Welcome to

Aakash + BYJU'S NOTES

Physical world and Units & Measurements

Origin of Science

Logic

Reasoning

Experimentation


Ancient philosophers

- Science is a systematic way of understanding various phenomena of nature that we observe based on logic, reasoning, and experimentation.

- Ancient philosophers also used logic and reasoning, but they lacked scientific approach.

Origin of Science

- Stone is heavier than water, so it sinks. Air bubbles and fire rise through water because air is lighter than water. Whereas wood floats in water.
- Initially these phenomenon are observed and later through the scientific experiments, density of the object will make it to sink or float.

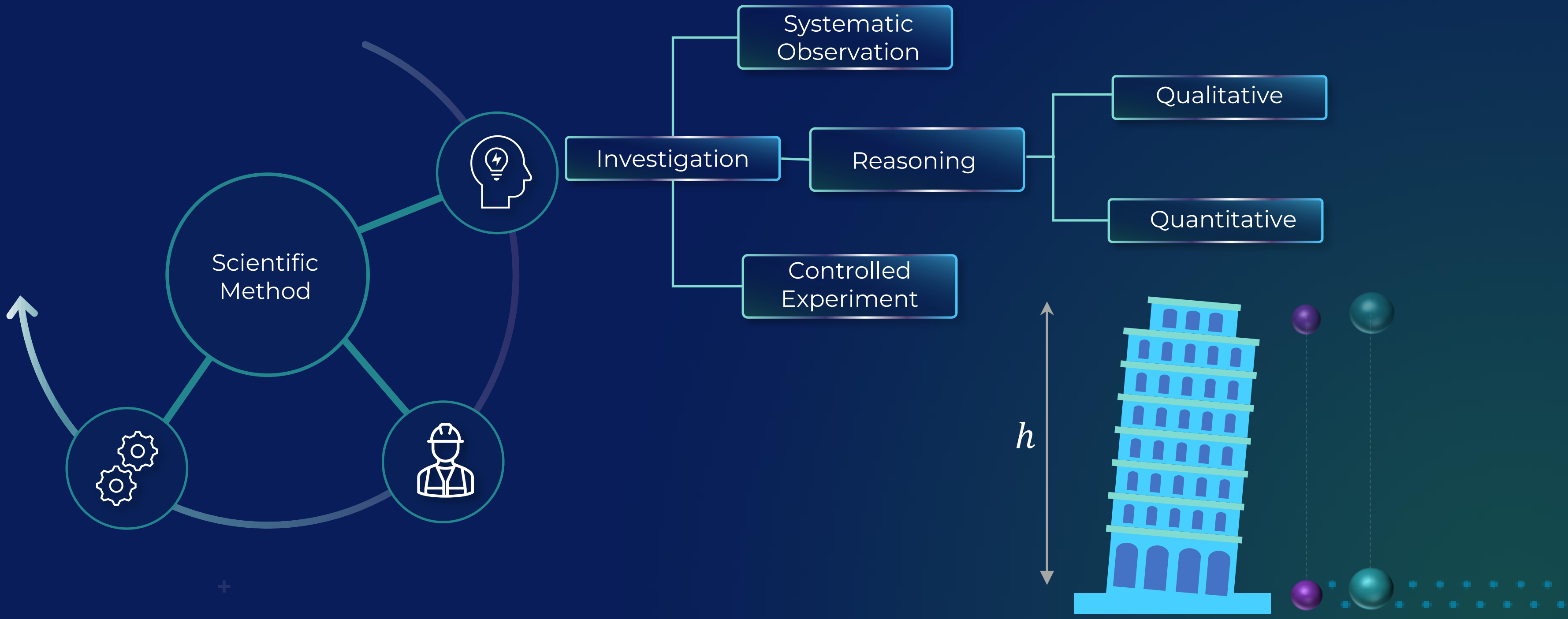
The word Science originates from the Latin verb Scientia, meaning 'to know'.



- Science is a systematic attempt to understand natural phenomena. This will further help to mankind to predict, modify and control the natural phenomena.
- This led the mankind to unraveling the secrets of nature and working of universe. Even today many discoveries are happening and it is further enhancing the knowledge of human race.

Scientific Method

A process with the help of which scientists try to investigate, verify or construct an accurate and reliable version of any natural phenomena.



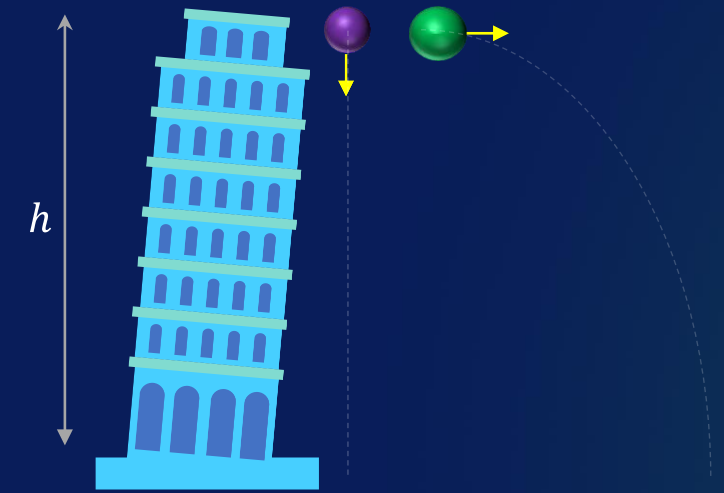
Scientific Method

A process with the help of which scientists try to investigate, verify or construct an accurate and reliable version of any natural phenomena.

Construction — Mathematical Modelling

Prediction

$$h = ut + \frac{1}{2}gt^2$$



Scientific Method

A process with the help of which scientists try to investigate, verify or construct an accurate and reliable version of any natural phenomena.

What is Physics?

Physics is the study of the physical world and the physical laws of nature.

Gravitation

Forces in nature (Drag)

Nuclear Fusion

Two approaches used by physics:

Unification

Attempt to explain various phenomena through the same laws

- Law of gravitation
- Electromagnetism

Reductionism

To derive the properties of a bigger, more complex, system from the properties and interactions of its constituent simpler parts.

- Thermodynamics

Scope of Physics

Physics is a basic discipline in the category of Natural Sciences, which also includes other disciplines like Chemistry and Biology.

Macroscopic

- Mechanics (11^{th})

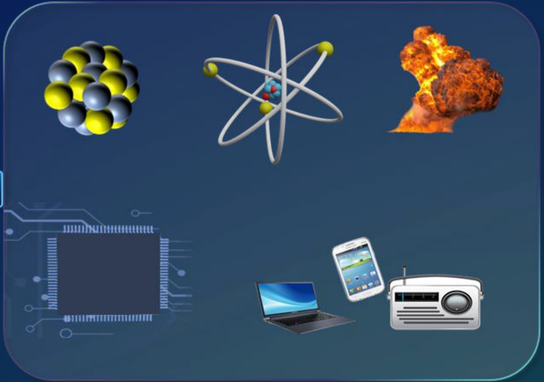
- Thermodynamics (11^{th})

- Electromagnetism (12^{th})

- Optics (12^{th})

Microscopic

- Quantum Physics



- The scope of physics is endless

Scale of length ($10^{-14}\,m$ to $10^{26}\,m$)

Scale of time ($10^{-22}\,s$ to $10^{18}\,s$)

Scale of mass ($10^{-30}\,kg$ to $10^{55}\,kg$)

Atomic and molecular phenomena are dealt with by

A

Newtonian mechanics deals with the motion of bodies

B

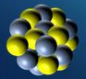
Fluid mechanics deals with mechanics of fluids

