

Welcome to



# Aakash



## BYJU'S NOTES

### Magnetism and Matter





# Introduction to Magnet



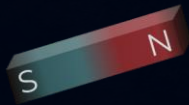
Magnet is a material that produces **magnetic field** and is able to **attract** materials like **iron**, **nickel** and **cobalt**.

## Types of Magnets

Permanent Magnets

Temporary Magnets

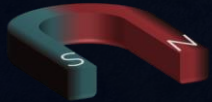
Electromagnets



Bar  
Magnet



Magnetic  
Needle

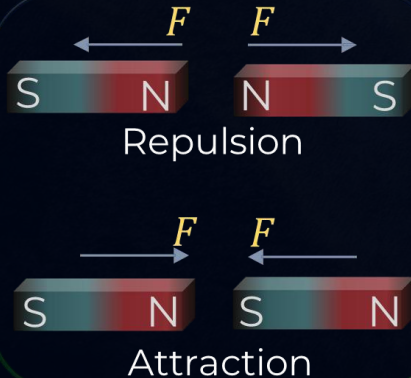
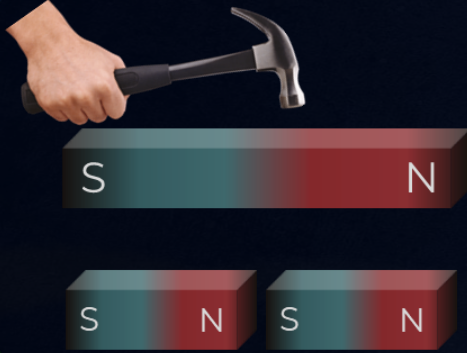


U-shaped  
Magnet





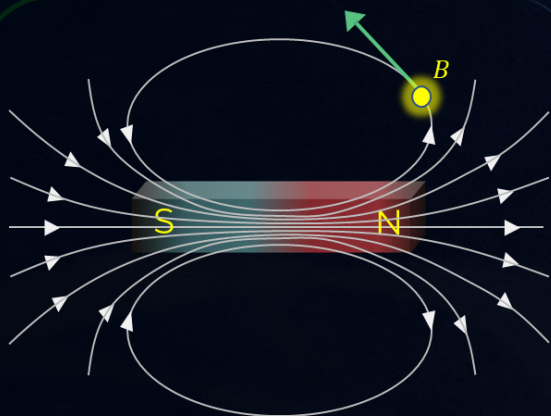
## Bar Magnet



- A **bar magnet** is a rectangular piece of object made up of **iron, steel** or any other substance that shows permanent **magnetic properties**.
- It produces its **own** persistent **magnetic field**.
- The **poles** of a bar magnet are situated near the **ends**.
- The bar magnet has two **poles**: North pole (**N**) and South pole (**S**).
- Magnetic **monopoles** do **not** exist.
- **Like** poles **repel** each other and **unlike** poles **attract** each other.
- Forces form **action-reaction pair** and act along the line joining the magnets.



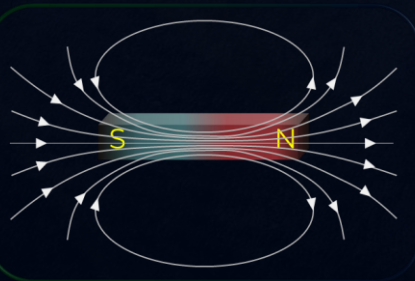
## Properties of Magnetic Field Lines



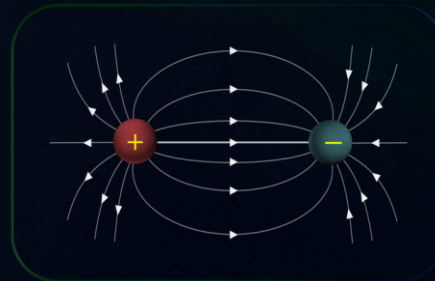
- Magnetic field lines **never intersect** with each other.
- Magnetic field lines appear to **originate** from the **North** pole and **terminate** at the **South** pole.
- **Inside** the magnet, the direction of the field lines is from the **South** pole to the **North** pole.
- The **tangent** at any point on the field line gives the **direction** of the field.
- The **density** of the field lines is directly proportional to the **strength** of field.

## Analogy between Bar Magnet and Electric Dipole

Magnetic Dipole



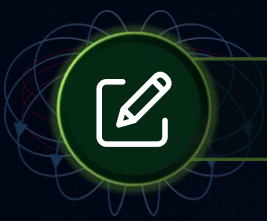
Electric Dipole



North pole  $\equiv +ve$  Charge  $= +m$

South pole  $\equiv -ve$  Charge  $= -m$





## Magnetic Dipole Moment

**Pole Strength ( $m$ ):**

The pole strength is equivalent to electric charge.

**Geometric Length ( $l_g$ ):**

It is the actual length of the magnet.

**Magnetic Length ( $l_m$ ):**

It is the distance between the two poles of the magnet.

$$l_m \approx 0.84 l_g$$

- A **bar magnet** is identical to a **magnetic dipole**.
- **Magnetic moment** is a **vector** quantity, and its direction is from **South** pole to **North** pole.



$$\vec{M} = m \vec{l}$$

$m$  = Pole Strength

$\vec{l}$  = Magnetic length

?

A straight steel wire of length ' $l$ ' has a magnetic moment ' $M$ '. It is bent into two different shapes as shown. Find the ratio of net magnetic moments of system A and system B.

system A



system B



Let us assume the pole strength of steel wire to be  $m$

$$M_A = ml' = m \left( 2 \frac{l}{2} \sin 30^\circ \right) = \frac{1}{2} ml = \frac{M}{2}$$

$$M_B = ml'' = m(2R) = m \left( \frac{2l}{\pi} \right) = \frac{2}{\pi} ml = \frac{2M}{\pi}$$

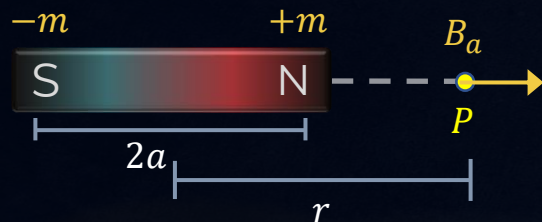
$$\frac{M_A}{M_B} = \frac{M/2}{2M/\pi} = \frac{\pi}{4}$$



## Magnetic field due to Magnetic dipole



Axial Point

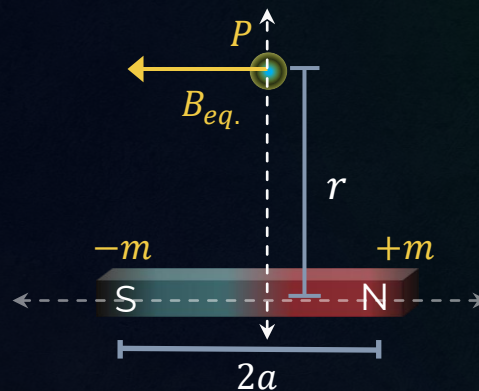


$$B_{ax} = \left( \frac{\mu_0}{4\pi} \right) \frac{2Mr}{(r^2 - a^2)^2}$$

For  $r \gg l$ , at any axial point

$$\vec{B}_{ax} = \left( \frac{\mu_0}{4\pi} \right) \frac{2\vec{M}}{r^3}$$

Equatorial Point

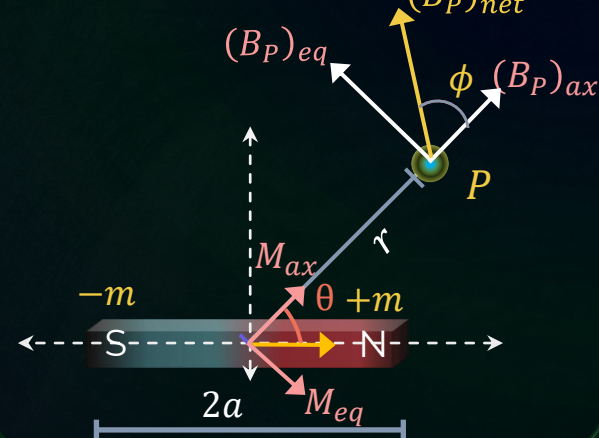


$$B_{eq} = \left( \frac{\mu_0}{4\pi} \right) \frac{M}{(r^2 + a^2)^{\frac{3}{2}}}$$

For far off points,  $r \gg l$

$$\vec{B}_{eq} = \left( \frac{\mu_0}{4\pi} \right) \frac{\vec{M}}{r^3}$$

General Point



$$|\vec{B}_{net}| = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3 \cos^2 \theta}$$

$$|\vec{B}_{net}| = \sqrt{B_{eq}^2 + B_{ax}^2} \quad \tan \phi = \left| \frac{B_{eq}}{B_{ax}} \right| = \frac{\tan \theta}{2}$$



?

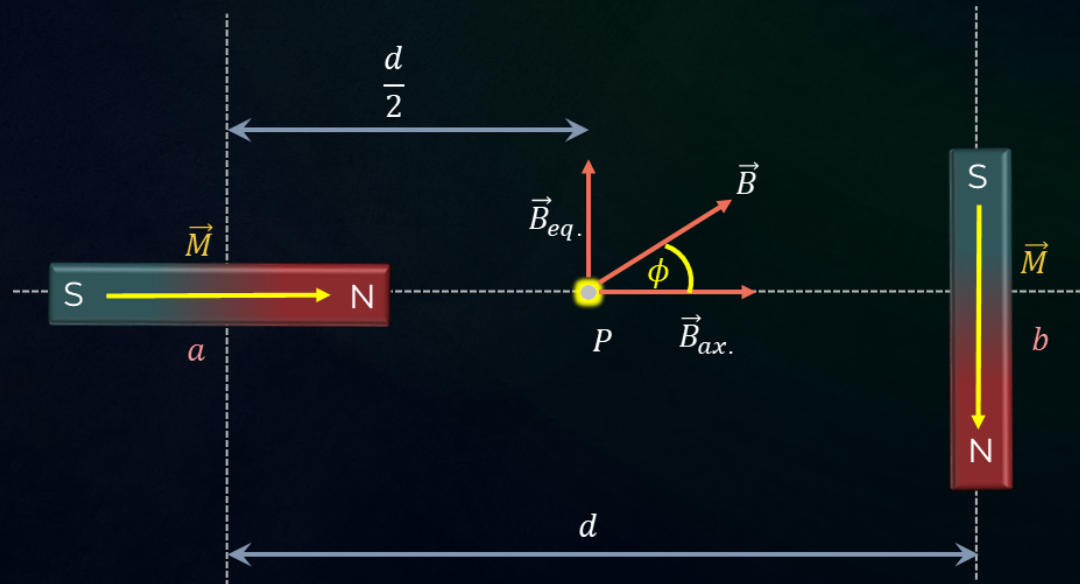
Two identical magnetic dipoles  $a$  and  $b$  of magnetic moments  $M$  each, are placed at a separation  $d$ , with their axes perpendicular to each other as shown. Find the magnetic field at the point  $P$  midway between the dipoles, in magnitude and direction.

$$\vec{B}_{ax.} = \left( \frac{\mu_0}{4\pi} \right) \frac{2\vec{M}}{r^3} = \frac{4\mu_0 M}{\pi d^3} = 2B_0$$

$$\vec{B}_{eq.} = - \left( \frac{\mu_0}{4\pi} \right) \frac{\vec{M}}{r^3} = \frac{2\mu_0 M}{\pi d^3} = B_0$$

$$|\vec{B}_{net}| = \sqrt{B_{eq}^2 + B_{ax}^2} = \sqrt{5}B_0 = \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

$$\tan \phi = \left| \frac{B_{eq}}{B_{ax}} \right| = \frac{B_0}{2B_0} = \frac{1}{2}$$

