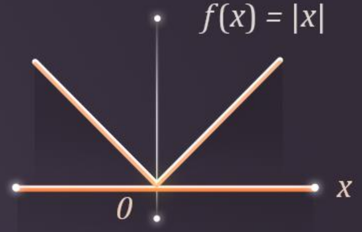
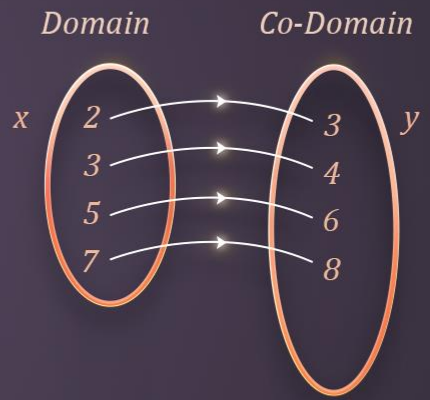
Welcome to

Aakash + BYJU'S LIVE

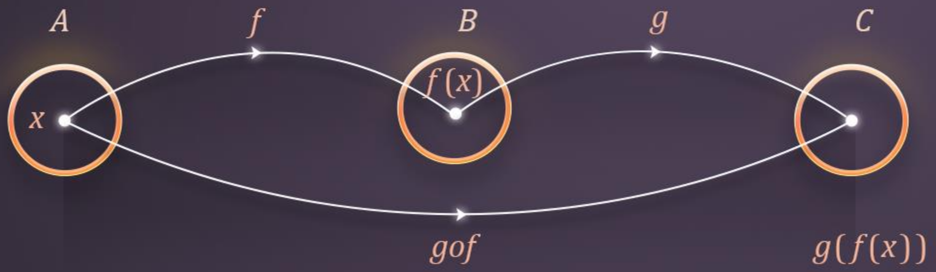
Relations & Functions II

$$y = ax^2 + bx + c$$





$A \rightarrow B$ co-domain



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Session 1

Introduction to Relations and Types of Relations

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Key Takeaways

Cartesian product of Sets:

Let A and B are two non-empty sets. The set of all ordered pairs (a, b) [where $a \in A$ and $b \in B$] is called Cartesian product of sets A and B.

- It is denoted by $A \times B$.
- If $n(A) = p,\ n(B) = q$, then the number of elements in cartesian product of sets is $n(A \times B) = p \times q$.

Example: $A = \{a, b, c\},\quad B = \{1, 2\}$

$$\Rightarrow A \times B = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$$

$$\Rightarrow n(A \times B) = 6 = n(A) \times n(B)$$

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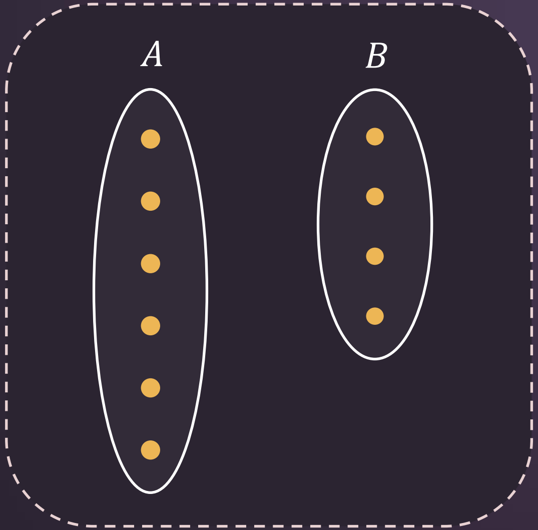
Key Takeaways

Relation:

Let A and B be two sets, then a relation R from A to B is a subset of $A \times B$.

- $R \subseteq A \times B$
- Number of relations = Number of subsets of $A \times B$
- If $n(A) = p, n(B) = q$, and $R: A \rightarrow B$, then number of relations = 2^{pq}



Example: $n(A) = 6,\ n(B) = 4$

$\Rightarrow n(A \times B) = n(A) \times n(B) = 6 \times 4 = 24$

Number of relations = Number of subsets of $A \times B$

$= 2^{24}$

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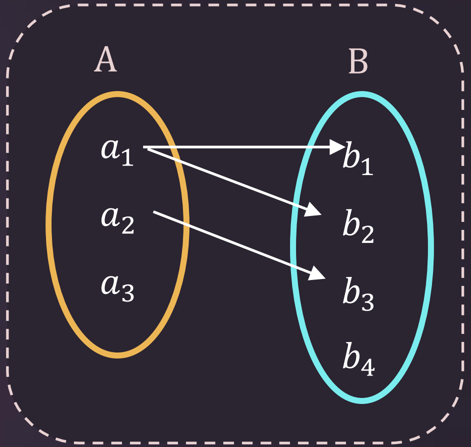
Domain and range of relation:

Let R be a relation defined from set A to set B.

Let $R = \{(a_1, b_1), (a_1, b_2), (a_2, b_3)\}$



- The set of all the first components of ordered pairs belonging to R is called domain of R.

  i.e., domain $\subseteq A$

- The set of all the second components of ordered pairs belonging to R is called range of R.

  i.e., Range $\subseteq B$

- Set B is called the co–domain of R.

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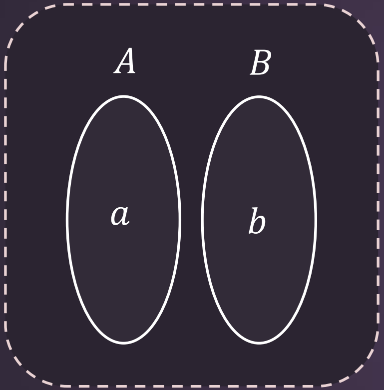
Inverse of a relation:

Let A and B are two sets and R be a relation from A to B, then the inverse of R is denoted by R^{-1} is a relation from B to A and is defined as:
$R^{-1} = \{(b, a), (a, b) \in R\}$



- Domain (R^{-1}) = Range of R
- Range (R^{-1}) = Domain of R

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If $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on set of integers $\mathbb{Z}$, then domain of R^{-1}.

A $\{-2, -1, 1, 2\}$

B $\{-1, 0, 1\}$

C $\{-2, -1, 0, 1, 2\}$

D $\{0, 1\}$

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If $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on set of integers $\mathbb{Z}$, then domain of R^{-1}.

Solution: $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + y^2 \leq 8\}$

Domain of R^{-1} = Range of R (values of y)

$x = 0, y^2 \leq 8/3 \quad \Rightarrow y \in \{-1, 0, 1\}$

$x = 1, y^2 \leq 7/3 \quad \Rightarrow y \in \{-1, 0, 1\}$

$x = 2, y^2 \leq 4/3 \quad \Rightarrow y \in \{-1, 0, 1\}$

$x = 3, y^2 \leq -1/3 \quad \Rightarrow y \in \phi$

$\therefore$ Domain of $R^{-1} = \{-1, 0, 1\}$

A $\{-2, -1, 1, 2\}$

B $\{-1, 0, 1\}$

C $\{-2, -1, 0, 1, 2\}$

D $\{0, 1\}$

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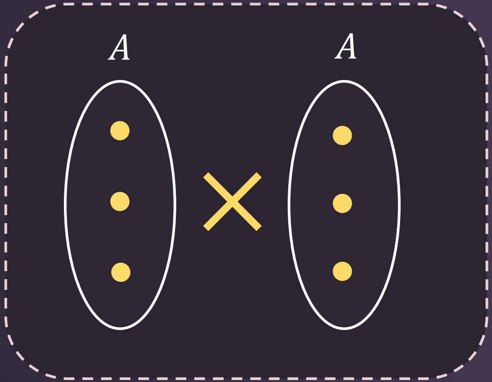
Void relation

A relation R on a set A is called a void or empty relation, if no element of set A is related to any element of A.

- $R = \phi$



Example: A = {students in boys' school}

Relation $R = \{(a, b) : b$ is sister of a & $a, b \in A\}$

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Universal relation

It is a relation in which each element of set A is related to every element of set A.

- $R = A \times A$

Example: $A = \{$set of all the students of a school$\}$

Relation $R = \{(a, b)$: difference between the heights of a & b is less than 10 meters, where $a, b \in A\}$

Explanation: It is obvious that the difference between the heights of any two students of the school has to be less than 10 m.

Therefore $(a, b) \in R$ for all $a, b \in A$.
$\Rightarrow R = A \times A$
$\therefore R$ is the universal-relation on set A.

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If A = {set of real numbers}, then check whether the relation $R = \{(a, b) : |a - b| \geq 0, a, b \in A\}$ is a universal relation or not?

Solution:

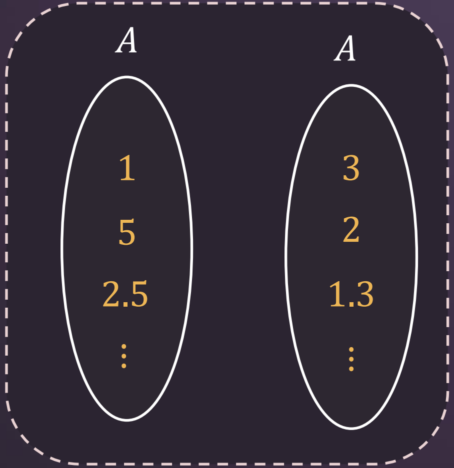
Given: $a \in \mathbb{R}$ & $b \in \mathbb{R}$

Since, the difference of two real number is a real number.

$a - b \rightarrow$ Real number

Absolute value of all real numbers ≥ 0

$|a - b| \geq 0$



$|1 - 3| \geq 0$

$|5 - 2| \geq 0$

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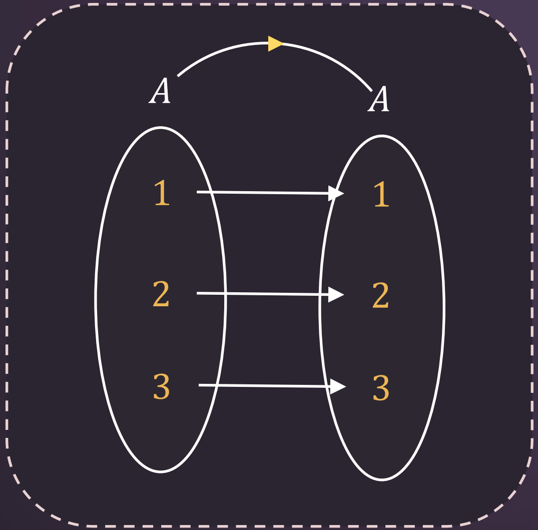
Key Takeaways

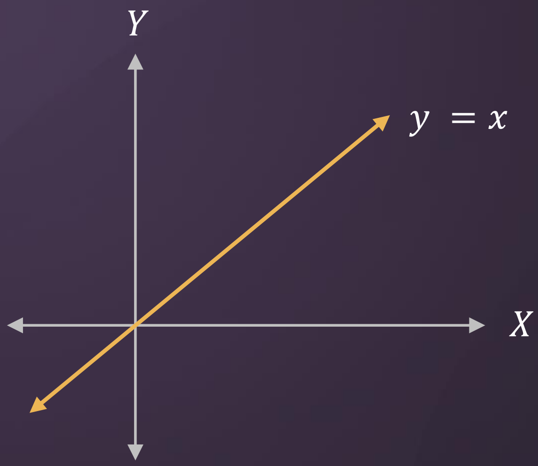
Identity relation:

Relation on set A is identity relation, if each and every element of A is related to itself only.

Example: A = {set of integers}

Relation $R = \{(a, b): a = b, a, b \in A\} = I_A$





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Key Takeaways

Reflexive relation:

A relation R defined on a set A is said to be reflexive if every element of A is related to itself.

- Relation R is reflexive if $(a, a) \in R \ \forall\ a \in A$ or $I \subseteq R$, where I is identity relation on A.

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A relation R defined on set of natural numbers, $R = \{(a, b) : a \text{ divides } b\}$, then R is a ________

Solution: $(a, b) \rightarrow a$ divides b

For being reflexive following condition must satisfy:

$(a, a) \Rightarrow a$ divides a, which is always true.

$\therefore$ R is a reflexive relation.

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$R = \{(1,1),(1,2),(2,2),(3,3)\}$ is:

A. Only identity

B. Only reflexive

C. Both a and b

D. None

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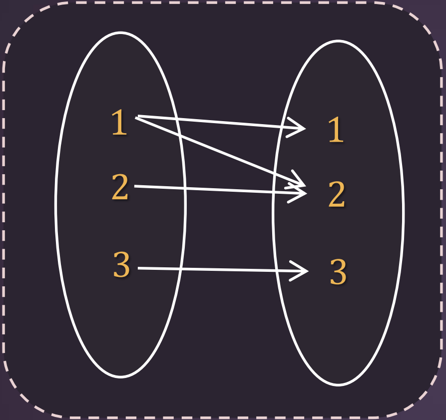
$R = \{(1,1),(1,2),(2,2),(3,3)\}$ is:

Solution:



$\therefore$ R is a reflexive relation

A Only identity

B Only reflexive

C Both a and b

D None

Key Takeaways

Symmetric relation:

A relation R on a set A is said to be a symmetric relation,
iff $(a, b) \in R \Rightarrow (b, a) \in R$.

$$\boxed{a\ R\ b \Rightarrow b\ R\ a, \forall\ (a, b) \in R}$$

Example: Consider a set $A = \{1,2,3\}$, which one is symmetric relation

$R_1 = \{(1, 1), (1, 2), (2, 1), (1, 3), (3, 1)\}$ Symmetric

$R_2 = \{(1, 1), (1, 2), (2,1), (1, 3)\}$ Not symmetric

$R_3 = \{(1, 1), (2, 2), (3, 3)\} = I_A$ Symmetric

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Important Note

- Number of Reflexive relation = $2^{n(n-1)}$
- Number of symmetric relation = $2^{\frac{n(n+1)}{2}}$

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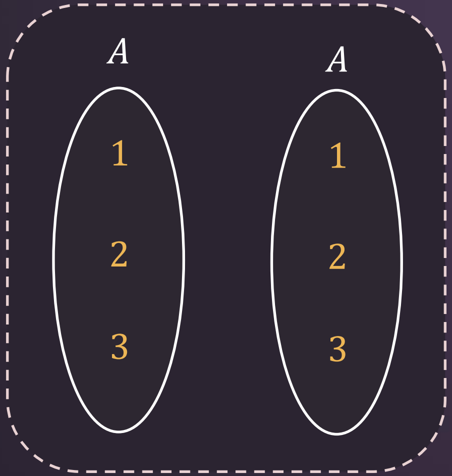
Key Takeaways

Transitive relation

A relation R on set A is said to be a transitive relation, iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R, \forall\, (a, b, c \in A)$.

$$\boxed{a\ R\ b \text{ and } b\ R\ c \Rightarrow a\ R\ c, a, b, c \in A}$$

Example: Consider a set $A = \{1, 2, 3\}$



$R_1 = \{(1, 2), (2, 3), (1, 3)\}$ Transitive

$1R2 \quad 2R3 \quad 1R3$

$R_2 = \{(1, 1), (1, 3), (3,2)\}$ Not transitive

$R_3 = \{(1, 1), (2, 2), (3, 3)\} = I_A$ Transitive

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Show that the relation R defined on the set of real number such that $R = \{(a, b) : a > b\}$ is transitive.

Solution:

Let $(a, b) \in \mathbb{R}$ and $(b, c) \in \mathbb{R}$

So $a > b$ and $b > c \Rightarrow a > c$

Thus $(a, c) \in \mathbb{R}$

$\therefore$ R is a transitive relation.

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Equivalence Relation

- A relation R on a set A is said to be equivalence relation on A iff,
    - If it is reflexive, i.e., $(a, a) \in R, \forall\ a \in A$
    - If it is symmetric, i.e., $(a, b) \in R \Rightarrow (b, a) \in R, \forall\ a, b \in A$
    - If it is transitive, i.e., $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R, \forall\ a, b, c \in A$
- Identity Relation is an Equivalence Relation.

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Key Takeaways

Note:

If a relation is reflexive, symmetric and transitive, then it is equivalence relation.

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Let T be the set of all triangles in a plane with R a relation given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

Solution:

Since every triangle is congruent to itself, $\Rightarrow R$ is reflexive

$(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2

$\Rightarrow T_2$ is congruent to T_1 $\Rightarrow R$ is symmetric

Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

$\Rightarrow T_1$ is congruent to T_2 and T_2 is congruent to T_3

$\Rightarrow T_1$ is congruent to T_3

$\Rightarrow R$ is transitive

Hence, R is an Equivalence Relation.

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Let $\mathbb{R}$ be the set of real numbers.

Statement 1 : $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y - x$ is an integer$\}$ is an equivalence relation on $\mathbb{R}$.

Statement 2 : $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation.

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A. Statement 1 is true, statement 2 is true and statement 2 is correct explanation of statement 1.

B. Statement 1 is true, statement 2 is true and statement 2 is not correct explanation of statement 1.

C. Statement 1 is true, statement 2 is false

D. Statement 1 is false, statement 2 is true

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Let $\mathbb{R}$ be the set of real numbers.

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Solution: $A = \{(x, y) \in \mathbb{R} \text{ X } \mathbb{R} : y - x$ is an integer$\}$

$(x, y) \in A \Rightarrow y - x$ is an integer $\Rightarrow x - x$ is an integer $\Rightarrow (x, x) \in A$

$\Rightarrow A$ is reflexive

$(x, y) \in A \Rightarrow y - x$ is an integer $\Rightarrow x - y$ is an integer $\Rightarrow (x, x) \in A$

$\Rightarrow A$ is symmetric

$(x, y) \in A$ and $(y, z) \in A$

$\Rightarrow y - x$ is an integer and $y - z$ is an integer

$\Rightarrow x - z$ is an integer $\Rightarrow (x, z) \in A \Rightarrow A$ is transitive

$\therefore A$ is an equivalence relation.

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Let $\mathbb{R}$ be the set of real numbers.

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Solution:

$B = \{(x, y) \in \mathbb{R}\,\mathrm{X}\,\mathbb{R} : x = \alpha y$ for some rational number $\alpha\}$

$(x, y) \in B \Rightarrow x = \alpha y \Rightarrow x = \alpha x$ for $\alpha = 1 \Rightarrow (x, x) \in B \Rightarrow B$ is reflexive

$(x, y) \in B \Rightarrow x = \alpha y$

Let $x = 0, y = 1$ Thus $\alpha = 0$

But $y \neq \beta x$ for any rational β

$\Rightarrow (y, x) \notin B$

$\Rightarrow B$ is asymmetric

$(x, y) \in B$ and $(y, z) \in B$

$\Rightarrow x = \alpha y$ and $y = \beta z$

$\Rightarrow x = \alpha\beta z$

$\Rightarrow (x, z) \in B$

$\Rightarrow B$ is transitive

$\therefore B$ is not an equivalence relation.

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Let $\mathbb{R}$ be the set of real numbers.

Statement 1 : $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y - x$ is an integer$\}$ is an equivalence relation on $\mathbb{R}$.

Statement 2 : $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation.

JEE Main 2011

A. Statement 1 is true, statement 2 is true and statement 2 is correct explanation of statement 1.

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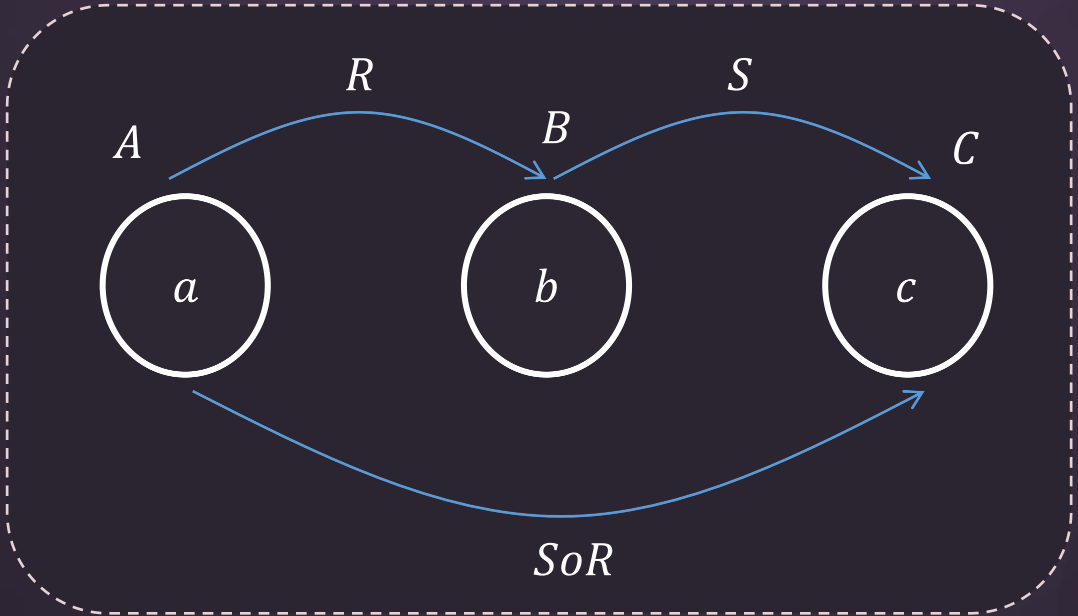
Composition of a Relation

The composition of two relations R & S (SoR) is a binary relation from A to C, if and only if there is $b \in B$ such that aRb & bSc where $a \in A$ & $c \in C$

Mathematically,

$$SoR = \{(a, c) | \exists\ b \in B : aRb \wedge bSc\}$$



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Session 2

Introduction to Function and Types of Functions

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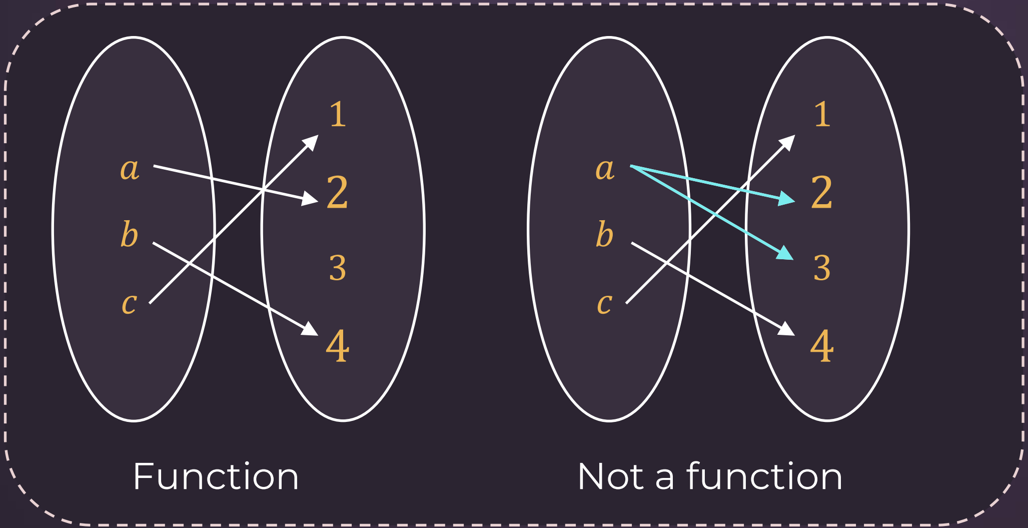
Key Takeaways

Function

A function is a relation defined from set A to set B such that each and every element of set A is uniquely related to an element of set B.

- It is denoted by $f : A \rightarrow B$

Example:



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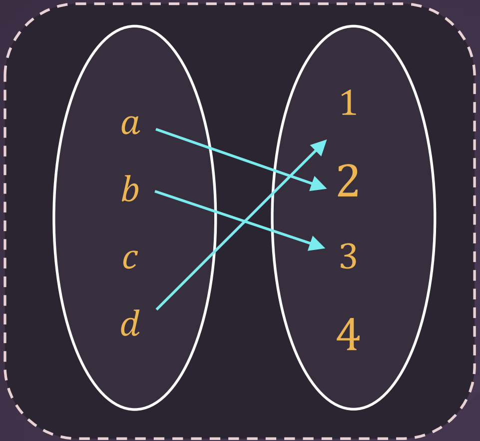
The following relation is a function. Yes or No?



A YES

B NO

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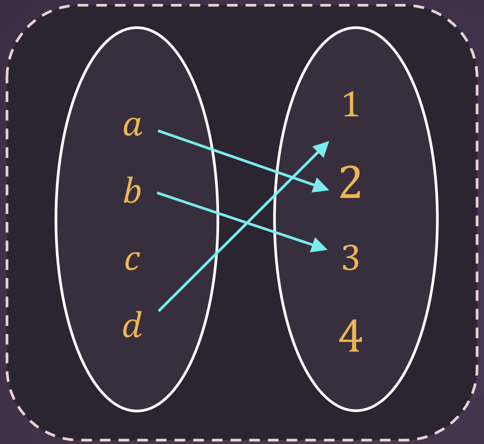
The following relation is a function. Yes or No?

Solution:



Answer is **No**.

For being function, every input should have unique output, here input c doesn't have any output.

A YES

B NO

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Domain, Range and Co-domain of function:

Domain : Values of set A for which function is defined.

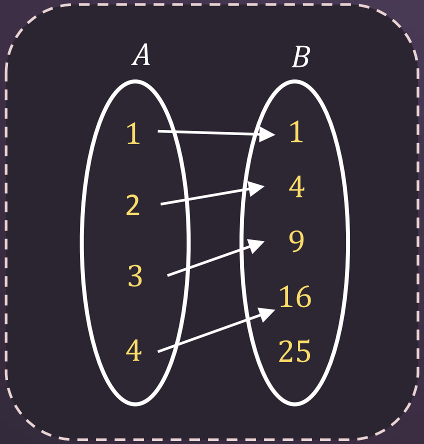
(Set of permissible inputs)

Range : All values that f takes (Range $\subseteq$ Co – domain).

(Set of output generated domain)

Co-domain : Set of all elements in set B.

Example:



Domain = $\{1, 2,3,4\}$

Range = $\{1, 4,9, 16\}$

Co-domain = $\{1, 4,9, 16,25\}$

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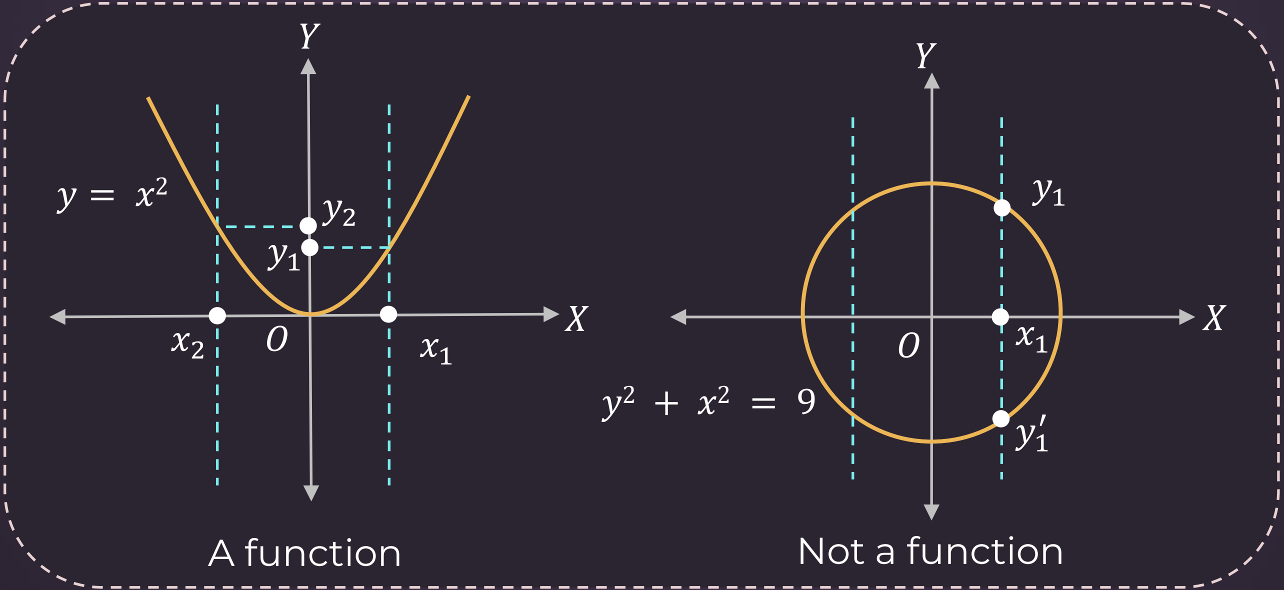
Key Takeaways

Vertical line test:

If any vertical line parallel to Y −axis intersect the curve on only one point, then it is a function. If it is intersecting more than one points, then it is not a function.



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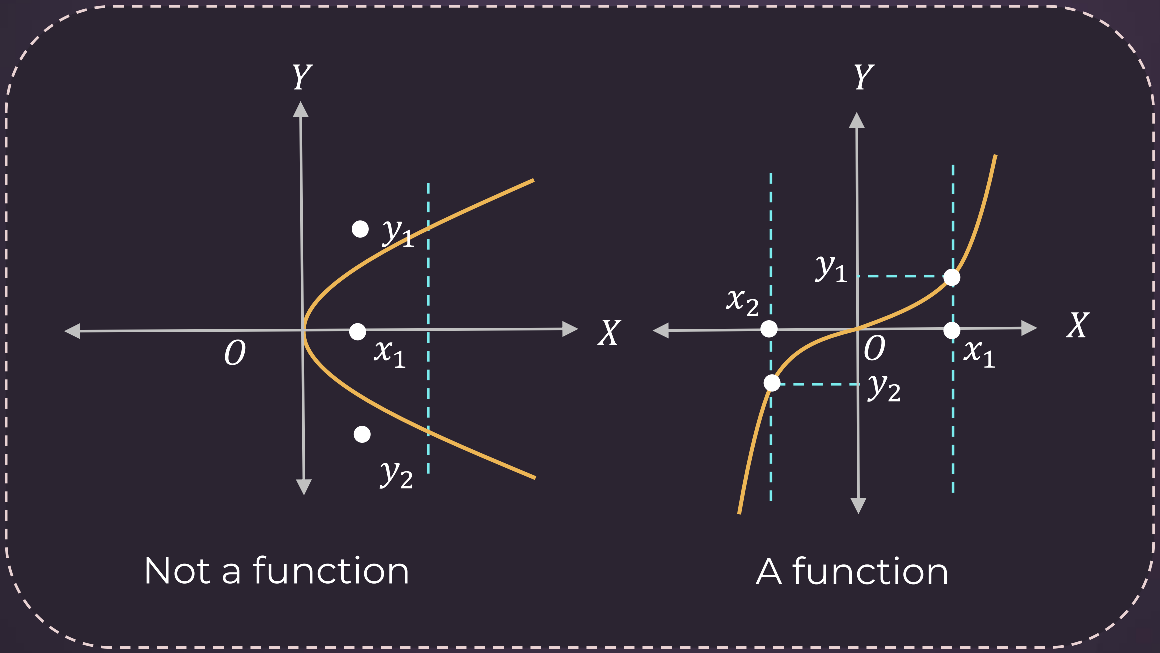
Key Takeaways

Vertical line test:

- $y^2 = x$
- $y = x^3$



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Key Takeaways

Real valued function:

A function which has either $\mathbb{R}$ or one of its subsets as its range, is called a real valued function. Further, if its domain is also either $\mathbb{R}$ or a subset of $\mathbb{R}$, is called a real function.

$R_f \subseteq \mathbb{R} \Rightarrow f$ is real valued function.

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Check whether $y^2 = e^{x^2+x}$ is function or not.

Solution:

For $x = 1$

$y^2 = e^{1+1} = e^2$

$y = \pm e$

We get two values of y for single value of x.

Hence, this is not a function.

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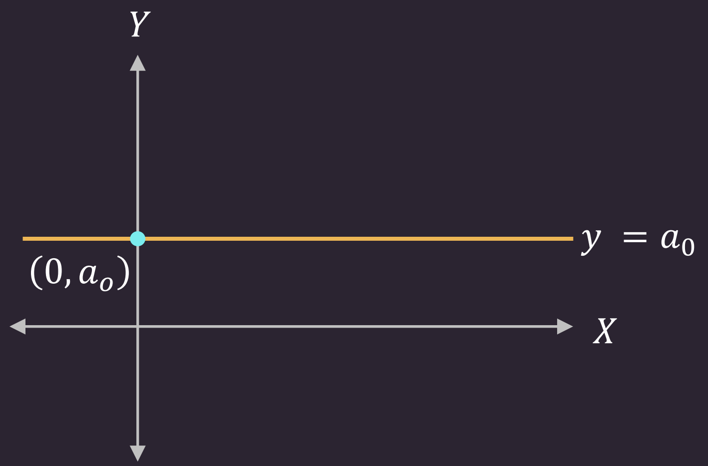
Key Takeaways

Polynomial function:

- Domain : $x \in \mathbb{R}$
- If $n = 0$, we get $P(x) = a_0$ (Constant Polynomial)

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \cdots, a_n \in \mathbb{R}, n \in \mathbb{W}$$



Domain : $\mathbb{R}$

Range : $\{a_0\}$

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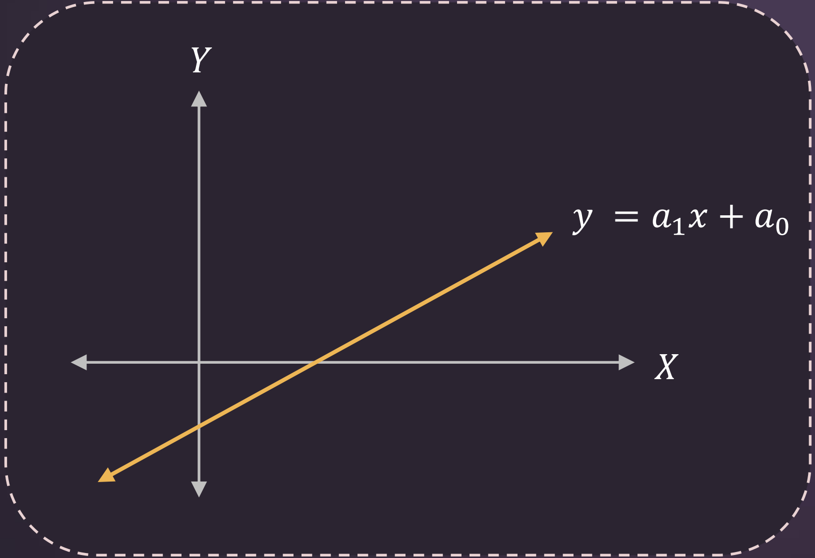
Key Takeaways

Polynomial function:

- Domain : $x \in \mathbb{R}$
- If $n = 1$, we get $P(x) = a_1\, x + a_0$

(Linear Polynomial)

$$P(x) = a_n\, x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \ldots, a_n \in \mathbb{R}, n \in \mathbb{W}$$



Domain : $\mathbb{R}$

Range : $\mathbb{R}$

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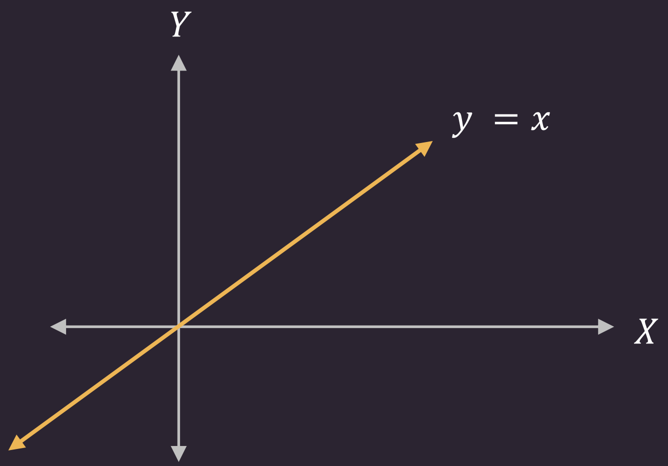
Key Takeaways

Identity function:

- $a_1 = 1,\ a_0 = 0$

  $p(x) = x$

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \ldots, a_n \in \mathbb{R}, n \in \mathbb{W}$$



Domain : $\mathbb{R}$

Range : $\mathbb{R}$

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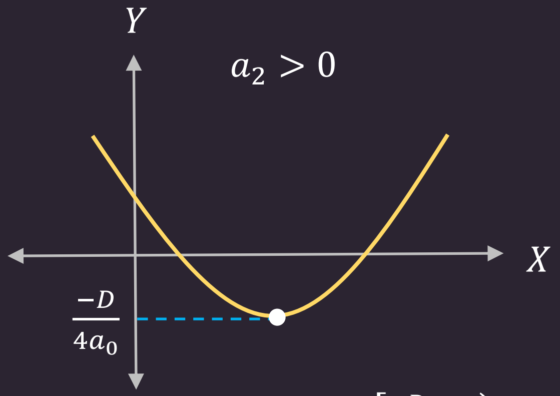
Key Takeaways

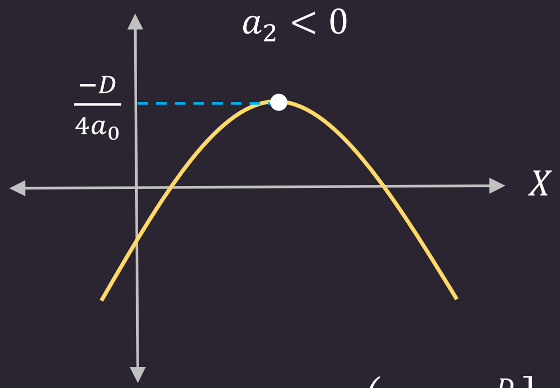
Polynomial function:

- Domain : $x \in \mathbb{R}$
- If $n = 2$, we get
  $P(x) = a_2x^2 + a_1x + a_0$
  (Quadratic Polynomial)

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \ldots, a_n \in \mathbb{R}, n \in \mathbb{W}$$



Range $\in \left[\frac{-D}{4a}, \infty\right)$



Range $\in \left(-\infty, -\frac{D}{4a}\right]$

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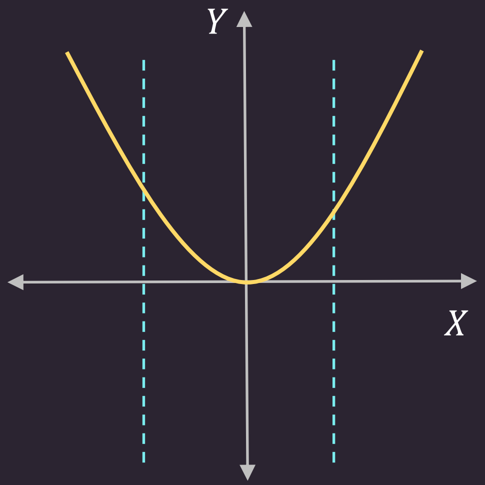
Key Takeaways

Polynomial function:

- Domain : $x \in \mathbb{R}$
- If n is even, $P(x)$ is called an even degree polynomial whose range is always a subset of $\mathbb{R}$.

