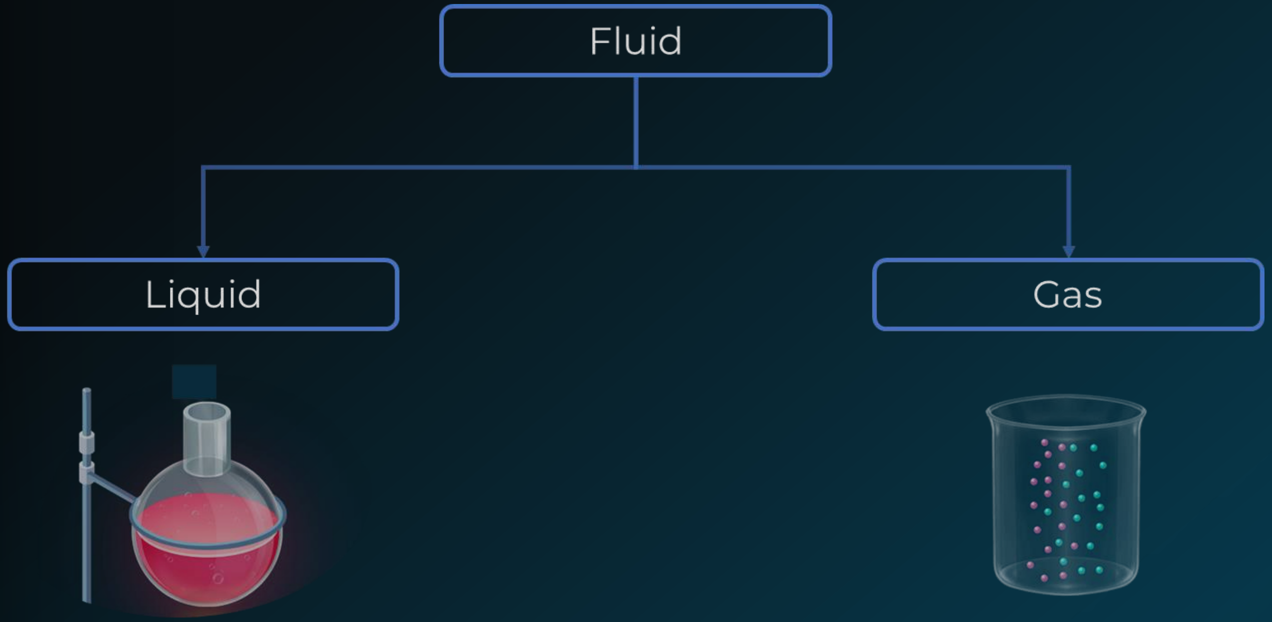
Welcome to

Aakash + BYJU'S NOTES

Mechanical properties of fluid

Fluid

- Substance that continually flows or deforms when subjected to a shear stress



- State of matter that cannot withstand or resist shear stress when it is at rest

Classification

Fluid Mechanics

- Fluid Statics
  - Fluid in a state of rest
  - (Velocity $v = 0$)
- Fluid Dynamics
  - Fluid in a state of motion
  - (Velocity $v \neq 0$)

Density

- Density of a substance or object is defined as its mass per unit volume

$$\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{V}$$

Standard values of density:

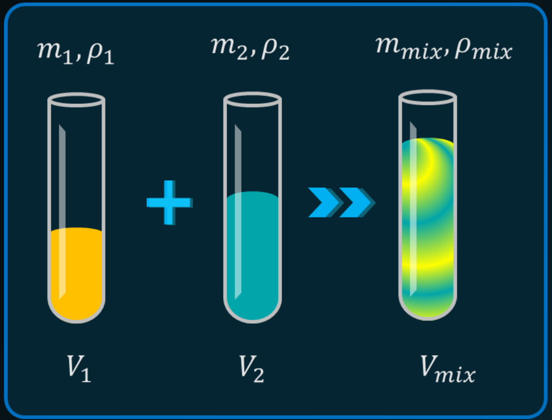
$$\rho_{water} = 1000\ kg\ m^{-3} = 1\ g\ cm^{-3}$$

$$\rho_{air} = 1.225\ kg\ m^{-3} = 0.001225\ g\ cm^{-3}$$

$$\rho_{Hg} = 13600\ kg\ m^{-3} = 13.6\ g\ cm^{-3}$$

- Scalar quantity
- SI unit $= kg\ m^{-3}$

Density of Mixture: Masses given



- For a mixture of non-reacting fluids

$$\rho_{mix} = \frac{m_{mix}}{V_{mix}}$$

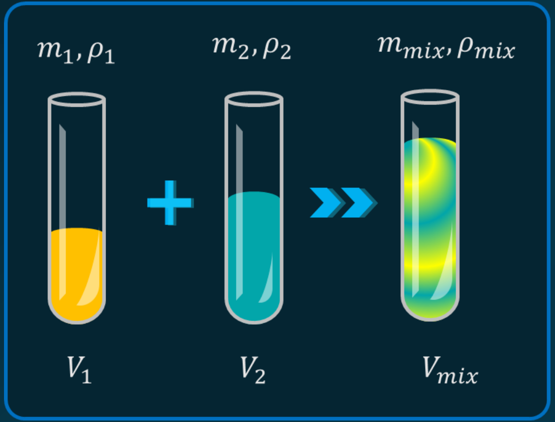
- When masses of fluids are given

$$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2}$$

$$\boxed{\rho_{mix} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}}$$

$$\boxed{\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}} \quad (\text{For } m_1 = m_2)$$

Density of Mixture: Volume given



- For a mixture of non-reacting fluids

$$\rho_{mix} = \frac{m_{mix}}{V_{mix}}$$

- When volumes of fluids are given

$$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2}$$

$$\boxed{\rho_{mix} = \frac{V_1\rho_1 + V_2\rho_2}{V_1 + V_2}}$$

$$\boxed{\rho_{mix} = \frac{\rho_1 + \rho_2}{2}} \quad (\text{For } V_1 = V_2)$$

Density of Mixture of n Fluids

$$\rho_{mix} = \frac{m_{mix}}{V_{mix}} = \frac{m_1 + m_2 + \cdots + m_n}{V_1 + V_2 + \cdots + V_n}$$

- Case 1: Masses given

$$\rho_{mix} = \frac{m_1 + m_2 + \cdots + m_n}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} + \cdots + \frac{m_n}{\rho_n}}$$

(For $m_1 = m_2 = \cdots = m_n$)

$$\frac{1}{\rho_{mix}} = \frac{1}{n}\left[\frac{1}{\rho_1} + \frac{1}{\rho_2} + \cdots . + \frac{1}{\rho_n}\right]$$

- Case 2: Volumes given

$$\rho_{mix} = \frac{V_1\rho_1 + V_2\rho_2 + \cdots + V_n\rho_n}{V_1 + V_2 + \cdots V_n}$$

(For $V_1 = V_2 = \cdots = V_n$)

$$\rho_{mix} = \frac{\rho_1 + \rho_2 + \cdots + \rho_n}{n}$$

Two liquids of densities ρ and 3ρ having volumes $3V$ and V are mixed. Find the density of the mixture.

Given: $\rho_1 = \rho,\ \rho_2 = 3\rho,\ V_1 = 3V,\ \ V_2 = V$

To Find: ρ_{mix}

Solution:

$$\rho_{mix} = \frac{m_{mix}}{V_{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

$$\rho_{mix} = \frac{\rho \times 3V + 3\rho \times V}{3V + V}$$

$$\boxed{\rho_{mix} = \frac{3}{2}\rho}$$

a. $\frac{2}{3}\rho$

b. $\frac{3}{2}\rho$

c. ρ

d. $\frac{5}{2}\rho$

Specific Gravity

- Ratio of the density of a substance to that of a standard substance (water at $4° \, C$)

$$SG = \frac{\text{Density of a substance}}{\text{Density of water at } 4° \, C}$$

- Also called Relative density (a dimensionless quantity)

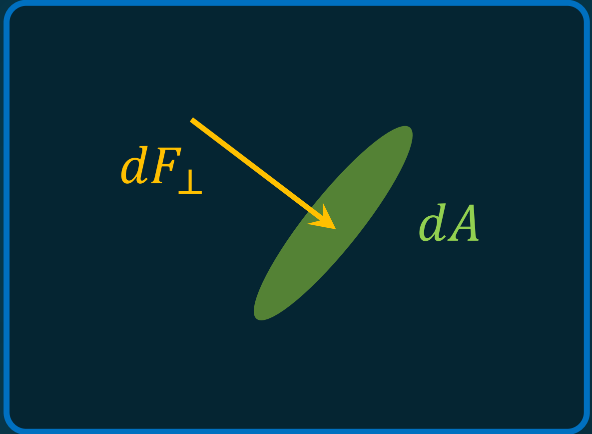
$$SG_{Hg} = \frac{\rho_{Hg}}{\rho_{water}} = \frac{13600 \, kg \, m^{-3}}{1000 \, kg \, m^{-3}}$$

$$SG_{Hg} = 13.6$$

Pressure

- Force applied per unit area normal to the surface over which that force is distributed

$$\text{Pressure } (P) = \frac{dF_{\perp}}{dA}$$



- Scalar quantity
- SI unit $= Nm^{-2}$ or Pascal (Pa)
- If a force F is uniformly distributed over a surface area A, then,

$$P = \frac{F_{\perp}}{A}$$

- Cutting objects (like scissors, knife, axe etc.) needs periodic sharpening

Atmospheric Pressure

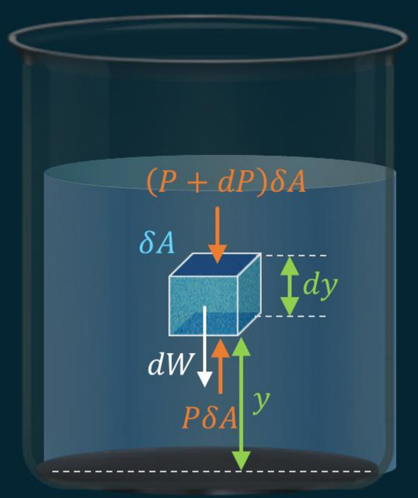
- Pressure due to the Earth's atmosphere
- Changes with weather and elevation
- Normal atmospheric pressure at sea level is $1.013 \times 10^5\ N/m^2$ or Pa
- $1\ atm = 1.013 \times 10^5\ Pa$
- $1\ bar = 10^5\ Pa$

Variation of Pressure with Depth

- The pressure at the free surface (exposed to atmosphere) of the liquid is P_{atm}

Consider a very small cubical element of fluid,

- $P_{atm} \approx 10^5\ Pa$



For equilibrium, $\sum F_y = 0$

$$\Rightarrow (P + dP)\delta A + dm\, g = P\delta A$$

$$P\delta A + dP\delta A + (\rho\delta A dy)g = P\delta A$$

$$\frac{dP}{dy} = -\rho g = \text{Pressure gradient}$$

- Pressure increases with depth

- The pressure increases linearly with the depth in addition to P_o

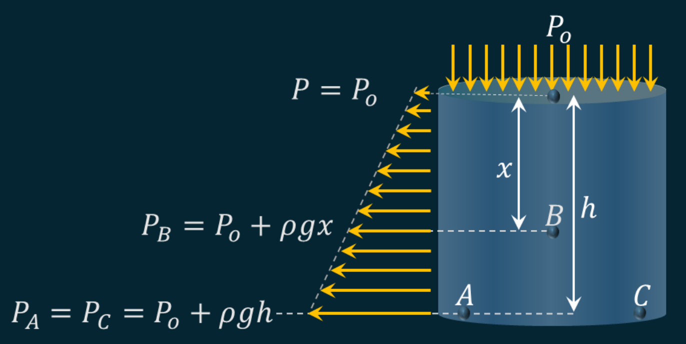
$$\left(\because \frac{dP}{dy} = -\rho g\right)$$

- Absolute pressure at a depth h

$$P(h) = P_o + \rho g h$$

$$P_{abs} = P_{atm} + P_{gauge}$$

Note: The term $\rho g h$ is called Gauge pressure



- Gauge pressure:

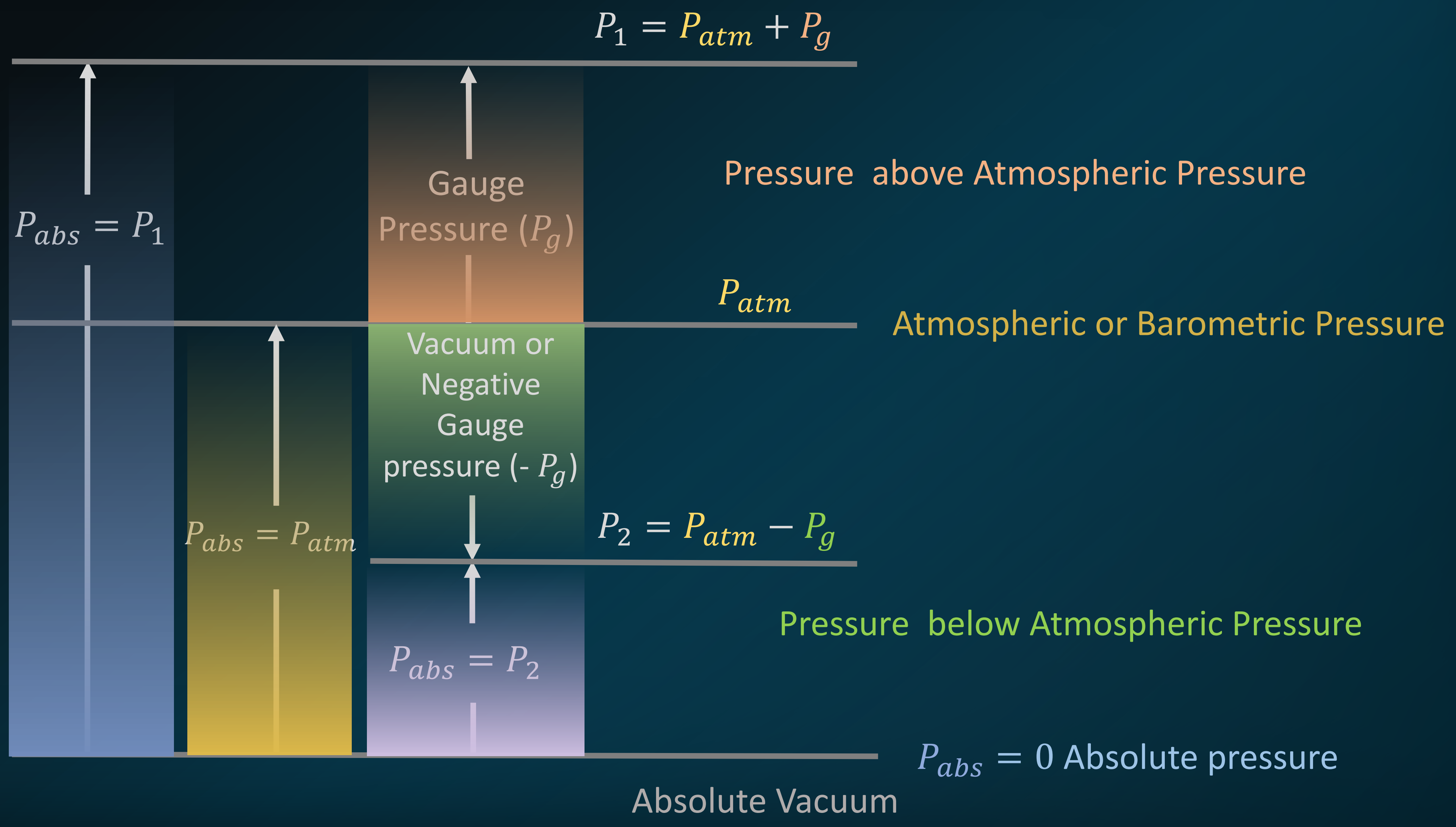
$$P(h) = P_o + \rho g h$$

Average Pressure on the wall of container:

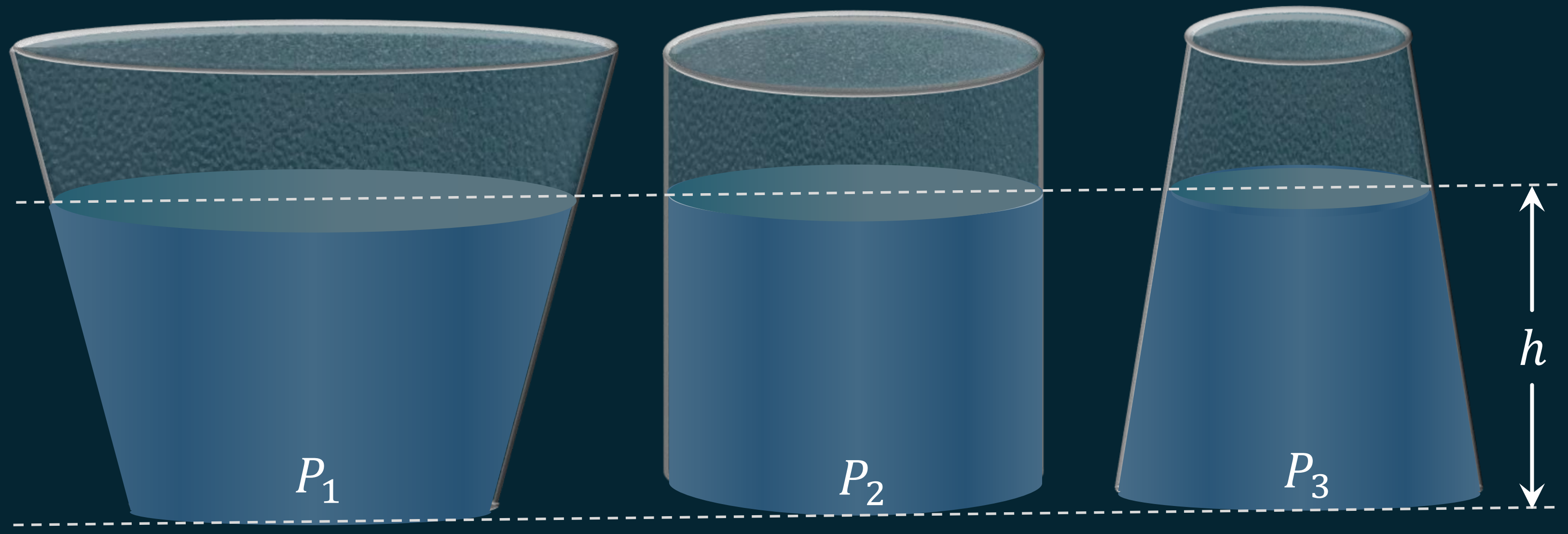
$$P_{avg} = \frac{P(h=0) + P(h=h)}{2}$$

$$P_{avg} = \frac{P_o + P_o + \rho g h}{2} = P_o + \frac{\rho g h}{2}$$

Variation of Pressure with Depth



Hydrostatic Paradox



$$P_1 = P_2 = P_3 = \rho g h + P_{atm}$$

- Independence from mass/shape of the fluid in a container is called Hydrostatic Paradox

Working of Barometer

- Measures the atmospheric pressure
- $76\ cm$ of height in Hg column corresponds to $1\ atm$ pressure.

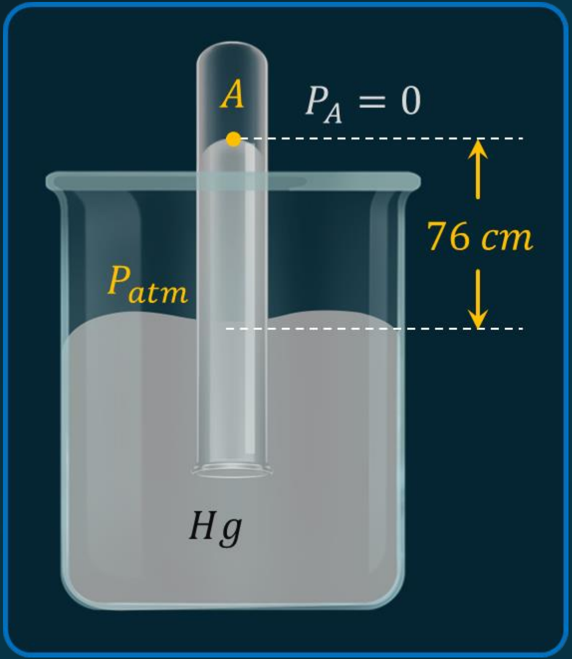
$$\rho g h = (13600)(9.8)(0.76)$$

$$= 1.01 \times 10^5\ Pa$$

$$= 1\ atm$$

- Units of pressure used in medicine and physiology:

$1\ torr = 133\ Pa = 1\ mm\ of\ Hg$



- $P_A = 0$, Due to Vaccum

- Barometer fluid:

Mercury is used as it is heavy
⇒ It gives $76\ cm$ rise for atmospheric pressure.

If water is used,

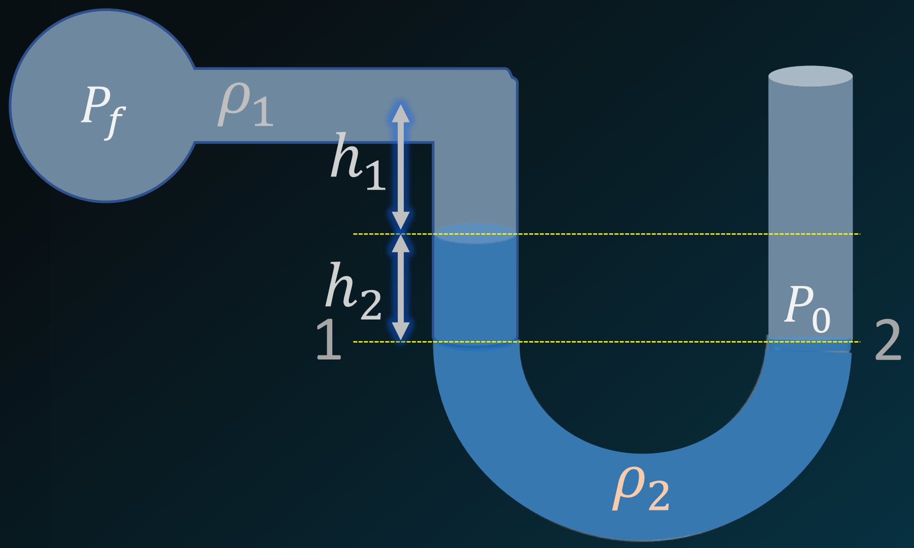
$$\rho_w h_w = \rho_{Hg} h_{Hg}$$

$$1000 \times h_w = 13600 \times 0.76$$

$$h_w = 13.6 \times 0.76 = 10.336\ m$$

- Low density fluids can be used for measuring low difference is pressure

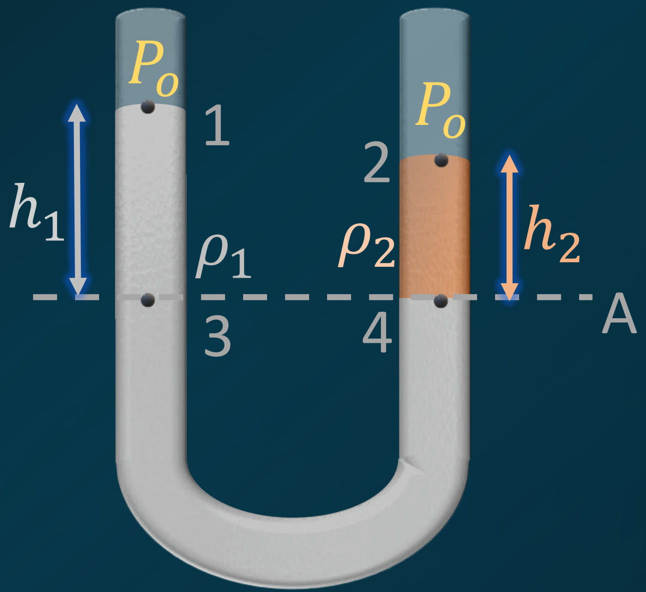
Manometer



Points 1 and 2 are at same height

$$P_1 = P_2$$

$$P_f + \rho_1 g h_1 + \rho_2 g h_2 = P_o$$



Points 3 and 4 are at same height

$$P_3 = P_4$$

$$P_o + \rho_1 g h_1 = P_o + \rho_2 g h_2$$

$$\rho_1 h_1 = \rho_2 h_2$$

A U-shaped tube is filled up to a height l by two different immiscible liquids of densities ρ and 3ρ separated by a valve as shown. If the valve is opened, find out the new height of the liquids in both columns.

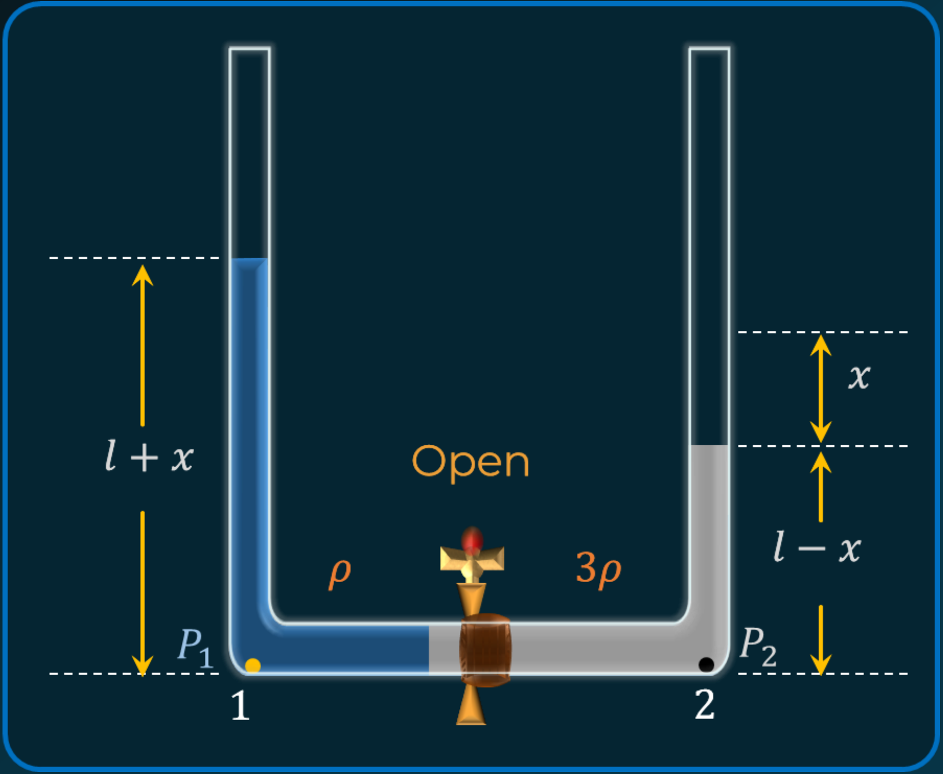
Solution: $P_1 = P_2$

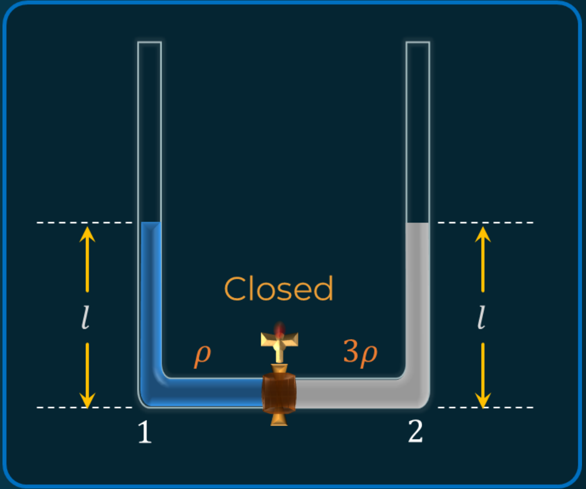
$$P_o + \rho g(l + x) = P_o + 3\rho g(l - x)$$

$$l + x = 3(l - x)$$

$$x = \frac{l}{2}$$

$$l + x = \frac{3l}{2}, \qquad l - x = \frac{l}{2}$$





a. $h_1 = \frac{l}{2}\,;\, h_2 = \frac{3l}{2}$

b. $h_1 = \frac{l}{2}\,;\, h_2 = \frac{l}{2}$

c. $h_1 = \frac{3l}{2}\,;\, h_2 = \frac{l}{2}$

d. $h_1 = \frac{l}{4}\,;\, h_2 = \frac{3l}{4}$

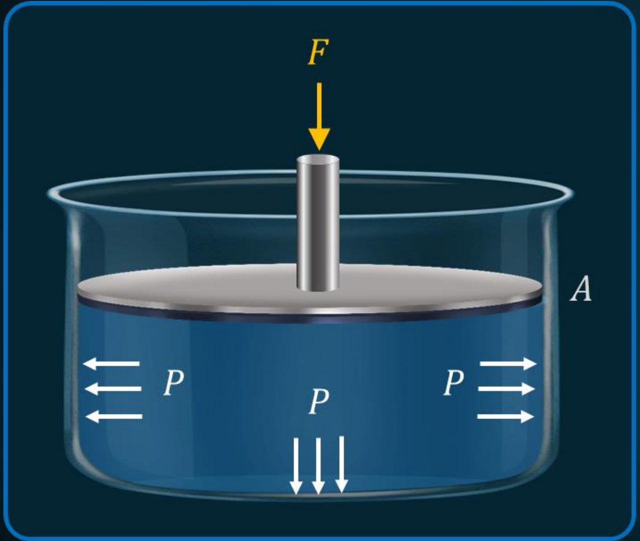
Pascal's Law

Statement:

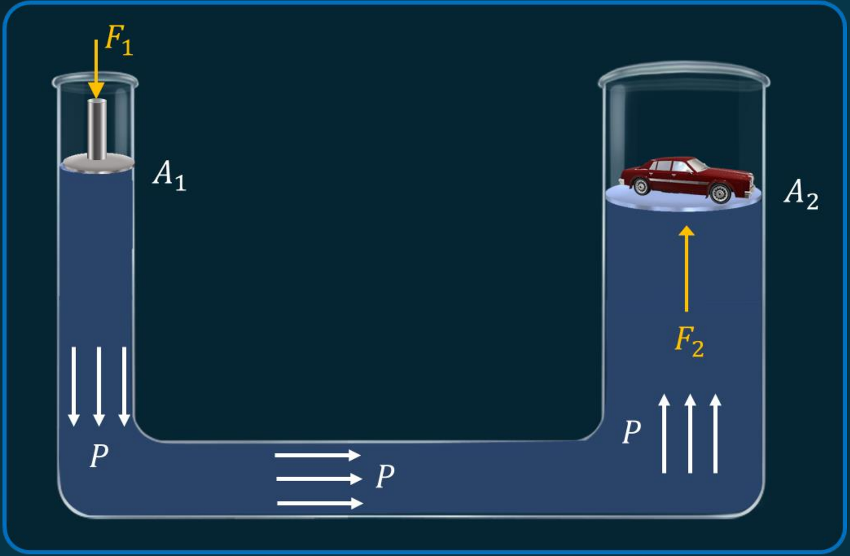
External pressure exerted on a non-compressible fluid in an enclosed vessel is distributed evenly throughout the fluid.

