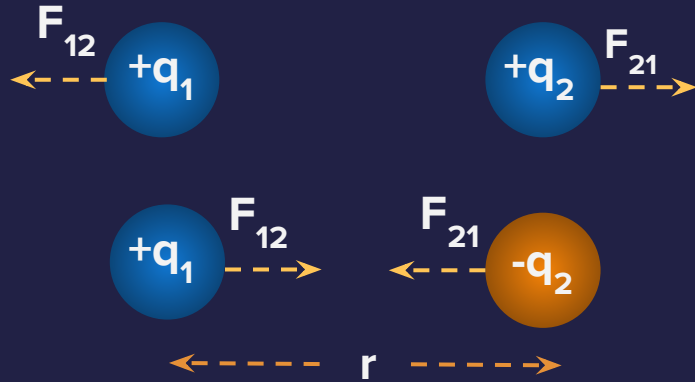
$$\varepsilon_0 = 8.854 \times 10^{-12}\frac{C}{Vm}$$

$\varepsilon_0 = $ **Permittivity of vacuum**

Potential Energy

$$P.E. = q \times V$$

q = Charge of the particle
V = Potential of surface

$$P.E. = \frac{1}{4\pi\varepsilon_0} \frac{q_1\, q_2}{r}$$

$$P.E. = K \frac{q_1\, q_2}{r}$$

Closest Distance Of Approach

When two charged particles of similar nature approach each other, the repulsion between them increases but due to initial kinetic energy, the particles come closer to each other (Recall the bombarding of positively charged alpha particle on gold foil where the alpha particle approaching positively charged nucleus of gold atom).
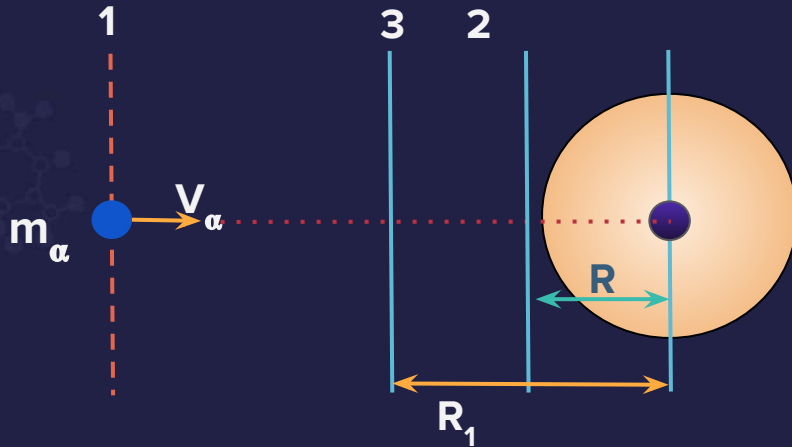
Closest distance of approach(in alpha particle scattering by gold nucleus),

$R = \sqrt{4KZe^2/m_\alpha} \, V_\alpha$

Closest Distance Of Approach
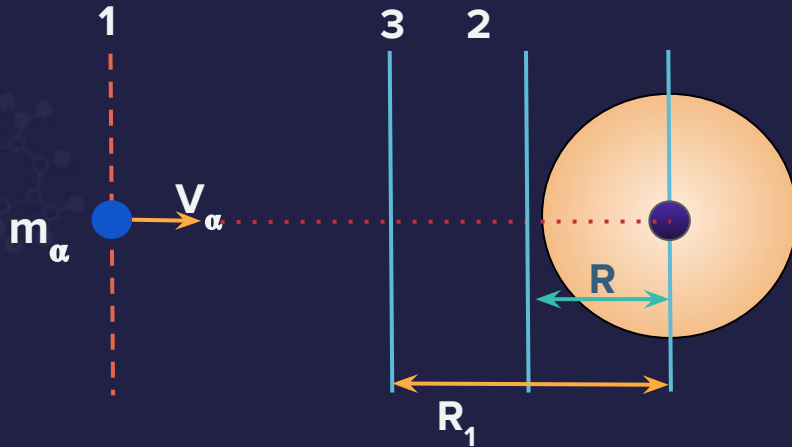
Closest distance of approach,

$R = \sqrt{4KZe^2/m_\alpha V_\alpha}$

But at a certain distance between them, the relative velocity becomes zero and after that due to repulsion, the particles starts going away from each other. This distance between the particles where velocity once becomes zero is called Closest distance of approach and can easily be calculated using conservation of energy concept.
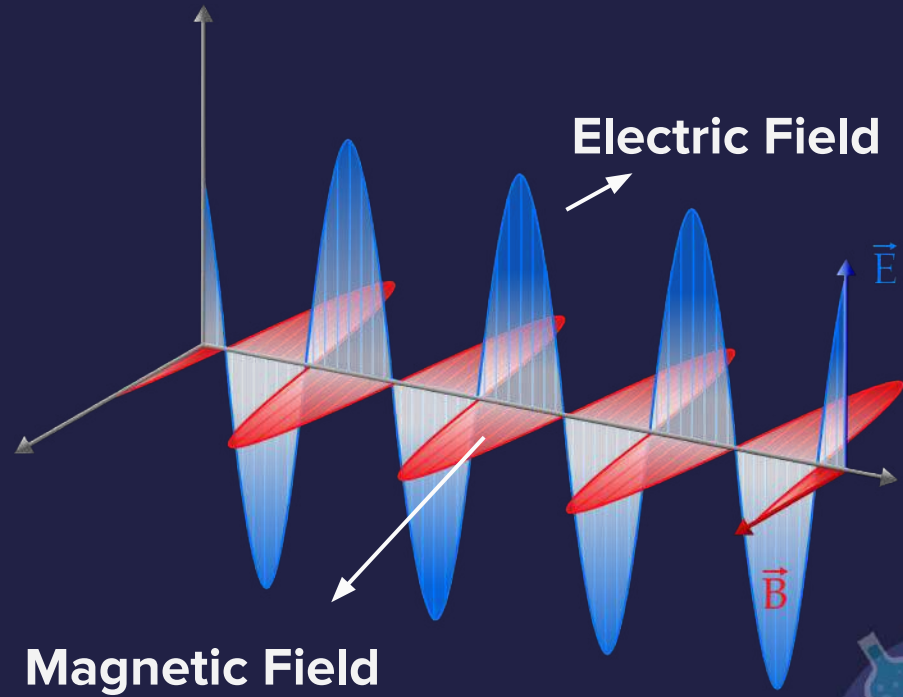
Properties of Electromagnetic Waves

Electric & magnetic field oscillate perpendicular to each other

Both oscillate perpendicular to the direction of propagation of wave

Do not require any medium for propagation

Can travel in vacuum

$c = 3 \times 10^8$ ms^{-1} (in vaccum)

Propagate at a constant speed i.e. with the speed of light (c)

Characteristics of Electromagnetic waves

Crest

Wavelength

Amplitude

Mean position

Trough

Velocity of the wave

Frequency: Number of times a wave oscillate from crest to trough per second

Characteristics of Electromagnetic waves

Wavelength (λ)

Distance between two consecutive crests or troughs

SI unit : m

Frequency (ν)

Number of waves passing a given point in one second
SI unit : Hertz (Hz), s^{-1}

Related to time period as:

$$\nu = \frac{1}{T}$$

Characteristics of Electromagnetic waves

Velocity (c or v)

Distance travelled by a wave in one second SI unit : ms^{-1}

Related to frequency (**ν**) & wavelength (λ) as:

$$c = \nu \lambda$$

Characteristics of Electromagnetic waves

Time Period (T)

Time taken to complete one oscillation

SI unit : s

Wavenumber (v)

Number of waves per unit length
SI unit : m^{-1}

Amplitude (A)

Height of the crest or the depth of the trough from the mean position
SI unit : m

Characteristics of Electromagnetic waves

Consists of radiations having different wavelength or frequency

$$\bar{\nu} = \frac{1}{\lambda}$$

$$\nu = c\,\bar{\nu}$$

$$\nu = \frac{1}{T}$$

Electromagnetic Spectrum

Electromagnetic radiations are arranged in the order of

Decreasing frequency or **Increasing wavelength**

Electromagnetic Spectrum

Short wavelength

Long wavelength

| Gamma rays | X-rays | UV | | Infrared | Microwave | Radio wave |

High frequency

Low frequency

Visible light
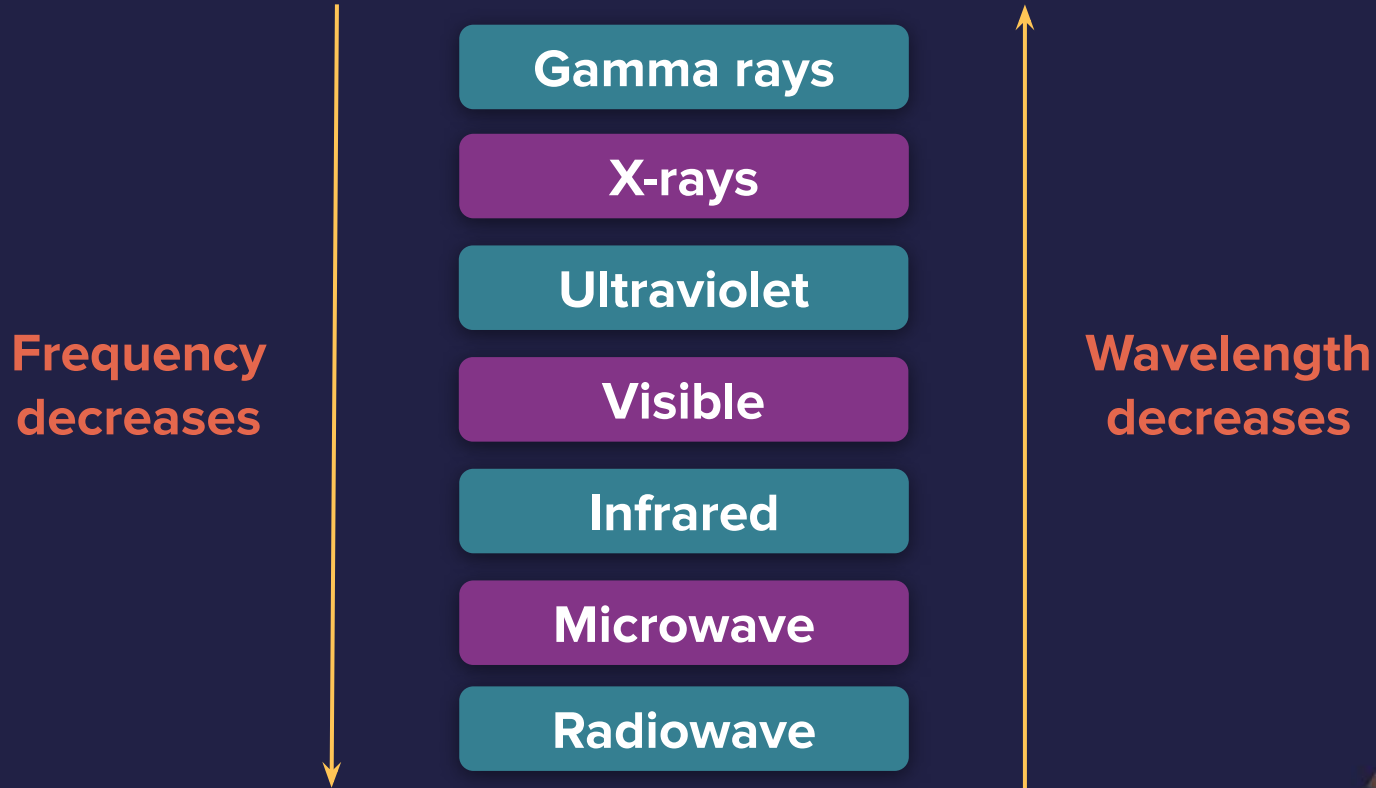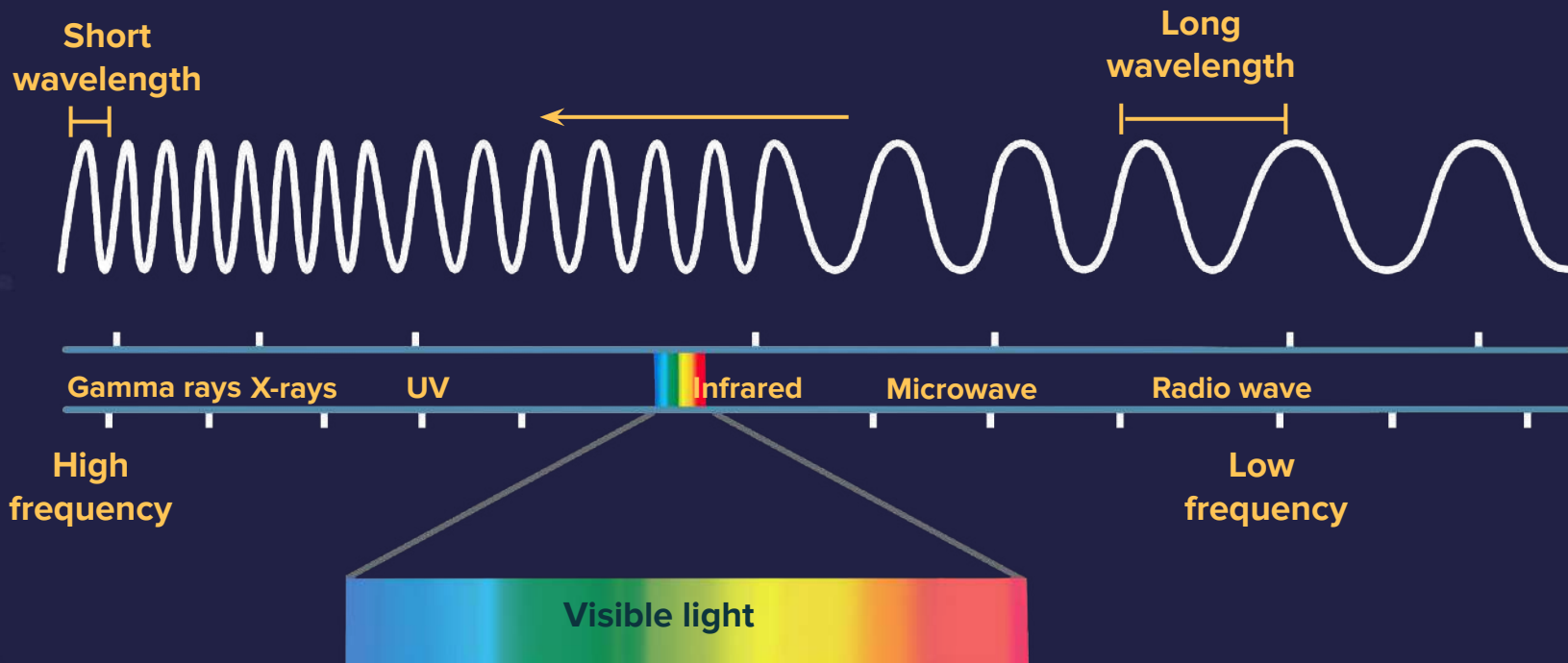
EM Radiation: Wave or Particle?

Wave nature of the EM radiation explains

Diffraction

Interference

EM Radiation: Wave or Particle?

Electromagnetic wave theory could not explain

Black-body radiation

Photoelectric effect

Variation of heat capacity of solids with temperature

Line spectrum of Hydrogen

Continuous vs Discrete

Mass **=** **N (mass of 1 water molecule)**

Where N ∈ + I

It seems that mass of water (or any other matter) can take any values (suppose we can go till 30 decimal points) and so we can say that mass has continuous range of values. But on microscopic level, we can observe that mass of water is always an integral multiple of 1 molecule of water. i.e., mass is quantized or we can say that quantization is a property of matter.

Back to EM waves
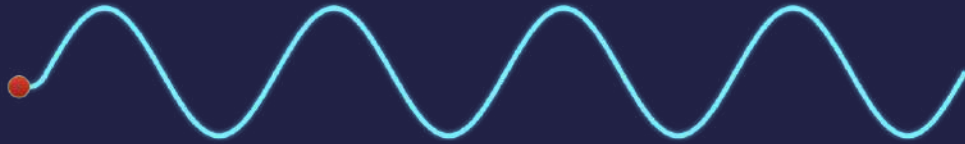
Low temperature
Low frequency
Longer wavelength
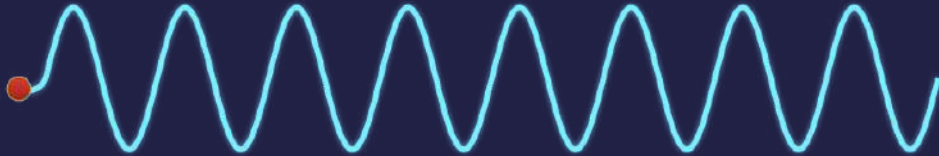
High temperature
High frequency
Shorter wavelength

What is a black body?

Idealized system

Absorbs & emits all frequencies

Absorbs regardless of the angle of incidence

Why the name, Black Body?

Visually Black body **VS** **Radiatively Black Body**
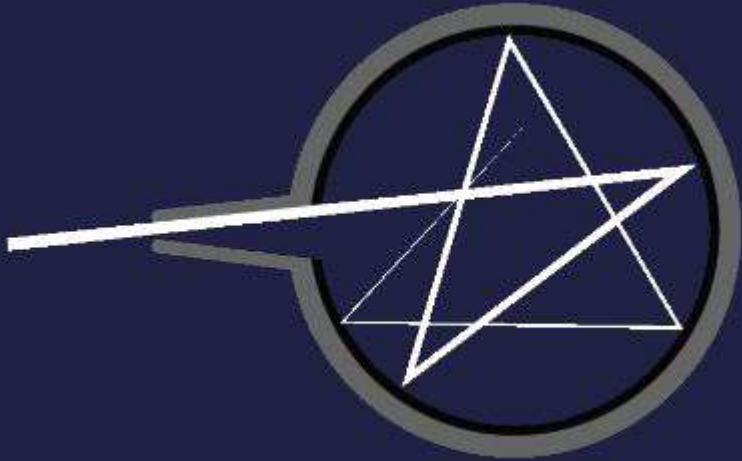
A true black body appears black because it is not reflecting any electromagnetic radiation.

However, everything you see to be black can not ber called as blackbody because there could be radiation coming out which is not in the visible range.

Wavelength-Intensity relationship

UV Visible

5000 K

Classical Theory
(5000 K)

4000 K

3000 K

Intensity
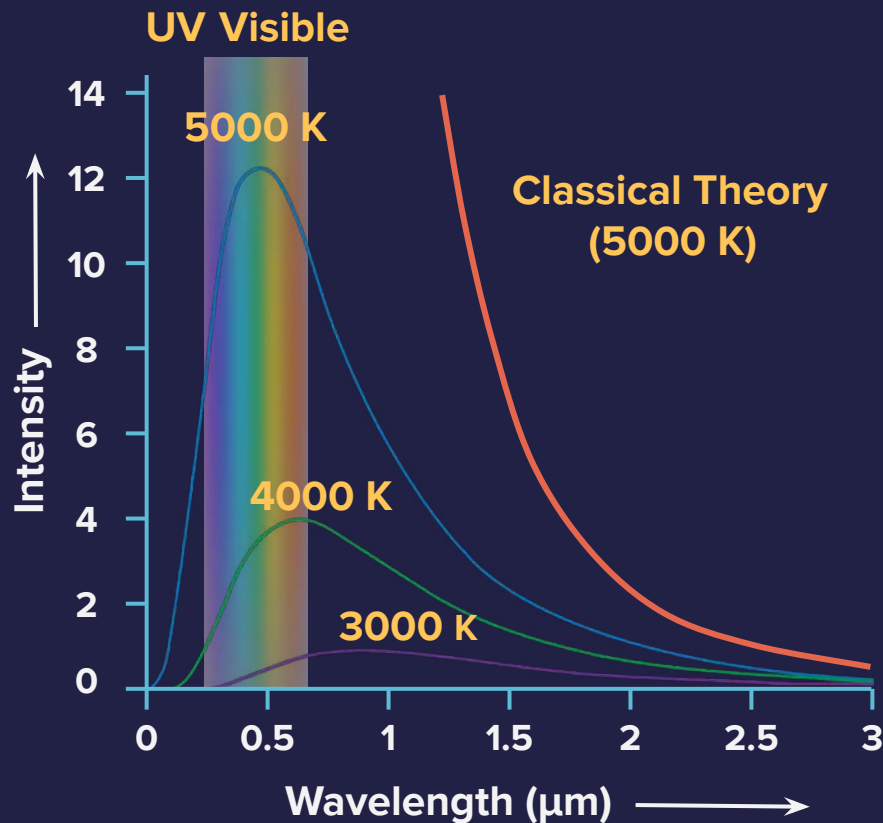
Wavelength (µm)

This graph shows intensity as a function of wavelengths emitted from a black body. It shows how bright it is at what wavelength. It shows quantization nature of energy and hence favours particle nature of light.

Planck's Quantum theory

The smallest packet or bundle of energy (quantum of radiation) is called a **photon.**

This is the smallest quantity of energy that can be emitted or absorbed in the form of EM radiation.

Quantum theory of Radiation

Energy (E) of a photon is proportional to its frequency (ν)

$$E \propto \nu_{\text{Radiation}}$$

$$E = nh\nu$$

$$E = h\nu = h\frac{c}{\lambda}$$

h = Planck's constant
= 6.626×10^{-34} Js

n = number of photons
= 0, 1, 2, 3, ...

One electron volt (eV)

Energy gained by an electron when it is accelerated from rest through a potential difference of 1 V

Important Conversions

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E \text{ (eV)} = \frac{12{,}400}{\lambda \text{ (Å)}}$$

$$E\left(\frac{kJ}{mole}\right) = E\left(\frac{eV}{particle}\right) \times 96.48$$

Photoelectric Effect

Phenomenon of electrons ejection	When a radiation of sufficient frequency falls on the metal surface
Photoelectrons	Electrons are ejected with the aid of light

Photoelectric Effect

Metal plate
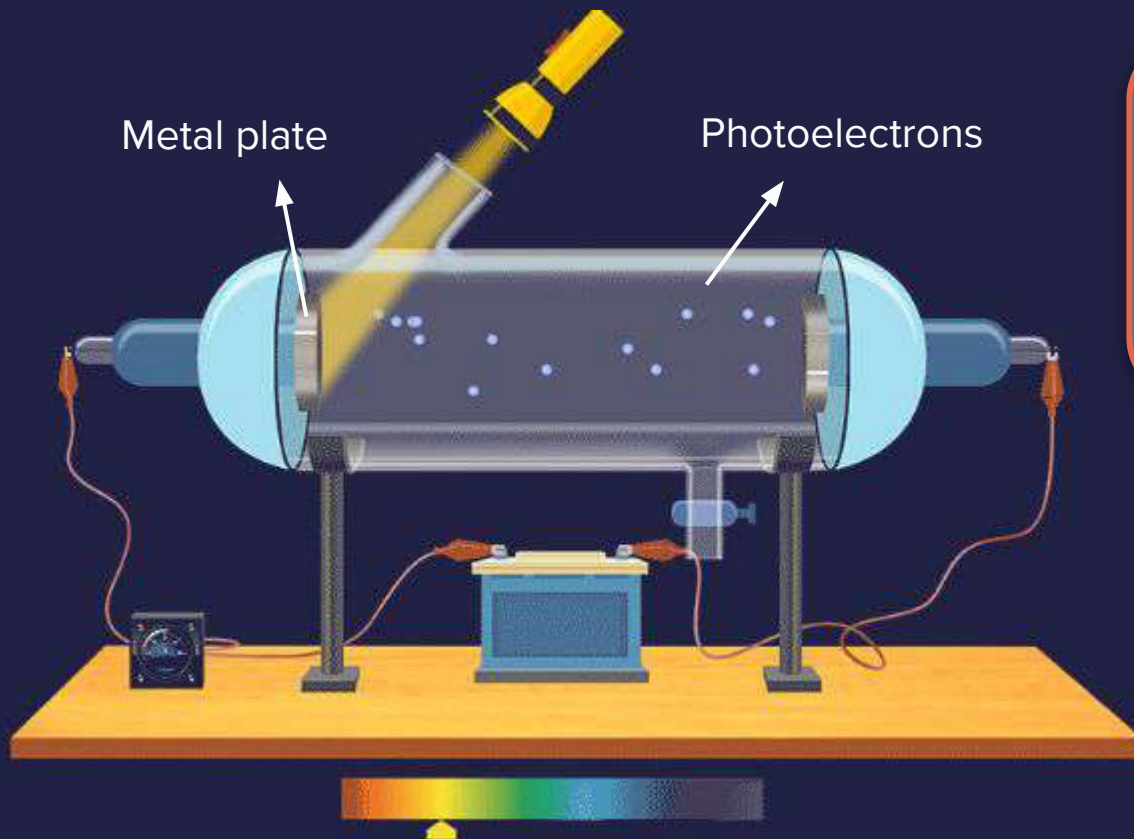
Photoelectrons

When radiation of sufficient energy falls on the metal plate, there starts emission of electrons called **photoelectrons.**

Observations

Electrons are ejected as soon as the beam of light of sufficient frequency strikes the metal surface

Instant transfer of energy to the electron when a photon of sufficient frequency strikes the metal atom

Threshold frequency (v_O)

Minimum frequency required to eject a photoelectron from a metal surface

Each metal has a characteristic threshold frequency

Observations

Number of electrons ejected $\propto$ **Intensity of light**

or

Brightness of light

Observations

No electron is ejected, regardless of the intensity of light

$\nu_{incident} < \nu_0$

Even at low light intensities, electrons are ejected immediately

$\nu_{incident} > \nu_0$

Particle nature of light

One photon is absorbed by only one electron in a single interaction. Not more than one photon can be absorbed by an electron.

If intense beam of light is used, large number of photons are available and large number of electrons are ejected. This observation shows particle nature of light.

Observations

When $\nu_{\text{incident}} > \nu_0$

$$\text{K.E.}_{\text{Ejected electron}} \propto \nu_{\text{Incident}}$$

Energy possessed by the photon ↑

Transfer of energy to the electron ↑

K.E. of the ejected electron ↑

Photoelectric Effect

$$0 \leq K.E._{\text{Ejected electrons}} \leq K.E._{\text{Max}}$$

K.E. is **independent** of the intensity of radiation

Striking photon's energy $=$ $h\nu$

Work function $=$ ϕ

Photoelectric Effect

$$E_{Photon} - \phi = K.E._{Max}$$

From the Law of Energy Conservation

$$E_{Incident} = \phi + K.E._{Max}$$

$$h\nu = h\nu_0 + \frac{1}{2} m_e v_{max}^2$$

m_e = mass of the electron

v_{max} = maximum velocity of the electron

Photoelectric Effect

K.E. of the ejected electron is given as

$$K.E._{Max} = h\nu - h\nu_0$$

$$K.E._{Max} = h\frac{c}{\lambda} - h\frac{c}{\lambda_o}$$

Plotting K.E. vs Frequency
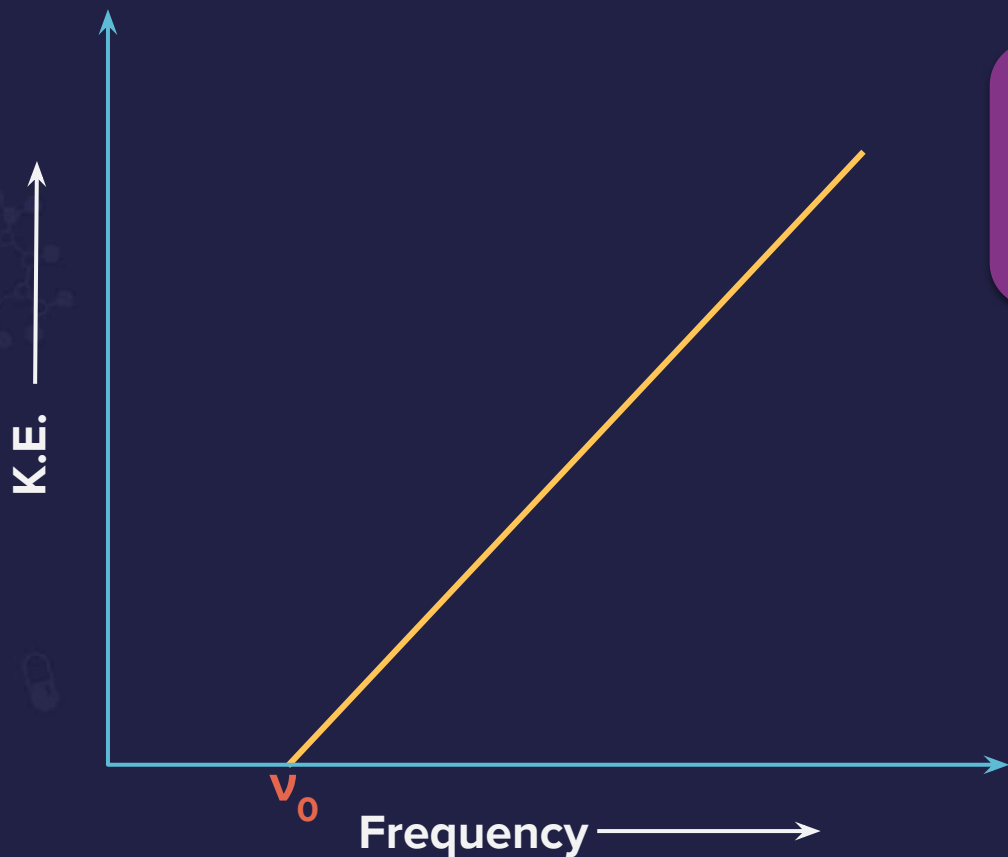
From the plot of kinetic energy vs frequency, it shows linear variation according to the equation:

$$\text{K.E.}_{Max} = h\nu - h\nu_0$$

Acceleration and Deceleration of Charged Particles

Acceleration:
If a positive charge moves from higher to lower potential (like an electron moves from cathode to anode) or a negative charge moves from lower to higher potential.
Deceleration is just opposite to the acceleration.

