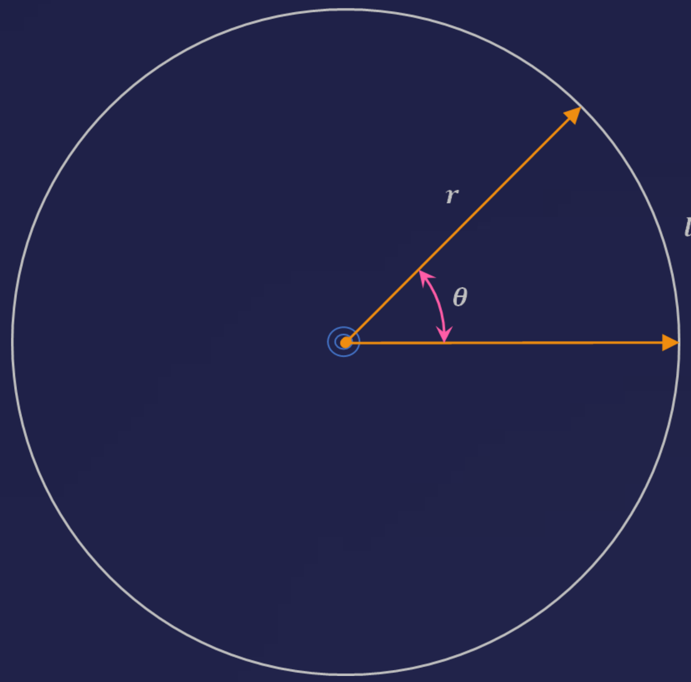
Welcome to

Aakash + BYJU'S NOTES

Orientation Sessions

Plane angle in radian and Conversion of degree to radian



$$\theta = \frac{\text{arc length}}{\text{radius of circle}} = \frac{l}{r}\ (rad)$$

degree to radian

$$\times \frac{2\pi}{360}$$

radian to degree

$$\times \frac{360}{2\pi}$$

$i)\ \ 540° = \frac{2\pi \times 540}{360}$

$= 3\pi\ rad$

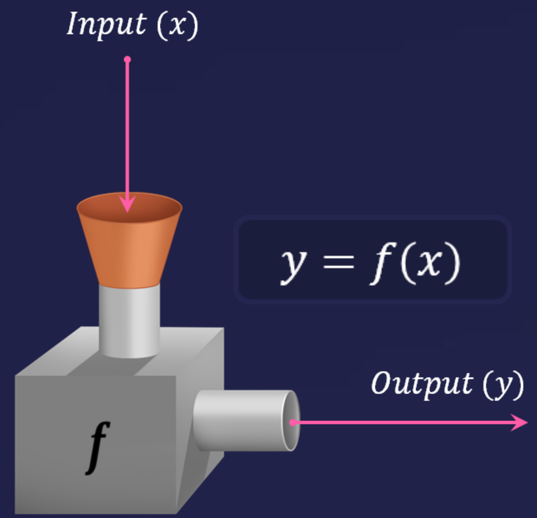
$ii)\ \ 210° = \frac{2\pi \times 210}{360}$

$= \frac{7}{6}\pi\ rad$

$iii)\ \ 270° = \frac{2\pi \times 270}{360}$

$= \frac{3}{2}\pi\ rad$

Functions

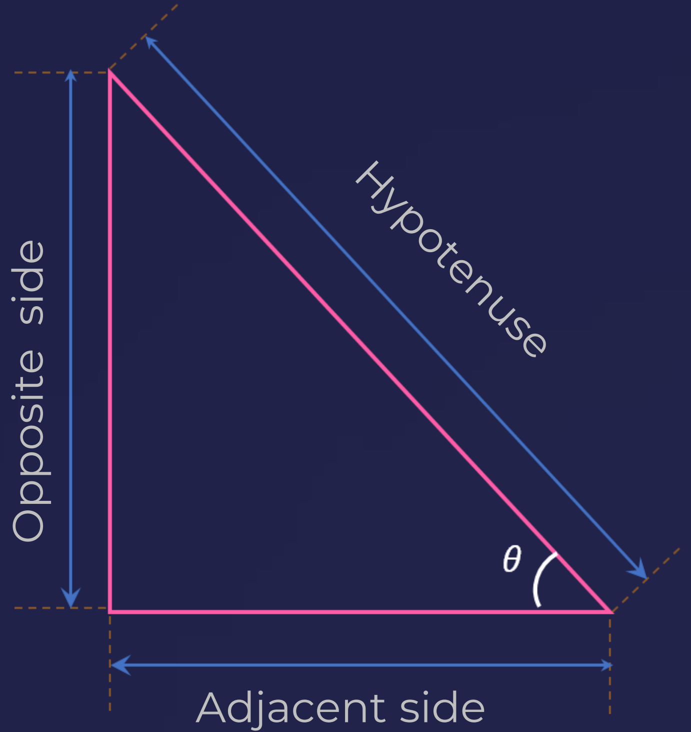


- Input variable is independent and output variable is dependent.
- A function is a mathematical relationship between independent and dependent variables

Functions

- Trigonometric
- Algebraic
- Exponential
- Logarithmic

Trigonometric Ratios



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{5}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$	∞

Trigonometric Ratios

$\sin(90° + \theta) = \cos\theta$

$\sin(180° - \theta) = \sin\theta$

$\cos(90° + \theta) = -\sin\theta$

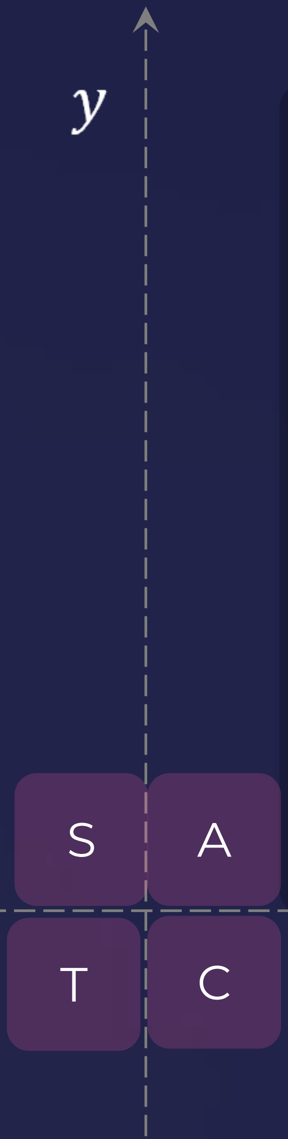
$\cos(180° - \theta) = -\cos\theta$

$\sec(180° - \theta) = -\sec\theta$

$\text{cosec}(180° - \theta) = \text{cosec}\,\theta$

$\tan(180° - \theta) = -\tan\theta$

$\cot(180° - \theta) = -\cot\theta$



$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\sec(90° - \theta) = \text{cosec}\,\theta$

$\text{cosec}(90° - \theta) = \sec\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

Trigonometric Formulae

- **Identities**
  - $\sin^2\theta + \cos^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \mathrm{cosec}^2\,\theta$
- **Additive**
  - $\sin(A \mp B) = \sin A\cos B \mp \cos A\sin B$
  - $\cos(A \mp B) = \cos A\cos B \pm \sin A\sin B$
  - $\tan(A \mp B) = \dfrac{\tan A \mp \tan B}{1 \pm \tan A\tan B}$
- **Multiple angle**
  - $\sin 2A = 2\sin A\cos A$
  - $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
  - $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Polynomials and Linear Function

$$y = f(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots\ldots a_nx^n$$

$\{n \in N\ , a_n \neq 0\}$

Constants: $a_0, a_1, a_2, a_3, \ldots, a_n$

Variables: $x^1, x^2, x^3, \ldots, x^n$

Degree of a polynomial = Highest power = Maximum number of solutions

$$y = f(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots\ldots a_nx^n$$

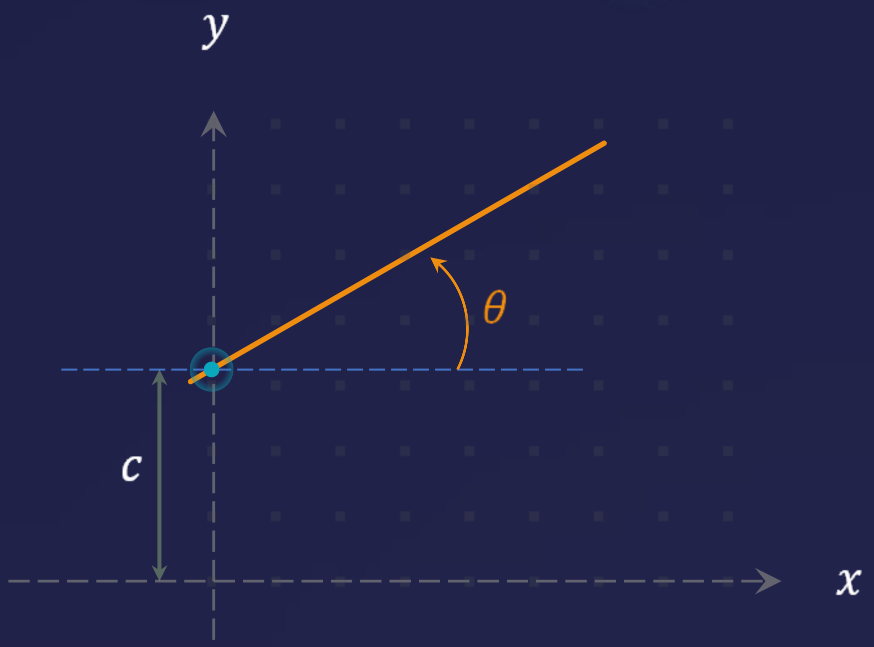
$\{n \in N\ , a_n \neq 0\}$

Constants: a_0, a_1

Variables: x^1

Degree of a polynomial = Highest power = Maximum number of solutions = 1

Graph of Linear Function

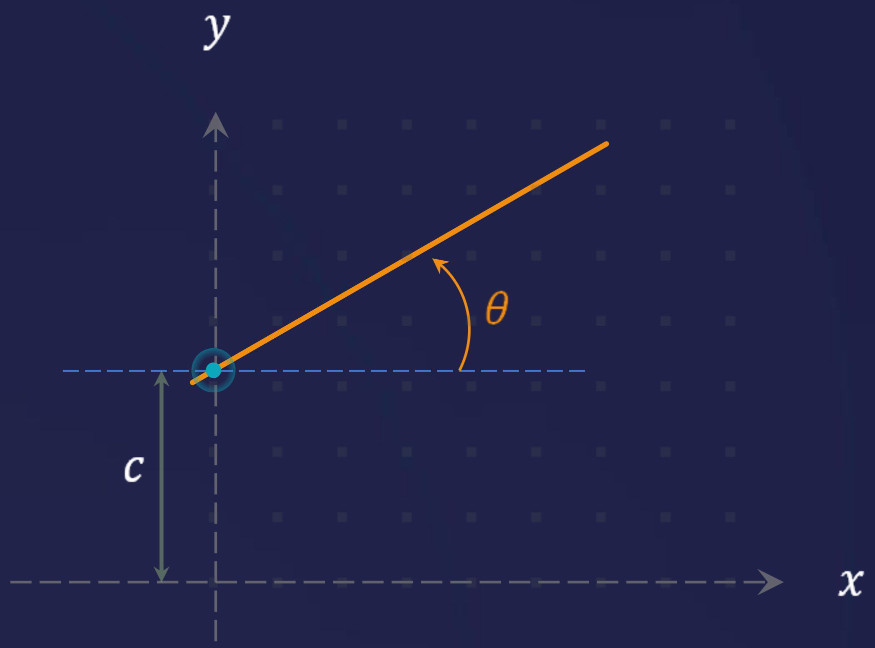


$y = a_0 + a_1 x^1$

$y = mx + c$

m = Slope

c = y - intercept



$y = mx + c$

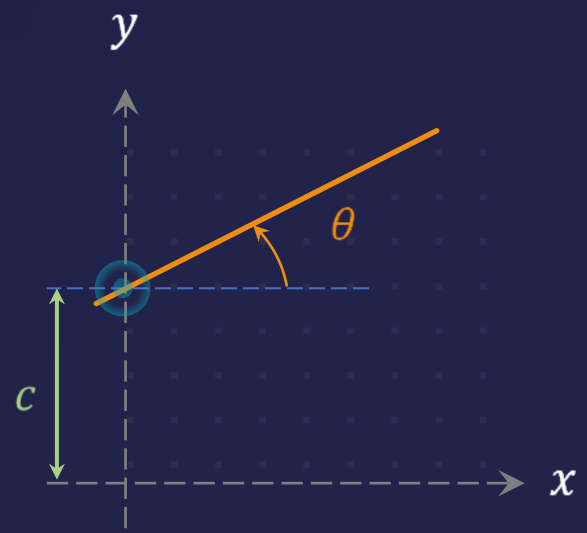
m = Slope = $\tan\theta$

c = y - intercept

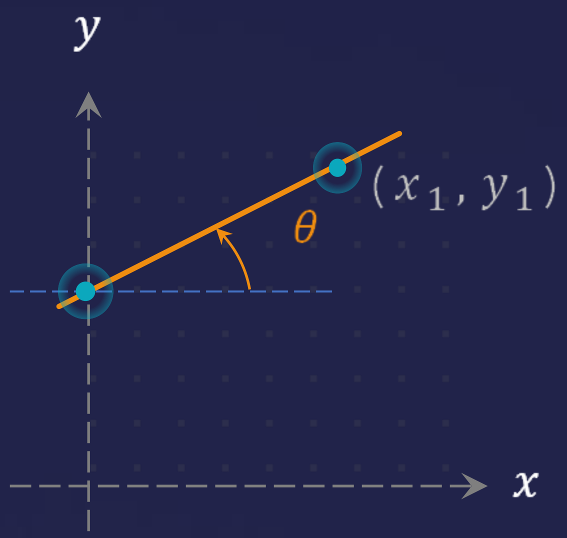
Slope - intercept form

Straight Line Equation

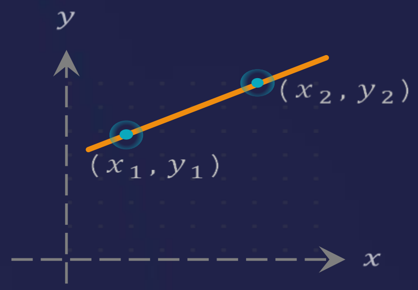
Slope - intercept	Point - slope	Two- Point



$$y = mx + c$$



$$(y - y_1) = m(x - x_1)$$



$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

Quadratic Function

Constants

$$y = f(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \;\dots\dots a_nx^n$$

$\{n \in N \;, a_n \neq 0\}$

Variables

Degree of a polynomial = Highest power = Maximum number of solutions = 2

The graphical representation of a Quadratic function is Parabola.

Quadratic equation: $ax^2 + bx + c = 0$

Root 1 :

$$\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Root 2 :

$$\beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots

$$\alpha + \beta = -\frac{b}{a}$$

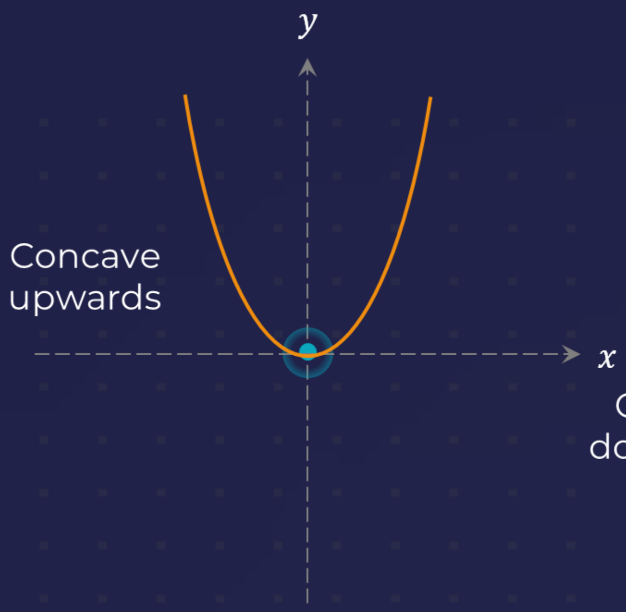
Difference of roots

$$|\beta - \alpha| = \frac{\sqrt{b^2 - 4ac}}{a}$$

Product of roots

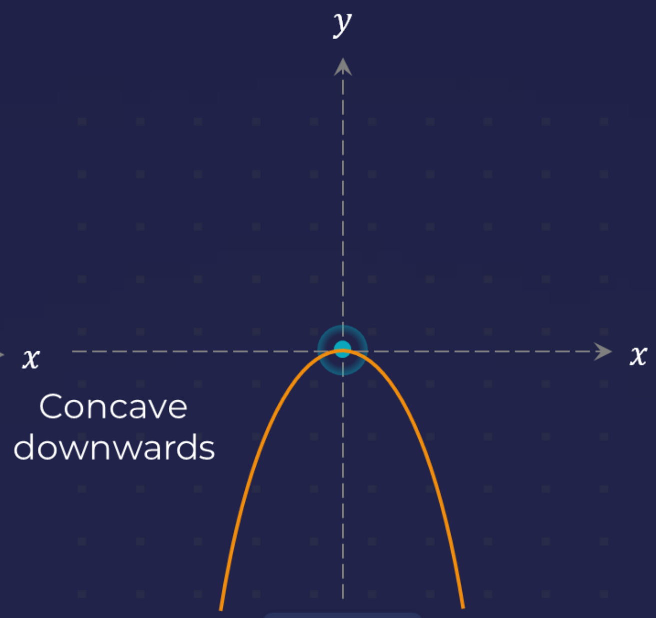
$$\alpha\beta = \frac{c}{a}$$

Parabola (Quadratic Curve)



