

Welcome to



Aakash



BYJU'S

NOTES

Ray Optics and Optical Instruments

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$P = \frac{1}{f}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$





Light Beam and Its Types



Ray: The straight-line path along which light travels in a homogenous medium called a “Ray”.

The arrow represents the direction of propagation of light.

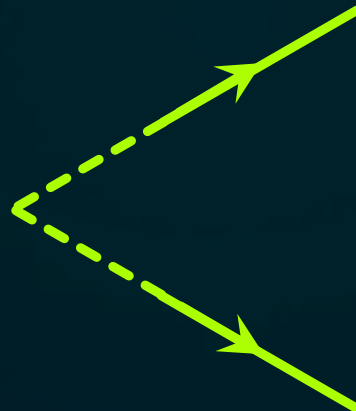
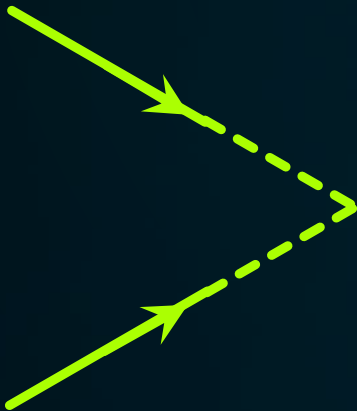


- A bundle or bunch of rays is called a light beam.

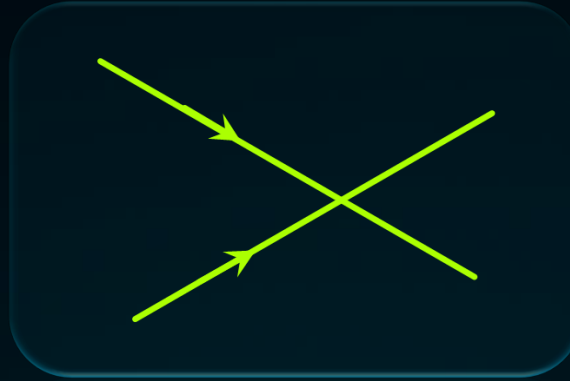
Convergent Beam

Divergent Beam

Parallel Beam



Principle of Mutual Independence of Rays

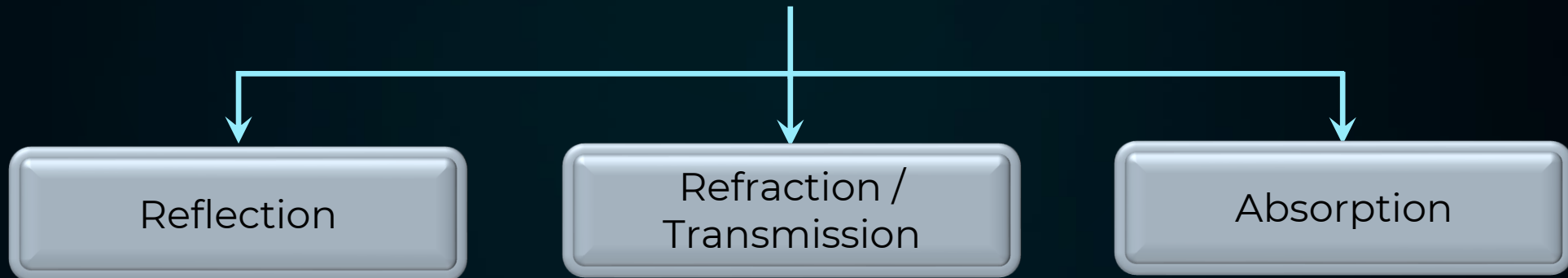


- **Statement:**

Rays do not disturb each other at an intersection. There is no modification and interaction when different rays pass through a point in space.

If Light Is Incident On a Medium

- If light is incident on medium, it can possess three phenomena:



- In general, **dark** coloured objects are **good absorbers**, **bright** coloured objects are **good reflectors** and **transparent** objects are **good transmitters**.



Introduction to Reflection and Properties of Light



- **Reflection:**

The phenomenon in which a light ray is **sent back** into the same medium from which it is coming.

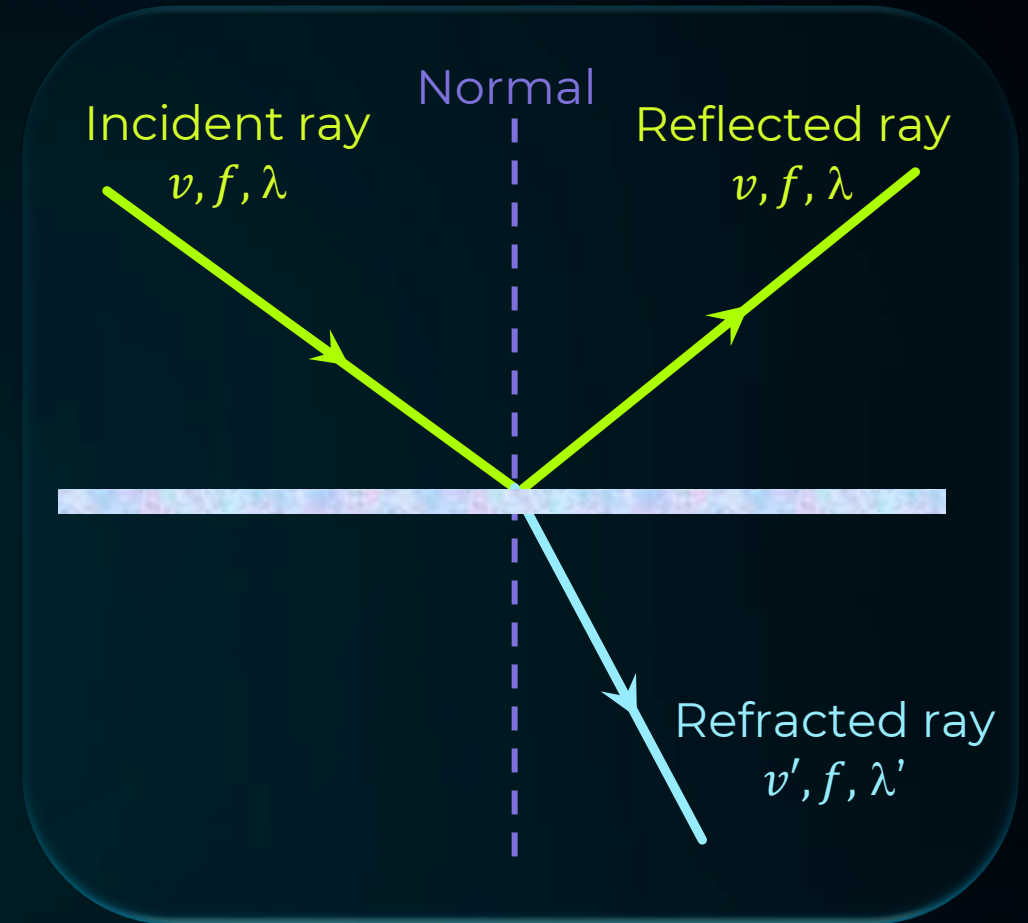
- During **reflection**:

- **No change** in frequency(f), wavelength(λ) and velocity(v).

- **Refraction:**

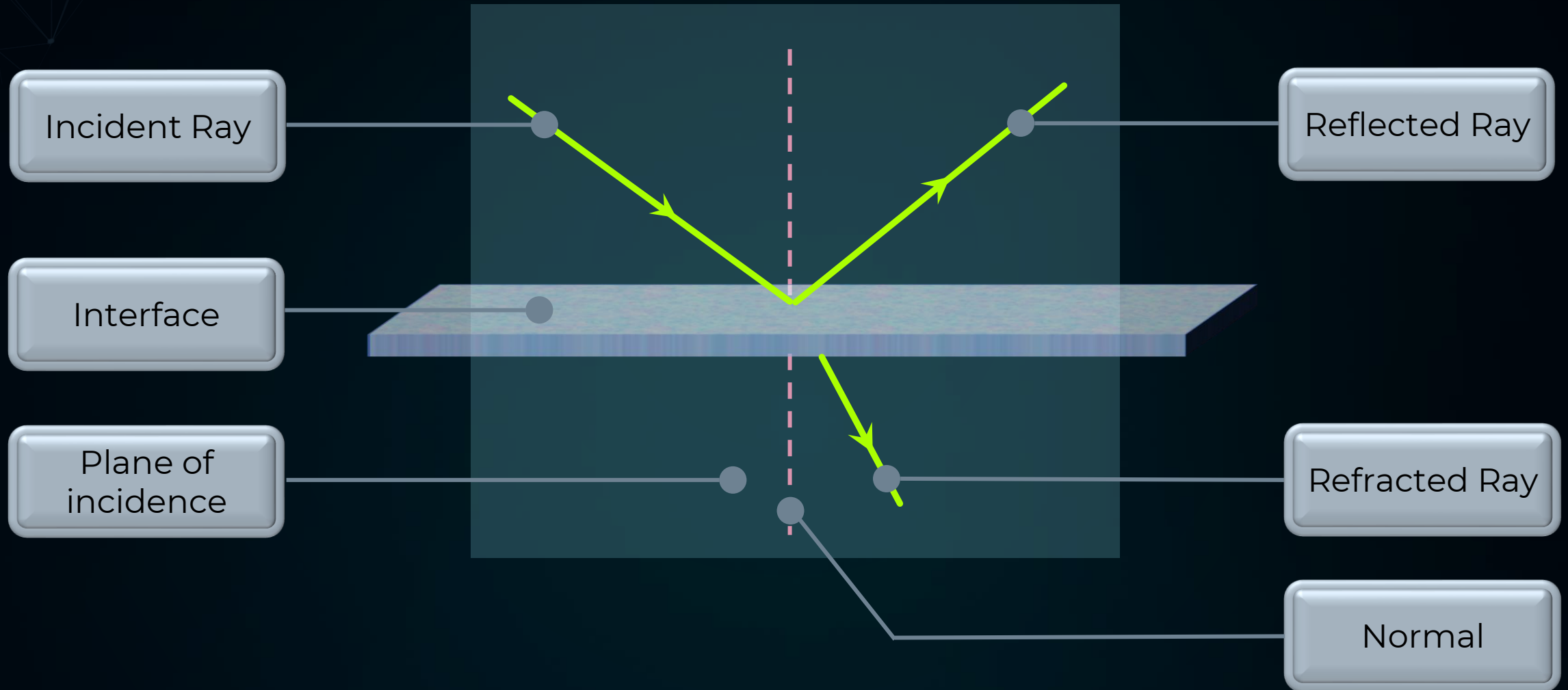
The phenomenon in which the incident ray **transmits** through the interface of two mediums and travels into the other medium with **different velocity**.

- During **refraction**:
- The **velocity** of the light **changes**, but the **frequency** remains **same** as that of the incident ray.
- So, the **wavelength** of the light **changes** as $v = f\lambda$.





Useful Nomenclature





Laws of Reflection



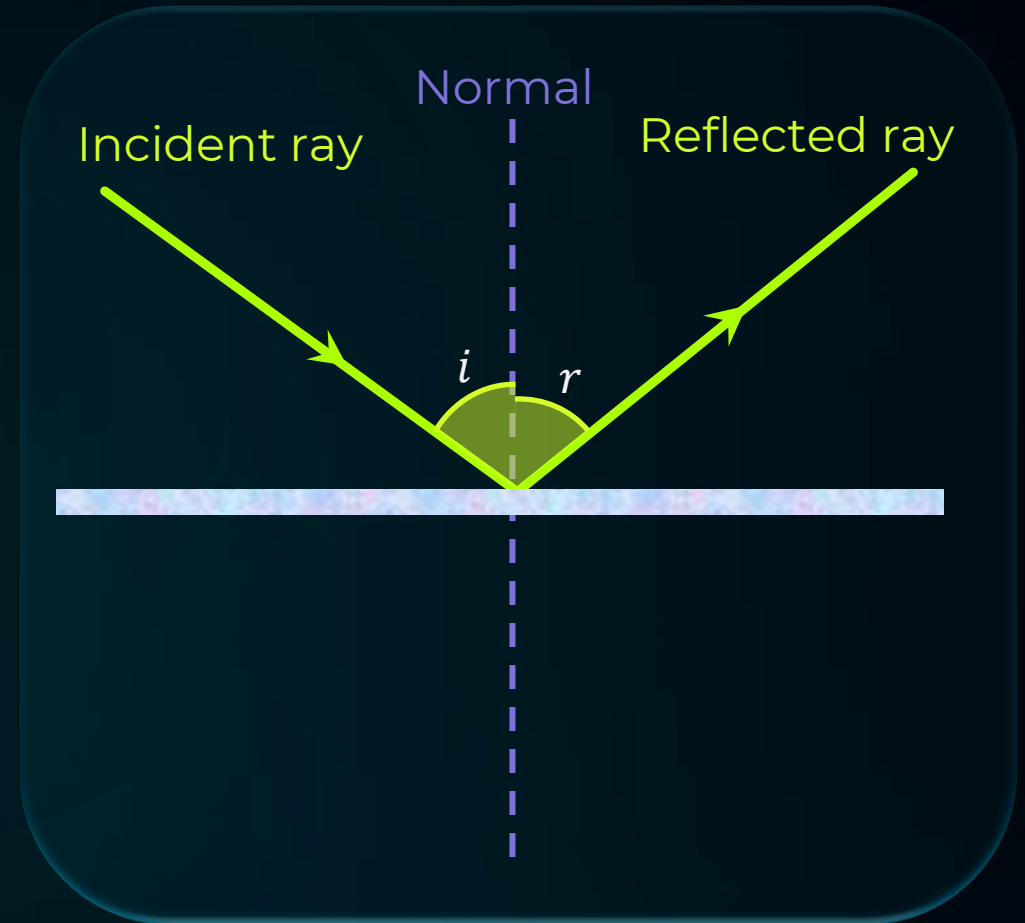
- Laws of reflection:
- The angle of incidence i is always equal to the angle of reflection r .

$$i = r$$

Note: i and r are measured w.r.t normal.

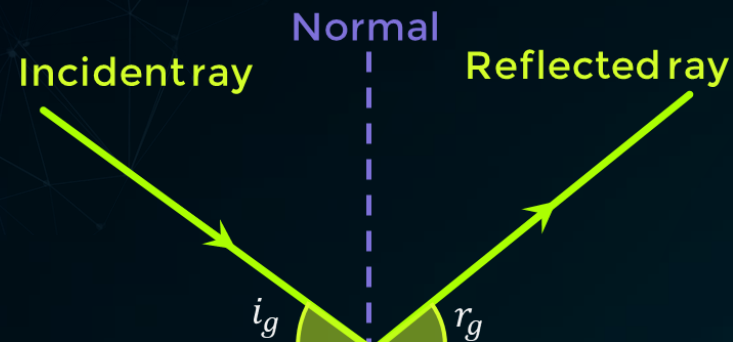
- The incident ray (\hat{I}), the reflected ray (\hat{R}), and the normal (\hat{N}) to the reflecting surface at the point of incidence are coplanar.

$$\hat{I} \cdot (\hat{N} \times \hat{R}) = 0$$





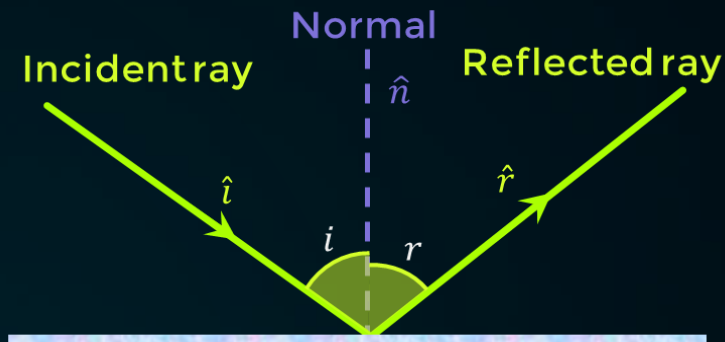
Glancing Angle



- Glancing angle of incidence (i_g):
Angle between incident ray and reflecting surface.
- Glancing angle of reflection (r_g):
Angle between reflected ray and reflecting surface.



Vector Law of Reflection



$$\hat{r} = \hat{i} - 2(\hat{i} \cdot \hat{n})\hat{n}$$

- If the unit vector of incident ray is \hat{i} , unit vector along the reflected ray is \hat{r} and unit vector along the normal is \hat{n} , then,



Vector Law of Reflection



- Proof:

Vector addition rule: $\vec{n} = (\hat{r} - \hat{i})$

Magnitude of \vec{n} :

$$|\vec{n}| = |\hat{r} - \hat{i}|$$

$$|\vec{n}| = |\hat{r}| \cos \theta + |-\hat{i}| \cos \theta$$

$$|\vec{n}| = 2 \cos \theta$$

Therefore,

$$\vec{n} = (\hat{r} - \hat{i}) = 2 \cos \theta \hat{n}$$

[Since $(\hat{r} - \hat{i})$ is along the normal]

We have, $(\hat{r} - \hat{i}) = 2 \cos \theta \hat{n}$

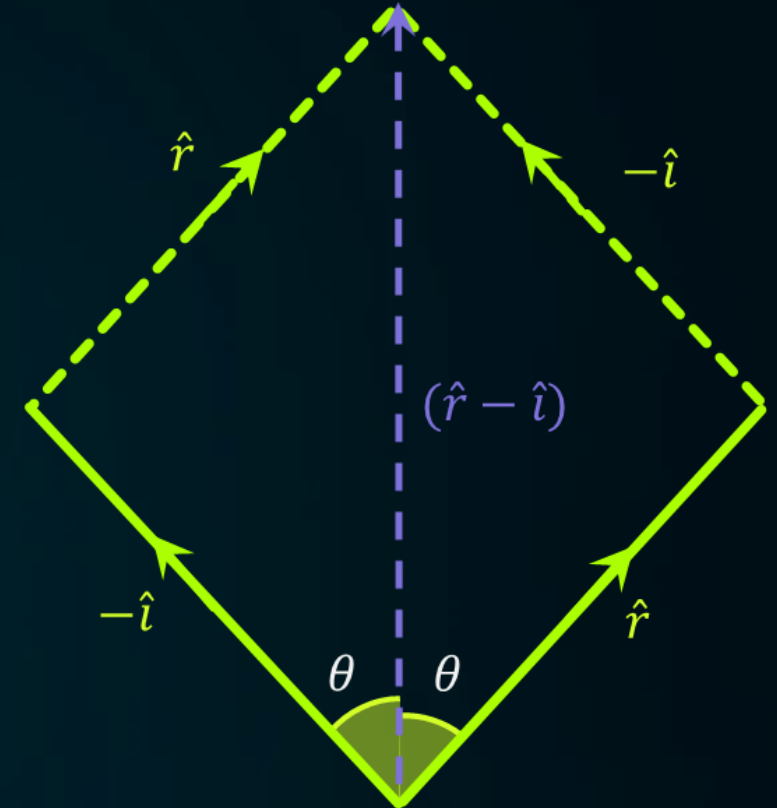
From figure,

$$-\hat{i} \cdot \hat{n} = \cos \theta$$

Substituting this value in the above equation, we get,

$$(\hat{r} - \hat{i}) = -2(\hat{i} \cdot \hat{n})\hat{n}$$

$$\hat{r} = \hat{i} - 2(\hat{i} \cdot \hat{n})\hat{n}$$



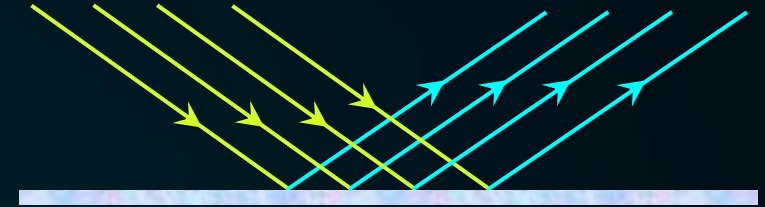


Regular and Diffused Reflection



- **Regular reflection:**

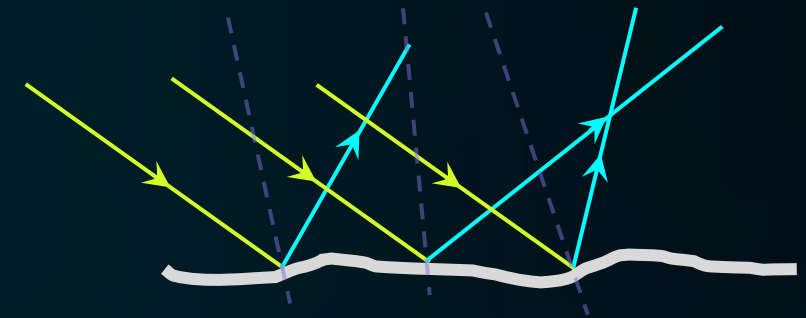
If a parallel beam of rays falls on a plane surface, the reflected beam of rays will also be parallel.



- **Diffused reflection:**

If a parallel beam of incident rays gets reflected from a rough & unpolished surface, the reflected rays will be scattered.

Note : Laws of reflection are valid in this case also.





A ray of light is incident on a circle, $x^2 + y^2 = R^2$, polished from inside as shown in the figure. Find the coordinates of the point at which the incident ray should fall, such that the reflected ray becomes **vertical**, after reflection.

Solution:

Normal at any point on a spherical surface is the line joining the centre of the spherical surface to that point.

$OS \Rightarrow$ Normal to the spherical surface.

Reflected rays become vertical after reflection. So,
 $\angle PAQ = 90^\circ$

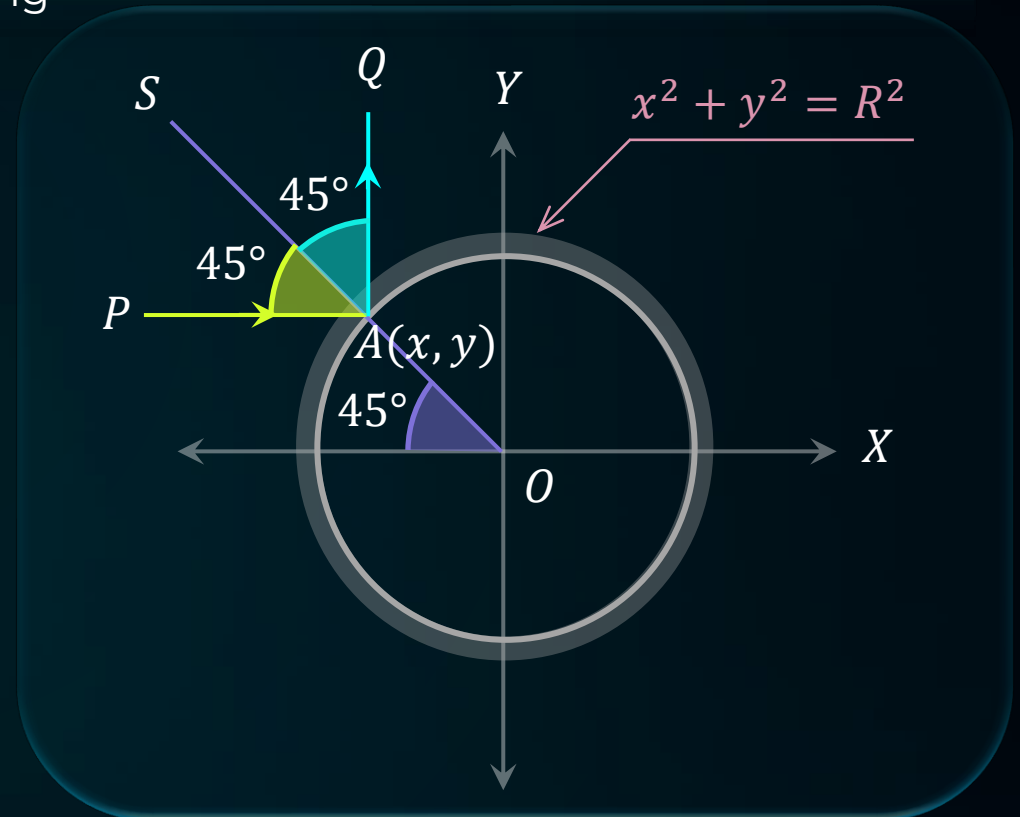
1st law of reflection: $\angle i = \angle r$

So, $\angle PAS = \angle SAQ = 45^\circ$

Co-ordinate of A is given by,

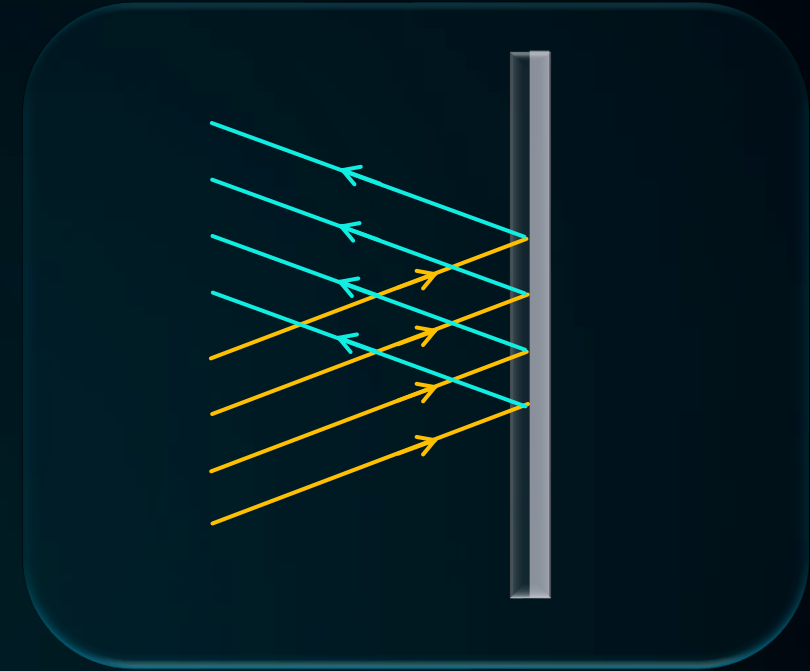
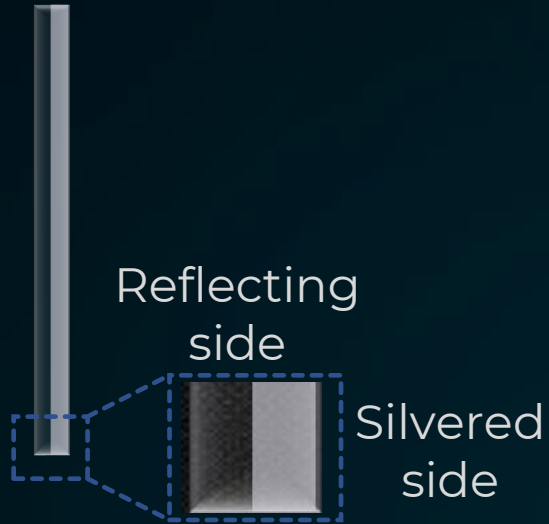
$$A(x, y) = (-R \cos 45^\circ, +R \sin 45^\circ) = \left[-\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}} \right]$$

$$(x, y) = \left(\frac{-R}{\sqrt{2}}, \frac{+R}{\sqrt{2}} \right)$$





Plane Mirror



- A plane mirror consists of two sides:
Silvered side: Coated with Aluminium/silver to enhance the reflectivity.
Reflecting side: Light falls on this part and gets reflected.
- When a parallel beam of light is incident on a plane mirror, the reflected part is also a parallel beam of light.

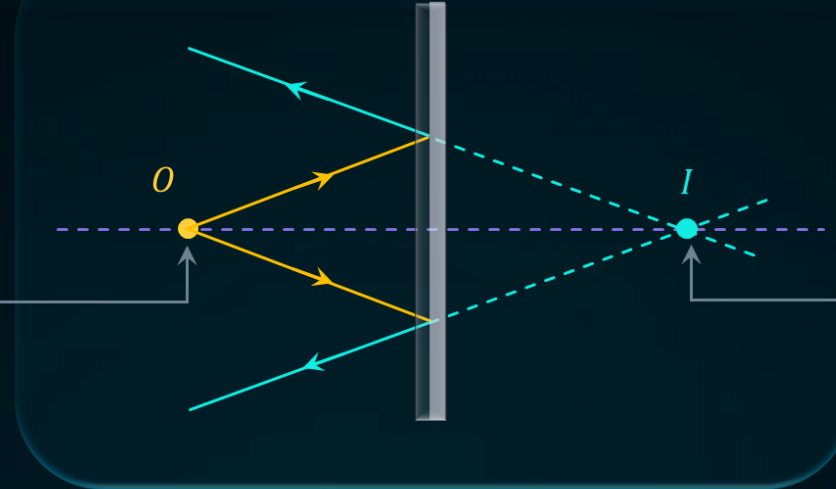


Object and Image for plane mirror



P.O.I of
incident ray:
Actual

Real Object



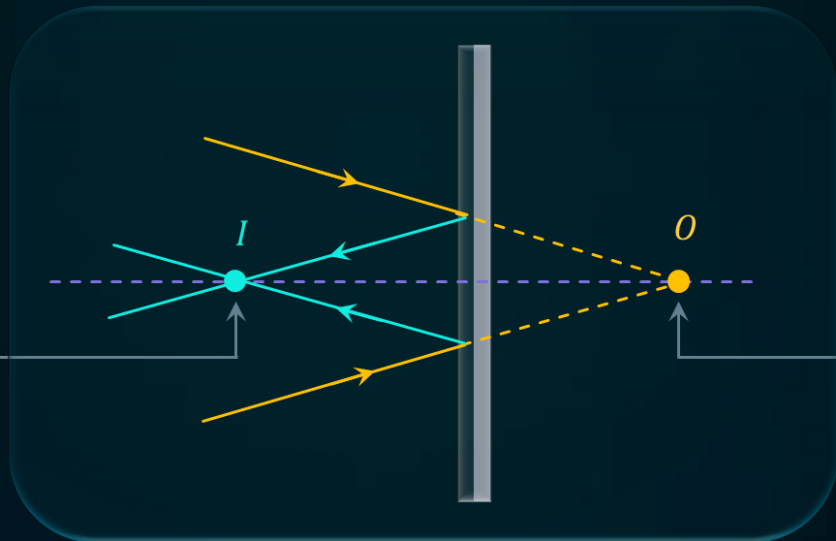
P.O.I of
reflected ray:
Apparent

Virtual Image

** P.O.I = Point of intersection

P.O.I of
reflected ray:
Actual

Real Image



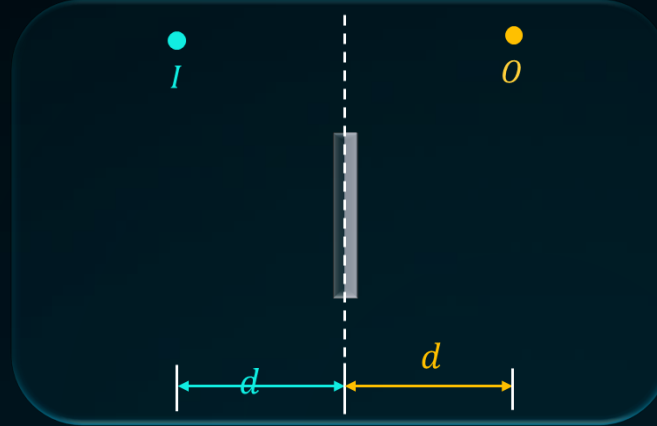
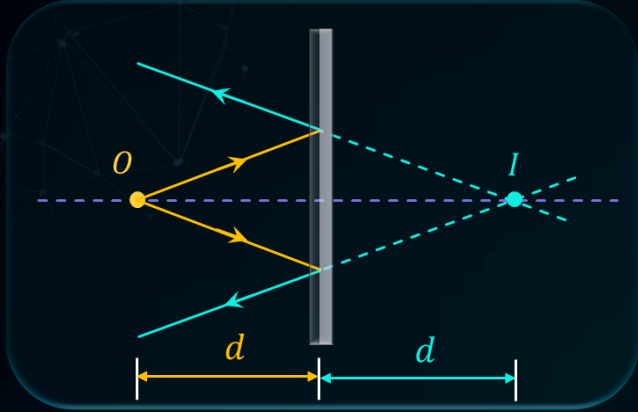
P.O.I of
incident ray:
Apparent

Virtual Object

** P.O.I = Point of intersection



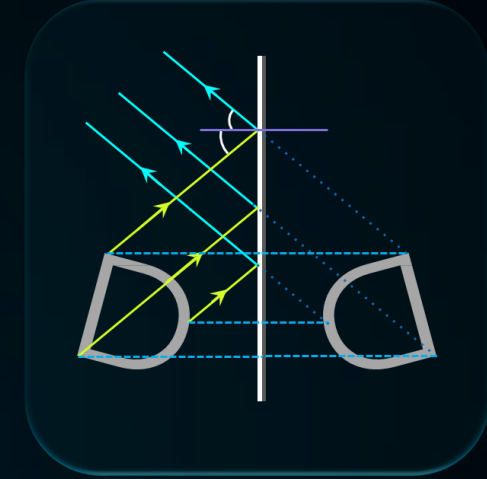
Properties of Reflection by Plane Mirror



- Object and image always are on opposite side.
- A plane mirror produces a virtual image of a real object, and vice-versa.
- Plane mirror is the **perpendicular bisector** of line joining the point object and its point image.



Lateral Inversion



- The image of an extended object made by the plane mirror undergoes **lateral inversion** or flipping of **left↔right** of the object itself.



Image of extended object by plane mirror

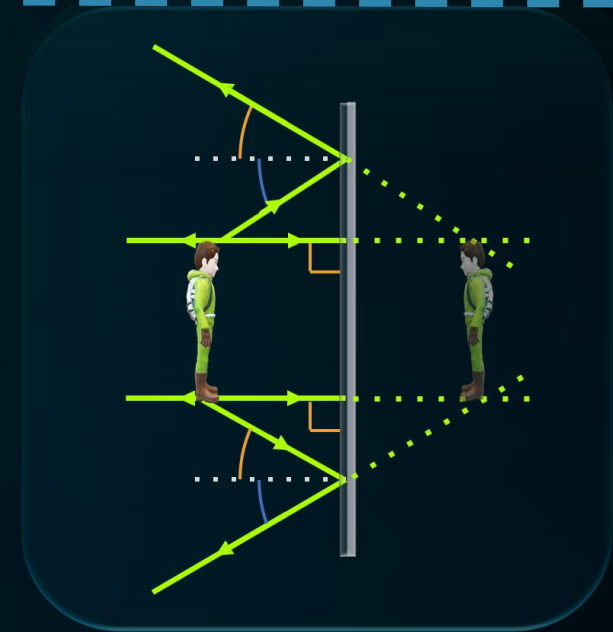


- Position of the image of an extended object can be obtained by considering two-point objects, at its extremities.
- Locate the images of these extreme points by using the principle, 'plane mirror is the perpendicular bisector of line joining the point object and its point image'.



The properties of image formed due to reflection of an extended object by a plane mirror is,

- Virtual
- Upright
- Unmagnified





Rotation of Plane Mirror



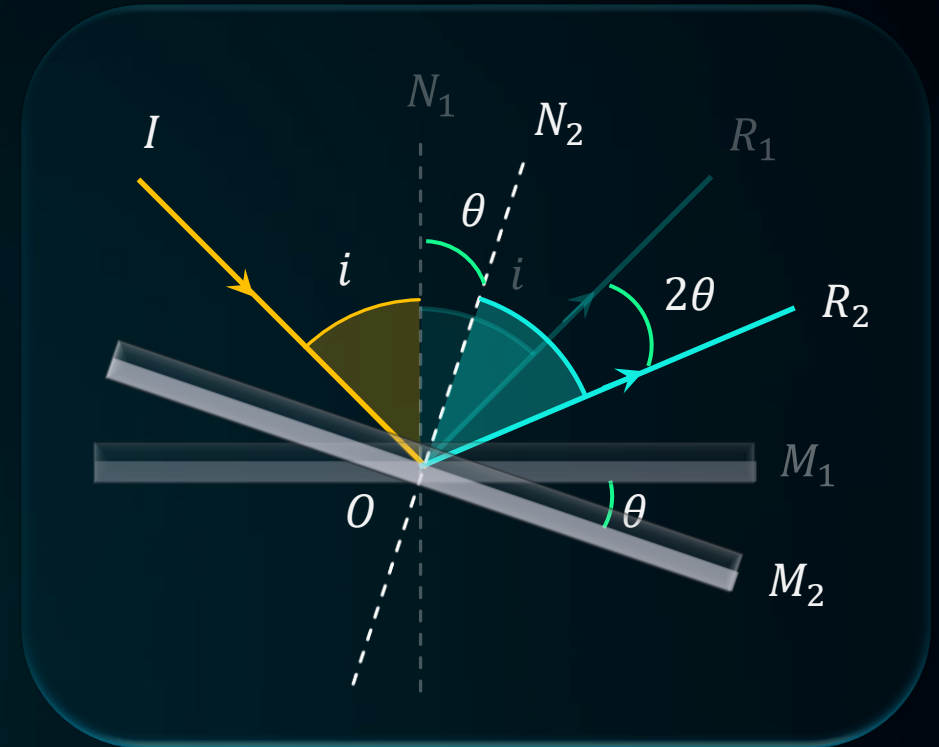
- If the mirror is rotated through an angle of θ , the reflected ray rotates through an angle of 2θ in the same direction.

$$\theta_R = 2\theta$$

θ_R is the angle of rotation of reflected ray.

- Relation between angular velocity of mirror ($\vec{\omega}$) and that of reflected ray ($\vec{\omega}_R$) is:

$$\vec{\omega}_R = 2\vec{\omega}$$





Incident Ray Is Rotated but Mirror Is Fixed



- Incident ray is rotated by an angle of θ without disturbing the mirror.



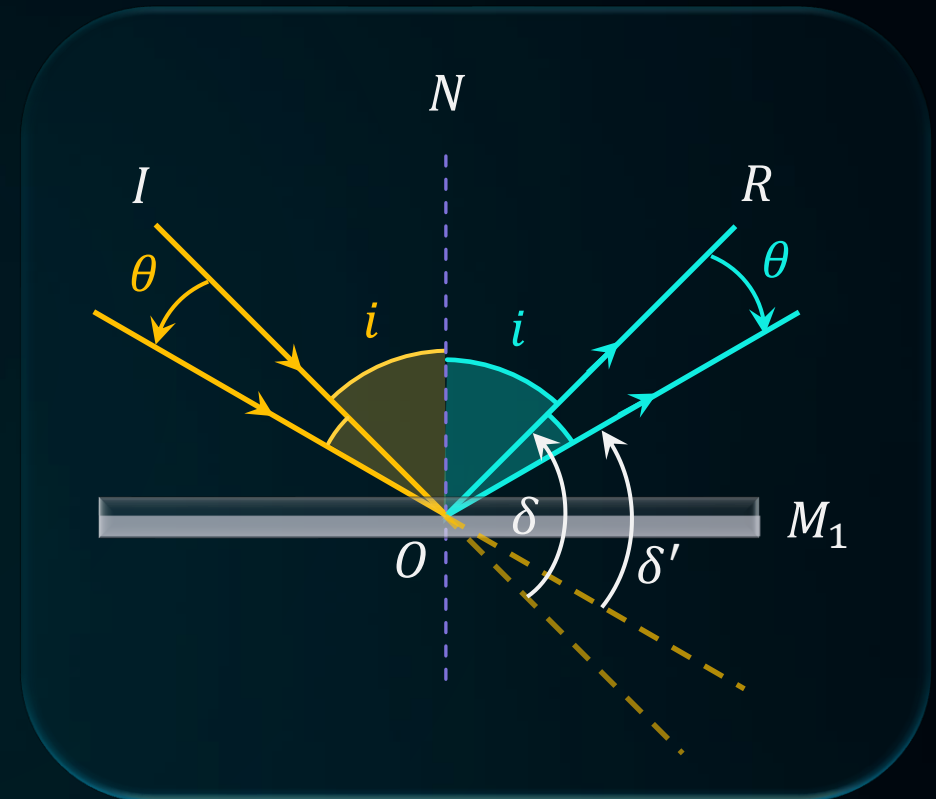
- Reflected ray will be rotated by angle θ also, but the direction of rotation will be opposite to each other.

- Deviation before rotation of incident ray:

$$\delta = \pi - 2i$$

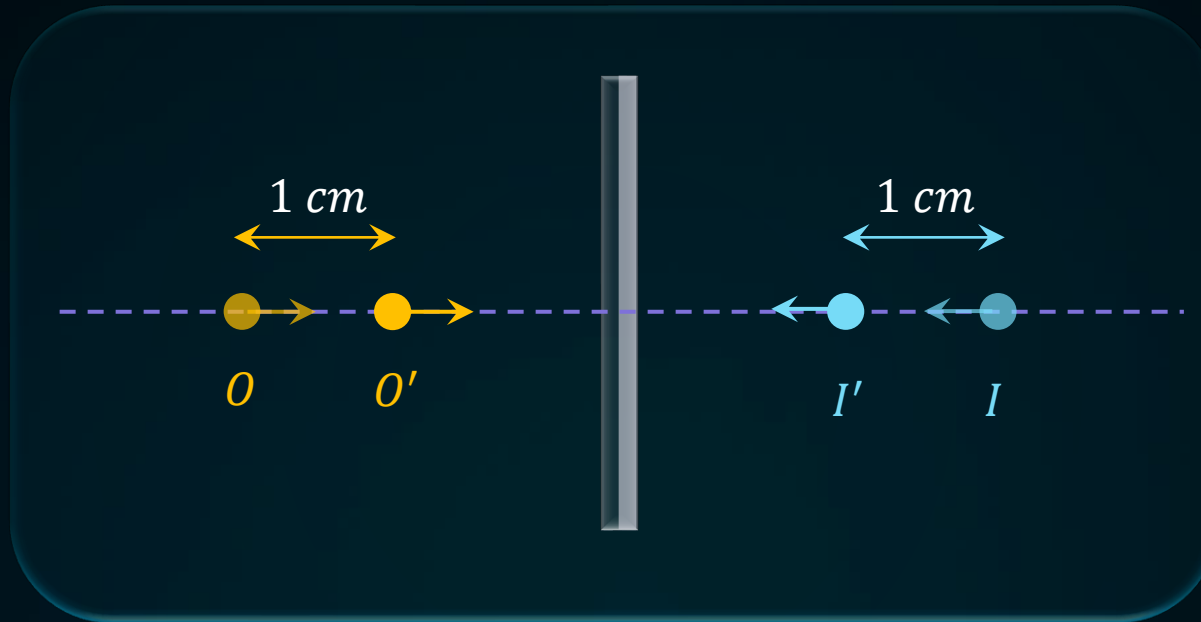
- Deviation after rotation of incident ray by θ :

$$\delta' = \delta - 2\theta$$





Speed Of Image of Moving Object



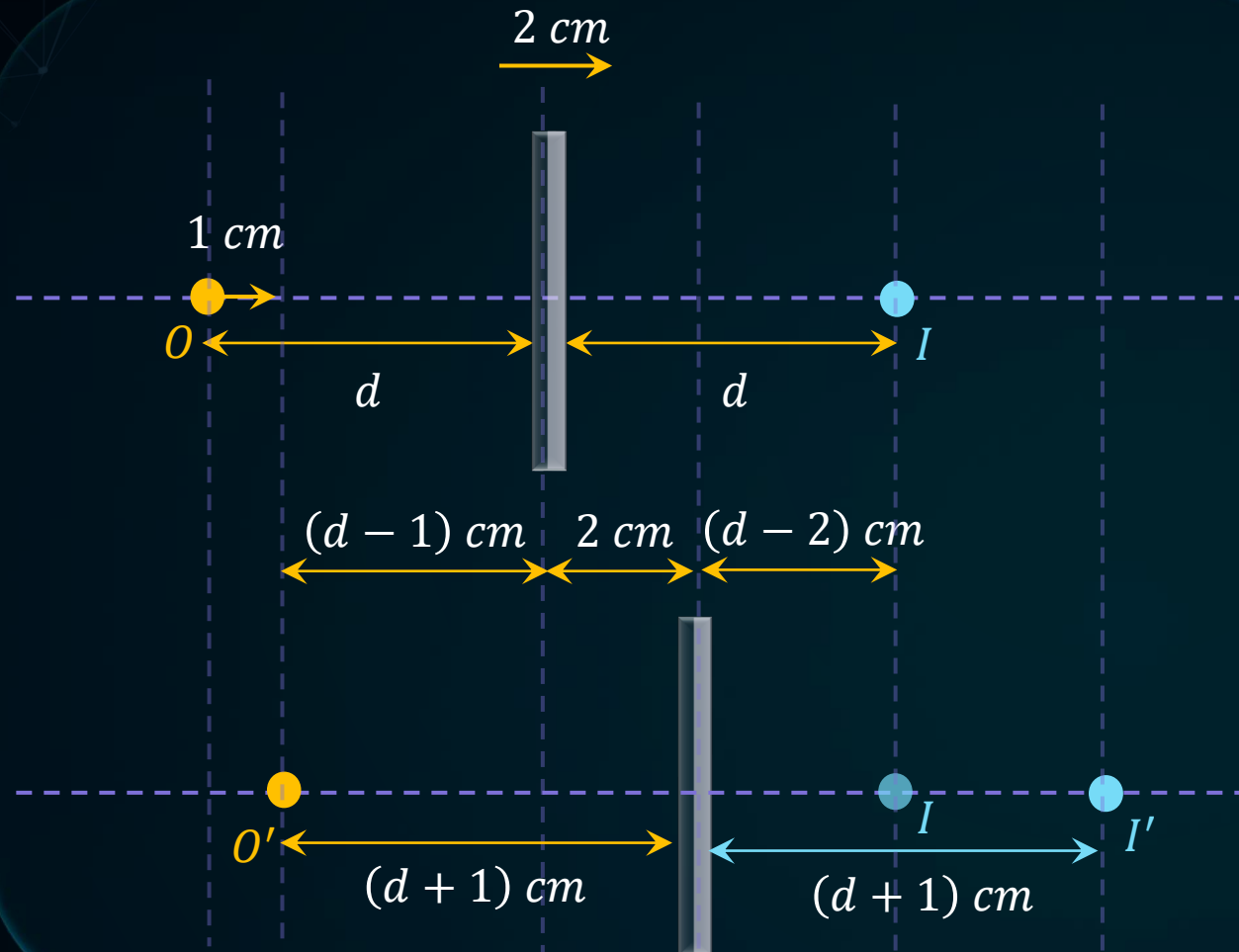
- Plane mirror is the perpendicular bisector of the line joining the object and image.



- If an object moves towards the stationary plane mirror, the image will also move towards the mirror and vice-versa.



Summary



- Mirror is moved to right by: 2 cm
- Object is moved to right by: 1 cm
- Image is moved by:
 $I'I = (d + 1) - (d - 2) = 3\text{ cm}$
(towards right)
- With respect to the plane mirror, the object distance and image distance should be same.



Summary



- In ground frame:

Object moves by: x_{OG} Mirror moves by: x_{MG}

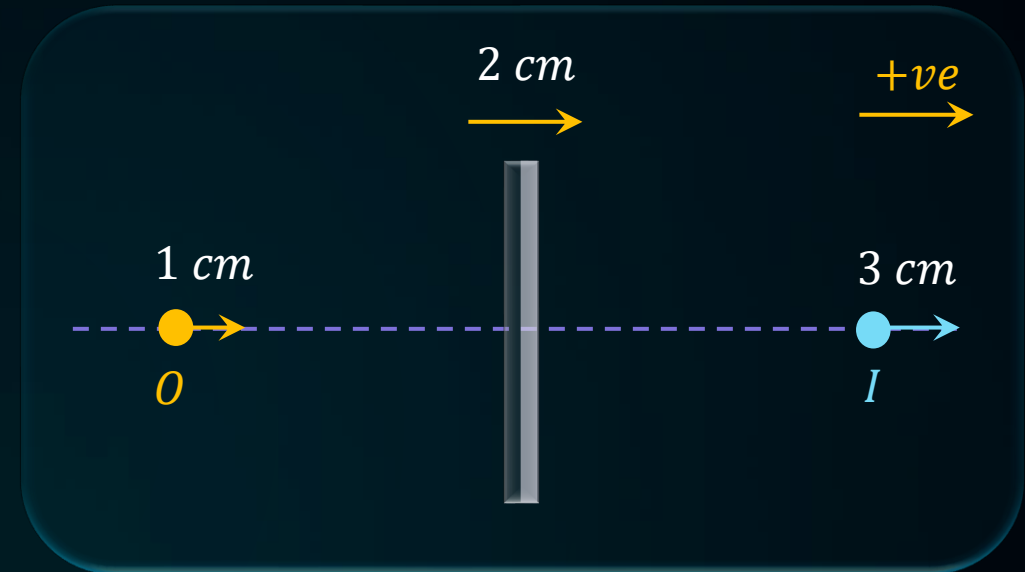
- In mirror frame: (Velocity of mirror = 0)

$$\vec{x}_{OM} = \vec{x}_{OG} - \vec{x}_{MG} \quad \vec{x}_{IM} = \vec{x}_{IG} - \vec{x}_{MG}$$

For plane mirror, motion of image is opposite to motion of object w.r.t mirror.

$$\vec{x}_{OM} = -\vec{x}_{IM}$$

$$\vec{x}_{MG} = \frac{\vec{x}_{OG} + \vec{x}_{IG}}{2}$$



For this case,

$$\vec{x}_{OG} = +1 \text{ cm} \quad \vec{x}_{MG} = +2 \text{ cm}$$

Hence,

$$\vec{x}_{IG} = 2\vec{x}_{MG} - \vec{x}_{OG} = 2(+2) - (+1) = +3 \text{ cm}$$

Image will move 3 cm towards right.



Summary



- Along x -axis:

Displacement: $\vec{x}_{OM} = -\vec{x}_{IM}$ (In mirror's frame of reference)

Velocity: $(\vec{v}_{OM})_x = -(\vec{v}_{IM})_x$

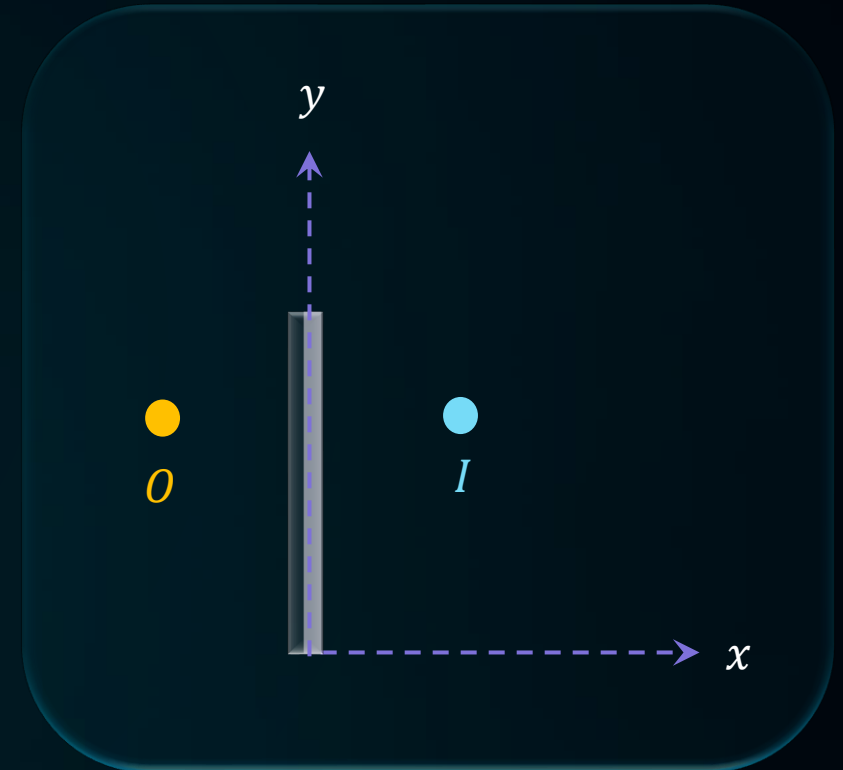
$$\vec{v}_{OG} - \vec{v}_{MG} = \vec{v}_{MG} - \vec{v}_{IG}$$

$$\vec{v}_{MG} = \frac{\vec{v}_{OG} + \vec{v}_{IG}}{2}$$

- Along y -axis:

Displacement: $\vec{y}_{OG} = \vec{y}_{IG}$ (In ground's frame of reference)

Velocity: $(\vec{v}_{OG})_y = (\vec{v}_{IG})_y$



?

There is a point object and a plane mirror. If the mirror is moved by **10 cm** away from the object, find the distance by which the image will move w.r.t ground.

Given:

1. Object is fixed
2. Mirror is moving away from object by 10 cm

To find: Shift of image

Solution:

$$\vec{x}_{OG} = 0 \quad \vec{x}_{MG} = +10 \text{ cm}$$

We have:

$$\vec{x}_{MG} = \frac{\vec{x}_{OG} + \vec{x}_{IG}}{2}$$

$$\vec{x}_{IG} = 2\vec{x}_{MG} - \vec{x}_{OG}$$

$$\vec{x}_{IG} = 2 \times (+10) - 0 = +20 \text{ cm}$$

Since the sign of the vector is positive, in the ground frame the image will be **shifted to the right** by **20 cm**.

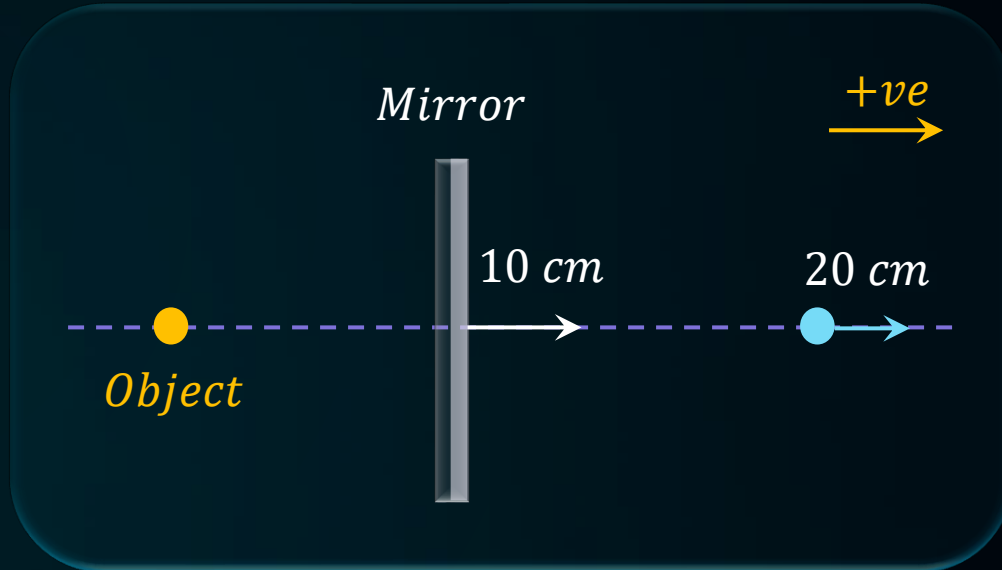
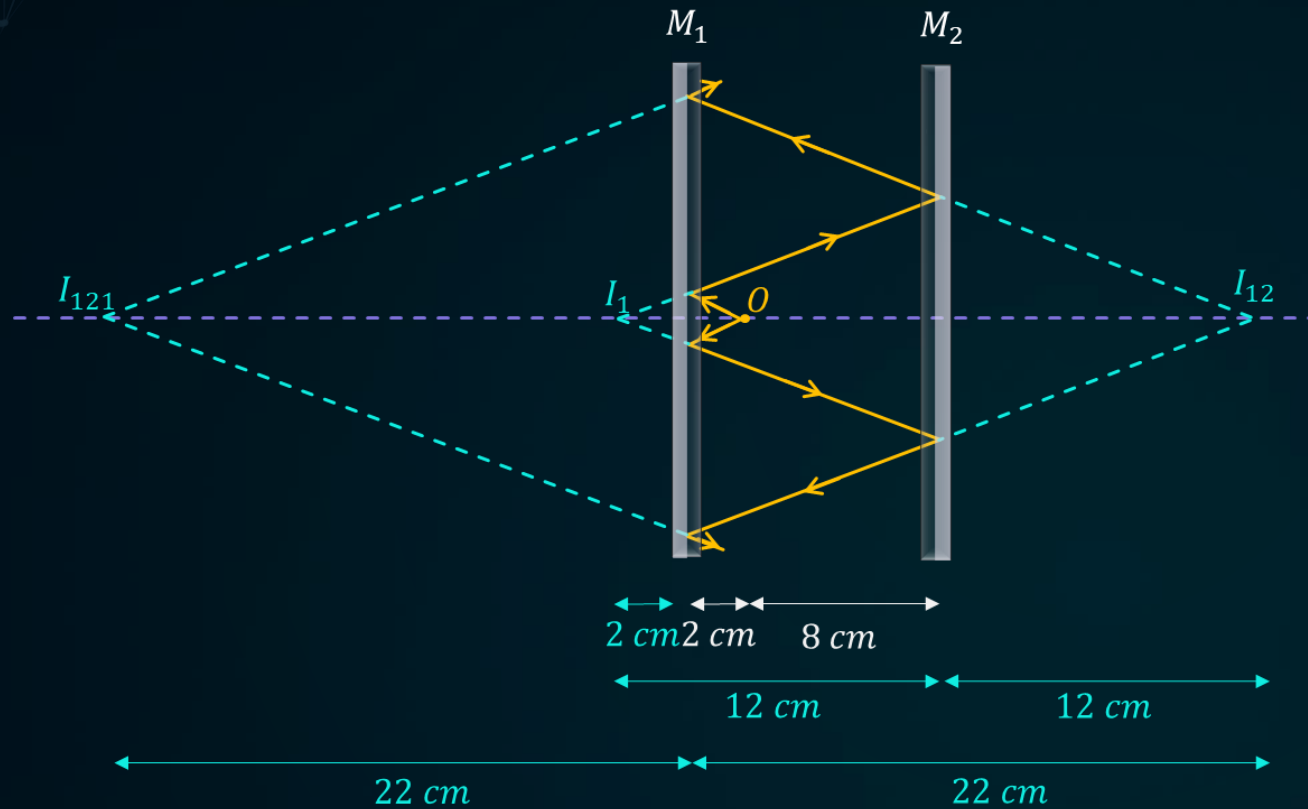




Image formation by parallel mirror

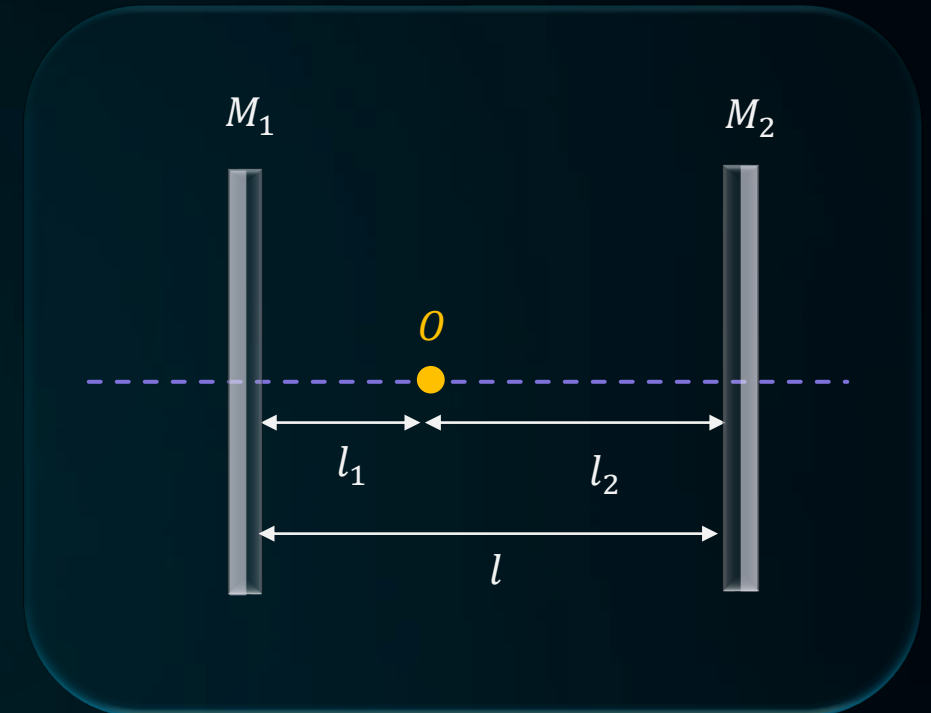


- Image formed by one mirror acts as the object for the other mirror.
- Infinite images are formed for this arrangement of the mirrors.



Trick to Find The Image Formed by Parallel Mirrors

No. of image	Image formed by M_1	Image formed by M_2
1	l_1	l_2
2	$l_2 + l$	$l_1 + l$
3	$l_1 + 2l$	$l_2 + 2l$
⋮	⋮	⋮





Trick to Find The Image Formed by Parallel Mirrors

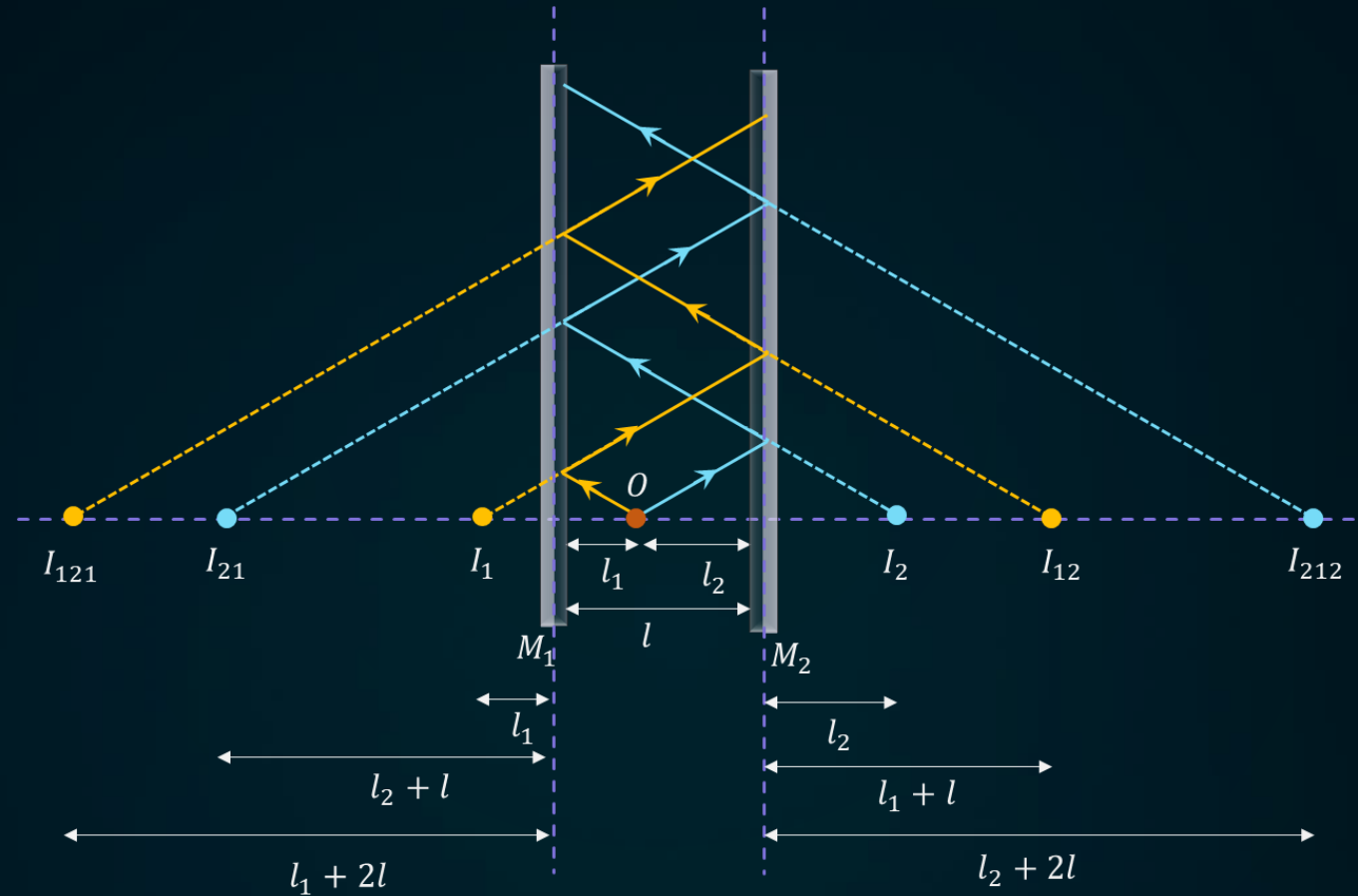
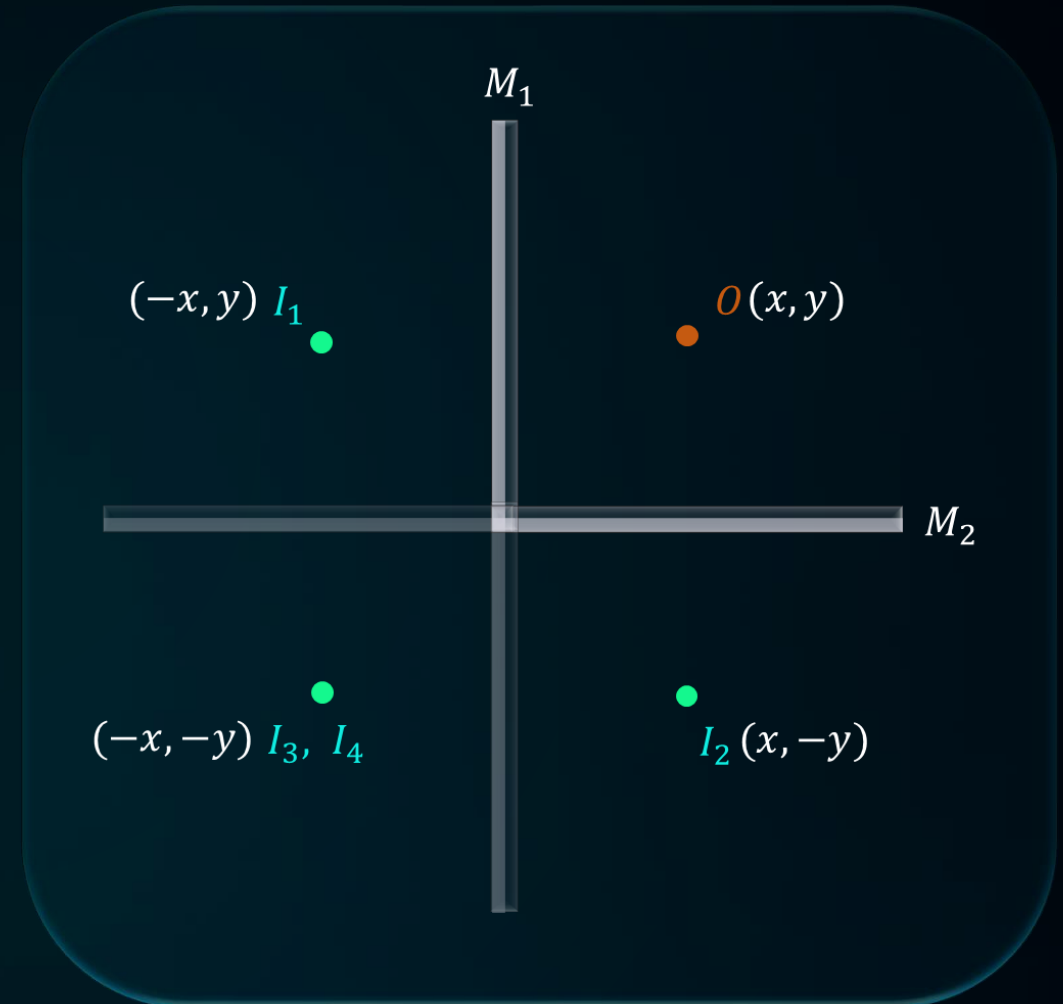




Image Formation by Perpendicular Mirrors



- I_1 is image of O formed by the mirror M_1 .
- I_2 is image of O formed by the mirror M_2 .
- I_3 is image of I_1 formed by the mirror M_2 .
- I_4 is image of I_2 formed by the mirror M_1 .
- The image I_3 and I_4 are formed at the same point.
- Total no. of images = 3



?