Stopping potential (V_s)

Minimum opposing potential required to stop the photoelectron having the maximum K.E.

$$eV_s = K.E._{max}$$

Accelerating potential voltage (V)

Voltage applied to increase the K.E. of an emitted electron

Minimum K.E. ← eV

K.E.$_{Max}$ + eV → Maximum K.E.
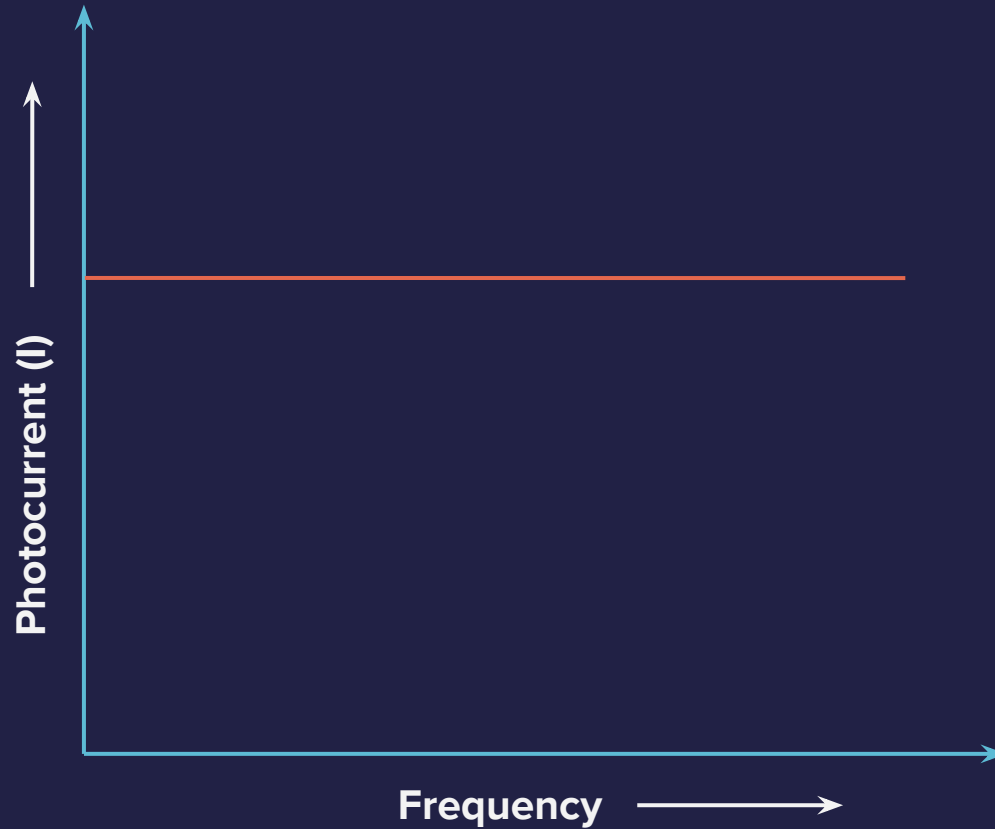
Photocurrent v/s Frequency of the Radiation

Photocurrent v/s Frequency of the Radiation

For $v > v_0$

K.E.
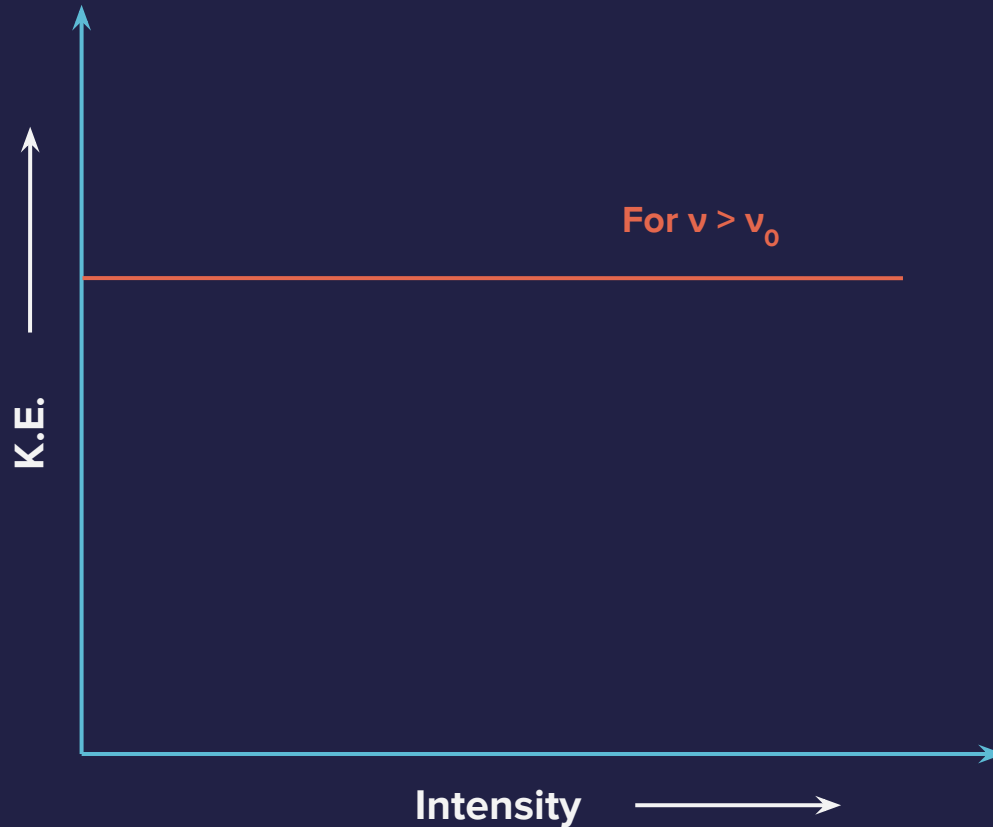
Intensity

Photocurrent vs Collector Plate Potential at different intensity of radiation

Photocurrent vs Collector Plate Potential at different intensity of radiation

$\nu_3 > \nu_2 > \nu_1$
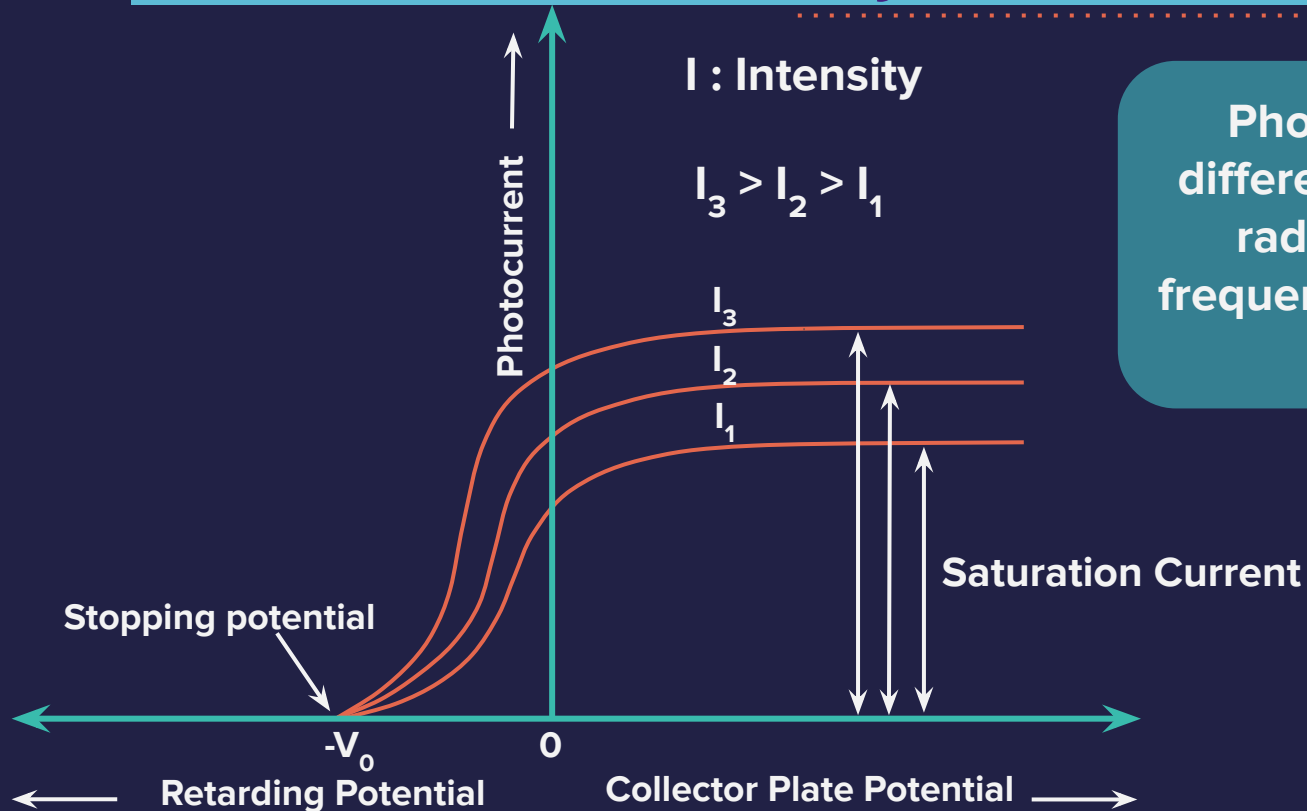
Photocurrent at different frequencies of radiation. Here intensity of radiation is same.

Bohr Atomic Model

Postulates

Stationary orbits

Concentric circular orbits around the nucleus

These orbits have fixed value of energy

Electrons revolve without radiating energy

n = 3

n = 2

n = 1

K   L   M
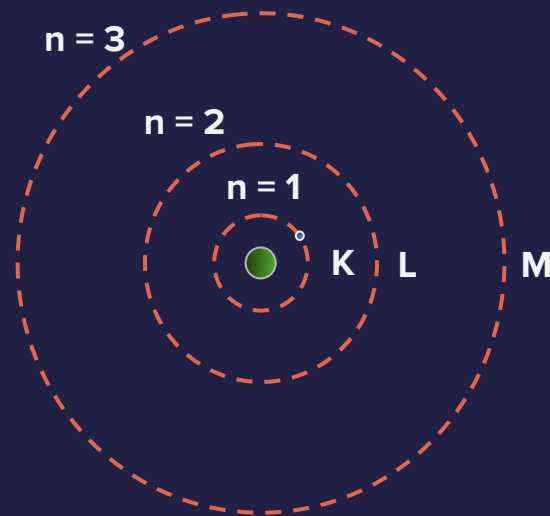
Stationary orbits **or** Energy states / levels

Postulates

Quantization of **Angular momentum**

$$mvr = \frac{nh}{2\pi}$$

$n = 1, 2, 3...$

Angular momentum of the electron in these orbits is always an integral multiple of $\frac{h}{2\pi}$

h	Planck's constant
m	Mass of electron
v	Velocity of electron
r	Radius of orbit

Postulates

Electron can jump from lower to higher orbit by absorbing energy in the form of photon

| Energy Absorbed | = | $E_3 - E_2$ |

Electrons can jump from higher to lower orbit by releasing energy in the form of photon

| Energy Released | = | $E_2 - E_1$ |

Energy (hν) absorbed

Energy (hν) released
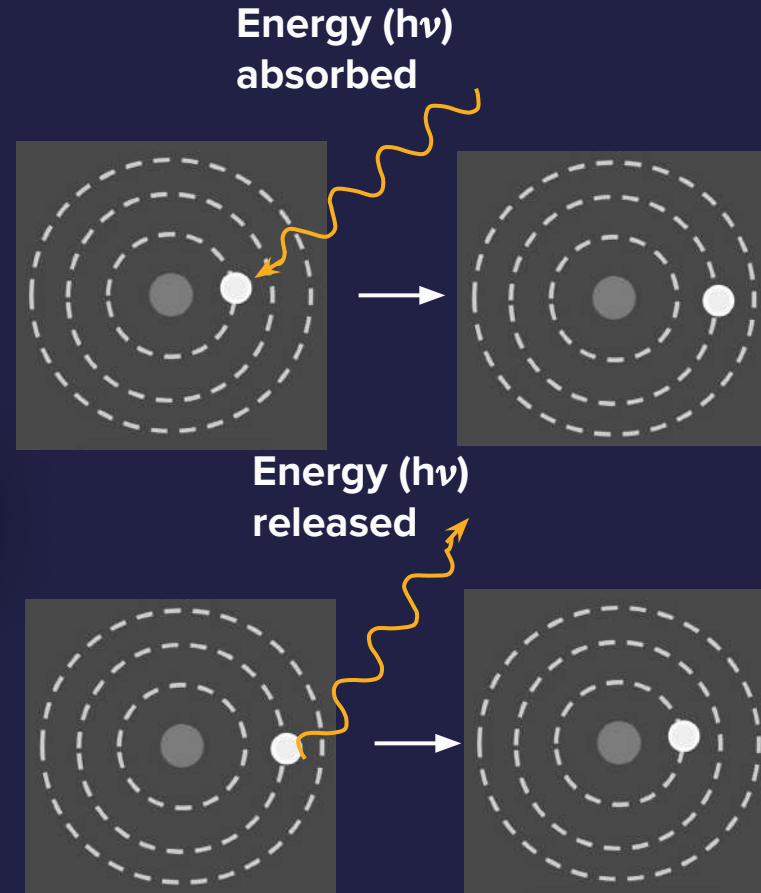
Postulates

Energy change does takes place in a discrete manner

$$\Delta E = E_{n_2} - E_{n_1}$$

n_1 — Initial energy state

n_2 — Final energy state

Bohr's Frequency Rule

Frequency (ν) of a **radiation absorbed**
or
emitted when a transition occurs

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

E_1 = Energy of **lower energy state**

E_2 = Energy of **higher energy state**

Bohr's Atomic Theory

Applicable only for **single electron species** like H, He$^+$, Li^{2+}, Be^{3+}

Mathematical Analysis

Calculating

Radius of Bohr orbit

Velocity of an electron in Bohr orbit

Time period of an electron in Bohr orbit

Frequency of an electron in Bohr orbit

Energy of an electron in Bohr orbit
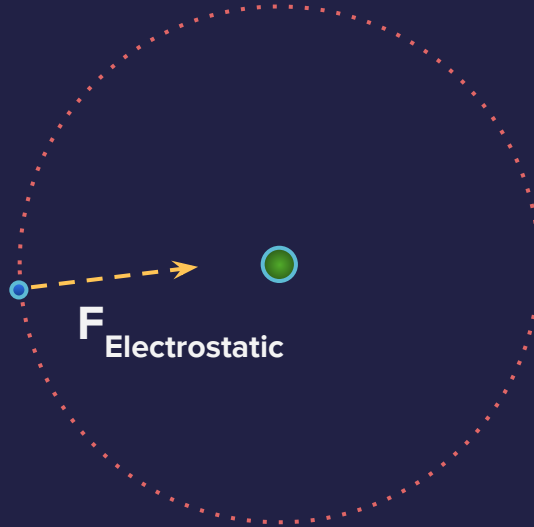
Calculating the radius of Bohr orbit

Equating both the forces,

$$F_{Centripetal} = F_{Electrostatic}$$

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

On rearranging,

$$v^2 = \frac{KZe^2}{mr}$$

i

Calculating the radius of Bohr orbit

According to Bohr's Postulates,

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi rm}$$

$$v^2 = \frac{n^2h^2}{4\pi^2r^2m^2} \quad \text{ii}$$

On comparing equation (i) and (ii),

$$\frac{KZe^2}{mr} = \frac{n^2h^2}{4\pi^2r^2m^2}$$

$$r = \frac{n^2h^2}{4\pi^2mKZe^2}$$

Calculating the radius of Bohr orbit

Putting the value of constants,

$$r_n = 0.529 \frac{n^2}{Z} \text{ Å}$$

$$r_n \propto n^2$$

r_n = Radius of nth Bohr orbit

n = Energy level

$$r_n \propto \frac{1}{Z}$$

Z = Atomic number

Calculation of velocity of an electron in Bohr orbit

Angular momentum of the electron revolving in the nth orbit

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi \, r \, m}$$ **i**

$$r = \frac{n^2h^2}{4\pi^2mKZe^2}$$ **ii**

Calculation of velocity of an electron in Bohr orbit

Putting equation (ii) in equation (i),

$$v = \frac{nh}{2\pi m} \times \frac{4\pi^2 mKZe^2}{n^2 h^2}$$

$$v = \frac{2\pi KZe^2}{nh}$$

$$v_n = 2.18 \times 10^6 \ \frac{Z}{n} \ ms^{-1}$$

v_n = Velocity of the electron in n^{th} Bohr orbit

Relation between v_n, n and Z

$$v_n \propto Z$$

$$v_n \propto \frac{1}{n}$$

Time period of Revolution (T)

Time period of revolution of an electron in its orbit

$$T = \frac{\text{Circumference}}{\text{Velocity}} = \frac{2\pi r_n}{v_n}$$

$$T = 1.5 \times 10^{-16} \, \frac{n^3}{Z^2} \, s$$

$$T \propto \frac{n^3}{Z^2}$$

Frequency of Revolution (f)

Frequency of revolution of an electron in its orbit

$$f = \frac{1}{T} = \frac{v_n}{2\pi r_n}$$

Putting the value of constants, r_n & v_n

$$f = 6.6 \times 10^{15} \frac{Z^2}{n^3} \text{ Hz}$$

$$f \propto \frac{Z^2}{n^3}$$

Calculation of Energy of an electron

Total energy (T.E.) of an electron revolving in a particular orbit

$$T.E. = K.E. + P.E.$$

K.E.	Kinetic energy
P.E.	Potential energy

$$K.E. = \frac{1}{2}mv^2$$

$$P.E. = -\frac{KZe^2}{r}$$

$$T.E. = \frac{mv^2}{2} + -\frac{KZe^2}{r}$$

Calculation of Energy of an electron

Centripetal force = **Electrostatic force**

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$$K.E. = \frac{mv^2}{2} = \frac{KZe^2}{2r}$$

Calculation of Energy of an electron

$$T.E. = K.E. + P.E.$$

$$T.E. = \frac{KZe^2}{2r} + -\frac{KZe^2}{r} = -\frac{KZe^2}{2r}$$

$$T.E. = -K.E. = \frac{1}{2}P.E.$$

Calculation of Energy of an electron

$$\text{T.E.} \ = \ -\frac{KZe^2}{2r}$$

Substituting the value of 'r' in the equation of T.E.

$$\text{T.E.} \ = \ -\frac{KZe^2}{2} \ \times \ \frac{4\pi^2 Ze^2 mK}{n^2 h^2} \ = \ -\frac{2\pi^2 Z^2 e^4 mK^2}{n^2 h^2}$$

$$\text{T.E.} \ = \ -\frac{2\pi^2 me^4 K^2}{h^2} \frac{Z^2}{n^2}$$

Calculation of Energy of an electron

Putting the value of constants we get:

$$\text{T.E.} = E_n = -13.6 \left[\frac{Z^2}{n^2}\right] \text{eV/atom}$$

Negative sign of T.E. shows attraction between electrons & nucleus.

Electron in an atom is more stable than a free electron

Energy of an electron

$$E_n \propto -\frac{Z^2}{n^2}$$

$Z \uparrow$ $E_n \downarrow$

$n \uparrow$ $E_n \uparrow$

Energy of an electron

$$E_K < E_L < E_M < E_N \quad \cdots\cdots\rightarrow \quad E_\infty = 0$$
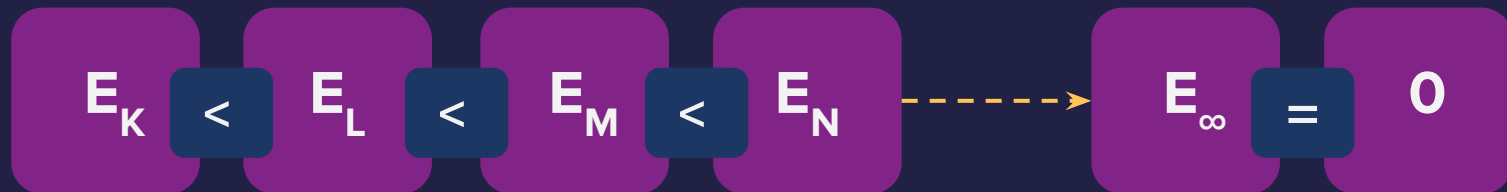
| Distance of electron from the nucleus | ↑ |

| Energy | ↑ |

Energy of an electron

$$E_n = -13.6 \, \frac{Z^2}{n^2} \text{ eV/atom}$$

$$E_n = -2.18 \times 10^{-11} \, \frac{Z^2}{n^2} \text{ erg/atom}$$

$$E_n = -313.6 \, \frac{Z^2}{n^2} \text{ kcal/mol}$$

$$E_n = -2.18 \times 10^{-11} \, \frac{Z^2}{n^2} \text{ erg/atom}$$

$$E_n = -313.6 \, \frac{Z^2}{n^2} \text{ kcal/mol}$$

Energy of an electron

At

$$n = \infty$$

$$T.E. = 0$$

$$P.E. = 0$$

$$K.E. = 0$$

Energy Difference

$$\Delta E = E_{n_2} - E_{n_1}$$

$$\Delta E\left(\frac{eV}{atom}\right) = \left(-13.6\,\frac{Z^2}{n_2^2}\right) - \left(-13.6\,\frac{Z^2}{n_1^2}\right)$$

$$\Delta E = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\frac{eV}{atom}$$

$$\Delta E = 2.18 \times 10^{-18}\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\frac{J}{atom}$$

Energy level diagram
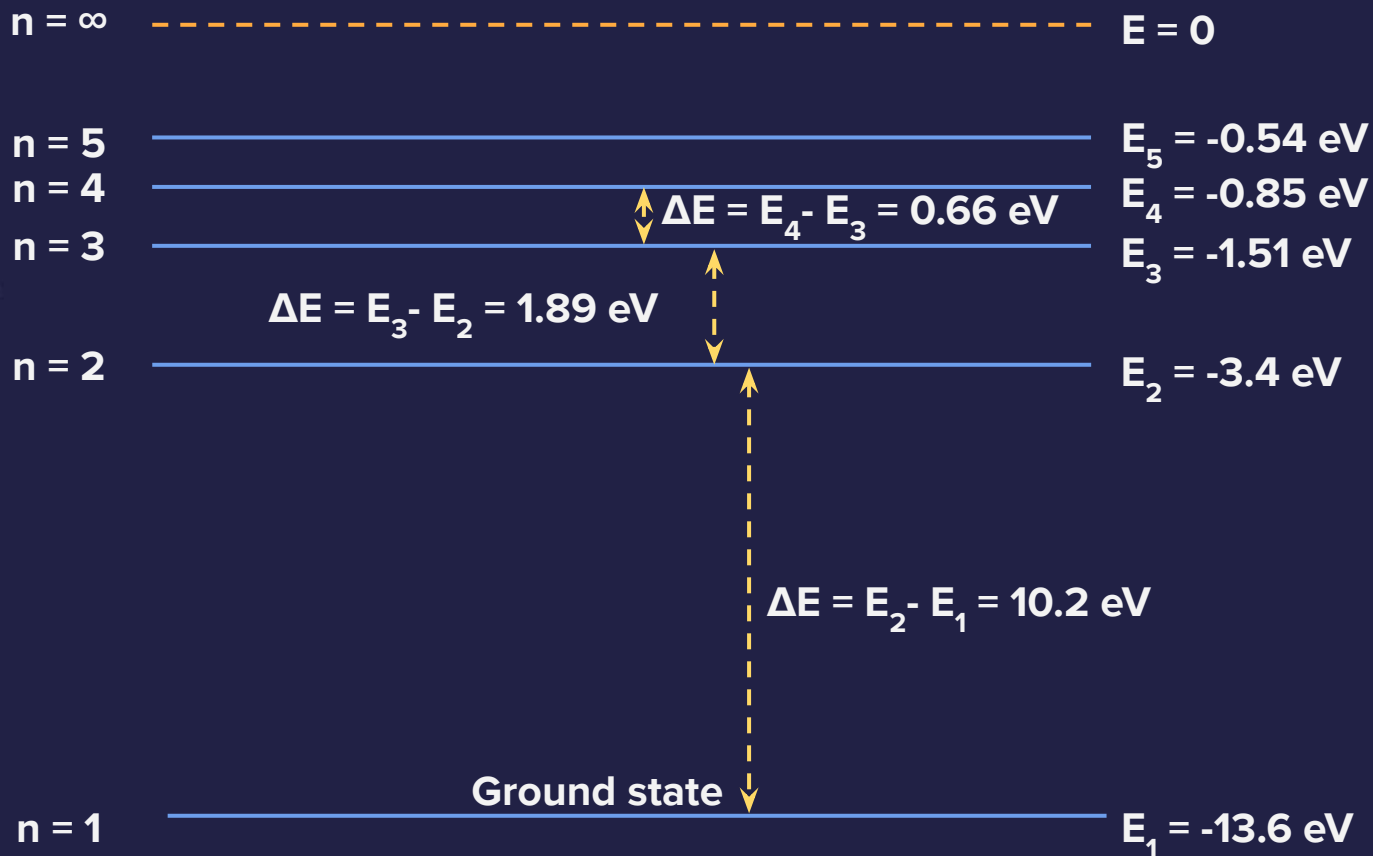
E = 0

n = ∞ -

n = 5 ─────────────────────

n = 4 ─────────────────────

n = 3 ─────────────────────

Higher energy states

n = 2 ─────────────────────

Ground state

n = 1 ─────────────────────

Energy level diagram

$n = \infty$ — — — — — — — — — — — — — — — $E = 0$

$n = 5$ ———————————————— $E_5 = -0.54$ eV

$n = 4$ ———————————————— $E_4 = -0.85$ eV

$\Delta E = E_4 - E_3 = 0.66$ eV

$n = 3$ ———————————————— $E_3 = -1.51$ eV

$\Delta E = E_3 - E_2 = 1.89$ eV

$n = 2$ ———————————————— $E_2 = -3.4$ eV

$\Delta E = E_2 - E_1 = 10.2$ eV

Ground state

$n = 1$ ———————————————— $E_1 = -13.6$ eV

Definition For Single Electron Systems

Ground state (G.S.)

Lowest energy state of any atom or ion   n = 1

G.S. energy of H-atom   -13.6 eV

G.S. energy of He^+ ion   -54.4 eV

Excited state

States of atom or ion other than the ground state — $n \neq 1$

$n = 2$ — **First excited state**

$n = 3$ — **Second excited state**

$n = m + 1$ — **m^{th} excited state**

Ionisation energy (I.E.)

Minimum energy required to remove an electron

from
n = 1 to n = ∞

$$\Delta E = 13.6 \, Z^2 \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right) \frac{eV}{atom}$$

Putting $n_2 = \infty$ & $n_1 = 1$

$$\Delta E = 13.6 \, Z^2 \frac{eV}{atom}$$

Ionisation energy (I.E.)

$$I.E._H = 13.6 \text{ eV}$$

$$I.E._{He^+} = 54.4 \text{ eV}$$

$$I.E._{Li^{2+}} = 122.4 \text{ eV}$$

Ionisation potential (I.P.)

Potential difference through which a free electron must be accelerated from rest

such that its **K.E. = I.E.**

I.P. $=$ 13.6 Z^2 V

$I.P._H$ $=$ 13.6 V

$I.P._{He^+}$ $=$ 54.4 V
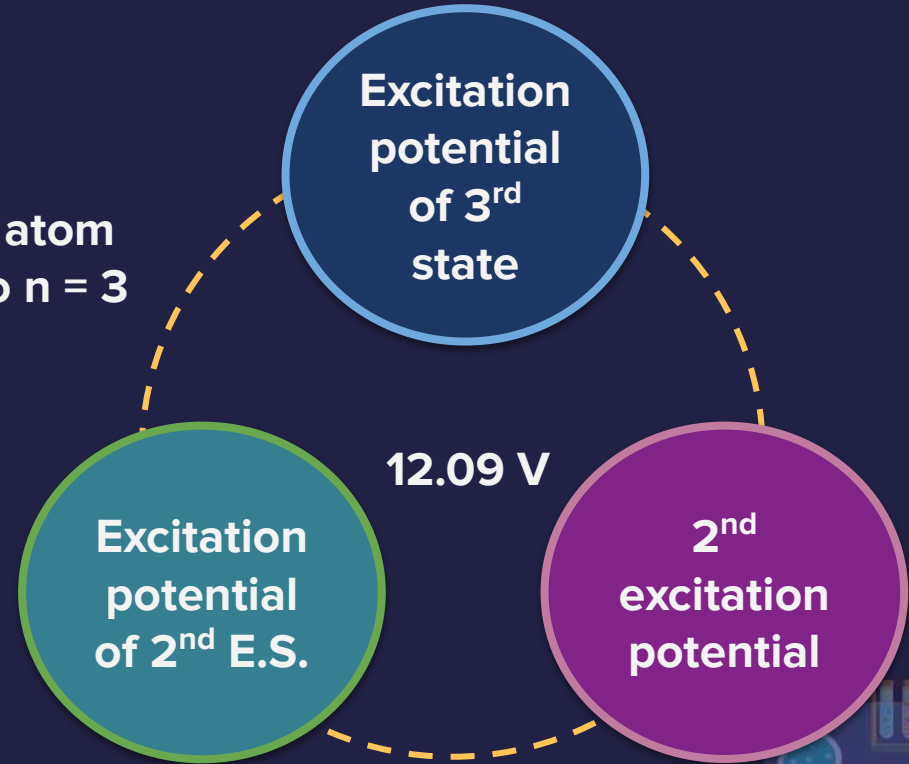
Binding or Separation energy

Energy required to
move an electron from any state to
$n = \infty$

B.E.~Ground state~ **=** I.E.~Atom or Ion~

Summary

Ground State	$n = 1$
Excited State	$n > 1$
Ionization Energy	From $n = 1$ to $n = \infty$
Binding Energy	From any state to $n = \infty$
Excitation Energy	From $n = 1$ to any other state

Spectroscopy

Spectroscopy Branch of science that deals with the study of spectra

Spectrograph/ Spectroscope Instrument used to separate radiation of different wavelengths

Spectrogram Spectrum of the given radiation

Emission Spectrum

Spectrum of radiation emitted by a substance

Emission Spectrum

Continuous

Discrete
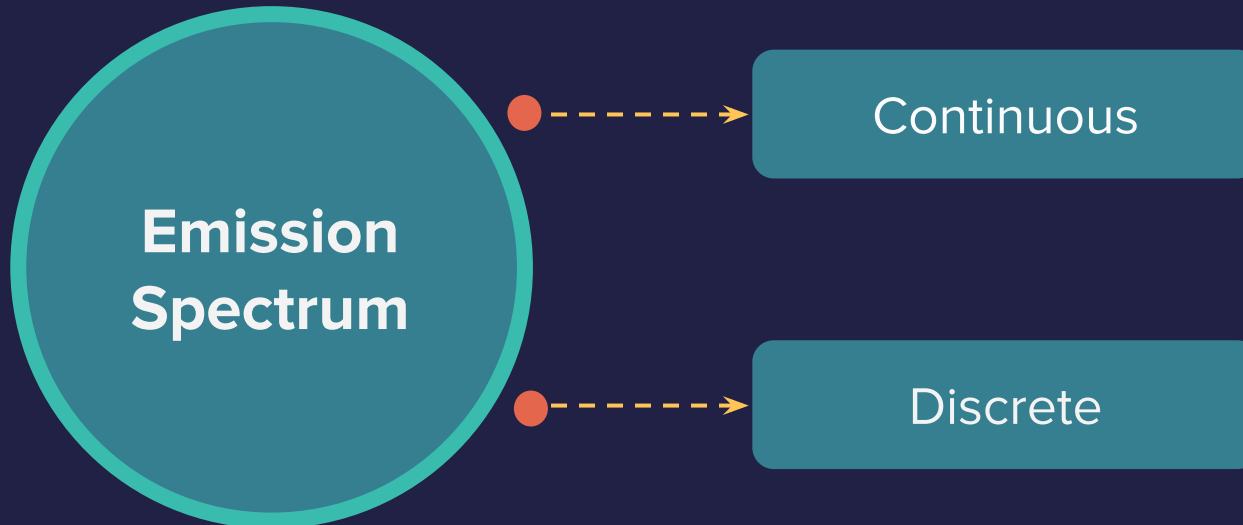
Continuous Spectrum

Continuous distribution of colours (VIBGYOR)
such that each colour merges into the next one

