Welcome to

Aakash + BYJU'S LIVE

Straight Lines

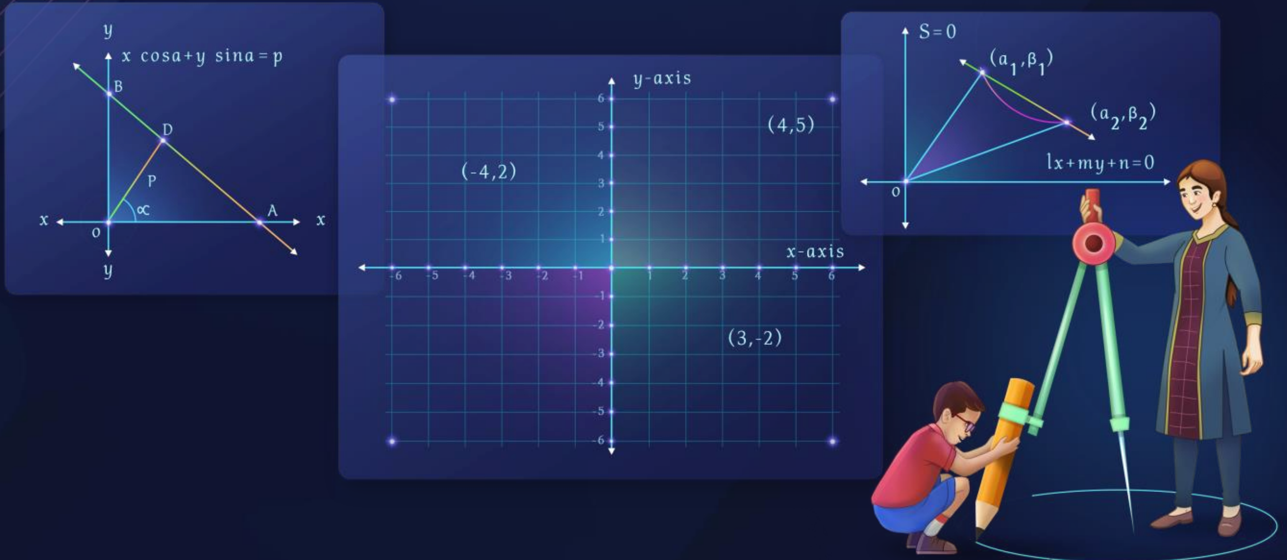


Table of Contents

Session 01 03
- Coordinate Plane 04
- Distance Formula 06
- Section Formula 16
- Area of Triangle 21
- Area of Quadrilateral. 23
- Area of Polygon 24

Session 02 26
- Polar Coordinates 27
- Centroid 34
- Incentre 39

Session 03 43
- Excentre 44
- Orthocenter 48
- Circumcenter 50
- Shifting of Origin 58
- Rotation of Axis 63

Session 04 67
- Locus 68
- Angle of inclination 75
- Slope of a line 79
- Calculation of Slope 86
- Condition for collinearity 89

Session 05 91
- Angle between two lines: 92
- Intercepts of a Line 98
- Slope - Intercept Form 104
- Slope Point Form 107
- Two Point Form 108
- Double Intercept Form 110

Session 06 115
- Normal Form 119
- Parametric Equation 124
- General Equation 134
- Slope Intercept Form 135
- Intercept Form 136

Session 07 137
- Internal Division 150
- External Division 152
- Distance of a point from a line 154
- Distance between parallel lines 159

Session 08 167
- Image of a point 168
- Reflection of a point about a Line 172
- Concurrency of Three Lines 180

Session 09 184
- Family of Line 185
- Angle Bisectors between the lines 193
- Angular Bisectors of Acute and Obtuse angles 195
- Bisector of the angle containing the given point 200

Session 10 204
- Pair of Straight Lines 205
- Homogeneous Equation 208
- Angle between Pair of Lines for non-homogenous equation 210
- Homogenization 221

Return to Top

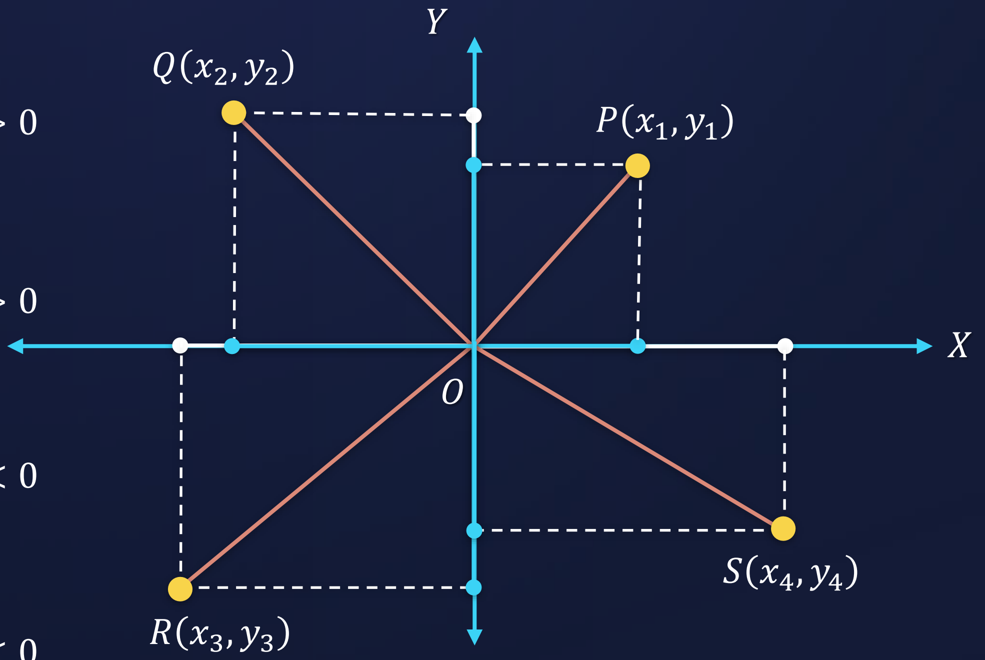
Session 01

Basics of coordinate Geometry

Return to Top

Coordinate Plane:

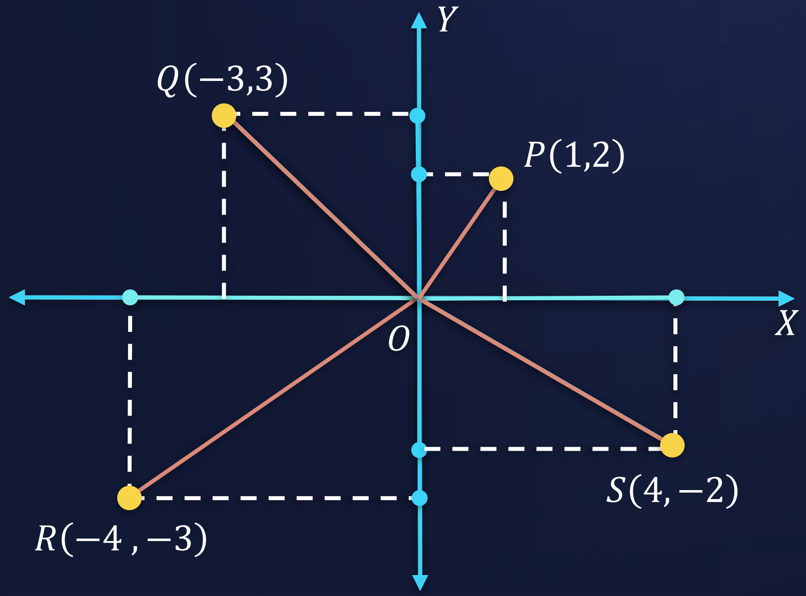
- 1^{st} Quadrant $x > 0; y > 0$
- 2^{nd} Quadrant $x < 0; y > 0$
- 3^{rd} Quadrant $x < 0; y < 0$
- 4^{th} Quadrant $x > 0; y < 0$