A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is

Solution:

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The reflected light from the mirror appears to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram.

$\Delta S'CD$ and $\Delta S'OB$ are similar triangle. So,

$$\frac{S'C}{S'O} = \frac{CD}{OB}$$

$$OB = \frac{CD \times S'O}{S'C} = \frac{\left(\frac{d}{2}\right) \times 3L}{L} \Rightarrow OB = \frac{3d}{2}$$

The distance over which the man can see the image S' is,

$$AB = AO + OB = 2OB = 3d$$

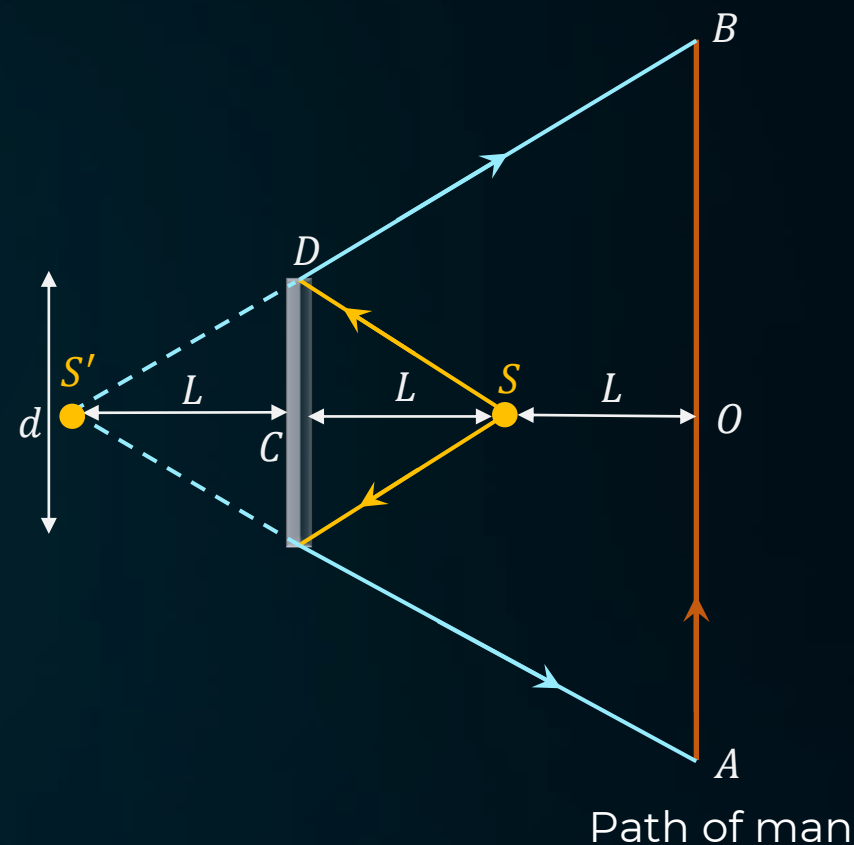




Image Formation by Two Inclined Mirrors



I_1 : Image of object at P due to mirror M_1

I_2 : Image of object at P due to mirror M_2

I_{21} : Image of object I_2 due to mirror M_1

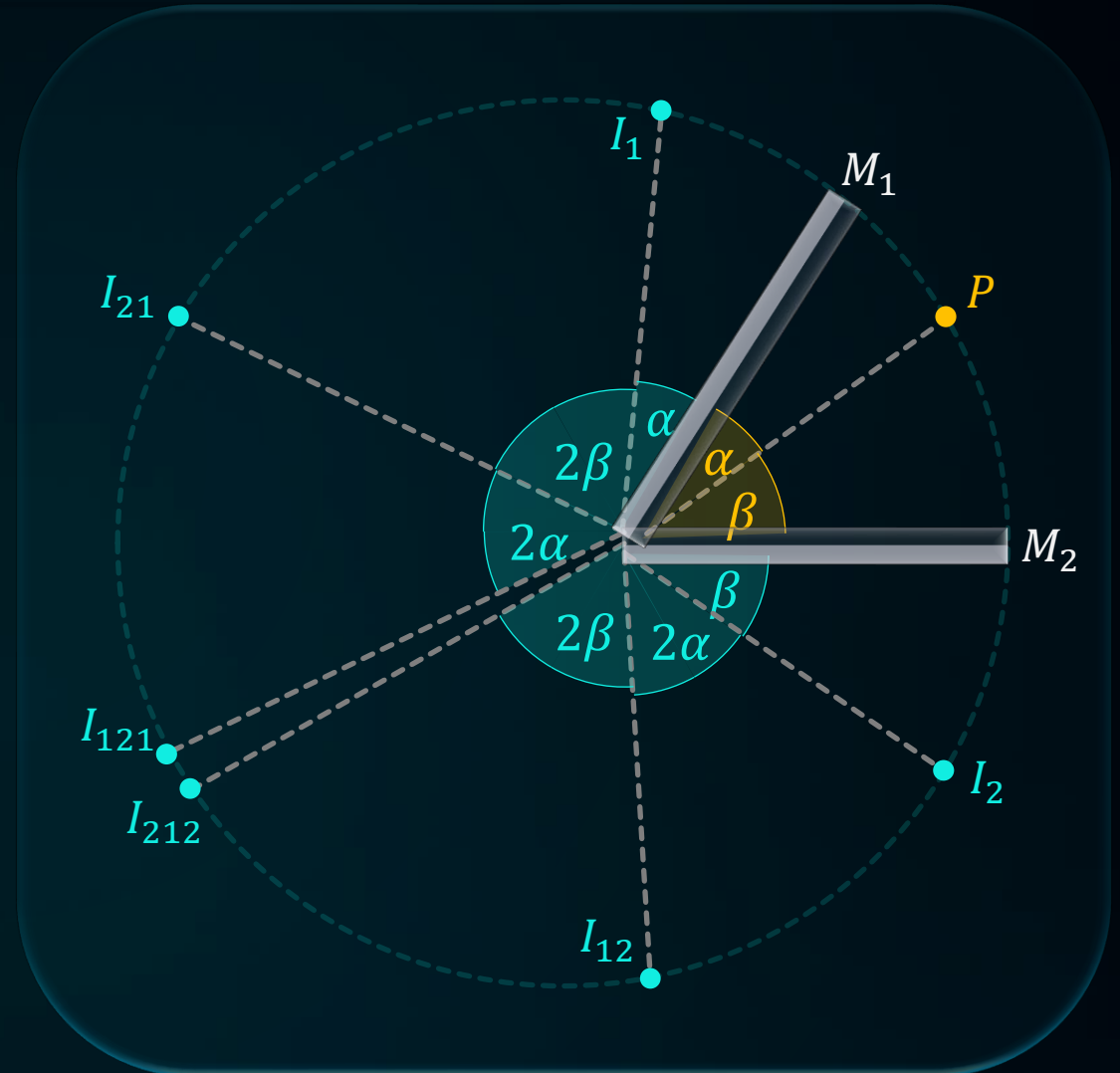
I_{12} : Image of object I_1 due to mirror M_2

I_{121} : Image of object I_{12} due to mirror M_1

I_{212} : Image of object I_{21} due to mirror M_2

Image formed by Mirror M_1 at an angle:	Image formed by Mirror M_2 at an angle:
α	β
$\alpha + 2\beta$	$2\alpha + \beta$
$3\alpha + 2\beta$	$2\alpha + 3\beta$

The mirror will not be able to form image if angle between the object and mirror becomes $\geq 180^\circ$.

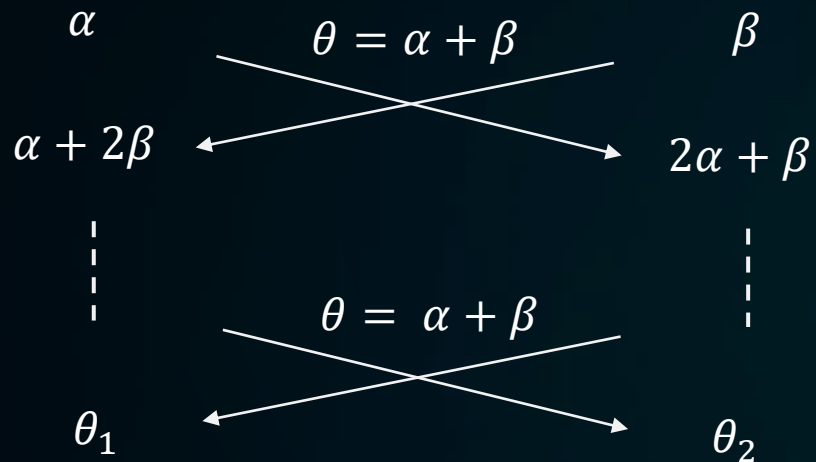




Overlapping of Images

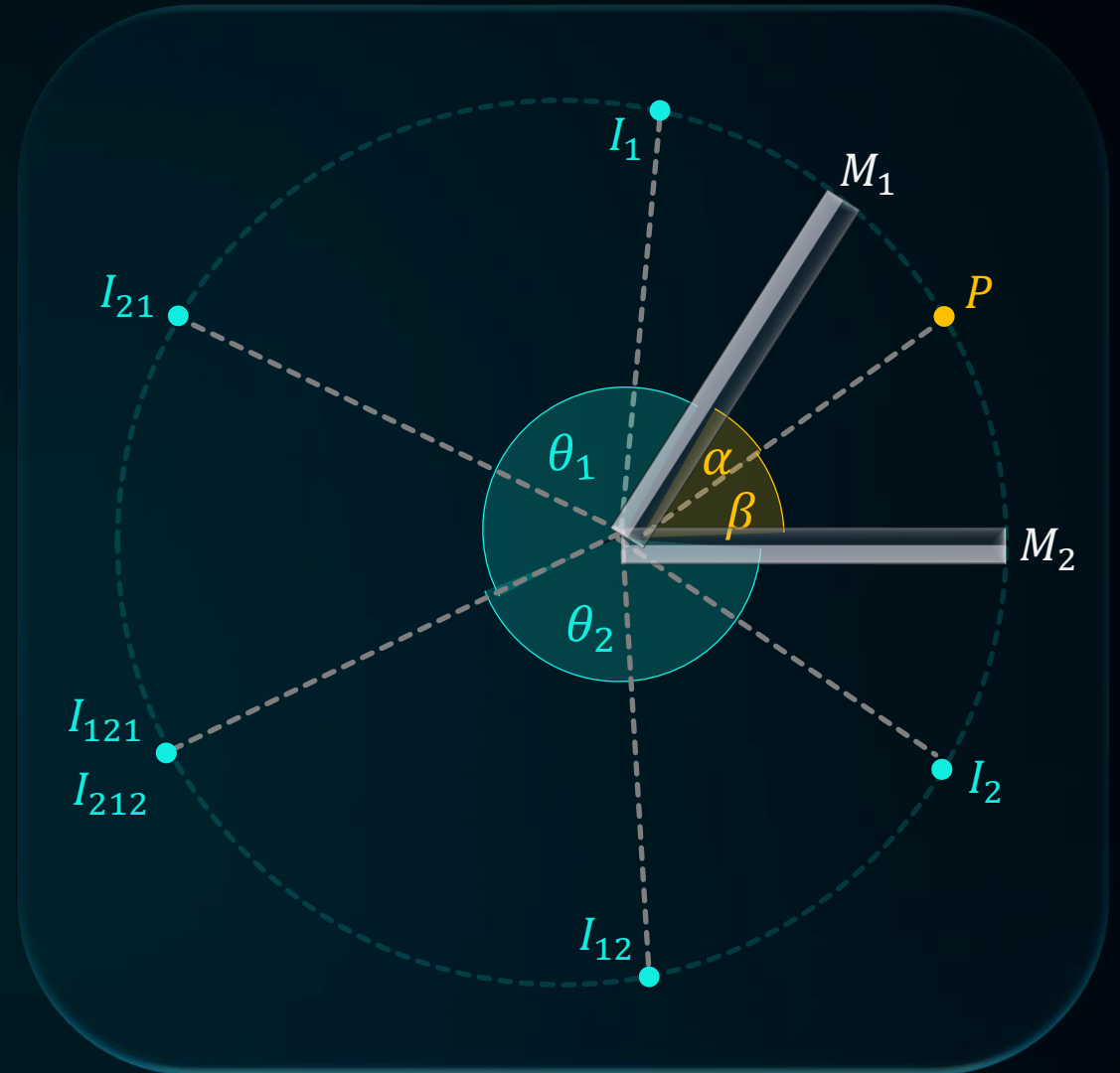


Image formed by
Mirror M_1 at an
angle:



Images are overlapped when:

$$\theta + \theta_1 + \theta_2 \geq 360^\circ$$



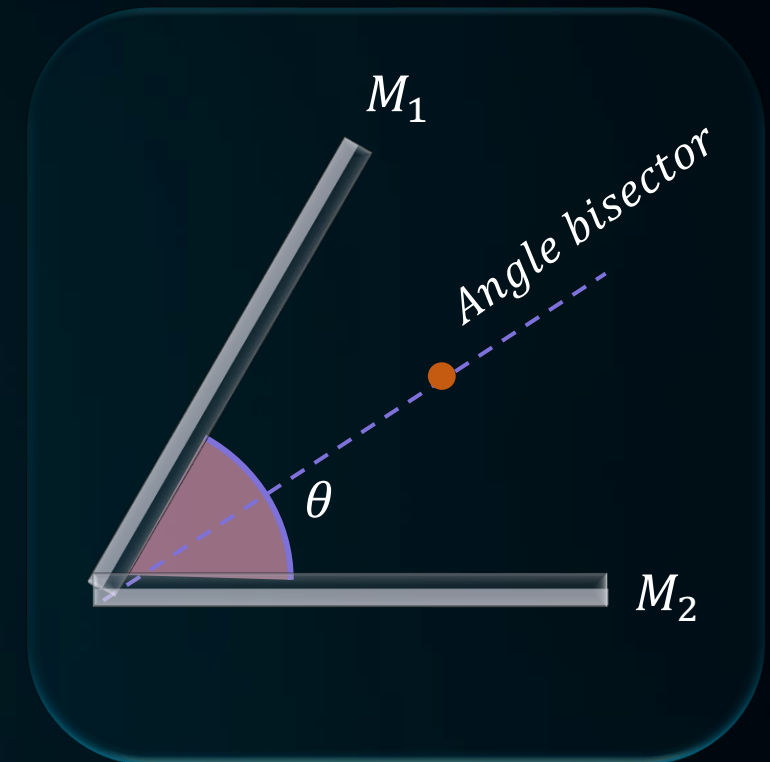


Number of Images due to Two Inclined Mirrors



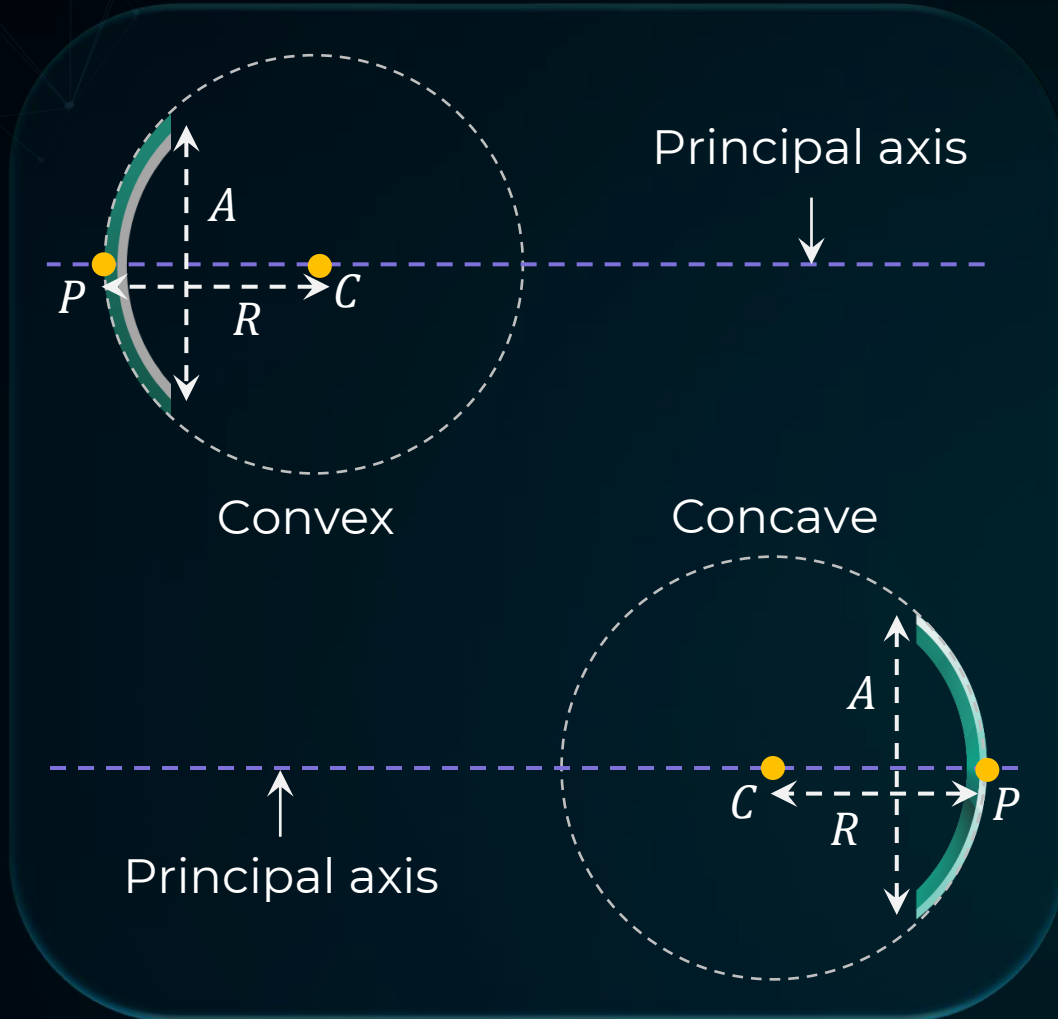
- Object is not placed at the angle bisector: **Non-symmetric**
- Object is placed at the angle bisector: **Symmetric**

$\frac{360^\circ}{\theta}$	Position of Object	Number of Images
Even integer	Symmetric and non-symmetric	$\frac{360^\circ}{\theta} - 1$
Odd integer	Symmetric	$\frac{360^\circ}{\theta} - 1$
Odd integer	Non-symmetric	$\frac{360^\circ}{\theta}$





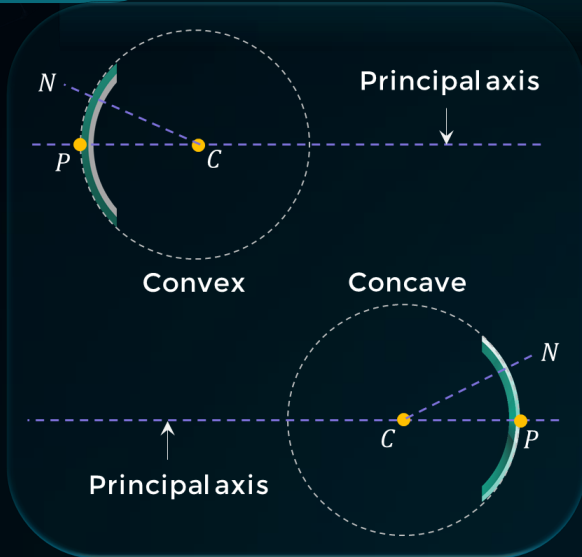
Introduction to Spherical Mirrors



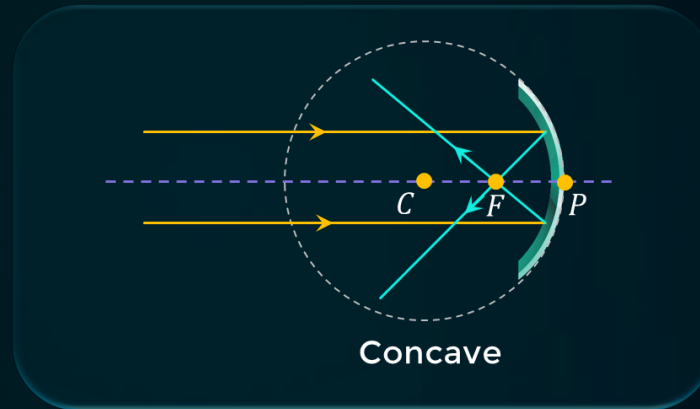
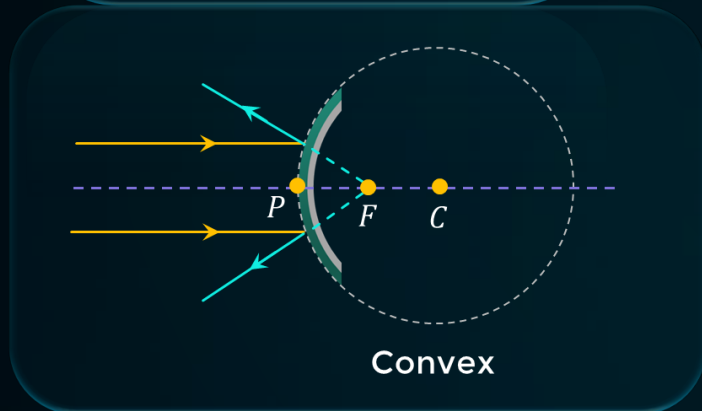
- **Center of curvature (C):** The centre of the sphere of which the spherical mirror is a part.
- **Radius of Curvature (R):** The radius of the sphere of which the spherical mirror is a part.
- **Pole (P):** The centre of the reflecting surface of the spherical mirror.
- **Aperture (A):** The part of a spherical mirror that is exposed to all the light rays which are incident on it is called the aperture of the spherical mirror.



Terminologies related to Spherical mirror



- **Principal axis:** The straight line joining the pole and centre of curvature.
- **Normal (CN):** Any straight line joining the mirror to its centre of curvature.



- **Principal focus (F):** Rays traveling parallel & close to the principal axis of a mirror after reflection pass through or appear to originate from a point called principal focus.
- **Focal length:** The distance of focus from the pole of spherical mirror.



Proof of $F = \frac{R}{2}$ and Concept of Paraxial Rays



$\Delta CQM \cong \Delta AQM$ (By Angle-Side-Angle)

$$CM = AM = \frac{CA}{2} = \frac{R}{2}$$

From ΔCMQ :

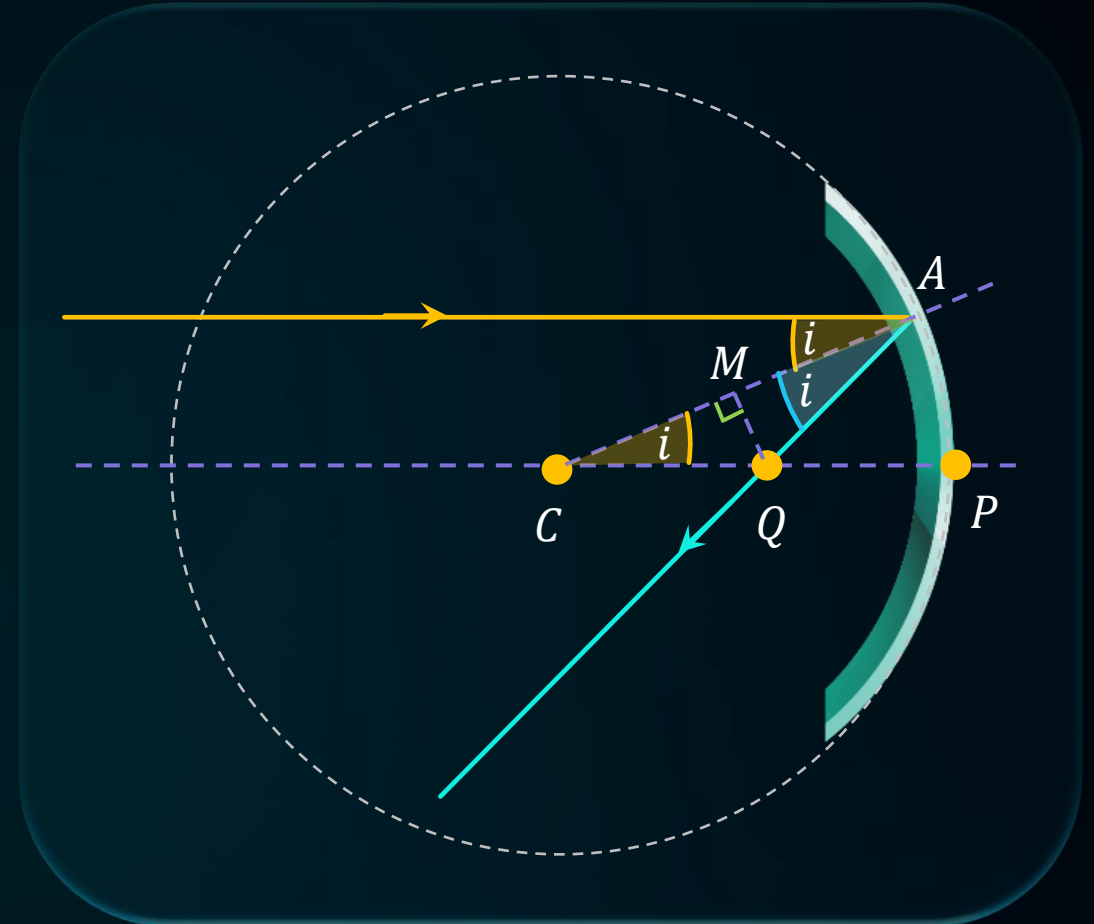
$$\cos \angle MCQ = \frac{CM}{CQ} \Rightarrow \cos i = \frac{(R/2)}{CQ} \Rightarrow CQ = \frac{R}{2 \cos i}$$

Therefore, focal length (F):

$$PQ = CP - CQ = R - \frac{R}{2 \cos i} = F$$

For small angle of incidence: $\cos i \approx 1$

$$F \approx R - \frac{R}{2} \approx \frac{R}{2}$$





Summary



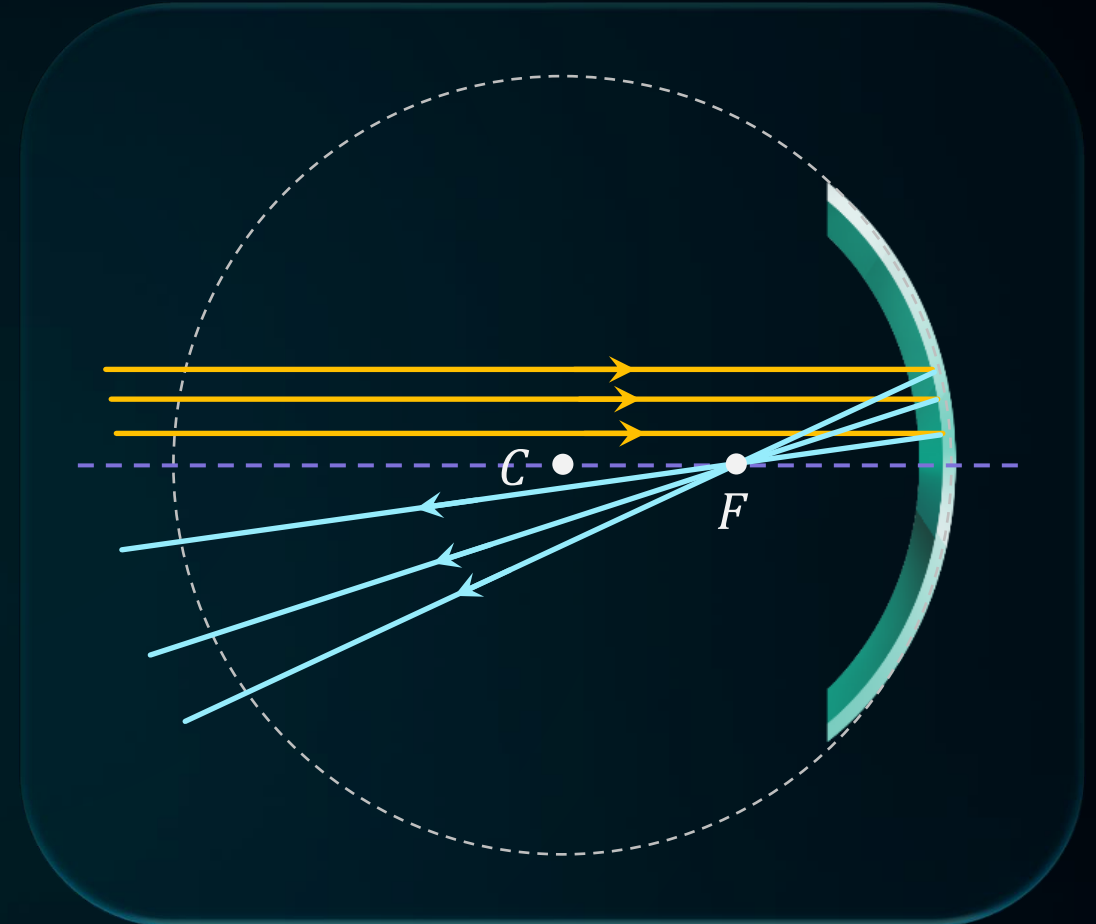
- Paraxial rays:

The rays that are close to the principal axis are known as paraxial rays and focus is defined only for these rays.

- To get paraxial rays in practical life, a mirror with small aperture is used.

- Assumptions in ray optics:

- 1) Neglect diffraction.
- 2) Assume paraxial rays if nothing is mentioned.

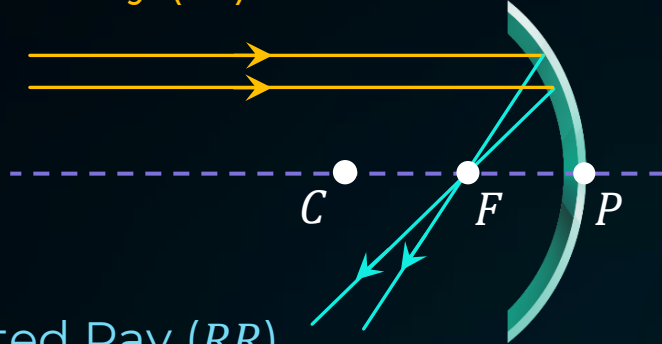




Ray Tracing and Sign Convention

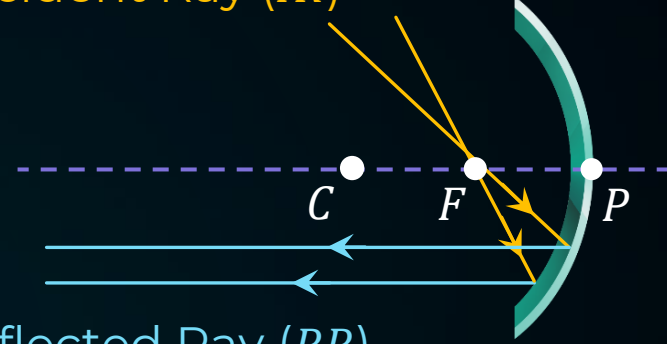


Incident Ray (*IR*)



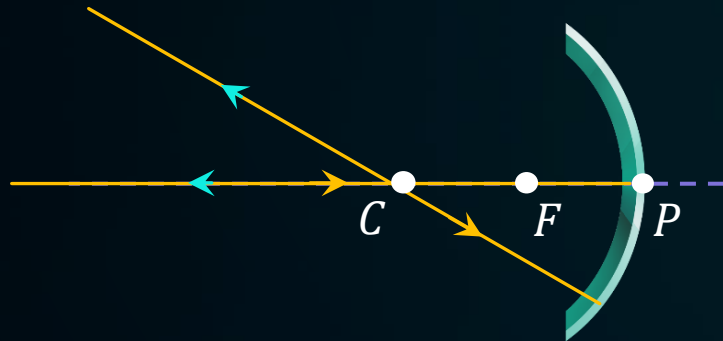
Reflected Ray (*RR*)

Incident Ray (*IR*)



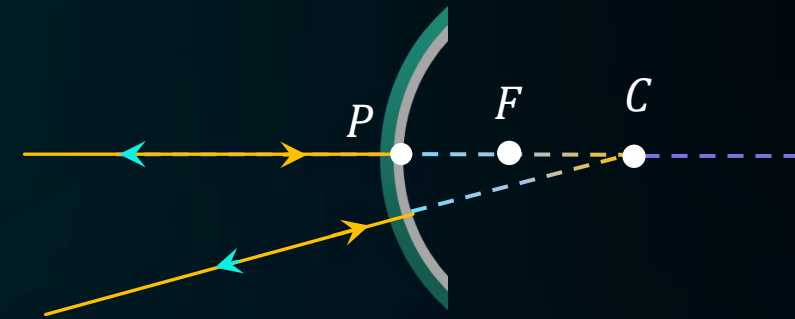
Reflected Ray (*RR*)

Incident Ray (*IR*): Passing through centre of curvature



Reflected Ray (*RR*): Retraces the path of incident ray

Incident Ray (*IR*): Appear to pass through centre



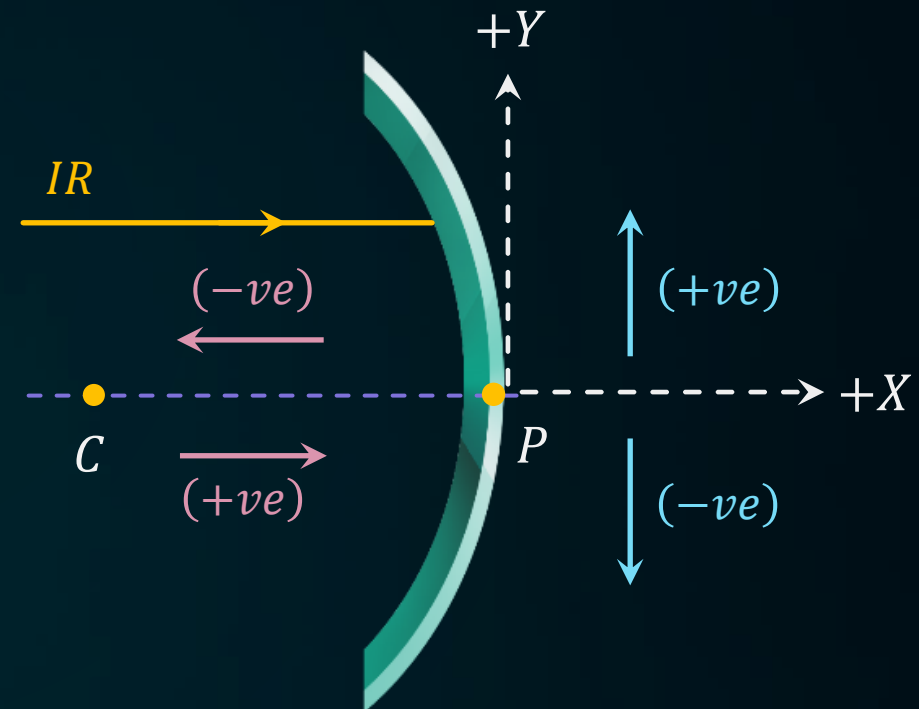
Reflected Ray (*RR*): Retraces the path of incident ray



Summary



- Pole (P) is taken as origin.
- X -axis along the Principal Axis.
- X -coordinate is taken positive along the incident light.
- Y -coordinate is taken positive above the Principal Axis.





Mirror Formula



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$$

u : X -coordinate of object

v : X -coordinate of image

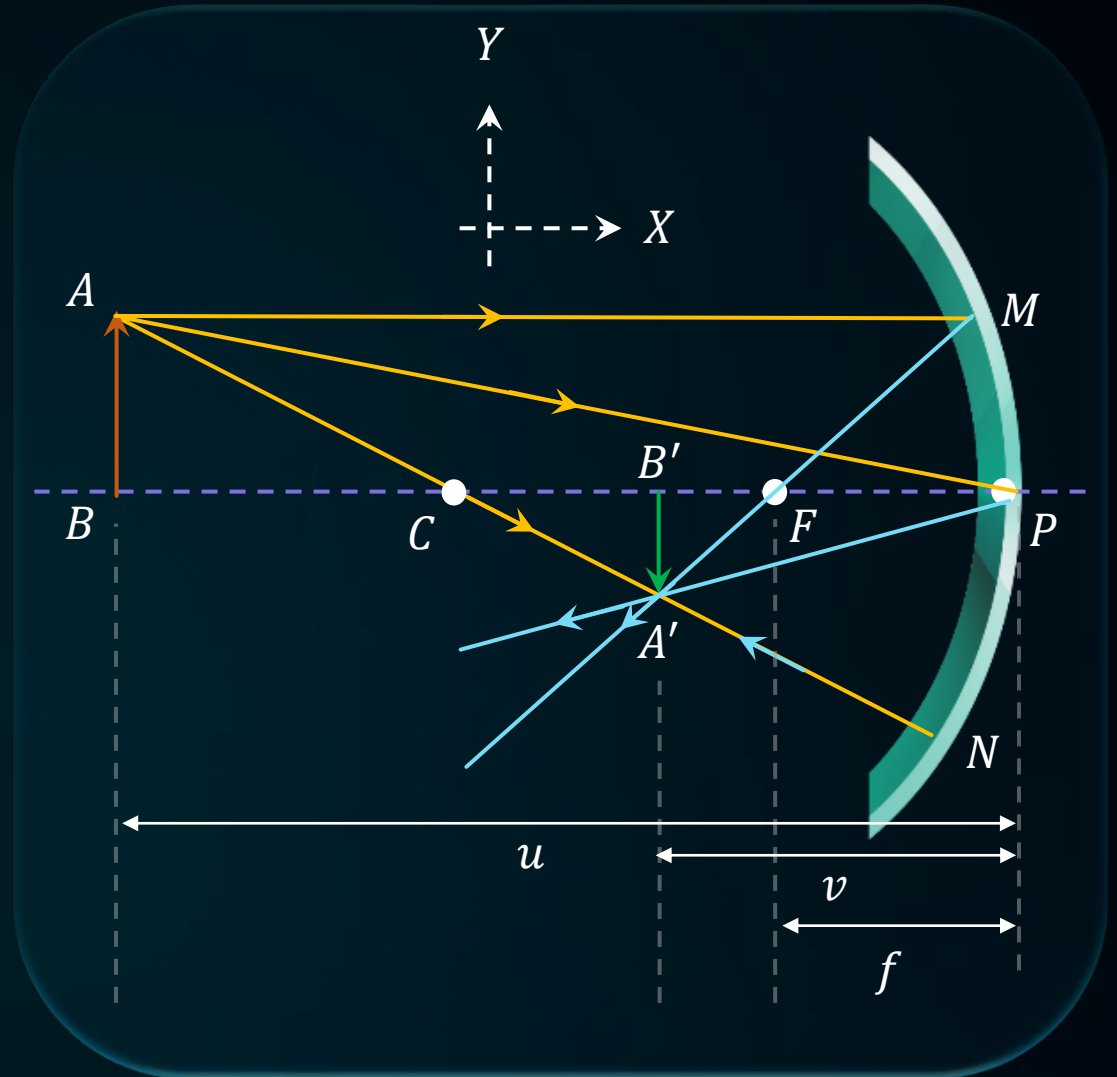
f : X -coordinate of focus

R : X -coordinate of centre of curvature

- If the distance of the object from the pole is x , then for the shown diagram:

$$u = -x$$

Distance is always positive, but coordinate can be negative.





Proof of Mirror Formula



Rays are paraxial $\rightarrow \alpha, \beta$ & γ are small

NP is negligible

From Triangle OMC : $\beta = \alpha + \theta$

From Triangle CMI : $\gamma = \beta + \theta$

$$2\beta = \alpha + \gamma$$

$$\alpha = \frac{MN}{NO} \approx \frac{MN}{PO} \approx \frac{MN}{u}$$

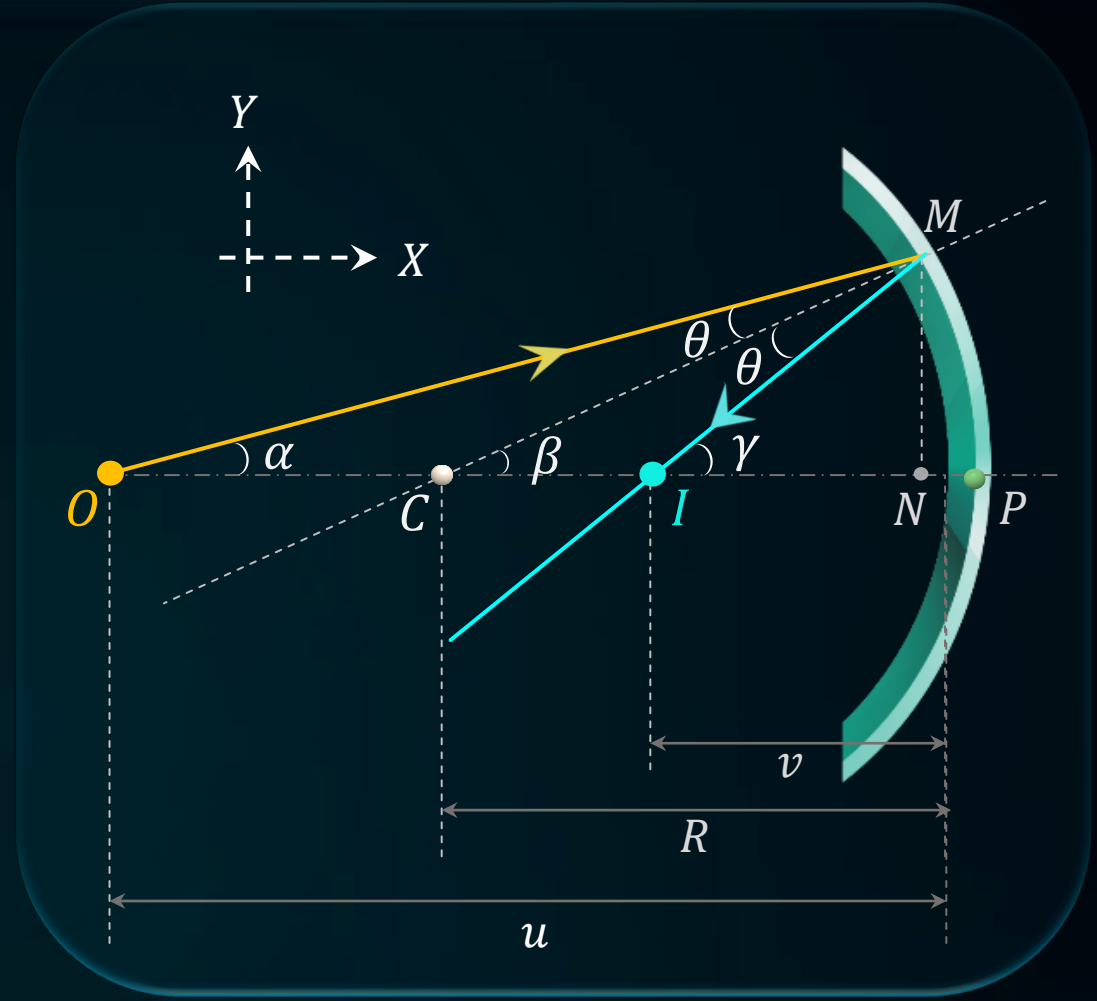
$$\beta = \frac{MN}{NC} \approx \frac{MN}{PC} \approx \frac{MN}{R}$$

$$\gamma = \frac{MN}{NI} \approx \frac{MN}{PI} \approx \frac{MN}{v}$$

$$2\beta = \alpha + \gamma$$

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$$



- The mirror formula is valid for both concave and convex mirror.



Demonstration of Sign Convention



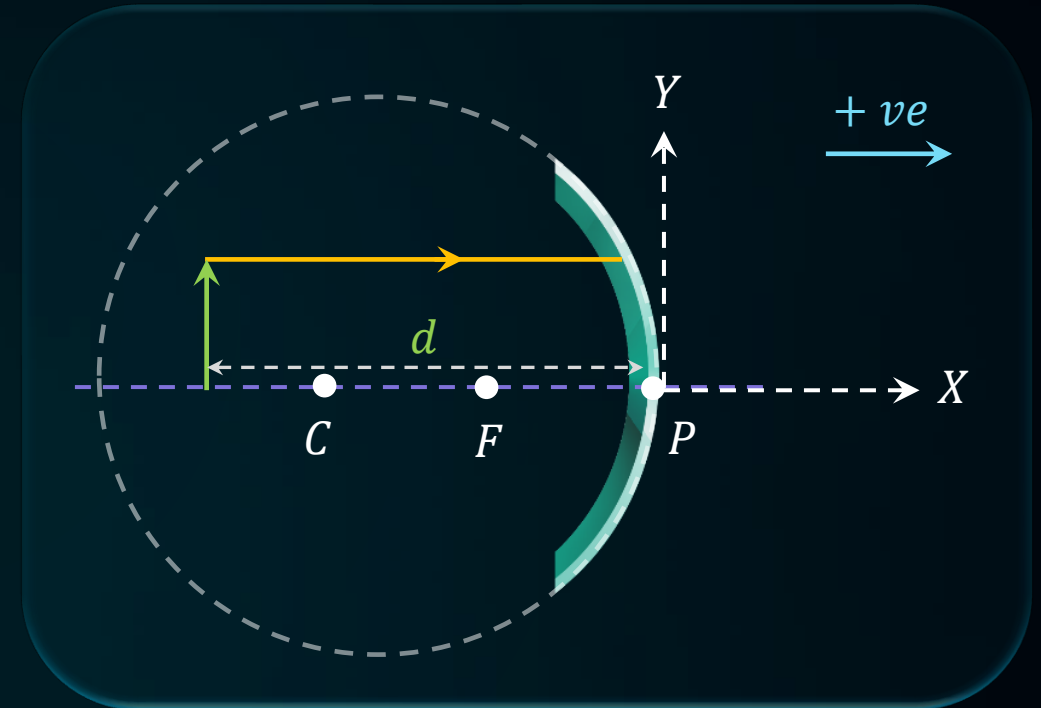
Case-1: Concave mirror

- Direction of incident ray is taken as $+ve$ direction.
- X -coordinate of focus and COC are always negative.
- Applying mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{(-d)} = \frac{1}{(-f)}$$

- If $v < 0$, then image is real, and if $v > 0$, the image is virtual.





Demonstration of Sign Convention



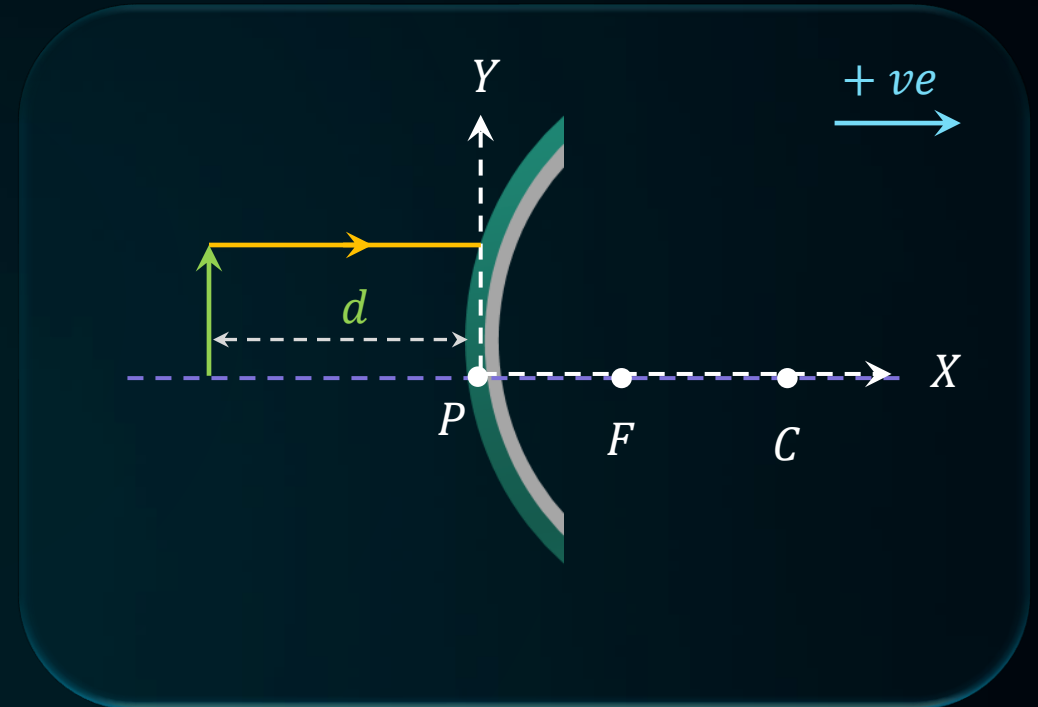
Case-2: Convex mirror

- X-coordinate of focus and COC are always positive.
- Applying mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{(-d)} = \frac{1}{(+f)}$$

- If $v < 0$, then image is real, and if $v > 0$, the image is virtual.





Lateral Magnification



- Lateral/Transverse magnification (m):

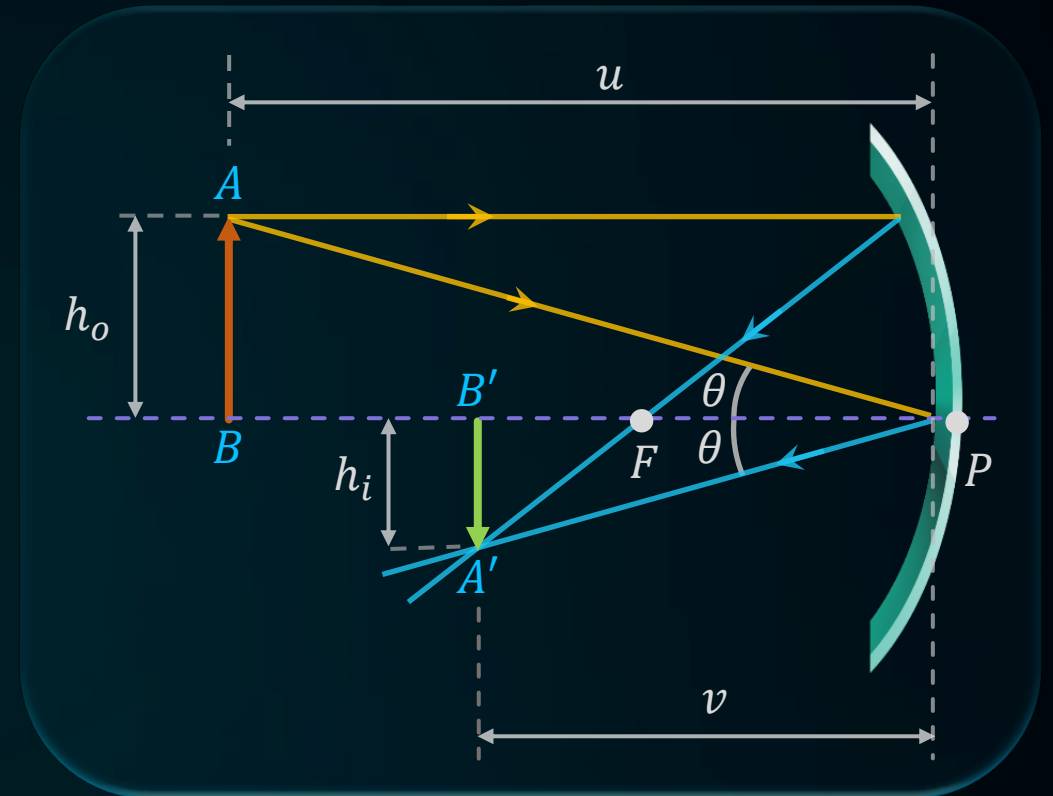
It is defined as the ratio of y -coordinate of image (h_i) to the y -coordinate of object (h_o).

$$m = \frac{y_i}{y_o} = -\frac{v}{u}$$

$$|m| = \frac{h_i}{h_o} = \left| \frac{v}{u} \right|$$

- m is positive: Erect image (object and image are on the same side of the principal axis)
- m is negative: Inverted image
- Alternate formula of Lateral/Transverse magnification (m):

$$m = \frac{f}{f - u} = \frac{f - v}{f}$$





Converging rays are incident on a convex mirror such that **their extensions** intersect **30 cm** behind the mirror on the optical axis. The reflected rays form a diverging beam such that their extensions intersect the principal axis **1.2 m** from the mirror. Determine the focal length of the mirror.

Given:

$$PO = 30 \text{ cm}$$

$$PI = 1.2 \text{ m} = 120 \text{ cm}$$

Solution:

X- coordinate of object: $u = +PO = +30 \text{ cm}$

X- coordinate of image: $v = +PI = +120 \text{ cm}$

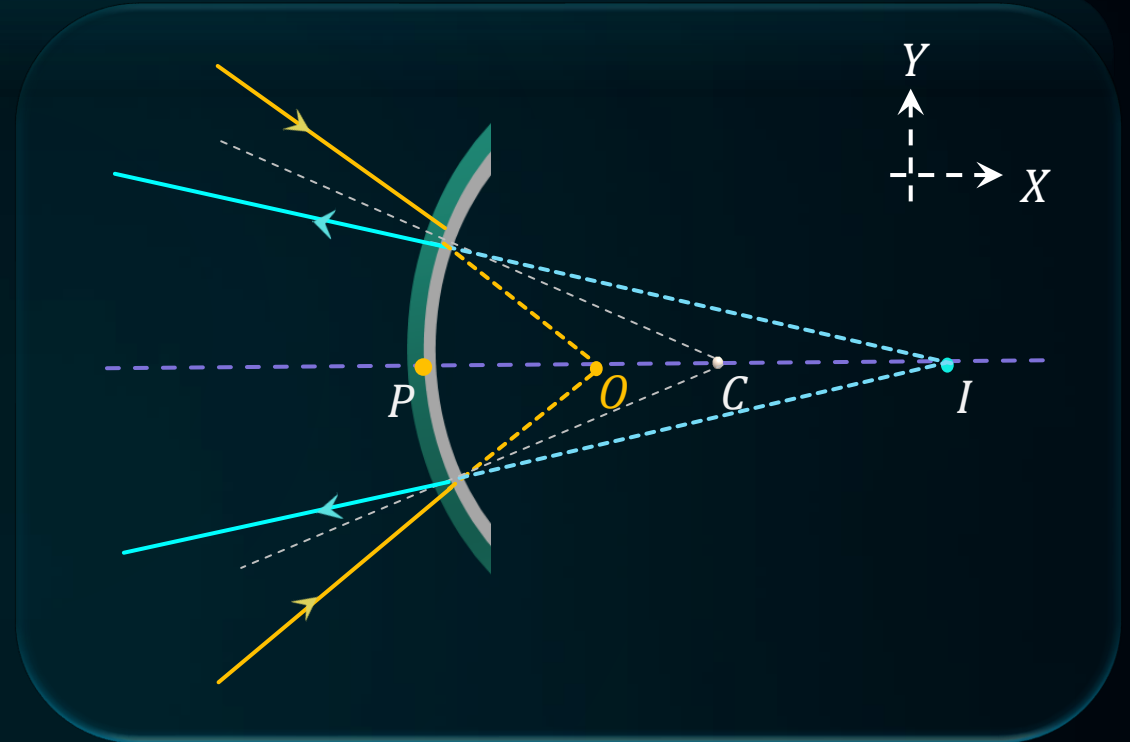
Applying Mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{(+120)} + \frac{1}{(+30)} = \frac{1}{f}$$

$$\frac{1 + 4}{120} = \frac{1}{f} \Rightarrow f = \frac{120}{5}$$

$$f = +24 \text{ cm}$$





A concave mirror for face viewing has focal length of 0.4 m . The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

JEE MAIN 2019

Given:

Focal length: $PF = 0.4\text{ m}$

Magnification: $m = +5$

Solution:

X-coordinate of focus: $f = -PF = -0.4\text{ m}$

$$m = -\frac{v}{u} = +5 \quad \Rightarrow \quad v = -5u$$

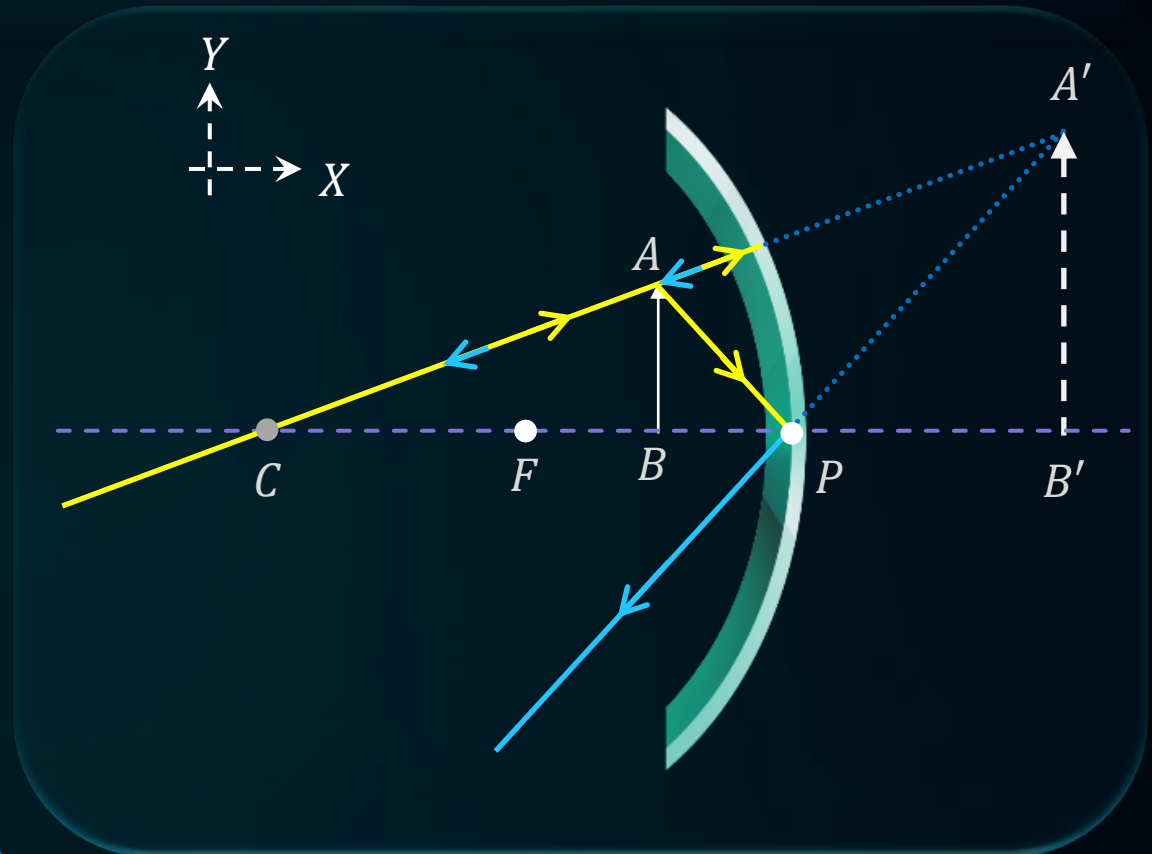
Applying Mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{(-5u)} + \frac{1}{(+u)} = -\frac{1}{0.4}$$

$$\frac{4}{5u} = -\frac{1}{0.4} \quad \Rightarrow \quad u = -\frac{1.6}{5}$$

$$u = -0.32\text{ m}$$





Point Object Is Not On Principal Axis



- For x -axis:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{x_i} + \frac{1}{x_o} = \frac{1}{f}$$



$$x_i = \frac{f x_o}{x_o - f}$$

- For y -axis:

$$m = \frac{y_i}{y_o} = -\frac{v}{u} = -\frac{x_i}{x_o}$$

$$y_i = -\frac{x_i}{x_o} y_o$$

$$y_i = -\left(\frac{f}{x_o - f}\right) y_o$$



$$y_i = \frac{f y_o}{f - x_o}$$

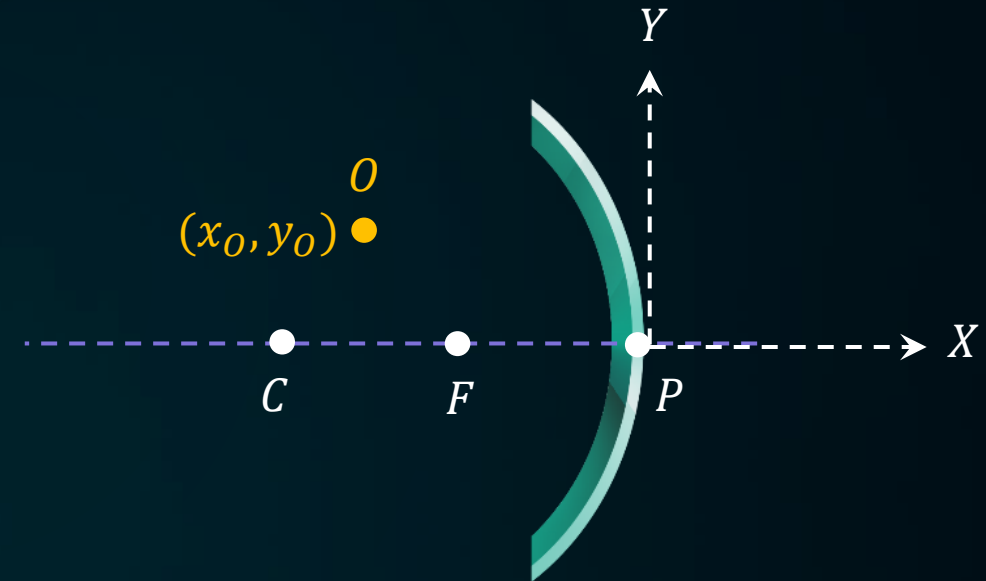
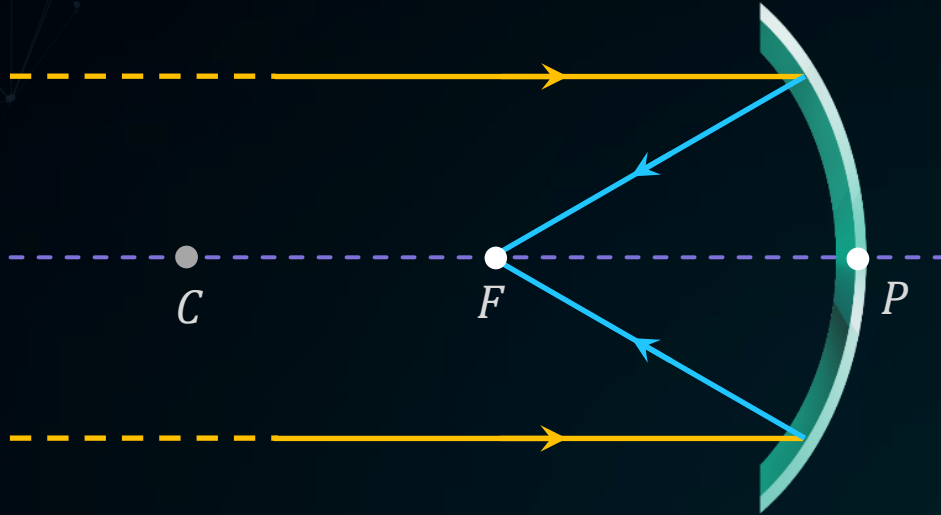




Image Formation by Concave Mirror

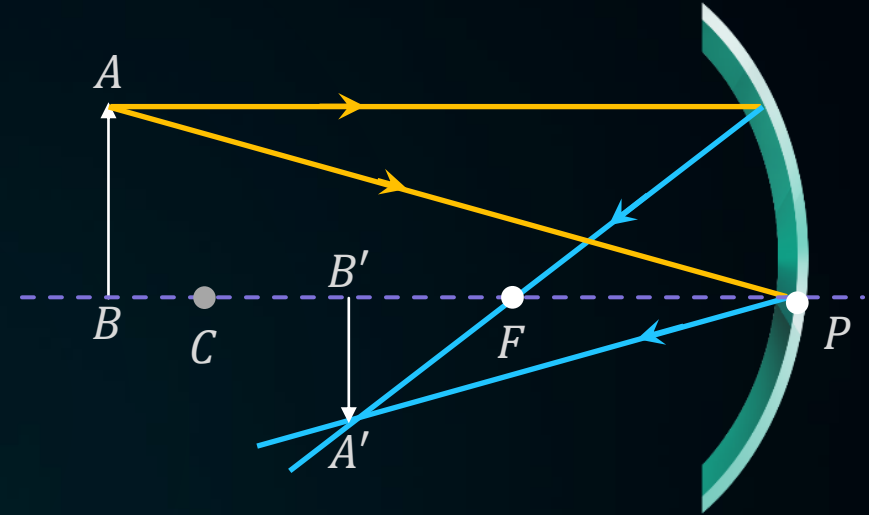


The object at infinity:

Image is formed at focus F .

Image formed is highly diminished or point size.

Image is real and inverted.



The object is placed beyond C :

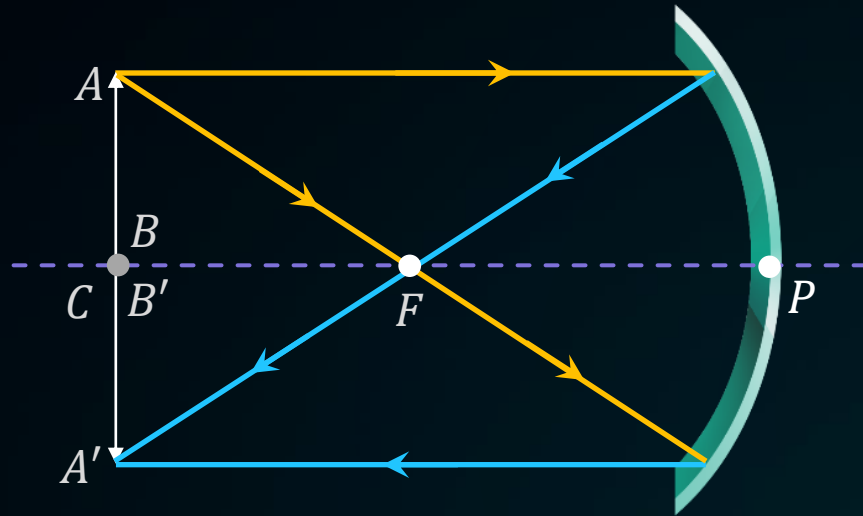
Image is formed between F and C .

Image formed is diminished.

Image is real and inverted.