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \ldots, a_n \in \mathbb{R}, n \in \mathbb{W}$$

- $y = x^2$



$x \to +\infty$
$y \to +\infty$

$x \to -\infty$
$y \to +\infty$

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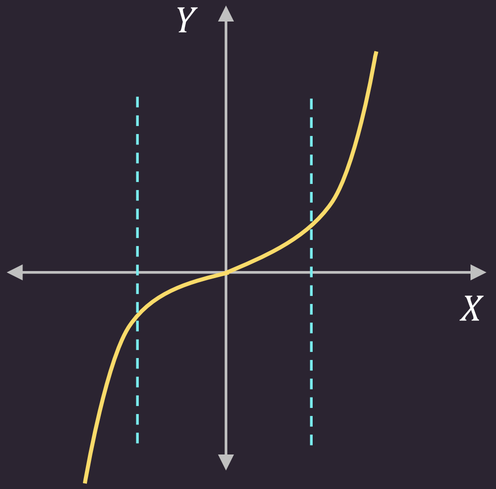
Key Takeaways

Polynomial function:

- Domain : $x \in \mathbb{R}$
- If n is odd, $P(x)$ is called an odd degree polynomial whose range is $\mathbb{R}$.

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$
$$a_0, a_1, \ldots, a_n \in \mathbb{R}, n \in \mathbb{W}$$

- $y = x^3$



$x \to +\infty$
$y \to +\infty$

$x \to -\infty$
$y \to -\infty$

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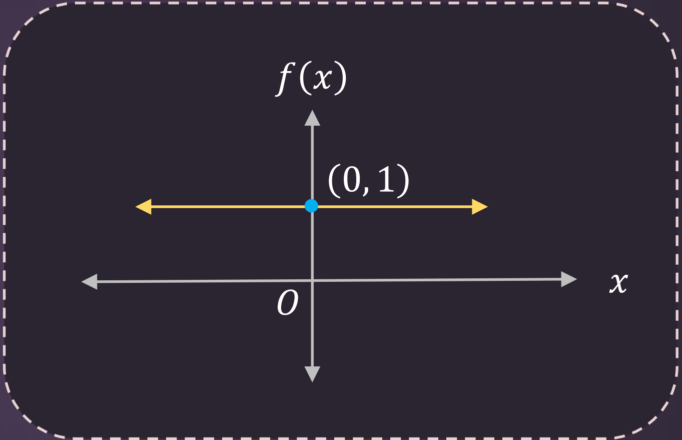
Find domain and range of function. $f(x) = \sin^2 x + \cos^2 x$

Solution:

$$f(x) = \sin^2 x + \cos^2 x = 1$$

$$D_f : x \in \mathbb{R}$$

$$R_f : y \in \{1\}$$



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Find range of the function $f(x) = x^2 + 4x + 3$

A $[-1, \infty)$

B $(0, \infty)$

C $[0, \infty)$

D $[3, \infty)$

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Find range of the function $f(x) = x^2 + 4x + 3$

Solution:

Given function:

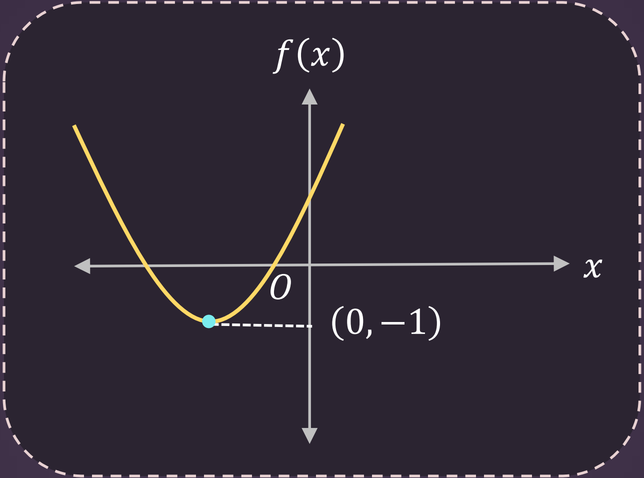
$f(x) = x^2 + 4x + 3$

$D_f: x \in \mathbb{R}$

$R_f: y \in \left[-\frac{D}{4a}, \infty\right)$

$a = 1, b = 4, c = 3$

$-\frac{D}{4a} = -\frac{(4)^2-4(1)(3)}{4\times1} = -\frac{4}{4} = -1$



A $[-1, \infty)$

B $(0, \infty)$

C $[0, \infty)$

D $[3, \infty)$

Hence, range of the function would be $[-1, \infty)$

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Key Takeaways

Rational Function:

- For $h(x) = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are functions of x
- Domain: Check domain of $f(x)$ and $g(x)$, & $g(x) \neq 0$
- If $f(x)$ & $g(x)$ is both are polynomials, then $h(x)$ is rational polynomial function.

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Find domain and range of $f(x) = \frac{x+1}{3x-5}$.

Solution:

Given: $f(x) = \frac{x+1}{3x-5}$

Domain: $3x - 5 \neq 0 \Rightarrow x \neq \frac{5}{3} \Rightarrow x \in \mathbb{R} - \left\{\frac{5}{3}\right\}$

Range: Let $f(x) = y = \frac{x+1}{3x-5}$ → Convert and make $'x'$ as a subject

$$\Rightarrow 3xy - 5y = x + 1$$

$$\Rightarrow x(3y - 1) = 5y + 1$$

$$\Rightarrow x = \frac{5y+1}{3y-1}$$

Since, x must be real.

$$\Rightarrow 3y - 1 \neq 0 \Rightarrow y \neq \frac{1}{3}$$

Range : $y \in \mathbb{R} - \left\{\frac{1}{3}\right\}$

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Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{x}{x^2+1}$, $x \in \mathbb{R}$. Then the range of f is:

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A $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$

B $\mathbb{R} - [-1, 1]$

C $(-1, 1) - \{0\}$

D $\left[-\frac{1}{2}, \frac{1}{2}\right]$

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Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x}{x^2+1}$, $x \in \mathbb{R}$. Then the range of f is:

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Solution:

Domain of $f(x)$ is $\mathbb{R}$

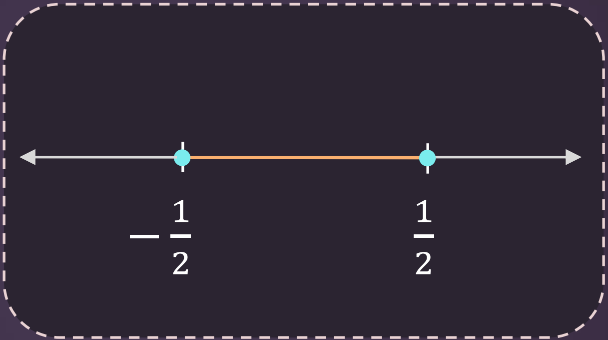
Let $y = \frac{x}{x^2+1} \Rightarrow yx^2 + y = x$

$$\Rightarrow \underbrace{yx^2 - x + y = 0}_{D \geq 0} \quad (\because x \in \mathbb{R})$$

$\Rightarrow 1 - 4y^2 \geq 0 \quad \Rightarrow 4y^2 - 1 \leq 0$

$\Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\therefore$ Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$



A. $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$

B. $\mathbb{R} - [-1, 1]$

C. $(-1, 1) - \{0\}$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

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Key Takeaways

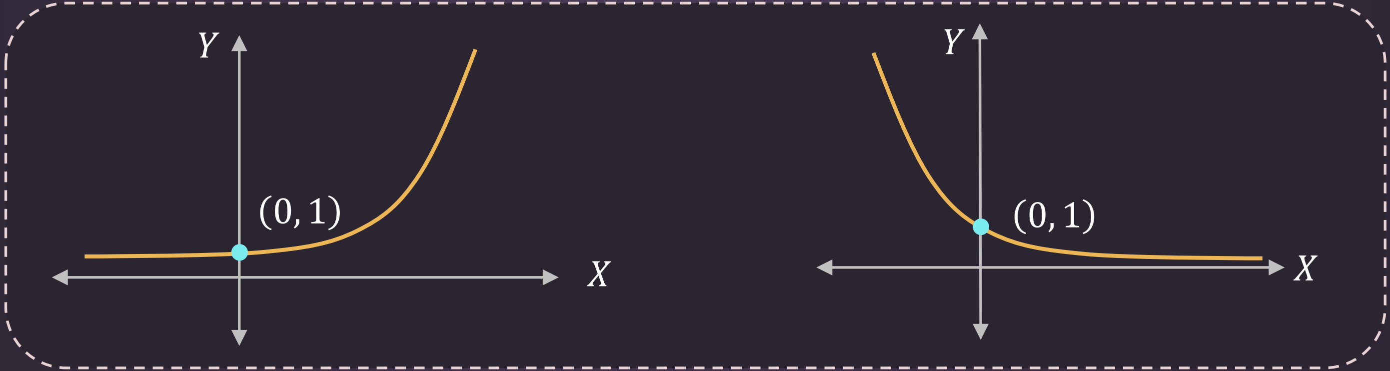
Exponential function:

$y = a^x,\ a > 0 \ \&\ a \neq 1$

- Domain : $x \in \mathbb{R}$
- Range : $y \in (0, \infty)$

Increasing function $(a > 1)$

Decreasing function $(0 < a < 1)$



Example: Find domain and range of $f(x)$, where $f(x) = e^{2x}$

We know $e > 1$

Domain: $x \in \mathbb{R}$ Range: $(0, \infty)$

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The range of $f(x) = e^x + 1$ is

A $[\mathbf{1}, \infty)$

B $(\mathbf{0}, \infty)$

C $[-\mathbf{1}, \infty)$

D $(\mathbf{1}, \infty)$

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The range of $f(x) = e^x + 1$ is

Solution: Range of e^x: $(0, \infty)$

So, range of $e^x + 1$: $(1, \infty)$

A $[1, \infty)$

B $(0, \infty)$

C $[-1, \infty)$

D $(1, \infty)$

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Session 3

Some more types of Functions

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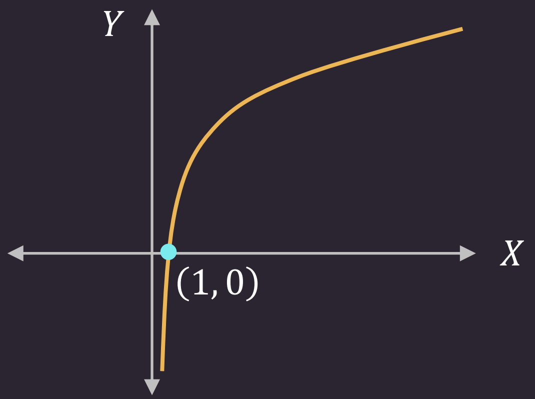
Key Takeaways

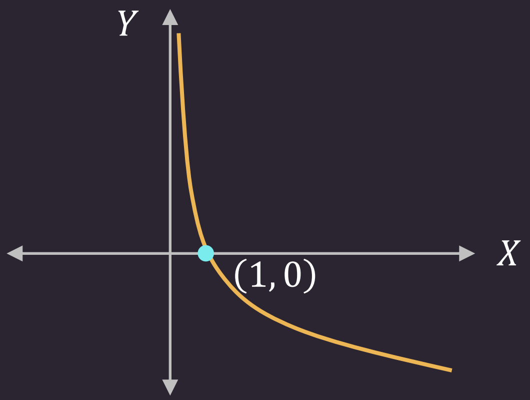
Logarithmic function:

$y = \log_a x \, , \, a > 0 \; \& \; a \neq 1$

- Domain : $x \in (0, \infty)$ or $\mathbb{R}^+$
- Range : $y \in (-\infty, \infty)$ or $\mathbb{R}$



Increasing function $(a > 1)$



Decreasing function $(0 < a < 1)$

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Key Takeaways

Logarithmic function:

$y = \log_a x$, $a > 0$ & $a \neq 1$

- Domain : $x \in (0, \infty)$ or $\mathbb{R}^+$
- Range : $y \in (-\infty, \infty)$ or $\mathbb{R}$

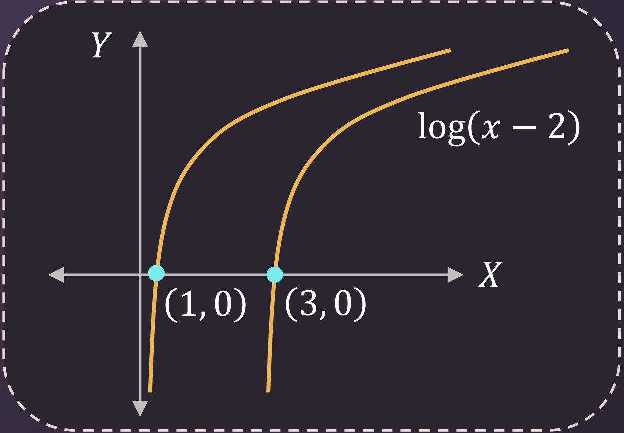
Increasing function $(a > 1)$

Decreasing function $(0 < a < 1)$

Example: Find domain and range of $f(x) = \log(x-2)$.

Solution: $f(x) = \log_{10}(x-2)$;
Domain: $x - 2 > 0 \Rightarrow x > 2$

$D_f = (2, \infty)$ Range: $y \in \mathbb{R}$



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The domain of the definition of the function

$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$ is :

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A. $(1, 2) \cup (2, \infty)$

B. $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

C. $(-1,0) \cup (1,2) \cup (2, \infty)$

D. $(-1, 0) \cup (1,2) \cup (3, \infty)$

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The domain of the definition of the function

$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$ is :

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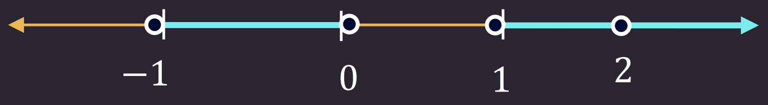
Solution:



$$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$$

$$4 - x^2 \neq 0 \quad \Rightarrow x \neq \pm 2 \cdots (i)$$

and $x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0$

$$\Rightarrow x \in (-1, 0) \cup (1, \infty) \cdots (ii)$$

From equation (i) and (ii)

$$\boxed{x \in (-1, 0) \cup (1, 2) \cup (2, \infty)}$$

A $(1, 2) \cup (2, \infty)$

B $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

C $(-1, 0) \cup (1, 2) \cup (2, \infty)$

D $(-1, 0) \cup (1, 2) \cup (3, \infty)$

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Key Takeaways

Note:

- For $h(x) = f(x)^{g(x)}$, to be defined for $f(x) > 0$, and normal condition for $g(x)$.

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Find domain of function $f(x) = \left(1 + \frac{3}{x}\right)^{\frac{1}{x-2}}$

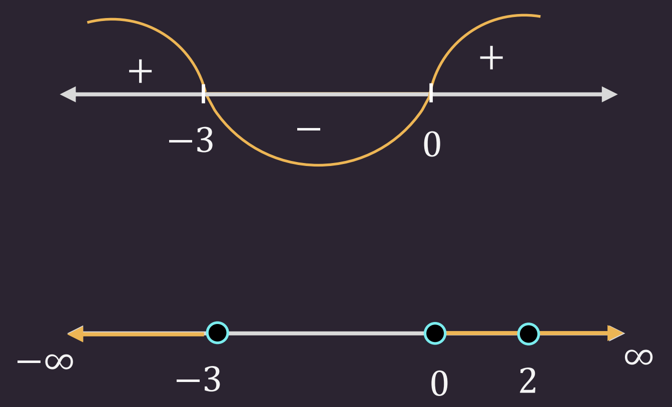
Solution:

$$f(x) = \left(1 + \frac{3}{x}\right)^{\frac{1}{x-2}}$$

$\left(1 + \frac{3}{x}\right) > 0$ and $x - 2 \neq 0$

$\Rightarrow x \in (-\infty, -3) \cup (0, \infty)$ and $x \neq 2$

$\Rightarrow x \in (-\infty, -3) \cup (0, 2) \cup (2, \infty)$



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Find domain and range of $f(x)$, where $f(x) = x^4 + x^2 + 4$.

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Solution: $f(x) = x^4 + x^2 + 4 = y$

Since $f(x)$ is a polynomial, it's domain is $\mathbb{R}$.

For range, $y = x^4 + x^2 + 4 = (x^2)^2 + 2 \times \frac{1}{2} \times x^2 + \frac{1}{4} - \frac{1}{4} + 4$

$$= \left(x^2 + \frac{1}{2}\right)^2 + \frac{15}{4}$$

Since, $x^2 \geq 0 \Rightarrow x^2 + \frac{1}{2} \geq \frac{1}{2}$

$\therefore y \geq \left(\frac{1}{2}\right)^2 + \frac{15}{4}$

$y \in [4, \infty)$

Alternate Method:
We know that, $x^2, x^4 \geq 0$
$\Rightarrow y \geq 4$

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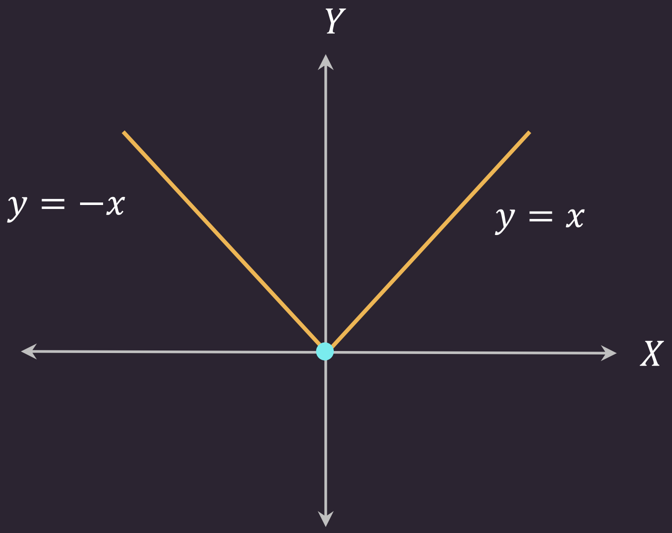
Key Takeaways

Modulus function

- $y = |x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

  Domain : $x \in \mathbb{R}$

  Range : $y \in [0, \infty)$



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Find the domain and the range of $f(x) = \frac{\sqrt{x^2}}{|x|}$.

Solution: $f(x) = \frac{\sqrt{x^2}}{|x|} = \frac{|x|}{|x|} = 1$ Where $x \neq 0$ $\quad \because \sqrt{(f(x))^2} = |f(x)|$

Domain : $x \in \mathbb{R} - \{0\}$

Range : $f(x) \in \{1\}$

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Find the range of the function $f(x) = 1 - |x - 2|$.

A $(-\infty, 1]$

B $(-\infty, 2]$

C $(1, \infty)$

D $\left(-\infty, \frac{1}{2}\right]$

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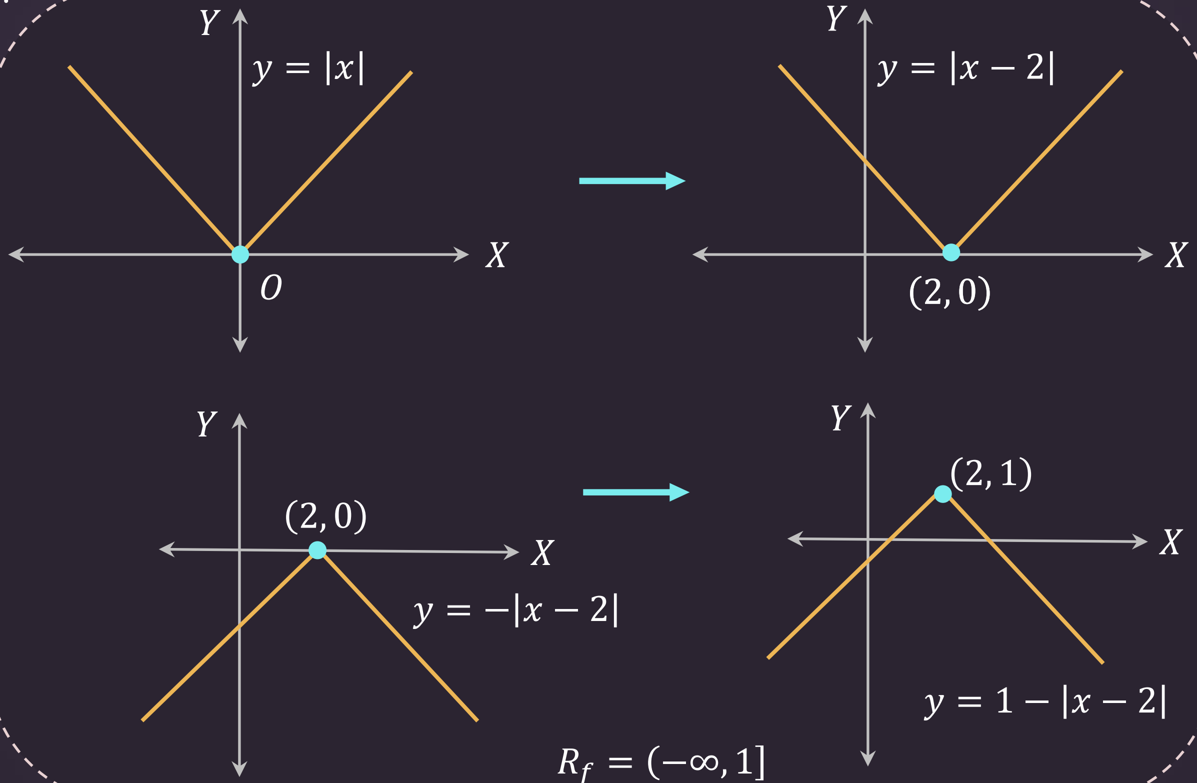
Find the range of the function $f(x) = 1 - |x - 2|$.

Solution:



$R_f = (-\infty, 1]$

A $(-\infty, 1]$

B $(-\infty, 2]$

C $(1, \infty)$

D $\left(-\infty, \frac{1}{2}\right]$

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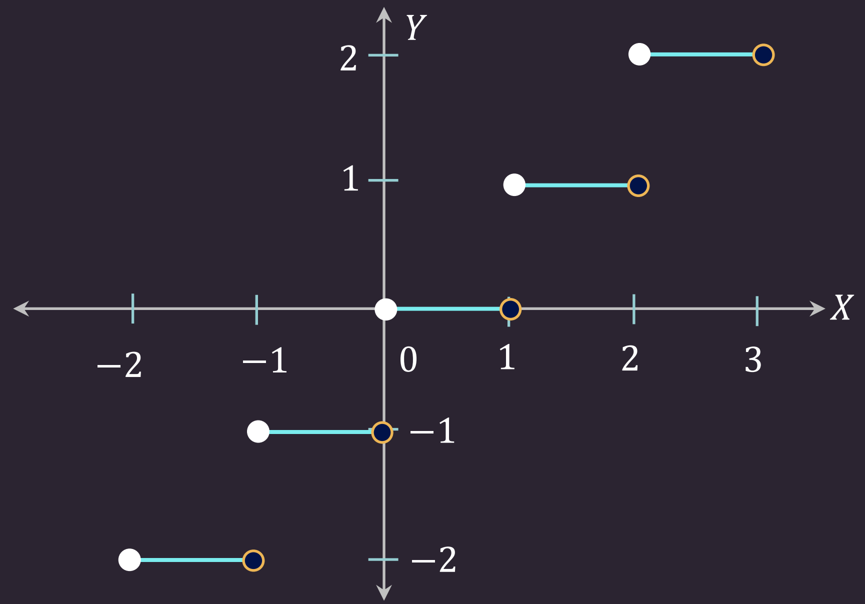
Key Takeaways

Greatest integer function(Step function)

- $y = [x]$ = Greatest Integer less than or equal to x

Domain : $x \in \mathbb{R}$ Range : $y \in \mathbb{Z}$



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If $[x] \leq -2$, then $x \in$

A $(-\infty, -2]$

B $(-\infty, 1]$

C $[-2, -1)$

D $(-\infty, -1)$

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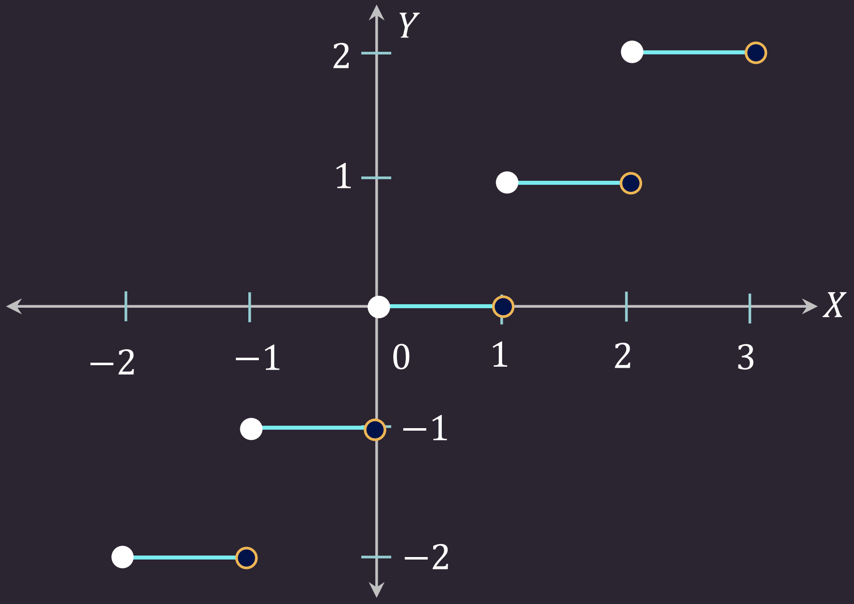
If $[x] \leq -2$, then $x \in$

Solution:

$[x] \leq -2$

$\Rightarrow x \in (-\infty, -1)$



A $(-\infty, -2]$

B $(-\infty, 1]$

C $[-2, -1)$

D $(-\infty, -1)$

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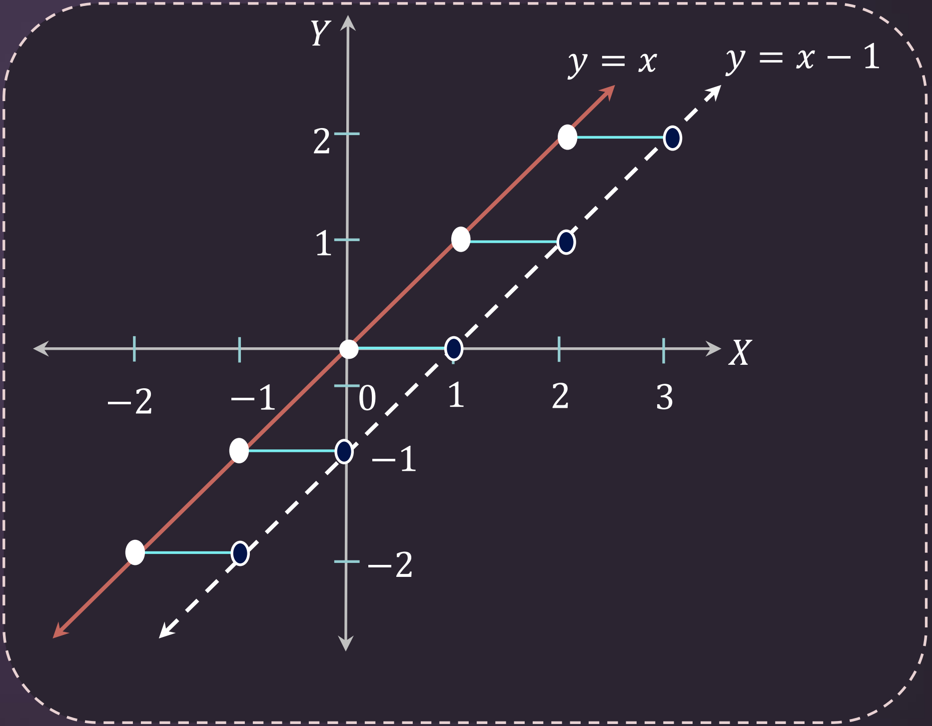
Key Takeaways

Greatest integer function

- $y = [x]$ = Greatest Integer less than or equal to x

Domain : $x \in \mathbb{R}$

Range : $y \in \mathbb{Z}$



Properties:

- $x - 1 < [x] \leq x$
- $[x + m] = [x] + m$; for $m \in \mathbb{I}$.
- $[x] + [-x] = \begin{cases} 0, x \in \mathbb{I} \\ -1, x \notin \mathbb{I} \end{cases}$

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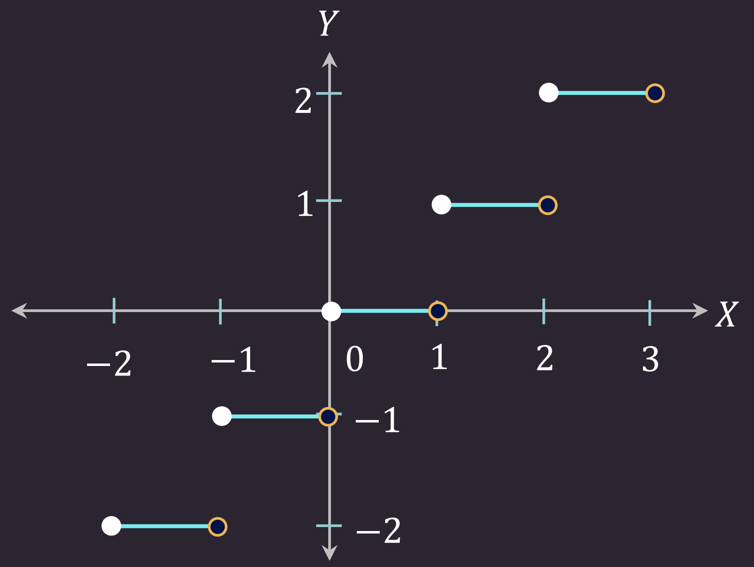
Find the domain and range of the function :

$f(x) = [x+1] + 1$, (where [.] denotes G.I.F)

Solution: $f(x) = [x+1] + 1 \Rightarrow f(x) = [x] + 2$

$[x+m] = [x] + m$; for $m \in \mathbb{I}$.

$y = [x]$



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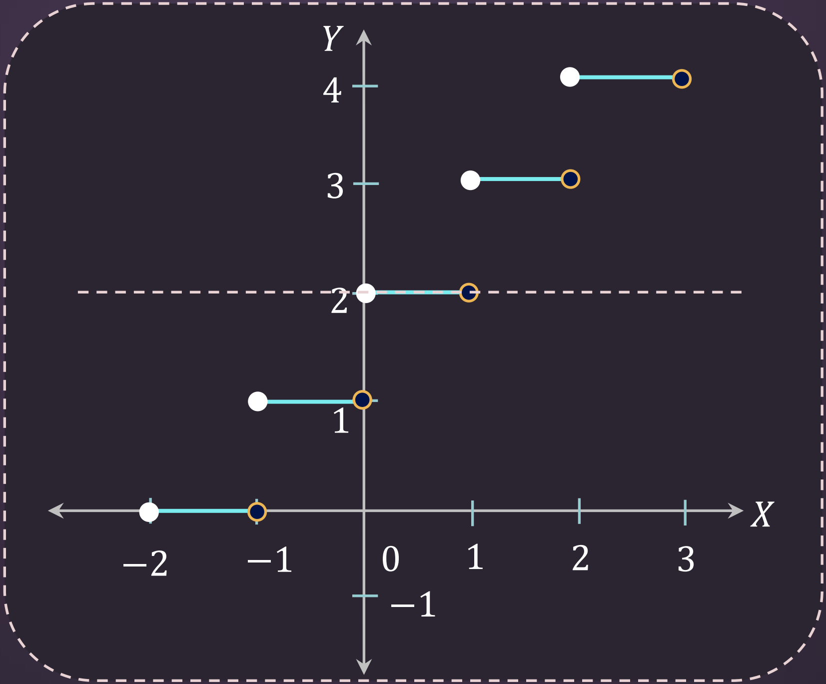
Find the domain and range of the function :

$f(x) = [x + 1] + 1$, (where [.] denotes G.I.F)

Solution: $f(x) = [x+1] + 1 \Rightarrow f(x) = [x] + 2$ $\quad y = [x] + 2$



Domain : $x \in \mathbb{R}$

Range : $\mathbb{Z}$

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Find the domain of $f(x) = \sqrt{1 - [x]^2}$, where [.] denotes G.I.F.

A $(1, 2)$

B $[-1, 2)$

C $[1, 2]$

D $(-1, 0)$

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Find the domain of $f(x) = \sqrt{1 - [x]^2}$, where [.] denotes G.I.F.

Solution:

$f(x) = \sqrt{1 - [x]^2}$

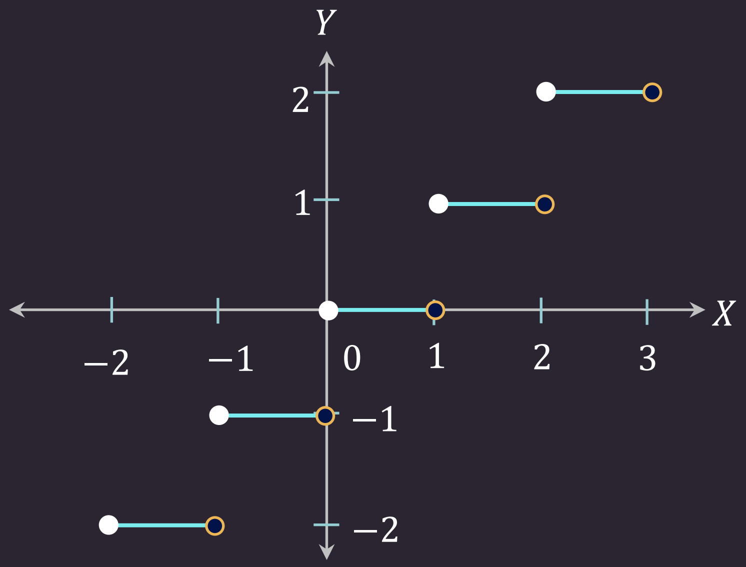
$1 - [x]^2 \geq 0$

$\Rightarrow [x]^2 - 1 \leq 0$

$\Rightarrow [x]^2 \leq 1$

$\Rightarrow -1 \leq [x] \leq 1$

$\Rightarrow x \in [-1, 2)$



A $(1, 2)$

B $[-1, 2)$

C $[1, 2]$

D $(-1, 0)$

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Find the range of the function :

$f(x) = x^{[x]}, x \in [1,3]$ (where $[x]$ denotes G.I.F.).

Solution: $f(x) = x^{[x]}, x \in [1,3]$ (where $[x]$ denotes G.I.F.).