Return to Top

Coordinate Plane:

Lattice Point- A point whose abscissa and ordinate both are integers

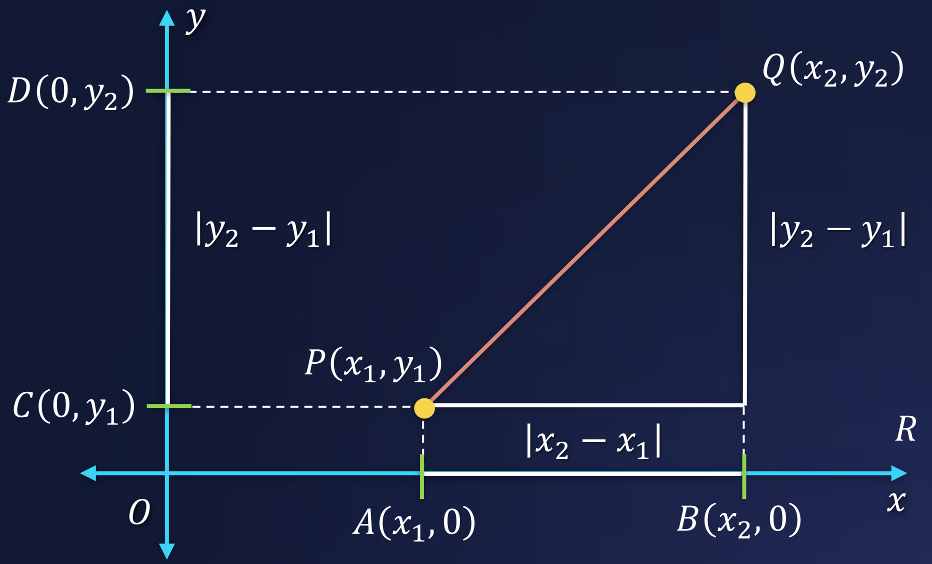


Return to Top

Key Takeaways

Distance Formula:

Distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$



[Return to Top](#)

Key Takeaways

Distance Formula:

Distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$

Using Pythagoras theorem,

$$PQ = \sqrt{PR^2 + QR^2}$$

$$\Rightarrow PQ = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$\boxed{PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Return to Top

The distance between points $(-3,4)$ and $(7,-6)$?

A. 10 units

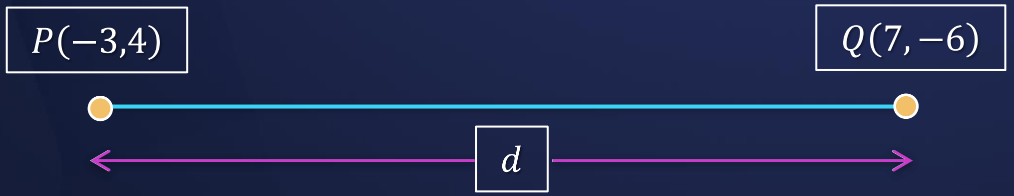
B. $10\sqrt{2}$ units

C. $5\sqrt{2}$ units

D. 5 units

Return to Top

The distance between points $(-3,4)$ and $(7,-6)$?



$$d = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7-(-3))^2 + (-6-4)^2}$$

$$= 10\sqrt{2}$$

$$\therefore d = 10\sqrt{2} \text{ units}$$

A 10 units

B $10\sqrt{2}$ units

C $5\sqrt{2}$ units

D 5 units

Return to Top

If $\frac{\pi}{2} < \alpha < \pi$, then the distance between the points $(\tan\alpha , 2)$ and $(0,1)$ is:

A. $\operatorname{cosec}\alpha$

B. $-\operatorname{cosec}\alpha$

C. $\sec\alpha$

D. $-\sec\alpha$

Return to Top

If $\frac{\pi}{2} < \alpha < \pi$, then the distance between the points $(\tan\alpha, 2)$ and $(0,1)$ is:

Let $P \equiv (\tan\alpha, 2)$, $Q \equiv (0,1)$

$$PQ = \sqrt{(\tan\alpha - 0)^2 + (2-1)^2}$$

$$= \sqrt{\tan^2\alpha + 1}$$

$$= \sqrt{\sec^2\alpha}$$

$$\because 1 + \tan^2\alpha = \sec^2\alpha$$

$$= |\sec\alpha|$$

$$\because \frac{\pi}{2} < \alpha < \pi$$

$$\boxed{\therefore PQ = -\sec\alpha}$$

A. $\operatorname{cosec}\alpha$

B. $-\operatorname{cosec}\alpha$

C. $\sec\alpha$

D. $-\sec\alpha$

Return to Top

A straight line through the point $A(3,4)$ is such that its intercept between the axes is bisected at A. Its equation is :

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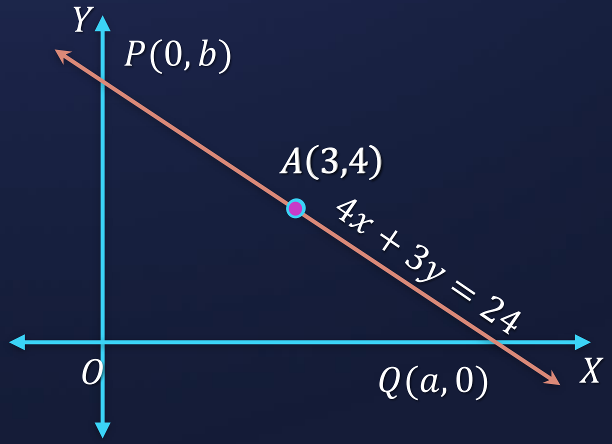
$A(3,4)$ is the mid-point of PQ

$\frac{a+0}{2} = 3$ & $\frac{0+b}{2} = 4$

$a = 6$ & $b = 8$

$\therefore$ Equation is $\frac{x}{6} + \frac{y}{8} = 1$

$4x + 3y = 24$



Return to Top

A triangle with vertices $(4,0)$, $(-1,-1)$ and $(3,5)$ is:

A. Isosceles

B. Equilateral

C. Right-angled

D. Right-angled Isosceles

Return to Top

A triangle with vertices $(4,0)$, $(-1,-1)$ and $(3,5)$ is:

Consider a ΔABC with $A(4,0), B(-1,-1)$ and $C(3,5)$

$$AB = \sqrt{(4-(-1))^2 + (0-(-1))^2} = \sqrt{26}$$

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2} = \sqrt{52}$$

$$AC = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$$

Here $AB = BC \Rightarrow \Delta ABC$ is Isosceles

Also, $AB^2 + AC^2 = BC^2 \Rightarrow \Delta ABC$ is Right-angled at A

$\therefore \Delta ABC$ is Right-angled Isosceles

A. Isosceles

B. Equilateral

C. Right-angled

D. Right-angled Isosceles

Return to Top

The quadrilateral formed by the points $P(-5,0), Q(-3,-1), R(-2,5)$ and $S(-4,6)$ is a:

A Rectangle

B Square

C Parallelogram

D Rhombus

Return to Top

The quadrilateral formed by the points $P(-5,0), Q(-3,-1), R(-2,5)$ and $S(-4,6)$ is a:

Given: $P(-5,0), Q(-3,-1), R(-2,5)$ and $S(-4,6)$

Now, finding the distances

$PQ = \sqrt{(-5+3)^2 + (0+1)^2} = \sqrt{5}$ units

$QR = \sqrt{(-3+2)^2 + (-1-5)^2} = \sqrt{37}$ units

$RS = \sqrt{(-2+4)^2 + (5-6)^2} = \sqrt{5}$ units

$SP = \sqrt{(-4+5)^2 + (6-0)^2} = \sqrt{37}$ units

So, opposite sides are equal in length,

now finding the diagonal lengths,

$PR = \sqrt{(-5+2)^2 + (0-5)^2} = \sqrt{34}$ units

$QS = \sqrt{(-3+4)^2 + (-1-6)^2} = \sqrt{50}$ units

As the diagonals are unequal so it is a parallelogram.