Image Formation by Concave Mirror

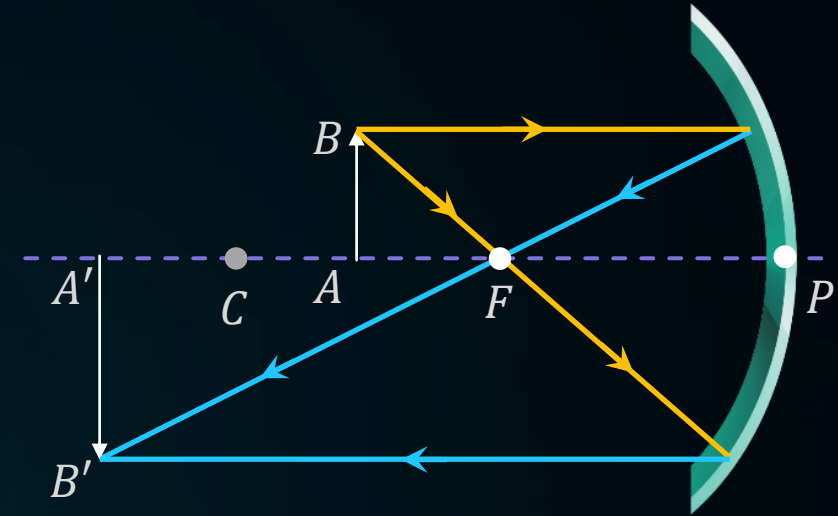


The object is at C :

Image is formed at C .

Image is of the same size as that of the object.

Image is real and inverted.



The object is between F and C :

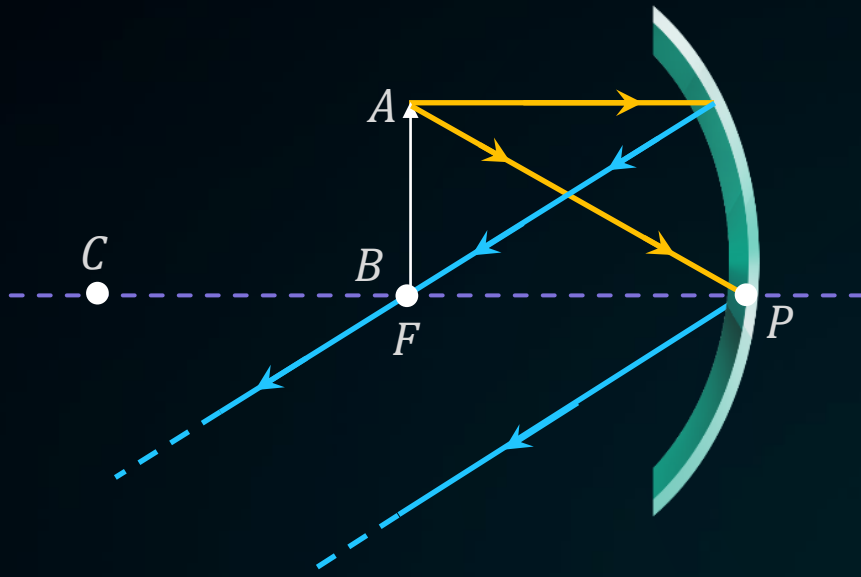
Image is formed beyond C .

Image formed is magnified.

Image is real and inverted.



Image Formation by Concave Mirror

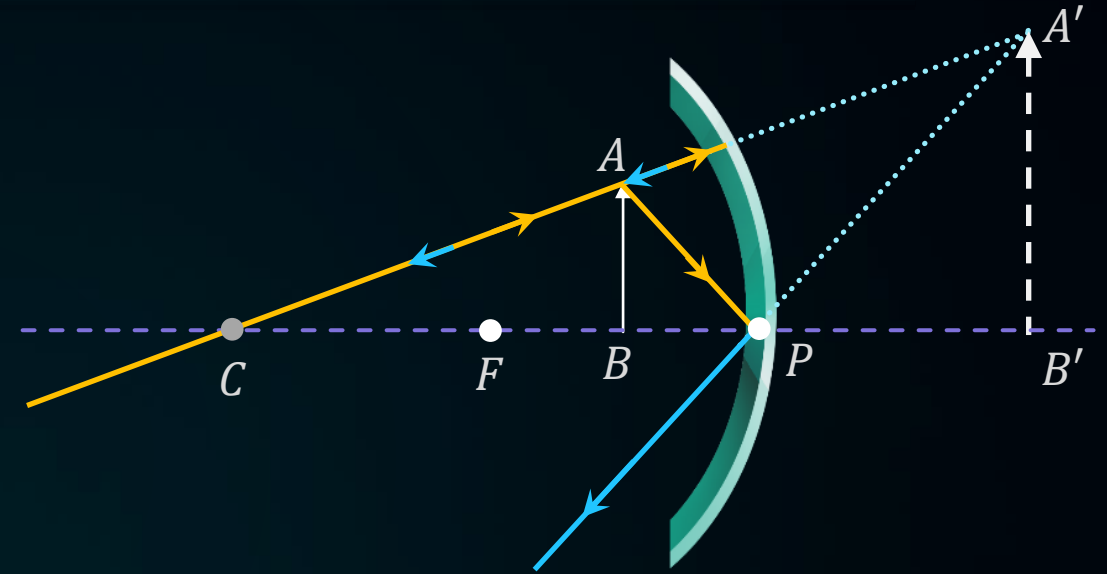


The object at F :

Image is formed at infinity.

Image formed is highly magnified.

Image is real and inverted.



The object placed between P and F :

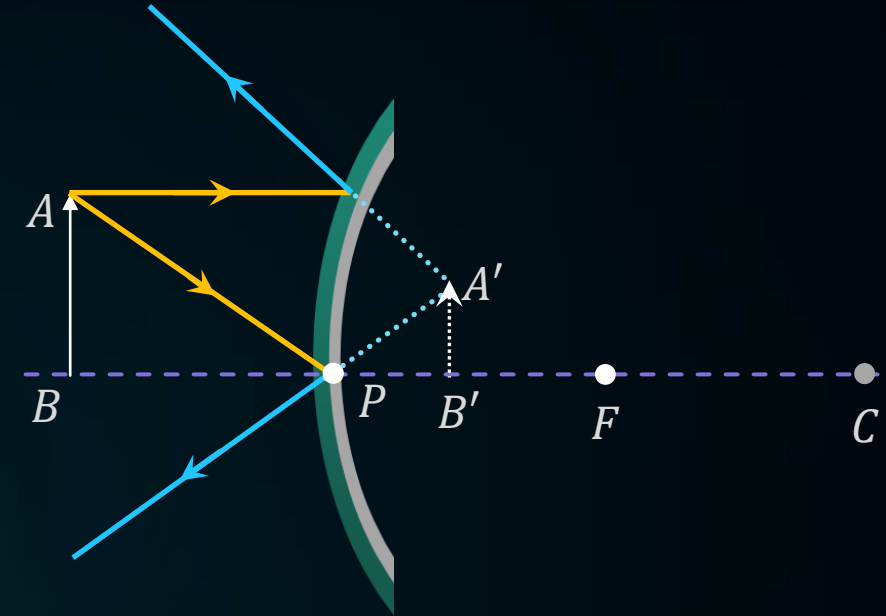
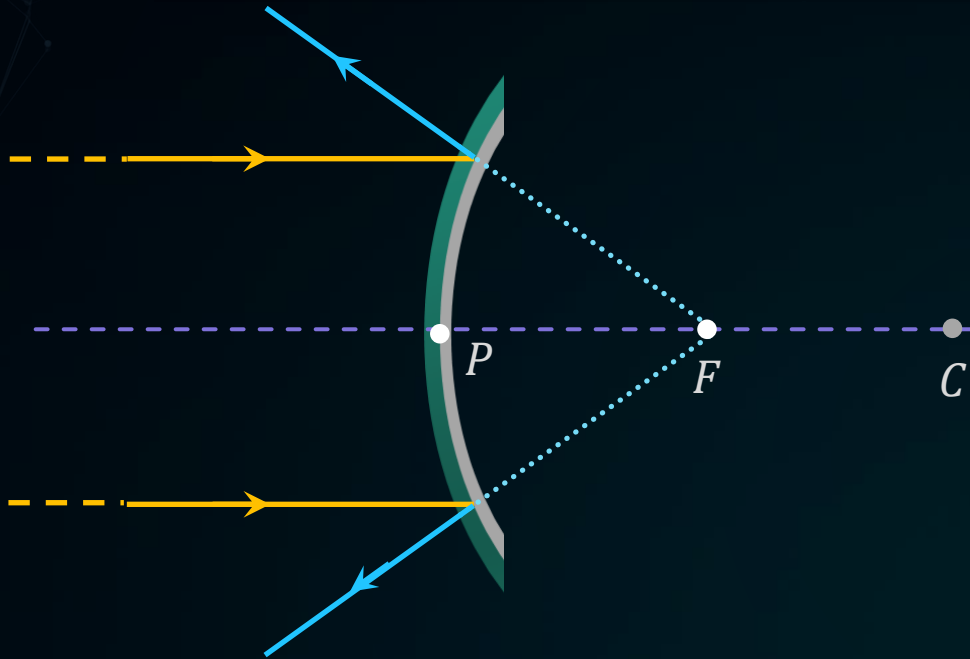
Image is formed behind the mirror.

Image formed is magnified.

Image is **virtual** and erect.



Image Formation by Convex Mirror



Position of Object	Position of Image	Nature of image	Size of the image
<i>At ∞</i>	<i>At F</i>	<i>Virtual and erect</i>	<i>Point size</i>
<i>At finite distance</i>	<i>Between F and P</i>	<i>Virtual and erect</i>	<i>Diminished</i>



v vs u graph: Concave mirror

Object	Image
$u = 0$	$v = 0$
$u = -f$	$v = \infty$
$u = -2f$	$v = -2f$
$-f < u < -2f$	$v > -2f$
$u > -2f$	$-f < v < -2f$

2nd Quadrant:

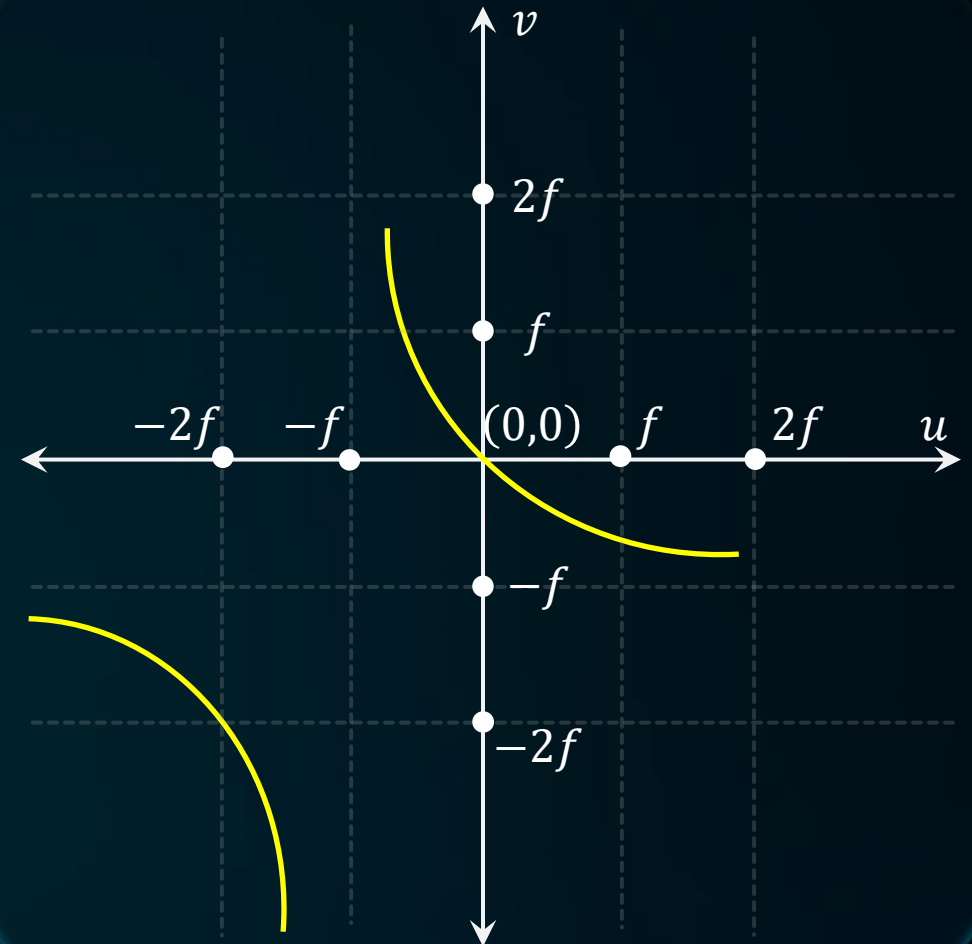
$u \rightarrow -ve, v \rightarrow +ve$ → Real object, virtual image

3rd Quadrant:

$u \rightarrow -ve, v \rightarrow -ve$ → Real object, real image

4th Quadrant:

$u \rightarrow +ve, v \rightarrow -ve$ → Virtual object, real image





$\frac{1}{v}$ vs $\frac{1}{u}$ Graph: Concave mirror



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$



$$\frac{1}{v} = -\frac{1}{u} - \frac{1}{f}$$

$$[y = -x - c]$$

(f is negative for concave mirror)

- 2nd Quadrant: ($u \rightarrow -ve, v \rightarrow +ve$)
Real object, virtual image
- 3rd Quadrant: ($u \rightarrow -ve, v \rightarrow -ve$)
Real object, real image
- 4th Quadrant: ($u \rightarrow +ve, v \rightarrow -ve$)
Virtual object, real image

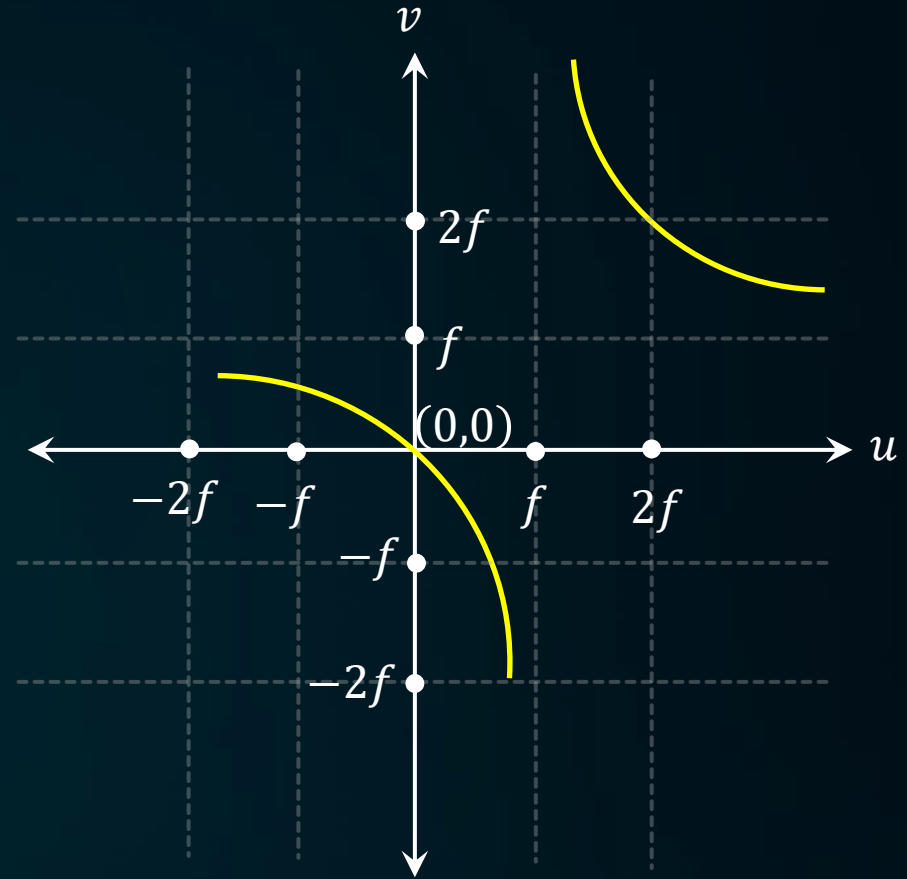




v vs u graph: Convex mirror



Object	Image
$u = 0$	$v = 0$
$u = f$	$v = \infty$
$u = 2f$	$v = 2f$
$f < u < 2f$	$v > 2f$
$u > 2f$	$f < v < 2f$



1st Quadrant:

$u \rightarrow +ve, v \rightarrow +ve$ → Virtual object, virtual image

2nd Quadrant:

$u \rightarrow -ve, v \rightarrow +ve$ → Real object, virtual image

4th Quadrant:

$u \rightarrow +ve, v \rightarrow -ve$ → Virtual object, real image



$\frac{1}{v}$ vs $\frac{1}{u}$ Graph: Convex mirror



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

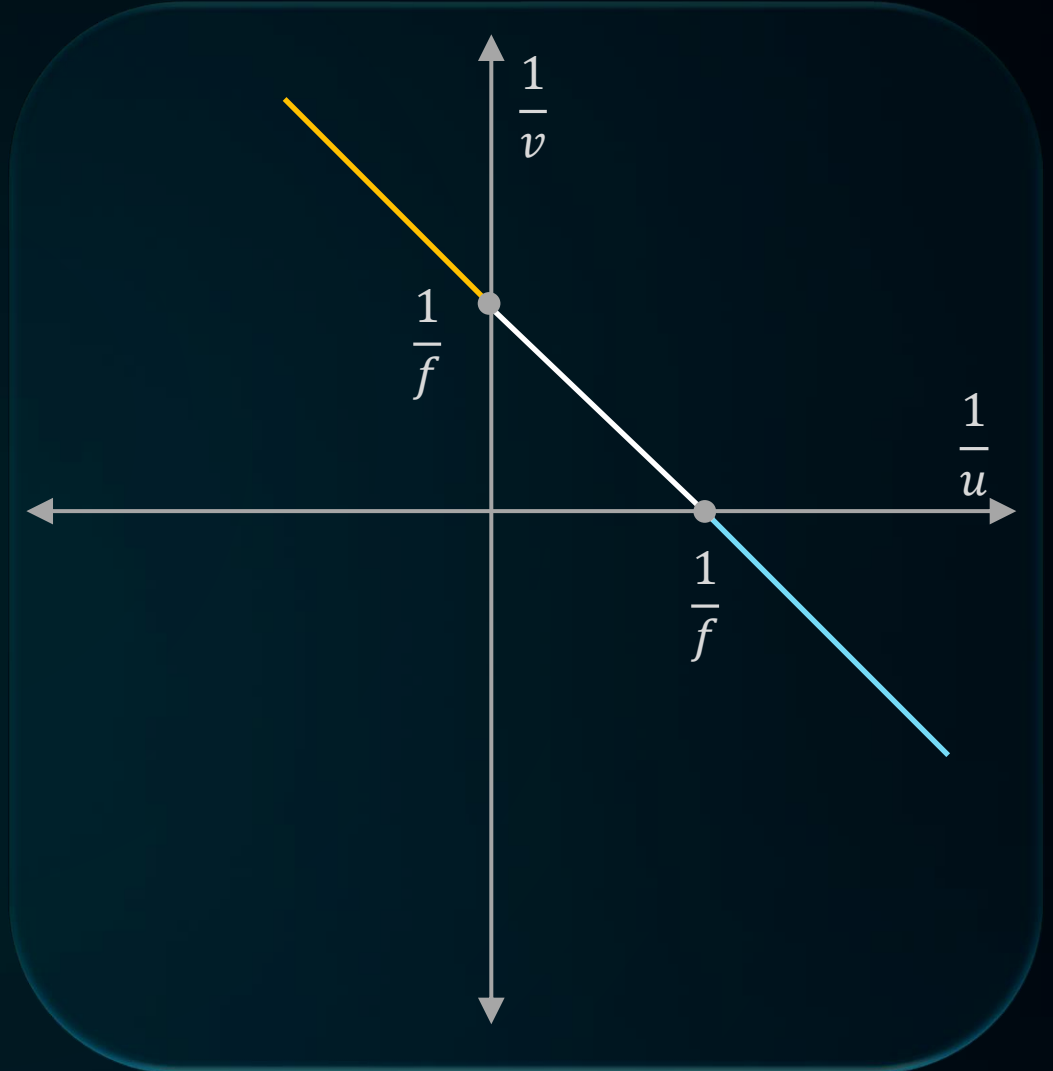


$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

$$[y = -x + c]$$

(f is positive for
convex mirror)

- 1st Quadrant: ($u \rightarrow +ve, v \rightarrow +ve$)
Virtual object, virtual image
- 2nd Quadrant: ($u \rightarrow -ve, v \rightarrow +ve$)
Real object, virtual image
- 4th Quadrant: ($u \rightarrow +ve, v \rightarrow -ve$)
Virtual object, real image





Motion Of the Image Perpendicular to principal axis



$$h_i = -\frac{v}{u} h_o$$

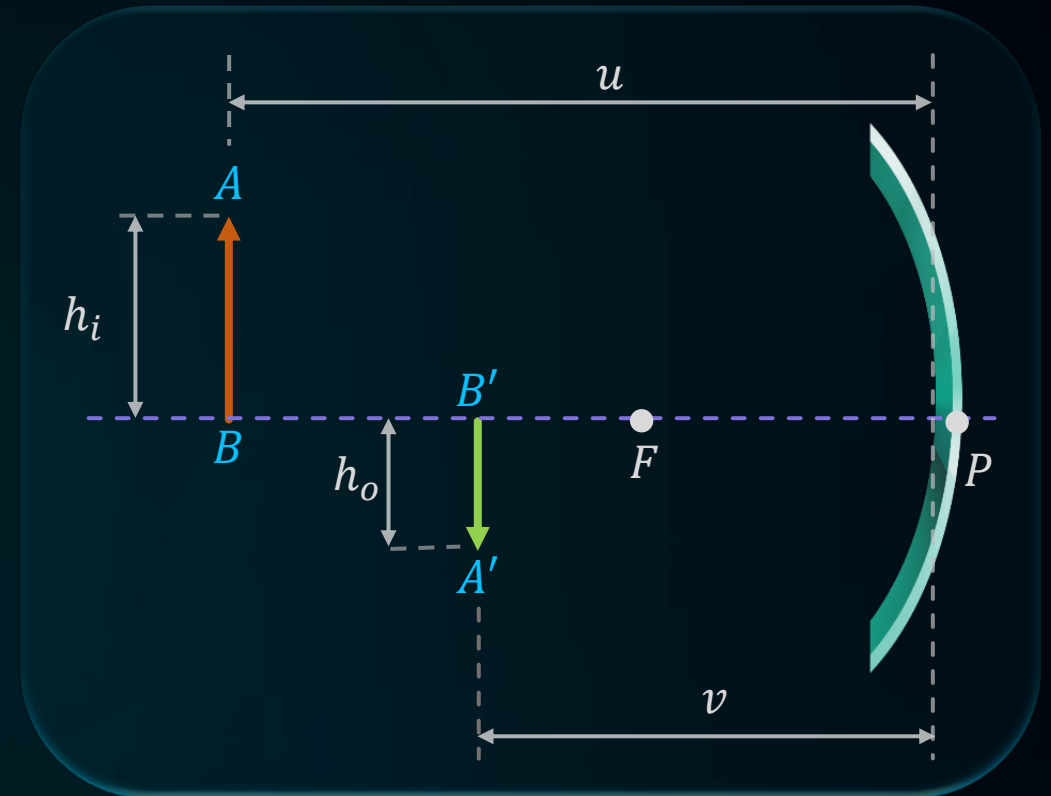


Differentiating w.r.t t

$$\frac{dh_i}{dt} = -\frac{v}{u} \frac{dh_o}{dt}$$

$\frac{dh_i}{dt}$ = Velocity of image perpendicular to principal axis

$\frac{dh_o}{dt}$ = Velocity of object perpendicular to principal axis





Motion Of the Image Parallel to principal axis



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

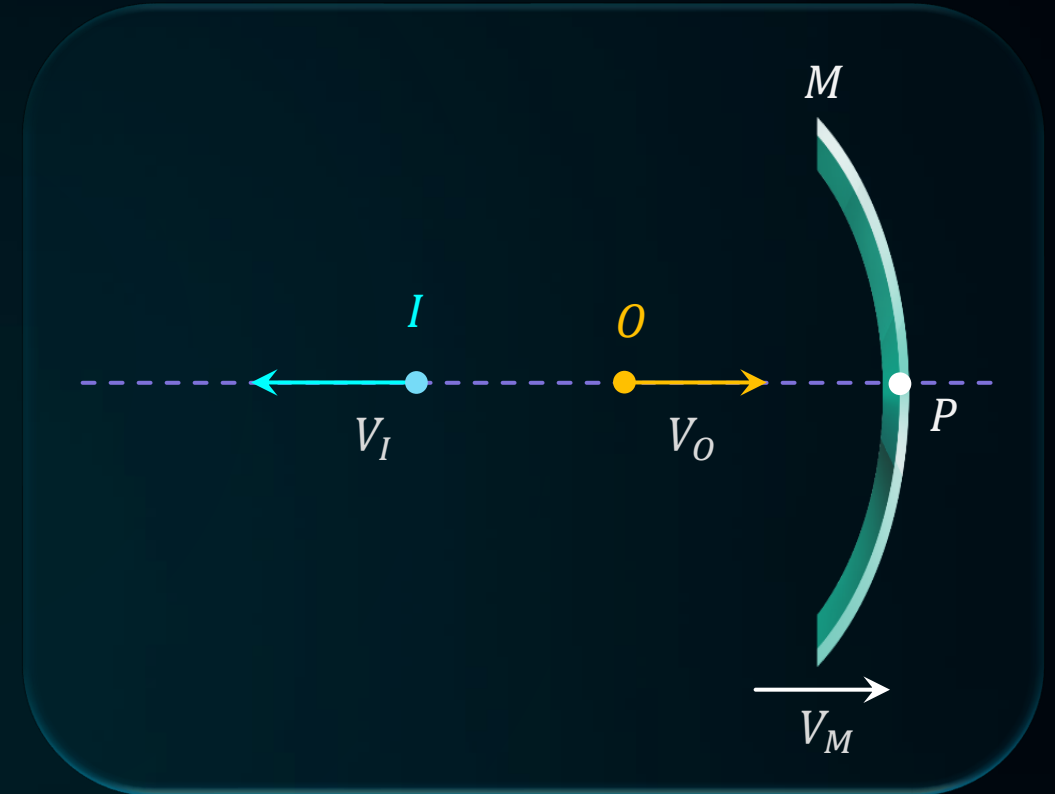


Differentiating w.r.t t

$$\vec{V}_{IM} = -\frac{v^2}{u^2}(\vec{V}_{OM})$$



$$\vec{V}_{IM} = -m^2(\vec{V}_{OM})$$



?

When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cm/s , find the speed (in cm/s) with which image moves at that instant.

JEE MAIN 2020

Given:

Object distance = 30 cm

Object's velocity = 9 cm/s

Image distance = 10 cm

To find: Image velocity

Solution:

Obeying sign convention:

$$u = -30\text{ cm} \quad v = -10\text{ cm} \quad \vec{V}_O = +9\text{ cm/s}$$

Velocity of mirror: $\vec{V}_M = 0$

We have: $\vec{V}_{IM} = -m^2 \vec{V}_{OM}$

$$(\vec{V}_I - \vec{V}_M) = -m^2 (\vec{V}_O - \vec{V}_M)$$

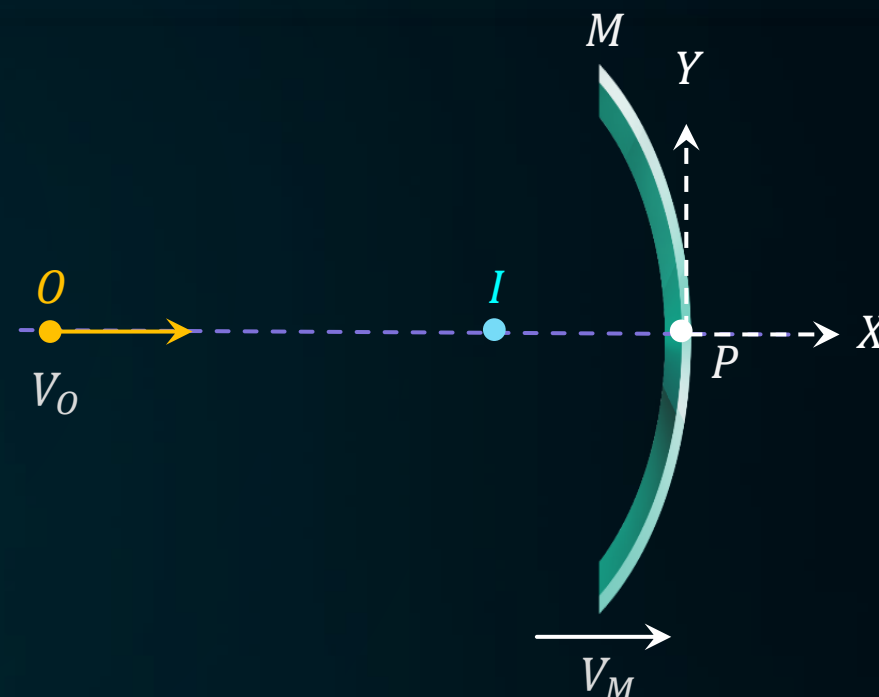
$$(\vec{V}_I - 0) = -\left(-\frac{v}{u}\right)^2 (\vec{V}_O - 0)$$

$$\vec{V}_I = -\left(-\frac{(-10)}{(-30)}\right)^2 \times (+9)\text{ cm/s}$$

$$\vec{V}_I = -\frac{1}{9} \times (+9)\text{ cm/s}$$

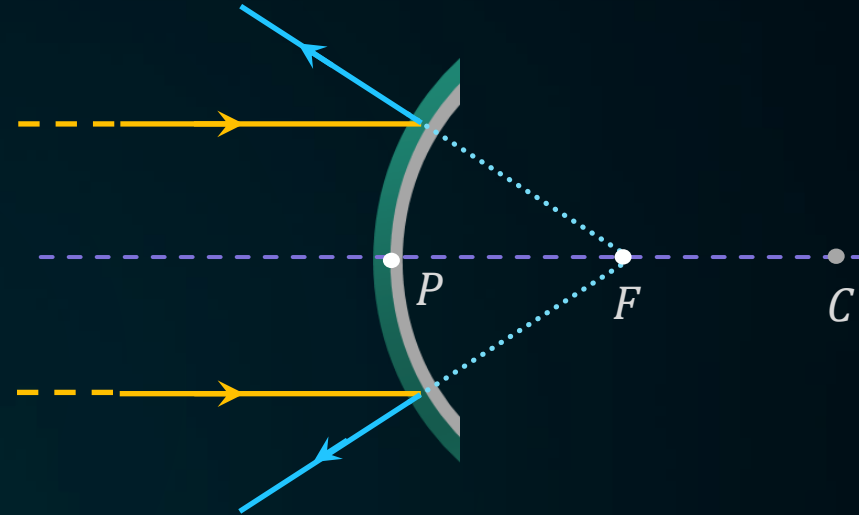
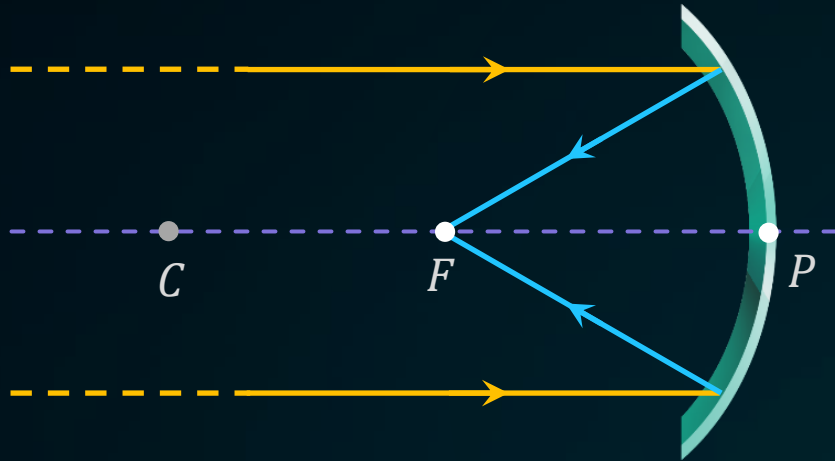
$$\vec{V}_I = -1\text{ cm/s}$$

Image velocity is 1 cm/s opposite to object velocity.





Power of mirror



- Power of a mirror is its ability to converge or diverge incident light.
- Power of mirror: $P = -\frac{1}{f}$
 - f = X-coordination of focus (in *meter*)
 - P = Power (in *dioptre*)



Longitudinal Magnification



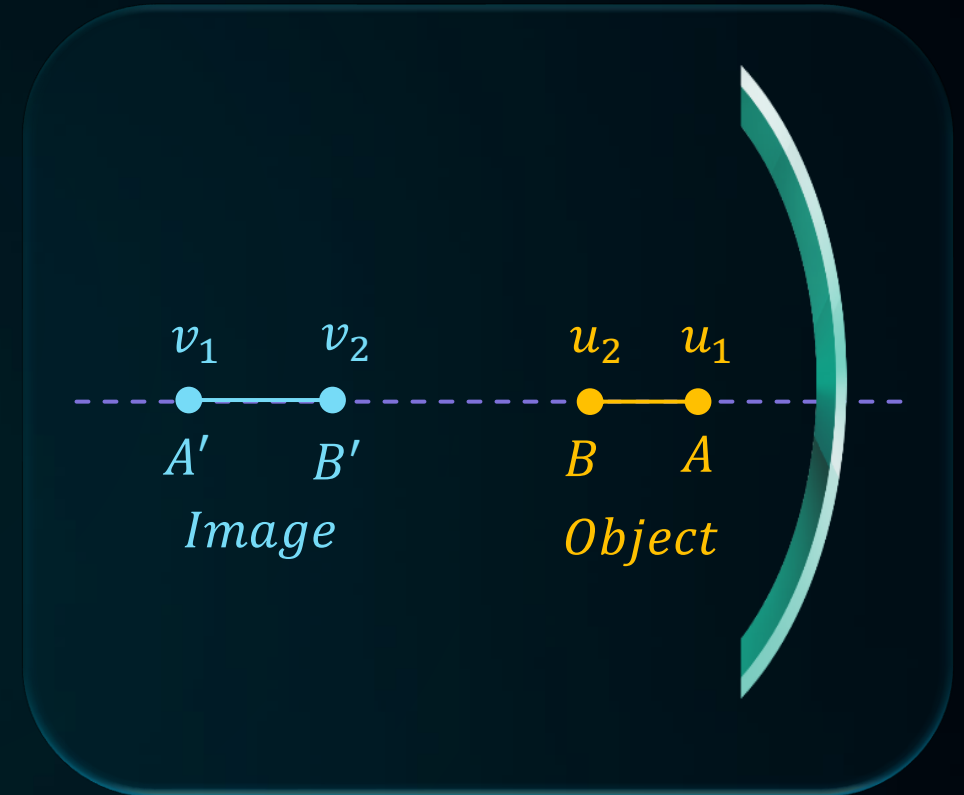
Longitudinal magnification:

$$m_L = \frac{v_2 - v_1}{u_2 - u_1}$$

When object size is very small
w.r.t its distance from pole

$$m_L = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = m^2$$

Negative sign indicates that image will be
inverted w.r.t object.



?

A point object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principal axis. Find the position of image. If object is shifted 1 mm towards the mirror along principal axis, find the shift in image.

Solution:

X-coordinate of object, $u = -60 \text{ cm}$

X-coordinate of focus, $f = -10 \text{ cm}$

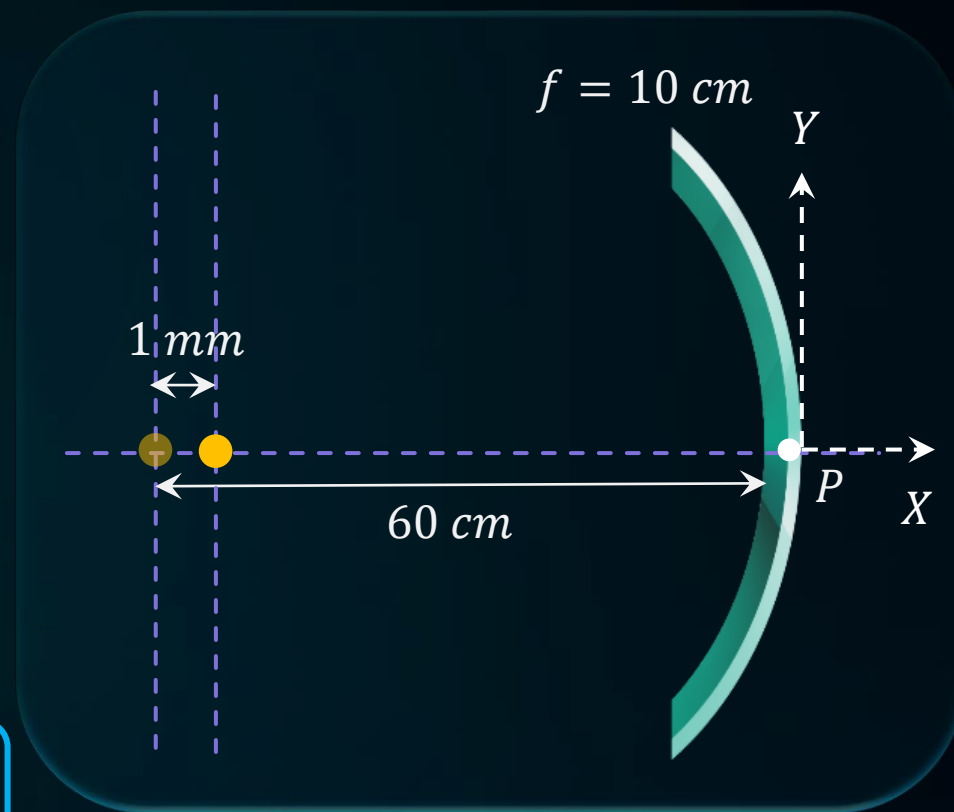
From mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{(-10)} - \frac{1}{(-60)} \Rightarrow \boxed{v = -12 \text{ cm}}$$

Shift of object: $du = +1 \text{ mm} \Rightarrow du = +0.1 \text{ cm}$

We have: $\frac{dv}{du} = -\left(\frac{v}{u}\right)^2$

Shift of image: $dv = -\left(\frac{v}{u}\right)^2 du \Rightarrow \boxed{dv = -0.004 \text{ cm}}$



?

The elevator is going up with an acceleration of 2 m/s^2 and the focal length of the convex mirror is 12 cm . All the surfaces are smooth, and the pulley is light. The mass-pulley system is released from rest (with respect to the elevator) at $t = 0$ when the distance of B from the mirror is 42 cm . Find the distance between the image of the block B and the mirror at $t = 0.2 \text{ s}$. Take $g = 10 \text{ m/s}^2$. (Given that mass of both A and B are equal)

$t = 0.2 \text{ s}$ $\uparrow a = 2 \text{ m/s}^2$

Solution:

- Upward acceleration of the elevator: $a = 2 \text{ m/s}^2$
- Focal length of the convex mirror: $f = 12 \text{ cm}$
- The mass-pulley system is released from rest with respect to the elevator.

From FBD of block A (in lift's frame): $T = ma'$

From FBD of block B (in lift's frame):

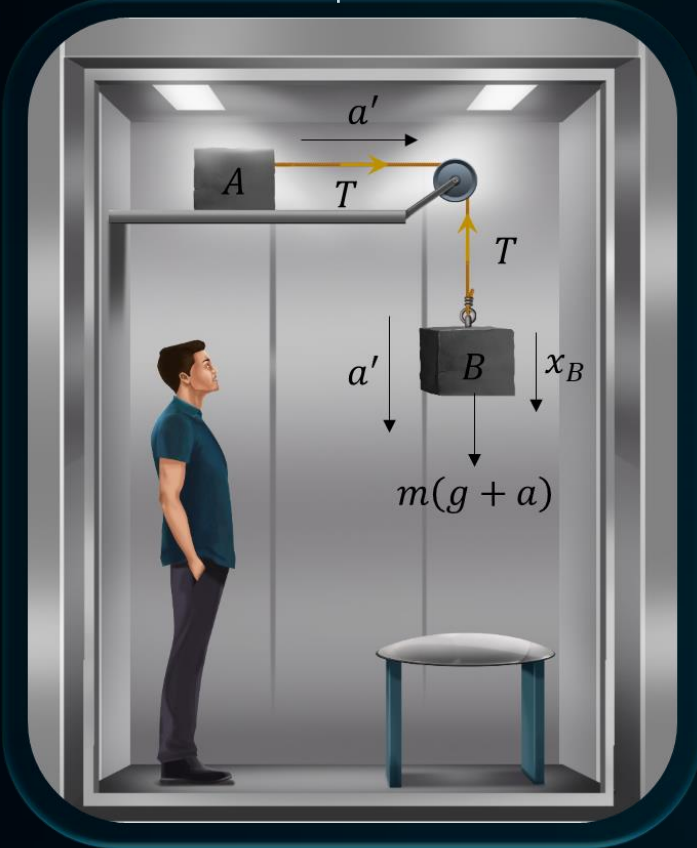
$$m(g + a) - T = ma'$$

$$a' = \frac{g + a}{2} = \frac{10 + 2}{2} = 6 \text{ m/s}^2$$

Displacement of block B in time $t = 0.2 \text{ s}$:

$$x_B = u_B t + \frac{1}{2} a_B t^2$$

$$x_B = \frac{1}{2} \times 6 \times (0.2)^2 \text{ m} \quad \Rightarrow \quad x_B = \frac{12}{100} \text{ m} = 12 \text{ cm}$$



$$t = 0.2 \text{ s} \quad \uparrow a = 2 \text{ m/s}^2$$

Distance of block B from mirror at $t = 0.2 \text{ s}$:

$$u = 42 - 12 = 30 \text{ cm}$$

Applying mirror formula:

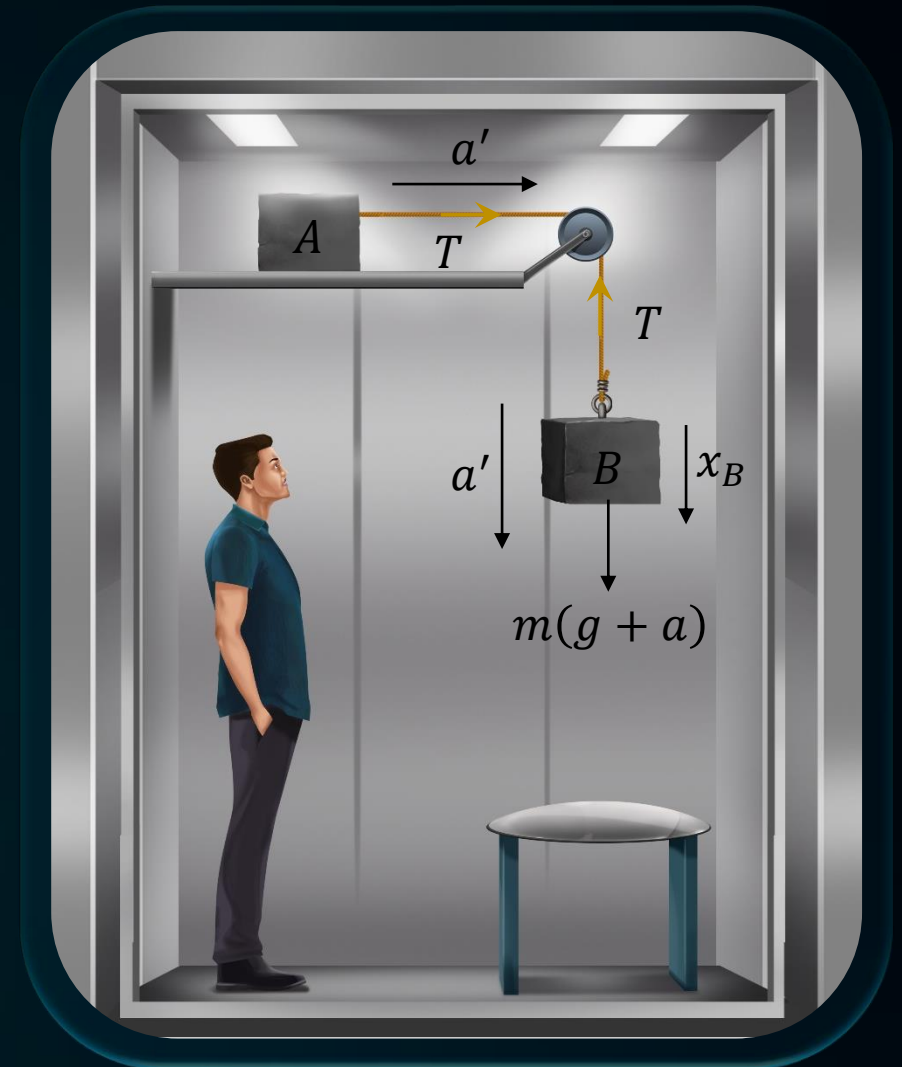
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{(-30)} + \frac{1}{v} = \frac{1}{(+12)}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{2 + 5}{60} = \frac{7}{60} \Rightarrow v = \frac{60}{7} \text{ cm}$$

$$v = 8.57 \text{ cm}$$



?

You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm . The radius of curvature of the mirror would then be :

JEE MAIN 2015

Solution:

X-coordinate of object, $u = -10\text{ cm}$

The person views the magnified image at 25 cm . Thus, the distance of the image from the mirror is $(25 - 10) = 15\text{ cm}$

X-coordinate of image, $v = +15\text{ cm}$

Apply mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

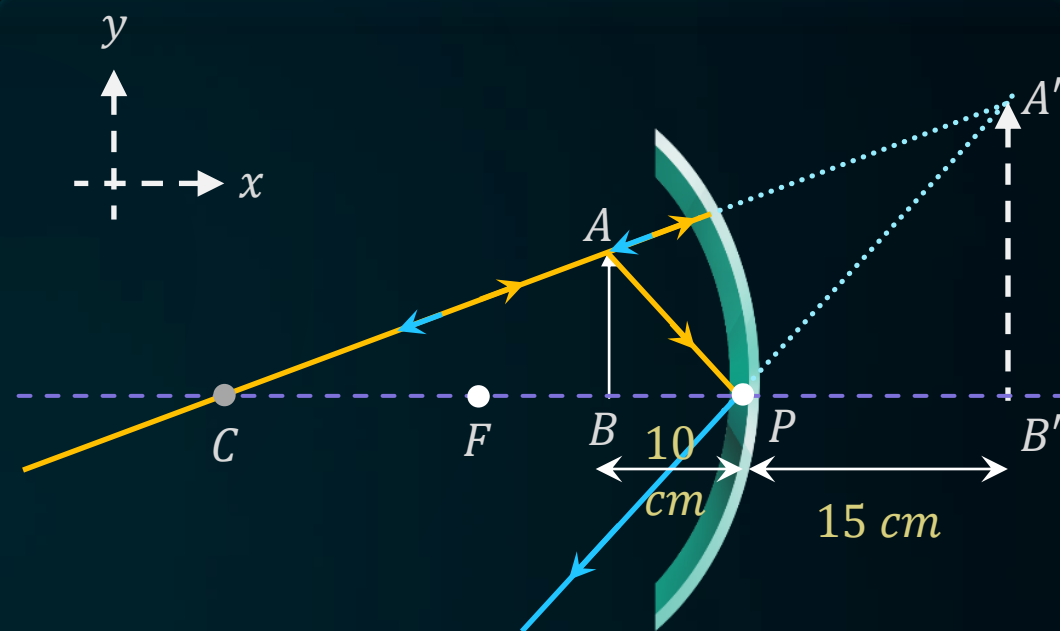
$$\frac{1}{(-10)} + \frac{1}{(+15)} = \frac{1}{f}$$

$$\frac{-3 + 2}{30} = \frac{1}{f}$$

$$f = -30\text{ cm}$$

Radius of curvature: $R = 2f$

$$R = -60\text{ cm}$$



?

Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A . The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is *JEE MAIN 2002*

Solution:

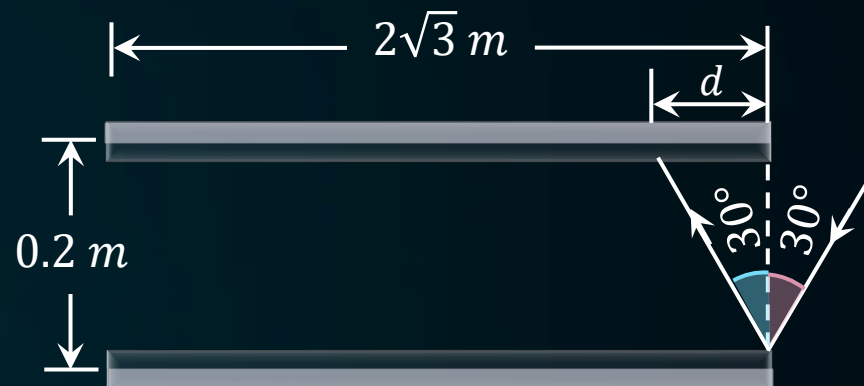
From figure:

$$d = 0.2 \tan 30^\circ$$

$$d = \frac{0.2}{\sqrt{3}} m$$

The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is:

$$n = \frac{2\sqrt{3}}{d} = \frac{2\sqrt{3}}{0.2} \times \sqrt{3} = 30$$

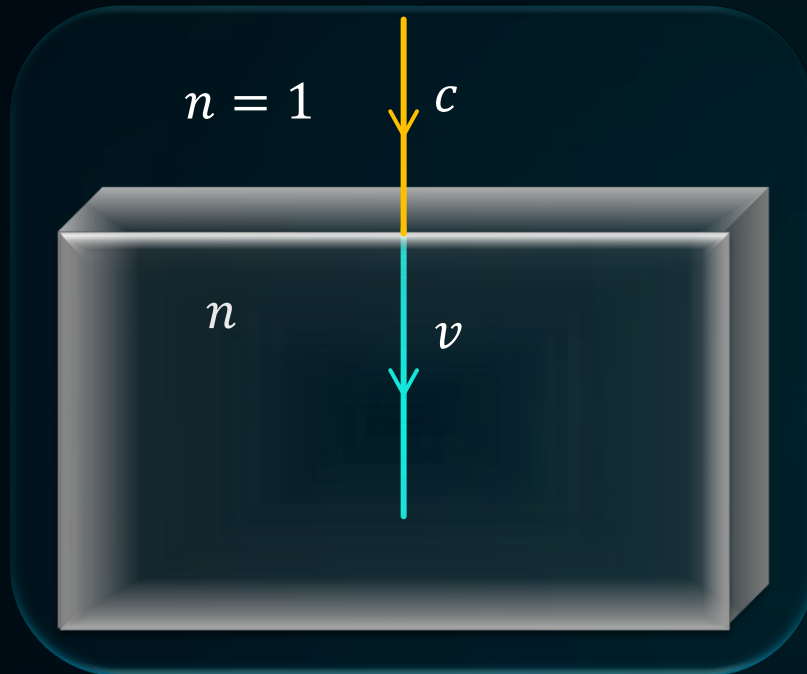




Refraction and Refractive Index



Refraction: **Change in properties of light** when it passes through mediums of different **optical densities**.



Optically denser → **Lower speed of light**

Optically rarer → **Higher speed of light**

Refractive Index (n): Ratio of **speed of light in vacuum(c)** to speed of light in a medium(v).

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

$$c = 3 \times 10^8 \text{ m/s}$$



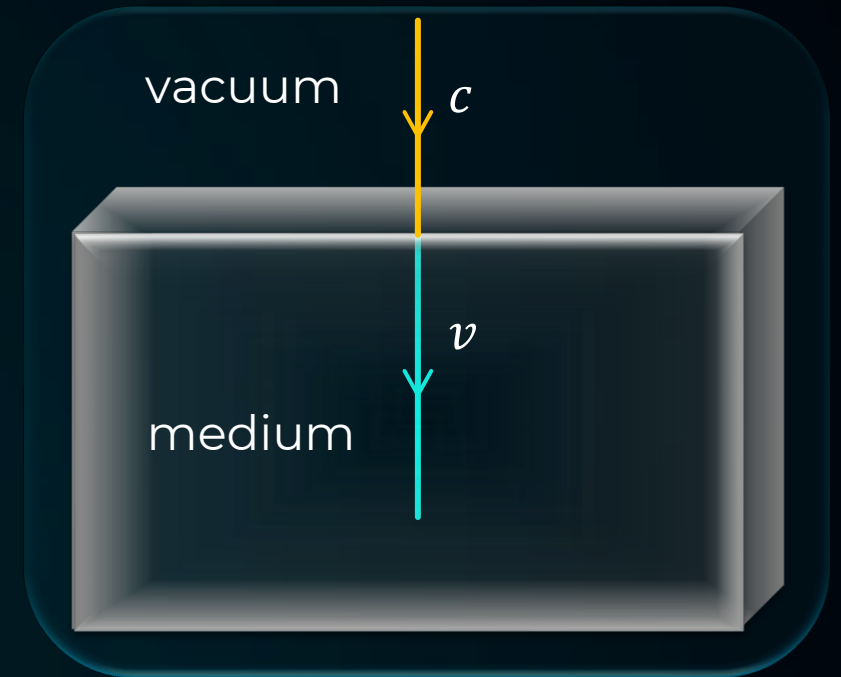
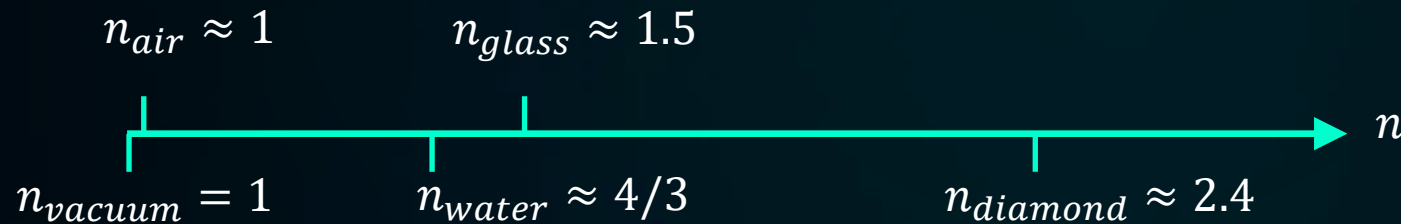
Absolute Refractive Index



Absolute Refractive Index (n): Ratio of **speed of light in vacuum(c)** to speed of light in a medium(v).

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

$$c = 3 \times 10^8 \text{ m/s}$$

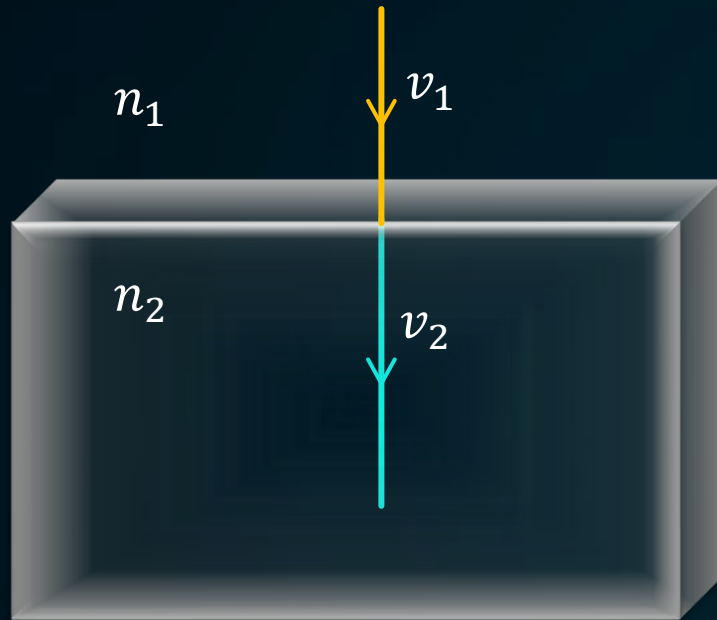




Relative Refractive Index



When light travels from medium 1 to medium 2, then the refractive index of the second medium with respect to the first medium is known as the relative refractive index.

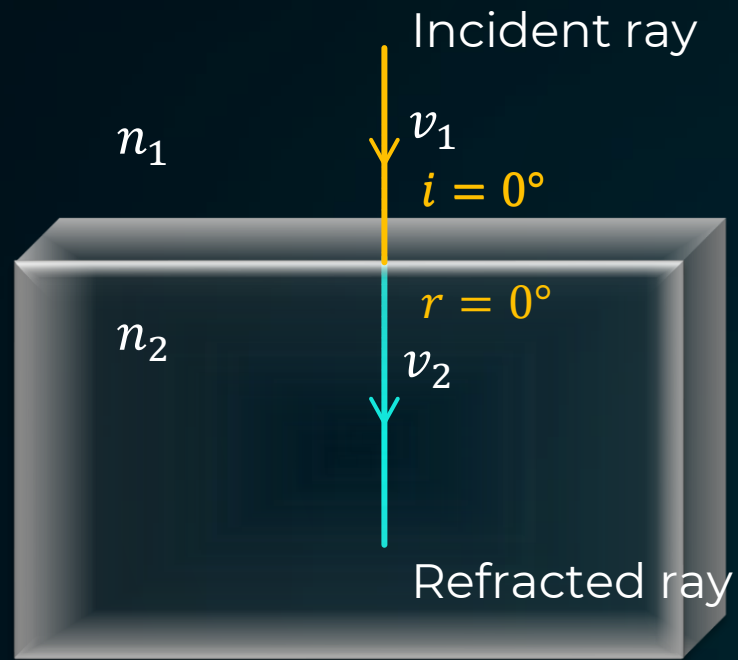


Relative Refractive Index(n_{21})

$${}_1n_2 = n_{21} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$



Normal Incidence



i = Angle of incidence

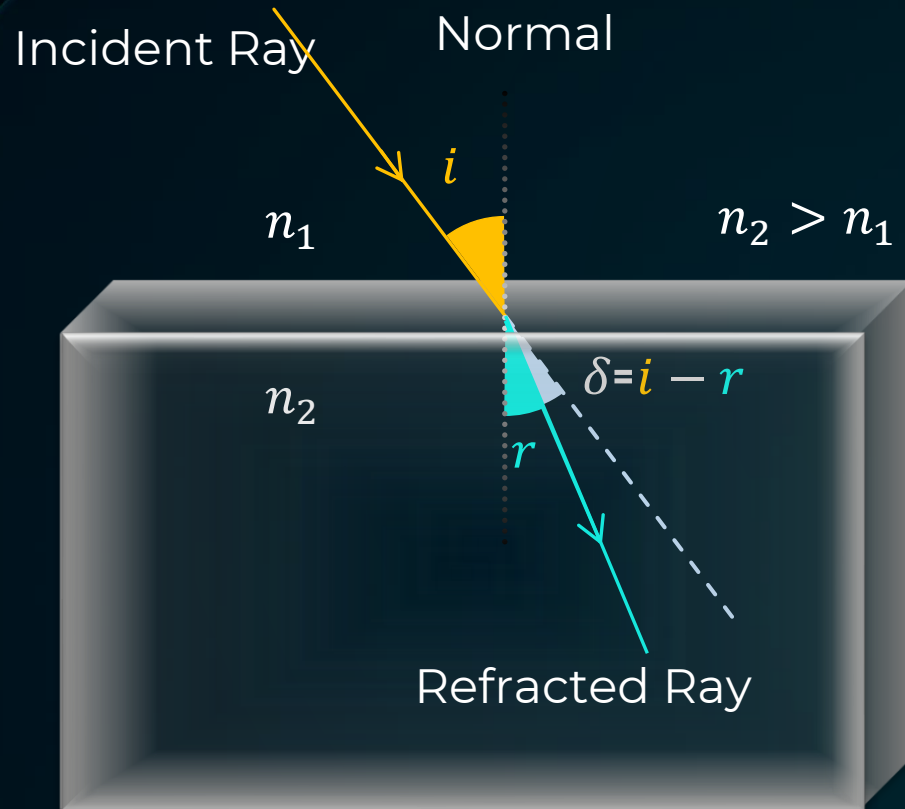
r = Angle of refraction

δ = Deviation = 0°

- Deviation is 0° for a normal incident light ray.



Oblique Incidence

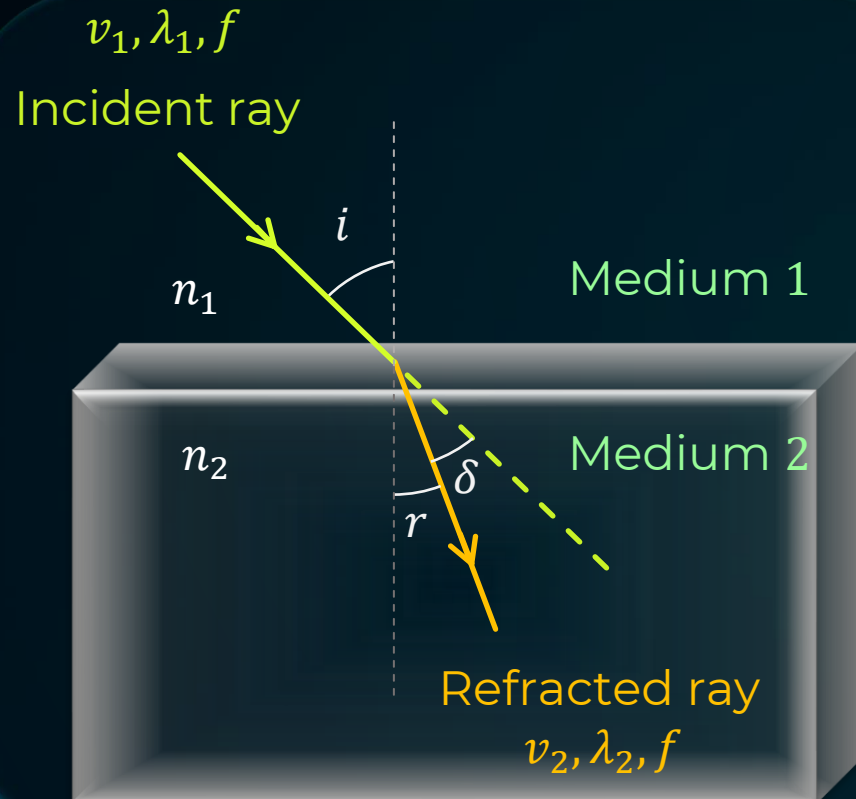


- **Angle of refraction (r):** The angle that the refracted ray makes with the normal.
 - **Angle of incidence (i):** The angle that the incident ray makes with the normal.
- $\delta = \text{Deviation} = i - r$
- Obliquely incident ray bends towards the normal when refracted from optically rarer to optically denser medium and vice-versa.



Laws of Refraction

- The incident ray, the normal to any refracting surface at the point of incidence and the refracted ray **all lie in a same plane**.
- Snell's Law: For any given pair of media, the ratio of sine of the angle of incidence(i) to the sine of the angle of refraction(r) is a constant and is known as the relative refractive index.



$$n_1 \sin i = n_2 \sin r$$

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

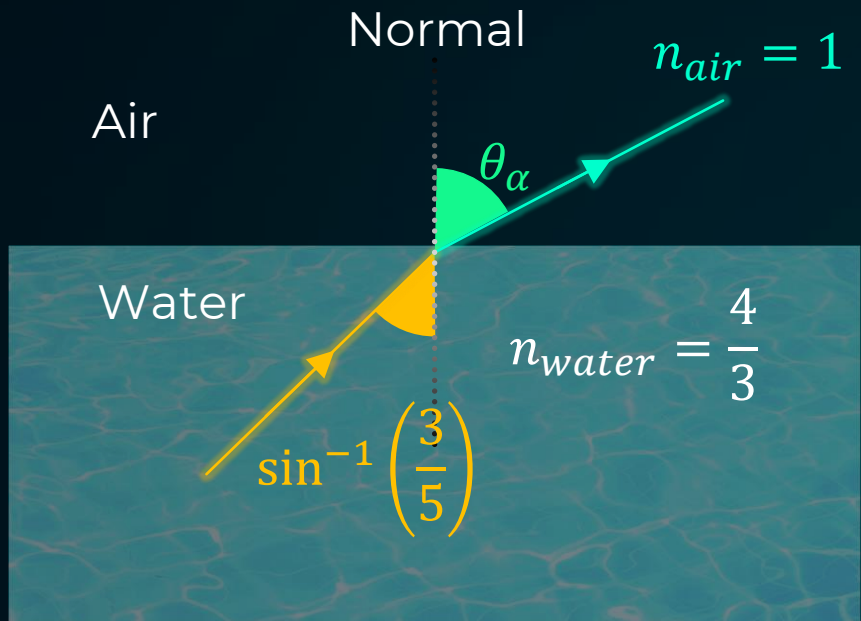
- When light travels from one medium to another medium, its velocity and wavelength changes but its frequency remains the same.

$$f_1 = f_2$$

- $n_1 < n_2 \Rightarrow v_1 > v_2$: Light bends towards the normal.
- $n_2 < n_1 \Rightarrow v_2 > v_1$: Light bends away from the normal.

?

Find the angle θ_α made by the light ray when it gets refracted from water to air, as shown in the figure.



Solution:

Apply Snell's law

$$n_i \sin i = n_r \sin r$$

$$\frac{4}{3} \times \sin\left(\sin^{-1}\left(\frac{3}{5}\right)\right) = 1 \times \sin\theta_\alpha$$

$$\sin\theta_\alpha = \frac{4}{5}$$

$$\theta_\alpha = 53^\circ$$



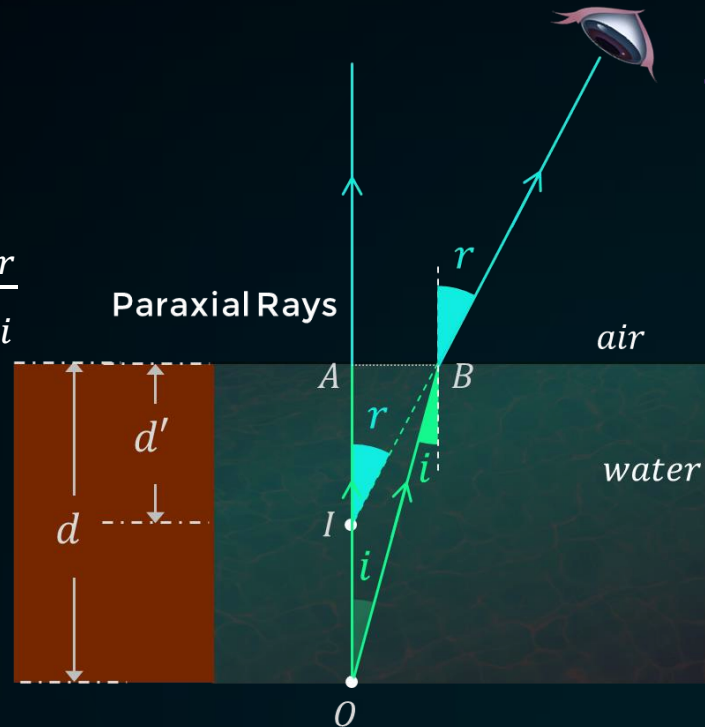
Apparent Depth



$$\frac{\tan(i)}{\tan(r)} = \frac{d'}{d} \approx \frac{\sin(i)}{\sin(r)} = \frac{n_r}{n_i}$$

$$d' = d \left(\frac{n_r}{n_i} \right)$$

$$d' = \frac{d}{\left(\frac{n_i}{n_r} \right)}$$



i and r are very small

- If object and observer are situated in **different mediums** then due to refraction, the object appears to be displaced from its real position.

d = Real Depth

d' = Apparent Depth

n_1 = Refractive index of the medium from which the light ray is coming

n_2 = Refractive index of the medium in which the light ray is going after refraction

- Rays should be paraxial.
- Both d and d' are measured from interface.
- Both object and Image will always be on the same side of interface.
- Rays are going from $n_i \rightarrow n_r$.