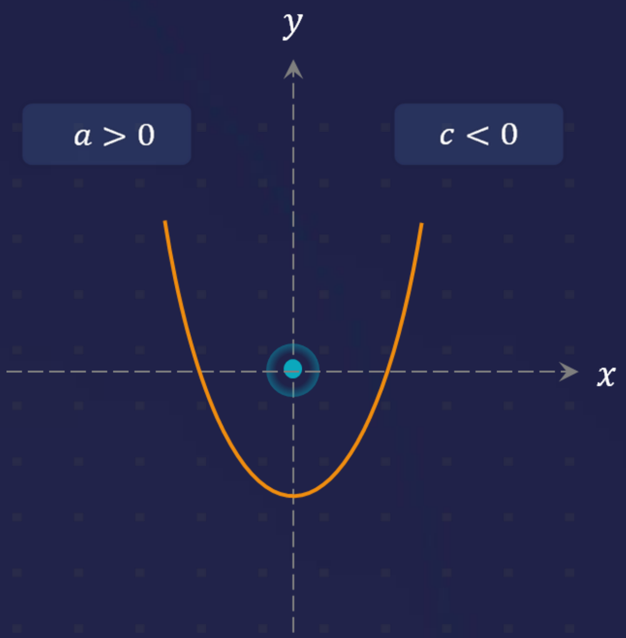
$a > 0$

$y = ax^2$

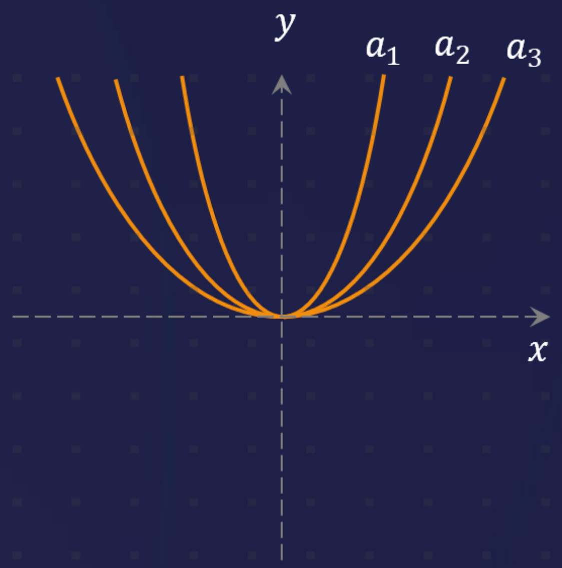


$a < 0$

$y = ax^2$



$y = ax^2 + c$



$a_1 < a_2 < a_3$

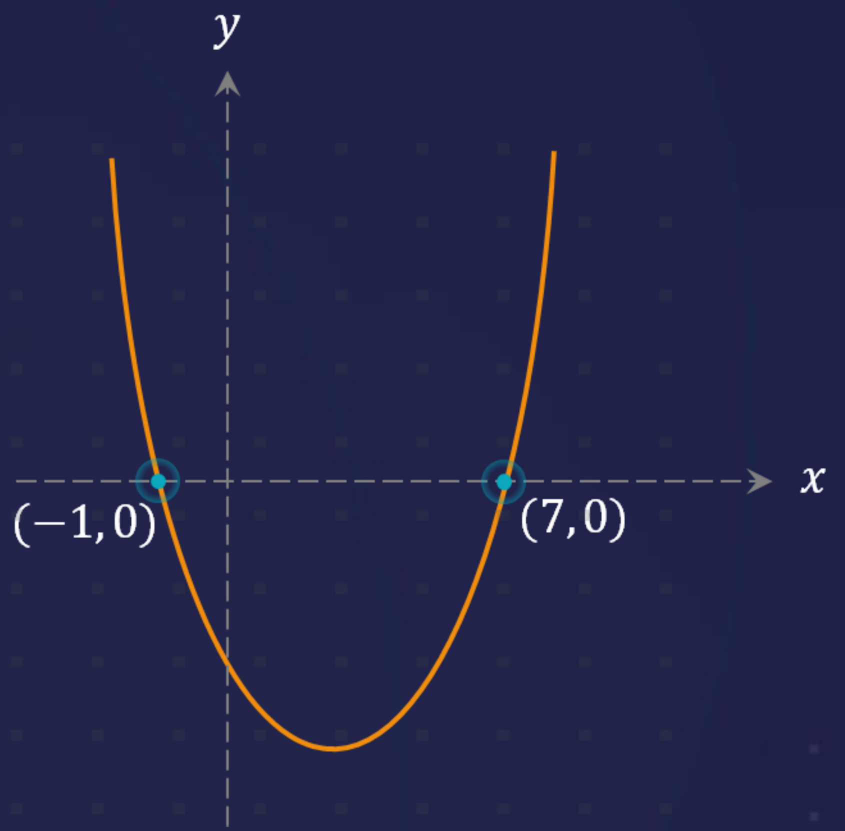
Find the roots of the Quadratic Equation, $y = x^2 - 6x - 7$

A 0 and 1

B 7 and −1

C 1 and −7

D 2 and 3



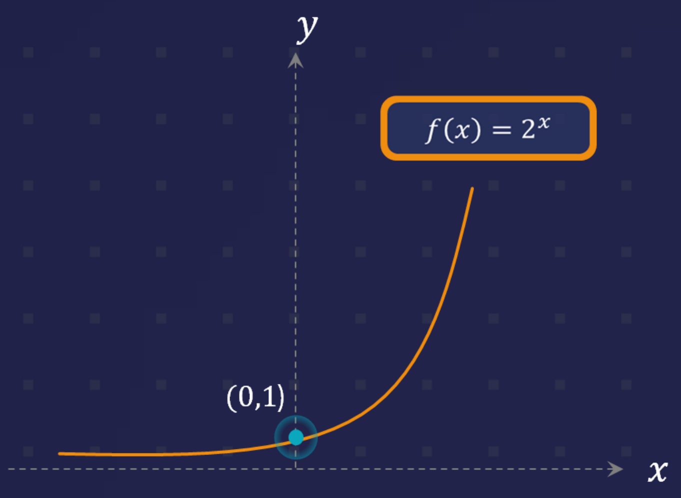
Exponential Functions and their Graph

$$y = f(x) = b^x$$

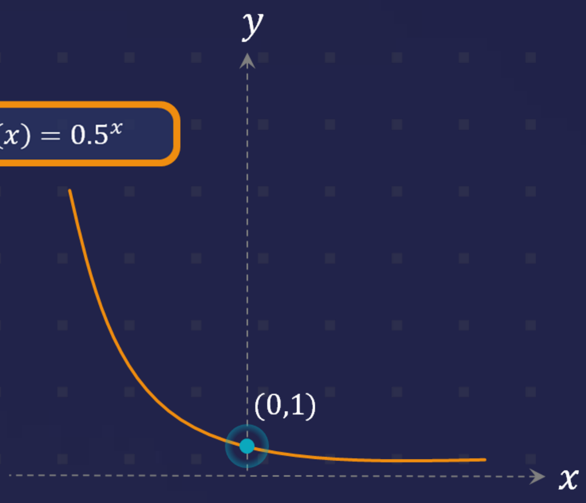
x – Index/ exponent [x ∈ real number]

b – base (Constant) [$b > 0$ and $b \neq 1$]

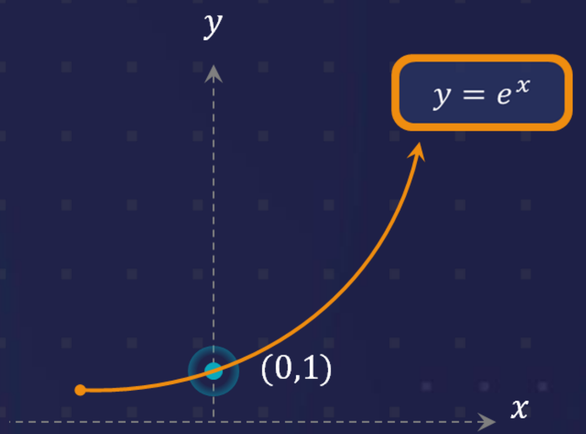
y – Output



Growth



Decay



Euler's number $e \approx 2.718$

Logarithmic Functions

Popular log functions

Natural Logarithm

base = e,

$f(x) = log_e(x)$

$= ln(x)$

Common logarithm

base = 10,

$f(x) = log_{10}(x)$

$y = log_a(x)$

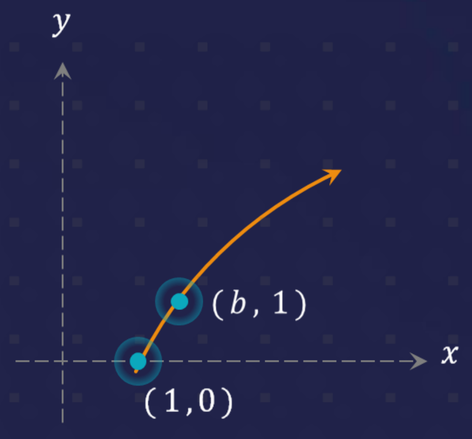
$x = a^y$

$a > 0$

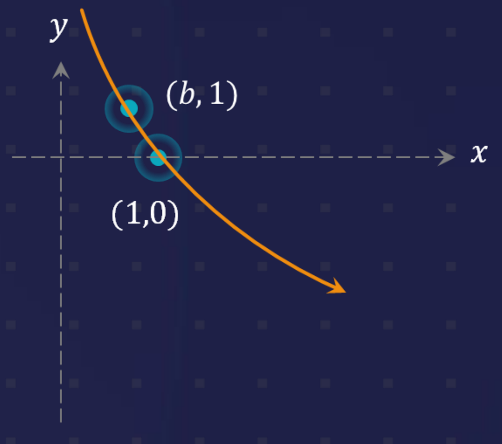
$a \neq 1$

$x > 0$

Graph of Logarithmic Function $y = log_b x$



$b > 1$



$0 < b < 1$

Properties of Logarithmic functions

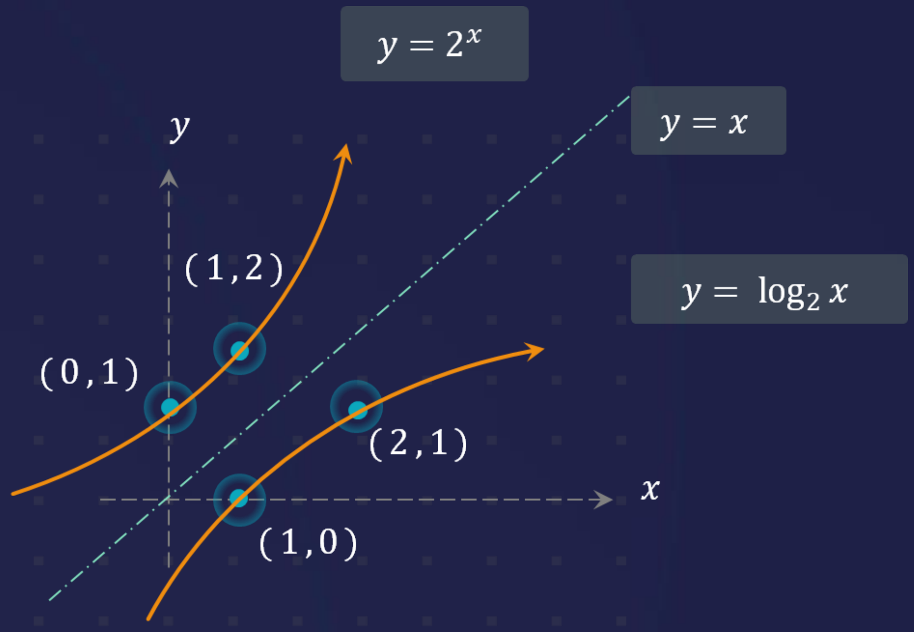
Product Rule $\log_b mn = \log_b m + \log_b n$

Quotient Rule $\log_b \frac{m}{n} = \log_b m - \log_b n$

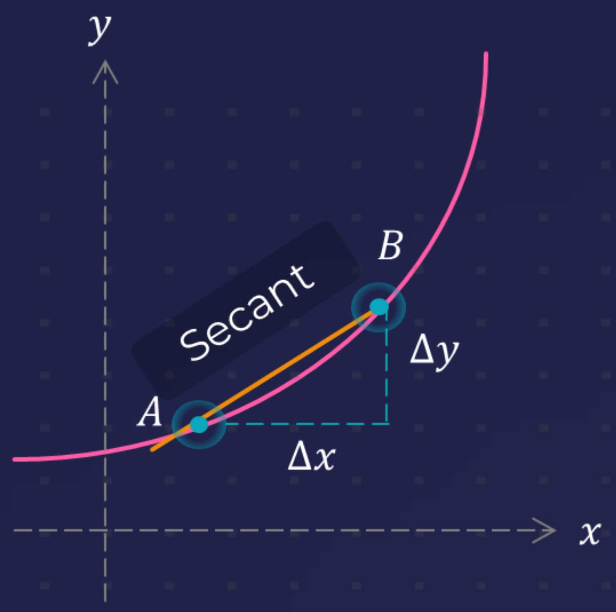
Power Rule $\log_b m^p = p.\log_b m$

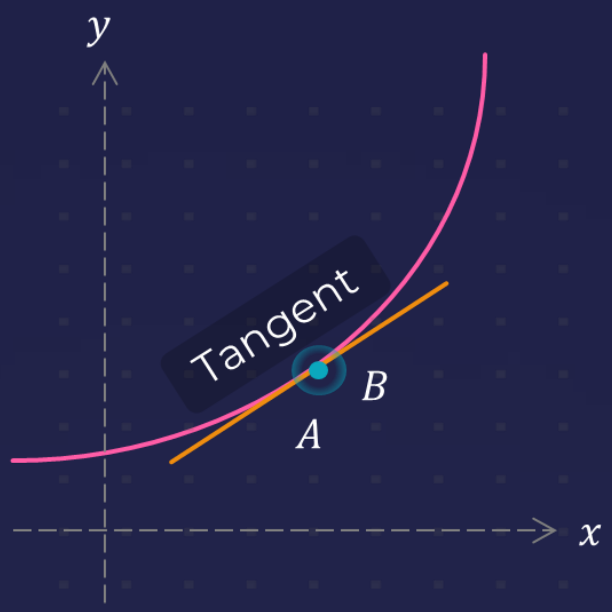
$\log_b 1 = 0$

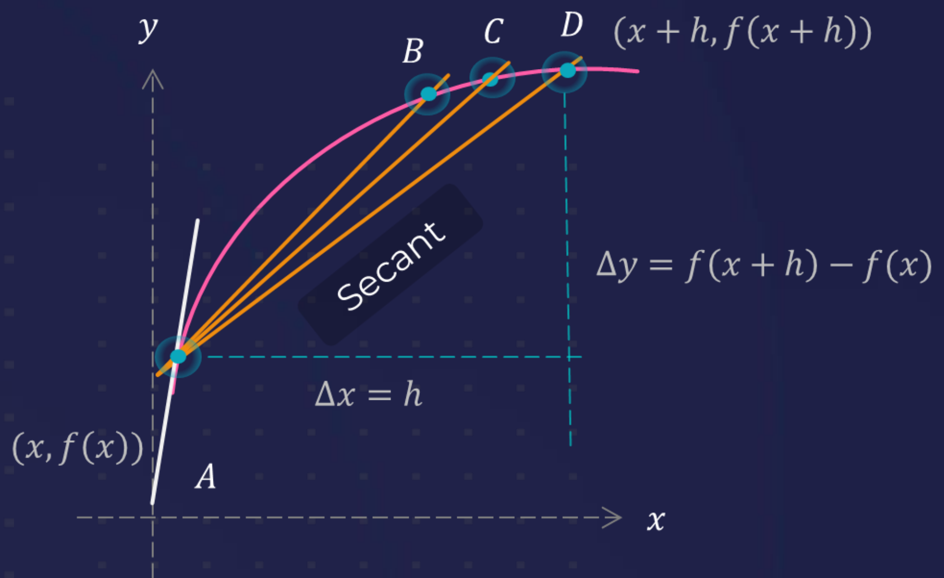
$\log_b b = 1$



Limiting case of a secant







$$\text{slope } (m) = \frac{\Delta y}{\Delta x}$$

$$\text{slope } (m) = \frac{dy}{dx}$$

$$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x}\right) = \lim_{\Delta x \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x}\right)$$

A limiting case of secant which intersects the curve at two infinitesimally close points is called a tangent.

Slope of the tangent $= \frac{dy}{dx} = y' = f'(x)$

Differentiation of Functions

Polynomial Functions

Power Rule:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Trigonometric Functions

Example of Trignometric fuction:

$$\frac{d}{dx}(\sin x) = \cos x$$

Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

Logarithmic Functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Differentiation of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

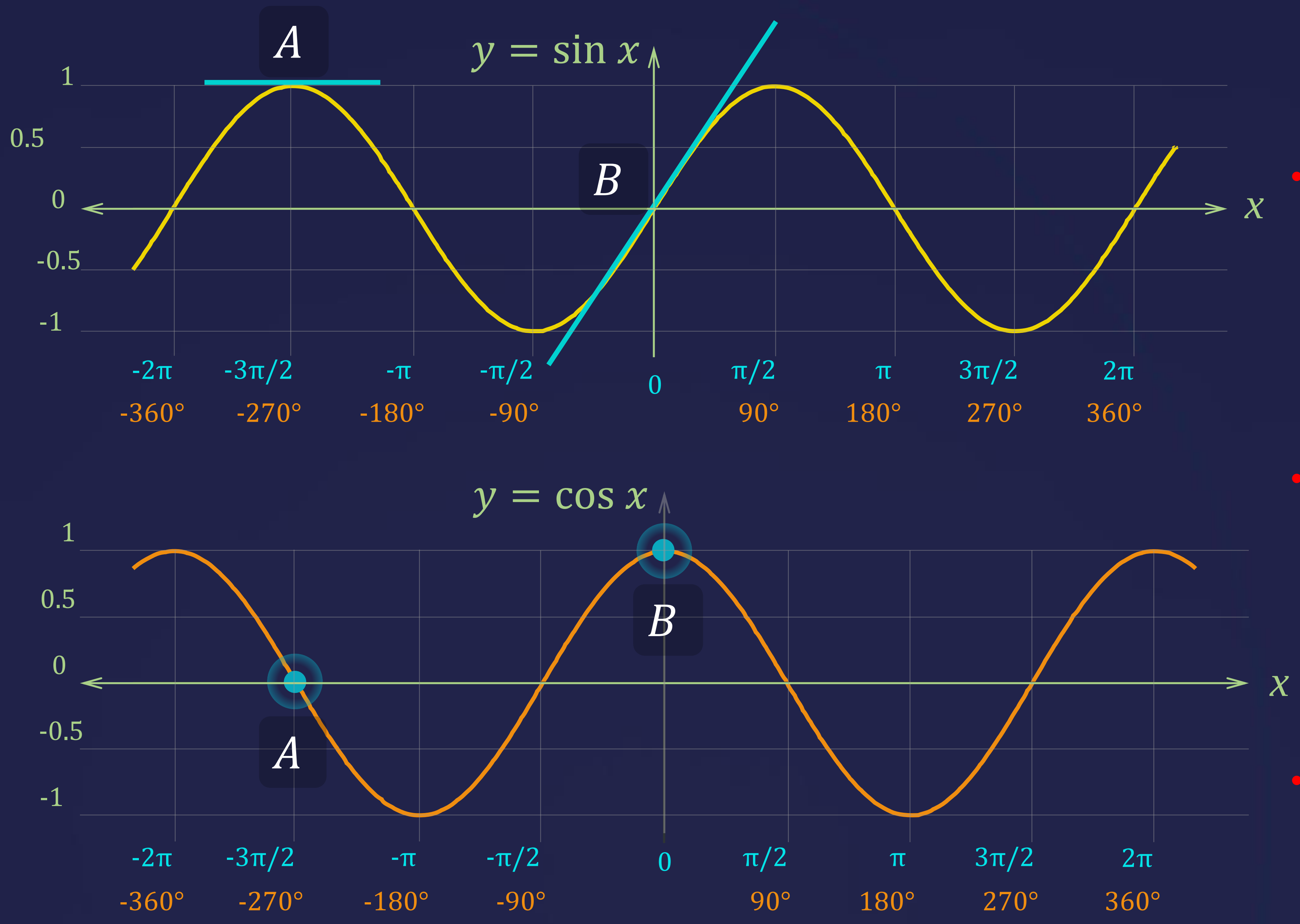
$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$



- Differentiation of a function is nothing but finding slope of curve at that point.
- Differentiation of a sin function at any point is a cosine function.
- Example at Position A and B

Properties of Derivatives

Derivative of a constant term a :

$$\frac{d}{dx}[a] = 0$$

$$(a)' = 0$$

Derivative of a Function multiplied by a constant term a :

$$\frac{d}{dx}[af(x)] = a\frac{d[f(x)]}{dx}$$

$$(af)' = af'$$

Derivative of sum or difference of two functions :

$$\frac{d}{dx}[af(x) \pm bg(x)] = a\frac{df(x)}{dx} \pm b\frac{dg(x)}{dx}$$

$$(af \pm bg)' = af' \pm bg'$$

Differentiate $f(x) = 2\cos x - \tan 45° \sec x + 3$ with respect to x

$$\frac{d[f(x)]}{dx} = \frac{d(2\cos x - \tan 45° \sec x + 3)}{dx}$$

$$f'(x) = 2\,[\cos x]' - 1.\,[\sec x]' + [3]'$$

$$= -2\sin x - \sec x \,.\, \tan x$$

$$= -2\sin x - \sec x \,.\, \tan x$$

Hint:

$$(af)' = af'$$

$$(f \pm g)' = f' \pm g'$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

Application of Derivative and Second derivative

To find the rate of change of one quantity with respect to another

Rate of change of position with time $= \dfrac{dx}{dt} =$ velocity

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$f(x) = \sin(x)$$
$$f'(x) = \cos(x)$$
$$f''(x) = -\sin(x)$$

If the motion of a particle is represented by,

$$x = (t^3 + t^2 - t + 2)\ m.$$

Find the position, speed of the particle at $t = 2\ s$?