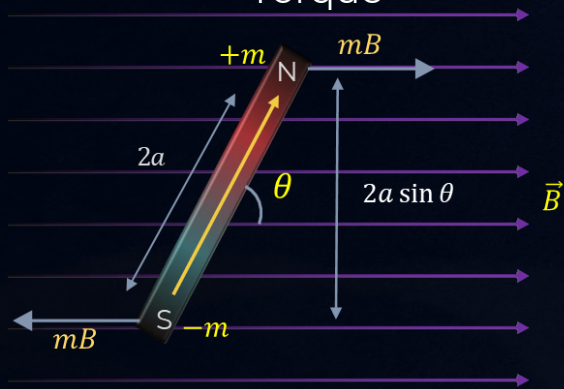




# Bar Magnet in Uniform External field



## Torque



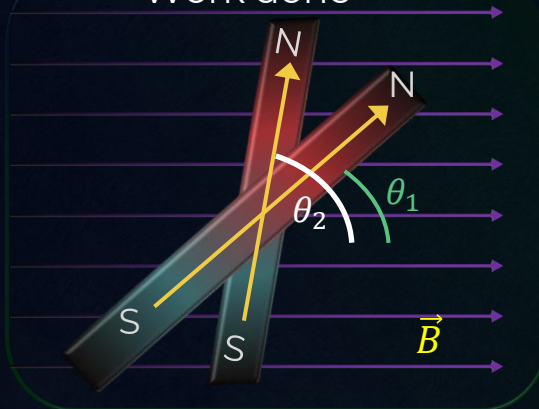
$$\tau = mB \times 2a \sin \theta$$

$$\tau = MB \sin \theta$$

$$\vec{F}_{net} = 0$$

$$\vec{\tau} = \vec{M} \times \vec{B}$$

## Work done



$$dW = \tau d\theta$$

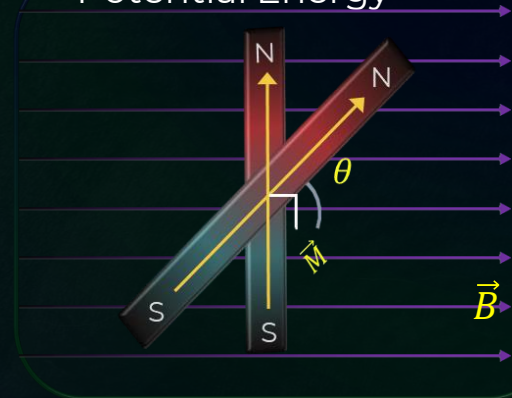
$$dW = MB \sin \theta d\theta$$

On integration,

$$W = -MB(\cos \theta_2 - \cos \theta_1)$$

$$W_{mag} = MB(\cos \theta_2 - \cos \theta_1)$$

## Potential Energy



$$\Delta U = -W_{mag}$$

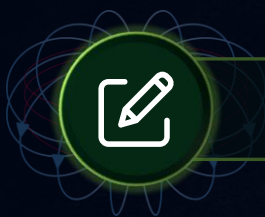
$$U_f - U_i = -MB(\cos \theta_2 - \cos \theta_1)$$

$$\theta_2 = \theta, \theta_1 = 90^\circ \text{ and } U_i = 0$$

$$U_f - 0 = -MB(\cos \theta_2 - \cos 90^\circ)$$

(Taking P.E. = 0 at  $\theta = 90^\circ$  as reference point)

$$U(\theta) = -MB(\cos \theta) = -\vec{M} \cdot \vec{B}$$



## Analogy between Electricity and Magnetism

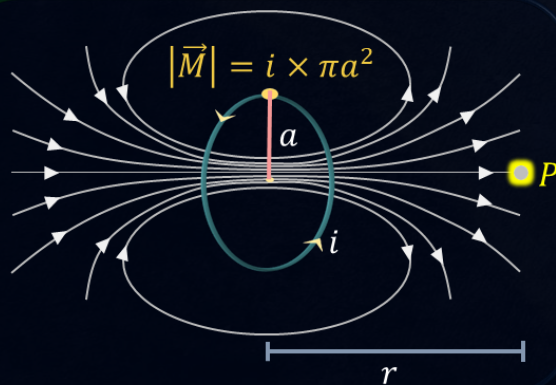
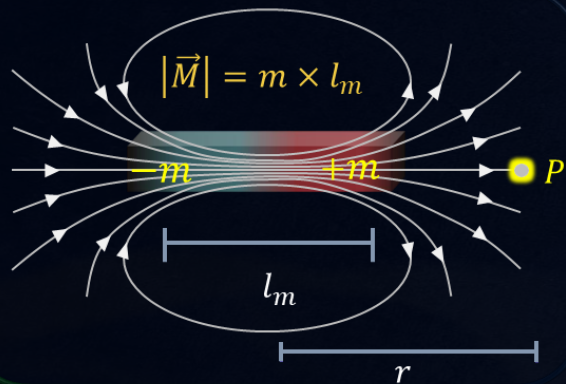


| Physical Quantity                   | Electrostatics   | Magnetism  |
|-------------------------------------|--|--|
|                                     | $1/\epsilon_0$   | $\mu_0$  |
| Dipole moment                       | $\vec{p}$  | $\vec{M}$  |
| Axial field for a dipole            | $\vec{E}_{ax} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$ | $\vec{B}_{ax} = \frac{\mu_0}{4\pi} \frac{2\vec{M}}{r^3}$ |
| Equatorial field for a dipole       | $\vec{E}_{eq} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3}$ | $\vec{B}_{eq} = -\frac{\mu_0}{4\pi} \frac{\vec{M}}{r^3}$ |
| In external field: Torque           | $\vec{p} \times \vec{E}$                                       | $\vec{M} \times \vec{B}$                                 |
| In external field: Potential Energy | $-\vec{p} \cdot \vec{E}$                                       | $-\vec{M} \cdot \vec{B}$                                 |

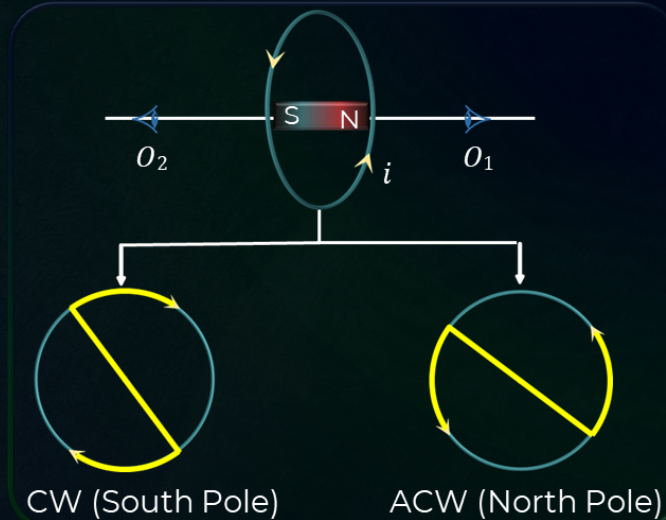




## Current carrying loop as Equivalent Dipole



$$\vec{B}_P = \frac{\mu_0}{4\pi} \frac{2\vec{M}}{r^3}$$



For observer 1, Current in loop is

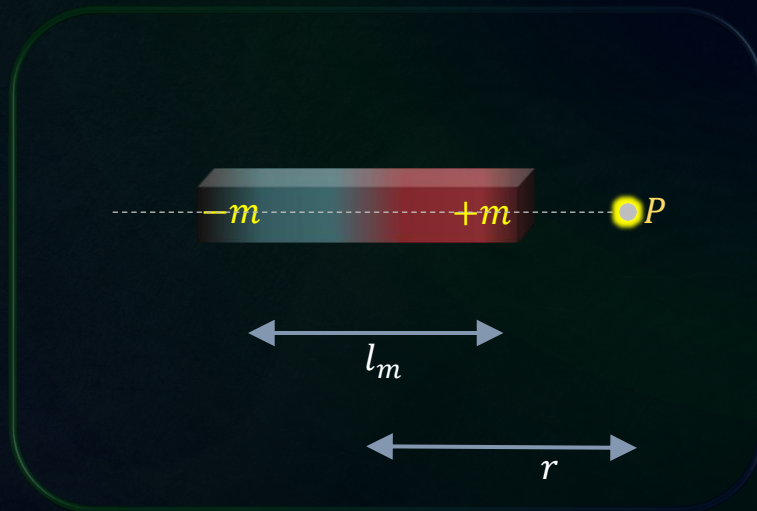
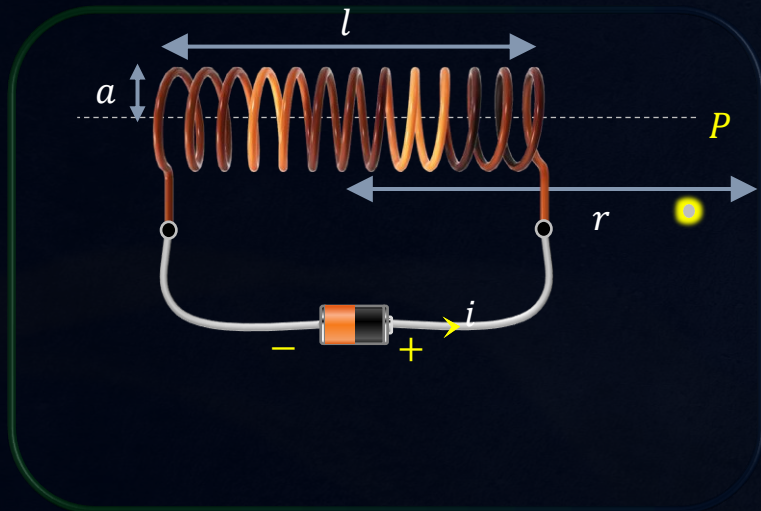
Anti-clockwise → North Pole

For observer 2, Current in loop is

Clockwise → South Pole



## Solenoid as Equivalent Bar Magnet



$$\vec{B}_P = \frac{\mu_0}{4\pi} \frac{2\vec{M}}{r^3}$$



?

A small bar magnet placed with its axis at  $30^\circ$  with an external field of  $0.06\text{ T}$  experiences a torque of  $0.018\text{ Nm}$ . The minimum work required to rotate it from its stable to unstable equilibrium position is:

Given:  $B = 0.06\text{ T}$ ,  $\tau = 0.018\text{ Nm}$ ,  $\theta = 30^\circ$

Solution:

$$\tau = 0.018 = MB \sin 30^\circ \Rightarrow M = 0.6\text{ Am}^2$$

- Position of stable equilibrium ( $\theta = 0^\circ$ )
- Position of unstable equilibrium ( $\theta = 180^\circ$ )



$$W = MB(\cos \theta_1 - \cos \theta_2) = MB(\cos 0^\circ - \cos 180^\circ)$$

$$W = 2MB = 2 \times 0.6 \times 0.06$$

$$W = 7.2 \times 10^{-2}\text{ J}$$



## Angular SHM of Dipole

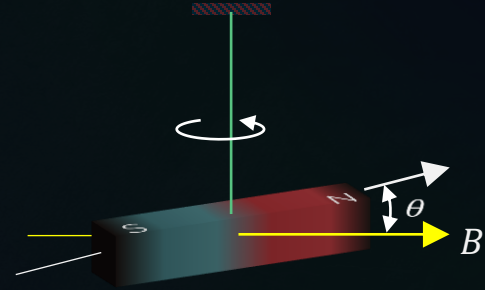
$$\tau = -MB \sin \theta = -MB\theta \quad [\text{for very small } \theta, \sin \theta \approx \theta]$$

$$I \frac{d^2\theta}{dt^2} = -MB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{MB}{I} \theta \quad \therefore \omega^2 = \frac{MB}{I}$$

So, dipole will execute angular SHM with time period :

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

Rotate by a small angular displacement  $\theta$



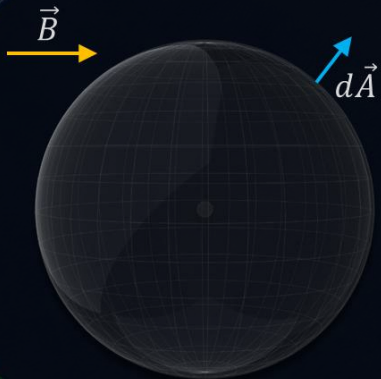
## Gauss's Law for Magnetism

It states that the net magnetic flux through any closed surface is zero.

$$\phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Reason:

- Magnetic monopoles do not exist
- There are no sources or sinks of magnetic field ( $\vec{B}$ )





?

The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is  $2\text{ s}$ . The magnet is cut along its length into **three equal** parts and these parts are then placed on each other with their like poles together. The **time period** of this combination will be.