?

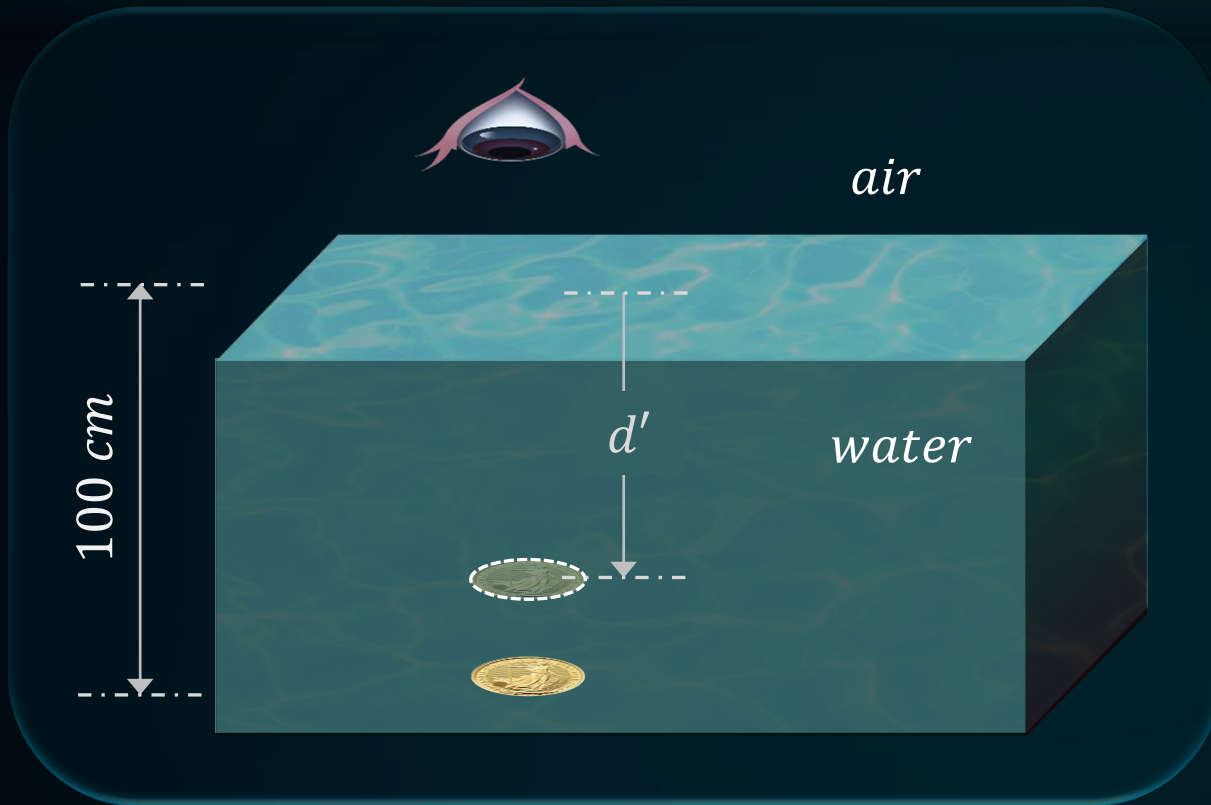
An object lies **100 cm** inside water. It is viewed from air nearly normally. Find the **apparent depth** of the object.

Solution:

$$d' = \frac{d}{n_{rel}}$$

$$d' = \frac{(100 \text{ cm})}{\frac{\left(\frac{4}{3}\right)}{1}}$$

$$d' = 75 \text{ cm}$$



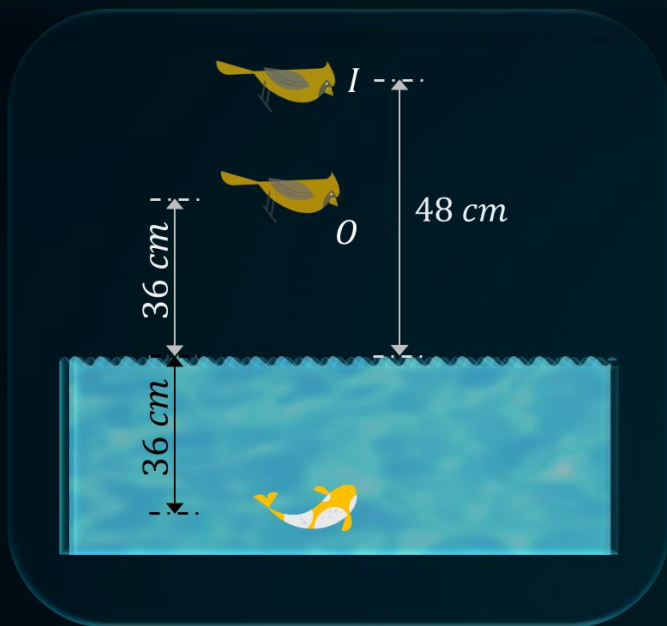
?

If the actual height of an eagle and the actual depth of a fish from the free surface of the water are 36 m and 36 m respectively as shown, find

- Find apparent height of the bird. At what distance will the bird appear to the fish?
- Find apparent depth of fish. At what distance will the fish appear to the bird?

Solution:

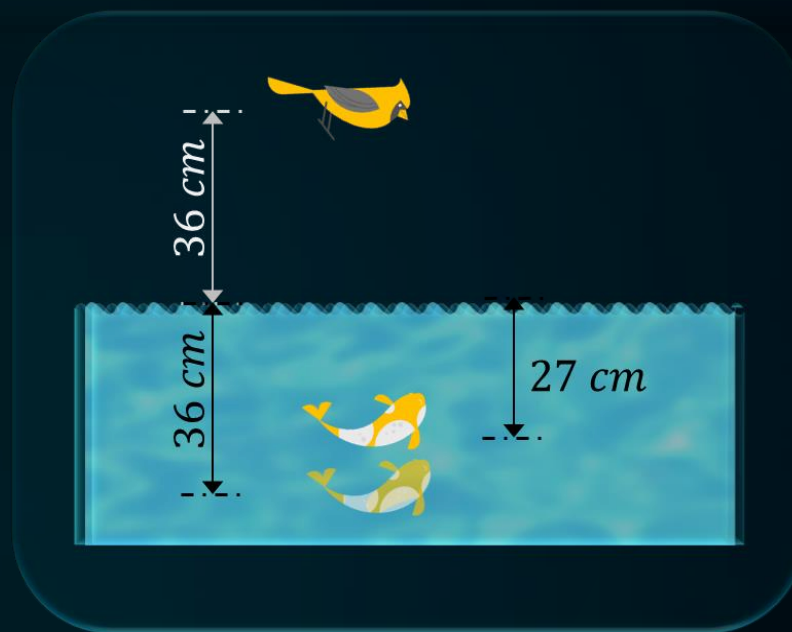
For Fish



$$(d')_B = \frac{36\text{ cm}}{\frac{1}{\left(\frac{4}{3}\right)}} = 48\text{ cm}$$

$$d_{BF} = 84\text{ cm}$$

For bird



$$(d')_F = \frac{36\text{ cm}}{\frac{\left(\frac{4}{3}\right)}{1}} = 27\text{ cm}$$

Fish will appear at-

$$d_{FB} = 36 + 27$$

$$d_{FB} = 63\text{ cm}$$

?

An observer can see through a small hole on the side of a jar (radius 15 cm) at a point height of 15 cm from the bottom. The hole is at a height of 45 cm . When the jar is filled with a liquid up to a height of 30 cm , the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, find the value of N . [Given $\sqrt{5/2} = 1.58$]

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Solution: $\left. \begin{array}{l} AE = BC = 15\text{ cm} \\ AB = CE = 30\text{ cm} \end{array} \right\} DE = CE - CD = 15\text{ cm}$

From $\triangle AED$:

$$\tan \angle ADE = \frac{AE}{DE} = \frac{15}{15} = 1 \Rightarrow \angle ADE = 45^\circ$$

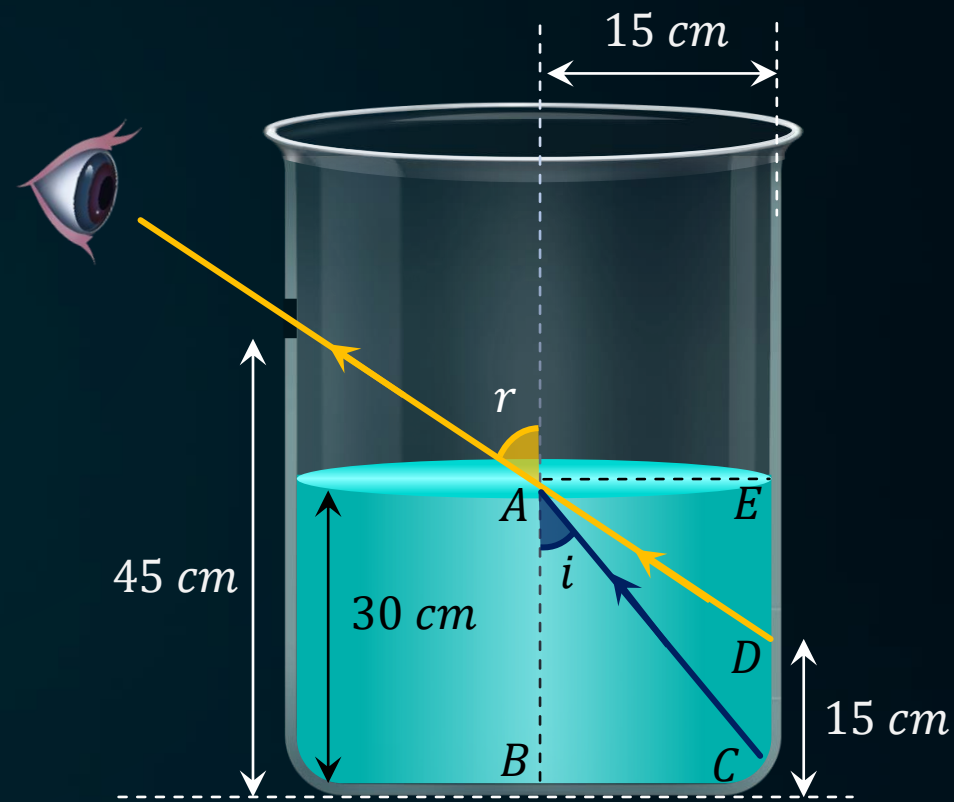
Therefore, $\angle r \equiv \angle ADE = 45^\circ$

$$\text{Now, } \sin i = \frac{BC}{AC} = \frac{15}{\sqrt{15^2 + 30^2}} = \frac{15}{15\sqrt{5}} = \frac{1}{\sqrt{5}}$$

Apply Snell's law:

$$\mu \times \sin i = 1 \times \sin r$$

$$\mu = \frac{\sin r}{\sin i} \Rightarrow \frac{N}{100} = \sqrt{\frac{5}{2}} \Rightarrow N = 158$$





Refraction Through Parallel Slab



- When R.I. of either side of the slab is same:

- For 1st refraction:

$$n_1 \sin i = n_2 \sin r$$

- For 2nd refraction:

$$n_2 \sin r = n_1 \sin e$$

$$i = e$$

From $\triangle ABN$:

$$\cos r = \frac{AN}{AB} = \frac{t}{AB}$$

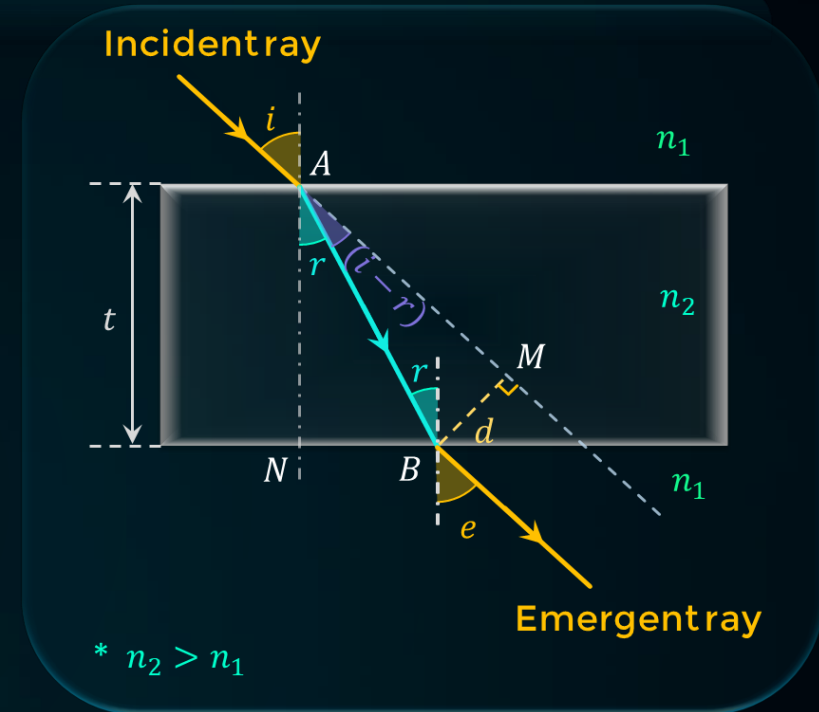
From $\triangle ABM$:

$$\sin(i - r) = \frac{BM}{AB} = \frac{d}{AB}$$

Equal AB from both equations

$$\frac{t}{\cos r} = \frac{d}{\sin(i - r)}$$

$$d = \frac{t \sin(i - r)}{\cos r}$$



- When ray of light incident on the parallel glass slab, the emergent ray **shifts laterally** to some distance. This shift is known as “**Lateral shift**”.



Minimum Lateral Shift



Rays incident normally continue undeviated.



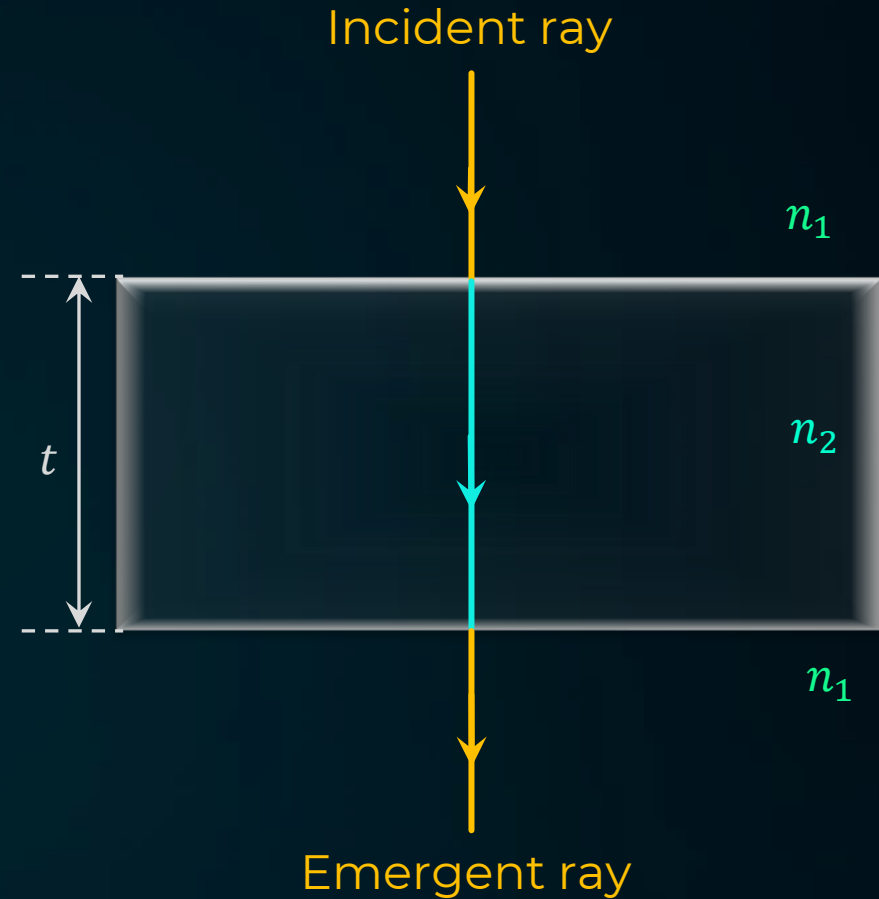
$$i = 0^\circ \text{ and } r = 0^\circ$$



$$d = \frac{t \sin(i - r)}{\cos r}$$

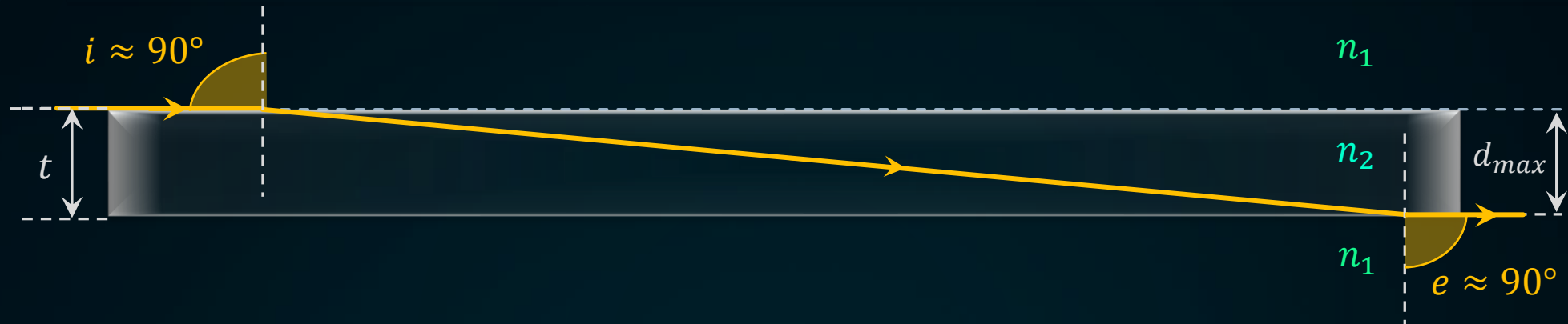


$$d_{min} = 0$$





Maximum Lateral Shift



** The slab is very long

- Maximum lateral shift occurs when:

$$i \approx 90^\circ$$

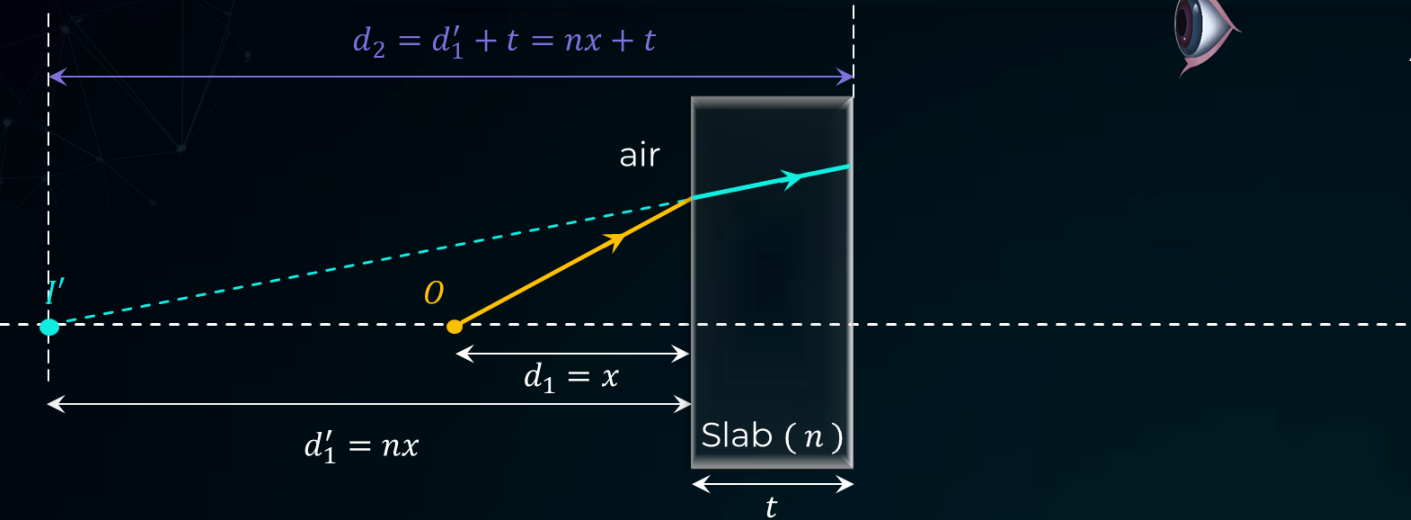
- Maximum lateral shift:

$$d_{max} = t$$

- When the R.I. of either side of the glass slab is same, the angle of emergence becomes equal to angle of incidence.

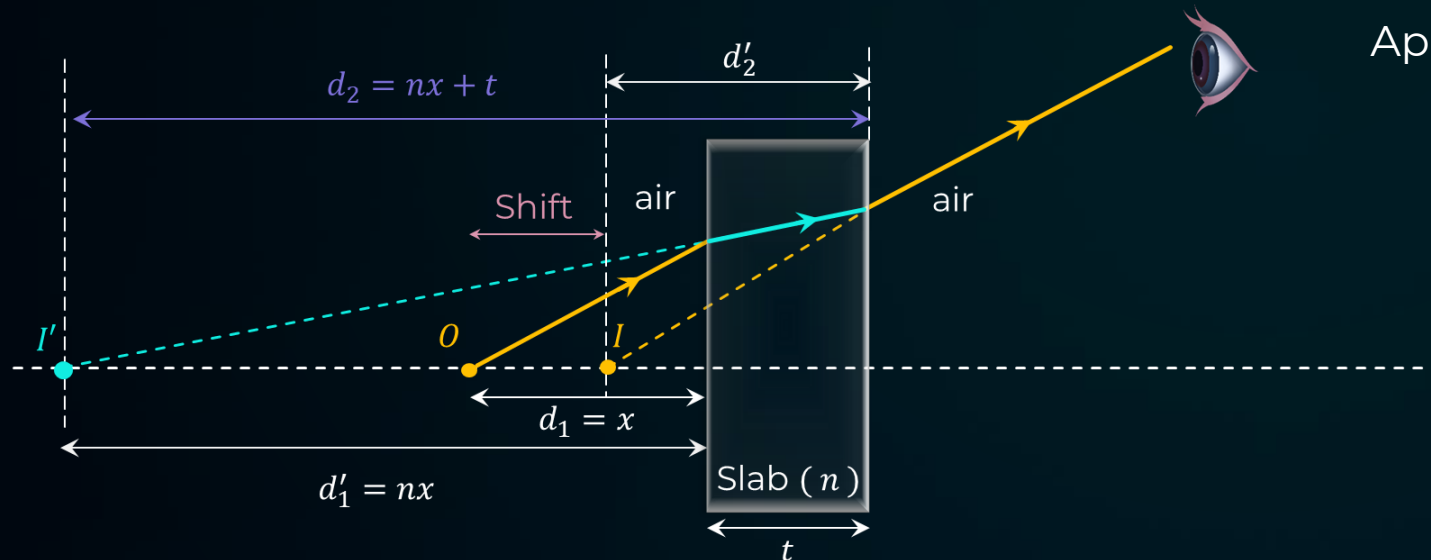


Normal Shift



Apparent position of image (I') after first refraction:

$$d'_1 = \frac{d_1}{\left(\frac{n_{\text{air}}}{n_{\text{slab}}}\right)} \Rightarrow d'_1 = \frac{x}{\left(\frac{1}{n}\right)} \Rightarrow d'_1 = nx$$



Apparent position of image (I) after second refraction:

$$d'_2 = \frac{d_2}{\left(\frac{n_{\text{slab}}}{n_{\text{air}}}\right)} \Rightarrow d'_2 = \frac{(nx + t)}{\left(\frac{n}{1}\right)} \Rightarrow d'_2 = x + \frac{t}{n}$$

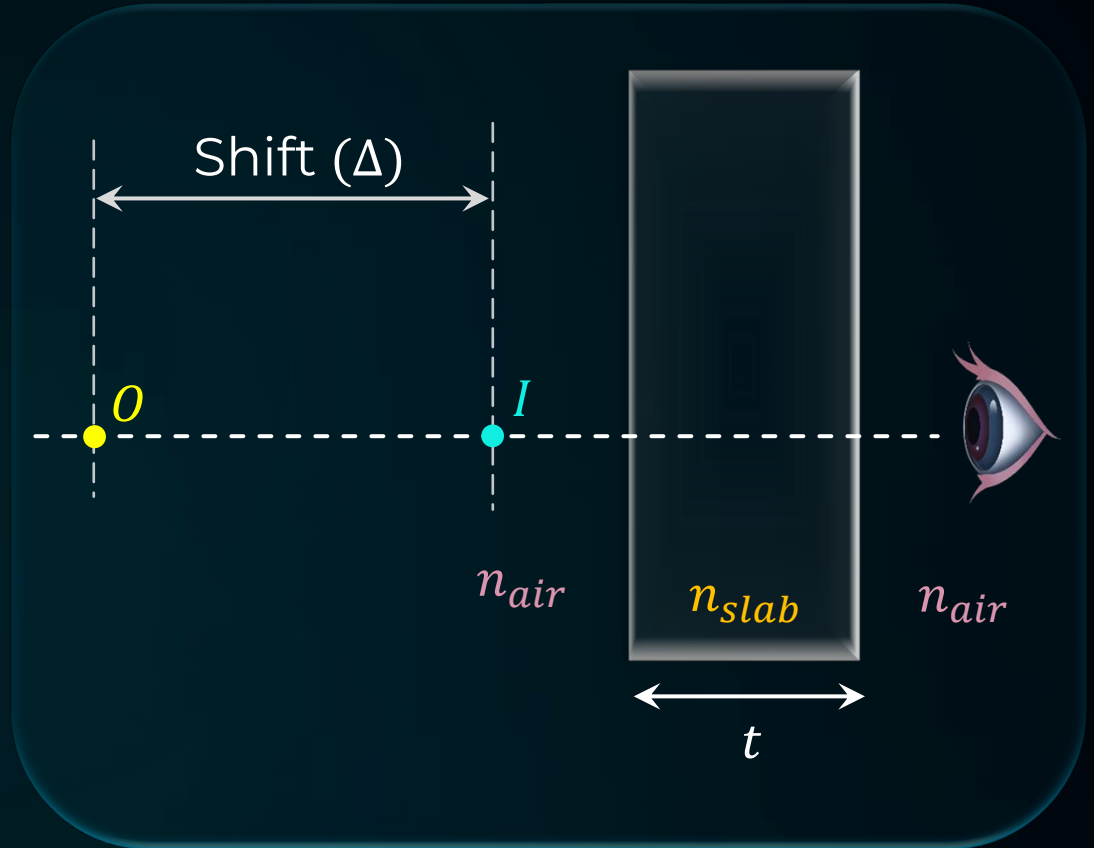
The shift of image: $\Delta = (x + t) - d'_2 = t \left(1 - \frac{1}{n}\right)$



Summary



- Rays should be **paraxial**.
- Shift is **independent** of object distance from the slab.
- Shift of image is measured from the object.
- The image is generally shifted towards the observer when a glass slab is placed in between object and observer (**both object and observer are in air**).





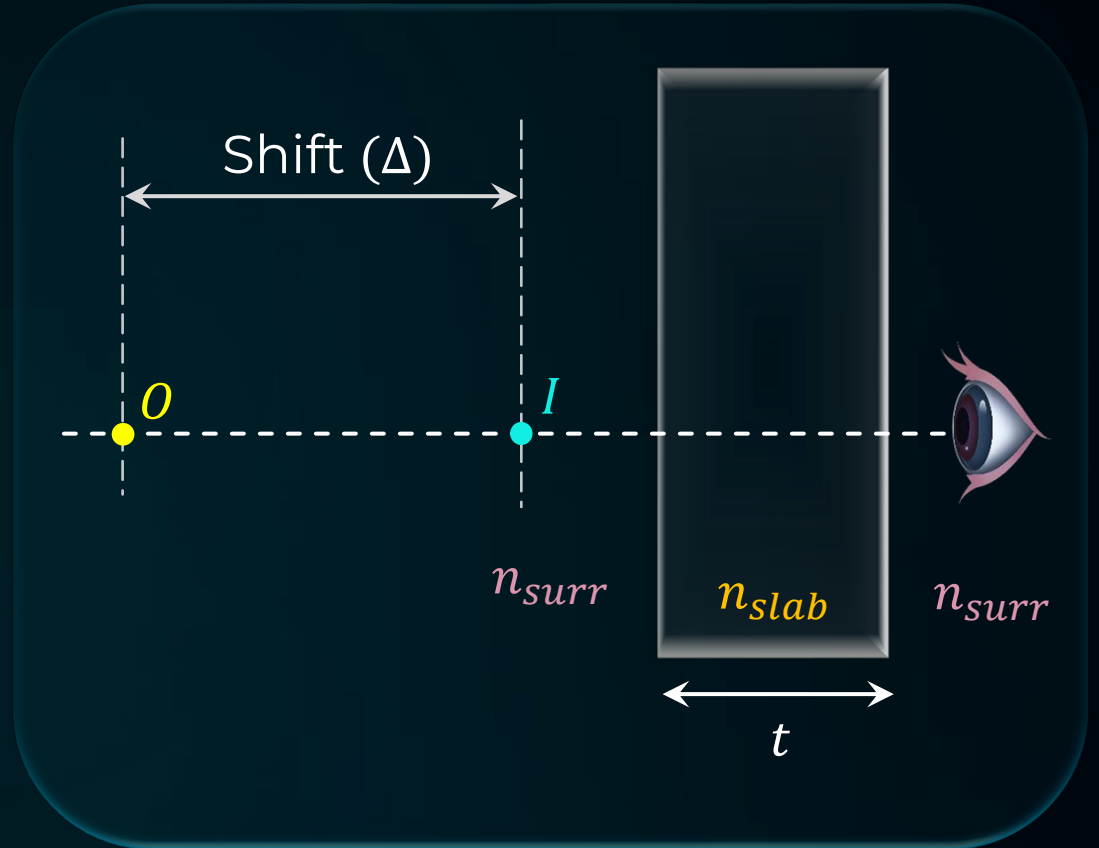
Normal Shift When Surrounding Of Slab Is Not Air



- When the R.I. of the surrounding is n_{surr} , the shift is given by,

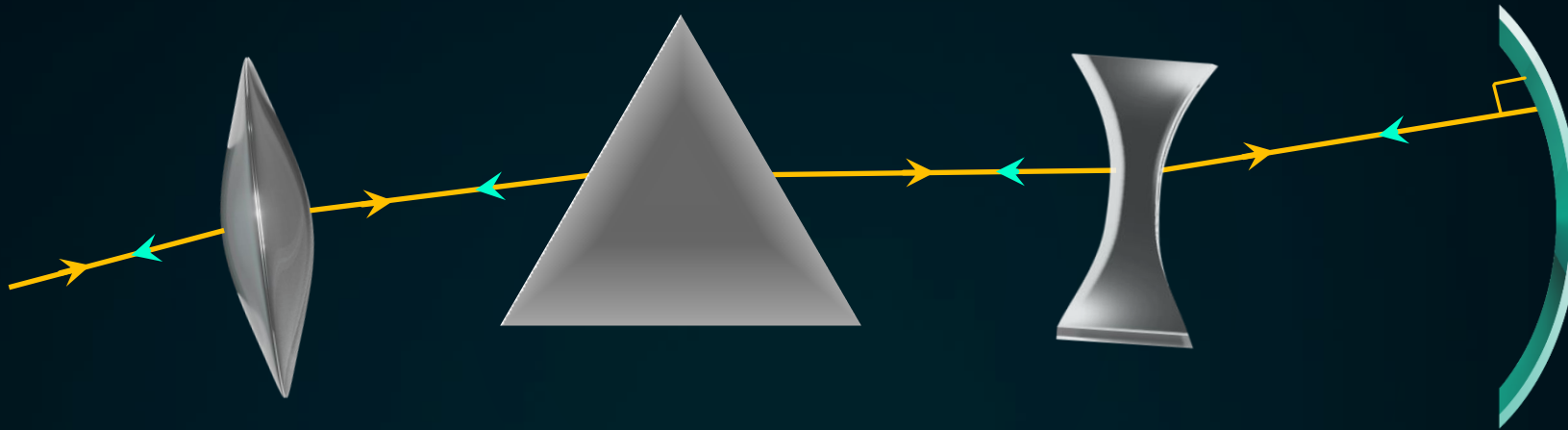
$$\Delta = t \left(1 - \frac{n_{surr}}{n_{slab}} \right)$$

- If $n_{surr} > n_{slab}$, $\Delta = -ve$. The image will be shifted away from observer.





Summary: Principle of Reversibility of Light



- If light falls on any reflecting surface (say plane mirror or spherical mirror) **normally** after a series of phenomena, then the light **retraces** its whole path after reflection, and this property of light is known as the **principle of reversibility of light**.



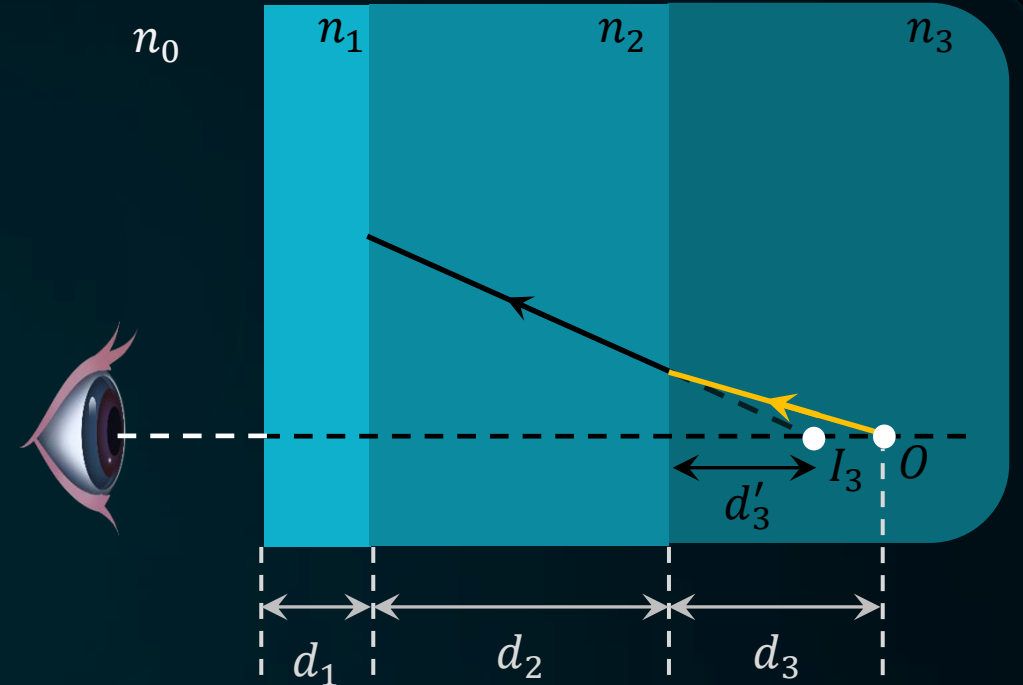
Object and Observer in Different Mediums



- The observer is in a medium of R.I. n_0 and the object is in a medium of R.I. n_3 .

1st refraction: From medium of R.I. n_3 to n_2

Apparent depth:
$$d'_3 = \frac{d_3}{\left(\frac{n_3}{n_2}\right)} = \left(\frac{n_2}{n_3}\right) d_3$$



** $n_0 < n_1 < n_2 < n_3$



Object and Observer in Different Mediums



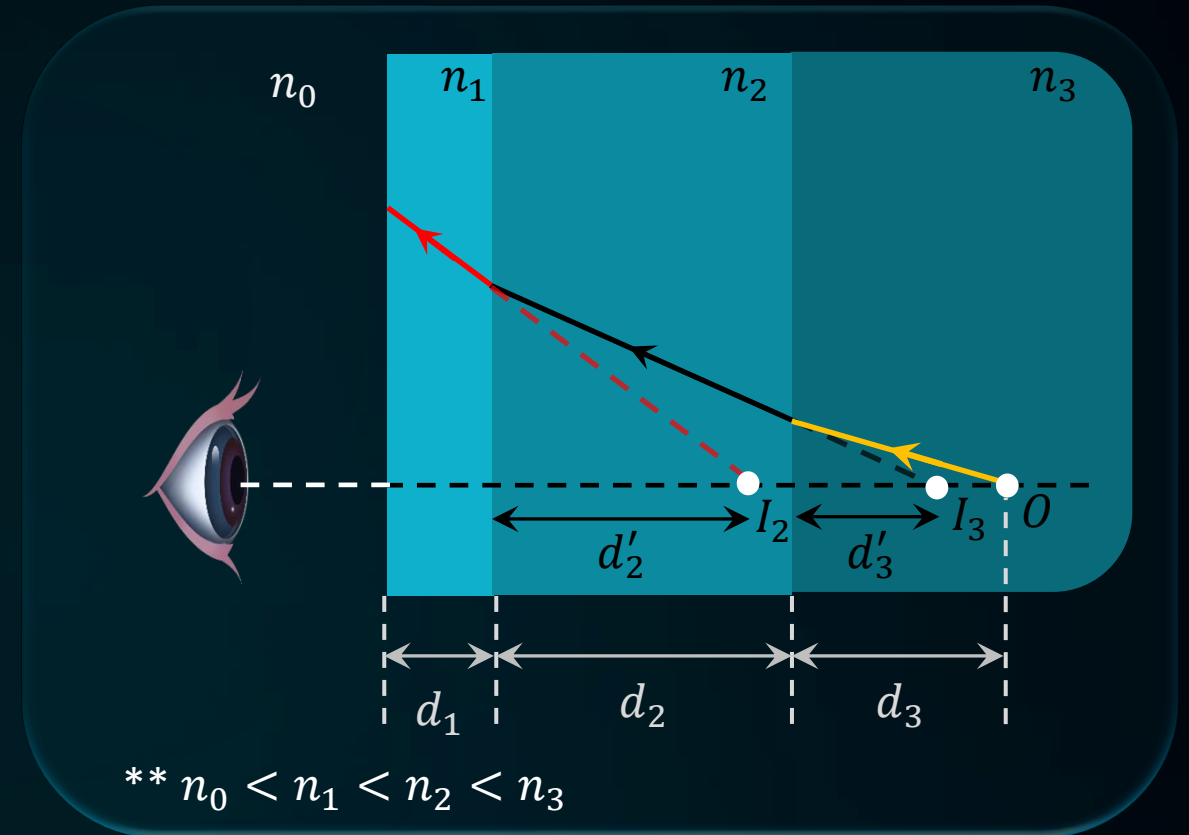
2nd refraction: From medium of R.I. n_2 to n_1

Image I_3 acts as an object for this refraction.

Distance of I_3 from the interface of n_2 and n_1 : $(d_2 + d'_3)$

Apparent depth: $d'_2 = \frac{(d_2 + d'_3)}{\left(\frac{n_2}{n_1}\right)}$

$$d'_2 = \left(\frac{n_1}{n_2}\right) d_2 + \left(\frac{n_1}{n_3}\right) d_3$$





Object and Observer in Different Mediums



3rd refraction: From medium of R.I. n_1 to n_0

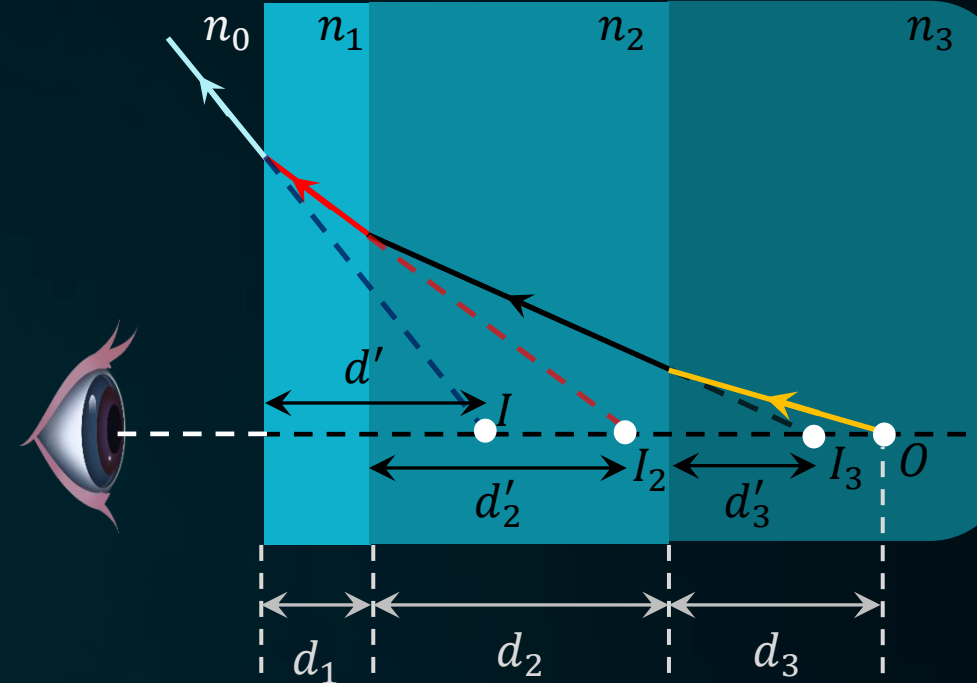
Image I_2 acts as an object for this refraction.

Distance of I_2 from the interface of

n_0 and n_1 : $(d_1 + d'_2)$

Apparent depth: $d' = \frac{(d_1 + d'_2)}{\left(\frac{n_1}{n_0}\right)}$

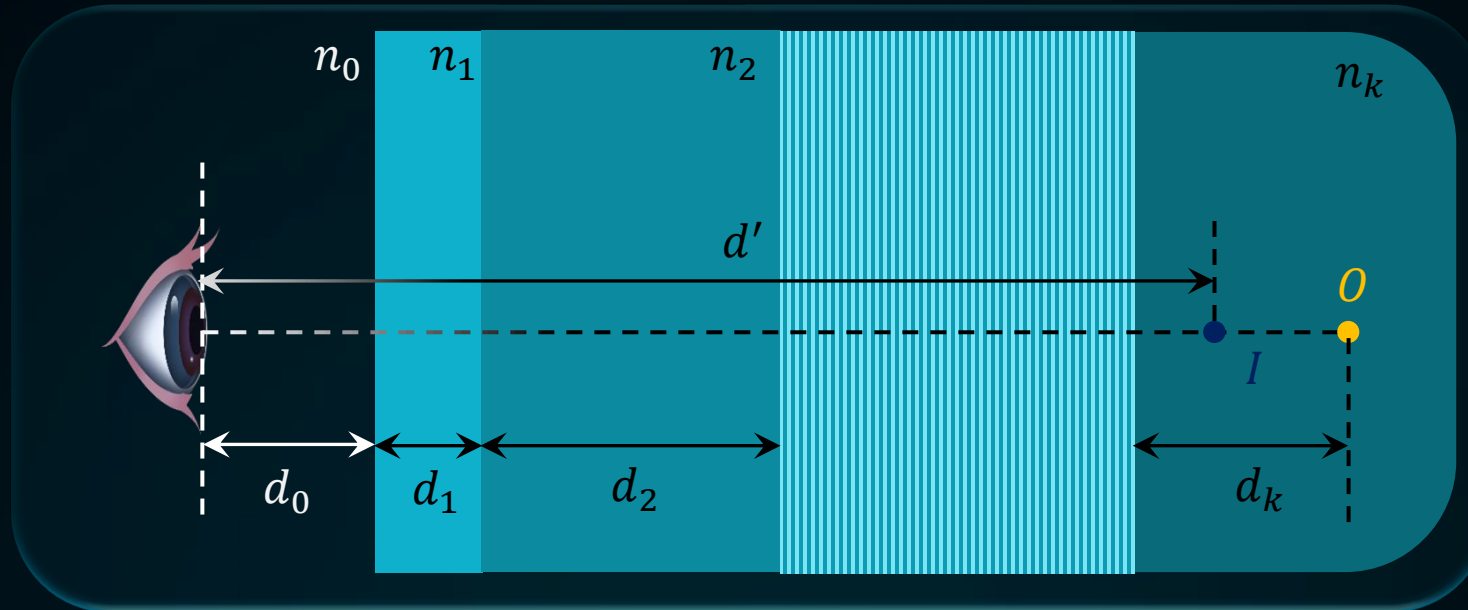
$$d' = \frac{d_1}{\left(\frac{n_1}{n_0}\right)} + \frac{d_2}{\left(\frac{n_2}{n_0}\right)} + \frac{d_3}{\left(\frac{n_3}{n_0}\right)}$$



** $n_0 < n_1 < n_2 < n_3$



Refraction of Light Through Composite Slab



$$d' = d_0 + \frac{d_1}{\left(\frac{n_1}{n_0}\right)} + \frac{d_2}{\left(\frac{n_2}{n_0}\right)} + \frac{d_3}{\left(\frac{n_3}{n_0}\right)} + \dots + \frac{d_k}{\left(\frac{n_k}{n_0}\right)}$$



$$d' = \sum_{i=0}^k \frac{d_i}{\left(\frac{n_i}{n_0}\right)}$$

?

k transparent slabs are arranged one over another. The refractive indices of the slabs are $n_1, n_2, n_3, \dots, n_k$ and the thicknesses are $t_1, t_2, t_3, \dots, t_k$. An object is seen through this combination with nearly perpendicular light. Find the equivalent refractive index of the system which will allow the image to be formed at the same place.

Solution:

R.I. of observer's medium is = 1

For the given system:

$$d' = \frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3} + \dots + \frac{t_k}{n_k}$$

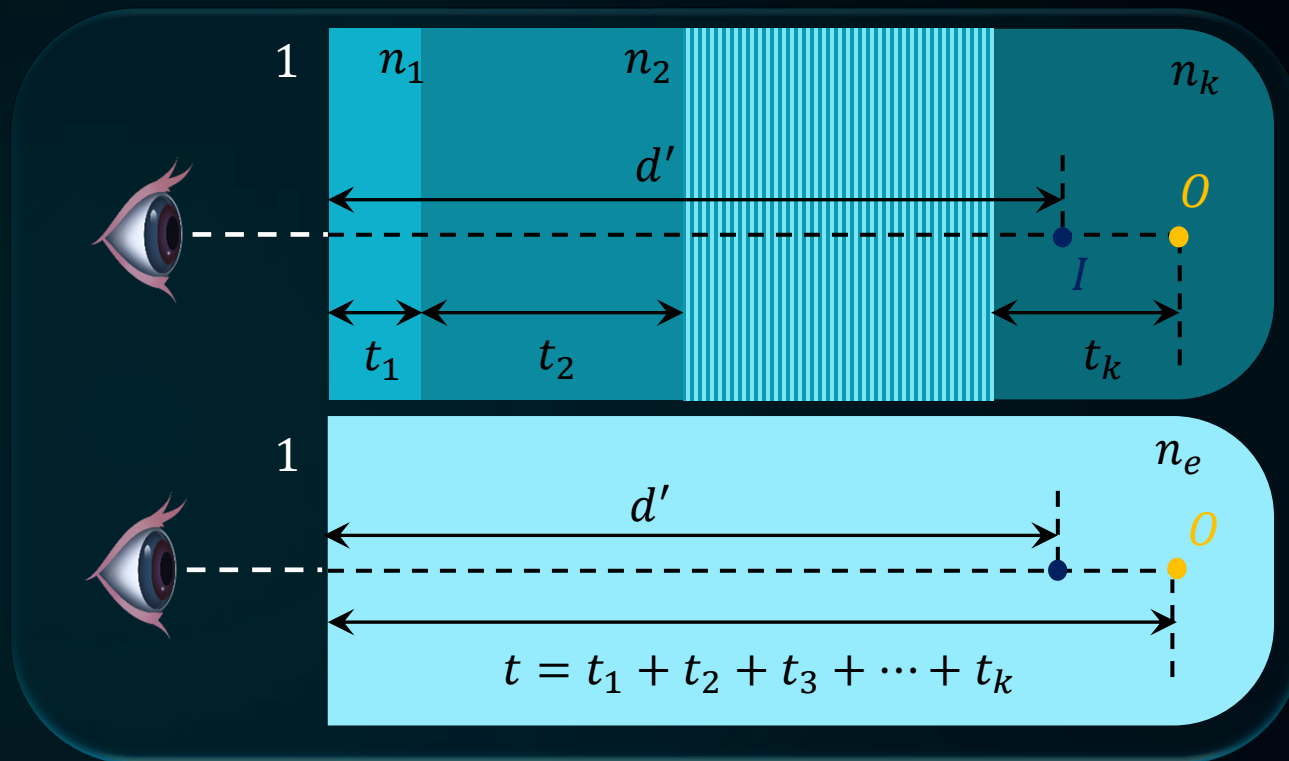
For the equivalent system:

$$d' = \frac{t_1 + t_2 + t_3 + \dots + t_k}{n_e}$$

Combining these two equations:

$$\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3} + \dots + \frac{t_k}{n_k} = \frac{t_1 + t_2 + t_3 + \dots + t_k}{n_e}$$

$$n_e = \frac{\sum_{i=1}^k t_i}{\sum_{i=1}^k \frac{t_i}{n_i}}$$



?

Find the lateral shift of light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45° .

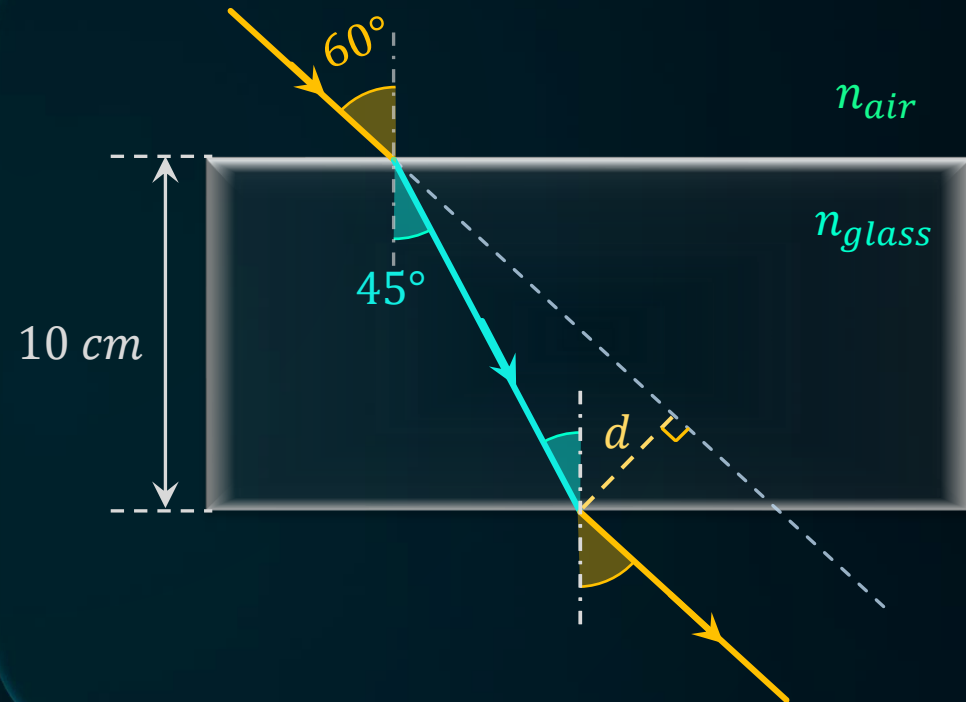
Solution:

$$i = 60^\circ \quad r = 45^\circ \quad t = 10\text{ cm}$$

$$d = \frac{t \sin(i - r)}{\cos r}$$

$$d = \frac{10 \times \sin 15^\circ}{\cos 45^\circ} = \frac{10 \times \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$d = 5(\sqrt{3} - 1)\text{ cm}$$





Refraction Through Multiple Parallel Slabs

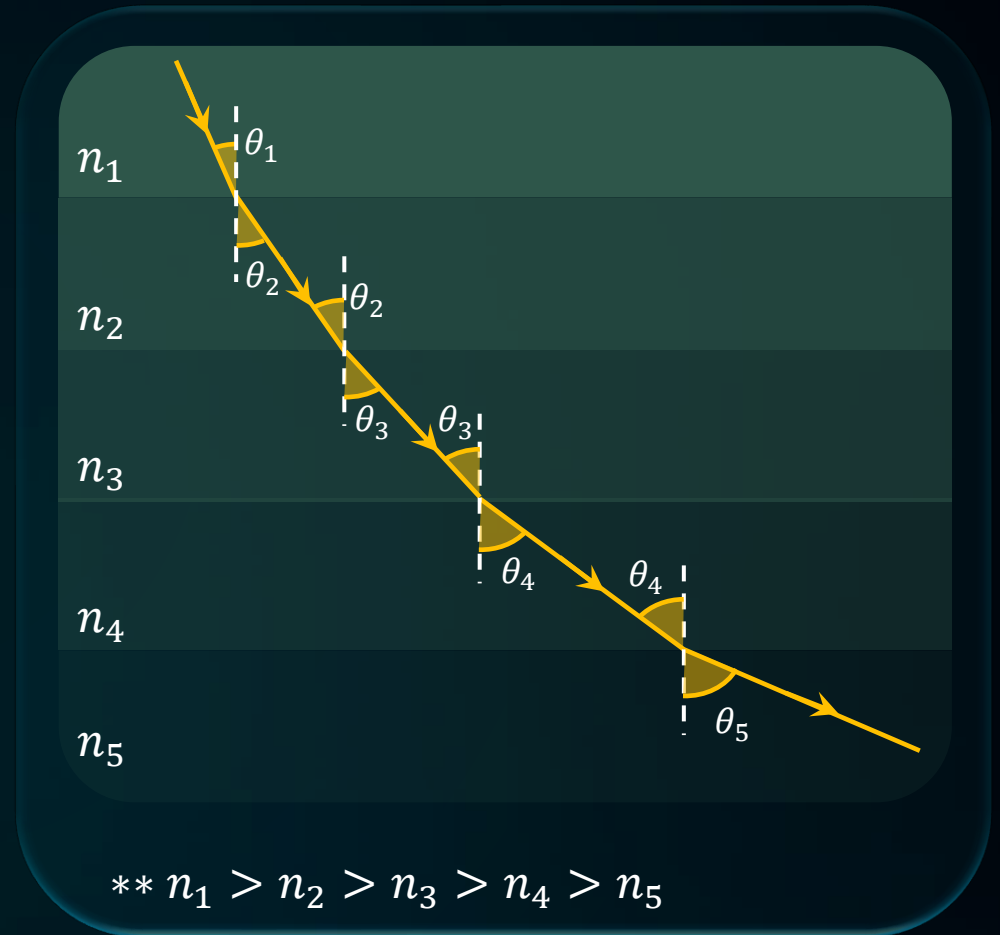


Applying Snell's law for consecutive mediums, we have:

$$n_1 \sin \theta_1 = n_5 \sin \theta_5$$

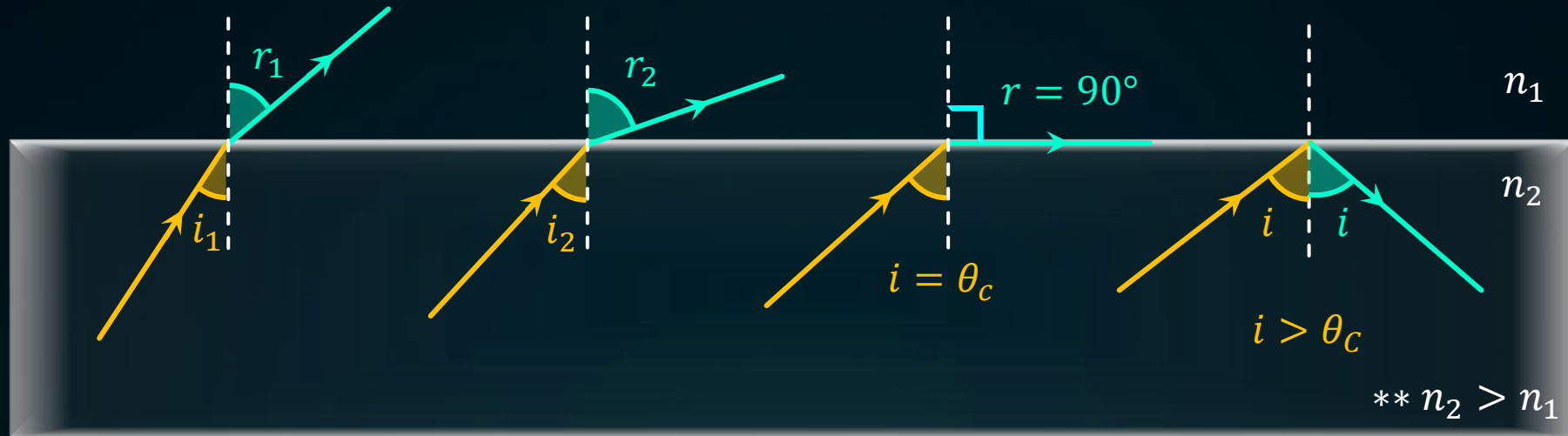
If number of slab is m :

$$n_1 \sin \theta_1 = n_m \sin \theta_m$$





Total Internal Reflection

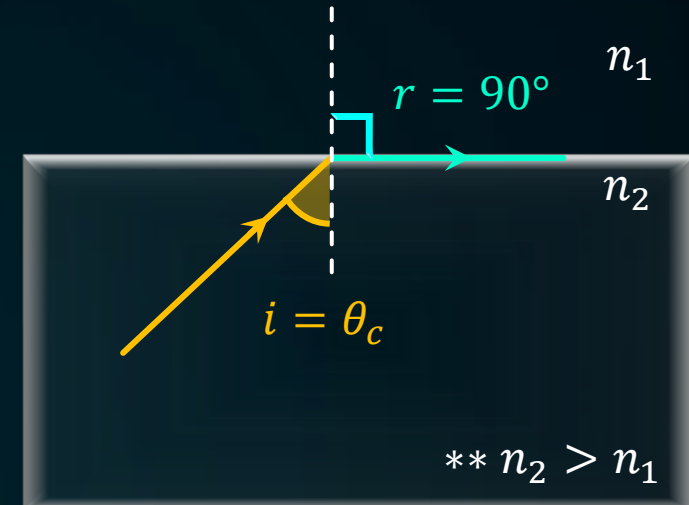
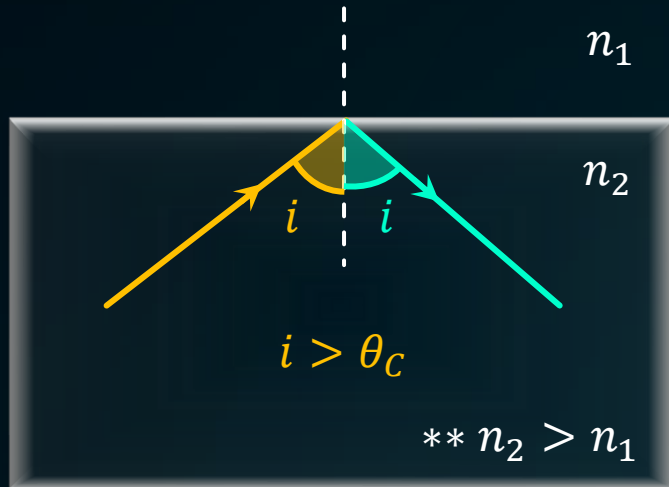


- Critical angle (θ_c): Angle of incidence for which angle of refraction becomes 90° .

Angle of incidence, $i = \theta_c$ \longrightarrow Grazing emergence, $r = 90^\circ$



Summary



- Conditions of **TIR**:
 - Light must travel from denser to rarer medium.
 - $i > \theta_c$

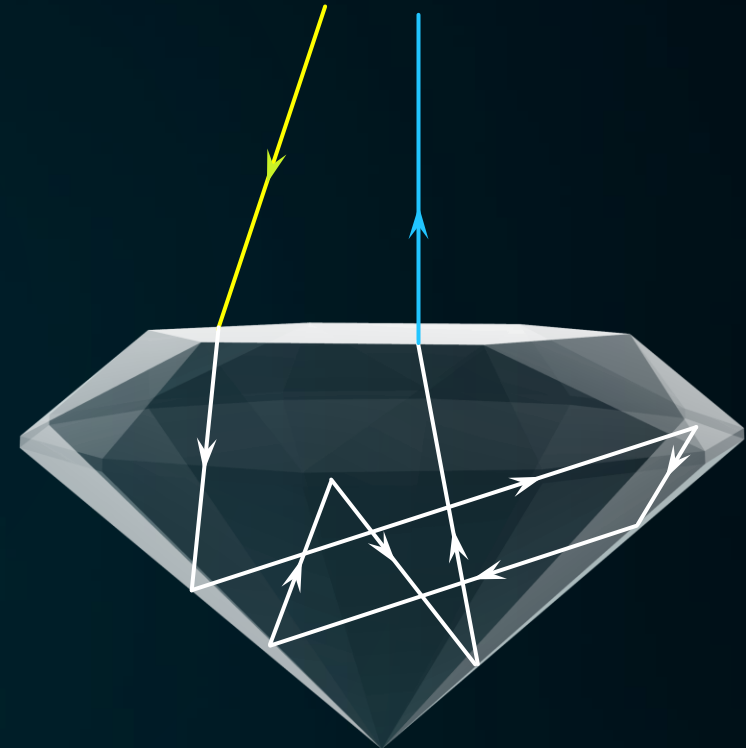
$$\theta_c = \sin^{-1} \left(\frac{n_1}{n_2} \right) = \sin^{-1} \left(\frac{n_{\text{rarer}}}{n_{\text{denser}}} \right)$$



Application of TIR: Sparkling of Diamond



- Total internal reflection is the main cause of the sparkling of diamonds.
- The refractive index of diamond with respect to air is **2.42**. Its critical angle is **24.41°** (for diamond-air interface).
- When light enters a diamond from any face at an angle greater than **24.41°** , it undergoes total internal reflection.
- Diamond surface is cut in such a way that rays come out of it from specific points only, after multiple reflections inside it.





The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be:

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Given:

Relative permittivity, $\epsilon_r = 3$

Relative permeability, $\mu_r = \frac{4}{3}$

To find: The critical angle

Solution:

We know: $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$ and $v = \frac{c}{n}$

Refractive index of the medium:

$$n = \sqrt{\epsilon_r \mu_r}$$

$$n = \sqrt{3 \times \left(\frac{4}{3}\right)} \rightarrow n = 2$$

Assume surrounding to be air, and light ray is going from denser to rarer.

Thus, $n_{rarer} = 1$ and $n_{denser} = n = 2$

The critical angle:

$$\theta_c = \sin^{-1} \left(\frac{n_{rarer}}{n_{denser}} \right)$$

$$\theta_c = \sin^{-1} \left(\frac{1}{2} \right)$$

$$\theta_c = 30^\circ$$

?

A fish looking up through the water ($n = 4/3$) sees the outside world. The fish's eyes are at depth d from the surface of the water. What should be the minimum surface area of the water so that the fish can see the outside world completely?

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Solution:

The fish will see the outside world completely if **grazing emergence** takes place as shown.

Angle of incidence at the water surface should be θ_c .

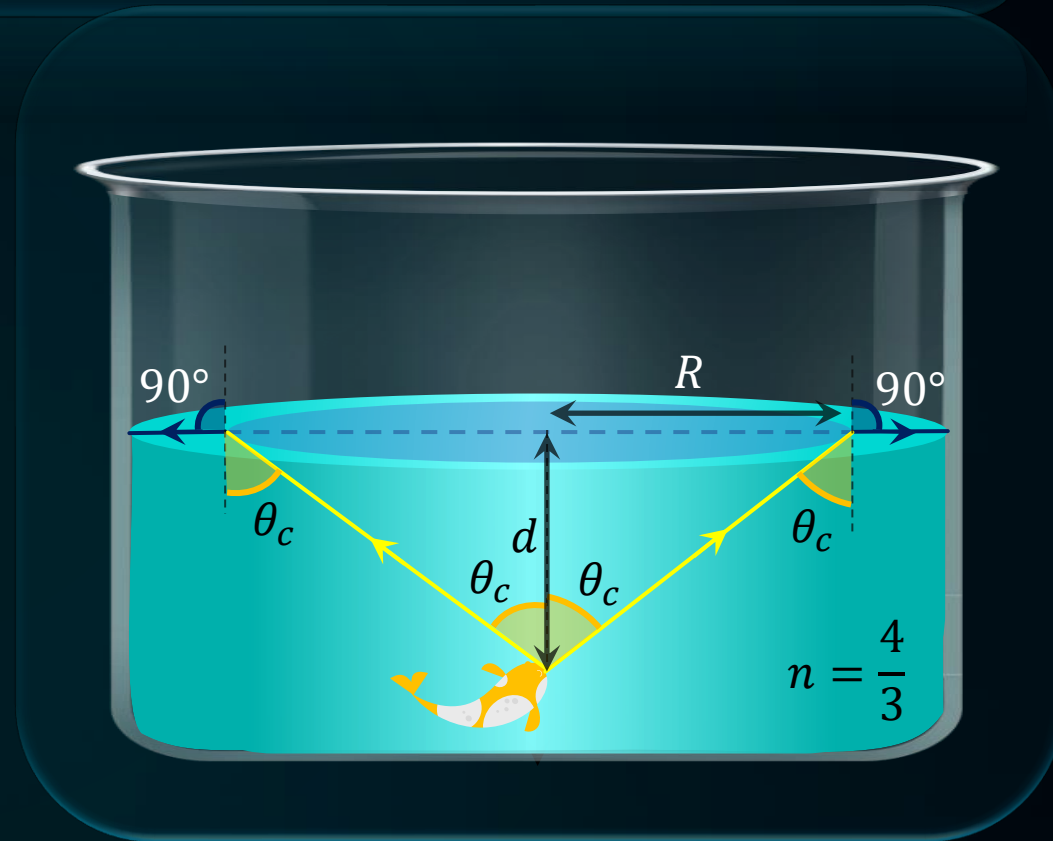
$$\sin \theta_c = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

Thus,

$$\tan \theta_c = \frac{\sin \theta_c}{\cos \theta_c} \rightarrow \frac{R}{d} = \frac{\frac{3}{4}}{\sqrt{1 - \left(\frac{3}{4}\right)^2}} \rightarrow R = \frac{3}{\sqrt{7}} d$$

Minimum surface area:

$$S = \frac{9}{7} \pi d^2$$



Trick:

$$R = \frac{d}{\sqrt{n^2 - 1}}$$

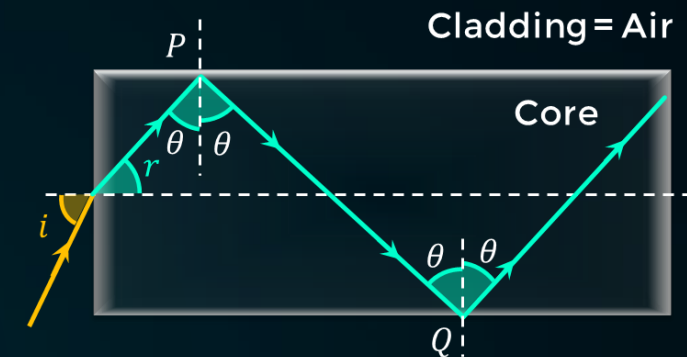
(When surrounding is air only)



Structure of Optical Fiber

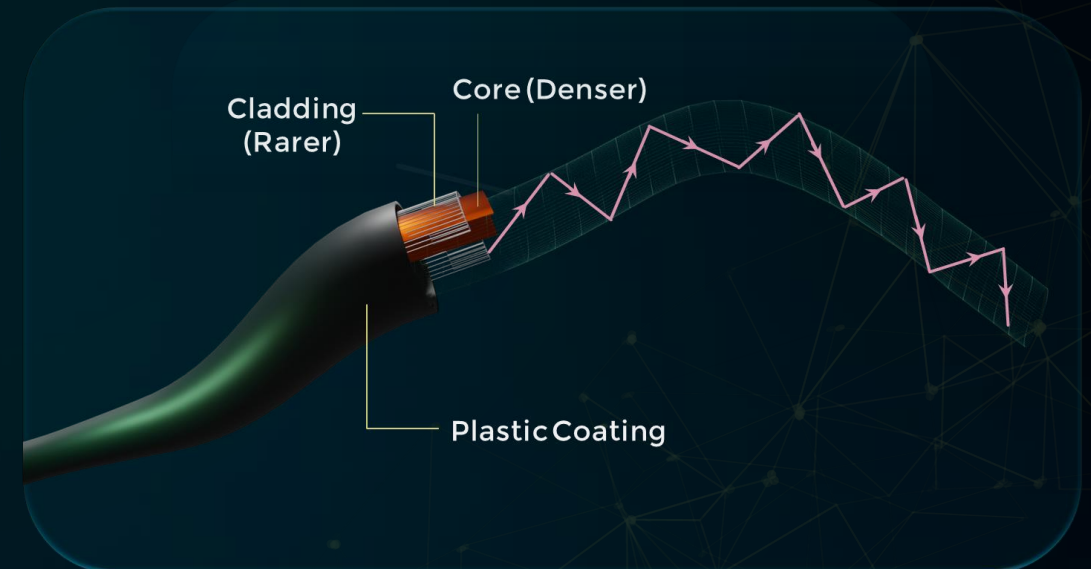


- To reduce lateral loss (refraction) in transmission of energy, total internal reflection is required.
- For TIR to take place at P , $\theta > \theta_c$.
- If $\theta > \theta_c$ is ensured at P , TIR will occur, and light will get trapped.