C

Applied mechanics deals with forces and its applications

D

Quantum mechanics is physics at atomic and subatomic level



A Newtonian Mechanics

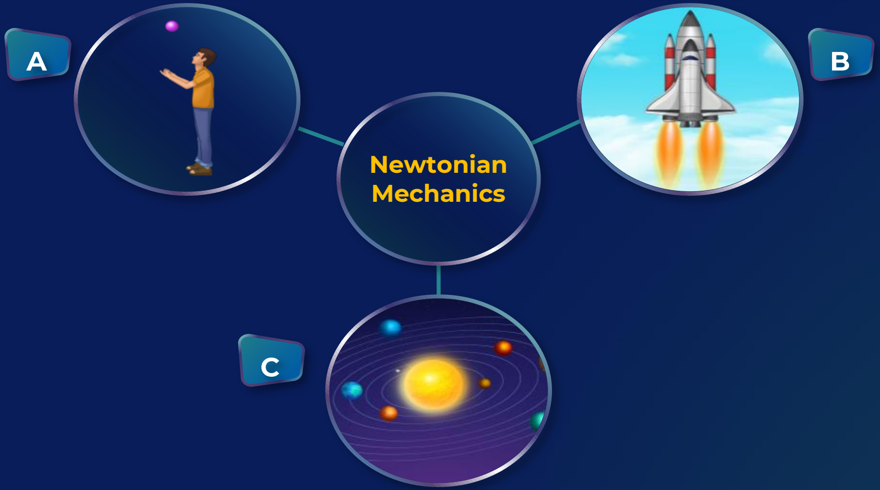
B Fluid Mechanics

C Applied Mechanics

D Quantum Mechanics

?

Newtonian mechanics could not explain



A Fall of bodies on earth

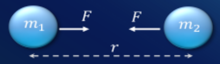
B Atomic phenomenon

C Movement of planets

D Flight of rockets

Fundamental forces in nature and towards unification

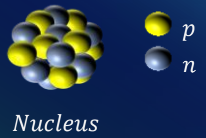
Gravitational Force



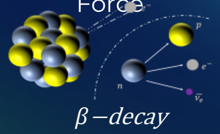
$$F = \frac{Gm_1m_2}{r^2}$$

Electromagnetic Force

Strong Nuclear Force



Weak Nuclear Force



Isaac Newton

Unified celestial and terrestrial mechanics; showed that the same laws of motion and the laws of gravitation apply to both the domains.

1687

H. C. Oersted

Showed that electric and magnetic phenomena are inseparable aspects of a unified domain

1820

Michael Faraday

Electromagnetism

1830

J.C. Maxwell

Unified electricity, magnetism and optics; showed that light is an electromagnetic wave

1873

Which among the following forces is relatively weakest?

Name	Relative Strength	Range	Operates among
Gravitational force	10^{-39}	Infinite	All objects in the universe
Weak nuclear force	10^{-13}	Very short ($\sim 10^{-16}$ m)	Some lighter elementary particles
EM force	10^{-2}	Infinite	Charged particles
Strong nuclear force	1	Short ($\sim 10^{-15}$ m)	Nucleons, heavier elementary particles

A. Electromagnetic Force

B. Gravitational Force

C. Strong Nuclear Force

D. Weak Nuclear Force

Conservation Laws

Conservation of Mechanical Energy

$$\Rightarrow E = U + K$$

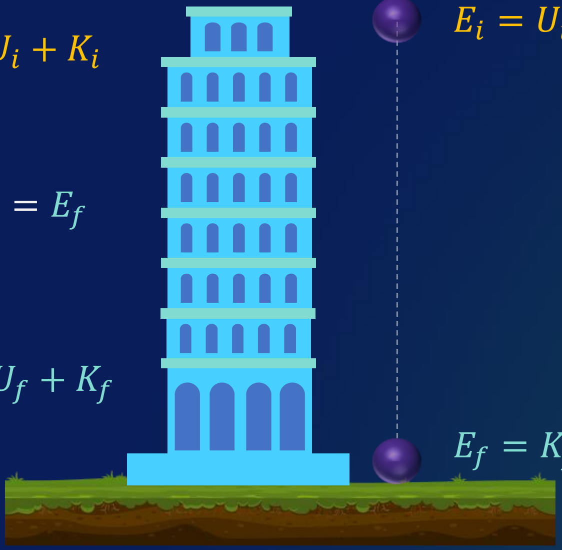
$$U = mgh\,,$$

$$K = \frac{1}{2}mv^2$$

$E_i = U_i + K_i$

$E_i = E_f$

$E_f = U_f + K_f$



At rest

$E_i = U_i$

$E_f = K_f$

Potential energy

Kinetic energy

Sound, Heat, Light etc,

Conservation Laws

Law of conservation of mass

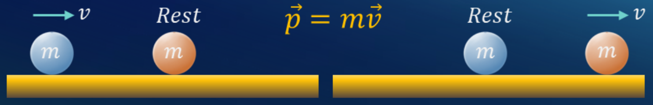
$$H_2 + O_2 \rightarrow H_2O$$

$4\ g \quad 32\ g \quad 36\ g$

Conservation of Linear Momentum

Linear momentum

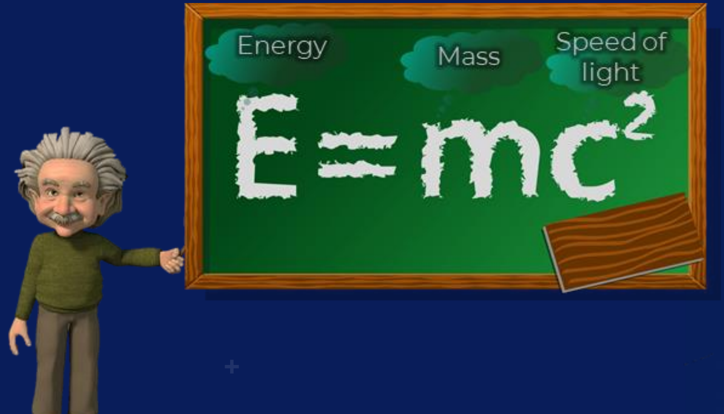
$\vec{p} = m\vec{v}$

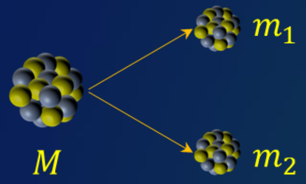


Before Collision

After Collision

Mass energy equivalence





Fission reaction

$$M > (m_1 + m_2)$$

Physical Quantities

- Represent Quantities from Physical World.
- They should be measurable directly or indirectly by an instrument.
- Can be expressed by a numerical value followed by a unit.

$$PQ = n \times u$$

Which of these are physical quantities ?