A. Rectangle

B. Square

C. Parallelogram

D. Rhombus

Return to Top

Key Takeaways

Section Formula:

Internal Division: $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ internally in the ratio $m : n$.

$$x = \frac{mx_2 + nx_1}{m + n}; \quad y = \frac{my_2 + ny_1}{m + n}$$

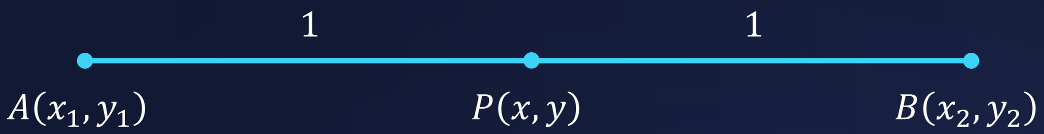
Return to Top

Key Takeaways

Mid-point of a line:

If P is the mid-point of AB

$\Rightarrow P$ divides AB in the ratio $1:1$



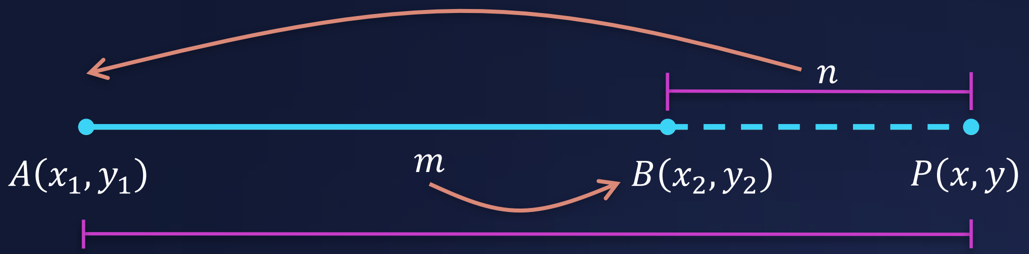
$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2}$$

Return to Top

Key Takeaways

Section Formula:

External Division: $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ externally in the ratio $m: n$.



$$x = \frac{mx_2 - nx_1}{m - n};\ y = \frac{my_2 - ny_1}{m - n}$$

Return to Top

Let the angular opposite points of a parallelogram be $(3, 4)$ and $(1, -2)$. Coordinates of remaining two points are $(6, 1)$ and (x, y). Compute (x, y):

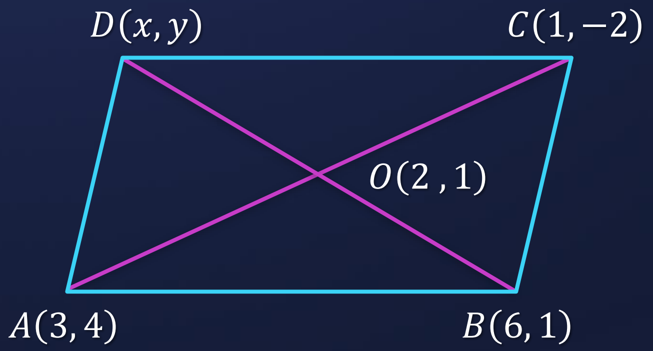
Diagonals of a parallelogram bisect each other

$$\Rightarrow O \equiv \left(\frac{3+1}{2}, \frac{4-2}{2}\right)$$

$$\equiv (2\,,1)$$

$$2 = \frac{x+6}{2}\ ;\ 1 = \frac{y+1}{2}$$

$$x = -2,\ y = 1.$$



Return to Top

Find the length of the median from vertex A of a triangle ΔABC whose vertices are $A\,(-1,3),\ B(1,-1)$ and $C(5,1)$

D is the mid-point of BC.

$$D \equiv \left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$$

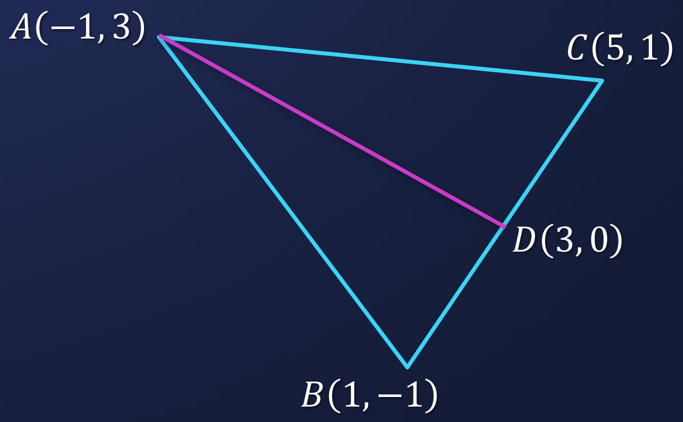
Using distance formula,

$$AD = \sqrt{(-1-3)^2 + (3-0)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$= 5$ units

$\therefore AD = 5$ units.



Return to Top

Key Takeaways

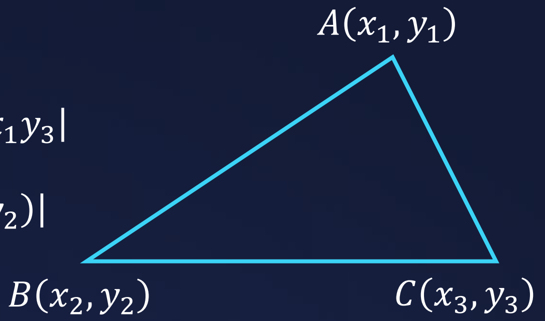
Area of Triangle:

$$\text{Area }(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\left|x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3\right|$$

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



Note:

If three points A, B, and C are collinear,

$Ar.(\Delta ABC) = 0$

Return to Top

Find the area of triangle whose vertices are $A(3,2), B(11,8)$, & $C(8,12)$.

Let $A \equiv (x_1, y_1) = (3, 2)$

$B \equiv (x_2, y_2) = (11, 8)$

$C \equiv (x_3, y_3) = (8, 12)$

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ 11 & 8 & 1 \\ 8 & 12 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|\{3(8-12) + 11(12-2) + 8(2-8)\}|$$

$$= \frac{1}{2}|\{-12 + 110 - 48\}|$$

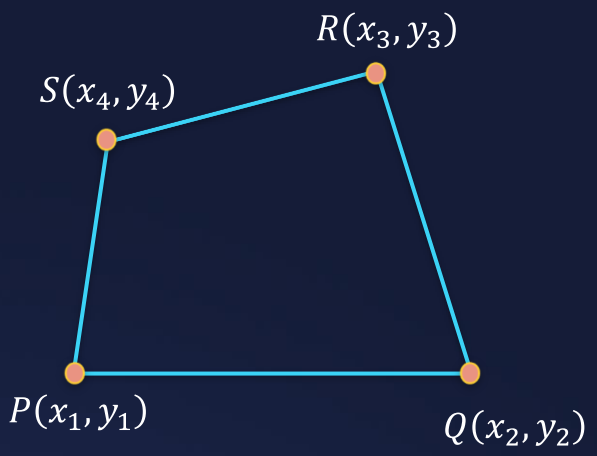
$$= \frac{1}{2}|\{-12 + 110 - 48\}| = 25$$

$\therefore$ Area of $\Delta ABC = 25$ sq. units

Return to Top

Key Takeaways

Area of Quadrilateral.



