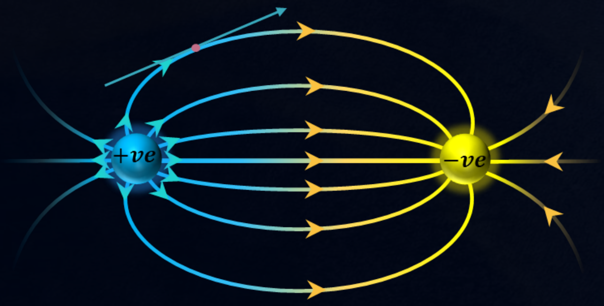
Welcome to

Aakash + BYJU'S NOTES

Moving Charges and Magnetism

Electric field Lines – Stationary Charges and Electromagnetism

Electric field Lines – Stationary Charges



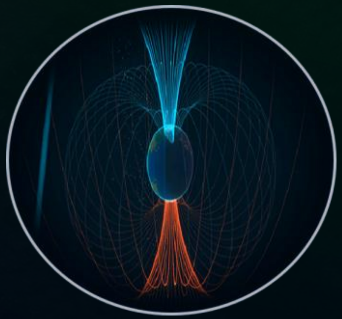
Electromagnetism

Electromagnetism is based on the principle that moving charges can produce electric as well as magnetic field.

Electrostatics

- Charges are at rest

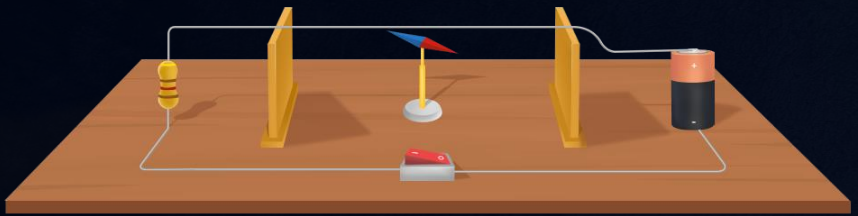
Electromagnetism



- Charges are moving

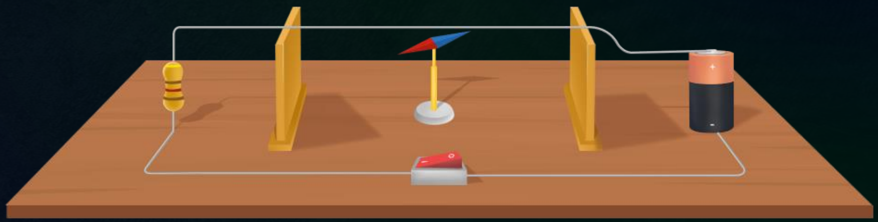
Oersted's experiment

When the switch is OFF



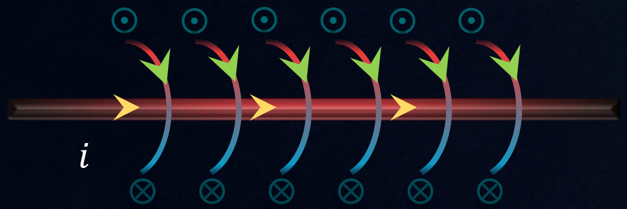
The needle of the magnetic compass directs towards the magnetic North and magnetic South poles of the earth.

When the switch is ON



Deflection in the needle of magnetic compass.

Magnetic Field Intensity



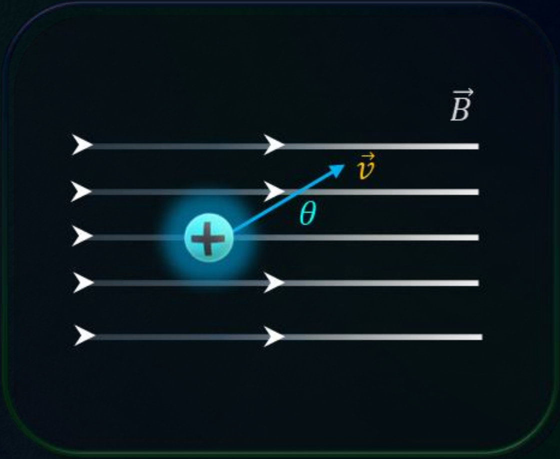
- The compass points tangentially to the imaginary circle drawn around the current carrying conductor.
- These imaginary circles are known as Magnetic Field Lines

Magnetic Force and its Direction

Magnetic force ($\vec{F}_m$) depends on the following:

$$\boxed{|\vec{F}_m| \propto q} \qquad \boxed{|\vec{F}_m| \propto |\vec{v}| \sin\theta}$$

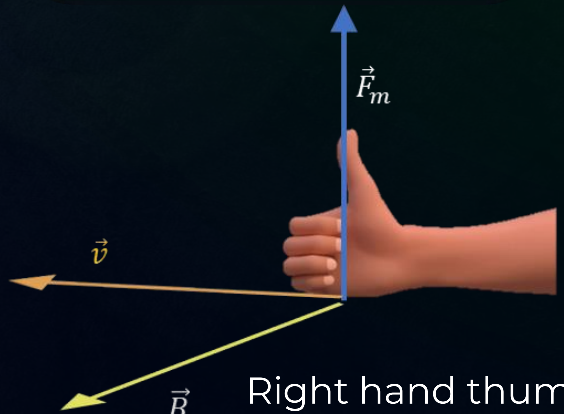
$$\boxed{|\vec{F}_m| \propto |\vec{B}|} \qquad \boxed{\vec{F}_m \perp \vec{B} \quad \vec{F}_m \perp \vec{v}}$$



Combining all these observations:

$$\boxed{\vec{F}_m = q(\vec{v} \times \vec{B})}$$

The direction of Magnetic force ($\vec{F}_m$) can be obtained using right hand thumb rule.



Right hand thumb rule

Note:
If q is positive, then the direction will be along $\vec{v} \times \vec{B}$.
But if q is negative, it will be the opposite.

Find the directions of the magnetic force, if the following configurations of magnetic field, direction of velocity, and sign of charge are given.

Direction of $(\vec{v})$: $-\hat{\imath}$
Direction of $(\vec{B})$: $-\hat{\jmath}$
Sign of q: $+ve$
Direction of $(\vec{F}_m)$: $\hat{k}$

Direction of $(\vec{v})$: $-\hat{\jmath}$
Direction of $(\vec{B})$: $-\hat{k}$
Sign of q: $-ve$
Direction of $(\vec{F}_m)$: $-\hat{\imath}$

Direction of $(\vec{v})$: $-\hat{\jmath}$
Direction of $(\vec{B})$: $+\hat{\jmath}$
Sign of q: $-ve$
Direction of $(\vec{F}_m)$: $null$

Direction of null vector is 'indeterminate'

A charge of $2.0\ \mu C$ moves with a speed of $2.0 \times 10^6 ms^{-1}$ along the positive x-axis. A magnetic field $\vec{B}$ of strength $(0.20\hat{j} + 0.40\hat{k})T$ exists in space. What is the magnetic force acting on the charge?

Solution, $\vec{F}_m = q(\vec{v} \times \vec{B})$

$$\vec{F}_m = qv\hat{\imath} \times (B_y\hat{j} + B_z\hat{k})$$

$$\vec{F}_m = qvB_y\hat{k} - qvB_z\hat{j}$$

$$= 2.0\ \mu C \times 2.0 \times 10^6\ m/s \times 0.20\ \hat{k} - 2.0\ \mu C \times 2.0 \times 10^6\ m/s \times 0.40\ \hat{j}$$

Simplifying we get,

$$\boxed{\vec{F}_m = (0.8\hat{k} - 1.6\hat{j})\ N}$$

A $(0.8\hat{k} - 1.6\hat{j})\ N$

C $(0.8\hat{k} - 0.8\hat{j})\ N$

B $(1.6\hat{k} - 1.6\hat{j})\ N$

D $(1.6\hat{k} - 0.8\hat{j})\ N$

Unit of Magnetic Field

$$\boxed{|\vec{B}| = \frac{|\vec{F}_m|_{max}}{q|\vec{v}|}}$$

The SI unit of magnetic field is Tesla (T)

$$1 \text{ Tesla} = 1\ \mathrm{T} = \frac{1N}{C-m/s} = \frac{1N}{A-m}$$

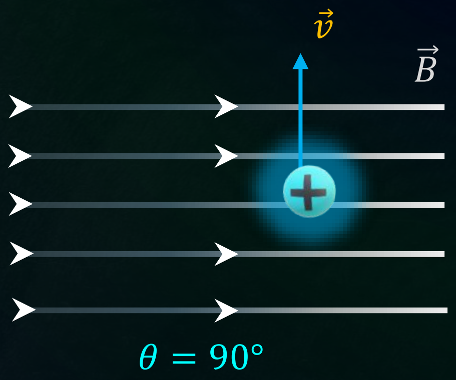
A smaller unit called Gauss is also used instead of Tesla

$$1 \text{ gauss} = 1\ G = 10^{-4} T$$

Gauss is the CGS unit of Magnetic Field.

Another commonly used unit is Wb/m^2

$$1\ T = \mathrm{Wb}/m^2$$



$$|\vec{F}_m|_{max} = q|\vec{v}||\vec{B}|$$

Work Done by a Magnetic Force

We know that $\vec{F}_m \perp \vec{v}$

Instantaneous power delivered is zero.

$$P = \vec{F}_m \cdot \vec{v} = 0$$

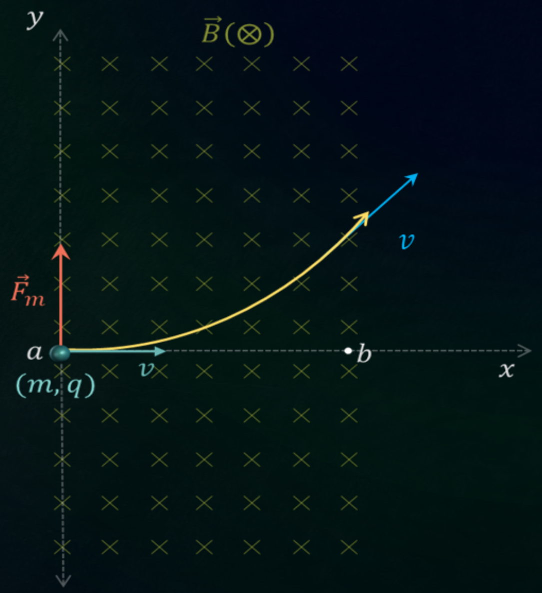
Work done by magnetic force,

$$\boxed{W_m = 0}$$

From Work-Energy Theorem, we know,

"If only magnetic force is acting on the system, then, the speed of the charged particle remains constant whereas the velocity may be variable since the direction of the particle might get changed during the motion"

If velocity of the particle is $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$, then the speed is defined as,



$$\left|\vec{F}_m\right|_{max} = q|\vec{v}||\vec{B}|$$

$$\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = constant}$$

Direction of Motion in Magnetic Field

Magnetic force ($\vec{F}_m$) depends on:

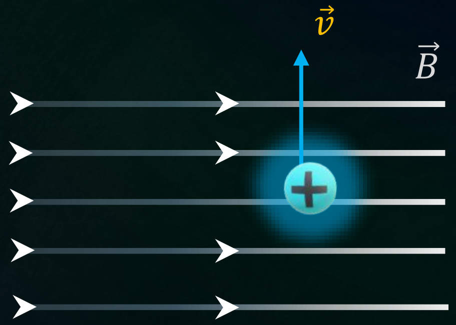
Angle between the velocity (v) and the magnetic field (B)

$\theta = 0°$

$\vec{F}_m = 0$

$\theta = 180°$

$\vec{F}_m = 0$



$\theta = 90°$

$\vec{F}_m = F_{max}$

There will be no deviation since the force acting is zero.

What will happen when the maximum force is acting on the particle, i.e., the force and velocity are perpendicular to each other?

Motion in Magnetic Field

When $\theta = 90°$, $|\vec{F}_m| = qvB$ $(\because \vec{F}_m \perp \vec{v}$ at every instant)

Radius of the circle (R) $\quad |\vec{F}_m| = qvB = \dfrac{mv^2}{R}$ $\quad \boxed{R = \dfrac{mv}{qB}}$

Also,

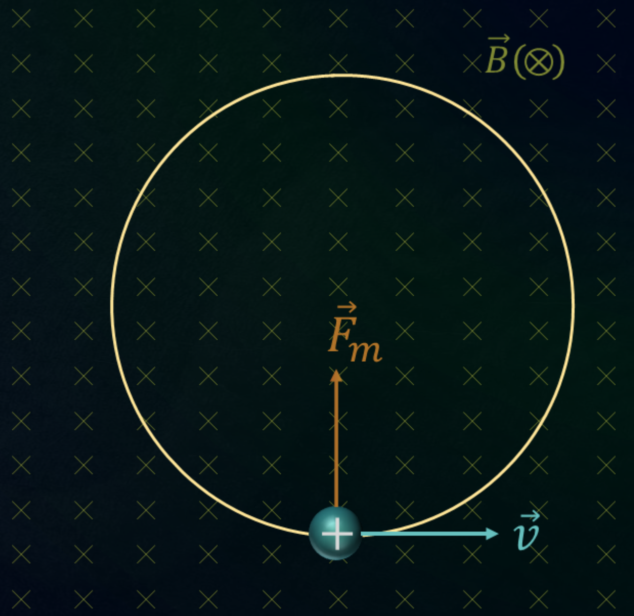
$$\boxed{R = \frac{v}{\alpha B}}$$

where α is the specific charge or charge per unit mass

Time Period, $\quad T = \dfrac{2\pi R}{v} = \left(\dfrac{2\pi}{v}\right)\left(\dfrac{mv}{qB}\right) \Rightarrow \boxed{T = \dfrac{2\pi m}{qB}}$

Frequency, $\quad f = \dfrac{1}{T} = \left(\dfrac{v}{2\pi}\right)\left(\dfrac{qB}{mv}\right) \Rightarrow \boxed{f = \dfrac{qB}{2\pi m}}$

Angular speed, $\quad \omega = \dfrac{v}{R} = v\left(\dfrac{qB}{mv}\right) \quad \boxed{\omega = \dfrac{qB}{m}}$



Uniform Circular Motion

Thus, T, f, ω are independent of speed(v) of the particle.

A proton, a deuteron, and an alpha particle moving with equal kinetic energies enter perpendicularly into a region of uniform magnetic field. If r_p, r_d and r_α are the respective radii of the circular paths, find the ratios r_p/r_d and r_p/r_α.

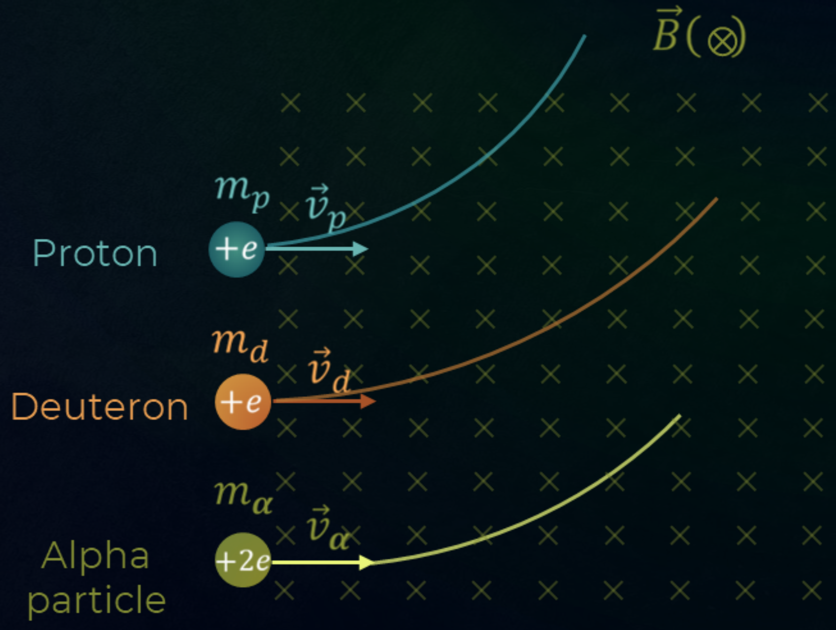
Solution

$$R = \frac{mv}{qB} \quad \Rightarrow \; R = \frac{\sqrt{2mK}}{qB}$$

$$R \propto \frac{\sqrt{m}}{q} \qquad (\because \text{ all other parameters are constant.})$$

$$\frac{r_p}{r_d} = \frac{\left(\frac{\sqrt{m_p}}{q_p}\right)}{\left(\frac{\sqrt{m_d}}{q_d}\right)} = \frac{\left(\frac{\sqrt{1}}{1}\right)}{\left(\frac{\sqrt{2}}{1}\right)} \Rightarrow \boxed{\frac{r_p}{r_d} = \frac{1}{\sqrt{2}}}$$

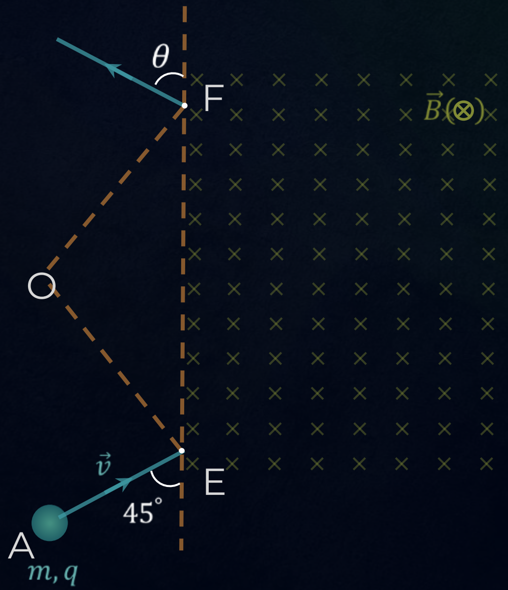
$$\frac{r_p}{r_\alpha} = \frac{\left(\frac{\sqrt{m_p}}{q_p}\right)}{\left(\frac{\sqrt{m_\alpha}}{q_\alpha}\right)} = \frac{\left(\frac{\sqrt{1}}{1}\right)}{\left(\frac{\sqrt{4}}{2}\right)} \Rightarrow \boxed{\frac{r_p}{r_d} = 1}$$



A particle of mass $m = 1.6 \times 10^{-27} kg$ and charge $q = 1.6 \times 10^{-19} C$ moves at a speed of $v = 1.0 \times 10^{7} m/s$. It enters a region of uniform magnetic field at a point E. The field has a strength of $1.0\ T$.

(a) The magnetic field is directed into the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(b) If the field is coming out of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.



Solution,

$|\vec{v}| = 1.0 \times 10^7 m/s \quad |\vec{B}| = 1.0\ T$

Since, $\vec{F}_m \perp \vec{v} \Rightarrow W = 0 \Rightarrow$ speed remains constant

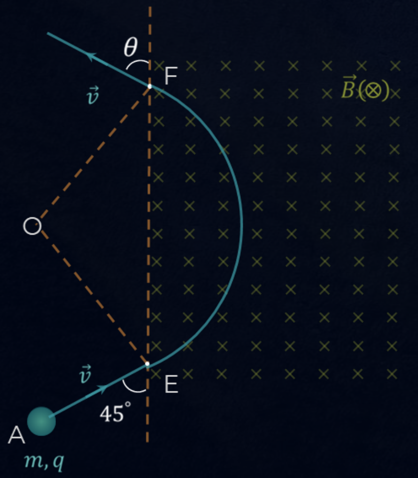
$\theta = 45°$

$$EF = 2R\cos 45° = 2\left(\frac{mv}{qB}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$EF = 2R\cos 45° = \sqrt{2}\left(\frac{1.6 \times 10^{-27} \times 1.0 \times 10^{7}}{1.6 \times 10^{-19} \times 1.0}\right)$$

$$\boxed{EF = 0.14\ m}$$

$$\boxed{\theta = 45°}$$



$|\vec{v}| = 1.0 \times 10^7 m/s \quad |\vec{B}| = 1.0\ T$

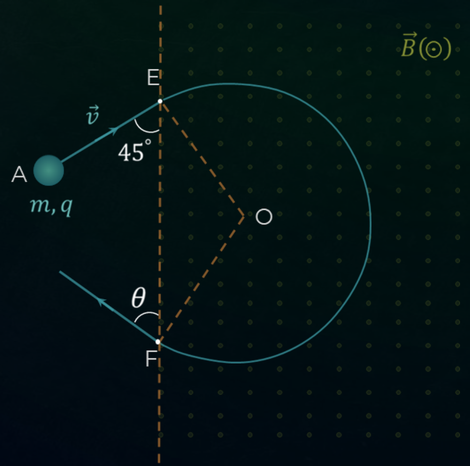
Since, $\vec{F}_m \perp \vec{v} \Rightarrow W = 0$

$$t_{EF} = \frac{2\pi - 2\theta}{v} R = \left(\frac{3\pi}{2v}\right)\left(\frac{mv}{qB}\right)$$

$$t_{EF} = \left(\frac{3\pi m}{2qB}\right) = \frac{3\pi}{2}\frac{1.6 \times 10^{-27}}{1.6 \times 10^{-19}}$$

$$t_{EF} = \frac{3\pi}{2} \times 10^{-8} s$$

$$\boxed{t_{EF} = 4.7 \times 10^{-8} s}$$



The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{\imath}$ travels along x-axis and enters the region of the magnetic field. Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle $30°$ to its initial velocity.