1. The density of brass is $8.4\ g/cm^3$
2. The relative density of alcohol is 0.8
3. The mass of an empty bottle is $21.8\ g$
4. The velocity of a moving train is $50\ km/h$

A. 1 and 4

B. $1, 3$ and 4

C. $1, 2, 3$ and 4

D. $1, 2$ and 3

Unit

Unit is a reference standard used to compare and measure a physical quantity.

$$PQ = n \times u$$

Numerical Value

unit

System of units	Mass – Length – Time
CGS	centimetre – gram – second
FPS	foot – pound – second
MKS	meter – kilogram – second

Unit

Prefixes	Factor	Prefixes	Factor
yotta	10^{24}	centi	10^{-2}
zetta	10^{21}	milli	10^{-3}
exa	10^{18}	micro	10^{-6}
peta	10^{15}	nano	10^{-9}
tera	10^{12}	pico	10^{-12}
giga	10^{9}	femto	10^{-15}
mega	10^{6}	atto	10^{-18}
kilo	10^{3}	zepto	10^{-21}
deci	10^{-1}	yocto	10^{-24}

Measurement of a Physical quantity

Measurement is the process of comparison of the given physical quantity with known standard quantity of the same nature.

$$Mass = \underbrace{5}_{\text{Numerical Value}} \times \underbrace{1\ Kg}_{\text{unit}}$$

- Measurement of a physical quantity is constant, irrespective of the system in which unit is selected.

$$nu = constant$$

$$n_1 u_1 = n_2 u_2 = constant$$

- Bigger the unit selected to measure the physical quantity, the smaller the values and vice-versa.

$$n \propto \frac{1}{u}$$

Types of Physical Quantities

Fundamental

- Length
- Mass
- Time
- Electric current
- Temperature
- Amount of substance
- Luminous intensity

Derived

- Area
- Volume
- Velocity
- Momentum
- Energy
- Weight
- Pressure

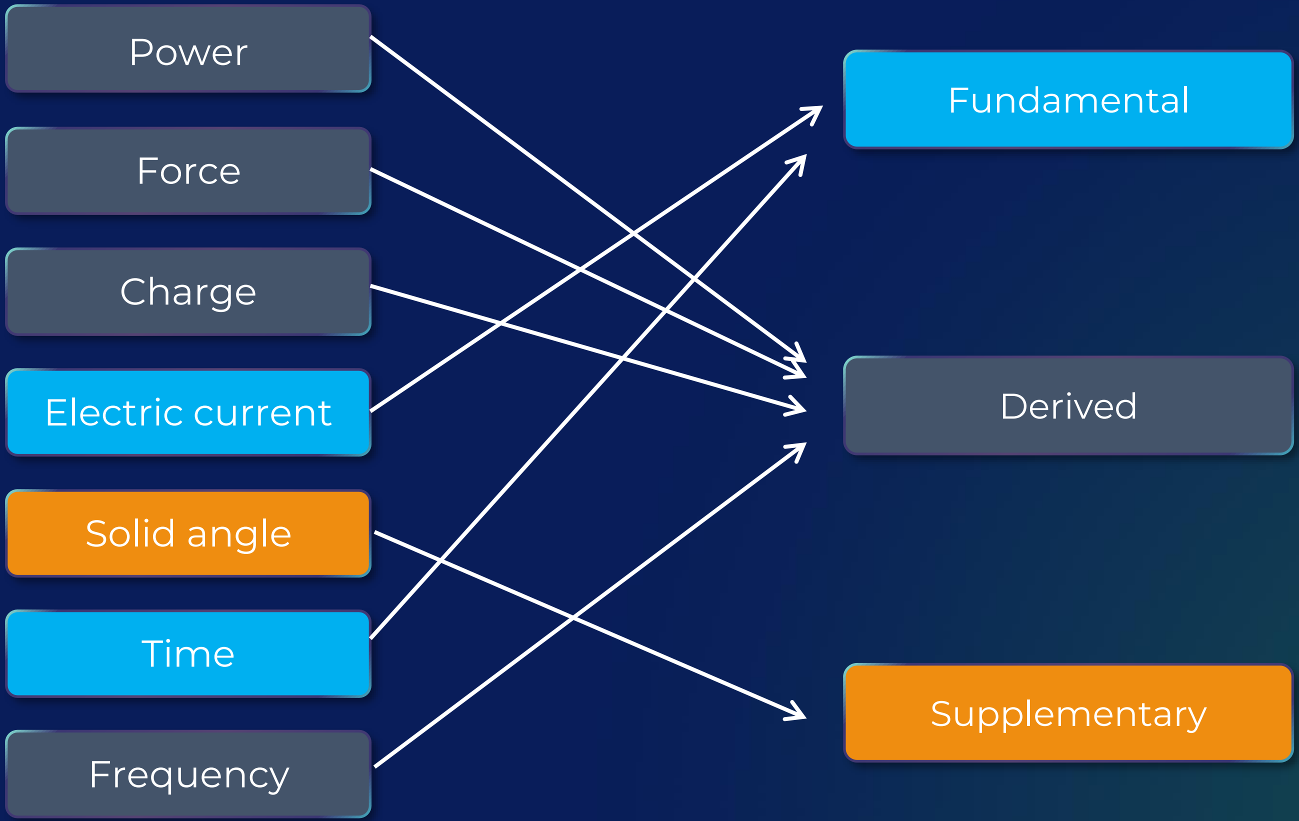
Supplementary

- Plane angle
- Solid angle

SI units

Fundamental Quantities	SI units
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Electric current	ampere (A)
Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Match the Following



Dimension

Dimensions are powers to which fundamental quantities are raised.

Physical Quantities	Dimension
Length	$[L]$
Mass	$[M]$
Time	$[T]$
Electric current	$[I]$
Temperature	$[K]$
Amount of substance	$[mol]$
Luminous intensity	$[cd]$

Physical Quantities	Dimension
Area	$[L^2]$
Velocity	$[LT^{-1}]$
Acceleration	$[LT^{-2}]$

Physical Quantity

- **Dimensional Physical Quantity**
  - **Dimensional Constant**
    - Eg: Plank's constant (h), Universal Gravitational constant
  - **Dimensional Variable**
    - Eg: Length, Time, Mass, Energy, Momentum
- **Dimensionless Physical Quantity**
  - **Dimensionless Constant**
    - Eg: $0, 1, 2, 3, \pi$, fine structure constant $\alpha = \frac{1}{137}$
  - **Dimensionless Variable**
    - Eg: Strain, Plane Angle, Solid Angle

Given the electrostatic force between two charges q_1 and q_2 separated by R is $F = k \times \frac{q_1 q_2}{R^2}$. Find the dimensional formula of constant k.

Given: $F = k \times \frac{q_1 q_2}{R^2}$

To find: The dimensional formulae of constant k

Solution:

$$[F] = MLT^{-2}$$

$$[q_1] = [q_2] = AT$$

$$[R] = L$$

$$\left[k \times \frac{q_1 q_2}{R^2}\right] = MLT^{-2}$$

$$\boldsymbol{[k] = ML^3T^{-4}A^{-2}}$$

Given the gravitational force between two objects of mass m_1 and m_2 separated by R is $F = G \times \frac{m_1 m_2}{R^2}$. Find the dimensional formula of gravitational constant G.

Given: $F = G \times \frac{m_1 m_2}{R^2}$

To find: The dimensional formulae of constant k

Solution:

$$[F] = MLT^{-2}$$

$$[m_1] = [m_2] = M$$

$$[R] = L$$

$$\left[G \times \frac{m_1 m_2}{R^2}\right] = MLT^{-2}$$

$$\boldsymbol{[k] = M^{-1}L^{3}T^{-2}}$$

Principle of Homogeneity

The magnitude of physical quantities may be added together or subtracted from one another only if they have same dimensions.

In a physical expression, the dimensions of all the terms must be identical.

$$\boxed{s} = \boxed{ut} + \boxed{\frac{1}{2}at^2}$$

$$[M^0LT^0] \qquad [M^0LT^0] \qquad [M^0LT^0]$$

Find the dimensions of α and β in the equation $v = \alpha + \beta t$, where v is the velocity and t is time.