Continuous Spectrum

Screen

Prism

Light Source

Line Spectrum

Ordered arrangement of lines of a particular wavelength separated by dark space
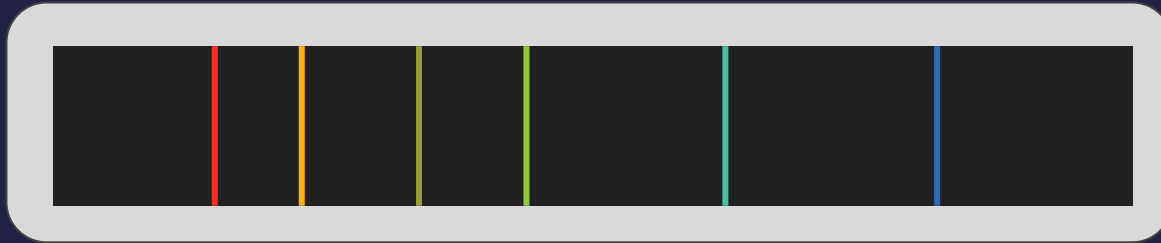
Line spectrum

Emission Spectra

Light emitted by gas

Gas in Excited State in Discharge Tube at low Pressure

Prism

Screen
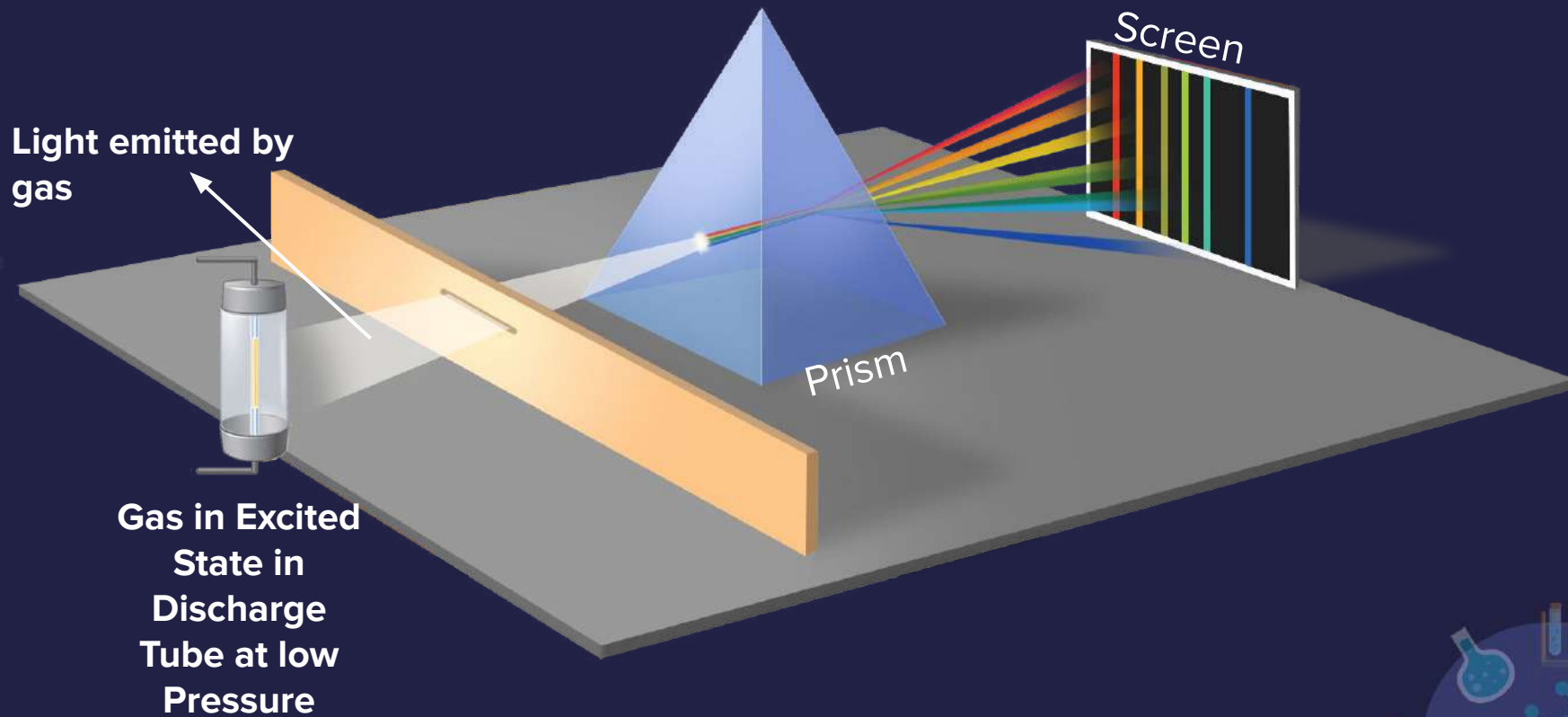
Application of Line Spectrum

Each element has a unique
line spectrum

Identification of unknown atoms

Band Spectrum

Continuous bands separated by some dark space

Molecular spectrum

Classification : Based on Origin

Spectrum

Emission

Absorption

Absorption Spectrum
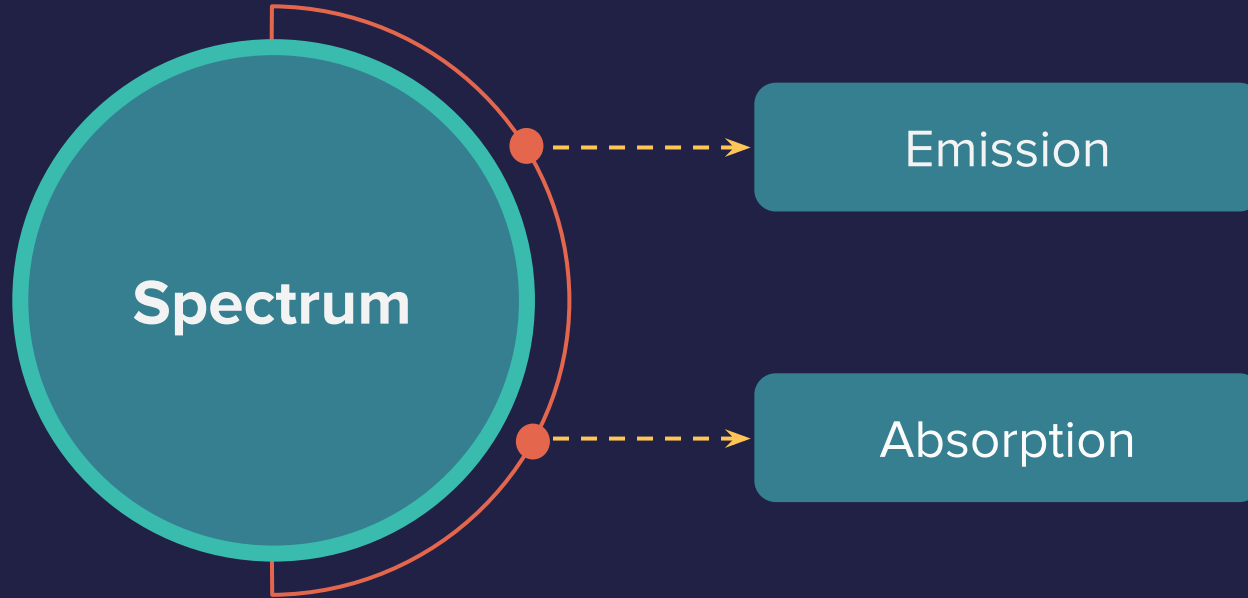
Screen

Prism
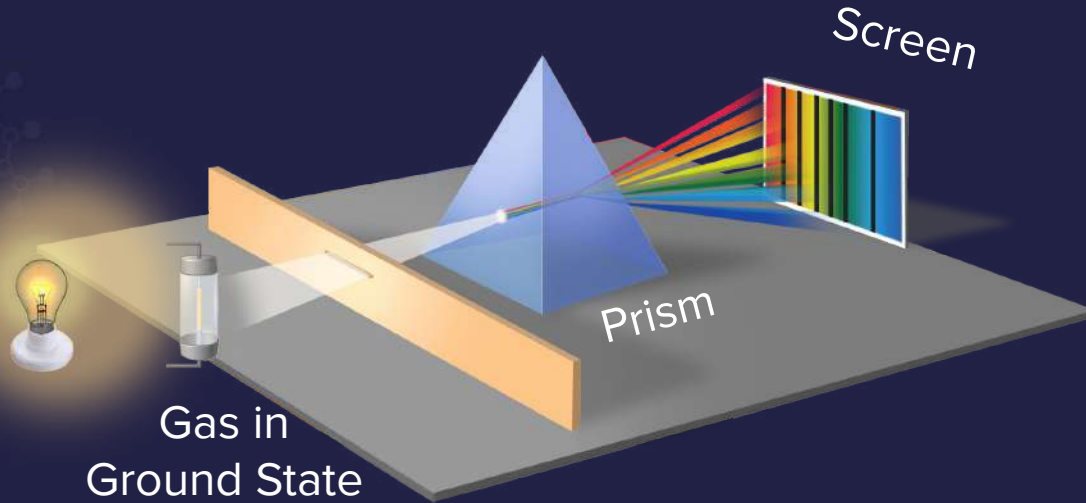
Gas in Ground State

The gas in ground state absorbs radiation of particular wavelengths and rediations of remaing wavelength passes through the gas which scatter through a prism and appears as bright lines on the ecreen. The dark lines shows the missing wavelengths which are already absorbed by the gas and no more available to pass through the prism.

Absorption Spectrum

Some dark lines in the
continuous spectrum

Represent absorbed radiations

Line Spectrum of Hydrogen

Emission Spectral Lines/
De-Excitation Series

Due to de-excitation of electron
from higher to lower orbit
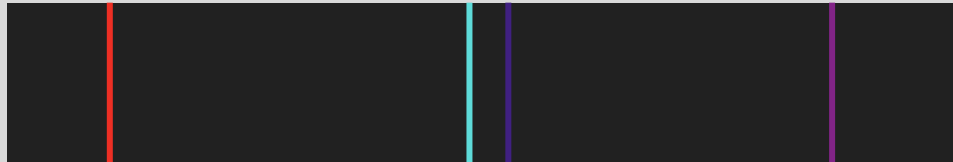
Energy Level Diagram for H atom
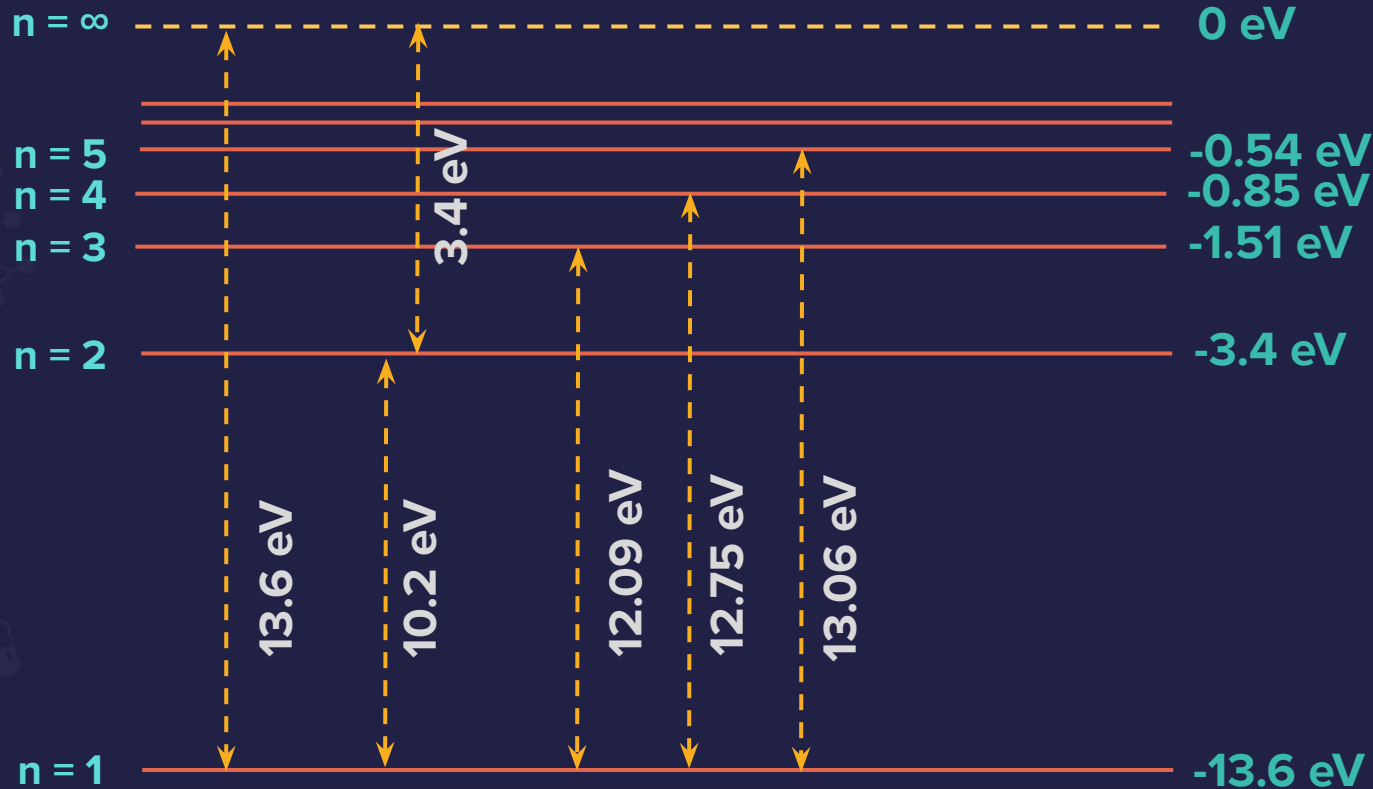
Rydberg's Formula

Electron makes transition from n_2 to n_1

$$\Delta E = \frac{hc}{\lambda} = 2.18 \times 10^{-18} \, Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \frac{J}{atom}$$

$$\frac{1}{\lambda} = \frac{2.18 \times 10^{-18} \, Z^2}{hc} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) m^{-1}$$

Rydberg's Formula

$$\frac{1}{\lambda} = 1.09678 \times 10^7 \times Z^2 \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right) m^{-1}$$

$$\frac{1}{\lambda} = 109678 \times Z^2 \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right) cm^{-1}$$

n_1 1, 2, 3, ...

n_2 $n_1 + 1$, $n_1 + 2$, ...

R_H Rydberg constant

$$R_H = 1.09678 \times 10^7 \ m^{-1} \approx 1.1 \times 10^7 \ m^{-1}$$

Rydberg's Formula

For any atom

$$\frac{1}{\lambda} = R_H Z^2 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

n_1	1, 2, 3, ...

n_2	$n_1 + 1, n_1 + 2, ...$

$$\lambda = \frac{912}{Z^2 \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right)} \, \text{Å}$$

Rydberg's Formula

For H atom

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right)$$

n_1	1, 2, 3, ...
n_2	$n_1 + 1$, $n_1 + 2$, ...

Spectral series of H atom

Lyman

Balmer

Paschen

Brackett

Pfund

Humphrey

Lyman Series

For an electron present in H atom

1st spectral series. Found in UV region by Lyman

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{1^2} - \frac{1}{n_2^{\,2}} \right)$$

$n_1 = 1$ (Final state)

$n_2 = 2, 3, 4 \ldots$
(Initial states, $n_2 > 1$)

Lyman Series

1st line of Lyman series (α line) — 2 → 1

2nd line of Lyman series (β line) — 3 → 1

Last line of Lyman series (Series limit) — ∞ → 1

$n_2 = \infty$

Wavelength of last line $= \dfrac{n_1^2}{R_H}$

$\lambda_{Lyman} = \dfrac{1}{R_H}$

Lyman Series

$$10.2 \text{ eV} \leq (\Delta E)_{Lyman} \leq 13.6 \text{ eV}$$

$$\frac{12400}{13.6} \text{ Å} \leq \lambda_{Lyman} \leq \frac{12400}{10.2} \text{ Å}$$

Lyman Series

$$\text{Longest wavelength} = \lambda_{longest} \text{ or } \lambda_{max} = \frac{12400}{\Delta E_{min}} \text{ Å}$$

$$\text{Shortest wavelength} = \lambda_{shortest} \text{ or } \lambda_{min} = \frac{12400}{\Delta E_{max}} \text{ Å}$$

Where, **E in eV**

Lyman Series

1st spectral line → λ_{max}

Last spectral line → λ_{min}

Series limit
Limiting/last line of any spectral series

Since $n_2 = \infty$

$$\bar{\nu}_{\text{last line}} = R_H$$
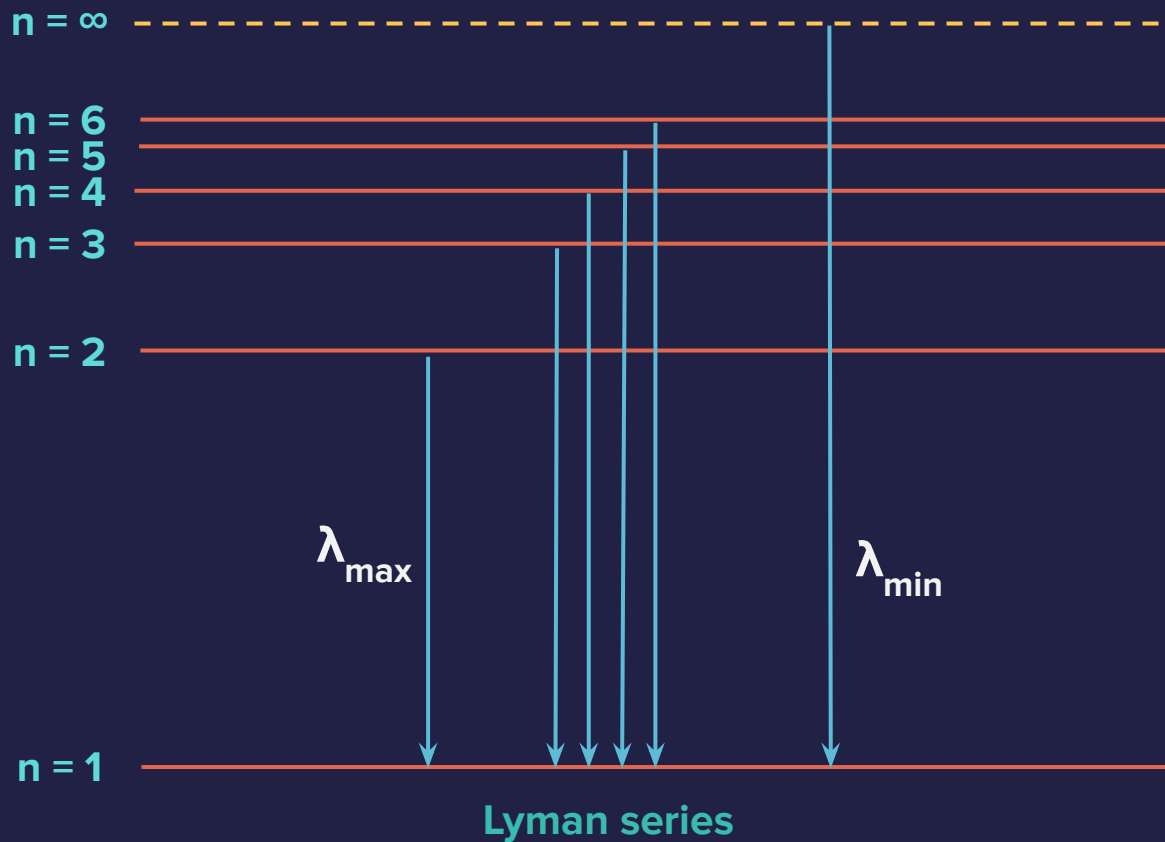
Balmer Series

2nd spectral series

Found in visible region by Balmer

For H atom

Only first 4 lines belongs to visible region

Rest belongs to UV region

Balmer Series

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right)$$

$n_1 = 2$ (Final state)

$n_2 = 3, 4, 5 \ldots$
(Initial states, $n_2 > 2$)

Balmer Series

n = ∞

n = 6
n = 5
n = 4
n = 3

λ_{max}

λ_{min}

n = 2
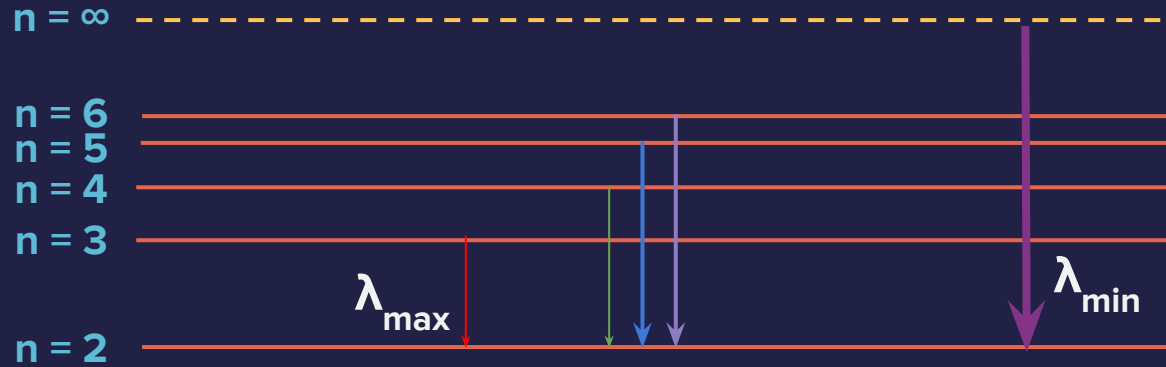
Balmer series

n = 1

Paschen Series

3rd spectral series; Found in IR region by Paschen

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{3^2} - \frac{1}{n_2^2} \right)$$

$n_1 = 3$ (Final state)

$n_2 = 4, 5, 6 \ldots$
(Initial states, $n_2 > 3$)
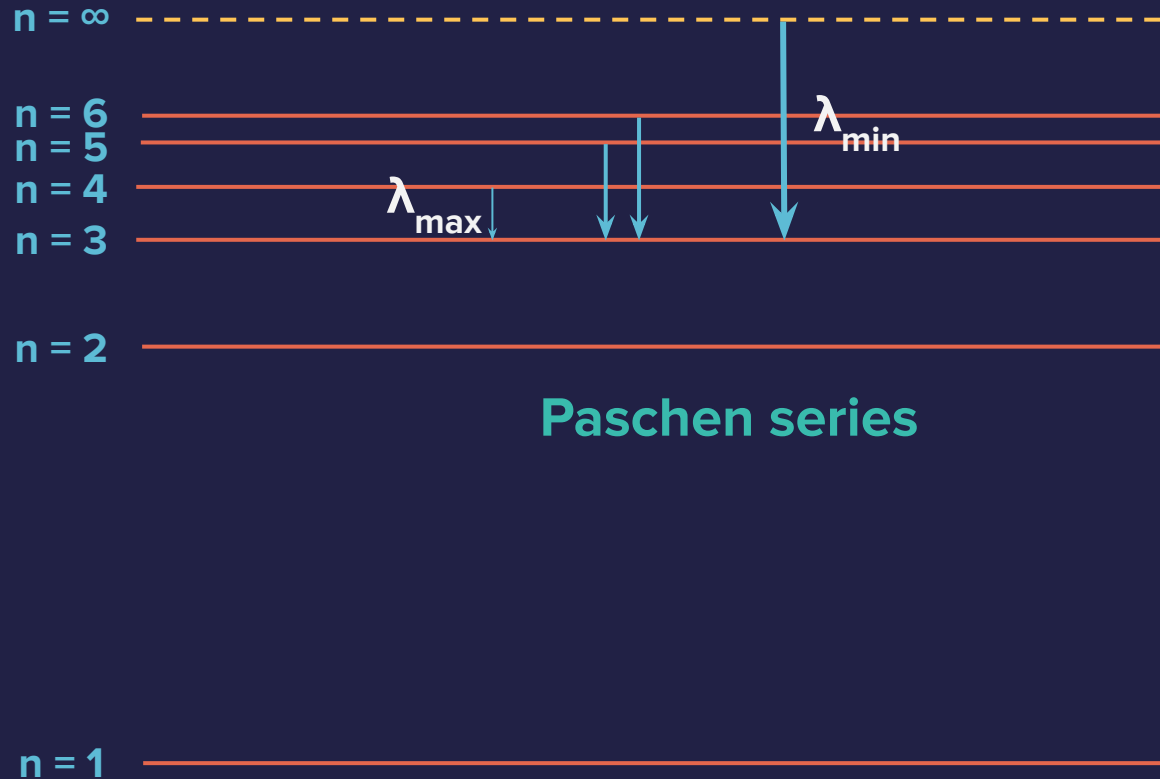
Brackett Series

4th spectral series; Found in IR region by Brackett

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{4^2} - \frac{1}{n_2^2} \right)$$

$n_1 = 4$ (Final state)

$n_2 = 5, 6, 7 \dots$
(Initial states, $n_2 > 4$)
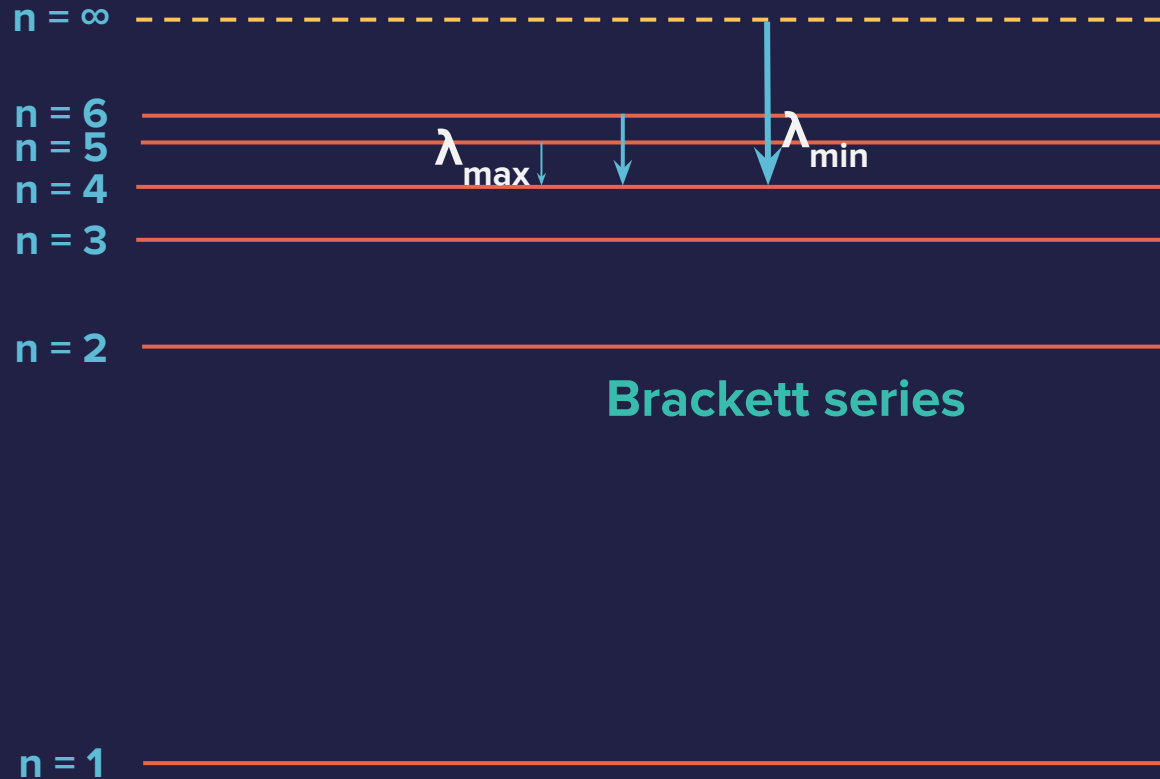
Pfund Series

5th spectral series; Found in IR region by Pfund

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{5^2} - \frac{1}{n_2^2} \right)$$

$n_1 = 5$ (Final state)

$n_2 = 6, 7, 8 \dots$
(Initial states, $n_2 > 5$)