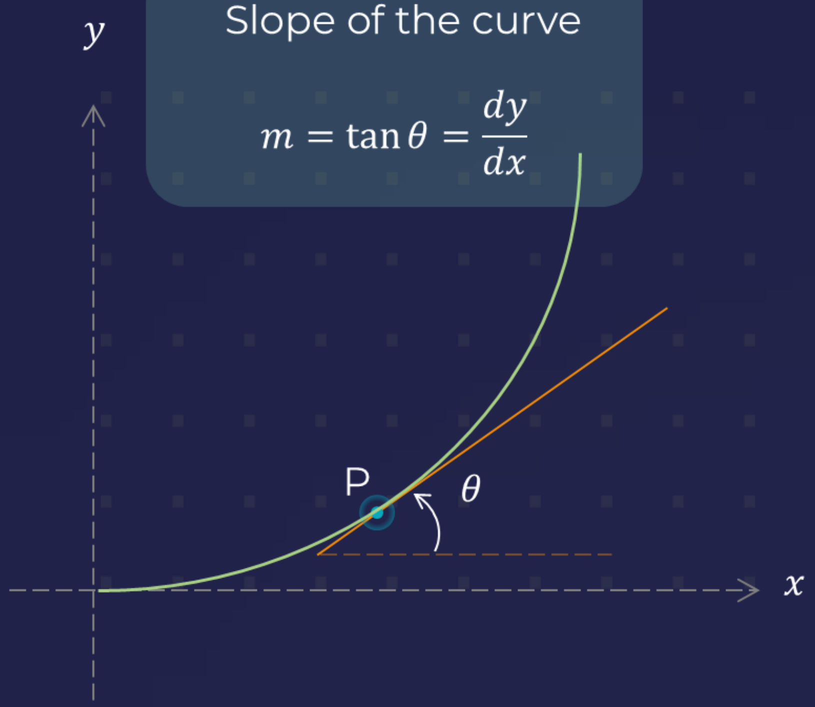
First Derivative

Position x	$t^3 + t^2 - t + 2$	$12\ m$
Speed $\frac{dx}{dt}$	$3t^2 + 2t^1 - 1 + 0$	$15\ m$

Slope of the curve and Increasing and Decreasing Functions

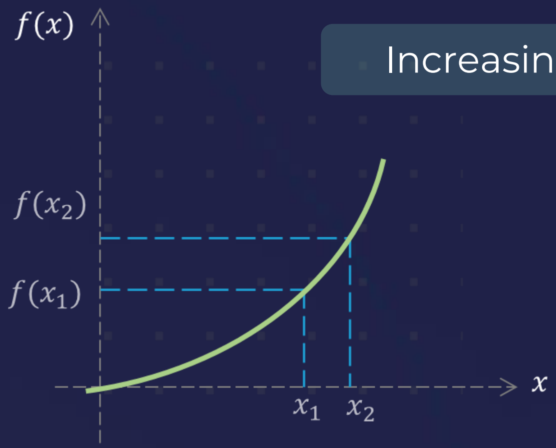
Slope of the curve

$$m = \tan\theta = \frac{dy}{dx}$$



Derivative of a function at a point gives slope of tangent at that point.

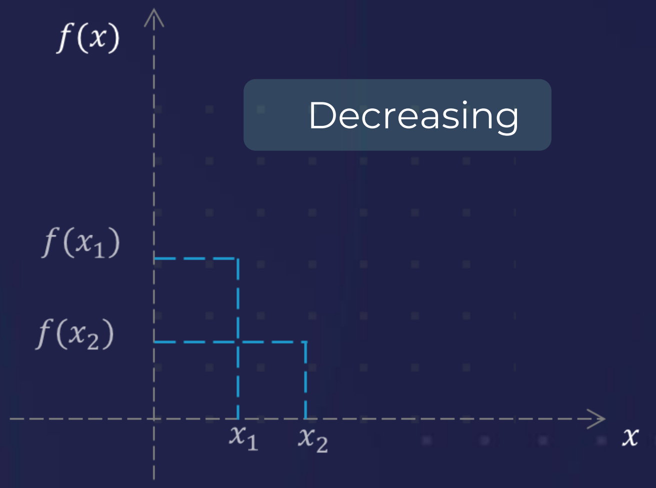
Increasing



$x_1 < x_2$

$f(x_1) < f(x_2)$

Decreasing



$x_1 < x_2$

$f(x_1) > f(x_2)$

Mention where the function is increasing or decreasing

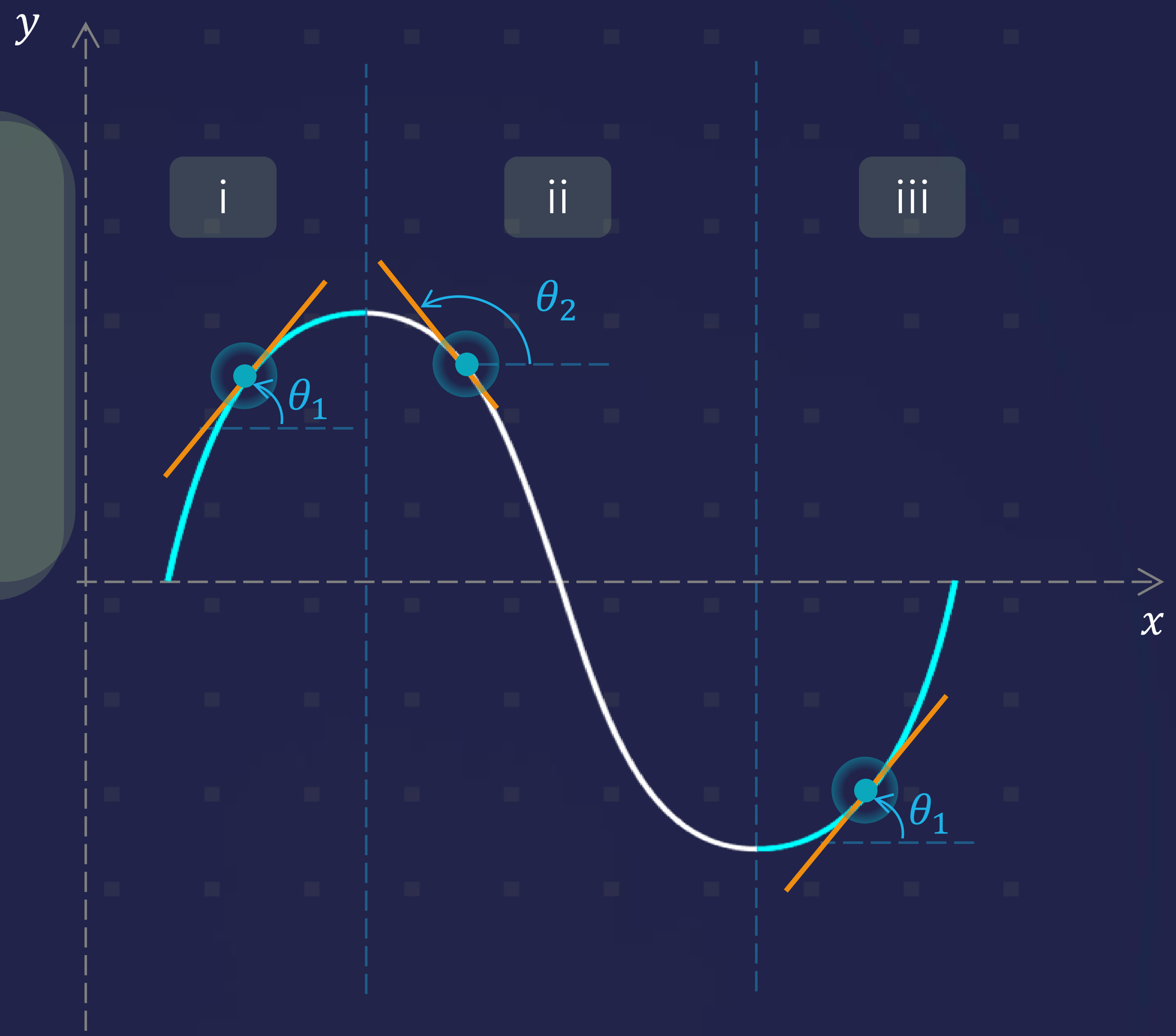
Increasing

$\theta < 90°$

$\tan\theta > 0$

$\frac{dy}{dx} > 0$

i iii



Decreasing

$\theta > 90°$

$\tan\theta < 0$

$\frac{dy}{dx} < 0$

ii

Find the slope of the tangent to the curve $y = x^2 - 5x + 4$ at point $(5, 4)$.

A	5
B	3
C	$2x - 5$
D	-15

Hint:

Slope of the tangent (m)

$$= \tan\theta = \frac{dy}{dx}$$

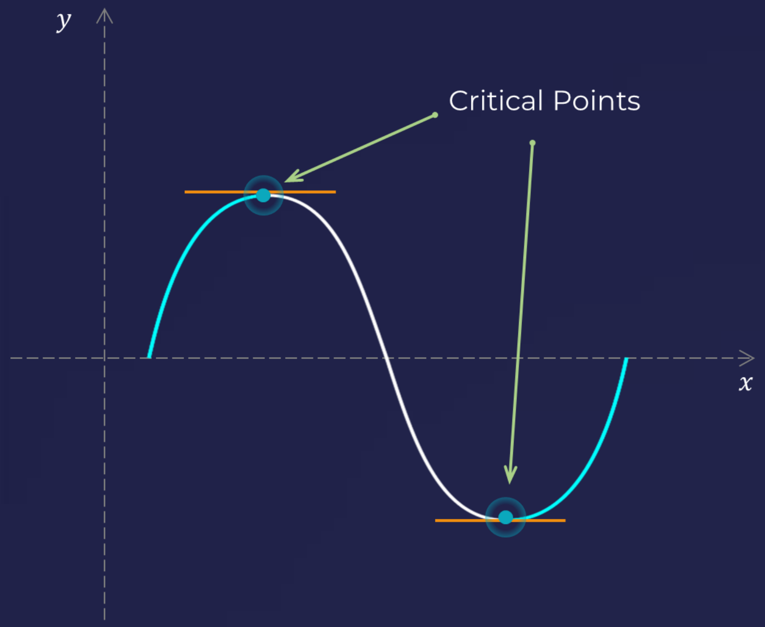
$$y = x^2 - 5x + 4$$

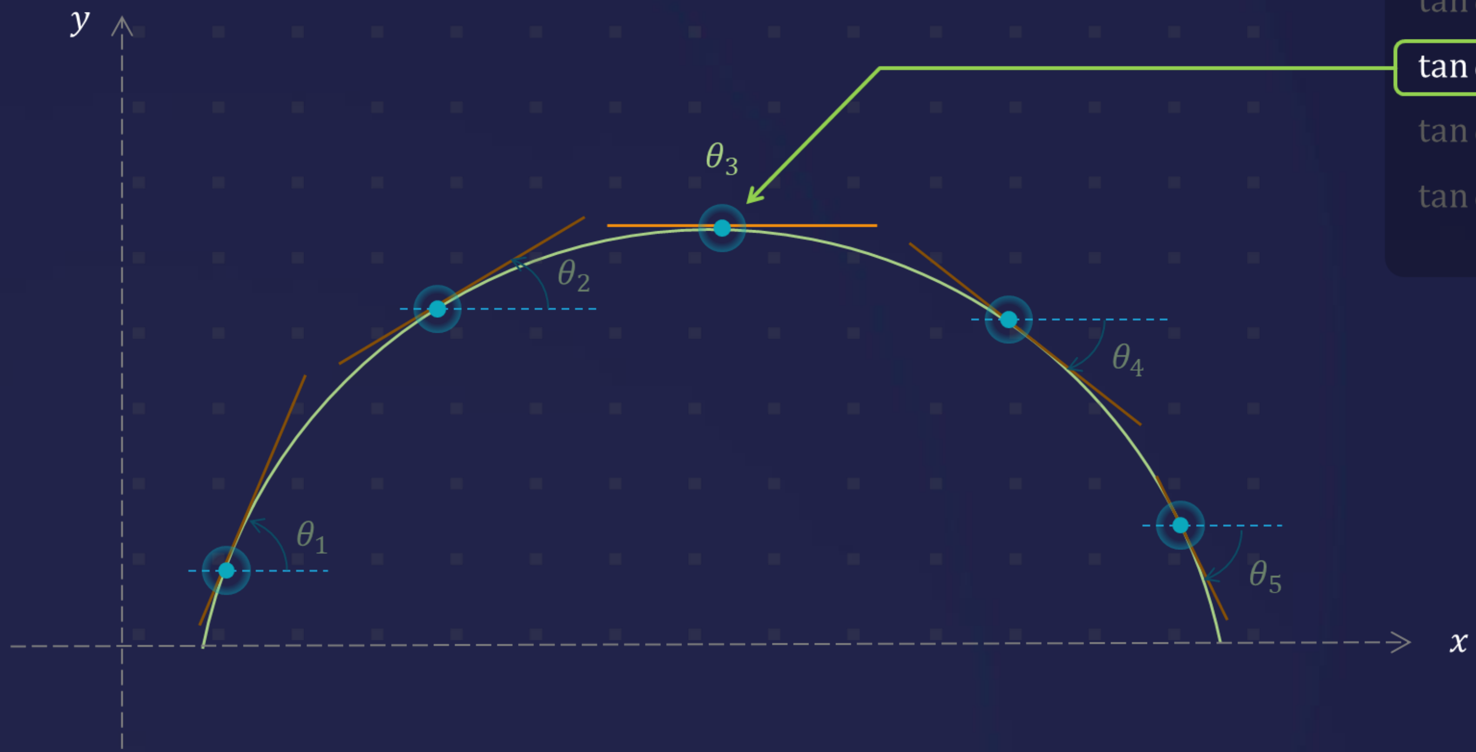
$\frac{dy}{dx}$ = 2x-5

= (2*5)-5

=5

Critical Points





$\tan\theta_1 = 1.73$

$\tan\theta_2 = 1$

$\tan\theta_3 = 0$

$\tan\theta_4 = -1$

$\tan\theta_5 = -1.73$

Slope of Tangent

at critical points

$\tan\theta = \tan 0° = 0$

Find the critical points of the function $f(x) = 2\sin x$?

$$\frac{df(x)}{dx} = 2\cos(x)$$

cos(x) is has a maximum and minimum value of 1,-1.
So, critical points of the function is -2,2

A $-1,1$

B $-2,2$

C $0, 1$

D $-3, 3$

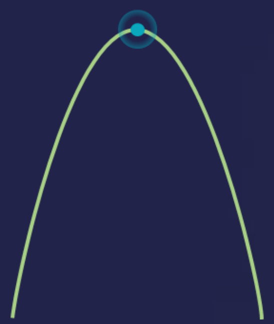
Maxima and Minima

Critical points

- Maxima
  - Global Maximum
  - Local Maximum
- Minima
  - Global Minimum
  - Local Minimum

$$y = f(x)$$

Critical Point/s $$\frac{dy}{dx} = 0$$



$$\frac{d^2y}{dx^2} < 0$$

Condition for Maxima

$$\frac{d^2y}{dx^2} > 0$$

Condition for Minima

Find the local maximum and minimum for the function $y = x^3 - 3x + 2$.

$$y = x^3 - 3x + 2$$

Critical Point's $(x = 1, -1)$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = -6 < 0$$

Maximum

(at $x = -1$)

$$\frac{d^2y}{dx^2} = 6 > 0$$

Minimum

(at $x = 1$)

A ball is thrown in the air. Its height at any time t is given by $h = 3 + 14t - 5t^2$. What is the maximum height attained by the ball ?

$$h = 3 + 14t - 5t^2$$

$$\frac{dh}{dt} = 14 - 10t = 0 \Rightarrow t = 1.4\ s$$

$$\frac{d^2h}{dt^2} = -10 < 0$$ $\Rightarrow$ There is a Maxima

$$h = 3 + 14(1.4) - 5(1.4)^2 = 12.8\ m$$

Product rule, Quotient Rule and Chain Rule

Product rule

$$\frac{d[f(x)\cdot g(x)]}{dx} = \frac{df(x)}{dx}\cdot g(x) + \frac{dg(x)}{dx}\cdot f(x)$$

$$(fg)' = f'g + fg'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{df(x)}{dx}\cdot g(x) - \frac{dg(x)}{dx}\cdot f(x)}{(g(x))^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain Rule

If $f = f\,(g);\ g = g\,(x)$

$$f'\,(x) = f'\,[g]\,g'\,(x)$$

Differentiate inner function

Differentiate outer function

$$\frac{df}{dx} = \frac{df}{dg}\cdot\frac{dg}{dx}$$

Differentiate: $f(x) = \frac{1}{1-x^2}$ w.r.t. to x using division rule.

$$f(x) = \frac{1}{1 - x^2}$$

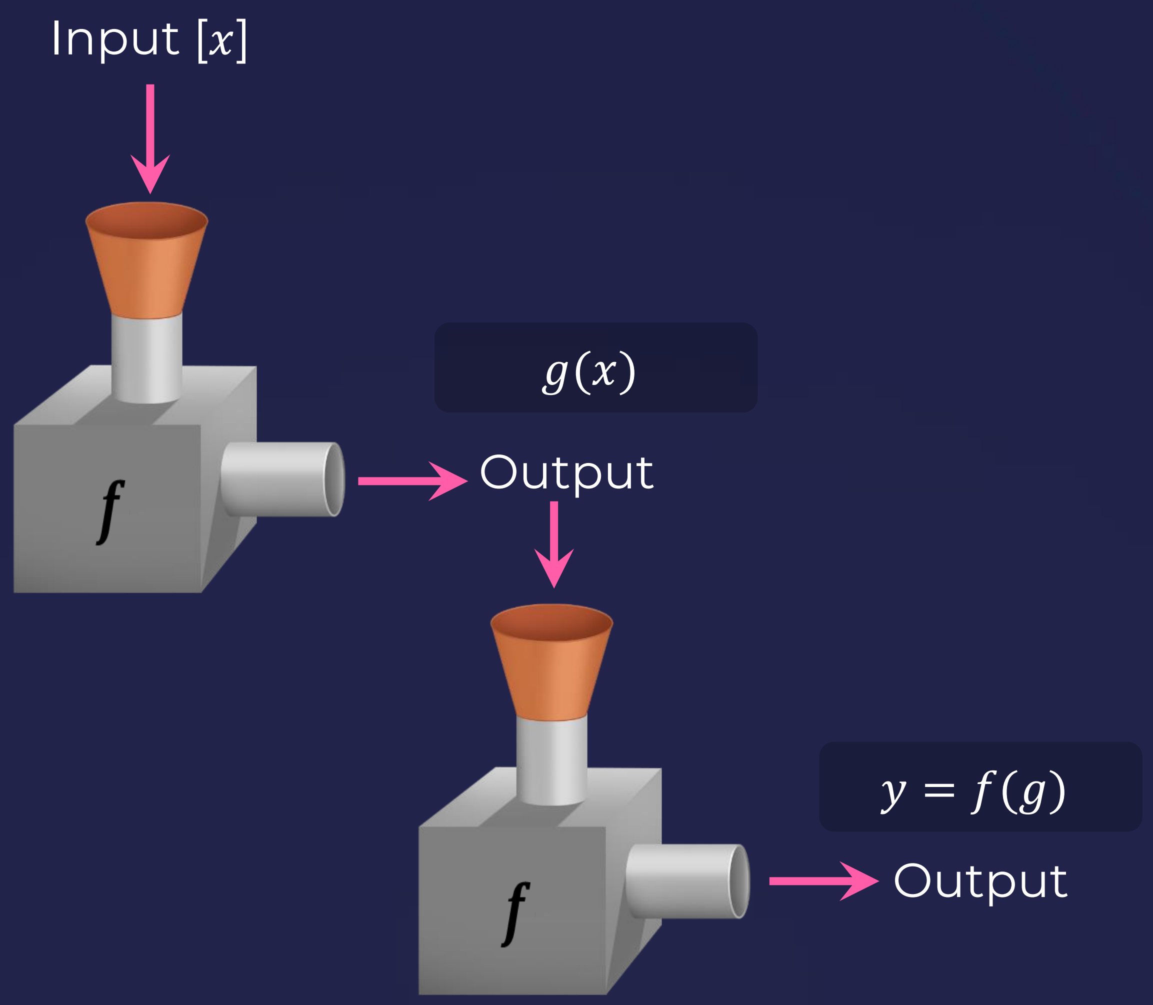
$$f'(x) = \frac{1 \times (-2x) - (1 - x^2) \times 0}{(1 - x^2)^2}$$

$$f'(x) = -\frac{2x}{\left(1 - x^2\right)^2}$$

Hint:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f(x)g'(x) - f'(x)g(x)}{(g(x))^2}$$