(a) $[\alpha] = [LT^{-1}]$ and $[\beta] = [LT^{-2}]$

(b) $[\alpha] = [LT^{-2}]$ and $[\beta] = [LT^{-1}]$

(c) $[\alpha] = [L^2T^{-1}]$ and $[\beta] = [LT^{-1}]$

(d) $[\alpha] = [LT^{-1}]$ and $[\beta] = [L^2T^{-2}]$

Solution: Dimension of α = Dimension of v = $[\alpha] = [LT^{-1}]$

Dimension of βt = Dimension of v = $[\beta] = [LT^{-2}]$

Find the dimensional formula of α in the equation $P = P_0 e^{-\alpha t^2}$, where P and P_0 are the pressures and t is time.

Given: $P = P_0 e^{-\alpha t^2}$

To find: The dimensional formula of α

Solution: Powers are dimensionless

Dimension of $\alpha t^2 = [M^0 L^0 T^0]$

$[\alpha] = [T^{-2}]$

Dimensional Analysis

1. Dimensional analysis is branch of physics that deals with the application of principle of homogeneity.
2. Physical quantities represented by symbols on both sides of a mathematical equation must have the same dimensions.
3. There are three major applications of dimensional analysis.

Uses of Dimensional Analysis

- To convert unit of a physical quantity from one system to another.

$$\boxed{\text{Numerical value} \times \text{unit} = \text{constant}}$$

- Let n_1 and n_2 be the numerical values of a physical quantity and u_1 and u_2 be the units respectively in two different system of units, then

$$n_1 u_1 = n_2 u_2$$

$$n_1\left[M_1^a L_1^b T_1^c\right] = n_2\left[M_2^a L_2^b T_2^c\right]$$

$$\boxed{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^C}$$

?

Acceleration due to gravity value, $g = 9.8\ m/s^2$. Express the g value in ft/s^2 (in FPS system). ($1\ m = 3.28\ ft$)

Solution:

$$[g] = [LT^{-2}]$$

$$\frac{1\ m/s^2}{1\ ft/s^2} = \frac{1\ m}{1\ ft} \times \frac{s^2}{s^2} = \frac{3.28\ ft}{ft} \times \frac{s^2}{s^2} = 3.28$$

$$1\ m/s^2 = 3.28\ ft/s^2$$

$$g = 9.8\ m/s^2 = 9.8 \times 3.28\ ft/s^2 = 32.144 \approx 32\ ft/s^2$$

$$\boxed{g = 32\ ft/s^2}$$

Find an expression for the time period (t) of a simple pendulum. The time period (t) may depend on i) mass (m) of the bob of the pendulum, ii) length (l) of the pendulum, iii) acceleration due to gravity (g) at the place where the bob is suspended. (Take k as constant of proportionality)

Let (i) $t \propto m^a$ (ii) $t \propto l^b$ (iii) $t \propto g^c$

Combining all three factors, we get

$t \propto m^a l^b g^c$ or $t = k m^a l^b g^c$

$$[M^0 L^0 T^1] = [M^a]\,[L^b]\,[(LT^{-2})^c] = [M^a L^{b+c} T^{-2c}]$$

On comparing the dimensions on both sides,

$a = 0,\quad b + c = 0,\quad -2c = 1,\quad \Rightarrow\quad a = 0,\ b = \frac{1}{2},\ c = -\frac{1}{2}$

$$t = k m^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

$$\boxed{t = k\sqrt{\frac{l}{g}}}$$

Given that the time period (t) of oscillation of a gas bubble from an explosion under water depends upon the static pressure (p), the density of water (d) and the total energy of explosion (E). Find a dimensional relation for t?

Solution:

$$t \propto p^a d^b E^c$$

$t = kp^a d^b E^c$ Where k is dimensionless constant

$$[T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

$$[T] = M^{a+b+c}\, L^{-a-3b+2c}\, T^{-2a-2c}$$

On comparing the dimensions on both sides,

$a + b + c = 0$ ....(1) $-a - 3b + 2c = 0$ .....(2) $-2a - 2c = 1$ ......(3)

$a = -\frac{5}{6}$ $b = \frac{1}{2}$ $c = -\frac{1}{3}$

$$\boxed{t = kp^{-\frac{5}{6}} d^{\frac{1}{2}} E^{\frac{1}{3}}}$$

Limitations of Dimensional Analysis

- A dimensionally correct equation need not be physically correct.

	$S = ut + at^2$	$S = ut + \frac{1}{2}at^2$
Dimensionally →	✓	✓
Physically →	✗	✓

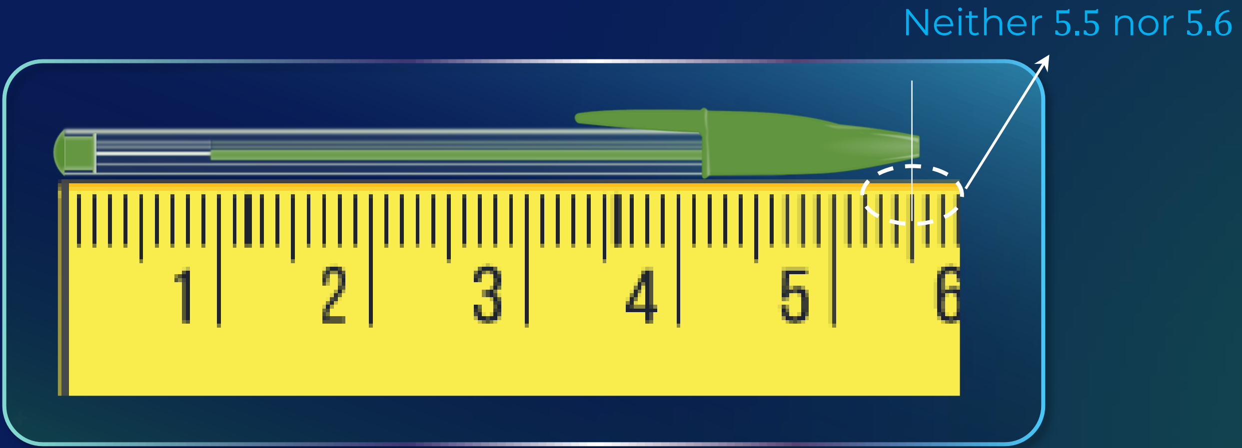
- Constant of proportionality (k) cannot be found out.

$$T = k\sqrt{\frac{l}{g}} \qquad k = ?$$

- Applicable only if relation is of product/ratio type. Fails in the case of addition, subtraction, power, exponential, logarithmic and trigonometric relations.

Error in Measurement

- The measuring process is essentially a process of comparison, in spite of our best efforts.
- The measured value of a quantity is always slightly different from its actual value or true value.
- The difference between the true value and the measured value of a quantity is called error of measurement.



Rules for counting Significant figures

1. All non-zero digits are significant.
2. All zeroes become significant if they appears between two non-digit numbers.
3. Leading zeroes or the zeroes placed to the left of the first non-zero digit are insignificant.
4. Zeroes placed to the right of the number after a decimal place are significant.
5. In exponential notation, the numerical portion gives the number of significant figures.

Count the number of significant figures in 8.25 , 100 , 7.00 , 3.238×10^9 respectively.

Solution:

8.25	$\longrightarrow$	8.25	(Three)
100	$\longrightarrow$	100	(One)
7.00	$\longrightarrow$	7.00	(Three)
3.238×10^9	$\longrightarrow$	3.238×10^9	(Four)

Number of significant figures are $3, 1, 3, 4$ respectively.

Count the number of significant figures in 8.25 , 100 , 7.00 , 3.238×10^{9} respectively.

A. 2 , 3 , 4 , 5

B. 1 , 2 , 3 , 4

C. 3 , 1 , 3 , 4

D. 2 , 3 , 1 , 4

Rules for Rounding off

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
2. If the digit to be dropped is more than 5, then the preceding digit is raised by one.
3. If the digit to be dropped is 5 followed by digits other than zeroes, then the preceding digit is raised by one.
4. If the digit to be dropped is 5 or 5 followed by zeroes, then preceding digit is left unchanged, if it is even.
5. If the digit to be dropped is 5 or 5 followed by zeroes, then preceding digit is raised by one, if it is odd.