Applications:

Hydraulic lift, Hydraulic brakes, Pumps, Calibration of pressure gauges, Paint sprayers, Hydraulic press etc.

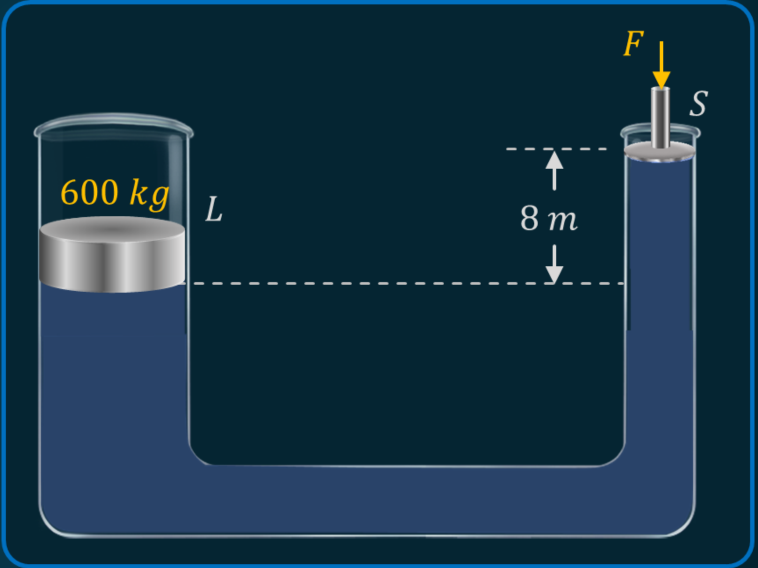


Hydraulic Lift



$$\text{Pressure } (P) = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

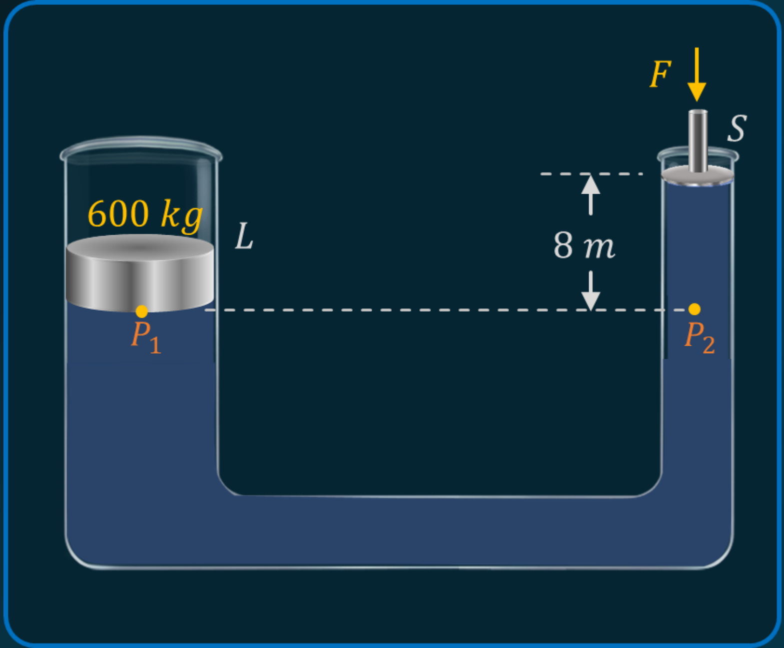
For the system shown in the figure, the cylinder on the left L has a mass of $600\ kg$ and a cross-sectional area of $800\ cm^2$. The piston on the right, S has a cross sectional area of $25\ cm^2$ and negligible weight. If the apparatus is filled with oil, what force is required to hold the system in equilibrium? $(g = 10\ m/s^2, \rho = 800\ kg/m^3)$



Solution:

By Pascal's Law,

Let A_S: Area of the piston
A_L: Area of the cylinder



$$\frac{F}{A_S} + \rho g h = \frac{m_L \times g}{A_L}$$

$$F = A_S\left(\frac{m_L \times g}{A_L} - \rho g h\right)$$

$$F = 25 \times 10^{-4}\left(\frac{6000}{800 \times 10^{-4}} - 800 \times 10 \times 8\right)$$

$$\boxed{F = 27.5\ N}$$

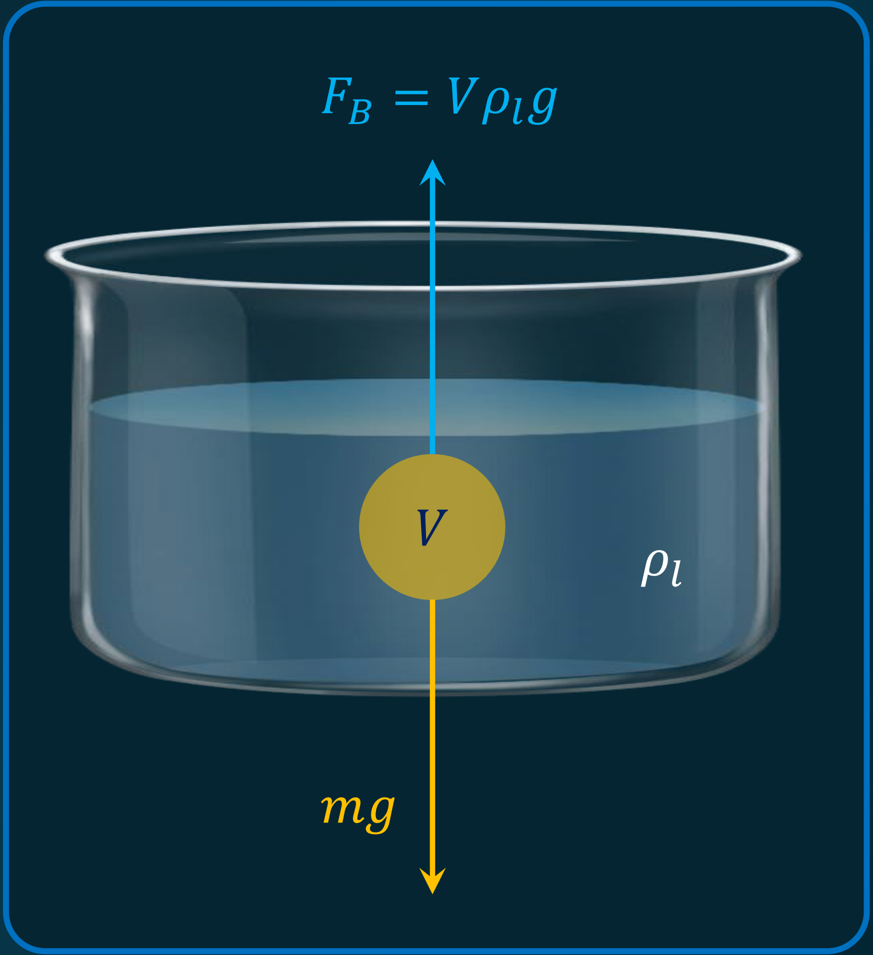
Archimedes' Principle

- The buoyant force exerted by the fluid on a partially/fully immersed body is equal to the weight of the fluid that it displaces
- The force acts in upward direction at the COM of displaced liquid

Mathematically,

$$\boxed{F_B = W_f = \rho_l V g}$$

F_B is also known as Buoyant Force



A wooden plank of length 1 m and uniform cross section is hinged at one end to the bottom of tank. The tank is filled with water up to a height of $0.5\ m$. The specific gravity of plank is 0.5. Find angle θ that the plank makes with vertical in equilibrium position (Exclude the case $\theta = 0°$)

Solution:

For equilibrium, $F_{net} = 0$ and $\tau_{net} = 0$

Mass of Plank:

$$m = (0.5\ \rho_w)Al$$

Weight of fluid displaced:

$$F_T = \rho_w g(l - x)A$$

Moment about O:

$$mg\frac{l}{2}\sin\theta = F_T\frac{(l-x)}{2}\sin\theta$$

$$mg\frac{l}{2}\sin\theta = F_T\frac{(l-x)}{2}\sin\theta$$

Substitute value of m and F_T

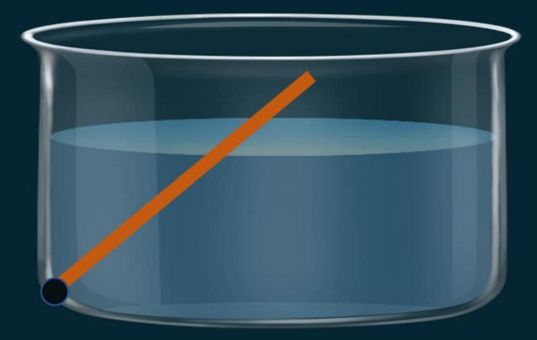
$$[(0.5\ \rho_w)Al] \times g\frac{l}{2}\sin\theta = [\rho_w g(l-x)A] \times \frac{(l-x)}{2}\sin\theta$$

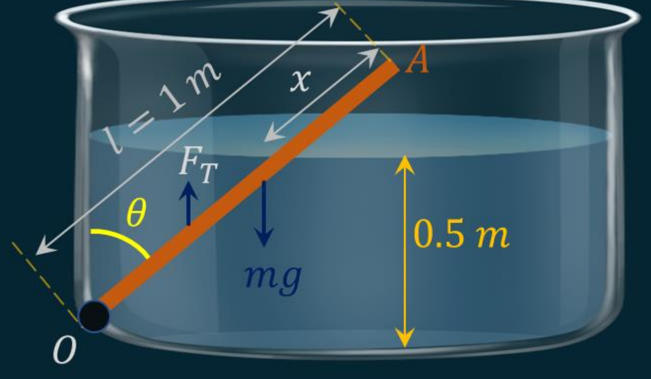
$$(0.5\ )\frac{1}{2} = \frac{(1-x)}{2} \times (1-x)$$

$$(1-x)^2 = 0.5 \quad \Rightarrow \quad \boxed{x = 0.293\ m}$$

From diagram,

$$\cos\theta = \frac{0.5}{1-x} = \frac{0.5}{0.707} = \quad 45° \ldots\ldots (Ans)$$





Law of Floatation

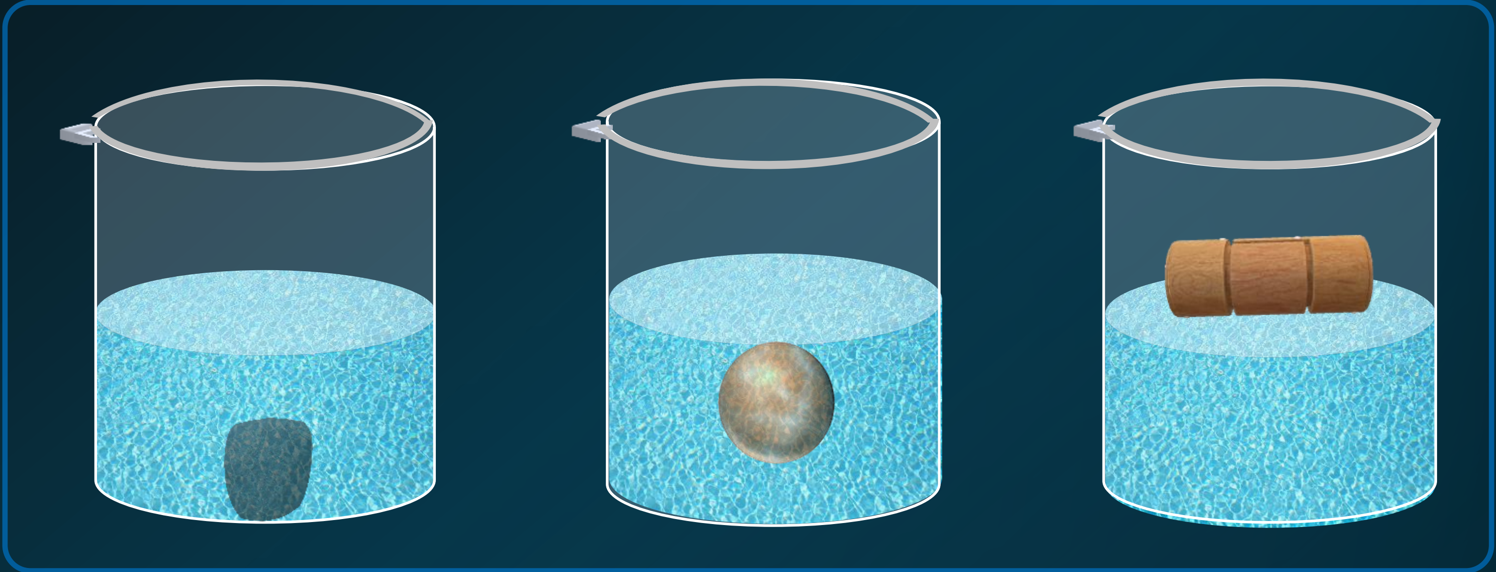
- Weight of the body,
  $W_s = \rho_s V_s g$

- Buoyant force on the body,
  $F_B = \rho_L V_s g$

- Apparent Weight,

$$W_{app} = W_{actual} - F_{buoyant}$$

$$W_{app} = \rho_s V g - \rho_L V g$$



$\rho_s > \rho_L$	$\rho_s = \rho_L$	$\rho_s < \rho_L$
$(W_s > F_B)$	$(W_s = F_B)$	$(W_s < F_B)$
$W_{app} = (\rho_s - \rho_L)gV$	$W_{app} = 0$	$W_{app} = 0$

An ornament weighing $36\ gram$ in air weighs only $34\ gram$ in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. The density of gold is $19.3\ gram/cm^3$ and that of copper is $8.9\ gram/cm^3$.