The magnetic field ($\vec{B}$) is represented by " " (dot). So, it means the field is coming out of plane of the screen i.e., along $+ve\ z$-axis.

When the particle just enters the magnetic field :

Velocity ($\vec{v}$) of the particle is along $+ve\ x$-axis.

The magnetic force ($\vec{F}_m$) of the particle of charge $+q$ will be along $(\vec{v} \times \vec{B}) = \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$$CP = CQ = r = \frac{mv_0}{qB_0}$$

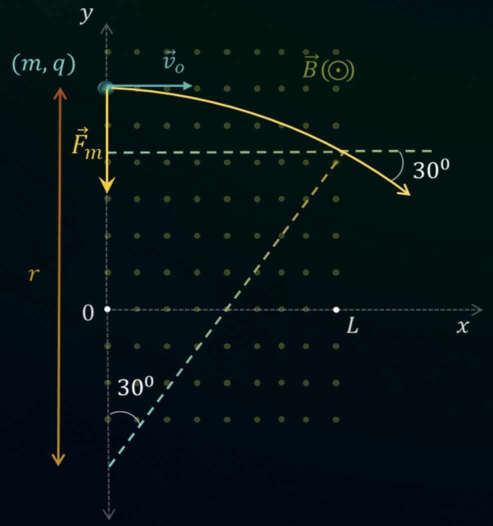
$$P'Q = L$$

$$\sin 30° = \frac{P'Q}{CQ} = \frac{1}{2}$$

$$\frac{L}{r} = \frac{1}{2}$$

$$L = \frac{r}{2}$$

$$\boxed{L = \frac{mv_0}{2qB_0}}$$



The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of the magnetic field. Neglect the gravity throughout the question.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto $2.1L$.

Condition of semi-circular path for the particle:

Width of the region of magnetic field > Radius of the semi-circular path

We have: $r = 2L$

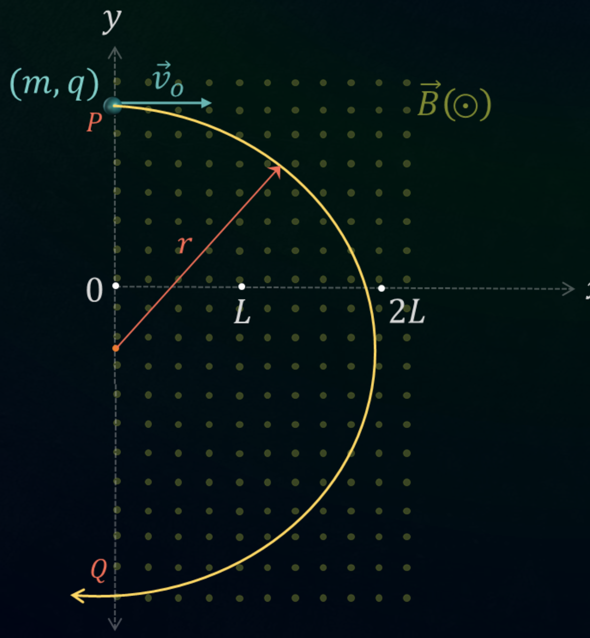
Given: $Width = 2.1L$

Condition satisfied

Final velocity : $\boxed{\vec{v}_f = \vec{v}_Q = -v_0\hat{i}}$

Time spent by the particle in the field = Time period to cover the semi-circle

$$t_{PQ} = \frac{T}{2} \rightarrow t_{PQ} = \frac{\left(2\pi m/qB_0\right)}{2} \rightarrow \boxed{t_{PQ} = \frac{\pi m}{qB_0}}$$



Two particles, each having a mass m are placed at a separation d in a uniform magnetic field $\vec{B}$ as shown. They have opposite charges of equal magnitude q. At time $t = 0$, the particles are projected towards each other, each with speed v. Suppose the Coulomb force between the charges is switched off.

(a) Find the maximum value v_m of the projection speed so that the two particles do not collide.

The Coulombic force between two charge particles is neglected.

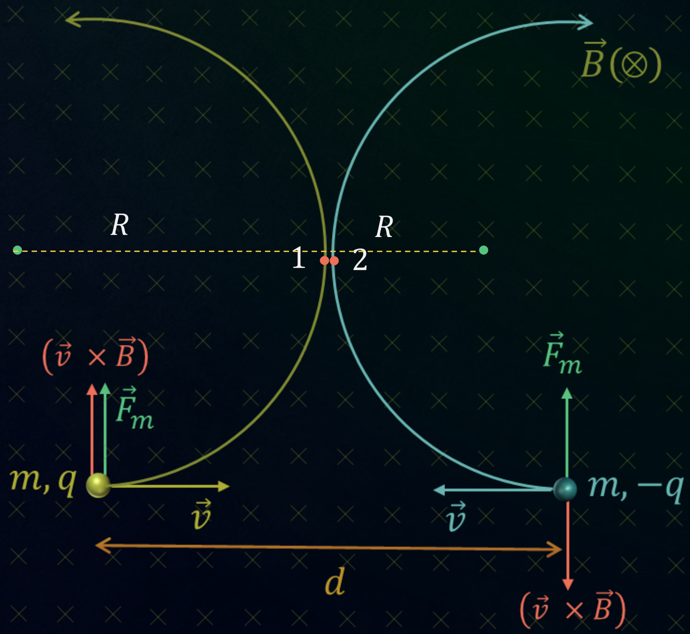
As particle's velocity increases, the radius of the semi-circular path also increases.

The particles will not collide if : $v < v_m$

At maximum speed v_m : $\boxed{d_{12} \to 0}$

$R + R = d \quad ; R = \frac{d}{2}$

$\frac{mv_m}{qB} = \frac{d}{2} \rightarrow \boxed{v_m = \frac{qBd}{2m}}$



Motion of a Charged Particle in a Uniform Magnetic Field

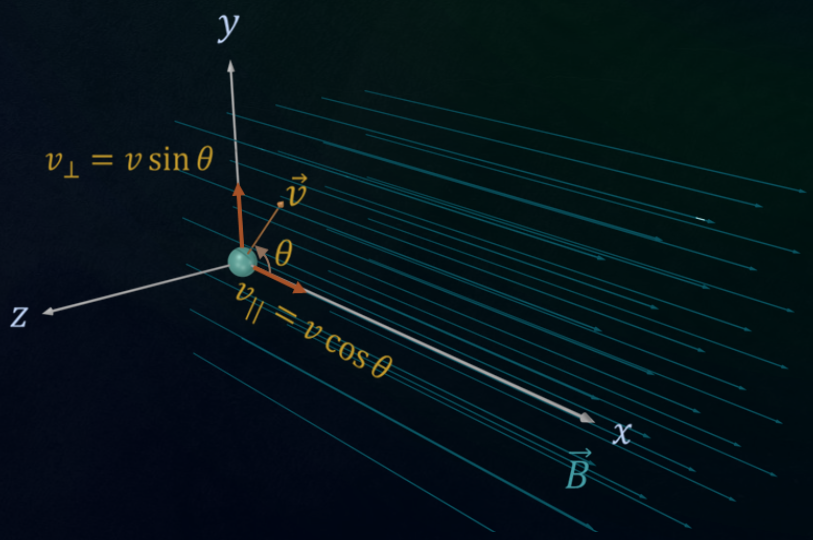
$\theta = 0°$	$\theta = 180°$	$\theta = 90°$	$\theta \neq 0°, 180°$ and $90°$
Straight line motion	Straight line motion	Uniform Circular motion	?

- Component of velocity perpendicular to the direction of magnetic field: $v_{\perp} = v \sin \theta$

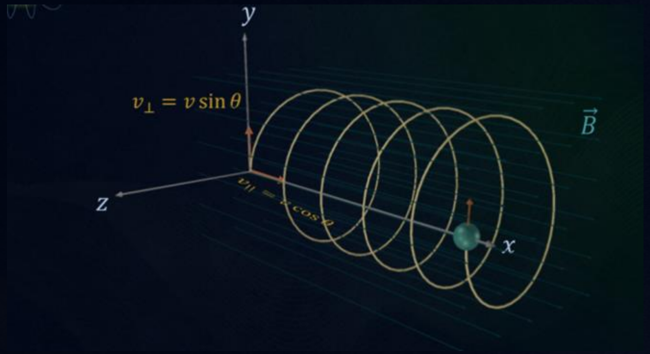
This is responsible for UCM of the particle.

- Component of velocity parallel to the direction of magnetic field: $v_{||} = v \cos \theta$

This is responsible for linear translational motion of the particle.



Motion of a Charged Particle in a Uniform Magnetic Field



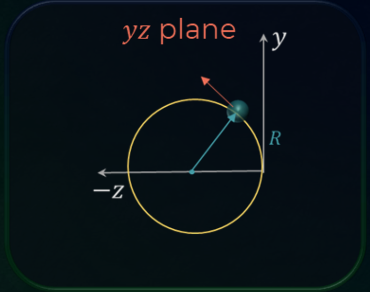
UCM of the particle

+

Linear translational motion of the particle

=

Helical motion of the particle



The radius of the helix

$$R = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

Time period

$$T = \frac{2\pi R}{v_{\perp}} = \frac{2\pi m}{qB}$$

Frequency

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

Pitch of helix

$$p = v_{\parallel} T = \frac{2\pi m v\cos\theta}{qB}$$

A beam of protons with a velocity of $4 \times 10^5\ ms^{-1}$ enters a region of uniform magnetic field of $0.3\ T$. The velocity makes an angle of $60°$ with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.

Solution :

Radius of the helical path : $R = \dfrac{mv \sin\theta}{qB}$

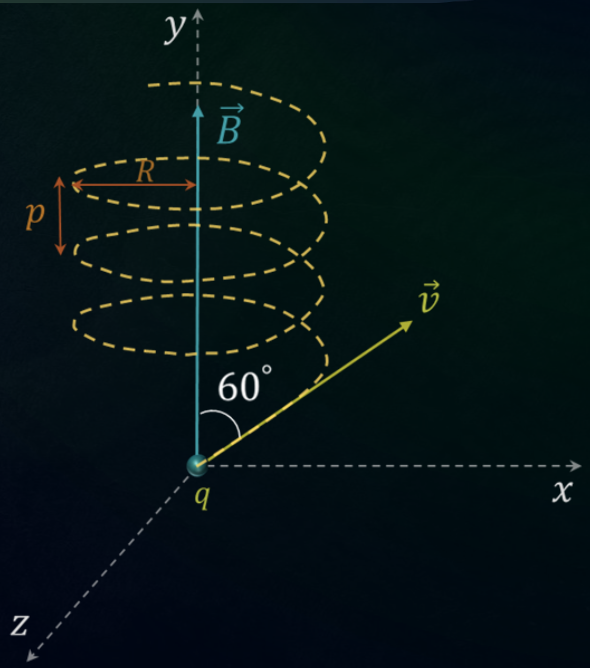
$$R = \frac{(1.67 \times 10^{-27})(4 \times 10^5) \sin 60°}{(1.6 \times 10^{-19})(0.3)}$$

$$\boxed{R = 1.2\ cm}$$

The pitch of the helical path : $p = \dfrac{2\pi mv \cos\theta}{qB}$

$$p = \frac{2\pi(1.67 \times 10^{-27})(4 \times 10^5) \cos 60°}{(1.6 \times 10^{-19})(0.3)}$$

$$\boxed{p = 4.4\ cm}$$



Electromagnetic Force on a Moving Charge; Lorenz Force

Electric force on the particle :

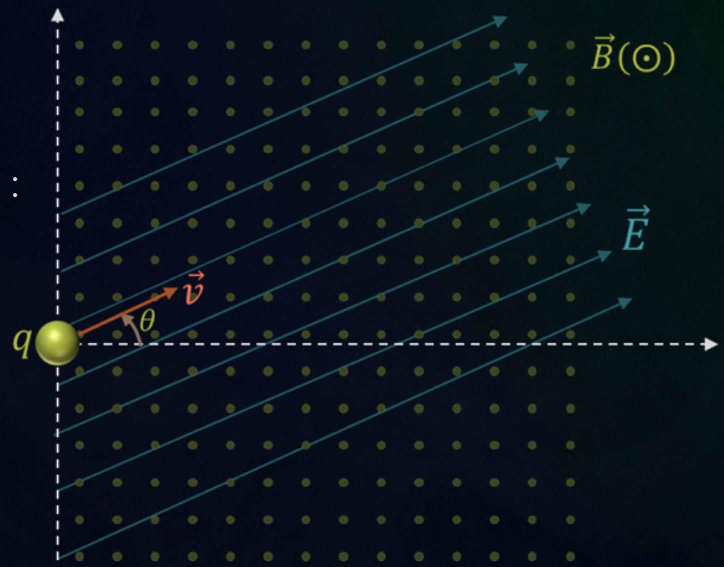
$\vec{F}_e = q\vec{E}$

Magnetic force on the particle :

$\vec{F}_m = q(\vec{v} \times \vec{B})$

Total force on the particle :

$\vec{F} = \vec{F}_e + \vec{F}_m$



When both electric field and magnetic field act simultaneously on a charged particle, then total force acting on the particle is the vector sum of electric force and magnetic force. This total force is known as "Lorentz force".

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

?

A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = -B_0\hat{j}$ and $\vec{E} = E_0\hat{k}$. Find the speed of the particle as a function of its z-coordinate.

Initial speed of the particle = 0 ⟶ Initial kinetic energy = 0

Final speed of the particle = v ⟶ Final kinetic energy = $\frac{1}{2}mv^2$

Work done on the particle by magnetic force, $W_m = 0$

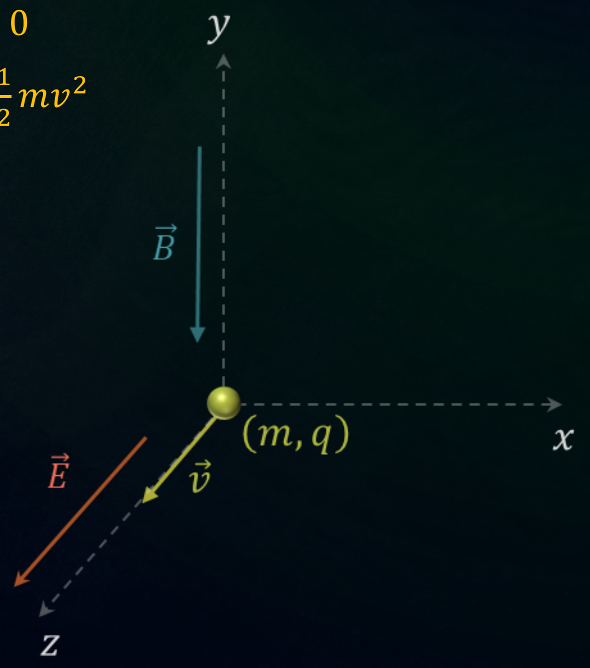
Work done on the particle by electric force, $W_e = \vec{F}_e \cdot z\hat{k} = qE_0z$

Apply work-energy theorem :

$$W_e + W_m = \Delta K.E. = \left(\frac{1}{2}mv^2 - 0\right)$$

$$qE_0z = \frac{1}{2}mv^2$$

$$\boxed{v = \sqrt{\frac{2qE_0z}{m}}}$$



Motion of a Charged Particle in a Uniform Electromagnetic Field

When $\vec{E}, \vec{B}$ and $\vec{v}$ all the three are collinear :

- The electrostatic force acting on the charge:

$$\vec{F}_e = q\vec{E}$$

- The direction of velocity of the particle and that of the magnetic field is along the same direction.

$$(\vec{v} \times \vec{B}) = 0$$

- The magnetic force on the charged particle on the particle :

$$\vec{F}_m = 0$$

- The total force i.e., Lorentz force acting on the charge:

$$\boxed{\vec{F} = \vec{F}_e = q\vec{E}}$$

- If the charge is positive, the charge will accelerate in the direction of electric field.

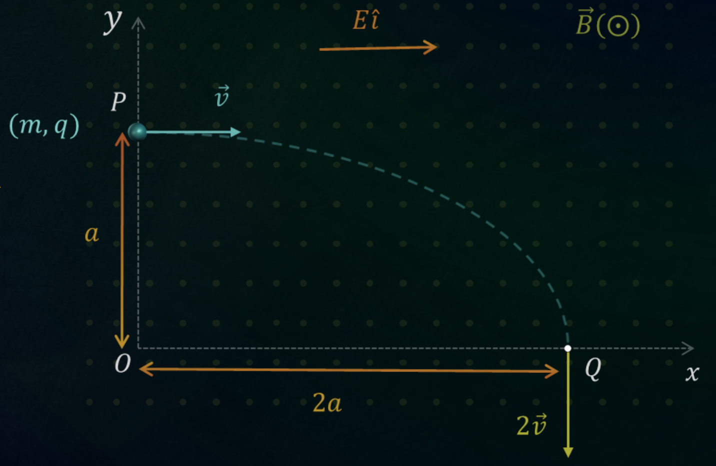
- If the charge is negative, the force on it will be opposite to its velocity and the charge will decelerate.

A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Find the magnitude of the electric field and the rate of work done by it at P.

Solution

Work done on the particle by magnetic force, $W_m = 0$

Work done on the particle by electric force, $W_e = \vec{F}_e \cdot 2a\hat{i} = 2qEa$



Apply Work-Energy theorem from P to Q :

$$W_e + W_m = \Delta K.E. = \left(\frac{1}{2}mv_Q^2 - \frac{1}{2}mv_P^2\right)$$

$$2qEa + 0 = \frac{1}{2}m(4v^2 - v^2)$$

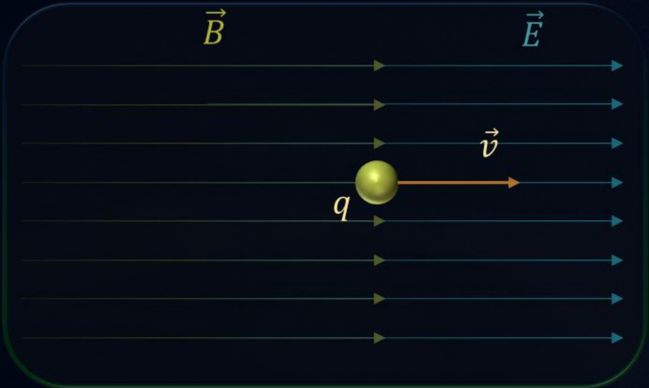
$$\boxed{E = \frac{3mv^2}{4qa}}$$

The rate of work done by electric field at P :

$$\frac{dW_e}{dt} = \vec{F}_e \cdot \vec{v}_P \qquad \frac{dW_e}{dt} = (qE\hat{i}) \cdot (v\hat{i}) \qquad \boxed{\frac{dW_e}{dt} = \frac{3}{4}\left(\frac{mv^3}{a}\right)}$$

Motion of a Charged Particle in a Uniform Electromagnetic Field

$\vec{E}, \vec{v}$, and $\vec{B}$ are Collinear

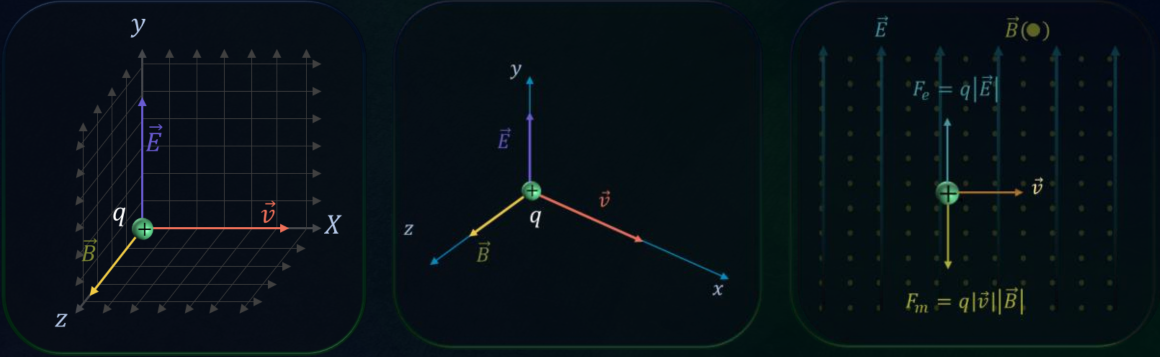


$$\vec{F}_e = q\vec{E}$$

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right) = 0$$

$$\boxed{\vec{F} = \vec{F}_e = q\vec{E}}$$

$\vec{E}, \vec{v}$, and $\vec{B}$ are Mutually Perpendicular



$$\left|\vec{F}\right| = q\left|\vec{E}\right| - q|\vec{v}|\left|\vec{B}\right|$$

Special Case:

$$\left|\vec{F}\right| = 0 \Rightarrow qE - qvB = 0$$

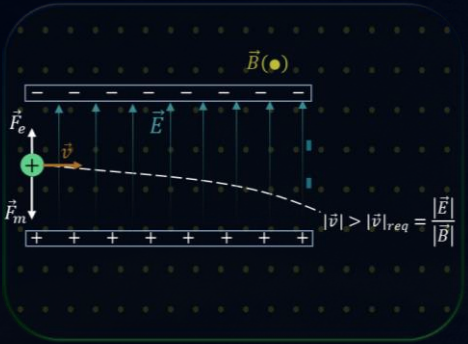
$$\boxed{\Rightarrow v = \frac{E}{B}}$$

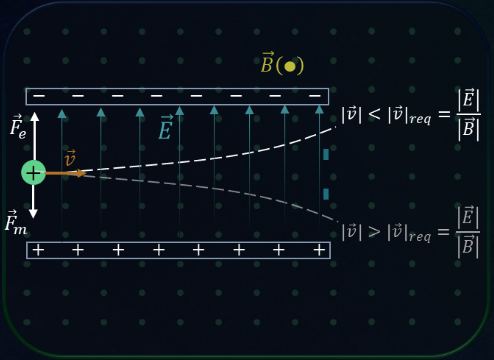
Conditions:

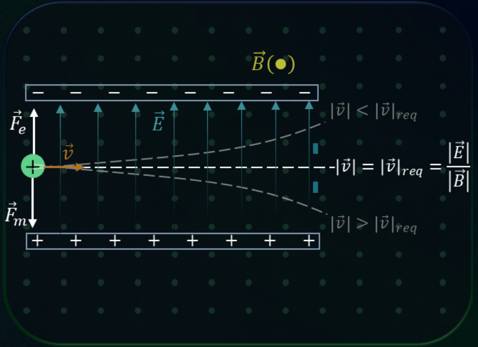
$\vec{E}, \vec{v}$, and $\vec{B}$ are mutually perpendicular

$\vec{E}$ and $\vec{B}$ are uniform

Velocity Selector







If $v > \frac{E}{B}$, the charged particle will move downward.

$$|\vec{F_m}| > |\vec{F_e}|$$

$$\Rightarrow qvB > qE$$

$$\Rightarrow v > \frac{E}{B}$$

If $v < \frac{E}{B}$, the charged particle will move upward.

$$|\vec{F_m}| < |\vec{F_e}|$$

$$\Rightarrow qvB < qE$$

$$\Rightarrow v < \frac{E}{B}$$

Selects charged particles of a particular velocity out of a beam containing charges moving with different velocities.