Use of Optical Fiber

- Optical fibres are extensively used for transmitting and receiving electrical signals, and optical signals.
- Optical fibres are also used in some decorative lamps used in household.





If Cladding is Not Air

- Angle of incidence at core: i
- From $\triangle ABC$: $r = 90^\circ - r'$
- Applying Snell's law at point A:

$$n_0 \sin i = n_d \sin r$$

$$n_0 \sin i = n_d \sin(90^\circ - r')$$

$$n_0 \sin i = n_d \cos r'$$

$$n_0 \sin i = n_d \sqrt{1 - \sin^2 r'}$$

$$n_0^2 \sin^2 i = n_d^2 (1 - \sin^2 r')$$

$$\sin r' = \frac{\sqrt{n_d^2 - n_0^2 \sin^2 i}}{n_d}$$

- For TIR at B: $r' \geq \theta_c$

$$\sin r' \geq \sin \theta_c$$

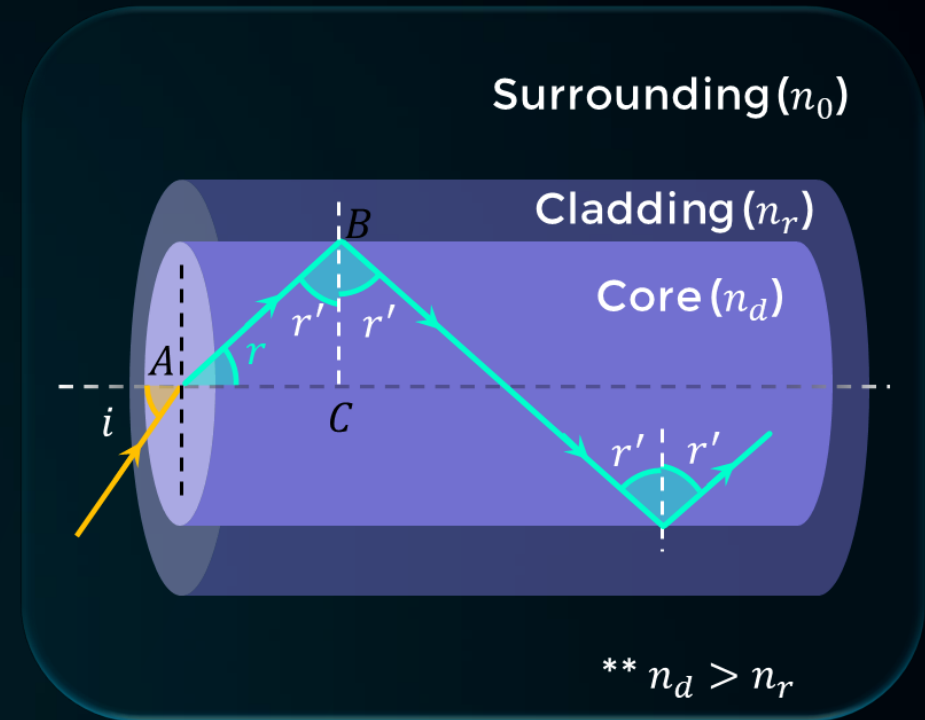
$$\sin r' \geq \left(\frac{n_r}{n_d} \right)$$

$$\frac{\sqrt{n_d^2 - n_0^2 \sin^2 i}}{n_d} \geq \left(\frac{n_r}{n_d} \right)$$

$$n_d^2 - n_0^2 \sin^2 i \geq n_r^2$$

$$\frac{n_d^2 - n_r^2}{n_0^2} \geq \sin^2 i$$

$$\sin i \leq \frac{\sqrt{n_d^2 - n_r^2}}{n_0}$$





If Cladding is Not Air

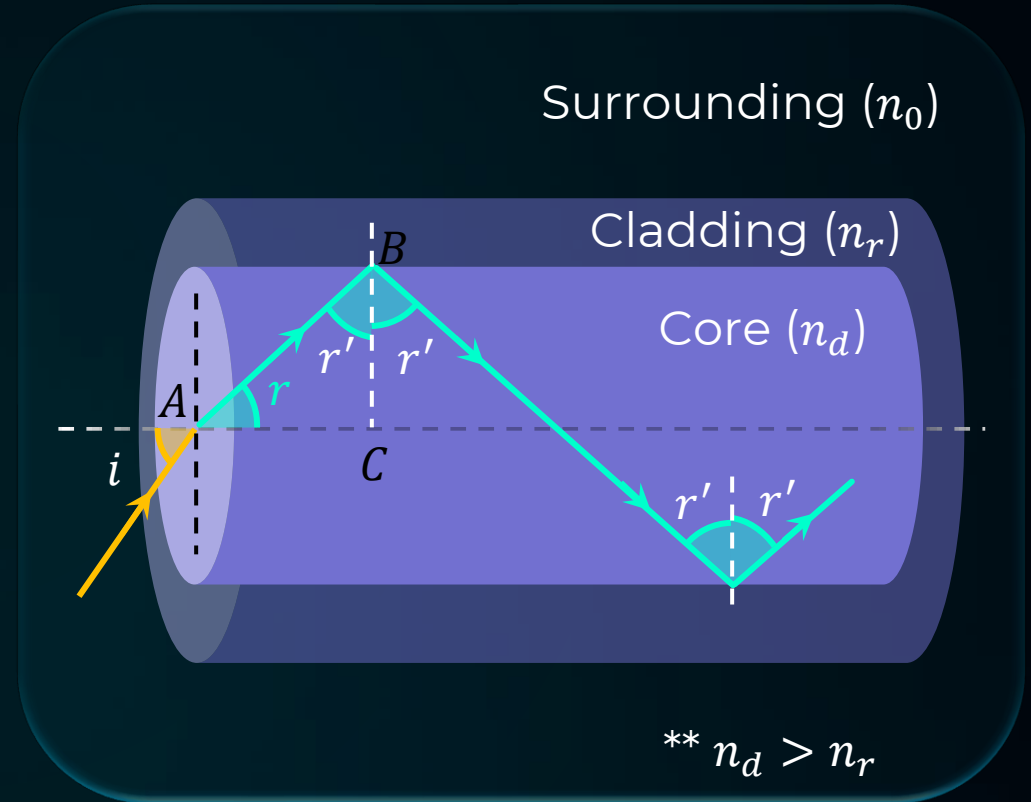


- We have: $\sin i \leq \frac{\sqrt{n_d^2 - n_r^2}}{n_0}$

$$i \leq \sin^{-1} \left[\frac{\sqrt{n_d^2 - n_r^2}}{n_0} \right]$$

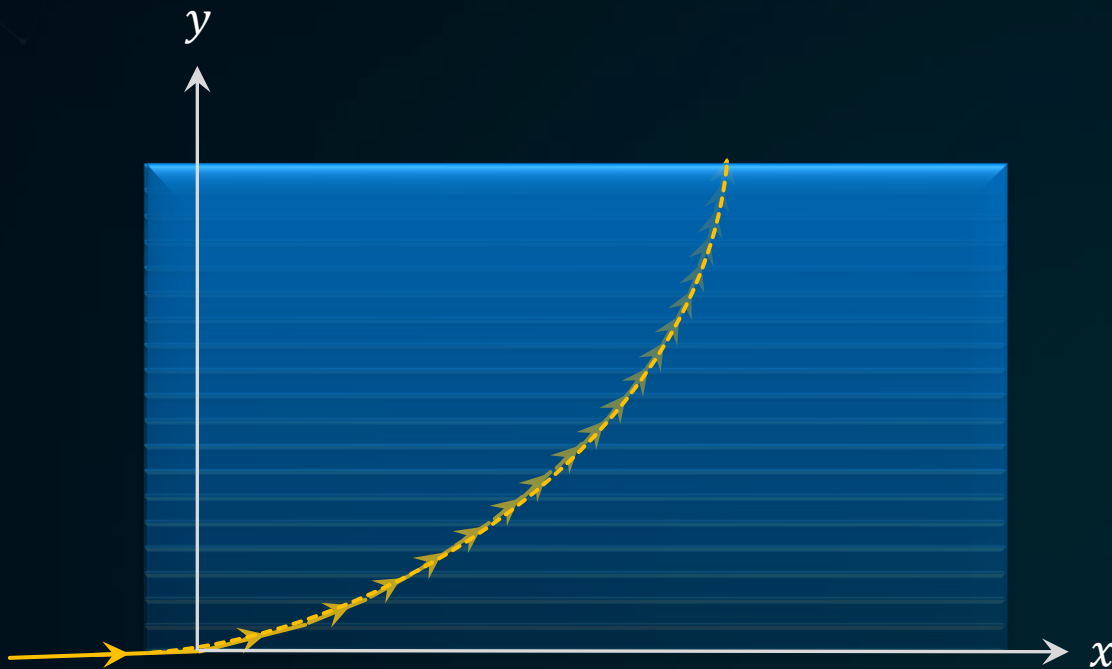
- The maximum angle at which the ray should enter into the core for the transmission through the optical fibre:

$$i_{max} = \sin^{-1} \left[\frac{\sqrt{n_d^2 - n_r^2}}{n_0} \right]$$





Visualization of the Path of Ray



- Light ray is travelling from air (rarer) to glass slab (denser).



- Light ray bends towards the normal (y-axis).

- Expression of the refractive index:

$$n(y \uparrow) = (Ky^{3/2} \uparrow + 1)^{1/2}$$

- As y increases, $n(y)$ increases.
- So, along y-axis, the slab gets denser.
- So, light ray travels from rarer to denser in each step, and hence, bends towards the normal.

?

A vessel of depth $2h$ is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be:

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Solution: The system can be considered as a composite slab.

Apparent depth:

$$d' = \sum_{i=1}^k \frac{d_i}{\left(\frac{n_i}{n_0}\right)}$$

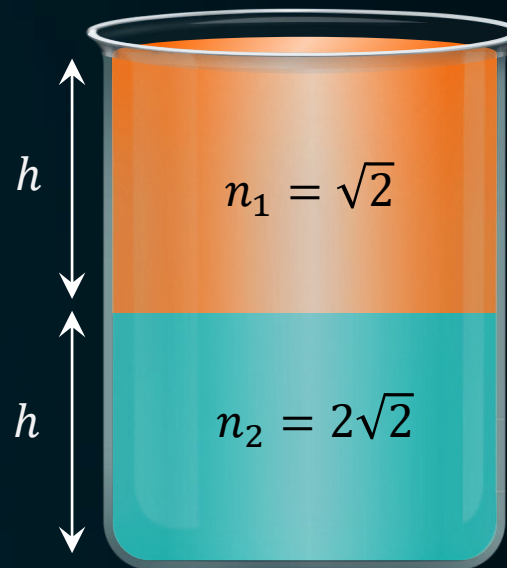
Here: $d_1 = h = d_2$

Assume surrounding to be air. So, $n_0 = 1$

Therefore,

$$d' = \frac{d_1}{n_1} + \frac{d_2}{n_2}$$

$$d' = \frac{h}{\sqrt{2}} + \frac{h}{2\sqrt{2}} \rightarrow d' = \frac{h}{\sqrt{2}} \left(\frac{1}{2} + 1 \right) \rightarrow d' = \frac{3}{4} h \sqrt{2}$$





Refraction through Spherical Surfaces



• From $\triangle OMC$,

$$\theta_1 = \alpha + \beta \quad (\theta_1 \text{ is the external angle of } \triangle OMC)$$

• From $\triangle MCI$,

$$\begin{aligned} \beta &= \theta_2 + \gamma \\ \theta_2 &= \beta - \gamma \end{aligned} \quad (\beta \text{ is the external angle of } \triangle MCI)$$

We have,

$$\tan \alpha = \frac{MN}{PO} \quad \tan \beta = \frac{MN}{PC} \quad \tan \gamma = \frac{MN}{PI}$$

(Considering N and P as the same point as the rays are paraxial)

Using Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

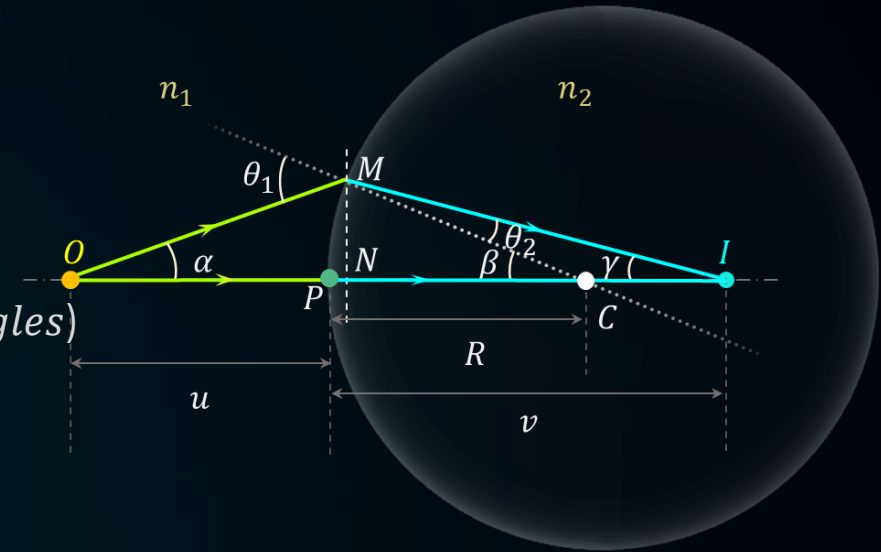
$$\Rightarrow n_1 \theta_1 = n_2 \theta_2 \quad (\because \theta_1 \text{ and } \theta_2 \text{ are small angles})$$

$$\sin \theta \approx \theta \approx \tan \theta \quad (\text{For small } \theta)$$

$$n_1(\tan \alpha + \tan \beta) = n_2(\tan \beta - \tan \gamma)$$

Taking in consideration the signs of u and v as per the sign convention:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$





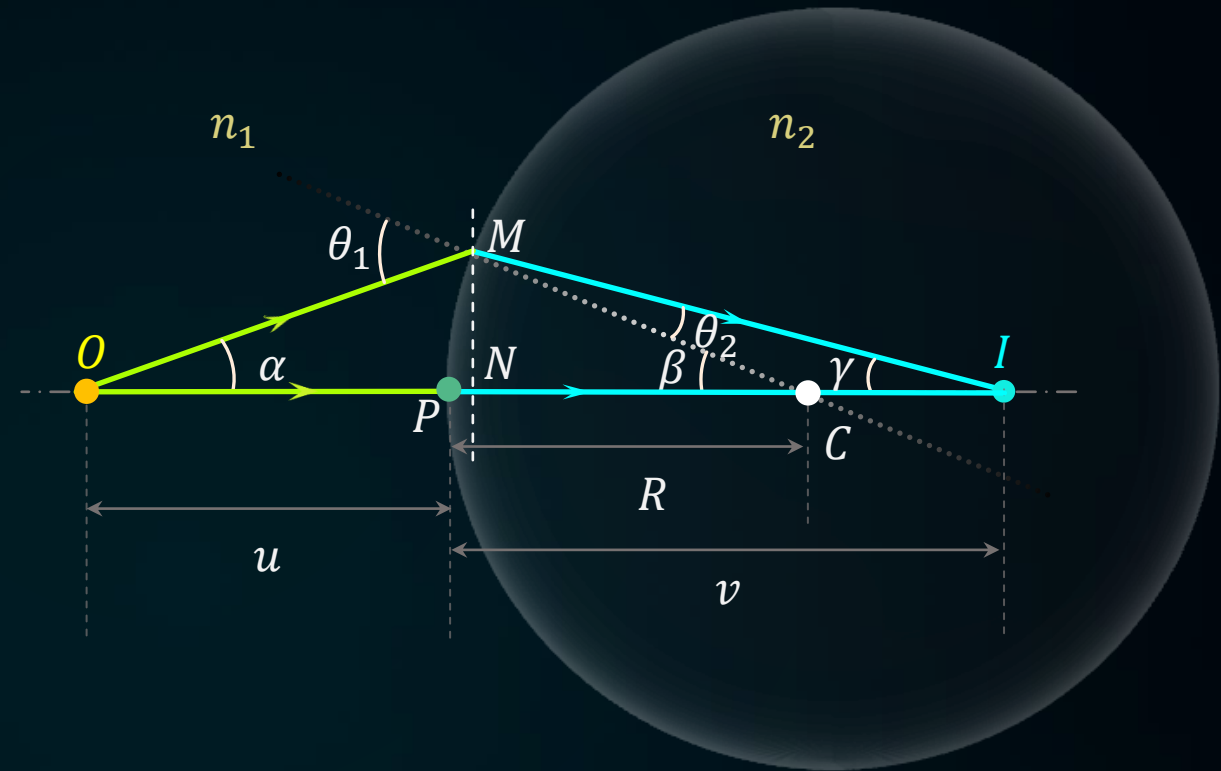
Refraction through Spherical Surfaces



General Result:

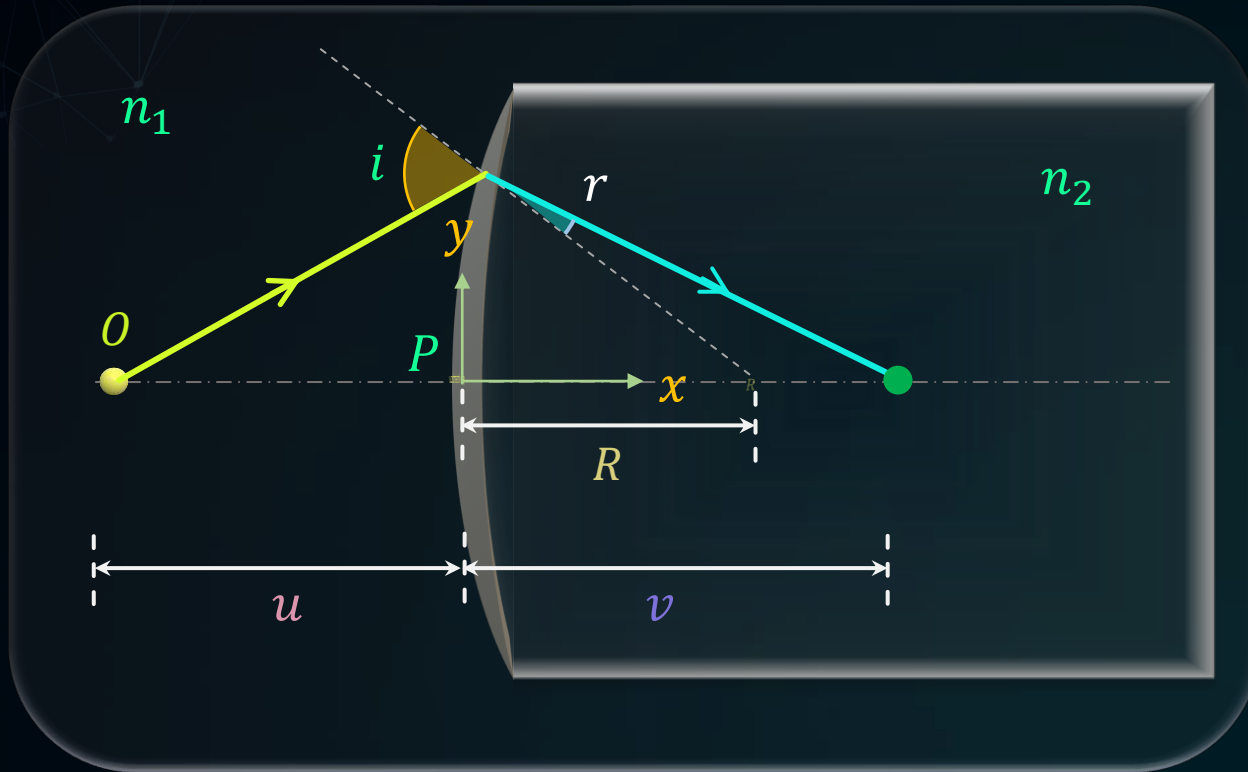
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- Pole is considered as origin and all the distance measured from it.
- The direction of incident ray is chosen as the positive direction.
- u, v, R are kept with sign.
- Only valid when rays are paraxial.
- Rays originate from medium of RI n_1 and goes into medium of RI n_2 .





Refraction through Spherical Surfaces



Cartesian sign convention

- Pole (P) is the origin.
- Principle axis is the x axis
- Direction of incident rays is taken as positive x direction.

Rays are coming from $n_1 \rightarrow n_2$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

u : x-coordinate of the object

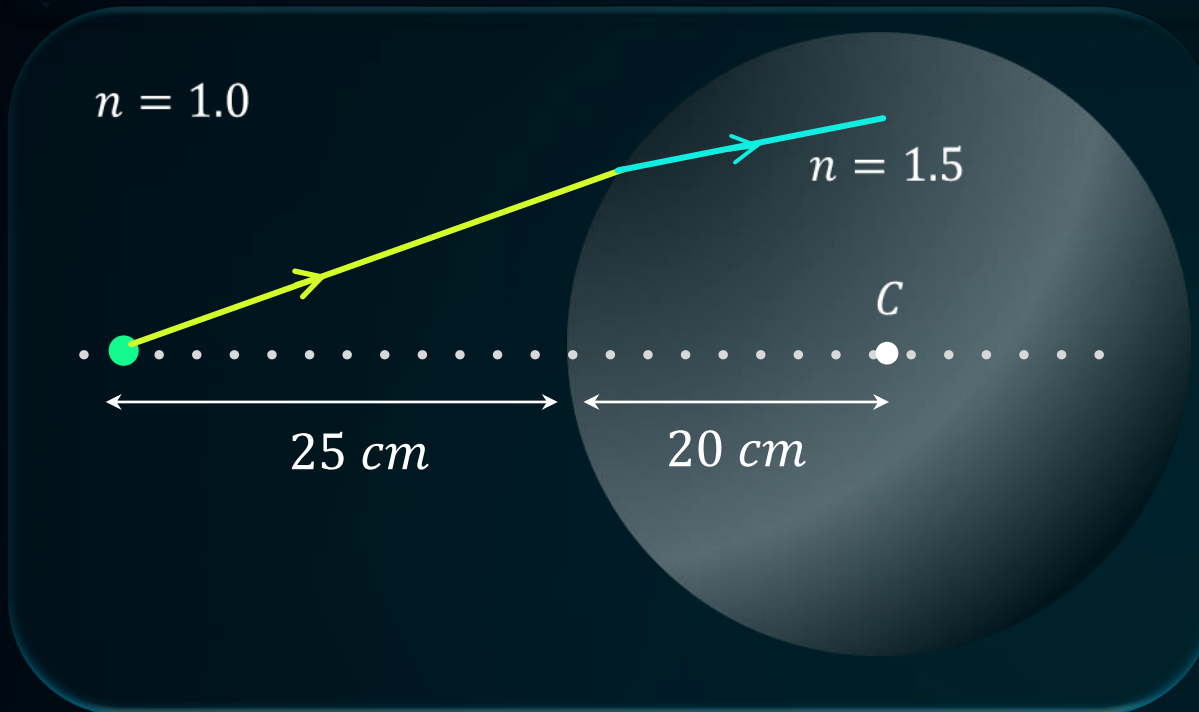
v : x-coordinate of the image

R : x-coordinate of centre of the curvature of the surface



Locate the **image** formed by **refraction** in the situation shown in the given figure.

Solution:



Rays are coming from $n_1 \rightarrow n_2$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{3/2}{v} - \frac{1}{-25} = \frac{(3/2 - 1)}{+20}$$

$$\frac{3}{2v} = \frac{1}{40} - \frac{1}{25}$$

$$\frac{3}{2v} = -\frac{3}{200}$$

$$v = -100\text{ cm}$$



Transverse Magnification



Using Snell's Law:

$$n_1 \sin \alpha = n_2 \sin \beta$$

For paraxial rays, α β are very small.

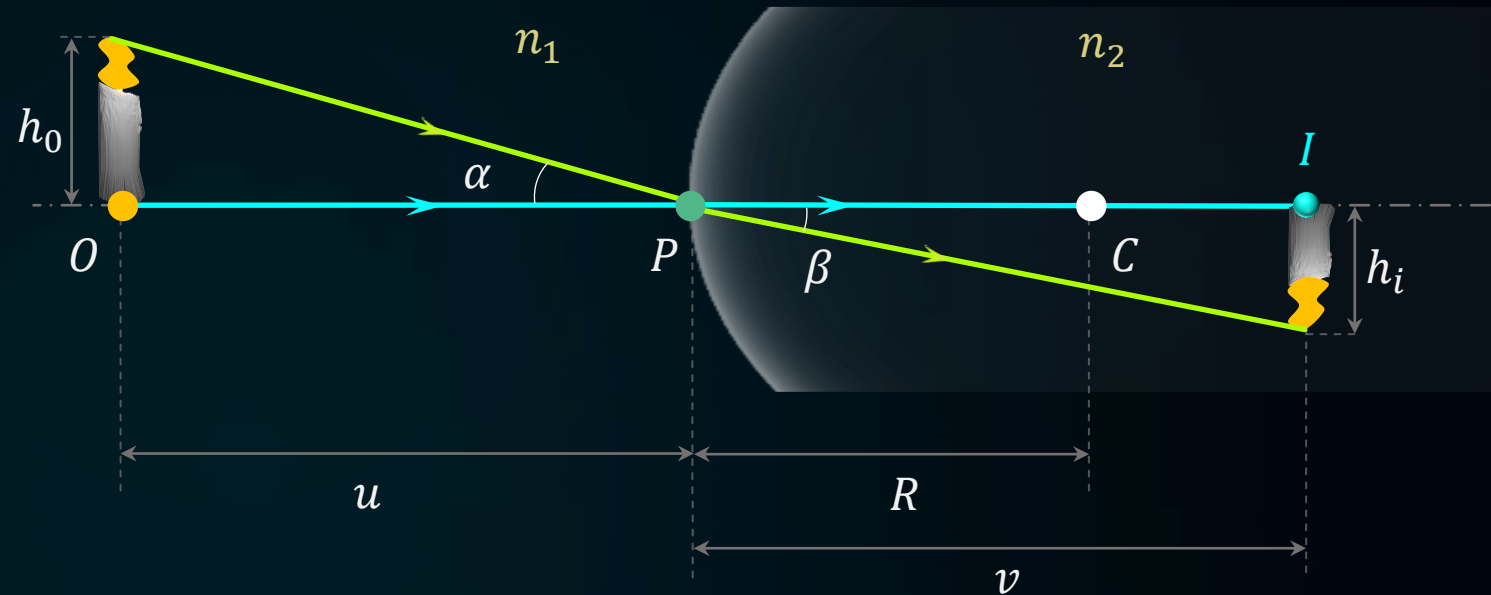
$$\sin \alpha \approx \tan \alpha$$

$$\sin \beta \approx \tan \beta$$

$$n_1 \tan \alpha = n_2 \tan \beta$$

$$n_1 \frac{h_0}{u} = n_2 \frac{h_i}{v}$$

$$m = \frac{h_i}{h_0} = \frac{n_1 v}{n_2 u}$$



Note: Always substitute h_i , h_0 , u and v with the proper sign.



Velocity of Image



General formula,

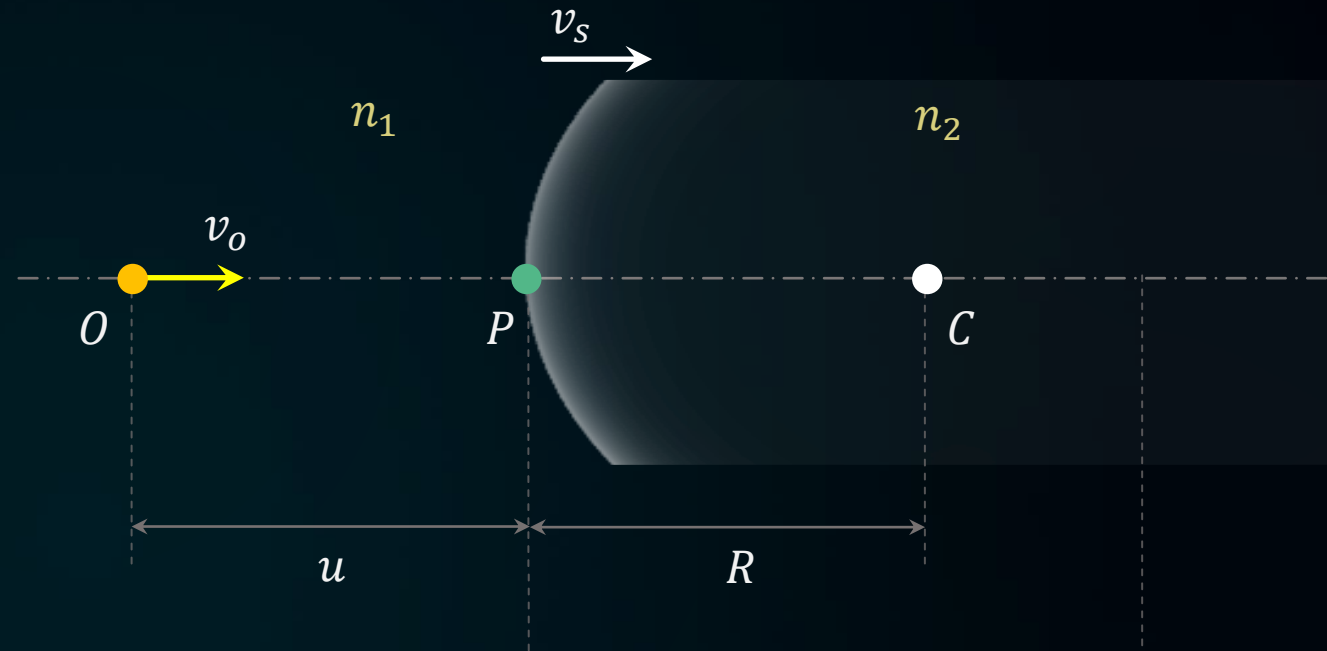
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Differentiating w.r.t. time,

$$-\frac{n_2}{v^2} \frac{dv}{dt} + \frac{n_1}{u^2} \frac{du}{dt} = 0$$

$$\frac{dv}{dt} = \frac{n_1}{n_2} \frac{v^2}{u^2} \frac{du}{dt}$$

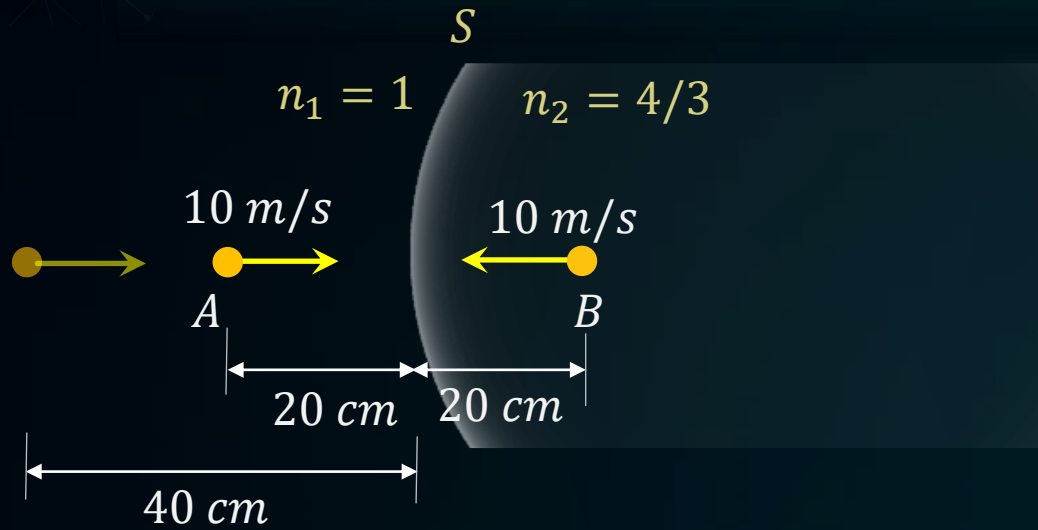
$$\vec{V}_{Is} = \frac{n_1}{n_2} \frac{v^2}{u^2} \vec{V}_{Os}$$



?

A spherical surface of radius of curvature $R = 20 \text{ cm}$ separates two media as shown in figure. Two particles A and B are moving with equal speed of 10 m/s as indicated. At a certain instant, the two particles are at equal distance of 20 cm from the spherical interface. At this instant relative velocity of A as seen from B is

Solution:



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{4}{3 \times v_1} + \frac{1}{20} = \frac{\left(\frac{4}{3} - 1\right)}{20} \Rightarrow v_1 = -40 \text{ cm}$$

$$\vec{V}_{A|S} = \frac{n_1}{n_2} \frac{v^2}{u^2} \vec{V}_{AS}$$

$$\vec{V}_A = \frac{1}{4/3} \times \frac{(-40)^2}{(-20)^2} \times 10$$

Image speed of particle A wrt surface = 30 m/s

Relative velocity of particle

A wrt $B = v_{AB} = 30 + 10 = 40 \text{ m/s}$

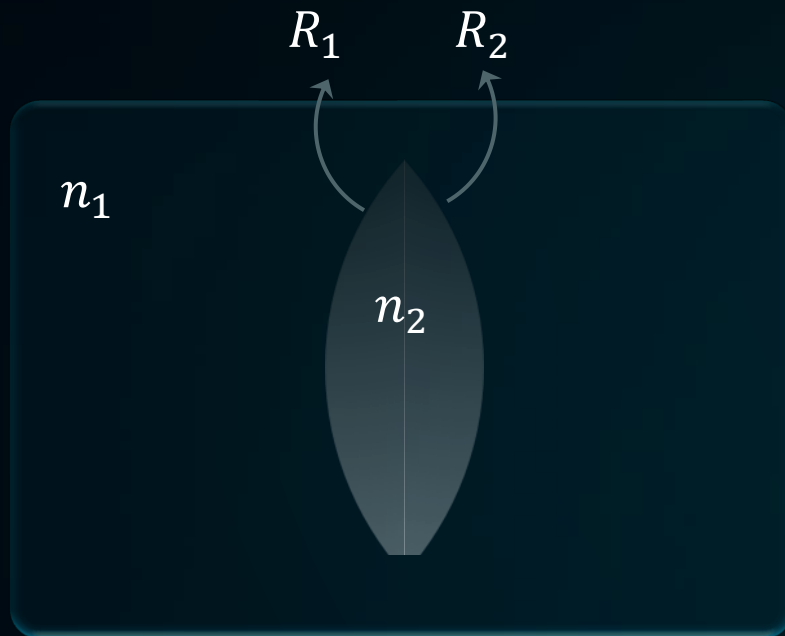
40 m/s towards right



Thin lenses

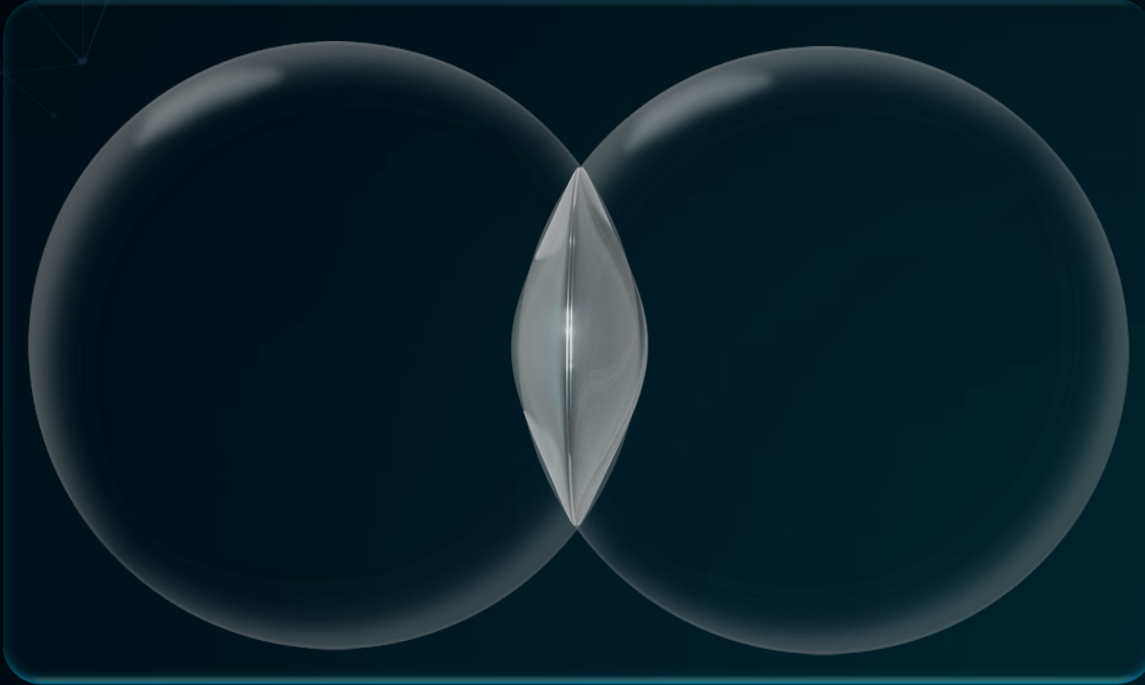


Thickness of a lens is significantly small as compared to other dimensions like distance of object/image.



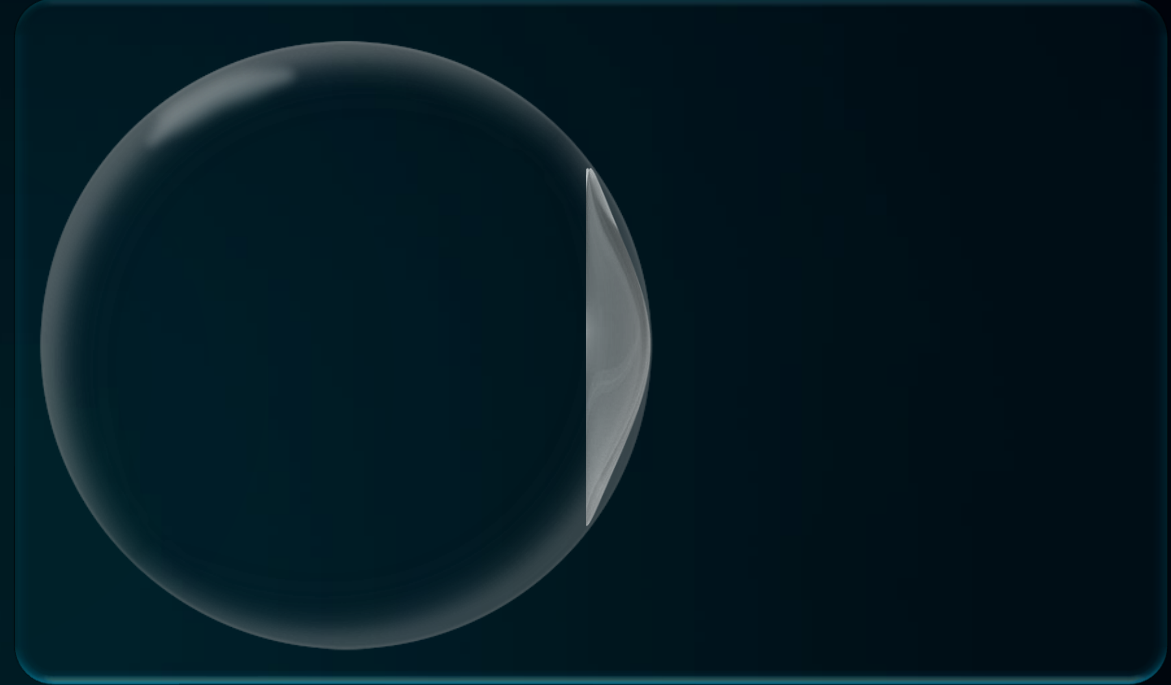
- Lens is bounded by two spherical surfaces
- These bounding surfaces can be convex, concave or plane
- Medium on either side is same

Bi-Convex Lens



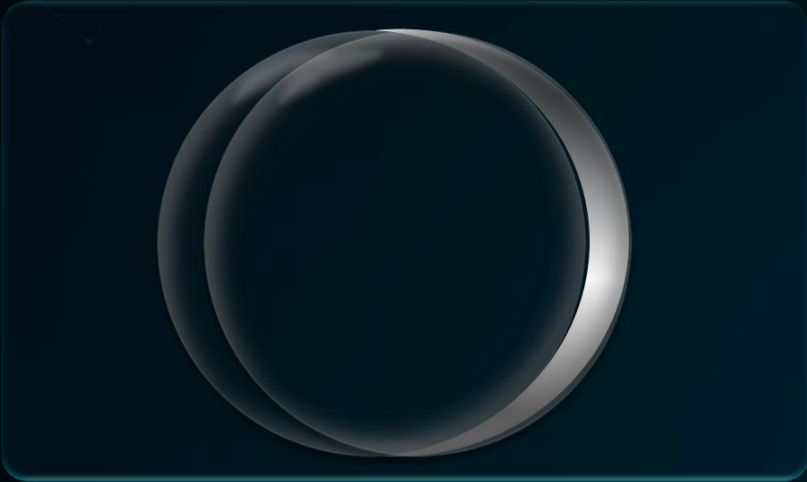
- Both surfaces are convex

Plano Convex Lens



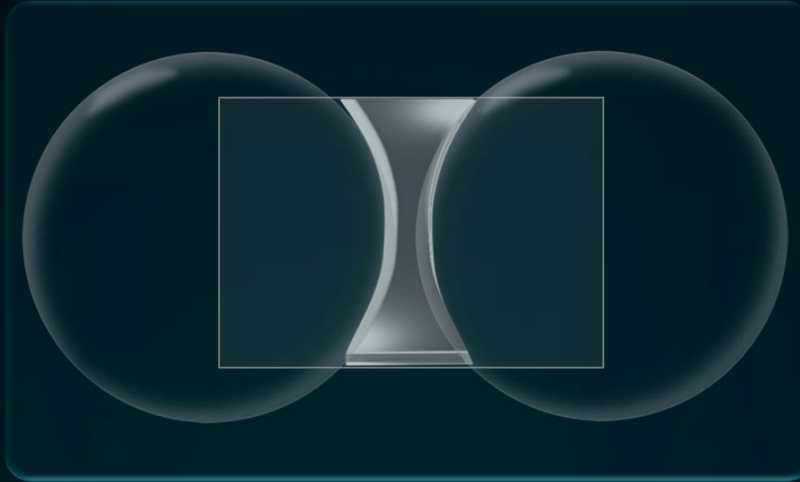
- One surface plane and other surface Convex

Concavo-Convex or Convexo-Concave Lens



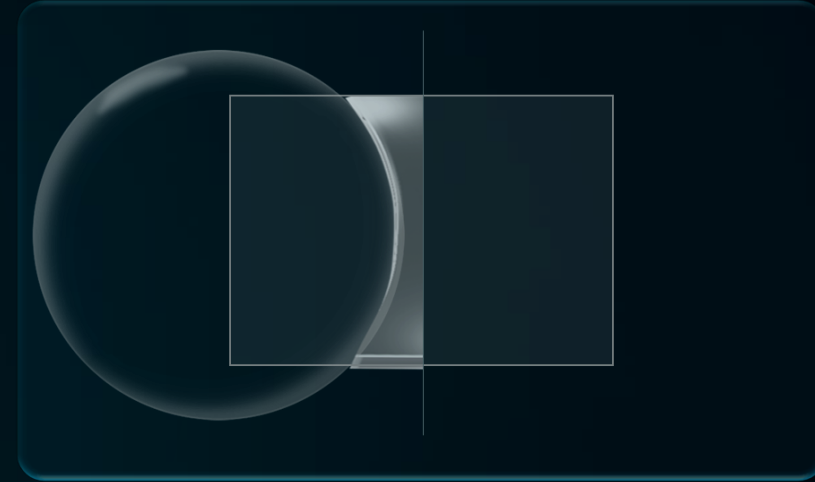
- One surface concave and other surface Convex

Bi-Concave Lens



- Both surfaces are concave

Plano Concave Lens



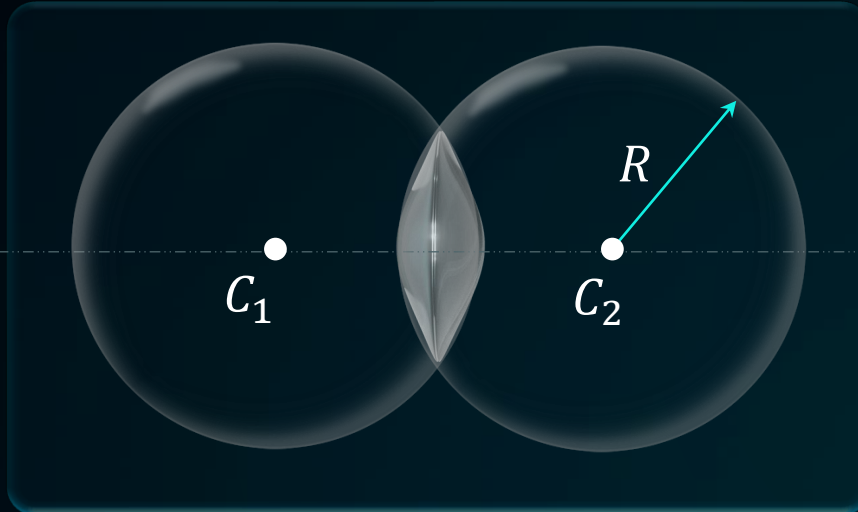
- One surface plane and other surface Concave



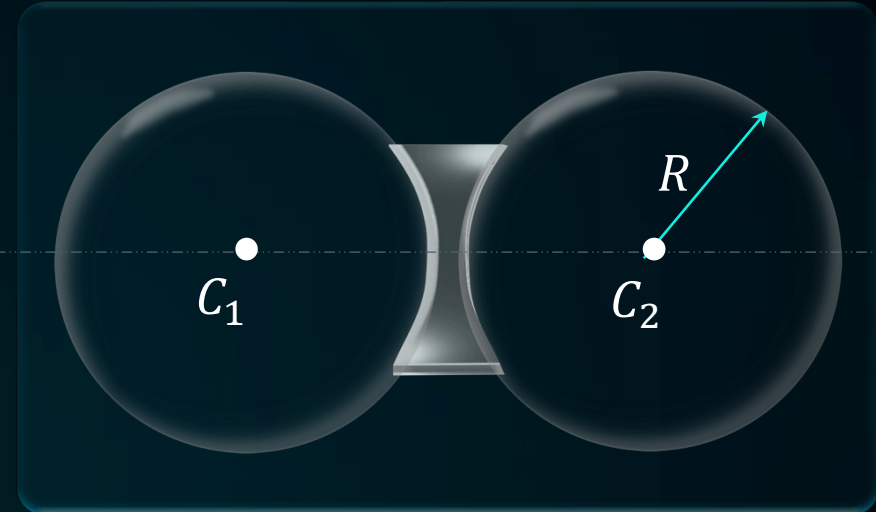
Convex and Concave Lens



Convex:



Concave:



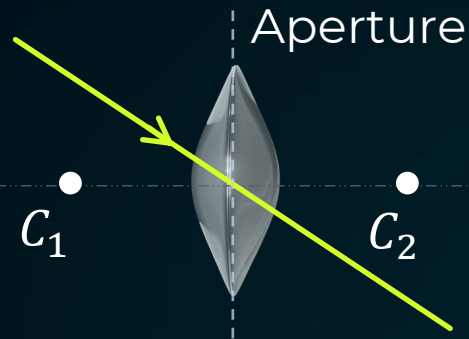
- **Centre of curvature:** The centre of spheres of which each surface of lens forms a part.
- **Radius of curvature:** The distance between the centre of lens and the centre of curvature.



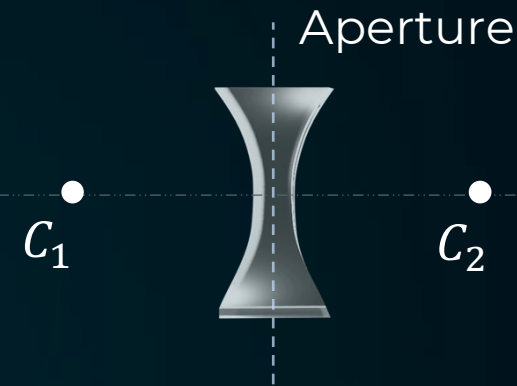
Convex and Concave Lens



Convex:



Concave:

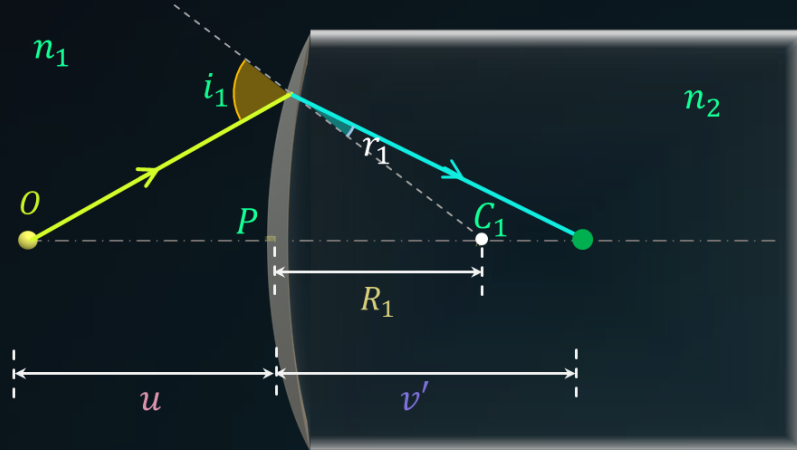


Principle
Axis

- **Optical Centre:** Central point of the lens through which a ray of light passes without any deviation.
- **Principal Axis:** Line joining Optical centre to centre of curvatures
- **Aperture:** Effective diameter of light-transmitting area of a lens.

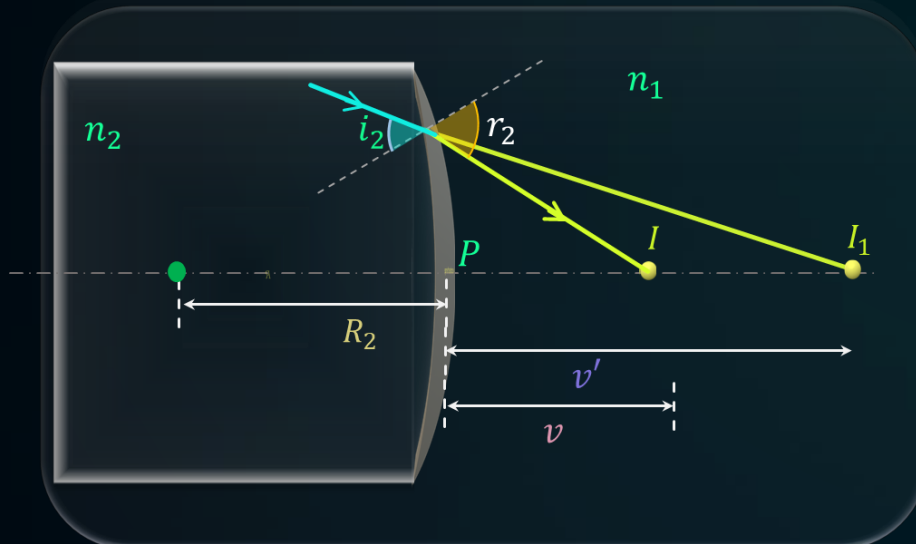


Lens Maker's Formula



First spherical refraction:

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1}$$

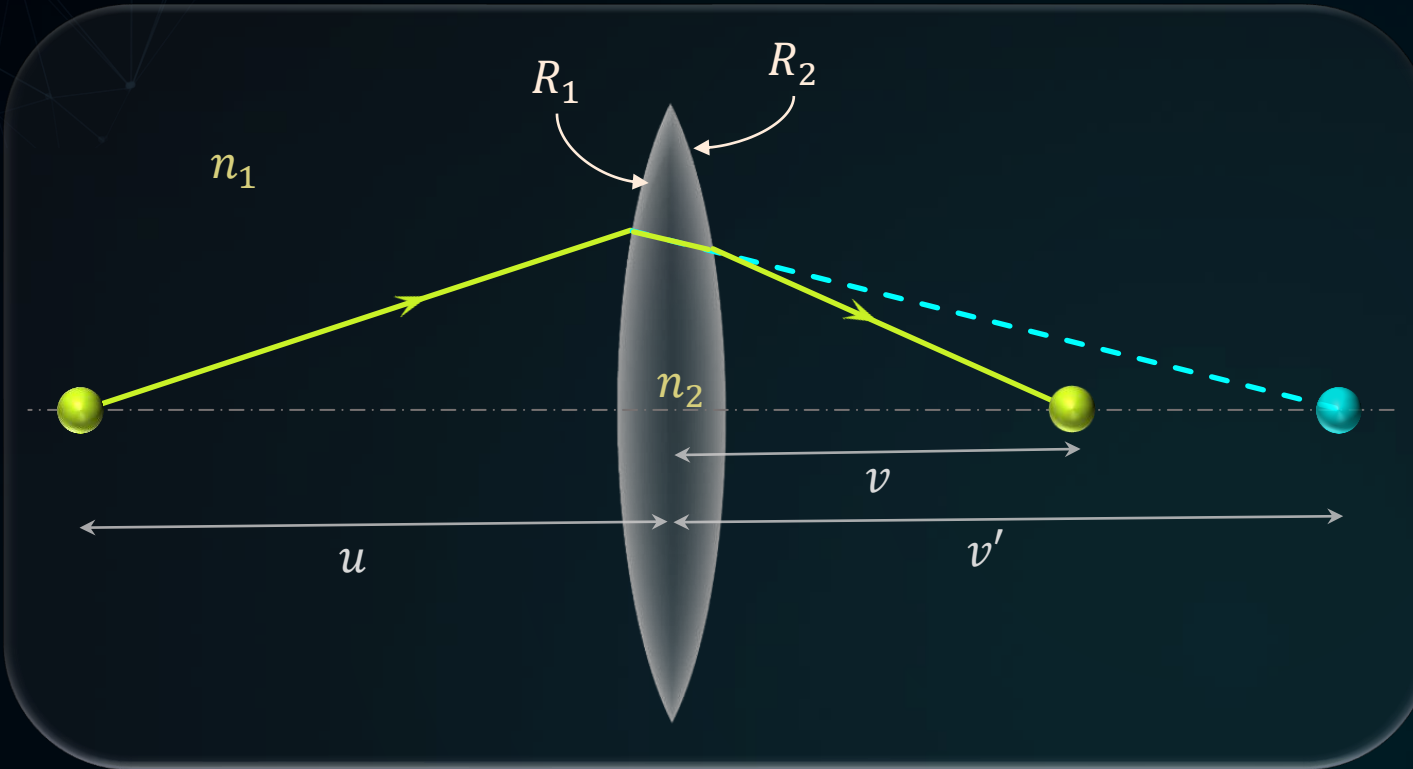


Second spherical refraction:

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_1 - n_2}{R_2}$$



Lens Maker's Formula



First spherical refraction:

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1}$$

Second spherical refraction:

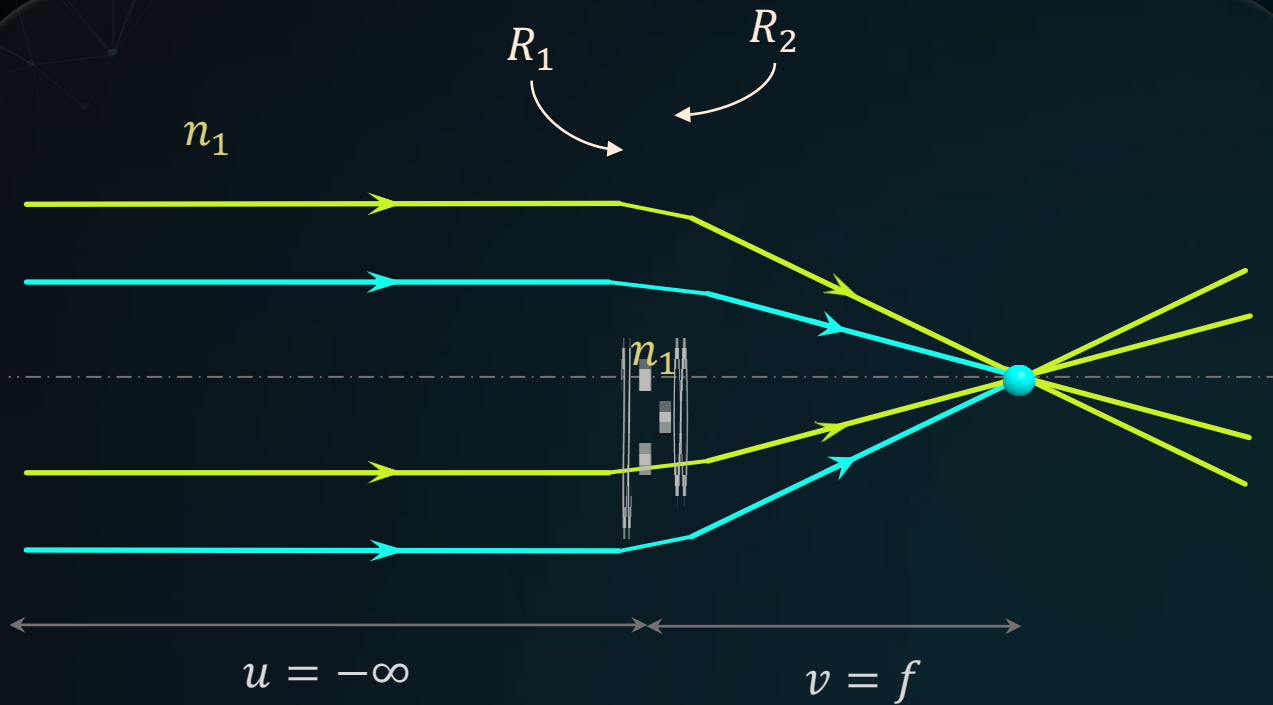
$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_1 - n_2}{R_2}$$

By adding both equations, we get General formula,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$



Lens Formula



General formula,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

When $u = -\infty$ & $v = f$,

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Lens formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$



Lens Maker's Formula



Factors on which focus depends :

$$\frac{1}{f} = (n_{rel} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

- f : Positive: Converging
- f : Negative: Diverging

When the **mediums** on both side of the lens are **same**:

- Convex: Converging $\Rightarrow \frac{1}{f} = (n_{rel} - 1) \left(\frac{1}{R_1} - \frac{1}{-R_2} \right)$
- Concave: Diverging $\Rightarrow \frac{1}{f} = (n_{rel} - 1) \left(\frac{1}{-R_1} - \frac{1}{R_2} \right)$

If $n_{rel} > 1$, Convex : Converging
Concave : Diverging

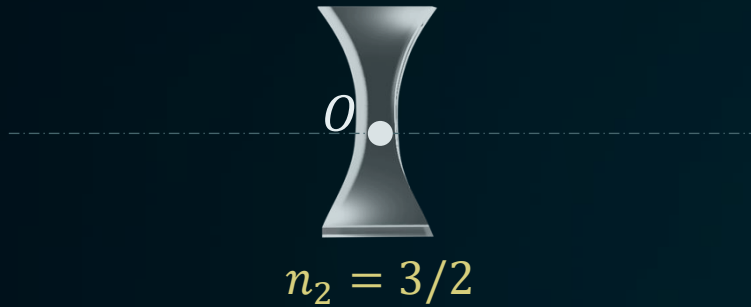
?

Given that the radius of curvature of both spherical surfaces is 10 cm , find the focus of the lens shown in the figure.

Solution:

$$n_1 = 1$$

$$n_1 = 1$$



$$n_2 = 3/2$$

Given: $R_1 = -10$, $R_2 = 10$
 $n_1 = 1$, $n_2 = 3/2$

$$\frac{1}{f} = (n_{rel} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = \left[\frac{(3/2)}{1} - 1 \right] \left[\frac{1}{-10} - \frac{1}{+10} \right]$$

$$\frac{1}{f} = \left[\frac{1}{2} \right] \left[\frac{-2}{10} \right]$$

$$f = -10\text{ cm}$$

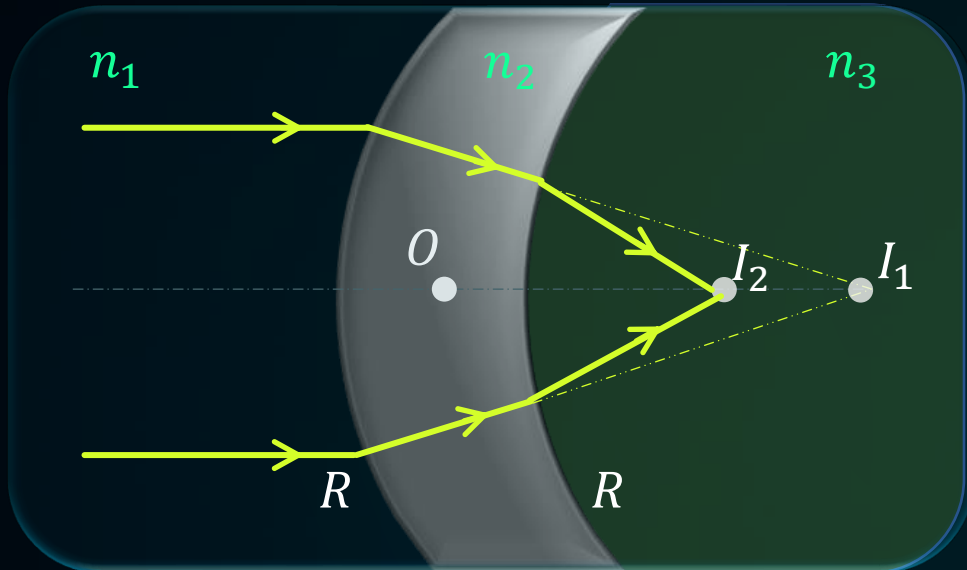
?

Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R .