Solution:

$\rho_{Au} = 19.3\ gram\ cm^{-3}$, $\rho_{Cu} = 8.9\ gram\ cm^{-3}$ (Given) $m_{actual} = 36\ gram$

Volume of the ornament $= V\ cm^3$ (Assume) $m_{app} = 34\ gram$

$$V = V_{Cu} + V_{Au}$$

$$m_{Cu} = x\ gram \quad \text{(Assume)}$$

$$m_{Au} = (36 - x)\ gram$$

$$V_{Cu} = \frac{x}{8.9}\ cm^3$$

$$V_{Au} = \frac{36 - x}{19.3}\ cm^3$$

$$W_{app} = W_{actual} - F_{buoyant}$$

$$F_{buoyant} = W_{actual} - W_{app}$$

$$V\rho_w g = m_{actual} g - m_{app} g$$

$$(V_{Cu} + V_{Au})\rho_w g = (36 - 34)g$$

$$\left(\frac{x}{8.9} + \frac{36 - x}{19.3}\right)(1) = 2$$

$$\Rightarrow x = 2.225\ gram$$

$$\boxed{m_{Cu} = x = 2.225\ gram}$$

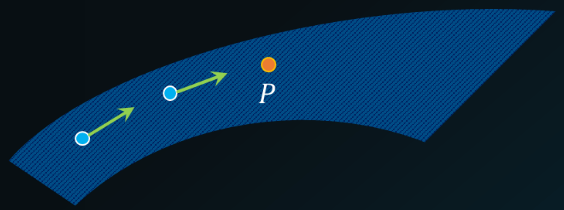
a.	$2.500\ gram$	c.	$5.000\ gram$
b.	$2.225\ gram$	d.	$5.500\ gram$

Classification

Fluid Mechanics

- Fluid Statics
  - Fluid in a state of rest
  - (Velocity $v = 0$)
- Fluid Dynamics
  - Fluid in a state of motion
  - (Velocity $v \neq 0$)

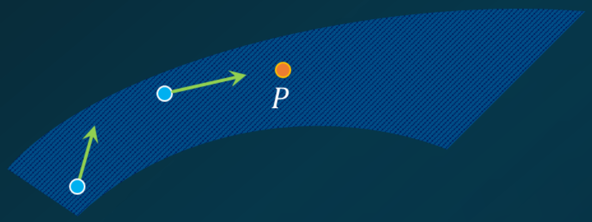
Steady vs Unsteady Flow



(Velocity of each particle at point P remains same)

Steady / Streamline Flow

- All the particles reaching a particular point have the same velocity at all time

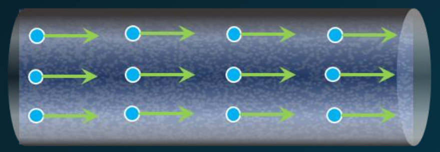


(Velocities of particles at point P are not the same)

Unsteady Flow

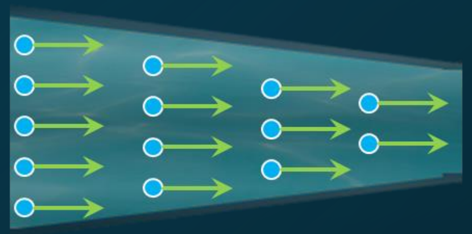
- All the particles reaching a particular point have different velocities at different time

Uniform vs Non-Uniform Flow



Uniform Flow

- Velocity at a time is constant at all points



Non-Uniform Flow

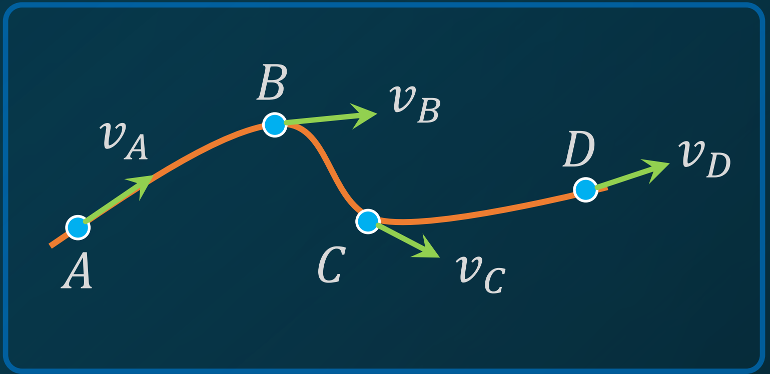
- Velocity at a time changes with location

Streamlines

- Path followed by a particle in steady flow is called streamline
- Tangent drawn to streamline at a point gives the direction of fluid velocity at that point

- Properties of Streamlines:

1. Tangent gives direction of velocity
2. Two streamlines can't cross
3. Fluid velocity is greater where streamlines are closely spaced



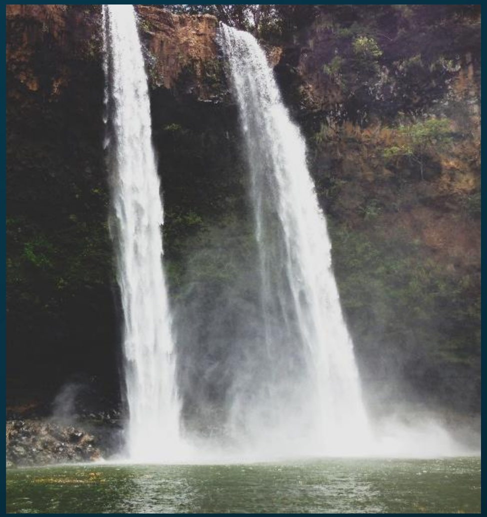
Laminar vs Turbulent Flow

Laminar flow

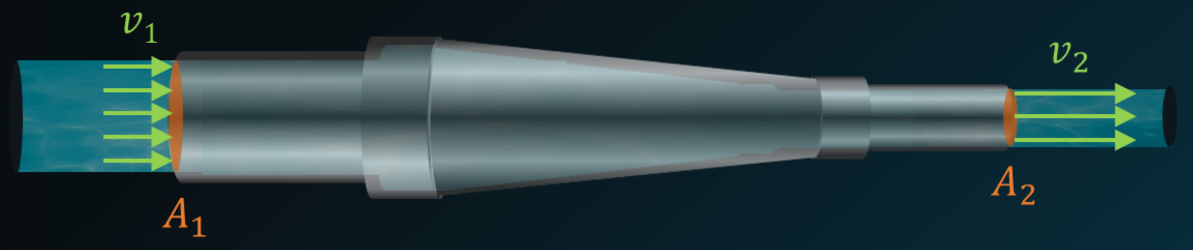
- Particles follow smooth paths in layers, without lateral mixing
- Particles move at reasonably low speeds

Turbulent flow

- Irregular, lateral, random, and chaotic movement of particles
- Laminar flow becomes turbulent after a certain increase in speed



Equation of Continuity



Incompressible fluid + Steady flow

Incompressible fluid + Steady flow

In time interval Δt,

Mass entering = Mass leaving

$$\rho_1 A_1 (v_1 \Delta t) = \rho_2 A_2 (v_2 \Delta t)$$

If ρ remains constant, i.e., for an incompressible fluid,

$$A_1 v_1 = A_2 v_2$$

$$\boxed{\text{Discharge, } Q\ (m^3 s^{-1}) = Av = \frac{dV}{dt}}$$

$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

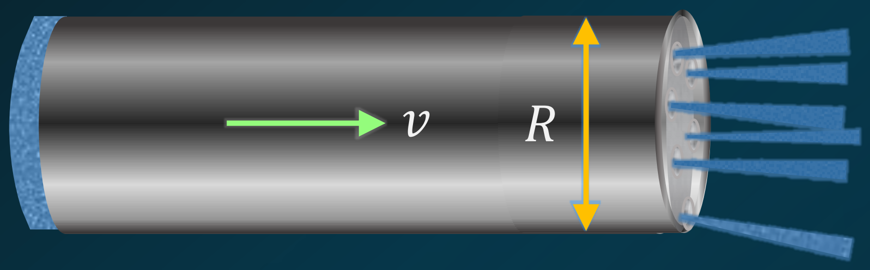
For multiple inlets/outlets,

$$\sum_{inlet} \rho_i A_i v_i = \sum_{outlet} \rho_i A_i v_i$$

$$\sum_{inlet} A_i v_i = \sum_{outlet} A_i v_i$$

The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is v, the speed of the ejection of the liquid through the holes is

Solution:



From continuity equation,

$$A_1 v_1 = A_2 v_2$$

$$\pi R^2 v = n\pi r^2 v_2$$

$$v_2 = \frac{\pi R^2 v}{n\pi r^2}$$

$$\boxed{v_2 = \frac{vR^2}{nr^2}}$$

a. $\frac{v^2 R}{nr}$

b. $\frac{vR^2}{nr^2}$

c. $\frac{vR^2}{n^2 r^2}$

d. $\frac{vR^2}{n^3 r^2}$

Bernoulli's Theorem

Mass of displaced infinitesimal element:

$$\Delta m = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t \quad (\because \text{Continuity equation})$$

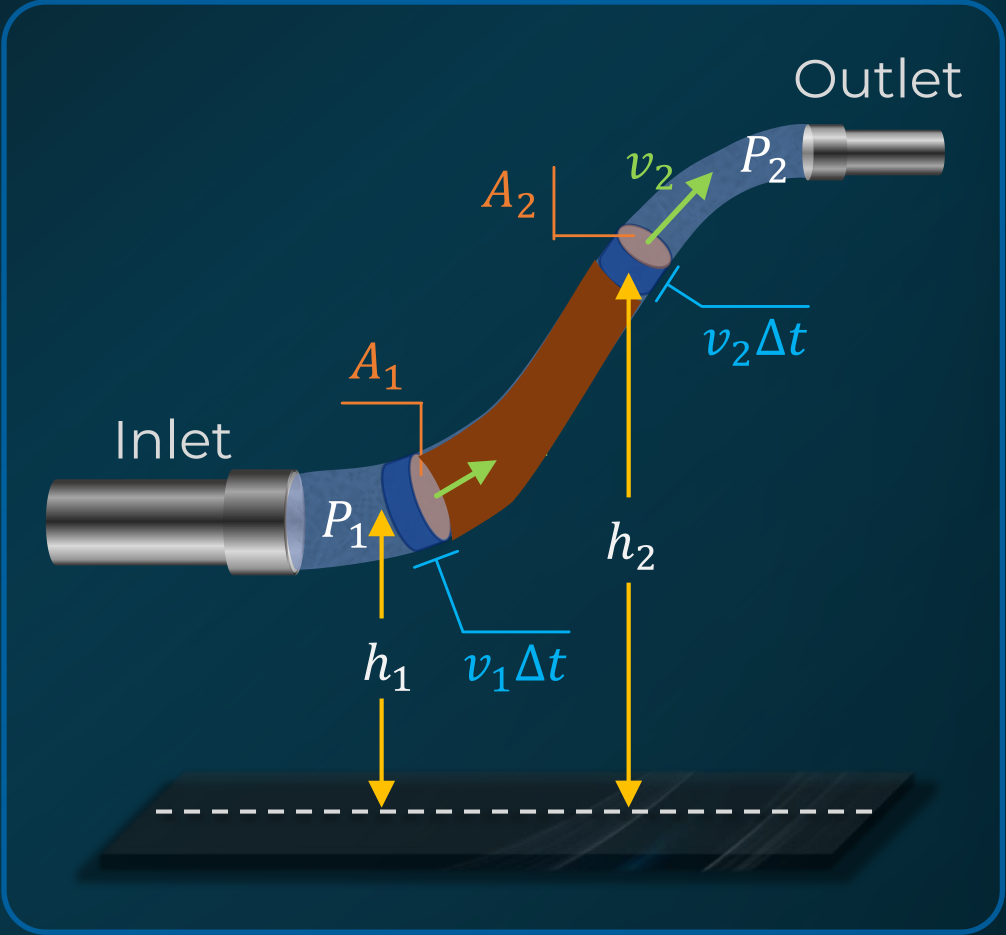
Applying Work-Energy theorem,

$$W_g + W_{pressure} = \Delta KE$$

$$-\Delta m g(h_2 - h_1) + P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t = \frac{1}{2}\Delta m(v_2^2 - v_1^2)$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{Constant}$$



Limitations:
- Flow must be steady and incompressible
- Fluid should be non viscous
- Thermal loss of energy is not accounted
- Centrifugal force at curved pipes is not considered

?