Selection is irrespective of their charge and mass.

$$|\vec{F_m}| = |\vec{F_e}| \Rightarrow qvB = qE \Rightarrow v = \frac{E}{B}$$

If $v = \frac{E}{B}$, the charged particle will move undeflected in the electromagnetic field.

The figure shows a velocity selector whose electric field is produced by a potential difference of 150 V across the two large parallel metal plates that are 4.5 cm apart. Find the magnetic field $\vec{B}$, so that a positively charged particle having a velocity of 3.25 km/s perpendicular to the fields will pass through the plates undeflected.

Calculate the magnitude of $\vec{B}$:

$$v = \frac{E}{B} \Rightarrow B = \frac{E}{v} = \frac{V/d}{v}$$

$$= \frac{150/0.045}{3250}$$

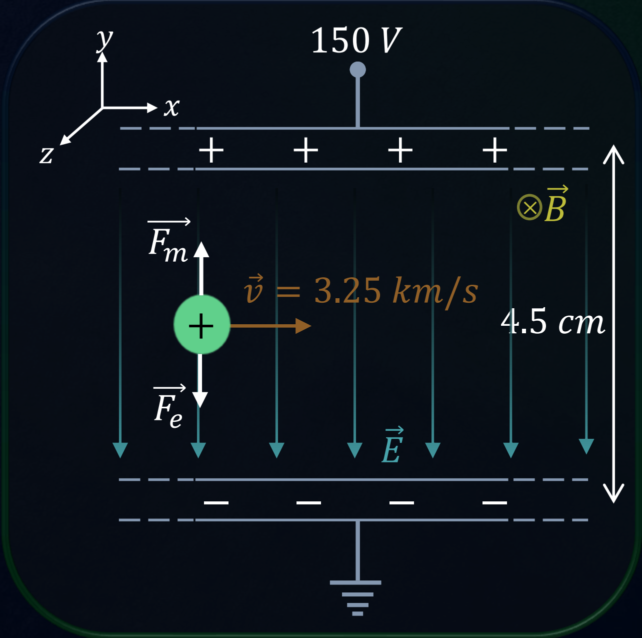
$$\Rightarrow \boxed{B = 1.02\ T}$$

Calculate the direction of $\vec{B}$:

$$\vec{F_m} \parallel \vec{v} \times \vec{B}$$

$\hat{j}$ $\quad$ $\hat{\imath}$ $\quad$ $-\hat{k}$

$$\hat{\imath} \times (-\hat{k}) = \hat{j}$$



A $1.02\ (-\hat{k})$

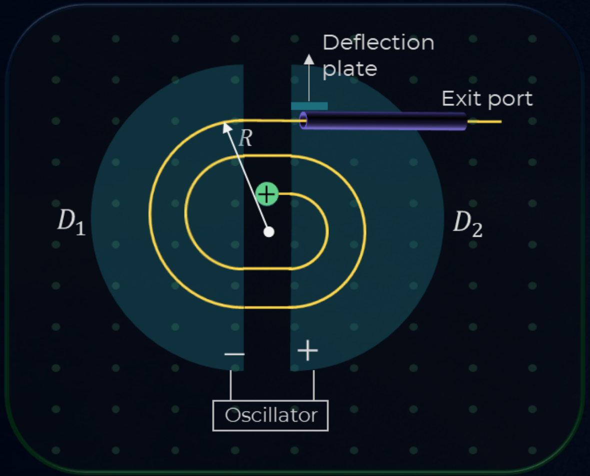
B $1.02\ \hat{k}$

C $1.20\ \hat{k}$

D $1.20\ (-\hat{k})$

Cyclotron

A machine that uses both electric and magnetic fields in combination to accelerate charged particles or ions to high energies



Cyclotron frequency

$$T = \frac{2\pi m}{qB} \Longrightarrow \boxed{f_c = \frac{qB}{2\pi m}}$$

f_c is independent of speed and radius of charged particle.

Kinetic energy imparted to charged particle

Radius of the trajectory at exit $= R$

We have, $R = \frac{mv}{qB}$

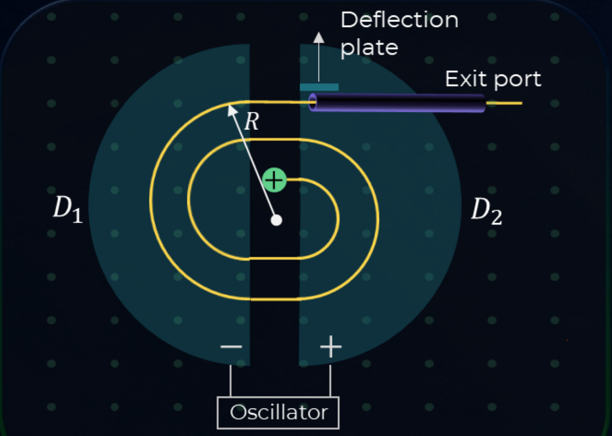
$\therefore$ Kinetic energy,

$$K = \frac{1}{2}mv^2$$

$$= \frac{m}{2}\left(\frac{qBR}{m}\right)^2$$

$$\boxed{K = \frac{q^2B^2R^2}{2m}}$$

A proton is accelerating in a cyclotron where the applied magnetic field is $2\ T$. If the potential gap is effectively $100\ kV$, then find the number of revolutions made by the proton between the "dees" to acquire kinetic energy of $20\ MeV$.



D_1 and D_2 are the "dees"

Potential gap, $V = 100\ kV$; Acquired kinetic energy, $K = 20\ MeV$

Energy acquired by the proton in one revolution,

$$E = 2 \times qV$$

$\therefore$ No. of revolutions,

$$N = \frac{K}{E} = \frac{20 \times 10^6 \times e}{2 \times e \times 10^5}$$

$$\boxed{\Rightarrow N = 100}$$

A) 100

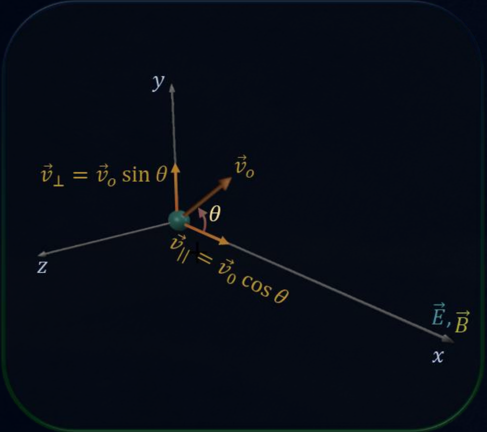
B) 150 150

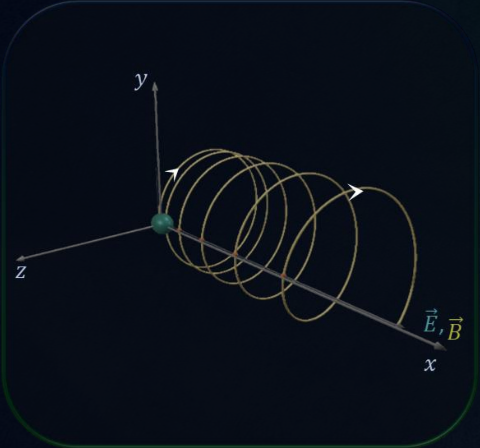
C) 200

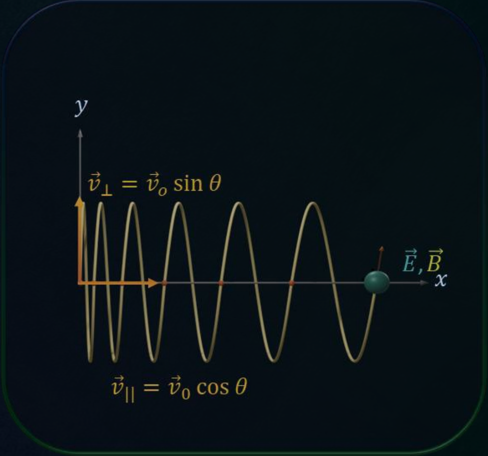
D) 300

Motion of a Charged Particle in a Uniform Electromagnetic Field

$\vec{E} \parallel \vec{B}$ and $\theta \neq 180^o, 0^o$







Along the x $axis$:

Force along x axis,

$$F_e = qE = ma_x$$

$$\Rightarrow \boxed{a_x = \frac{qE}{m}}$$

$$\therefore\ v_x = u_x + a_x t$$

$$\Rightarrow \boxed{v_x = v_o \cos\theta + a_x t}$$

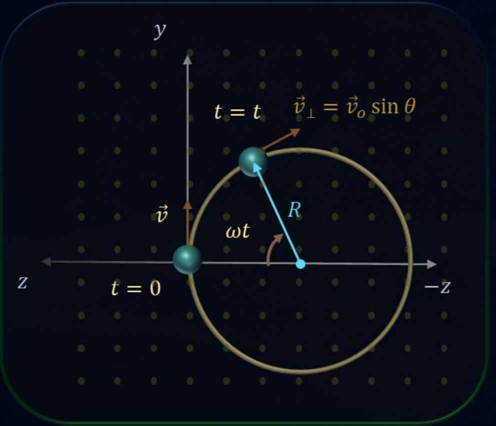
Magnetic force results in a helical motion.

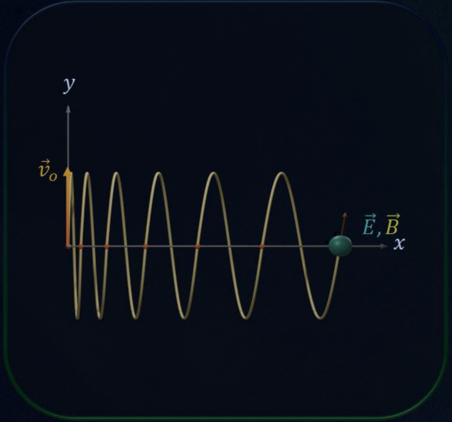
Electric force accelerates the particle along the x - direction.

$\therefore$ particle will undergo helical motion with variable pitch.

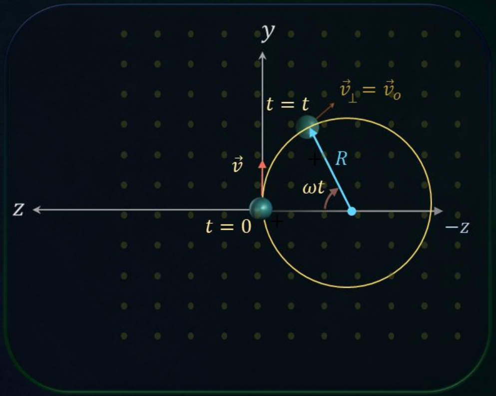
Motion of a Charged Particle in a Uniform Electromagnetic Field

$\vec{E} \parallel \vec{B}$ and $\theta \neq 180^o, 0^o$





$\vec{E} \parallel \vec{B}$ and $\theta = 90^o$



Along the x $axis$:

Force along x axis,

$$F_e = qE = ma_x$$

$$\Rightarrow \boxed{a_x = \frac{qE}{m}}$$

$$\therefore\ v_x = u_x + a_x t$$

$$\Rightarrow \boxed{v_x = a_x t}$$

In the yz $plane$:

Radius, $\boxed{R = \frac{mv_o \sin\theta}{qB}}$

Time period, $\boxed{T = \frac{2\pi m}{qB}}$

Acceleration,

$$\boxed{\vec{a}_y = \omega^2 R \sin \omega t\, (-\hat{j})}$$

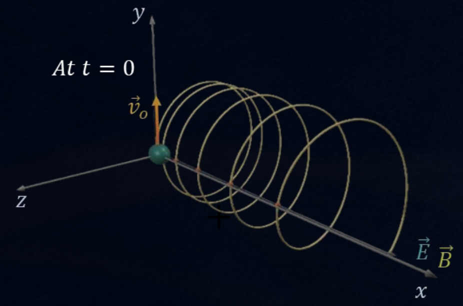
$$\boxed{\vec{a}_z = \omega^2 R \cos \omega t\, (-\hat{k})}$$

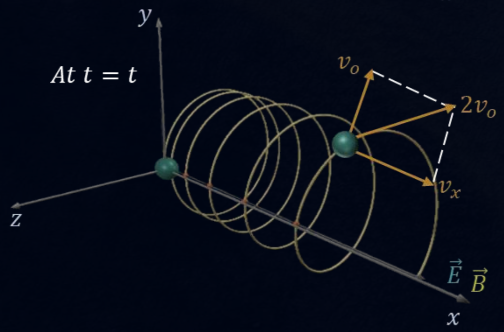
In the yz $plane$:

Radius, $\boxed{R = \frac{mv_o}{qB}}$

Time period, $\boxed{T = \frac{2\pi m}{qB}}$

A particle of mass m and charge q has an initial velocity $\vec{v} = v_o\hat{j}$. If an electric field $\vec{E} = E_o\hat{i}$ and magnetic field $\vec{B} = B_o\hat{i}$ act on the particle, its speed will double after a time





Initial velocity:

$u_x = 0 \qquad u_y = v_o$

Magnetic field do not change the magnitude of velocity.

After time t,

$$v = \sqrt{{v_x}^2 + ({v_y}^2 + {v_z}^2)}$$

$$\Rightarrow 2v_o = \sqrt{{v_x}^2 + {v_o}^2}$$

$$\Rightarrow v_x = \sqrt{3}v_o$$

But,

$$v_x = u_x + a_x t$$

$$\Rightarrow \sqrt{3}v_o = 0 + \frac{qE_o}{m}t$$

$$\Rightarrow \boxed{t = \frac{\sqrt{3}mv_o}{qE_o}}$$

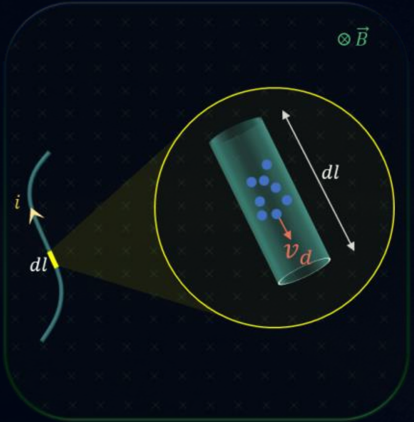
A $\frac{2mv_o}{qE_0}$

B $\frac{3mv_o}{qE_0}$

C $\frac{\sqrt{3}mv_o}{qE_0}$

D $\frac{\sqrt{2}mv_o}{qE_0}$

Magnetic Force on a Current Carrying Element



Force on one electron,

$$\overrightarrow{dF_e} = -e(\overrightarrow{v_d} \times \vec{B})$$

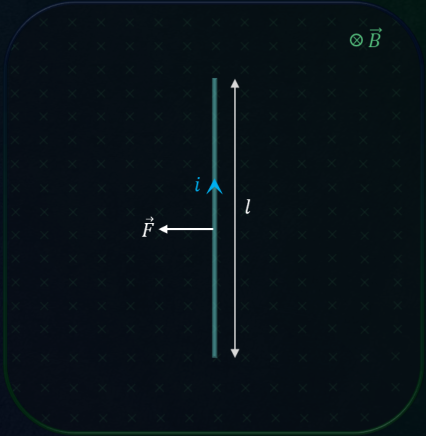
Total force on n electrons,

$$\overrightarrow{dF} = (nAdl)[-e(\overrightarrow{v_d} \times \vec{B})]$$

$$= (-neAdl)[(\overrightarrow{v_d} \times \vec{B})]$$

$$\Rightarrow \overrightarrow{dF} = i(\overrightarrow{dl} \times \vec{B})$$

$$[\because i = -neAv_d]$$



Force on a small element,

$$\overrightarrow{dF} = i(\overrightarrow{dl} \times \vec{B})$$

Integrating,

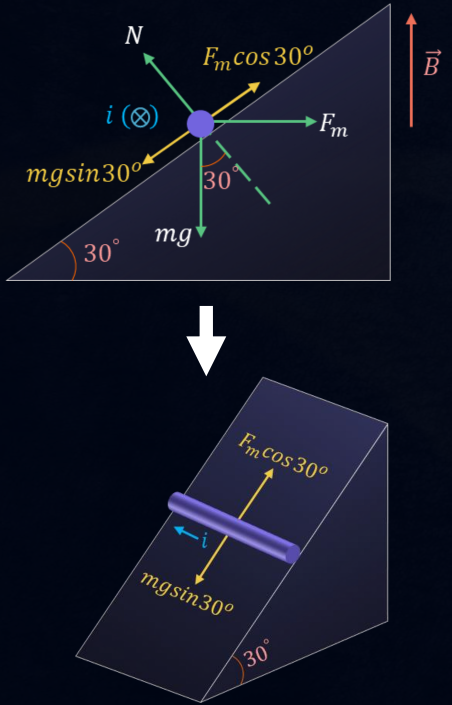
$$\vec{F} = \int \overrightarrow{dF}$$

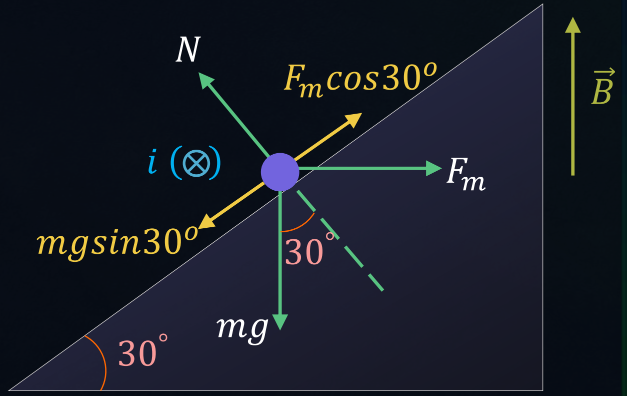
$$= \int i(\overrightarrow{dl} \times \vec{B})$$

$$\Rightarrow \boxed{\vec{F} = i(\vec{l} \times \vec{B})}$$

$[\because \vec{B}$ is uniform$]$

On a smooth inclined plane at 30° with the horizontal, a thin current carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field $\vec{B}$ of $0.15\ T$ in the vertical direction. For what value of current can the rod remain stationary? The mass per unit length of the rod is $0.30\ kg/m$. (Take $g = 9.8\ m/s^2$)





For equilibrium of the rod,

$$mgsin30^o = F_m cos30^o$$

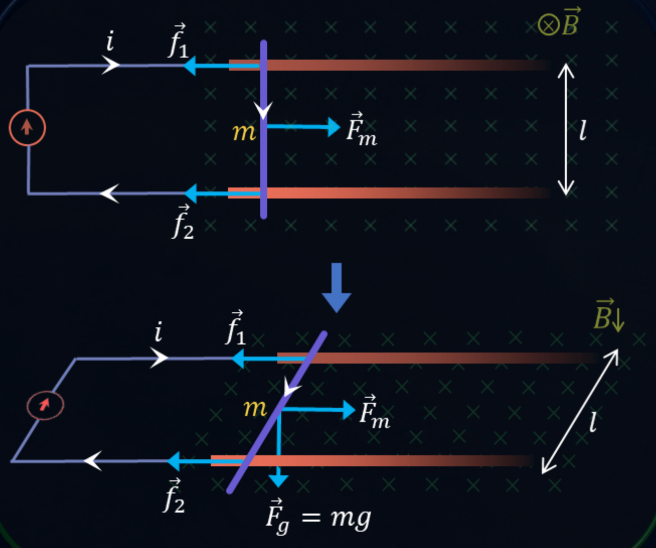
$$\Rightarrow mg \times \frac{1}{2} = ilB \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow i = \left(\frac{m}{l}\right) \times g \times \left(\frac{1}{\sqrt{3}B}\right)$$

$$= \frac{0.3 \times 9.8}{\sqrt{3} \times 0.15}$$

$$\Rightarrow \boxed{i = 11.3\ A}$$

Two long metal rails placed horizontally and parallel to each other are at a separation l. A uniform magnetic field $\vec{B}$ exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.
What should be the minimum value of μ which can prevent the wire from sliding on the rails?