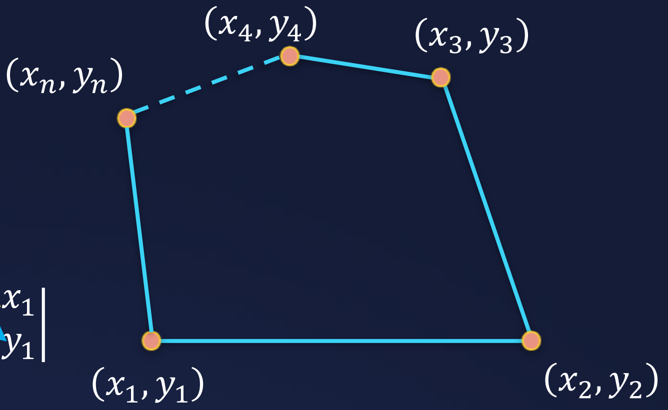
- $Ar.(PQRS) = \frac{1}{2}\left|\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{array}\right|$

- $Ar.(PQRS) = \frac{1}{2}|x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4|$

Return to Top

Key Takeaways

Area of Polygon.



- Area of polygon $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{vmatrix}$

$$= \frac{1}{2}\left|x_1 y_2 - x_2 y_1 + \cdots \cdots + x_n y_1 - x_1 y_n\right|$$

Return to Top

The area of pentagon with vertices $(1,1),(7,21),(12,2),(7,-3)$ and $(0,-3)$ taken in order is ?

Vertices of pentagon $\equiv (1,1),(7,21),(12,2),(7,-3)$ and $(0,-3)$

Area of Pentagon $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} 1 & 7 & 12 & 7 & 0 & 1 \\ 1 & 21 & 2 & -3 & -3 & 1 \end{vmatrix}$$

$\Rightarrow \Delta = \frac{1}{2}|(14 + (14 - 252) + (-36 - 14) + (-21) + (3)|$

$\Rightarrow \Delta = \frac{1}{2} \times |14 - 238 - 50 - 21 + 3|$

$= \frac{1}{2} \times |-309 + 17|$

$\Rightarrow \Delta = 146$ sq. units

Return to Top

Session 02

Polar coordinates and Geometrical centers

Return to Top

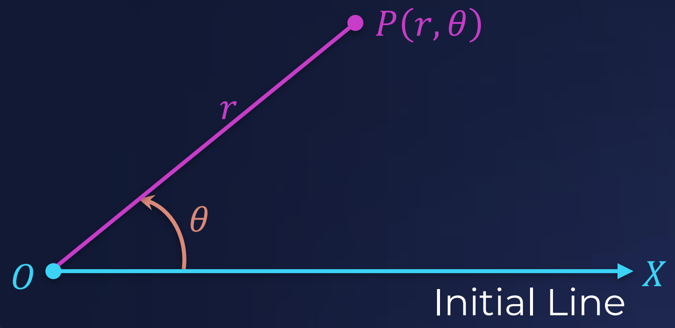
Key Takeaways

Polar Coordinates:

$OP = r$ (radius vector)

$\angle XOP = \theta$ (Vectorial angle)

where $\theta \in (-\pi, \pi]$



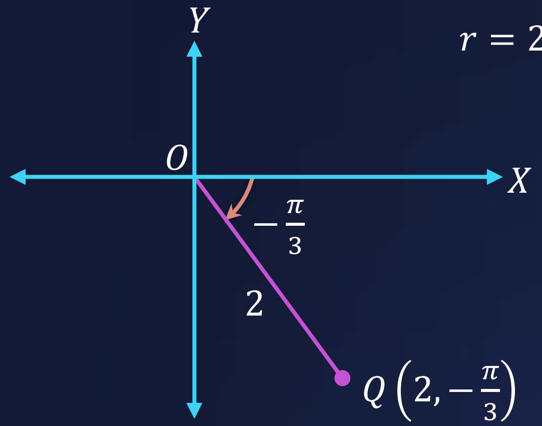
Polar coordinates of the point $P \equiv (r, \theta)$

Return to Top

Key Takeaways

Example:

To plot the point with polar coordinates $\left(2, -\frac{\pi}{3}\right)$ in the plane :



$r = 2, \theta = -\frac{\pi}{3}$

Return to Top

Key Takeaways

Relation between POLAR and CARTESIAN coordinates:

$P(x, y) \equiv$ Cartesian coordinates

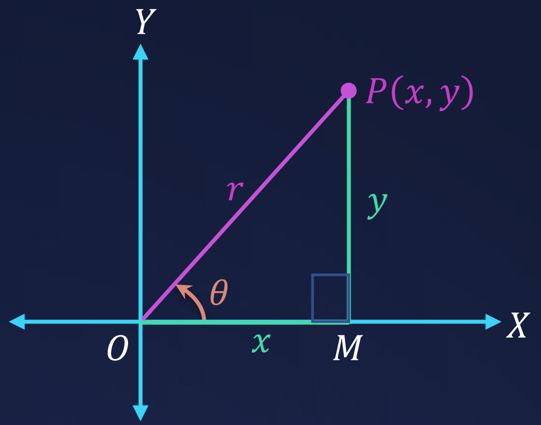
$(r, \theta) \equiv$ Polar coordinates

In ΔOMP,

$\Rightarrow r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$

$$\left.\begin{aligned} &\Rightarrow \sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta \\ &\Rightarrow \cos\theta = \frac{y}{r} \Rightarrow x = r\cos\theta \end{aligned}\right\} \text{Conversion formulas}$$

Also, $\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$



Return to Top

Key Takeaways

Relation between POLAR and CARTESIAN coordinates:

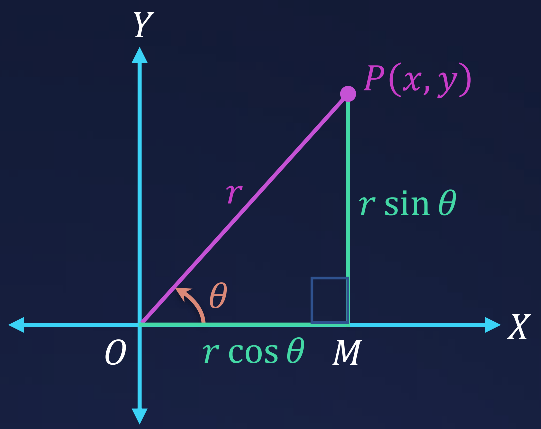
$$\left.\begin{aligned} y &= r\sin\theta \\ x &= r\cos\theta \end{aligned}\right\} \text{Conversion formulas}$$

$$r = \sqrt{x^2 + y^2}\,; \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad -\pi < \theta \le \pi$$

$$\therefore (x, y) \equiv (r\cos\theta\,, r\sin\theta)$$

P can be written in polar coordinates as :

$$P(r, \theta) = P\left(\sqrt{x^2 + y^2}, \tan^{-1}\left(\frac{y}{x}\right)\right)$$



Return to Top

Key Takeaways

Note:

x	y	θ
$+$	$-$	$\tan^{-}\left(\frac{y}{x}\right)$
$-$	$+$	$\pi - \tan^{-}\left(\frac{y}{x}\right)$
$-$	$+$	$-\pi + \tan^{-}\left(\frac{y}{x}\right)$
$+$	$-$	$-\tan^{-}\left(\frac{y}{x}\right)$

Return to Top

The polar coordinates of the points whose Cartesian coordinates are $(-3\,,3\,)$ is :