A horizontal pipeline carries water in a streamline flow. At a point along the tube where the cross-sectional area is $10^{-2}\ m^2$, the water speed is $2\ m/s$ and the pressure is $8000\ Pa$. The pressure of water at another point where cross-sectional area is $0.5 \times 10^{-2}\ m^2$ is

Given: $A_1 = 10^{-2}\ m^2$, $v_1 = 2\ m/s$, $P_1 = 8000\ Pa$,

Solution:

$v_2 = 4\ m/s$ $\quad [\because A_1 v_1 = A_2 v_2]$

Applying Bernoulli's theorem,

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$P_2 = 8000 + \frac{1}{2} \times 1000 \times (2^2 - 4^2)$$

$$P_2 = 8000 + 500 \times (-12)$$

$$\boxed{P_2 = 2000\ Pa}$$

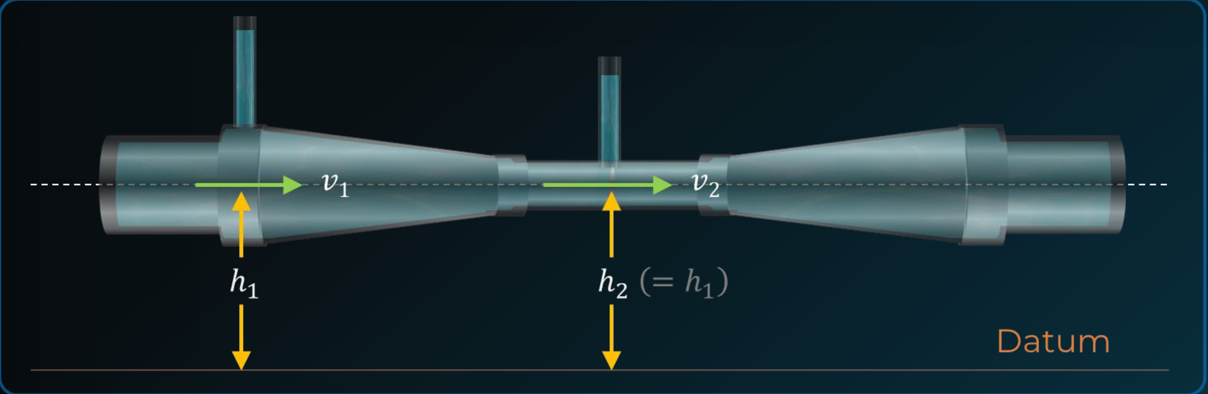
a. $4000\ Pa$

b. $1000\ Pa$

c. $2000\ Pa$

d. $3000\ Pa$

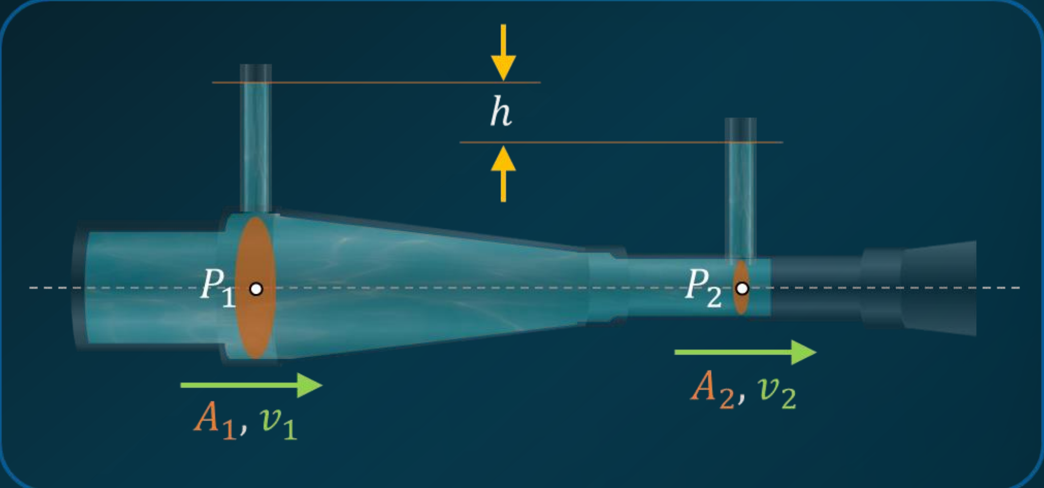
Venturimeter



A lab device that can be used to measure the flow rate

$$A_1 v_1 = A_2 v_2 \qquad \text{(Continuity equation)}$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \qquad \text{(Bernoulli's equation)}$$



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\rho g h = \frac{1}{2}\rho\left[\left(\frac{A_1 v_1}{A_2}\right)^2 - v_1^2\right]$$

$$2gh = v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

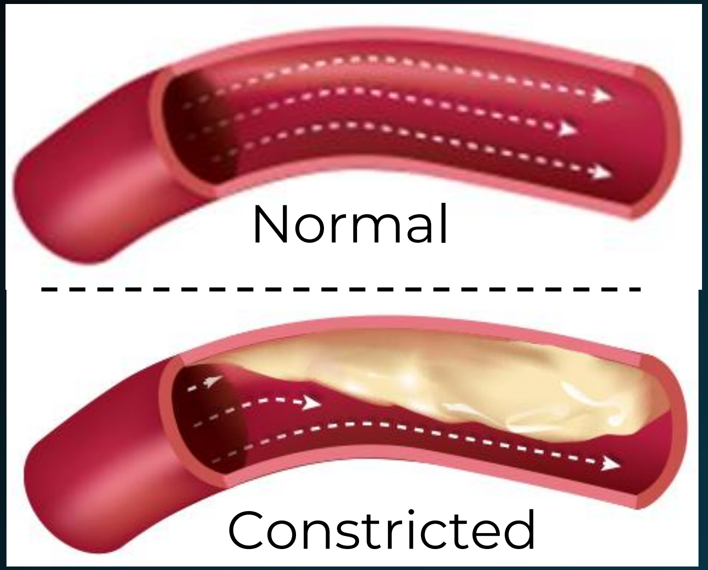
Rate of flow $= Q = A_1 v_1$

$$\boxed{Q = A_1\sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

Applications of Bernoulli's Theorem

Blood Flow and Heart Attack

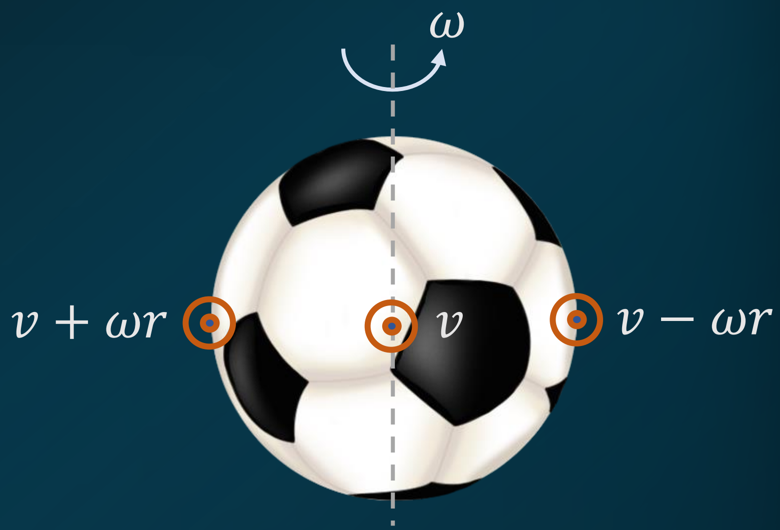
Inner walls of artery gets constricted due to deposition of plaque



Due to pressure difference across walls, artery breaks

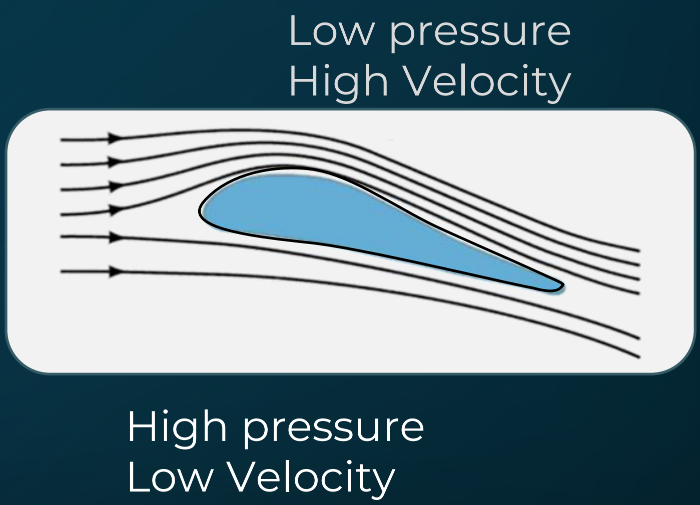
Dynamic Lift and Magnus Effect

The force which acts on a body such as an airplane wing hydro fall or spinning Ball by virtue of its motion through a fluid



Aero foil and Lift on Aircraft Wing

The shape that creates more up thrust and experiences less drag



Speed of Efflux

Let,

$A =$ Cross sectional area of the tank

$a =$ Cross sectional area of the orifice

$av = AV$ (Equation of continuity)

$$V = \frac{av}{A}$$

Applying Bernoulli's equation between the cross sections 1 and 2,

$$P_o + \frac{1}{2}\rho V^2 + \rho g H = P_o + \frac{1}{2}\rho v^2 + \rho g(H - h)$$

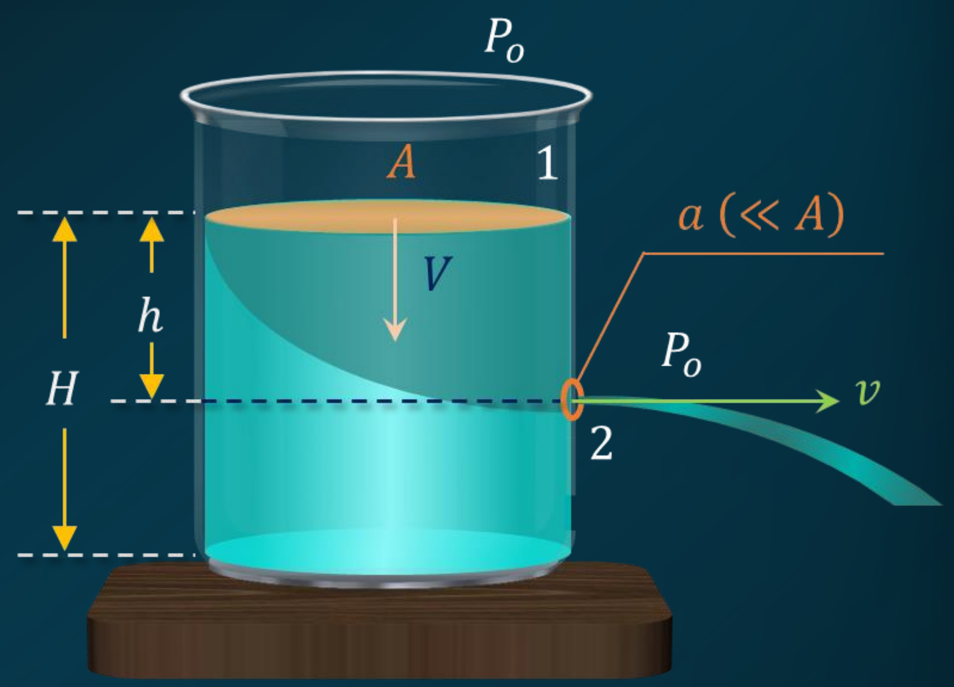
$$\rho g[H - (H - h)] = \frac{1}{2}\rho(v^2 - V^2)$$

$$2gh = v^2 - V^2$$

$$2gh = v^2 \underbrace{\left[1 - \left(\frac{a}{A}\right)^2\right]}_{\approx 1} \qquad \left(\because V = \frac{av}{A}\right)$$

$a\ (\ll A)$

$$\boxed{v = \sqrt{2gh}} \Rightarrow v \propto \sqrt{h}$$

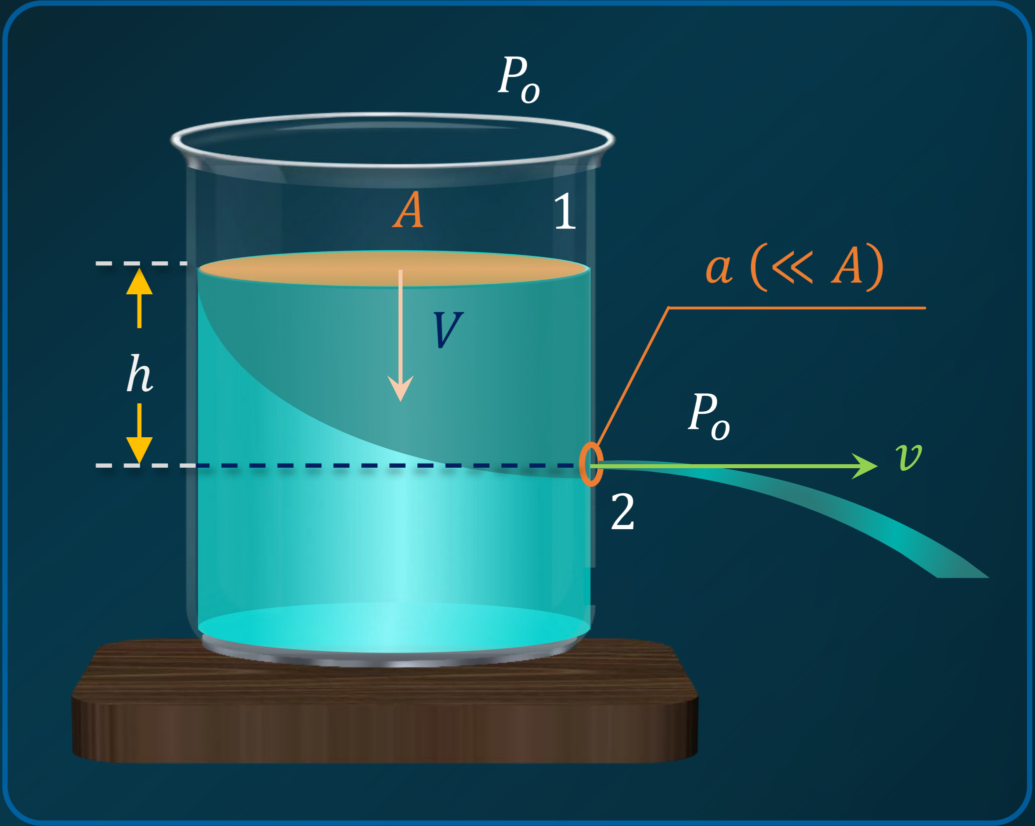


Torricelli's Theorem

The speed of flow of a liquid from an orifice is equal to the speed that it would attain if falling freely a distance equal to the height of the free surface of the liquid above the orifice.

$$v = \sqrt{2gh}$$

- Applicable only if $a \ll A$
- h is the depth of orifice



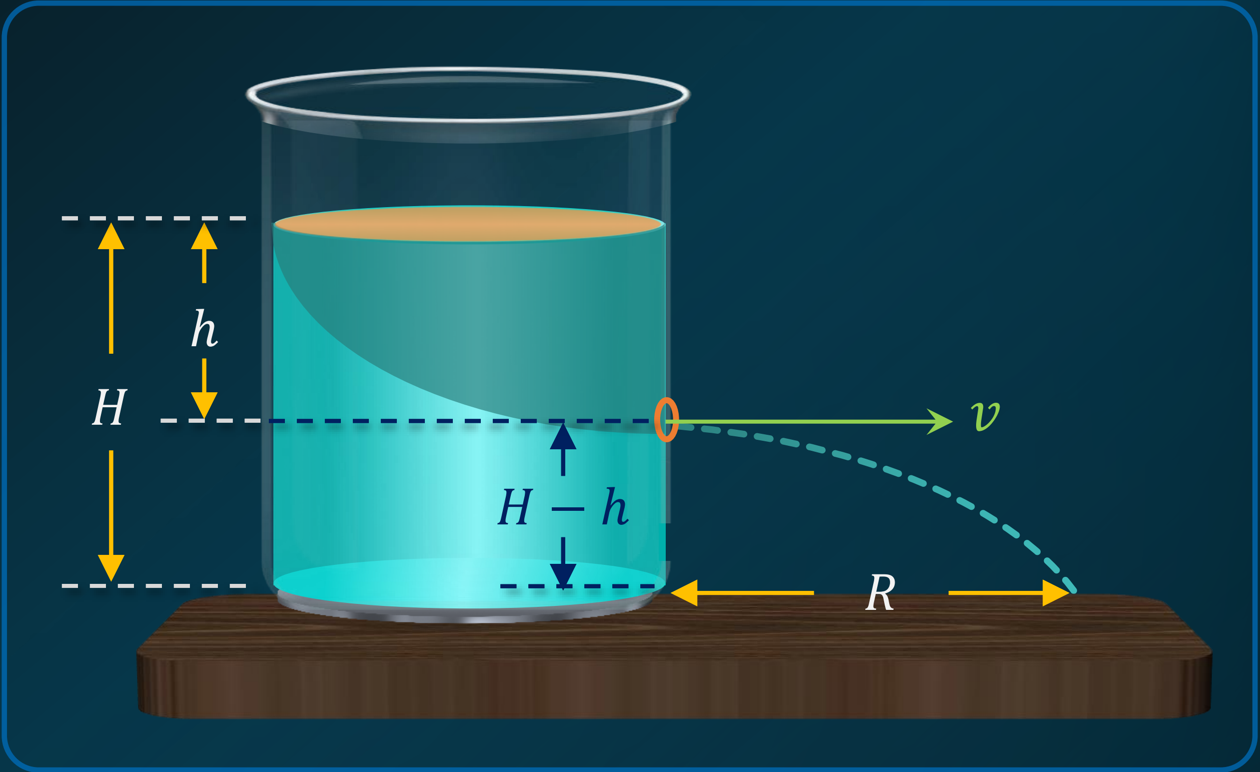
Range of Efflux

R = [Speed of efflux] × [Time taken]