1



Given:  $T = 2\text{ s}$      $T = 2\pi \sqrt{\frac{I}{MB}}$     Where  $I = \frac{1}{12} m_b l^2$

**Solution:** When the magnet is cut into three pieces the pole strength ( $m$ ) will **remain the same** and

Moment of inertia of new magnet,  $I' = \frac{1}{12} \left( \frac{m_b}{3} \right) \left( \frac{l}{3} \right)^2 \times 3 = \frac{I}{9}$

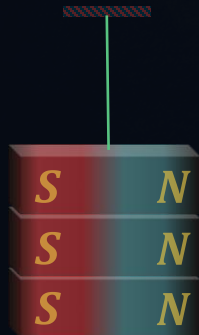
Magnetic moment ( $M$ ) =  $ml$

$\therefore$  New magnetic moment,  $M' = m \left( \frac{l}{3} \right) \times 3 = ml = M$

$\therefore$  New time period,  $T' = 2\pi \sqrt{\frac{I'}{M'B}} = 2\pi \sqrt{\frac{I}{9MB}} = T/3$

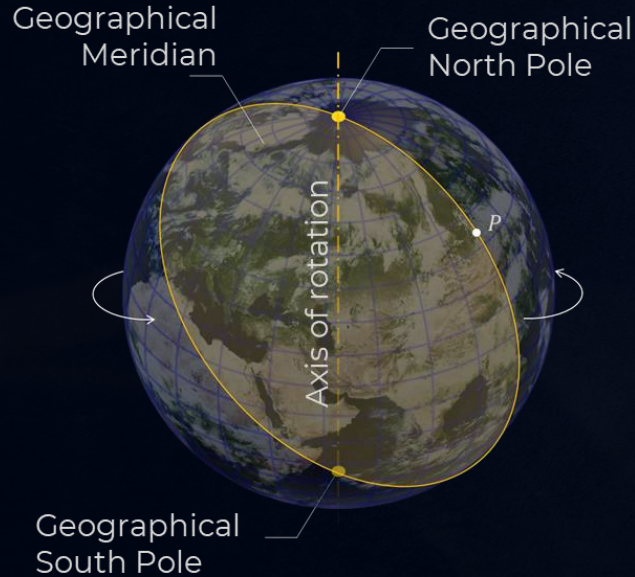
$T' = \frac{T}{3} = \frac{2}{3}\text{ s}$

2

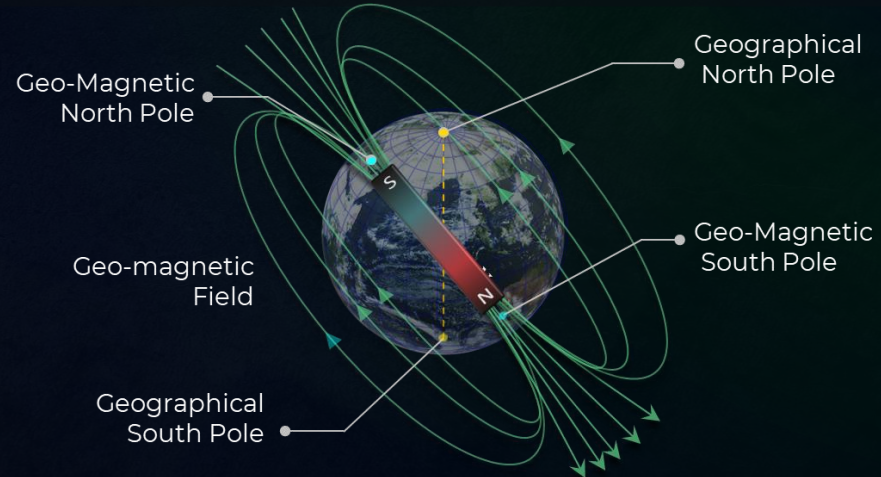




# Earth's Magnetism



A plane passing through the geographical poles and a given point  $P$  on the earth's surface is called the **Geographical Meridian** at the point  $P$



## Magnetic Elements of Earth

Magnetic declination

Magnetic inclination(Dip)

Horizontal component of  
earth's magnetic field





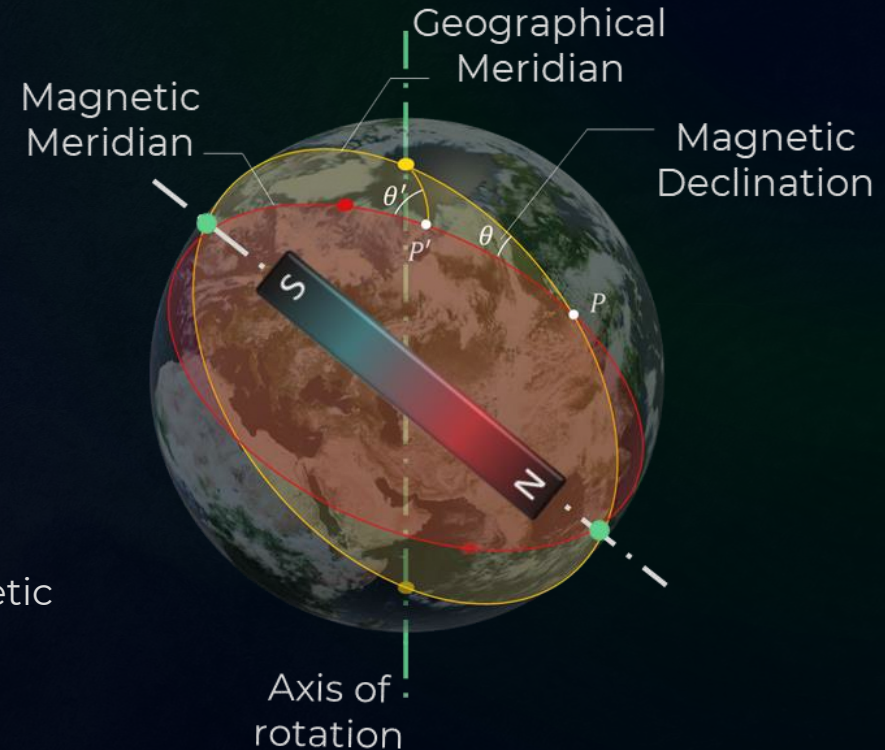
## Magnetic Declination



The angle made by the magnetic meridian at a point with the geographical meridian is called **Declination** at that point.

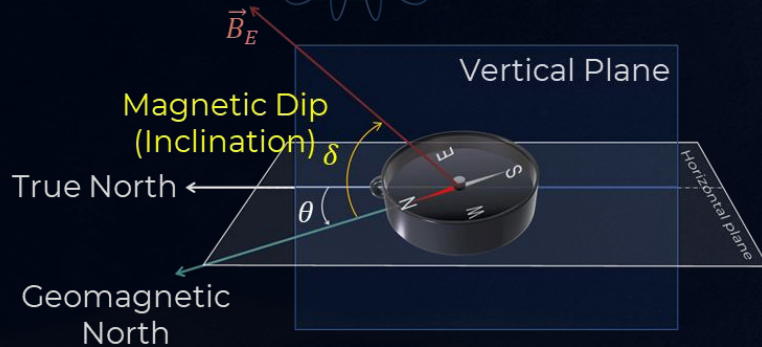
$\theta$  : Angle of declination

$\theta' > \theta$  ( As we move closer to the Pole, Magnetic Declination increases )

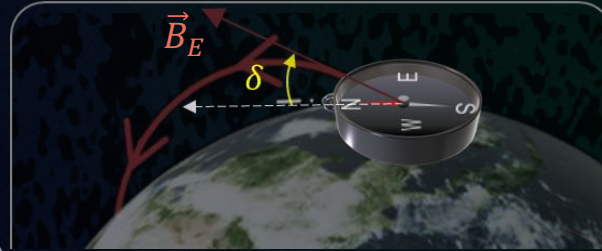




## Magnetic Inclination (Dip)

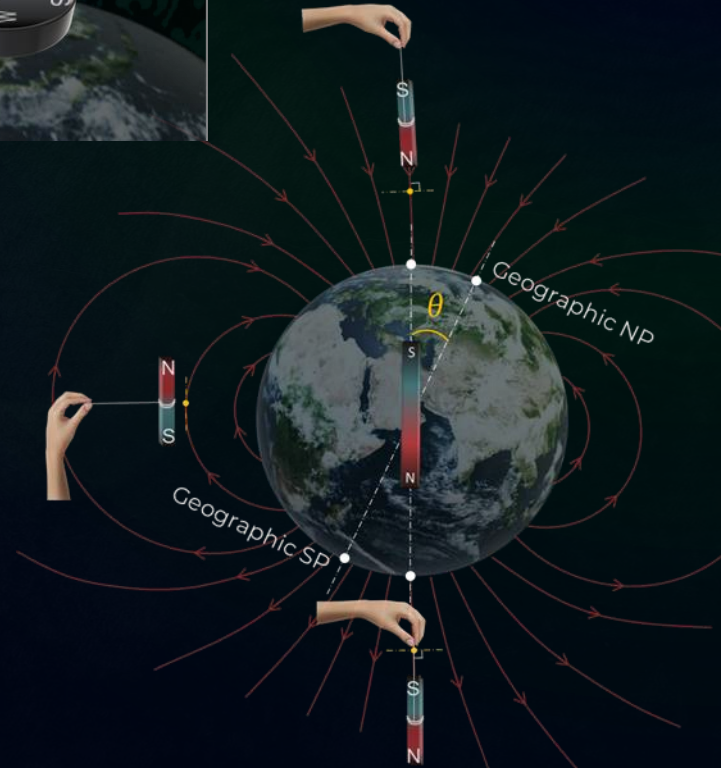


$\delta$  : Angle of dip



The angle made by the Earth's magnetic field with the horizontal direction in the magnetic meridian is called **Inclination or Dip** at that point.

- Near magnetic north pole angle of dip is **90°**.
- Near magnetic south pole angle of dip is **90°**.
- Near magnetic equator, dip is **zero**.





?

The earth's magnetic field at geomagnetic poles has a magnitude  $6.2 \times 10^{-5} \text{ T}$ . Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of  $135^\circ$  with the axis of the earth's assumed magnetic dipole. What is the **inclination (dip)** at this point?

$$|B_p| = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3(\cos \theta)^2}$$

At  $\theta = 0^\circ \rightarrow B_p = 6.2 \times 10^{-5} \text{ T}$

$$\frac{\mu_0 M}{4\pi r^3} = 3.1 \times 10^{-5} \text{ T}$$

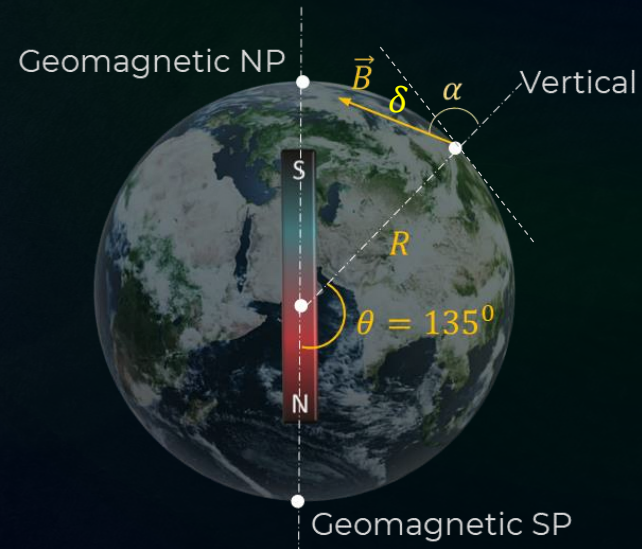
At  $\theta = 135^\circ \rightarrow B = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3/2}$

$$B = 4.9 \times 10^{-5} \text{ T}$$

$$\tan \alpha = \frac{\tan \theta}{2} = \frac{\tan 135^\circ}{2} = -0.5$$

$$\alpha = 153^\circ$$

$$\delta = 153^\circ - 90^\circ = 63^\circ$$





## Horizontal component of earth's magnetic field



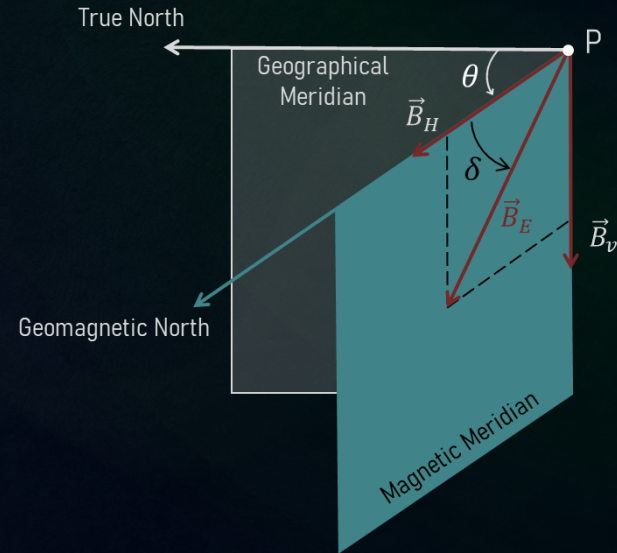
- Magnetic needle gives the direction of **Horizontal Component** of earth's magnetic field.

$$B_E = B_H / \cos \delta$$

$$\delta = \tan^{-1} \frac{B_V}{B_H}$$

- Tangent Law**

$$B_V = B_H \tan \delta$$





?

A magnet makes **10** oscillations per minute at a place where the angle of dip is **45°** and the total intensity is **0.4 gauss**. Calculate the **frequency of oscillations** at another place where the angle of dip is **60°** and the intensity of the field is equal to **0.5 gauss**.

In case of angular SHM of a magnet in magnetic field :

$$T = 2\pi \sqrt{\frac{I}{MB_H}} = 2\pi \sqrt{\frac{I}{MB_E \cos \theta}}$$

For a given magnet at two different places, we would have :

$$\frac{T_2}{T_1} = \sqrt{\frac{B_1 \cos \theta_1}{B_2 \cos \theta_2}}$$

Given :

$$B_1 = 0.4 \text{ gauss} \quad \theta_1 = 45^\circ$$

$$B_2 = 0.5 \text{ gauss} \quad \theta_2 = 60^\circ$$

$$T_1 = \frac{60}{10} = 6 \text{ sec}$$

$$\frac{T_2}{6} = \sqrt{\frac{0.4 \times \cos 45^\circ}{0.5 \times \cos 60^\circ}}$$

$$T_2 = 6.38 \text{ sec}$$

$$\frac{1}{f_2} = 6.38 \text{ sec}$$

$$f_2 = 0.157 \text{ Hz}$$

?