Composite Function



Composite function – example

Where , $f(g) = \sin(g)$

$g(x) = x^2$

It can also be written as ,

$f[g(x)] = \sin(x^2)$

Differentiate: $g(t) = \frac{1}{(4t^2-3t+2)^2}$ w.r.t. x using chain rule.

$$g(t) = \frac{1}{(4t^2 - 3t + 2)^2}$$

Put , $x = 4t^2 - 3t + 2$

$$g(t) = \frac{1}{(x)^2} \qquad \Rightarrow g'(t) = -\frac{2}{x^3}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow g'(t) = -\frac{2}{x^3}(8t - 3)$$

$$\Rightarrow g'(t) = \frac{-2(8t-3)}{(4t^2 - 3t + 2)^3}$$

Hint:

$f'(x) = f'(g) \cdot g'(x)$

Differentiate:
$f(x) = \sin(x^2 + 5) + e^x \tan x$ w.r.t. x.

$$f(x) = \sin(x^2 + 5) + e^x \tan x$$

Using Chain rule and Product rule:

$$f'(x) = \cos(x^2 + 5) \cdot \frac{d}{dx}(x^2 + 5)$$

$$+ \frac{d}{dx}(e^x) \cdot \tan x + e^x \cdot \frac{d}{dx}(\tan x)$$

$$f'(x) = \mathbf{2x\cos(x^2 + 5) + e^x(\sec^2 x + \tan x)}$$

$$\frac{d}{dx}(tan\, x) = sec^2\, x$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Integration

$$\int f(x)\, dx$$

- Integral symbol: $\int$
- Function to integrate (Integrand): $f(x)$
- Variable w.r.t. which function is to be integrated: x (in dx)
- $\int f(x)\, dx$: Integral of f

Integration of Functions

- Algebraic
- Exponential
- Trigonometric

Integration

Integration of Functions

Algebraic

This is also called as Indefinite integration

$$\frac{d(x^n)}{dx} = nx^{n-1} \quad \Big| \quad \int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \quad \Big| \quad \int \frac{1}{x} \, dx = \ln x + C$$

Exponential

$$\frac{d(e^x)}{dx} = e^x \quad \Big| \quad \int e^x \, dx = e^x + C$$

Trigonometric

$$\frac{d(\sin x)}{dx} = \cos x \quad \Big| \quad \int \cos x \, dx = \sin x + C$$

$$\frac{d(\cos x)}{dx} = -\sin x \quad \Big| \quad \int \sin x \, dx = -\cos x + C$$

$$\int \sqrt{x^3}dx = ?$$

Using the formula of integration

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \sqrt{x^3}dx = \frac{2}{5}\, x^{\frac{5}{2}} = \frac{2}{5}\sqrt{x^5} + C$$

Integration of Algebraic expressions

Derivative of any constant term as a sum in original function becomes zero. Hence, adding C to the result of integration. C gives the most general case.

$$\frac{d}{dx}(x^2) = 2x$$

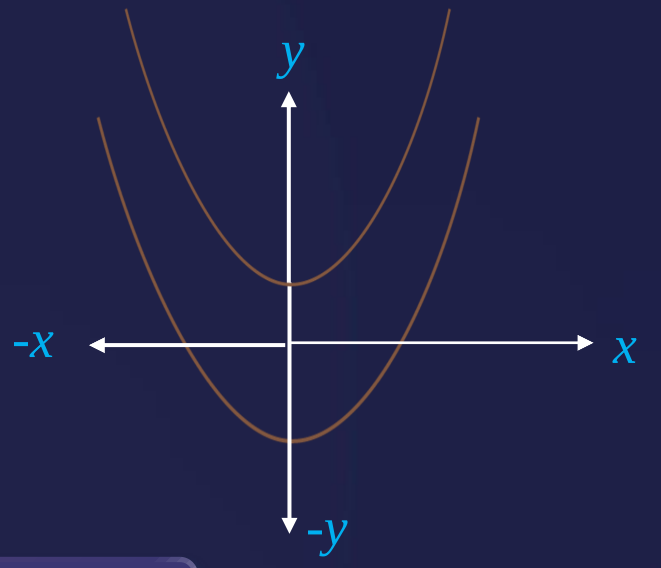
$$\frac{d}{dx}(x^2 + 1) = 2x$$

$$\frac{d}{dx}(x^2 - 1) = 2x$$

If $\frac{dy}{dx} = 2x \Rightarrow y = x^2 + C$

↓

Family of curves



Properties of Integration

$$\int a\, f(x)\, dx = a \int f(x)\, dx$$

$$\int [af(x) + bg(x)]dx = a \int f(x)\, dx + b \int g(x)dx$$

Important Formulae in Indefinite integration

$\frac{d(\tan x)}{dx} = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d(\sec x)}{dx} = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
$\frac{d(\cot x)}{dx} = -\text{cosec}^2 x$	$\int \text{cosec}^2 x \, dx = -\cot x + C$
$\frac{d(\text{cosec} x)}{dx} = -\text{cosec} x \cot x$	$\int \text{cosec} x \cot x \, dx = -\text{cosec} x + C$

$$\int \left(4x^3 + \frac{1}{x} + \sin x\right) dx = ?$$

$$\int \left(4x^3 + \frac{1}{x} + \sin x\right) dx = x^4 + \ln(x) - \cos(x) + c$$

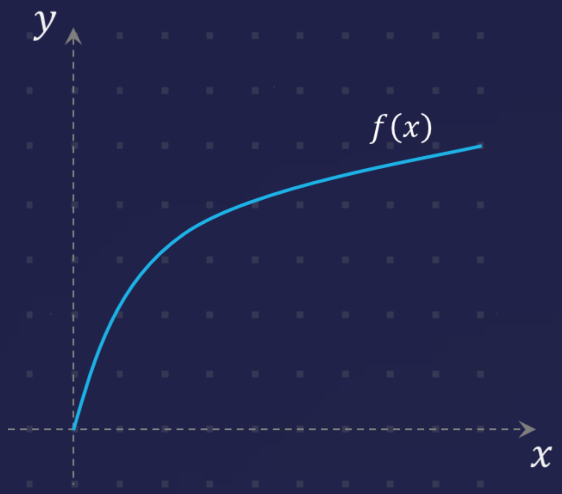
A $4x^4 + \ln(x) + \cos x + C$

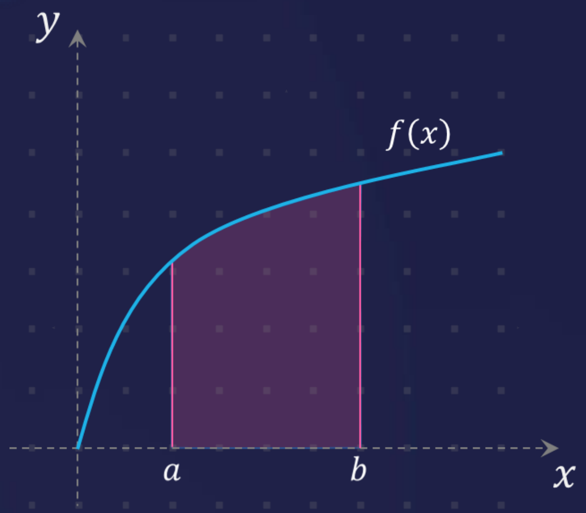
B $4x^4 + \ln(x) + \cos x$

C $x^4 + \ln(x) - \cos x + C$

D $x^4 + \ln(x) - \cos x$

Geometrical Meaning of Integration

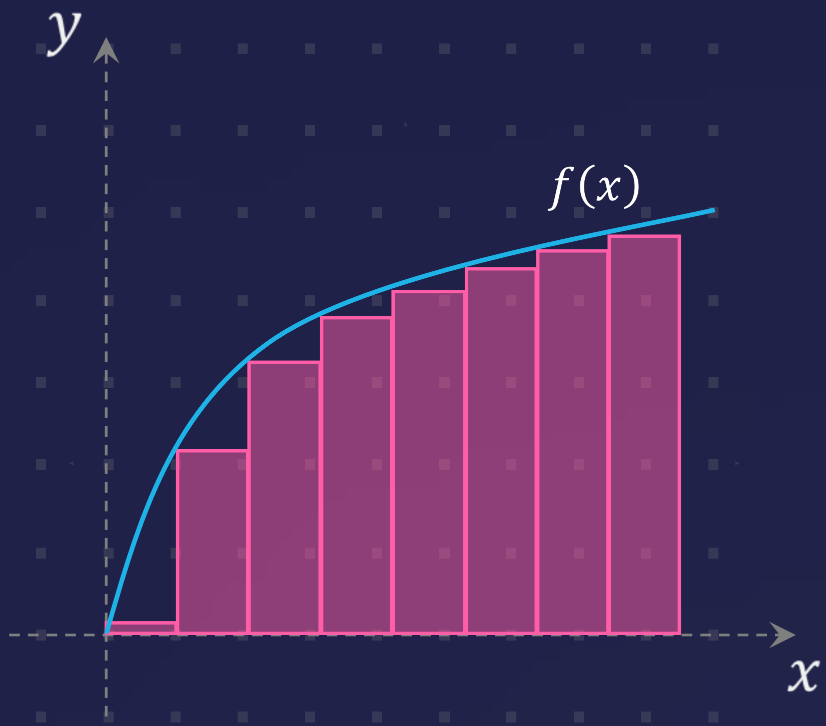


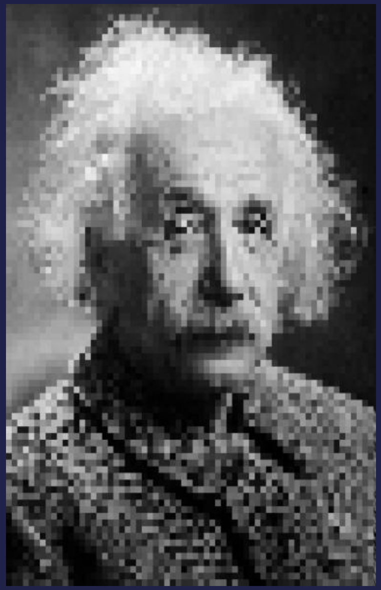


$\int f(x)dx \rightarrow$ Area under the curve

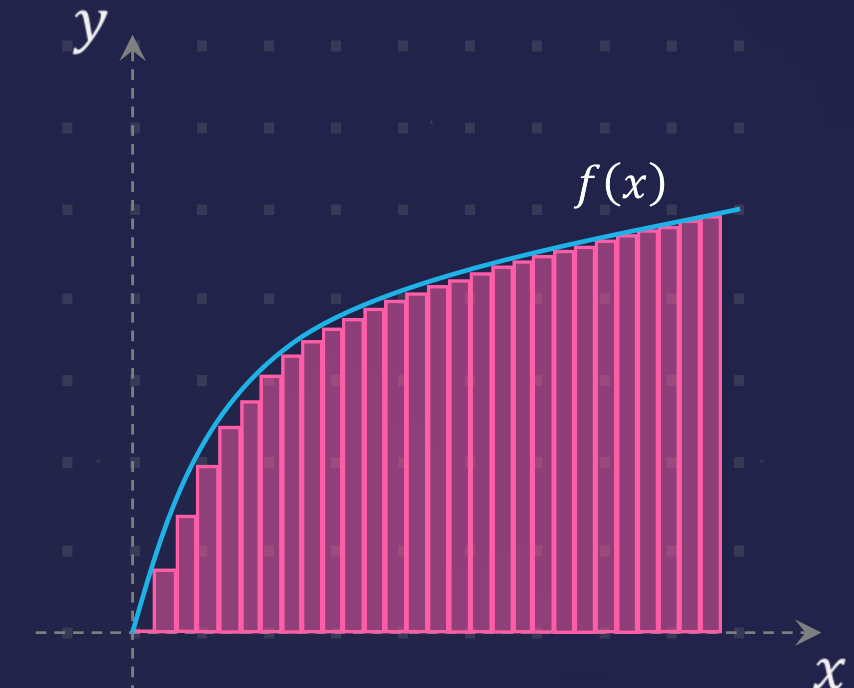
$\int f(x)dx \rightarrow$ Area under the curve from $x = a$ to $x = b$

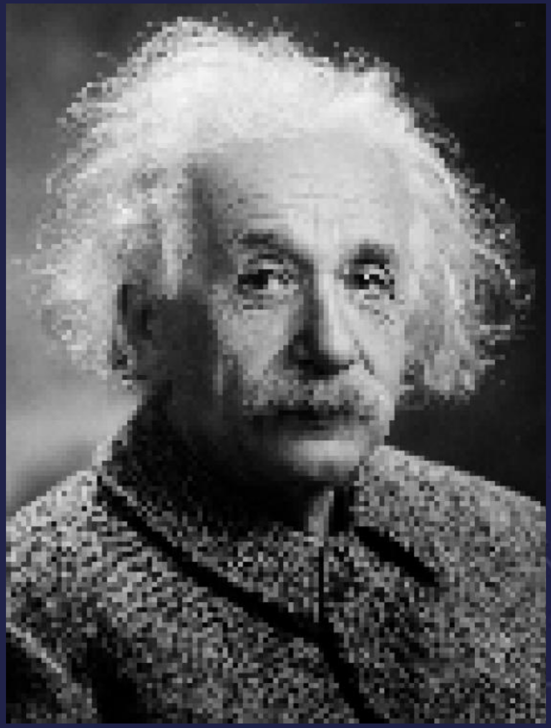
Such a process is called Definite integration.



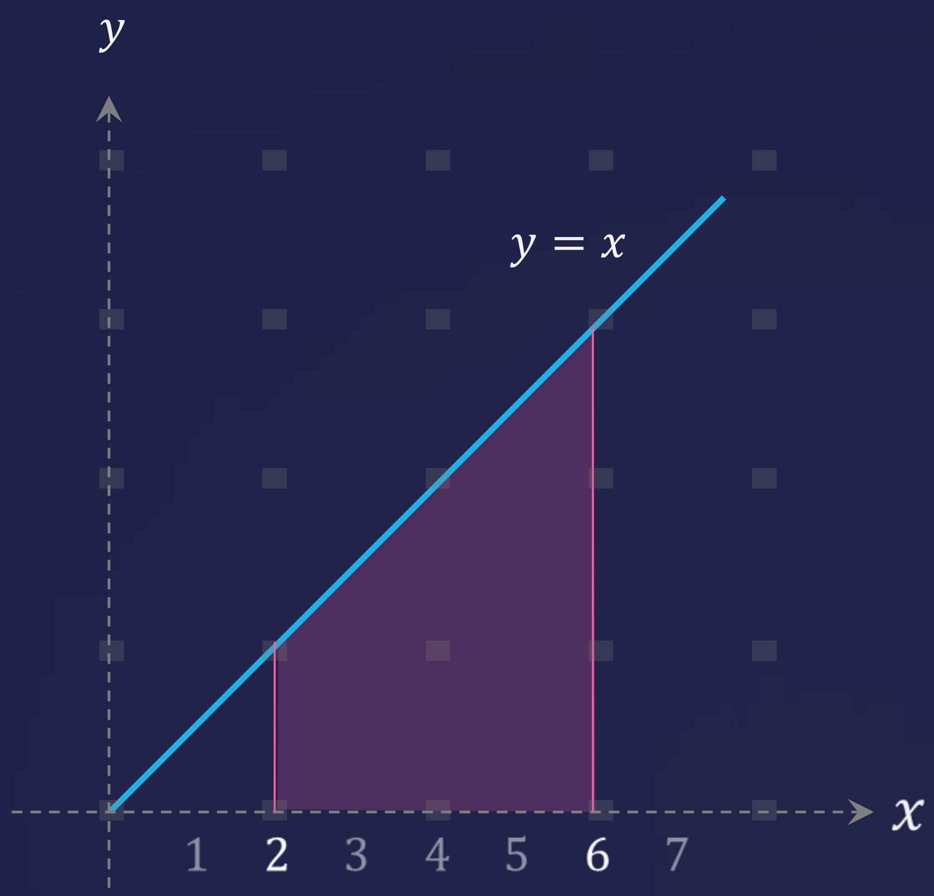


- In the adjacent picture less number of pixels are there and the picture quality is low.
- Below picture more the pixels, better the picture quality.
- Likewise, Smaller the element, less will be the error in measuring area, which is mathematically integration





Find the area bounded by the lines $y = x$, $y = 0$, $x = 2$ and $x = 6$



$$\text{Area of Trapezium} = \frac{h(a+b)}{2}$$

$$= \frac{4(2+6)}{2} = 16$$

$$\text{Area of Trapezium} = \int_2^6 x\,dx$$

$$= \left[\frac{x^2}{2}\right]_2^6 = 16$$

Evaluate the integral: $\int_{-\pi/2}^{\pi/2} \cos x \, dx$

$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = \sin x \Big|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = 2$$

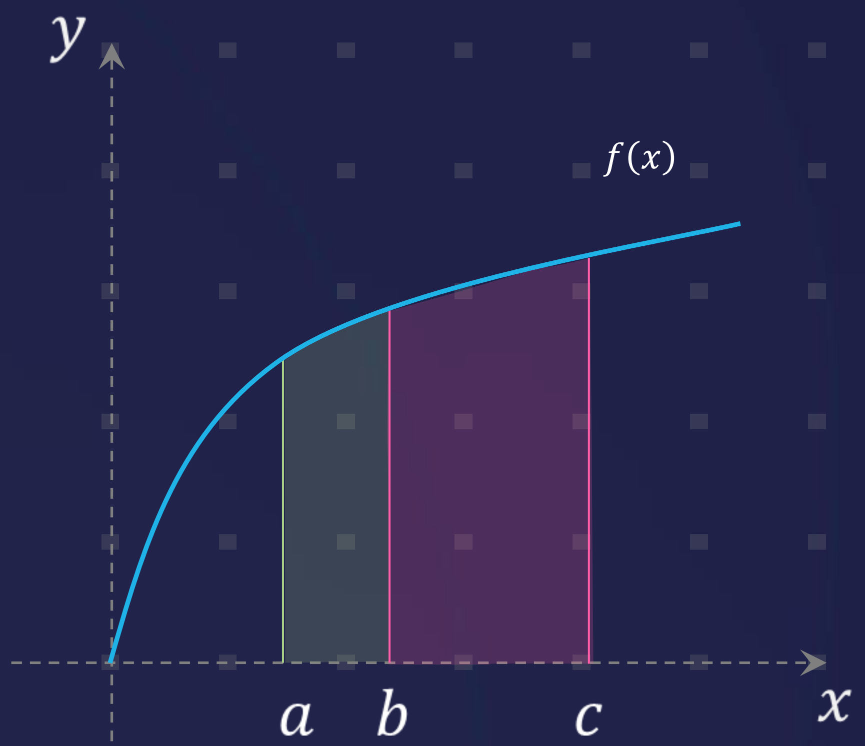
$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = 2$$