Pfund Series

n = ∞

n = 6
n = 5
n = 4
n = 3

n = 2

λ_{max}

λ_{min}

Pfund series

n = 1
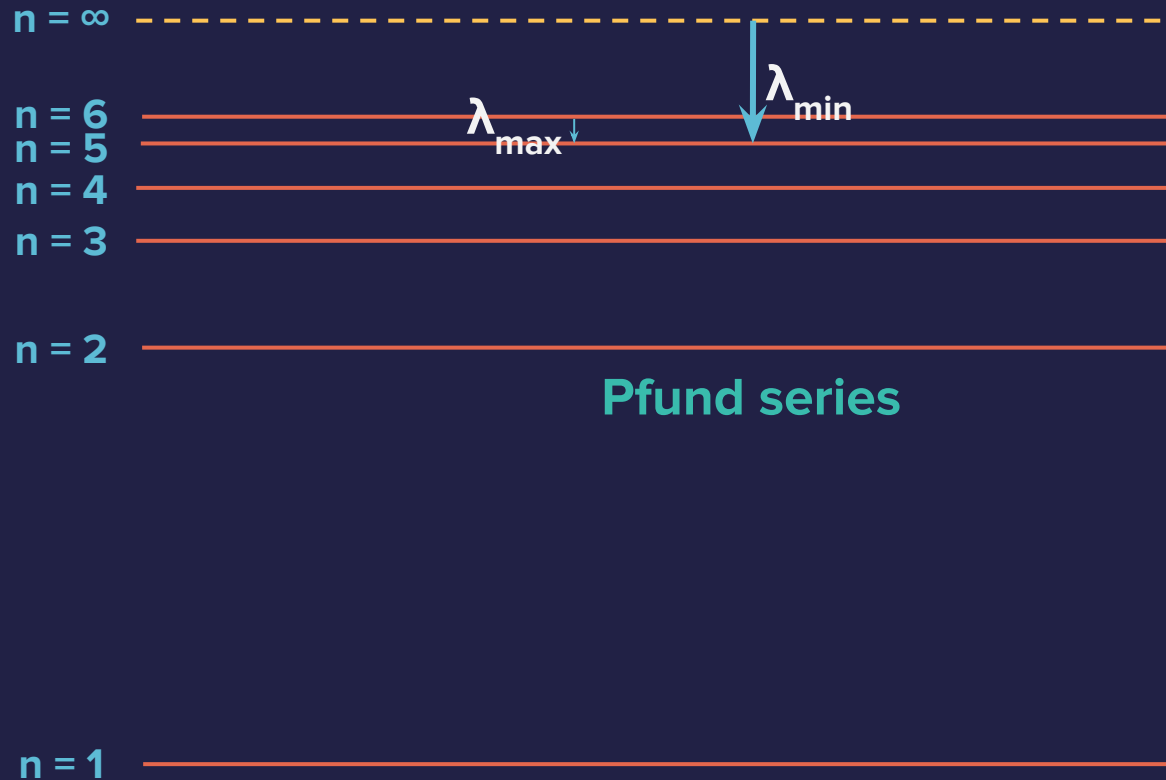
Humphrey Series

6th spectral series; Found in IR region by Humphrey

$$\frac{1}{\lambda} = R_H \times \left(\frac{1}{6^2} - \frac{1}{n_2{}^2} \right)$$

n_1 = 6 (Final state)

n_2 = 7, 8, 9 ...
(Initial states, n_2 > 6)
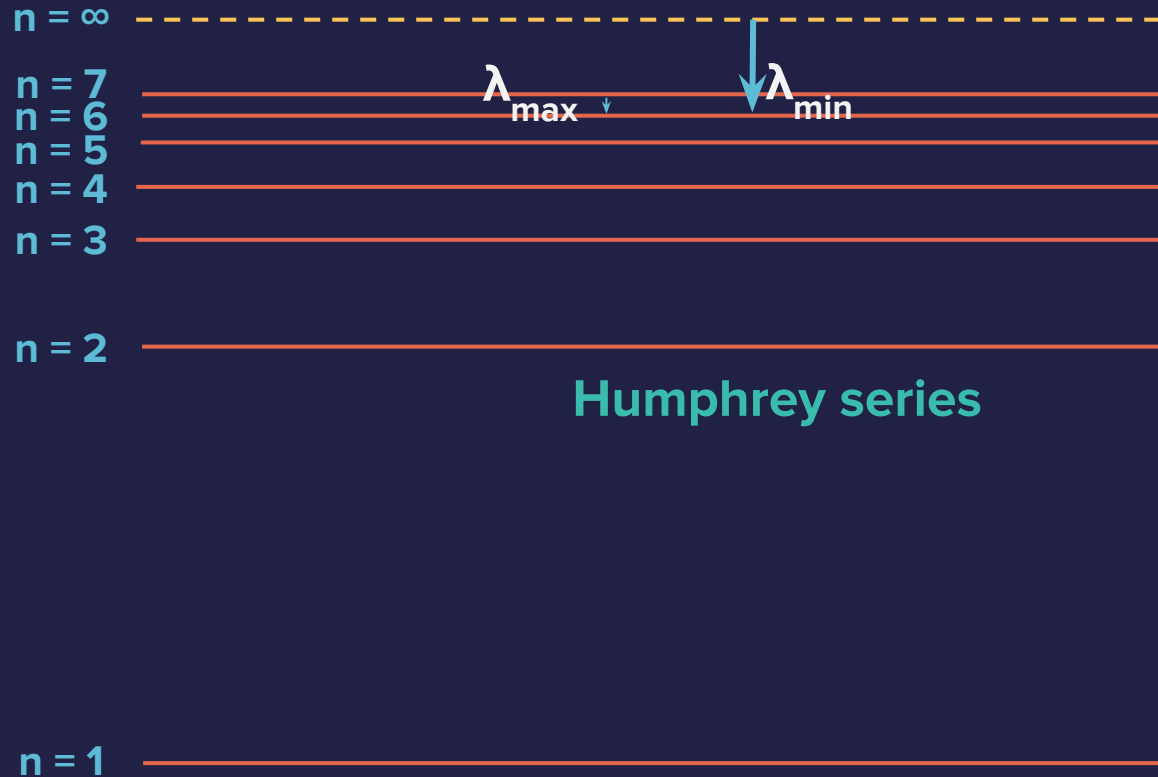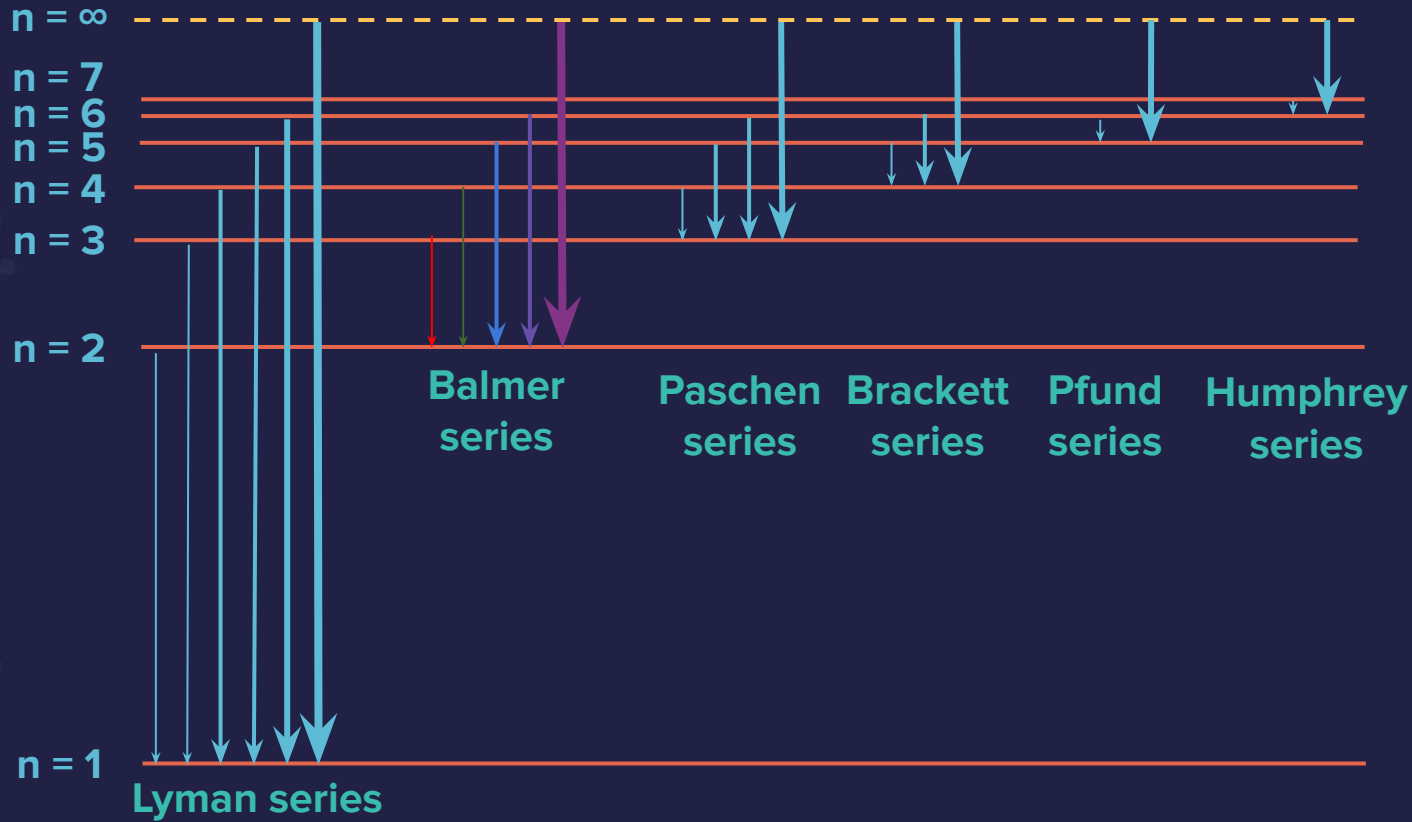
Maximum Number of Spectral Lines

Maximum number of lines

=

Maximum number of different types of photons

Maximum Lines for the transition from 4 to 1

4 to 3
4 to 2   3 to 2
4 to 1   3 to 1   2 to 1

n = 4

n = 3

n = 2

n = 1
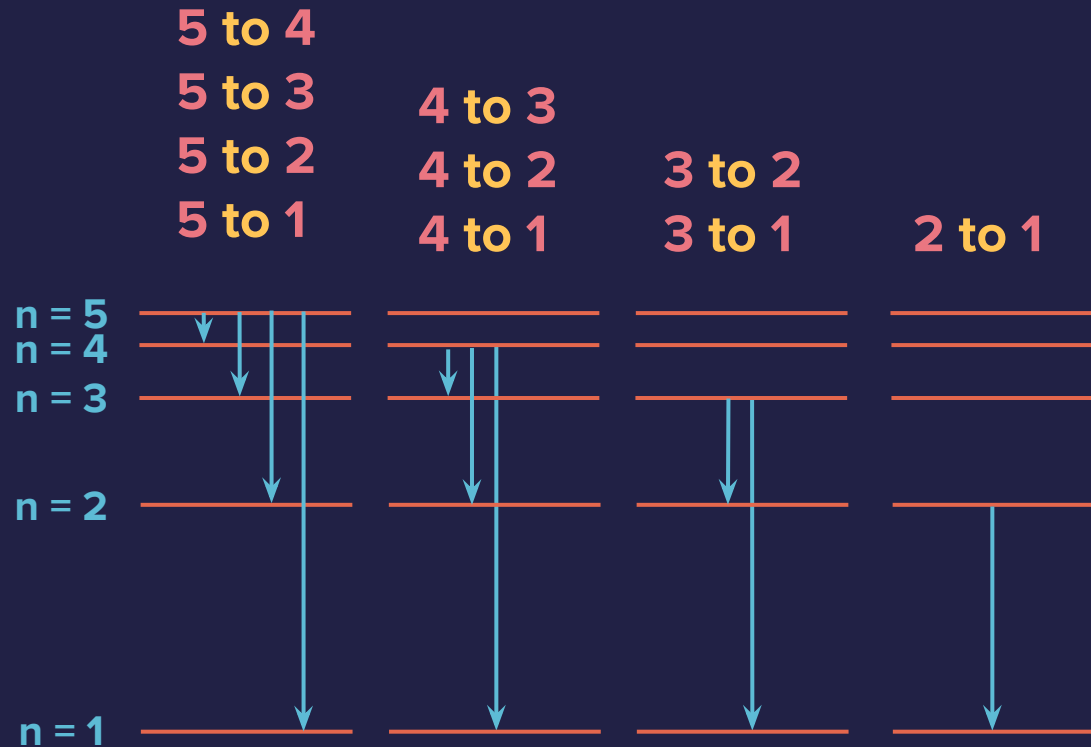
Maximum Number of Lines

$$\text{Spectral Lines} = \frac{(n_H - n_L + 1)(n_H - n_L)}{2} = \frac{(\Delta n + 1)(\Delta n)}{2}$$

Δn	$n_H - n_L$
n_H	Higher energy level
n_L	Lower energy level

Maximum Number of Lines

For transition upto n = 1 or n = n_{Series}

In Lyman series ($n_{Higher} > 1$) = n_{Higher} - 1

In Balmer series ($n_{Higher} > 2$) = n_{Higher} - 2

In Paschen series ($n_{Higher} > 3$) = n_{Higher} - 3

For a particular series

Spectral lines = n_{Higher} - n_{Series}

Maximum Number of Lines

In Brackett series $(n_{Higher} > 4)$	=	$n_{Higher} - 4$
In Pfund series $(n_{Higher} > 5)$	=	$n_{Higher} - 5$
In Humphrey series $(n_{Higher} > 6)$	=	$n_{Higher} - 6$

Number of spectral lines in Lyman series from 4th shell

$$= n_H - 1 = 4 - 1 = 3$$

n = 4

n = 3

n = 2

n = 1

Example

4 to 3
3 to 2
2 to 1   4 to 2   3 to 1   4 to 1

n = 4

n = 3

n = 2

n = 1

1st Atom   2nd Atom   3rd Atom   4th Atom
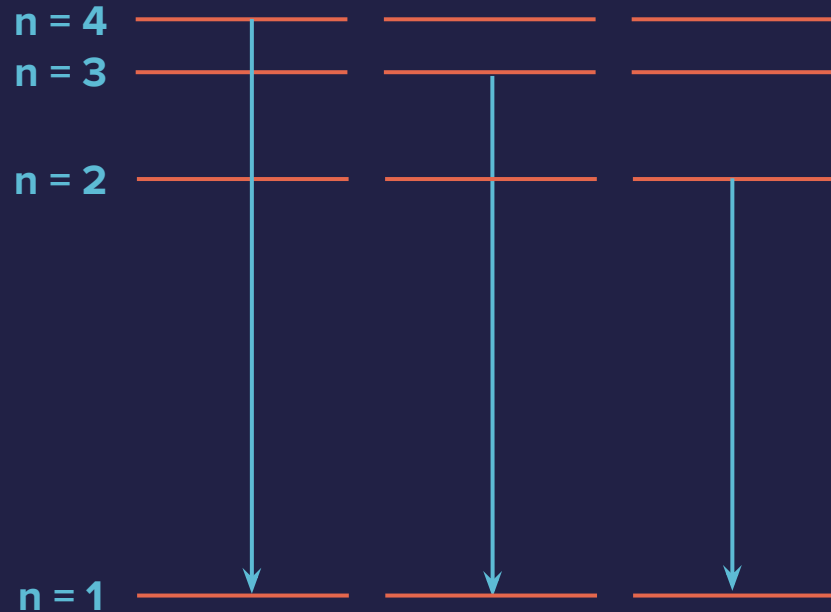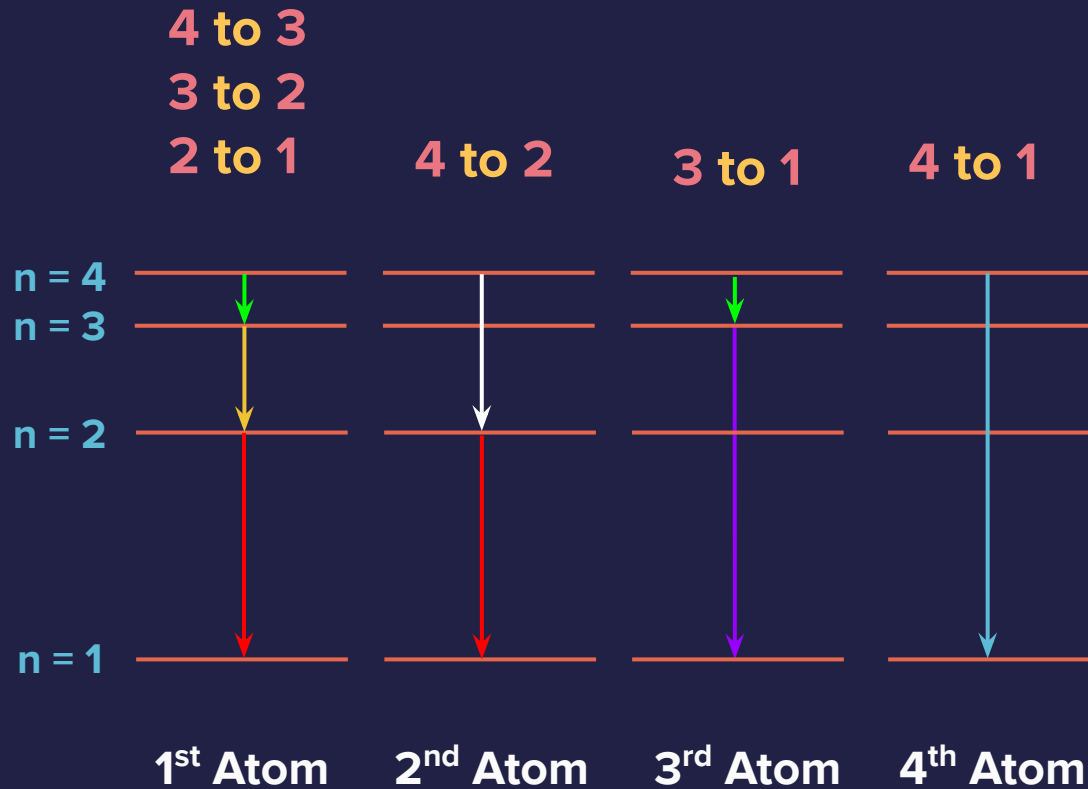
Pathway to Quantum Mechanical Model

Important Developments

Dual Nature of Matter

Heisenberg's Uncertainty Principle
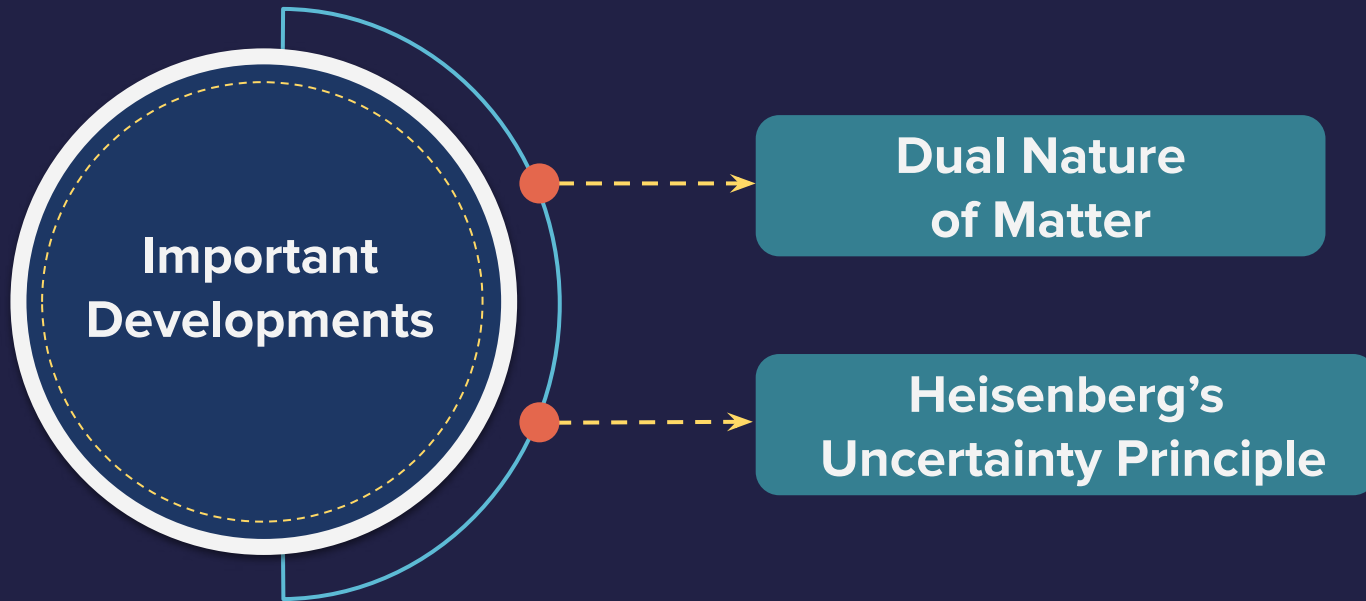
These developments shows that electron has sufficient wave character and it can not be assumed as a particle only. So, its position or trajectory can not be determined as shown by Bohr model.

Dual Nature of Matter

de Broglie proposed that **particle** has dual nature

Particle nature

Wave nature

Louis de Broglie

Einstein suggested that **light** has dual nature i.e., particle nature as well as wave nature.

de Broglie Hypothesis
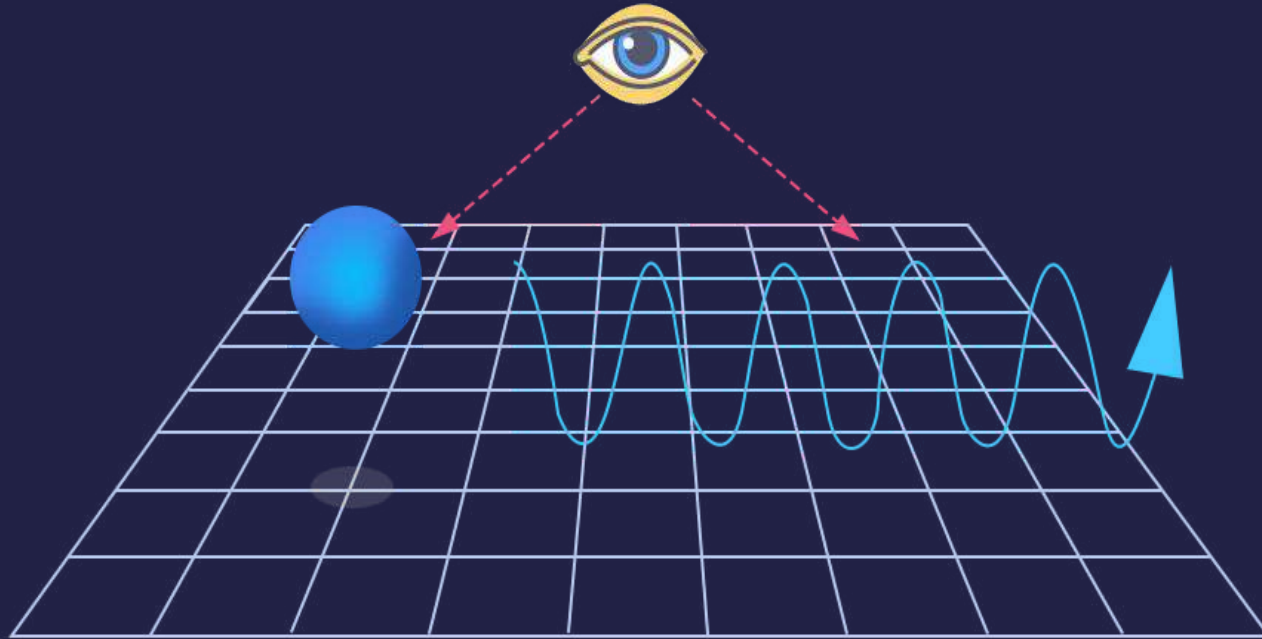
de Broglie Hypothesis

$$\lambda = \frac{h}{p}$$

Wave associated with **moving particles**

How $\lambda = \dfrac{h}{p}$?

de Broglie Hypothesis

Planck's equation

$$E = \dfrac{hc}{\lambda}$$

Einstein's Mass Energy relationship

$$E = mc^2$$

de Broglie Hypothesis

Equating both

$$\frac{hc}{\lambda} = mc^2$$

For photon

$$\lambda = \frac{h}{mc}$$

By same analogy, de Broglie proposed

For matter

$$\lambda = \frac{h}{mv}$$

de Broglie Wavelength (λ)

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

p	**Momentum of particle**
v	**Velocity of particle**
m	**Mass of particle**
h	**Planck's constant**

de Broglie Wavelength (λ)

$$\lambda = \frac{h}{p}$$
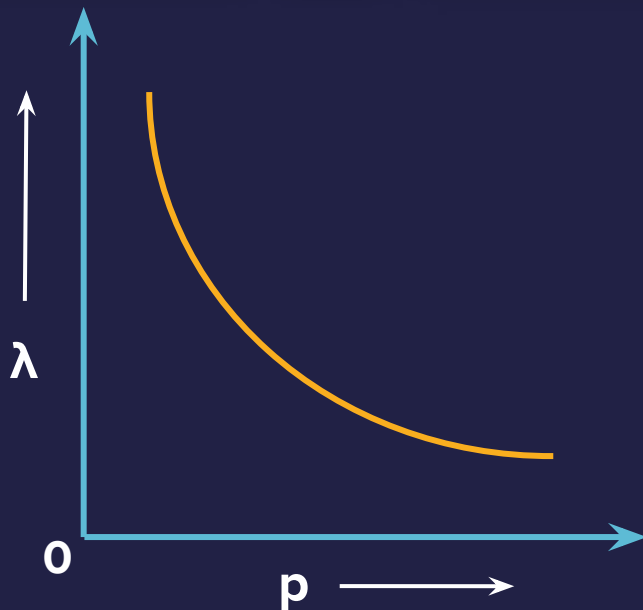
Relativistic Mass

$$m = \frac{m_o}{\sqrt{1 - \left(\dfrac{v}{c}\right)^2}}$$

v — Velocity w.r.t. the observer

c — Velocity of light

m — Dynamic mass

m_o — Rest mass

Relativistic Mass

$$m_0 = m \times \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

If v = c

$$m_0 = 0$$

Rest mass of photon is zero

Davisson and Germer's Experiment

Experimental verification of **de Broglie's prediction**

It was observed that an **electron beam** undergoes **diffraction**

Wavelength of a ball & an electron!

de Broglie wavelength: $\lambda = \dfrac{h}{mv}$

Cricket ball

m = **150 g**
v = **25 m s^{-1}**

$$\lambda = \frac{6.626 \times 10^{-34}}{(150 \times 10^{-3}) \times 25}$$

λ = **1.767 × 10^{-34} m**

λ is **insignificant.**

Electron

m = **9.1 × 10^{-31} kg**
v = **2 × 10^3 m s^{-1}**

$$\lambda = \frac{6.626 \times 10^{-34}}{(9.1 \times 10^{-31}) \times 2 \times 10^3}$$

λ = 0.364 × 10^{-6} m
= **364 nm**

λ is **significant.**
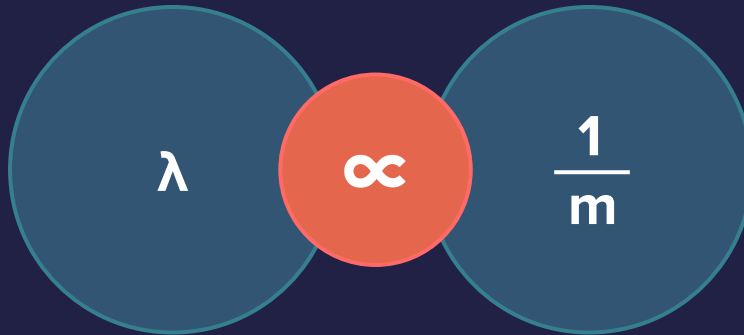
de Broglie's Equation & Kinetic Energy

$$K.E. = \frac{1}{2}mv^2$$

Multiplying both sides by m & rearranging

$$m^2v^2 = 2\,K.E. \times m$$

$$mv = \sqrt{2\,K.E. \times m}$$

de Broglie's Equation & Kinetic Energy

de Broglie equation

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Since

$$mv = \sqrt{2\ \text{K.E.}\ \text{x}\ m}$$

$$\lambda = \frac{h}{\sqrt{2\ \text{K.E.}\ \times\ m}}$$

de Broglie's Equation & Kinetic Energy

A charged particle accelerated from rest across a **potential difference of V**

$$|\text{K.E.}| = |qV|$$

$$\therefore \quad mv = \sqrt{2m \times \text{K.E.}}$$

$$\therefore \quad mv = \sqrt{2m \times qV}$$

de Broglie's Equation & Kinetic Energy

$$\lambda = \frac{h}{\sqrt{2m \times q\,V}}$$

λ — Wavelength (m)	**q** — Charge on a particle (C)
m — Mass of charged particle (kg)	**h** — Planck's constant (Js)

Electron as a Wave

In phase

λ

r
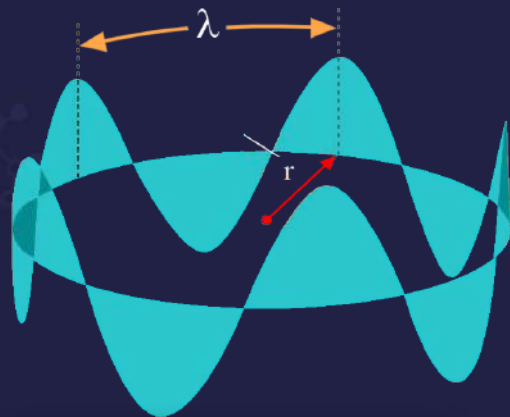
Electron exist

**Circumference,
2πr = nλ**

Out of phase

r
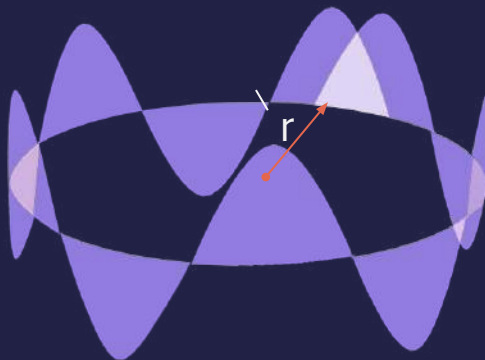
Electron don't exist

**Circumference,
2πr ≠ nλ**

n = Number of waves made in Bohr's orbit

An integral number of complete wavelengths must fit around the circumference of the orbit.

Electron as a Wave

When electrons are in phase,

$$2\pi r = \frac{nh}{mv}$$

Bohr's Postulate verified

$$mvr = \frac{nh}{2\pi}$$

n = Energy level

Heisenberg's Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot m \cdot \Delta v \geq \frac{h}{4\pi}$$

Δx Uncertainty in position	**Δp** Uncertainty in momentum
m Mass of particle	**Δv** Uncertainty in velocity

Principle of Optics

If a light (wavelength 'λ') is used to locate the position of a particle, then

Minimum error in the position measurement (Δx) = ± λ

Heisenberg's Uncertainty Principle

Since Δx $=$ λ

For accurate position $\Delta x \longrightarrow 0$ $\lambda \longrightarrow 0$

For a photon

E $=$ $\dfrac{hc}{\lambda}$

$\lambda \longrightarrow 0$ $E \longrightarrow \infty$

Heisenberg's Uncertainty Principle

High energy photon strikes particle → $\Delta p \uparrow$

Similarly

For accurate momentum → $\Delta x \uparrow$

Heisenberg's Uncertainty Principle

For an electron

$$\Delta x \,.\, m \,.\, \Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \,.\, \Delta v = \frac{6.626 \times 10^{-34} \text{ Js}}{4 \times 3.14 \times 9.1 \times 10^{-31} \text{ kg}}$$

$$\Delta x \,.\, \Delta v \approx 10^{-4} \text{ m}^2\text{s}^{-1}$$

Heisenberg's Uncertainty Principle

If $\Delta x = 10^{-8}$ m then $\Delta v = 10^4$ ms^{-1}

| Position | High accuracy | Δx is small |

| Velocity | Uncertain | Δv is large |

If $\Delta v = 10^{-8}$ ms^{-1} then $\Delta x = 10^4$ m

| Velocity | High accuracy | Δv is small |

| Position | Uncertain | Δx is large |

Conclusion:

Heisenberg's uncertainty principle is meaningless for bigger particles.
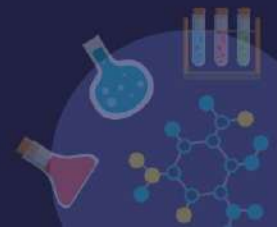
Energy - Time Variant of H.U.P.