$f(x) = x^{[x]}, x \in [1,3]$

Case 1: $x \in [1,2)$

$f(x) = x \ (\because [x] = 1)$

$f(x) \in [1,2) \cdots (i)$

Case 2: $x \in [2,3)$

$f(x) = x^2 \ (\because [x] = 2)$

$f(x) \in [4,9) \cdots (ii)$

Case 3: $x = 3$

$f(x) = x^3 \ (\because [x] = 3)$

$f(x) \in \{27\} \cdots (iii)$

$(i) \cup (ii) \cup (iii)$

$f(x) \in [1,2) \cup [4,9) \cup \{27\}$

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Session 4

Fractional part function, Signum function and One – one and Many-one function

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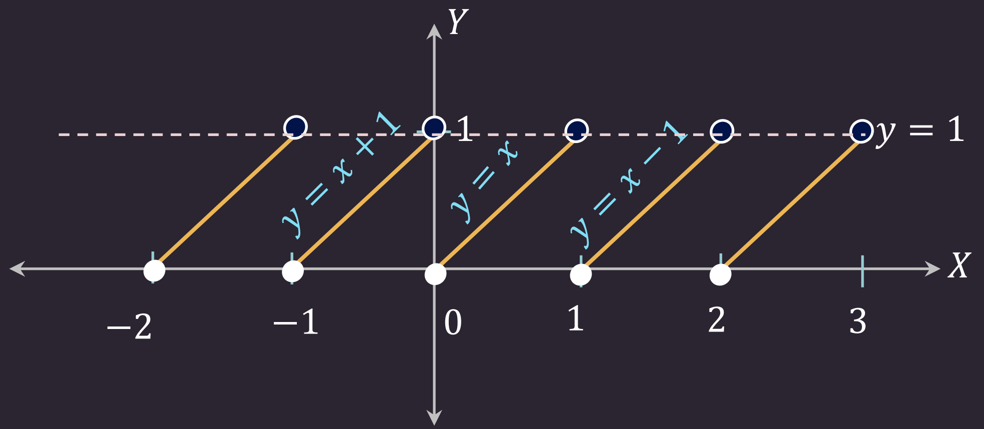
Key Takeaways

Fractional Part Function

- $y = \{x\} = x - [x]$

Domain : $x \in \mathbb{R}$ Range : $y \in [0,1)$



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What is the fractional part of 1.53?

A. 1

B. 0.53

C. 0.47

D. -0.53

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What is the fractional part of 1.53?

Solution:

$$y = \{x\} = x - [x]$$

$$= 1.53 - 1 = 0.53$$

A 1

B 0.53

C 0.47

D -0.53

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Key Takeaways

Fractional Part Function

- $y = \{x\} = x - [x]$

Domain : $x \in \mathbb{R}$ Range : $y \in [0,1)$

Properties:

- $\{x + n\} = \{x\}, n \in \mathbb{I}$
- $\{x\} + \{-x\} = \begin{cases} 0, x \in \mathbb{I} \\ 1, x \notin \mathbb{I} \end{cases}$

Examples:

$$\{1.25\} = 1.25 - [1.25]$$
$$= -1.25 - 1$$
$$= 0.25$$

$$\{-1.25\} = -1.25 - [-1.25]$$
$$= -1.25 - (-2)$$
$$= -1.25 + 2 = 0.75$$

$$\{1.25\} + \{-1.25\} = 1$$

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Find the domain and range of the function :

$f(x) = 2\{x+1\} + 3$, (where $\{.\}$ denotes fractional part function).

Solution: $f(x) = 2\{x+1\} + 3 \Rightarrow f(x) = 2\{x\} + 3$

$\boxed{\{x+n\} = \{x\}, n \in \mathbb{I}}$

$0 \leq \{x\} < 1$

$0 \leq 2\{x\} < 2$

$0 + 3 \leq 2\{x\} + 3 < 2 + 3$

$3 \leq f(x) < 5$

Domain : $x \in \mathbb{R}$

Range : $f(x) \in [3,5)$

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Find the range of the function : $f(x) = \frac{\{x\}}{1+\{x\}}$, (where $\{.\}$ denotes fractional part function).

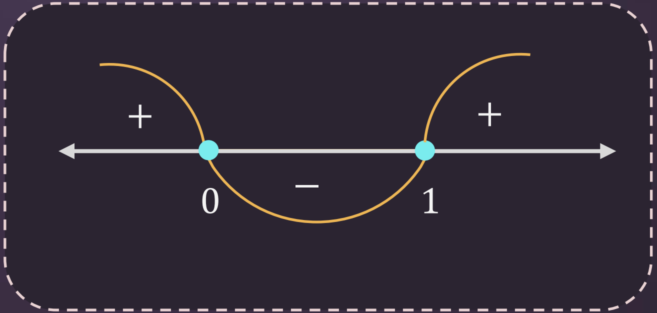
Solution: Let $y = f(x) = \dfrac{\{x\}}{1+\{x\}}$

On cross multiplying,

$$y(1+\{x\}) = \{x\} \Rightarrow y + y\{x\} = \{x\}$$

$$\Rightarrow \{x\} = \frac{y}{1-y} \quad (\because \{x\} \in [0,1)) \quad \Rightarrow 0 \leq \frac{y}{1-y} < 1$$

$$\frac{y}{1-y} \geq 0 \Rightarrow \frac{y}{y-1} \leq 0$$



$$y \in [0,1) \longrightarrow (I)$$

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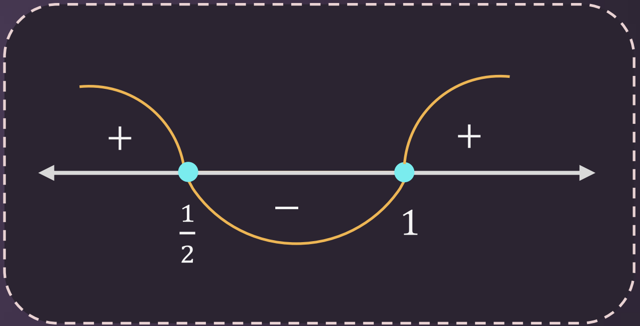
Find the range of the function : $f(x) = \frac{\{x\}}{1+\{x\}}$, (where $\{.\}$ denotes fractional part function).

Solution:

$$0 \le \frac{y}{1-y} < 1$$

$$\Rightarrow \frac{y}{1-y} < 1 \quad \Rightarrow \frac{y}{1-y} - 1 < 0$$

$$\Rightarrow \frac{2y-1}{1-y} < 0 \Rightarrow \frac{2y-1}{y-1} > 0$$



$$y \in \left(-\infty, \frac{1}{2}\right) \cup (1,\ \infty) \longrightarrow (II)$$

By $(I) \cap (II)$ we get:

$$y \in \left[0, \frac{1}{2}\right)$$

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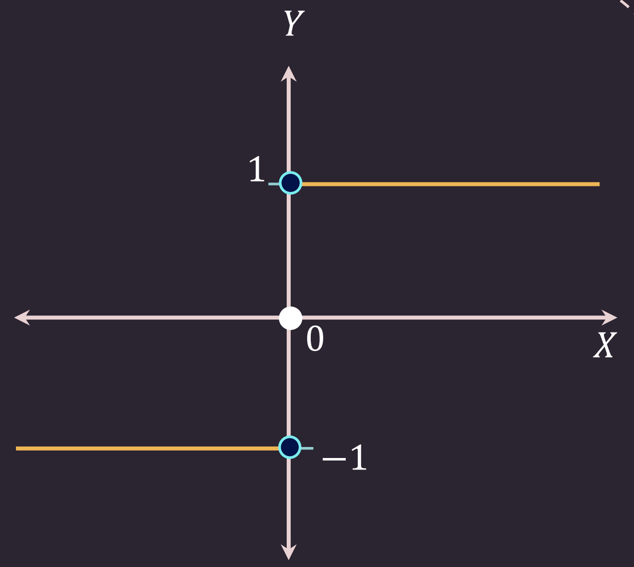
Key Takeaways

Signum Function

- $$y = \operatorname{sgn}(x) = \begin{cases} \dfrac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases} = \begin{cases} 1, x > 0 \\ -1, x < 0 \\ 0, x = 0 \end{cases}$$

- Domain : $x \in \mathbb{R}$ Range : $y \in \{-1,0,1\}$

- $\operatorname{sgn}(\operatorname{sgn}(\operatorname{sgn} \cdots \cdots \cdots (\operatorname{sgn} x) = \operatorname{sgn}(x)$



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Find the domain and range of the function : $f(x) = \operatorname{sgn}\left(\frac{x^3+x^2}{x+1}\right)$

Solution:

$$f(x) = \operatorname{sgn}\left(\frac{x^3 + x^2}{x + 1}\right)$$

$$\Rightarrow f(x) = \operatorname{sgn}\left(\frac{x^2(x + 1)}{x + 1}\right)$$

Domain : $x \in \mathbb{R} - \{-1\}$

$$\Rightarrow f(x) = \operatorname{sgn}(x^2)$$

Thus, $f(x) \in \{0,1\}$ $\quad (\because x^2 \geq 0)$

If $x^2 > 0 \Rightarrow f(x) = \operatorname{sgn}(x^2) = 1$

If $x^2 = 0 \Rightarrow f(x) = \operatorname{sgn}(x^2) = 0$

Range : $f(x) \in \{0,1\}$

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One input - one output

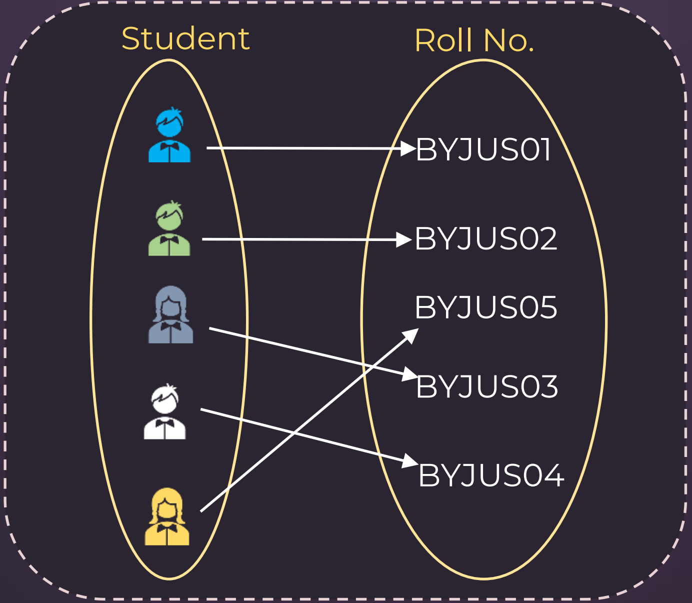
Name	Kishor
Roll no.	BYJUS01
Score	92%

Name	Arya
Roll no.	BYJUS02
Score	93%

Name	Roohi
Roll no.	BYJUS03
Score	95%

Name	Ayan
Roll no.	BYJUS04
Score	92%

Name	Alia
Roll no.	BYJUS05
Score	93%



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Many inputs - one output

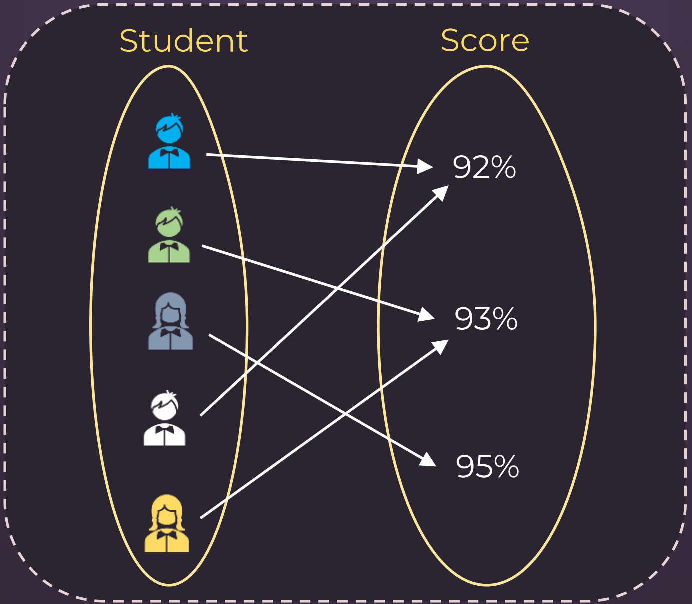
Name	Kishor
Roll no.	BYJUS01
Score	92%

Name	Arya
Roll no.	BYJUS02
Score	93%

Name	Roohi
Roll no.	BYJUS03
Score	95%

Name	Ayan
Roll no.	BYJUS04
Score	92%

Name	Alia
Roll no.	BYJUS05
Score	93%



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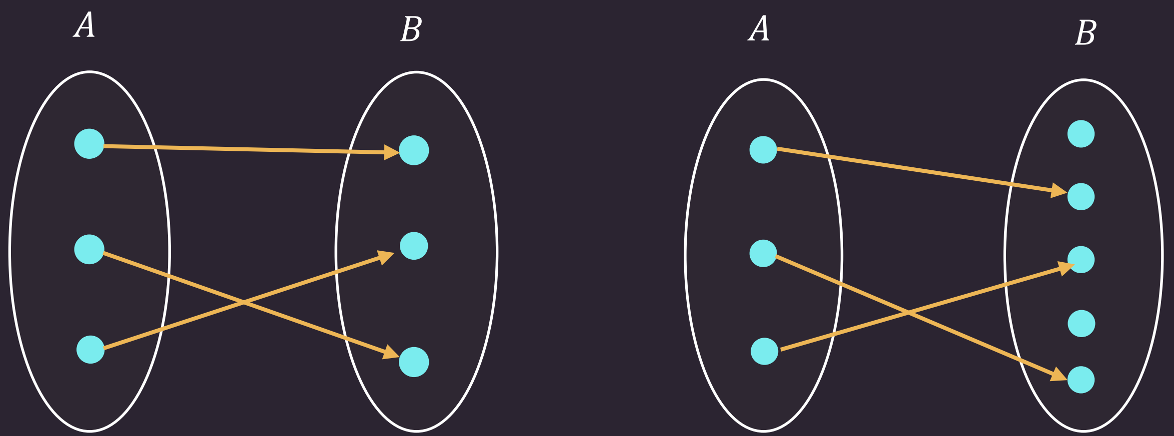
Key Takeaways

One – one function (Injective function/ Injective mapping) :

A function $f: A \rightarrow B$ is said to be a one-one function if different elements of set A have different f images in set B.



Both are example of one-one function

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Key Takeaways

Methods to determine whether a function is ONE-ONE or NOT:

For $x_1, x_2 \in A$ and $f(x_1), f(x_2) \in B$

$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$

Example:

A function $f: \mathbb{R} \to \mathbb{R}$ such that

$f(x) = 3x + 5$

Suppose for some $x_1, x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$\Rightarrow 3x_1 + 5 = 3x_2 + 5$

$\Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one.

$f(x) = x^2$

Suppose for some $x_1, x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow x_1 = x_2$ or $x_1 = -x_2$

$\therefore f(x)$ is not one-one.

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Check whether the given function $f(x)$ is one-one or many one: $f(x) = x^2 + x + 2$

Solution: Suppose for some $x_1, x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + x_1 + 2 = x_2^2 + x_2 + 2$

$\Rightarrow x_1^2 - x_2^2 + x_1 - x_2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) + x_1 - x_2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ or $x_1 + x_2 = -1$

We get two conclusions here

Which indicates that many such x_1 & x_2 are possible

$\therefore f(x)$ is many-one function

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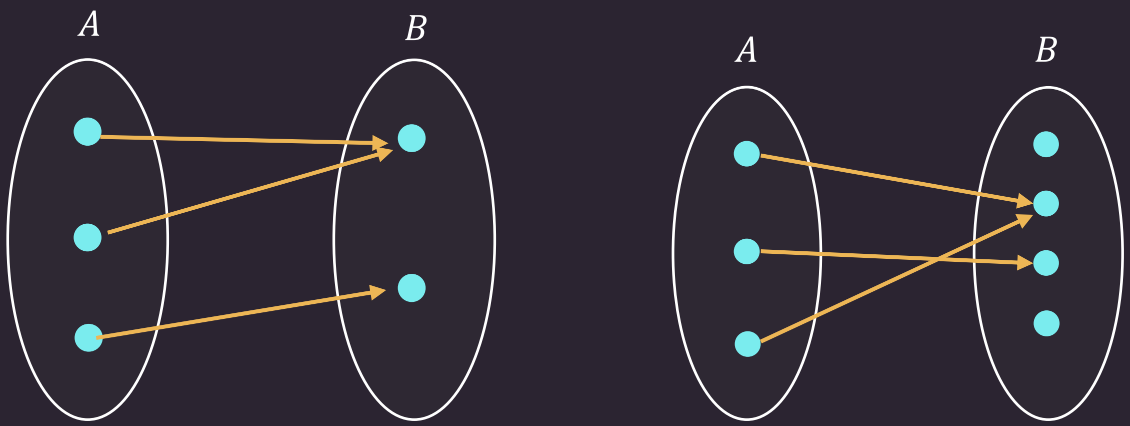
Key Takeaways

Many one function :

A function $f: A \rightarrow B$ is said to be a many-one function if there exist at least two or more elements of set A that have same f image in B.



Both are example of many one function

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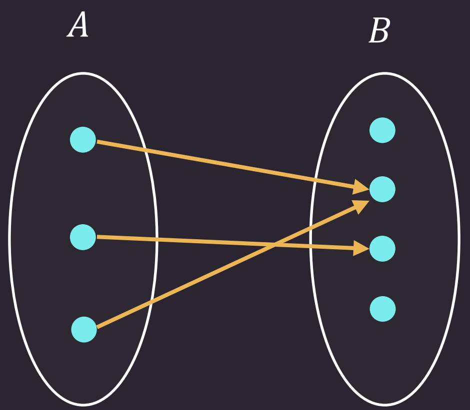
Key Takeaways

Methods to determine whether a function is ONE-ONE or MANY ONE :

A function $f: A \to B$ is many one iff there exists atleast two elements $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$
$(f(x_1), f(x_2) \in B$ but $x_1 \neq x_2)$



Many one function

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Session 5

Methods to Find Whether a Function is One-One or not, Number of Functions and Number of One-One mappings

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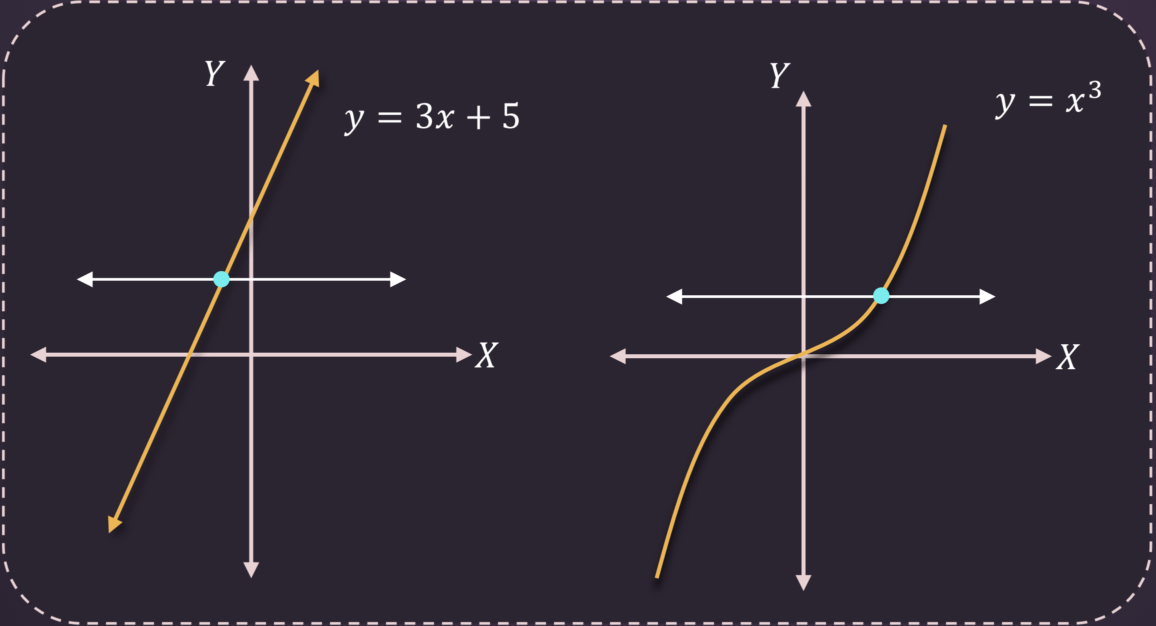
Key Takeaways

Methods to determine whether a function is ONE-ONE or MANY ONE :

Horizontal line test : If we draw straight lines parallel to x –axis, and they cut the graph of the function at exactly one point, then the function is ONE-ONE.



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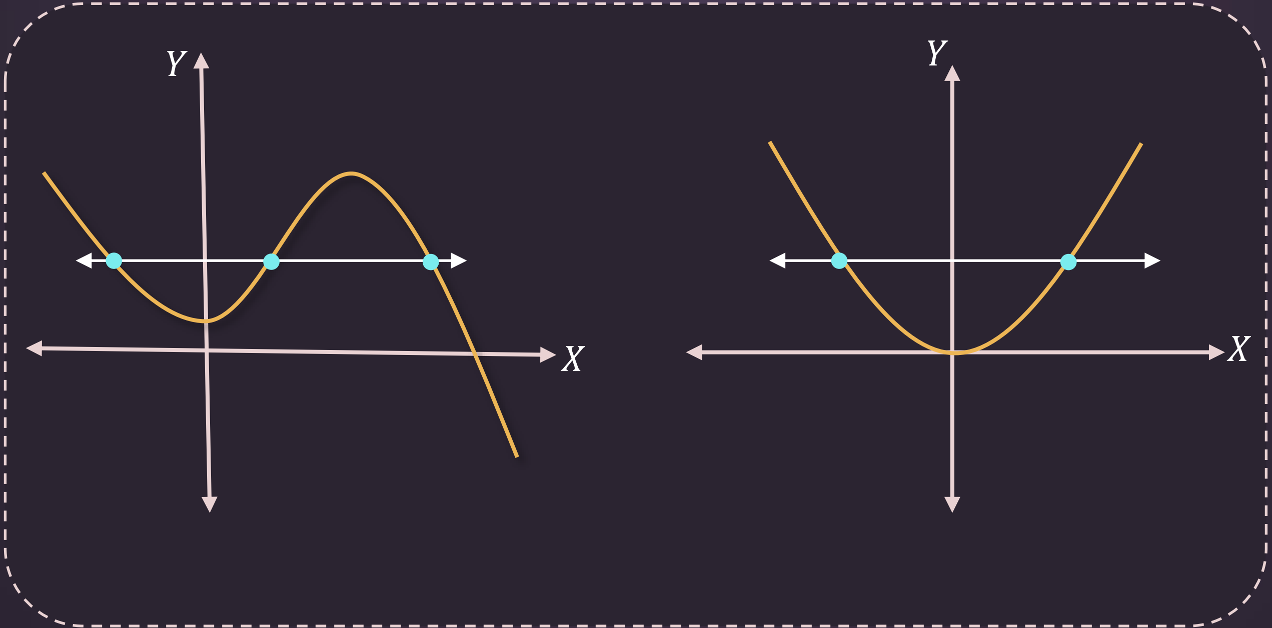
Key Takeaways

Methods to determine whether a function is ONE-ONE or MANY ONE :

Horizontal line test : If there exists a straight lines parallel to x –axis, which cuts the graph of the function at atleast two points, then the function is MANY-ONE.



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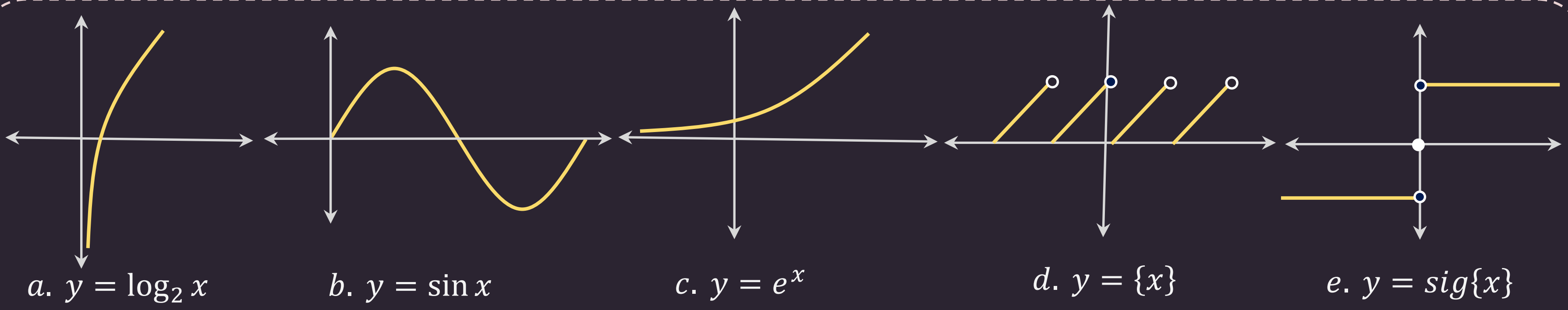
Choose the correct option:



A (a), (b) & (e) are one-one mapping

B (a) & (e) are many-one mapping

C (a) & (c) are one-one mapping

D None

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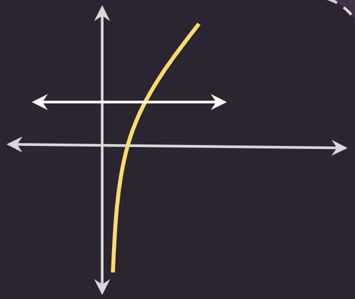
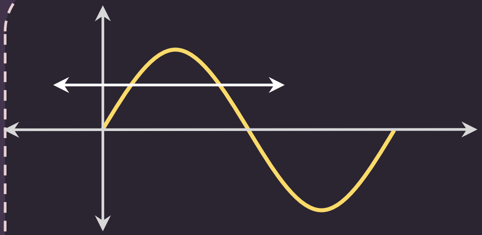
Choose the correct option:

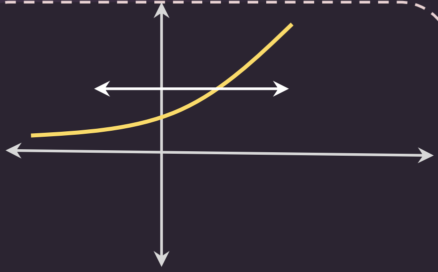
Solution:



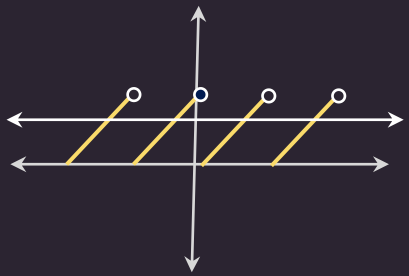
$a.\ y = \log_2 x$



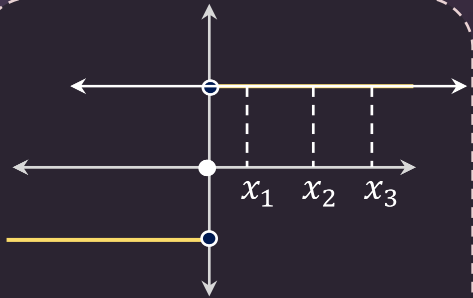
$b.\ y = \sin x$



$c.\ y = e^x$



$d.\ y = \{x\}$



$e.\ y = sig\{x\}$

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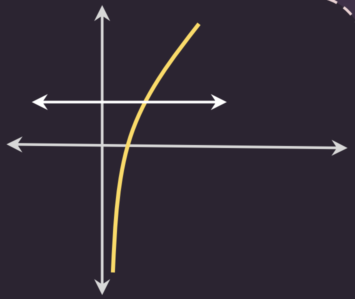
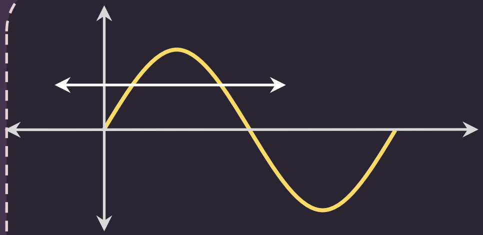
Choose the correct option:

Solution: Exponents and logarithmic functions are one-one.



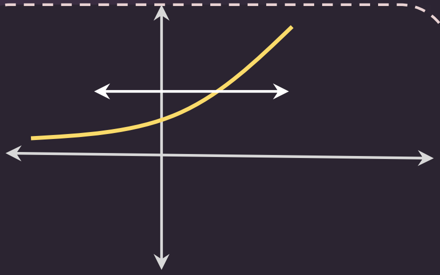
a. $y = \log_2 x$

One-One



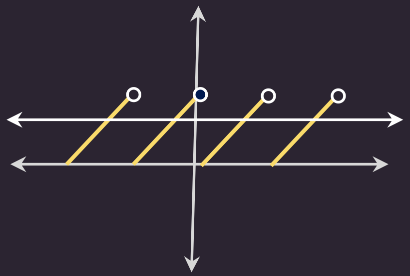
b. $y = \sin x$

Many-One



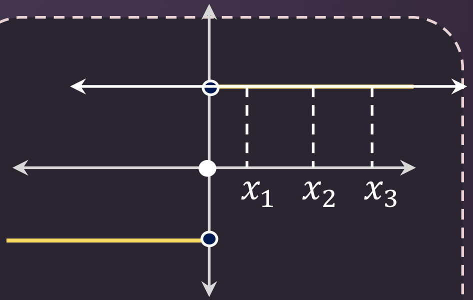
c. $y = e^x$

One-One



d. $y = \{x\}$

Many-One



e. $y = sig\{x\}$

Many-One

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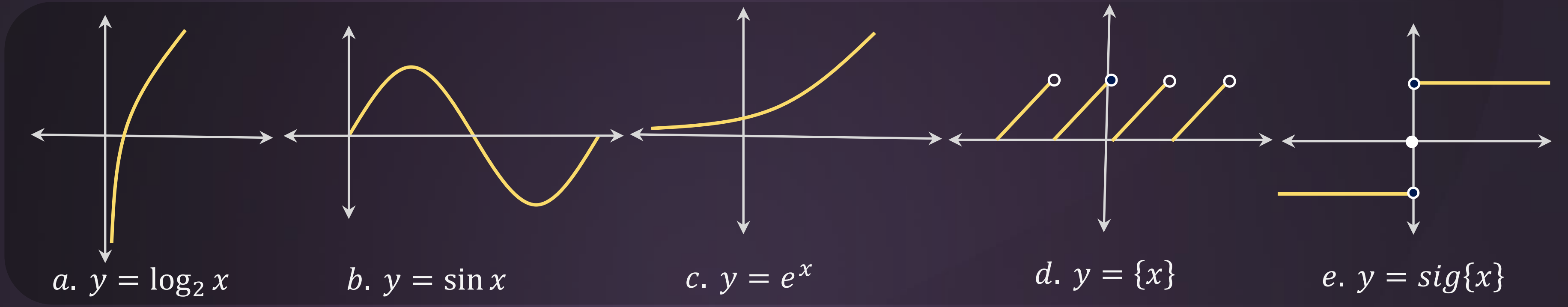
Choose the correct option:



A (a), (b) & (e) are one-one mapping

B (a) & (e) are many-one mapping

C (a) & (c) are one-one mapping

D None

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Identify the following functions as One-one or Many-one: $f(x) = \sqrt{1 - e^{\left(\frac{1}{x} - 1\right)}}$

Solution: Suppose for some $x_1, x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$\Rightarrow \sqrt{1 - e^{\left(\frac{1}{x_1} - 1\right)}} = \sqrt{1 - e^{\left(\frac{1}{x_2} - 1\right)}}$

On squaring both sides:

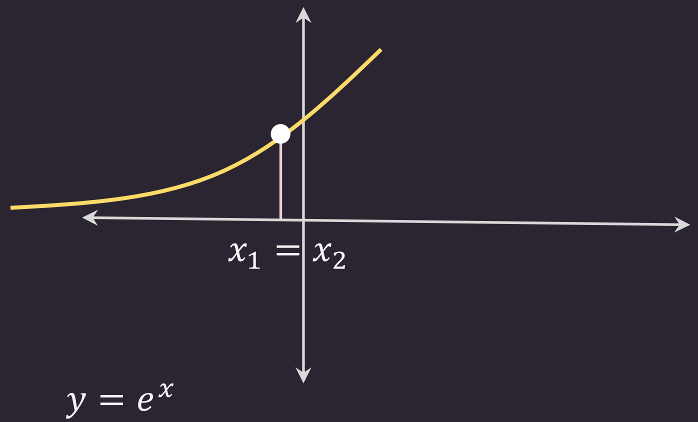
$\Rightarrow 1 - e^{\left(\frac{1}{x_1} - 1\right)} = 1 - e^{\left(\frac{1}{x_2} - 1\right)}$

$\Rightarrow e^{\left(\frac{1}{x_1} - 1\right)} = e^{\left(\frac{1}{x_2} - 1\right)}$

$\Rightarrow e^{\frac{1}{x_1}} = e^{\frac{1}{x_2}}$

$\Rightarrow x_1 = x_2$

Hence, One-one



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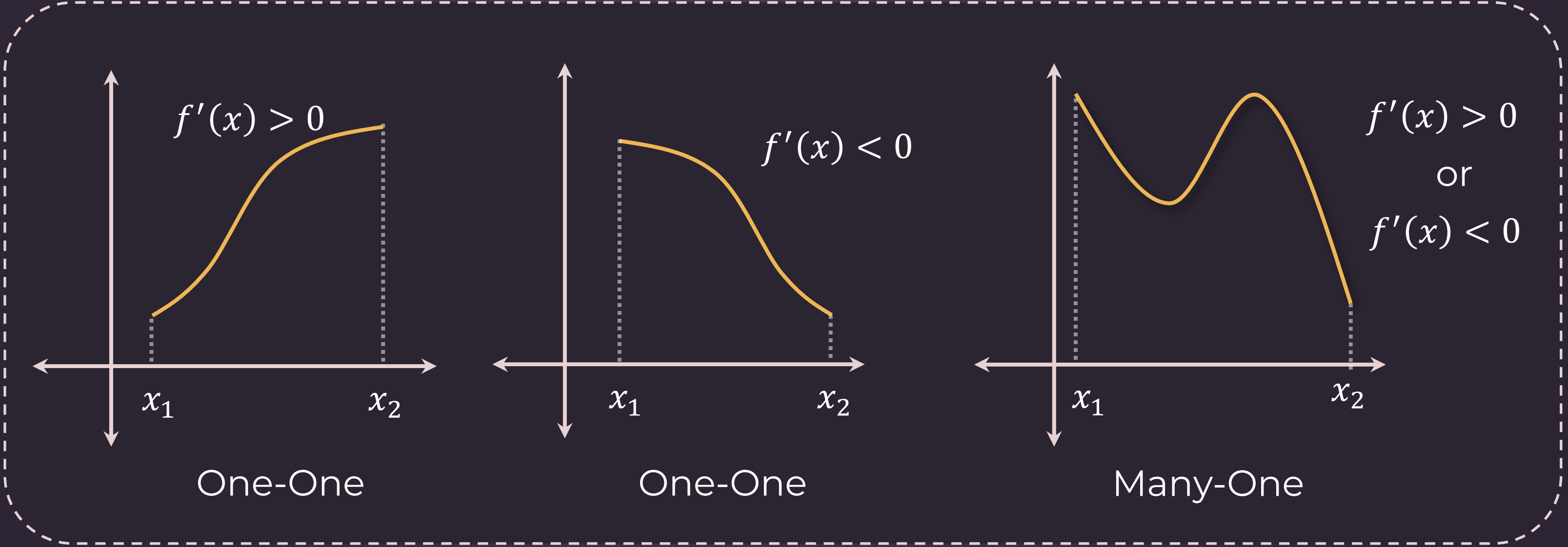
Key Takeaways

Methods to determine whether a function is ONE-ONE or MANY ONE :

Any function which is either increasing or decreasing in given domain is one-one, otherwise many Many-one.