Properties of definite Integration

1) $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

2) $\int_a^a f(x)dx = 0$

3) $\int_a^b f(x)dx = -\int_b^a f(x)dx$



$$\int_1^4 \frac{1}{2\sqrt{x}}\,dx + \int_4^1 \frac{1}{2\sqrt{x}}\,dx = ?$$

$$\int_1^4 \frac{1}{2\sqrt{x}}\,dx + \int_4^1 \frac{1}{2\sqrt{x}}\,dx = [\sqrt{x}]_1^4 + [\sqrt{x}]_4^1 = 0$$

A 0

B 4

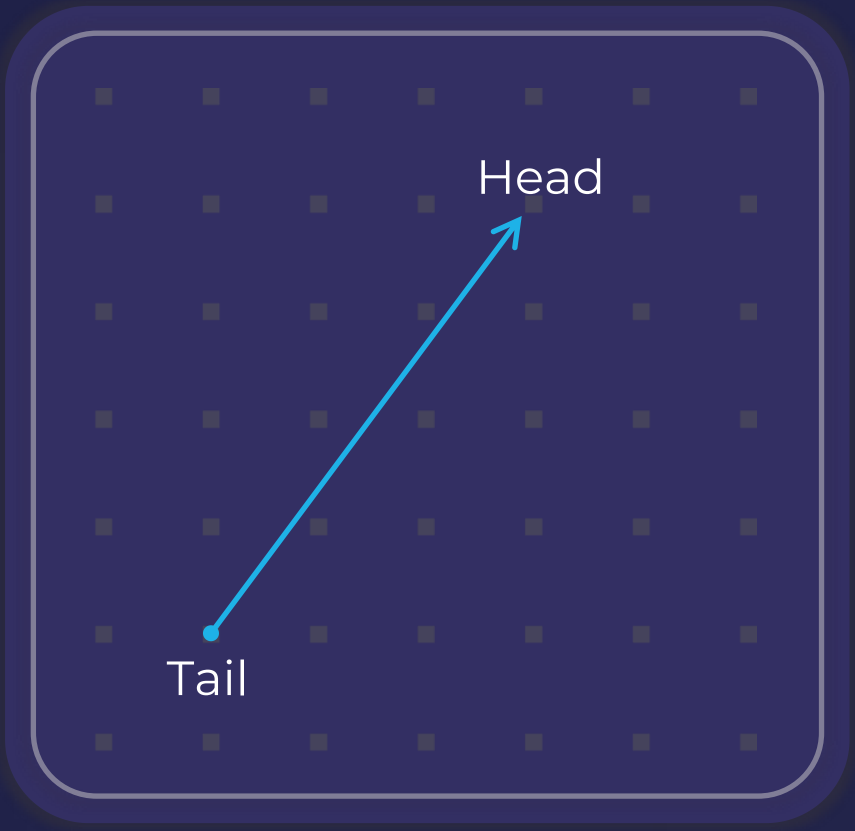
C 2

D 3

Vector

A physical quantity is a vector when,

- It has a magnitude and a unit
- It has a direction



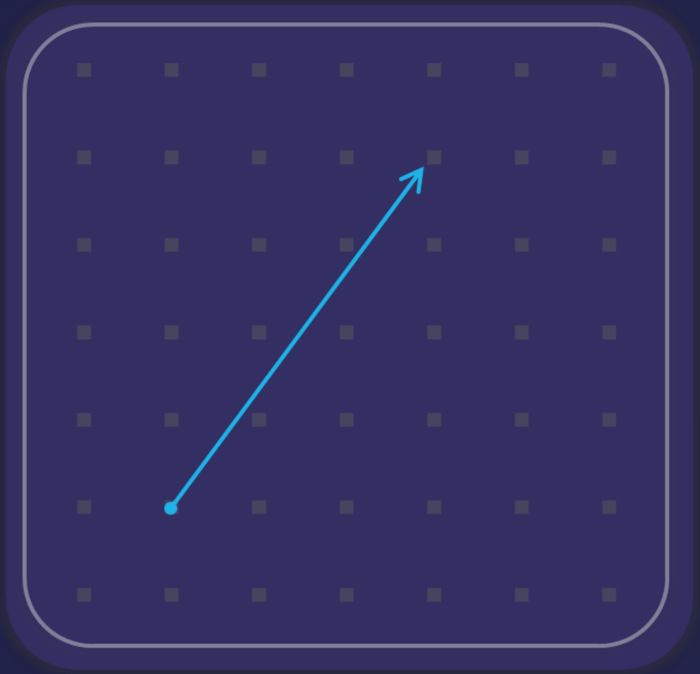
Identify the Vector quantities

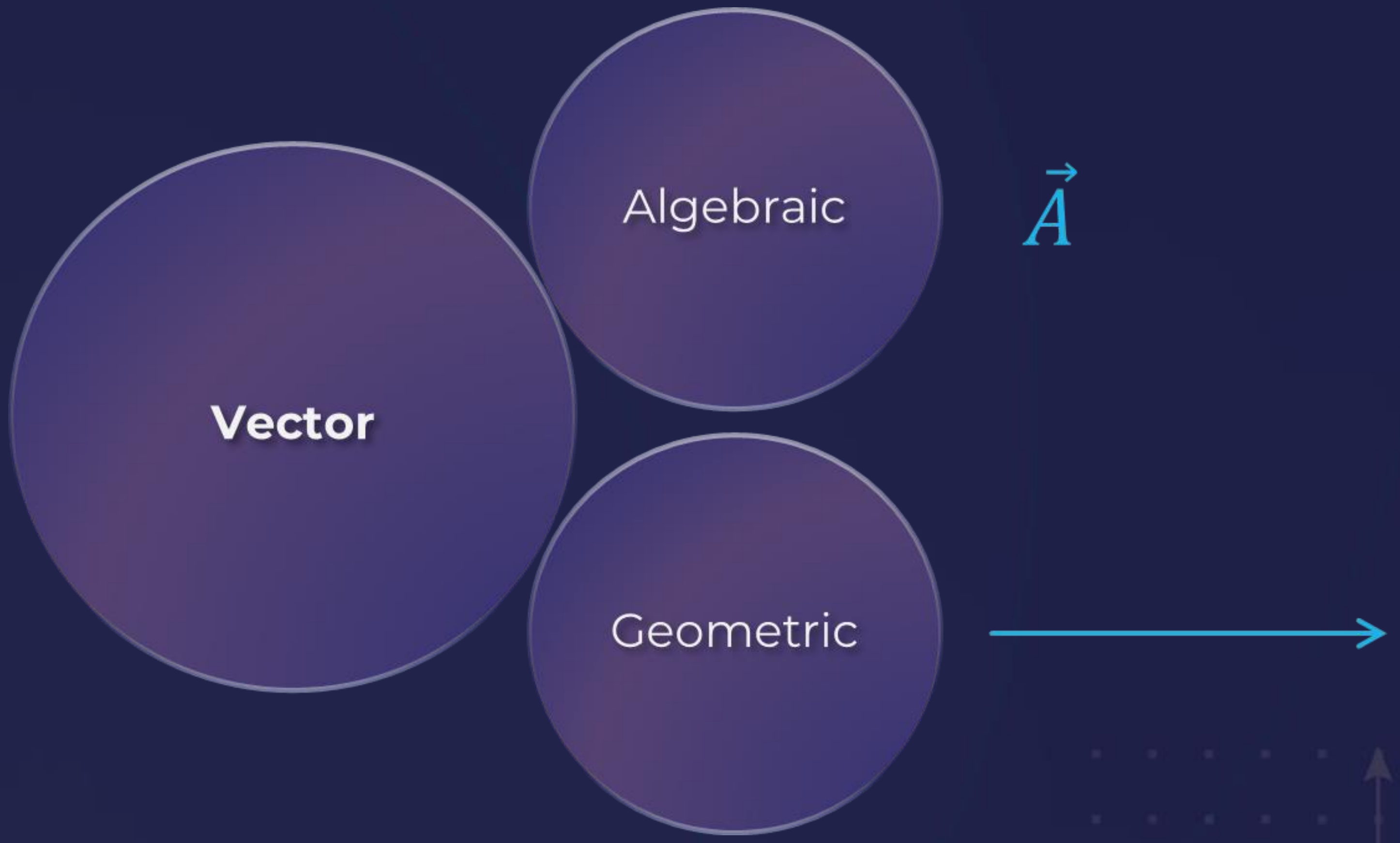
Mass	Speed
Temperature	Velocity
Time	Acceleration
Volume	Electric Current

Definition and How to represent a vector

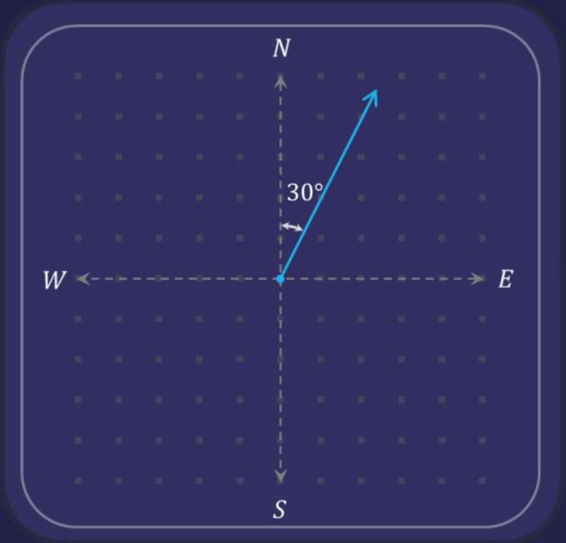
A physical quantity is a vector when,

- It has a magnitude and a unit
- It has a direction
- It obeys laws of vector algebra

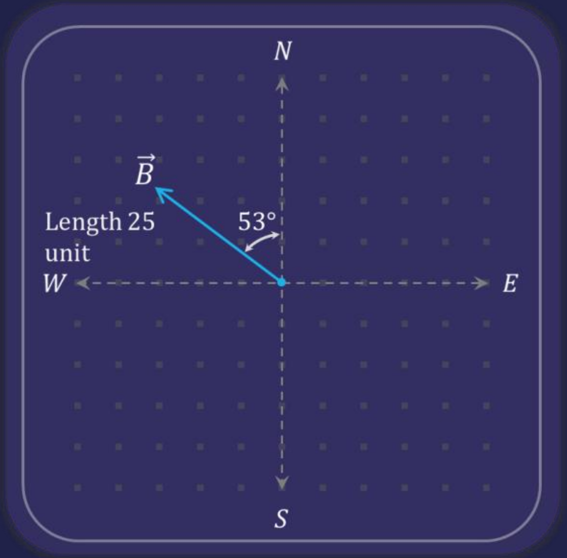




Representation of a Vector



Length 5 unit
$30° \; E$ of N

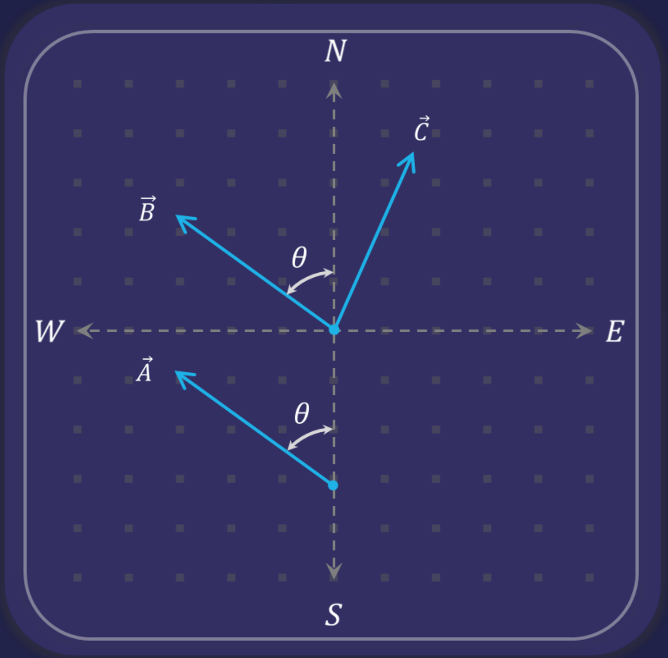


Length 25 unit

$53° \; W \; of \; N$

$37° \; N \; of \; W$

Are these vectors different?



Assume that all vectors have the same magnitude and are of the same physical quantity (Say Force)

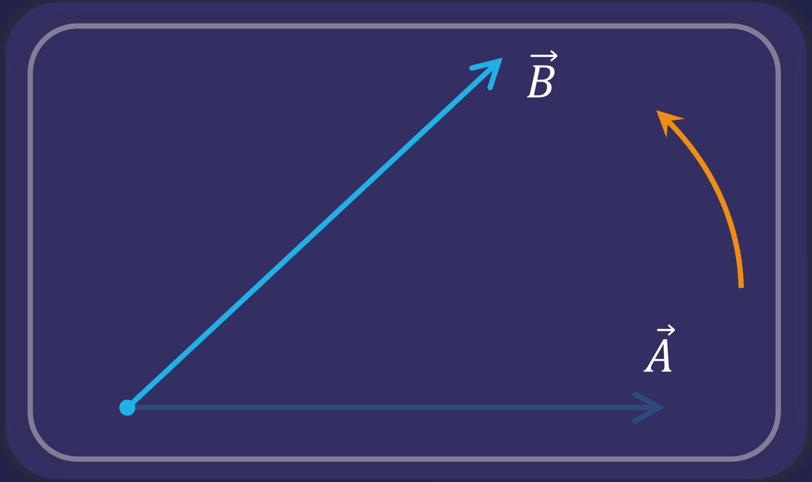
$\vec{A} = \vec{B}$

Properties of Vectors

- If a vector is displaced parallel to itself, then it does not change in magnitude or direction.

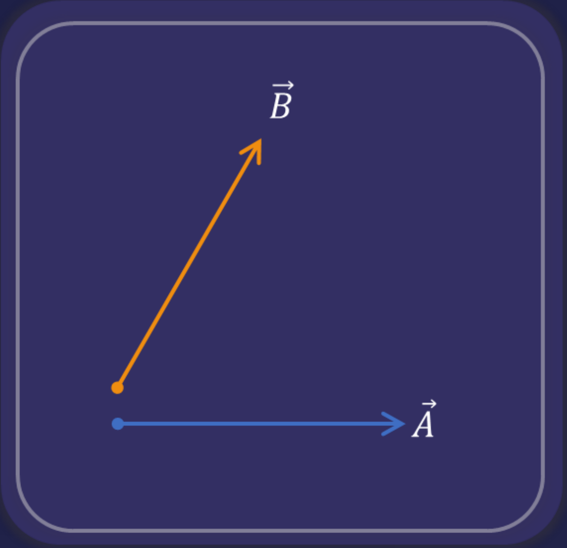
$\vec{A} = \vec{B}$

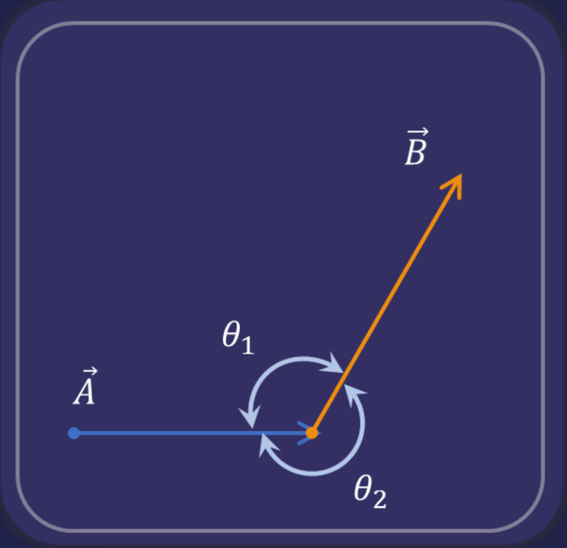
- If a vector is rotated through an angle, other than integer multiple of 2π (or 360°), then the vector changes.

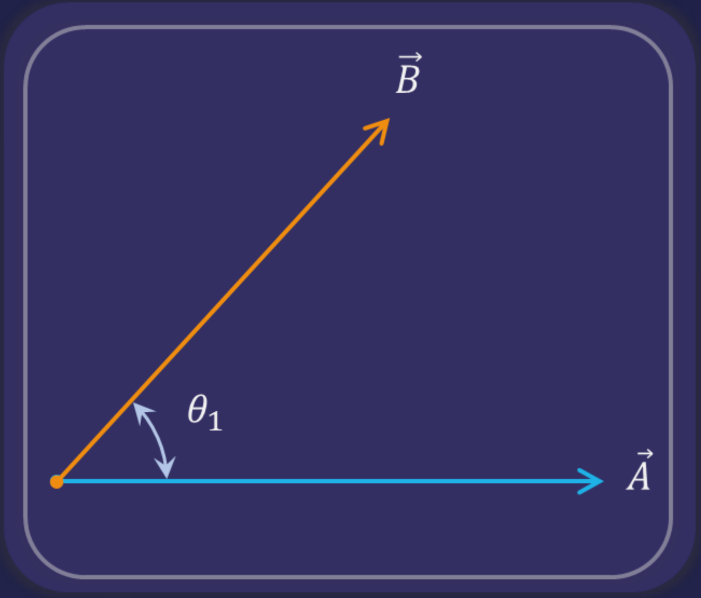


$\vec{A} \neq \vec{B}$

How to measure angle between vectors?