A magnetic needle having magnetic moment  $10 \text{ Am}^2$  and length  $2.0 \text{ cm}$  is clamped at its center in such a way that it can rotate in vertical east-west plane. A horizontal force towards east is applied at the north pole to keep the needle fixed at an angle of  $30^\circ$  with the vertical. Find the magnitude of the applied force. The vertical component of the earth's magnetic field is  $40 \mu\text{T}$ .

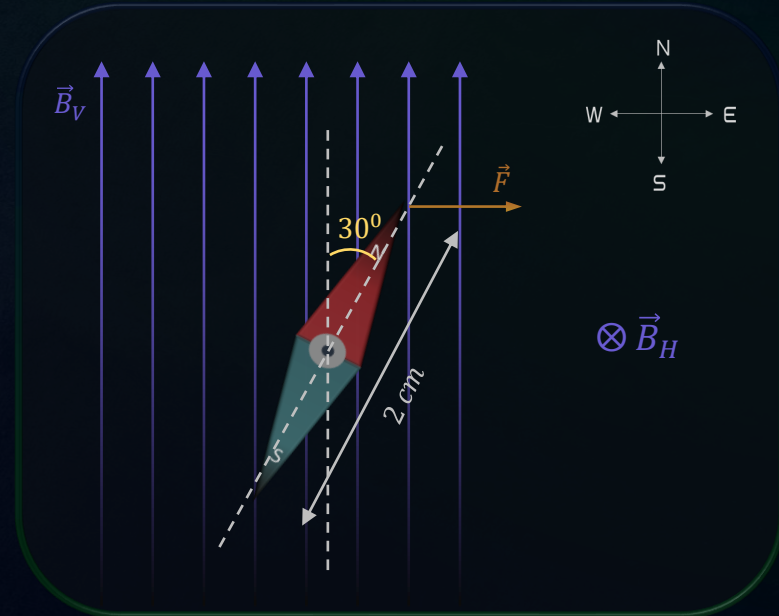
- $\vec{B}_H$  doesn't produce any torque on the needle.
- For rotational equilibrium :

$$|\vec{M} \times \vec{B}_V| = |\vec{r} \times \vec{F}|$$

$$MB_V \sin 30^\circ = rF \sin 60^\circ$$

$$10 \times (40 \times 10^{-6}) \times \frac{1}{2} = \frac{2 \times 10^{-2}}{2} \times F \times \frac{\sqrt{3}}{2}$$

$$F = 2.3 \times 10^{-2} \text{ N}$$





?

A bar magnet of magnetic moment  $2.0 \text{ Am}^2$  is free to rotate about a vertical axis through its center. The magnet is released from the east-west position. Find the kinetic energy of the magnet as it takes the north-south position. The horizontal component of the earth's magnetic field is  $B_H = 25 \mu\text{T}$ .

- Change in potential energy due to change in orientation of bar magnet :

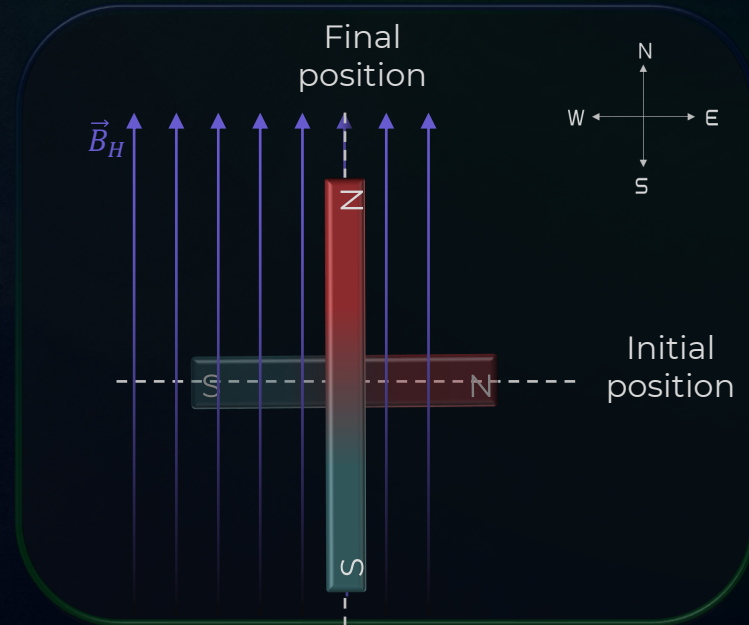
$$\Delta U = -MB [\cos \theta_f - \cos \theta_i]$$

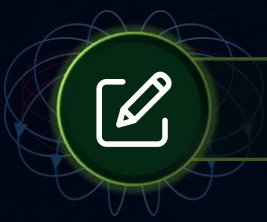
$$\Delta U = -2 \times 25 \times 10^{-6} [\cos 0^\circ - \cos 90^\circ]$$

$$\Delta U = -50 \mu\text{J}$$

- Loss in potential energy = Gain in kinetic energy
- So, kinetic energy of the magnet as it takes to north-south position is,

$$K = 50 \mu\text{J}$$

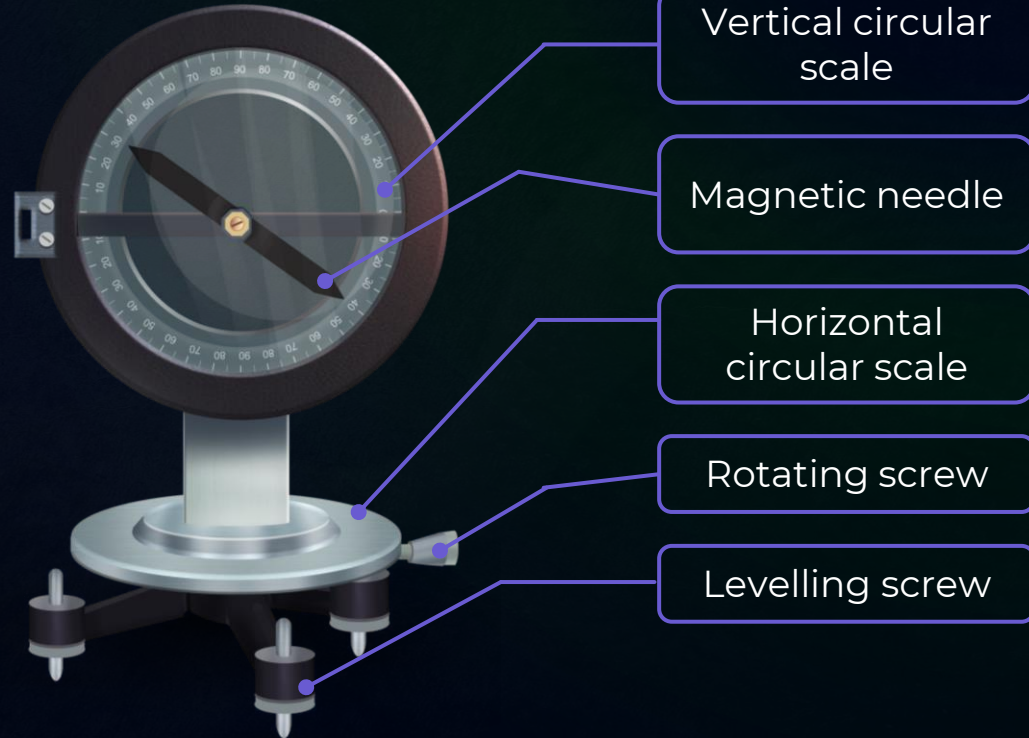




## Dip Circle : Experimental Setup



- Vertical circular scale can be rotated using the rotating screw and the angle of rotation is measured with the help of horizontal circular scale.
- Vertical circular scale is divided into four quadrants.
- Each quadrant graduates from  $0^\circ$  to  $90^\circ$  with  $0^\circ - 0^\circ$  in horizontal and  $90^\circ - 90^\circ$  in vertical.
- The horizontal circular scale is graduated from  $0^\circ$  to  $360^\circ$ .







## Steps To Measure True Dip



### Step -1

The angular position of the vertical circular scale should be such that the needle remains in  $90^\circ - 90^\circ$  position.

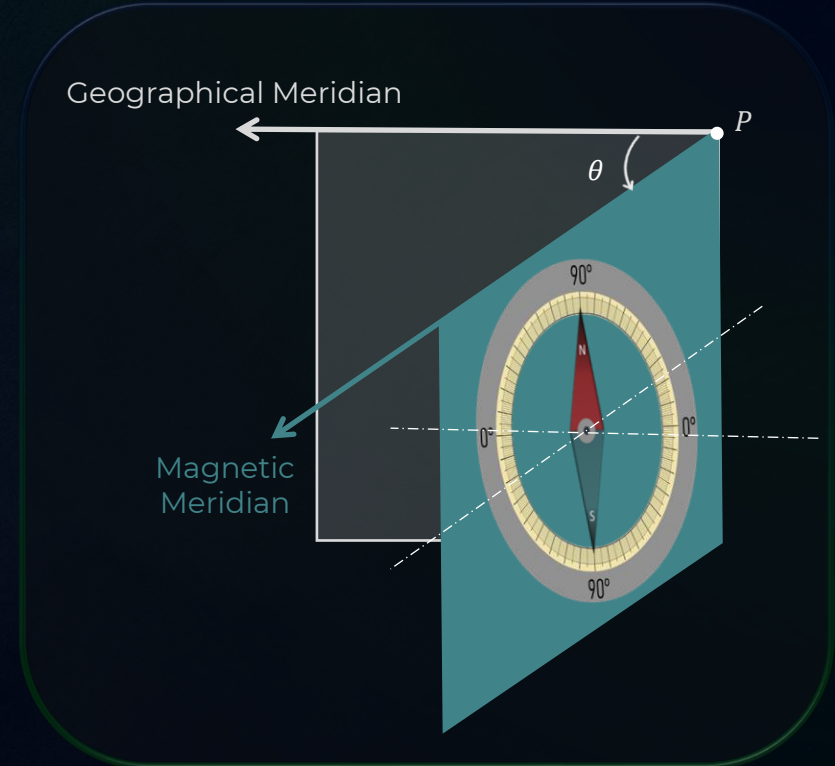
For this position of the needle,  $\vec{B}_H$  doesn't produce any force on it because  $\vec{B}_H$  is perpendicular to the needle.

### Step -2

The vertical circular scale needs to rotate by  $90^\circ$  so that it becomes parallel to magnetic meridian.

- Consequently, the needle will deflect from  $90^\circ - 90^\circ$  position and it will get aligned along the direction of Earth's magnetic field.

The angle  $\delta$  with horizontal is "Angle of dip" and this is "true dip".





## Apparent Dip



$\alpha$  : Angle between the plane of Dip circle and Magnetic Meridian

$\delta'$  : Apparent Dip

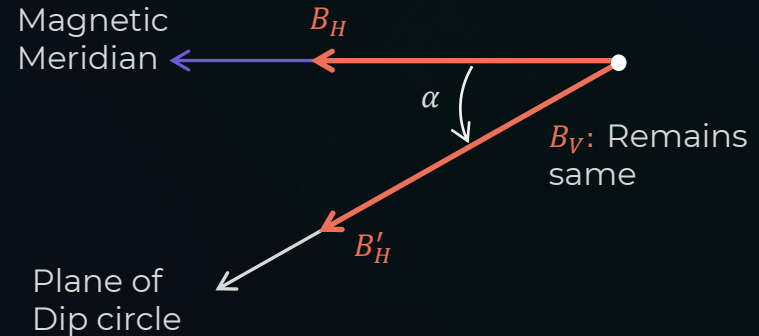
$$\tan \delta = \frac{B_V}{B_H} \quad \tan \delta' = \frac{B_V}{B'_H} \quad B'_H = B_H \cos \alpha$$

$$\cot \delta = \frac{B_H}{B_V}$$

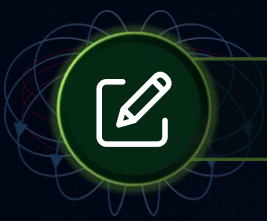
$$\cot \delta = \frac{B'_H}{B_V} \frac{1}{\cos \alpha} = \cot \delta' \frac{1}{\cos \alpha}$$

$$\cot \delta' = \cot \delta \cdot \cos \alpha$$

$$\tan \delta = \tan \delta' \cdot \cos \alpha$$







## Tangent Law



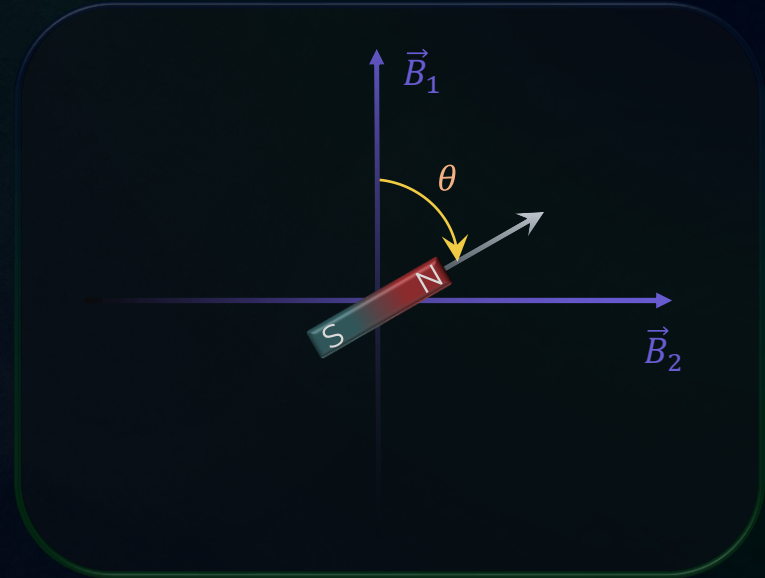
- A dipole is free to rotate in two mutually perpendicular uniform fields  $\vec{B}_1$  and  $\vec{B}_2$ .
- Magnetic moment of the dipole :  $\vec{M}$
- For rotational equilibrium :