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Determine whether a function $f(x) = \sin x + 5x$ is ONE-ONE or MANY-ONE

Solution:

$$f(x) = \sin x + 5x$$

$$f'(x) = \cos x - 5 \;\; < 0$$

$\Rightarrow$ Always decreasing $\rightarrow$ one-one

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Determine whether a function $f(x) = x^3 + x^2 + x + 1$ is ONE-ONE or MANY-ONE

Solution: $f(x) = x^3 + x^2 + x + 1$

$f'(x) = 3x^2 + 2x + 1$

$D = 2^2 - 4(3 \times 1) = -8 < 0$

Hence $f'(x) > 0$ always

$\Rightarrow f(x)$ is always increasing $\rightarrow$ one-one

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Key Takeaways

Number of functions :

Let a function $f: A \to B$

$n(A) = 4, \; n(B) = 5$

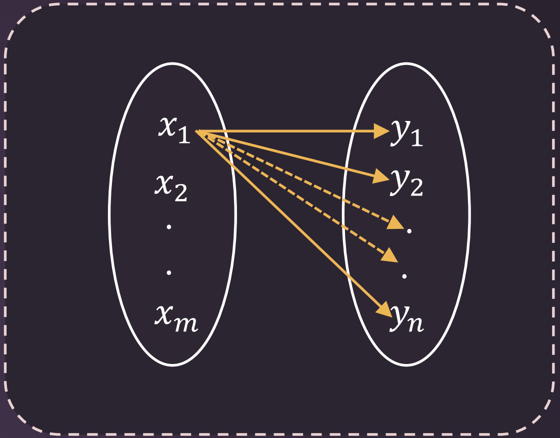
Thus, total number of function from A to B

$\Rightarrow 5 \cdot 5 \cdots 5$ (4 times)$= 5^4$

If $n(A) = m, n(B) = n \; (m < n)$

Thus, total number of functions from A to B

$= n \cdot n \cdot n \cdots n(m$ times$) = n^m$



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Key Takeaways

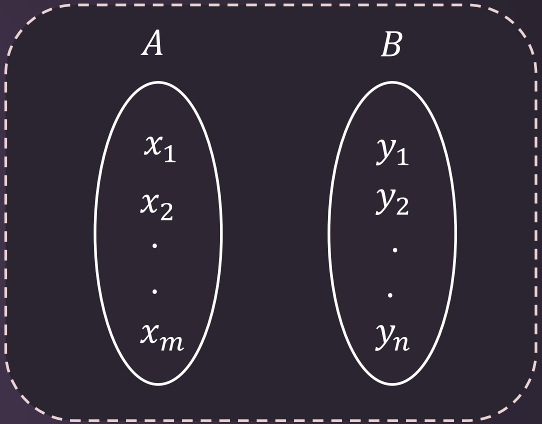
Number of ONE-ONE Mappings :

Let a function $f: A \to B$

$n(A) = 4,\ n(B) = 5$

Thus total number of function from A to B

$\Rightarrow 5(5-1)(5-2)\cdots(5-4+1) = {}^{5}P_4$



Thus, number of mappings

$$\Rightarrow \underbrace{n(n-1)(n-2)\cdots(n-m+1)}_{{}^{n}P_m,\ \text{if } n \geq m \qquad 0,\ \text{if } n < m} = n^m$$

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Number of MANY-ONE mappings :

Number of Many-ONE Function

= (Total Number of Functions) – (Number of One-One Functions)

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If $A = \{1, 2, 3, 4\}$, then the number of functions on set A, which are not ONE-ONE is:

A 240

B 248

C 232

D 256

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If $A = \{1, 2, 3, 4\}$, then the number of functions on set A, which are not ONE-ONE is:

Solution:

Number of many one functions

= Total number of functions−Number of ONE-ONE functions

$$= 4^4 - {}^4\mathrm{P}_4 \cdot 4^4$$

$$= 256 - 24$$

$$= 232$$

A. 240

B. 248

C. 232

D. 256

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Let $A=\{a,b,c\}$ and $B=\{1,2,3,4\}$. Then the number of elements in the set $C=\{f: A \rightarrow B \mid 2 \in f(A)$ and f is not one-one $\}$ is ______.

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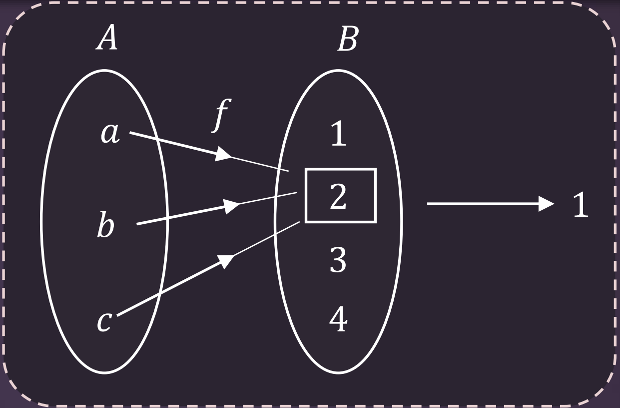
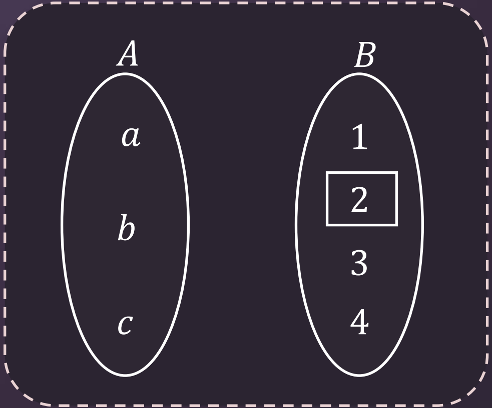
Solution:

Only one Image



When all element (a,b,c) are related to only one image

Only two Image and 2 has to be there



$^3C_1\{2^3-2\}$

To select one more image From $\{1,3,4\}$

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Let $A=\{a,b,c\}$ and $B=\{1,2,3,4\}$. Then the number of elements in the set $C=\{f: A\to B | 2\in f(A)$ and f is not one-one $\}$ is ______.

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Solution: Only one Image- 1

Only two Image and 2 has to be there- ${}^{3}C_{1}\,\{2^{3}-2\}=18$

The number of elements in set $C=1+18=19$

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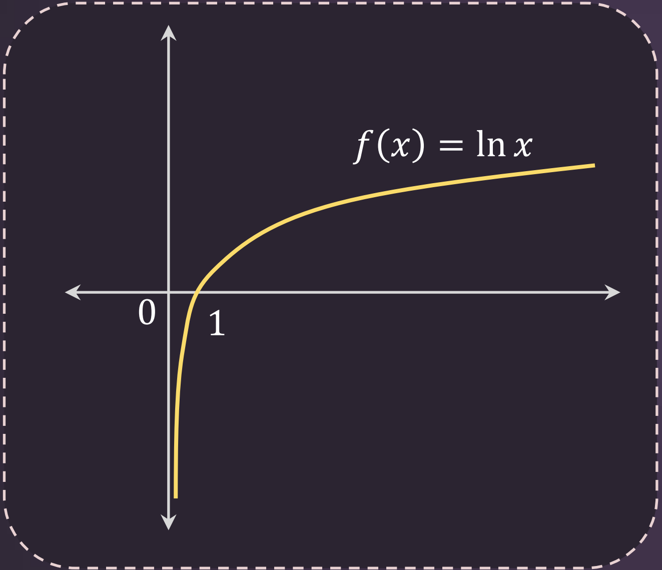
Determine whether the following function is ONE-ONE or MANY-ONE:

$f(x) = \ln x$

Solution:



Any function which is either increasing or decreasing in the whole domain is one-one, otherwise many-one.

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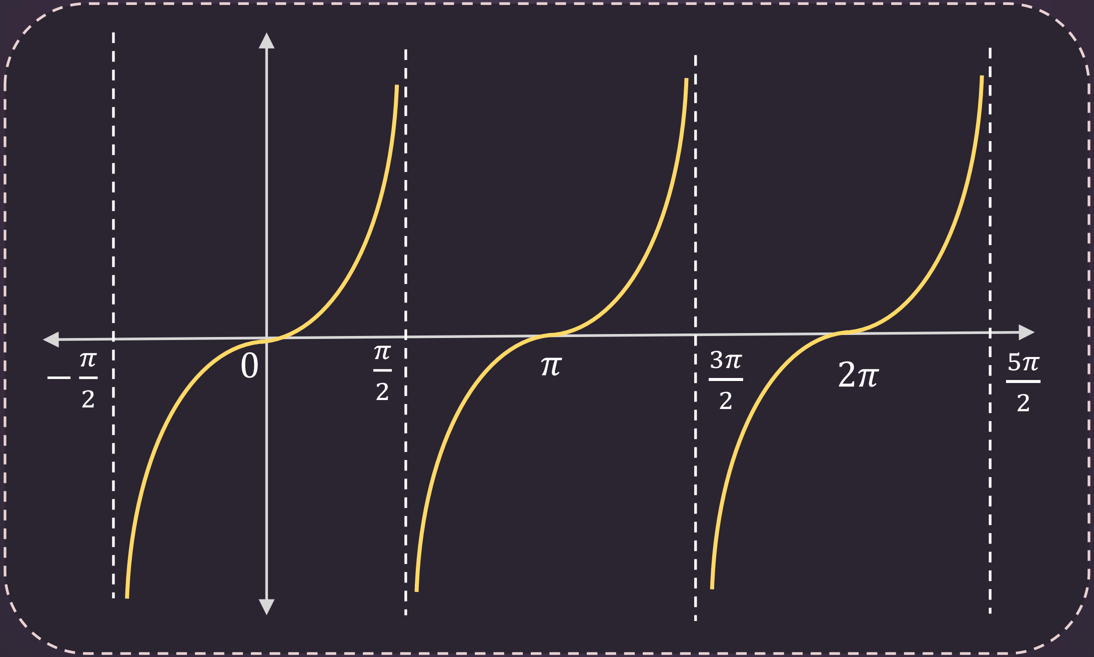
Identify the following function as One-One or Many-One:

$f(x) = 2\tan x\,;\left(\frac{\pi}{2},\frac{3\pi}{2}\right) \to R$

Solution:



One-One Function.

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Session 6

Onto & Into Functions

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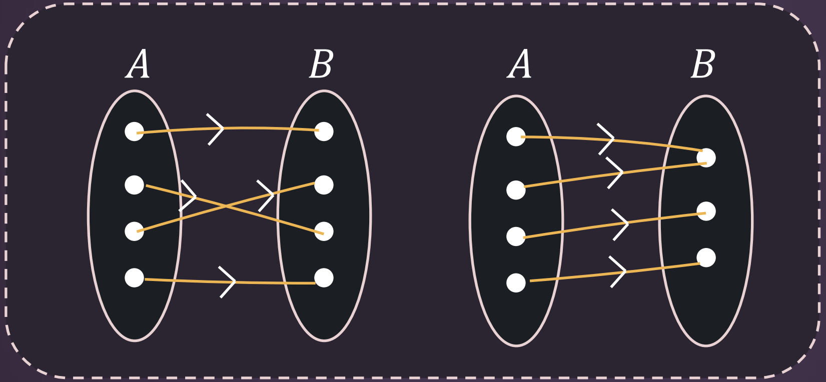
Key Takeaways

Onto function (surjective mapping)

If the function $f : A \rightarrow B$ is such that each element in B (co-domain) must have at least one pre-image in A, then we say that f is a function of A 'onto' B.



- Or, if range of f = Co – domain of f.
- $f : A \rightarrow B$ is surjective iff $\forall\, b \in B$, there exists some $a \in A$ such that $f(a) = b$.
- If not given, co-domain of function is taken as R

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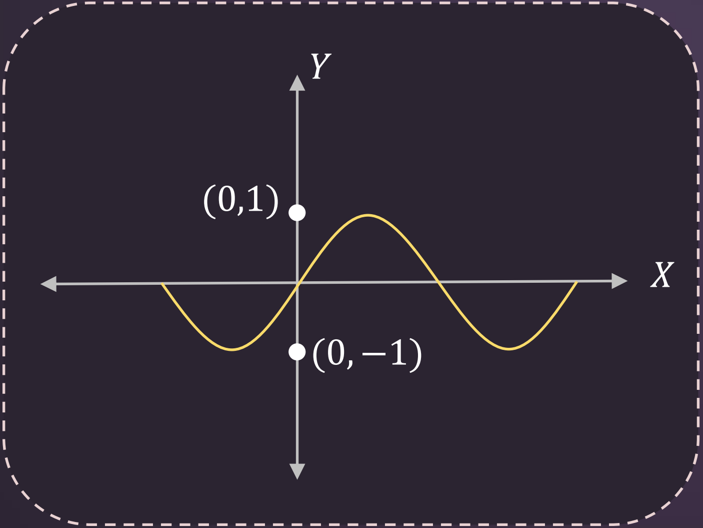
Key Takeaways

Onto function (surjective mapping)

If the function $f: A \to B$ is such that each element in B (co-domain) must have at least one pre-image in A, then we say that f is a function of A 'onto' B.

Example: $f(x) = \sin x : R \to [-1,1]$



Onto Function

Range : $[-1, 1]$

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Check $f : R \to [-1, 2]$ given by $f = \cos x$ is onto function or not.

Solution:

$f(x) = \cos x : R \to [-1,2]$

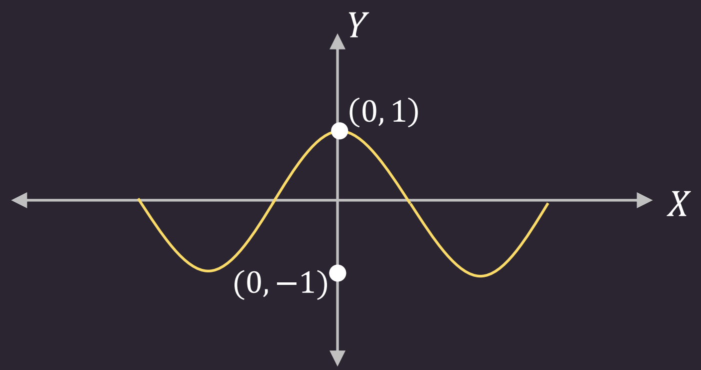
Range of $f(x) = \cos x$ is $[-1, 1]$

But given co-domain is $[-1, 2]$

Here, Range $\subset$ Co-domain

$\Rightarrow [-1, 1] \subset [-1, 2]$

Hence $f(x)$ is not onto Function



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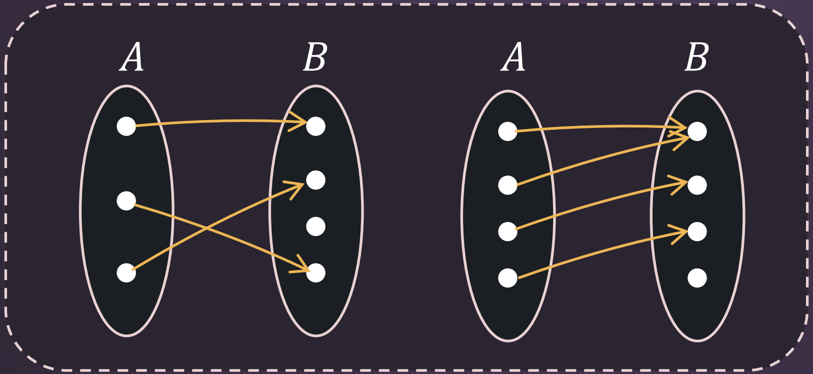
Key Takeaways

Into function

- If the function $f: A \to B$ is such that there exists at least one element in B (co-domain) which is not the image of any element in domain (A), then f is 'into'.

- For an into function range of $f \neq$ Co – domain of f and Range of $f \subset$ Co – domain of f.



- If a function is onto, it cannot be into and vice – versa.

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Key Takeaways

Into function

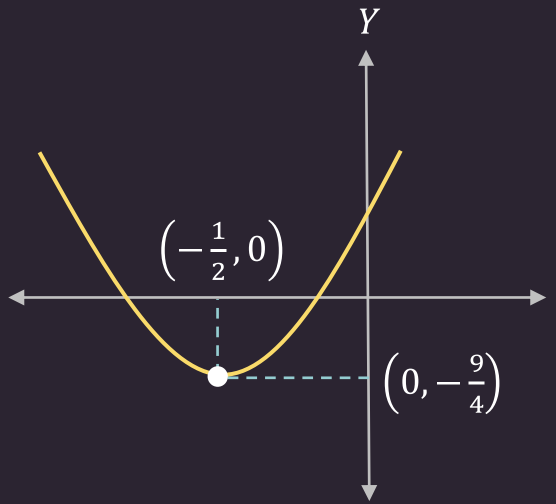
Example: $f(x) = x^2 + x - 2\,, x \in \mathbb{R}$

Solution:

Range of $f(x) = \left[-\frac{9}{4}, \infty\right)$

Thus, range $\neq$ co-domain

$\therefore$ INTO Function



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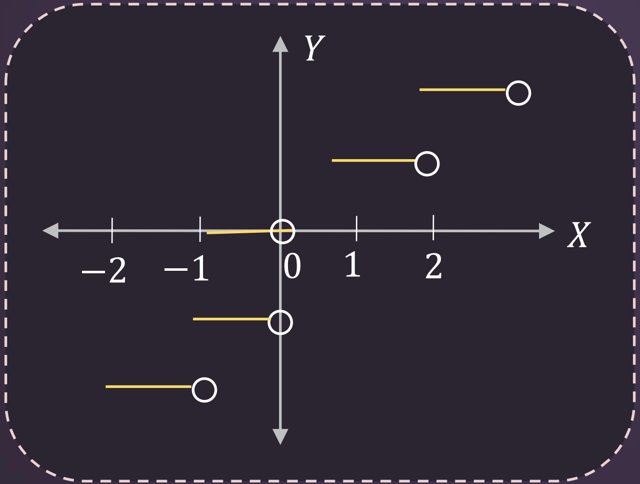
Check whether the following functions are into function or not

(i) $f(x) = [x]$, where [ ] denotes greatest integer function

(ii) $g: \mathbb{R} \to [0, 1)$ given by $g(x) = \{x\}$ where { } represents fractional part function

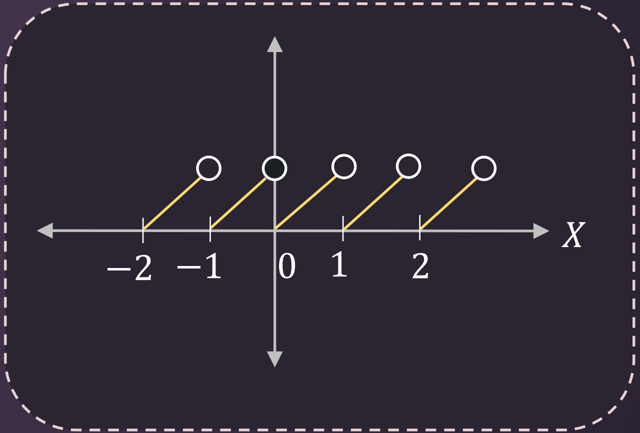
Solution: (i) Here, co-domain = R Range = Z



Range ⊆ Co-domain

⇒ $f(x)$ is into function

(ii) Here, Range of $g(x) = [0, 1)$



Range = Co-domain

$g(x)$ is onto function

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Bijection Function

If $f: A \rightarrow B$ is both an injective and a surjective function, then f is said to be bijection or one to one and onto function from A to B.

- If A, B are finite sets and $f: A \rightarrow B$ is a bijective function, then $n(A) = n(B)$
- If A, B are finite sets and $n(A) = n(B)$ then number of bijective functions defined from A to B is $n(A)!$

Note:

A function can be of one of these four types :

- One-one, onto (injective and surjective) also called as Bijective functions.
- One – one, into (injective but not surjective)
- Many – one, onto (surjective but not injective)
- Many – one, into (neither surjective nor injective)

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If the function $f : \mathbb{R} - \{-1,1\} \to A$, defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to :

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Solution:

$$f(x) = y = \frac{x^2}{1-x^2}$$

$$\Rightarrow y - yx^2 = x^2$$

$$\Rightarrow x^2 = \frac{y}{1+y} \quad (\because x^2 \geq 0)$$

$$\Rightarrow \frac{y}{1+y} \geq 0$$

$$\Rightarrow y \in (-\infty, -1) \cup [0, \infty)$$

$$\therefore A = \mathbb{R} - [-1, 0)$$

A $\mathbb{R} - [-1, 0)$

B $\mathbb{R} - (-1, 0)$

C $\mathbb{R} - \{-1\}$

D $[0, \infty)$

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If $f: \mathbb{R} \to [a, b]$, $f(x) = 2\sin x - 2\sqrt{3}\cos x + 1$ is onto function, then the value of $b - a$ is

Solution:

$$f(x) = 2\sin x - 2\sqrt{3}\cos x + 1$$

$$\left(\because a\cos\theta + b\sin\theta \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]\right)$$

$$\Rightarrow f(x) \in [-3, 5]$$

Thus, $B = [-3, 5]$

$$b - a = 8$$

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$f(x) = \sin\left(\frac{\pi x}{2}\right) : [-1, 1] \rightarrow [-1, 1]$ is ____.

A. One-one, onto Function

B. Many-one, onto Function

C. One-one, into Function

D. Many-one, into Function

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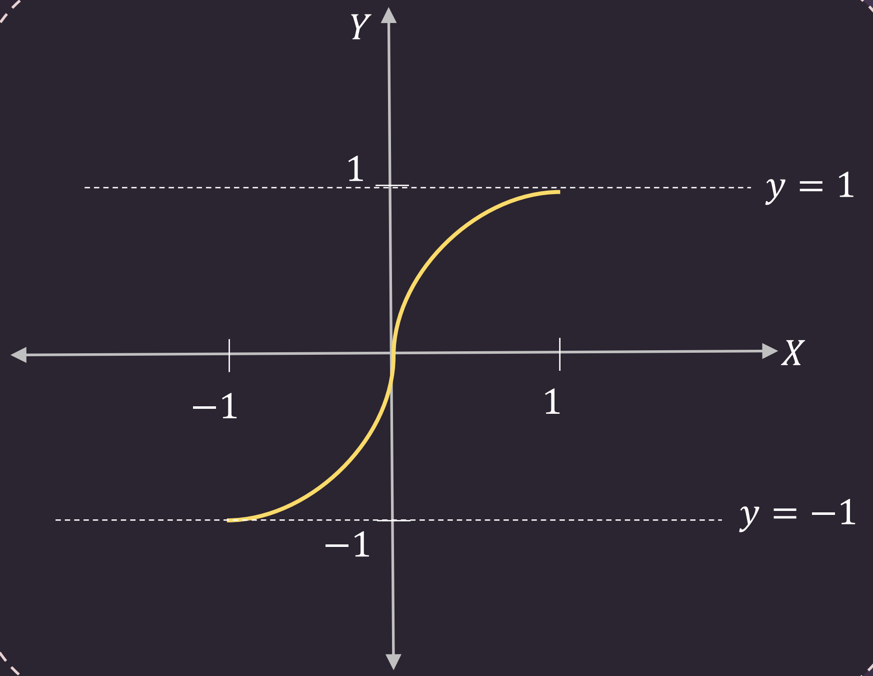
$f(x) = \sin\left(\frac{\pi x}{2}\right) : [-1, 1] \to [-1, 1]$ is ____.

Solution:



Range = Co-domain ⇒ Onto

∴ One-one, onto

A One-one, onto Function

B Many-one, onto Function

C One-one, into Function

D Many-one, into Function

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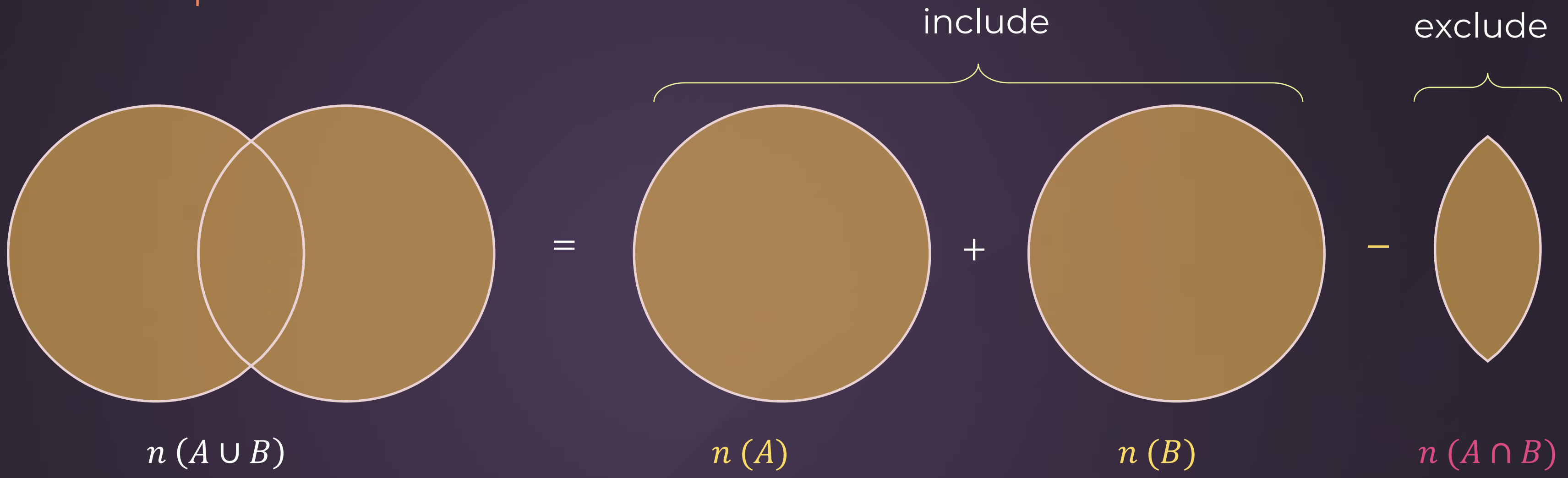
Key Takeaways

Principle of inclusion and exclusion



$$n\,(A \cup B) = n\,(A) + n\,(B) - n\,(A \cap B)$$

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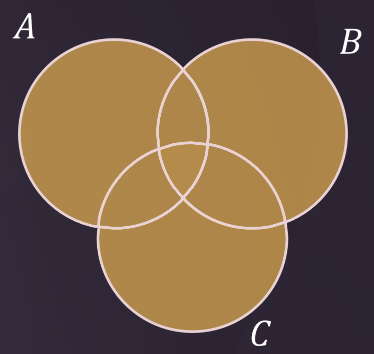
Key Takeaways

Principle of inclusion and exclusion



$$n(A \cup B \cup C)$$

$$= \underbrace{n(A) + n(B) + n(C)}_{\text{include}} - \underbrace{n(A \cap B) - n(A \cap C) - n(B \cap C)}_{\text{exclude}} + \underbrace{n(A \cap B \cap C)}_{\text{include}}$$

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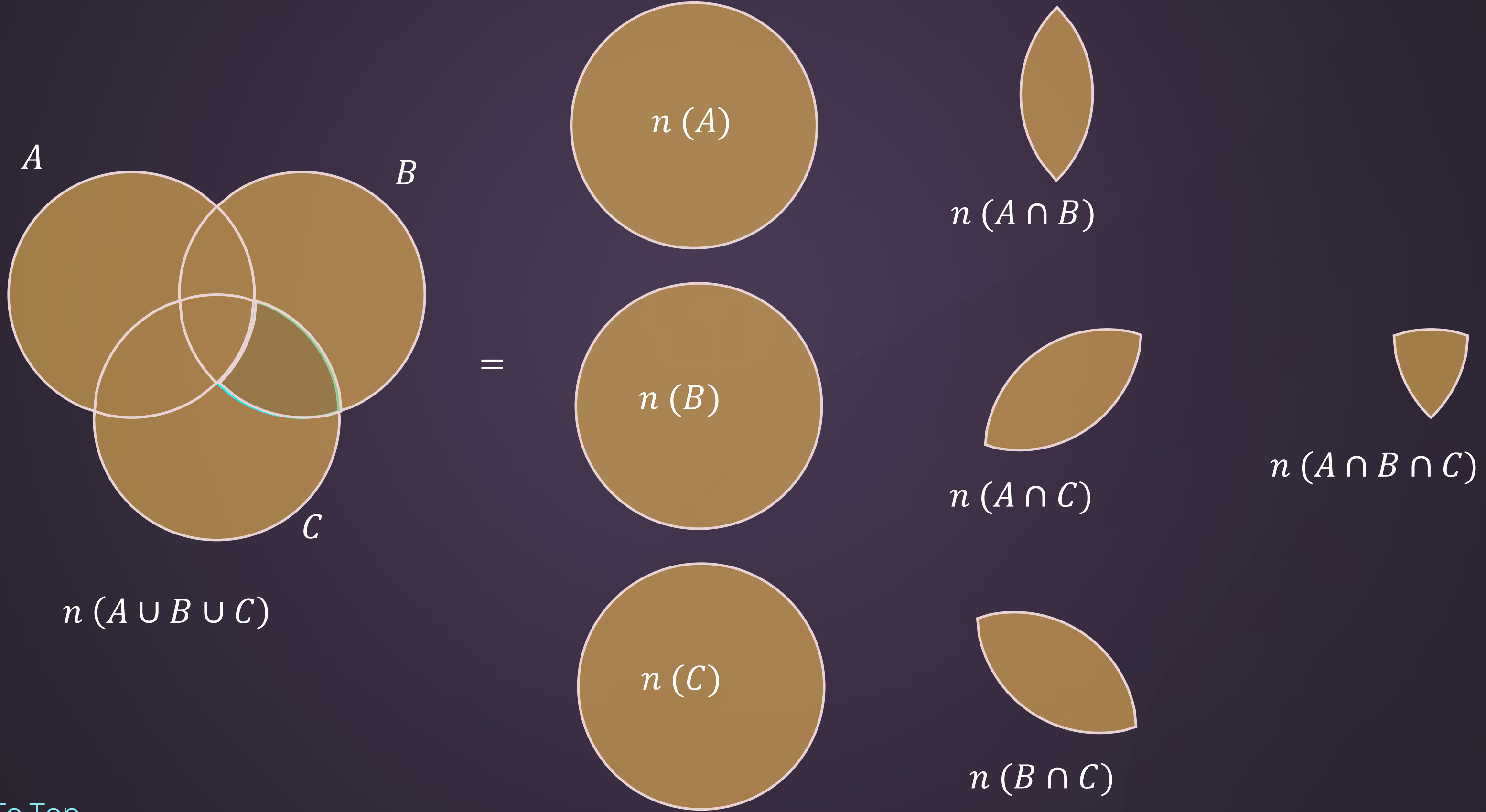
Key Takeaways

Principle of inclusion and exclusion



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Key Takeaways

Principle of inclusion and exclusion

$n(A_i) =$ Total functions when y_i excluded

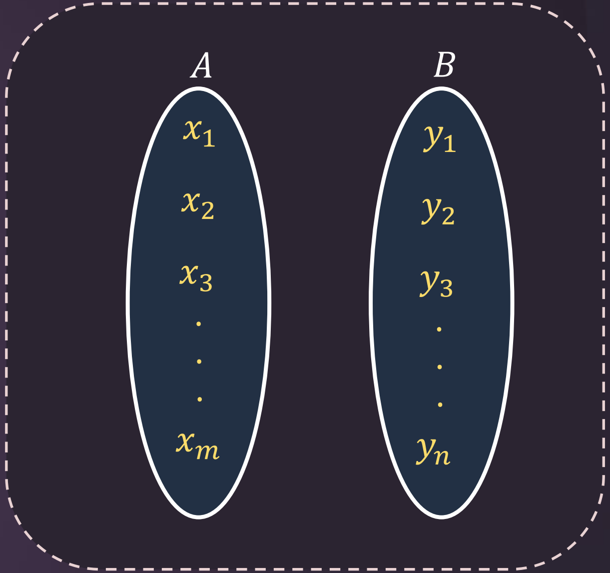
$n(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)$

$=$ Total functions where atleast one of element excluded

$$= \sum n(A_i) - \sum n(A_i \cap A_j) + \sum n(A_i \cap A_j \cap A_k) - \cdots$$

$$\cdots + (-1)^n n(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n)$$

$$= {}^nC_1 (n-1)^m - {}^nC_2 (n-2)^m + {}^nC_3 (n-3)^m - \cdots$$



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Key Takeaways

Principle of inclusion and exclusion

$n(A_i)$ = Total functions when y_i excluded

$n(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)$ = Total functions where atleast one of element excluded

$$= \sum n\left(A_i\right) - \sum n\left(A_i \cap A_j\right) + \sum n\left(A_i \cap A_j \cap A_k\right) - \cdots$$

$$\cdots + (-1)^n \, n\left(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n\right)$$

$$= \; {}^nC_1\,(n-1)^m \; - {}^nC_2\,(n-2)^m \; + {}^nC_3\,(n-3)^m - \cdots$$

Number of onto functions = Total functions $- \, n(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)$

$$= n^m - \left({}^nC_1\,(n-1)^m - {}^nC_2\,(n-2)^m + \cdots\right)$$

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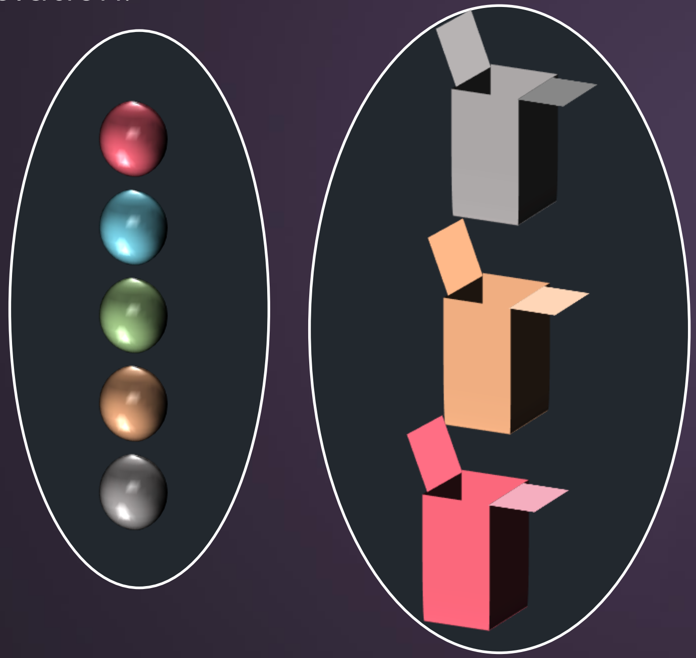
In how many ways can 5 distinct balls be distributed into 3 distinct boxes such that

(i) any number of balls can go in any number of boxes

(ii) Each box has atleast one ball in it.

Solution:



- Total ways in which all balls can go in boxes:

$= 3 \times 3 \times 3 \times 3 \times 3$

$= 3^5$

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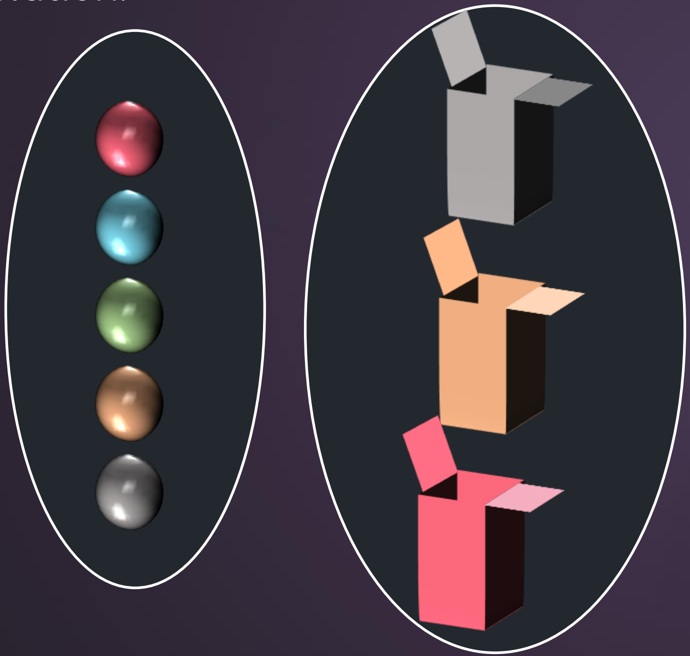
In how many ways can 5 distinct balls be distributed into 3 distinct boxes such that

(i) any number of balls can go in any number of boxes

(ii) Each box has atleast one ball in it.

Solution:



Number of onto functions $= 3^5 - {}^3C_1\, 2^5 + {}^3C_2\, 1^5$

$\boxed{= 150}$

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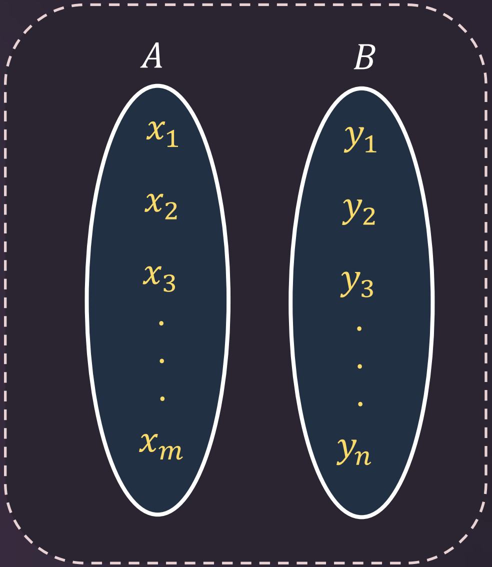
Principle of inclusion and exclusion

Number of onto functions $= \text{Total functions} - n(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)$

$$= n^m - \left({}^nC_1\,(n-1)^m - {}^nC_2\,(n-2)^m + \cdots\right)$$



Number of onto functions
$$= \begin{cases} n^m - \left({}^nC_1\,(n-1)^m - {}^nC_2\,(n-2)^m + \cdots\right), & (m > n) \\ n!\,, & (m = n) \\ 0\,, & (m < n) \end{cases}$$

Number of into functions $=$ (Total number of functions) $-$ (Number of onto functions)

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Number of Into functions that can be defined from A to B if $n(A) = 5$ and $n(B) = 3$ is

A. 3^5

B. 150

C. 5^3

D. 93

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Number of Into functions that can be defined from A to B if $n(A) = 5$ and $n(B) = 3$ is

A 3^5

B 150

C 5^3

D 93

Solution:

$n(A), n(B)$

Number of functions from A to $B = 3^5 = 243$

Number of onto functions from A to $B = 3^5 = 243$

$= 3^5 - {}^5C_1 2^5 + {}^5C_2 1^5 = 150$

$\therefore$ Total number of into functions

$= 243 - 150 = 93$

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Session 7

Even-Odd Functions and Composite Functions

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Key Takeaways

Even Function

- If $f(-x) = f(x)\ \forall\ x$ in domain of 'f', then f is said to be an even function.