$B \equiv (-3\,,3\,) \equiv (x, y)$

Let the polar coordinates of $B(-3,3)$ be $Q(r,\theta)$

$r = \sqrt{x^2 + y^2}$

$\Rightarrow r = \sqrt{(-3)^2 + 3^2}$

$= \sqrt{18}$

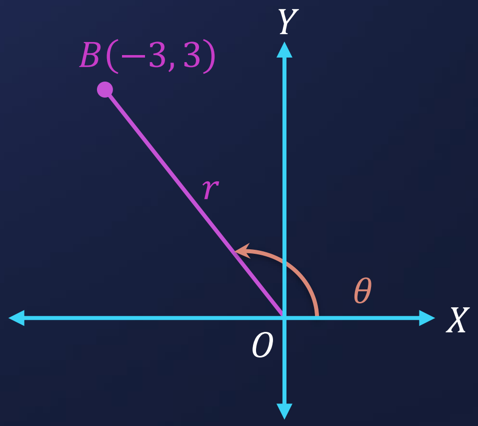
$= 3\sqrt{2}$

$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$

$\Rightarrow \alpha = \tan^{-1}\left|\frac{3}{-3}\right|$

$= \tan^{-1}1$

$= \frac{\pi}{4}$



But $B(-3,3)$ lies in 2$^{\text{nd}}$ Quadrant

$\Rightarrow \theta = \pi - \alpha = \pi - \frac{\pi}{3} = \frac{3\pi}{4}$

$\therefore$ In polar coordinates $B(-3,3) = B\left(3\sqrt{2}, \frac{3\pi}{4}\right)$

Return to Top

The Cartesian coordinates of the points whose polar coordinates are $(5\sqrt{2}, \frac{\pi}{4})$ is:

$$\left.\begin{aligned} y &= r\sin\theta \\ x &= r\cos\theta \end{aligned}\right\} \text{Conversion formulas}$$

$$r = 5\sqrt{2},\ \theta = \frac{\pi}{4}$$

$$x = 5\sqrt{2}\cos\frac{\pi}{4} = 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5$$

$$y = 5\sqrt{2}\sin\frac{\pi}{4} = 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5$$

$\therefore$ In Cartesian coordinates $\left(5\sqrt{2}, \frac{\pi}{4}\right) = (5, 5)$

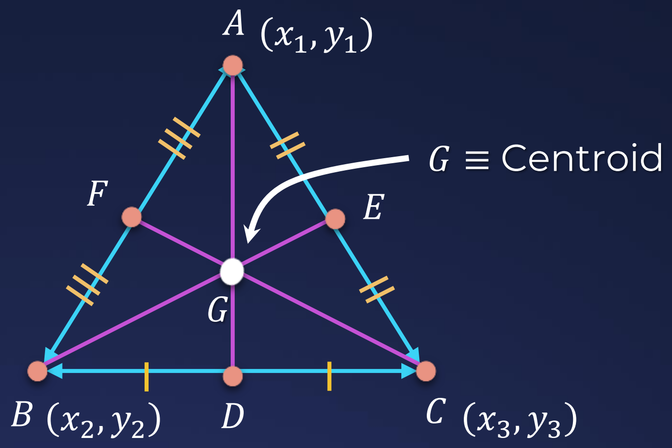
Return to Top

Key Takeaways

Centroid

- The point of concurrency of the medians of a triangle.

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$



Return to Top

Two vertices of a triangle are $(-1, 4)$ and $(5, 2)$. If $(0, -3)$ is its centroid then third vertex is :

A. $(-4, -15)$

B. $(3, 7)$

C. $(4, 15)$

D. $(-4, 14)$

Return to Top

Two vertices of a triangle are $(-1,4)$ and $(5,2)$. If $(0,-3)$ is its centroid then third vertex is :

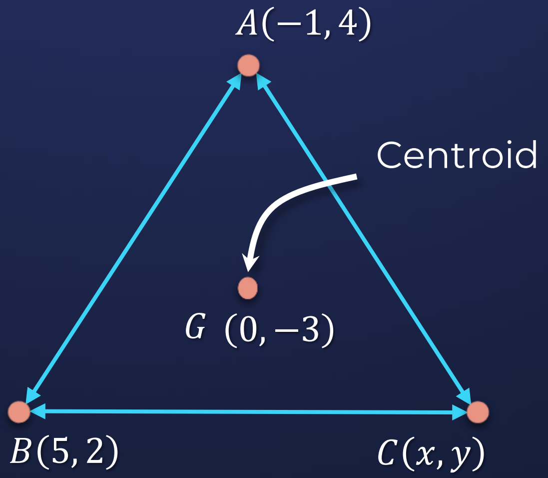
$P(0,-3)$ is the centroid

$$\Rightarrow (0,-3) \equiv \left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right)$$

$$\Rightarrow (0,-3) \equiv \left(\frac{x+4}{3}, \frac{y+6}{3}\right)$$

$\frac{x+4}{3} = 0$	$\frac{y+6}{3} = -3$
$\Rightarrow x = -4$	$\Rightarrow y = -15$

$\therefore$ Coordinates of $C \equiv (-4,-15)$



A $(-4,-15)$

B $(3,7)$

C $(4,15)$

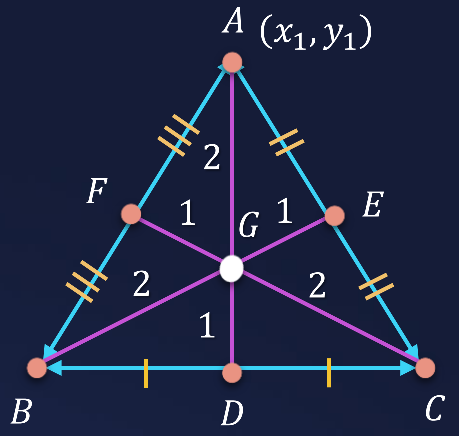
D $(-4,14)$

Return to Top

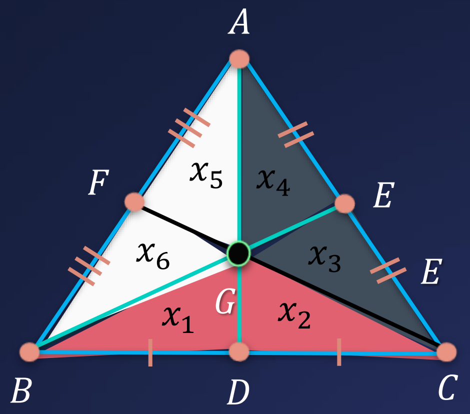
Key Takeaways

Features of Centroid

- Centroid divides the median in the ratio 2 : 1.



- All three medians together divide a triangle into six equal parts.



$$
\begin{array}{rl}
 & x_1 + x_5 + x_6 = x_2 + x_3 + x_4 \\
+ & x_1 + x_2 + x_3 = x_4 + x_5 + x_6 \\
\hline
 & x_1 = x_4
\end{array}
$$

Return to Top

Let $A(1,0), B(6,2)$ and $C\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$ is ______

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Given: P is a point inside the ΔABC such that

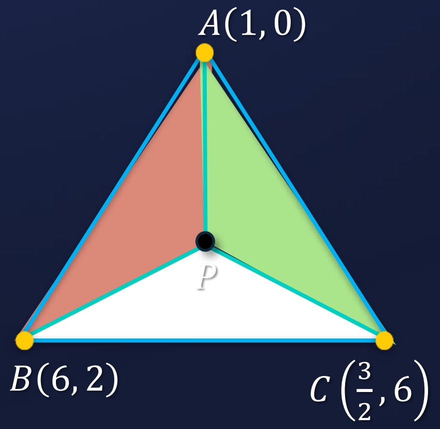
$$Ar.(\Delta APC) = Ar.(\Delta APB) = Ar.(\Delta BPC)$$

$\Rightarrow P$ will be centroid of ΔABC

$$\therefore P \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{2}\right) \equiv \left(\frac{1+6+\frac{3}{2}}{3}, \frac{0+2+6}{3}\right)$$

$$\Rightarrow P \equiv \left(\frac{17}{6}, \frac{8}{3}\right)$$

Given, $Q\left(-\frac{7}{6},-\frac{1}{3}\right)$



$$\Rightarrow PQ = \sqrt{\left(\frac{17}{6}-\left(-\frac{7}{6}\right)\right)^2+\left(\frac{8}{3}-\left(-\frac{1}{3}\right)\right)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