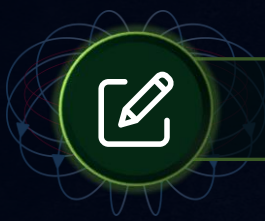
$$\vec{M} \times \vec{B}_1 = \vec{M} \times \vec{B}_2$$

$$MB_1 \sin \theta = MB_2 \sin(90^\circ - \theta)$$

$$MB_1 \sin \theta = MB_2 \cos \theta$$

$$B_2 = B_1 \tan \theta$$





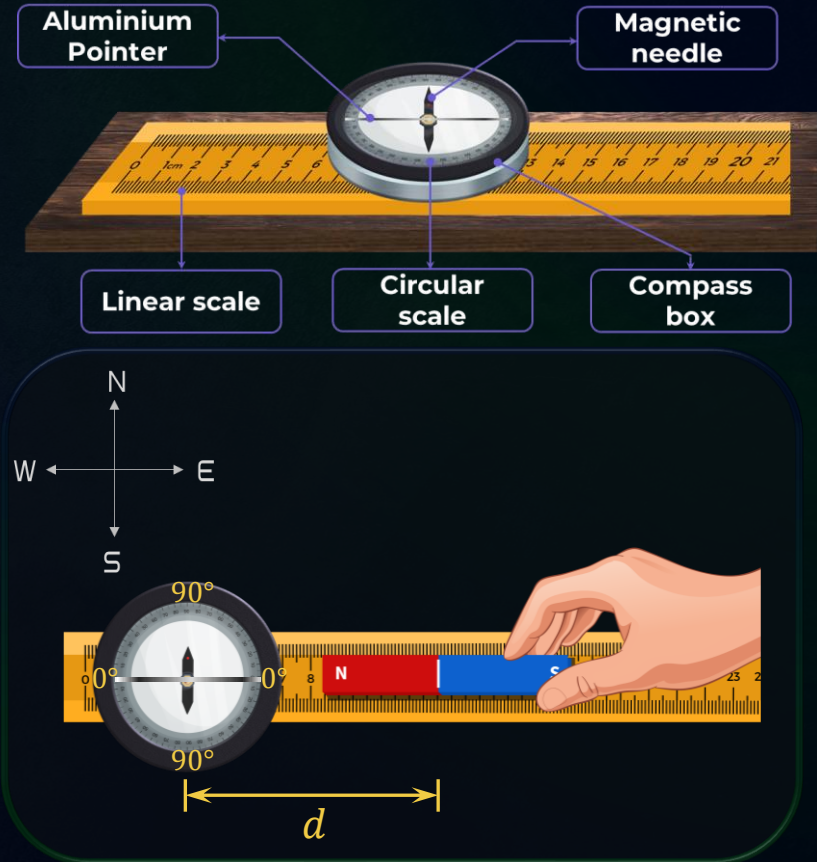
## Deflection Magnetometer : Tan A position



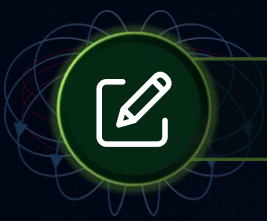
- In this position, the **arms of the magnetometer** is set along **East-West** direction i.e., **perpendicular to the magnetic meridian**.
- The external bar magnet is placed with its length parallel to the arm.
- Magnetic field due to a bar magnet at far off point on its axis :

$$B_{bar} = \frac{\mu_0}{4\pi} \left( \frac{2M}{d^3} \right)$$

- The magnetic field  $B_{bar}$  due to the bar magnet and Earth's horizontal magnetic field  $B_H$  are perpendicular to the each other.







## Deflection Magnetometer : Tan A position



- **Tangent Law** for this situation yields :

$$B_{bar} = B_H \tan \theta$$

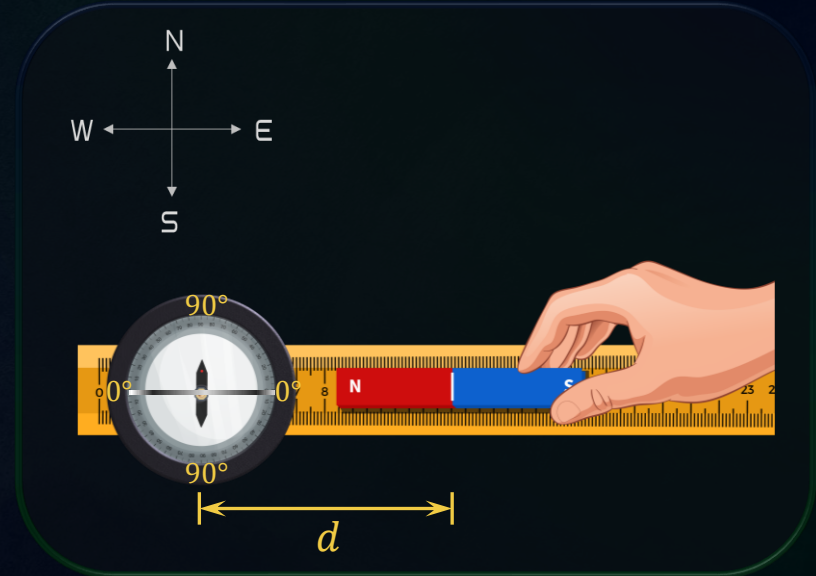
$$\frac{\mu_0}{4\pi} \left( \frac{2M}{d^3} \right) = B_H \tan \theta$$

- **Dipole moment** of the bar magnet :

$$M = \frac{4\pi B_H d^3 \tan \theta}{\mu_0 2}$$

- For two different bar magnets :

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2} \times \left( \frac{d_1}{d_2} \right)^3$$





## Deflection Magnetometer : Tan B position

- In this position, the **arms of the magnetometer** is set along **North-South** direction i.e., **along to the magnetic meridian**.
- The external bar magnet is placed with its length **perpendicular** to the arm.
- Magnetic field due to a bar magnet at far off point on the equatorial line :

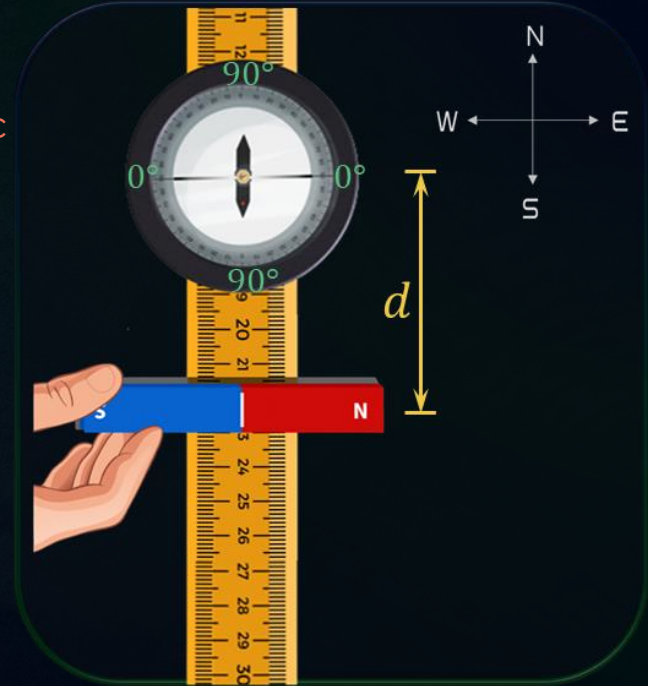
$$B_{bar} = \frac{\mu_0}{4\pi} \left( \frac{M}{d^3} \right)$$

- Tangent Law** for this situation yields :

$$B_{bar} = B_H \tan \theta \quad \frac{\mu_0}{4\pi} \left( \frac{M}{d^3} \right) = B_H \tan \theta$$

- Dipole moment** of the bar magnet :

$$M = \frac{4\pi}{\mu_0} B_H d^3 \tan \theta$$



- For two different bar magnets :

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2} \times \left( \frac{d_1}{d_2} \right)^3$$

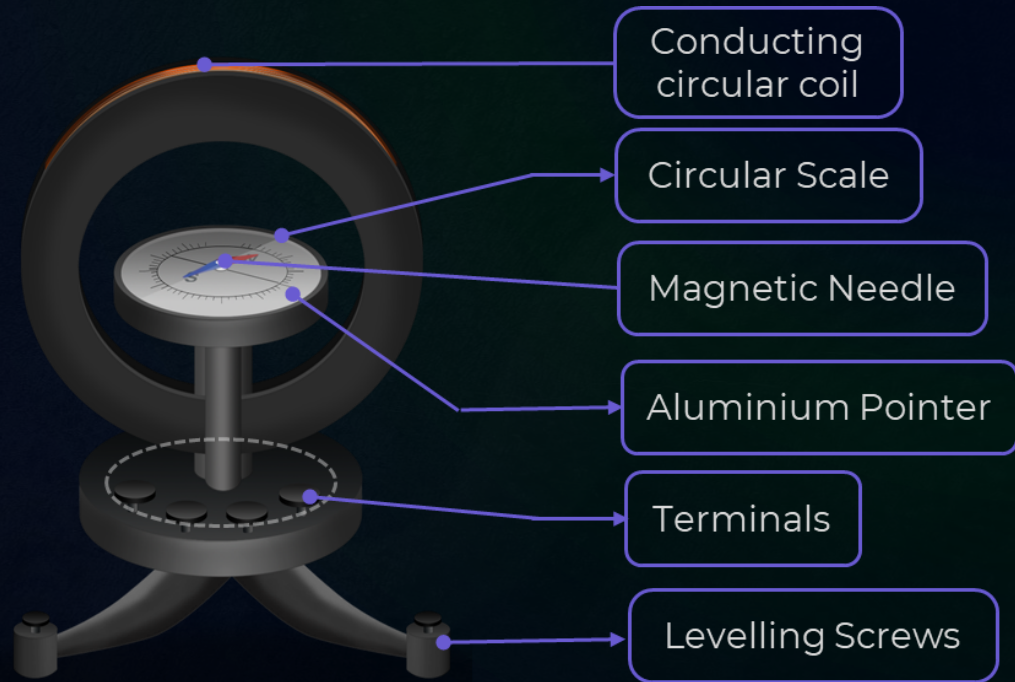




## Tangent Galvanometer : Experimental Setup



- It is an instrument used for the measurement of **electric current**.
- It uses the **Earth's magnetic field** to determine the value of the unknown current flowing through the coil.
- It works on the principle of '**tangent law of magnetism**'.





## Tangent Galvanometer



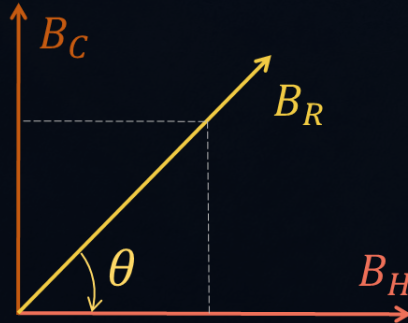
### Working :

- To find the plane of **magnetic meridian**, the compass box is rotated in such a way that the plane of the coil is **parallel** to the compass needle.
- When the current is passed through the coil, the needle points towards **net magnetic field**.