Example: $f(x) = \cos x$

$$f(-x) = \cos(-x) = \cos x = f(x)$$

Example: $f(x) = |x|$

$$f(-x) = |-x| = |-1 \times x| = |-1| \times |x| = |x| = f(x)$$

Example: $f(x) = x^2 + 3$

$$f(-x) = (-x)^2 + 3 = x^2 + 3 = f(x)$$

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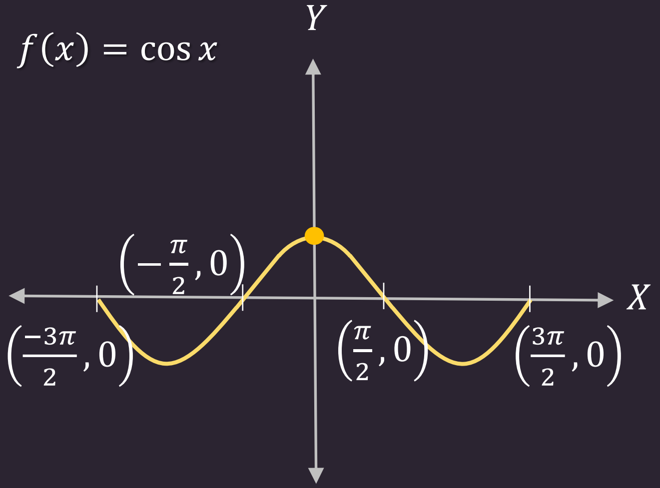
Key Takeaways

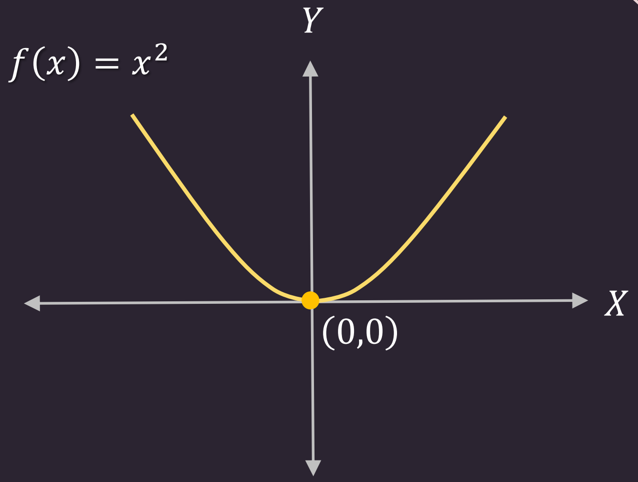
Even Function

- If $f(-x) = f(x)\ \forall\ x$ in domain of 'f', then f is said to be an even function.
- The graph of every even function is symmetric about the y– axis.

Example:



Example:



Key Takeaways

Odd Function

- If $f(-x) = -f(x)\ \forall\ x$ in domain of 'f', then f is said to be an odd function.

Example: $f(x) = x$

$$f(-x) = -x = -f(x)$$

Example: $f(x) = \sin x$

$$f(-x) = \sin(-x) = -\sin x = -f(x)$$

Example: $f(x) = \tan x$

$$f(-x) = \tan(-x) = -\tan x = -f(x)$$

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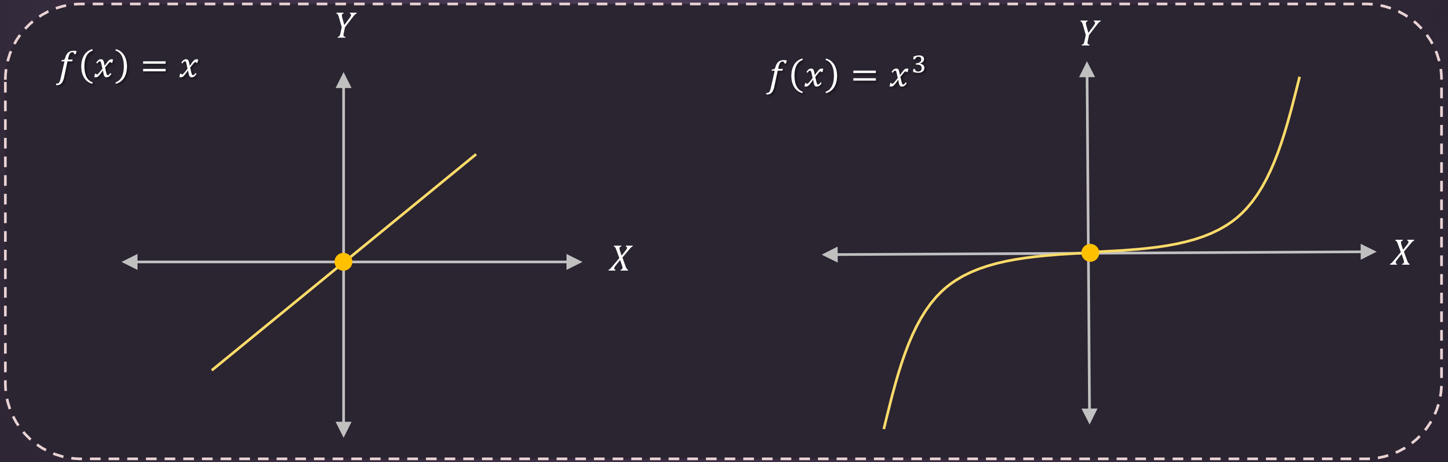
Key Takeaways

Odd Function

- If $f(-x) = -f(x)\ \forall\ x$ in domain of 'f', then f is said to be an odd function.
- The graph of an odd function is symmetric about the origin.

Example: Example:



- If an odd function is defined at $x = 0$, then $f(0) = 0$.

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Identify whether the function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$, is even or not ?

Solution: $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$

$$f(-x) = \frac{-x}{e^{-x} - 1} - \frac{x}{2} + 1 = \frac{-xe^x}{1 - e^x} - \frac{x}{2} + 1$$

$$= \frac{xe^x}{e^x - 1} - \frac{x}{2} + 1 = \frac{x(e^x - 1) + x}{e^x - 1} - \frac{x}{2} + 1$$

$$= x + \frac{x}{e^x - 1} - \frac{x}{2} + 1$$

$$= \frac{x}{e^x - 1} + \frac{x}{2} + 1$$

$\Rightarrow f(x) = f(-x)$ $\boxed{\therefore \text{Even function}}$

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Find whether the following function is even / odd or none : $f(x) = \ln\left(\frac{1+x}{1-x}\right), |x| < 1$

Solution:

$$f(x) = \ln\left(\frac{1+x}{1-x}\right), |x| < 1$$

$$f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

$$f(-x) = \ln\left(\frac{1-x}{1+x}\right) = -\ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow f(-x) = -f(x)$$

$$\boxed{\text{Hence the function is odd}}$$

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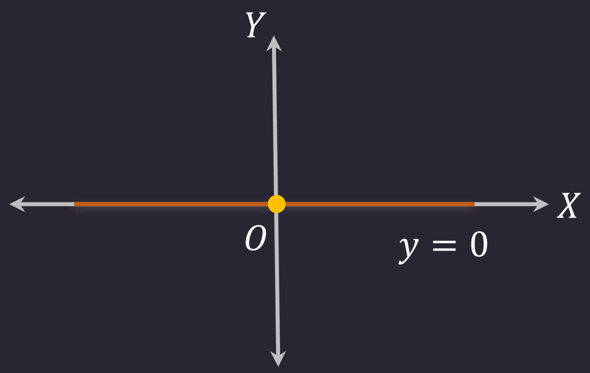
Key Takeaways

Properties of Even/Odd Function

- Some functions may neither be even nor odd.

  Example: $f(x) = 3x + 2$

- The only function which is defined on the entire number line and is even as well as odd is $f(x) = 0$.



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Key Takeaways

Properties of Even/Odd Function

- All functions (whose domain is symmetric about origin) can be expressed as sum of an even and an odd function

$$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{even}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odd}}$$

Example: Let a function $f(x) = x + e^x$, express it as sum of an even and an odd function

$$f(x) = x + e^x$$

$$\therefore f(x) = \frac{(x+e^x)+(-x+e^{-x})}{2} + \frac{(x+e^x)-(-x+e^{-x})}{2}$$

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Let $f(x) = a^x \ (a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals :

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A. $2f_1(x+y)\ f_1(x-y)$

B. $2f_1(x)\ f_1(y)$

C. $2f_1(x)\ f_2(y)$

D. $2f_1(x+y)\ f_2(x-y)$

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Let $f(x) = a^x \ (a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals :

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Solution:

$$f(x) = a^x \qquad f(x) = f_1(x) + f_2(x)$$

$$f_1(x+y) + f_1(x-y) = \frac{a^{x+y}+a^{-(x+y)}}{2} + \frac{a^{x-y}+a^{-(x-y)}}{2}$$

$$= \frac{a^x(a^y+a^{-y}) + a^{-x}(a^y+a^{-y})}{2}$$

$$= \frac{(a^y+a^{-y})(a^x+a^{-x})}{2}$$

$$= \frac{2f_1(y).2f_1(x)}{2}$$

$$= 2f_1(x).f_1(y)$$

$$\boxed{\therefore f_1(x+y) + f_1(x-y) = 2f_1(x)\ f_1(y)}$$

A) $2f_1(x+y)\ f_1(x-y)$

B) $2f_1(x)\ f_1(y)$

C) $2f_1(x)\ f_2(y)$

D) $2f_1(x+y)\ f_2(x-y)$

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Key Takeaways

Properties of Even/Odd Function

- $f(x) = x^2$, $g(x) = |x|$

  $f(x) = x^2 \downarrow$ Even, $g(x) = |x| \downarrow$ Even

$$h(x) = f(x) + g(x) = x^2 + |x|$$

$$h(-x) = (-x)^2 + |-x|$$

$$= (x)^2 + |x| = h(x) \rightarrow \text{Even}$$

$$h(x) = f(x) \times g(x) = x^2 \times |x|$$

$$h(-x) = (-x)^2 \times |-x|$$

$$= (x)^2 \times |x| = h(x) \rightarrow \text{Even}$$

f	g	$f \pm g$	$f . g$	$f/g (g \neq 0)$
Even	Even	Even	Even	Even

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Key Takeaways

Properties of Even/Odd Function

- $f(x) = x, g(x) = \sin x$

$$\underset{\text{odd}}{f(x) = x}, \quad \underset{\text{odd}}{g(x) = \sin x}$$

$$h(x) = f(x) + g(x) = x + \sin x$$

$$h(-x) = -x - \sin x$$

$$= -h(x) \rightarrow \text{odd}$$

$$p(x) = f(x) \times g(x) = x \times \sin x$$

$$p(-x) = (-x) \times (-\sin x)$$

$$= p(x) \rightarrow \text{even}$$

f	g	$f \pm g$	$f . g$	$f/g (g \neq 0)$
Even	Even	Even	Even	Even
Odd	Odd	Odd	Even	Even

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Key Takeaways

Properties of Even/Odd Function

- $f(x) = x^2, g(x) = x$

$$\underset{\text{even}}{\downarrow} \qquad \underset{\text{odd}}{\downarrow}$$

f	g	$f \pm g$	$f . g$	$f/g(g \neq 0)$
Even	Even	Even	Even	Even
Odd	Odd	Odd	Even	Even
Even	Odd	NENO	Odd	Odd

$$h(x) = f(x) + g(x) = x^2 + x$$

$$h(-x) = (-x)^2 - x$$

$$\left.\begin{aligned} &= x^2 - x \neq h(x) \\ &\neq -h(x) \end{aligned}\right\} \text{Neither even nor odd}$$

$$p(x) = f(x) \times g(x) = x^2 \times x$$

$$p(-x) = (-x)^2 \times (-x)$$

$$= -p(x) \rightarrow \text{odd}$$

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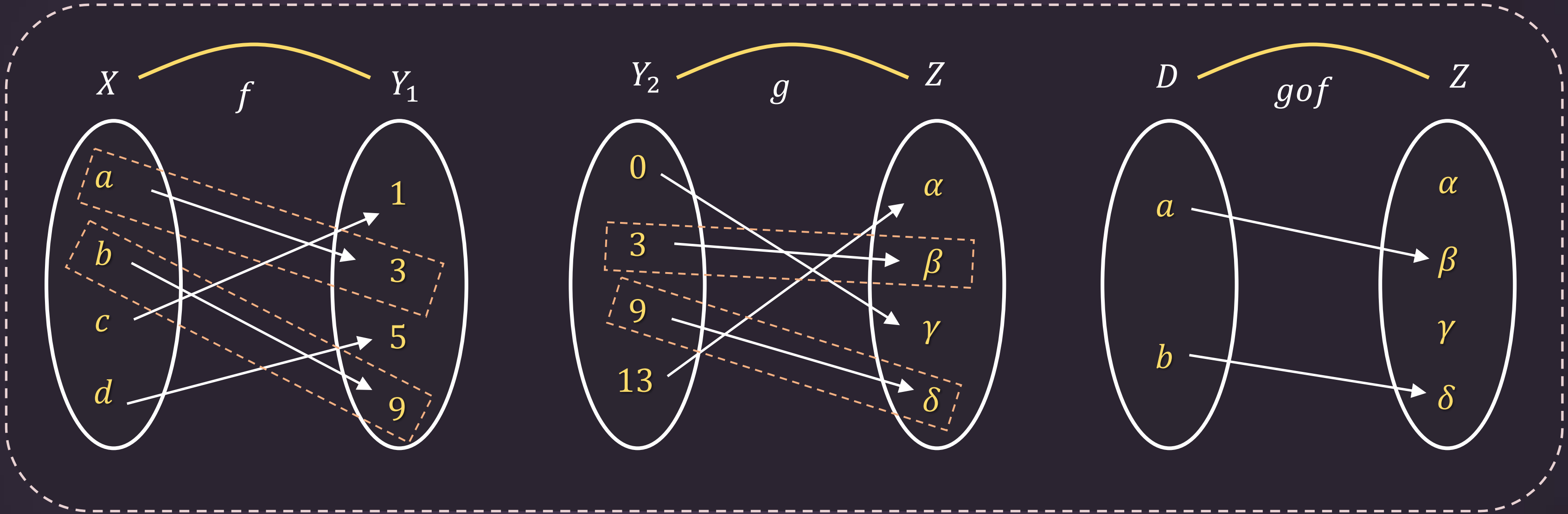
Key Takeaways

Composite Functions $\quad f : X \to Y_1 \qquad g : Y_2 \to Z$



- Here $g(f(a)) = \beta \quad g(f(c)) = g(1) =$ not defined

  $g(f(b)) = \delta \quad g(f(d)) = g(5) =$ not defined

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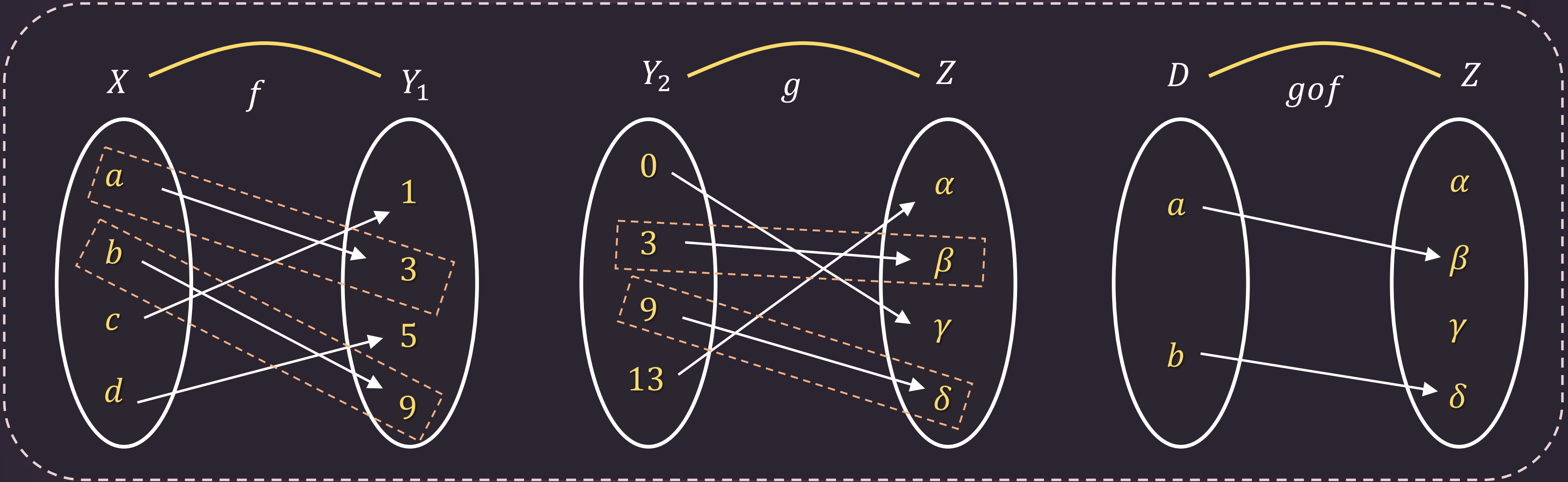
Key Takeaways

Composite Functions

$f : X \to Y_1$ $g: Y_2 \to Z$ $R_f \subseteq D_g$



So, $g(f(x))$ is defined for only those values of x for which range of f is a subset of domain of g.

$\therefore$ $f: X \to Y_1$ and $g: Y_2 \to Z$ be two functions and D is set of x such that if $x \in X$, then $f(x) \in Y_2$

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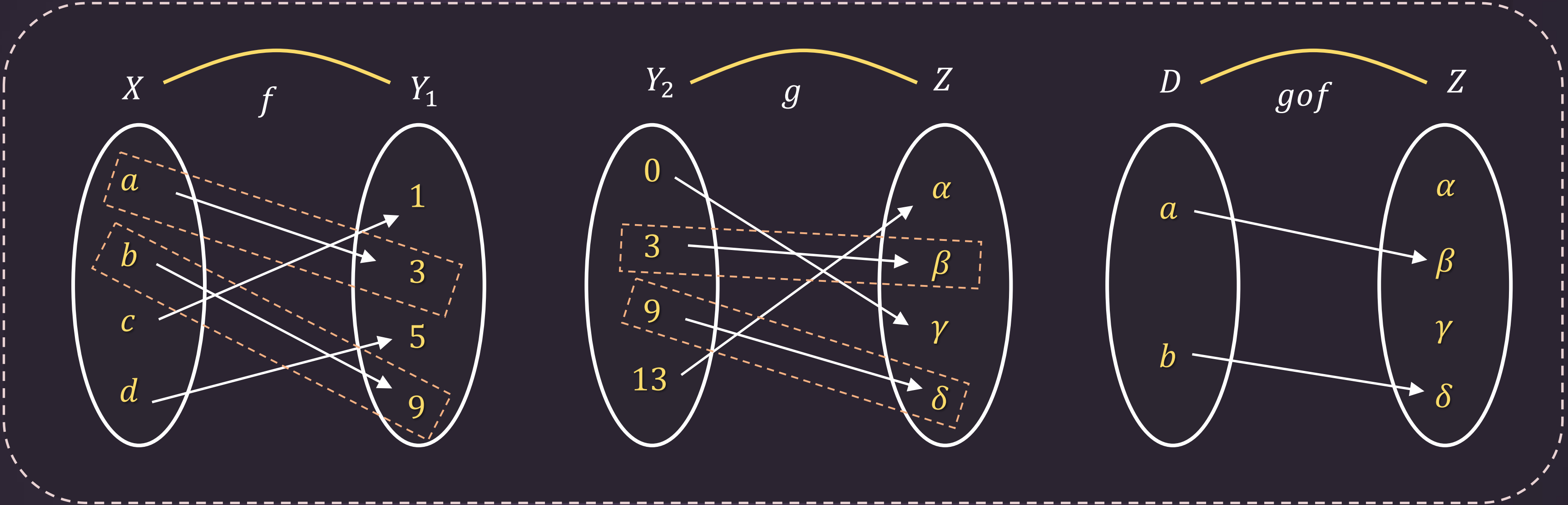
Key Takeaways

Composite Functions



If $D \neq \emptyset$, then the function h defined by $h(x) = g(f(x))$ is called composite function of g and f and is denoted by gof. It is also called as function of a function.

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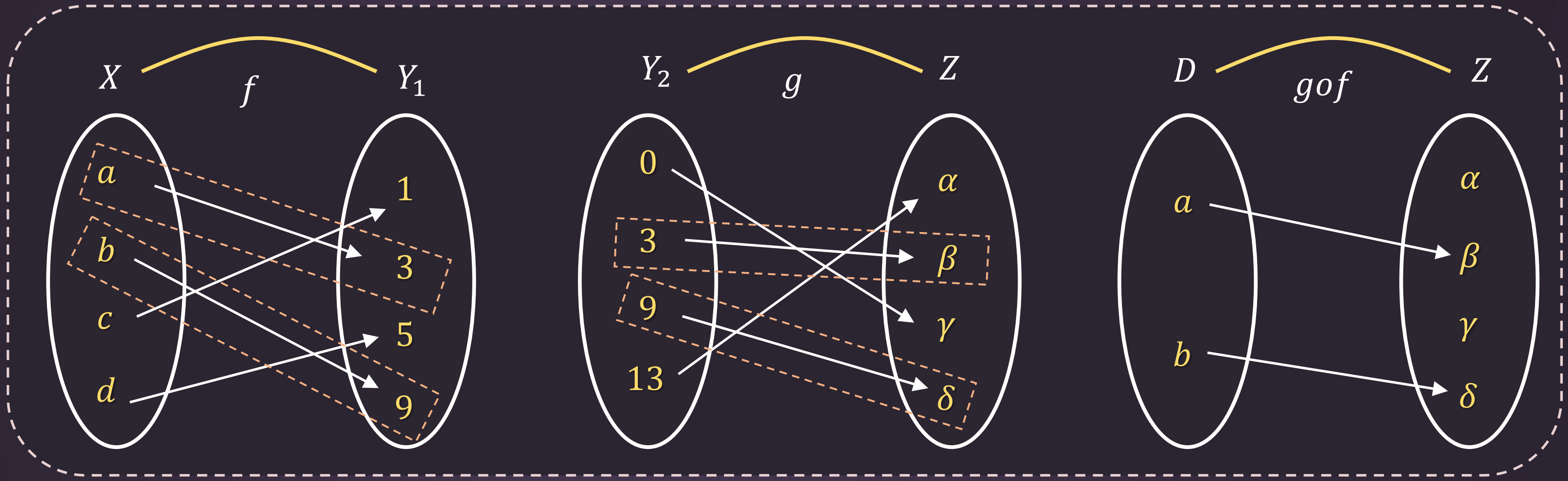
Key Takeaways

Composite Functions

$D_{gof} : \{a , b\}$ $R_{gof} : \{\beta , \delta\}$



Note : Domain of $g\,of$ is D which is subset of X (the domain of f).
Range of $g\,of$ is a subset of range of g. If $D = X$, then $f(x) \subseteq Y_2$

Pictorially , $gof(x)$ can be viewed as -

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(i) Two functions f and g defined from $\mathbb{R} \to \mathbb{R}$ such that $f(x) = x + 1$, $g(x) = x + 2$, then find a) $g\big(f(x)\big)$ b) $f\big(g(x)\big)$

(ii) Two functions f and g defined from $\mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2$, $g(x) = x + 1$, then show that $f\big(g(x)\big) \neq g\big(f(x)\big)$

Solution: (i) $g\big(f(x)\big) = \big(f(x)\big) + 2 = (x + 1) + 2 = (x + 3)$

$$f\big(g(x)\big) = \big(g(x)\big) + 1 = (x + 2) + 1 = (x + 3)$$

$$\Rightarrow gof(x) = fog(x)$$

$$(ii)\ g\big(f(x)\big) = \big(f(x)\big) + 1 = x^2 + 1$$

$$f\big(g(x)\big) = \big(g(x)\big)^2 = (x + 1)^2 = x^2 + 2x + 1$$

$$\Rightarrow gof(x) \neq fog(x)$$

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Composite Functions

Note :

The composition of functions are not commutative in general i.e., two functions f and g are such that if fog and gof are both defined, then in general $fog \neq gof$.

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If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

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A. $2f(x)$

B. $(f(x))^2$

C. $2f(x^2)$

D. $-2f(x)$

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If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

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Solution: $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$ Let $g(x) = \frac{2x}{1+x^2}$

Then $f(g(x)) = \log_e\left(\frac{1-g(x)}{1+g(x)}\right)$

$$= \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right) = \log_e\left(\frac{(1-x)^2}{(1+x)^2}\right)$$

$$\therefore\ f(g(x)) = 2\log_e\left(\frac{1-x}{1+x}\right) = 2f(x)$$

A $2f(x)$

B $(f(x))^2$

C $2f(x^2)$

D $-2f(x)$

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Key Takeaways

Composite Functions

The composition of functions are associative i.e. if three functions f, g, h are such that $fo(goh)$ and $(fog)oh$ are defined , then $fo(goh) = (fog)oh$

Example: Let $f(x) = x, g(x) = \sin x$, $h(x) = e^x$, domain of f, g, h is $\mathbb{R}$

$$fo(goh)(x) = fo\big(g(\mathrm{e}^x)\big) = f(\sin e^x) = \sin e^x$$

$$(fog)oh(x) = (\ \sin\big(h(x)\big) = \sin e^x$$

$$\therefore\ fo(goh) = (fog)oh$$

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Session 8

Composite Functions and Periodic Functions

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Rahul wishes to order a laptop from BuyCart.

He has two coupons with the following discounts.

1) 30% off on your first purchase.

2) Rs. 5000 off on your first purchase.

If he can avail both the coupons, which coupon will he apply first and why ?

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BUY CART

All | laptop

Hello, Rahul
Account & Lists

Returns
& Orders

Hello, Rahul

Trending

- Best Sellers
- New Releases
- Movers and Shakers

Digital Devices

- Echo & Alexa
- Fire TV
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Shop By Department

- Mobiles, Computers
- TV, Appliances
- Gadgets

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HT Studypad Core i3 15.6 HD Thin and Light Laptop (8GB/1TB HDD/Windows 11/MS Office 2021/2Yr Warranty/Platinum Grey/1.7Kg)

665 ratings | 93 answered questions

M.R.P.: ~~₹57,290.00~~
Price: ₹ 50000

EMI starts at ₹1,647. No Cost EMI available EMI options

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Upto ₹1,500.00 discount on select Credit Cards
2 offers ›

Cashback
Get rewards up t
₹2000 on card
approval + 5% ba
4 offers ›

Add to Cart

Buy Now

Secure transaction

Add gift options

SAVE 30% OFF
your first purchase
View Details

₹5000 OFF
your first purchase
View Details

BUY CART

All | laptop

Hello, Rahul
Account & Lists

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& Orders

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Trending

- Best Sellers
- New Releases
- Movers and Shakers

Digital Devices

- Echo & Alexa
- Fire TV
- Videos

Shop By Department

- Mobiles, Computers
- TV, Appliances
- Gadgets

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Add to Cart | Buy Now

Secure transaction

Add gift options

SAVE 30% OFF
your first purchase

View Details

₹5000 OFF
your first purchase

View Details

Let x = Price of laptop

$$f(x) = 0.70x\,; \quad g(x) = x - 5000$$

Option 1 :

$$h(x) = f(g(x))$$

Option 2 :

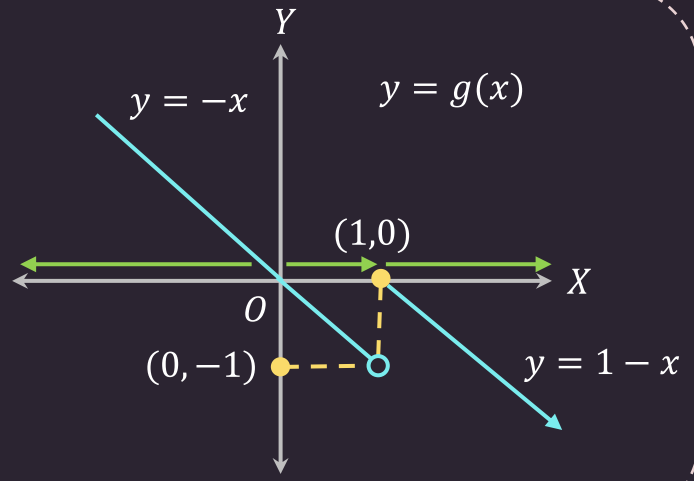
$$k(x) = g(f(x))$$

If $f(x)=\begin{cases}1-x\ ,x\le 0\\ x^2\ ,x>0\end{cases}$ $g(x)=\begin{cases}-x\ ,x<1\\ 1-x\ ,x\ge 1\end{cases}$, then find $fog(x)$

Solution:

$$fog(x)=\begin{cases}1-g(x)\ \ ,g(x)\le 0\\ (g(x))^2,g(x)>0\end{cases}$$

$$fog(x)=\begin{cases}1-(-x)\ ,x\in[0,1)\\ 1-(1-x),\ x\ge 1\\ (-x)^2,x<0\end{cases}$$



$$\therefore\ fog(x)=\begin{cases}(x)^2,x\in(-\infty,0)\\ 1+x\ ,x\in[0,1)\\ x,\ x\in[1,\infty)\end{cases}$$

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Key Takeaways

Properties of Composite Function

- If f and g are one – one , then gof if defined will be one – one.
- If f and g are bijections and gof is defined , then gof will be a bijection iff range of f is equal to domain of g.

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Key Takeaways

Periodic Functions :

- Mathematically, a function $f(x)$ is said to be periodic function if $\exists$ a positive real number T, such that

$$\boxed{f(x+T) = f(x), \forall\, x \in \text{domain of } 'f'\ ; T > 0}$$

- Here T is called period of function f and smallest value of T is called fundamental period.

Note :

> Domain of periodic function should not be restricted (bounded).

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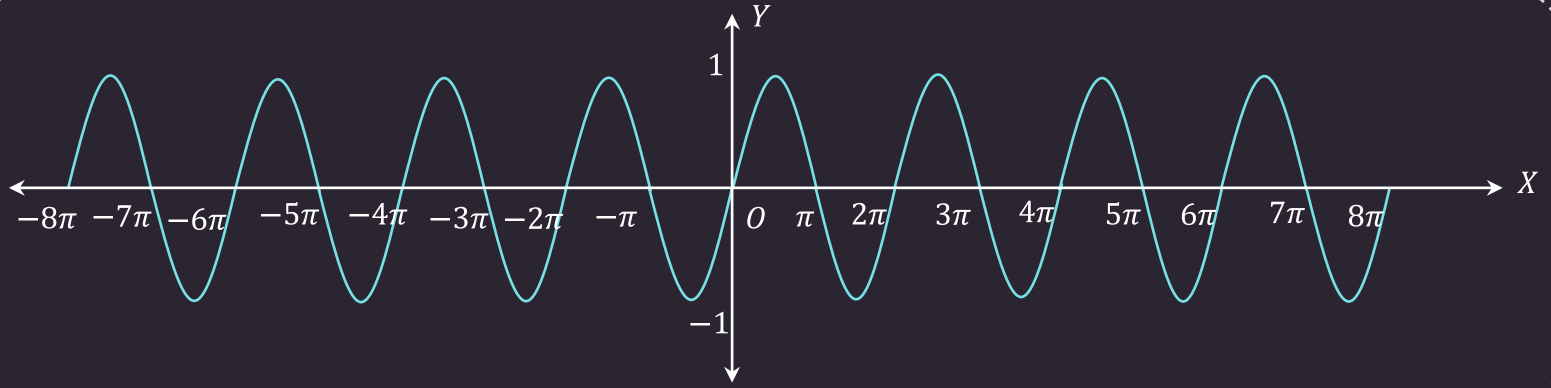
Key Takeaways

Periodic Functions :

- Example : $f(x) = \sin x$



$$f(x+T) = f(x) \Rightarrow \sin(x+T) = \sin x$$

$$\Rightarrow \sin(x+T) - \sin x = 0 \Rightarrow 2\sin\left(\frac{T}{2}\right)\cos\left(\frac{2x+T}{2}\right) = 0$$

$$\Rightarrow \frac{T}{2} = n\pi \Rightarrow T = 2n\pi, n \in \mathbb{I}$$ Thus, fundamental period $= 2\pi$

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Periodic Functions

Note :

If a function is dis-continuous, it’s discontinuity should repeat after a particular interval for the function to be periodic.

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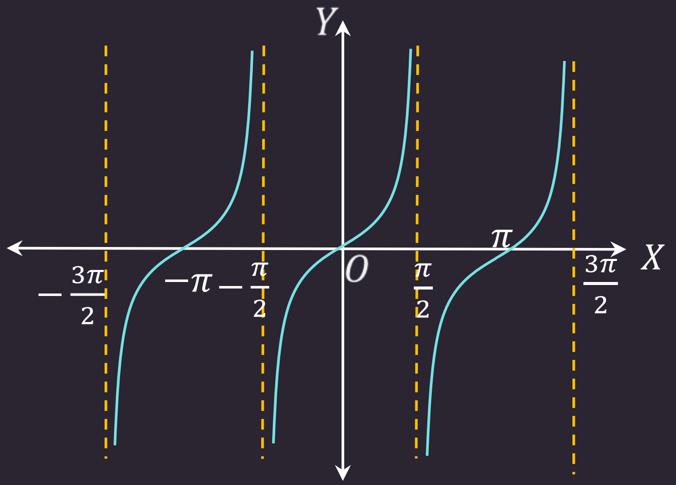
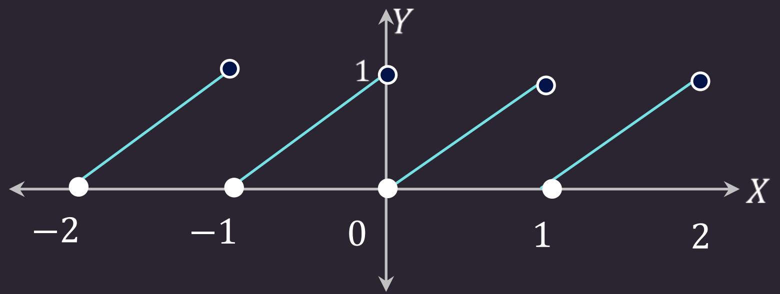
Find the period of function.

i) $f(x) = \tan x$ *ii)* $f(x) = \{x\}$ where $\{.\}$ denotes fractional part function.

Solution: *i)* $f(x+T) = f(x)$ *ii)* $f(x) = \{x\}$





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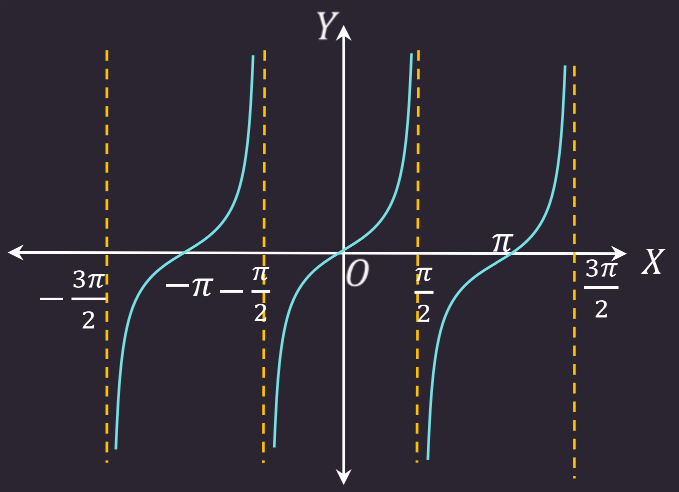
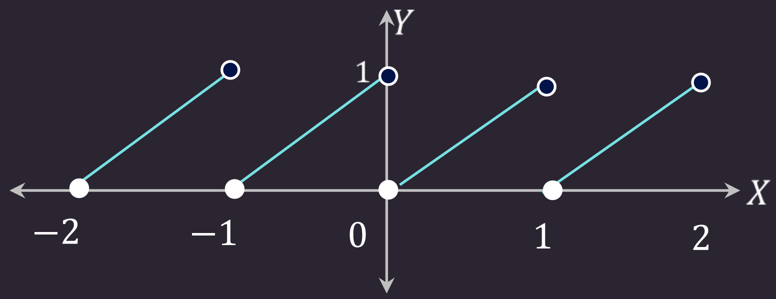
Find the period of function.