Magnetic force:

$$\overrightarrow{dF_m} = i(\overrightarrow{dl} \times \vec{B})$$

$$\Rightarrow \overrightarrow{F_m} = i(\vec{l} \times \vec{B})$$

$$\Rightarrow F_m = ilB\sin 90^o$$

$$\Rightarrow F_m = ilB$$

Static friction:

$$f_1 + f_2 = f_s \leq \mu mg$$

Condition for no sliding:

$$F_m = f_s$$

$$\Rightarrow F_m \leq \mu mg$$

$$\Rightarrow ilB \leq \mu mg$$

$$\Rightarrow \mu \geq \frac{ilB}{mg} \quad \Rightarrow \boxed{\mu_{max} = \frac{ilB}{mg}}$$

A) $\frac{mg}{ilB}$

B) $\frac{mgl}{iB}$

C) $\frac{iB}{mgl}$

D) $\frac{ilB}{mg}$

?

A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then,

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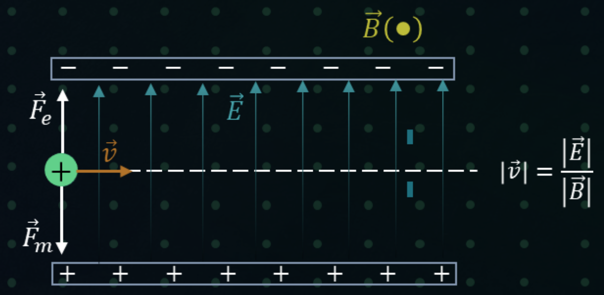
For undeflected motion of the charge particle, the required condition of velocity of the charge particle is,

$$v = \frac{E}{B}$$

The direction of velocity $\vec{v}$ should also be parallel to $(\vec{E} \times \vec{B})$. So,



$$v = \left|\frac{\vec{E} \times \vec{B}}{B^2}\right|$$

$$v = \frac{EB \sin 90^\circ}{B^2} \quad [\text{Since } \vec{E} \perp \vec{B}]$$

$$v = \frac{E}{B}$$

A $\vec{v} = \frac{\vec{B} \times \vec{E}}{E^2}$

B $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$

C $\vec{v} = \frac{\vec{B} \times \vec{E}}{B^2}$

D $\vec{v} = \frac{\vec{E} \times \vec{B}}{E^2}$

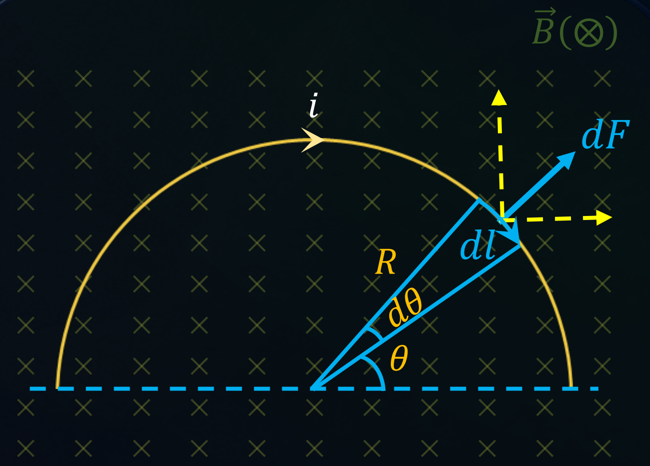
Find the magnetic force on a current carrying semi-circular wire.

Consider a small element of length dl.

The magnetic force on the element :

$$|d\vec{F}| = i|d\vec{l} \times \vec{B}| = i(Rd\theta)B$$

The horizontal component of dF from symmetrical wire elements get cancelled out in pair and hence, net force on the wire will be due to vertical component only.



So, net magnetic force on the current carrying semi-circular wire,

$$F = \int dF \sin\theta$$

$$F = \int (i(Rd\theta)B) \sin\theta$$

$$F = iRB \int_0^{\pi} \sin\theta \; d\theta$$

$$F = iRB[-\cos\theta]_0^{\pi}$$

$$\boxed{F = 2iRB}$$

Net force on the wire will be in the upward direction.

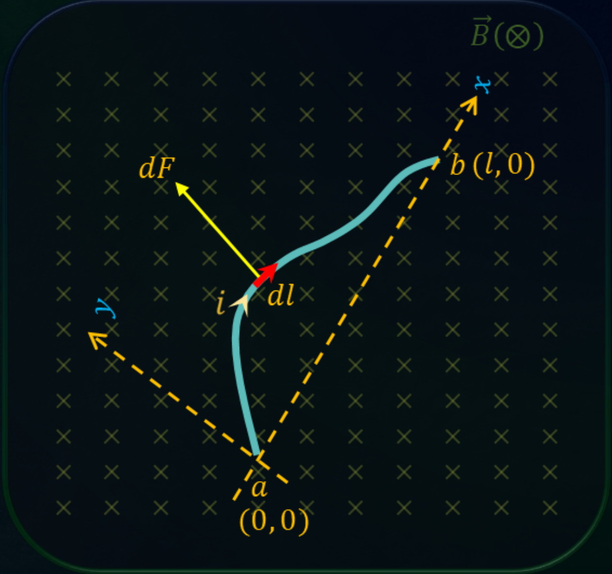
Magnetic Force on arbitrary shaped wire

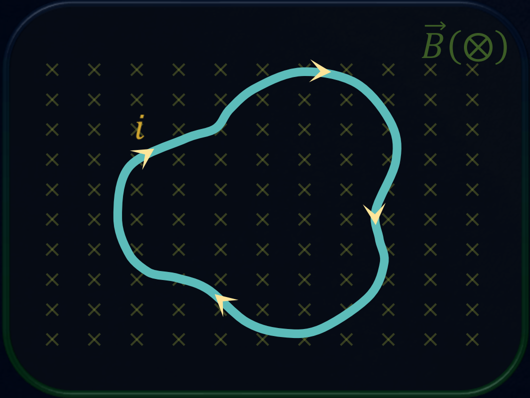
- The magnetic force on any arbitrary shaped current carrying wire is:

$$\boxed{\vec{F} = i(\vec{l} \times \vec{B})}$$

Magnetic field should be uniform

Where $\vec{l}$ is the vectorial length of the wire obtained from the coordinates of two ends of the wire and its direction is same as the current.





- Since the current carrying loop is a closed loop, the vectorial length of the wire : $\vec{l} = 0$.
- Therefore, the magnetic force on a closed current carrying loop placed in a uniform magnetic field is:

$$\boxed{\vec{F} = 0}$$

A wire, carrying a current i, is bent and kept in the x-y plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A uniform magnetic field $\vec{B}$ exists in the z-direction. Find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$.

The magnetic force on any arbitrary shaped current carrying wire placed in a uniform magnetic field is :

$$\boxed{\vec{F} = i(\vec{l} \times \vec{B})}$$

The vectorial length of the portion of the wire between $x = 0$ and $x = \lambda$ is :

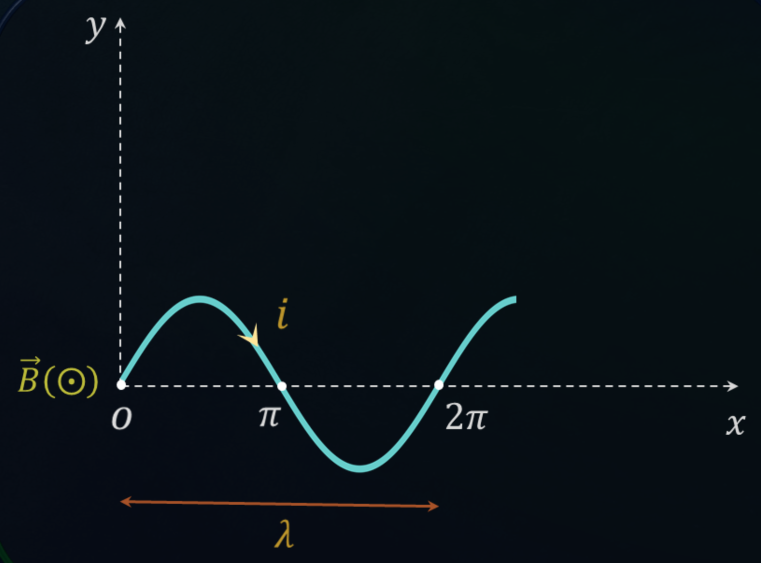
$$\vec{l} = \lambda\hat{\imath}$$

The magnetic field : $\vec{B} = B\hat{k}$

So, the magnitude of the magnetic force is :

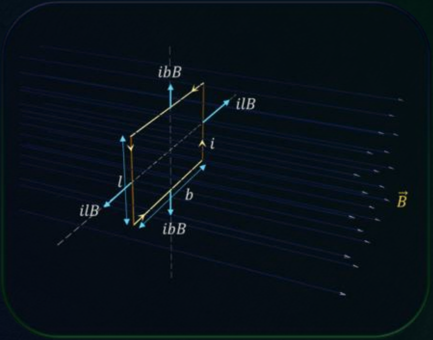
$$|\vec{F}| = |i(\vec{l} \times \vec{B})| = i|\lambda\hat{\imath} \times B\hat{k}| = i|\lambda B(-\hat{\jmath})| = i\lambda B$$

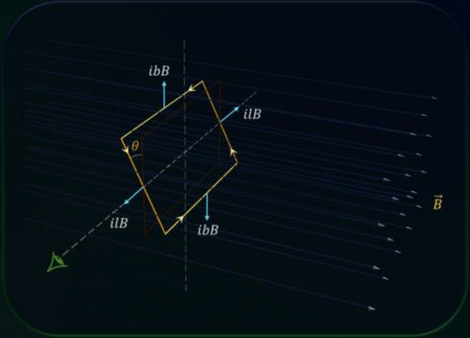
$$\boxed{|\vec{F}| = i\lambda B}$$



Current carrying loop in Magnetic Field

- If we rotate the loop in its own plane by some angle, it will remain in that new orientation and this configuration of the loop is its neutral equilibrium position.
- At this orientation of the loop, all the opposite pair of forces act along the same line, opposite in direction and equal in magnitude.
- So, net magnetic force and the net torque on the loop are zero.
- When the loop is rotated by an angle θ, as shown, the pair of forces each having magnitude ibB forms a couple, and a torque is produced in the loop.





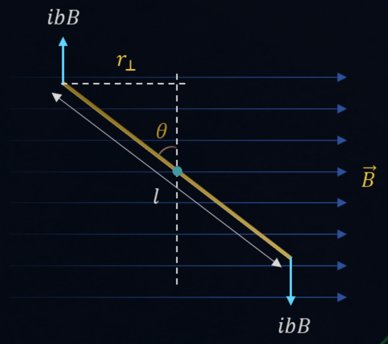
Current carrying loop in Magnetic Field

Torque on the loop :

$\tau = i(lb)B \sin\theta$

$\tau = iAB \sin\theta$

$\boxed{\vec{\tau} = i\vec{A} \times \vec{B}}$

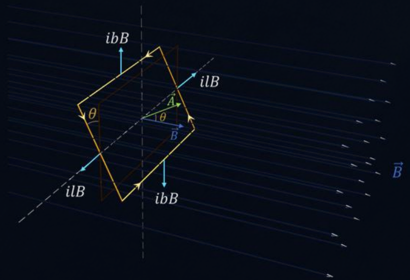


Magnetic dipole moment of the current carrying loop :

$\boxed{\vec{\mu} = i\vec{A}}$

Torque on the loop :

$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$



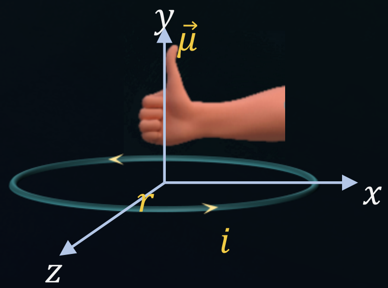
Magnetic dipole moment of the current carrying loop :

Magnitude of magnetic dipole moment :

$$\mu = iA = i\pi r^2$$

Right hand thumb rule gives the direction of magnetic dipole moment along y-axis.

Magnetic dipole moment : $\boxed{\vec{\mu} = i\pi r^2 \hat{j}}$



Find the magnitude of magnetic moment of the current carrying loop $ABCDEFA$. Each side of the loop is $10\ cm$ long and current in the loop is $i = 2\ A$.

Magnitude of magnetic moment of the loop $ABEFA$
$=$ Magnitude of magnetic moment of the loop $BCDEB$

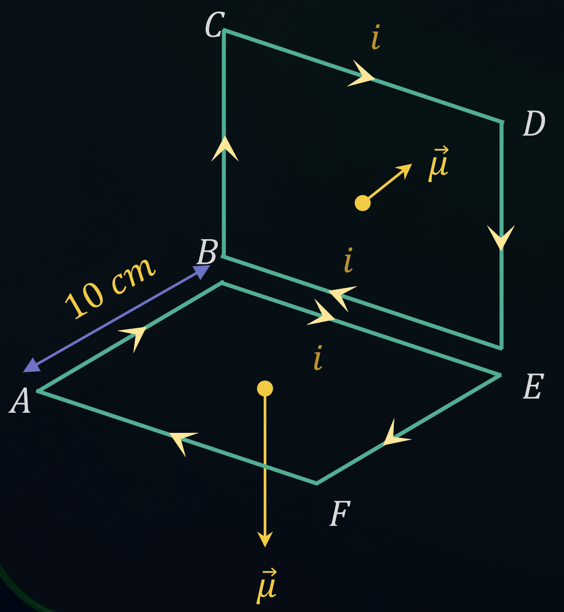
Magnetic dipole moment each loop :

$$\mu = il^2 = 2 \times \left(\frac{10}{100}\right)^2 = \frac{2}{100} = \frac{1}{50}\ Am^2$$

Net magnetic dipole moment :

$$\mu_{net} = \sqrt{\mu^2 + \mu^2} = \sqrt{2}\mu = \frac{\sqrt{2}}{50}\ Am^2$$

$$\boxed{\mu_{net} = \frac{1}{25\sqrt{2}}\ Am^2}$$



A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

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Area of the loop is :

Area of two circles of radius $\frac{a}{2}$ + Area of a square of side length a

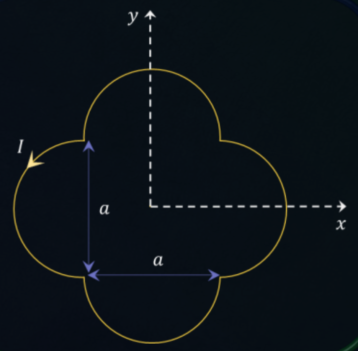
$$A = 2 \times \pi \left(\frac{a}{2}\right)^2 + a^2$$

$$A = \frac{\pi a^2}{2} + a^2 = \left(\frac{\pi}{2} + 1\right) a^2$$

Magnitude of the magnetic moment of the loop is :

$$\mu = IA = \left(\frac{\pi}{2} + 1\right) a^2 I$$

Direction of current suggests the direction of magnetic moment should be along $\hat{k}$.



A) $a^2 I\,\hat{k}$

B) $-\left(\frac{\pi}{2} + 1\right) a^2 I\,\hat{k}$

C) $\left(\frac{\pi}{2} + 1\right) a^2 I\,\hat{k}$

D) $-(2\pi + 1)a^2 I\,\hat{k}$

$$\boxed{\vec{\mu} = \left(\frac{\pi}{2} + 1\right) a^2 I\,\hat{k}}$$

A uniform, constant magnetic field $\vec{B}$ is directed at an angle of 45^o to the x-axis in the xy-plane. $PQRS$ is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.
What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field ?

- Magnetic field :

$$\vec{B} = B\cos 45°\,\hat{\imath} + B\sin 45°\,\hat{\jmath}$$

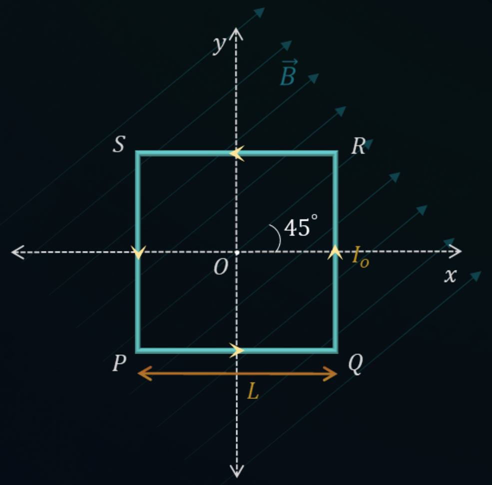
$$\vec{B} = \frac{B}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$$

- Magnetic moment :

$$\vec{\mu} = I_0 L^2 \odot = I_0 L^2 \hat{k}$$

- Torque : $\vec{\tau} = \vec{\mu} \times \vec{B} = \dfrac{I_0 L^2 B}{\sqrt{2}} \hat{k} \times (\hat{\imath} + \hat{\jmath})$

$$\boxed{\vec{\tau} = \frac{I_0 L^2 B}{\sqrt{2}}(\hat{\jmath} - \hat{\imath})}$$



A rectangular coil of size $3.0\ cm \times 4.0\ cm$ and having 100 turns, is pivoted about the z-axis as shown. The coil carries an electric current of $2.0\ A$ and a magnetic field of $1.0\ T$ is present along the y-axis, Find the torque acting on the coil if the side in the x-y plane makes an angle $\theta = 37^o$ with the x-axis.

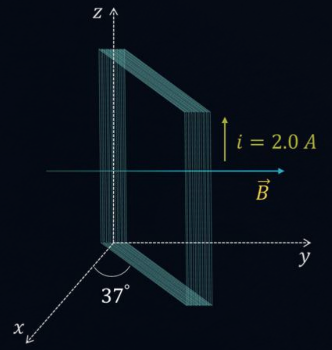
Solution :

Magnitude of the magnetic moment :

$$\mu = niA$$

$$\mu = 100 \times 2 \times \left(\frac{3}{100}\right) \times \left(\frac{4}{100}\right)\ Am^2$$

$$\mu = \frac{6}{25}\ Am^2$$



The magnetic moment in vector form :

$$\vec{\mu} = \mu \cos 53°\,\hat{\imath} - \mu \sin 53°\,\hat{\jmath}$$

$$\vec{\mu} = \frac{3\mu}{5}\hat{\imath} - \frac{4\mu}{5}\hat{\jmath}$$

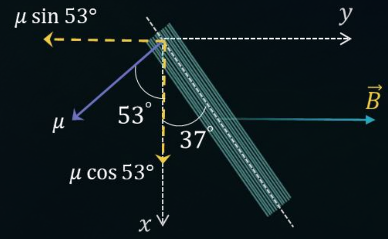
Magnetic field : $\vec{B} = |\vec{B}|\hat{\jmath} = 1\,\hat{\jmath}$

The torque is :

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \left(\frac{3\mu}{5}\hat{\imath} - \frac{4\mu}{5}\hat{\jmath}\right) \times 1\hat{\jmath}$$

$$\vec{\tau} = \frac{3\mu}{5}\hat{k} = \frac{3}{5} \times \frac{6}{25}\ \hat{k}$$

$$|\vec{\tau}| = \frac{18}{125} = 0.14\ Nm$$



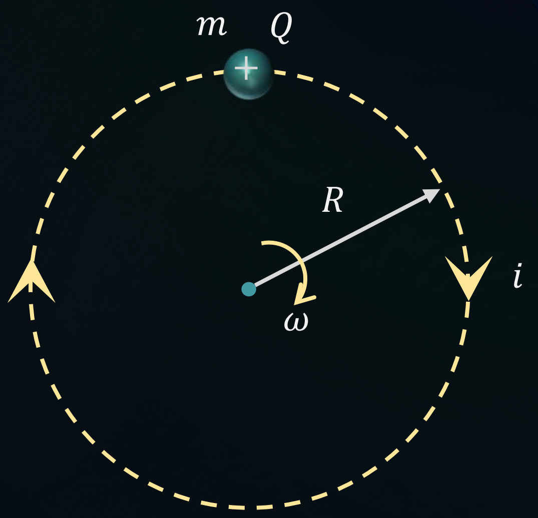
x-y plane (Top view)

$$\boxed{|\vec{\tau}| = 0.14\ Nm}$$

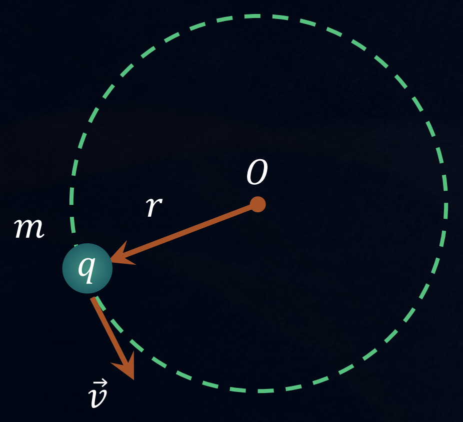
Revolving Charged Particles

- Revolution of charged particle constitutes a current.
- Current : $i = \frac{Q}{T} = \frac{Q\omega}{2\pi}$
- Magnetic moment : $\vec{\mu} = i\vec{A} = \left(\frac{Q\omega}{2\pi}\right)\pi R^2 \otimes$
- Angular momentum : $\vec{L} = mR^2\omega \otimes$

$$\boxed{\vec{\mu} = \frac{Q}{2m}\vec{L}}$$



Gyromagnetic Ratio



Magnetic Moment of rotating charged particle

$$M = \frac{qvr}{2}$$

Angular Momentum of rotating charged particle

$$L = mvr$$

Gyromagnetic Ratio

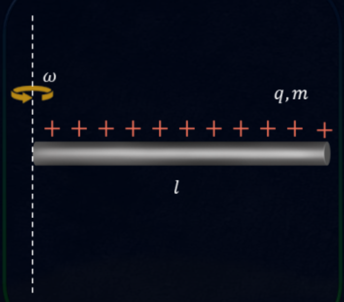
$$\gamma = \frac{M}{L} = \frac{q}{2m} = \frac{total\ charge}{2 \times\ total\ mass}$$

Note:

This ratio is a constant for all rotating uniformly charged bodies.