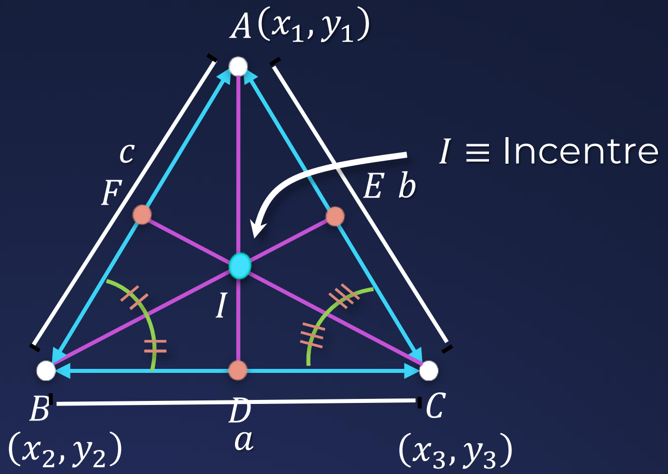
Return to Top

Key Takeaways

Incentre

- The point of concurrency of internal angle bisectors of a triangle.

$$I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$



Return to Top

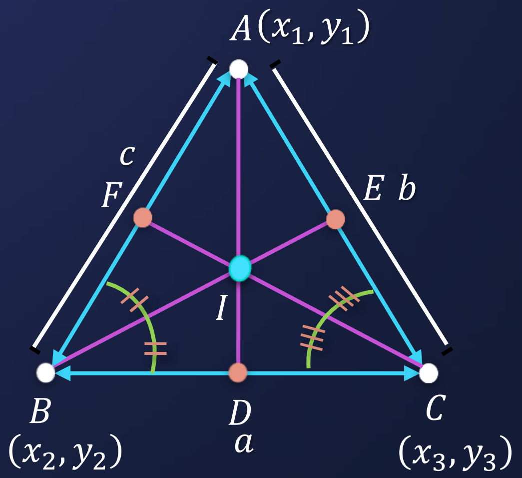
Features of Incentre

- Angular bisector divides opposite side in the ratio of other two sides (Angular bisector theorem)

$$\frac{BD}{DC} = \frac{c}{b}$$

$$\frac{AE}{EC} = \frac{c}{a}$$

$$\frac{AF}{FB} = \frac{b}{a}$$



- Ratio in which the incentre divides the internal angle bisectors:

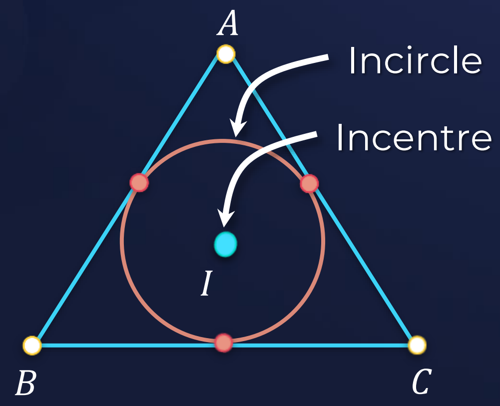
$$\frac{AI}{ID} = \frac{b+c}{a}$$

$$\frac{BI}{IE} = \frac{c+a}{b} \qquad \frac{CI}{IF} = \frac{a+b}{c}$$

Return to Top

Features of Incentre

- The largest circle contained in a triangle is called the Inscribed circle or the incircle of the triangle.



[Return to Top](#)

If the vertices of a triangle are $(4,-2)$, $(-2,4)$ and $(5,5)$, then find its incentre.

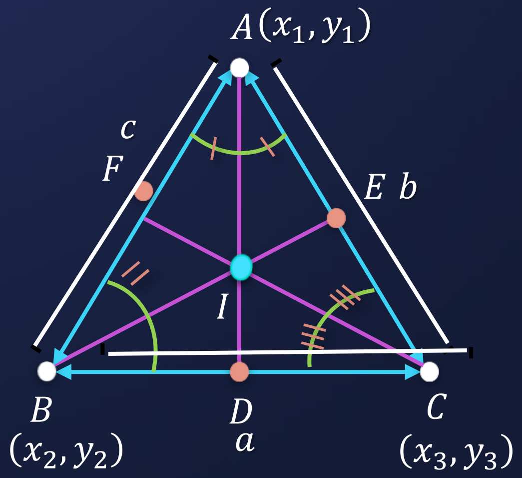
$$I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

Using distance formula

$$a = \sqrt{(5+2)^2+(5-4)^2} = \sqrt{50} = 5\sqrt{2}$$

$$b = \sqrt{(5-4)^2+(5+2)^2} = \sqrt{50} = 5\sqrt{2}$$

$$c = \sqrt{(4+2)^2+(-2-4)^2} = \sqrt{72} = 6\sqrt{2}$$



$$\equiv \left(\frac{4\times5\sqrt{2}+(-2)\times5\sqrt{2}+5\times6\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}, \frac{-2\times5\sqrt{2}+4\times5\sqrt{2}+5\times6\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}\right)$$

$$\equiv \left(\frac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{16\sqrt{2}}, \frac{-10\sqrt{2}+20\sqrt{2}+30\sqrt{2}}{16\sqrt{2}}\right) \equiv \left(\frac{40\sqrt{2}}{16\sqrt{2}}, \frac{40\sqrt{2}}{16\sqrt{2}}\right)$$

$$\equiv \left(\frac{5}{2}, \frac{5}{2}\right) \qquad \therefore \text{ Incentre} \equiv \left(\frac{5}{2}, \frac{5}{2}\right)$$

Return to Top

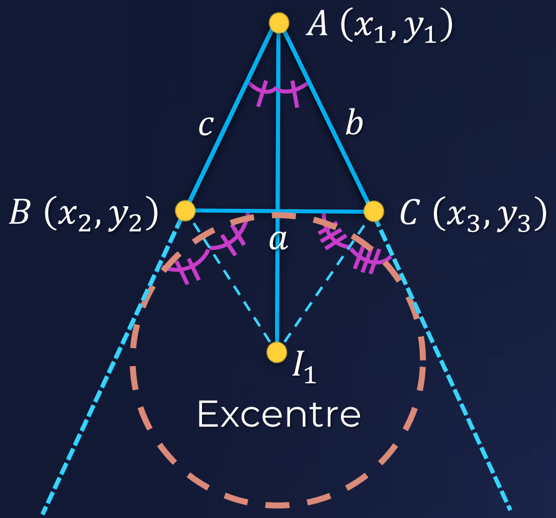
Session 03

Transformation and Rotation of Axis

Return to Top

Key Takeaways

Excentre:

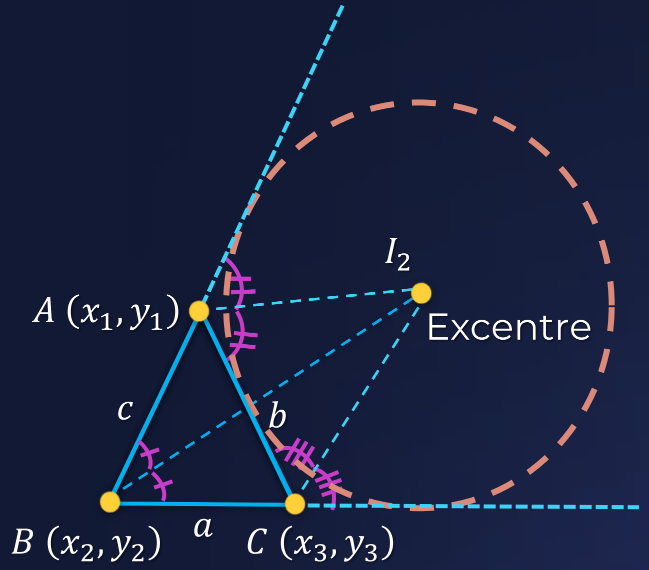


$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

Return to Top

Key Takeaways

Excentre:

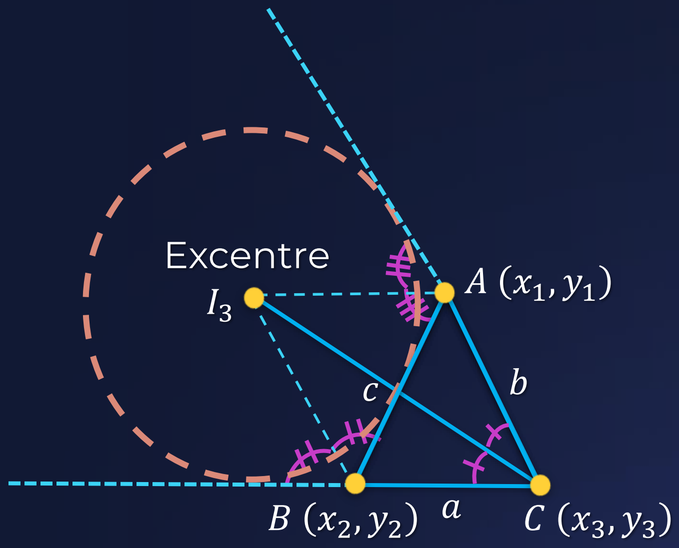


$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right)$$

Return to Top

Key Takeaways

Excentre:



$$I_3 \equiv \left(\frac{ax_1+bx_2-cx_3}{a+b-c},\ \frac{ay_1+by_2-cy_3}{a+b-c}\right)$$

Return to Top

If the coordinates of the vertices of the triangle ABC are (4,0), (2,8), $(0,-6)$ respectively then find excentre opposite to vertex A.

Steps for finding a, b, c ;

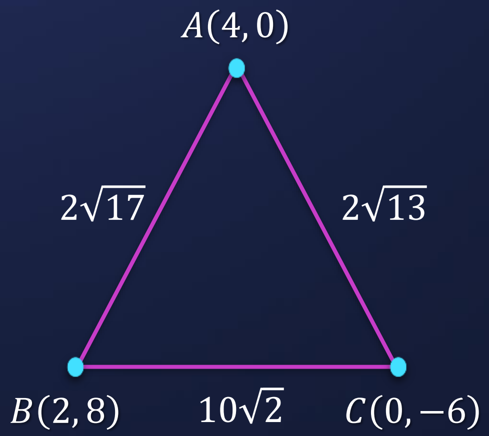
By using distance formula

$$BC = a = \sqrt{(0-2)^2 + (-6-8)^2} = 10\sqrt{2}$$

$$AC = b = \sqrt{(0-4)^2 + (-6-0)^2} = 2\sqrt{13}$$

$$AB = c = \sqrt{(2-4)^2 + (8-0)^2} = 2\sqrt{17}$$

$$a = 10\sqrt{2},\ b = 2\sqrt{13},\ c = 2\sqrt{17}$$



Hence, excentre opposite to vertex A

$$\because I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

$$= \left(\frac{-10\sqrt{2}(4) + 2\sqrt{13}(2) + 2\sqrt{17}(0)}{-10\sqrt{2} + 2\sqrt{13} + 2\sqrt{17}}, \frac{-10\sqrt{2}(0) + 2\sqrt{13}(8) + 2\sqrt{17}(-6)}{-10\sqrt{2} + 2\sqrt{13} + 2\sqrt{17}}\right)$$

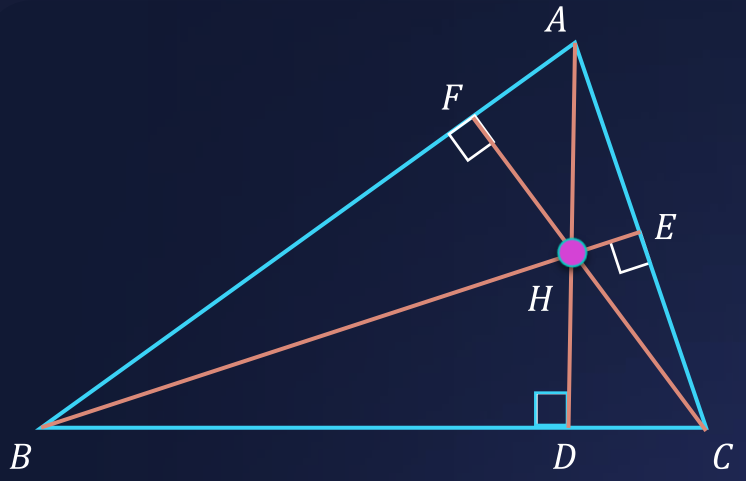
$$= \left(\frac{-20\sqrt{2} + 2\sqrt{13}}{-5\sqrt{2} + \sqrt{13} + \sqrt{17}}, \frac{8\sqrt{13} - 6\sqrt{17}}{-5\sqrt{2} + \sqrt{13} + \sqrt{17}}\right)$$

Return to Top

Key Takeaways

Orthocenter:

The point of concurrency of altitudes of a triangle



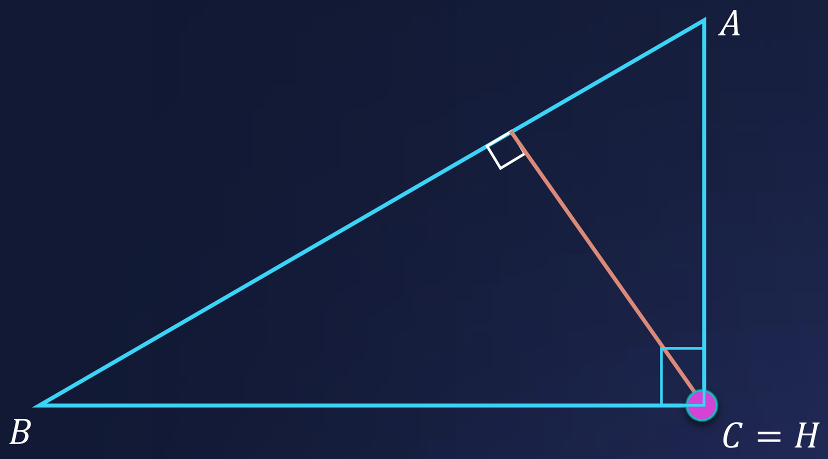
$H \equiv$ Orthocentre

Return to Top

Key Takeaways

Feature of Orthocenter:

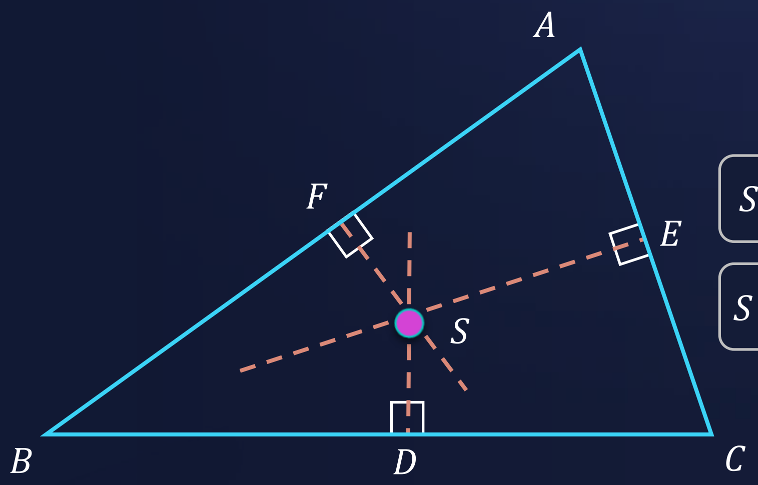
If $\triangle ABC$ is a right angled triangle, then, orthocenter coincides with the right angular vertex.