$i)\ f(x) = \tan x$ $\quad ii)\ f(x) = \{x\}$ where $\{.\}$ denotes fractional part function.

Solution: $i)\ f(x+T) = f(x)$ $\quad \tan(x+\pi) = \tan x$ $\quad ii)\ f(x) = \{x\}$



Period is π



Period is 1

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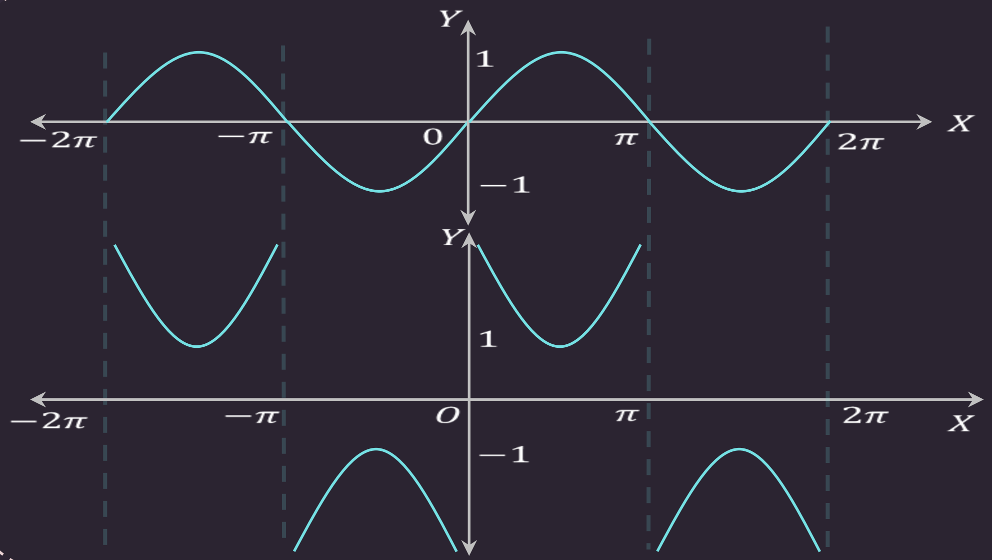
Key Takeaways

Properties of Periodic Functions :

- If a function $f(x)$ has a period T, then $\frac{1}{f(x)}, \left(f(x)\right)^n \ (n \in \mathbb{N}), |f(x)|, \sqrt{f(x)}$ also has a period T (T may or may not be fundamental period.)

Example : $y = \operatorname{cosec} x$



Fundamental period $= 2\pi$

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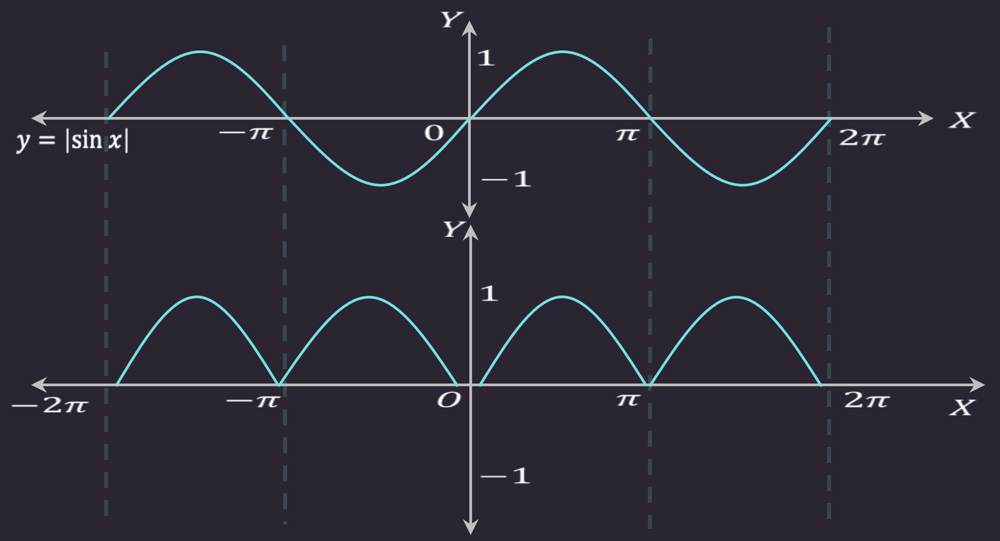
Key Takeaways

Example : $y = |\sin x|$

Fundamental period $= \pi$



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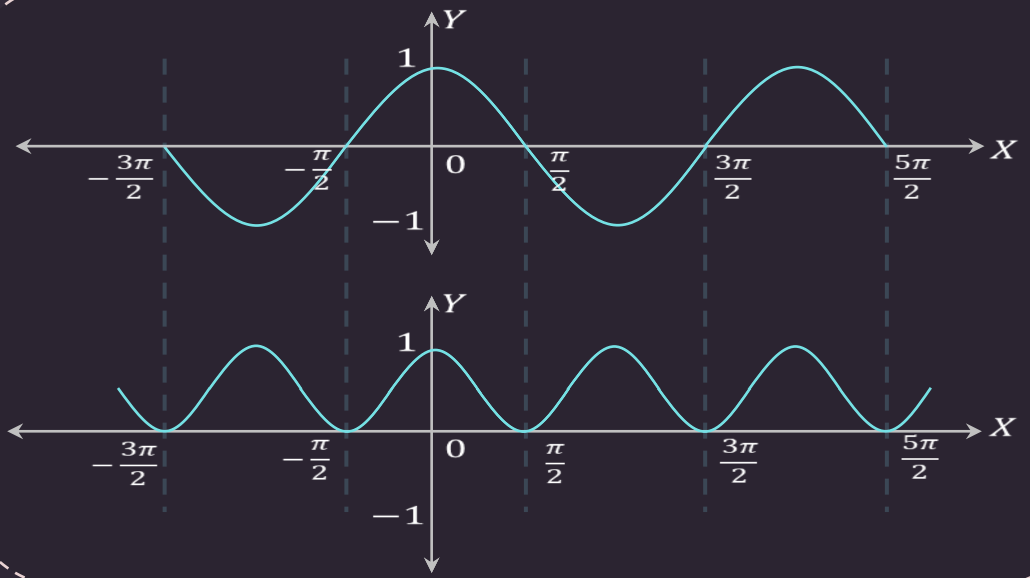
Key Takeaways

Example : $y = \cos^2 x$

Fundamental period $= \pi$



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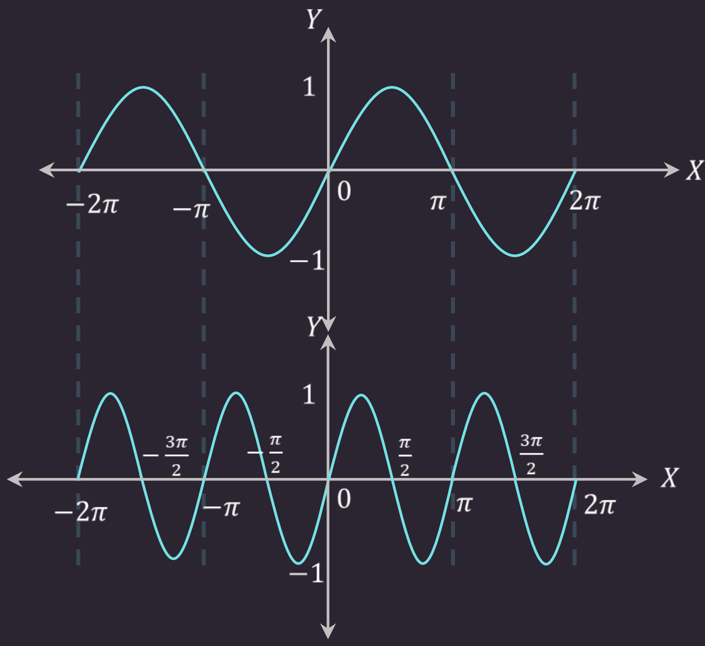
Key Takeaways

Properties of Periodic Functions :

- If a function $f(x)$ has a period T, then $f(ax+b)$ has the period $\frac{T}{|a|}$.
- For $y = \sin x$, fundamental period $= 2\pi$
- For $y = \sin 2x$, fundamental period $= \pi$



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Properties of Periodic Functions

- Every constant function defined for unbounded domain is always periodic with no fundamental period.

Example :

- $f(x) = \sin^2 x + \cos^2 x$, domain is $\mathbb{R}$

$\Rightarrow f(x) = 1$

Periodic with no fundamental period.

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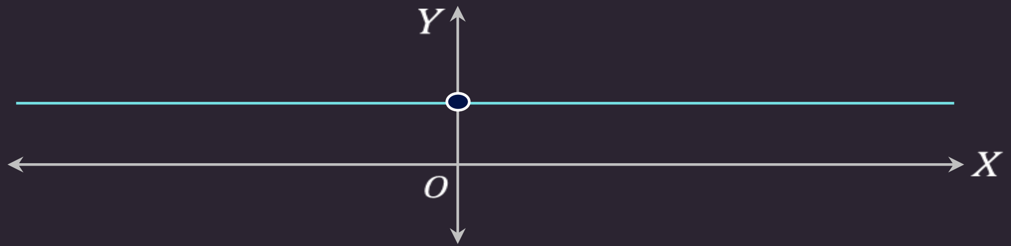
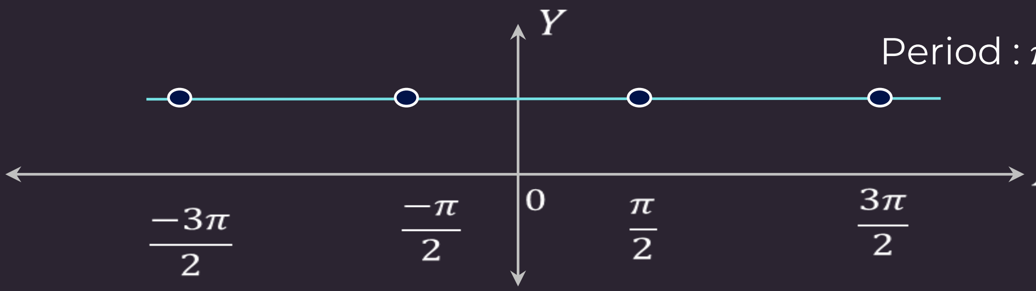
Find the period of function. $i)\ f(x) = x \cdot \frac{1}{x}$ $\quad ii)\ f(x) = \cos x \cdot \sec x$

Solution: $\quad i)\ f(x) = x \cdot \frac{1}{x}$ (domain $x \in \mathbb{R} - \{0\}$)



Not periodic

$ii)\ f(x) = \cos x \cdot \sec x$ (domain $x \in \mathbb{R} - \left\{(2n+1)\left(\frac{\pi}{2}\right), n \in \mathbb{Z}\right\}$)



Period : π

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Fundamental period of $y = \left\{\frac{x}{3}\right\}$, where $\{\cdot\}$ denotes fractional part function is

A. 2

B. $\frac{1}{2}$

C. 3

D. $\frac{1}{3}$

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Fundamental period of $y = \left\{\frac{x}{3}\right\}$, where $\{\cdot\}$ denotes fractional part function is

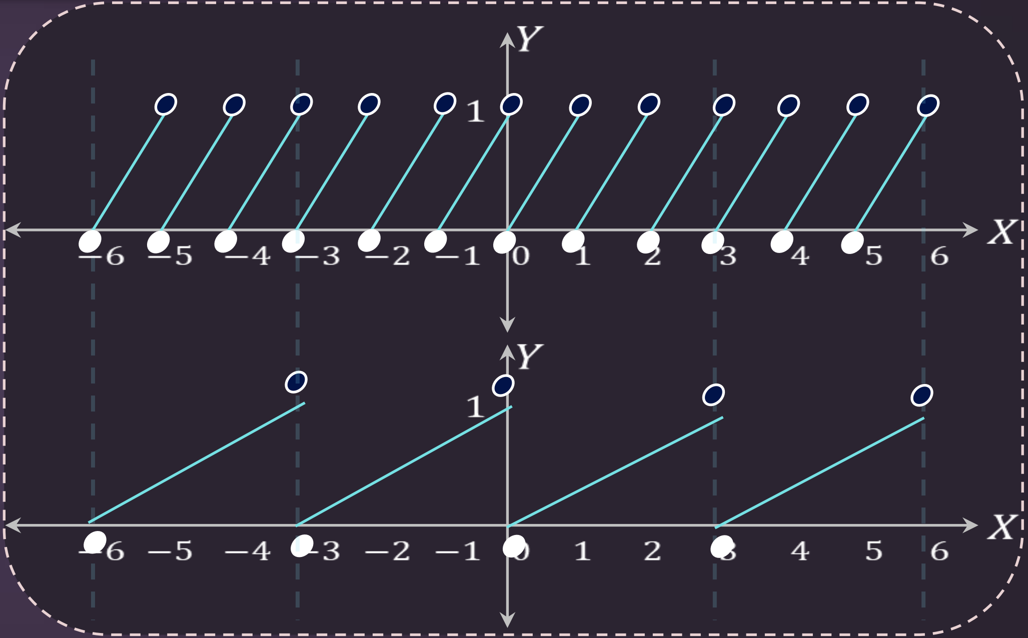
Solution:

If a function $f(x)$ has a period T,

then $f(ax+b)$ has the period $\frac{T}{|a|}$.

For $\{x\}$, fundamental period $= 1$

For $\left\{\frac{x}{3}\right\}$, fundamental period $= 3$



A) 2

B) $\frac{1}{2}$

C) 3

D) $\frac{1}{3}$

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Session 9

Inverse Functions & Binary operations

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Key Takeaways

Properties of Periodic Functions :

- If $f(x)$ has a period T_1 and $g(x)$ has a period T_2, then

  $f(x) \pm g(x), f(x) \cdot g(x)$ or $\frac{f(x)}{g(x)}$ is L.C.M of T_1 and T_2 (provided L.C.M exists).

  L.C.M of $\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{\text{L.C.M }(a,c)}{\text{H.C.F }(b,d)}$

  However, L.C.M need not be fundamental period.

- If L.C.M does not exists, then $f(x) \pm g(x), f(x) \cdot g(x)$ or $\frac{f(x)}{g(x)}$

  is non-periodic or aperiodic.

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Find the period of function. $i)\ f(x) = \sin\left(\frac{3x}{2}\right) + \cos\left(\frac{9x}{4}\right)$ $ii)\ f(x) = |\sin x| + |\cos x|$

Solution:

$i)\ f(ax+b)$ has the period $\frac{T}{|a|}$

$$\sin\frac{3x}{2} \to T_1 = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$$

$$\cos\frac{9x}{4} \to T_2 = \frac{2\pi}{\frac{9}{4}} = \frac{8\pi}{9}$$

L.C.M of $\frac{4\pi}{3}, \frac{8\pi}{9}$ $\Rightarrow$ L.C.M of $\left(\frac{4}{3}, \frac{8}{9}\right)\pi$

$$\boxed{\left(\frac{\text{L.C.M }(4,8)}{\text{H.C.F }(3,9)}\right)\pi = \frac{8}{3}\pi}$$

$ii)$ Period of $f(x) \pm g(x)$ is L.C.M of (T_1, T_2)

L.C.M of $(\pi, \pi) = \pi$

$\frac{\pi}{2}$ may also be period.

$$f\left(x+\frac{\pi}{2}\right) = \left|\sin\left(x+\frac{\pi}{2}\right)\right| + \left|\cos\left(x+\frac{\pi}{2}\right)\right|$$

$$= |\cos x| + |-\sin x|$$

$$= f(x)$$

$\boxed{\text{Period is } \frac{\pi}{2}.}$

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Key Takeaways

Properties of Periodic Functions :

- If g is a function such that gof is defined on the domain of f and f is periodic with T, then gof is also periodic with T as one of its period.

Example:

- $h(x) = \{\cos x\}$, where $\{\cdot\}$ is fractional part function

  Let $f(x) = \cos x\,, g(x) = \{x\}$ then $h(x) = g\big(f(x)\big)$, period 2π

- $h(x) = \cos\{x\}$, where $\{\cdot\}$ is fractional part function.

  Let $f(x) = \cos x\,, g(x) = \{x\}$ then $h(x) = f\big(g(x)\big)$, period 1

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Key Takeaways

Properties of Periodic Functions :

- If g is a function such that gof is defined on the domain of f and f is periodic with T, then gof is also periodic with T as one of its period.

Note :

- If g is a function such that gof is defined on the domain of f and f is aperiodic, then gof may or may not be periodic.

Example :

$h(x) = \cos(x + \sin x)$

$h(x) = h(x + 2\pi)$

$\Rightarrow$ period of $h(x)$ is 2π

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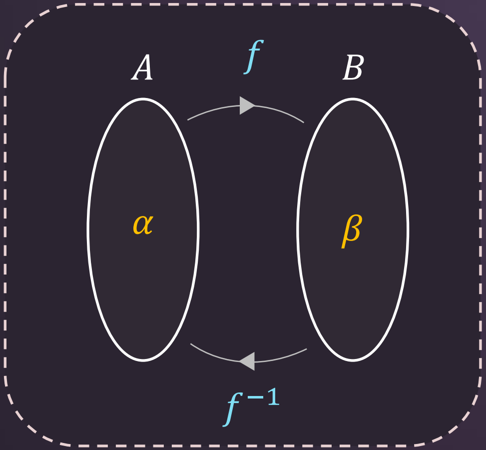
Key Takeaways

Inverse Function

Let $y = f(x): A \to B$ be a one – one and onto function , i.e. a bijection , then there will always exist a bijective function $x = g(y): B \to A$ such that if (α, β) is an element of f, (β, α) will be an element of g and the functions $f(x)$ and $g(x)$ are said to be inverse of each other.

- $g = f^{-1}: B \to A = \{(f(x), x) | (x, f(x)) \in f\}$



$(\alpha, \beta) \in f$

Then

$(\beta, \alpha) \in f^{-1}$

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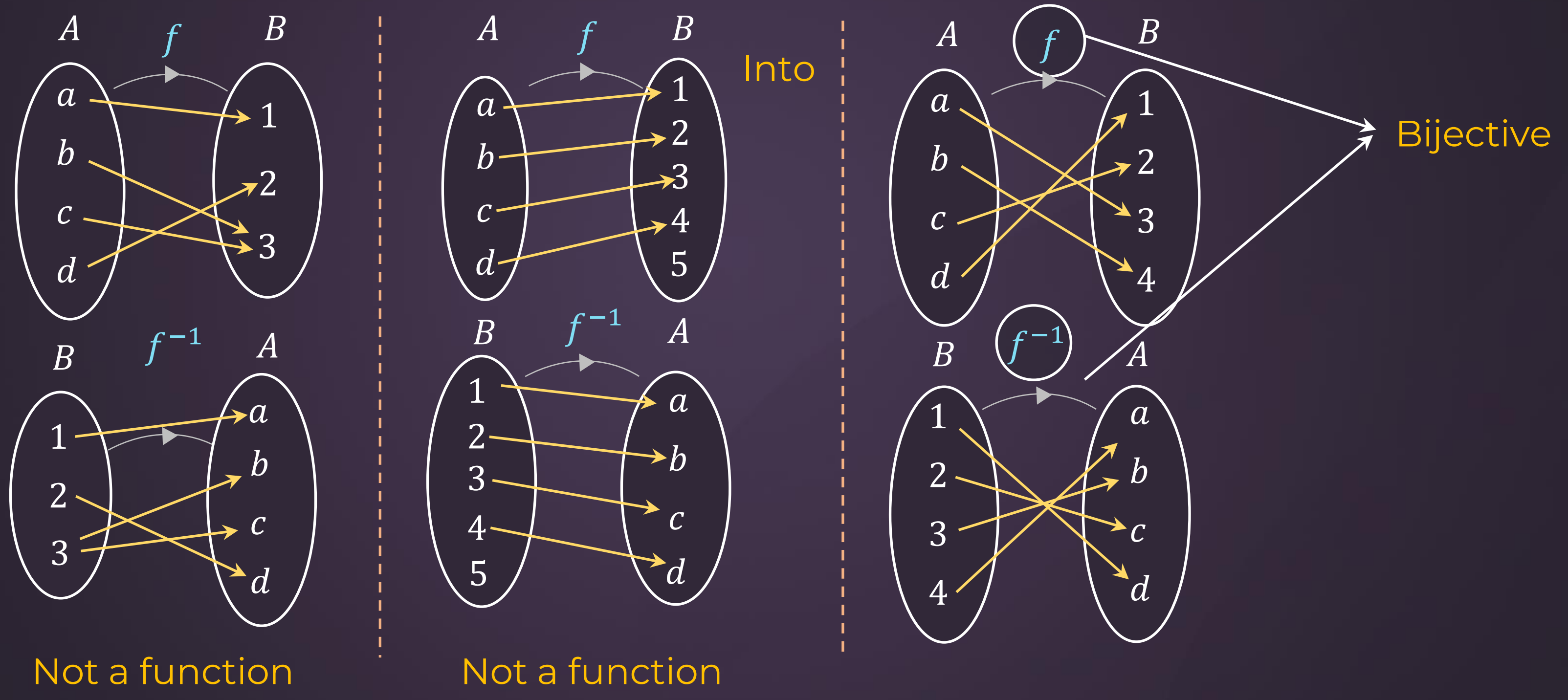
Key Takeaways

Inverse Function

- Why function must be bijective for it to be invertible?



- Inverse of a bijection is unique and also a bijection.

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Key Takeaways

Inverse Function

- To find inverse :

(i) For $y = f(x)$, express x in terms of y

Example : $y = e^x$

$x = \ln y$

(ii) In $x = g(y)$, replace y by x in g to get inverse.

$y = \ln x = f^{-1}(x)$

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$f(x) = \frac{2x+3}{4} : \mathbb{R} \to \mathbb{R}$, then find it's inverse.

Solution: Let $f(x) = y = \frac{2x+3}{4}$

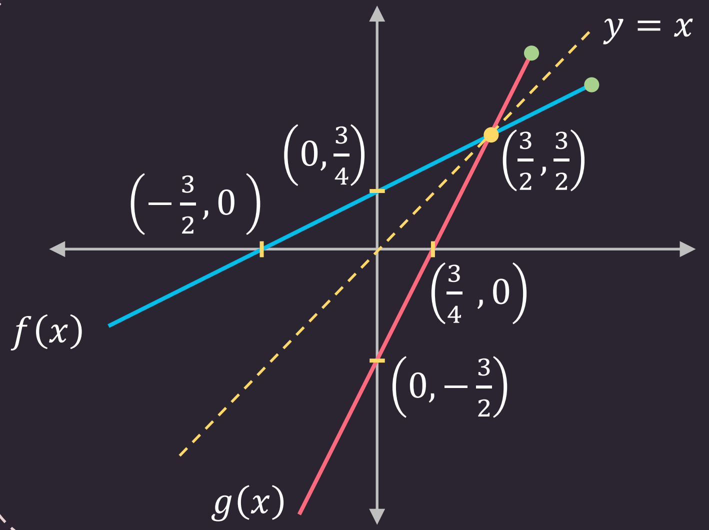
$$\Rightarrow x = \frac{4y-3}{2} = g(y)$$

$$\therefore g(x) = f^{-1}(x) = \frac{4x-3}{2} : \mathbb{R} \to \mathbb{R}$$

To find inverse :

For $y = f(x)$, express x in terms of y

In $x = g(y)$, replace y by x in g to get inverse.



Function and its inverse are symmetric about $y = x$

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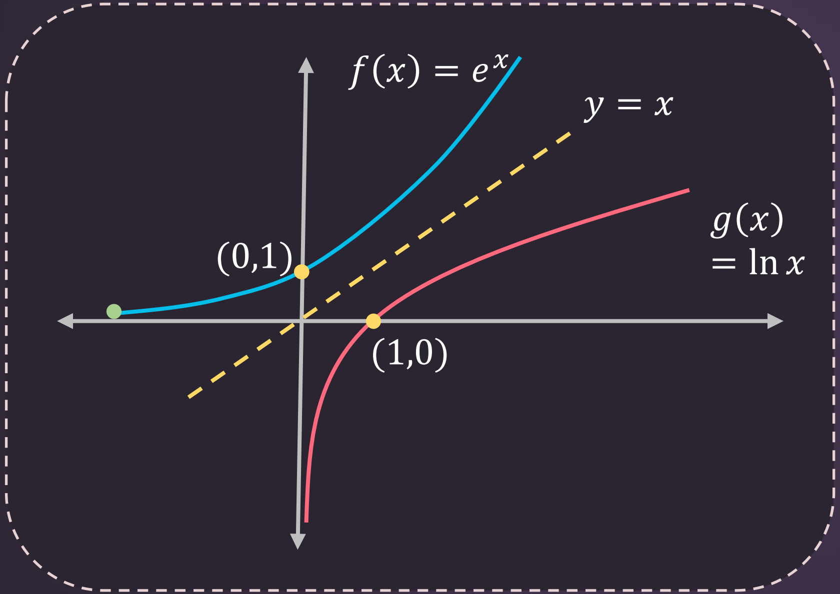
Inverse Function

Example: $f(x) = e^x, g(x) = \ln x$



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If $f(x) = x^2 + x + 1 : [0, \infty) \to [1, \infty)$, find its inverse.

Solution:

Since $f(x)$ is bijective.

Let $y = x^2 + x + 1 \quad \Rightarrow x^2 + x + 1 - y = 0$

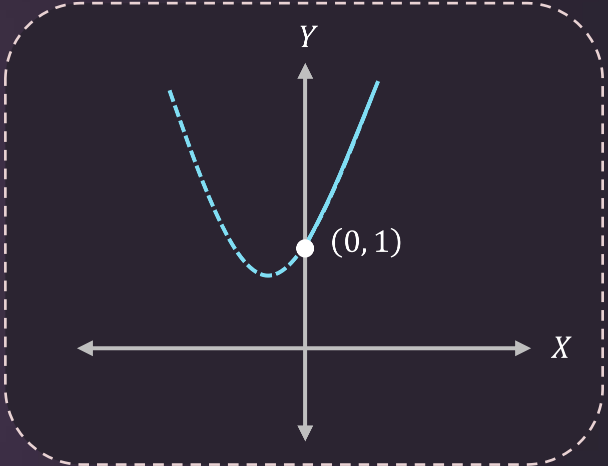
Solving for x,

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-4(1-y)}}{2} = \frac{-1 \pm \sqrt{4y-3}}{2}$$

But since inverse of a function is unique,

$$\Rightarrow x = \frac{-1 + \sqrt{4y-3}}{2} = g(y)$$

$$\therefore f^{-1}(x) = \frac{-1 + \sqrt{4x-3}}{2} : [1, \infty) \to [0, \infty)$$



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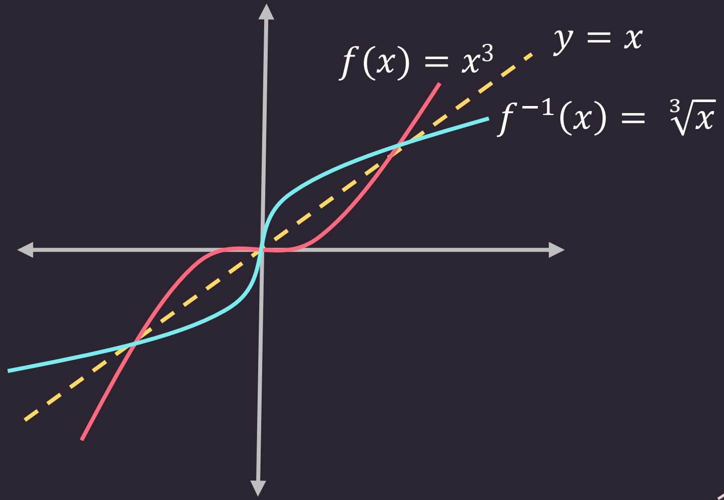
Key Takeaways

Properties of Inverse Function

- The graphs of f and g are the mirror images of each other about the line $y = x$.
- If functions f and f^{-1} intersect , then at least one point of intersection lie on the line $y = x$.



$$f(x) = x^3 \Rightarrow f^{-1}(x) = \sqrt[3]{x}$$

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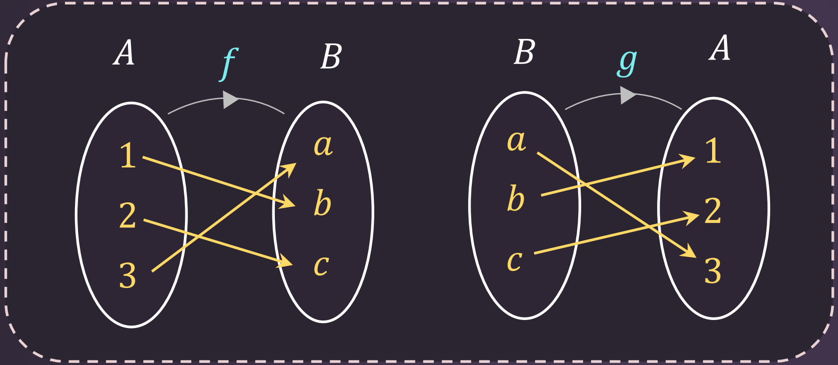
Key Takeaways

Properties of Inverse Function

- If f and g are inverse of each other, then $fog = gof = x$.



$$f\big(g(a)\big) = f(3) = a$$

However, fog and gof can be equal even if f and g are not inverse of each other, but in that case $fog = gof \neq x$

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Key Takeaways

Properties of Inverse Function

However, fog and gof can be equal even if f and g are not inverse of each other , but in that case $fog = gof \neq x$

Example: $f(x) = x + 2, g(x) = x + 1$

Then , $fog(x) = (x + 1) + 2 = x + 3$

And , $gof(x) = (x + 2) + 1 = x + 3, \quad \Rightarrow fog = gof \neq x$

but f and g are non inverse of each other.

- If f and g are two bijections, $f: A \rightarrow B, g: B \rightarrow C$, then inverse of gof exists and

$$(gof)^{-1} = f^{-1}og^{-1}$$

Binary Operation:

Definition:

A binary operation $*$ on a set A is a function $*: A \times A \to A$.

Denoted as $*\,(a, b) \to a * b$

Example: Show that addition is a binary operation on R,

but division is not a binary operation.

Solution: $+: R \times R \to R$ is given by $+(a, b) \to a + b$, is a function on R

$\div: R \times R \to R$ is given by $\div\,(a, b) \to \frac{a}{b}$, is not a function on R

and not a binary operation as for $b = 0$, $\frac{a}{0}$ is not defined.

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Properties of Binary Operation:

(i) Commutative:

A binary operation $*$ on a set X is called commutative if $a * b = b * a$ for every $a, b \in X$.

Example: Addition is commutative on R, but subtraction is not.

Solution: $a + b = b + a \rightarrow$ commutative

but $a - b \neq b - a \rightarrow$ not commutative

(ii) Associative:

A binary operation $*$ is said to be associative

$(a * b) * c = a * (b * c), \forall\, a, b, c \in A.$

Example: $(8 + 5) + 3 = 8 + (5 + 3)$ associative

$(8 - 5) - 3 \neq 8 - (5 - 3)$ not associative

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Properties of Binary Operation:

(iii) Identity:

Given a binary operation $*: A \times A \to A$, an element $e \in A$, if it exists, is called identity for the operation if $a * e = a = e * a,\ \forall\, a \in A$

Note: i. 0 is identity for addition on R

ii. 1 is identity for multiplication on R

(iv) Inverse:

Given a binary operation $*: A \times A \to A$, with identity element e in A, an element $a \in A$, is said to be invertible w.r.t $*$, if there exists an element b in A such that $a * b = e = b * a$ and b is called inverse of a and is denoted by a^{-1}.

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Properties of Binary Operation:

Note: *i.* $-a$ is inverse of a for addition operation on R.

$$a + (-a) = 0 = (-a) + a$$

ii. $\frac{1}{a}$ is inverse of $a(a \neq 0)$ for multiplication operation on $R - \{0\}$.

$$a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$$

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Let $*$ be a binary operation on $Q - \{-1\}$, defined by $a * b = a + b + ab$ for all $a, b \in Q - \{-1\}$, then:
(i) Show that $*$ is both commutative and associative on $Q - \{-1\}$
(ii) Find the identity element in $Q - \{-1\}$
(iii) Show that every element of $Q - \{-1\}$ is invertible.
Also, find inverse of an arbitrary element.

Solution:

Given $a * b = a + b + ab$.

First, we must check commutativity of $*$

Let $a, b \in Q - \{-1\}$

Then $a * b = a + b + ab$

$= b + a + ba$

$= b * a$

Therefore, $a * b = b * a, \ \forall\, a, b \in Q - \{-1\}$

Now, we have to prove associativity of $*$

Let $a, b, c \in Q - \{-1\}$, then

$a * (b * c) = a * (b + c + bc) = a + (b + c + bc) + a(b + c + bc)$

$= a + b + c + ab + bc + ac + abc$

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Let $*$ be a binary operation on $Q-\{-1\}$, defined by $a*b=a+b+ab$ for all $a, b \in Q-\{-1\}$, then:

(i) Show that $*$ is both commutative and associative on $Q-\{-1\}$

(ii) Find the identity element in $Q-\{-1\}$

(iii) Show that every element of $Q-\{-1\}$ is invertible.

Also, find inverse of an arbitrary element.

Solution: $(a*b)*c=(a+b+ab)*c$

$=a+b+ab+c+(a+b+ab)c$

$=a+b+c+ab+bc+ac+abc$

Therefore, $a*(b*c)=(a*b)*c, \forall\, a, b, c \in Q-\{-1\}$

Thus, $*$ is associative on $Q-\{-1\}$.

(ii) Let e be the identity element in $Q-\{-1\}$ with respect to $*$ such that

$a*e=a=e*a, \quad \forall\, a \in Q-\{-1\}$

$a*e=a$ and $e*a=a, \ \forall\, a \in Q-\{-1\}$

$a+e+ae=a$ and $e+a+ea=a, \forall\, a \in Q-\{-1\}$

$e+ae=0$ and $e+ea=0, \forall\, a \in Q-\{-1\}$

$e(1+a)=0$ and $e(1+a)=0, \forall\, a \in Q-\{-1\}$

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Let $*$ be a binary operation on $Q-\{-1\}$, defined by $a*b=a+b+ab$ for all $a, b \in Q-\{-1\}$, then:

(i) Show that $*$ is both commutative and associative on $Q-\{-1\}$

(ii) Find the identity element in $Q-\{-1\}$

(iii) Show that every element of $Q-\{-1\}$ is invertible. Also, find inverse of an arbitrary element.

Solution:

$e=0, \forall\, a \in Q-\{-1\}$ [because $a \neq -1$]

Thus, 0 is the identity element

in $Q-\{-1\}$ with respect to $*$.

(iii) Let $a \in Q-\{-1\}$ and $b \in Q-\{-1\}$

be the inverse of a. Then,

$a*b=e=b*a$

$a*b=e$ and $b*a=e$

$a+b+ab=0$ and $b+a+ba=0$

$b(1+a)=-a, \forall\, a \in Q-\{-1\}$

$b=-\frac{a}{1+a} \forall\, a \in Q-\{-1\}$ [because $a \neq -1$]

$b=-\frac{a}{1+a}$ is the inverse of $a \in Q-\{-1\}$

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Session 10

Functional Equations and Transformation of Graphs

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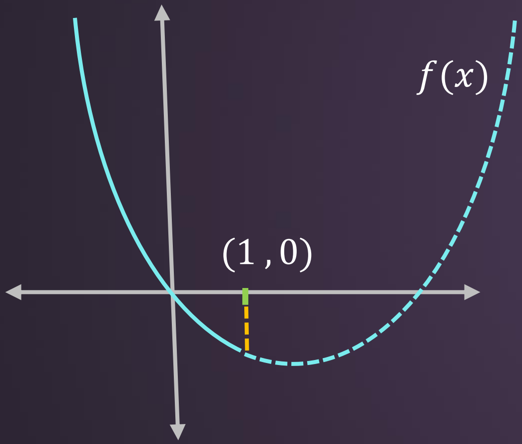
Find the solution of equation $x^2 - 3x = \frac{3-\sqrt{9+4x}}{2}$, $x \in (-\infty, 1]$.