Step 1:

- Place vectors tail to tail or head to head

Step 2:

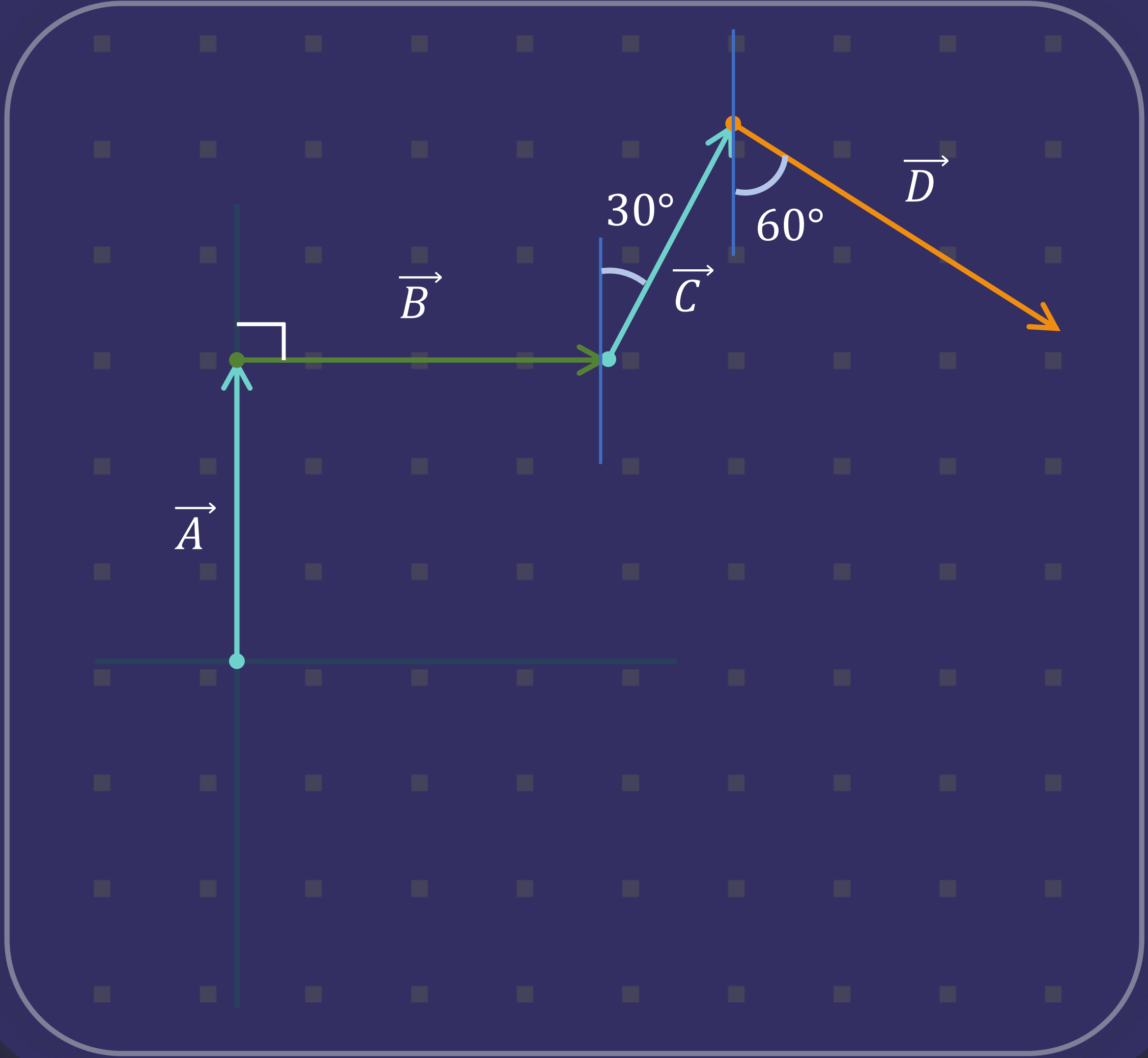
- Measure the smaller angle i.e. angle which is less than 180° between them

Find the angle between the vectors

(i) $\vec{A}$ and $\vec{B}$ is 90°

(ii) $\vec{A}$ and $\vec{C}$ is 30°

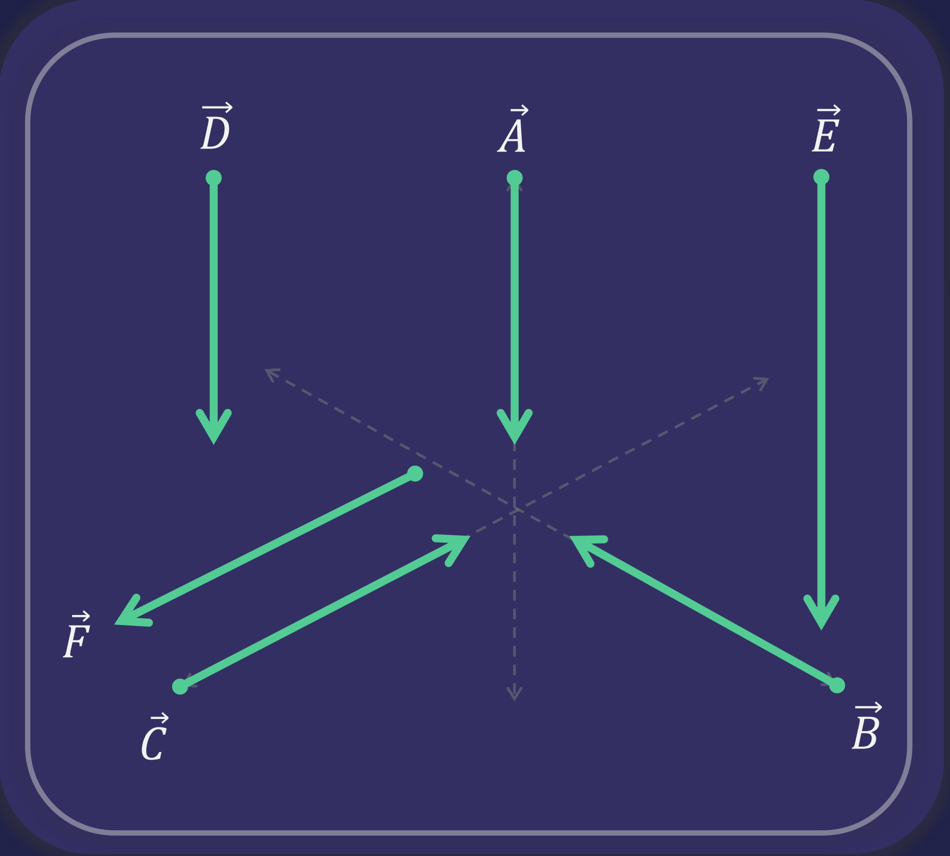
(iii) $\vec{A}$ and $\vec{D}$ is 120°



Types of Vectors

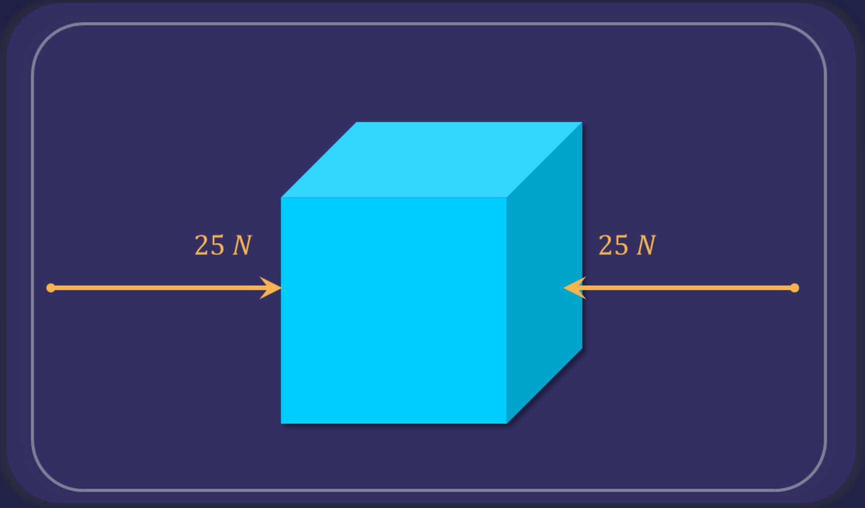
Collinear	Lie on the same line
Parallel	Vectors along lines parallel to each other
Negative	Equal in magnitude but opposite in direction
Equal	Same magnitude and along the same direction
Null or zero	Zero magnitude

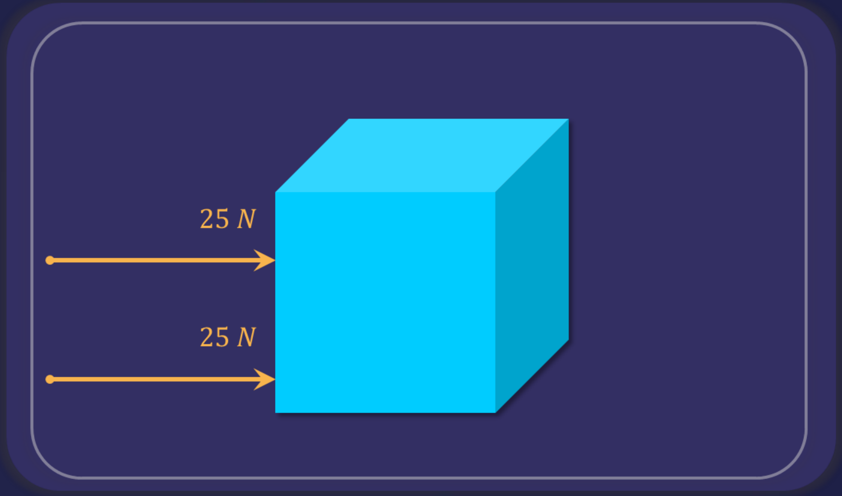
From the following diagram, write (i) a pair of equal vectors (ii) a vector, which is negative of $\vec{F}$



Equal Vectors	$\vec{A}$ & $\vec{D}$
Negative of $\vec{F}$	$\vec{C}$

Find the net force on the block



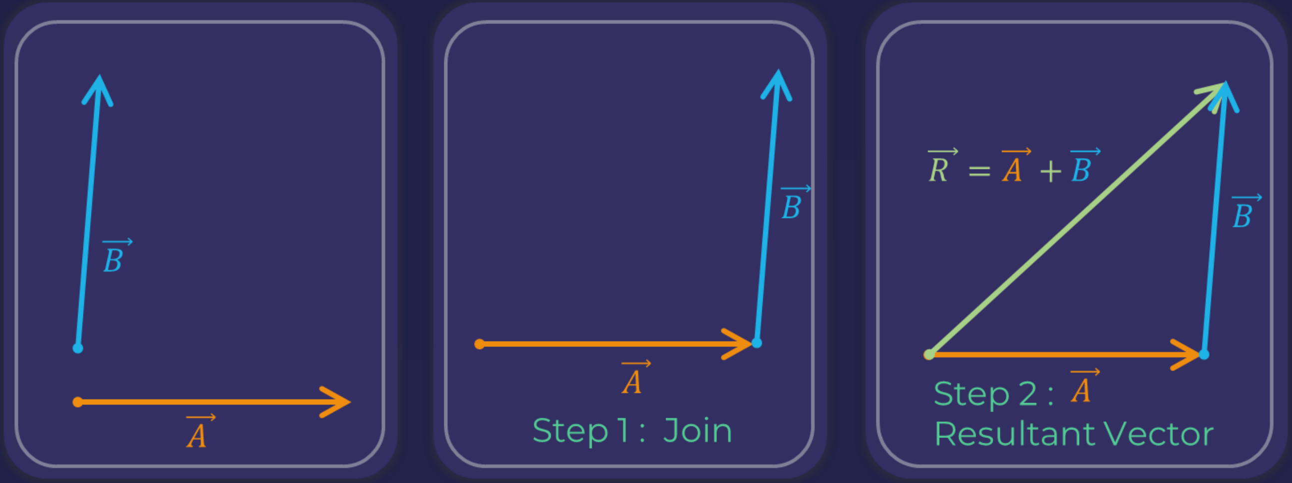


Force on the block is 0

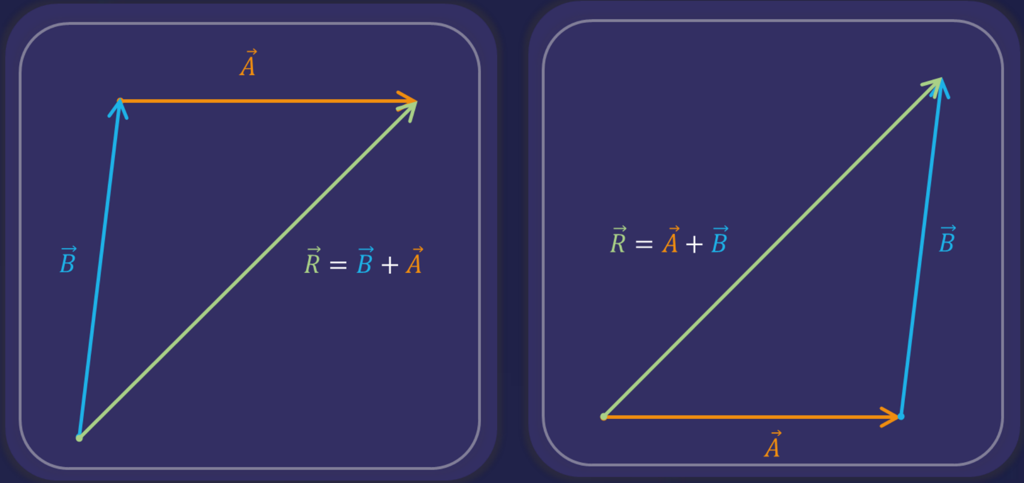
Force on the block is 50 N

Triangle law of vector addition

When two Vectors are aligned head to tail, their "vector sum" or "resultant vector" is represented by the third side of the completed triangle in the opposite order



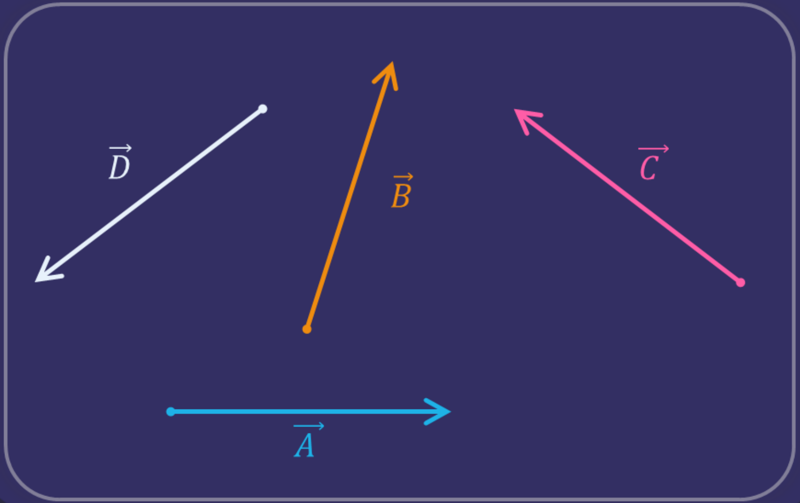
Commutative law

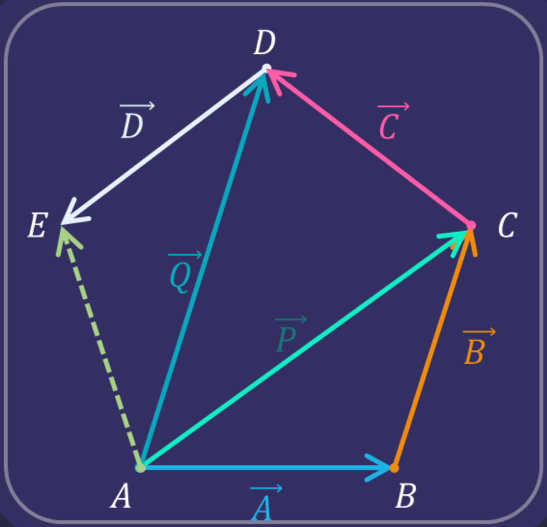


$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

From the following diagram, find , $\vec{A}+\vec{B}+\vec{C}+\vec{D}=$

Arranging all vectors in head-to-tail manner.
According to polygon law, $\overrightarrow{AE}$ is their resultant.





Using Triangle Law in ΔABC, ΔACD & ΔADE

$$\vec{A}+\vec{B}=\vec{P}$$

$$\vec{P}+\vec{C}=\vec{Q}$$

$$\vec{Q}+\vec{D}=\vec{E}$$

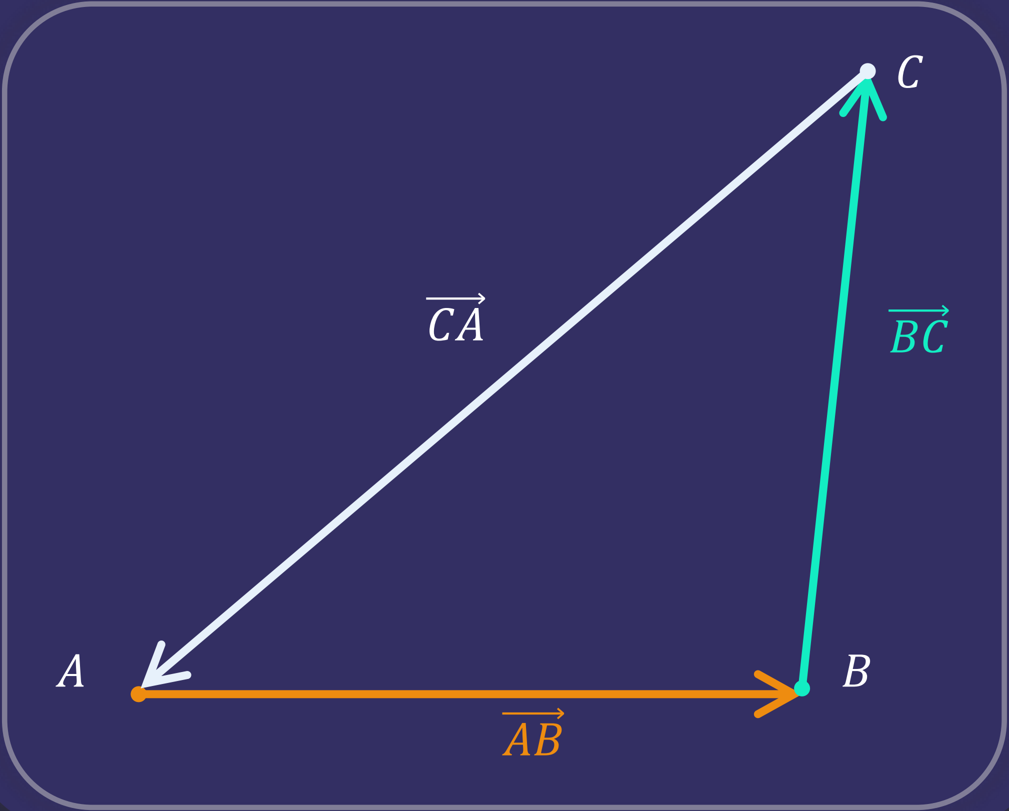
$$\vec{A}+\vec{B}+\vec{C}+\vec{D}=\vec{E}$$

If several vectors are represented in head-to-tail manner so that they form an incomplete polygon, then the remaining side of the polygon, drawn from tail of first vector to the head of last vector, is their resultant vector.