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Multiplied and divided by Δt

$$\frac{\Delta p}{\Delta t} \cdot \Delta x \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\frac{\Delta p}{\Delta t} = \textbf{Rate of change in momentum = F}$$

Energy - Time Variant of H.U.P.

$$F \cdot \Delta x \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

ΔE   **Uncertainty in Energy**

Δt   **Uncertainty in Time**

Significance of the Uncertainty Principle

1 Not an instrumental error, rather conceptual error

2 Rules out the existence of definite paths of electrons

3 Introduced concept of probability of finding the electrons

Precise statements of **position** & **momentum** of an electron replaced with probability

Forms the basis of **Quantum Mechanical Model of atom**

Limitations of Bohr Model

1 Could not explain the line spectra of atoms containing more than one electron

2 Could not explain the presence of doublet i.e. two closely spaced lines
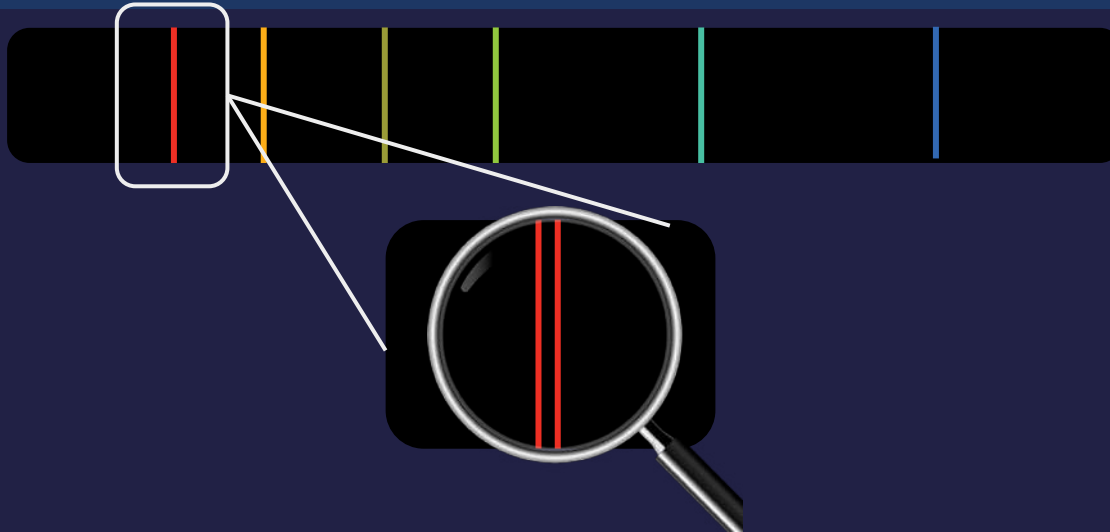
Limitations of Bohr Model

3 Unable to explain the splitting of spectral lines in the presence of magnetic field (**Zeeman effect**) and electric field (**Stark effect**).

4 No conclusion was given for the principle of quantisation of angular momentum

5 Unable to explain de Broglie's concept & Heisenberg's Uncertainty Principle

Quantum Mechanical Model

Schrodinger Wave Equation (SWE)

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0$$

- **x, y, z = Cartesian coordinates**

- **Ψ = Amplitude of the electron wave or Wave function**

- **h = Planck's constant**

- **V = Potential energy of the electron**

- **E = Total energy of the electron**

Wavefunction

SWE is solved to get values of **Ψ** and their corresponding **energies**

Ψ

A function that contains all the **dynamical information** about a system

SWE can be solved for H like species more easily in
Spherical polar coordinates
(r , θ , φ)

Spherical Coordinate System

x	=	r sinθ cosϕ
y	=	r sinθ sinϕ
z	=	r cosθ

z = r cosθ
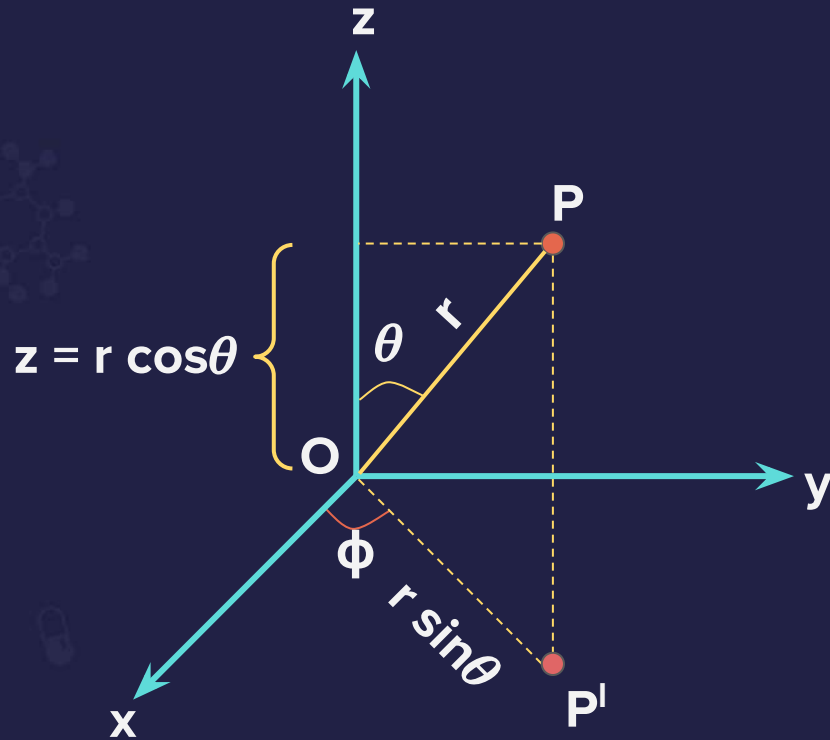
Wavefunction

When SWE is solved for **H like species**, the obtained values of Ψ could be factorized into one containing only 'r' and the other containing (θ, ϕ).

Wave function		Radial part of wave function		Angular part of wave function
$\Psi (r, \theta, \phi)$	=	$\Psi (r)$	×	$\Psi (\theta, \phi)$
		n, l		l, m$_l$

Schrodinger Wave Equation

Ψ corresponds to **atomic orbital**

Characterized by a **set of quantum numbers**

First three quantum numbers (n, l, m) were derived from Schrodinger equation. The spin quantum number added later.

n	*l*	m*l*	s
✔	✔	✔	x

What is an Orbital?

Orbital → 3D region around the nucleus → Probability of finding an electron is maximum
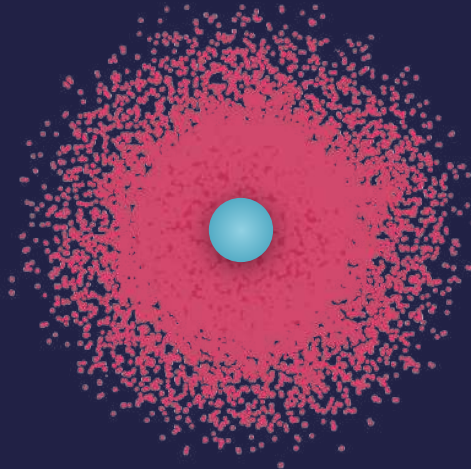
What is an Orbital?

Defines the probability of finding an electron

Does not define a definite path of electrons
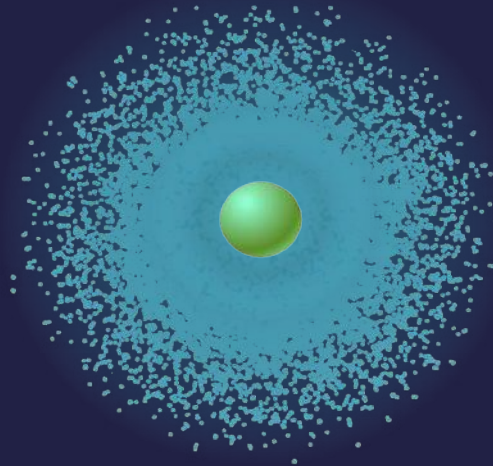
Quantum Numbers

Set of **four numbers** required to **define an electron** in an atom completely

1. Principal Quantum Number (n)

2. Azimuthal Quantum number (l)

3. Magnetic Quantum Number (m_l)

4. Spin Quantum Number (s)

Principal Quantum Number (n)

1 Designates the **shell** to which the electron belongs

2 Signifies energy level for **single electron species**

3 Accounts for the **main lines** in the atomic spectrum

Proposed by Niels Bohr

Principal Quantum Number (n)

Describes the **size** of electron wave & the **total energy** of the electron

n = 1, 2, 3...

Represented as K, L, M, N,...

Angular momentum in any shell $= \dfrac{nh}{2\pi}$

Azimuthal Quantum Number (l)

1 Designates the **subshell** to which the electron belongs

2 Energy of the orbital in **multielectron species** (both n&l)

3 Accounts for the **fine lines** in atomic spectrum

Proposed by Sommerfeld

Azimuthal Quantum Number (l)

Describes the **3-D shape** of the **orbital** or the electron cloud

Also known as

Subsidiary Quantum Number

Orbital Angular momentum Quantum Number

For a given value of Principal Quantum Number (n)

| l | = | 0 to (n - 1) |

Boundary Surface Diagram

Encloses the 3D region where probability of finding electrons is maximum

Example
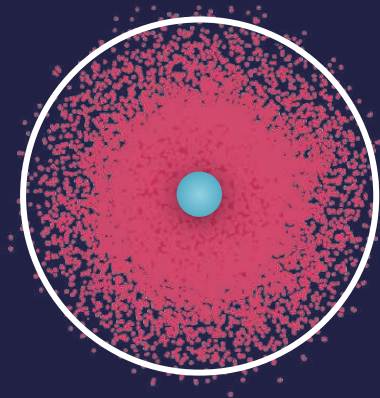
Shape : **Spherical**

Classification of Orbitals
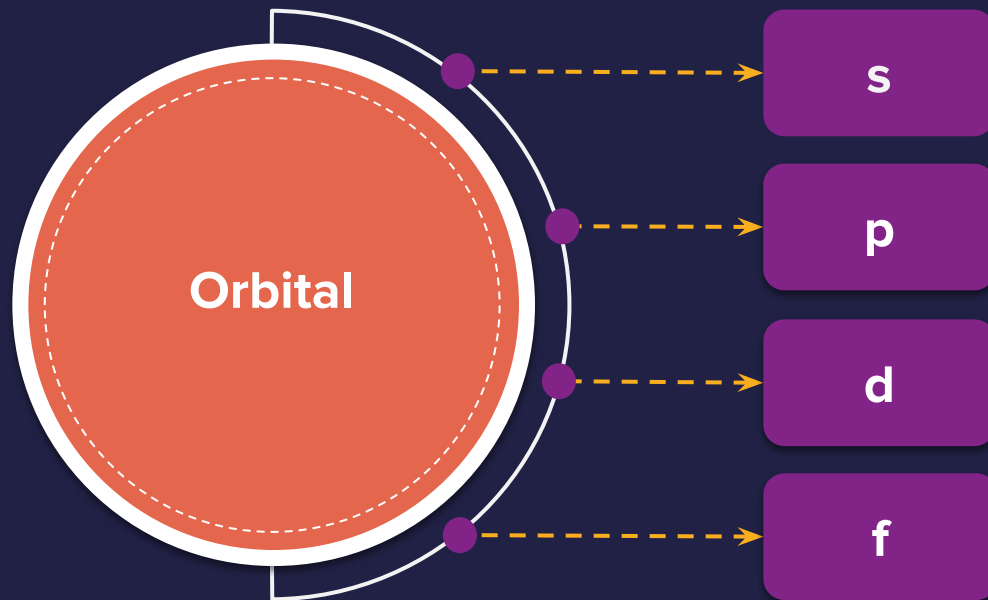
Shape of Orbitals

s - orbital
Shape : **Spherical**
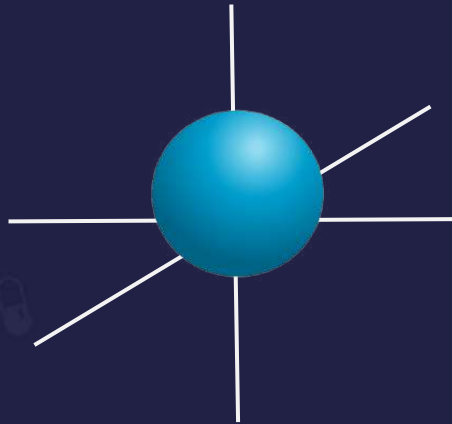
p - orbital
Shape : **Dumb bell**
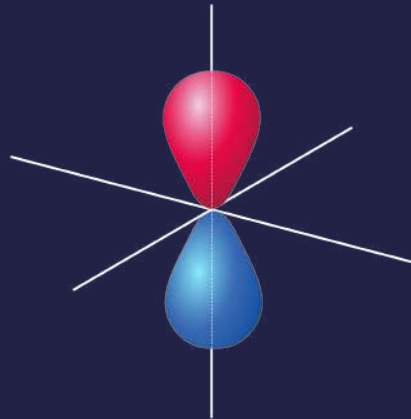
Shape : **Double dumb bell**
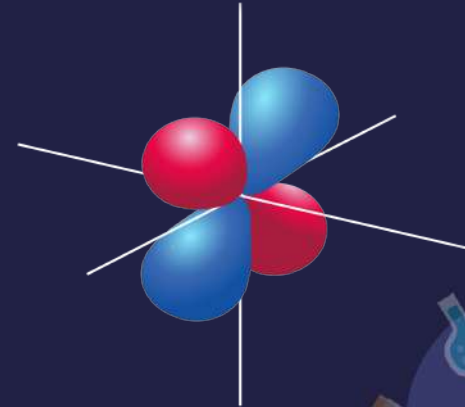
Probability Density Plots

s orbital
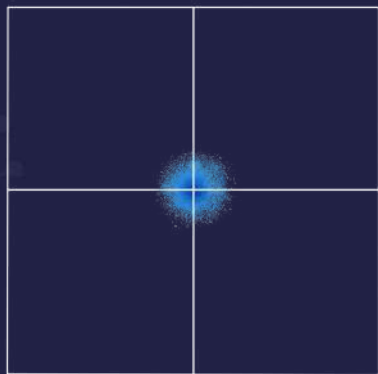
p orbital
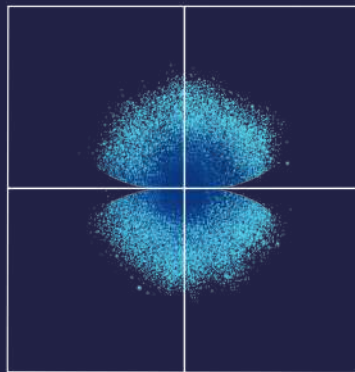
d orbital
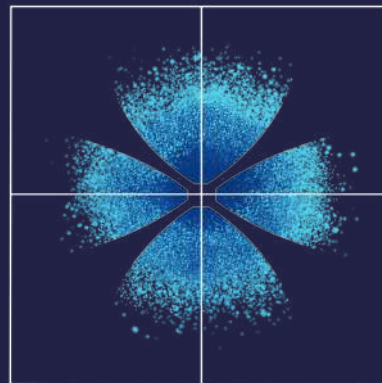
Shapes of Orbitals

Orbital	Shape
s	Spherical
p	Dumb bell
d	Double dumb bell
f	Leaf like / Complicated

Subshell

Collection of **similar shaped orbitals** of same **n**.

Subshell

l	Subshell	Description
0	s	Sharp
1	p	Principal
2	d	Diffused
3	f	Fundamental
4	g	Generalised

Subshell Representations

Number of subshells in the nth shell

n	l	Subshell notation
1	0	1s
2	0, 1	2s, 2p
3	0, 1, 2	3s, 3p, 3d
4	0, 1, 2, 3	4s, 4p, 4d, 4f

Azimuthal Quantum Number (l)

$$\text{Orbital angular momentum (L)} = \sqrt{l\,(l+1)}\ \hbar$$

$$\hbar = \frac{h}{2\pi}$$

Subshell	Orbital angular momentum
s	0
p	$\sqrt{2}\ \hbar$
d	$\sqrt{6}\ \hbar$
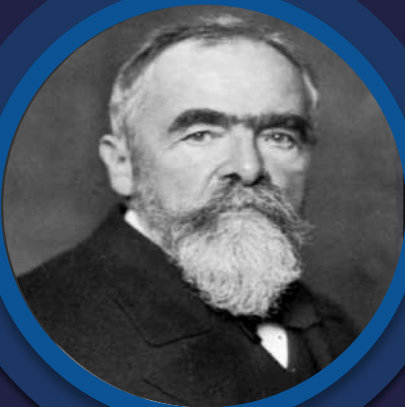
Magnetic Quantum Number (m_l)

Can have values from **- l to + l** including **zero**

Each value corresponds to an **orbital**

For d subshel, l = 2

m_l = -2, -1, 0, 1, 2

Magnetic Quantum Number (m_l)

Maximum number of orbitals in a subshell = $2l + 1$

Subshell	Number of orbitals
s	1
p	3 (p_x, p_y, p_z)
d	5 (d_{xy}, d_{yz}, d_{zx}, $d_{x^2-y^2}$, d_{z^2})
f	7

s - orbital

Shape : **Spherical**
Non-directional in nature
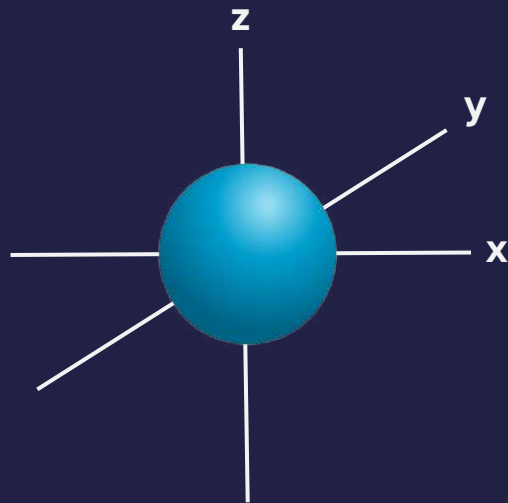
p - orbital

Shape : **Dumb bell**
Directional in nature

p_z

p_y

p_x

d - orbital

z y x **d$_{yz}$**

z y x **d$_{xy}$**
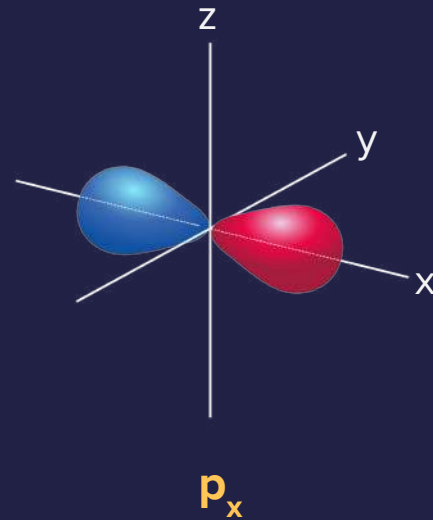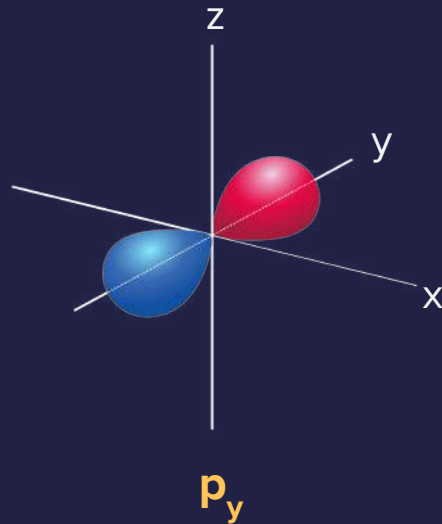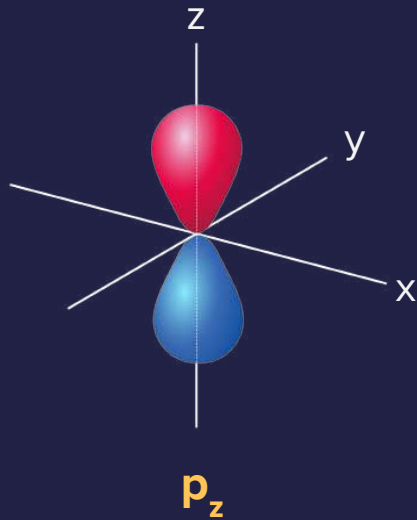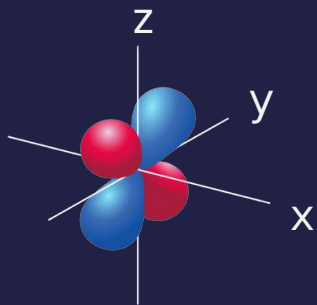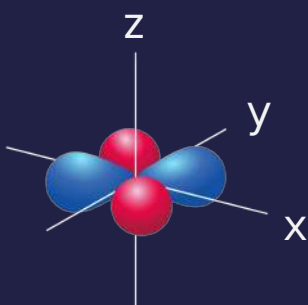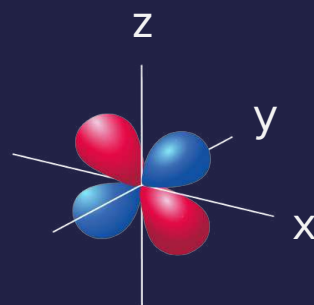
z y x **d$_{xz}$**

Non axial d-orbitals

Shape : **Dumb bell**
Directional in nature

z y x **d$_{z^2}$**
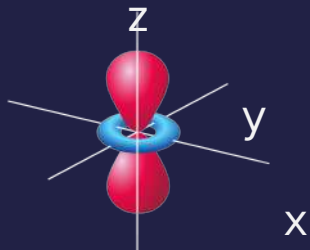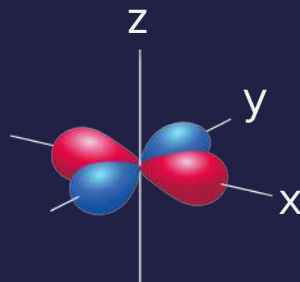
z y x **d$_{x^2- y^2}$**

Axial d-orbitals

An **orbitals** can accommodate maximum of **2 electrons**.

Maximum number of electrons in a subshell	=	$2(2l + 1)$

Subshell	s	p	d	f
l	0	1	2	3
Number of electrons	2	6	10	14

Spin Quantum Number (s or m_s)

Presence of **two closely spaced lines** in **atomic spectrum**

↓

Spin of an **electron**

$$s = +\frac{1}{2}$$

$$s = -\frac{1}{2}$$

Proposed by George Uhlenbeck (left) and Samuel Goudsmit (Right)

Spin Quantum Number (s)

Spin magnetic moment (μ) **=** $\sqrt{n(n+2)}$ **B.M.**

n = number of unpaired electron

Spin Quantum Number (s)

Spin angular momentum	=	$\dfrac{h}{2\pi}\sqrt{s(s+1)}$
Maximum Spin of an atom (S)	=	$\dfrac{1}{2}\,n$
Spin multiplicity	=	$2\,|S| + 1$

Orbit and Orbital

Orbit	Orbital
Well defined circular path around the nucleus where electrons revolve	**3D region** around the nucleus where electrons are most likely to be found
Maximum number of electrons in n^{th} orbit is $\mathbf{2n^2}$	**Cannot accommodate** more than **two electrons**

Orbit and Orbital

Orbit	Orbital
Not in accordance with Heisenberg's Uncertainty Principle	**In accordance** with Heisenberg's Uncertainty Principle
Designated as **K, L, M, N, ...**	Designated as **s, p, d, f, ...**

Rules for Filling Electrons in Orbitals
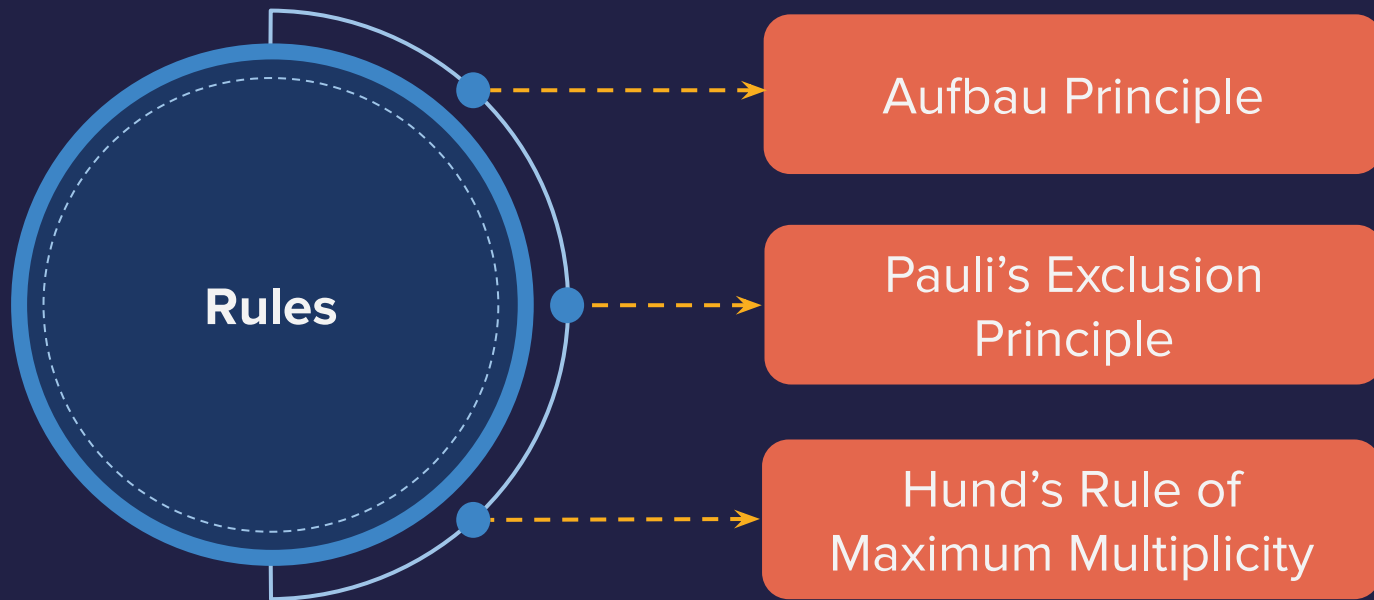
Aufbau Principle

Electrons are filled in various orbitals in order of their increasing energies

Energies of Subshells of H-like Species

Energy of **single electron species** depends only on the **Principal Quantum Number**

1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f < ...

Order of Energy

Energies of Subshells of H-like Species

Energy →

4s   4p   4d   4f

3s   3p   3d

2s   2p

1s

Degenerate Orbitals: Orbitals that have same energy.

Example: In the given diagram 3s, 3p and 3d are degenerate orbitals.
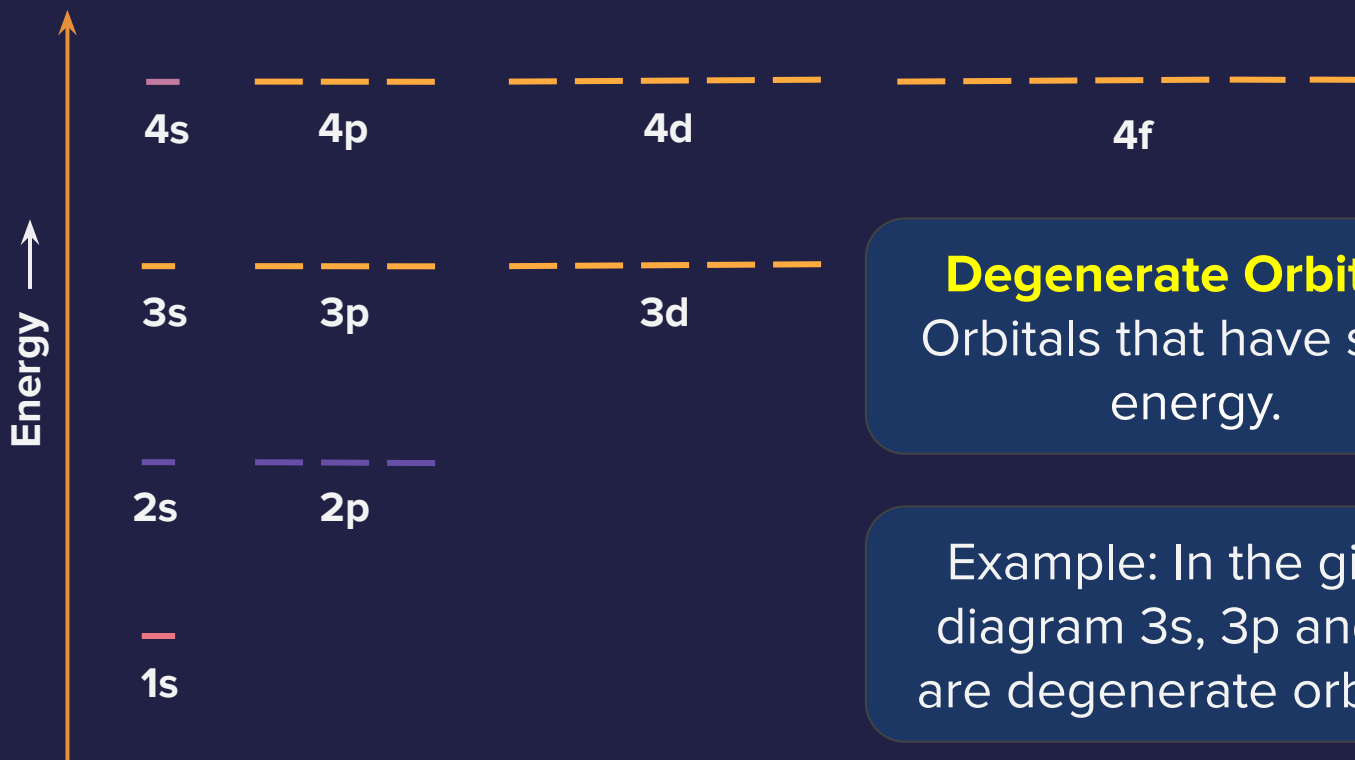
Energy of Subshells of Multi-electron Species

Different subshells have different energy which depends on:

Principal Quantum Number

Azimuthal Quantum Number

(n + *l*) rule or Bohr-Bury's Rule

Lower value of **(n + *l*)** → **Lower** will be **energy of subshell**

Two subshells with **same (n + *l*)** value → **Subshell** with **lower 'n'** value has **lower energy**