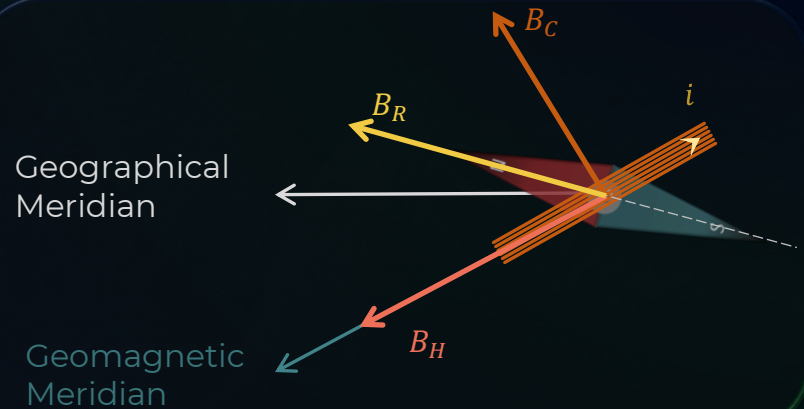
Resultant magnetic field :

$$B_R = \sqrt{B_C^2 + B_H^2}$$

$$\tan \theta = \frac{B_C}{B_H}$$



$B_C$  = Magnetic field due to conducting coil  
 $B_H$  = Magnetic field due to Earth



Geographical Meridian

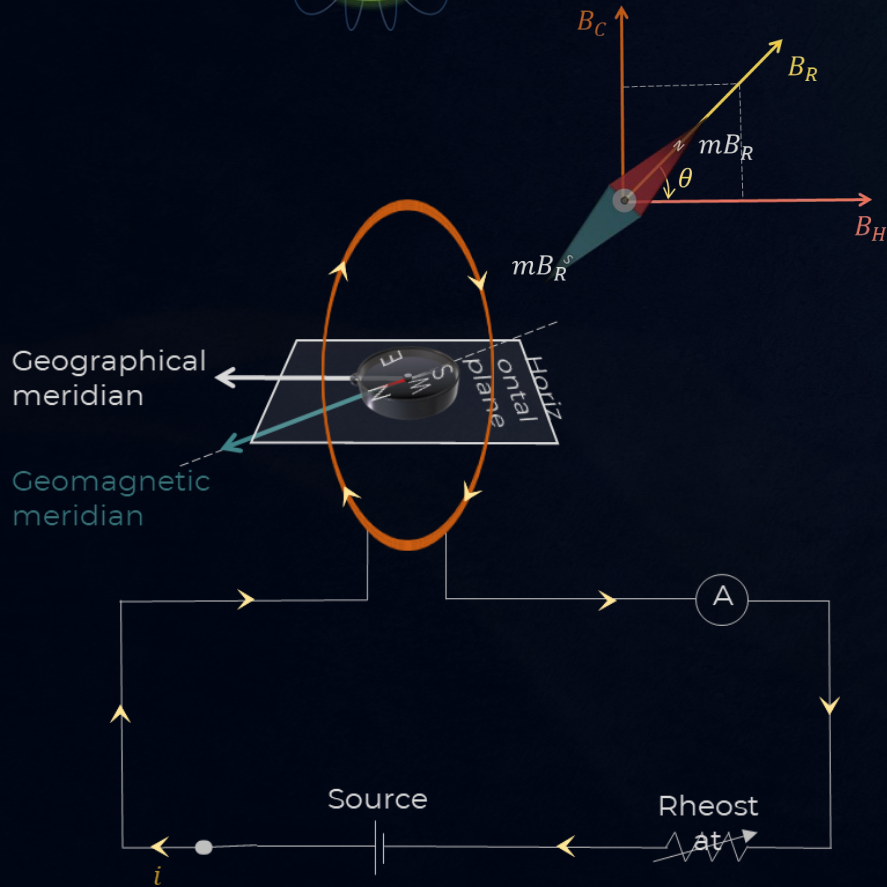
Geomagnetic Meridian

- $\theta$  can be measured on the circular scale.





## Tangent Galvanometer : Measurement of Current



$$B_C = B_H \tan \theta$$

$$\frac{\mu_0 n i}{2R} = B_H \tan \theta$$

$n$  = No. of turns in the coil

$R$  = Radius of the coil

$$i = \frac{B_H \tan \theta \cdot 2R}{\mu_0 n}$$

$$i = K \tan \theta$$

$$K = \frac{2B_H R}{\mu_0 n} = \text{Reduction factor}$$

?

Two tangent galvanometers differ only in the matter of number of turns in the coil. On passing current through the two joined in series, the first shows the deflection of  $35^\circ$  and the other shows  $45^\circ$  deflection. Compute the ratio of their number of turns. Take  $\tan 35^\circ = 0.7$

**Solution :**

As the two tangent galvanometers are connected in series, they carry the same current i.e.,  $I_1 = I_2$

$$I_1 = \frac{2rH}{\mu_0 n_1} \tan \theta_1 \quad I_2 = \frac{2rH}{\mu_0 n_2} \tan \theta_2$$

$$\therefore \frac{2rH}{\mu_0 n_1} \tan \theta_1 = \frac{2rH}{\mu_0 n_2} \tan \theta_2$$

$$\frac{n_1}{n_2} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\tan 35^\circ}{\tan 45^\circ} = \frac{0.7}{1}$$

$$\frac{n_1}{n_2} = 0.7$$



?

The values of **apparent angle of dip** at two places measured in two mutually perpendicular planes are **30°** and **45°**. Determine the **true angle of dip** at the place.

**Solution :** Apparent dip at plane  $P_1$  :  $\delta_1 = 30^\circ$

Apparent dip at plane  $P_2$  :  $\delta_2 = 45^\circ$

True dip =  $\delta$

$$\tan \delta_1 = \frac{\tan \delta}{\cos \theta} \quad \Rightarrow \quad \cos \theta = \frac{\tan \delta}{\tan \delta_1}$$

$$\tan \delta_2 = \frac{\tan \delta}{\cos(90^\circ - \theta)} = \frac{\tan \delta}{\sin \theta} \quad \Rightarrow \quad \sin \theta = \frac{\tan \delta}{\tan \delta_2}$$

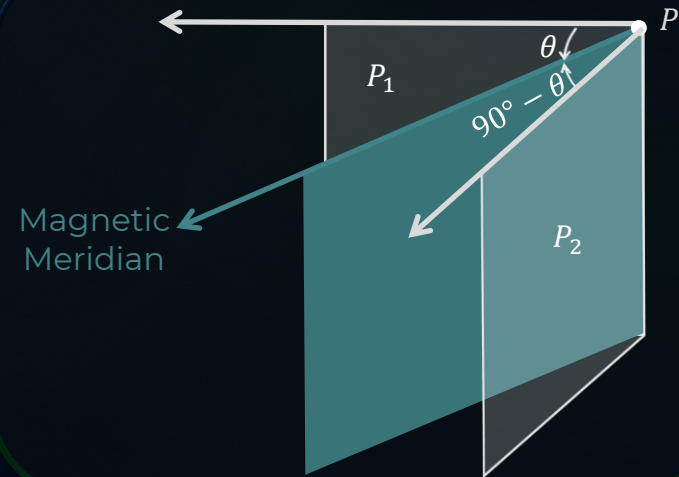
We know :  $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{\tan^2 \delta}{\tan^2 \delta_1} + \frac{\tan^2 \delta}{\tan^2 \delta_2} = 1$$

$$\cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$$

$$\cot^2 30^\circ + \cot^2 45^\circ = \cot^2 \delta$$

$$\cot \delta = 2$$



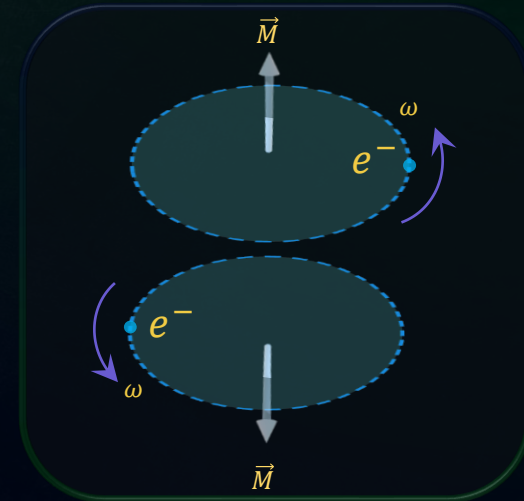
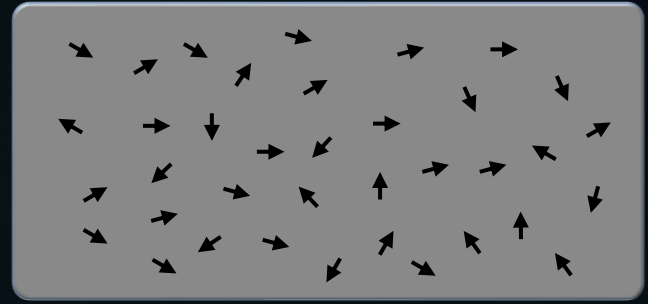
$$\delta = \tan^{-1} \left( \frac{1}{2} \right)$$



## Magnetic Property



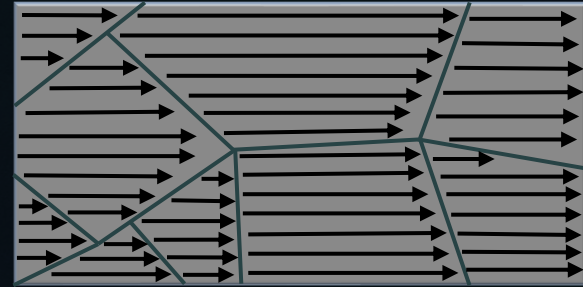
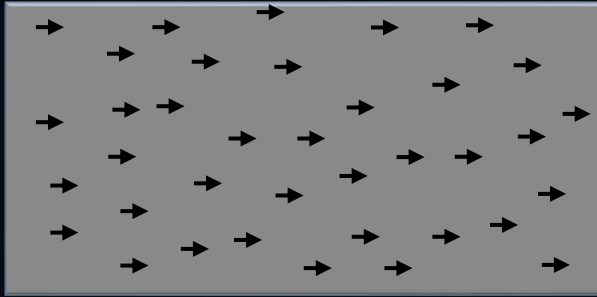
- Every charged particle has an intrinsic property known as “Spin angular momentum”.
- Magnetic moment of an atom is due to :
  - Orbital motion of  $e^-$ .
  - Magnetic moment due to spin angular moment of  $e^-$ .
  - Magnetic moment of nucleus.
- Net magnetic moment ( $\vec{M}$ ) of an atom is a **vector sum** of magnetic moments.
- Paired electrons cancels the magnetic moment of each other.
- Atoms having only paired electrons can't show magnetic property.
- Atoms having unpaired electrons do have magnetic moment individually, but they are randomly oriented canceling the effect. So, net magnetic moment  $\sum \vec{M} = 0$ .







## Magnetic Property



- When atoms having unpaired electrons get aligned in a particular direction, net magnetic moment  $\sum \vec{M} \neq 0$  and the material shows magnetic property.
- A cluster of atoms having unpaired electrons is known as “Domain” in which the magnetic moment of the atoms are aligned in same direction.
- When all the domains are aligned in a particular direction, the material shows magnetic property.

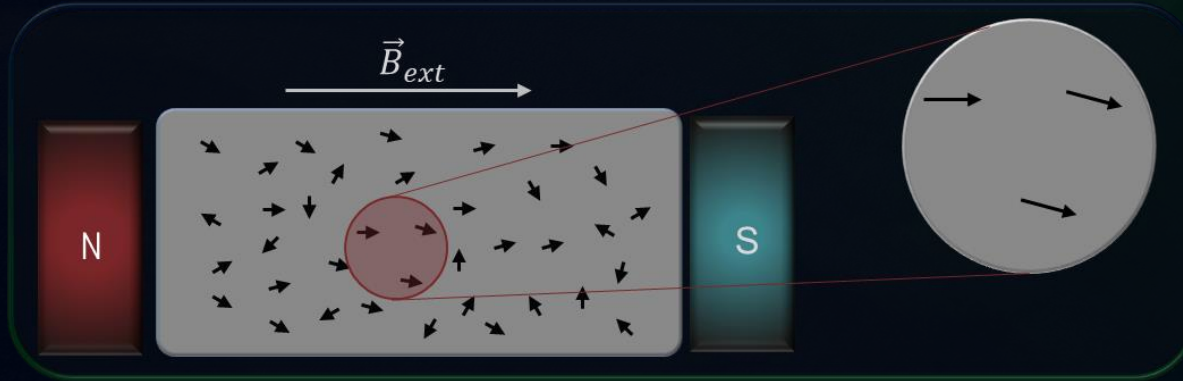


## Magnetization of Materials



Random alignment of dipoles

$$\vec{M} = 0$$



Applied external  
magnetic field



Dipoles get aligned in the  
direction of external field

$$\vec{M} \neq 0$$





## Intensity of Magnetization ( $\vec{I}$ )

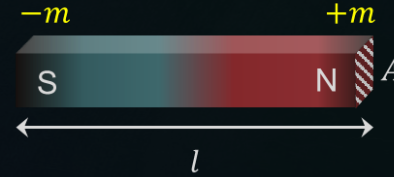


- Intensity of Magnetization :  
Net magnetic moment per unit volume

$$\vec{I} = \frac{\vec{M}_{net}}{V}$$

- SI unit: *ampere metre*<sup>-1</sup> ( $Am^{-1}$ )
- $\vec{I}$  depends on nature of **material** and **temperature**.