Polygon Law

Sum of all the vectors in cyclic order is a **zero vector**

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

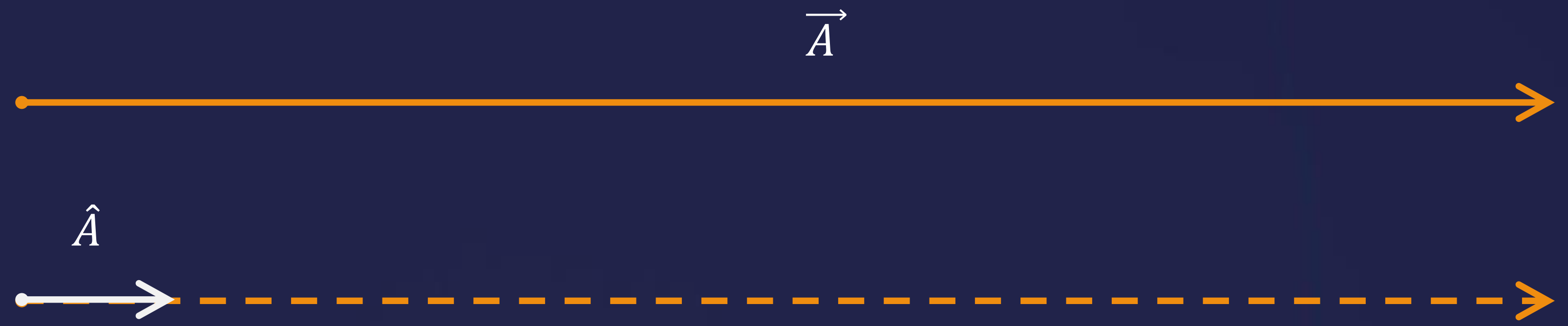


Unit Vector

Vector

Magnitude | Direction

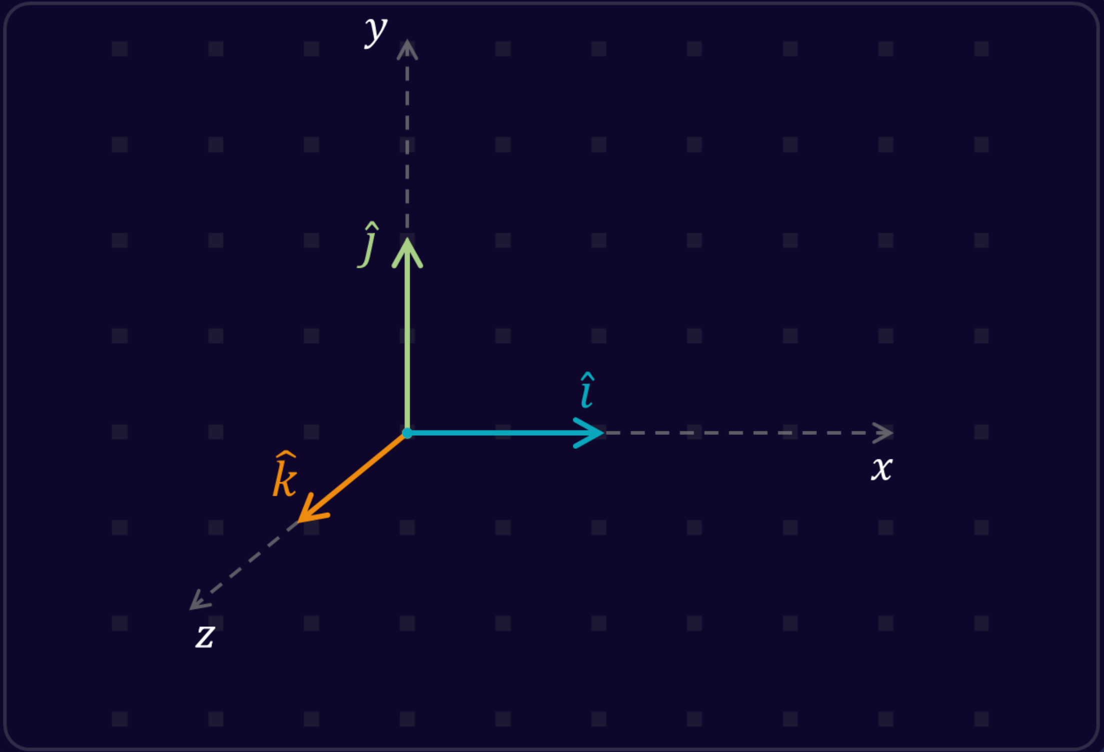
Vector divided by its own magnitude is a vector with unit magnitude and direction along the parent vector.

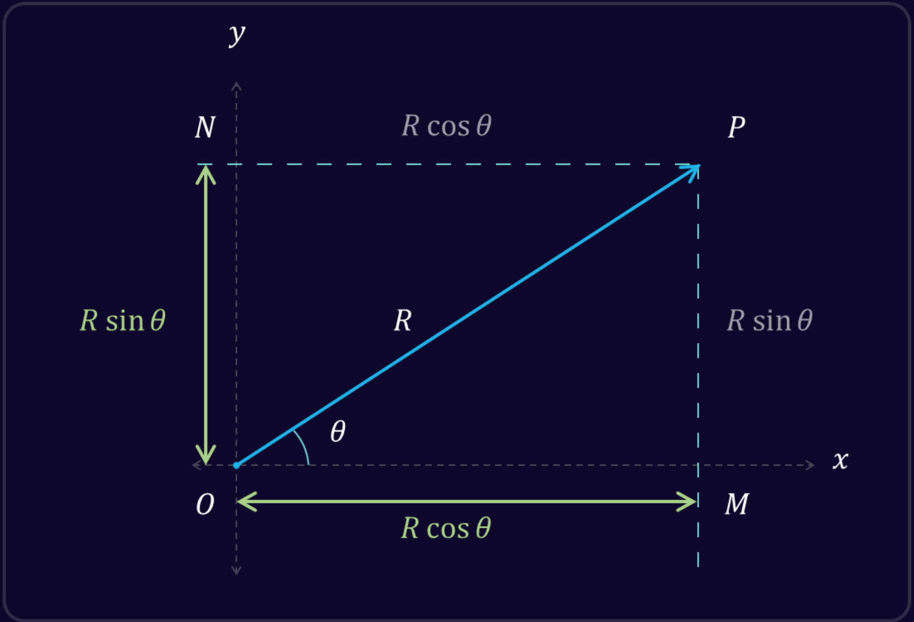


$$\frac{\vec{A}}{|\vec{A}|} = \hat{A}$$

$$\vec{A} = |\vec{A}|\hat{A}$$

Resolution of a vector and Special unit vectors defined along coordinate axes



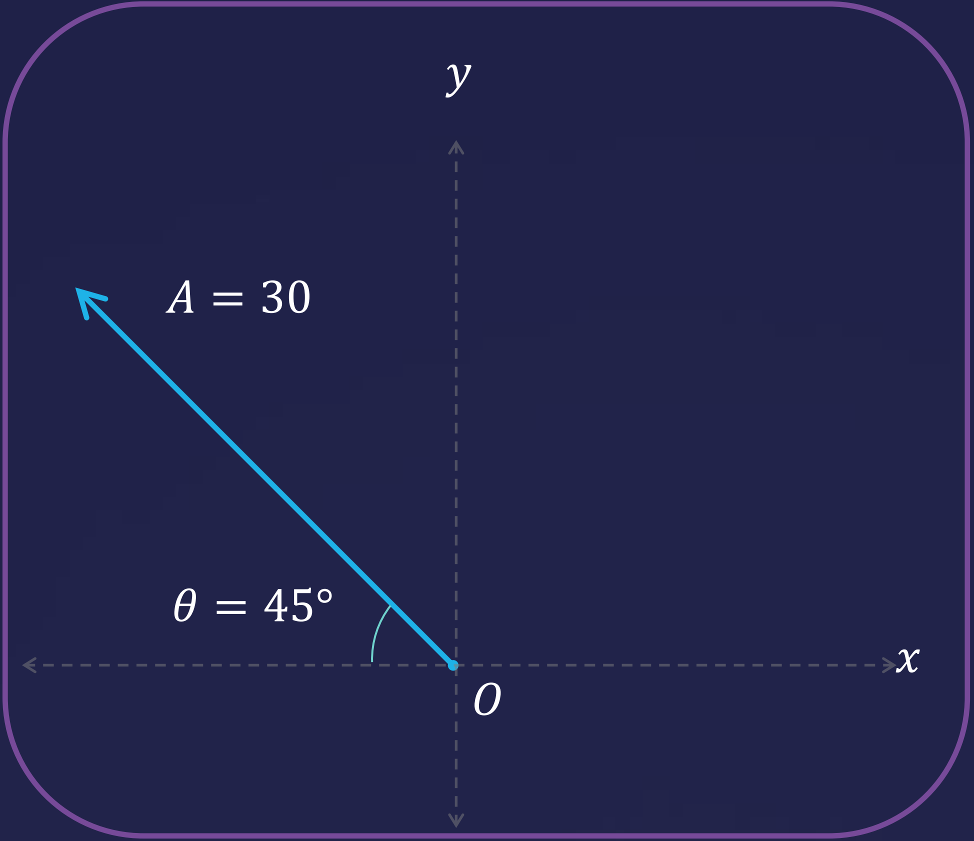


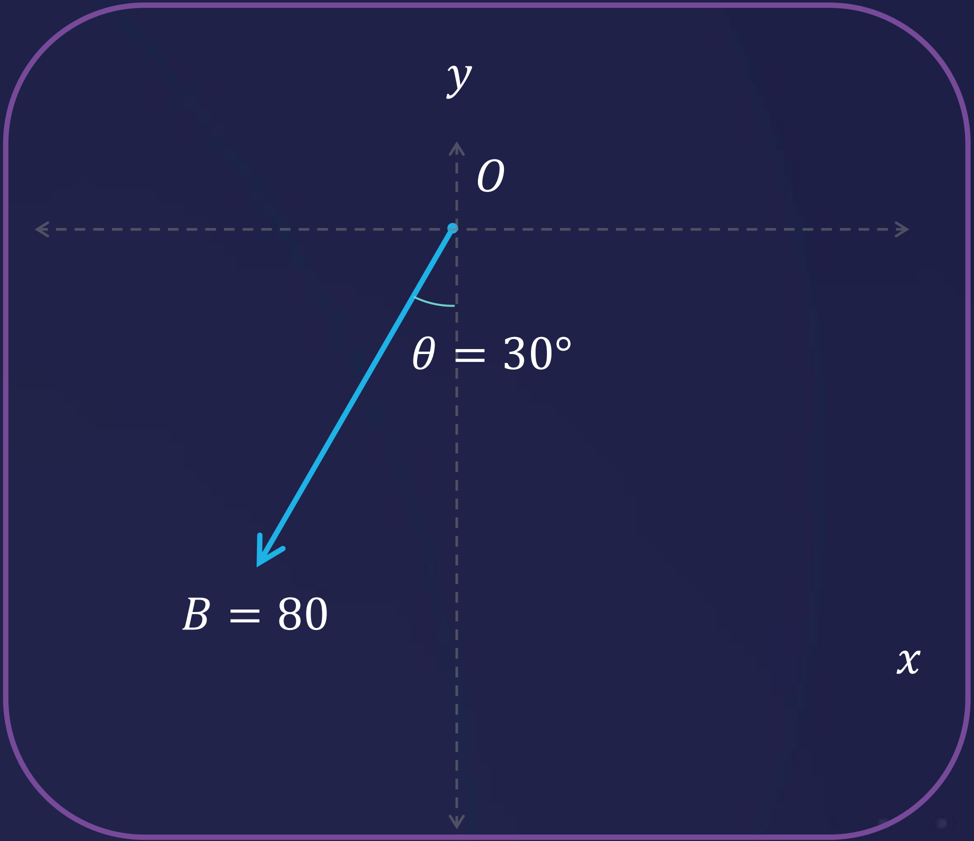
$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

$$\vec{R} = R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}$$

Fill in the Blanks

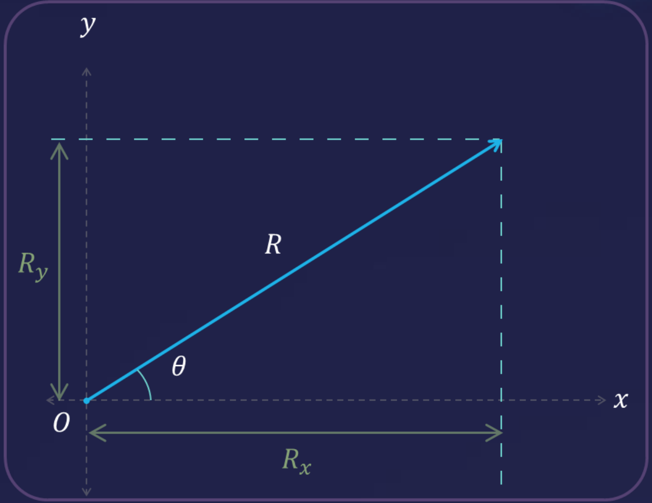




$\vec{A} = \boxed{-15\sqrt{2}}\,\hat{\imath} + \boxed{15\sqrt{2}}\,\hat{\jmath}$

$\vec{B} = \boxed{-40}\,\hat{\imath} + \boxed{-40\sqrt{3}}\,\hat{\jmath}$

Magnitude and Direction (2D and 3D)



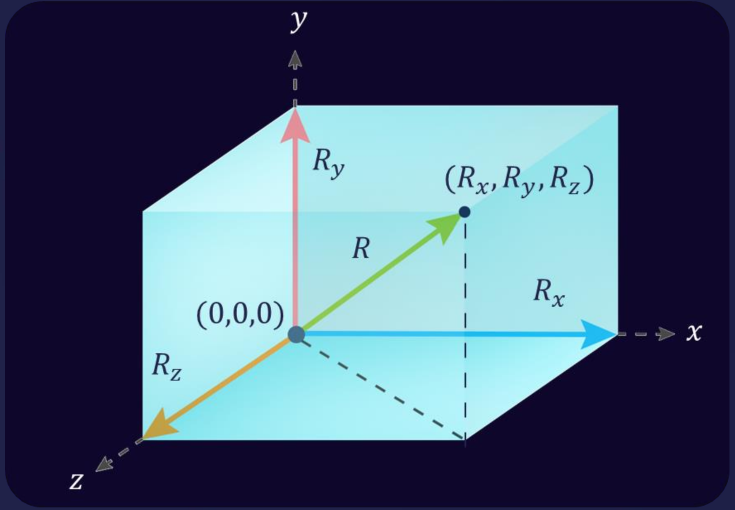
$$\vec{R} = R_x\,\hat{\imath} + R_y\,\hat{\jmath}$$

Magnitude

$$|\vec{R}| = \sqrt{{R_x}^2 + {R_y}^2}$$

Direction

$$\tan\theta = \frac{R_y}{R_x}$$



$$\vec{R} = R_x\,\hat{\imath} + R_y\,\hat{\jmath} + R_z\,\hat{k}$$

Magnitude

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

The unit vector along $\vec{A} = 2\hat{i} + 3\hat{j}$ is...

Solution :

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

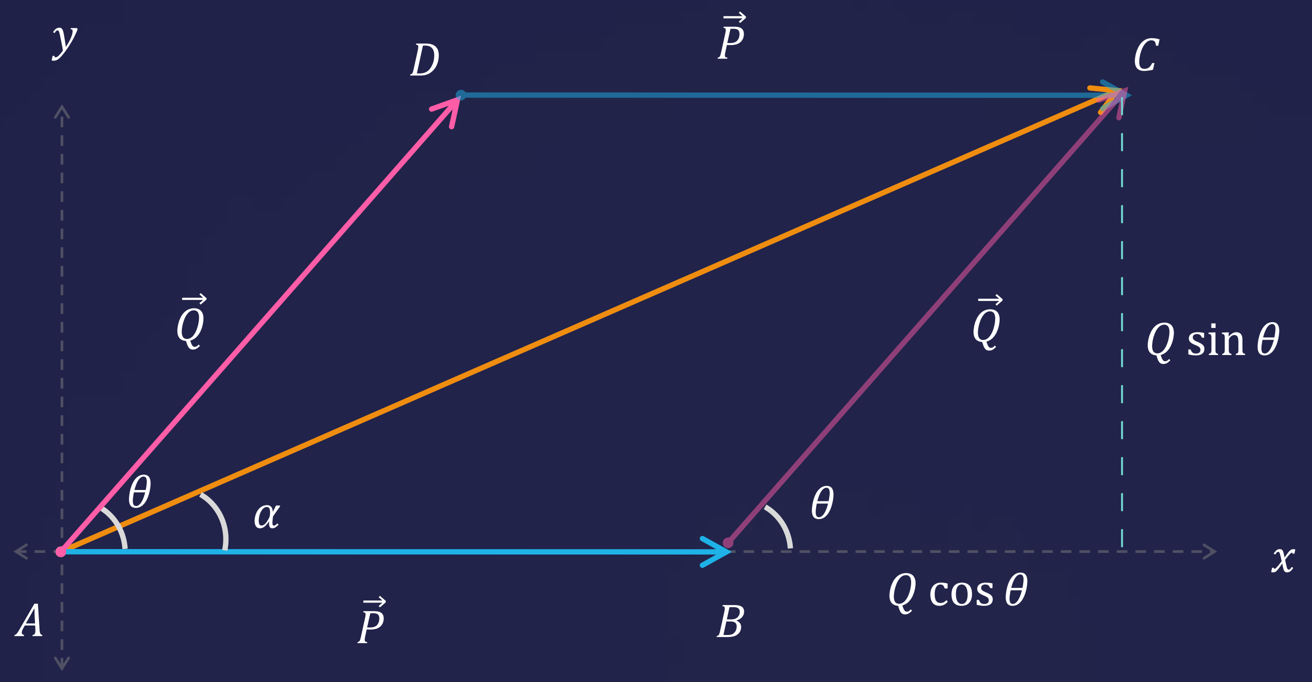
$$|\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\hat{A} = \frac{2\hat{i}}{\sqrt{13}} + \frac{3\hat{j}}{\sqrt{13}}$$

Parallelogram Law

If two vectors are oriented coinitial, representing two adjacent sides of a parallelogram, then their resultant is represented by the included diagonal of the completed parallelogram.

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD} = \vec{P} + \vec{Q}$$



Magnitude of the Resultant

$$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Direction of resultant

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

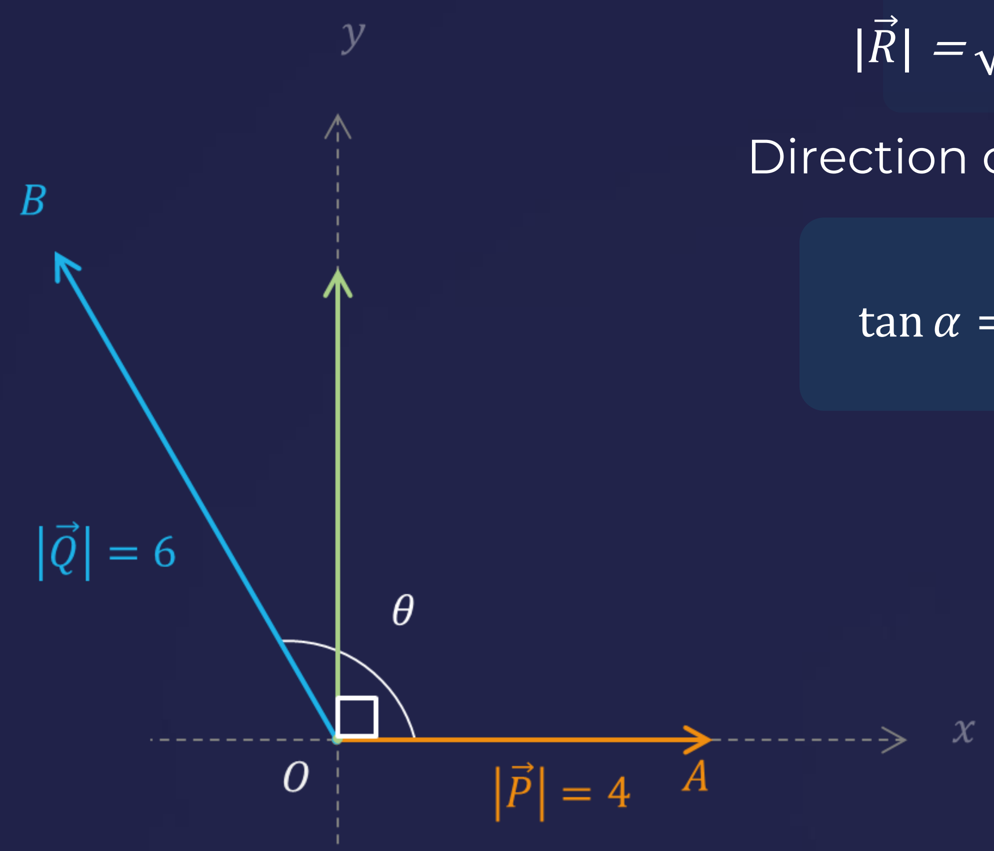
The resultant of vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$ as shown in the figure. Find the angle AOB.