Comparison of orbital energy

1s	<	2s
2s	<	2p
5p	>	4d
2p	<	3p
3d	<	4p

Energies of Subshells of Multi electron Species

Energy →

4p

3d

4s

3p

3s

2p

2s

1s
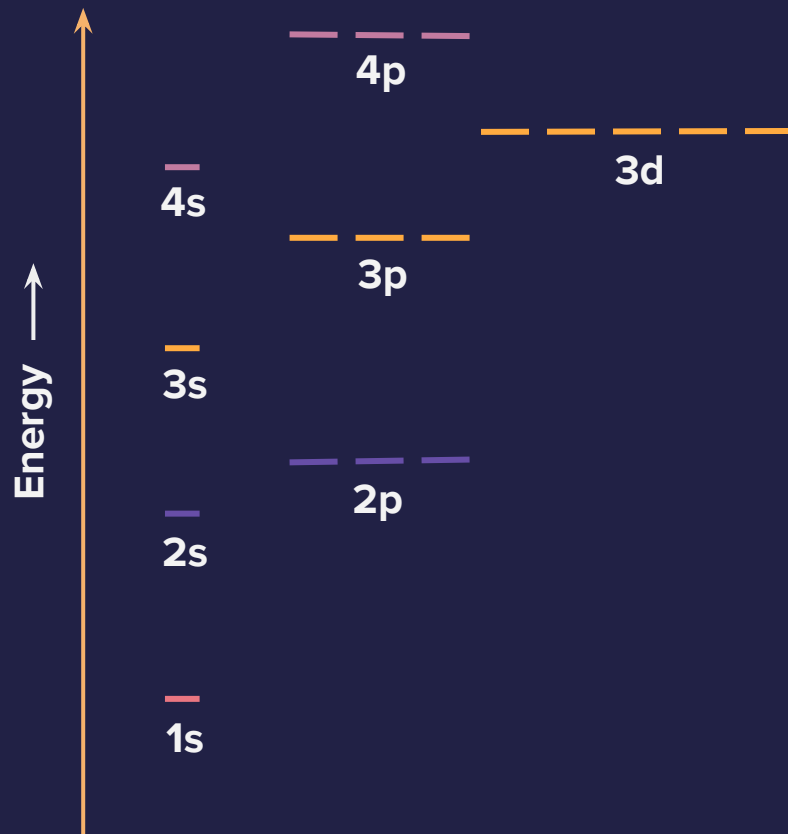
Energies of Subshells of Multi electron Species
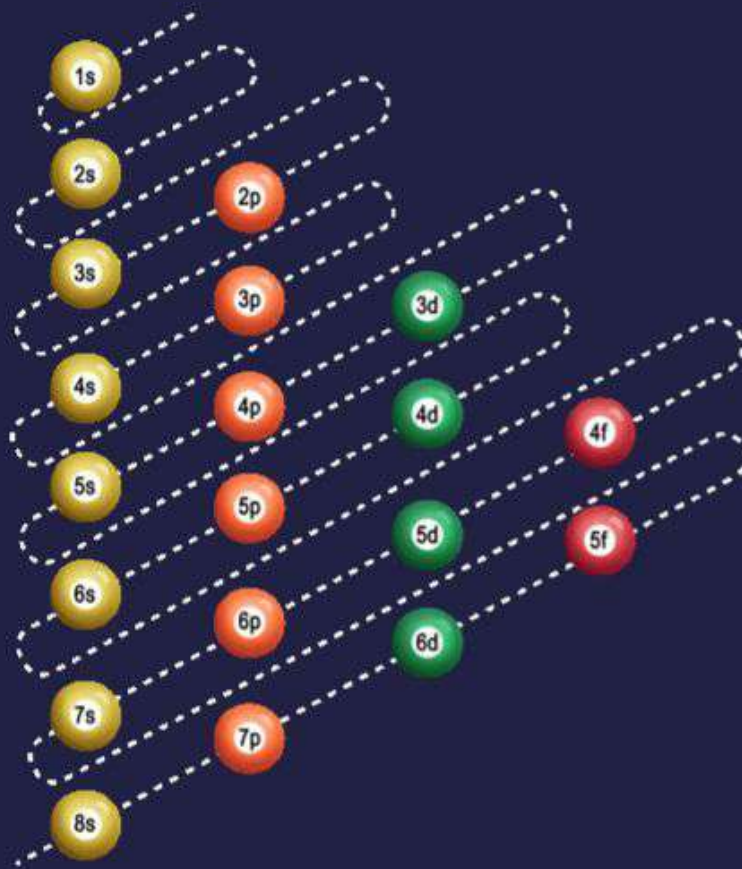
H-like v/s Multi-electron species

H-like species	Multi-electron species
Energy of a subshell depends on **'n' only**.	Energy of a subshell depends on **(n + l)**.
Only **attractive forces** are present between the **nucleus** and the **electron**.	Electrons experience **attractive forces** towards the **nucleus** as well as **repulsive forces** from other **electrons**.
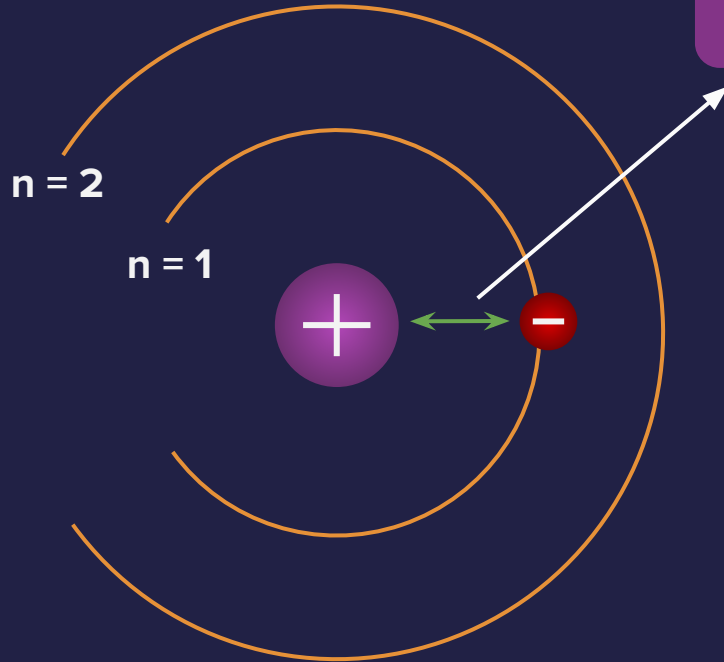
Multiple-electron species
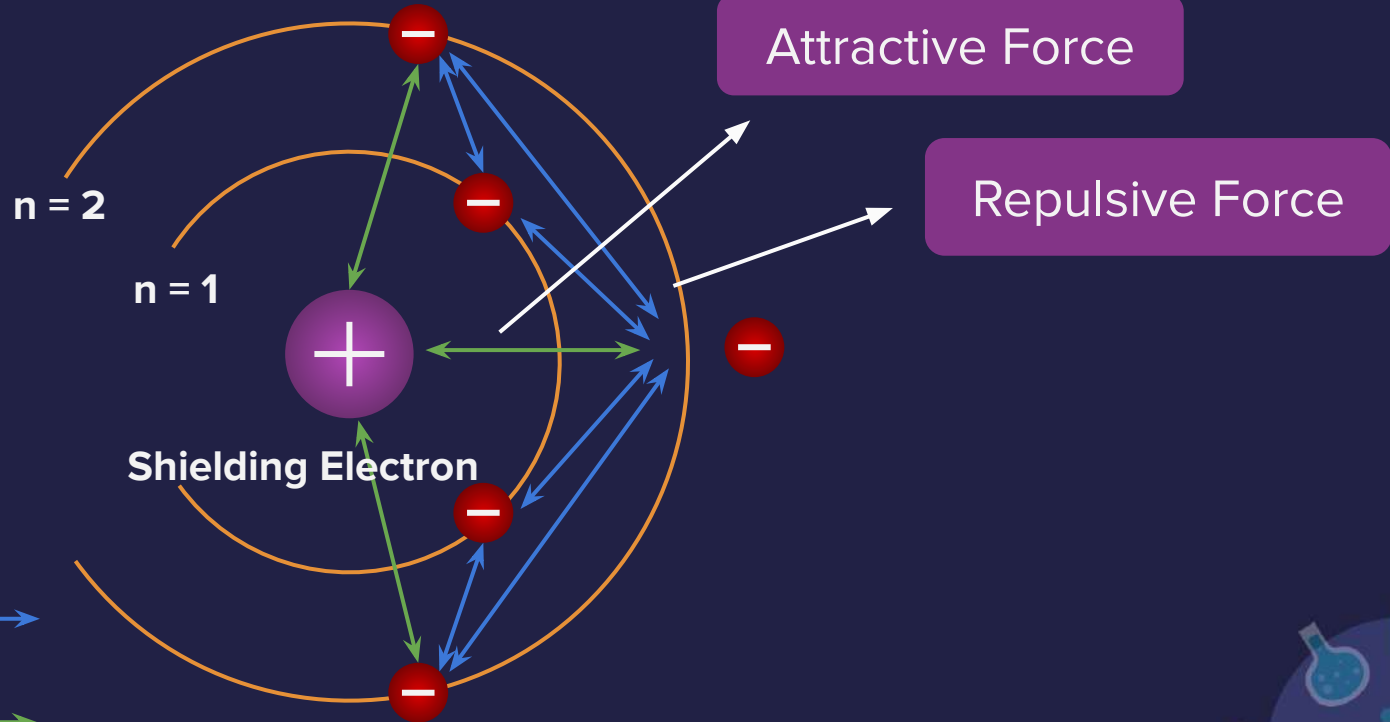
Pauli's Exclusion Principle

Restrict the **filling** of number of **electrons** in an **orbital**

Wrong

Right

Subshell electron capacity

s^2 ⇅

p^6 ⇅ ⇅ ⇅

d^{10} ⇅ ⇅ ⇅ ⇅ ⇅

f^{14} ⇅ ⇅ ⇅ ⇅ ⇅ ⇅ ⇅

Hund's Rule of Maximum Multiplicity

Friedrich Hund

Hund's Rule of Maximum Multiplicity

No **electron pairing** takes place in the orbitals in a subshell

Until each orbital is occupied by **1 electron** with **parallel spin**

Hund's rule is an **empirical rule**

Determines the **lowest energy arrangement** of electrons

Why Maximum multiplicity?

Maximum spin of an atom (S)	=	$\frac{1}{2} \times n$
Spin Multiplicity (S.M.)	=	2S + 1

Spin Multiplicity (S.M.) ↑ - - → **Stability** ↑

Electrons with **parallel spins**

Repel each other

Have a **tendency to stay apart**

Atom **shrink** slightly

Electron-nucleus interaction is **improved**

Electronic Configuration

Distribution of electrons in orbitals of an atom
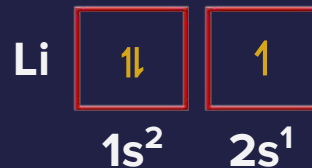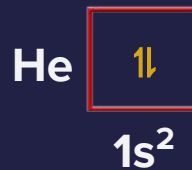
H $\boxed{\uparrow}$

$1s^1$

He $\boxed{\uparrow\downarrow}$

$1s^2$

Li $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow}$

$1s^2$ $\quad$ $2s^1$

Electronic Configuration of Various Elements

Be

⇅	⇅
$1s^2$	$2s^2$

B

⇅	⇅	↑		
$1s^2$	$2s^2$		$2p^1$	

C

⇅	⇅	↑	↑	
$1s^2$	$2s^2$		$2p^2$	

N

⇅	⇅	↑	↑	↑
$1s^2$	$2s^2$		$2p^3$	

O

⇅	⇅	⇅	↑	↑
$1s^2$	$2s^2$		$2p^4$	

F

⇅	⇅	⇅	⇅	↑
$1s^2$	$2s^2$		$2p^5$	
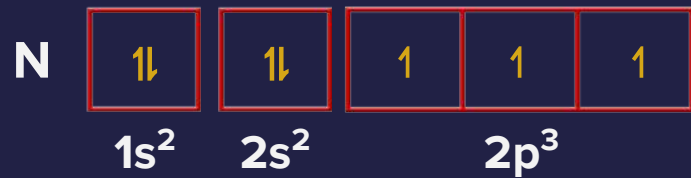
Electronic Configuration of Various Elements
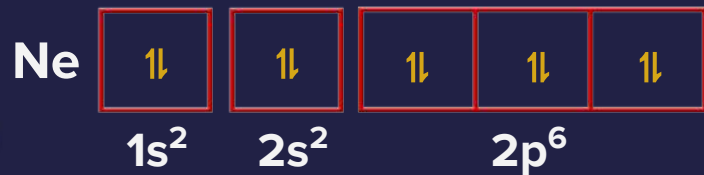
Simplified Electronic Configuration

Configuration of Sodium:
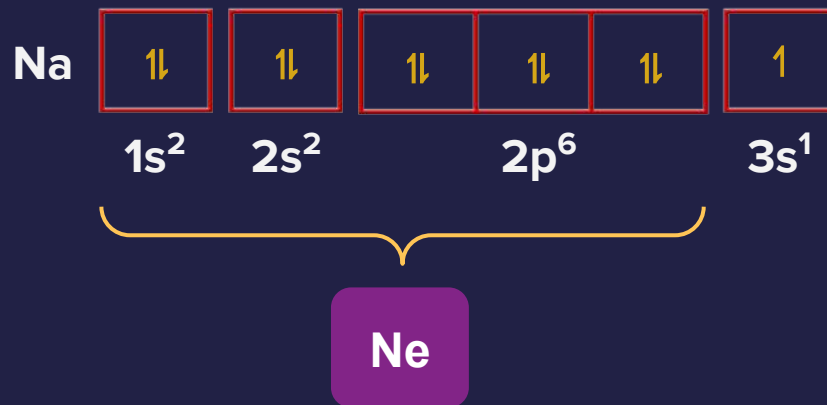
Simplified configuration:

Electronic Configuration

a) $_{21}$Sc → $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^1$

→ $[Ar]\ 4s^2\ 3d^1$

→ $[Ar]\ 3d^1\ 4s^2$

1						1↓

$3d^1$ **$4s^2$**

45
$_{21}$**Sc**
Scandium
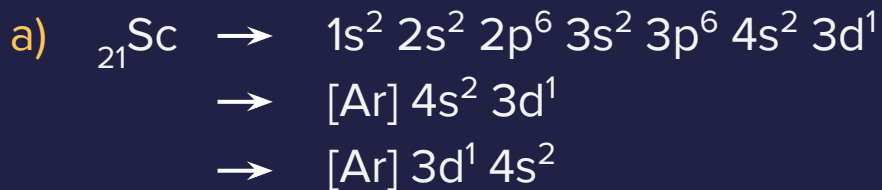
Number of unpaired electrons	1

Total spin	$+\dfrac{1}{2}$ or $-\dfrac{1}{2}$

Electronic Configuration

b) $_{26}$Fe $\rightarrow$ 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6

$\rightarrow$ [Ar] 4s^2 3d^6

$\rightarrow$ [Ar] 3d^6 4s^2

⇅	↑	↑	↑	↑		⇅

3d^6   **4s^2**

56
26 **Fe**
Iron
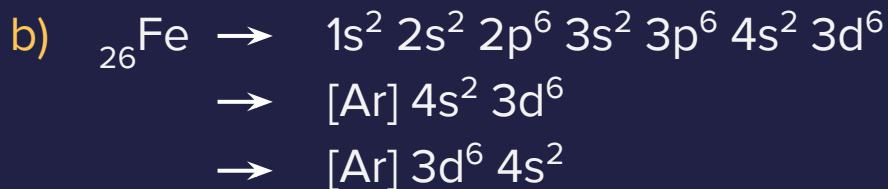
Number of unpaired electrons	4
Total spin	$+ \dfrac{4}{2}$ or $- \dfrac{4}{2}$

Exceptions

$_{24}$Cr   [Ar] $4s^2$ $3d^4$ Not Correct

$_{24}$Cr   [Ar] $4s^1$ $3d^5$ Correct

52 Cr 24

d^5 is more stable than d^4 configuration

Exceptions

$_{29}$**Cu**   **[Ar] 4s^2 3d^9**   Not Correct

$_{29}$**Cu**   **[Ar] 4s^1 3d^{10}**   Correct

63 Cu 29

d^{10} is more stable than d^9 configuration

Half-filled & Fully Filled Orbitals

Exactly **half filled** & **fully filled** orbitals make the configuration more stable

p^3, p^6, d^5, d^{10}, f^7 & f^{14} configurations are stable

Stability of **half filled** & **fully filled** orbitals

Symmetry

Exchange Energy

Symmetry

Symmetrical distribution of electrons

Symmetry leads to stability

Consequently, their **shielding** of one another is **relatively small**

Electrons are more **strongly attracted** by the nucleus

Electrons in the **same subshell**

Equal energy

Different spatial distribution

Have **less energy and more stability**

Exchange Energy

Energy released when **two or more electrons** with the **same spin** in the **degenerate** orbitals

Tends to **exchange** their **positions**

Number of exchanges that can take place is **maximum**

When subshell is either **half filled or fully filled**.

Exchange Energy
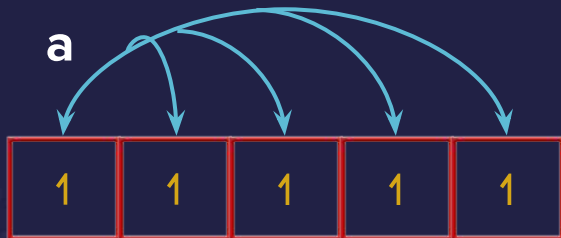
4 exchange by electron 'a'
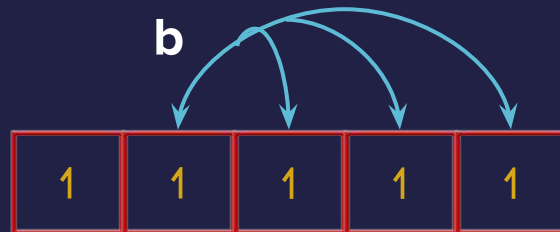
3 exchange by electron 'b'
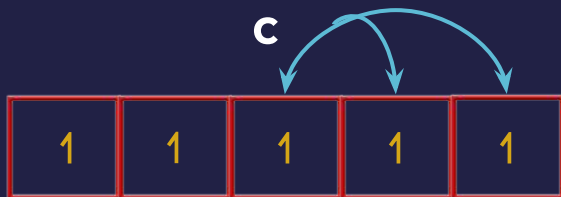
2 exchange by electron 'c'
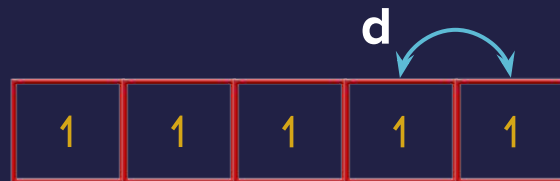
1 exchange by electron 'd'

Electronic Configuration of Ions

Ions

Cations

Anions

Electronic Configuration of Cations

Formed by **removing outermost electron** from a **neutral atom**

Na

| $1s^2$ | $2s^2$ | $2p^6$ | $3s^1$ |

Na^+

| $1s^2$ | $2s^2$ | $2p^6$ | $3s^0$ |
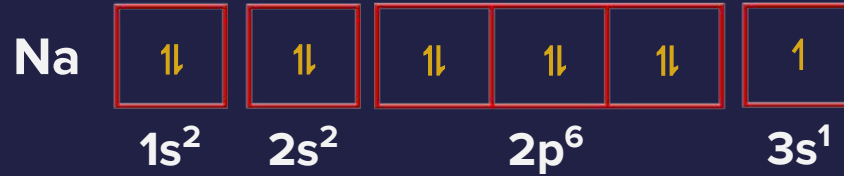
Electronic Configuration of Cations

In **d-block** metals

electrons are **first removed** from **ns** orbital, then from the penultimate **(n-1)d** orbital

Examples

Fe: [Ar] $3d^6$ $4s^2$ or

Ar

| ⇅ | ↑ | ↑ | ↑ | ↑ | | ⇅ |

$3d^6$ **$4s^2$**

Fe^{2+}: [Ar] $3d^6$ $4s^0$ or

Ar

| ⇅ | ↑ | ↑ | ↑ | ↑ | | |

$3d^6$ **$4s^0$**

Fe^{3+}: [Ar] $3d^5$ $4s^0$ or

Ar

| ↑ | ↑ | ↑ | ↑ | ↑ | | |

$3d^5$ **$4s^0$**

Examples

$_{29}$Cu : **Ar** | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ↑

$3s^2$ $3p^6$ $3d^{10}$ $4s^1$

$_{29}$Cu$^+$: **Ar** | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ | ⇅ |

$3s^2$ $3p^6$ $3d^{10}$ $4s^0$
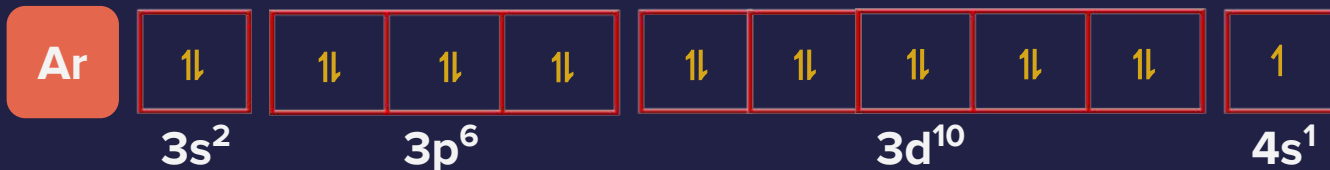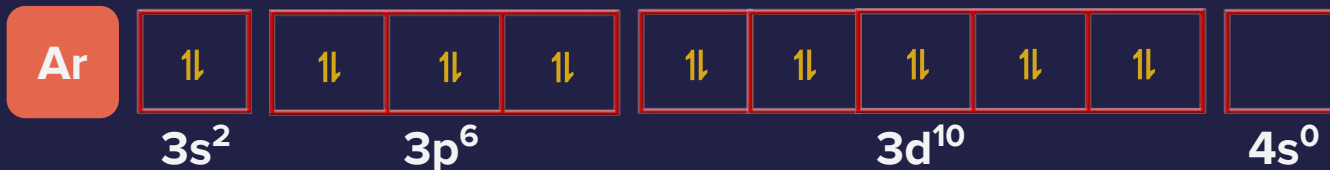
18 electrons   Pseudo inert gas configuration

Electronic Configuration of Anions

Formed by **adding electrons to a neutral atom** according to the 3 rules **(Pauli's, Aufbau & Hund's rule)**

F

$1s^2$ $2s^2$ $2p^5$

F $^-$

$1s^2$ $2s^2$ $2p^6$

Schrodinger Wave Equation (SWE)

Based on the **dual nature of matter**

Describes the behavior of **electron** around the nucleus

Schrodinger Wave Equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0$$

- x, y, z = Cartesian coordinates

  Ψ = Amplitude of the electron wave or Wave function

- h = Planck's constant

- V = Potential energy of the electron

- E = Total energy of the electron
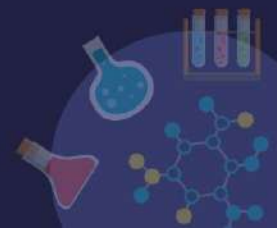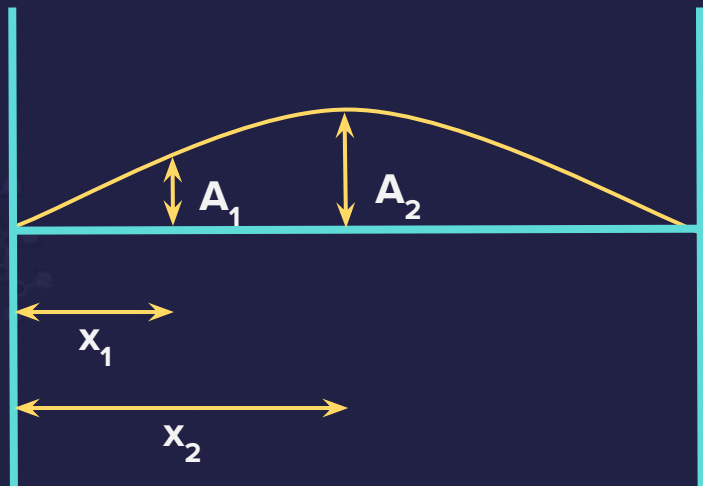
Wavefunction

Ψ corresponds to the allowed solutions of SWE

Ψ contains all the information related to the **motion of an electron** in an atom

Amplitude of a standing wave is a function of x

Similarly, Ψ is a **function of coordinates**

Wavefunction

Wave function

Radial part of wave function

Angular part of wave function

$$\Psi\,(r, \theta, \phi) = \Psi\,(r) \times \Psi\,(\theta, \phi)$$

n, l

l, m$_l$

Physical significance of Ψ^2

Maxwell's wave theory

Intensity of wave	$\propto$	Square of amplitude

Max Born suggested that

Ψ^2 - - - → Probability of finding an electron per unit volume or **probability density**

Probability density

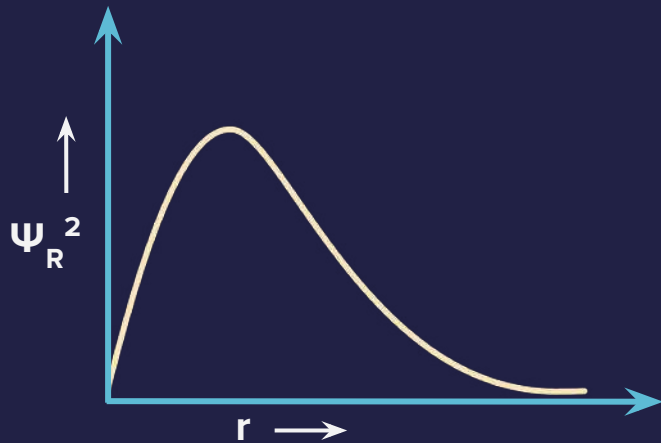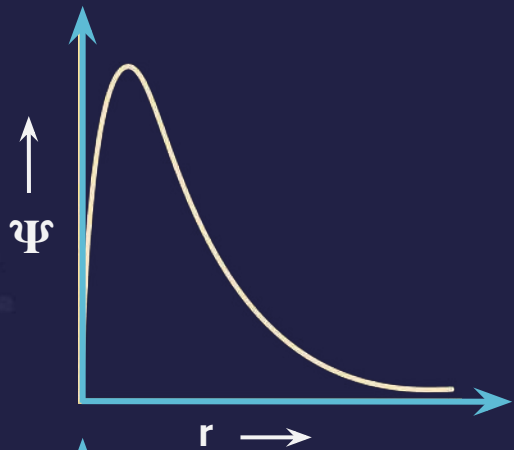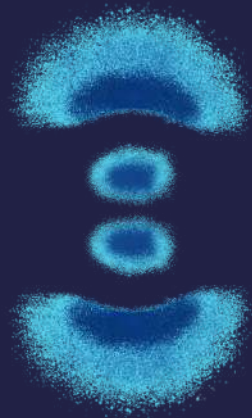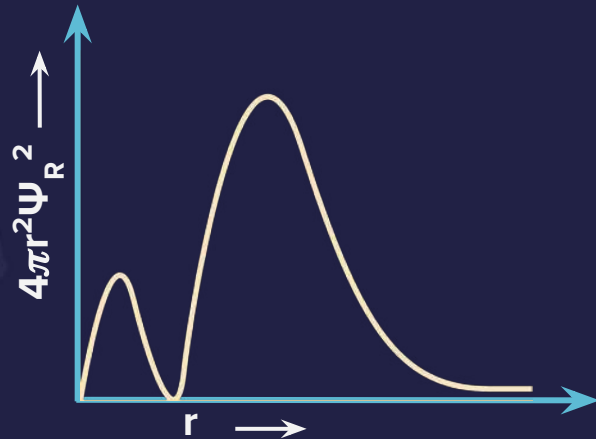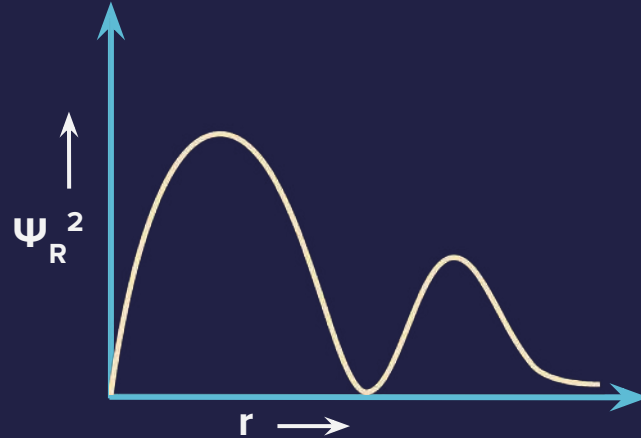| $\Psi(r,\theta,\phi)$	=	$\Psi(r)$	×	$\Psi(\theta,\phi)$
$\Psi^2(r,\theta,\phi)$	=	$\Psi^2(r)$	×	$\Psi^2(\theta,\phi)$

Probability density

Radial probability density

Angular probability density

Radial Distribution Function

Radial probability of finding an **electron** in a shell of thickness 'dr' at a radial distance 'r'

$=$

$\Psi_R{}^2 \times dV$

This probability which is independent of direction is called radial probability and is equal to $[4\pi r^2 dr R^2]$.
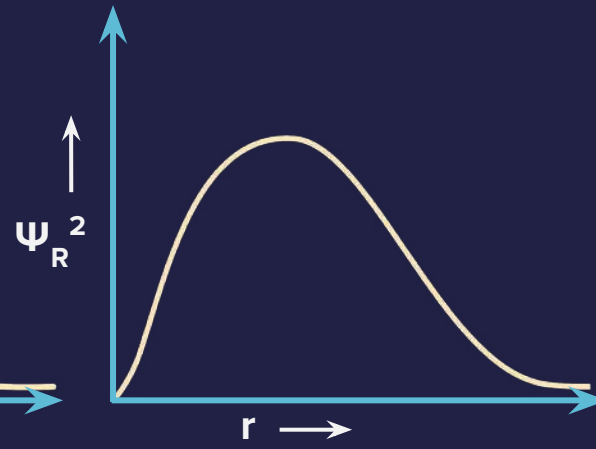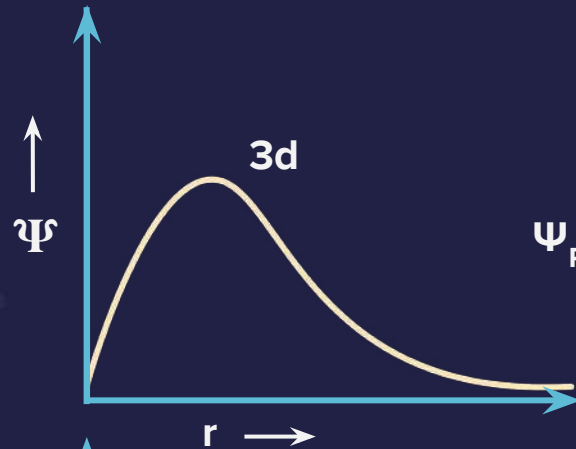
It gives the probability of finding the electron at a distance r from the nucleus regardless of direction.
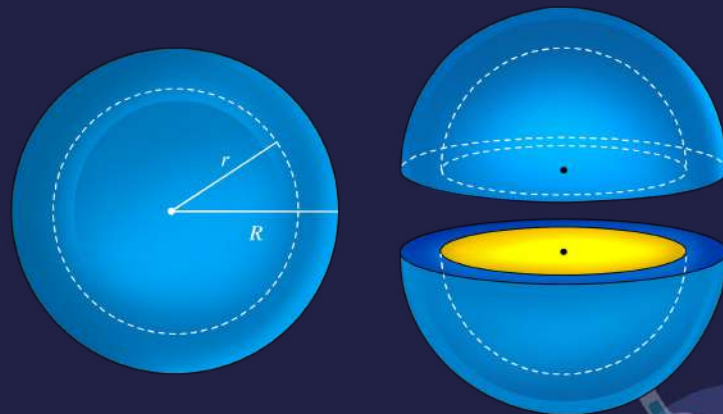
Radial Distribution Function

$$dV = \frac{4}{3}\pi(r + dr)^3 - \frac{4}{3}\pi r^3 = 4\pi r^2 \cdot dr$$

$$\text{Radial probability density on a layer} \times \text{Volume of that layer} = \Psi_R^{\,2} \cdot 4\pi r^2 \cdot dr$$

$$\text{Radial distribution function} = \Psi_R^{\,2} \cdot 4\pi r^2$$

Radial Distribution Function
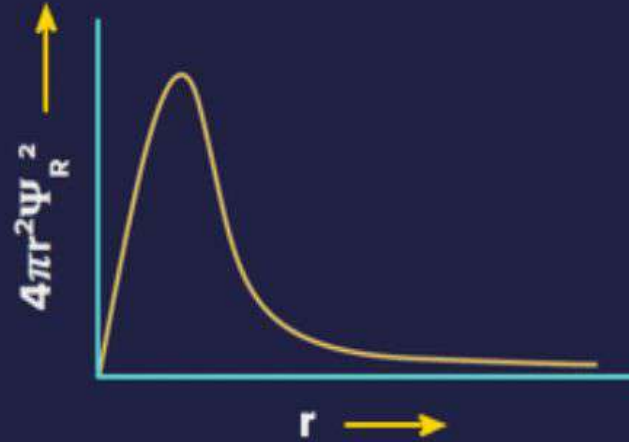
Radial wave function	→	Ψ_R
Radial probability density	→	$\Psi_R{}^2$
Radial probability distribution	→	$4\pi r^2 \Psi_R{}^2$

Region where the **probability density** is **zero** i.e. where the **probability of finding** an **electron** is **zero**

$$\Psi^2.dV \quad = \quad 0$$

∴ dV can't be zero

$$\Rightarrow \quad \Psi^2 \quad = \quad 0$$

Node

$$\Psi(r,\theta,\phi) = \Psi(r) \times \Psi(\theta,\phi) = 0$$

$$\Rightarrow \Psi(r) = 0 \quad \text{or} \quad \Psi(\theta,\phi) = 0$$

Radial Node

Spherical region around nucleus | Ψ_R or $\Psi_R{}^2$ is zero

Number of **radial nodes** in an **orbital** = $n - l - 1$

Angular Node

Plane or a surface passing through the **nucleus**

$\Psi_{\theta, \Phi}$ or $\Psi^2_{\theta, \Phi}$ is zero

Number of **angular nodes** in an **orbital** = l

Nodes

Total number of nodes $=$ Radial nodes $(n - l - 1)$ $+$ Angular nodes (l) $=$ $n - 1$

What are these radial and angular nodes?