- Intensity of Magnetization for bar magnet :



$$|\vec{I}| = \left| \frac{\vec{M}_{net}}{V} \right| = \frac{m \times l}{A \times l} = \frac{m}{A}$$

- $\vec{I}$  = pole strength per unit cross sectional area.



## Magnetic intensity ( $\vec{H}$ )



In order to magnetise a magnetic material, it is kept in an external magnetic field  $B_{ext}$ . The ratio of the magnetising field to the permeability of free space is known as “Magnetic intensity”.

$$\vec{H} = \frac{\vec{B}_{ext}}{\mu_0}$$

- It is a measure of ability of an external magnetic field to magnetize a material medium.
- SI unit : *ampere metre*<sup>-1</sup> ( $Am^{-1}$ )

- Net magnetic field inside the material :  $\vec{B} = \vec{B}_{ext} + \vec{B}_{int}$
- Internal factor :  $\vec{B}_{int} = \mu_0 \vec{I}$  (Due to alignment of dipoles)
- External factor :  $\vec{B}_{ext} = \mu_0 \vec{H}$  (Due to magnetic intensity)

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{I}$$

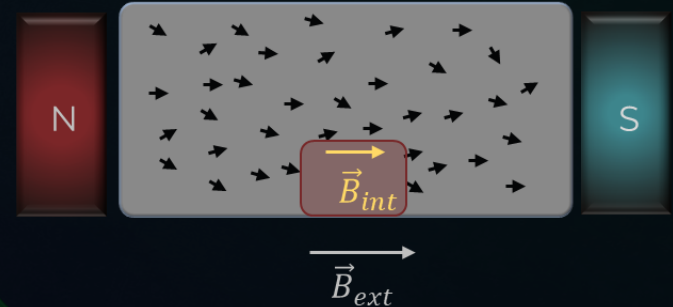
Or

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{I}$$

- If no magnetic material is present,  $\vec{I} = 0$ , in that case;

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

- $\vec{H}$  is a measure of external magnetic field. So, it is independent of nature of medium.





?

The magnetic intensity  $H$  at the centre of a long solenoid carrying a current of  $2.0\text{ A}$  is found to be  $1500\text{ Am}^{-1}$ . Find the number of turns per centimetre of the solenoid?

**Solution :**

The solenoid is long and no magnetic material is present at its core.

Therefore, magnetic field at the centre :

$$B_0 = \mu_0 ni$$

$$\mu_0 H = \mu_0 ni$$

$$H = ni$$

$$1500 = n \times 2$$

$$n = 750 \text{ turns/metre}$$

$$n = 7.5 \text{ turns/cm}$$

## Magnetic Susceptibility

It is defined as the ratio of the magnitude of “Intensity of magnetization ( $\vec{I}$ )” to that of “Magnetic intensity ( $\vec{H}$ )”.

$$\chi = \frac{I}{H}$$

- It is a **dimensionless** and **unitless scalar quantity**.
- The physical significance of magnetic susceptibility is that **it is the degree of ease with which a magnetic material can be magnetised**. A material with higher value of  $\chi$  can easily be magnetized.
- $\chi$  depends on nature of **material** and **temperature**.

## Magnetic Permeability



It is defined as the ratio of the magnitude of “**total magnetic field ( $\vec{B}$ )**” inside the material to that of “**Magnetic intensity ( $\vec{H}$ )**”.

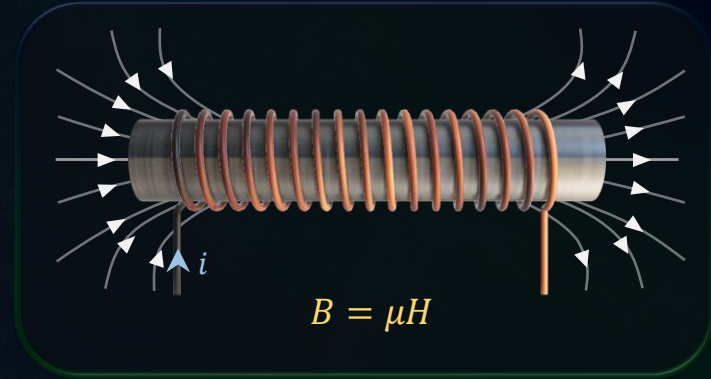
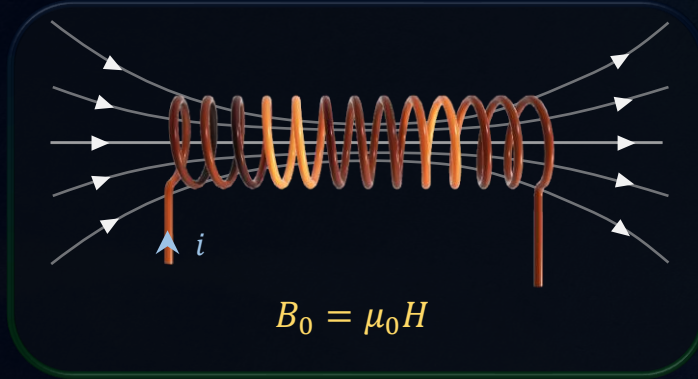
$$\mu = \frac{B}{H}$$

- It is a **scalar quantity**.
- **SI Unit** :  $Wb/Am^{-1}$
- **Dimension** :  $[MLT^{-2}A^{-2}]$
- The physical significance of magnetic permeability is that **it measures the extent to which a magnetizing field can penetrate or permeate a given material**.





## Relative Permeability



- Relative permeability :

It is defined as the ratio of permeability of a medium ( $\mu$ ) to permeability of free space ( $\mu_0$ ).

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$

- $\mu_r$  is the factor by which magnetic field increases when a material is introduced.

?

An iron rod of volume  $10^{-3} \text{ m}^3$  and relative permeability **1000** is placed as core in a solenoid with **10 turns/cm**. If a current of **0.5 A** is passed through the solenoid, then the **magnetic moment of the rod** will be :

*JEE MAIN 2020*

For a solenoid having  $n$  turns per unit length and carrying a current  $i$ , we have :

$$H = ni$$

We have :

$$B = \mu_0(H + I)$$

$$\mu H = \mu_0 H + \mu_0 I$$

$$\mu_0 I = \mu H - \mu_0 H$$

$$I = \left( \frac{\mu}{\mu_0} - 1 \right) H$$

$$I = (\mu_r - 1)ni$$

The magnetic moment of the rod :

$$M = I \times V$$

$$M = (\mu_r - 1)ni \times V$$

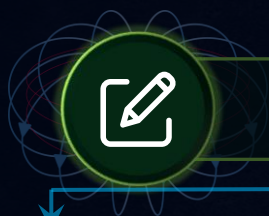
$$M = (1000 - 1) \times 1000 \times 0.5 \times 10^{-3}$$

$$M = (1000 - 1) \times 1000 \times 0.5 \times 10^{-3}$$

$$M = 499.5 \approx 500 \text{ Am}^2$$

$$M = 5 \times 10^2 \text{ Am}^2$$





# Magnetic Materials

Diamagnetic  
Materials

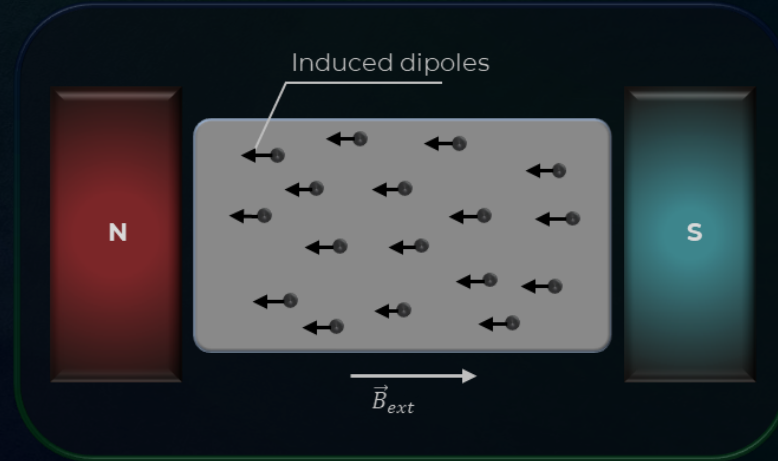
Paramagnetic  
Materials

Ferromagnetic  
Materials

- Diamagnetic materials have paired electrons only. So, the resultant magnetic moment becomes zero. Hence, diamagnetic materials do not have permanent magnetic dipoles.
- Diamagnetic material is placed in external magnetic field



Dipoles are induced but in opposite direction to  $\vec{B}_{ext}$ .

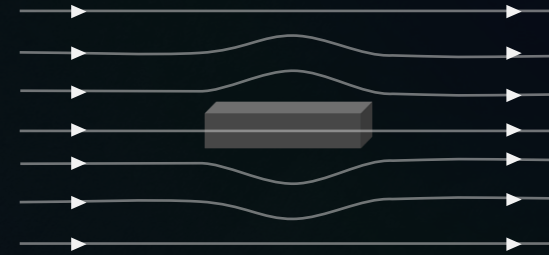




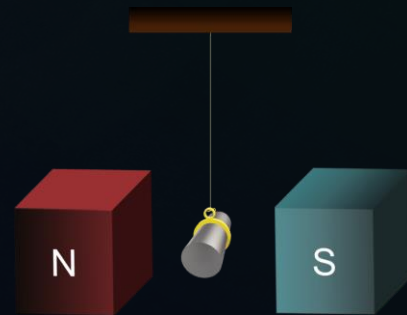
# Diamagnetic Materials



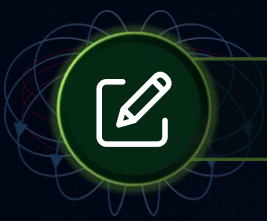
- Magnetic susceptibility :  $\chi < 0$  (small and negative)
- Relative permeability :  $\mu_r < 1$
- The magnetic field lines are **weakly repelled** by diamagnetic materials.
- A diamagnetic rod sets itself **perpendicular** to the field as field is stronger at poles (align itself along weaker field).



Diamagnetic substance







## Superconductor

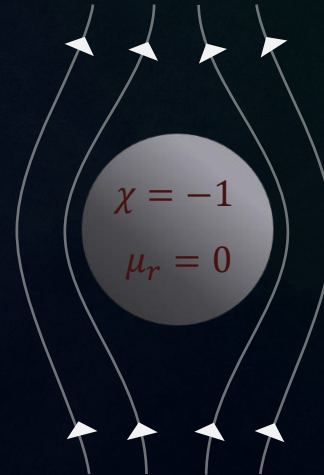


- Diamagnetic materials when cooled to very low temperatures exhibit perfect diamagnetism.
- They also show perfect conductivity and thus called Superconductors.
- These superconductors expel the pre-established external field.



Diamagnetic material

Cooled to very low  
temperatures



Superconductor

MEISSNER EFFECT



# Paramagnetic Materials



- The substances which get **feebly magnetized** when placed in external magnetic field, are known as “**Paramagnetic substances**”. E.g.,  $O_2$  (at STP),  $Na$ ,  $FeO$ ,  $Al$ .
- The atoms of these substances have some **finite dipole moment**.
- Total magnetic moment of the substance is **zero** due to **random orientation** in the absence of external magnetic field.
- In presence of external magnetic field, **dipoles get aligned in the direction of external magnetic field**.



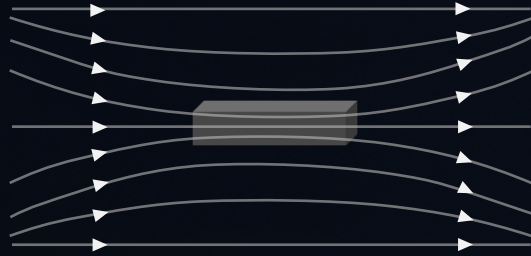
Net magnetization becomes non-zero.







# Paramagnetic Materials



Paramagnetic substance

- Magnetic susceptibility :  $\chi > 0$  (small and positive)
- Relative permeability :  $\mu_r > 1$
- The magnetic field lines become **denser** inside paramagnetic materials when placed in external magnetic field.



- A paramagnetic rod sets itself **parallel** to the field as the field is stronger at poles.

?