This result is used for calculating γ for various charge configurations.

Magnetic Moment of Rotating Charged shapes



For any rotating charge distribution

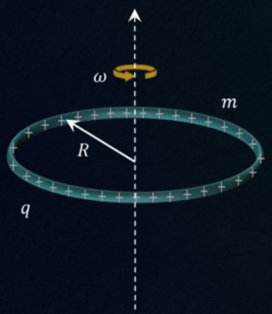
$$\frac{M}{L} = \frac{q}{2m} = constant$$

$$\Rightarrow M = L\frac{q}{2m}$$

For given configuration,

$$L = \frac{ml^2}{3}\omega$$

$$\boxed{M = \frac{q\omega L^2}{6}}$$



For any rotating charge distribution

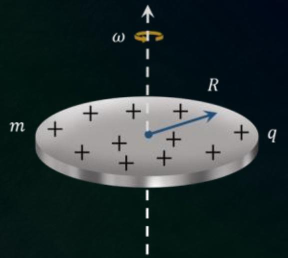
$$\frac{M}{L} = \frac{q}{2m} = constant$$

$$\Rightarrow M = L\frac{q}{2m}$$

For given configuration,

$$L = mR^2\omega$$

$$\boxed{M = \frac{q\omega L^2}{2}}$$



For any rotating charge distribution

$$\frac{M}{L} = \frac{q}{2m} = constant$$

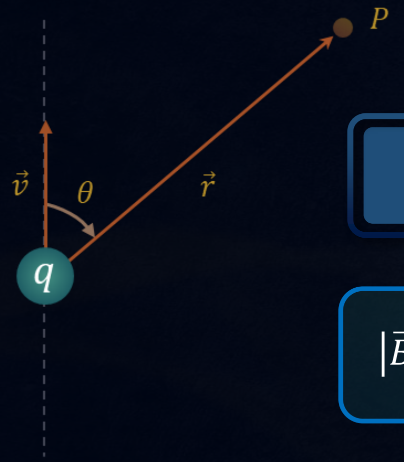
$$\Rightarrow M = L\frac{q}{2m}$$

For given configuration,

$$L = \frac{mR^2}{2}\omega$$

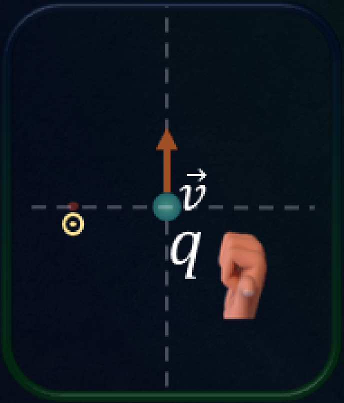
$$\boxed{M = \frac{q\omega L^2}{4}}$$

Magnetic Field due to Moving Charge



Scalar Form

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$



Vector Form

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$$

μ_0 is magnetic permeability of free space .

For free space, $\frac{\mu_0}{4\pi} = 10^{-7}\frac{T-m}{A}$

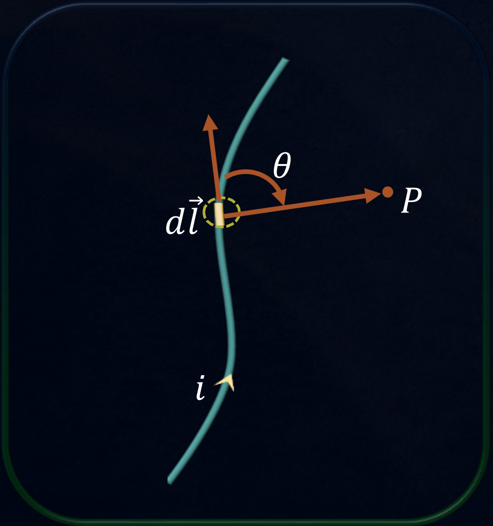
Direction of Magnetic Field:

- If q is positive $\longrightarrow$ $(\hat{v}\times\hat{r})$
- If q is negative $\longrightarrow$ opposite of $(\hat{v}\times\hat{r})$

BIOT-SAVART LAW

Biot-Savart's law is the fundamental law of the magnetics.

It gives the magnetic field by a small current element and is based on the experimental facts.



The magnetic field due to current element

Scalar Form:

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{q\ idl\sin\theta}{r^2}$$

Vector Form:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i(d\vec{l}\times\vec{r})}{r^3}$$

Note:

Direction of $\vec{B}$ is given by $(d\vec{l}\times\vec{r})$ using Right hand rule.

Steps to calculate Magnetic Field

Step: 1

Find out magnetic field due to small current element of wire

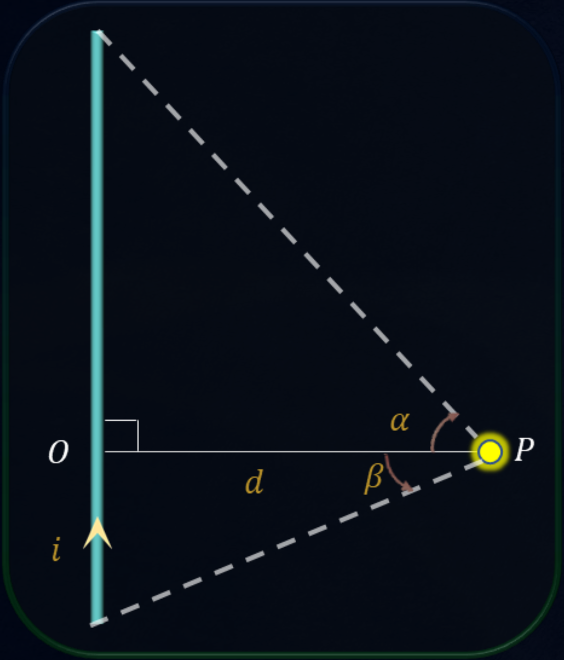
Step: 2

Convert the expression of the magnetic field into one single variable

Step: 3

Finally integrate the expression

Magnetic field due to Straight Finite Current Carrying Wire



The value of magnetic field, B is given by:

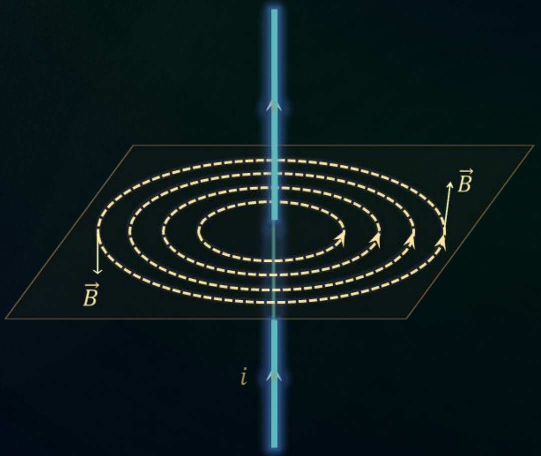
$$B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)$$

Note:

Only the magnitude of α and β should be put in the above formula.

Magnetic field lines of Force due to Straight Current Wire

Right hand thumb rule



Concentric circles

Consider a square loop $ABDC$ with edge-length a. The resistance of the wire ABD is r and that of ACD is $2r$. Find the magnetic field B at the centre of the loop assuming uniform wires.

Solution:

From current division rule

$$i_{ABD} = i \times \frac{2r}{(2r+r)} = \frac{2}{3} i$$

$$i_{ACD} = i \times \frac{r}{(2r+r)} = \frac{1}{3} i$$

$$\vec{B}_O = \vec{B}_{AB} + \vec{B}_{BD} + \vec{B}_{AC} + \vec{B}_{CD}$$

$$\vec{B}_{AB} = \vec{B}_{BD} = B_{AB} \otimes$$

$$\vec{B}_{AC} = \vec{B}_{CD} = B_{AC} \odot$$

$$B_O = 2(B_{AB} - B_{AC})$$

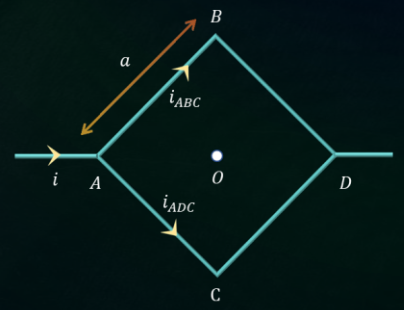
$$\boxed{B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)}$$

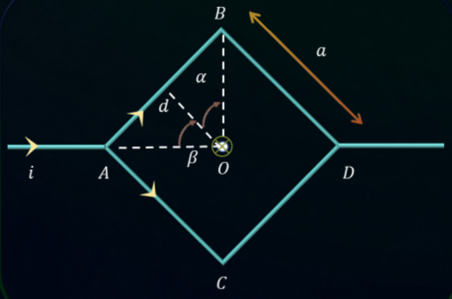
$$\alpha = 45^\circ; \beta = 45^\circ; d = \frac{a}{2}$$

$$B_{AB} = \frac{\mu_0}{4\pi}\frac{\frac{2i}{3}}{\frac{a}{2}}(\sin 45^\circ + \sin 45^\circ) = \frac{4\sqrt{2}}{3}\frac{\mu_0}{4\pi}\frac{i}{a}$$

$$B_{AC} = \frac{\mu_0}{4\pi}\frac{\frac{i}{3}}{\frac{a}{2}}(\sin 45^\circ + \sin 45^\circ) = \frac{2\sqrt{2}}{3}\frac{\mu_0}{4\pi}\frac{i}{a}$$

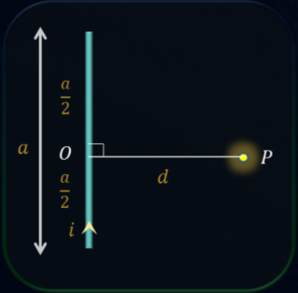
$$\boxed{B_O = \frac{\sqrt{2}}{3}\frac{\mu_0}{\pi}\frac{i}{a}}$$





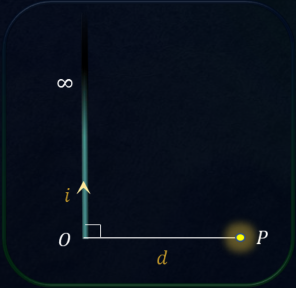
Special Cases

On perpendicular bisector



$$B = \frac{\mu_0 i a}{(2\pi d)\sqrt{a + 4d^2}}$$

Semi-infinite wire

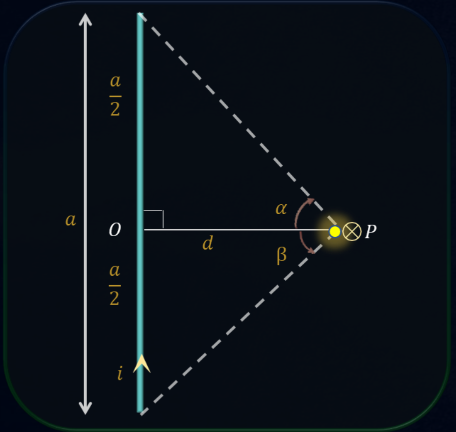


$$B = \frac{\mu_0}{4\pi}\frac{i}{d}$$

Infinite wire

$$B = \frac{\mu_0}{2\pi}\frac{i}{d}$$

On Perpendicular Bisector



The B due to finite wire at perpendicular distance, d from the wire is:

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)}$$

For given case

P is on the perpendicular bisector

$\alpha = \beta = \theta$ (assume)

For given configuration,

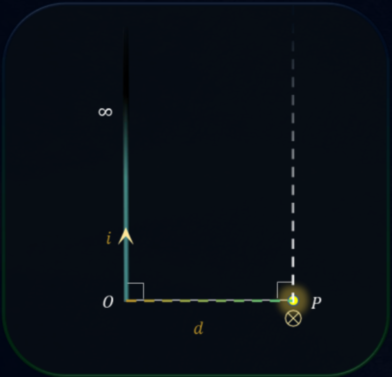
$$\sin\theta = \frac{a}{2\sqrt{d^2 + \frac{a^2}{4}}} = \frac{a}{\sqrt{4d^2 + a^2}}$$

Substitute the value of $sin\,\theta$, we get

$$\boxed{B = \frac{\mu_0 i a}{(2\pi d)\sqrt{a + 4d^2}}}$$

Semi-Infinite wire and Infinite wire

Semi-Infinite



The B due to a finite wire at a distance d is:

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)}$$

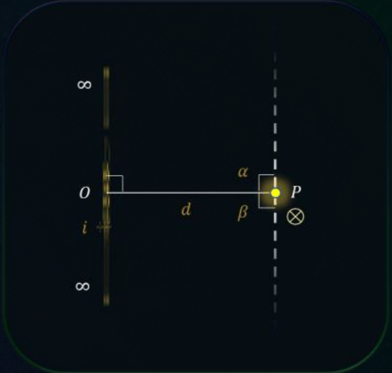
For semi-infinite wire:

$$\alpha = \frac{\pi}{2}; \beta = 0$$

Therefore,

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{i}{d}}$$

Infinite



For infinite wire:

$$\alpha = \frac{\pi}{2}; \beta = \frac{\pi}{2}; sin\frac{\pi}{2} = 1$$

Therefore, the magnetic field at point P :

$$B = \frac{\mu_0}{4\pi}\frac{i}{d}[1+1]$$

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{i}{d}}$$

A long, straight wire carries a current i. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire is projected towards it with a velocity $\vec{v}$. Find the minimum separation between the wire and the particle.

Particle move in xy plane

At $t = 0s$ $\quad \vec{v} = v(-\hat{\imath}) \quad x = x_0$

At $t = t^*$ $\quad \vec{v} = v(-\hat{\jmath}) \quad x = x_{min}$

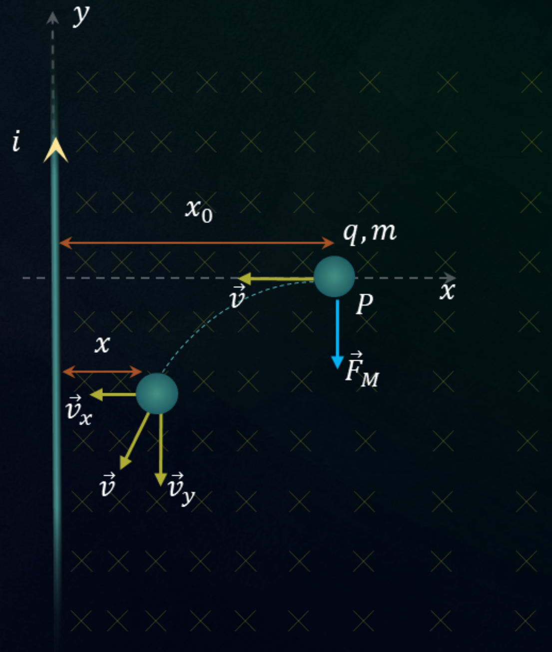
Magnetic field and velocity at any x will be:

$$\vec{B}_x = \frac{\mu_0 i}{2\pi x}(-\hat{k}) \qquad \vec{v} = -v_x\hat{\imath} - v_y\hat{\jmath}$$

Therefore, the magnetic force on the charged particle at the instant of time t will be

$$\vec{F}_M = q\vec{v} \times \vec{B} = q\left[-(v_x\hat{\imath} + v_y\hat{\jmath}) \times \frac{\mu_0 i}{2\pi x}(-\hat{k})\right]$$

$$\boxed{\vec{F}_M = \frac{\mu_0 iq}{2\pi x}[v_y\hat{\imath} - v_x\hat{\jmath}]}$$



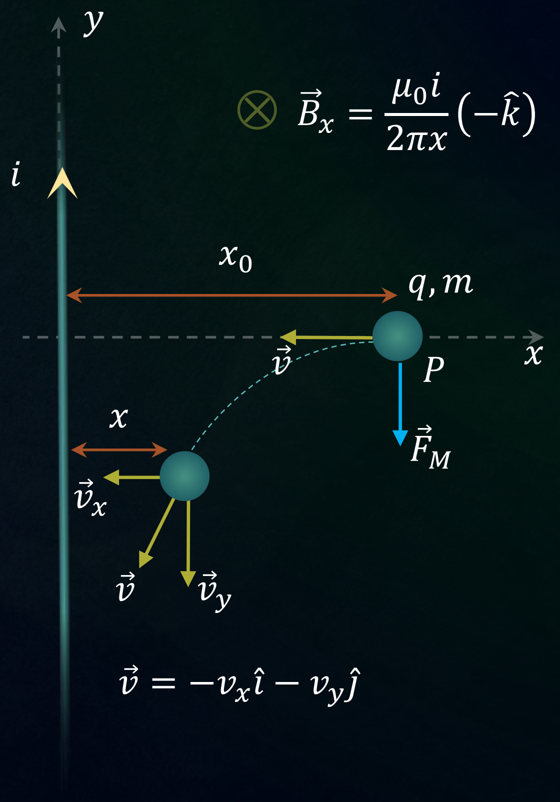
$$\vec{F}_M = \frac{\mu_0 i q}{2\pi x}[v_y\hat{i} - v_x\hat{j}]$$

$$a_x = v_x\frac{dv_x}{dx} = \frac{\mu_0 i q}{2\pi m}\left(\frac{v_y}{x}\right) \qquad a_y = v_y\frac{dv_y}{dy} = -\frac{\mu_0 i q}{2\pi m}\left(\frac{v_x}{x}\right)$$

$$v^2 = v_x^2 + v_y^2 \quad \Rightarrow \quad \boxed{v_x dv_x = -v_y dv_y}$$

$$\int_0^v dv_y = -\frac{\mu_0 i q}{2\pi m}\left(\int_{x_0}^{x_{min}}\frac{dx}{x}\right)$$

$$\boxed{x_{min} = x_0 e^{-\frac{2\pi m v}{\mu_0 q i}}}$$



Force between two parallel current-carrying straight conductors

Magnetic field at a distance d from current carrying wire i_1 :

$$\overrightarrow{B_1}(d) = \frac{\mu_0 i_1}{2\pi d} \otimes$$

Force on element dl in the current carrying wire i_2 :

$$d\vec{F} = i_2 d\vec{l} \times \overrightarrow{B_1}$$

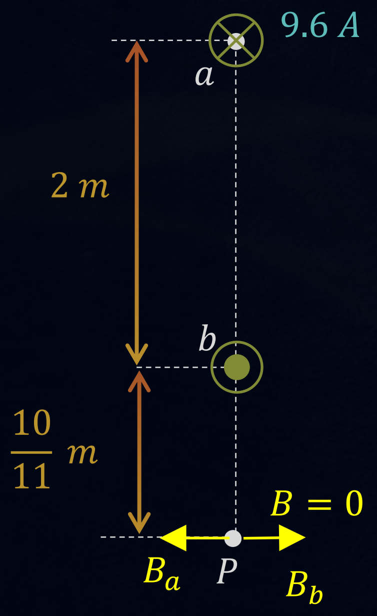
$$dF = (i_2 dl)\left(\frac{\mu_0 i_1}{2\pi d}\right)$$

$$\boxed{\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}}$$ ➜ Interactive magnetic force between two current carrying parallel wire per unit length

Two long, straight wires a and b are $2.0\ m$ apart, perpendicular to the plane of the paper. The wire a carries a current of $9.6\ A$ directed into the plane of the figure. The magnetic field at the point P at a distance of $\frac{10}{11}\ m$ from the wire b is zero.