Solution:

Let $f(x) = y = x^2 - 3x$



Let $f(x) = y = x^2 - 3x$

$\Rightarrow x^2 - 3x - y = 0$

$\Rightarrow x = \frac{3-\sqrt{9+4y}}{2}$ Then, $f^{-1}(x) = \frac{3-\sqrt{9+4x}}{2}$

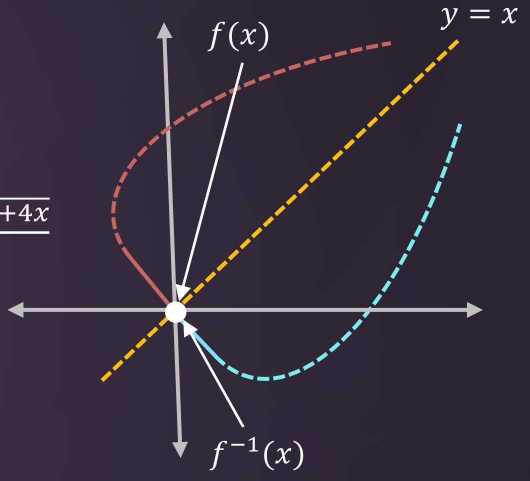
Since, $f(x) = f^{-1}(x) = x$

So, $x^2 - 3x = x$

$\Rightarrow x = 0, 4$

But, acc. to given domain

$\boxed{x = 0}$



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Key Takeaways

Functional Equations

If x, y are independent real variable, then

- $f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx, k \in \mathbb{R}$.
- $f(x+y) = f(x).f(y) \Rightarrow f(x) = a^{kx}, k \in \mathbb{R}$.
- $f(xy) = f(x) + f(y) \Rightarrow f(x) = k\log_a x, k \in \mathbb{R}, a > 0, a \neq 1$.
- $f(xy) = f(x).f(y) \Rightarrow f(x) = x^n, n \in \mathbb{R}$.
- If $f(x)$ is a polynomial of degree 'n' , such that

  $f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = 1 \pm x^n$

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If $f(x)$ is a polynomial function such that $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, such that $f(3) = -26$. Then $f(4) = ?$

A 64

B -65

C -63

D 65

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If $f(x)$ is a polynomial function such that $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, such that $f(3) = -26$. Then $f(4) = ?$

Solution:

$$\boxed{f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)} \Rightarrow f(x) = 1 \pm x^n$$

$$\Rightarrow f(3) = -26 \Rightarrow 1 \pm 3^n = -26$$

$3^n = -27$ ✗

$-3^n = -27$ ✓

$$\Rightarrow -3^n = -27 \quad \Rightarrow n = 3$$

$$\therefore f(x) = 1 - x^3$$

$$\boxed{f(4) = -63}$$

A. 64

B. -65

C. -63

D. 65

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If a function $f(x)$ satisfies the relation $f(x+y) = f(x) + f(y)$, where $x, y \in \mathbb{R}$ and $f(1) = 4$. Then find the value of $\sum_{r=1}^{10} f(r) =?$

A. 100

B. 220

C. 160

D. 300

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If a function $f(x)$ satisfies the relation $f(x+y) = f(x) + f(y)$, where $x, y \in \mathbb{R}$ and $f(1) = 4$. Then find the value of $\sum_{r=1}^{10} f(r) = ?$

Solution:

$$\boxed{f(x+y) = f(x) + f(y)} \quad \Rightarrow f(x) = kx$$

$$\Rightarrow f(1) = 4 = k$$

$$\therefore \sum_{r=1}^{10} f(r) = \sum_{r=1}^{10} 4r = 4\sum_{r=1}^{10} r$$

$$\boxed{= 220}$$

A. 100

B. 220

C. 160

D. 300

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For $x \in \mathbb{R} - \{0\}$, the function $f(x)$ satisfies $f(x) + 2f(1-x) = \frac{1}{x}$. Find the value of $f(2)$.

A. $-\frac{5}{6}$

B. $\frac{1}{2}$

C. -2

D. $\frac{3}{4}$

Solution: $f(x) + 2f(1-x) = \frac{1}{x}$

Put $x = 2$ $\Rightarrow f(2) + 2f(-1) = \frac{1}{2} \cdots (i)$

Put $x = -1$ $\Rightarrow f(-1) + 2f(2) = -1 \cdots (ii)$

By (i) and (ii)

$$
\begin{array}{rcl}
2f(-1) + 4f(2) &=& -2 \\
-\,f(2) + 2f(-1) &=& -\,\frac{1}{2} \\
\hline
3f(2) &=& -\frac{5}{2}
\end{array}
\quad \Rightarrow f(2) = -\frac{5}{6}
$$

$$\boxed{f(2) = -\frac{5}{6}}$$

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Let the function $f:[0,1] \to R$ be defined by $f(x) = \frac{4^x}{4^x+2}$

Then the value of $f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \cdots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$ is___.

Solution: $f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$

$$= \frac{4^x}{4^x+2} + \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4^x}{4^x+2} + \frac{4}{4+2\cdot 4^x}$$

$$= \frac{4^x}{4^x+2} + \frac{2}{4^x+2}$$

$\therefore f(x) + f(1-x) = 1$

$$\Rightarrow f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \cdots + f\left(\frac{20}{40}\right) + \cdots + f\left(\frac{39}{40}\right) = 19 + f\left(\frac{20}{40}\right)$$

$$\Rightarrow f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \cdots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right) = 19 + f\left(\frac{20}{40}\right) - f\left(\frac{1}{2}\right)$$

$$\boxed{= 19}$$

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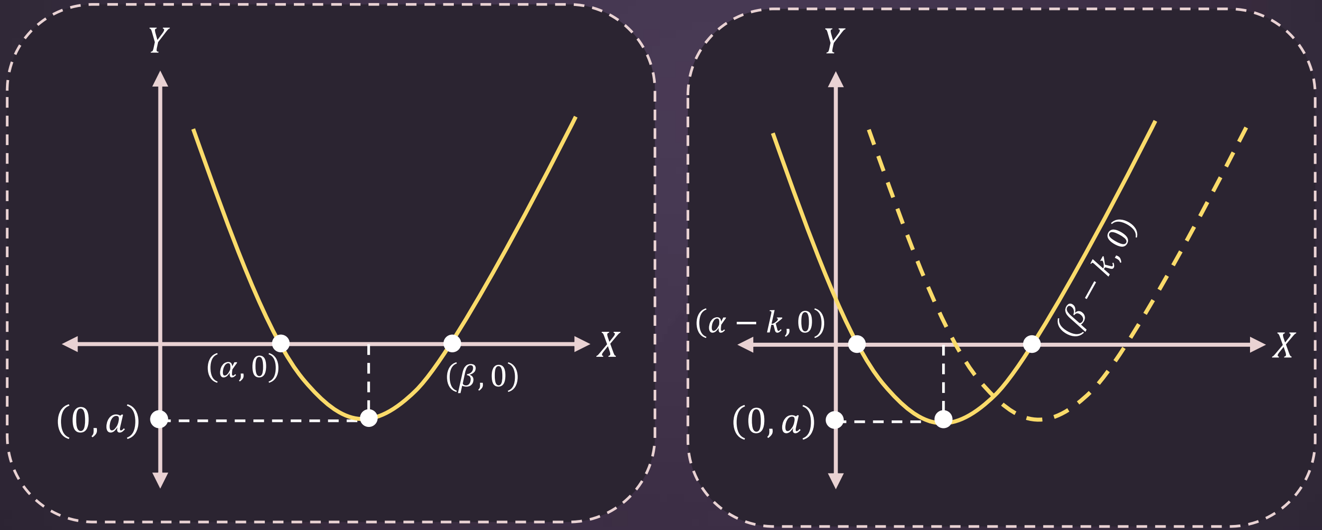
Key Takeaways

Transformation of graphs (horizontal shifts):

- Let $y = f(x)$

  $y = f(x+k), k > 0$ (graph goes to left by $'k'$ units)



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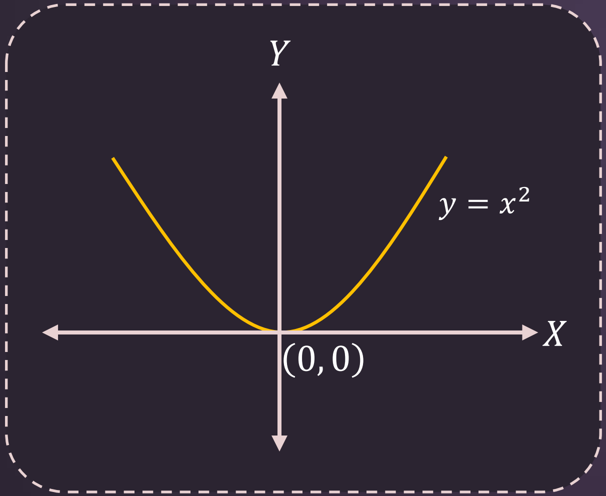
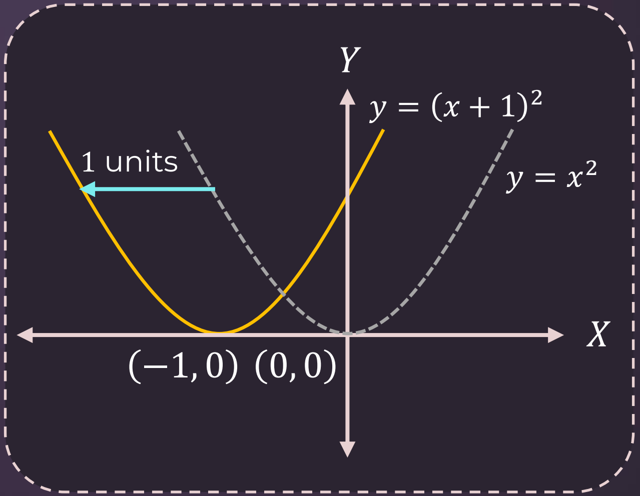
Plot the following curve:

$(i)\ y = (x+1)^2$ $\qquad$ $(ii)\ y = (x-2)^2$

Solution: (i) For $y = f(x+k), k > 0$ graph shift k units toward left from $y = f(x)$ graph





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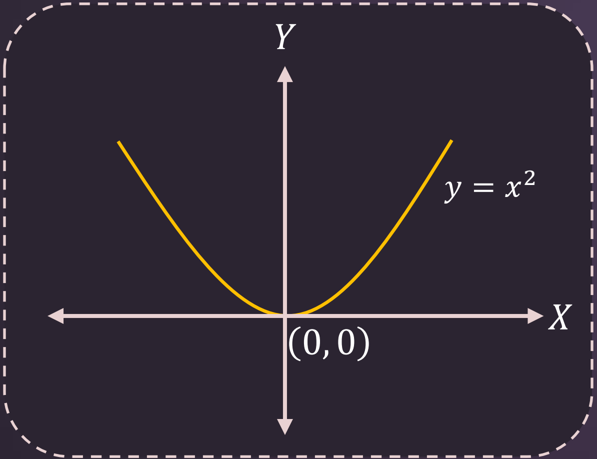
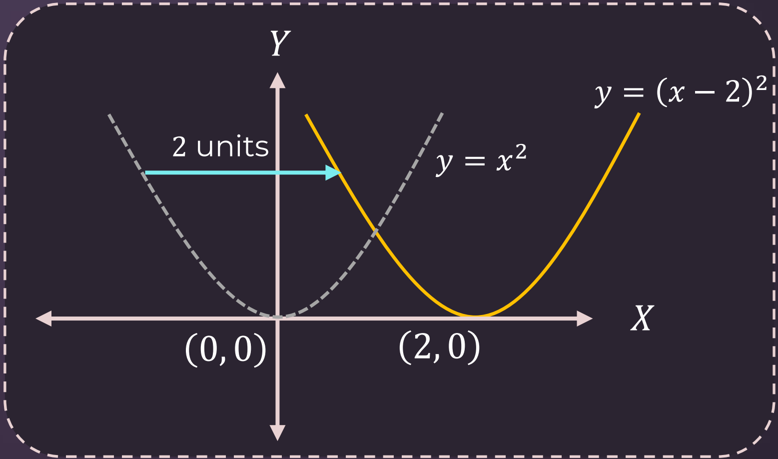
Plot the following curve:

$(i)\ y = (x+1)^2$ $\qquad$ $(ii)\ y = (x-2)^2$

Solution: $(ii)\ y = (x-2)^2 = \left(x + (-2)\right)^2$

Here graph shift 2 units toward right





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For $y = f(x - k), k > 0$ graph shift k units towards right horizontally from $y = f(x)$ graph.

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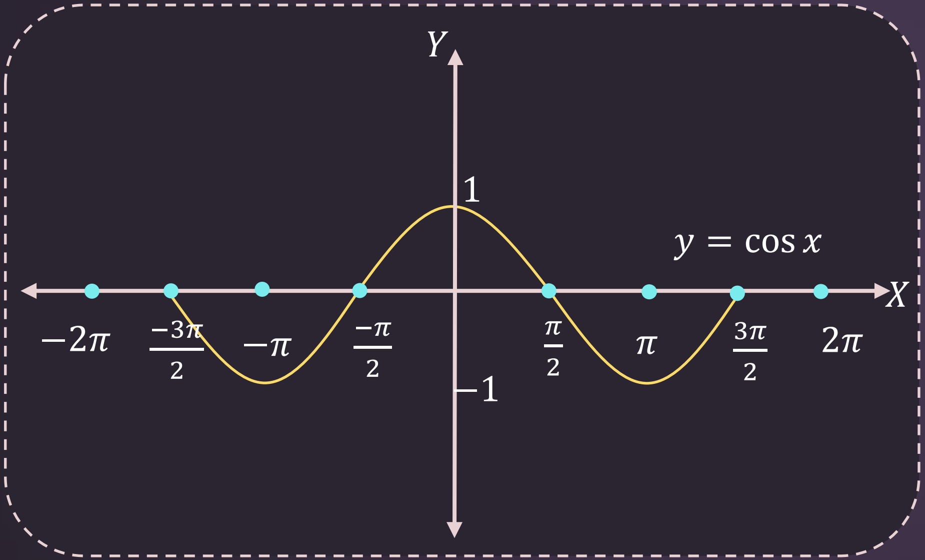
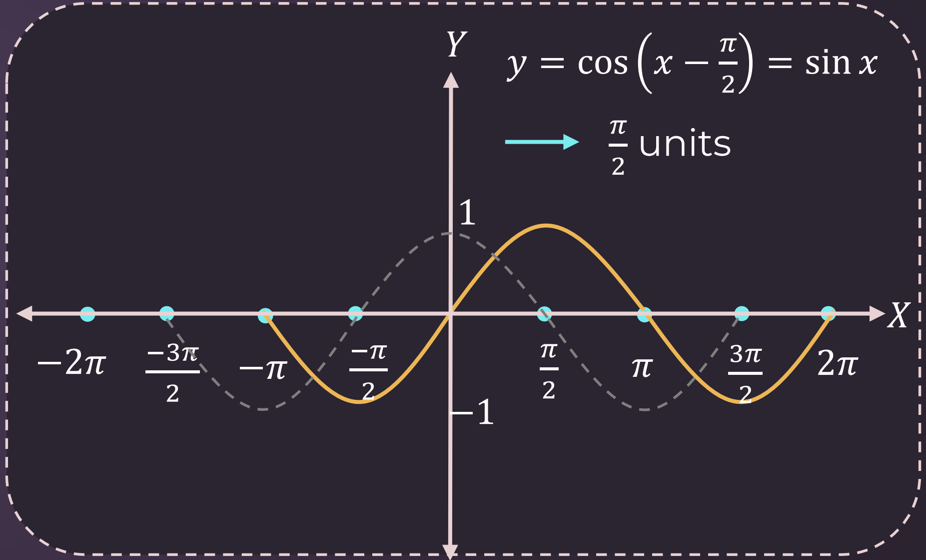
Plot the curve of function $y = \cos\left(x - \frac{\pi}{2}\right)$ using transformations

Solution:





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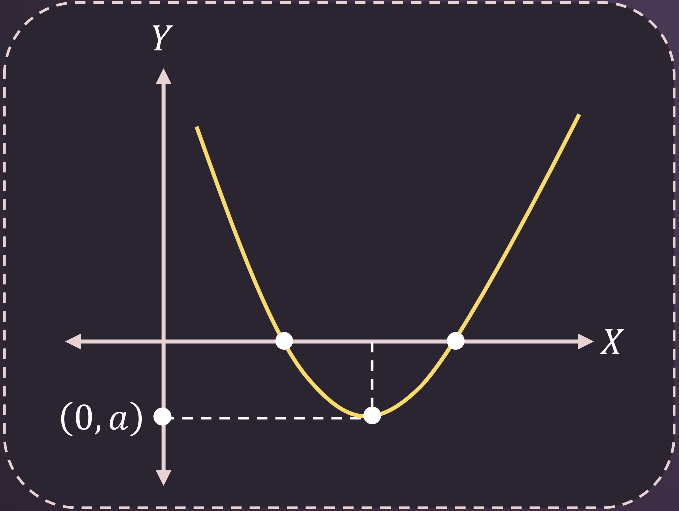
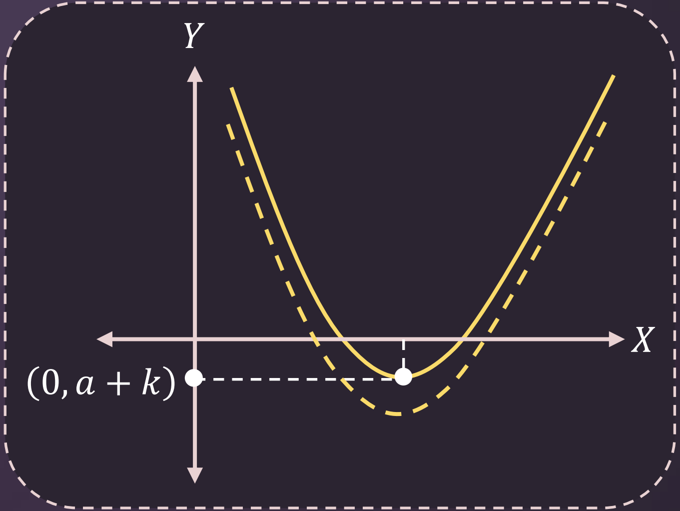
Key Takeaways

Transformation of graphs (Vertical shifts):

- Let $y = f(x)$

  $y = f(x) + k, k > 0$ (graph goes to up by $'k'$ units)





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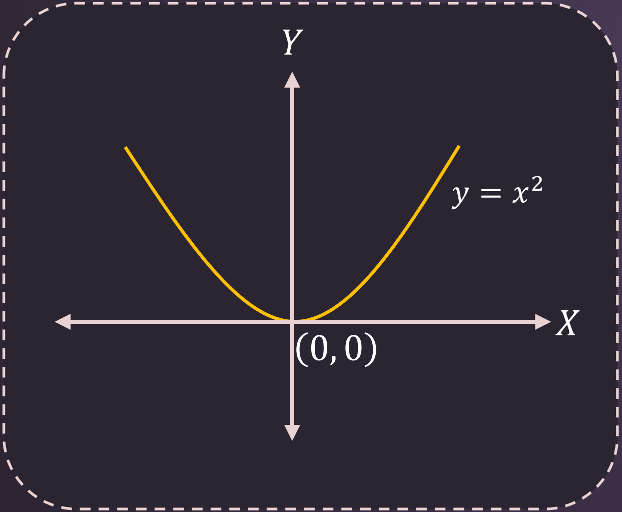
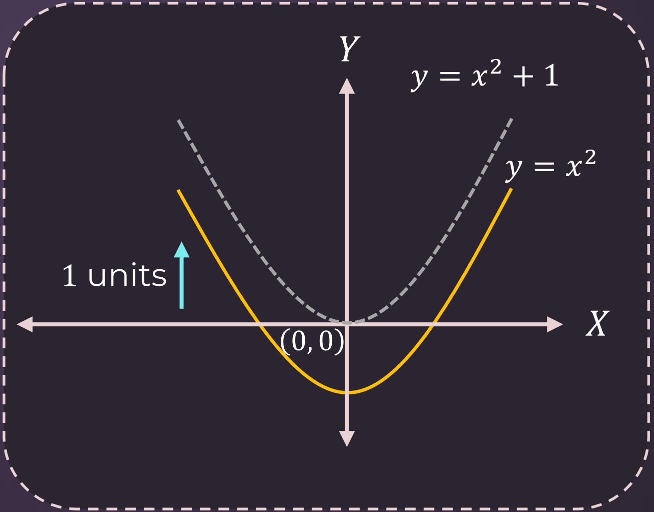
Plot the following curves:

$(i)\ y = x^2 + 1$ $(ii)\ y = x^2 - 2$

Solution: (i) For $y = f(x) + k\ , k > 0$ graph shift k units toward down from $y = f(x)$ graph





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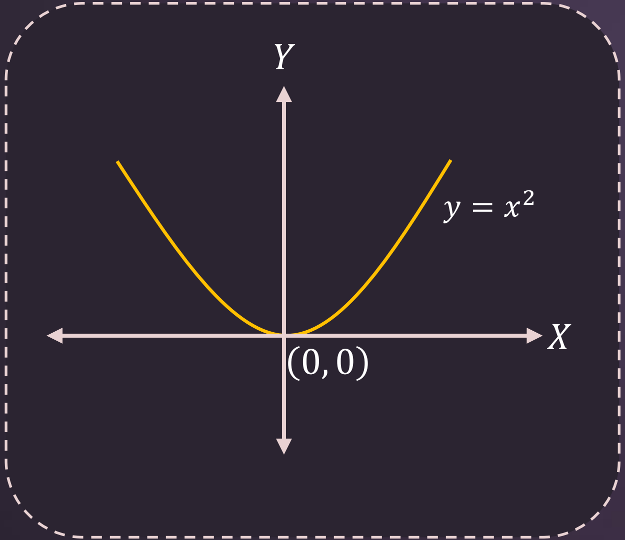
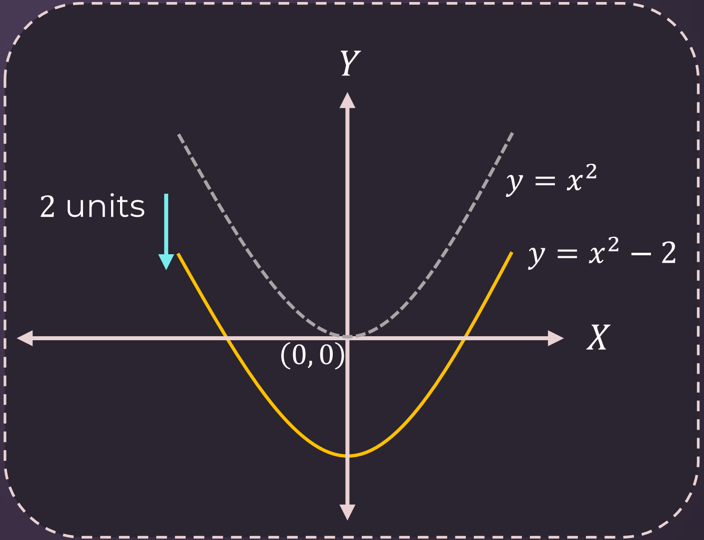
Plot the following curves:

$(i)\ y = x^2 + 1$ $(ii)\ y = x^2 - 2$

Solution: $(ii)\ y = x^2 - 2$

Here graph shift 2 units upward





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For $y = f(x) - k,\ k > 0$ graph of $y = f(x)$ will shift k units downwards.

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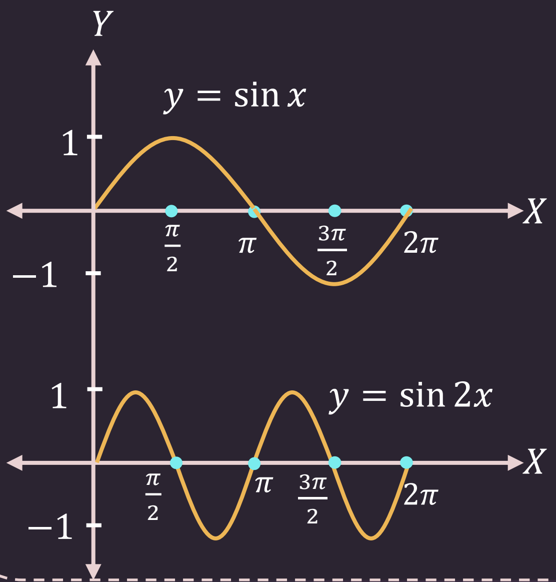
Key Takeaways

Transformation of graphs (horizontal stretch):

- Let $y = f(x)$

  $y = f(kx),\ k > 1$(points on x-axis divided by $'k'$units)

Example:



Two loops in 0 to 2π

Two loops in 0 to π

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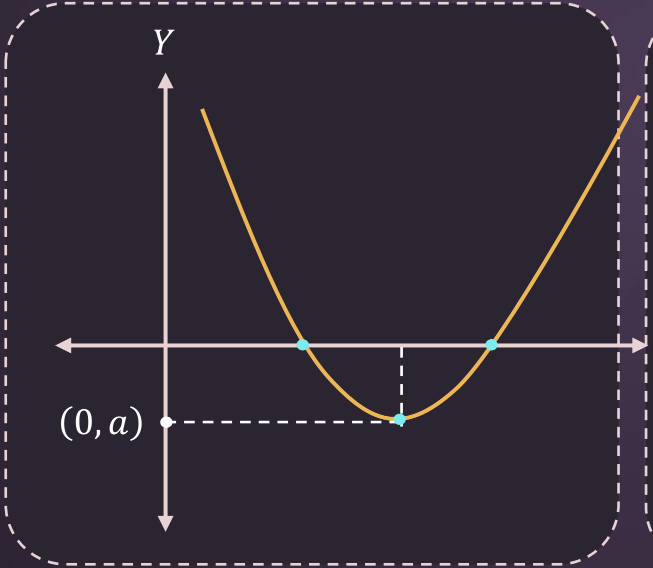
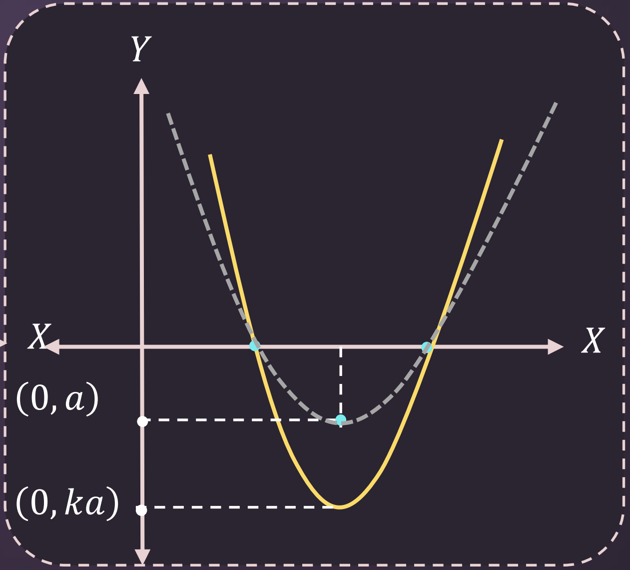
Key Takeaways

Transformation of graphs (Vertical stretch):

- Let $y = f(x)$

  $y = k \cdot f(x), k > 1$ (Point on y–axis is multiplied by $'k'$ units)





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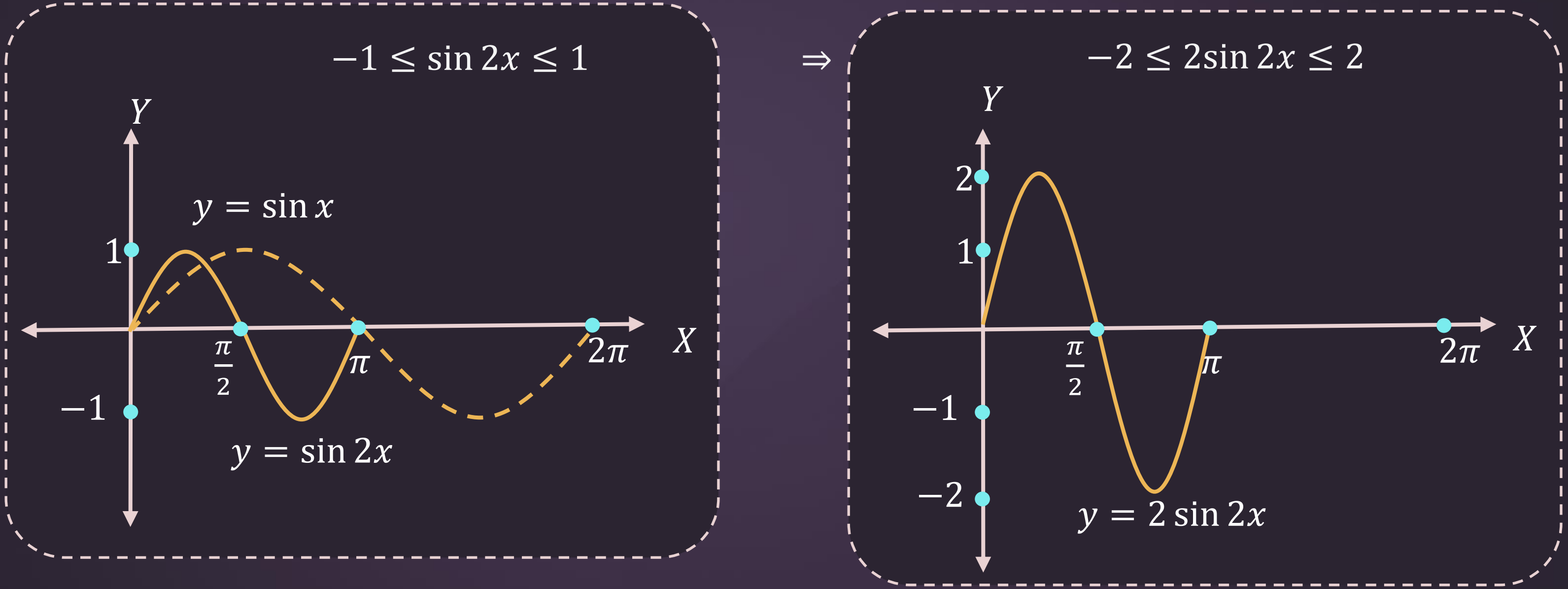
Plot graph of the following functions: $y = 2\sin 2x$

Solution:



Period of $\sin 2x$ = Period of $2\sin 2x = \pi$

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Session 11

Playing with Graphs

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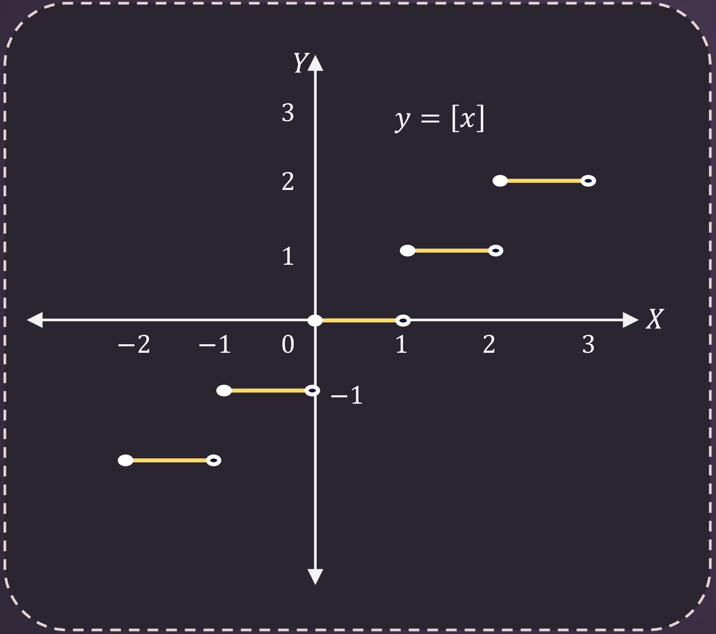
Plot the following curves for $x \in R$: $(i)\ y = 1 + [x]$ $(ii)\ y = x + [x]$

[ ] denotes G.I.F.

Solution: $(i)\ y = 1 + [x]$

1. Make the plot of the graph $[x]$



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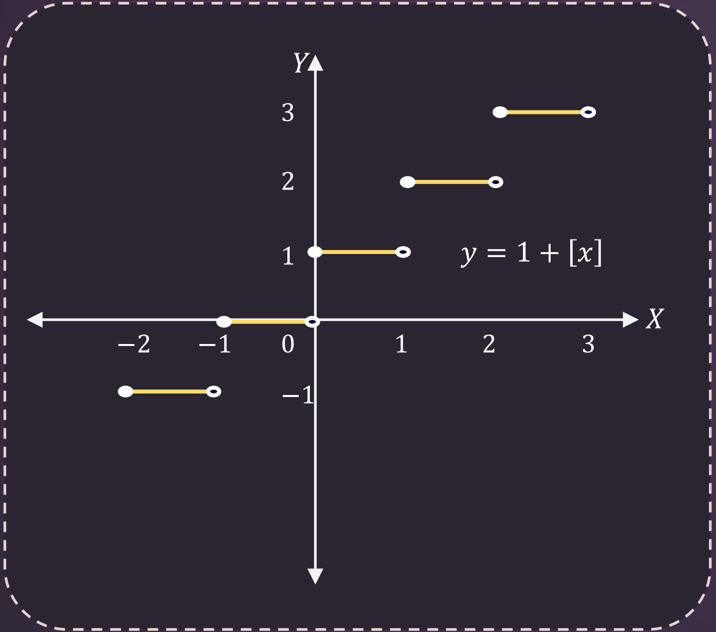
Plot the following curves for $x \in R$: $(i)\ y = 1 + [x]$ $(ii)\ y = x + [x]$
[ ] denotes G.I.F.

Solution: $(i)\ y = 1 + [x]$

2. Now, up the graph by 1.



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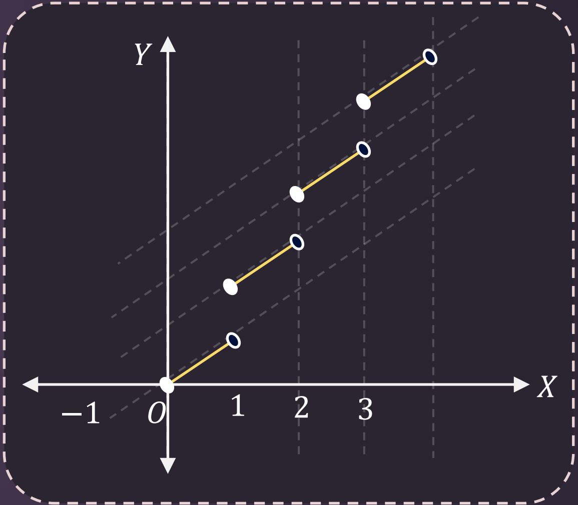
Plot the following curves for $x \in R$: $(i)\ y = 1 + [x]$ $(ii)\ y = x + [x]$

[ ] denotes G.I.F.