Round off the following numbers to 1 decimal
6.87 , 7.43 , 3.250 , 4.750 respectively.

Solution:

6.87 $\longrightarrow$ 6.87 $\longrightarrow$ 6.9

7.43 $\longrightarrow$ 7.43 $\longrightarrow$ 7.4

3.250 $\longrightarrow$ 3.250 $\longrightarrow$ 3.2

4.750 $\longrightarrow$ 4.750 $\longrightarrow$ 4.8

Rounded off numbers are 6.9 , 7.4 , 3.2 , 4.8

Significant figures in Calculation

Case 1: Addition/Subtraction

The final result should retain as many decimal places as there are in the original number with the least decimal places.

Example:

$$\begin{array}{r} 33.3 \\ 3.11 \\ +\ 0.313 \\ \hline 36.7 \end{array} \qquad \begin{array}{r} 14.37 \\ -\ 0.31 \\ \hline 14.06 \end{array}$$

Significant figures in Calculation

Case 2: Multiplication/Division

The final result should retain as many decimal places as there are in the original number with the least decimal places.

Example:

Given that distance, $d = 25.5\ m$ and time, $t = 5\ s$. Find the speed?

Solution: $d = 25.5\ m$, $t = 5\ s$

Upto 1 decimal place

Speed, $s = \frac{d}{t} = \frac{25.5\ m}{5\ s} = 5.1\ m/s = \mathbf{5\ m/s}$

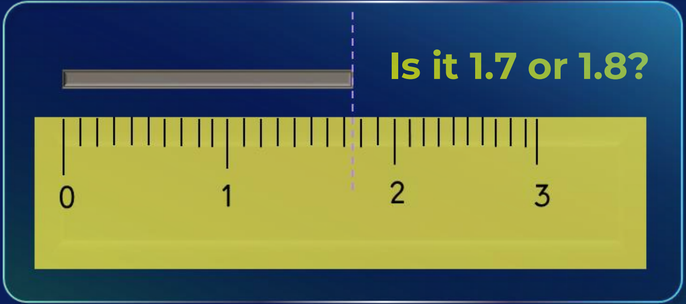
Upto 0 decimal place

Upto 0 decimal place

Errors in the Measurement

The difference between the true value and the measured value of a quantity is called error in measurement.

$$\text{Error} = \text{True Value} - \text{Measured Value}$$



- The measuring process is essentially a process of comparison.
- In spite of our best efforts, the measured value of quantity is always slightly different from its actual value or true value.
- Any measurement made without the knowledge of the degree of uncertainty is meaningless.

Types of Errors

- Types of Error
  - Random Errors
  - Systematic Errors
    - Instrumental
    - Procedural
    - Personal

Random Error

- Errors which occur due to unknown and unpredictable changes in experiment are known as Random errors.

Reason

- Random and unpredictable fluctuations in experimental conditions
- Personal (unbiased) errors by the observer taking readings

Least Count Error: Error associated with the resolution of the instrument. Within a limited size; it occurs with both systematic and random errors

Systematic Error

- Errors that tend to be in one direction, either positive or negative. They are identifiable and can be fixed.

Types of Systematic Error

Instrumental	Experimental Technique	Personal
• Due to imperfect design or calibration of the measuring instrument • zero error in the instrument	• Due to flawed experimental procedure inaccuracies are generated	• Due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations

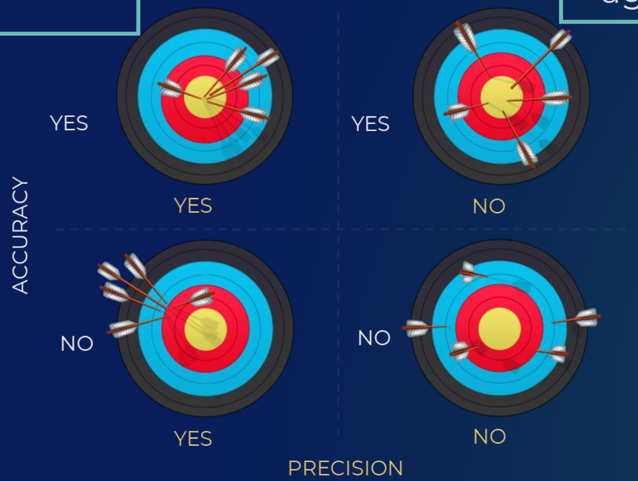
Accuracy and Precision

Accuracy

- It is the ability of the instrument to measure as close as possible to the true value.
- It could be defined as the level of correctness of a measurement to its true value.
- It represents how closely the results agree with the standard value.

Precision

- It could be defined as the sharp exactness of the measurement.
- The closeness of two or more measurements to each other is known as the precision of the measurement.
- It represents how closely results agree with one another.



Absolute Error

- The magnitude of the difference between the true value and the measured value of a quantity.

Let the physical quantity be measured n times.

Let the measured values be $a_1, a_2, a_3 \ldots a_n$.

The arithmetic mean of these values is:

$$\boxed{a_m = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}}$$

Usually, a_m is taken as the true value of the quantity.

Absolute Error

- The errors in the measured values of the quantity:

$$\Delta a_1 = a_1 - a_m$$
$$\Delta a_2 = a_2 - a_m$$
$$.$$
$$.$$
$$\Delta a_n = a_n - a_m$$

- The errors can be positive or negative. But absolute errors |Δa| will always be positive.

- Mean absolute error: Arithmetic mean of the magnitudes of absolute errors.

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ \ldots . + |\Delta a_n|}{n}$$

- Result of the measurement:

$$a = a_m \pm \overline{\Delta a} \quad \begin{cases} a = a_m + \overline{\Delta a} \\ a = a_m - \overline{\Delta a} \end{cases}$$

Relative and Percentage Error

- Relative Error

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\overline{\Delta a}}{a_m}$$

- Percentage Error

$$\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100$$

In a series of successive measurements in an experiment, the readings of the length of a rod are found to be $10.3\ m$, $10.6\ m$, $9.4\ m$, $9.7\ m$, $10.2\ m$ and $9.8\ m$. Find the percentage error in the measurement.

Solution:

Arithmetic Mean:

$$a_m = \frac{a_1 + a_2 + a_3 + \ \dots + a_n}{n} = \frac{10.3 + 10.6 + 9.4 + 9.7 + 10.2 + 9.8}{6} = 10\ cm$$

Mean Absolute Error:

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ \dots + |\Delta a_n|}{n}$$

$$= \frac{|10 - 10.3| + |10 - 10.6| + |10 - 9.4| + |10 - 9.7| + |10 - 10.2| + |10 - 9.8|}{6} = \frac{2.2}{6} = 0.366 = 0.4\ m$$

Percentage Error:

$$\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100 = \frac{0.4}{10} \times 100 \ \boxed{= 4\ \%}$$

In a series of successive measurements in an experiment, the reading of the period of oscillation of a simple pendulum were found to be $2.63\ s$, $2.56\ s$, $2.42\ s$, $2.71\ s$ and $2.80\ s$. Find the time period of simple pendulum?