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$R = \sqrt{4h(H-h)}$$

$$\boxed{R = 2\sqrt{h(H-h)}}$$



Maximum Range

We know, Range of liquid

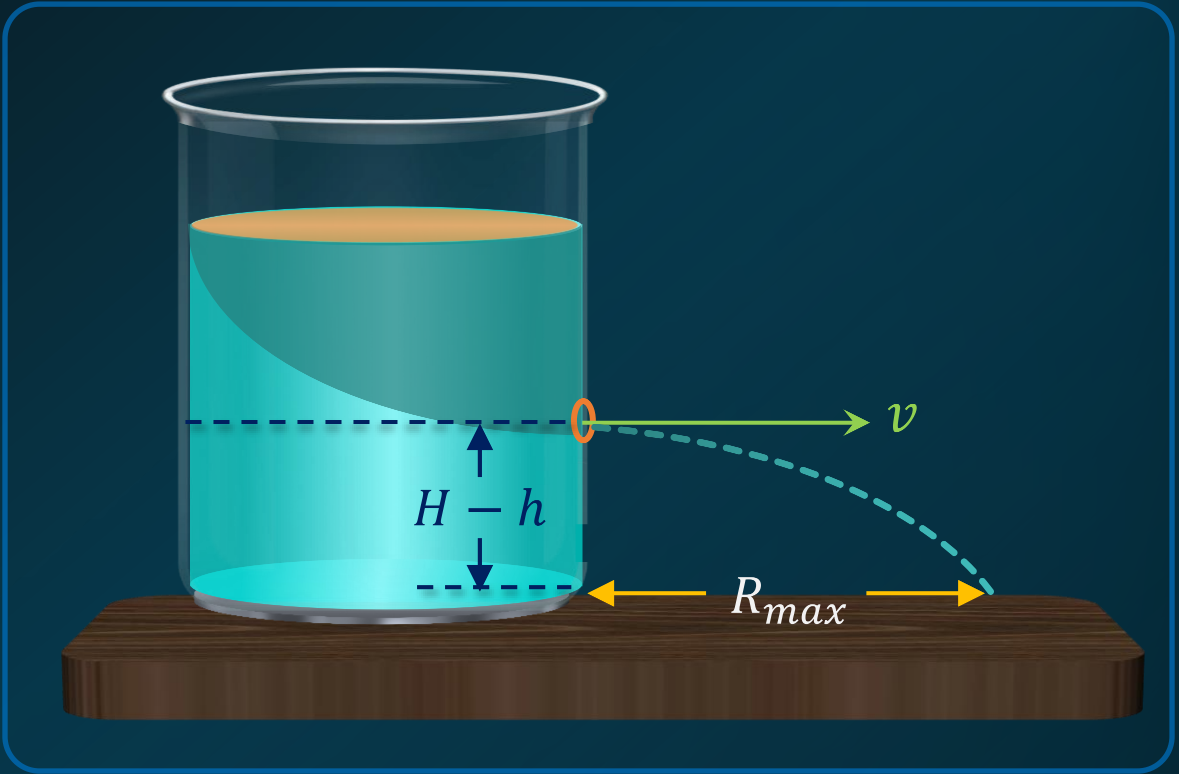
$$R = 2\sqrt{h(H-h)}$$

$$R^2 = 4h(H-h)$$

$$\frac{dR^2}{dh} = 4\frac{d[h(H-h)]}{dh} = 0$$

$$h(-1) + (H-h) = 0 \qquad \Rightarrow h = \frac{H}{2}$$

$$R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)}$$



$$\boxed{R_{max} = H}$$

Maximum range occurs only at an instant when $h = H/2$

Time Required to Empty the Tank

Discharge through tank:

$$av = AV$$

$$a\sqrt{2gh} = A\left(-\frac{dh}{dt}\right)$$

$$\frac{a\sqrt{2g}}{A}\,dt = -\frac{dh}{\sqrt{h}}$$

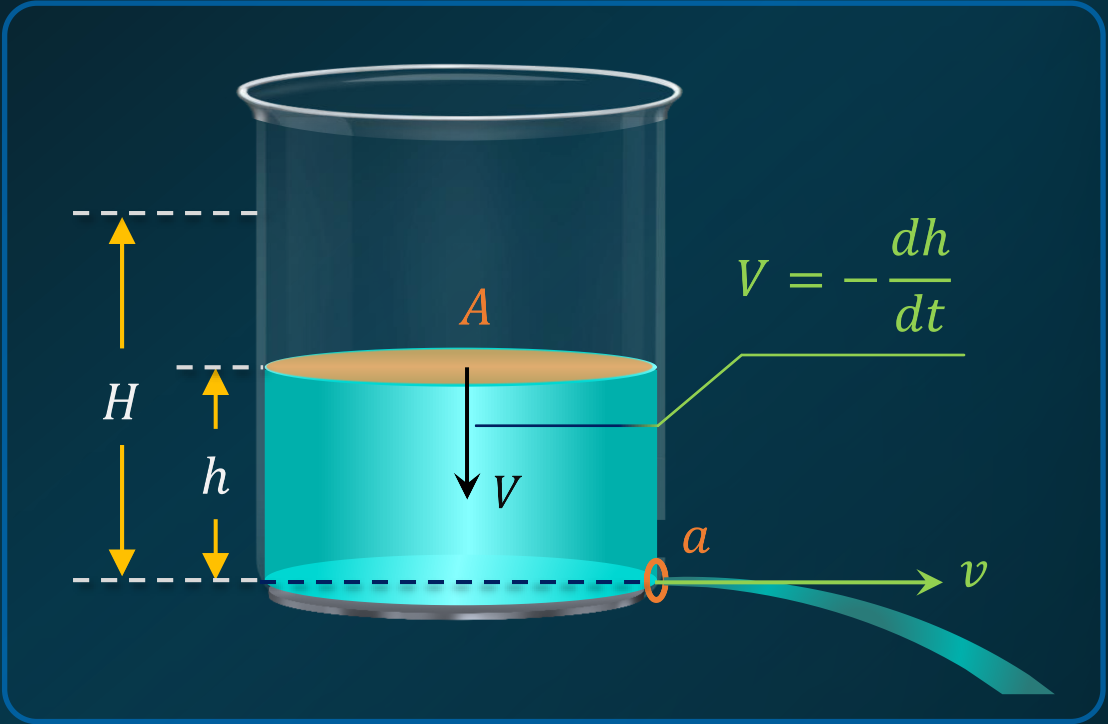
$$-\int_{H}^{0}\frac{dh}{\sqrt{h}} = \frac{a}{A}\sqrt{2g}\int_{0}^{T}dt$$

$$-\int_{H}^{0}\frac{dh}{\sqrt{h}} = \frac{a}{A}\sqrt{2g}\int_{0}^{T}dt$$

$$-\left[2\sqrt{h}\right]_{H}^{0} = \frac{a}{A}\sqrt{2g}\,[t]_{0}^{T}$$

$$2\sqrt{H} = \frac{a}{A}\sqrt{2g}\,[T]$$

$$\boxed{T = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$



Ideal Fluid

Non-viscous

Incompressible ($d\rho = 0$)

Ideal fluid

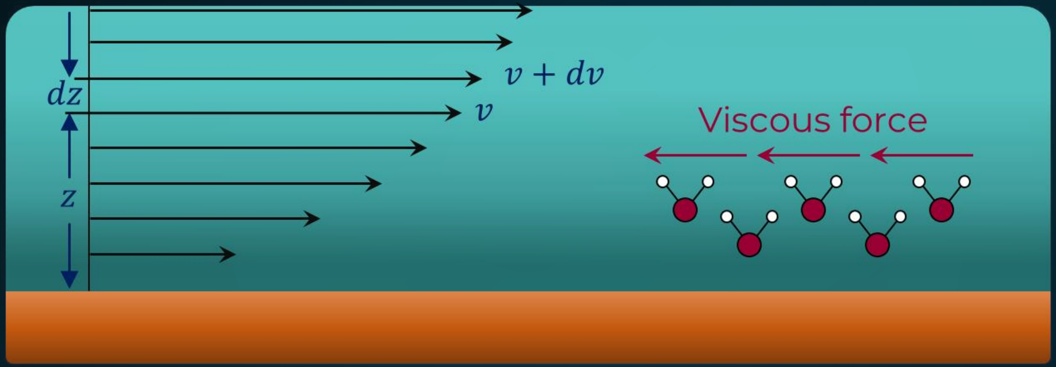
Steady

Laminar

Ideal/Tubular flow

Viscosity

- Internal friction between the fluid layers in motion
- Due to molecular cohesion in fluids
- Plays a key role in operations that require the transport of fluid from one source to another



Velocity gradient $= \dfrac{dv}{dz} = \dfrac{d(Shear\ strain)}{dt} = \dfrac{d\theta}{dt}$

Force:

$$F_v = -\eta A \frac{dv}{dz}$$ (Viscous force)

Shear Stress:

$$\tau = \frac{F_v}{A} = -\eta \frac{dv}{dz}$$

Where,

η = Coefficient of viscosity

A = Area of contact

$\dfrac{dv}{dz}$ = Velocity gradient

Coefficient of Viscosity

$$F_v = -\eta A \frac{dv}{dz}$$ (Viscous force)

$$\eta = -\frac{\left(F_v/A\right)}{\left(dv/dz\right)} \longrightarrow \frac{N/m^2}{\left(\frac{m/s}{m}\right)} \longrightarrow \frac{Ns}{m^2} \text{ or } Pa \cdot s$$ (Also called 1 decapoise)

$$\eta = -\frac{\text{Shear stress}}{\text{Velocity gradient}}$$

poise = CGS unit $= dyne \dfrac{s}{cm^2}$

Relative viscosity ($\eta_r = \frac{\eta}{\eta_0}$): Ratio of viscosity of a general solution to viscosity of standard solvent

Viscosity depends on:

1. Temperature

For liquids: $\eta = \eta_0 \left(\dfrac{1}{1 + \alpha T + \beta T^2}\right)$ η_0 = Viscosity at 0 ℃

For Gases: $\eta = \eta_0 \left(\dfrac{aT_0 + C}{aT + C}\right)\left(\dfrac{T}{T_0}\right)^{3/2}$

2. Pressure: Increases for liquids and decreases for gases with increase in pressure
3. Nature of fluid,
4. Velocity gradient

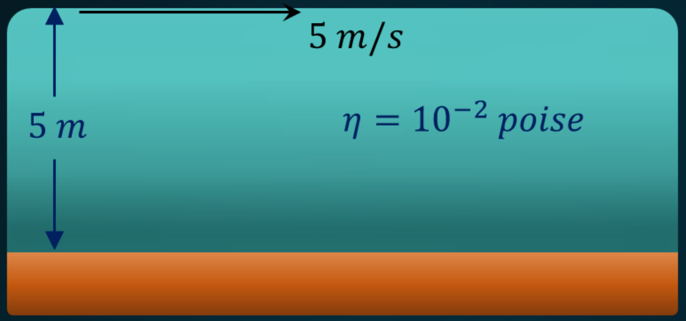
?

The speed of water in a river is $18\ km/h$ near the surface. If the river is $5\ m$ deep, then find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water is $10^{-2}\ poise$.

Given:

$v = 18\ km/h = 5\ m/s, \quad z = 5\ m$

$\eta = 10^{-2}\ poise = 10^{-3}\ Ns/m^2$



Solution:

$$F_v = \left| -\eta A \frac{\Delta v}{\Delta z} \right| \quad \text{(Viscous force)}$$

$$\frac{F_v}{A} = \left| -\eta \frac{\Delta v}{\Delta z} \right| \quad \text{(Shear stress)}$$

$$\sigma_v = 10^{-3} \times \left(\frac{5-0}{5-0} \right)$$

$$\boxed{\sigma_v = 10^{-3}\ N/m^2}$$

a. $10^{-2}\ Nm^{-2}$

b. $10^{-3}\ Nm^{-2}$

c. $10^{-4}\ Nm^{-2}$

d. $5 \times 10^{-4}\ Nm^{-2}$

Stoke's Law

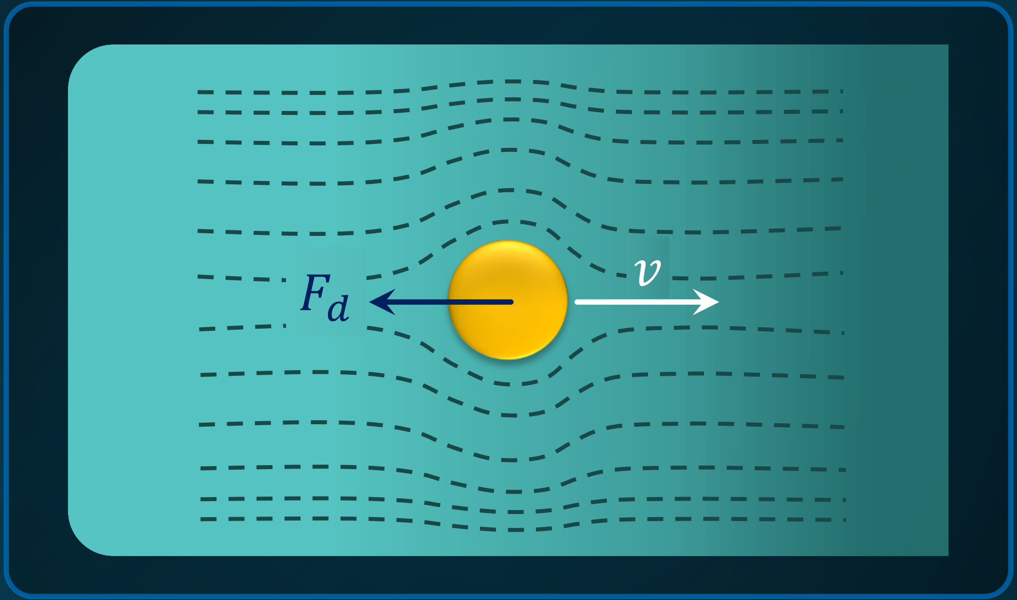
- Fluid particles exert drag force F_d on the solid object, opposing its motion in the medium