A paramagnetic substance in the form of a cube with sides  $1\text{ cm}$  has a magnetic dipole moment of  $20 \times 10^{-6}\text{ Am}^2$  when a magnetic intensity of  $60 \times 10^3\text{ Am}^{-1}$  is applied. Its magnetic susceptibility will be :

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Side length of the paramagnetic cube :

$$a = 1\text{ cm} = 10^{-2}\text{ m}$$

Volume of the paramagnetic cube:

$$V = a^3 = 10^{-6}\text{ m}^3$$

Magnetic dipole moment of the cube:

$$M = 20 \times 10^{-6}\text{ Am}^2$$

So, intensity of magnetization is,

$$I = \frac{M}{V} = \frac{20 \times 10^{-6}}{10^{-6}} = 20\text{ Am}^{-1}$$

Magnetic intensity of the cube:

$$H = 60 \times 10^3\text{ Am}^{-1}$$

Magnetic susceptibility :

$$\chi = \frac{I}{H}$$

$$\chi = \frac{20}{60 \times 10^3}$$

$$\chi = \frac{1}{3} \times 10^{-3}$$

$$\chi = 3.3 \times 10^{-4}$$





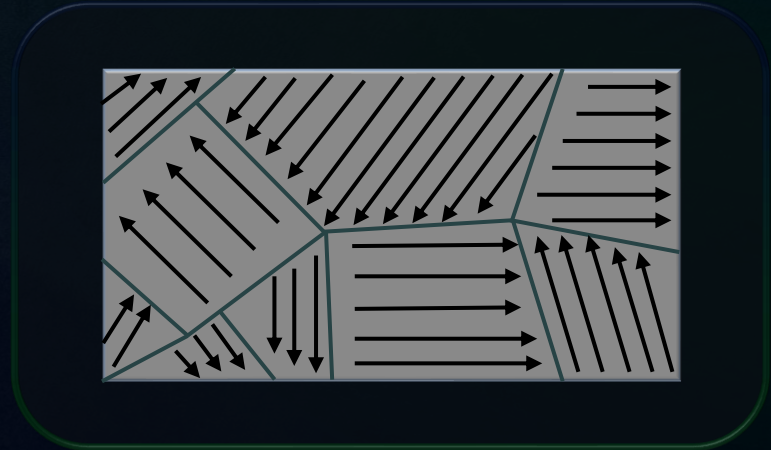
# Ferromagnetic Materials



- The substances which get **strongly magnetized** when placed in an external magnetic field, are known as “**Ferromagnetic substances**”. E.g., *Fe, Co, Ni* and other alloys.
- Atoms of these substances have **permanent dipole moment**.
- **Domain** : It is a **macroscopic volume over which** the individual dipole moment interacts with one another so that they **spontaneously** align themselves in a **common direction**.
- In the absence of external magnetic field, the domains are randomly oriented.

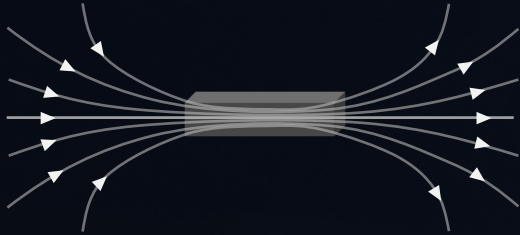


**Net magnetic moment is zero.**



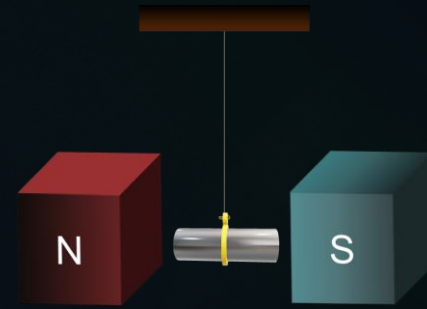


# Ferromagnetic Materials



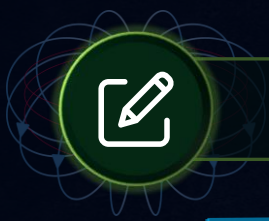
Ferromagnetic substance

- Magnetic susceptibility :  $\chi \gg 1$
- Relative permeability :  $\mu_r \gg 1$
- The magnetic field lines **tend to crowd** into ferromagnetic materials.



- A ferromagnetic rod **quickly** sets itself **parallel** to the field as the field is stronger at poles.





# Magnetic Materials



Diamagnetic  
Materials

Paramagnetic  
Materials

Ferromagnetic  
Materials

| Diamagnetic           | Paramagnetic        | Ferromagnetic   |
|-----------------------|---------------------|-----------------|
| $-1 \leq \chi \leq 0$ | $0 < \chi < k$      | $\chi \gg 1$    |
| $0 \leq \mu_r \leq 1$ | $1 < \mu_r < 1 + k$ | $\mu_r \gg 1$   |
| $\mu < \mu_0$         | $\mu > \mu_0$       | $\mu \gg \mu_0$ |

$k$  = Small positive number



## Curie's law



- For paramagnetic substances :

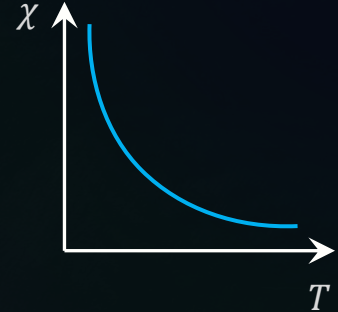
Magnetic susceptibility of a paramagnetic substance is inversely proportional to absolute temperature.

$$\chi \propto \frac{1}{T}$$

Or

$$\chi = \frac{C}{T}$$

C: Curie constant



- Curie's temperature :

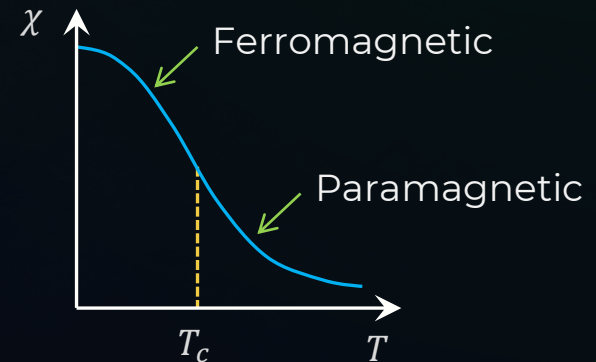
The temperature of transition from ferromagnetic to paramagnetic is called the Curie temperature ( $T_c$ ).

- For ferromagnetic substances :

At temperature above the curie temperature, a ferromagnetic substance becomes an ordinary paramagnetic substance whose magnetic susceptibility obeys the following law :

$$\chi = \frac{C}{T - T_c}$$

C: Curie constant



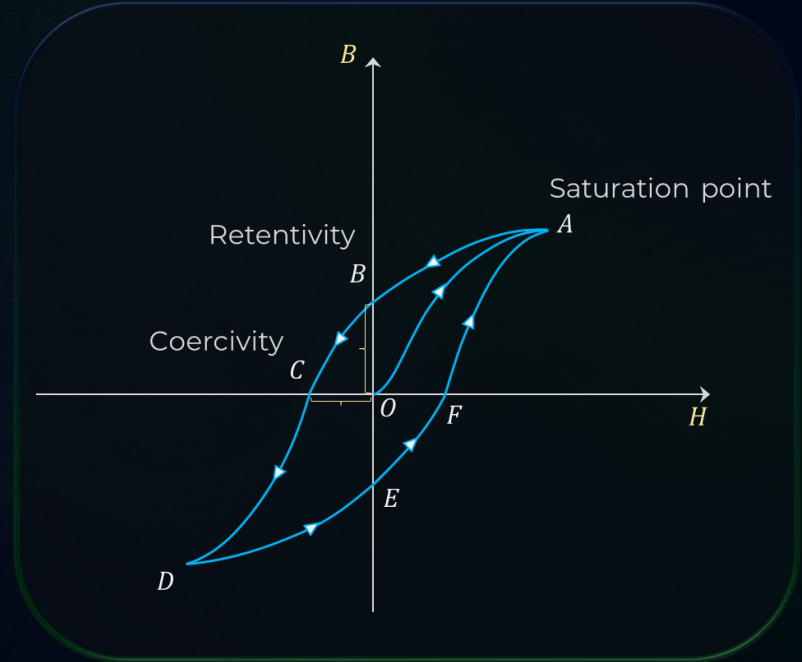




# Magnetic Hysteresis



- **Saturation point** : At this point, domains of ferro-magnetic material are aligned due to application of external magnetic field, and the material becomes magnetically saturated.
- **Retentivity** : The value ( $OB$ ) of  $B$  at  $H = 0$  is called “Retentivity” or “Remanence”. This is due to the fact that domains are not completely randomized even though  $H = 0$ .
- **Coercivity** : The value ( $OC$ ) of magnetizing field  $H$  needed to reduce net magnetic field  $B$  to zero is called “Coercivity”.

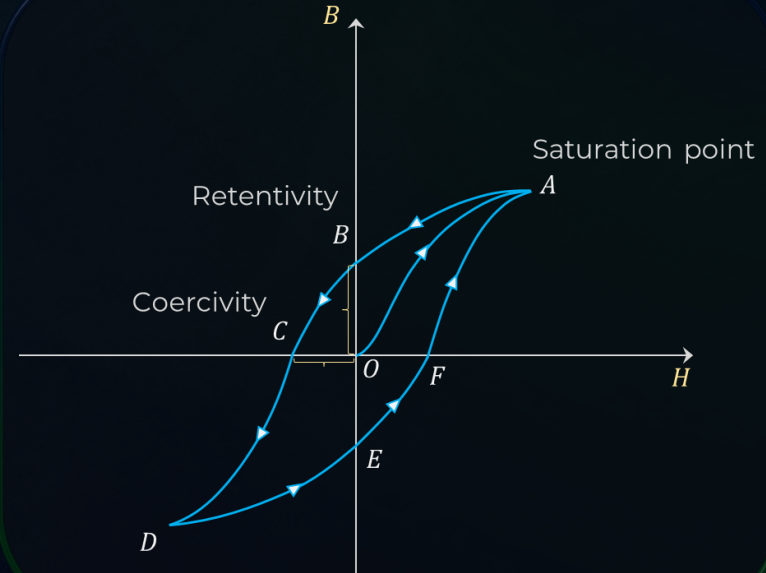




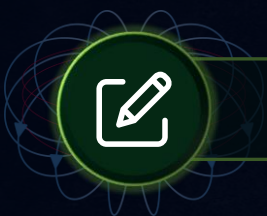
# Magnetic Hysteresis



- When the **net magnetic field  $B$**  of ferro-magnetic substances is plotted against **magnetic intensity  $H$**  for a complete cycle of magnetization and demagnetization, the resulting loop is called “**Hysteresis loop**”.
- **Hysteresis loss** : Energy lost in form of heat during a complete cycle of magnetization and demagnetization.
- **Area of hysteresis loop  $\propto$  Thermal energy developed** per unit volume of the material in a hysteresis cycle.
- For a complete cycle of magnetization and demagnetization, the **net magnetic field  $B$**  lags behind the magnetic intensity or magnetizing field  $H$ .







## Hard and Soft Magnets

- **Hard magnets :**

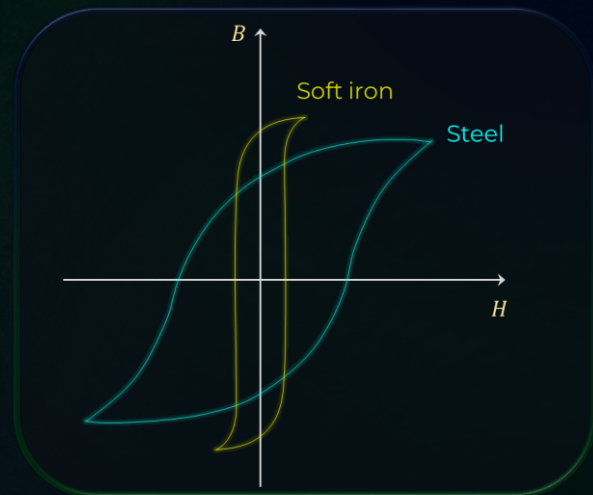
The ferromagnetic materials which **retain magnetisation for a long period of time** are hard magnetic materials or hard ferromagnets.

**Examples :** Steel, Alnico and naturally occurring lodestone.

- **Soft magnets :**

The ferromagnetic materials which **retain magnetisation as long as the external field persists** are called soft magnetic materials or soft ferromagnets.

**Examples :** Soft iron.



| Quantity  | Steel     | Soft iron |
|---|-----------|-----------|
| Magnetic intensity ( $\vec{H}$ ) for saturation | Very high | Low       |
| Retentivity                                     | Low       | High      |
| Coercivity                                      | Very high | Low       |
| Hysteresis loss                                 | High      | Low       |

## Permanent Magnet

- The substances which at room temperature retain their magnetization (i.e., ferromagnetic property) for long period of time are known as “Permanent magnets”. E.g., Steel, alnico, cobalt steel and ticonal.
- Substances should have :
  - High retentivity  
(so that the magnet becomes strong)
  - High coercivity  
(so that it couldn't be demagnetized easily)
  - High permeability.
- Uses: In electric clocks, microphones, speakers, generators, motors.

## Electromagnet



- An electromagnet is a temporary strong magnet and is just a solenoid with its winding on a material having low retentivity and high permeability. E.g., Soft Iron, aluminium.
- Core material should have :
  - Low retentivity
  - Low coercivity
  - High permeability
- Uses: In electric bells, transformer cores, loudspeakers and telephone diaphragms.