Arithmetic Mean:

$$a_m = \frac{a_1 + a_2 + a_3 + \ \ldots . + a_n}{n}$$

$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= 2.62\ s$$

Mean Absolute Error:

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ \ldots . + |\Delta a_n|}{n}$$

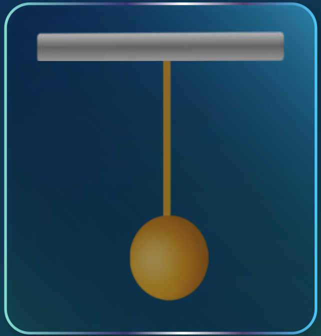
$$= \frac{|2.62 - 2.63| + |2.62 - 2.56| + |2.62 - 2.42| + |2.62 - 2.71| + |2.62 - 2.80|}{5}$$

$$= \frac{0.54}{6} = 0.108 = 0.11\ s$$

Percentage Error:

$$\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100 = \frac{0.11}{2.62} \times 100 = 4\ \%$$

Time period of simple pendulum: $\boldsymbol{T = 2.6\ s \pm 4\%}$



Error in Sum and Difference

Suppose $x = a + b$

The maximum absolute error in x:
$\Delta x = (\Delta a + \Delta b)$

Suppose $x = a - b$

The maximum absolute error in x:
$\Delta x = (\Delta a + \Delta b)$

The percentage error in the value of x:

$$\boxed{\frac{(\Delta a + \Delta b)}{a + b} \times 100}$$

Where,

Δa = Absolute error in measurement of a

Δb = Absolute error in measurement of b

Δx = Absolute error in measurement of x

Error in the quantities always gets added irrespective of their addition or subtraction.

The lengths of the two rods are recorded as $25.2 \pm 0.1\ cm$ and $16.8 \pm 0.1\ cm$. Find the sum and difference of the lengths of the two rods with the limits of errors.

Sum of the lengths:

$$\text{Sum} = (l_1 + l_2) \pm (\Delta l_1 + \Delta l_2)$$

$$= (25.2 + 16.8) \pm (0.1 + 0.1)$$

$$= \boxed{42.0 \pm 0.2\ cm}$$

Difference of the lengths:

$$\text{Difference} = (l_1 - l_2) \pm (\Delta l_1 + \Delta l_2)$$

$$= (25.2 - 16.8) \pm (0.1 + 0.1)$$

$$= \boxed{8.4 \pm 0.2\ cm}$$

Error in Product and Division

Suppose $x = a \times b$

The maximum fractional error in x:

$$\frac{\Delta x}{x} = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Suppose $x = \dfrac{a}{b}$

The maximum fractional error in x:

$$\frac{\Delta x}{x} = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Where,

Δa = Absolute error in measurement of a

Δb = Absolute error in measurement of b

Δx = Absolute error in measurement of x

If the length and breadth of a plane are $(40.0 \pm 0.2)\ cm$ and $(20.0 \pm 0.1)\ cm$. Find the Area of the plane.

Solution:

Area of the plane:

$$A = l \times b$$

$$= 40.0 \times 20.0$$

$$= 800\ cm^2$$

Fractional Error in area:

$$\frac{\Delta A}{A} = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)$$

$$= \frac{0.2}{40} + \frac{0.1}{20}$$

$$= 0.01$$

$$\Delta A = 800 \times 0.01$$

$$= 8$$

Area with Limits of Error:

Area $= A \pm \Delta A$

$$= \boxed{800 \pm 8\ cm^2}$$

Error in Quantity raised to Power

Suppose $x = \frac{a^n}{b^m}$

The maximum fractional error in x:

$$\boxed{\frac{\Delta x}{x} = \left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)}$$

Where,

Δa = Absolute error in measurement of a

Δb = Absolute error in measurement of b

Δx = Absolute error in measurement of x

?

In an experiment, four quantities a, b, c and d are measured with percentage error $1\ \%, 2\ \%, 3\ \%$ and $4\ \%$ respectively. Quantity P is calculated as $P = a^3b^2/cd$. Find the percentage error in P.

Solution:

Quantity P: $P = a^3b^2c^{-1}d^{-1}$

Percentage error in P:

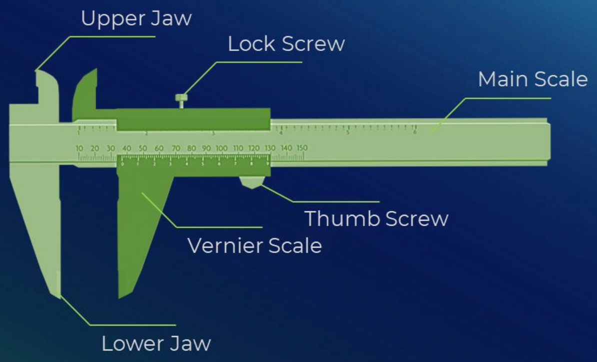
$$\frac{\Delta P}{P} \times 100 = 3 \times \frac{\Delta a}{a} \times 100 + 2 \times \frac{\Delta b}{b} \times 100 + 1 \times \frac{\Delta c}{c} \times 100 + 1 \times \frac{\Delta d}{d} \times 100$$

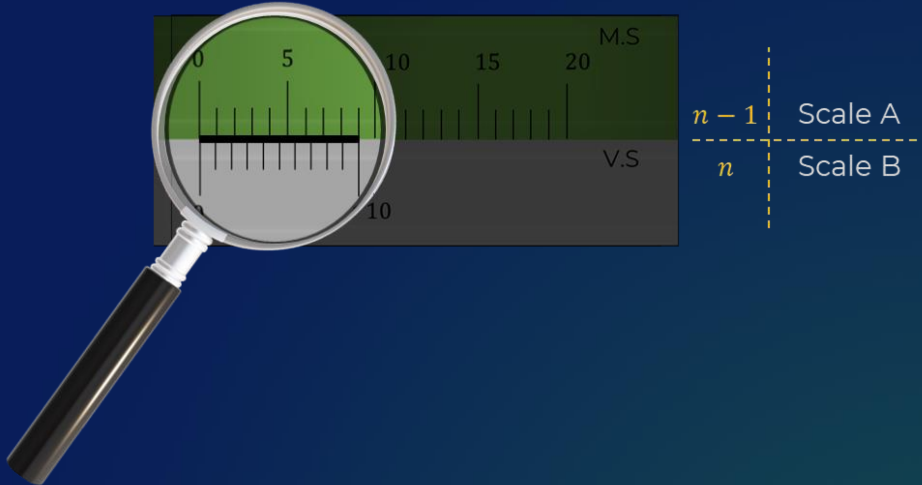
$$= (3 \times 1\ \%) + (2 \times 2\ \%) + (1 \times 3\ \%) + (1 \times 4\ \%)$$

$$= \boxed{14\ \%}$$

Vernier Callipers

Principle: Vernier callipers uses the principle of alignment of line segments to determine a more accurate reading.





$n-1$ divisions of Scale A = n divisions of Scale B

$n-1$ (Scale A) = n (Scale B)

Scale B = $\frac{n-1}{n}$ (Scale A)

Vernier Callipers

Least Count: The smallest measurement that can be taken accurately with an instrument is called as its least count.