- F_d depends on the shape and size of the solid, its speed and the coefficient of viscosity of the fluid

- For a spherical object,

$F_d = 6\pi\eta r v$ (Drag force)

$\vec{F_d} = -6\pi\eta r\vec{v}$ Acts opposite to velocity



Solid vs Fluid Friction

Solid Friction	Fluid Friction
Independent of surface area in contact	Depends on surface area in contact
Depends on normal reaction	Independent of normal reaction
Occurs between solid surfaces in contact	Occurs between fluid layers or solid and fluid in contact
Independent of relative velocity between surfaces in contact	Depends on relative velocity between fluid layers

Terminal Velocity

When the object achieves terminal speed v_t,

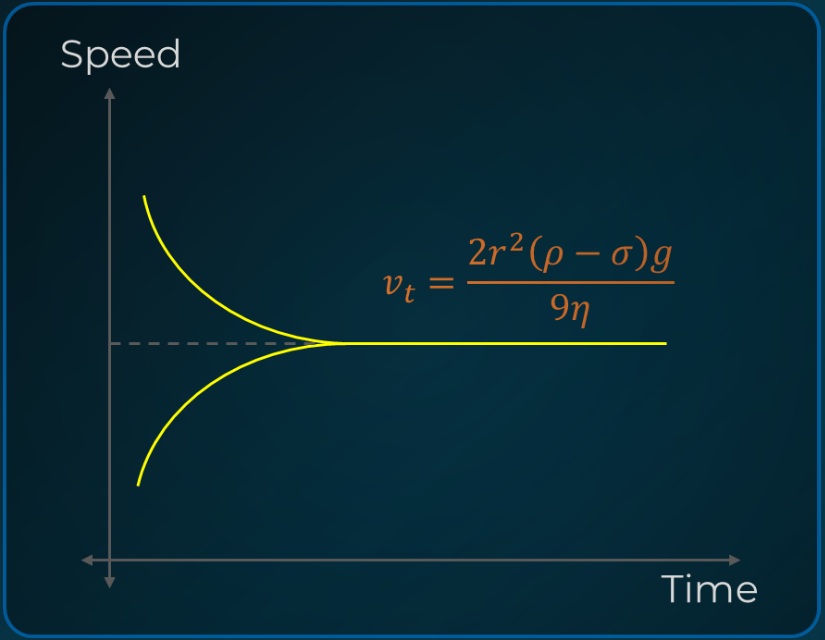
$$F_d + F_B = W$$

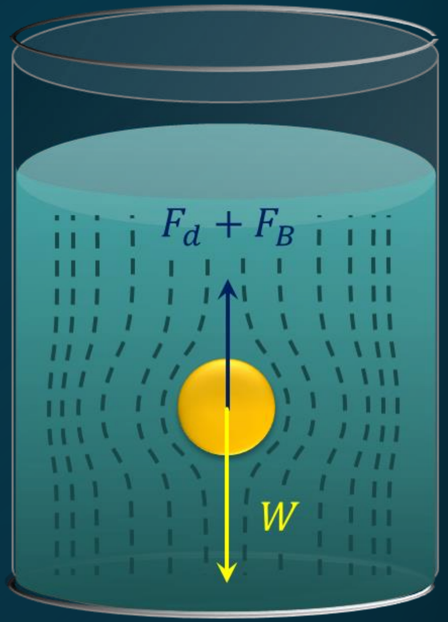
$$F_d = W - F_B$$

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$





ρ = Density of solid body

σ = Density of fluid

Eight spherical raindrops of equal size are falling vertically through air with a terminal speed of $1\,m/s$. What would be the terminal speed if these drops were to coalesce to form a single spherical drop?

Solution:

Terminal velocity,

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta} \qquad \Rightarrow v_T \propto r^2$$

Let r: Radius of small rain drops
R: Radius of large drop

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$R^3 = 8r^3 \Rightarrow \quad R = 2r$$

$$\frac{(v_T)_i}{(v_T)_f} = \frac{r^2}{R^2}$$

$$(v_T)_f = \frac{R^2}{r^2}(v_T)_i$$

$$(v_T)_f = \frac{4r^2}{r^2}(1)$$

$$\boxed{(v_T)_f = 4\,m/s}$$

a. $1\,m/s$

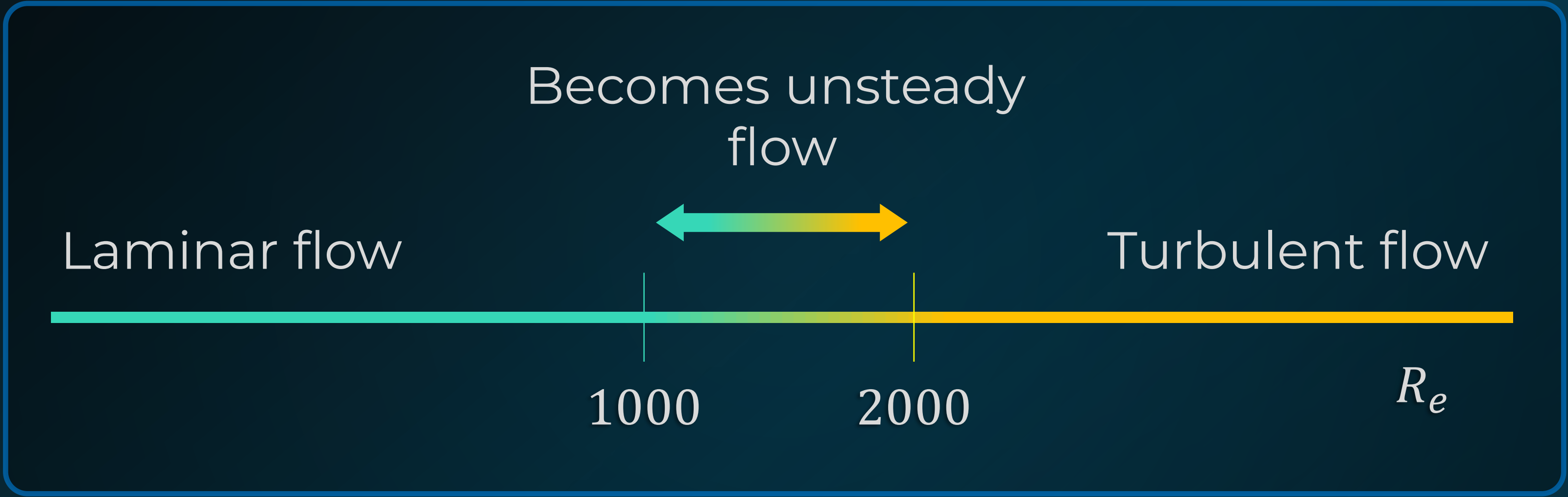
b. $2\,m/s$

c. $4\,m/s$

d. $8\,m/s$

Reynold's Number

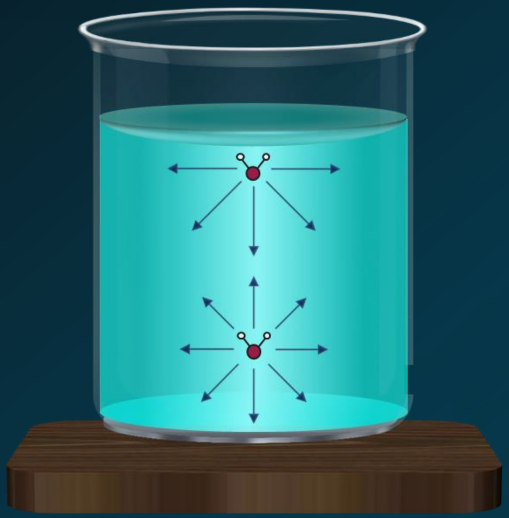
- Ratio of Inertia to viscous force
- Dimensionless number indicating type of flow



$$R_e = \text{Reynolds Number} = \frac{\rho v D}{\eta}$$

Surface Tension

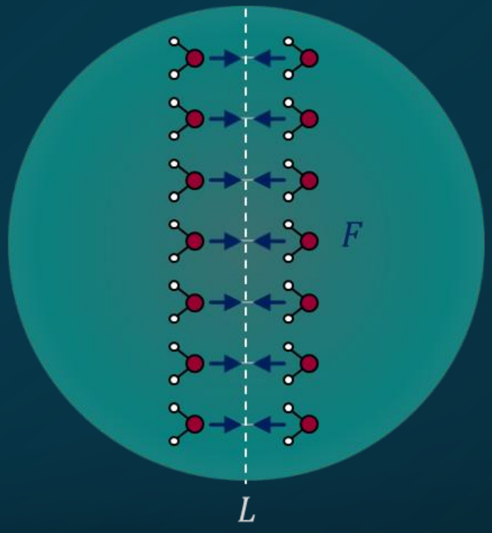
- Molecules near the free surface, experience effective attractive forces in downward direction
- Surface tension:
  - Tendency of a liquid at rest
  - Free surface behaves like a stretched membrane under tension and tries to occupy as smaller area as possible



- Surface energy per unit area
- Force per unit length in the plane of interface between plane of liquid and any other plane

$$S = \frac{\text{Force } (F)}{\text{Length } (L)}$$

- Scalar quantity
- Forces molecules outward ⇒ Makes surface behave as elastic membrane



- SI unit: N/m
- CGS unit: $dyne/cm$

Work Done by Surface Tension

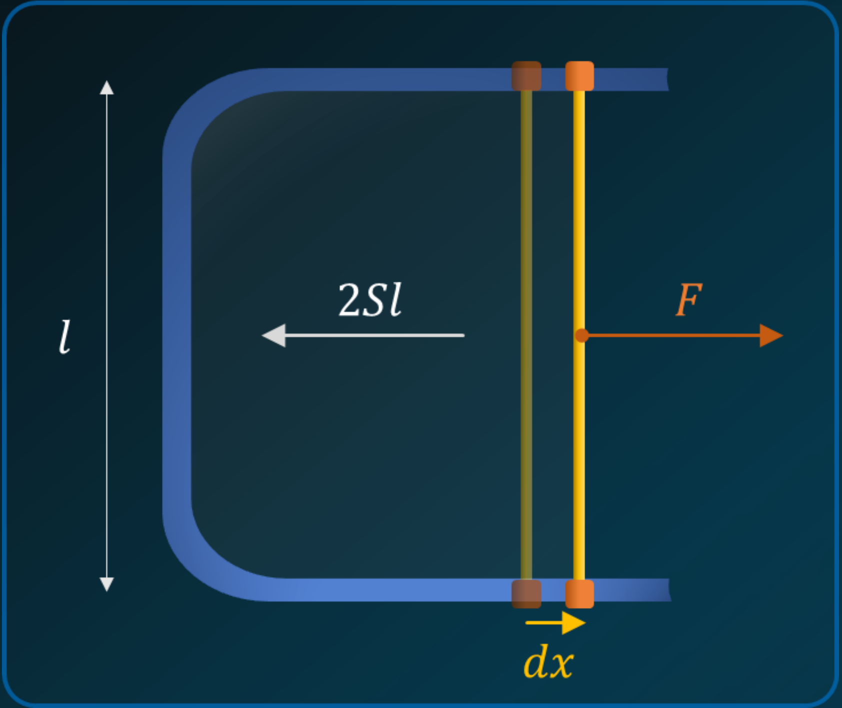
Work done in stretching the surface through length dx:

$$(dW)_{ext} = F\,dx$$

$$= 2Sl\,dx$$

$$= S \cdot 2l dx$$

$$= S\,dA$$



- Molecules near the free surface have some additional energy due to their positions
- Work done against attractive forces in acquiring such positions is stored as the surface energy (U)

$$S = \frac{(dW)_{ext}}{dA} = \frac{dU}{dA}$$

(Provided $\Delta KE = 0$)

$$\boxed{S = \frac{(dW)_{ext}}{dA}}$$

= External work required per unit change in area of the liquid (Quasi-static)

How much work will be done in increasing the radius of a soap bubble from r_1 to r_2? The surface tension of the soap solution is S.

Solution:

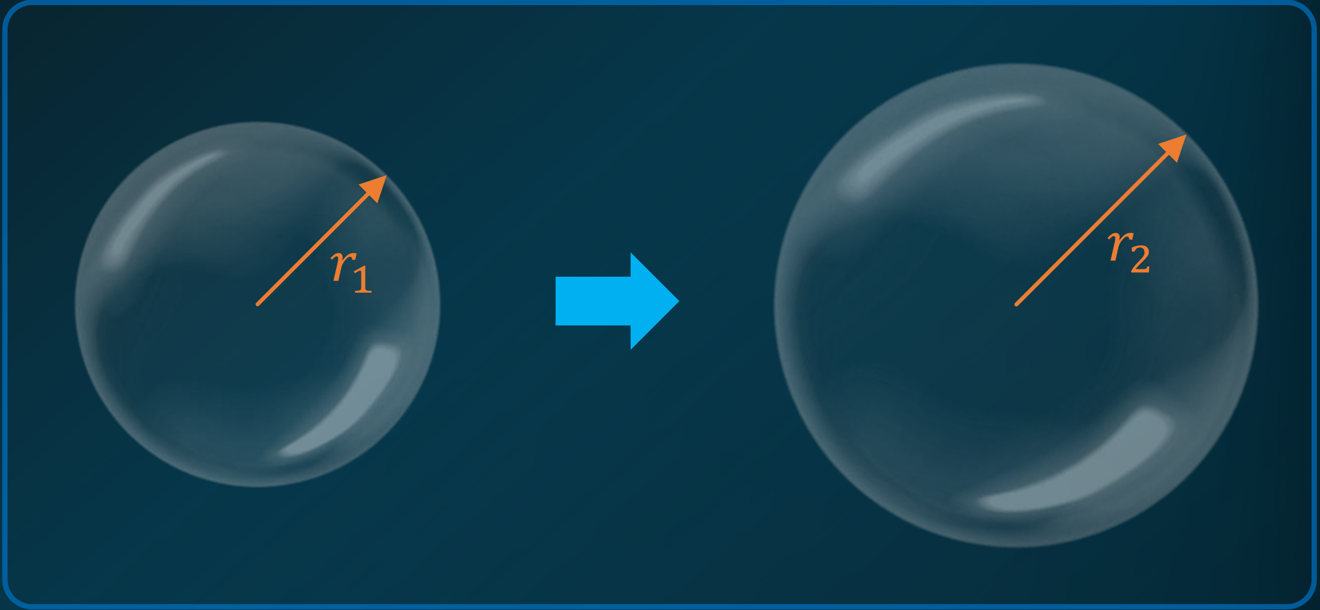
Work done in inflating the bubble is given by,