(a) The magnitude and direction of the current in b.

(b) The force per unit length on the wire b.



(a) the magnitude and direction of the current in b

$$\vec{B}_P = \vec{B}_a + \vec{B}_b = 0 \Rightarrow \vec{B}_a = -\vec{B}_b$$

$$|\vec{B}_a| = |\vec{B}_b| \Rightarrow \frac{\mu_0 i_a}{2\pi\left(2+\frac{10}{11}\right)} = \frac{\mu_0 i_b}{2\pi \frac{10}{11}} \Rightarrow \frac{11}{32} i_a = \frac{11}{10} i_b$$

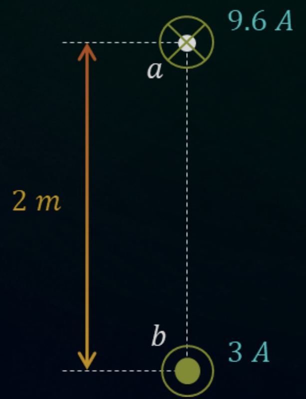
$$\boxed{i_b = 3\ A \ \odot}$$

(b) The force per unit length on the wire b.

$$\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\frac{dF}{dl} = \frac{4\pi\, 10^{-7}}{2\pi(2)}(9.6 \times 3)$$

$$\boxed{\frac{dF}{dl} \approx 2.9 \times 10^{-6}\ N/m}$$



Two long wires, carrying currents i_1 and i_2 are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length dl of the second wire situated at a distance l from the first wire.

The magnetic field at the location of dl due to the first wire is,

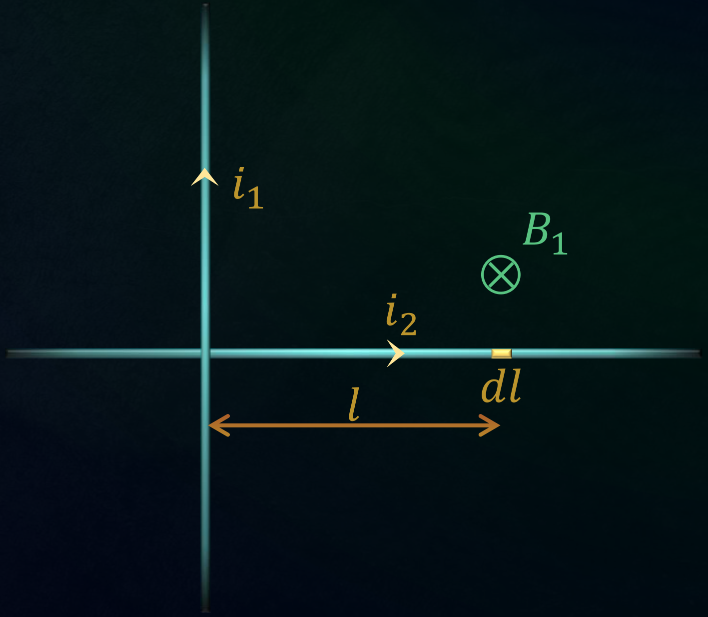
$$\overrightarrow{B_1}(l) = \frac{\mu_0 i_1}{2\pi l}$$

Therefore, the magnetic force on a small length dl of the second wire situated at a distance l from the first wire is given by,

$$d\vec{F} = i_2 d\vec{l} \times \overrightarrow{B_1}$$

$$dF = (i_2 dl)\left(\frac{\mu_0 i_1}{2\pi l}\right)$$

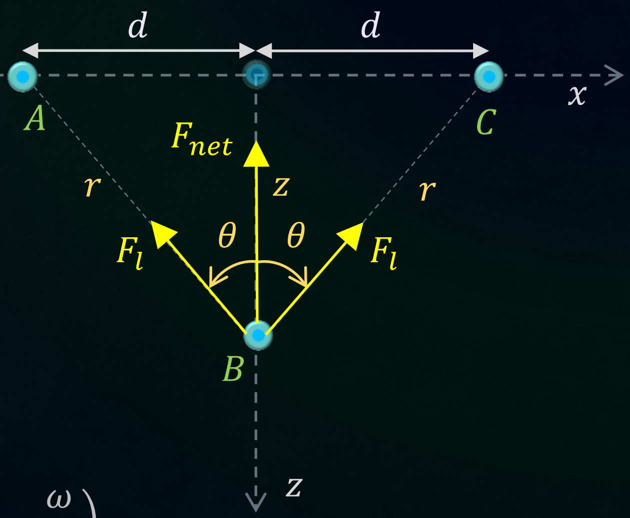
$$\boxed{dF = \frac{\mu_0 i_1 i_2 dl}{2\pi l}}$$



(b) If the central wire is displaced along the z direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wire is λ, find the frequency of oscillation.

Magnetic force between two current carrying parallel wire per unit length (F_l)

$$F_l = \frac{dF}{dl} = \frac{\mu_0 i^2}{2\pi r}$$



Net force on wire B is (F_{net}) $\quad (\therefore F_{net} = 2F_l \cos\theta)$

$$F_{net} = \frac{\mu_0 i^2}{\pi(d^2+z^2)} z$$ Restoring in nature

$(z \ll d \quad \Rightarrow \quad d^2 + z^2 \approx d^2)$

$$\boxed{F_{net} \approx \frac{\mu_0 i^2}{\pi d^2} z}$$

$$dF = F_{net}(dl) = dm\, a = \lambda\, dl\, a$$

$$a = \frac{\mu_0 i^2}{\pi \lambda d^2} z \quad (\therefore a = -\omega^2 z)$$

($-Ve$ indicate restoring)

$$\omega = \frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} \qquad \left(\therefore f = \frac{\omega}{2\pi}\right)$$

$$\boxed{f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}}$$

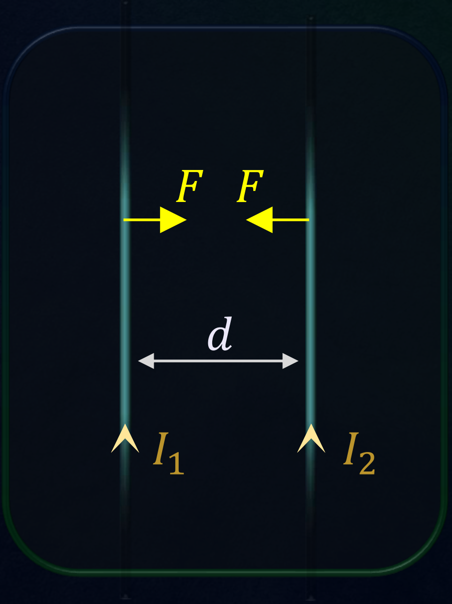
Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is

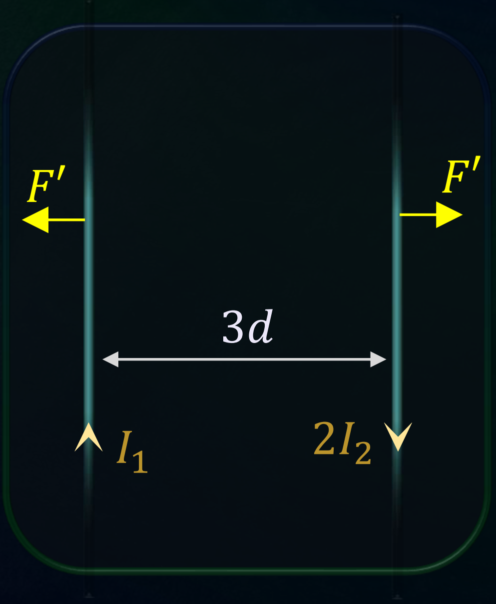
Initially $F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} L$

Finally $F' = \frac{\mu_0}{2\pi} \frac{I_1(-2I_2)}{3d} L$

$\therefore \frac{F'}{F} = -\frac{2}{3}$

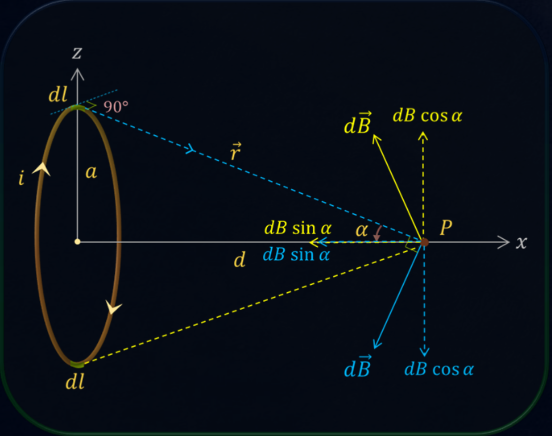
$$\boxed{F' = -\frac{2F}{3}}$$





Magnetic Field due to a Circular Loop of Current

- The current in the coil is in clockwise direction.
- The plane of the screen is axial plane of the circular loop.
- At the top most point of the loop, current goes into the axial plane.
- At the bottom most point of the loop, current comes out of the axial plane.



- Magnetic field at P due to an element dl :

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin 90^\circ}{r^2}$$

$$\boxed{dB = \left(\frac{\mu_0 i(dl)}{4\pi r^2}\right)}$$

- Net magnetic field at P :

$$\boxed{B = \int dB\sin\alpha}$$

- Due to circular symmetry of the coil, magnetic field at P is,

$$B = \int dB\sin\alpha$$

$$B = \int \left(\frac{\mu_0 i(dl)}{4\pi r^2}\right)\left(\frac{a}{r}\right)$$

$$B = \frac{\mu_0 i a^2}{2r^3}$$

$$\boxed{B = \frac{\mu_0 i a^2}{2(d^2+a^2)^{3/2}}}$$

Magnetic field at the centre of current carrying circular loop:

$$B = \frac{\mu_0 i a^2}{2(d^2+a^2)^{3/2}} \xrightarrow{d\,=\,0} \boxed{B = \frac{\mu_0 i}{2a}}$$

Magnetic Field due to a Circular Loop of Current

The magnitude of magnetic field at an axial point far off the centre of a uniformly charged circular loop :

$$\boxed{B = \frac{\mu_0 i a^2}{2d^3}} \rightarrow \boxed{B = \frac{2\left(\frac{\mu_0}{4\pi}\right) i(\pi a^2)}{d^3}}$$

The magnitude of electric field at an axial point far off the centre of a uniformly charged circular loop :

$$\boxed{E = \frac{2Kp}{d^3}}$$

"$K = \frac{1}{4\pi\varepsilon_0}$" of electric field $\equiv$ $\frac{\mu_0}{4\pi}$ of magnetic field

p = Electric dipole moment $\equiv$ $i(\pi a^2)$ = Magnetic dipole moment of the circular loop

A circular coil of 200 turns has a radius of $10\ cm$ and carries a current of $2.0\ A$. Find the distance d from the centre along the axis of the coil where the field $\vec{B}$ drop to half its value at the centre. $(\sqrt[3]{4} = 1.5874\ ...)$

The field at point P due to the coil: $B_p = \dfrac{\mu_0 i N a^2}{2(d^2 + a^2)^{3/2}}$

The field at the centre of the coil: $B_c = \dfrac{\mu_0 N i}{2a}$

According to question :

$$B_p = \frac{B_C}{2}$$

$$\frac{a^2}{(d^2 + a^2)^{3/2}} = \frac{1}{2a}$$

$$(d^2 + a^2)^3 = 4a^6$$

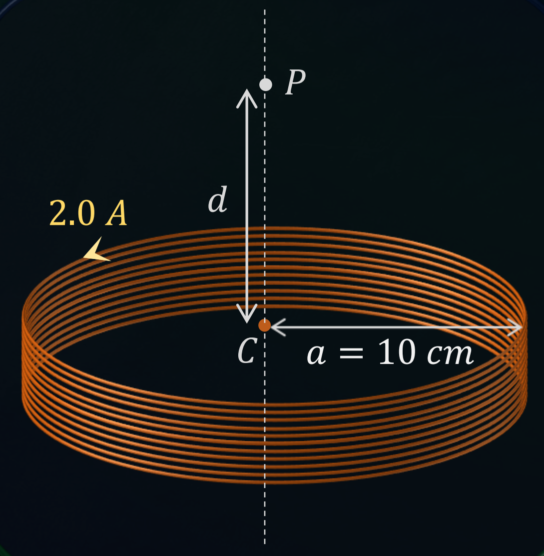
$$(d^2 + a^2) = \sqrt[3]{4}a^2$$

$$d^2 = (\sqrt[3]{4} - 1)a^2$$

$$d^2 = (1.5874 - 1) \times 100\ \ cm^2$$

$$d^2 = 58.74\ \ cm^2$$

$$\boxed{d = 7.66\ \ cm}$$



Two circular coils of radii $5.0\ cm$ and $10\ cm$ carry equal currents of $2.0\ A$. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centres coincide. Find the magnitude of the magnetic field $\vec{B}$ at the common centre of the coils if the currents in the coils are in the same sense.

Solution :

Magnetic field at the centre of a coil of radius a having N turns and i current is :

$$B = \frac{\mu_0 N i}{2a}$$

The sense of current through the coil gives the direction of magnetic field along $-ve\ z$-axis i.e., along $-\hat{k}$ for both the coils.

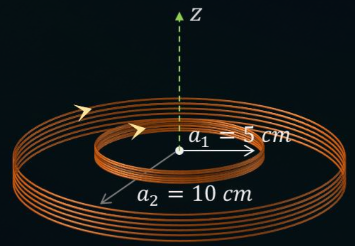
Magnetic field due to coil 1 :

$$\vec{B}_1 = \frac{\mu_0 n_1 i_1}{2a_1}(-\hat{k})$$

Magnetic field due to coil 2 :

$$\vec{B}_2 = \frac{\mu_0 n_2 i_2}{2a_2}(-\hat{k})$$

Net magnetic field at the centre :



$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B} = \frac{\mu_0 n i}{2a}(-\hat{k}) + \frac{\mu_0 (2n) i}{2(2a)}(-\hat{k})$$

$$\vec{B} = \frac{\mu_0 n i}{2a}(-\hat{k}) + \frac{\mu_0 (2n) i}{2(2a)}(-\hat{k})$$

$$\vec{B} = 2 \times \frac{\mu_0 n i}{2a}(-\hat{k})$$

$$\vec{B} = \frac{\mu_0 n i}{a}(-\hat{k})$$

$$\vec{B} = \frac{(4\pi \times 10^{-7}) \times 50 \times 2}{5 \times 10^{-2}}(-\hat{k})$$

$$\boxed{\vec{B} = 8\pi \times 10^{-4}(-\hat{k})\ \ Tesla}$$

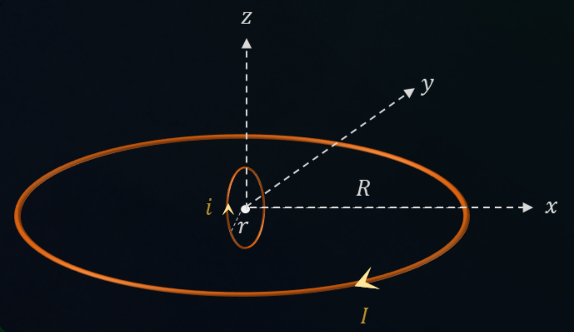
A circular loop of radius R carries a current I. Another circular loop of radius $r\ (<< R)$ carries a current i and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

The radius of the smaller loop is so small compared to bigger loop that the magnetic field through the smaller loop due to the bigger loop is assumed as uniform.

Torque acting on the smaller loop :

$$\vec{\tau} = \vec{\mu} \times \vec{B} = [i(\pi r^2)(-\hat{\imath})] \times \left[\frac{\mu_0 I}{2R}(-\hat{k})\right]$$

$$\boxed{\vec{\tau} = \frac{\mu_0 \pi i I r^2}{2R}(-\hat{\jmath})}$$

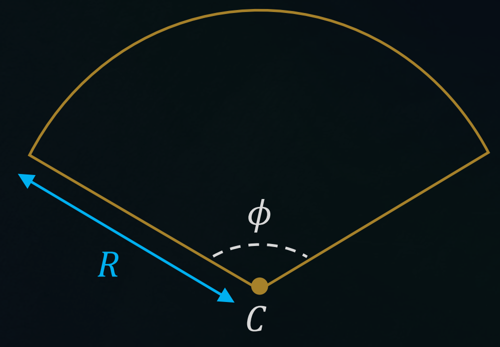


Magnetic Field due to an Arc at its Centre

Magnetic field at the centre of a current carrying circular loop :

$$\boxed{B = \frac{\mu_0 i}{2R}}$$

A circle makes an angle 2π at its centre.



Apply unitary method :

$$\theta = 2\pi \longrightarrow B = \frac{\mu_0 i}{2R}$$

$$\theta = 1 \longrightarrow B = \frac{1}{2\pi}\left(\frac{\mu_0 i}{2R}\right)$$

$$\theta = \phi \longrightarrow B = \frac{\phi}{2\pi}\left(\frac{\mu_0 i}{2R}\right)$$

$$\boxed{B_C = \frac{\phi}{2\pi}\left(\frac{\mu_0 i}{2R}\right)}$$

Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field $\vec{B}$ at the centre O.

The point O lies on the straight line extending from wire DA and CB.

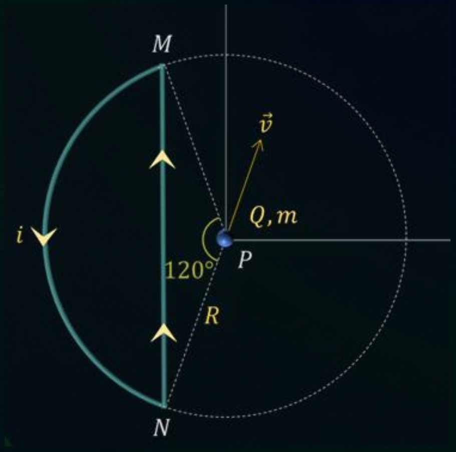
Magnetic field at point O due to wire DA and CB is zero.

Magnetic field at point O	
Due to arc DC	$B_{DC} = \frac{\theta}{2\pi}\left(\frac{\mu_0 i}{2b}\right) \otimes$
Due to arc BA	$B_{BA} = \frac{\theta}{2\pi}\left(\frac{\mu_0 i}{2a}\right) \odot$

- $a < b \Rightarrow \frac{1}{a} > \frac{1}{b} \Rightarrow$ Net magnetic field at point O :

$$\boxed{\vec{B}_o = \frac{\mu_0 i\theta}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right] \odot}$$

A wire loop carrying a current i is placed in the xy plane as shown. If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP, find its instantaneous acceleration.