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Solution:

Rays are coming from $n_1 \rightarrow n_2 \rightarrow n_3$



$$n_1 < n_2 < n_3$$

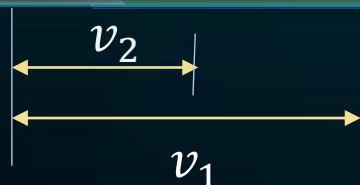


Image after first refraction (v_1):

$$\frac{n_2}{v_1} - \frac{n_1}{-\infty} = \frac{n_2 - n_1}{+R} \dots (i)$$

Image after second refraction (v_2):

$$\frac{n_3}{v_2} - \frac{n_2}{v_1} = \frac{n_3 - n_2}{+R} \dots (ii)$$

Adding (i) and (ii)

$$\frac{n_3}{v_2} = \frac{n_3 - n_1}{R} \Rightarrow v_2 = \frac{n_3 R}{n_3 - n_1}$$

Final image is formed at the focus when incident rays are parallel

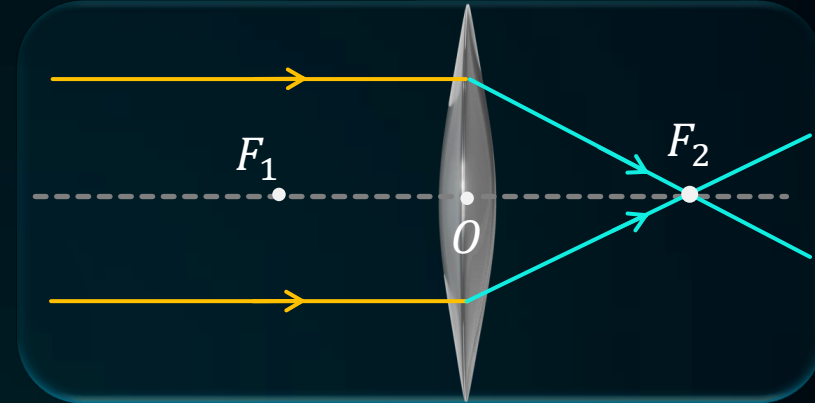
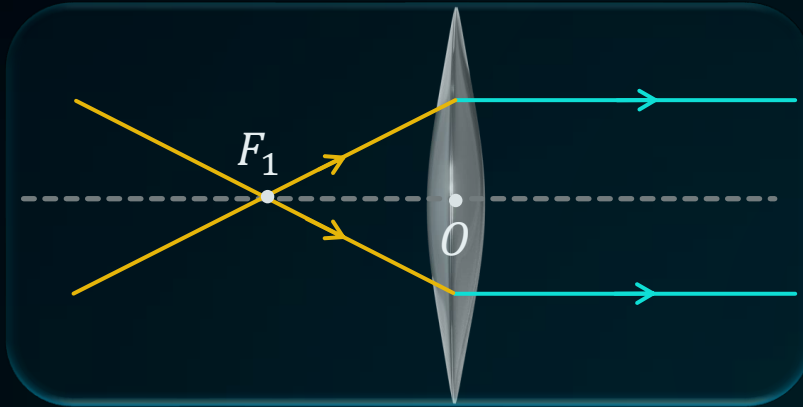
$$f = \frac{n_3 R}{n_3 - n_1}$$



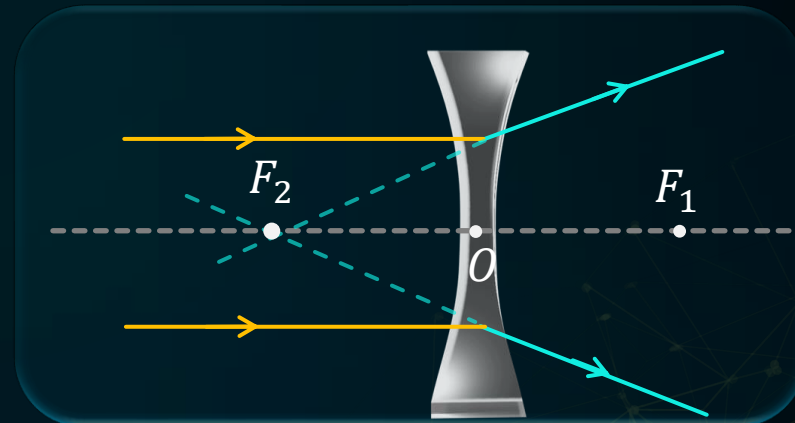
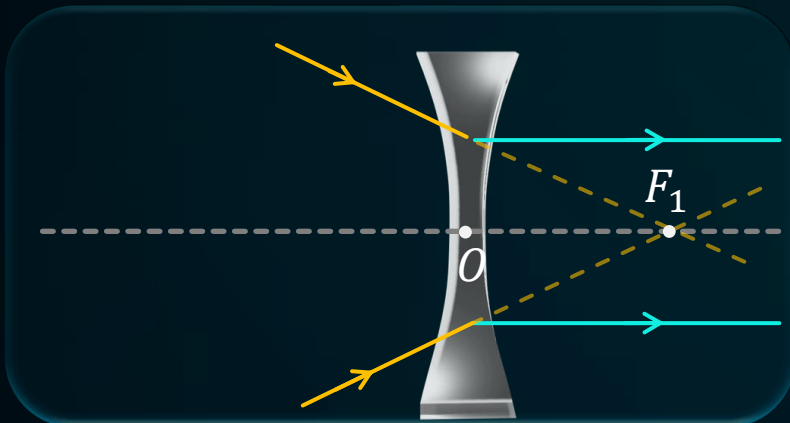
Foci of Lens



Convex lens



Concave lens

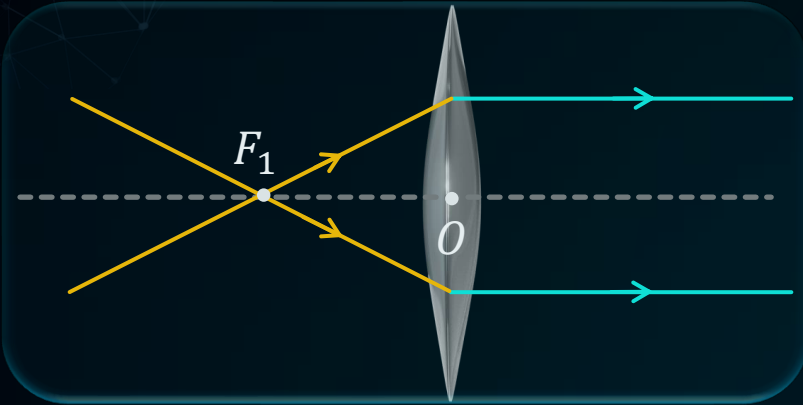




Foci of Lens



Convex lens



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (\text{Lens formula})$$

$$\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$$

$$u = -f$$

$$v = \infty$$

$$u = -f$$

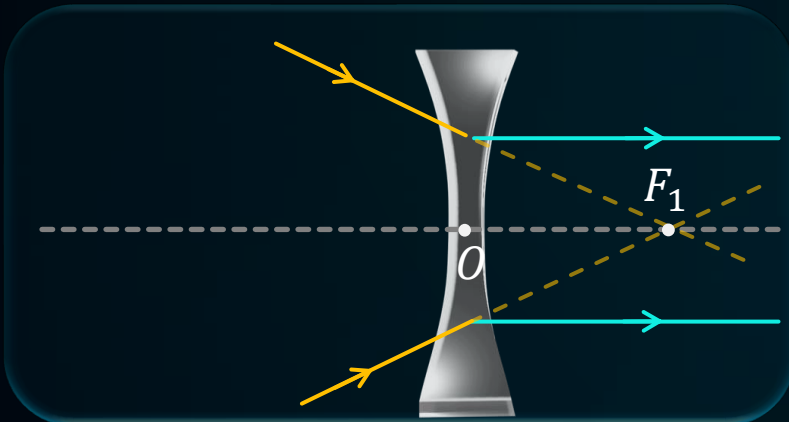
$$v = \infty$$

- **First principal focus:** point on the principal axis of the lens at which if an object is placed, the image is formed at infinity.

$$u = f$$

$$v = \infty$$

Concave lens



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (\text{Lens formula})$$

$$\frac{1}{\infty} - \frac{1}{u} = \frac{1}{-f}$$

$$u = f$$

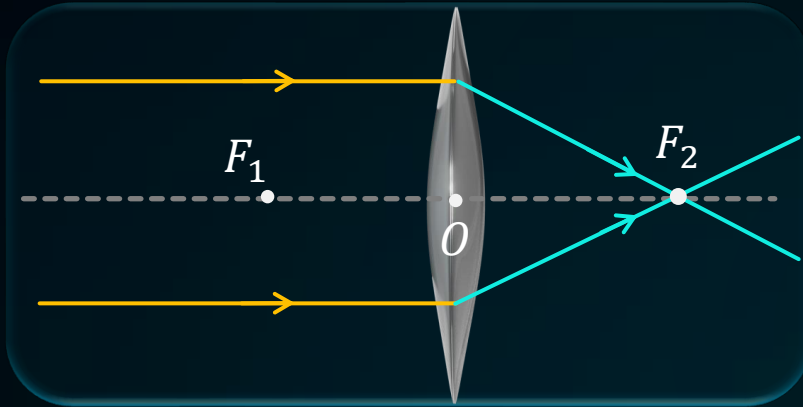
$$v = \infty$$



Foci of Lens



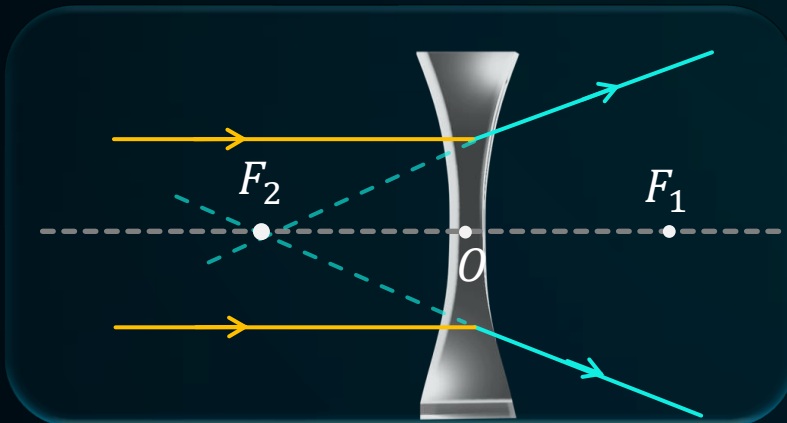
Convex lens



$$u = -\infty$$

$$v = +f$$

Concave lens



$$u = -\infty$$

$$v = -f$$

- **Second principal focus:** point on the principal axis of the lens where the image is formed when the parallel rays from an object at infinity falls on it.

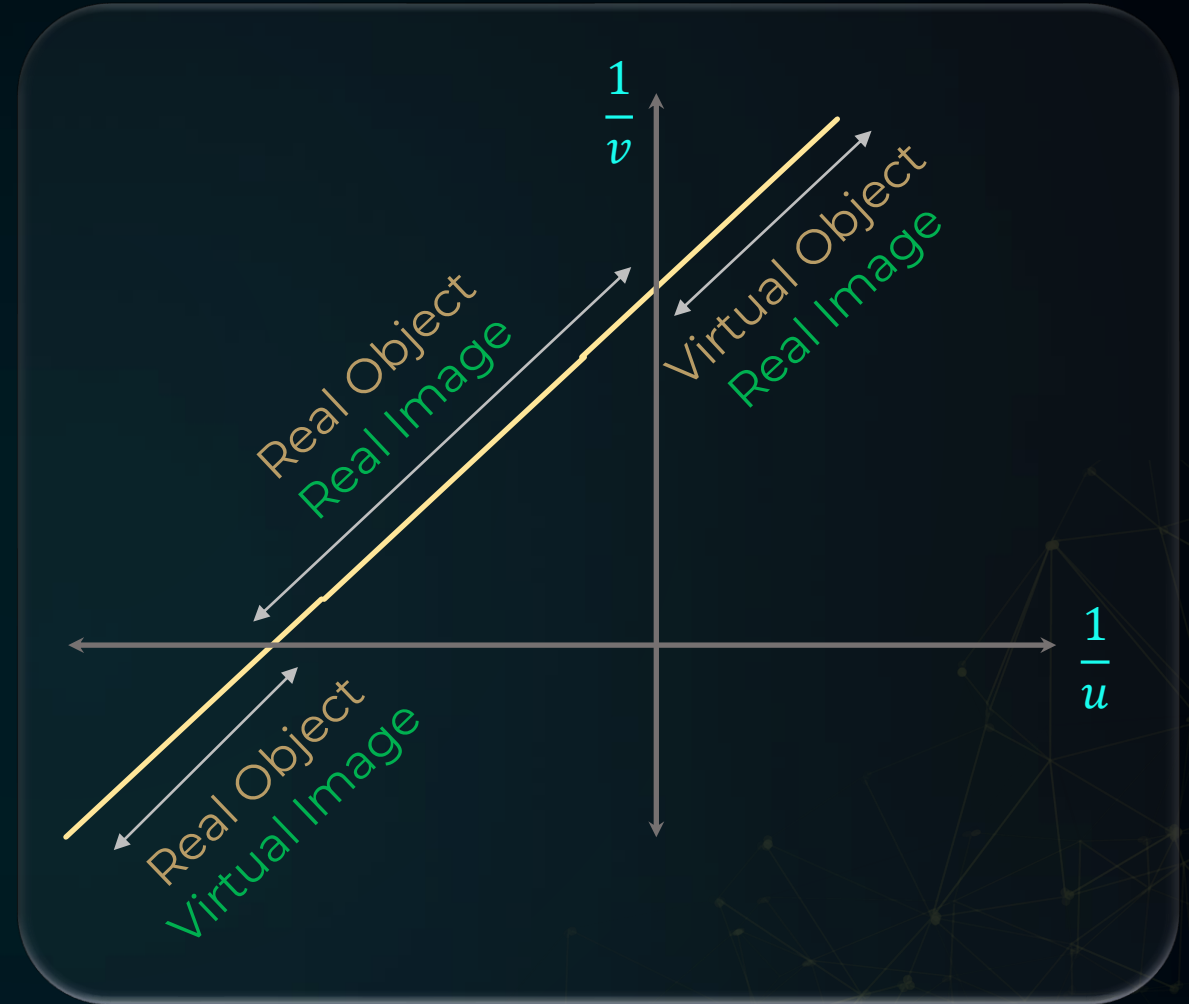


Graphical Representation: Converging Lens



We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- Let's choose $\frac{1}{f} = C$. Now by choosing $\frac{1}{u} = x$ and $\frac{1}{v} = y$. We get,
 $y - x = C$.
- This represents a straight line with slope of -1 .
- $v = -ve$, if $x > f$





Graphical Representation: Converging Lens



We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- A graph between coordinates of object and image is shown in the figure.

Coordinate:

Object Image

- $(-\infty, f)$
- $(-f, \infty)$
- $(0, 0)$
- $(+\infty, f)$

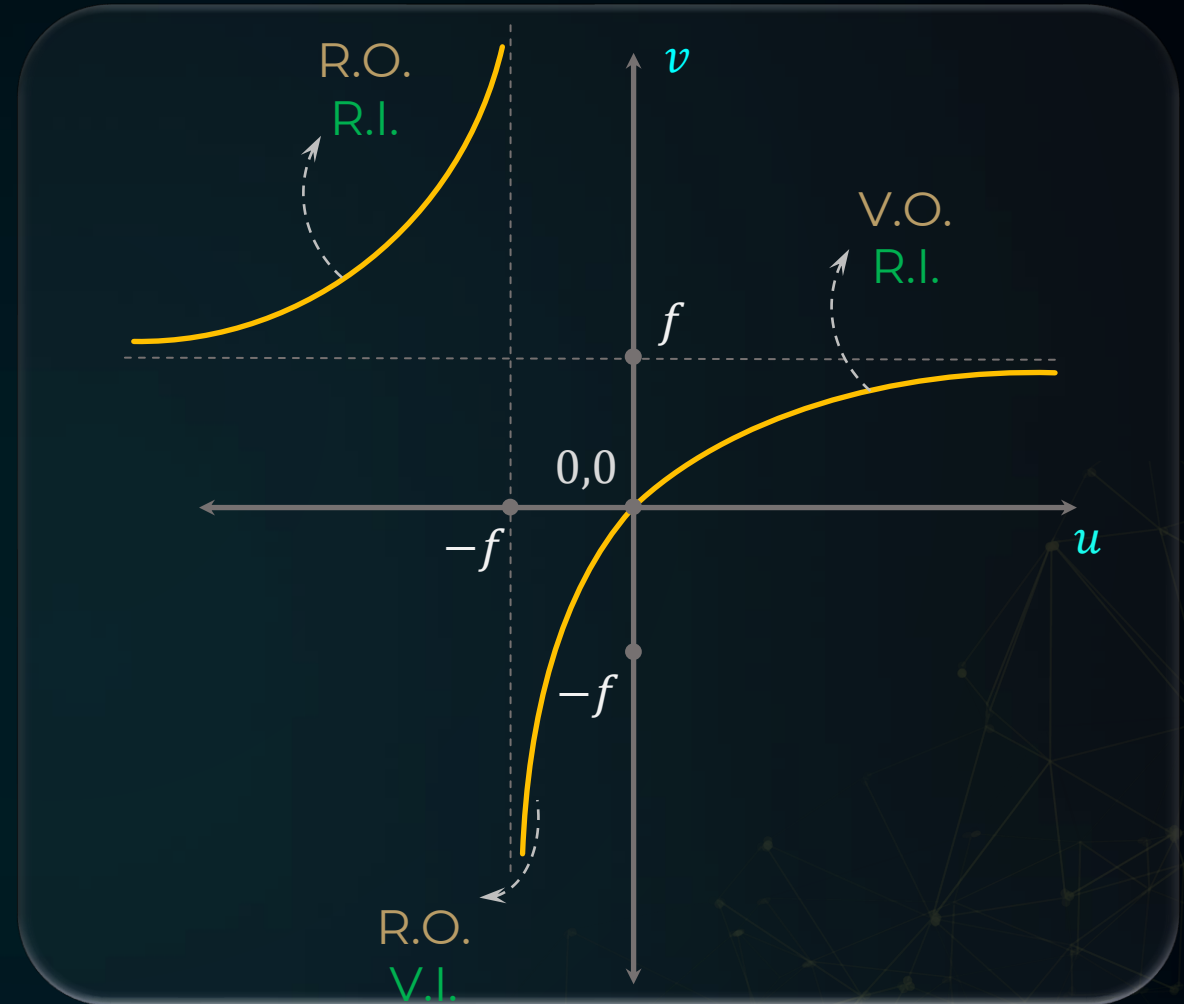


Image Formation by Convex Lens

Position of Object	Position of Image	Nature of Image	Size of Image
At $-\infty$	At F_2	Real, inverted	Highly diminished
Beyond $2F_1$	Between F_2 and $2F_2$	Real, inverted	Diminished
At $2F_1$	At $2F_2$	Real, inverted	Same size
Between $2F_1$ and F_1	Beyond $2F_2$	Real, inverted	Enlarged
At F_1	At ∞	Real, inverted	Highly enlarged
Between F_1 and O	Behind the object	Virtual, erect	Enlarged

Image Formation by Concave Lens

Position of Object	Position of Image	Nature of Image	Size of Image
At $-\infty$	At F_2	Virtual, erect	Highly diminished
Anywhere else	Between O and F_1	Virtual, erect	Diminished



Graphical Representation: Diverging Lens



We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- Since f is constant, Let's choose $\frac{1}{f} = C$.
Now by choosing $\frac{1}{u} = x$ and $\frac{1}{v} = y$.
We get, $y - x = C$.
- This represents a straight line with a positive slope.





Graphical Representation: Diverging Lens



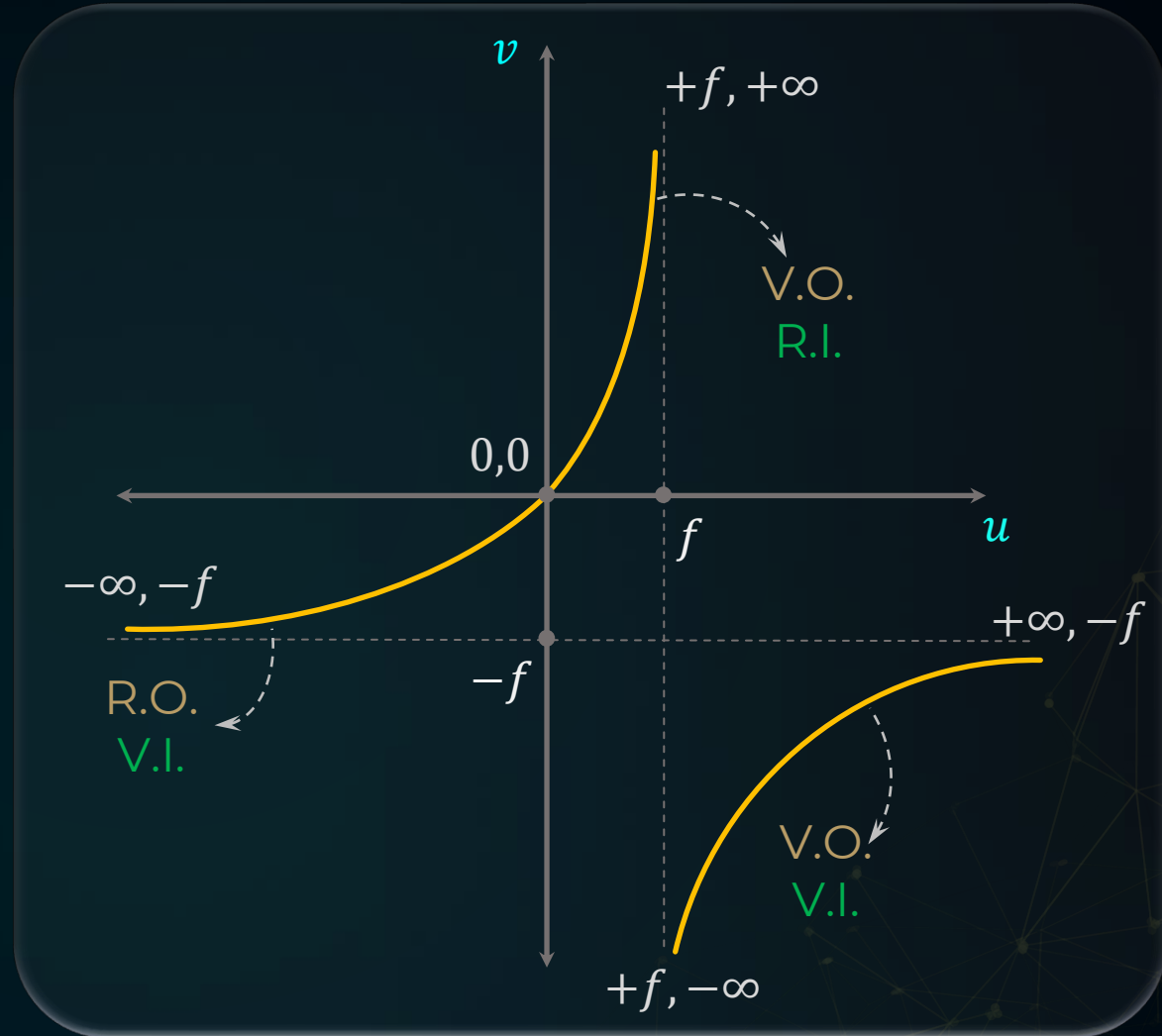
We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- Since f is constant, if we plot the “ v ” vs “ u ” graph, then the graph will be a hyperbola.

Coordinate:

Object Image

- $(-\infty, -f)$
- $(0,0)$
- $(+f, +\infty)$
- $(+f, -\infty)$
- $(+\infty, -f)$





Transverse Magnification

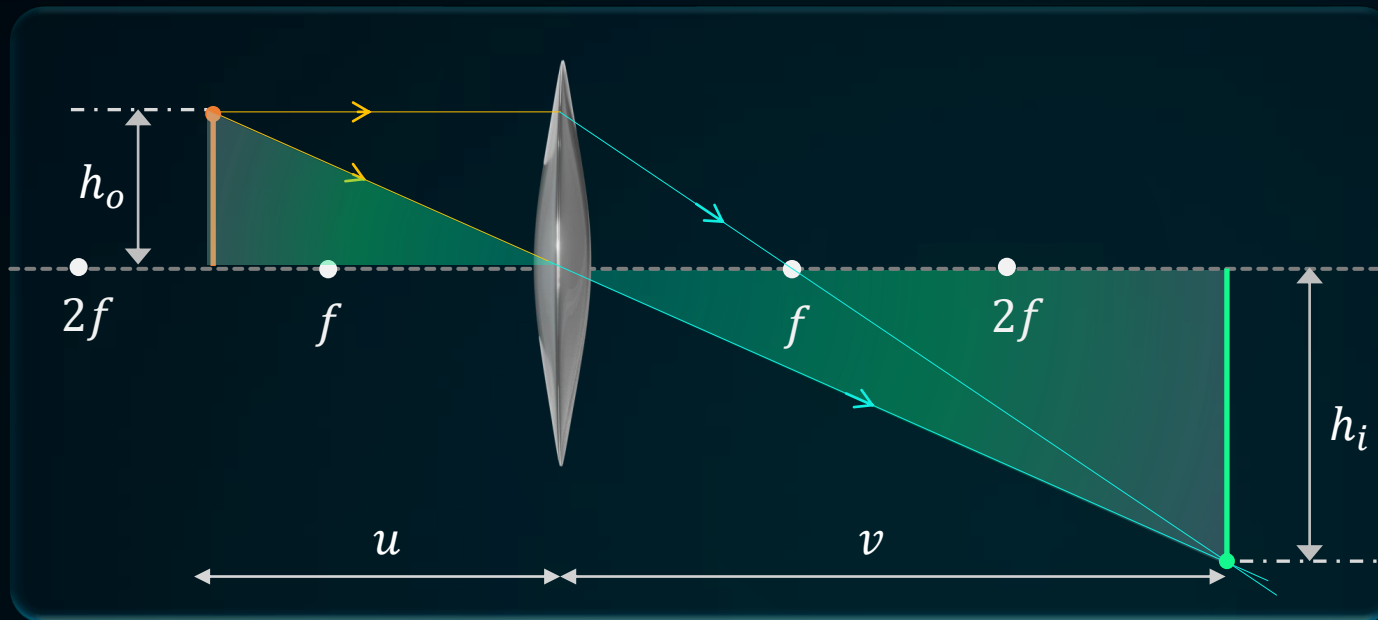


The **ratio** of the **image** height to the **object** height (perpendicular to principal axis)

For mirror:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow m = \frac{y_i}{y_o} = -\frac{v}{u}$$



Transverse magnification,

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$



Longitudinal Magnification



Longitudinal magnification is calculated when the object is placed parallel to the principal axis. It is defined as the ratio of the **length of the image** to the **length of the object**.

For a Very small length of Object

Using lens formula,

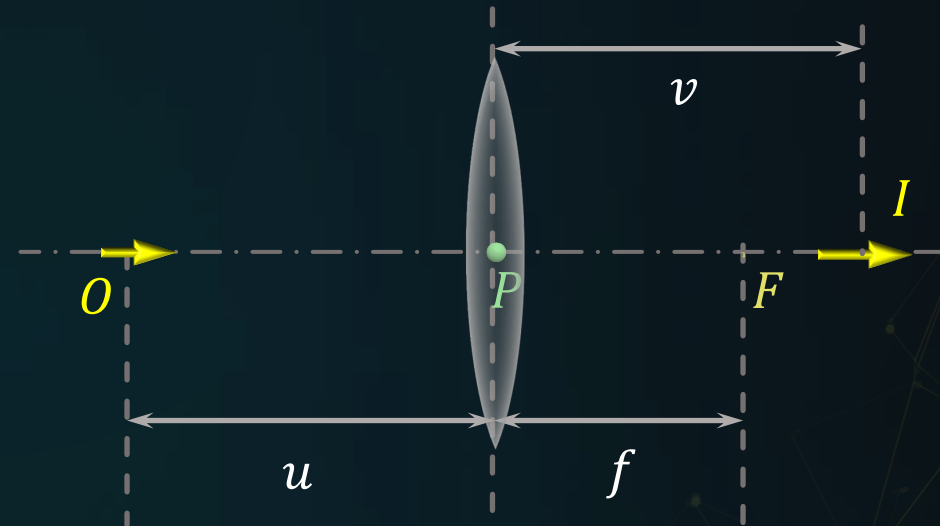
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Differentiating both sides, we get

$$-\frac{1}{v^2} dv + \frac{1}{u^2} du = 0$$

$$\frac{dv}{du} = \frac{v^2}{u^2}$$

dv = Length of the image
 du = Length of the object



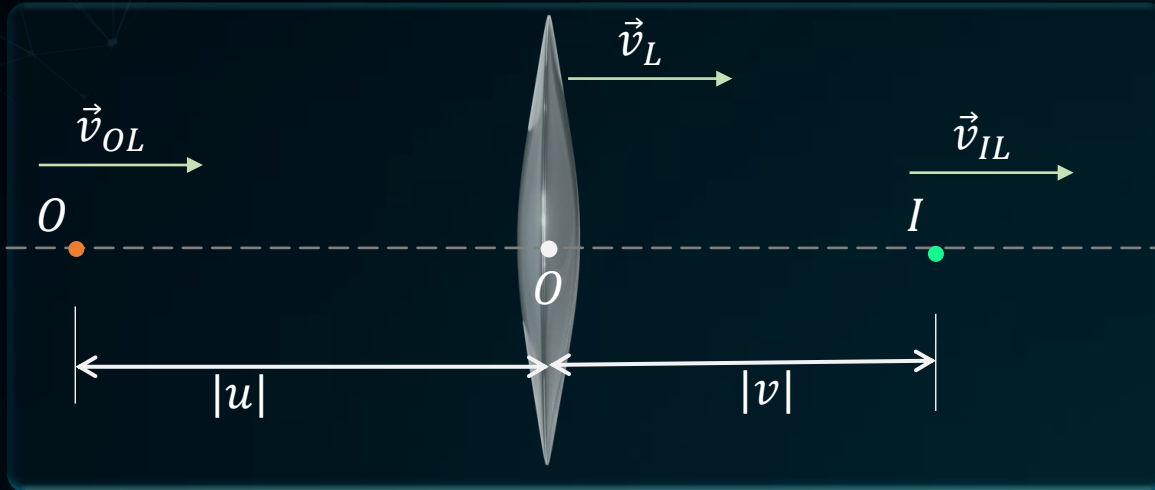
$(du \ll u)$



Velocity of Image and Object



Object moving along principle axis:



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\left(-\frac{1}{v^2}\right)\frac{dv}{dt} + \left(\frac{1}{u^2}\right)\frac{du}{dt} = 0$$

$$\frac{dv}{dt} = \frac{v^2}{u^2} \frac{du}{dt}$$

- W.r.t lens, both object and image move in same direction

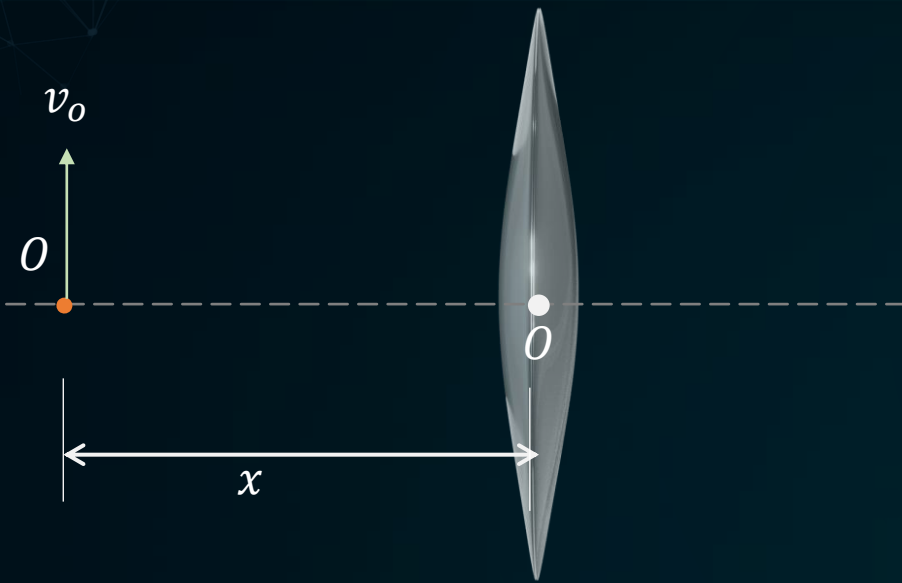
$$\vec{v}_{IL} = \frac{v^2}{u^2} \vec{v}_{OL} = m^2 \vec{v}_{OL}$$



Velocity of Image and Object



Object moving \perp^{ar} to principle axis:



$$m = \frac{y_i}{y_o} = \frac{f}{f + u}$$

$$\Rightarrow y_i = \left(\frac{f}{f + u} \right) y_o$$

$$\Rightarrow \frac{dy_i}{dt} = \left(\frac{f}{f + u} \right) \frac{dy_o}{dt}$$

$$\frac{dy_i}{dt} = \left(\frac{f}{f - x} \right) \frac{dy_o}{dt}$$



Velocity of Image and Object

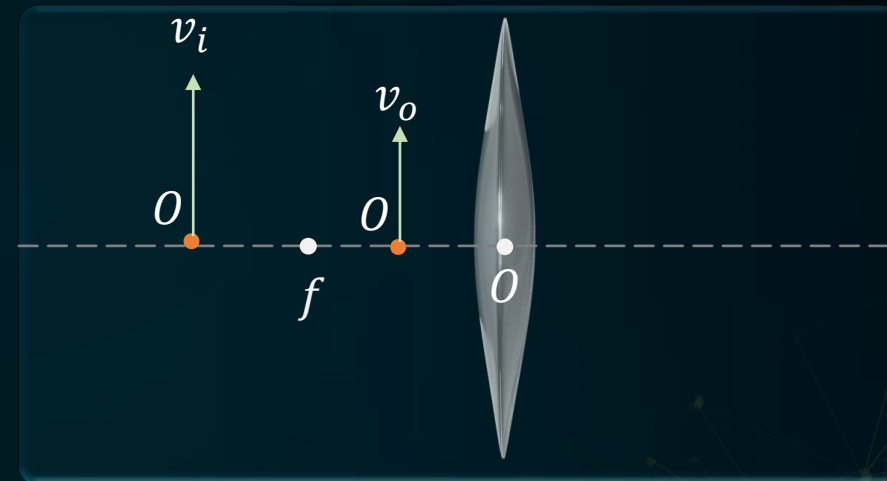
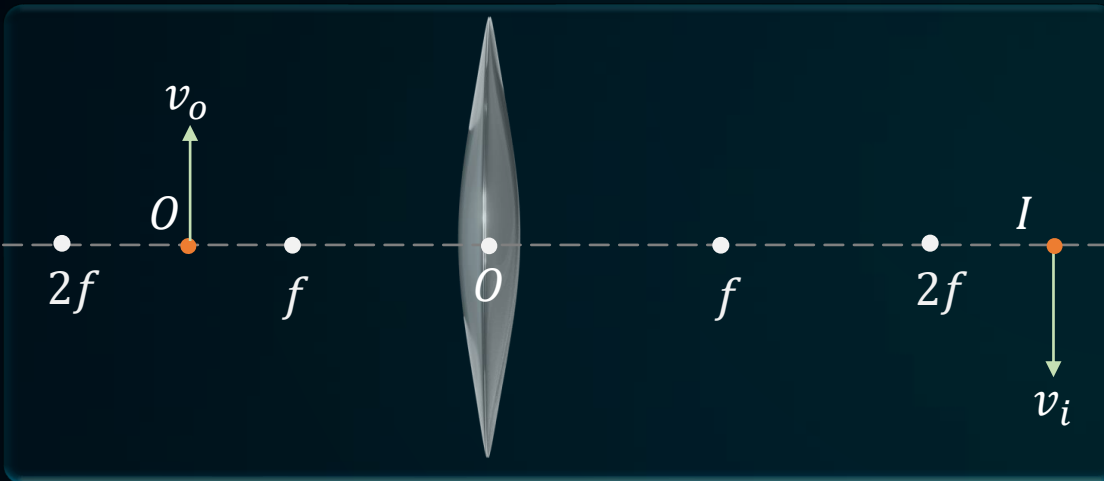


Object moving \perp^{ar} to principle axis:

$$\frac{dy_i}{dt} = \left(\frac{f}{f - x} \right) \frac{dy_o}{dt}$$

If $x > f$, $f - x < 0$, then v_i and v_o are antiparallel

If $x < f$, $f - x > 0$, then v_i and v_o are parallel





Optical Power



Power is the **degree** to which a lens, mirror, or other optical system **converges** or **diverges** light.

For lens

$$P_l = \frac{1}{f_l}$$

For mirror

$$P_m = -\frac{1}{f_m}$$

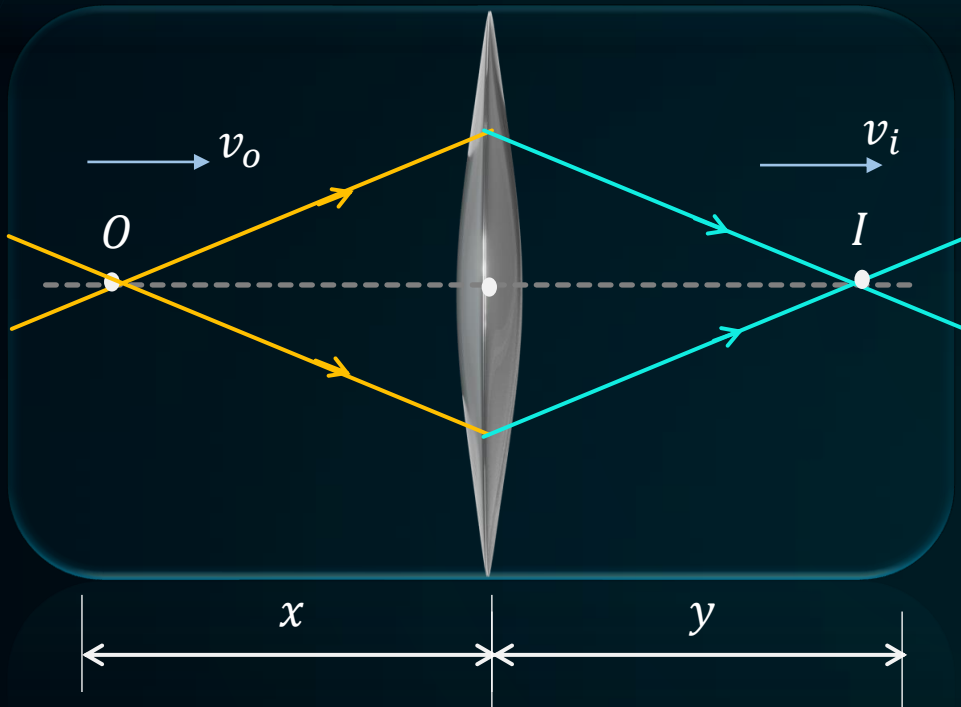
SI unit: Diopter (m^{-1})

For converging lens: $P = +ve$

For diverging lens: $P = -ve$

? Prove that for a convex lens, minimum distance between real object and real image is $4f$

Solution:



At minimum separation (d_{min})

$$\vec{v}_{seperation} = 0 \Rightarrow v_i = v_o$$

$$\vec{v}_{IL} = m^2 \vec{v}_{OL}$$

$$m^2 = 1$$

$$m^2 = \pm 1$$

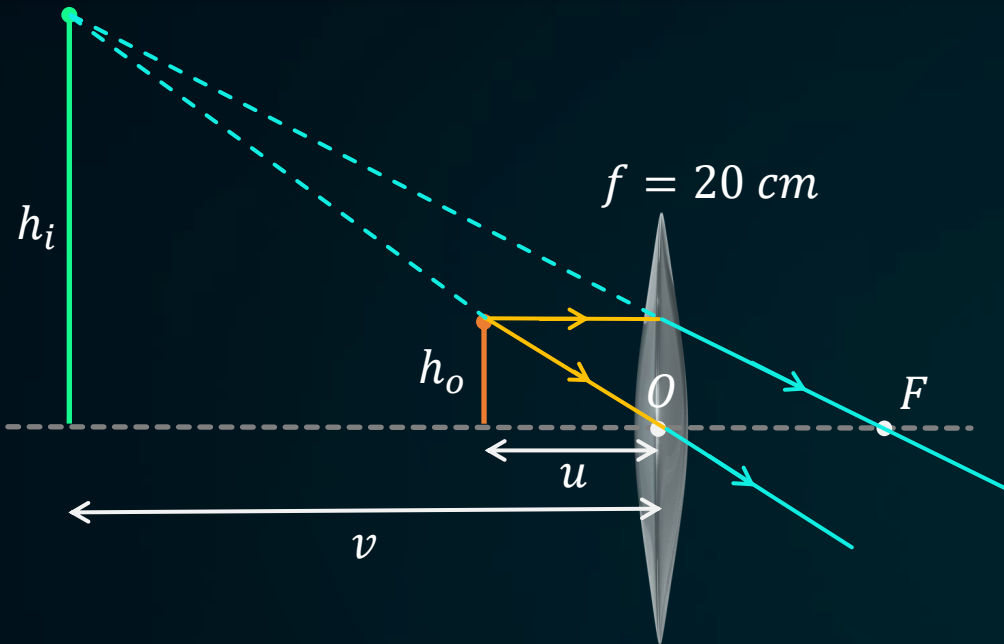
$$\Rightarrow |v| = |u| = 2f$$

$$d_{min} = 4f$$

?

A **5 diopter** lens forms a virtual image which is **4** times the object placed perpendicularly on the principal axis of the lens. Find the distance of the object from the lens.

Solution:



Given: $P = +5 D$

$$\frac{h_i}{h_o} = 4 \Rightarrow u = -d, \quad v = -4d$$

For lens:

$$P_l = \frac{1}{f_l} \Rightarrow f = \frac{1}{5} m = 20 cm \quad (d < 20 cm)$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-4d} - \frac{1}{-d} = \frac{1}{20} \Rightarrow 60 = 4d$$

$$d = 15 cm \quad u = -15 cm$$

?

A diverging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed coaxially at a separation of 5 cm . Where should an object be placed so that a real image is formed at the object itself?

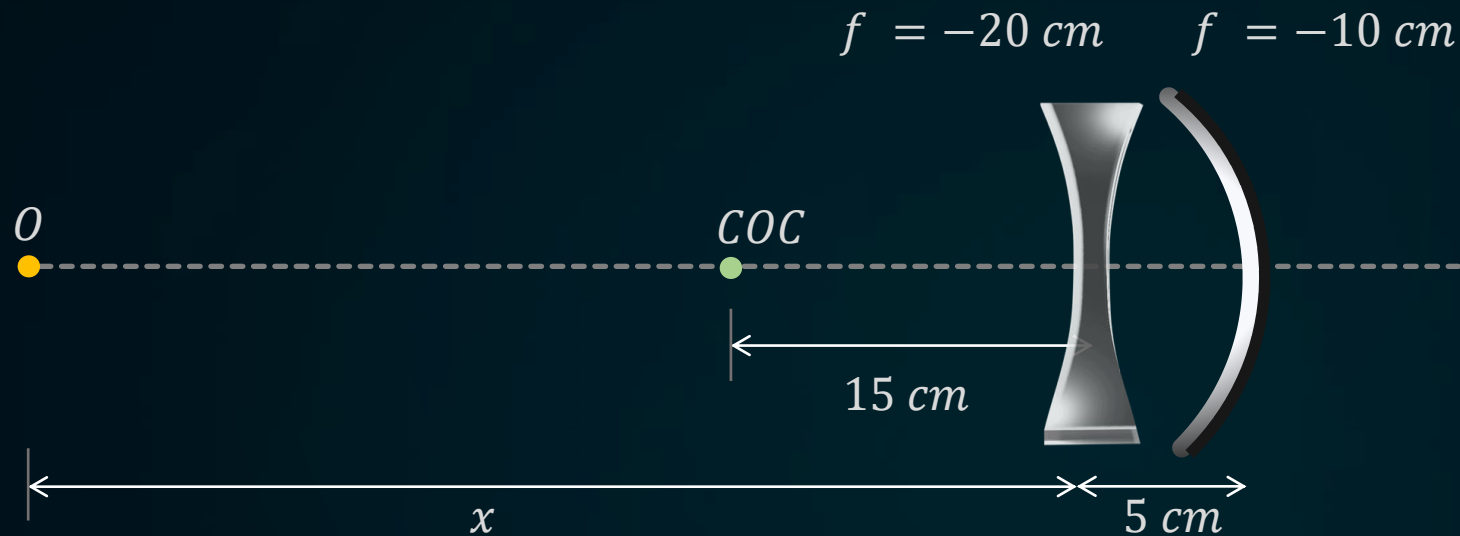


Image formed by a concave lens should be at COC of the concave mirror

Solution:

For mirror:

$$f = 10\text{ cm}$$

$$R = 2f = 20\text{ cm}$$

For lens: $v = -15\text{ cm}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-15} - \frac{1}{-x} = \frac{1}{-20} \Rightarrow x = 60\text{ cm}$$

$$x = 60\text{ cm}$$

?

A converging lens and a diverging mirror are placed at a separation of **15 cm**. The focal length of the lens is **25 cm** and that of the mirror is **40 cm**. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis?

Solution:

For convex mirror: $u = -x$, $v = +10 \text{ cm}$

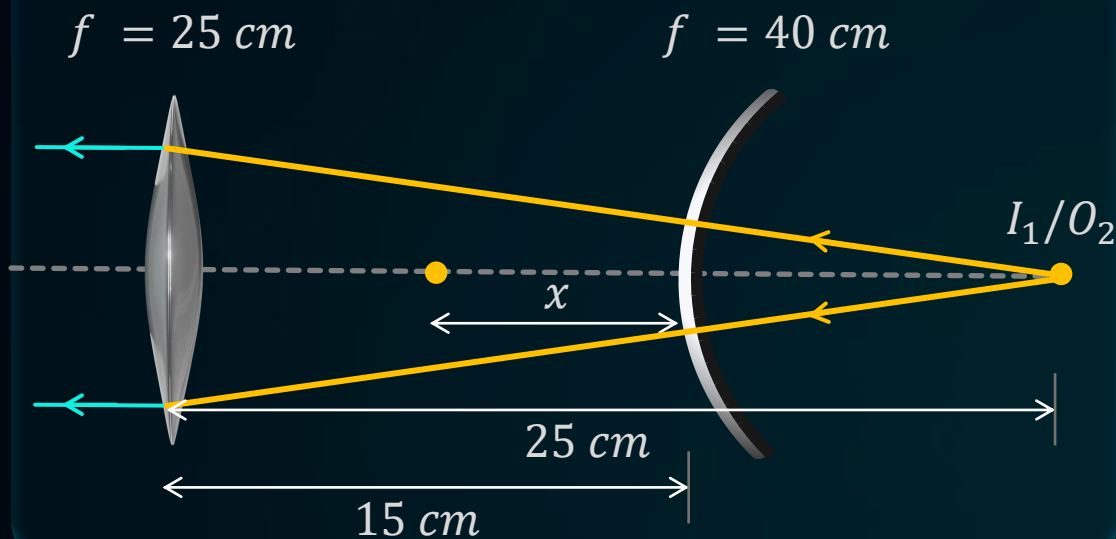
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{+10} + \frac{1}{-x} = \frac{1}{40}$$

$$\frac{1}{x} = \frac{1}{10} - \frac{1}{40} \Rightarrow x = \frac{40}{3} \text{ cm}$$

13.33 cm from the mirror

1.67 cm from the lens





Cutting of lenses



Case-I: A convex lens is cut into two halves by a plane perpendicular to principal axis.

Air($\mu = 1$)

μ_l



$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right)$$

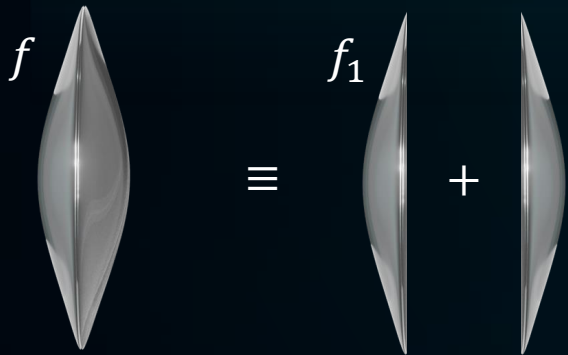
$$\Rightarrow f = \frac{R}{2(\mu - 1)}$$

$$\frac{1}{f_1} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-\infty} \right)$$

$$\Rightarrow \frac{1}{f_1} = (\mu - 1) \left(\frac{1}{R} \right)$$

$$\Rightarrow f_1 = \frac{2R}{2(\mu - 1)}$$

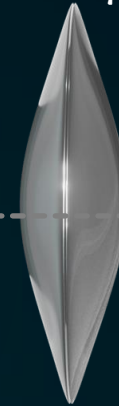
$$f_1 = 2f$$



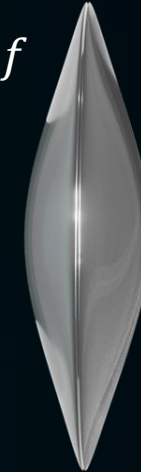
Case-II: A convex lens is cut into two halves by a plane parallel to principal axis.

Air($\mu = 1$)

μ_l



f



\equiv

+

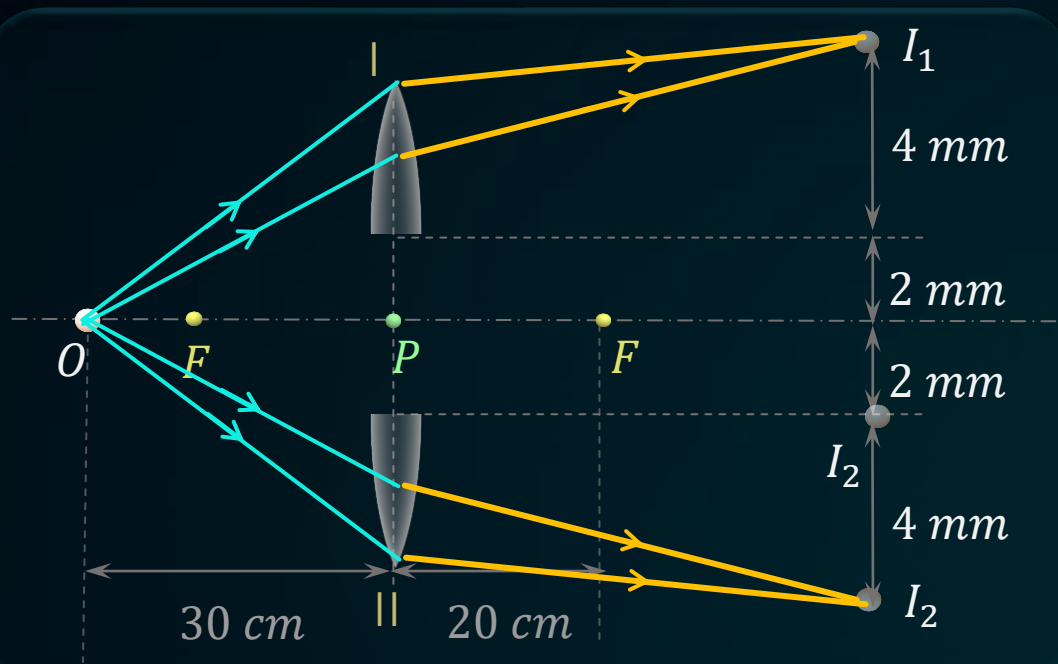


$$f_1 = f$$

?

A convex lens of focal length 20 cm is cut parallel to the principal axis into two equal halves as shown and the parts are moved 2 mm away from either side of the principal axis. Find the separation between the images of the object O formed by these parts.

Solution:



1st Part: $u = -30\text{ cm}, v = +60\text{ cm}, h_o = -2\text{ mm}$

2nd Part: $u = -30\text{ cm}, v = +60\text{ cm}, h_o = +2\text{ mm}$

For 1st Part: The image is at I_1

$$\frac{h_1}{h_o} = \frac{v}{u}$$

$$\Rightarrow h_1 = \frac{v}{u} h_o = \frac{60}{-30} (-2)$$

$$h_1 = +4\text{ mm}$$

For 2nd Part: The image is at I_2

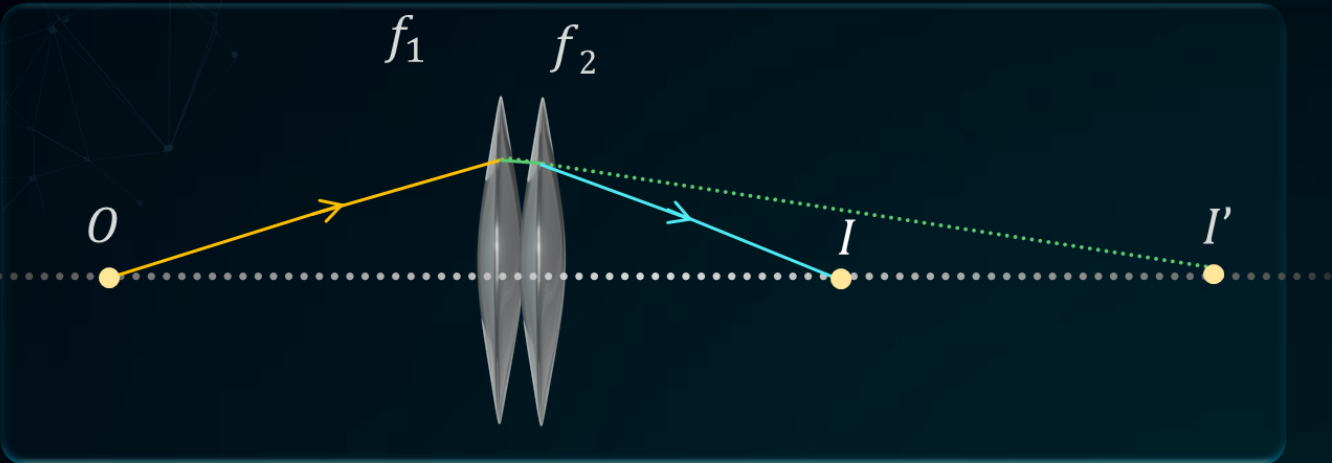
$$\Rightarrow h_2 = \frac{v}{u} h_o = \frac{60}{-30} (+2)$$

$$h_2 = -4\text{ mm}$$

Distance between I_1 & $I_2 = 12\text{ mm}$



Thin Lenses in Contact



If, $P_{eq} > 0 \Rightarrow$ System is converging

If, $P_{eq} < 0 \Rightarrow$ System is diverging

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2 \quad (\text{With sign})$$

Equivalent focal length of lenses is: $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

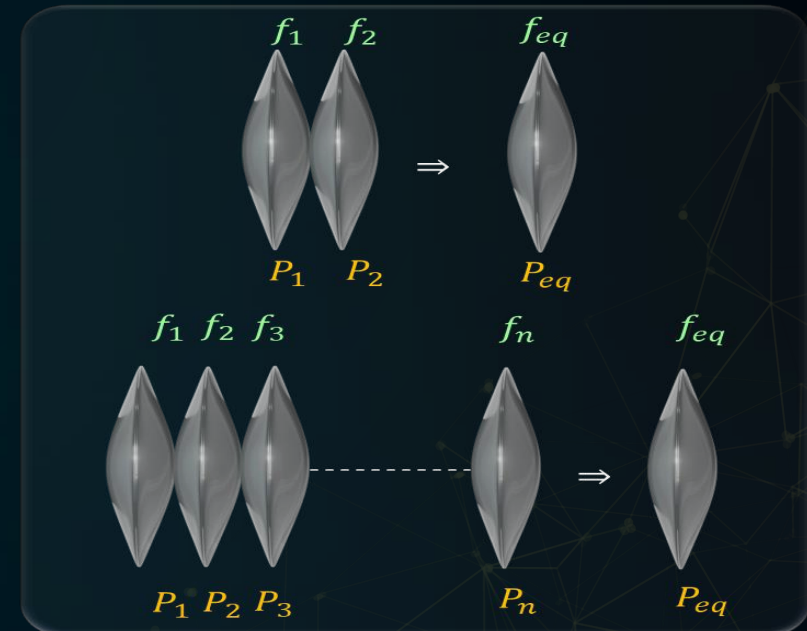
The equivalent power of lenses is: $P_{eq} = P_1 + P_2$

The equivalent focal length of n -lenses is,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_n}$$

The equivalent power of n -lenses is,

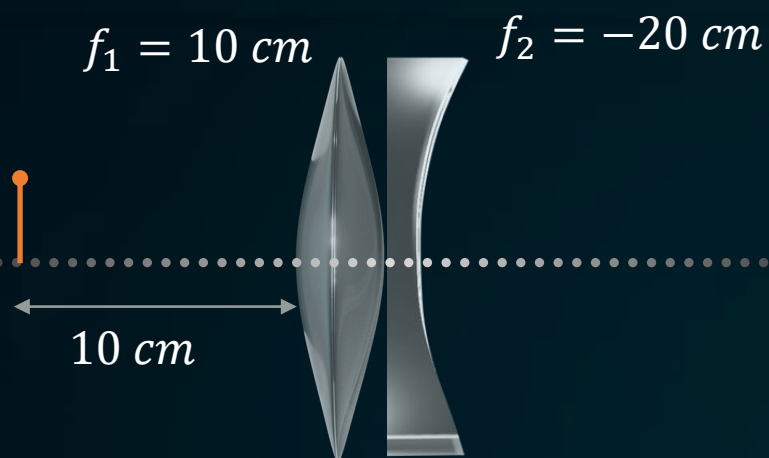
$$P_{eq} = P_1 + P_2 + \dots + P_n$$





Find the **lateral magnification** produced by the combination of lenses shown in the figure.

Solution:



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{f_{eq}} = \frac{1}{10} + \frac{1}{-20}$$

$$f_{eq} = +20 \text{ cm}$$

Lenses in contact:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{v} - \frac{1}{-10} = \frac{1}{+20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{10} \Rightarrow \frac{1}{v} = -\frac{1}{20} \Rightarrow v = -20 \text{ cm}$$

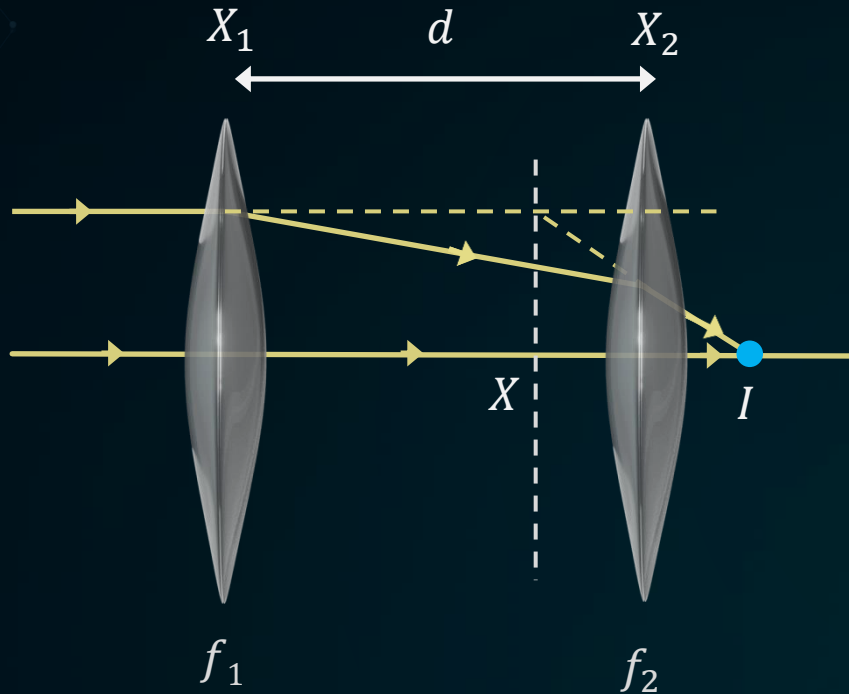
Magnification:

$$m = \frac{y_I}{y_O} = +\frac{v}{u} \Rightarrow m = \frac{+(-20)}{(-10)}$$

$$m = 2$$



Thin Lenses Separated by a Distance



$XI \rightarrow$ Equivalent focal length (f_{eq})

$X \rightarrow$ Position of equivalent lens

$d \rightarrow$ Distance between two lenses

- The **equivalent** focal length is given by:

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

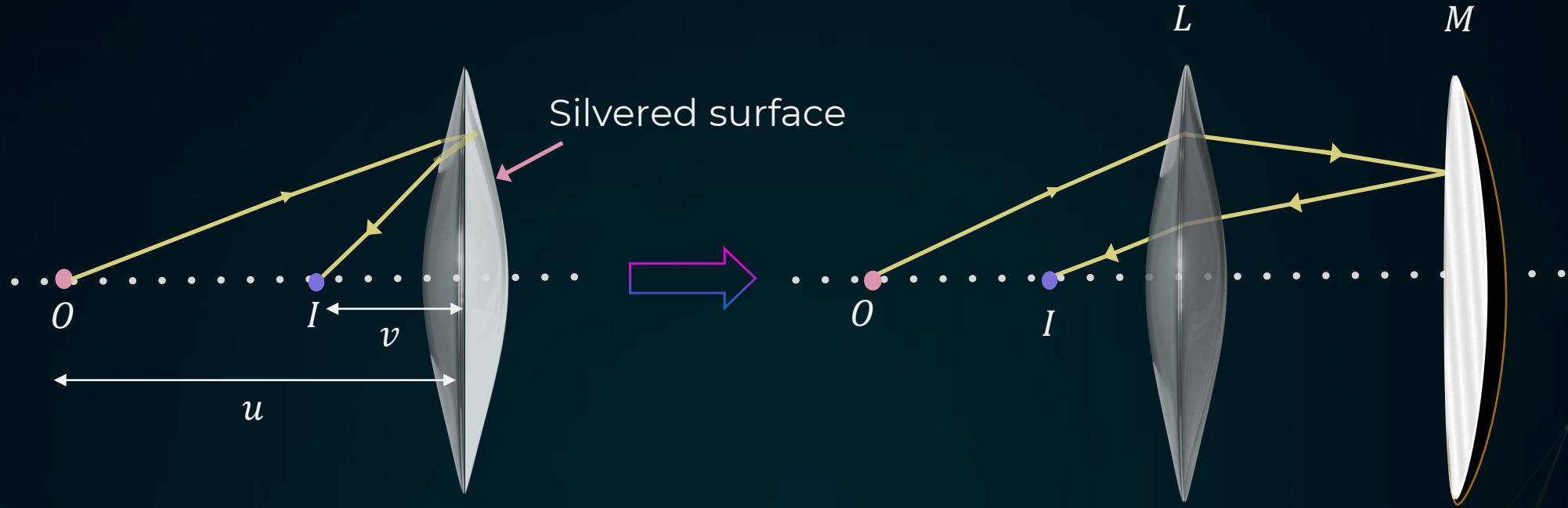
- The **position** of equivalent lens from lens 2 (X_2X) is given by,

$$X_2X = \frac{d f_{eq}}{f_1}$$

- Note: These formulae are valid only for the special case of **parallel incident** beam.
- If the object is at finite distance, the image distance should be calculated by using lens formula for two lenses separately.



Silvering of Lens

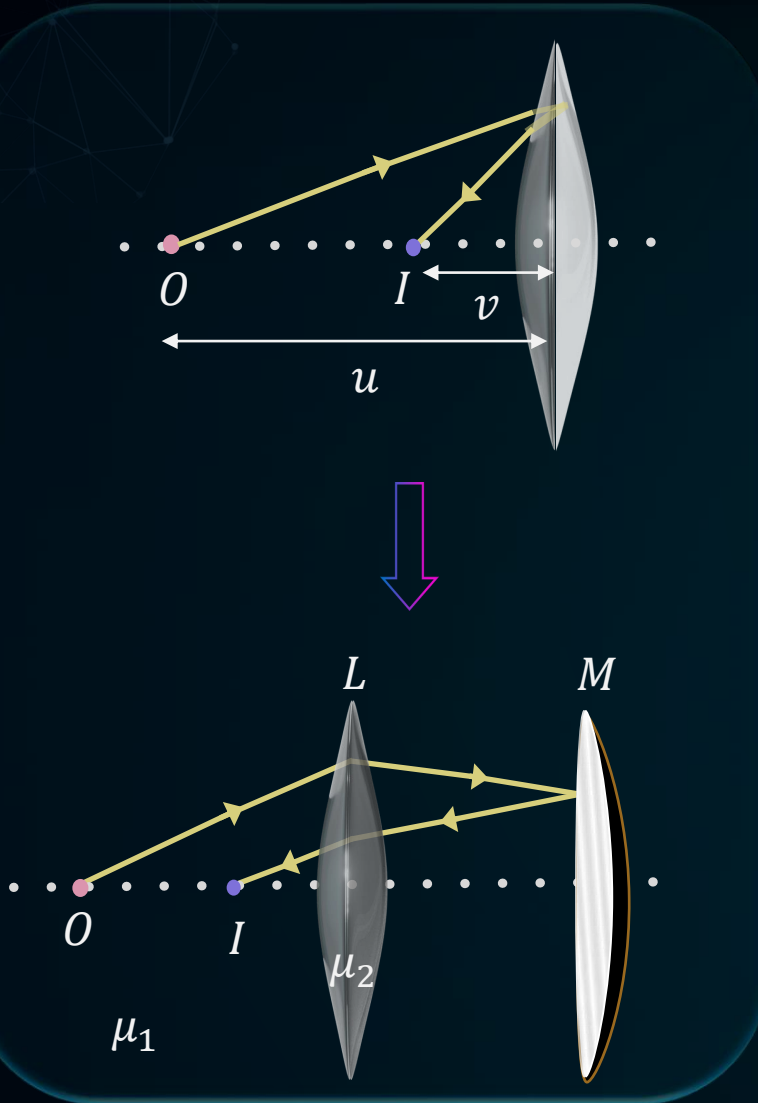


- The overall system behaves as a **mirror**.

- The equivalent focal length of the system: $\frac{1}{F_{eq}} = \frac{1}{u} + \frac{1}{v}$



Silvering of Lens



- The equivalent **power** of the system is:

$$P_{eq} = P_L + P_M + P_L$$

Where:

- $P_{eq} = -\frac{1}{F_{eq}}$ (system acts as a mirror)
- $P_L = \frac{1}{f_L}$ (For Lens)
- $P_M = -\frac{1}{f_M}$ (For silvered surface)
- The equivalent **focal length** of the system is:

$$-\frac{1}{F_{eq}} = \frac{1}{f_L} + \left(-\frac{1}{f_M}\right) + \frac{1}{f_L}$$

- The effective focal length of the system is:

$$-\frac{1}{F_{eq}} = \frac{1}{f_L} + \left(-\frac{1}{f_M}\right) + \frac{1}{f_L}$$

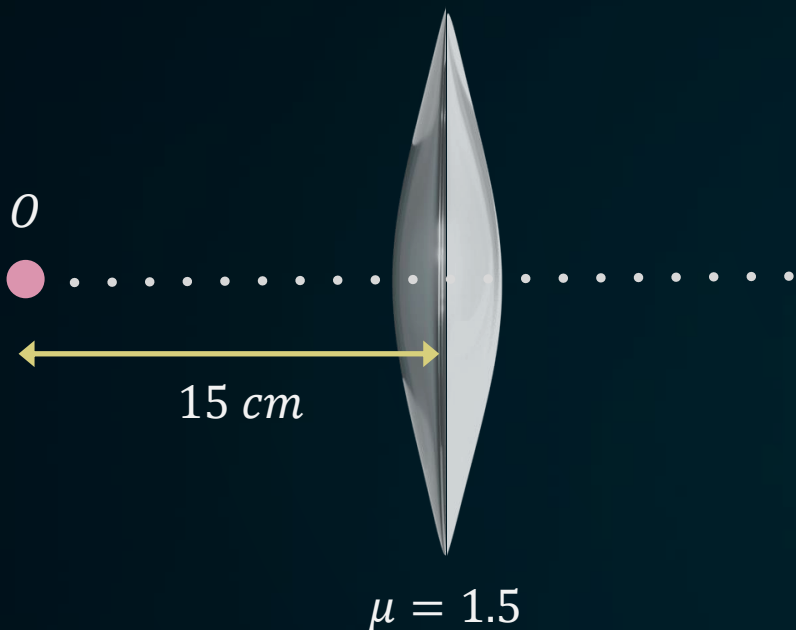
Where:

- $f_M = \frac{R_m}{2}$
(For Mirror)
- $f_L = \left(\frac{\mu_2}{\mu_1} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
(Lens Maker's Formula)
- $\frac{1}{F_{eq}} = \frac{1}{u} + \frac{1}{v}$
(system acts as a mirror)

?

An equiconvex lens ($\mu = 1.5$) has radius of curvature 10 cm . One of its surface is silvered. An object is placed at 15 cm from the lens. Find the position of the image formed by the lens after silvering.

Solution:



Given: $\mu = 1.5$; $R_1 = R_2 = 10 \text{ cm}$
 $u = -15 \text{ cm}$

The effective focal length after silvering is given by,

$$-\frac{1}{F_e} = \frac{1}{f_L} + \left(-\frac{1}{f_m}\right) + \frac{1}{f_L}$$

$$\frac{1}{f_L} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

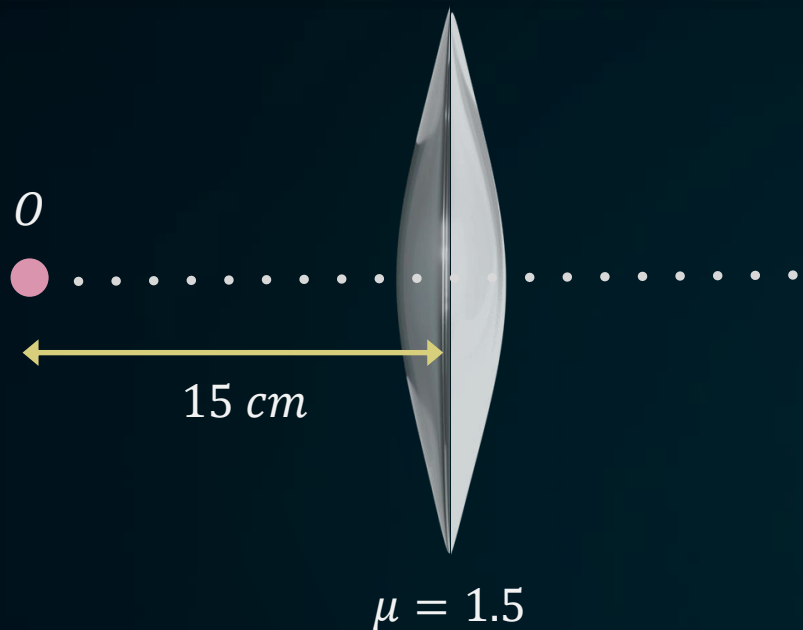
$$\frac{1}{f_L} = 0.5 \left(\frac{1}{10} - \frac{1}{(-10)} \right) = \frac{1}{10} \text{ cm}^{-1} \Rightarrow \frac{1}{F_e} = -\frac{4}{10} \text{ cm}^{-1}$$

$$f_m = \frac{R_2}{2} \Rightarrow \frac{1}{f_m} = -\frac{2}{10} \text{ cm}^{-1}$$

$$-\frac{1}{F_e} = \frac{1}{10} + \frac{2}{10} + \frac{1}{10}$$



An equiconvex lens ($\mu = 1.5$) has radius of curvature 10 cm . One of its surface is silvered. An object is placed at 15 cm from the lens. Find the position of the image formed by the lens after silvering.