$$W_{ext} = \Delta U$$

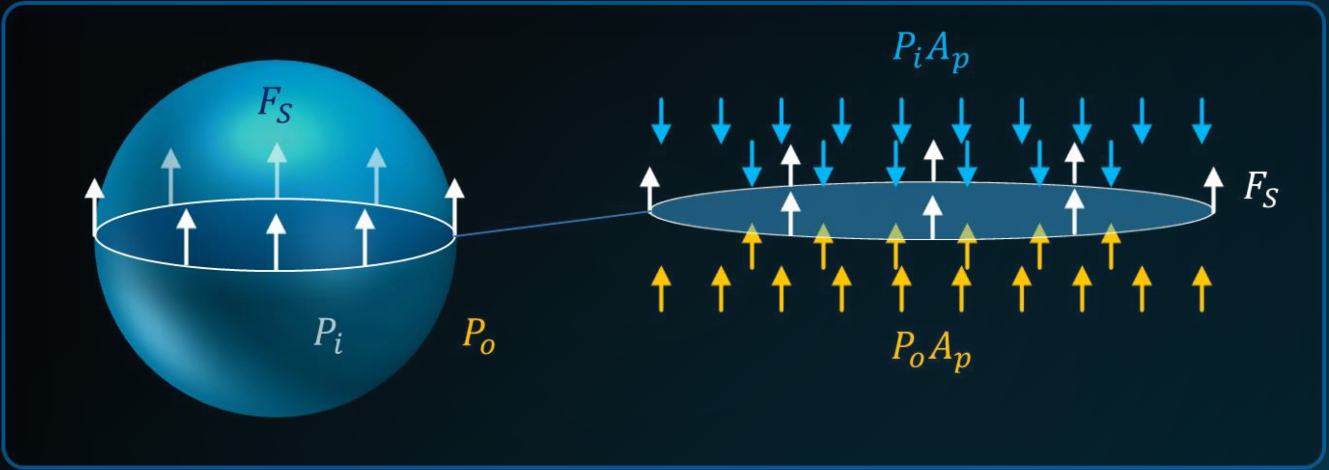
$$= 2S\Delta A$$

$$= 2S(4\pi r_2^2 - 4\pi r_1^2)$$

$$\boxed{W_{ext} = 8\pi S(r_2^2 - r_1^2)}$$



Excess Pressure inside Drop



- Net horizontal force on the cross section: Inner and outward pressure forces (zero)
- Forces in vertical direction: F_S, P_iA_p and P_oA_p

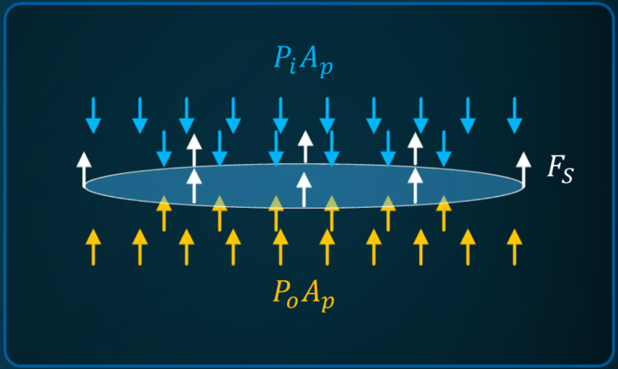
Force balance in the vertical direction:

$$P_iA_p = F_S + P_oA_p$$

$$(P_i - P_o)A_p = F_S$$

$$(P_i - P_o)\pi R^2 = S(2\pi R)$$

$$P_i - P_o = \frac{2S}{R}$$



= Excess pressure inside water drop in air

= Excess pressure inside an air bubble in water

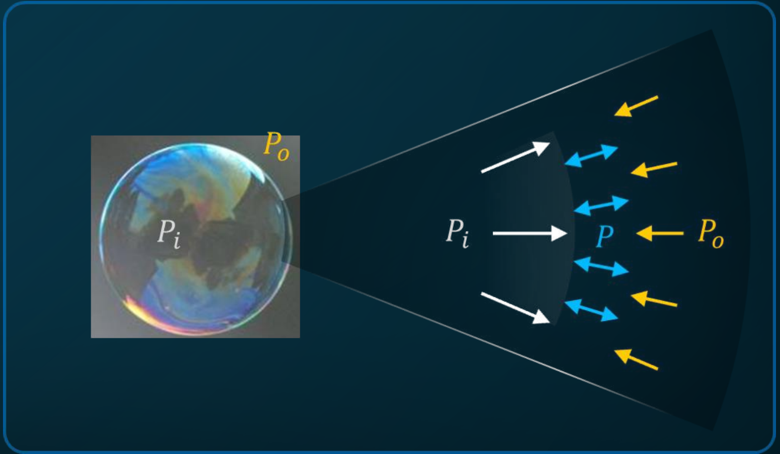
Excess Pressure inside Bubble

Unlike water drop, a soap bubble has two different surfaces in contact with the air.

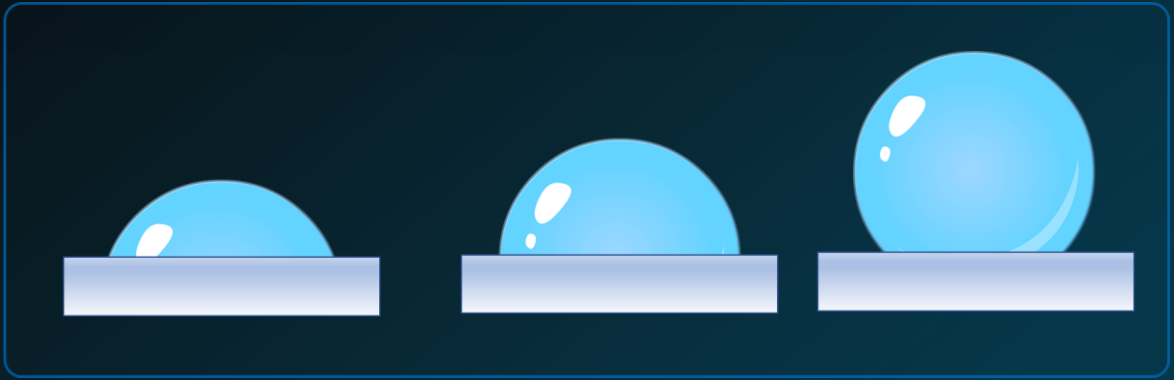
$$P_i - P = \frac{2S}{R} \qquad \text{(Inner)}$$

$$P - P_o = \frac{2S}{R} \qquad \text{(Outer)}$$

$$P_i - P_o = \frac{4S}{R}$$



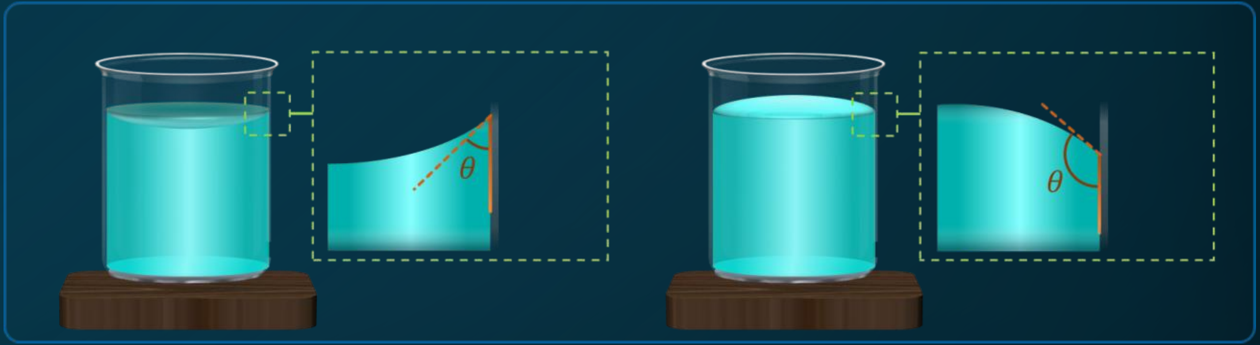
Wettability



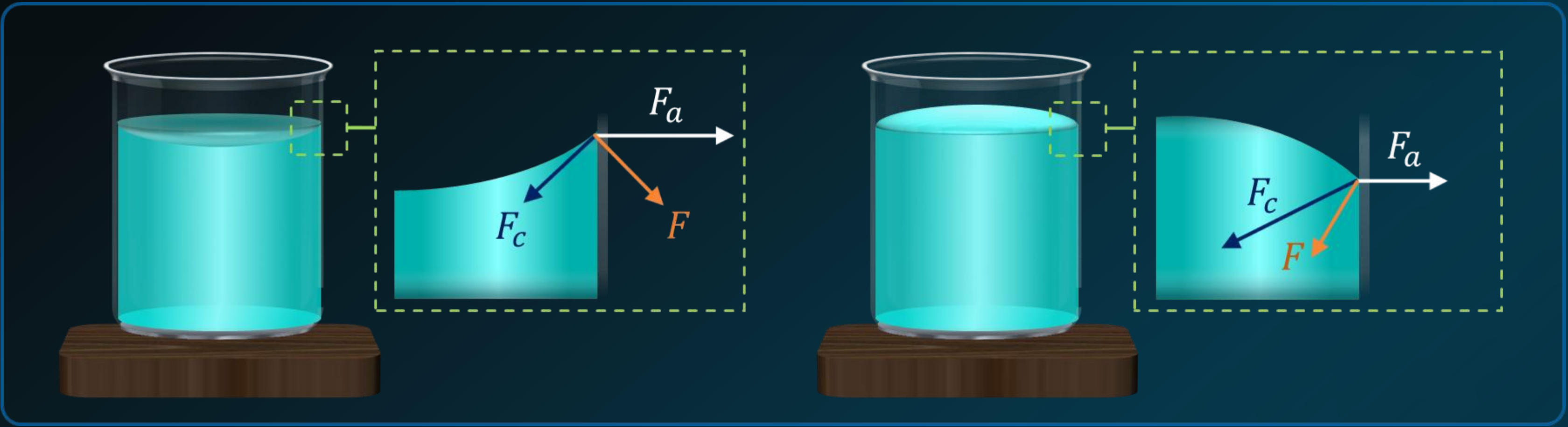
- Ability of a liquid to maintain contact with a solid surface

Contact Angle

- Angle that the tangent to liquid surface at the point of contact makes with the solid surface inside the liquid
- Contact angle/shape of meniscus is directly related to the relative strength of the cohesive and adhesive forces
- When cohesive forces dominate, meniscus is convex
- Similarly, when adhesive forces are larger, the meniscus is concave



Shape of Meniscus



- If θ is acute

  $S_{sa} = S_{sl} + S_{la}\cos\theta$

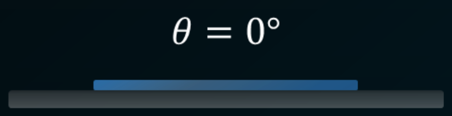
  $\Rightarrow S_{sa} > S_{sl}$

- If θ is obtuse

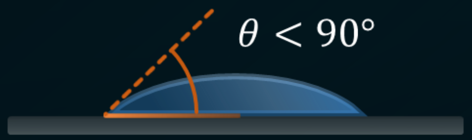
  $S_{sl} = S_{sa} + S_{la}\cos(\pi - \theta)$

  $\Rightarrow S_{sl} > S_{sa}$

Contact Angle

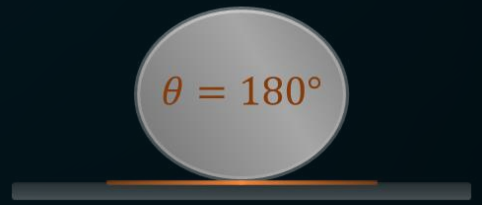


Perfect wetting



Good wetting

Bad wetting



Perfectly non-wetting

Capillarity

- Phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid

Force balance in the vertical direction,

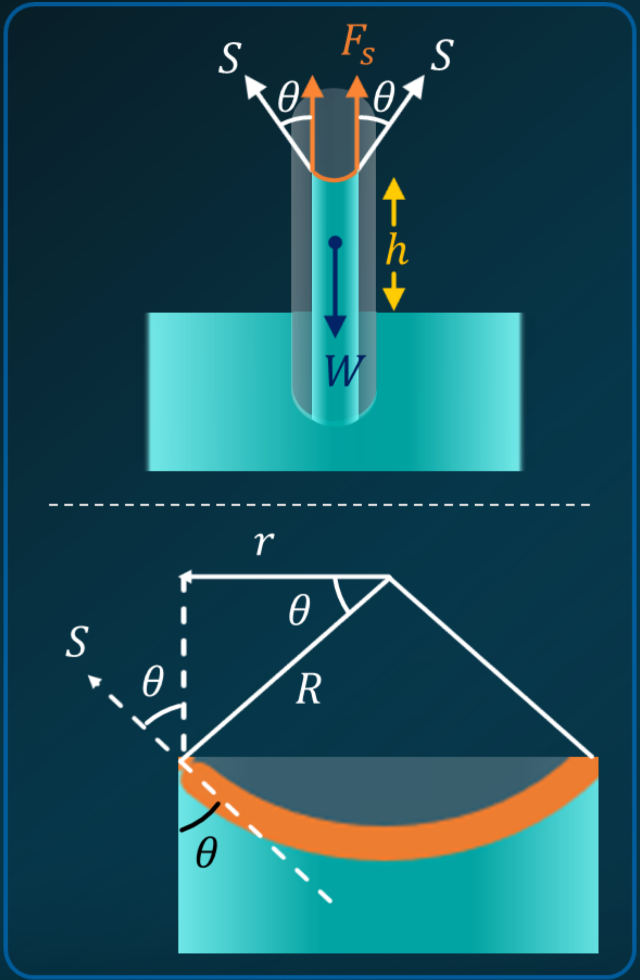
$$F_s = W$$

$$S\cos\theta\,(2\pi r) = \rho(\pi r^2 h)g$$

Capillary rise,

When tube height is insufficient, radius of meniscus changes such that,

$$h\frac{r}{\cos\theta} = hR = \frac{2S}{\rho g} = Constant$$



- When capillarity experiment is performed in accelerated frame,

$$h = \frac{2S\cos\theta}{\rho g_{eff} r}$$

Where,

$g_{eff} = (g - a)$ for downward motion

$= (g + a)$ for upward motion

- If experiment is performed in gravity free space or free fall,

$g = 0 \Rightarrow$ liquid overflows