Magnitude of the Resultant

$$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Direction of resultant

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$



$$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\tan 90^\circ = \frac{6\sin\theta}{4 + 6\cos\theta}$$

$$\theta = \cos^{-1}\left(-\frac{2}{3}\right)$$

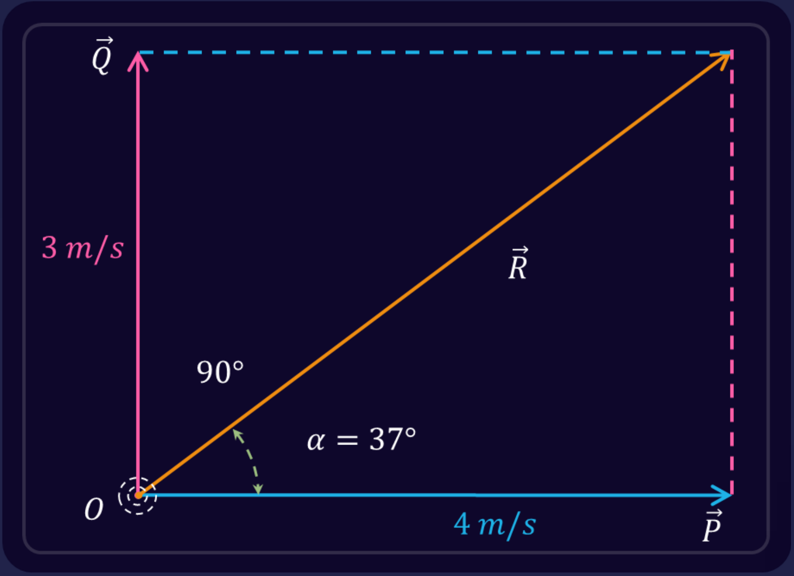
A river is flowing at $4\,m/s$. A girl swims with velocity $3\,m/s$ perpendicular to the direction of flow. Find the resultant velocity of the girl ?

Magnitude of the Resultant :

$$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Direction of resultant :

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$



Here, $P = 4\,ms^{-1}, Q = 3\,ms^{-1}, \theta = 90^o$

$$|\vec{R}| = \sqrt{16 + 9 + (2 \times 4 \times 3 \times \cos 90^o)}$$
$$= \sqrt{25} = 5\,ms^{-1}$$

$$\tan\alpha = \frac{3 \times \sin 90^o}{4 + 3\cos 90^o} = \frac{3}{4} \Rightarrow \alpha = 37^o$$

$$|R| = 5\,ms^{-1};\ \alpha = 37°$$

If the magnitude of the resultant of two vectors of equal magnitudes is equal to the magnitude of either of the vectors, then find the angle between two vectors?

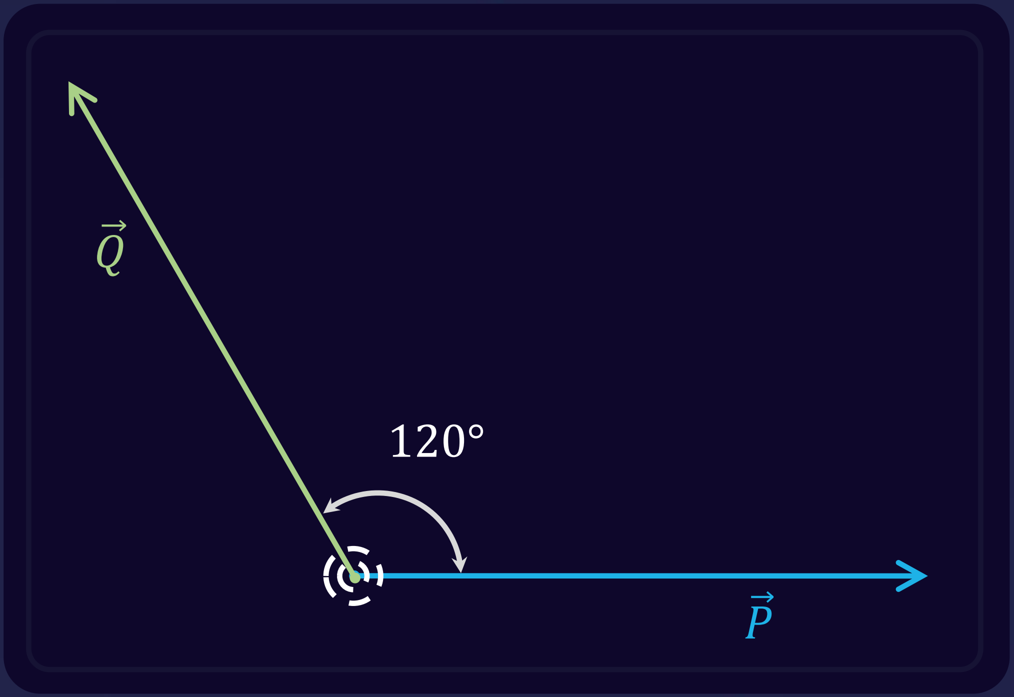
$$|\vec{P}| = |\vec{Q}| = |\vec{R}| = x$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$x^2 = x^2 + x^2 + 2x^2\cos\theta$$

$$\cos\theta = -\frac{1}{2}$$

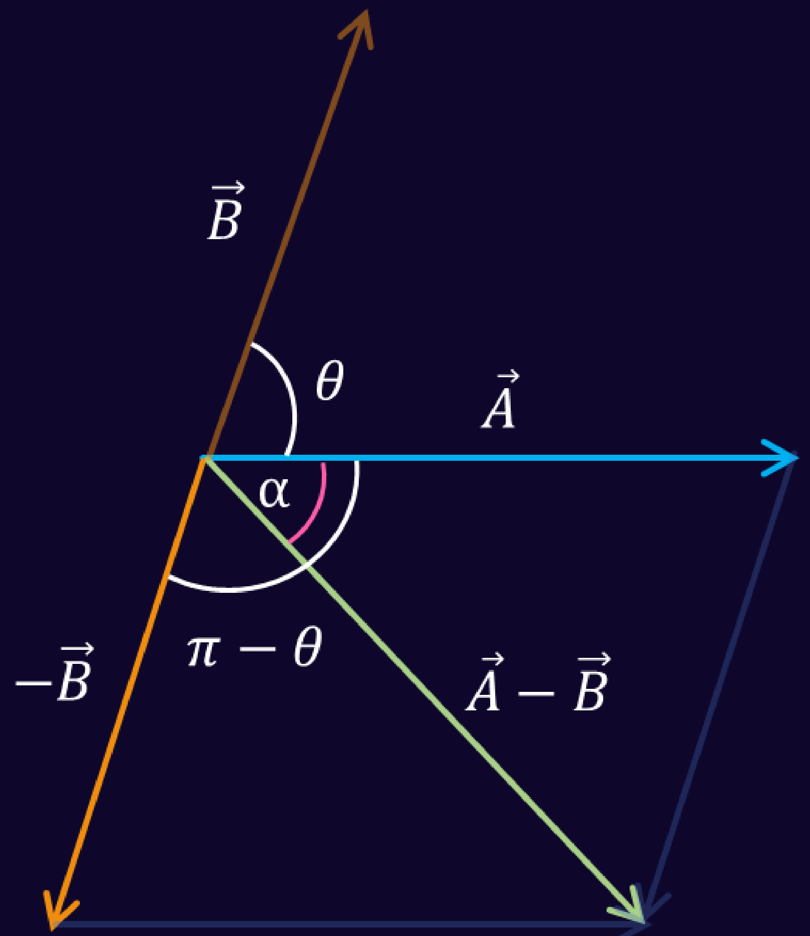
$$\theta = 120°$$



Vector Substraction

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$



Steps to add and subtract more than two Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

Addition

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\vec{A}$	A_x	A_y	A_z
$\vec{B}$	B_x	B_y	B_z
$\vec{C}$	C_x	C_y	C_z

$$\vec{A} + \vec{B} + \vec{C} = (A_x+B_x+C_x)\hat{i} + (A_y+B_y+C_y)\hat{j} + (A_z+B_z+C_z)\hat{k}$$

Subtraction

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\vec{A}$	A_x	A_y	A_z
$\vec{B}$	B_x	B_y	B_z
$\vec{C}$	C_x	C_y	C_z

$$\vec{A} - \vec{B} - \vec{C} = (A_x-B_x-C_x)\hat{i} + (A_y-B_y-C_y)\hat{j} + (A_z-B_z-C_z)\hat{k}$$

If magnitude of the sum of two vectors is equal to the magnitude of difference of the two vectors, then the angle between these vectors is,

Solution : $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

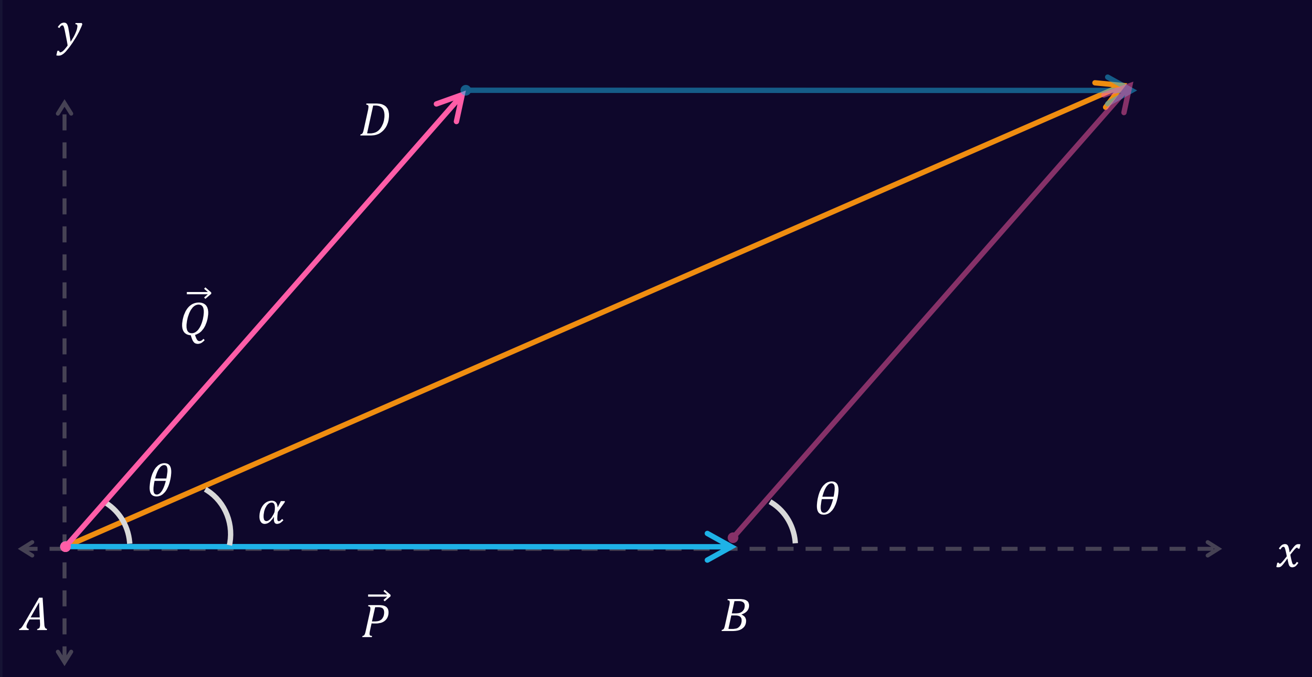
$$\sqrt{A^2+B^2+2AB\ cos\ \theta} = \sqrt{A^2+B^2-2AB\ cos\ \theta}$$

$$\Rightarrow 4AB\cos\theta = 0$$

$$\Rightarrow \theta = 90^o$$

Range of the resultant vector

Range of magnitude $|P - Q| \leq |\vec{R}| \leq P + Q$



If $\vec{P}$ of magnitude 5 unit and $\vec{Q}$ of magnitude of 7 unit are added, then which of the following can not be the magnitude of the resultant vector?

Solution : Range of magnitude

$$|P - Q| \leq |\vec{R}| \leq P + Q$$

$$|5 - 7| \leq |\vec{R}| \leq 5 + 7$$

$$2 \leq |\vec{R}| \leq 12$$

Find the vector $\vec{A} + 2\vec{B} + \vec{C}$, where $\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ and $\vec{C} = \hat{\imath} - \hat{\jmath} - \hat{k}$ and also find its magnitude.

Solution :

$$\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$2\vec{B} = 6\hat{\imath} + 8\hat{\jmath} - 4\hat{k}$$

$$\vec{C} = \hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\vec{D} = \vec{A} + 2\vec{B} + \vec{C} = 9\hat{\imath} + 6\hat{\jmath} - 4\hat{k}$$

$$|\vec{D}| = \sqrt{9^2 + 6^2 + 4^2} = \sqrt{133}$$

$$|\vec{D}| = \sqrt{133}$$

$$\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k},$$

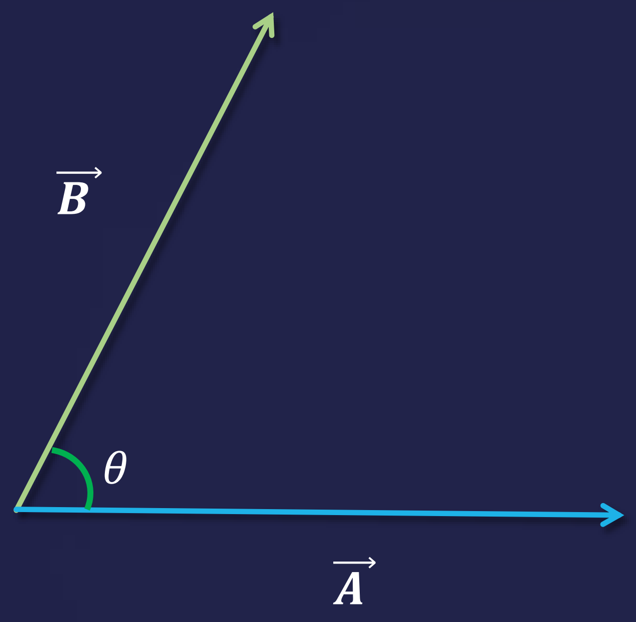
$$\vec{B} = 3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$$

$$\vec{C} = \hat{\imath} - \hat{\jmath} - \hat{k}$$

Dot Product and Cross Product

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted by $\vec{A}.\vec{B}$ is defined as the product of their magnitudes with cosine of the angle between them.

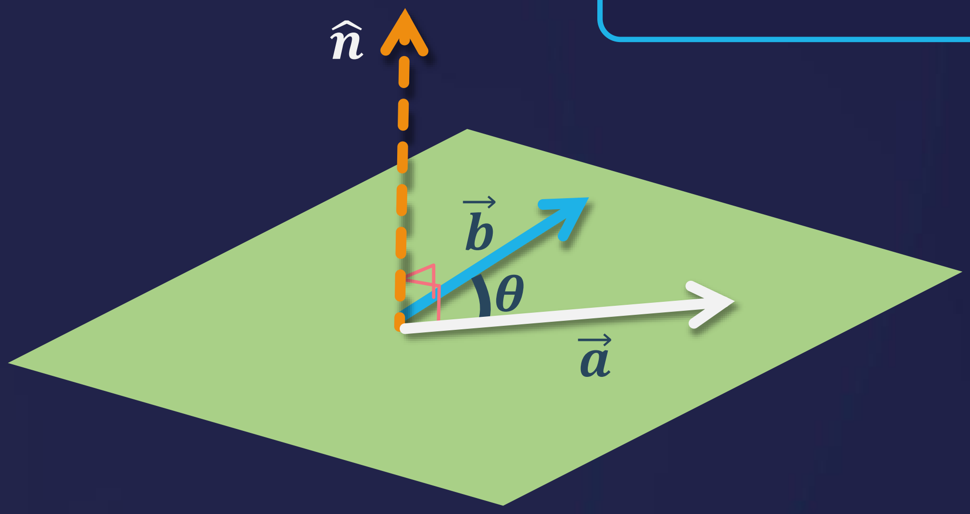
$$\vec{A}.\vec{B} = AB\cos\theta$$



A vector in magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them and in direction perpendicular to the plane containing the two vectors considered.

$\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ both and θ is angle $(0 \le \theta \le \pi)$ between two vectors $\vec{a}$ and $\vec{b}$.

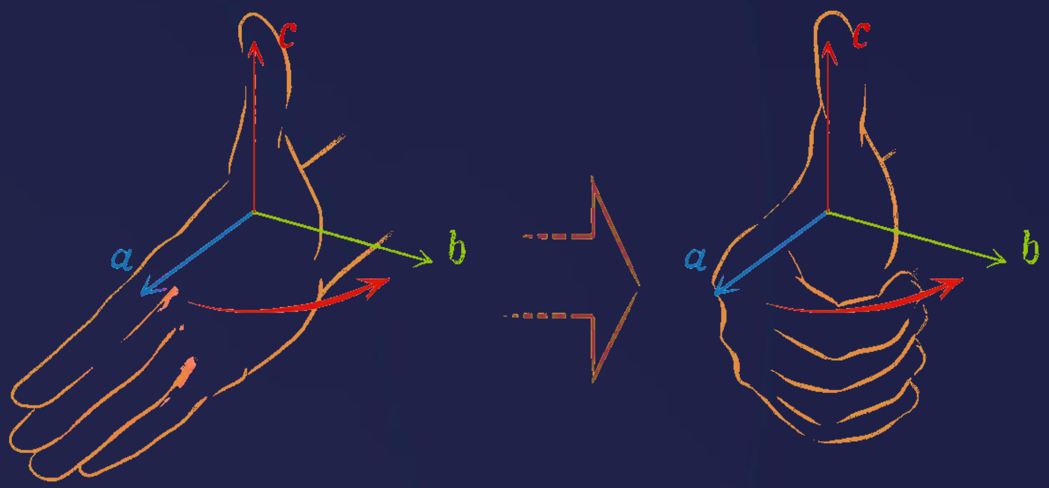
$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$



Right hand thumb rule

To find the direction of $\vec{a} \times \vec{b}$

- Draw $\vec{a}$ and $\vec{b}$ such that they are coinitial.
- Place the stretched right palm perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that the fingers are along $\vec{a}$ and when the fingers are closed, they go towards $\vec{b}$.
- The direction in which thumb points gives the direction of $\vec{c} = \vec{a} \times \vec{b}$.



If $|\vec{A}| = 2, |\vec{B}| = 3$ and the angle between $\vec{A}$ and $\vec{B}$ is $60°$, then find $\vec{A} \cdot \vec{B}$.

Given : $|\vec{A}| = 2, |\vec{B}| = 3$ and $\theta = 60^o$

To find : $\vec{A} \cdot \vec{B}$

Solution :

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$\vec{A}.\vec{B} = 2 \times 3 \times \cos 60^o = 3$$

$$\vec{A} \cdot \vec{B} = 3$$

Find the magnitude and direction of $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$, where $\vec{A}$ & $\vec{B}$ lie in $x - y$ plane.

Given : $|\vec{A}| = 5, |\vec{B}| = 4$ and $\theta = 30^o$

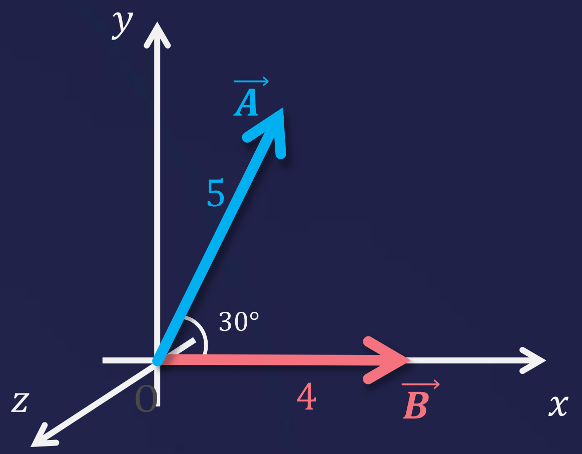
To find : $\vec{A} \cdot \vec{B}$

Solution :

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = |\vec{A}||\vec{B}| \sin\theta$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = 5 \times 4 \times \sin 30$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = 10$$



$(\hat{n} = \hat{k})$

$\vec{A} \times \vec{B} = 10\hat{k}$

$\vec{B} \times \vec{A} = -10\hat{k}$