Dark coloured regions show increased electron density

Radial node
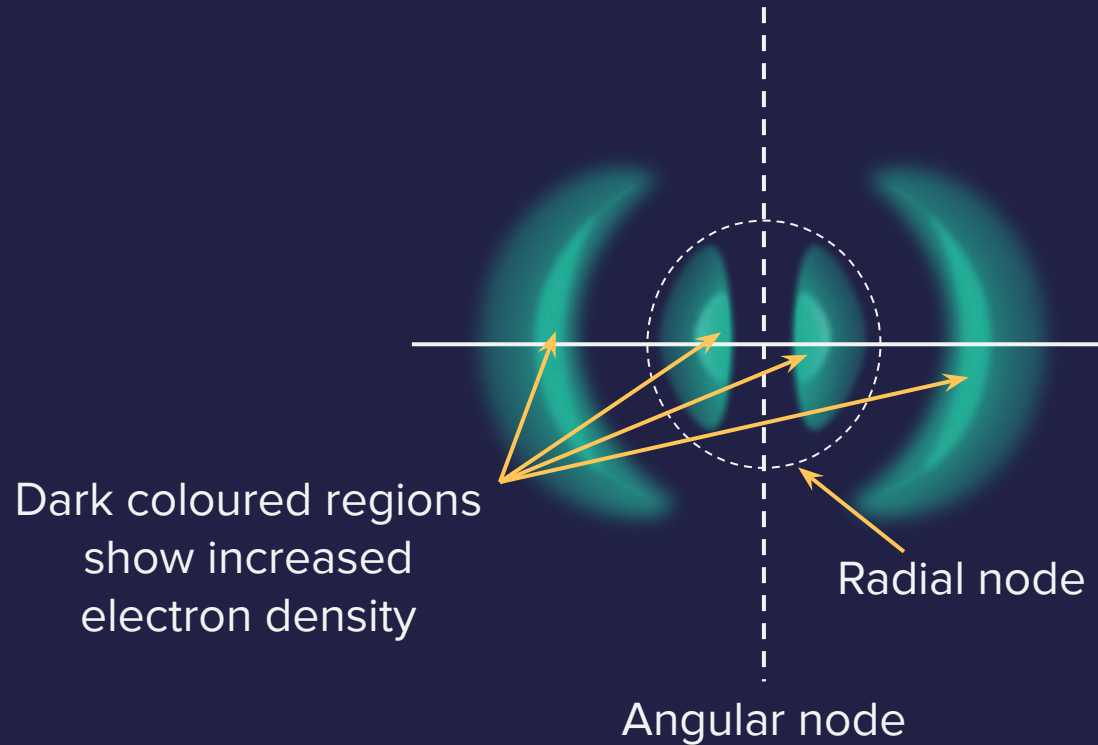
Angular node

Radial Node vs Angular Node

Radial node	Angular node
Spherical regions where the probability of finding an electron is **zero**.	**Flat planes** or **cones** where the probability of finding an electron is **zero**.
Have **fixed radii**.	Have **fixed angles**.
Number of radial nodes is given by (**n - l - 1**)	Number of angular nodes is given by (**l**)

Nodes of 's' orbitals

Node

Nodes

1s

2s

3s
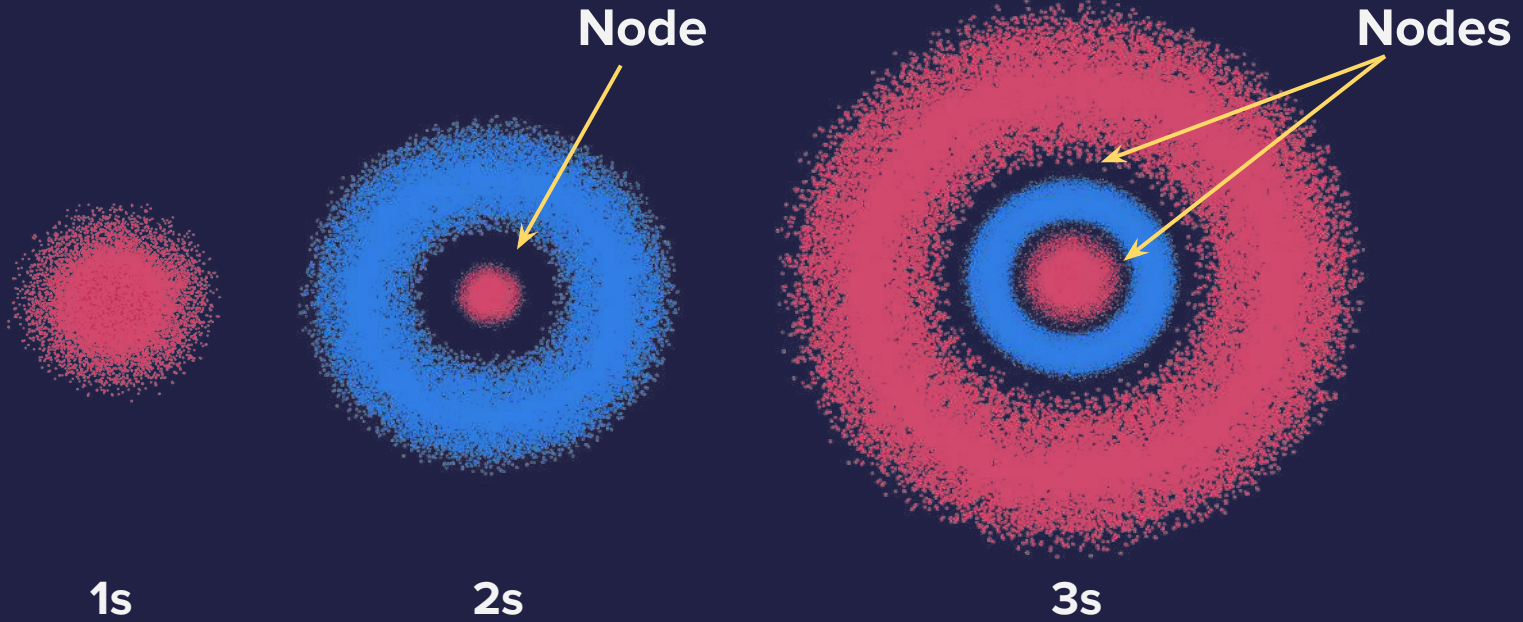
Angular Nodes of 'p' orbitals
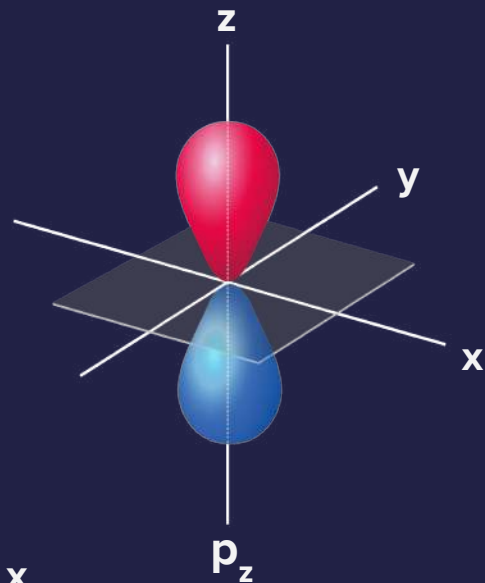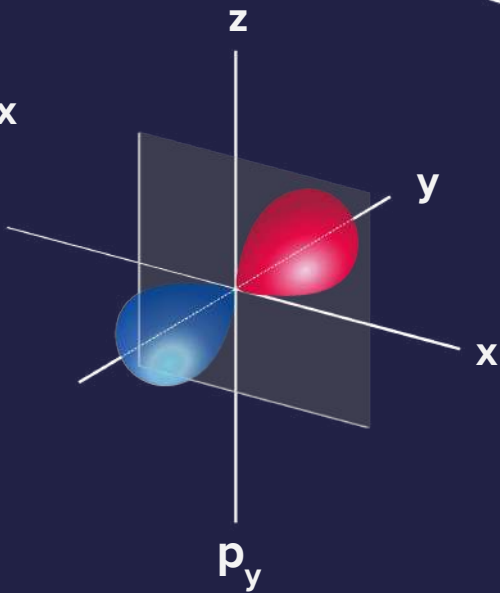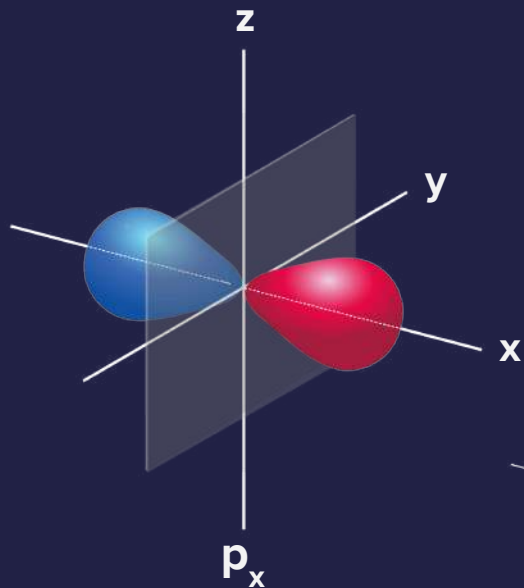
Angular Nodes of 'd' orbitals

d_{zx}

d_{yz}

d_{xy}

Angular Nodes of 'd' orbitals

$d_{x^2 - y^2}$

d_{z^2}
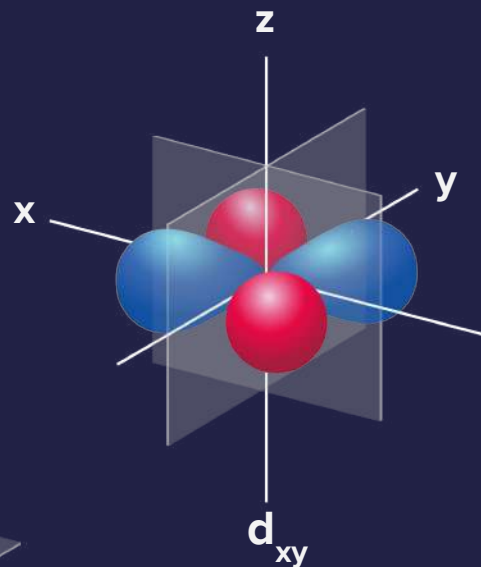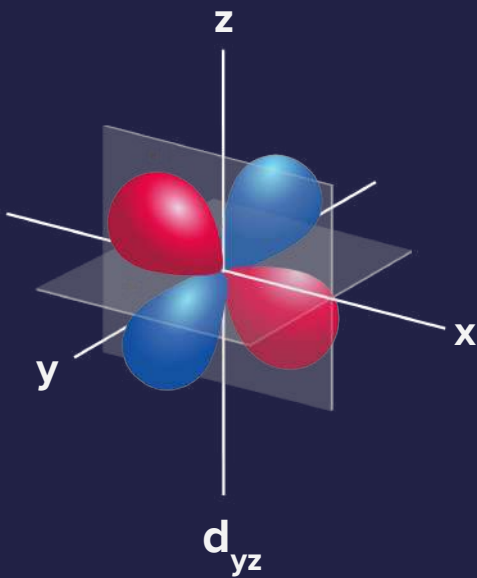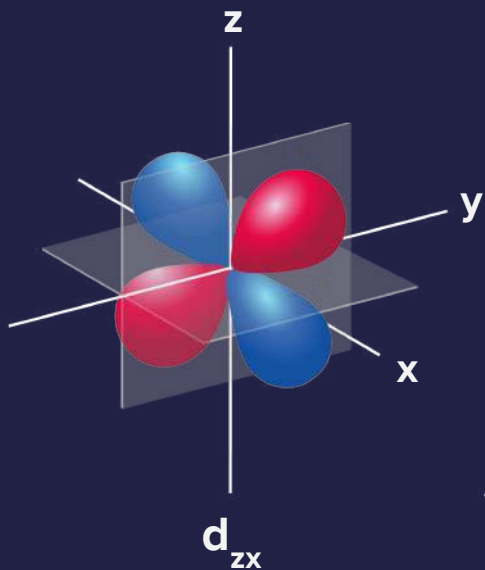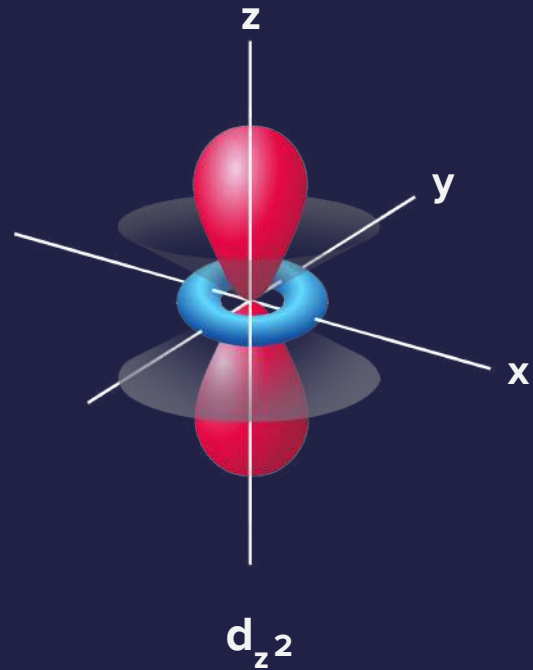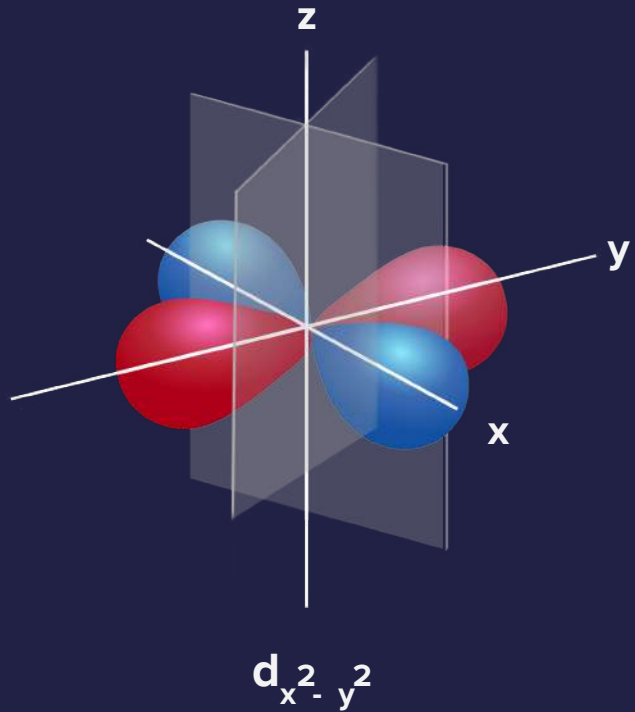
Comparison of Penetration Power

Additional maximas in **3s curve**

↓

Electron in **3s** spends **maximum time near the nucleus** compared to **3p and 3d.**

↓

Penetration power : 3s > 3p > 3d

Probability Curves

Plots

- Radial Wave function (Ψ) against r

- Radial Probability density (Ψ^2) against r

- Radial Probability distribution ($4\pi r^2 \Psi_R^2$) against r

s-orbital

Density of electron cloud is **spherical**

Probability density is **maximum** at the **nucleus** and **decreases** at **large distance**

Radial Probability of 1s
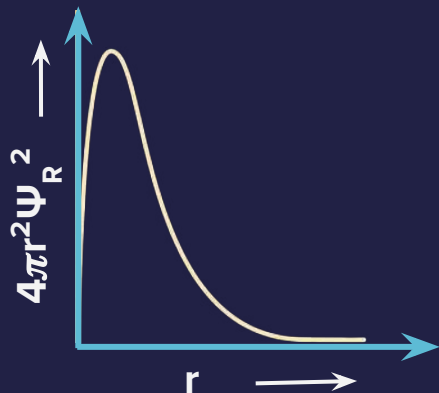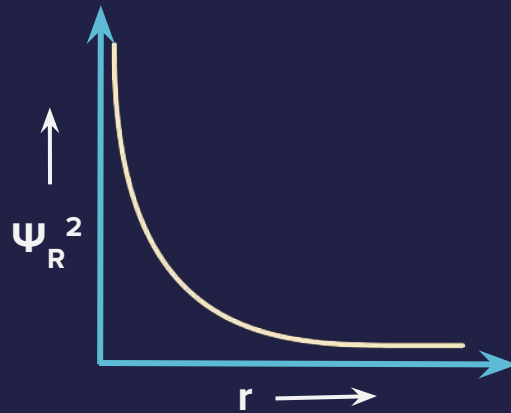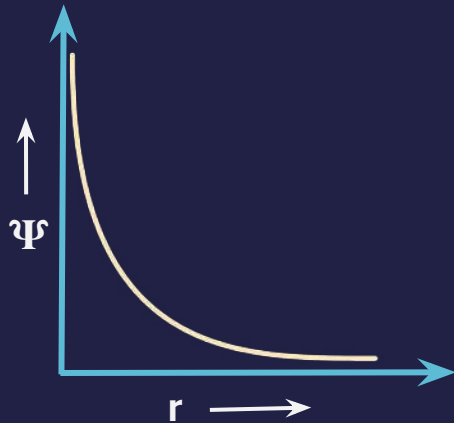
Radial Probability of 2s
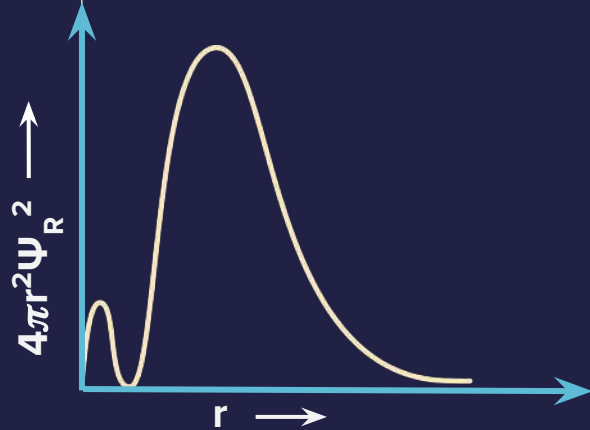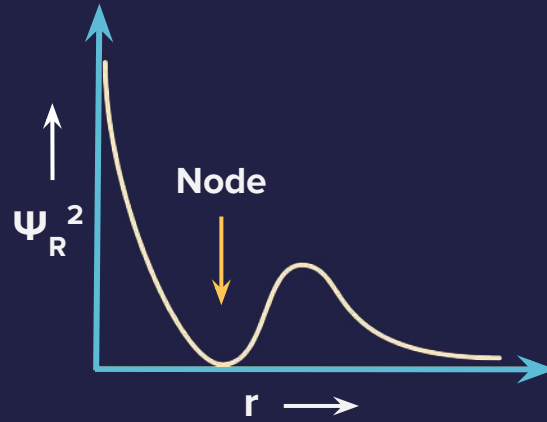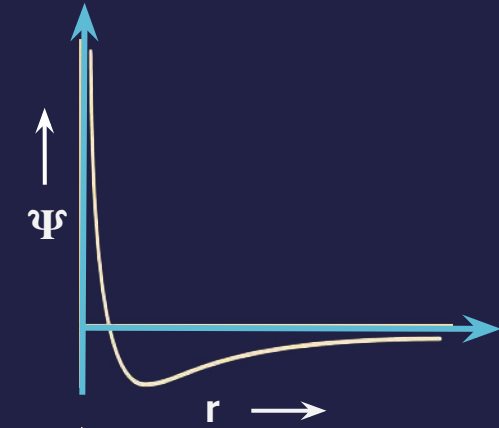
Radial Probability of 3s
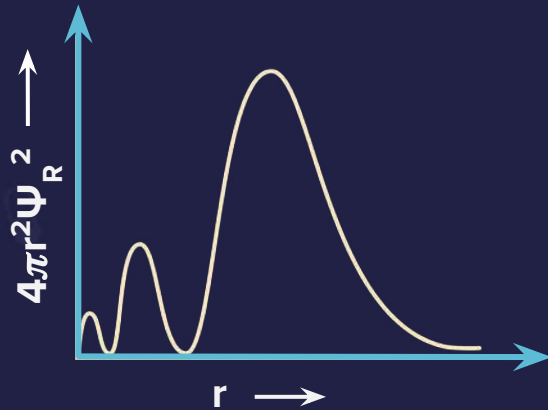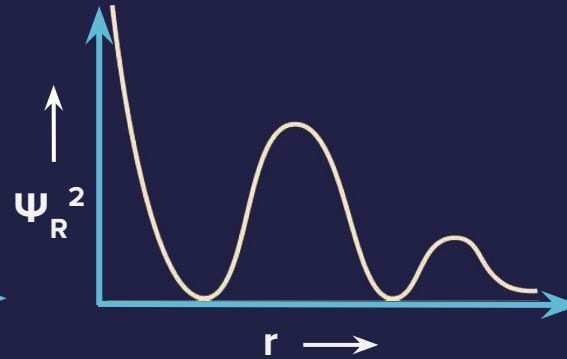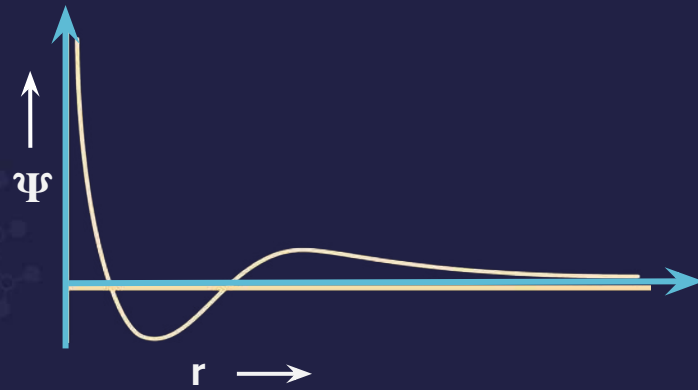
Radial Probability of 2p

Radial Probability of 3p

3p

Ψ

$r \longrightarrow$

$\Psi_R^{\ 2}$

$r \longrightarrow$

$4\pi r^2 \Psi_R^{\ 2}$

$r \longrightarrow$
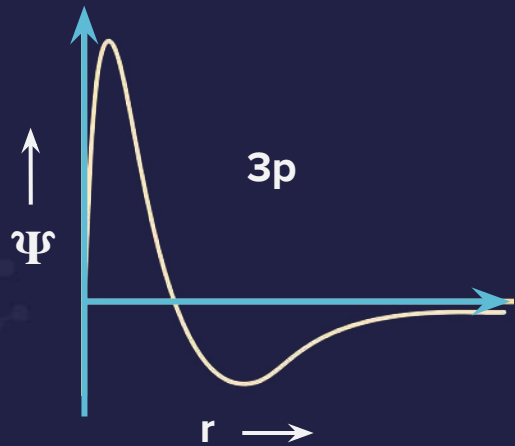
Radial Probability of 3d

Analysis: Ψ Plots

1s
2p
3p
3d
2s
3s
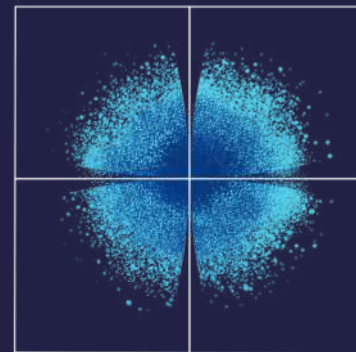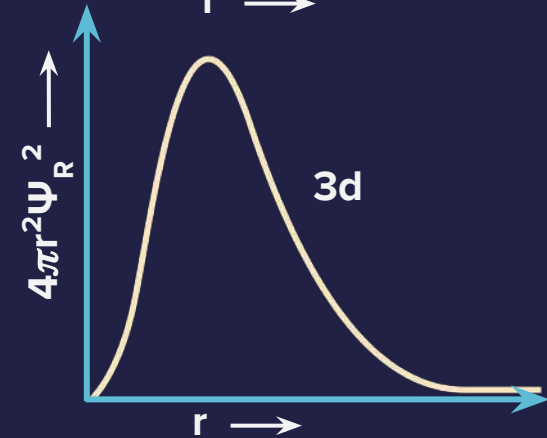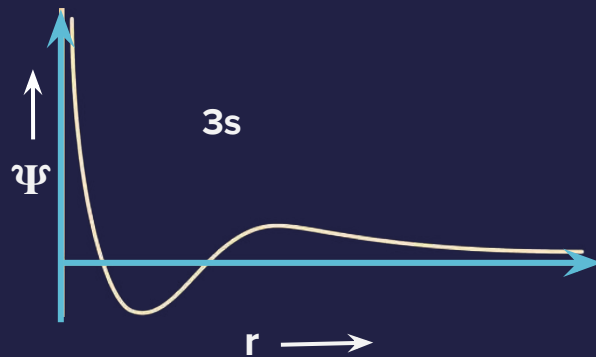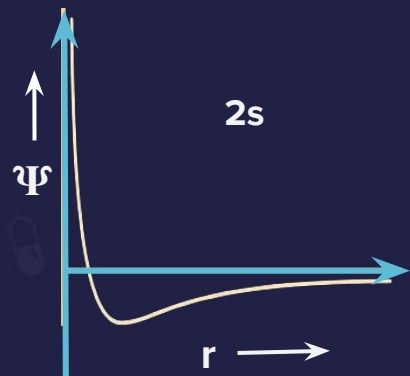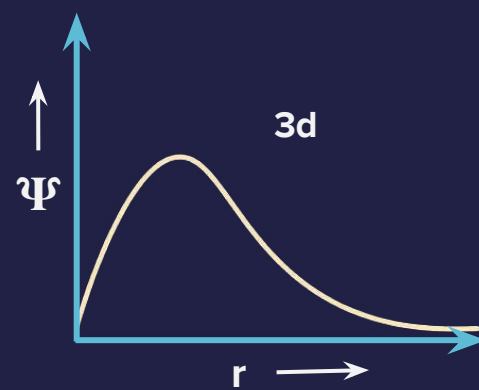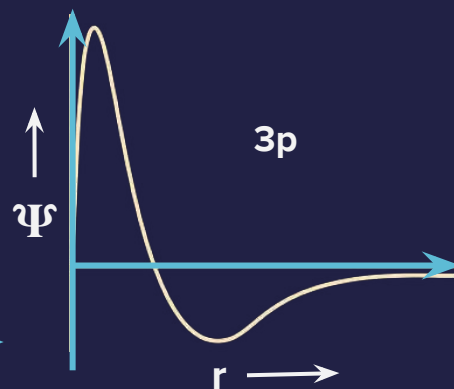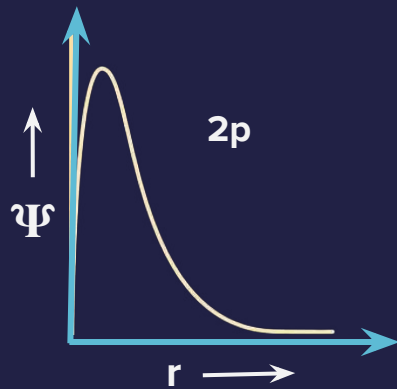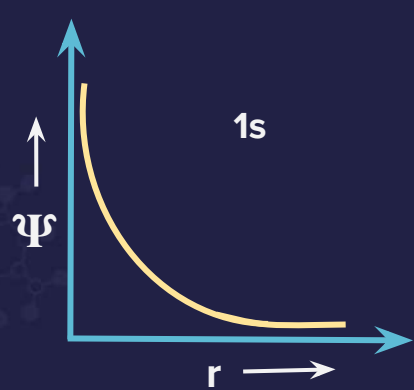
Analysis: Ψ² Plots