Given: $\mu = 1.5$; $R_1 = R_2 = 10 \text{ cm}$
 $u = -15 \text{ cm}$

Solution:

The effective focal length after silvering is given by,

$$\Rightarrow \frac{1}{F_e} = -\frac{4}{10} \text{ cm}^{-1}$$

After silvering, the system acts as a mirror, therefore, using mirror formula,

$$\frac{1}{F_e} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{v} = +\frac{1}{15} - \frac{4}{10} = -\frac{10}{30} \text{ cm}^{-1}$$

$$v = -3 \text{ cm}$$



Optical Power of Lens-Mirror Combination



Incident light



Lens-Mirror combination



Refraction
through
lens



Reflection
on mirror



Refraction
through
lens



P_l



P_m



P_l

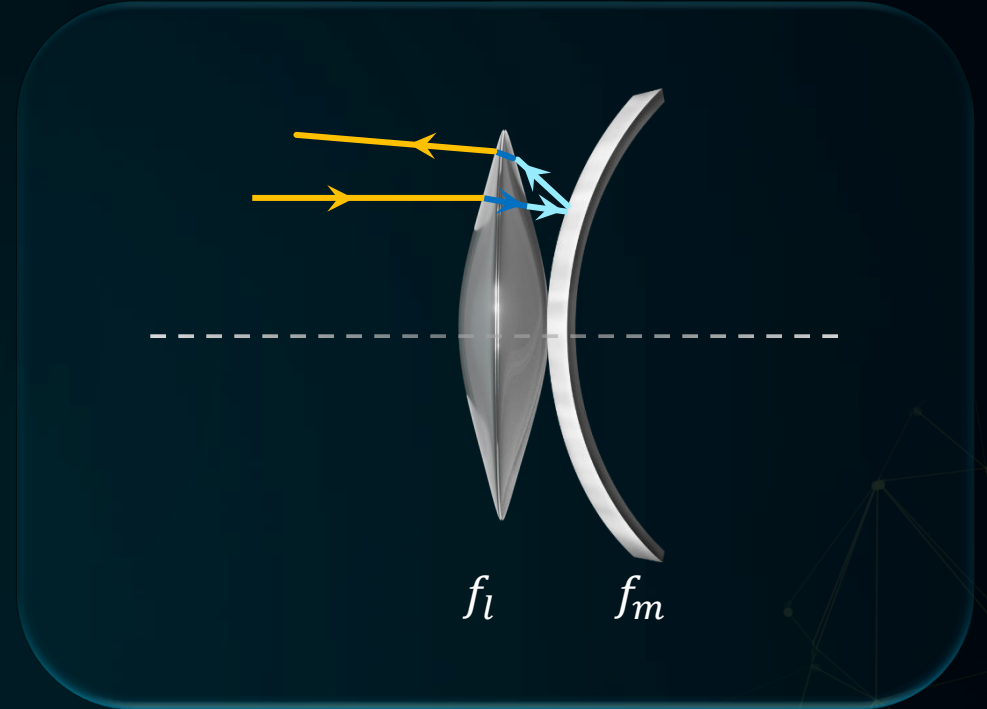


Equivalent power:

$$P_{eq} = 2P_l + P_m$$



$$-\frac{1}{f_{eq}} = \frac{2}{f_l} + \left(-\frac{1}{f_m}\right)$$



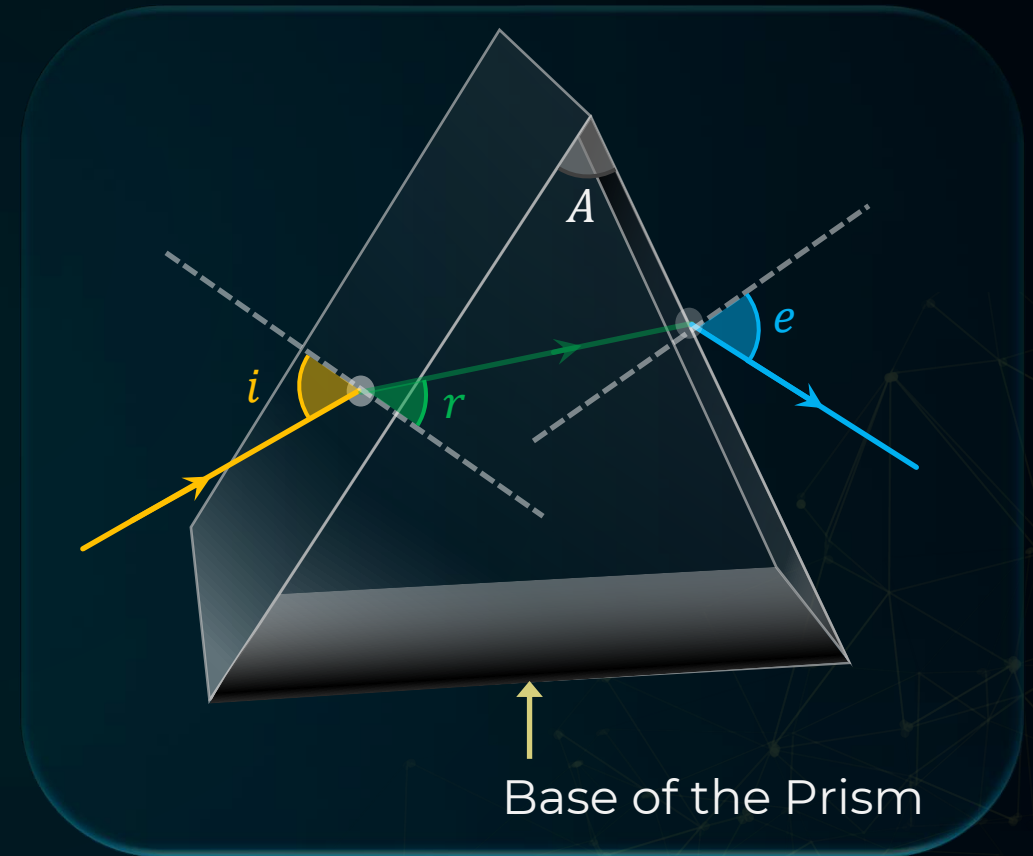


Introduction to Prism



A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a **prism**.

- $A \rightarrow$ **Angle of the Prism (or refracting angle)** is defined as the angle between two surfaces through which the light passes.
- $i \rightarrow$ Angle of incidence
- $r \rightarrow$ Angle of refraction
- $e \rightarrow$ Angle of emergence





Summary



PR and RQ are normal to two refracting surfaces.

$$\angle OPR = 90^\circ = \angle OQR$$

From the quadrilateral $OPRQ$:

$$A + \angle PRQ = 180^\circ \dots\dots\dots (1)$$

From $\triangle PQR$:

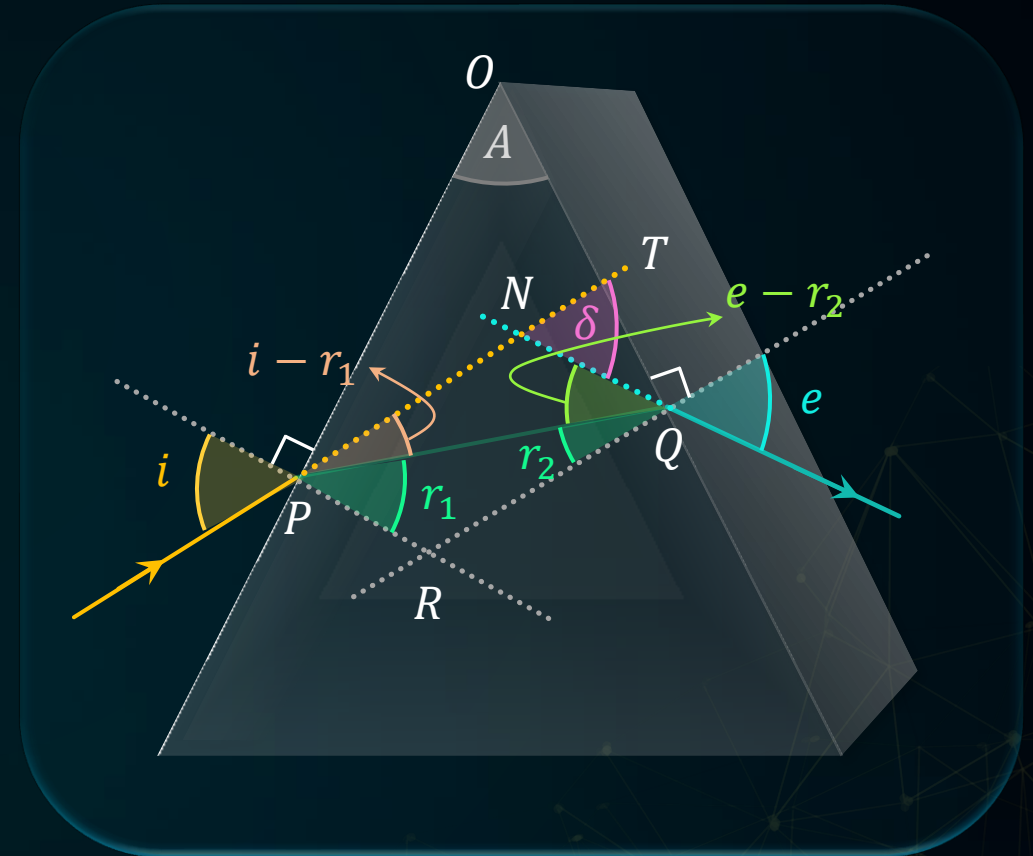
$$\angle PRQ = 180^\circ - (r_1 + r_2) \dots\dots\dots (2)$$

Combining equation (1) and (2):

$$A = (r_1 + r_2)$$

From external properties of $\triangle PNQ$:

$$\delta = (i - r_1) + (e - r_2) \rightarrow \delta = (i + e) - A$$





Variation of δ with i

If i and e are interchanged, the angle of deviation remains same, due to the principle of reversibility of light.

- Using Snell's law on the 1st refracting surface:

$$r_1 = \sin^{-1} \left(\frac{1}{n} \sin i \right)$$

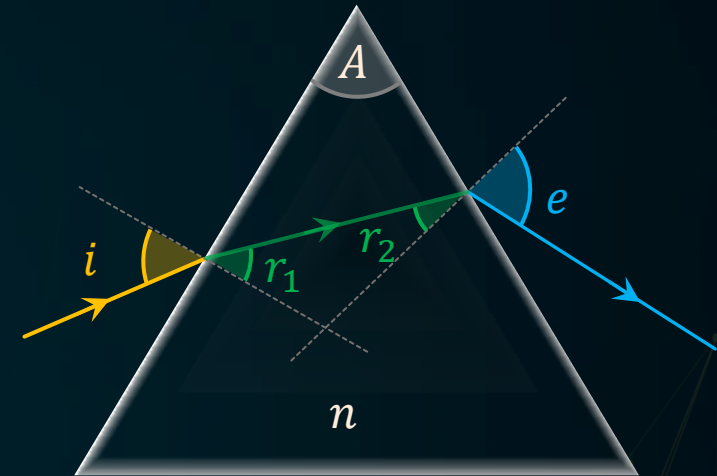
- $A = (r_1 + r_2) \Rightarrow r_2 = (A - r_1)$
- Using Snell's law on the 2nd refracting surface:

$$n \sin r_2 = \sin e$$

$$e = \sin^{-1} (n \sin (A - r_1))$$

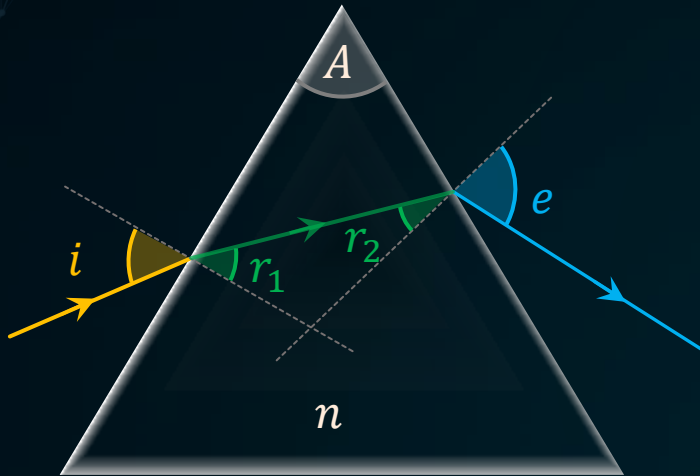
$$e = \sin^{-1} \left(n \sin \left(A - \sin^{-1} \left(\frac{\sin i}{n} \right) \right) \right)$$

$$\delta_{net} = i + \sin^{-1} \left(n \sin \left(A - \sin^{-1} \left(\frac{\sin i}{n} \right) \right) \right) - A$$

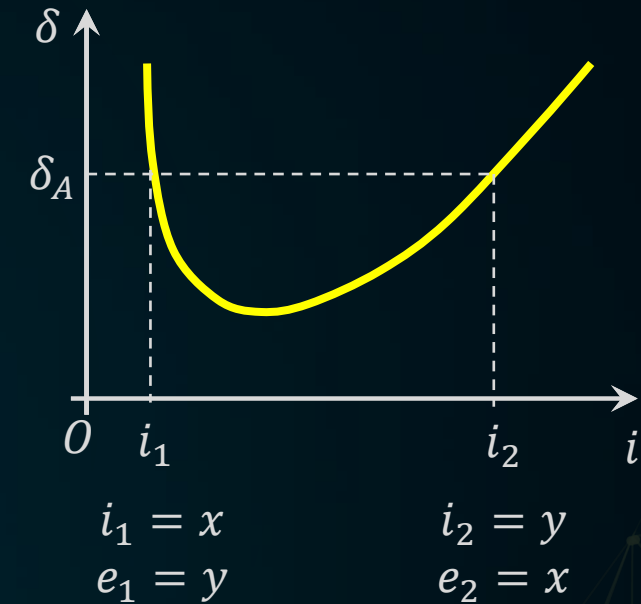




Variation of δ with i



$$\delta_{net} = i + e - A$$



- The deviation is same for two angles of incidence (i_1 and i_2).
- As δ remains the same, an increase in angle of incidence causes angle of emergence to decrease and vice-versa.



Summary



For δ_{min} : $i = e \rightarrow r_1 = r_2 = \frac{A}{2}$

$$\delta = i + e - A$$

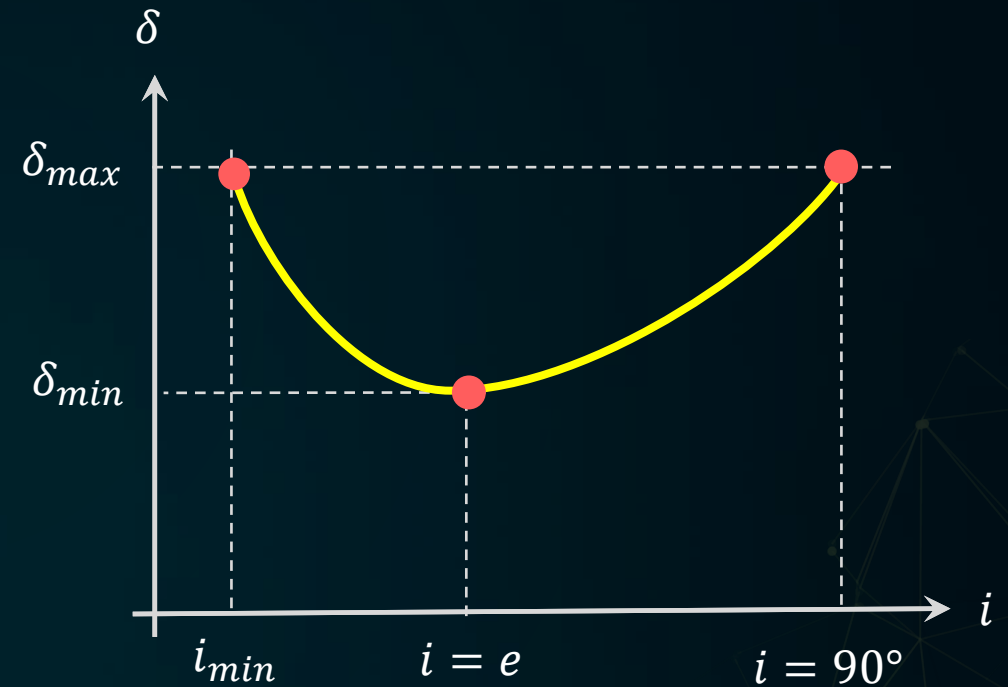
$$\delta_{min} = i + i - A$$

$$i = \frac{\delta_{min} + A}{2}$$

Using Snell's law for r_1 :

$$1 \times \sin i = n \times \sin r_1 \rightarrow \sin\left(\frac{A + \delta_{min}}{2}\right) = n \sin\left(\frac{A}{2}\right)$$

$$n = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$





Thin Prism

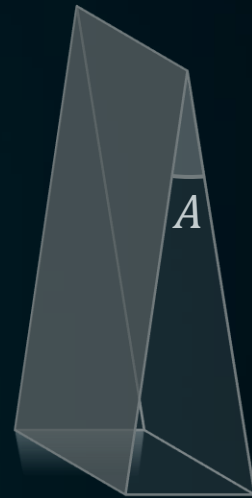


- A prism is taken as thin when the angle of prism is small.
- Generally, A is less than 10° for a thin prism.
- We have:

$$n = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$n = \frac{\left(\frac{A + \delta_{min}}{2}\right)}{\left(\frac{A}{2}\right)}$$

$$\delta_{min} = (n - 1)A$$



?

A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is:

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Solution:

Shape of prism: Equilateral triangle



$$A = 60^\circ$$

For minimum deviation: $r_1 = r_2 = \frac{A}{2} = 30^\circ$

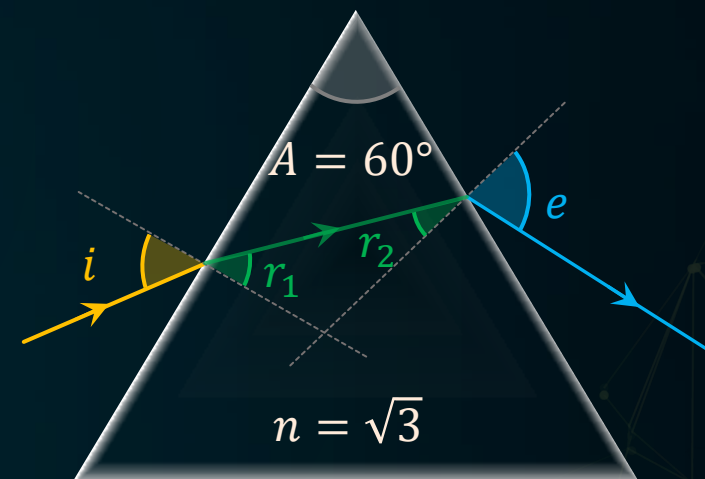
From Snell's law of refraction:

$$n_1 \times \sin i = n_2 \times \sin r_1$$

$$1 \times \sin i = \sqrt{3} \times \sin 30^\circ$$

$$\sin i = \frac{\sqrt{3}}{2}$$

$$i = 60^\circ$$





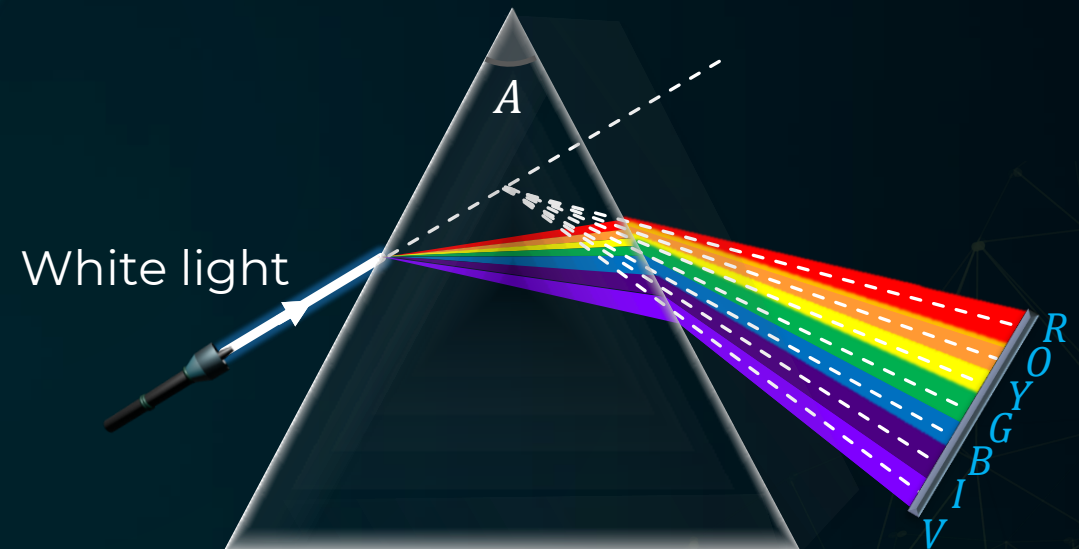
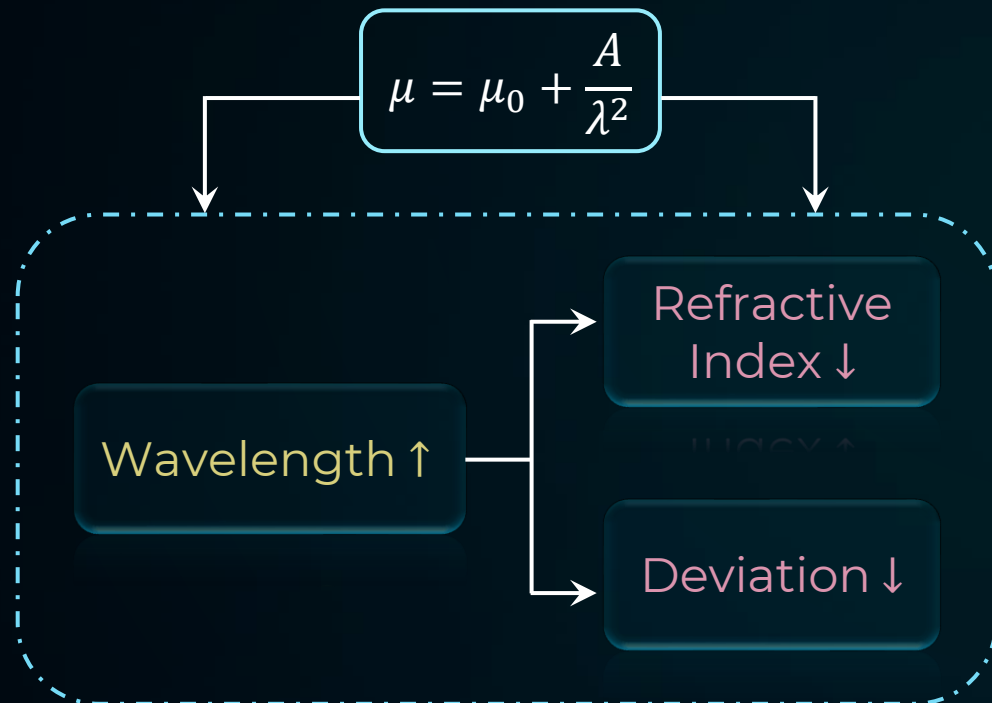
Dispersion of Light



- **Dispersion of Light:** The phenomenon of separation of different constituent colors of light while passing through a transparent medium.

- **Cauchy's Formula:**

Refractive index of a medium depends upon the wavelength of incident light:





Deviation



- **Average Deviation:**

The overall deviation of the white light beam is the deviation of the yellow light as this deviation is roughly the average of all deviations.

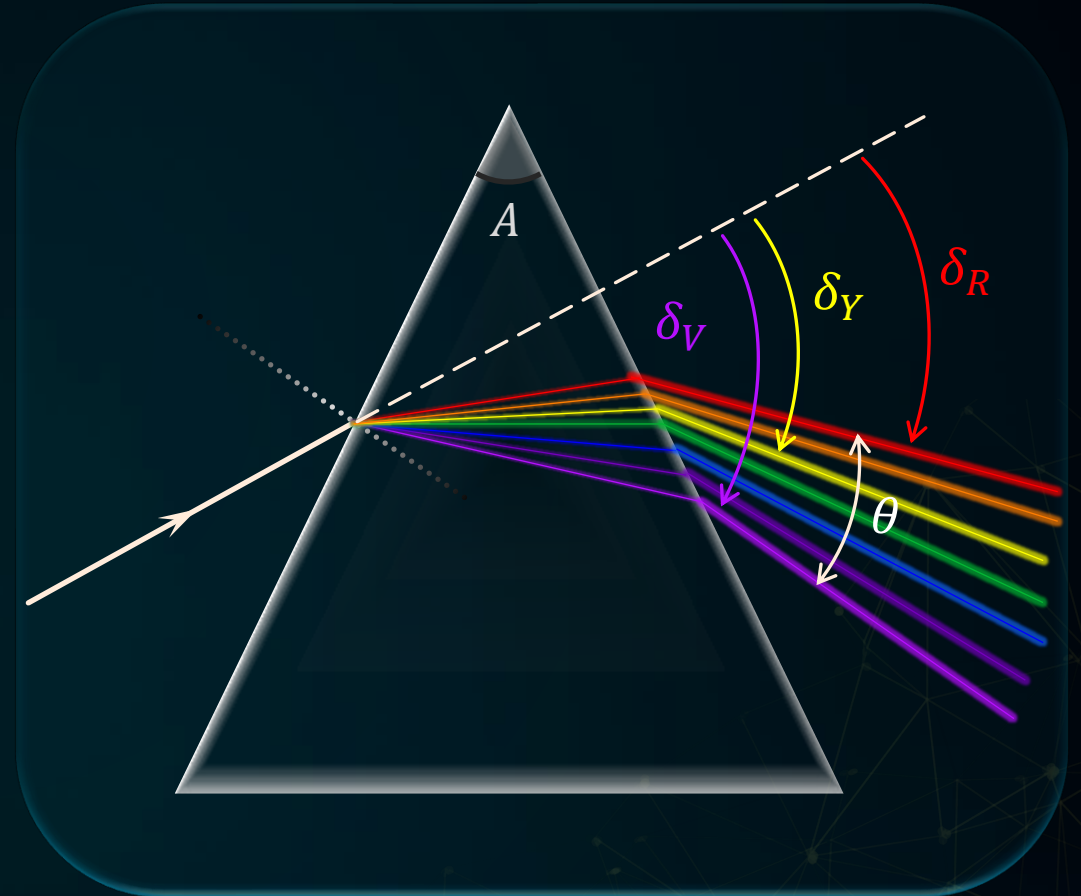
- **Angular Dispersion:**

The difference in the angles of deviation of two extreme colors of the white light i.e., $\theta = (\delta_V - \delta_R) = (\mu_V - \mu_R)A$.

- **Dispersive Power:**

The ratio of angular dispersion to the average deviation.

$$\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{\mu_V - \mu_R}{\mu_Y - 1} \quad (\text{Only if } A \text{ is small})$$





Combinations of Prisms



Refracting angles of two prisms: A, A'

Dispersive power of two prisms: ω, ω'

Angle of deviation due to:

prism 1: δ_1 (clockwise)

prism 2: δ_2 (Anti-clockwise)

Average deviation due to:

prism 1: $\delta_1 = (\mu_Y - 1)A$

prism 2: $\delta_2 = (\mu'_Y - 1)A'$

Angular dispersion due to:

prism 1: $\theta_1 = (\mu_V - \mu_R)A$

prism 2: $\theta_2 = (\mu'_V - \mu'_R)A'$

Net average deviation:

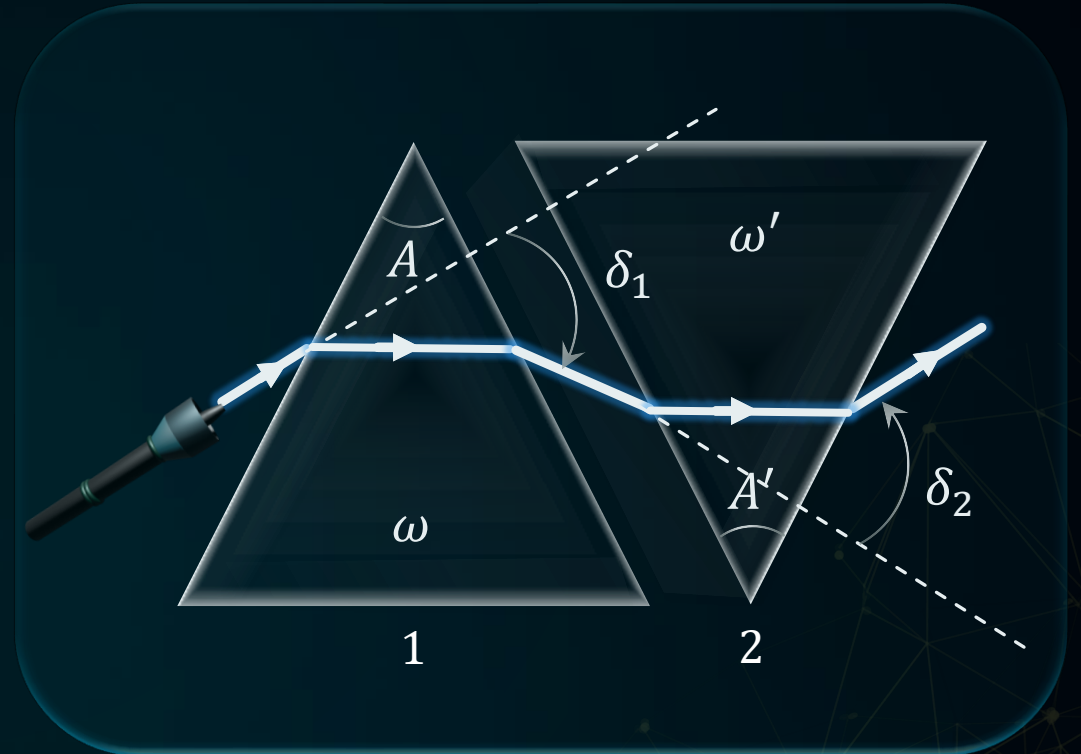
$$\delta = \delta_1 - \delta_2$$

$$\delta = (\mu_Y - 1)A - (\mu'_Y - 1)A'$$

Net angular dispersion:

$$\theta = \theta_1 - \theta_2$$

$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A'$$





Combination of Prisms: No average deviation

$$\delta = (\mu_Y - 1)A - (\mu'_Y - 1)A'$$



No average deviation

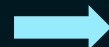
$$(\mu_Y - 1)A = (\mu'_Y - 1)A'$$

Dispersive power of:

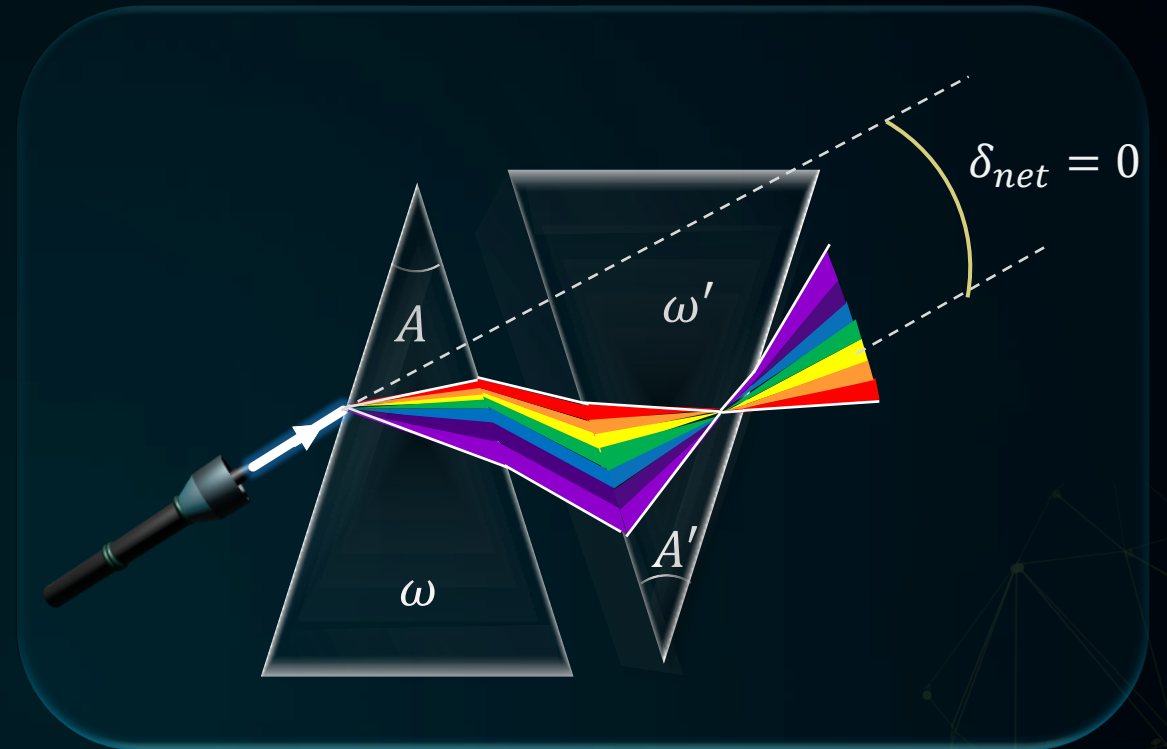
$$\text{prism 1: } \omega = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)}$$

$$\text{prism 2: } \omega' = \frac{(\mu'_V - \mu'_R)}{(\mu'_Y - 1)}$$

$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A'$$



$$\theta = (\mu_Y - 1)A(\omega - \omega')$$





Combination of Prisms: No angular dispersion



$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A'$$



No angular dispersion

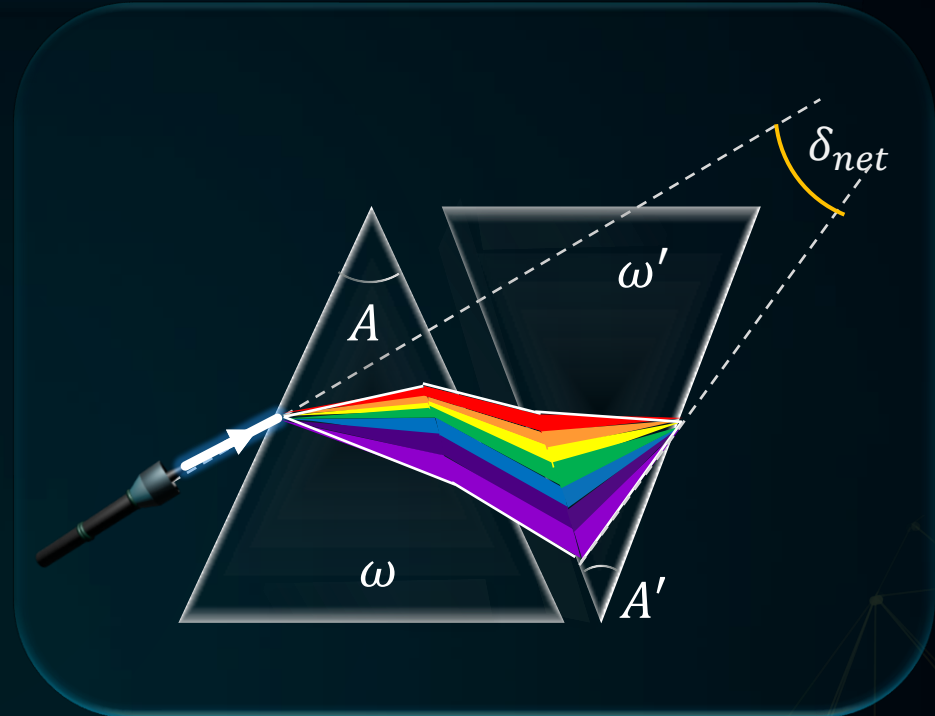
$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

Dispersive power of:

$$\text{prism 1: } \omega = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)}$$

$$\text{prism 2: } \omega' = \frac{(\mu'_V - \mu'_R)}{(\mu'_Y - 1)}$$

$$A\omega(\mu_Y - 1) = A'\omega'(\mu'_Y - 1) \rightarrow \frac{A(\mu_Y - 1)}{A'(\mu'_Y - 1)} = \frac{\omega'}{\omega}$$



$$\delta = (\mu_Y - 1)A - (\mu'_Y - 1)A'$$

$$\delta = (\mu_Y - 1)A \left(1 - \frac{\omega'}{\omega} \right)$$



Summary



- **Rainbow:**

It is the combined effect of dispersion, reflection and refraction of sunlight by spherical water droplets of rain.

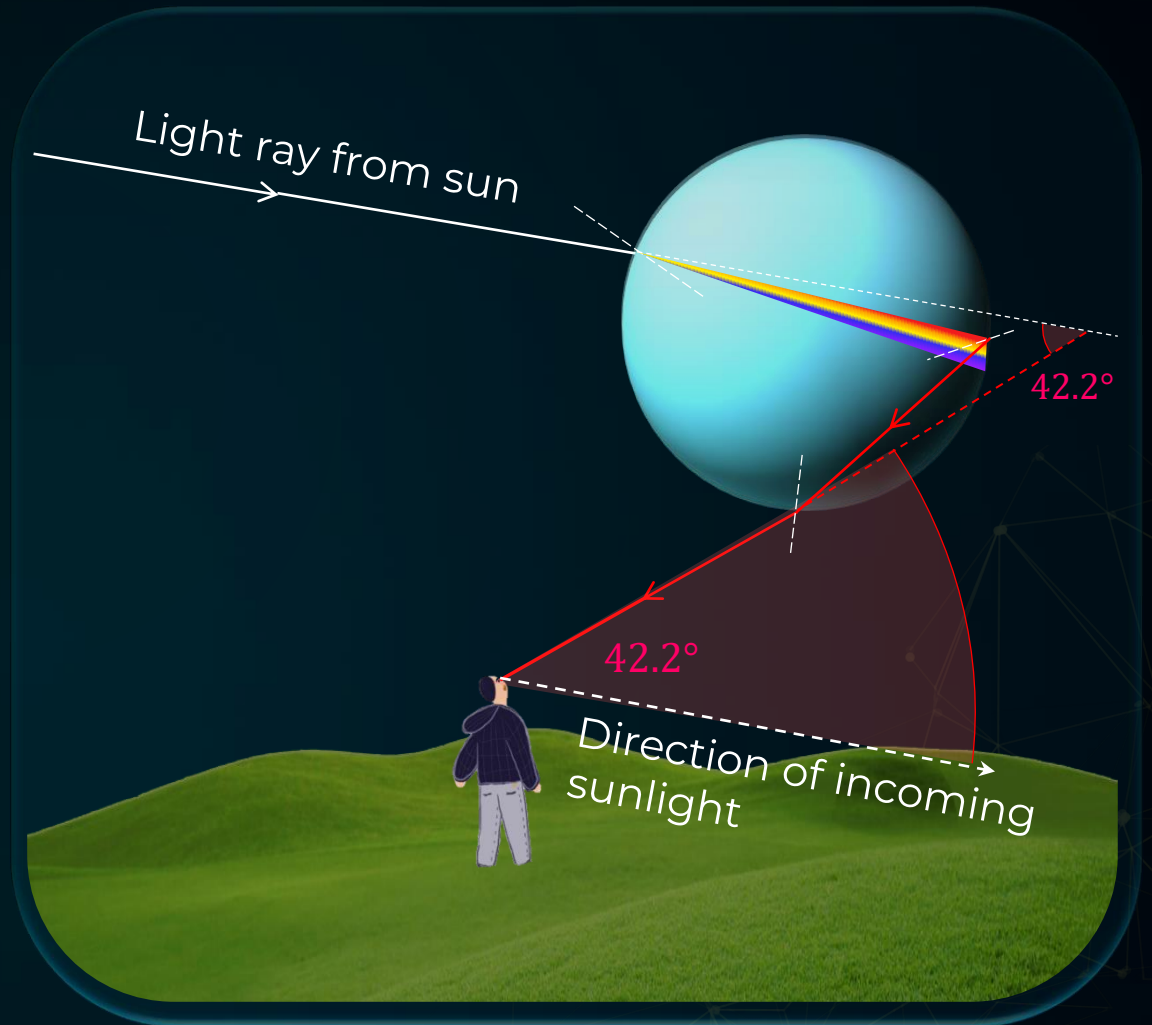
An observer can see the rainbow only when his back is towards the sun.

- **Primary Rainbow:**

It is a three step process:

Refraction with dispersion → Reflection → Refraction.

- Red light with max intensity emerges at an angle 42.2° with respect to the direction of incoming sunlight.
- Violet light with max intensity emerges at an angle 40.6° with respect to the direction of incoming sunlight.
- The rays except red and violet falls in between 42.2° and 40.6° .
- The red light will be at top and the violet light will be at bottom.

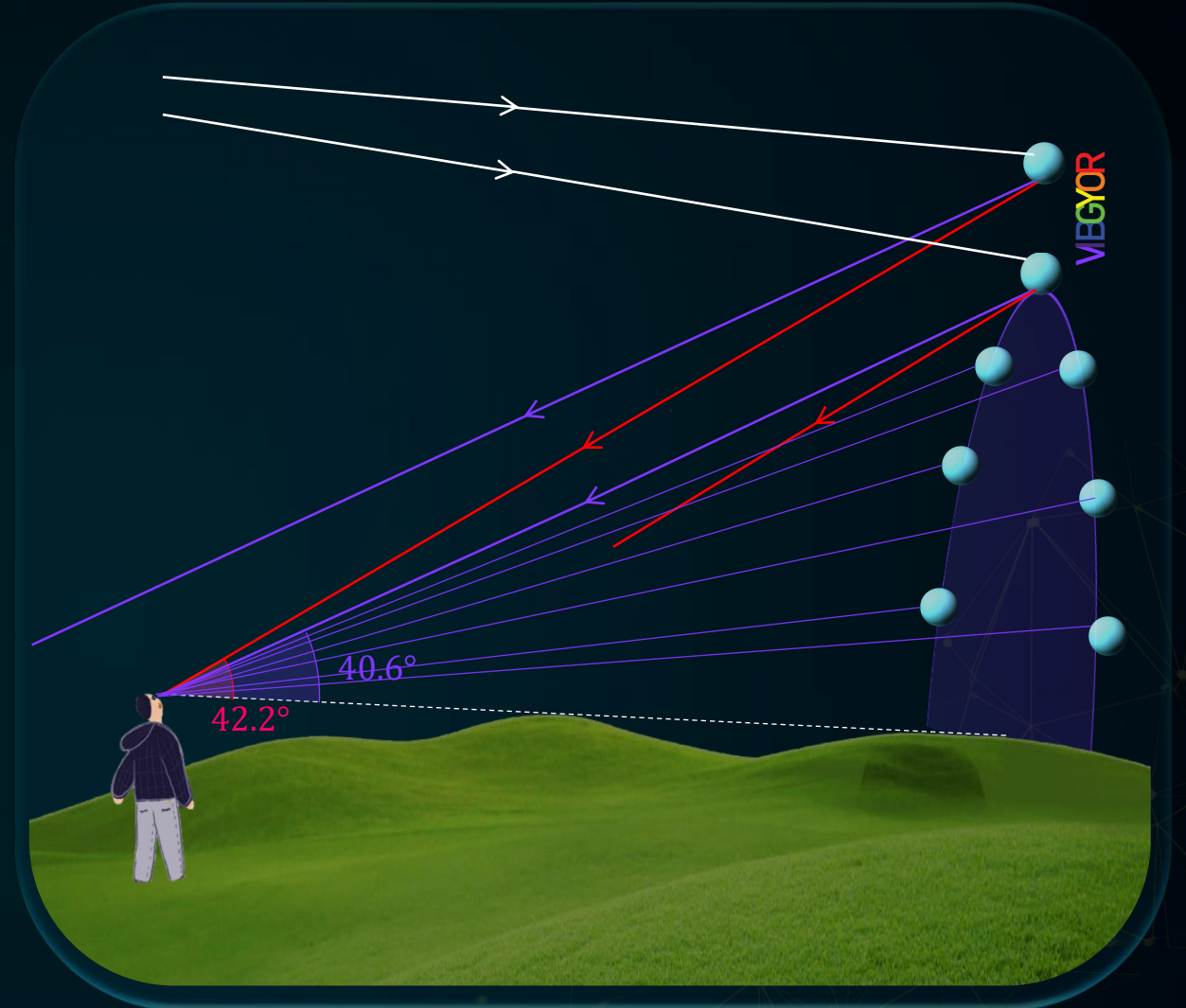




Why Rainbows Are Round?



- During or after rain, there are infinite number of water droplets.
- The observer can see violet light with maximum intensity in every direction if his/her eye make an angle 40.6° with the direction of incoming sunlight.
- The same is true for red light for an angle 42.2° .
- A circle or circular arc can subtend equal angle at some external point for each point on the circumference of the circle. Thus, the rainbow have the circular shape.



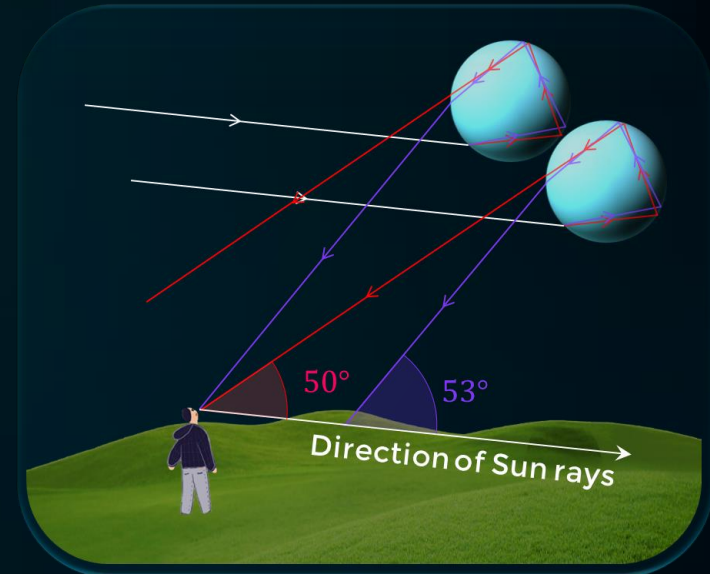
Primary Rainbow

- Red light with max. intensity emerges at 42.2° .
- Violet light with max. intensity emerges at 40.6° .
- For angle above 42.2° , we can see the normal colour sky.
- For angle less than 40.6° , a mixture of all colour (VIBGYOR) is present and hence, it appears as white to the observer. Thus, the core of rainbow is white.



Secondary Rainbow

- Secondary Rainbow:
It is a four-step process:
Refraction with dispersion → Reflection → Reflection → Refraction.
- Red light with max intensity emerges at an angle 50° and violet light with max intensity emerges at an angle 53° with respect to the direction of incoming sunlight.
- In secondary rainbow, violet will be at top and red will be at bottom.



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A beam of light consisting of red, green and blue colours is incident on a **right-angled** prism. The refractive index of the material of the prism for the red, green and blue colours are **1.39**, **1.44** and **1.47** respectively. The prism will:

Given: $\mu_{red} = 1.39, \mu_{green} = 1.44, \mu_{blue} = 1.47$

Solution: $\mu_{prism} = \frac{1}{\sin i_c} \Rightarrow \sin i_c = \frac{1}{\mu_{prism}}$

For total internal reflection

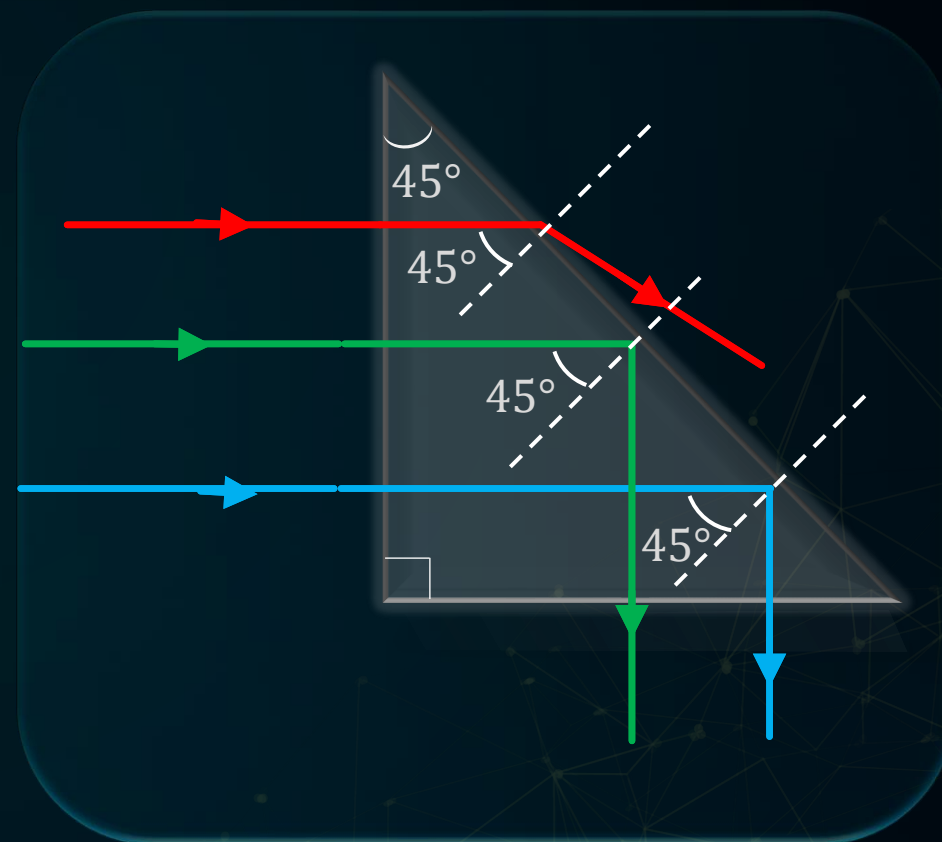
Angle of incidence > critical angle

$$i > i_c \Rightarrow \sin i > \sin i_c$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} > \frac{1}{\mu_{prism}} \quad (i = 45^\circ)$$

$$\mu_{prism} > \sqrt{2} \Rightarrow \mu_{prism} > 1.414$$

Since RI of green and blue beam is greater than 1.414, these colours will suffer **total internal reflection**. Only red colour beam will refract.



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A thin prism having refracting angle 10° is made of glass of refractive index 1.42 . This prism is combined with another thin prism of glass of refractive index 1.7 . This combination produces **dispersion without deviation**. The refracting angle of second prism should be:

Given: $A_1 = 10^\circ, \mu_1 = 1.42, \mu_2 = 1.7$

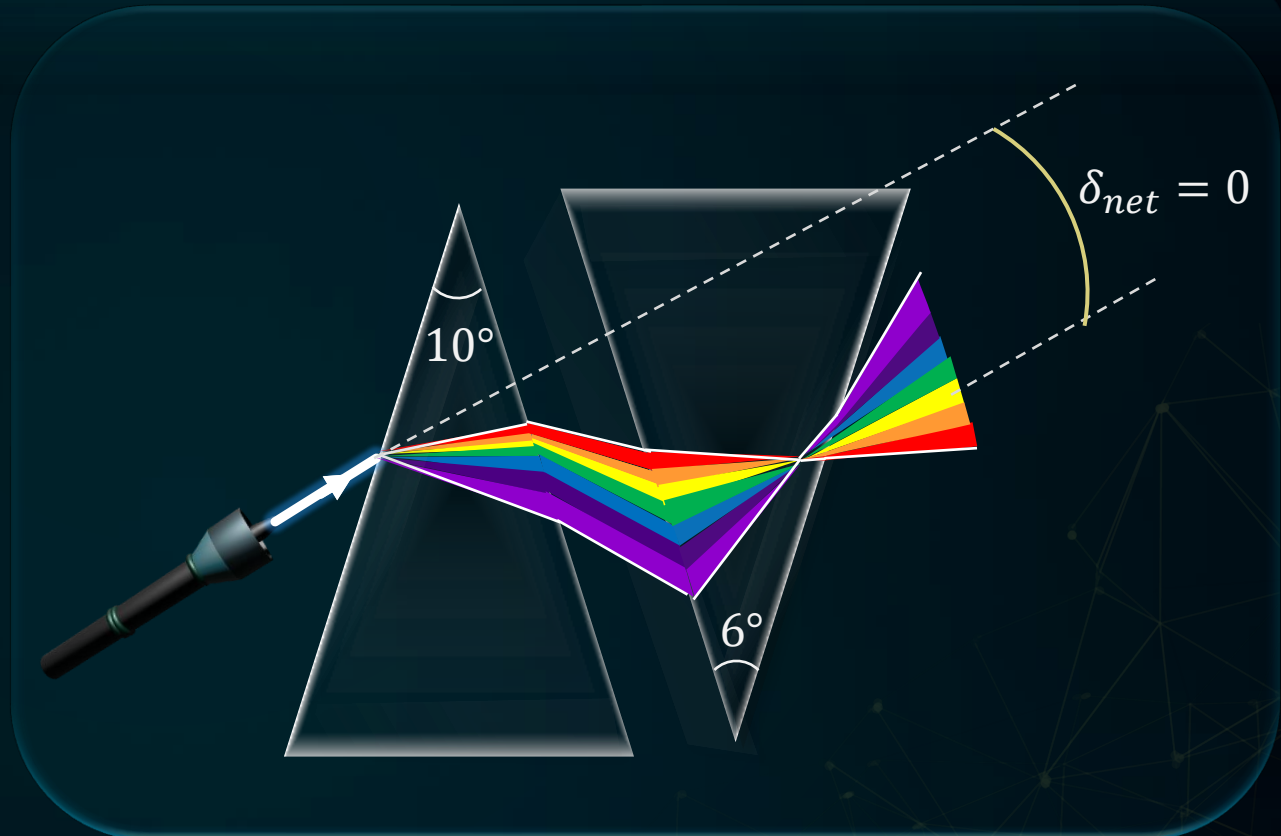
To find: A_2

Solution: From the condition of dispersion without deviation:

$$A_2 = \frac{(\mu_1 - 1)}{(\mu_2 - 1)} A_1$$

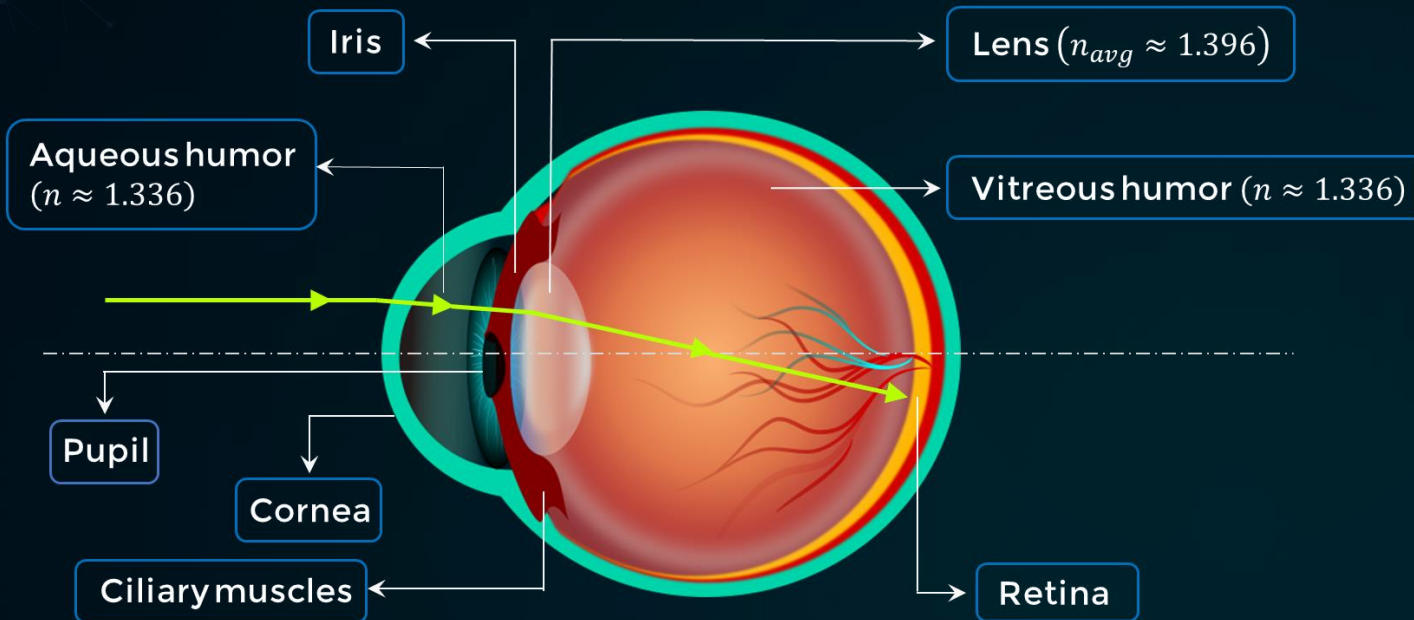
$$A_2 = \frac{(1.42 - 1)}{(1.7 - 1)} 10^\circ$$

$$A_2 = 6^\circ$$





Human Eye



Near Point:

The **minimum** distance at which an eye can see objects distinctly and clearly without getting tired. For a normal human eye near point is **25 cm**.

Far Point:

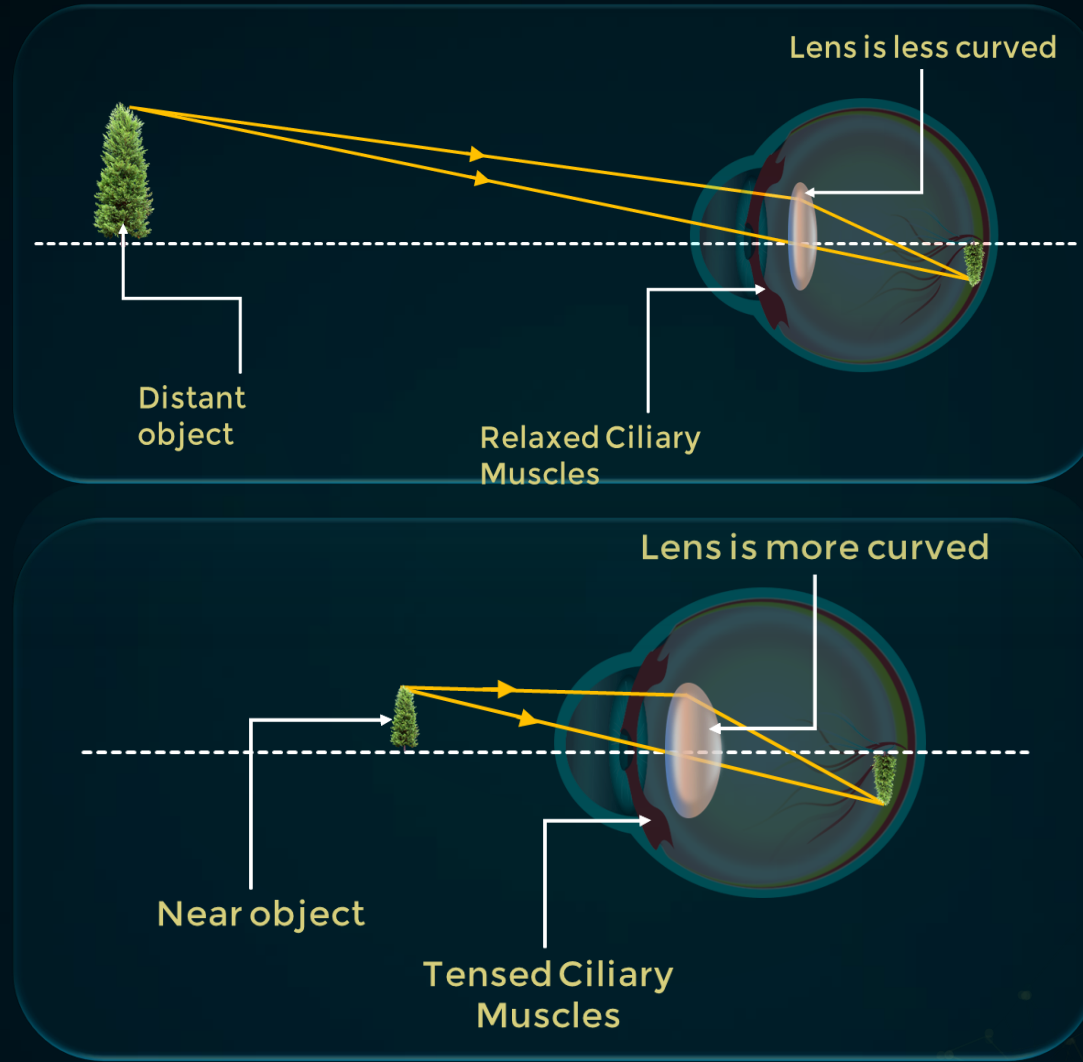
The **maximum** distance at which an eye can see objects distinctly and clearly without getting tired. For a normal human eye far point is **infinity**.



Accommodation

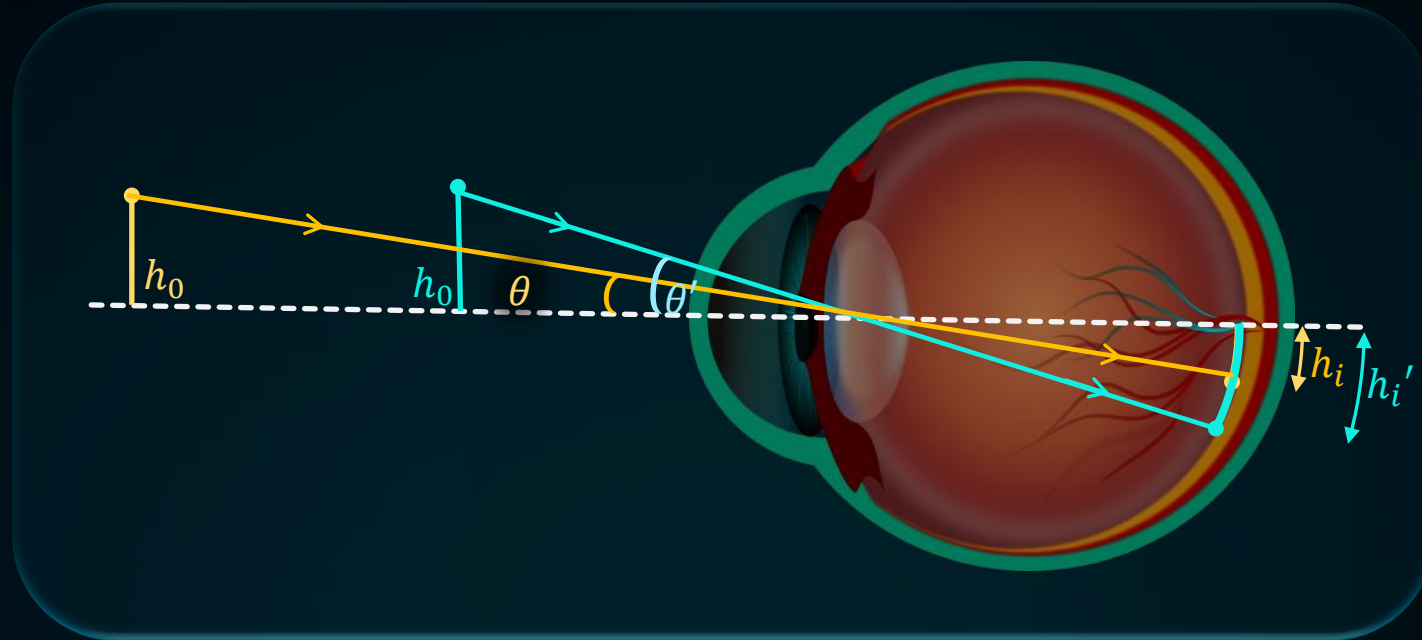


The process of adjusting the focal length of the eye lens by the ciliary muscles, in order to produce a sharp image on the retina, is called **accommodation**.





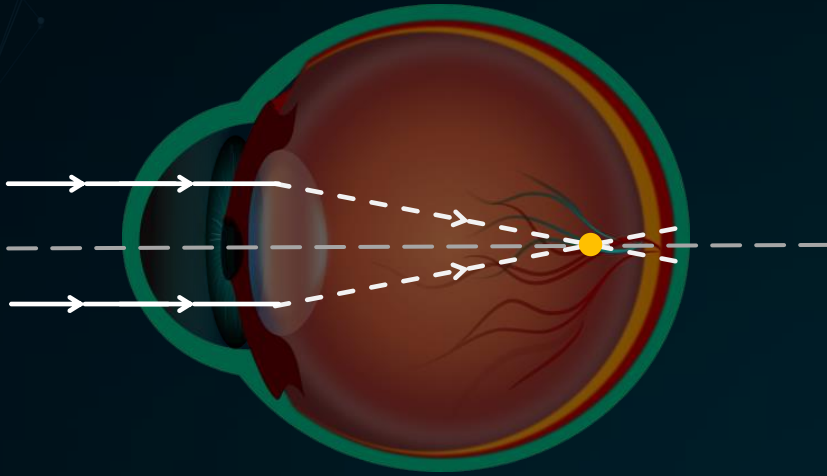
Apparent size of an Object



- The **angle** subtended by an object on the eye lens is called **visual angle** (θ).
- Apparent size of an object is **directly** proportional to **visual angle**.
- For an object Visual angle is **maximum** at **near point**. Therefore, apparent size of object is maximum at near point.

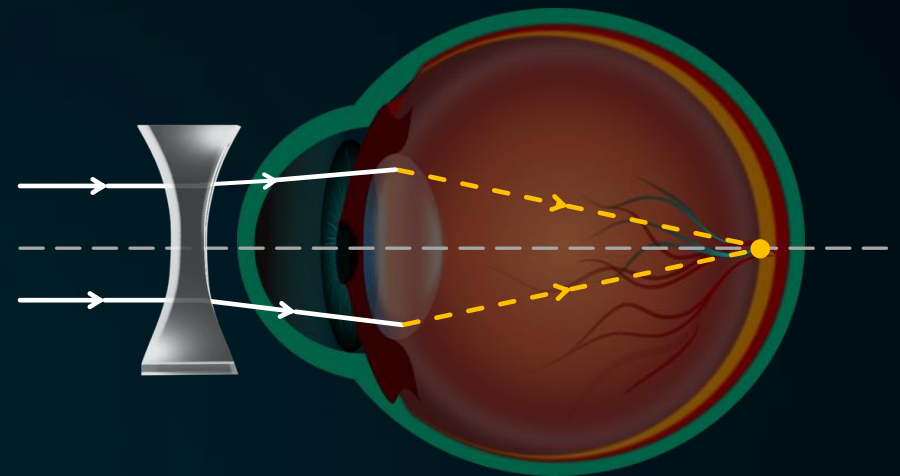


Myopia (or) Near Sightedness



Myopic eye

- Far-off objects appear blur and near-by objects are clearly visible.
- Image is formed in front of Retina.
- Caused due to thickening of eye lens.