- The acceleration of the particle : $\vec{a} = \frac{\vec{F}_m}{m} = \frac{Q(\vec{v} \times \vec{B})}{m}$
- The circular arc MN subtends an angle $120° \equiv \frac{2\pi}{3}$ at point P . So, magnetic field at P due to arc :

$$\vec{B}_{arc} = \frac{\left(\mu_0 i/_{2R}\right)}{2\pi} \times \frac{2\pi}{3} \odot \qquad = \frac{\mu_0 i}{6R}\hat{k}$$

- Magnetic field at P due to the straight wire NM :

$$\vec{B}_{straight\ wire} = \frac{\mu_0 i}{4\pi(R\cos 60°)}[\sin 60° + \sin 60°] \otimes \Rightarrow \vec{B}_{straight\ wire} = \frac{\sqrt{3}\mu_0 i}{2\pi R}(-\hat{k})$$

- The net magnetic field at P :

$$\vec{B} = \vec{B}_{straight\ wire} + \vec{B}_{arc}$$

$$\boxed{\vec{B} = \frac{\mu_0 i}{R}\left[\frac{1}{6} - \frac{\sqrt{3}}{2\pi}\right]\hat{k}}$$

- The velocity of the particle at P :

$$\vec{v} = v\cos 60°\,\hat{\imath} + v\sin 60°\,\hat{\jmath}$$

$$\boxed{\vec{v} = \frac{v}{2}[\hat{\imath} + \sqrt{3}\hat{\jmath}]}$$

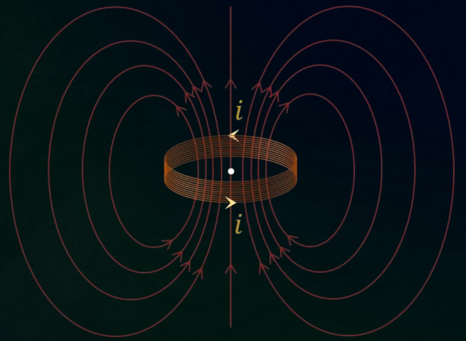
- The acceleration of the particle at P :

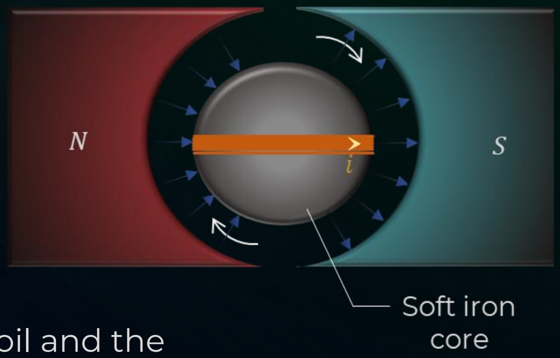
$$\vec{a} = \frac{Q}{m}(\vec{v} \times \vec{B}) = \frac{Q}{m}\left(\frac{v}{2}(\hat{\imath} + \sqrt{3}\hat{\jmath}) \times \frac{\mu_0 i}{R}\left[\frac{1}{6} - \frac{\sqrt{3}}{2\pi}\right]\hat{k}\right)$$

$$\boxed{|\vec{a}| = \frac{Qv}{m}\frac{\mu_0 i}{6R}\left[1 - \frac{3\sqrt{3}}{\pi}\right]}$$

Moving Coil Galvanometer

- The moving coil galvanometer consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field.
- A current carrying coil acts as a magnetic dipole. So, when it is placed in an external magnetic field, it will experience a torque and hence, it will rotate.
- The coil is wrapped on a cylindrical soft iron core which enables the field to remain radial in all position of the coil.
- The field remains parallel to the plane of the coil at all positions. So, angle between area vector of the coil and the magnetic field vector is, $\theta = 90°$





- Torque on the coil :

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\tau} = (ni\vec{A}) \times \vec{B}$$

$$|\vec{\tau}| = niAB\sin\theta = niAB$$

n = No. of turns in the coil

A = Area of the coil

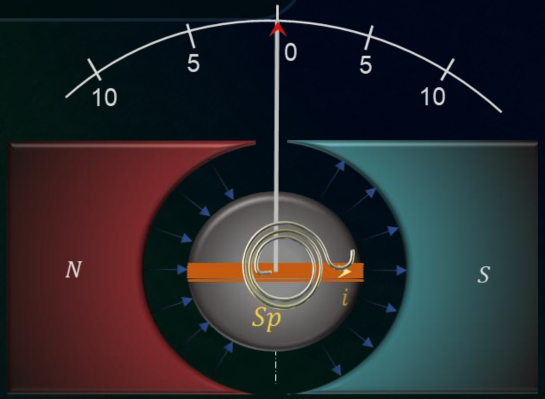
θ = Angle between area vector of the coil and the magnetic field vector

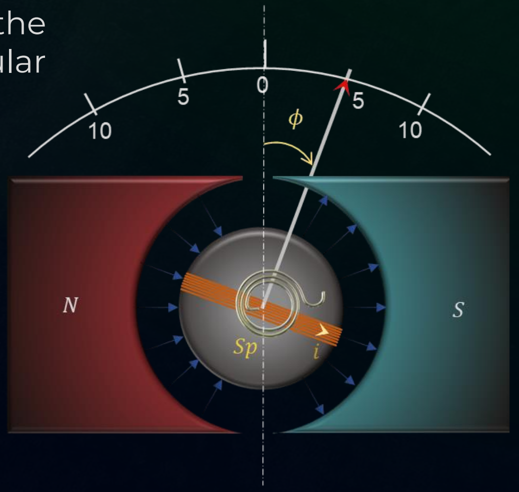
Moving Coil Galvanometer

- An indicator is attached with the coil, and it gets deflected as the coil rotates.
- The coil will keep on rotating as long as current exists through it and indicator will not come back to its original position even if the current is made zero.
- As a remedy to this problem, a spring Sp is introduced.
- When the indicator turns, spring gets coiled and produces a restoring torque.
- The spring Sp provides the restoring torque $\tau_R = K\phi$ that balances the torque due to magnetic field $\tau = niAB$, resulting in a steady angular deflection ϕ of the indicator.
- In equilibrium :

$$K\phi = niAB$$

$$\boxed{i = \frac{K}{nAB}\phi}$$





Gauss's Law

Statement :

The flux of the net electric field through a closed surface is equal to the net charge enclosed by the surface divided by ε_o.

Mathematical form of Gauss's law :

$$\boxed{\phi = \oint \vec{E} \, . \, \overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}}$$

$\longrightarrow$ $\vec{E}$ is due to all charges.

- The closed surface or the periphery of a volume on which Gauss's law is applied is known as the

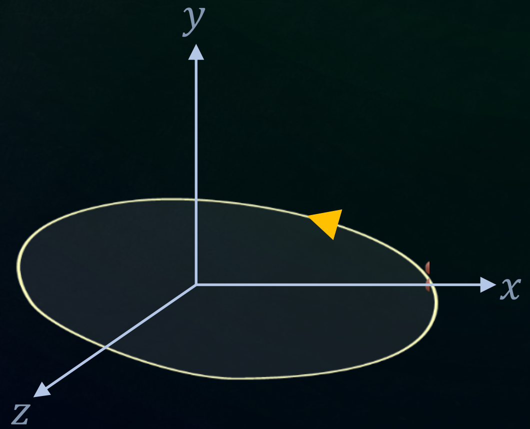
Ampere's Circuital Law and Sign Convention

- Line integral or circulation of $\vec{B}$ along a closed loop :

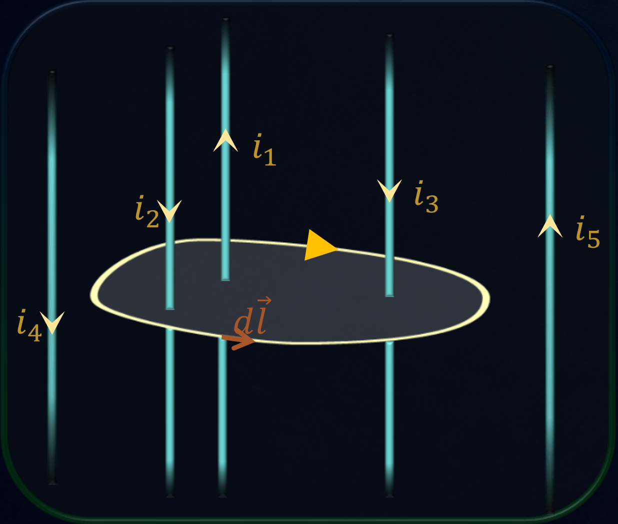
$$\oint \vec{B}\,.\,dl = \mu_0(\,I_{enclosed}\,)$$

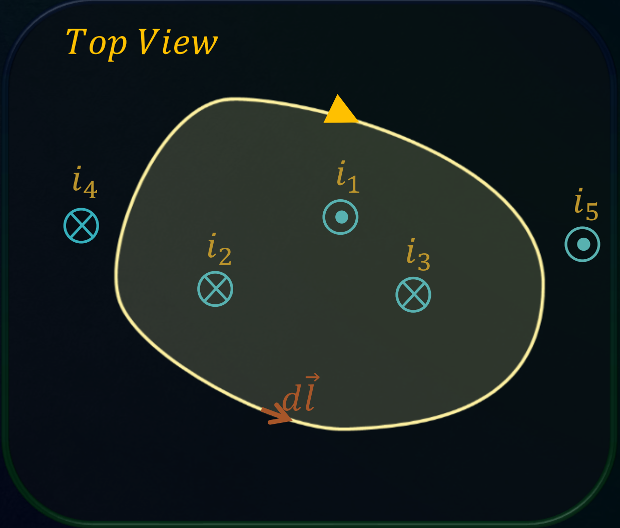
μ_0 = permeability of free space

Fold your right-hand's fingers along the integration path, then the erect thumb gives the positive current direction.



Find out line integral of $\vec{B}.\vec{dl}$ for the given loop?





$$\oint \vec{B}.d\vec{l} = \mu_0(i_2 + i_3 - i_1)$$

Validity of Ampere's Law

Ampere's law is useful in cases of highly symmetrical current distribution.

Examples:

Infinite long wire

(Thin & Thick)

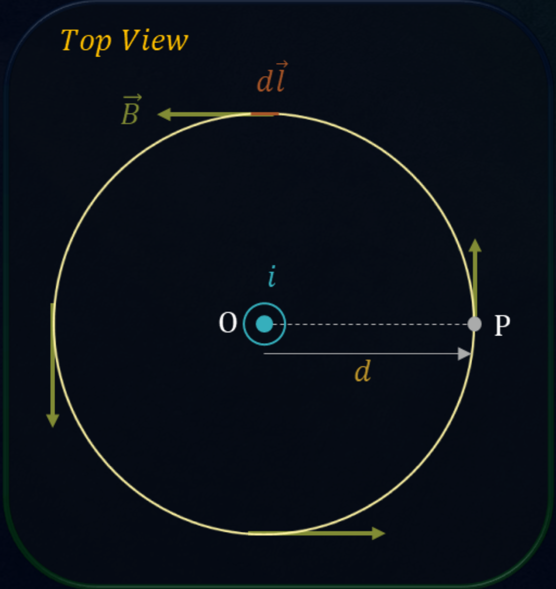
Ring

Solenoid

Toroid

Application of Ampere's Law

Magnetic field due to long straight current carrying wire

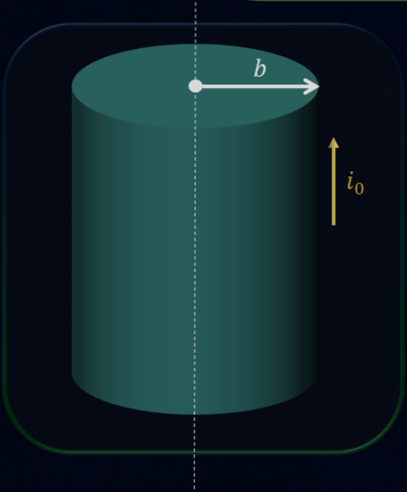


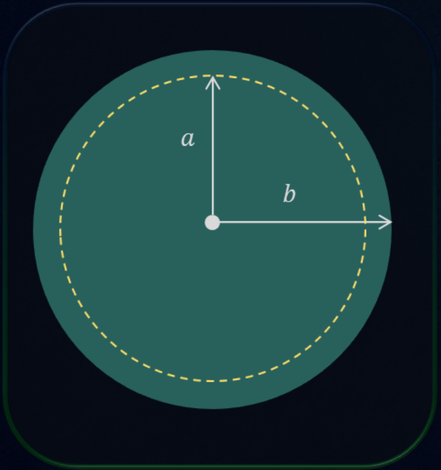
$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\sum I)$$

$$\Rightarrow B(2\pi d) = \mu_0 i$$

$$\boxed{|\vec{B}| = \frac{\mu_0 i}{2\pi d}}$$

A long, cylindrical wire of radius b carries a current i_0 distributed uniformly over its cross section. Find the magnitude of the magnetic field at a point inside the wire at a distance a from the axis.





Applying Ampere's Law

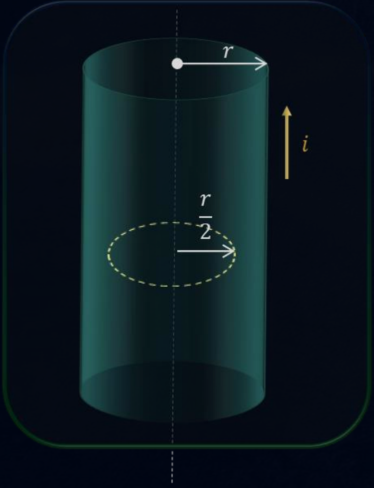
$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B(2\pi a) = \mu_0\left(\frac{i}{\pi b^2} \times \pi a^2\right)$$

$$\boxed{\Rightarrow B(a < b) = \frac{\mu_0 i a}{2\pi b^2}}$$

A thin but long, hollow, cylindrical tube of radius r carries a current i along its length. Find the magnitude of the magnetic field at a distance $\frac{r}{2}$ from the surface.

(a) inside the tube (b) outside the tube



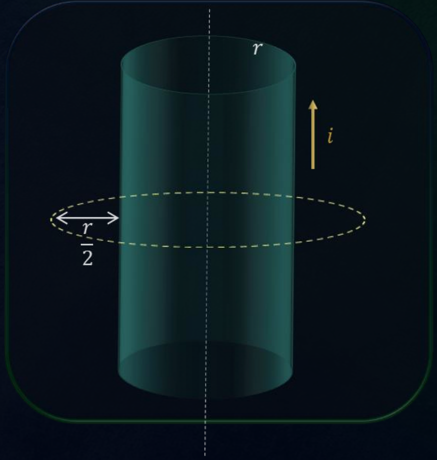
(a) Inside the tube

Applying Ampere's Law

$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B\left(\frac{2\pi r}{2}\right) = \mu_0(0)$$

$$\boxed{\Rightarrow B(inside) = 0}$$



(b) Outside the tube

Applying Ampere's Law

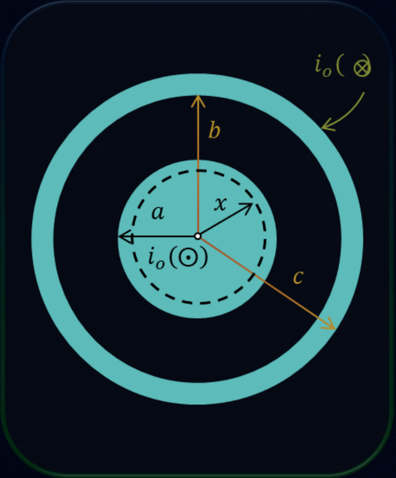
$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B\left(\frac{2\pi \times 3r}{2}\right) = \mu_0(i)$$

$$\boxed{\Rightarrow B = \frac{\mu_0 i}{3\pi r}}$$

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where,
(a) $x < a$ (Assume that the current density is uniform in the inner wire and also uniform in the outer shell)
(b) $a < x < b$ (Assume that the current density is uniform in the inner wire and also uniform in the outer shell)

(a) $x < a$



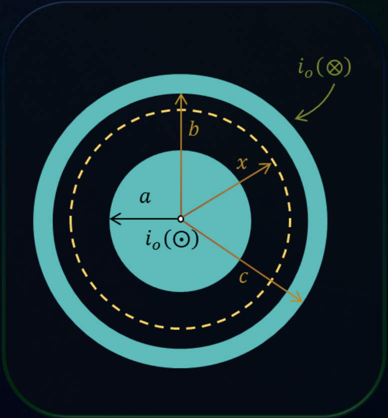
Applying Ampere's Law

$$\oint \vec{B}.d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B(2\pi x) = \mu_0\left(\frac{i_0}{\pi a^2} \times \pi x^2\right)$$

$$\boxed{B(x < a) = \frac{\mu_0 i_0}{2\pi a^2}\, x}$$

(b) $a < x < b$



Applying Ampere's Law

$$\oint \vec{B}.d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B(2\pi x) = \mu_0(i_0)$$

$$\boxed{\Rightarrow B(a < x < b) = \frac{\mu_0 i_0}{2\pi x}}$$

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where,

(c) $b < x < c$ (Assume that the current density is uniform in the inner wire and also uniform in the outer shell)

(d) $x > c$ (Assume that the current density is uniform in the inner wire and also uniform in the outer shell)

(c) $b < x < c$

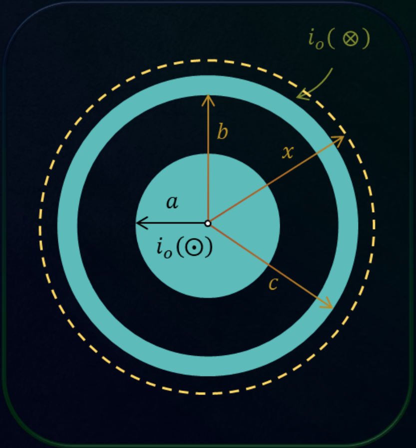
Applying Ampere's Law

$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\sum I)$$

$$\Rightarrow B(2\pi x) = \mu_0\left(i_0 - \frac{i_0}{\pi(c^2 - b^2)} \times \pi(x^2 - b^2)\right)$$

$$\boxed{\Rightarrow B(b < x < c) = \frac{\mu_0 i_0 (c^2 - x^2)}{2\pi x(c^2 - b^2)}}$$

(d) $x > c$



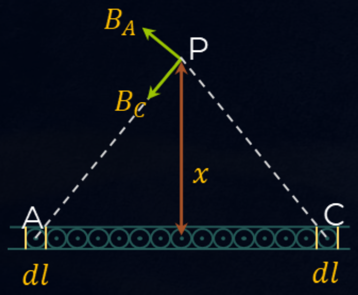
Applying Ampere's Law

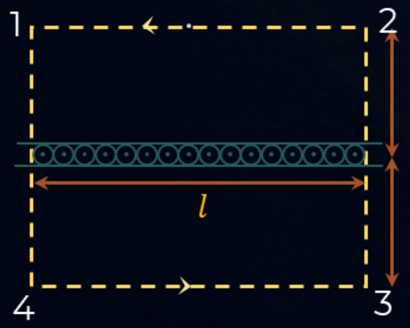
$$\oint \vec{B}\,.\,d\vec{l} = \mu_0(\sum I)$$

$$\Rightarrow B(2\pi x) = \mu_0(0)$$

$$\boxed{B(x > c) = 0}$$

Cross section of a large metal sheet is carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point at a distance x from the metal sheet.





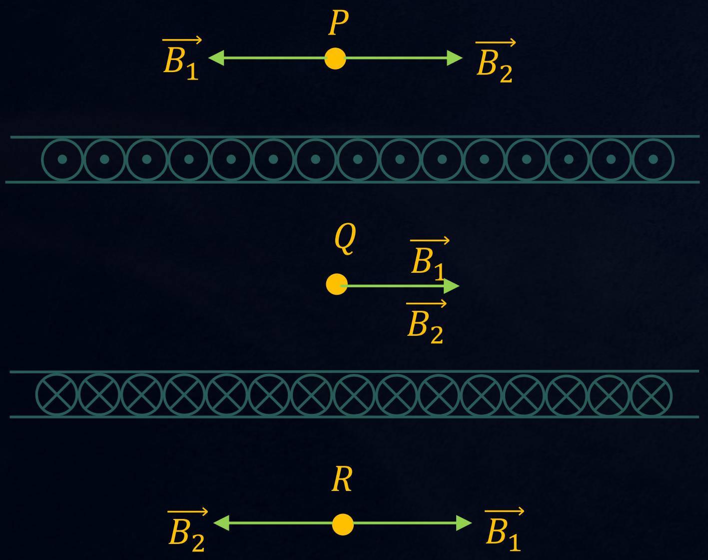
$$\left(\oint \vec{B}.d\vec{l}\right)_{12} + \left(\oint \vec{B}.d\vec{l}\right)_{23} + \left(\oint \vec{B}.d\vec{l}\right)_{34} + \left(\oint \vec{B}.d\vec{l}\right)_{41} = \mu_0(\sum I)$$

$$\Rightarrow\ Bl + 0 + Bl + 0 = \mu_0(\sum K\, dl)$$

$$\Rightarrow 2Bl = \mu_0 K(\sum dl)$$

$$\boxed{B = \frac{\mu_0 K}{2}}$$

Two large metal sheets carry surface currents as shown. The current through a strip of width dl is Kdl where K is a constant. Find the magnetic field $\vec{B}$ at the point P, Q and R.



$$|\vec{B_1}| = |\vec{B_2}| = \frac{\mu_0 K}{2}$$

At point P

$$B_{net} = 0$$

At point R

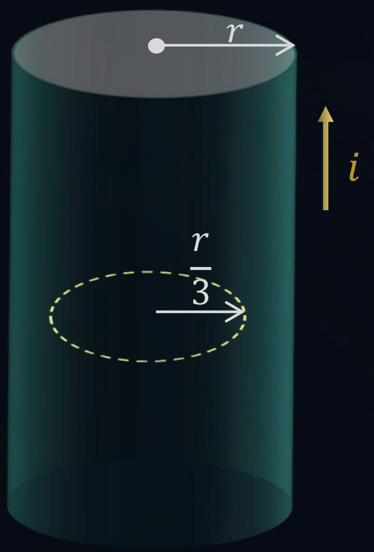
$$B_{net} = 0$$

At point Q

$$|B_{net}| = \mu_0 K$$

A long, straight wire of radius a carries a current distributed uniformly over its cross section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and $2a$, respectively from the axis of the wire is

JEE Main



(a) Inside the tube

Suppose current density is J

Applying Ampere's Law

$$\oint \vec{B}.d\vec{l} = \mu_0(\Sigma I)$$

$$\Rightarrow B\left(\frac{2\pi a}{3}\right) = \mu_0\left(J \times \frac{\pi a^2}{9}\right)$$

$$\boxed{B(inside) = \frac{\mu_0 J a}{6}}$$

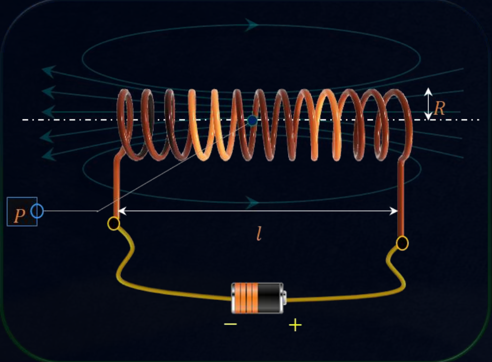
A $\frac{2}{3}$

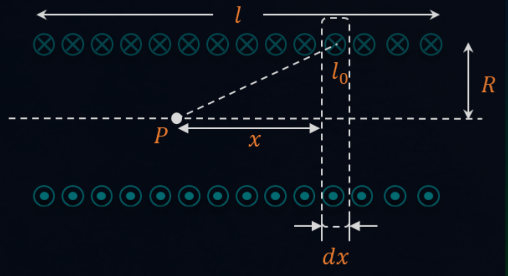
B $\frac{3}{2}$

C $\frac{1}{2}$

D 2

Solenoid





The solenoid is a composition of infinite number of rings provided the distance between each ring is very small.