Analysis: $\Psi^2.4\pi r^2$ Plots

$(\Psi_R{}^2)\, 4\pi r^2$

r →

$(\Psi_R{}^2)\, 4\pi r^2$

r →

$(\Psi_R{}^2)\, 4\pi r^2$

r →

Node

Nodes

1s

2s

3s

Ψ^2

Ψ^2 ← Node

Ψ^2 Nodes

r

r

r

1s

2s

3s
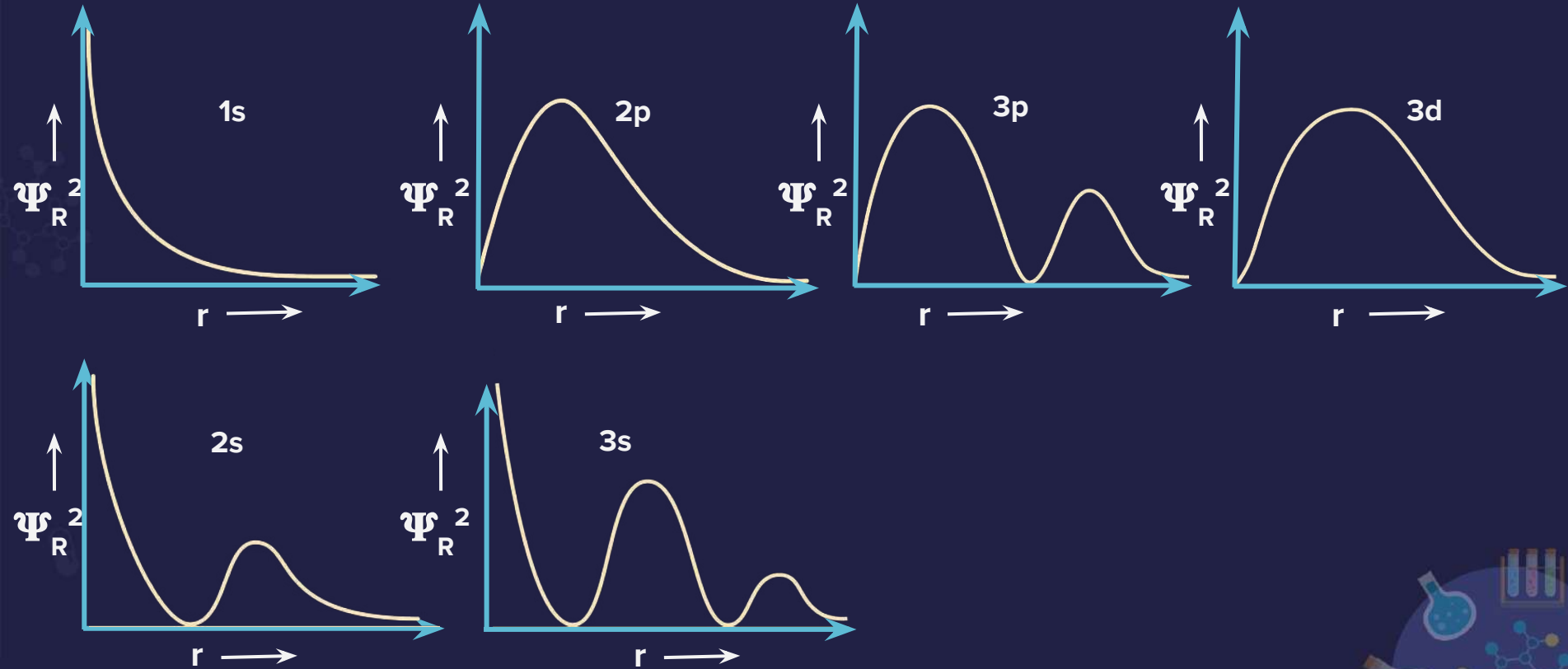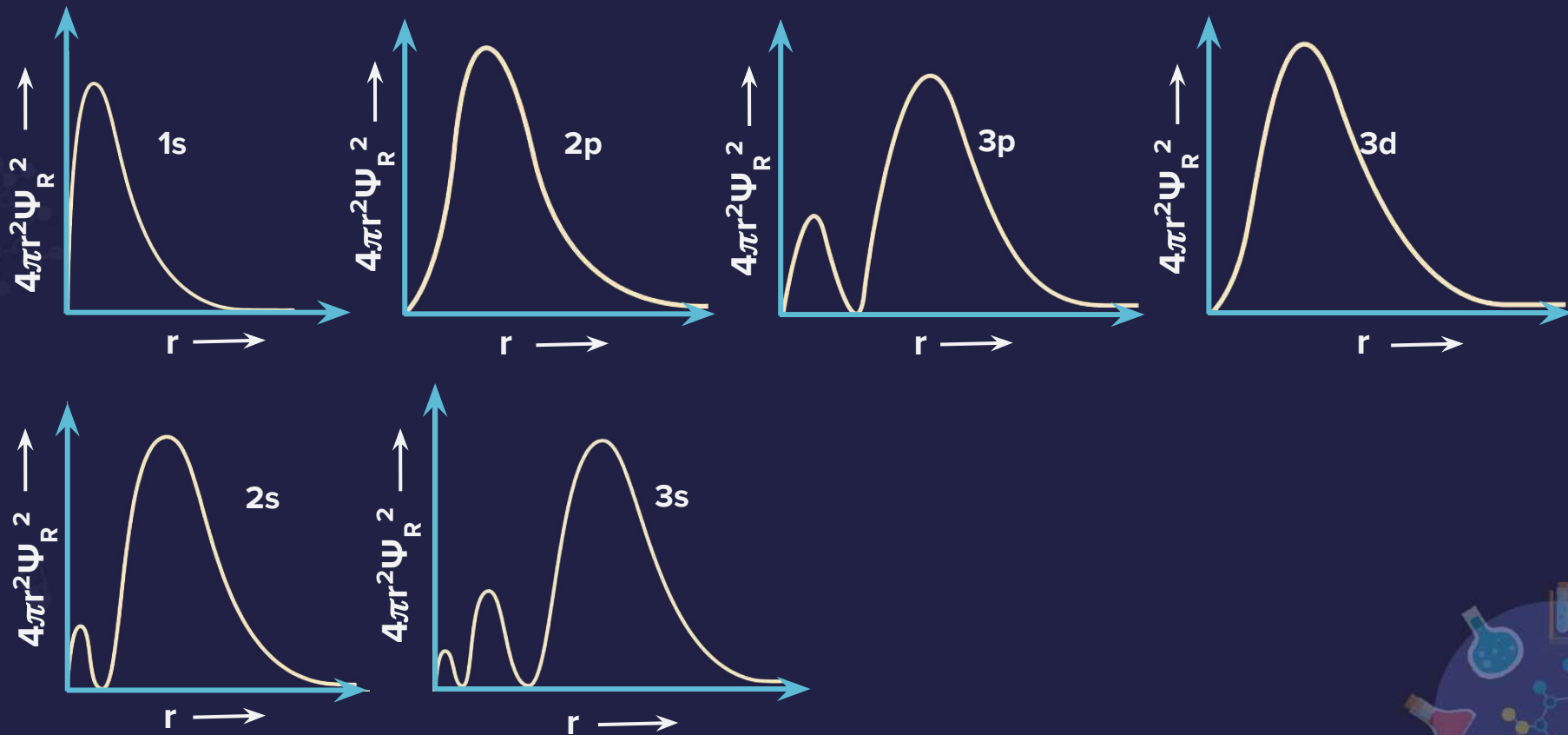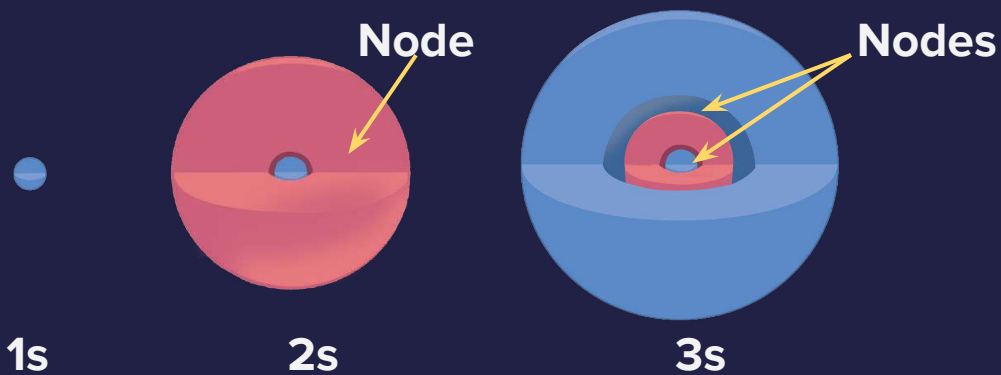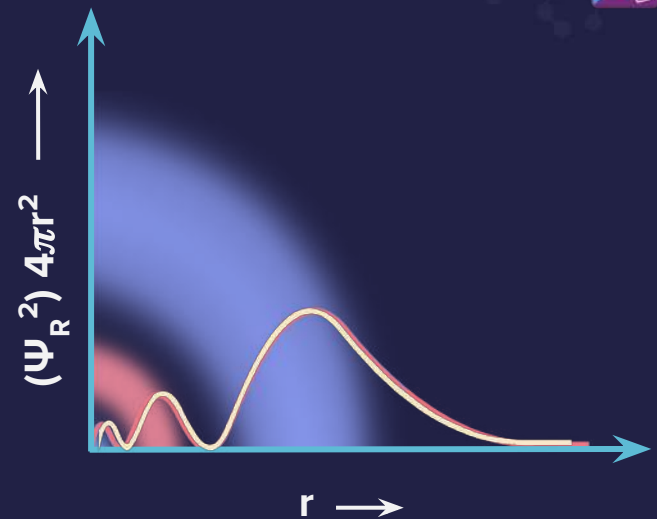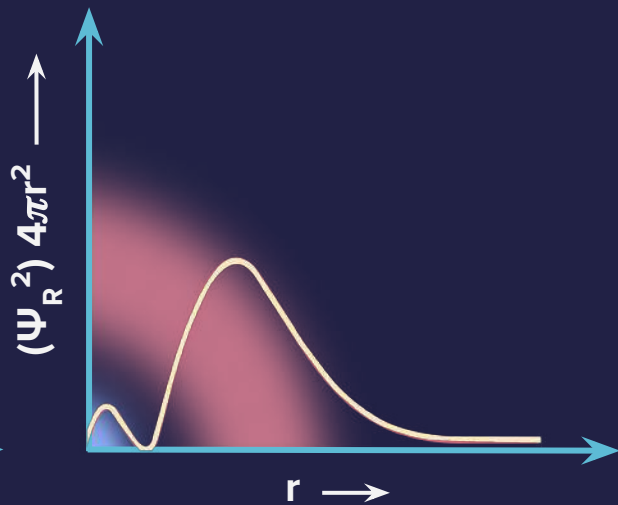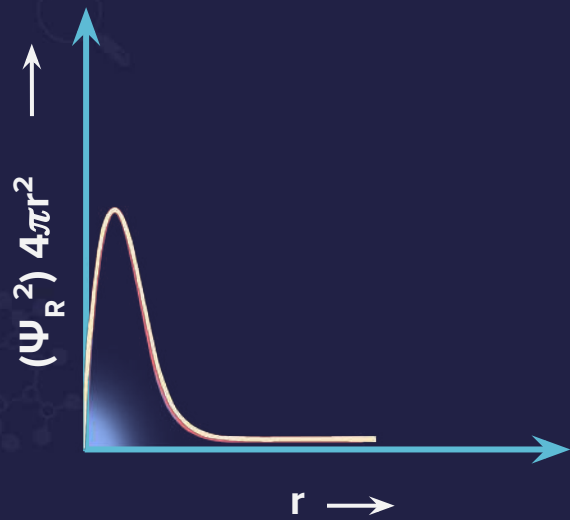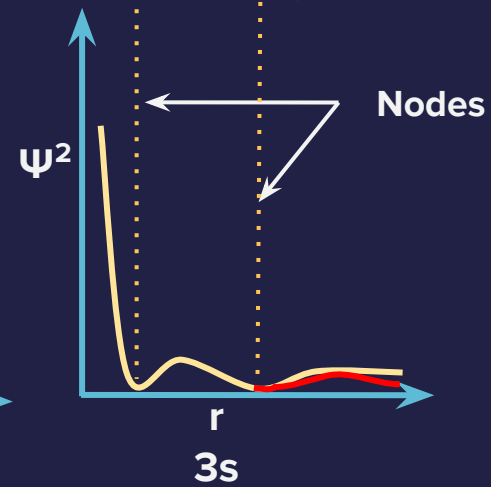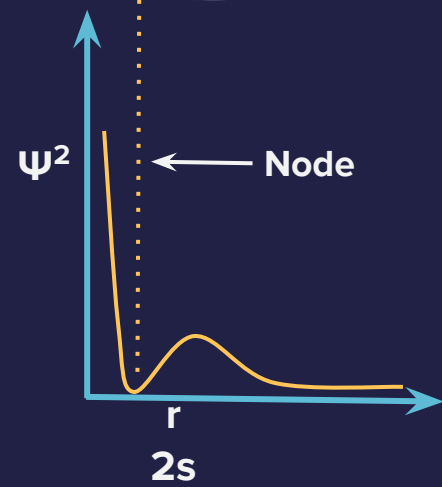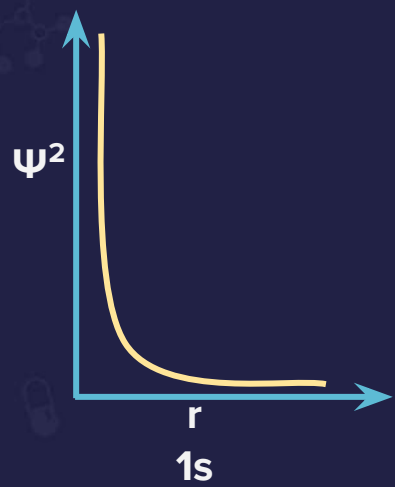
Radial Wavefunction of Hydrogenic Atoms

$$R_{nl}(r) = \left(\frac{2Z}{na_0}\right)^{3/2} \left(\frac{(n-l-1)!}{2n\left[(n+l)!\right]^3}\right)^{1/2} e^{-Zr/na_0} \left(\frac{2Zr}{na_0}\right)^{l} L_{n-l-1}^{2l+1}\left(\frac{2Zr}{na_0}\right)$$

Laguerre Polynomial

$$L_{n-l-1}^{2l+1}(\varrho) = \sum_{i=0}^{n-l-1} \frac{(-1)^i \left[(n+l)! \right]^2 \varrho^i}{i! \, (n-l-1-i)! \, (2l+1+i)!}$$

where $\varrho = 2kr$ and $k_n = \dfrac{Z}{a_0 n}$

1s orbital

$$R(1s) = K_1 \sigma e^{-\sigma/2}$$

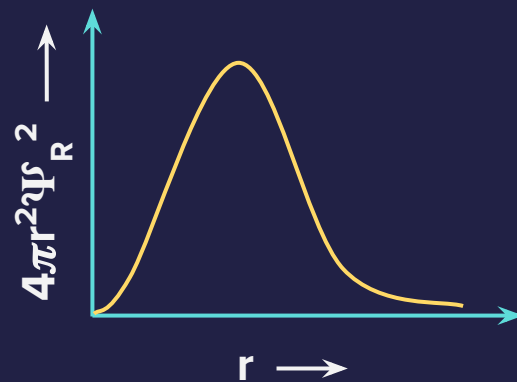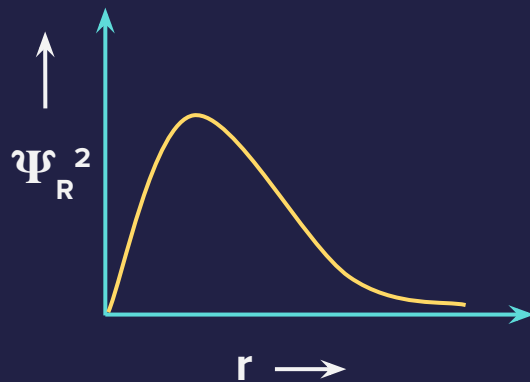where, $\sigma = \dfrac{2Zr}{n\, a_0}$

Number of radial nodes $= n - l - 1 = 1 - 0 - 1 = 0$

Radial Probability of 1s
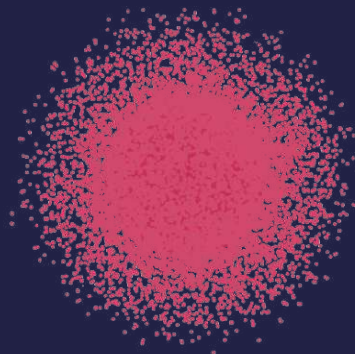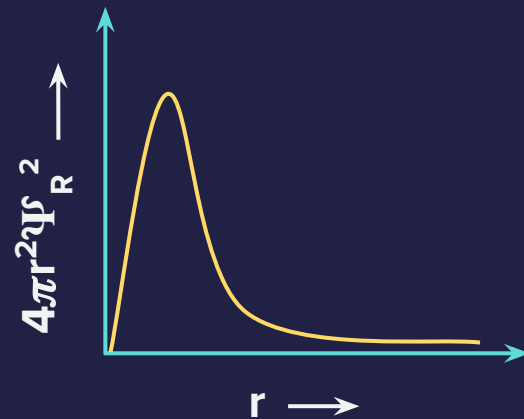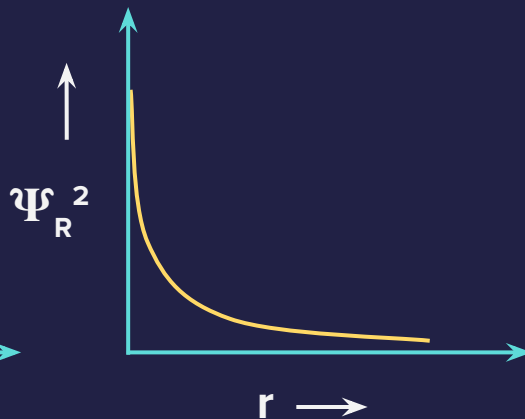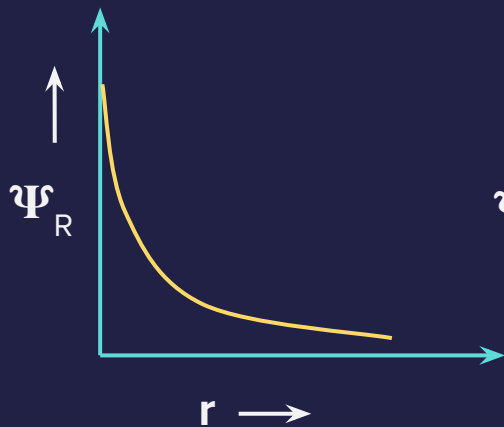
2s orbital

$$R(2s) = K_2(2 - \sigma)\, e^{-\sigma/2}$$

Number of radial nodes $= n - l - 1 = 2 - 0 - 1 = 1$

Radial Probability of 2s

Ψ_R

$r \longrightarrow$

$\Psi_R{}^2$

Node

$r \longrightarrow$

$4\pi r^2 \Psi_R{}^2$

$r \longrightarrow$
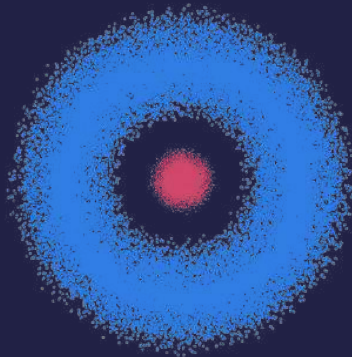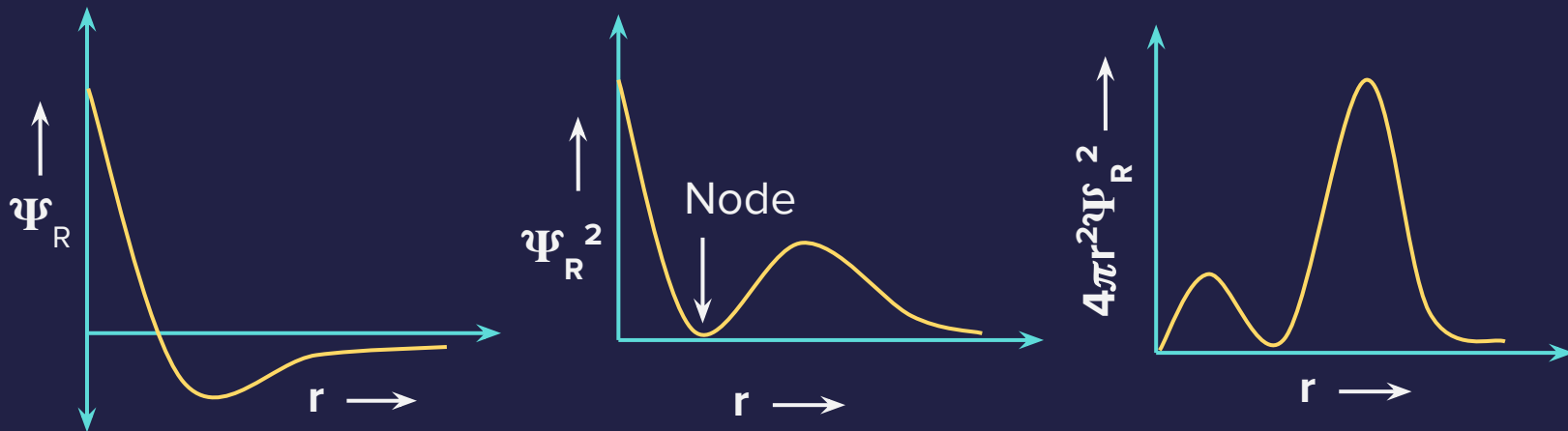
p-orbitals

p-orbitals consist of **lobes**
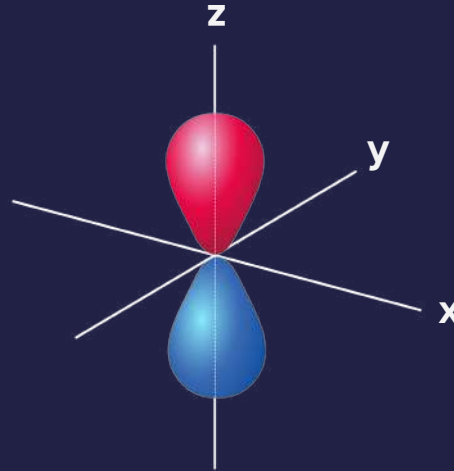
Dumb bell Shaped

2p orbital

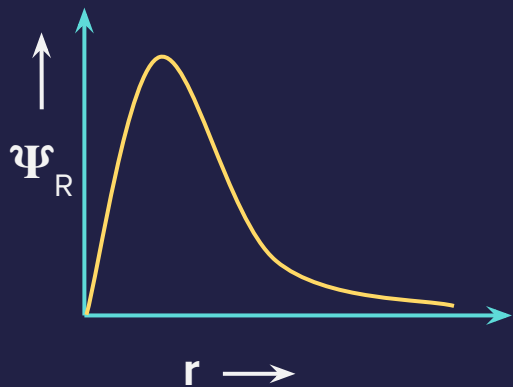$$R(2p) = K_3 \sigma e^{-\sigma/2}$$

Number of radial nodes $= n - l - 1 = 2 - 1 - 1 = 0$

Radial Probability of 2p
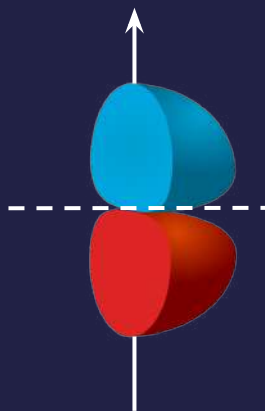
3p orbital

$$R(3p) = K_4(4 - \sigma)\,\sigma\,e^{-\sigma/2}$$
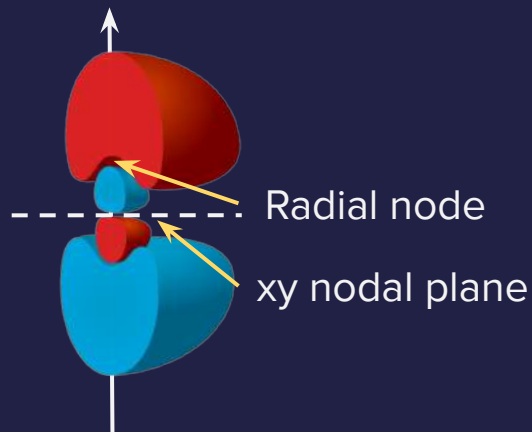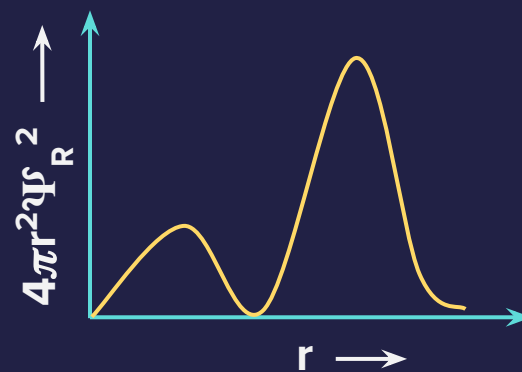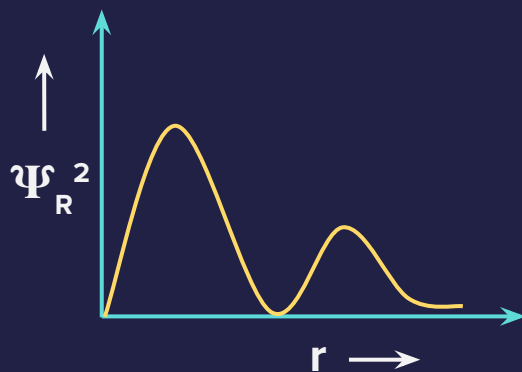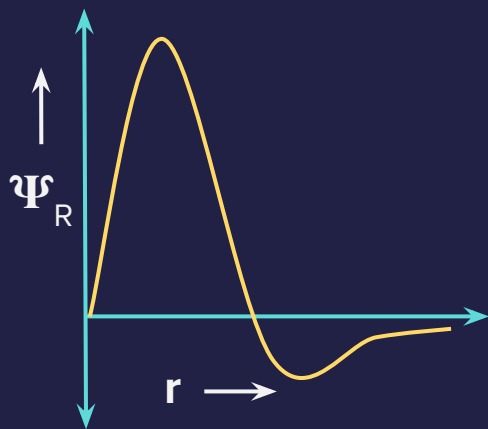
Number of radial nodes $= n - l - 1 = 3 - 1 - 1 = 1$

Radial Probability of 3p

Radial node

xy nodal plane

Nodes in p-orbital

$3p_z$ orbital

$2p_z$ orbital

z

z

Radial node

xy nodal plane
(Angular node)

xy nodal plane
(Angular node)
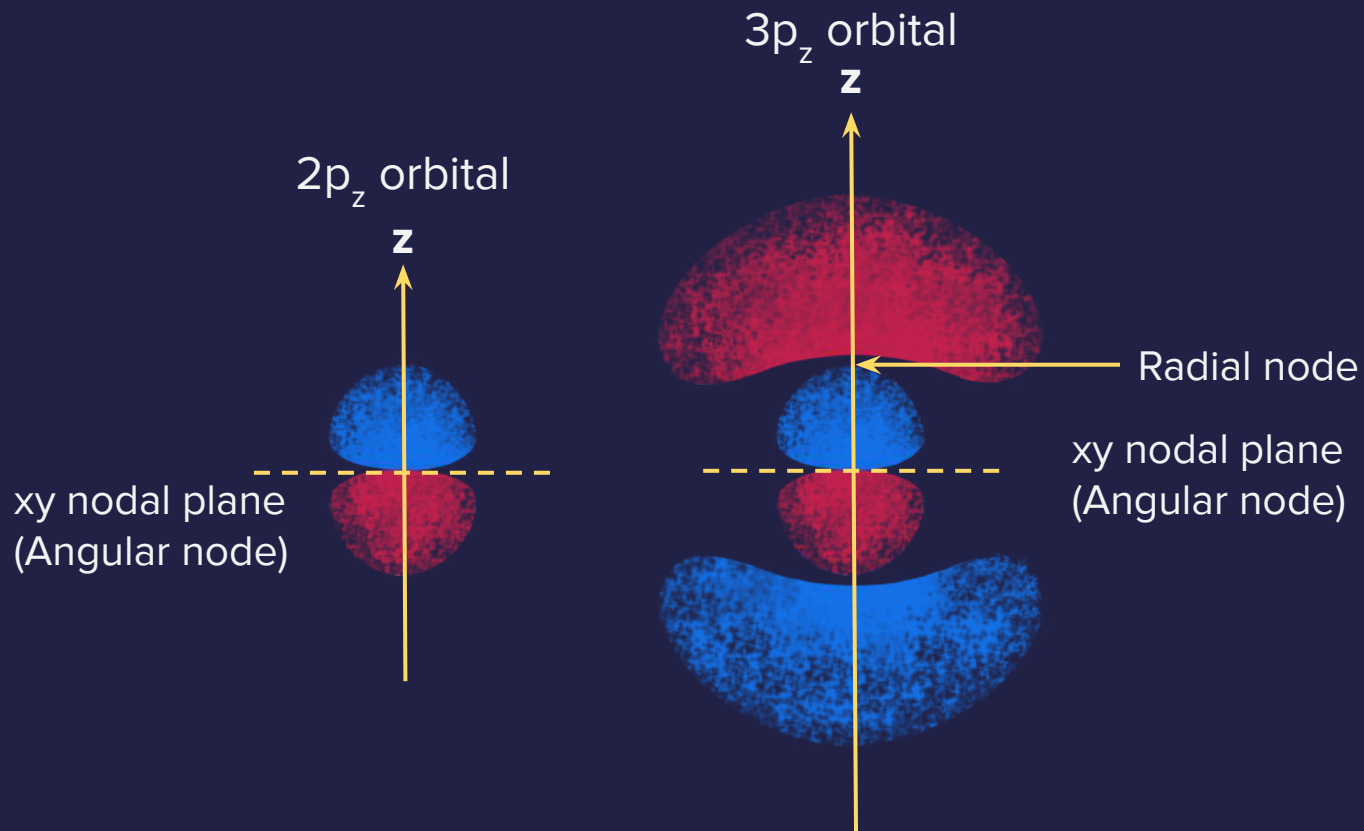
Nodes in d-orbitals
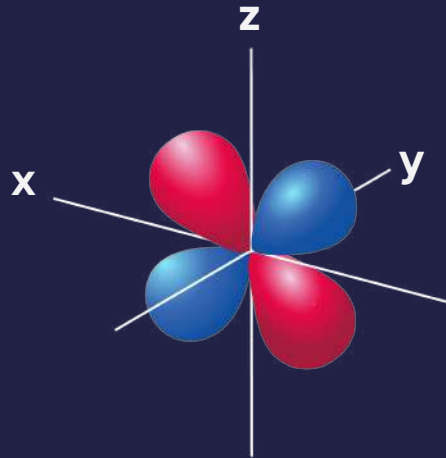
Double Dumb bell Shaped

3d orbital

$$R(3d) = K_5 \sigma^2 e^{-\sigma/2}$$

Number of radial nodes $= n - l - 1 = 3 - 2 - 1 = 0$