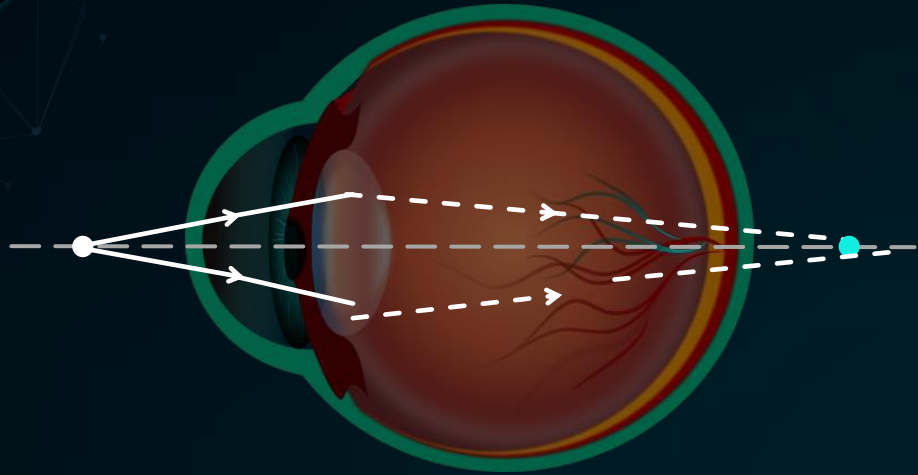
Corrected eye

- Myopia is corrected using concave lens of appropriate focal length.
- Power of lens: $P = \frac{1}{f} = -\frac{1}{x}$

Where x is the maximum distance that can be seen by a myopic eye.

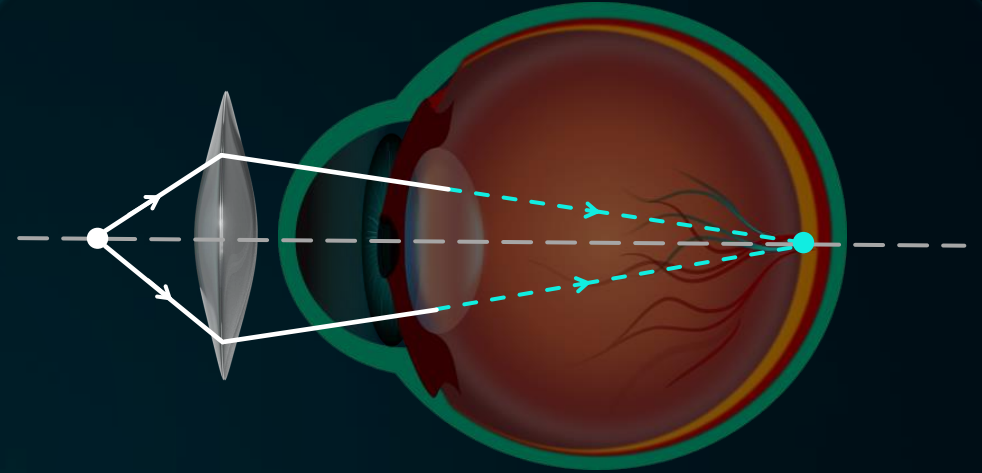


Hypermetropia (or) Far Sightedness



Hypermetropic eye

- Near-by objects appear blur and far-off objects are clearly visible.
- Image is formed behind of Retina.
- Caused due to thinning of eye lens.



Corrected eye

- Hypermetropia is corrected using convex lens of appropriate focal length.

- Power of lens: $P = \frac{1}{f} = \frac{y - D}{yD}$

Where y is the distance of near point of defective eye.

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A man suffering from myopia can read a book placed at 10 cm distance. For reading the book at a distance of 60 cm with relaxed vision, focal length of the lens required will be?

Given: $u = -60\text{ cm}, v = -10\text{ cm}$

To find: Focal length of lens

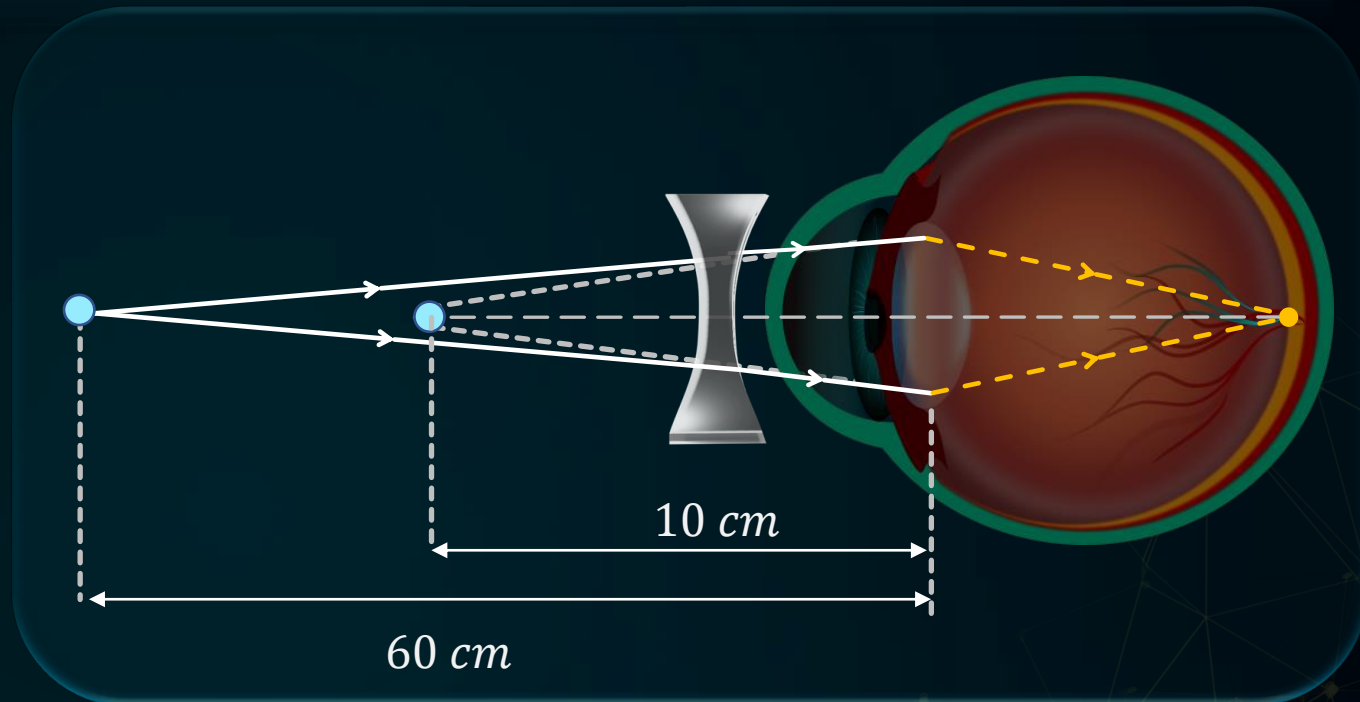
Solution: Lens formula is given by:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-10} - \frac{1}{-60}$$

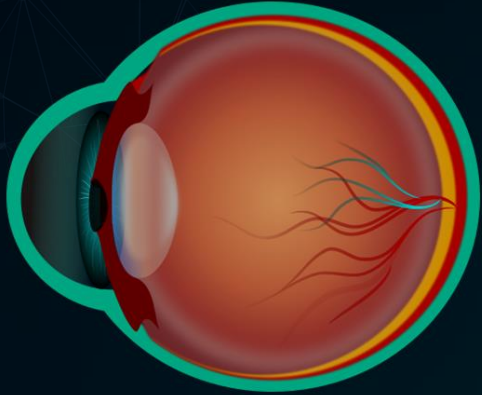
$$\frac{1}{f} = \frac{-6 + 1}{60} = \frac{-5}{60}$$

$\therefore f = -12\text{ cm}$ {negative sign indicates concave lens}

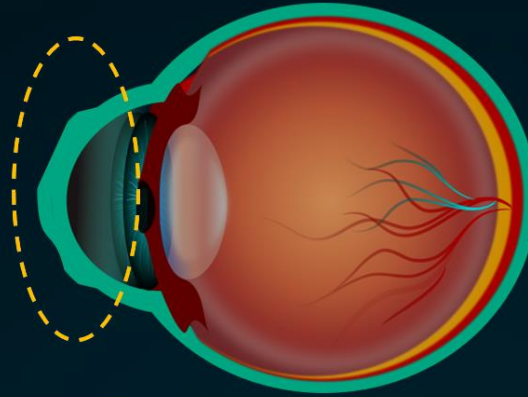




Astigmatism



Normal Eye



Eye with
Astigmatism

- The eye develops different curvatures in different planes.
- Eye has inability to see in all the directions equally well.

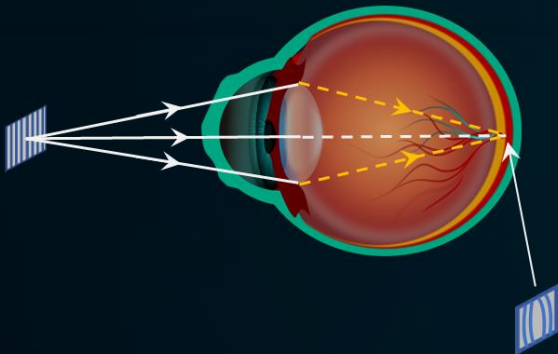


Image as formed on the retina

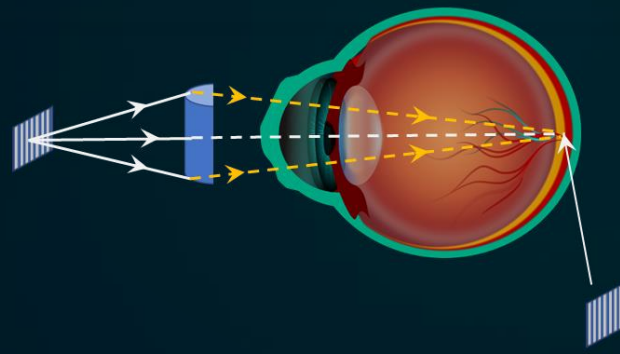


Image as formed on the retina

- Astigmatism is corrected using cylindrical lenses.

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For a normal eye, the cornea of eye provides a converging power of 40 D and the least converging power of the eye lens behind the cornea is 20 D . Using this information, the distance between the **retina** and the **eye lens** of the eye can be estimated to be:

Given: $P_{lens} = 20\text{ D} ; P_{cornea} = 40\text{ D}$

To find: f_{eff}

Solution: $P_{eff} = P_{lens} + P_{cornea}$

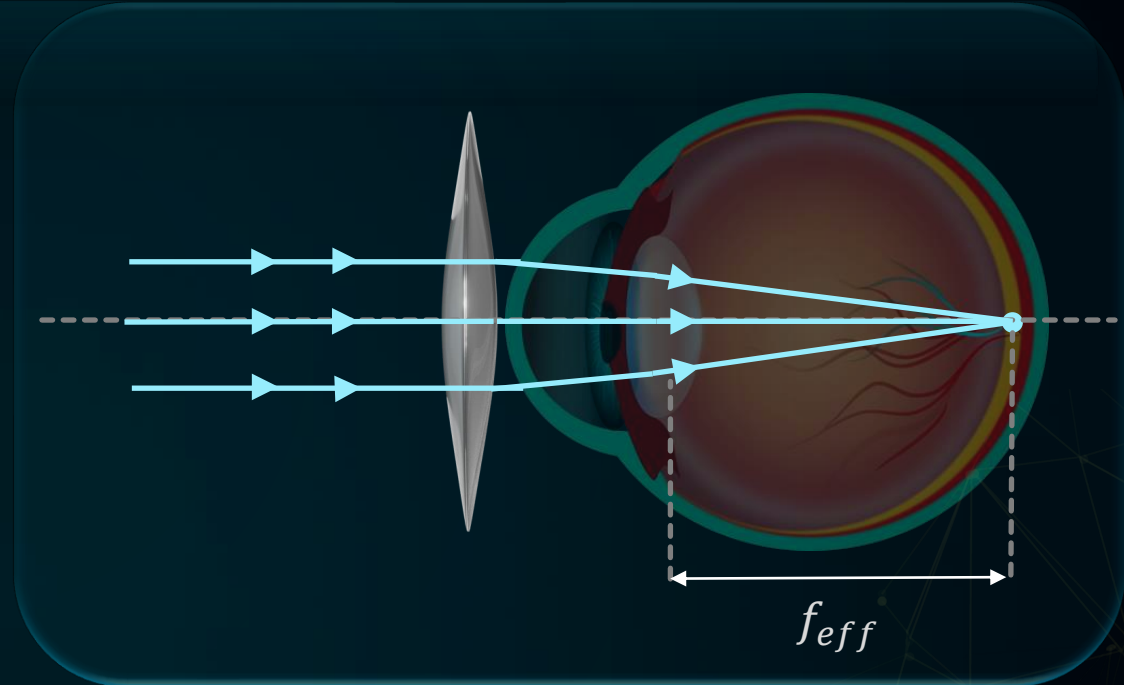
$$P_{eff} = 20 + 40 = 60\text{ D}$$

Power is given by:

$$P = \frac{100}{f(\text{in cm})}$$

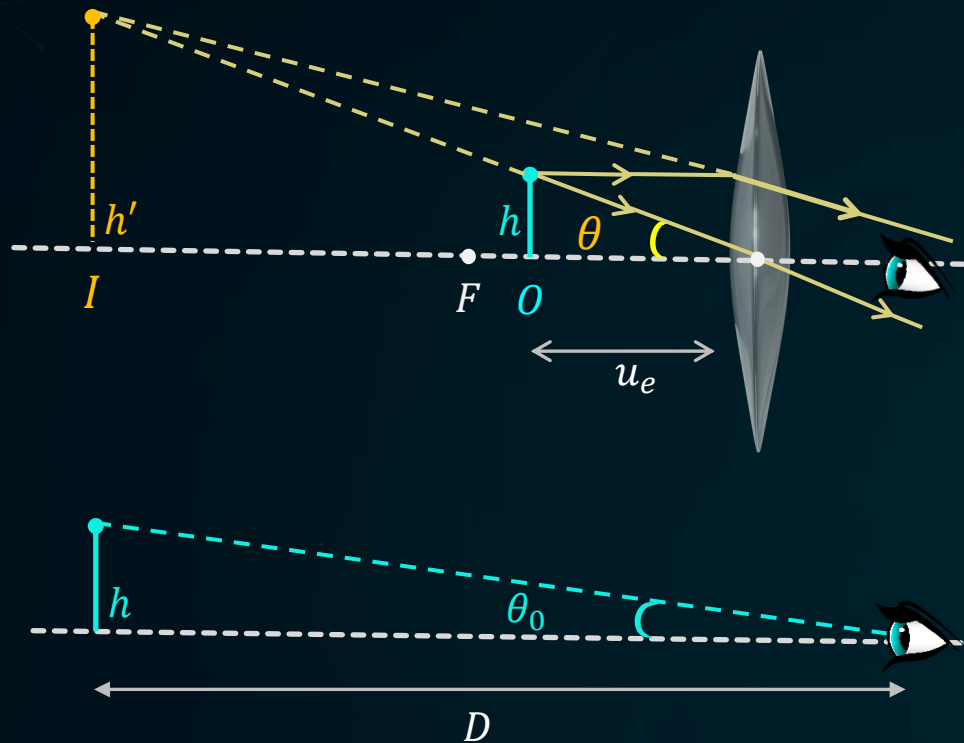
$$60 = \frac{100}{f_{eff}}$$

$$f_{eff} = \frac{100}{60} = 1.67\text{ cm}$$





Simple Microscope



- A simple microscope is a **bi-convex** lens.
- Object is placed between **focus** and **optical center** to get a **virtual magnified** image.
- For a simple microscope $f < D$.

$$m = \frac{\theta}{\theta_o}$$

$\theta = \frac{h}{u_e}$

$\theta_o = \frac{h}{D}$

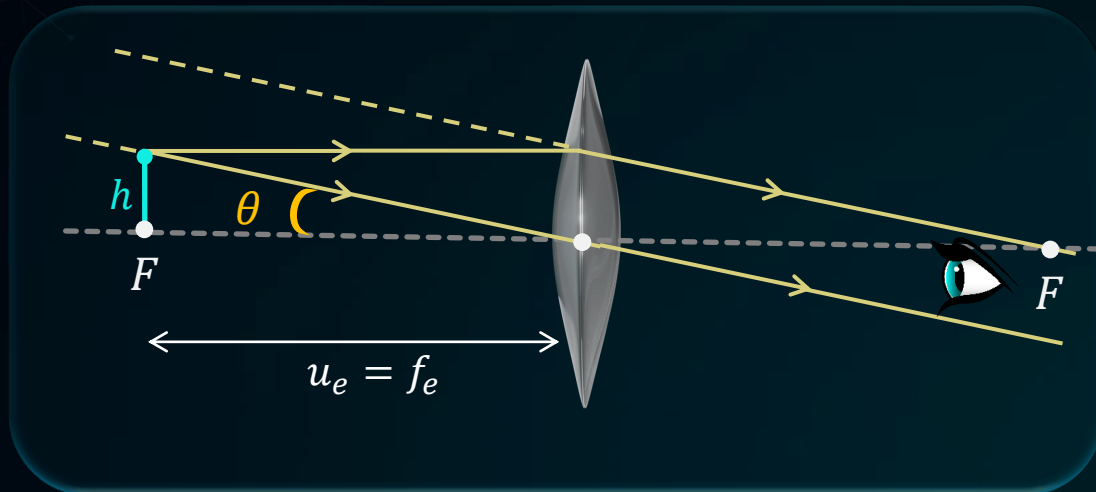
$m = \frac{\theta}{\theta_o} = \frac{D}{u_e}$



Special Cases of Angular Magnification



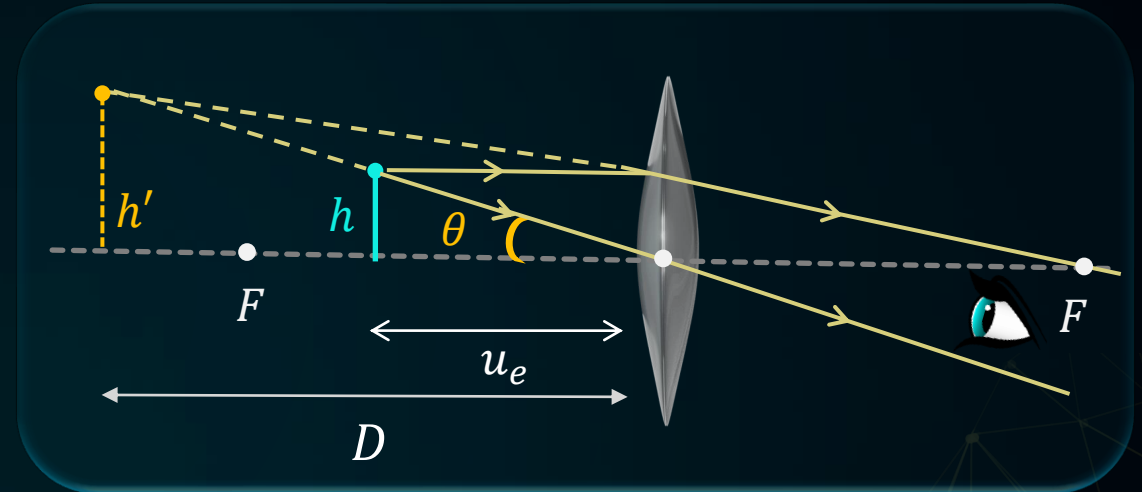
Image formed at Far point



$$m = \frac{D}{u_e} \quad (u_e = f_e)$$

$$m = \frac{D}{f_e}$$

Image formed at LDDV



$$m = \frac{D}{u_e} \quad \left(u_e = \frac{f_e D}{f_e + D} \right)$$

$$m = 1 + \frac{D}{f_e}$$

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A person uses a lens of power $P = +3D$ to normalize vision. What is the near point of hypermetropic eye?

Given: $u = \text{near point of normal eye} = -25 \text{ cm}$

To Find: Near point of eye

Solution: $f = \frac{1}{P} = \frac{1}{3} \text{ m} = \frac{100}{3} \text{ cm}$

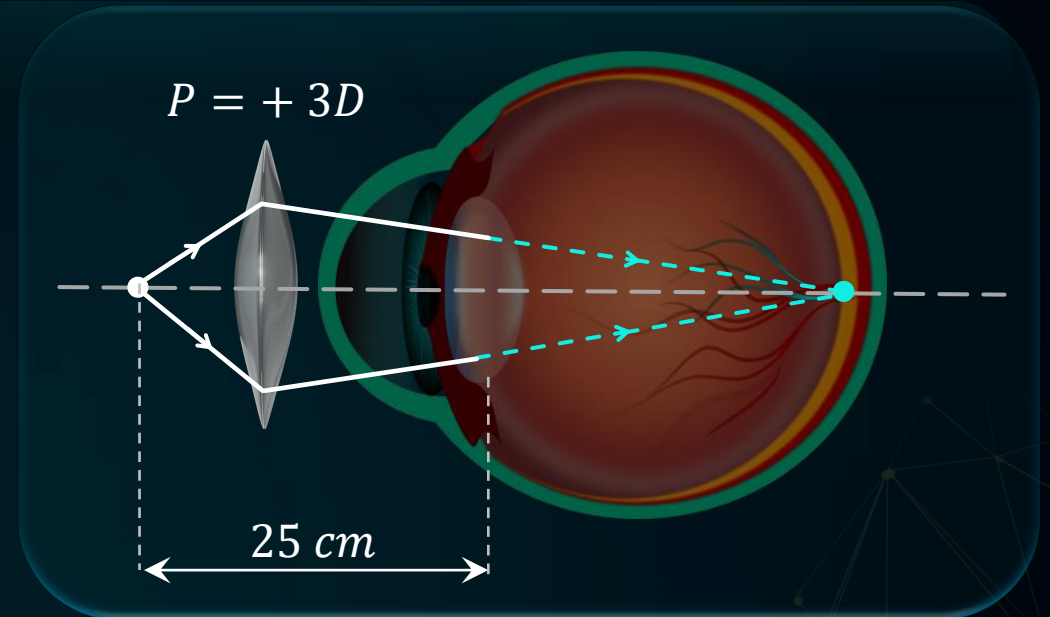
Apply lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{3}{100} = \frac{1}{v} - \frac{1}{-25}$$

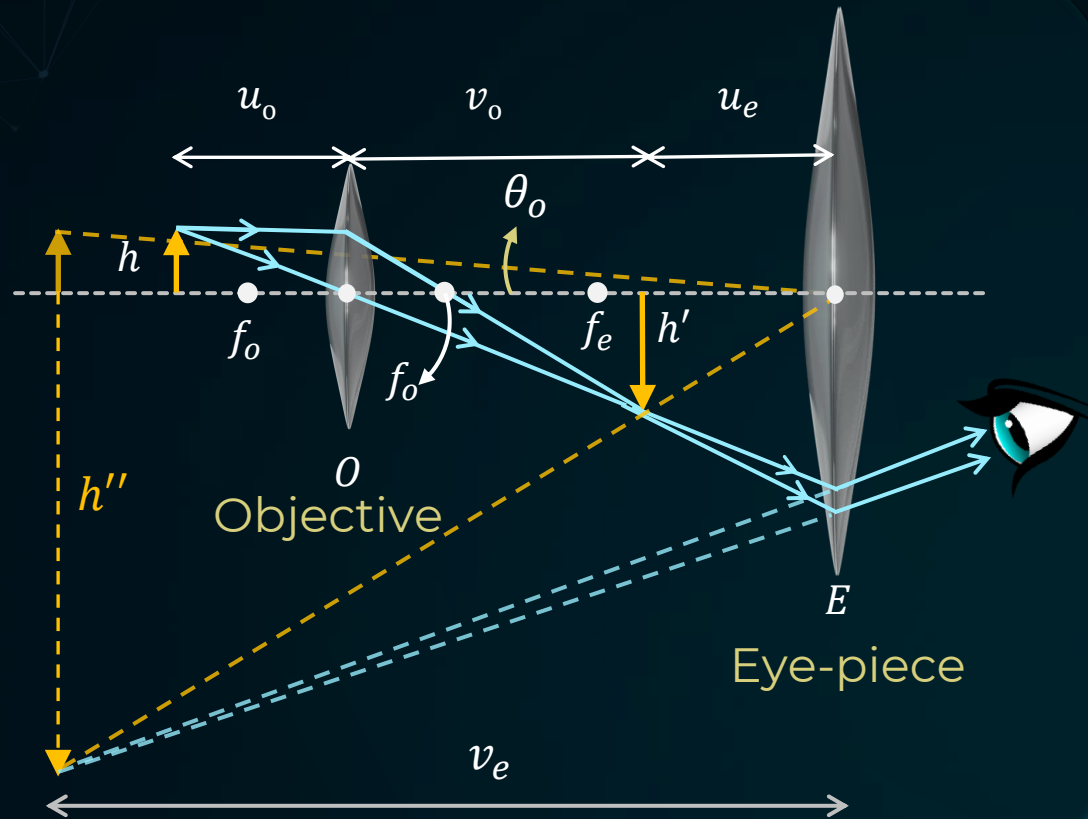
$$\frac{1}{v} = \frac{3}{100} - \frac{1}{25}$$

$$v = -100 \text{ cm}$$





Compound Microscope



- Objective: Closer to Object
- Eye-piece: Closer to Eye

- The image formed by the objective acts as the **object** for eyepiece.
- The length of compound microscope:

$$L = v_o + u_e$$

- Angular Magnification:

$$m = \frac{\theta}{\theta_o}$$
$$\theta = \frac{h'}{u_e}$$
$$\theta_o = \frac{h}{D}$$

$$m = \left(\frac{h'}{h} \right) \left(\frac{D}{u_e} \right)$$



Normal Adjustment



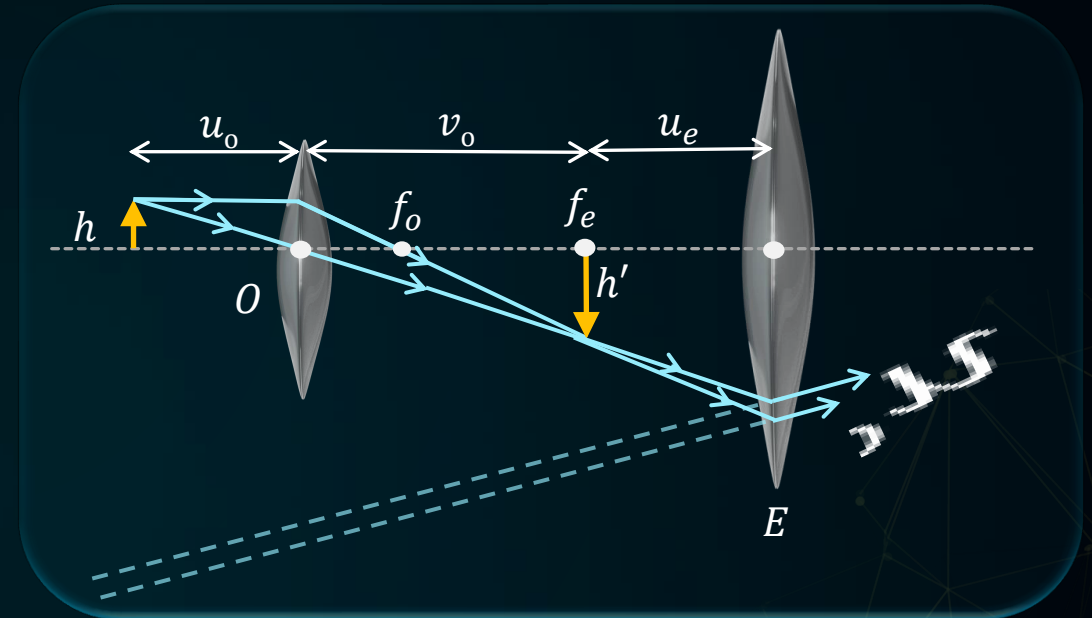
- The final image is formed at infinity.
- So, the image formed by objective lens should be at the focus of eyepiece.

$$u_e = f_e$$

- Magnifying power:

$$m = \left(\frac{v_o}{u_o} \right) \left(\frac{D}{u_e} \right)$$

$$m = \left(\frac{v_o}{u_o} \right) \left(\frac{D}{f_e} \right)$$





LDDV Adjustment



- The final image is formed at near point / LDDV.
- For eyepiece: $u = -u_e$ $v = -D$

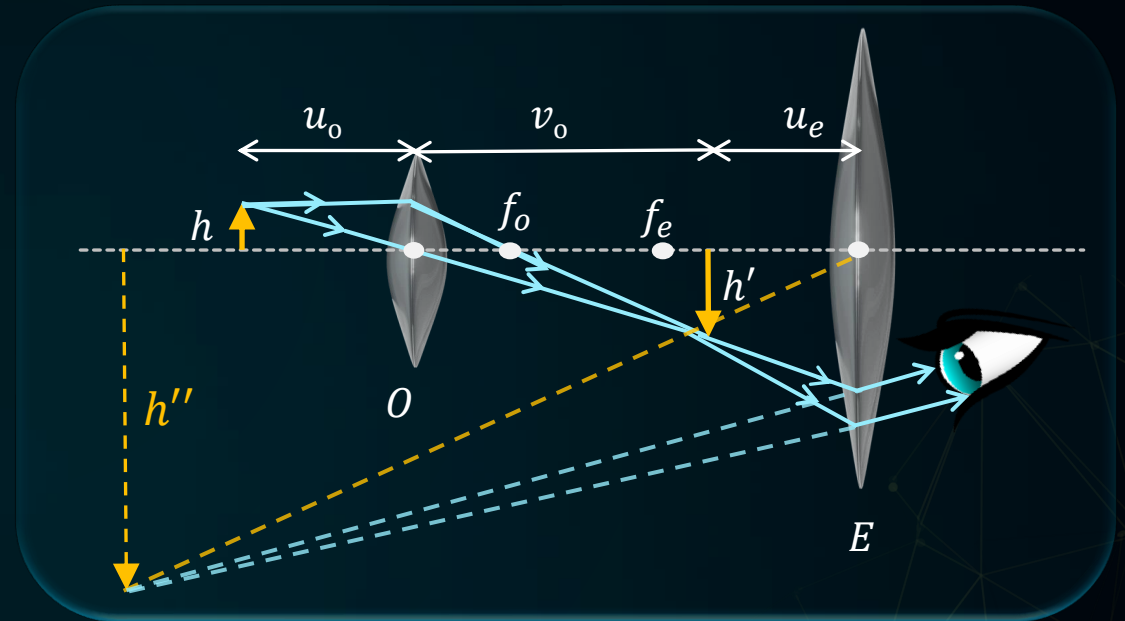
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{(-D)} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

$$\Rightarrow u_e = \frac{f_e D}{f_e + D}$$

$$\Rightarrow \frac{D}{u_e} = 1 + \frac{D}{f_e}$$

- Magnifying power:

$$m = \left(\frac{v_o}{u_o} \right) \left(\frac{D}{u_e} \right) \Rightarrow m = \left(\frac{v_o}{u_o} \right) \left(1 + \frac{D}{f_e} \right)$$





When f_o is very small



Apply **lens formula** to objective lens:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

Assuming $v_o \approx l$ and $\frac{v_o}{f_o} \gg 1$, we get:

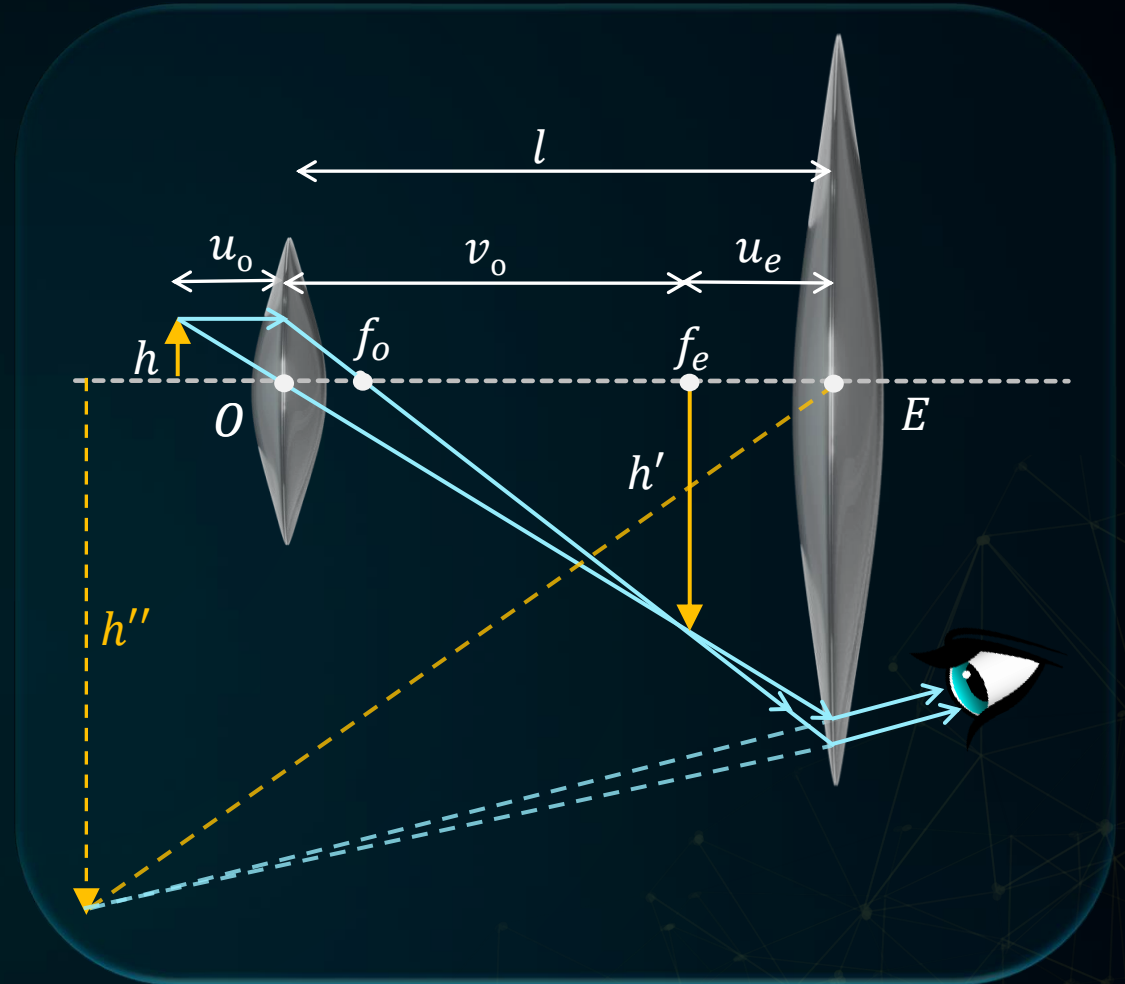
$$\frac{v_o}{u_o} = -\frac{l}{f_o}$$

For the **Normal** adjustment:

$$m = -\left(\frac{l}{f_o}\right)\left(\frac{D}{f_e}\right)$$

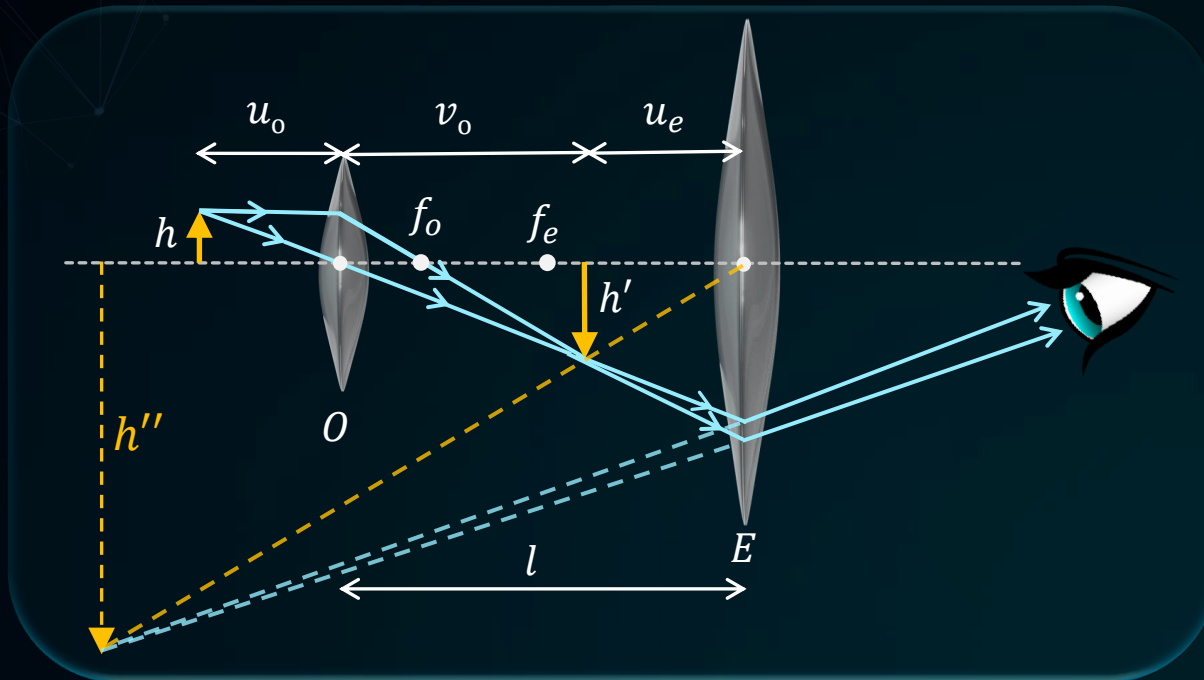
For **LDDV** adjustment:

$$m = -\left(\frac{l}{f_o}\right)\left(1 + \frac{D}{f_e}\right)$$





When f_o is very small



Apply **lens formula** to objective lens:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$1 - \frac{v_o}{u_o} = \frac{v_o}{f_o}$$

$$\frac{v_o}{u_o} = 1 - \frac{v_o}{f_o}$$

$$\frac{v_o}{u_o} = -\frac{v_o}{f_o}$$

$$(v_o \approx l) \quad \left(\frac{v_o}{f_o} \gg 1 \right)$$

$$\frac{v_o}{u_o} = -\frac{l}{f_o}$$

?

A compound microscope consists of an objective of focal length 1 cm and an eye-piece of focal length 5 cm . An object is placed at 0.5 cm from the objective. What should be the separation between the lenses so that the microscope projects an inverted real image of the object on a screen 30 cm behind the eye-piece?

To find: l

Solution: Apply lens formula to the eye-piece:

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{5} = \frac{1}{30} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = \frac{1}{30} - \frac{1}{5} \Rightarrow u_e = -6\text{ cm}$$

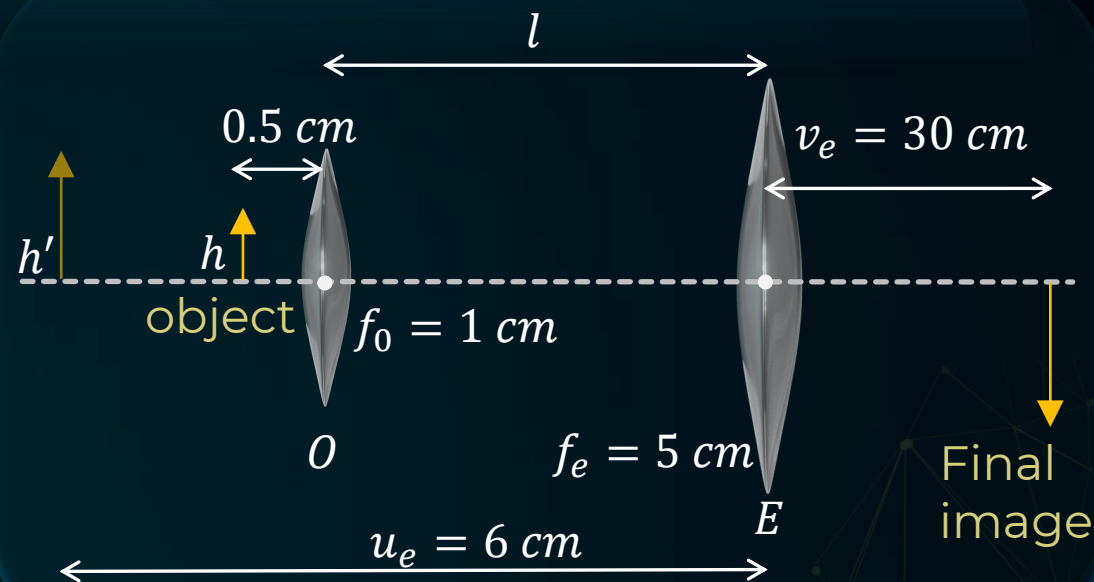
Apply lens formula to the objective:

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow v_o = -1\text{ cm}$$

The separation between lenses is:

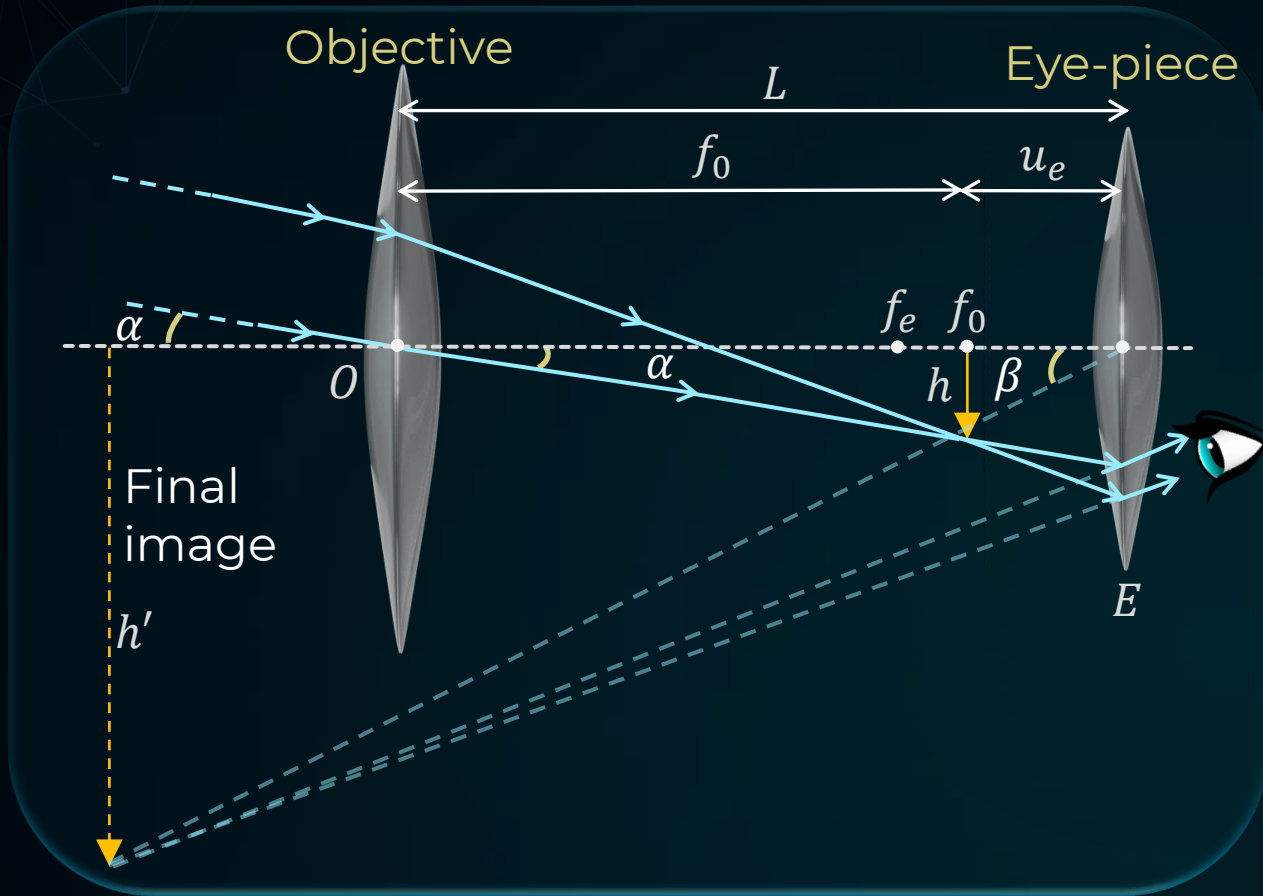
$$l = |u_e| - |v_o| = 6 - 1 = 5\text{ cm}$$

$$l = 5\text{ cm}$$





Astronomical Telescope-Magnification



Objective (larger focal length and aperture)

Eye-piece (smaller focal length and aperture)

Angle subtended by **object** at objective:

$$\alpha = \frac{h}{f_o}$$

Angle subtended by **final image** at eyepiece:

$$\beta = \frac{h}{u_e}$$

Magnification of telescope:

$$m = \frac{\beta}{\alpha} = -\frac{f_o}{u_e}$$

Length of telescope:

$$L = f_o + u_e$$



Normal Adjustment



- The final image is formed at infinity.
- So, the image formed by objective lens should be at the focus of eyepiece.

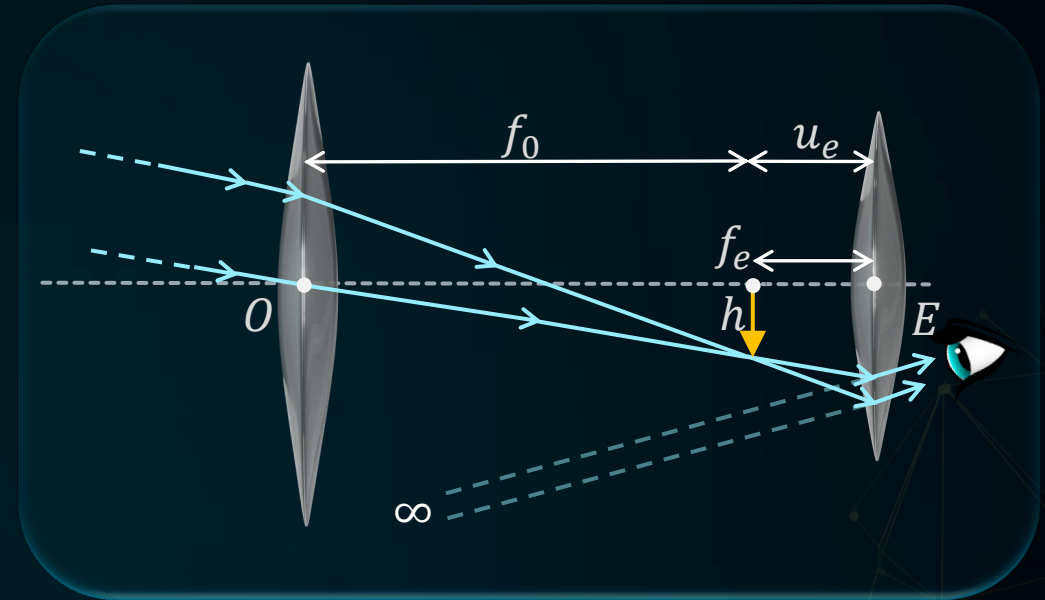
$$u_e = f_e$$

- Magnifying power:

$$m = -\left(\frac{f_o}{u_e}\right) \Rightarrow m = -\left(\frac{f_o}{f_e}\right)$$

- Length of the telescope:

$$L = f_o + u_e \Rightarrow L = f_o + f_e$$





LDDV adjustment



- The final image is formed at near point / LDDV.

- For eyepiece: $u = -u_e$ $v = -D$

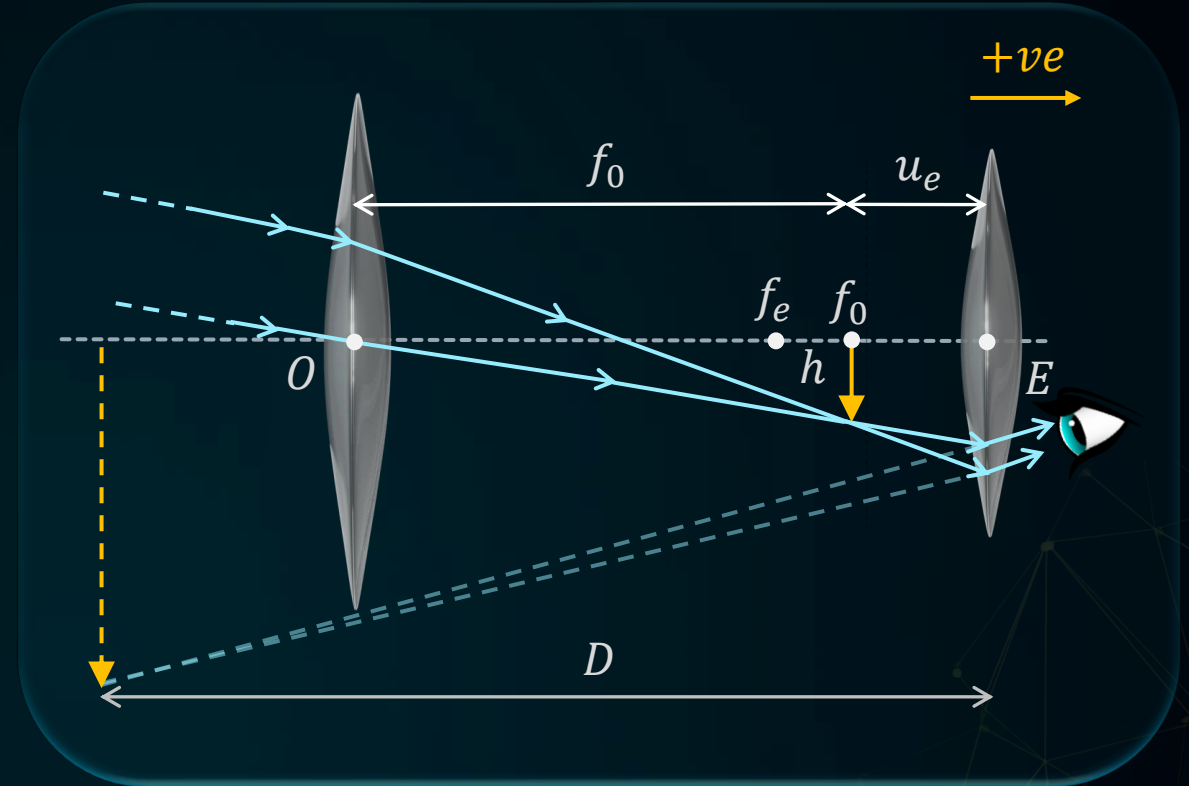
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{(-D)} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$
$$\Rightarrow u_e = \frac{f_e D}{f_e + D}$$

- Magnifying power:

$$m = -\left(\frac{f_o}{u_e}\right) \Rightarrow m = -\left(\frac{f_o}{f_e}\right)\left(1 + \frac{f_e}{D}\right)$$

- Length of the telescope: $L = f_o + u_e \Rightarrow$

$$L = f_o + \frac{f_e D}{f_e + D}$$





An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eye-piece.

Solution:

Magnification produced: $|m| = \frac{f_o}{f_e} = 50$

$$\Rightarrow f_o = 50f_e$$

The length of telescope is:

$$L = f_o + f_e = 102 \text{ cm}$$

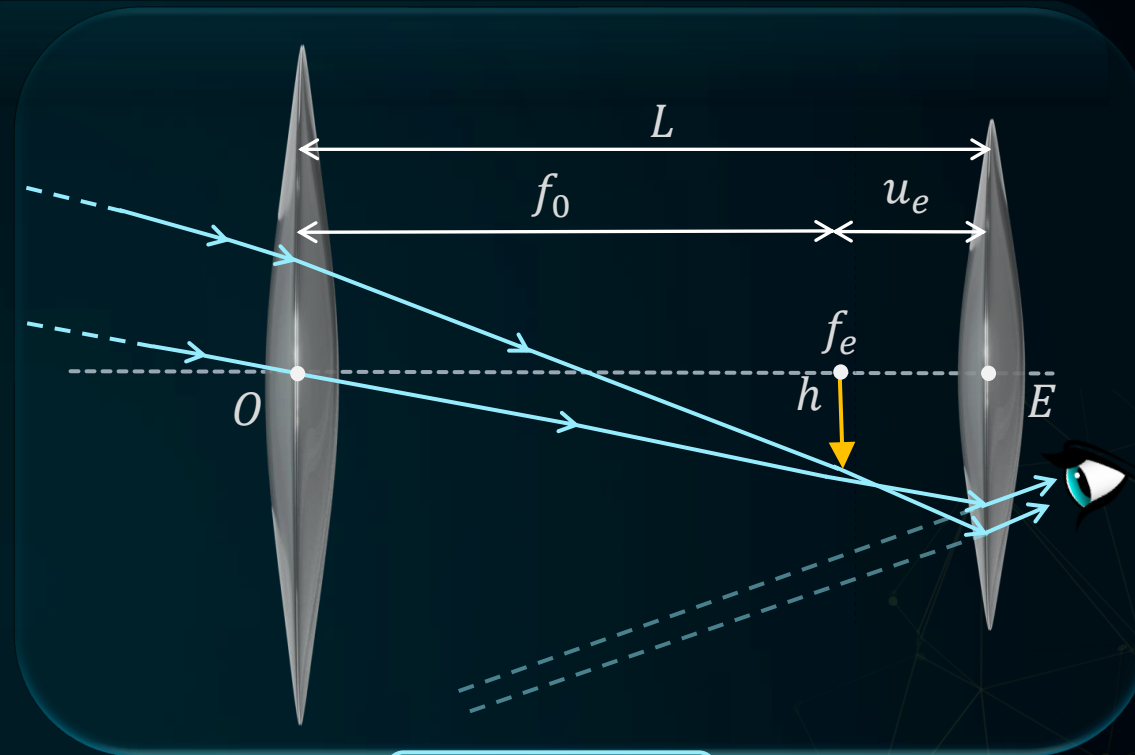
$$\Rightarrow 50f_e + f_e = 51f_e = 102 \text{ cm}$$

$$\Rightarrow f_e = 2 \text{ cm} = +0.02 \text{ m}$$

$$\Rightarrow f_o = +100 \text{ cm} = 1 \text{ m}$$

The power of eye-piece: $P_e = \frac{1}{f_e} = \frac{1}{0.02} = +50 \text{ D}$

The power of objective: $P_o = \frac{1}{f_o} = \frac{1}{1} = +1 \text{ D}$

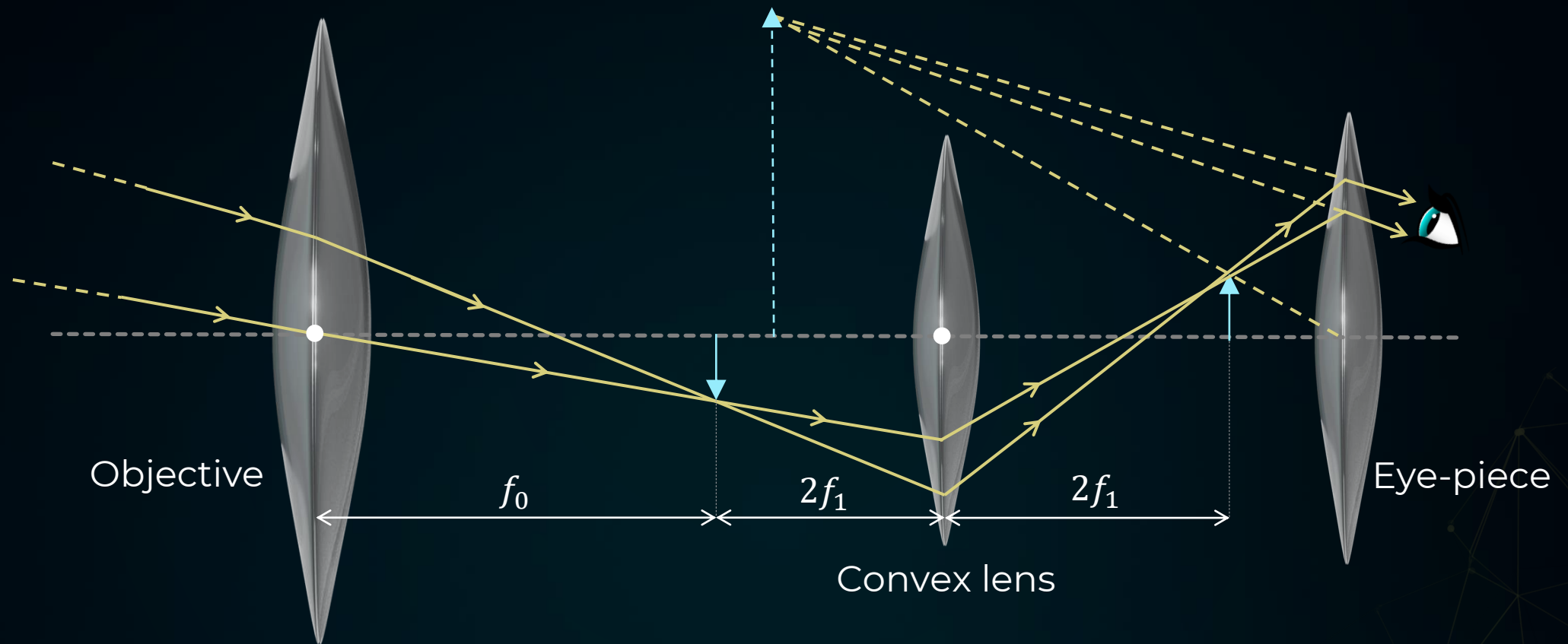


$$P_e = +50 \text{ D}$$

$$P_o = +1 \text{ D}$$



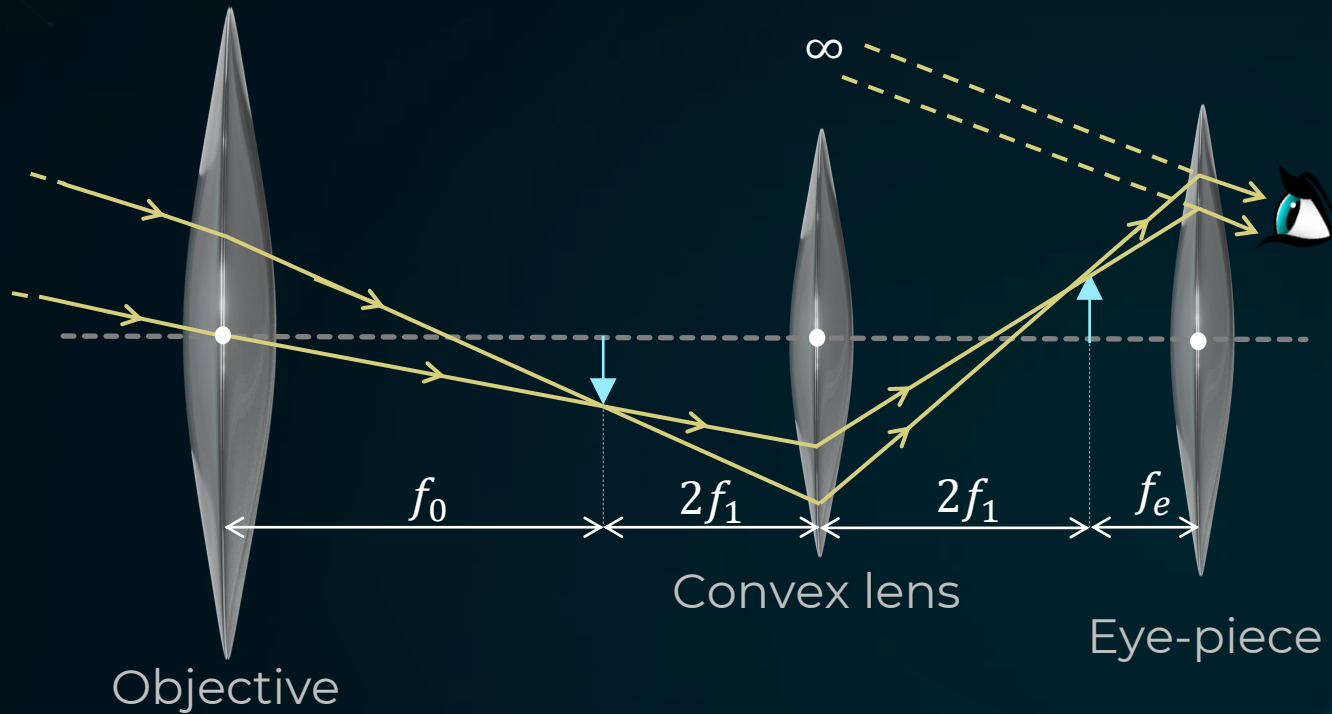
Terrestrial Telescope-Ray Diagram



- Astronomical Telescope is **not** suitable for viewing objects on earth.
- A **convex** lens is used in terrestrial telescope to produce erect image.



Terrestrial Telescope – Normal Adjustment



- Final image is formed at **infinity**.

- Magnifying power:

$$m = \frac{f_o}{f_e}$$

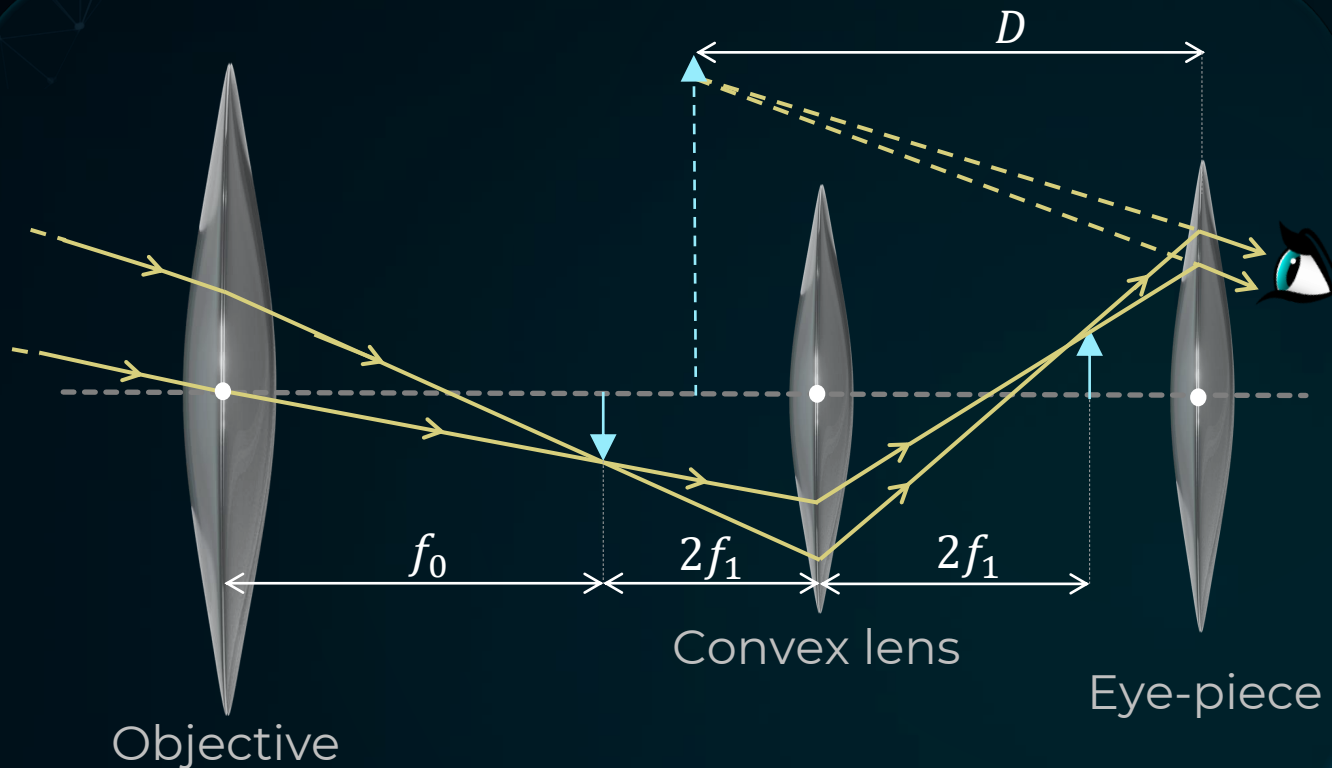
- Length of telescope:

$$L = f_o + 4f_1 + f_e$$

(f_1 is focal length of convex lens)



Terrestrial Telescope – LDDV Adjustment



- Final image is formed at LDDV.
- Magnifying power:

$$m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

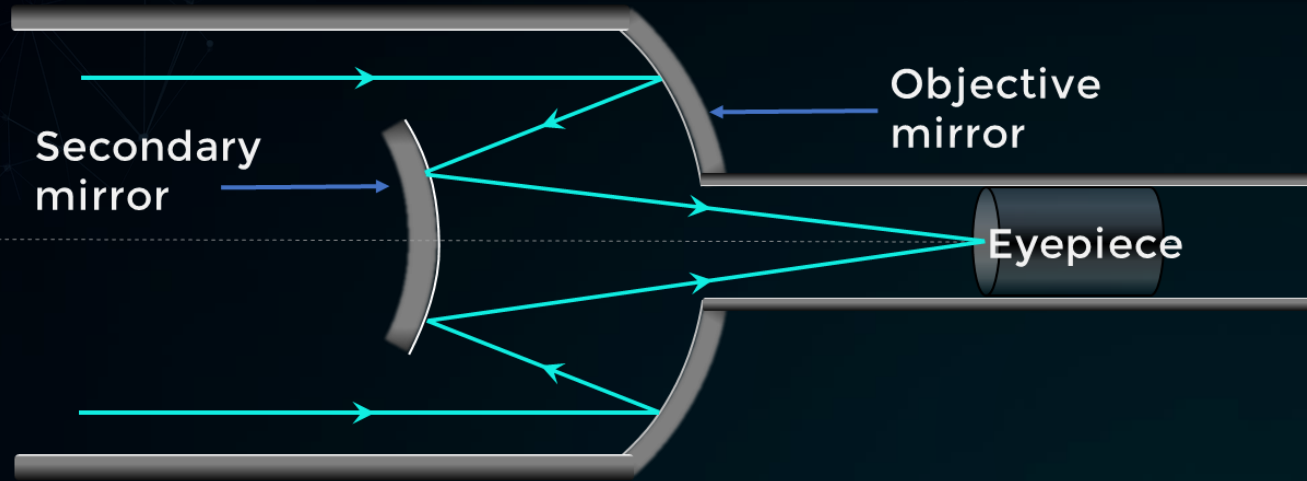
- Length of telescope:

$$L = f_o + 4f_1 + \left(\frac{f_e D}{f_e + D} \right)$$

(f_1 is focal length of convex lens)



Cassegrain Telescope



Cassegrain telescope is a **reflecting** telescope.

Advantages of Cassegrain Telescope

- There is **no** chromatic aberration in a mirrors.
- If a **parabolic** reflecting surface is chosen, **spherical aberration** is also removed.
- Mechanical support is much **less** of a problem.
- The **largest** reflecting telescope in the world are the pair of **reck telescopes** in Hawaii, USA, with a reflector of **10 m** in diameter.