It is used to produce uniform magnetic field.

P is a point on the axis of the solenoid.

Magnetic field at P due to elemental ring

$$B = \frac{\mu_0 i_0 R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

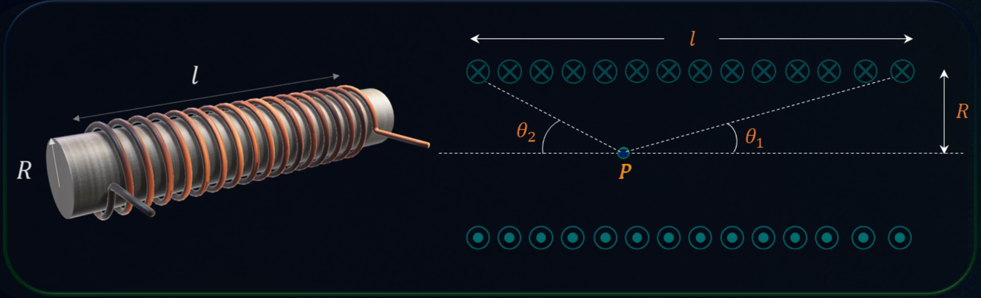
Let n = number of turns per unit length

$$n = \frac{dN}{dl}$$

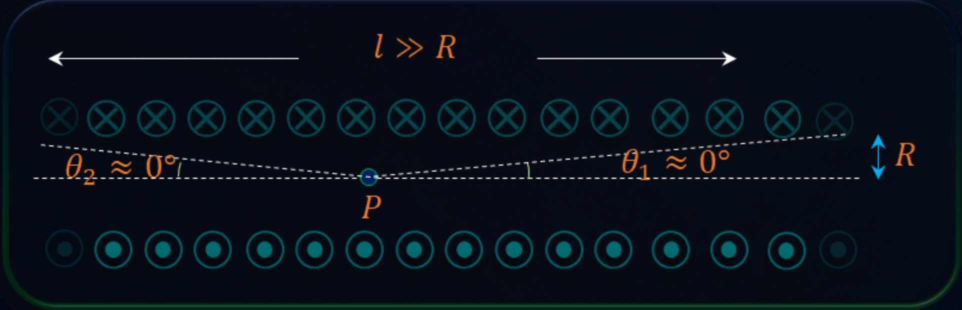
$$i_0 = N(i) = \frac{dN}{dl}\, dx\, i \quad = (n\, dx)\, i$$

$$\boxed{dB = \frac{\mu_0 n i (dx) R^2}{2(R^2 + x^2)^{\frac{3}{2}}}}$$

Ideal Solonoid and Magnetic Field at an axial point P



$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 + \cos\theta_1]$$

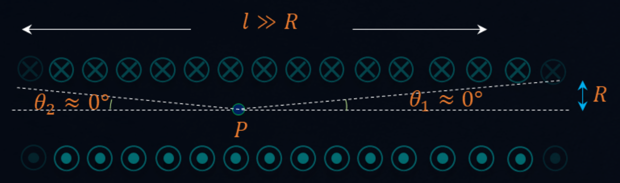


A solenoid can be called as an ideal solenoid if the following conditions are satisfied:

- $l \gg R \Rightarrow$ long solenoid
- $n = N/l$ is a very large number

  $\Rightarrow$ wire is very closely wound.

Ideal Solenoid



A solenoid can be called as an ideal solenoid if the following conditions are satisfied:

- $l \gg R \Rightarrow$ long solenoid
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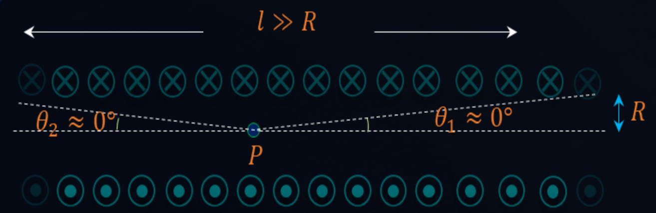


Magnetic field on the axis of a solenoid:

$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 + \cos\theta_1]$$

For ideal solenoid, $l \gg R$.

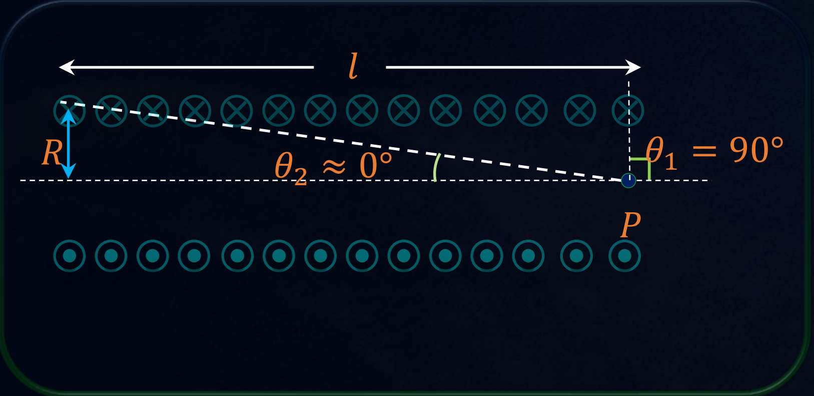
$\Rightarrow$ For any point P on axis,

$\theta_1 \approx 0°$ and $\theta_2 \approx 0°$

$$\Rightarrow B = \frac{\mu_0 n i}{2}[\cos 0° + \cos 0°]$$

$$\boxed{B = \mu_0 n i}$$

Ideal Solenoid: Field at the end point on axis



Magnetic field on the axis of a solenoid:

$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 + \cos\theta_1]$$

At the end point on axis,

$$\Rightarrow \theta_1 = 90° \text{ and } \theta_2 \approx 0°$$

$$\Rightarrow B = \frac{\mu_0 n i}{2}[\cos 0° + \cos 90°]$$

$$\boxed{B = \frac{\mu_0 n i}{2}}$$

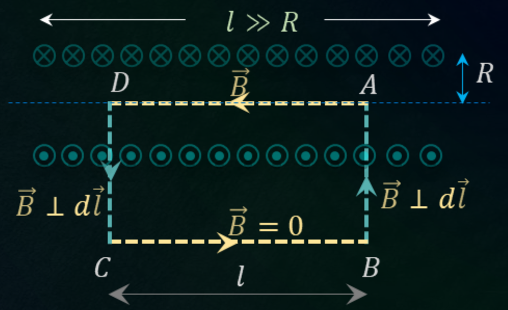
Ideal Solenoid-Ampere's law

- Consider *ABCDA* as Amperian loop.
- Divide the loop in parts to write field individually.

$$\left[\oint \vec{B}.\vec{dl}\right]_{AB} = \left[\oint \vec{B}.\vec{dl}\right]_{DC} = 0 \quad [\because \vec{B} \perp d\vec{l}]$$

$$\left[\oint \vec{B}.\vec{dl}\right]_{AD} = Bl \quad [\because \vec{B} \parallel d\vec{l}]$$

$$\left[\oint \vec{B}.\vec{dl}\right]_{AD} = 0 \quad [\because \vec{B}_{outside} = 0]$$



- Using Ampere's law,

$$\left[\oint \vec{B}.\vec{dl}\right]_{ABCDA} = \mu_0 i_{in}$$

$$\Rightarrow 0 + 0 + Bl + 0 = \mu_0 Ni$$

$$\boxed{B = \frac{\mu_0 Ni}{l} = \mu_0 ni}$$

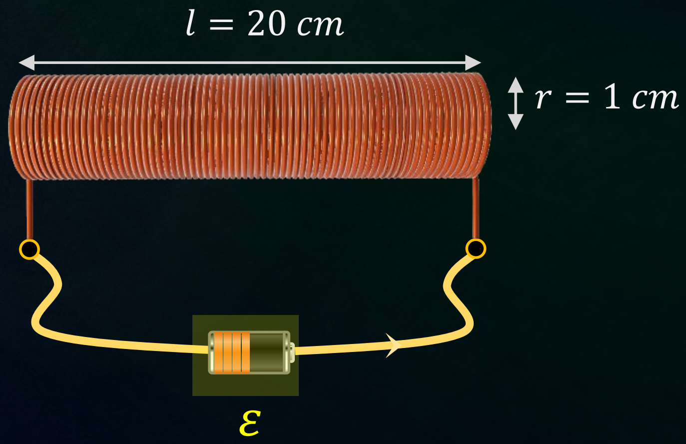
A copper wire having resistance $0.01\ ohm$ in each meter is used to wind $400\ turns$ solenoid of a radius of $1.0\ cm$ and length $20\ cm$. Find the emf of a battery which when connected across the solenoid will cause a magnetic field of $1.0 \times 10^{-2}\ T$ near the centre of the solenoid.

Solution: $R_s = \left(\frac{dR}{dl}\right)(N)(2\pi r)$ $\quad i = \frac{\varepsilon}{R_s}$

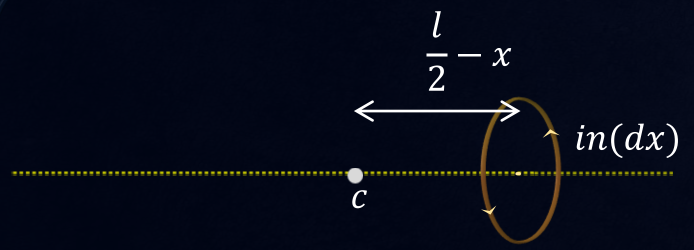
$$B = \mu_0 n i = \mu_0 \left(\frac{N}{l}\right)\left(\frac{\varepsilon}{\left(\frac{dR}{dl}\right)(2\pi r)(N)}\right)$$

$$\Rightarrow 10^{-2} = \frac{(4\pi \times 10^{-7})(\varepsilon)(10)}{(10^{-2})(2\pi)(10^{-2})(2)}$$

$$\boxed{\Rightarrow \varepsilon = 1\ V}$$



A tightly-wound solenoid of a radius a and length l has n turns per unit length. It carries an electric current i. Consider a length dx of the solenoid at a distance x from one end. This contains $n\,dx$ turns and may be approximated as a circular current $in(dx)$.
(a) Write the magnetic field at the centre of the solenoid due to this circular current. Integrate this expression under proper limits to find the magnetic field at the centre of the solenoid.



Here, Elementary magnetic field is

$$dB = \frac{\mu_0 (ni)(dx)a^2}{2\left(a^2 + \left(\frac{l}{2} - x\right)^2\right)^{\frac{3}{2}}}$$

$$dB = \frac{\mu_0 ni\,(dx)}{2a\left(1 + \left(\frac{l - 2x}{2a}\right)^2\right)^{\frac{3}{2}}}$$

Let's integrate both side

$$\Rightarrow B = \int_0^l \frac{\mu_0 ni\,(dx)}{2a\left(1+\left(\frac{l-2x}{2a}\right)^2\right)^{\frac{3}{2}}}$$

Let-

$$(l-2x) = 2a\tan\theta$$

$$dx = -a\sec^2\theta\,(d\theta)$$

$$I = \int -\frac{a\sec^2\theta\,(d\theta)}{(1+\tan^2\theta)^{\frac{3}{2}}}$$

$$= -a\int \cos\theta\, d\theta$$

$$\Rightarrow I = -a\sin\theta$$

$$\Rightarrow I = -\frac{a(l-2x)}{\sqrt{(l-2x)^2+4a^2}}$$

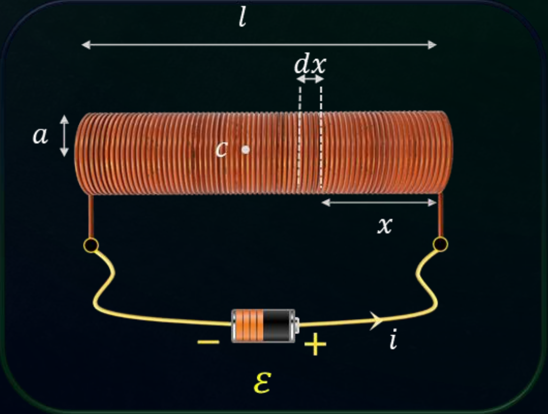
$$\Rightarrow B = \frac{\mu_0 ni}{2a}\left[\frac{a(2x-l)}{\sqrt{(l-2x)^2+4a^2}}\right]_0^l$$

$$\Rightarrow B = \frac{\mu_0 ni}{2a}\left[\frac{al}{\sqrt{l^2+4a^2}} - \frac{(-al)}{\sqrt{l^2+4a^2}}\right]$$

$$\Rightarrow B = \frac{\mu_0 ni}{2a}\left[\frac{2al}{\sqrt{l^2+4a^2}}\right]$$

$$\Rightarrow B = \frac{\mu_0 nil}{\sqrt{l^2+4a^2}}$$

$$\boxed{B = \frac{\mu_0 ni}{\sqrt{1+\left(\frac{2a}{l}\right)^2}}}$$



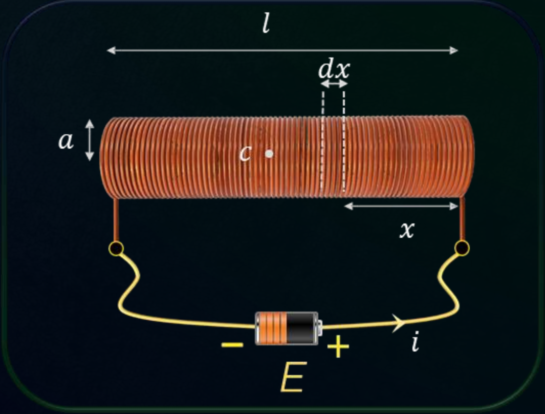
A tightly-wound solenoid of a radius a and length l has n turns per unit length. It carries an electric current i. Consider a length dx of the solenoid at a distance x from one end. This contains $n\,dx$ turns and may be approximated as a circular current $in(dx)$.

(b) Verify that if $l \gg a$, the field tends to $B = \mu_0 ni$ and if $l \ll a$, the field tends to $B = \frac{\mu_0 nil}{2a}$.

$$B = \frac{\mu_0 ni}{\sqrt{1 + \left(\frac{2a}{l}\right)^2}} = \frac{\mu_0 nil}{2a\sqrt{\left(\frac{l}{2a}\right)^2 + 1}}$$

Case: $l \gg a$ $\qquad \frac{2a}{l} \to 0 \qquad \boxed{B = \mu_0 ni}$

Case: $a \gg l$ $\qquad \frac{l}{2a} \to 0 \qquad \boxed{B = \frac{\mu_0 nil}{2a}}$



Magnetic Field due to a Toroid

- Using Ampere's circuital law,

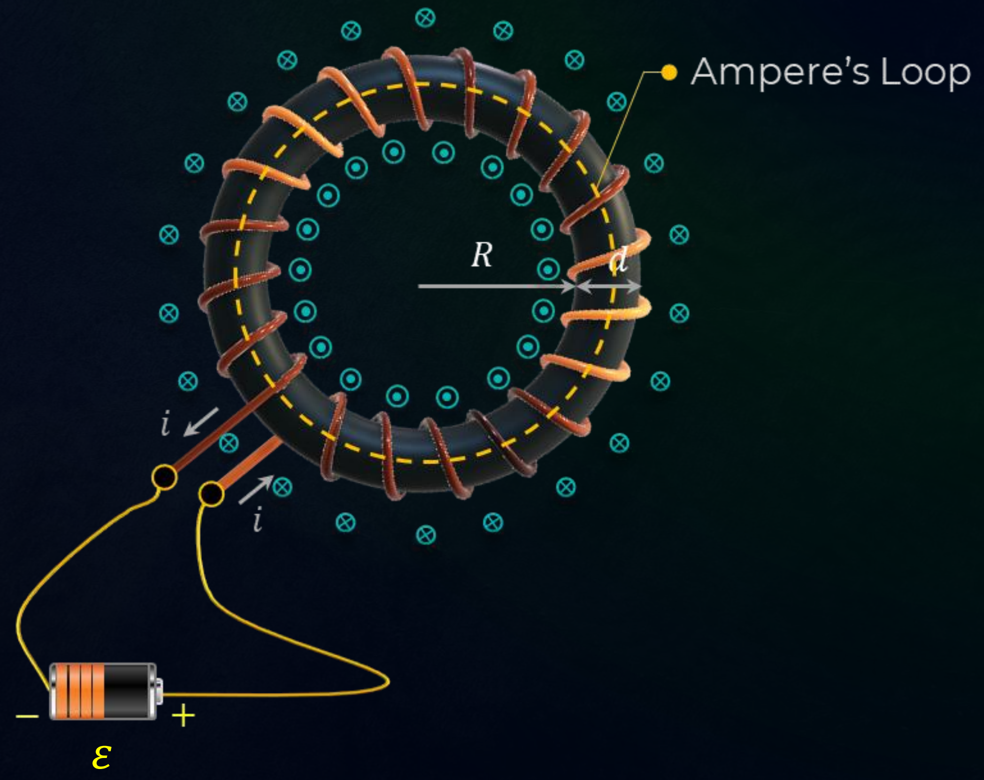
$$\oint \vec{B}.\vec{dl} = \mu_0\, i_{in}$$

$$\boxed{\Rightarrow B = \frac{\mu_0 N i}{2\pi r} = \mu_0 n i}$$

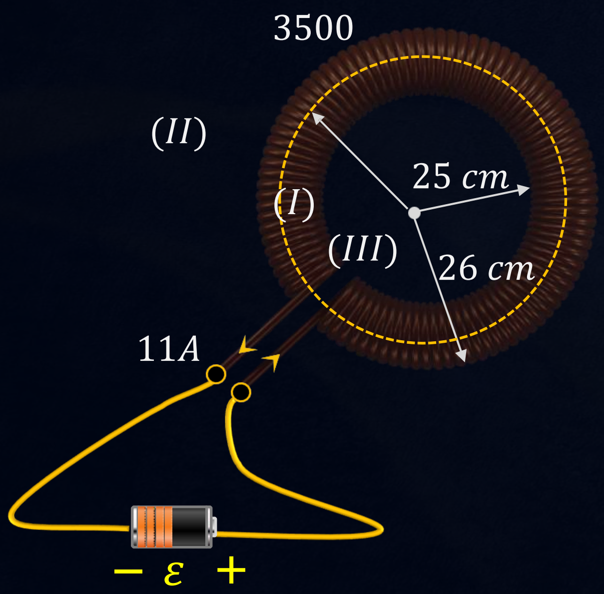
- Here,

N = Total number of turns

$$n = \frac{N}{2\pi r}$$



A toroid has inner radius $25\ cm$ and outer radius $26\ cm$, with the 3500 turns and $11\ A$ current flowing through it. Find the magnetic field in the regions $(I), (II)$ and (III).



$$B_{II} = B_{III} = 0$$

$$R = \frac{25 + 26}{2} = 25.5\ cm$$

For exact calculation use $25.5\ cm$

For approx. calculation use $25\ cm$

$$B = \frac{\mu_0 n i}{2\pi R} = \frac{4\pi \times 10^{-7}(3500)(11)(100)}{2\pi(25)}$$

$$\Rightarrow B = 30.8 \times 10^{-3} T = 0.0308\ T$$

$$\boxed{\Rightarrow B = 0.0308\ T}$$

A tightly-wound, long solenoid carries a current of $2.0\ A$. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of $1.0 \times 10^8\ rev\ s^{-1}$. Find the number of turns per meter in the solenoid.

$$B_{solenoid} = \mu_0 ni$$

$$R = \frac{mv}{qB}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

$$f = \frac{1}{T} = \frac{qB}{2\pi m} = \frac{q}{2\pi m}(\mu_0 ni)$$

$$f = \frac{q\mu_0 ni}{2\pi m}$$

$$\Rightarrow 10^8 = \frac{(1.6 \times 10^{-19})(4\pi \times 10^{-7})(n)(2)}{2\pi(9.1 \times 10^{-31})}$$

$$\Rightarrow n \simeq 1.42 \times 10^3$$

$$\boxed{\Rightarrow n \simeq 1420\ turns\ m^{-1}}$$

A. 1360

B. 1420

C. 1580

D. 1240