Least Count = 1 M.S.D – 1 V.S.D

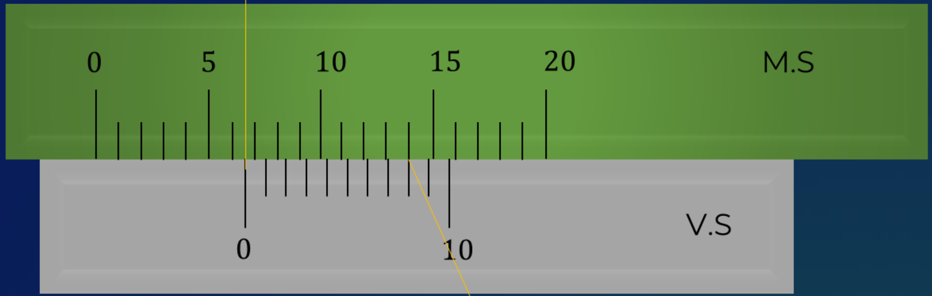
1 V.S.D = $\frac{n-1}{n}$ 1 M.S.D

Least Count = $\frac{1}{n}$ M.S.D

$$\text{Least Count} = \frac{\text{Value of 1 M.S.D}}{\text{No. of divisions on vernier scale}}$$

Reading of Vernier Callipers:

0 of vernier is in between 6 and 7.
Main scale reading is $6.0\ mm$



8 Coincides with marking on main scale .Vernier scale reading is 0.8

Total = $6.0 + 8 \times 0.1 = 6.8\ mm$

Least Count of Vernier Callipers

Least Count:

Now, suppose 'n' V.S.D coincides with 'm' M.S.D.

$$n \text{ V.S.D} = m \text{ M.S.D}$$

$$\text{V.S.D} = \left(\frac{m}{n}\right) \text{ M.S.D}$$

$$\text{L.C} = 1 \text{ M.S.D} - 1 \text{ V.S.D}$$

$$= 1 \text{ M.S.D} - \left(\frac{m}{n}\right) \text{M.S.D}$$

$$\boxed{L.C = M.S.D \left(1 - \frac{m}{n}\right)}$$

A vernier caliper has $1\ mm$ marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. Find the least count of the vernier caliper ?

Given: Main scale reading = $1\ mm$

To find: Least count of the vernier callipers

Formula: Least Count = M.S.D $\left(1 - \frac{m}{n}\right)$

Solution: Main scale reading = $1\ mm$

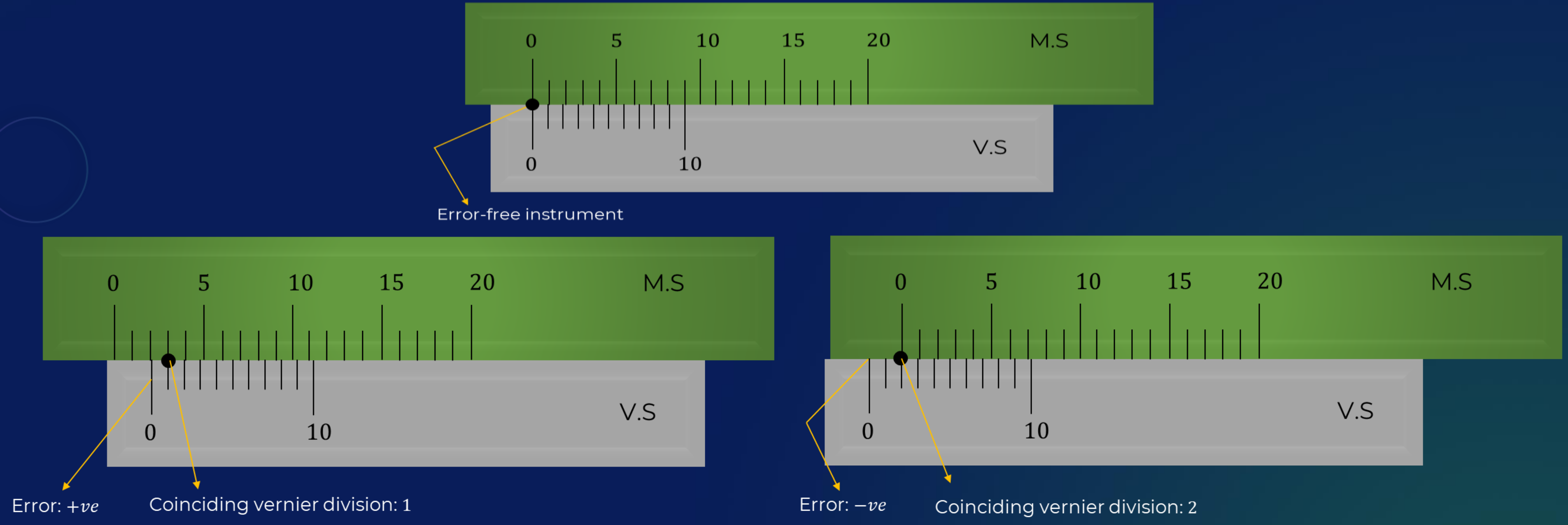
20 V.S.D $= 16$ M.S.D

$\Rightarrow$ V.S.D $= \left(\frac{16}{20}\right)$ M.S.D = 0.8 M.S.D = 0.8 mm

Least Count = M.S.D − V.S.D = 1−0.8 mm = **0.2 mm**

Zero Error

If the zero of the main scale does not coincide with the zero of the vernier scale, then the instrument is said to have zero error.



Correct reading = Measured reading $\pm$ Zero error

Zero Error = Coinciding vernier division with main scale × L.C

Find the zero error in the scale given.

Solution: Vernier scale is ahead of main scale, so the scale has positive zero error.

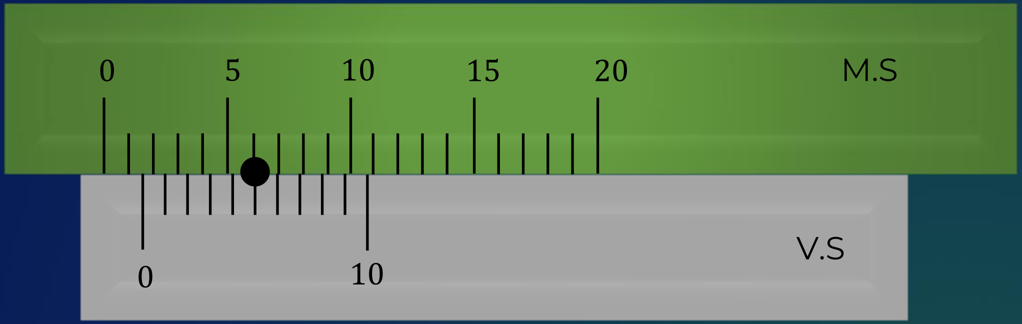
Coinciding vernier division is 5

Zero Error = Coinciding vernier division with main scale $\times$ L.C

L.C $= 0.1\ mm$

Zero Error $= +5 \times 0.1\ mm$

Zero Error $= +0.5\ mm$



The diameter of a steel ball is measured using a no zero error vernier callipers which has divisions of $0.05\ cm$ on its main scale (MS) and 16 divisions of its vernier scale (VS) match 12 divisions on the main scale. The measurements for a ball is given as (MSR – $0.5\ cm$, VSD – 8):

Solution:

To find the least count:

16 V.S.D = 12 M.S.D

V.S.D = $\frac{12}{16} \times 0.05\ cm$

V.S.D = $0.0375\ cm$

L.C = M.S.D – V.S.D

L.C $= (0.05 - 0.0375)\ cm$

L.C $= 0.0125\ cm$

Total reading = M.S.R + V.S.R

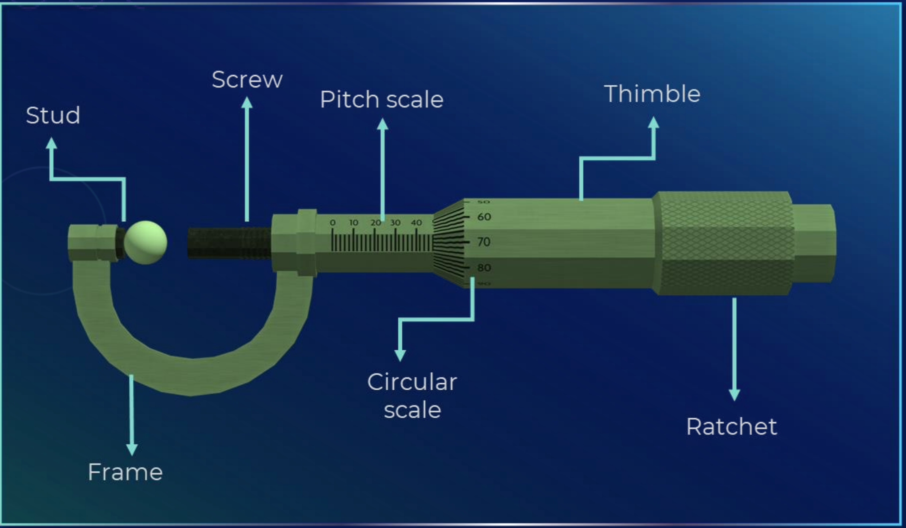
V.S.R = Coinciding division of V.S × L.C

V.S.R $= 8 \times 0.0125\ cm$

V.S.R $= 0.1\ cm$

Total reading $= 0.5\ cm + 0.1\ cm = 0.6\ cm$

Construction of Screw Gauge



Principle:

By rotating the screw head , we get the linear movement of the main scale.

Pitch:

Distance moved by screw due to one complete rotation.

$$\text{Pitch of the screw} = \frac{\text{Distance moved by the screw}}{\text{No.of full rotations given}}$$

Least Count: The least count of the screw is defined as the distance moved by the tip of the screw when turned through one division of the circular scale.

$$\text{Least Count} = \frac{\text{Pitch}}{\text{No. of circular scale Division}}$$

If a screw moves by $1\ mm$ in 2 rotations and its circular scale has 500 divisions, then find the Pitch and Least Count of the screw gauge?

Given: Screw moves by $1\ mm$ in 2 rotations & circular scale has 500 divisions.

To find: Pitch and Least count

Formula:

$$\text{Pitch of the screw} = \frac{\text{Distance moved by the screw}}{\text{No.of full rotations given}}$$

$$\text{Least Count} = \frac{\text{Pitch}}{\text{No. of circular scale Division}}$$

Solution:

$$\text{Pitch of the screw} = \frac{1\ mm}{2} = 0.5\ mm$$

$$\text{Least Count} = \frac{0.5\ mm}{500}$$

$$\boxed{\textbf{Least Count} = 0.001\ mm}$$

Screw Gauge

Least Count:

$$\text{Least Count} = \frac{\text{Pitch}}{\text{No. of circular scale Divisions}}$$

Reading = Main scale reading + Circular scale reading

= Reading on M.S just before the head of C.S + L.C × Coinciding division of C.S with base line.

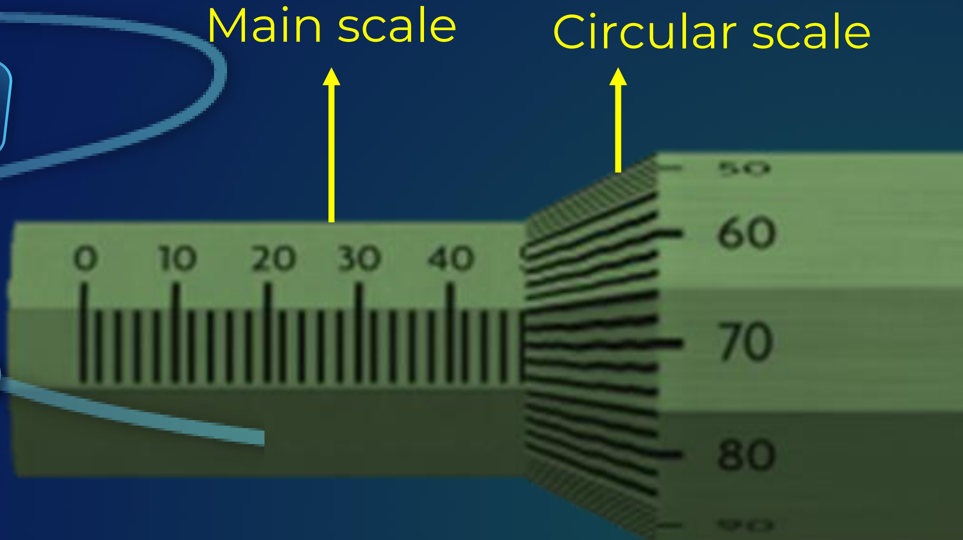
Reading Screw Gauge

01 Rotate circular scale such that stud & the screw touch each other.

02 Make sure that the zero on the Circular scale coincides with the base line of Main scale and the head of C.S. coincides with zero of Main scale.

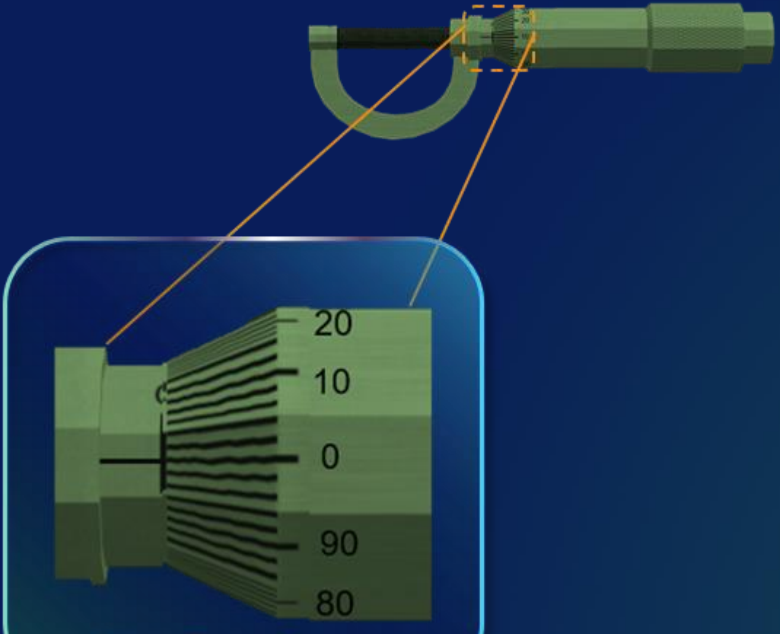
03 Put the object to be measured between the stud and the screw.

04 Note down Main scale reading and Circular scale reading.



Zero Error

If zero of circular scale does not coincide with base line (at initial condition before starting measurement, when spindle and stud touch each other), then it is said that there is a zero error in screw gauge.



Non Zero Error

Types of Zero Error

Positive error:
Zero on C.S is below base line

Negative error:
Zero on C.S is above base line

Positive error = L.C × Coinciding division of C.S with baseline.

Correct reading = Reading – Positive zero error

Negative error = L.C × Coinciding division of C.S with baseline.

Correct reading = Reading + Negative zero error

?

The pitch of the screw gauge is $1\ mm$ and there are 100 divisions on the circular scale. A student measures the diameter of a wire, and he gets the main scale reading as $5\ mm$ and circular divisions as 25. If the screw gauge has a positive zero error of $0.03\ mm$, the correct value of diameter will be:

Solution: Least Count = $\dfrac{\text{Pitch}}{\text{No. of circular scale Division}}$

$$\text{L.C} = \frac{1\ mm}{100}$$

Correct Reading= (M.S.R+ C.R×L.C) – Z.E

Correct Reading = $(5\ mm + 25 \times 0.01\ mm) - 0.03\ mm$

$$\boxed{\text{Correct Diameter} = 5.22\ mm}$$