Solution: $(ii)\ y = x + [x]$

$x \in [0\,,1)$	$y = x + 0$
$x \in [1\,,2)$	$y = x + 1$
$x \in [2\,,3)$	$y = x + 2$
$x \in [-1\,,0)$	$y = x - 1$



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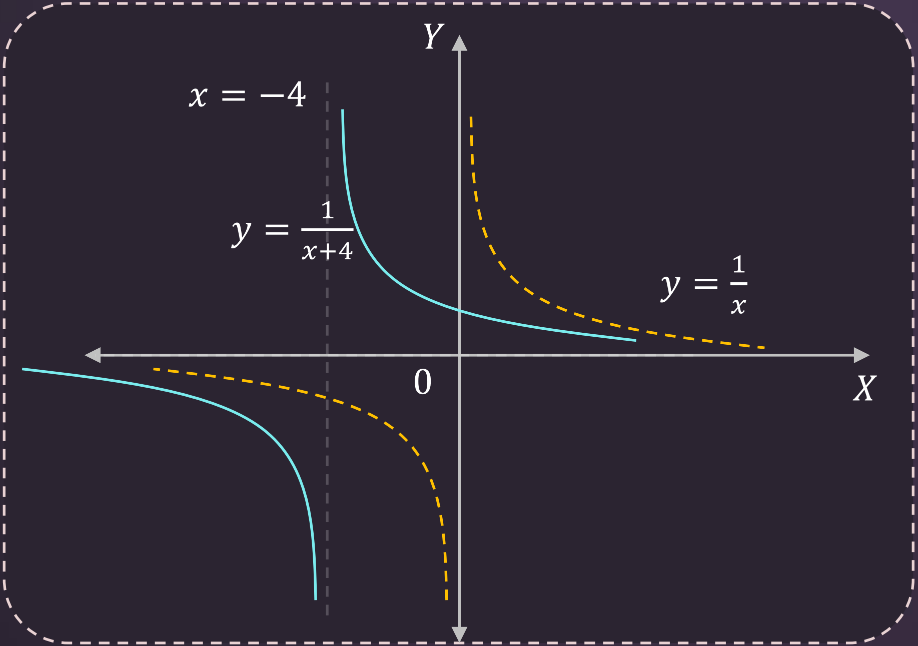
Plot graph of the following functions. $(i)\ y = \frac{1}{x+4}$ $(ii)\ y = \frac{1}{x+4} + 3$

Solution: $i)\ y = \frac{1}{x+4}$

Shift $y = \frac{1}{x}$ at $x = -4$



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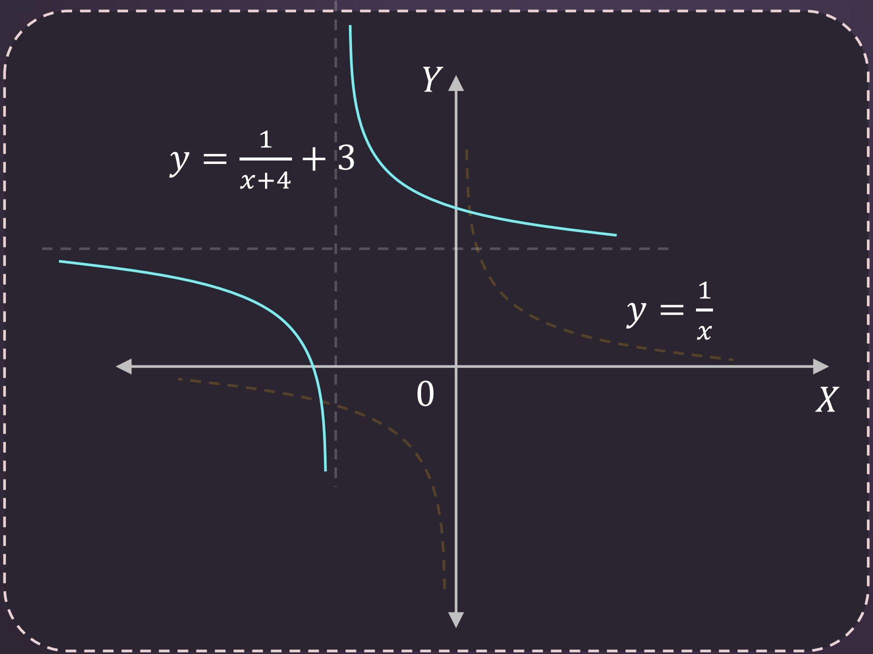
Plot graph of the following functions. $(i)\ y=\frac{1}{x+4}$ $(ii)\ y=\frac{1}{x+4}+3$

Solution: $ii)\ y=\frac{1}{x+4}+3$

Shift $y=\frac{1}{x}$ at $x=-4$



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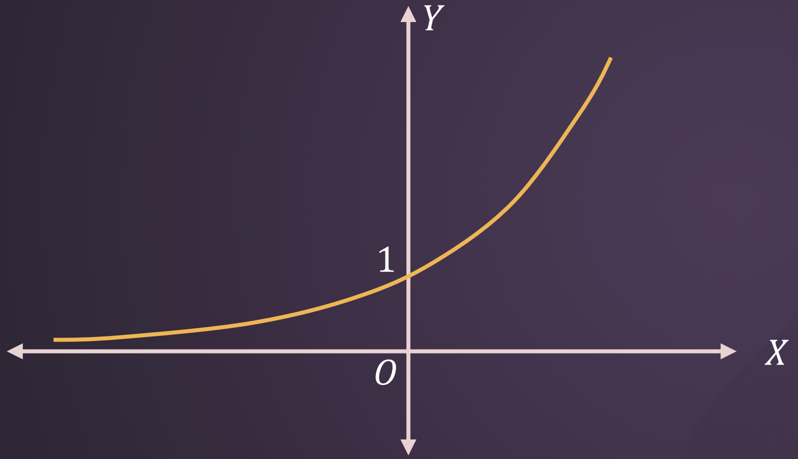
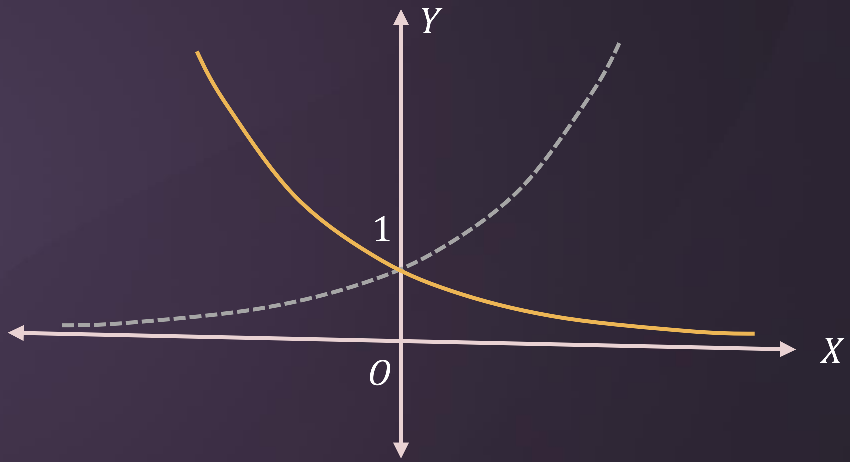
Key Takeaways

Transformation of graphs

- Let $y = f(x)$



$y = f(-x)$, (mirrored about y –axis)



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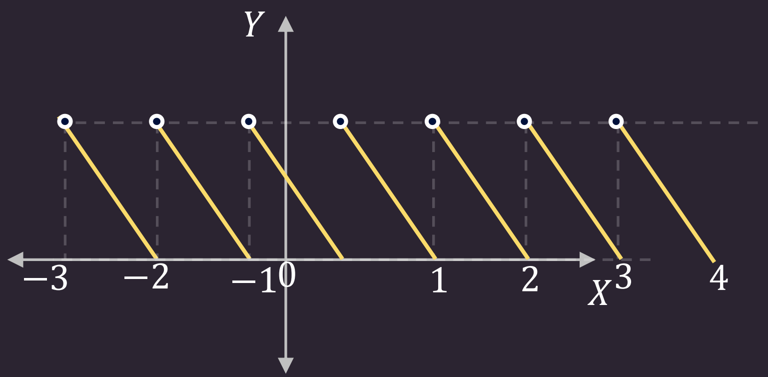
Plot the curve $\{-x\}$,
Where { } denotes fractional part function

Solution: $y = \{-x\}$



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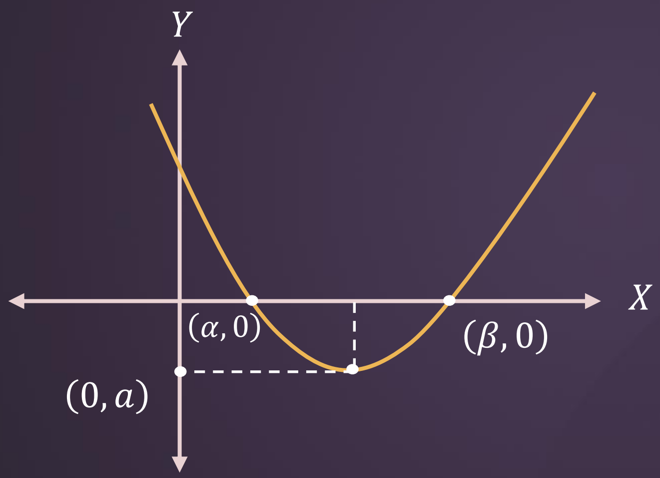
Key Takeaways

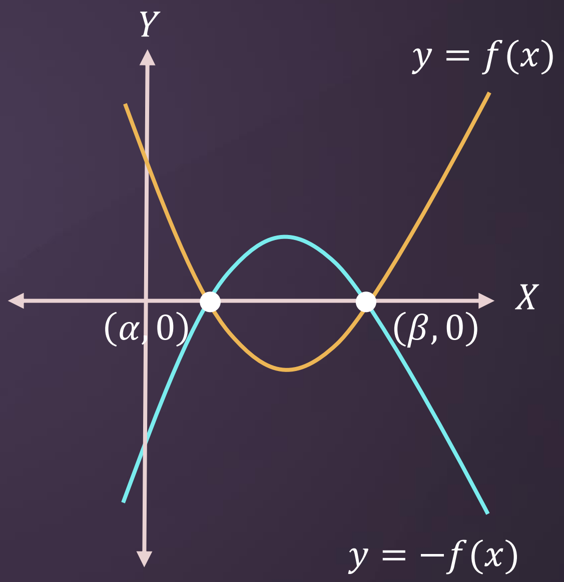
Transformation of graphs:

- Let $y = f(x)$



$y = -f(x)$, (mirrored about x –axis)

Values of y, multiplied by -1



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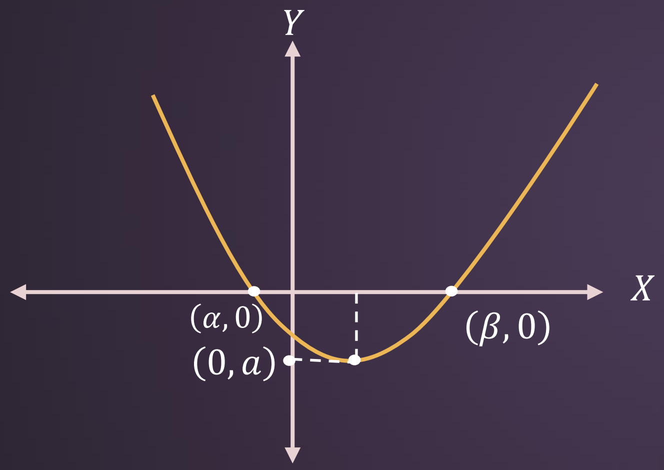
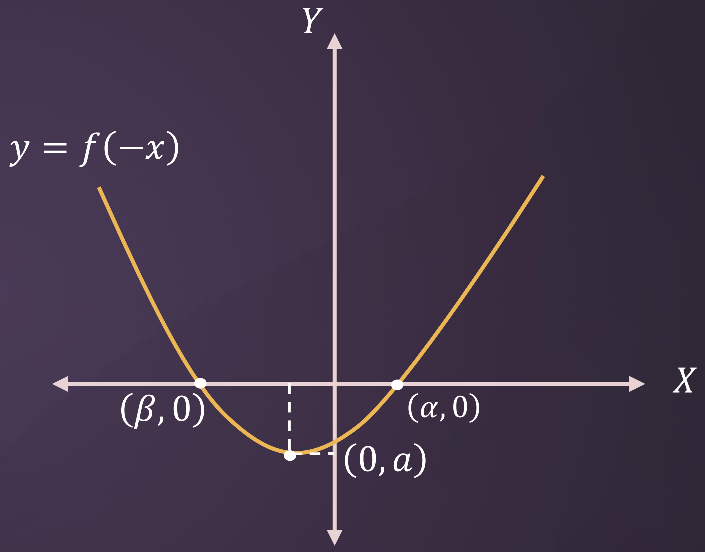
Key Takeaways

$y = -f(-x)$ transformation from $y = f(x)$:

- Let $y = f(x)$





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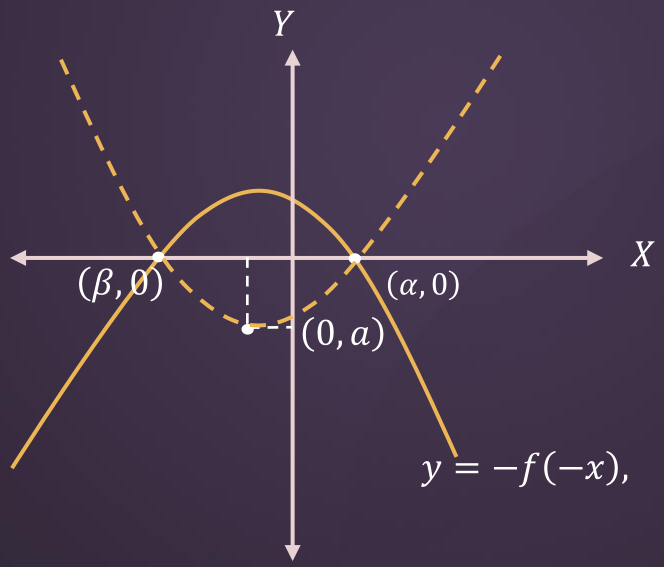
Key Takeaways

$y = -f(-x)$ transformation from $y = f(x)$:

$$y = -f(-x),$$



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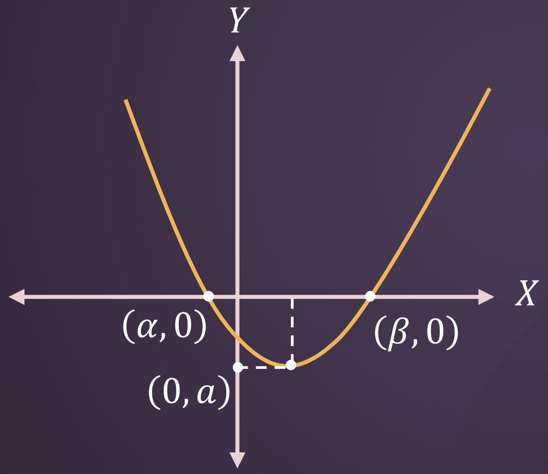
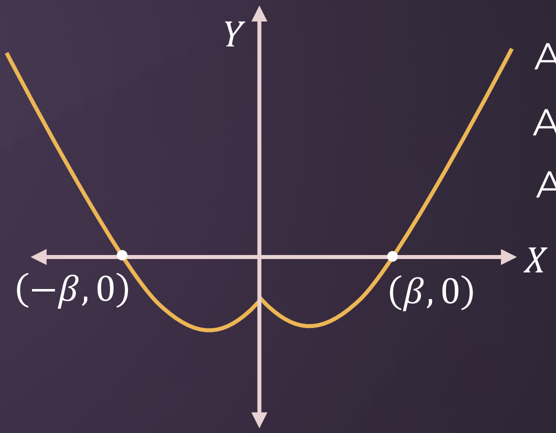
Key Takeaways

Transformation of graphs:

- Let $y = f(x)$



$y = f(|x|)$ (image of f for +ve x, about y –axis)



At $x = 1, y = f(1)$

At $x = -1, y = f(1)$

At $x = 2, y = f(2)$

At $x = -2, y = f(2)$

$y = f(|x|)$ is an even function

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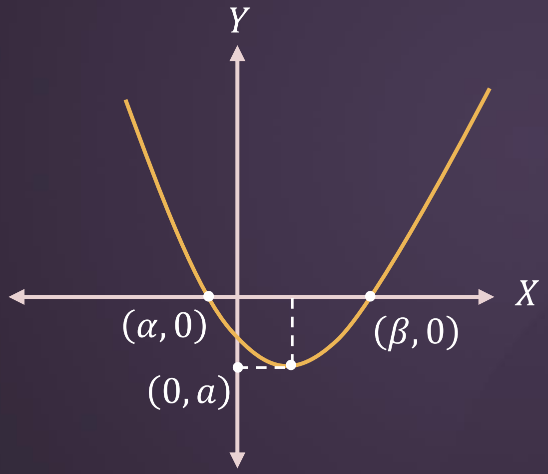
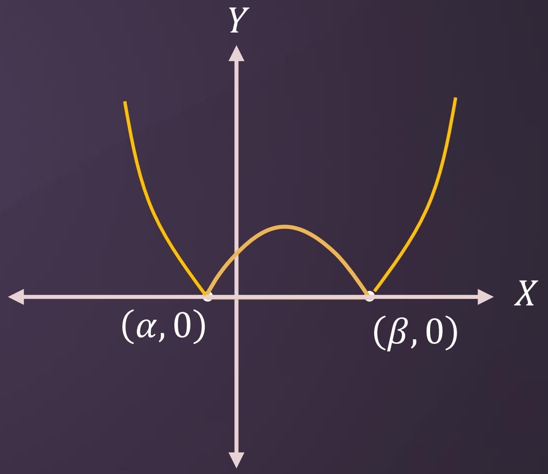
Key Takeaways

Transformation of graphs:

- Let $y = f(x)$



$y = |f(x)|$ (–ve y –axis portion flipped about x –axis)



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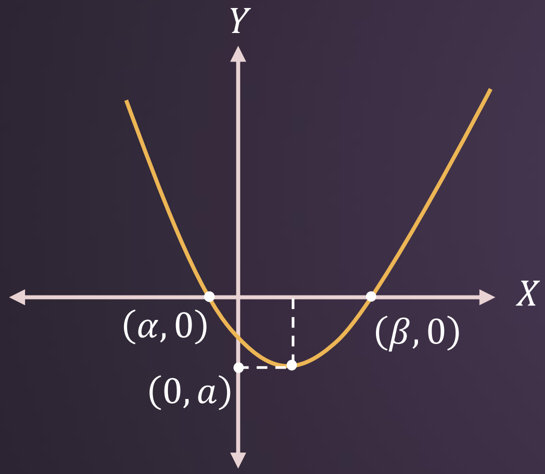
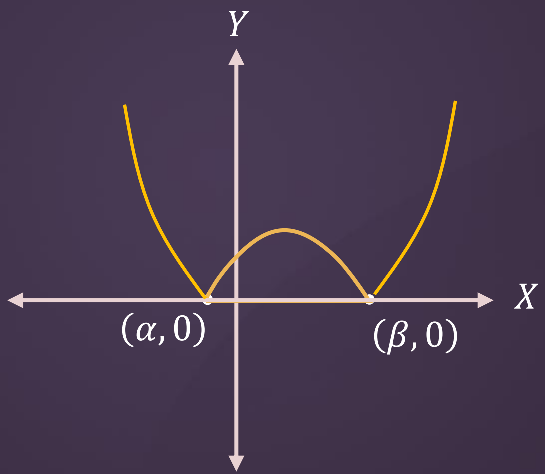
Key Takeaways

Transformation of graphs:

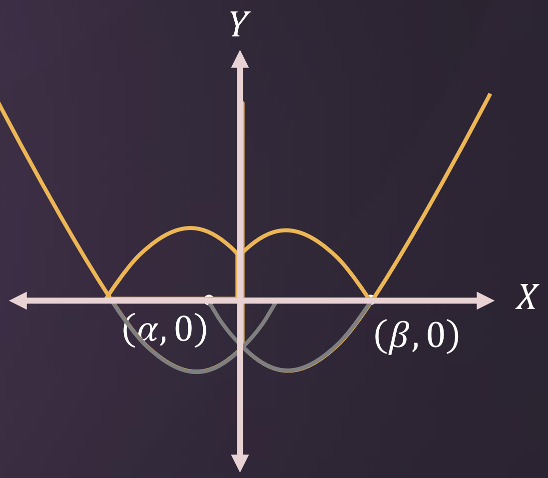
- Let $y = f(x)$



$y = |f(x)|$



$y = |f(|x|)|$ (+ve x axis portion of $f(|x|)$ flipped about y –axis)



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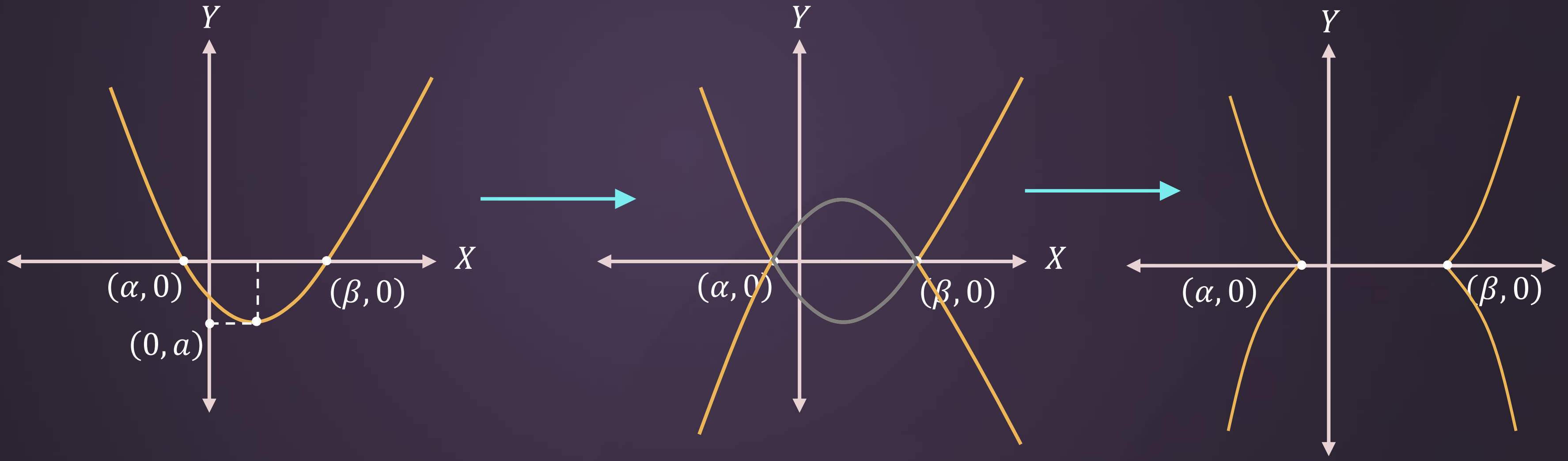
Key Takeaways

$|y| = f(x)$ transformation from $y = f(x)$:

$y = f(x)$ $\qquad$ $|y| = f(x)$



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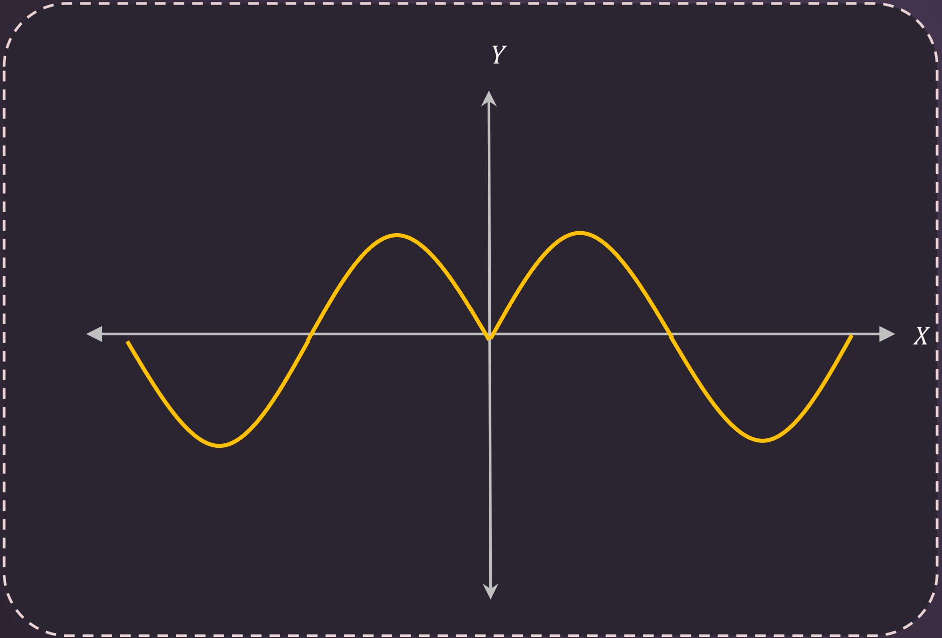
Plot graphs of the function $(i)\ y = \sin|x|$ $(ii) y = \left|(x-2)^{\frac{1}{3}}\right|$ $(iii)\ |y| = \ln x$

Solution:

$(i)\ y = \sin|x|$



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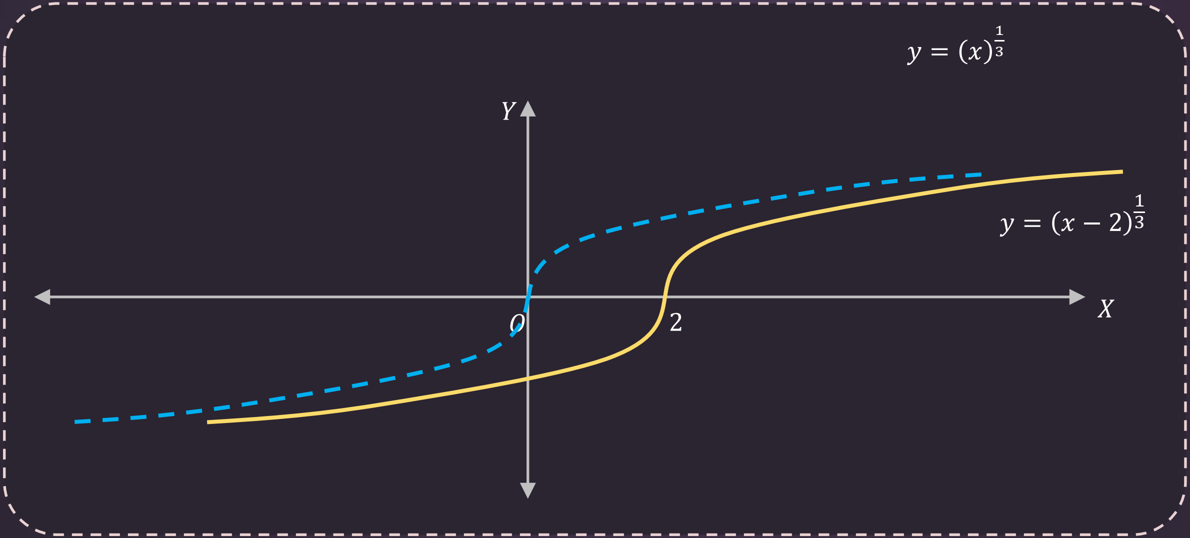
Plot graphs of the function $(i)\ y = \sin|x|$ $(ii) y = \left|(x-2)^{\frac{1}{3}}\right|$ $(iii)\ |y| = \ln x$

Solution: $(ii)\ y = \left|(x-2)^{\frac{1}{3}}\right|$

Shift $y = x^{\frac{1}{3}}$ at $x = 2$



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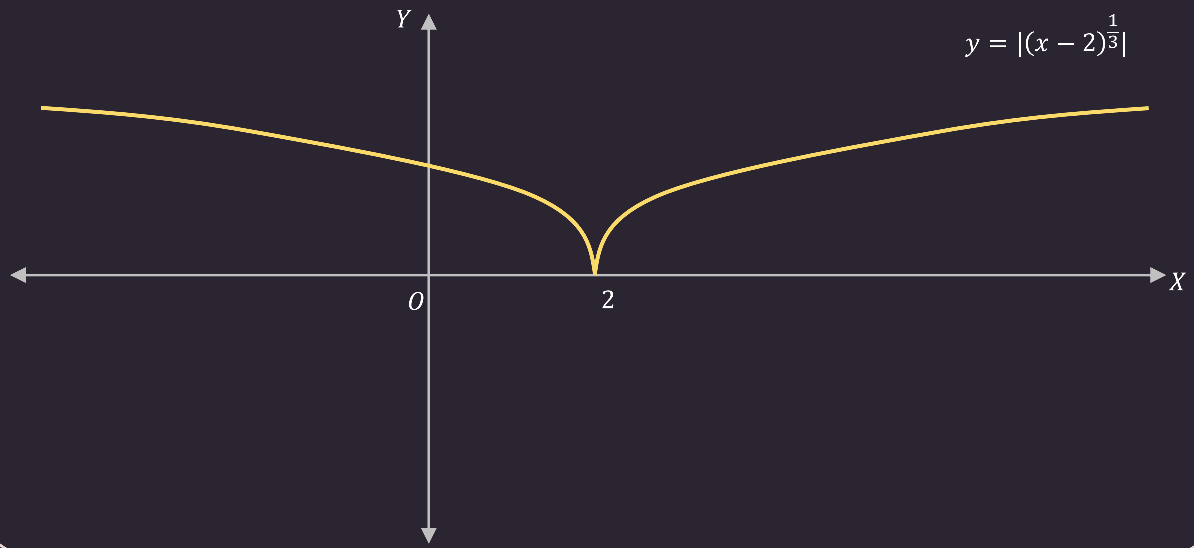
Plot graphs of the function $(i)\ y = \sin|x|$ $(ii) y = \left|(x-2)^{\frac{1}{3}}\right|$ $(iii)\ |y| = \ln x$

Solution: Now, draw graph for $y = \left|(x-2)^{\frac{1}{3}}\right|$ at $x = 2$



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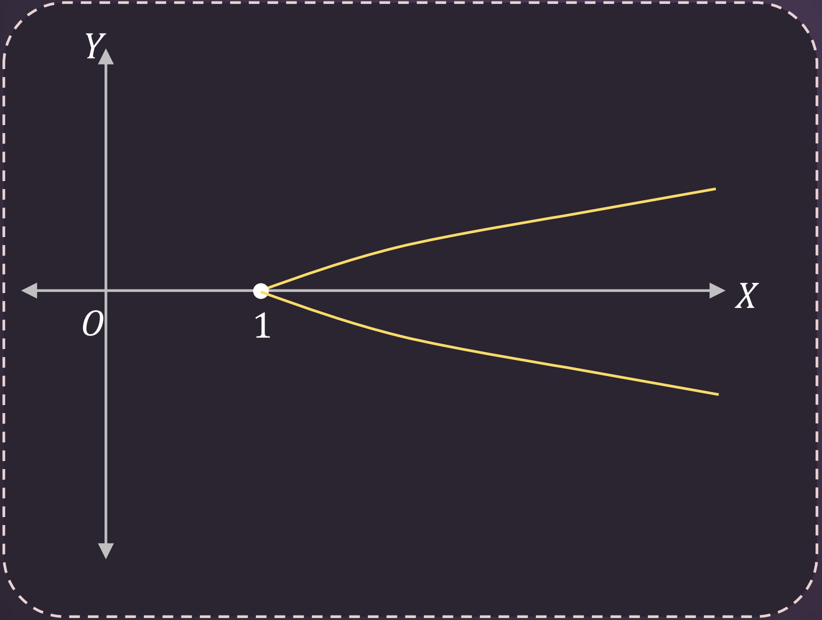
Plot graphs of the function $(i)\ y = \sin|x|$ $(ii) y = \left|(x-2)^{\frac{1}{3}}\right|$ $(iii)\ |y| = \ln x$

Solution:

$(iii)\ |y| = \ln x$



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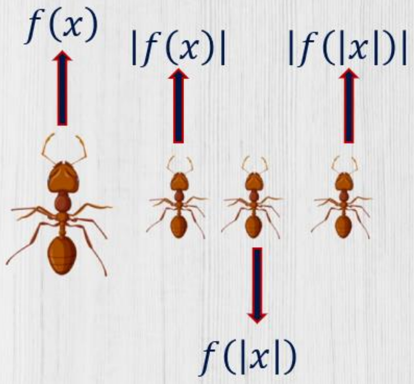
Key Takeaways



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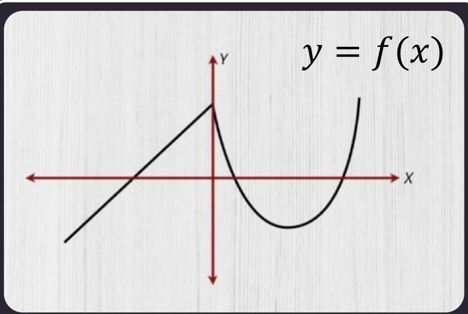
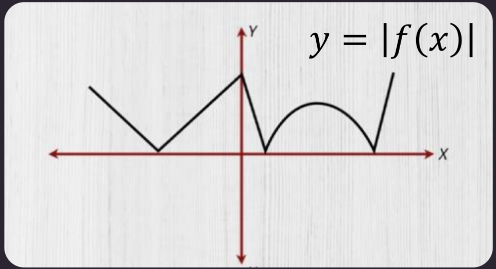
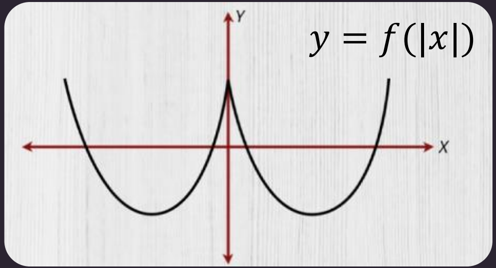
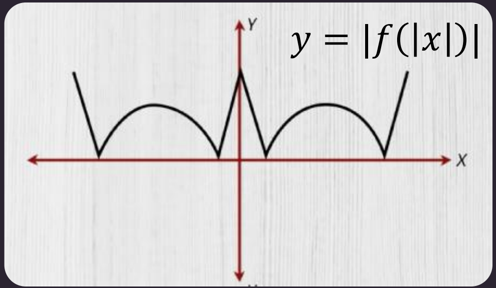
Key Takeaways









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Number of solutions of two curves $y = f(x)$ & $y = g(x)$ is number of intersection points for 2 curves $y = f(x)$ & $g(x)$

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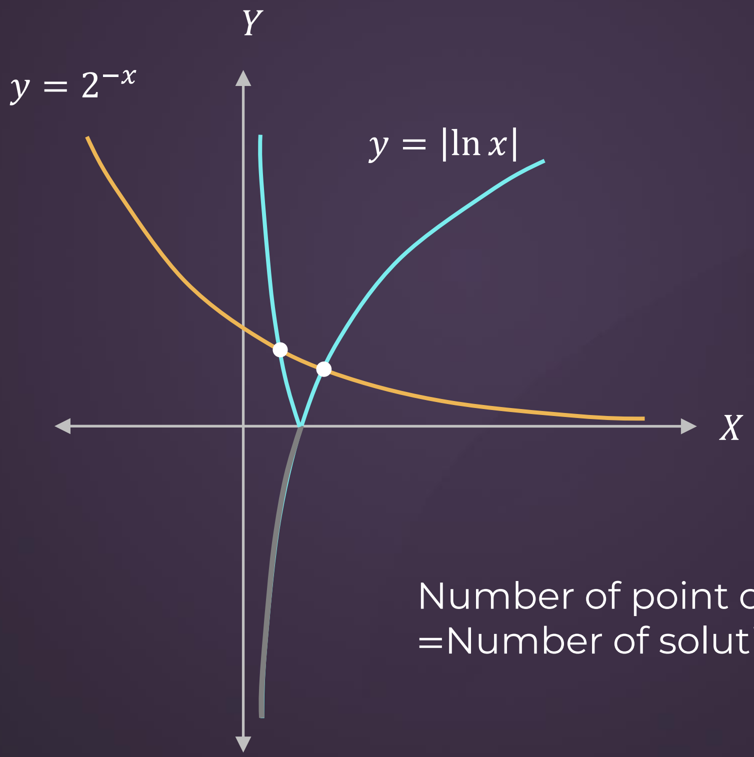
Find the number of solutions for $|\ln x| = 2^{-x}$

Solution:



Number of point of intersections

=Number of solutions= 2

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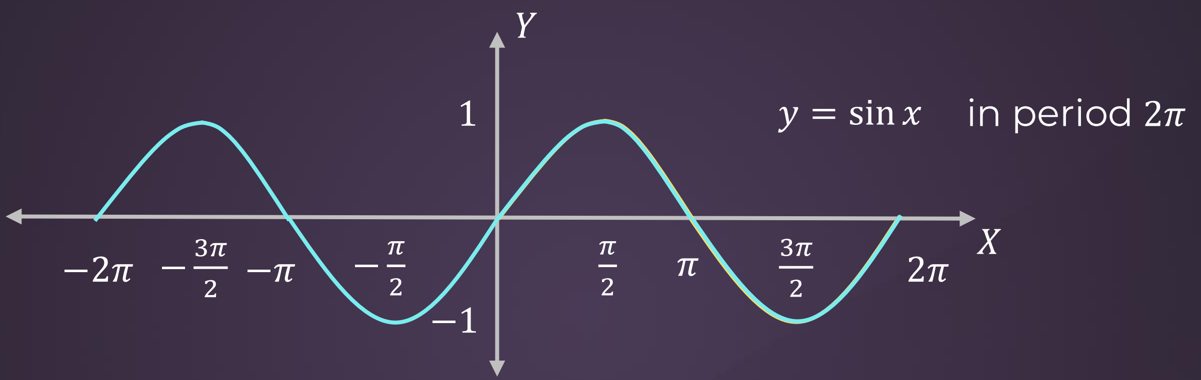
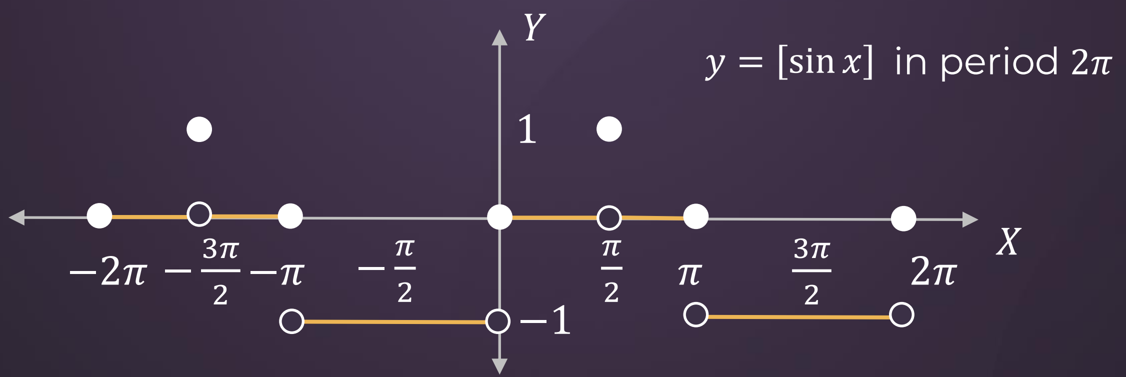
Plot the curve of $y = [\sin x]$:

Solution:





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Thank You

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