[Return to Top](#)

Circumcenter:

The point of concurrency of perpendicular side bisectors of a triangle.



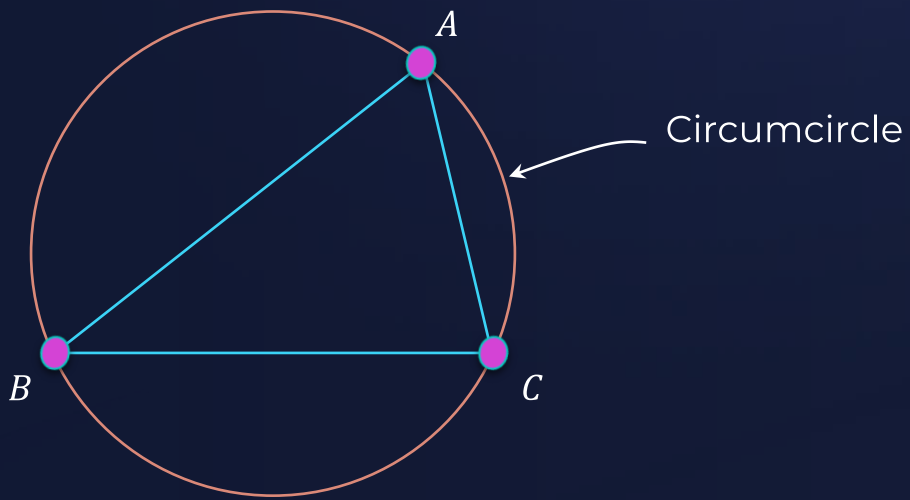
$S \equiv$ Circumcentre

$$S \equiv \left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

Return to Top

Feature of Circumcenter:

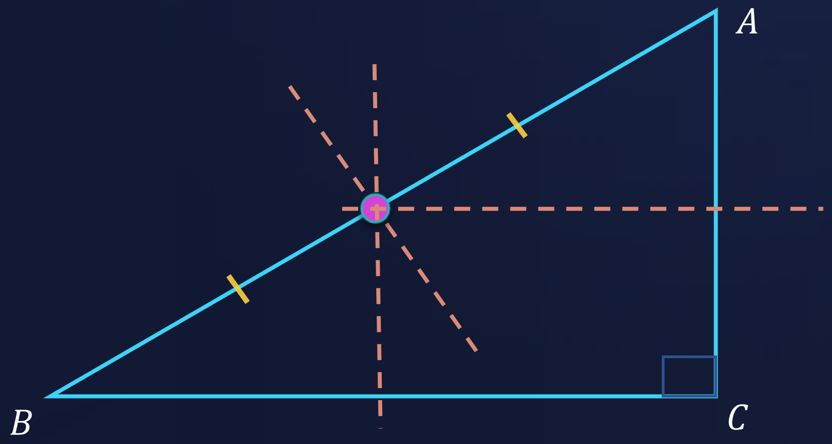
The circle circumscribing the vertices of the triangle is called the Circumcircle of the triangle.



Return to Top

Feature of Circumcenter:

In a right angled triangle, circumcenter lies on the mid-point of the hypotenuse.



Return to Top

Feature of Circumcenter:

In a triangle, Circumcenter 'S', Centroid 'G' and Orthocenter 'H' are collinear. G divides SH in the ratio $1:2$

$$\boxed{SG : GH = 1 : 2 \; i.e. \frac{SG}{GH} = \frac{1}{2}}$$

Return to Top

Let the orthocenter and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcenter of this triangle, then the radius of the circle having line segment AC as diameter is :

A $2\sqrt{5}$

B $\frac{3\sqrt{5}}{2}$

C $3\sqrt{\frac{5}{2}}$

D $\sqrt{10}$

Return to Top

Let the orthocenter and centroid of a triangle be $A\,(-3\,,5)$ and $B(3\,,3)$ respectively. If C is the circumcenter of this triangle , then the radius of the circle having line segment AC as diameter is :

Given $A \equiv$ Orthocentre $(-3,5)$

$B \equiv$ Centroid $(3,3)$

$C \equiv$ Circumcentre

Return to Top

Let the orthocenter and centroid of a triangle be $A\,(-3\,,5)$ and $B(3\,,3)$ respectively. If C is the circumcenter of this triangle , then the radius of the circle having line segment AC as diameter is :

$$\Rightarrow \frac{BC}{AB} = \frac{1}{2}$$

Adding 1 both sides :

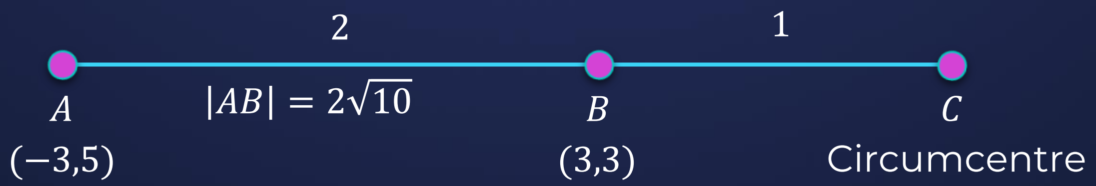
$$\Rightarrow \frac{BC}{AB} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\Rightarrow \frac{BC+AB}{AB} = \frac{3}{2} \Rightarrow \frac{AC}{AB} = \frac{3}{2}$$

$$\Rightarrow |AB| = \sqrt{(3+3)^2 + (3-5)^2} = \sqrt{40} = 2\sqrt{10}$$

Return to Top

Let the orthocenter and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcenter of this triangle, then the radius of the circle having line segment AC as diameter is :



$$\Rightarrow \frac{AC}{AB} = \frac{3}{2}$$

$$\Rightarrow AC = \frac{3}{2}AB \Rightarrow AC = \frac{3}{2}2\sqrt{10} = 3\sqrt{10}$$

$\therefore$ Radius of the circle r with AC as diameter

$$r = \frac{AC}{2} = \frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}$$

A. $2\sqrt{5}$

B. $3\sqrt{\frac{5}{2}}$

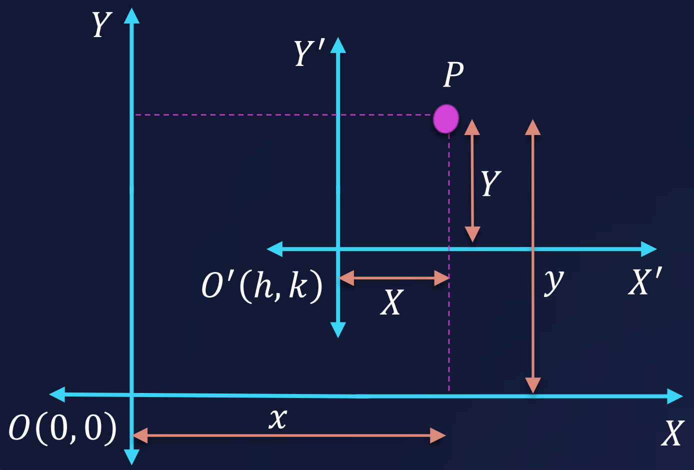
C. $3\sqrt{\frac{5}{2}}$

D. $\sqrt{10}$

Return to Top

Key Takeaways

Shifting of Origin:



OX & OY → Original Coordinate Axes
$O'X'$ & $O'Y'$ → Shifted Coordinate Axes

$P(x, y)$ coordinate of point P with respect to O.
$P(X, Y)$ coordinate of point P with respect to O'.

Return to Top

Key Takeaways

Shifting of Origin:

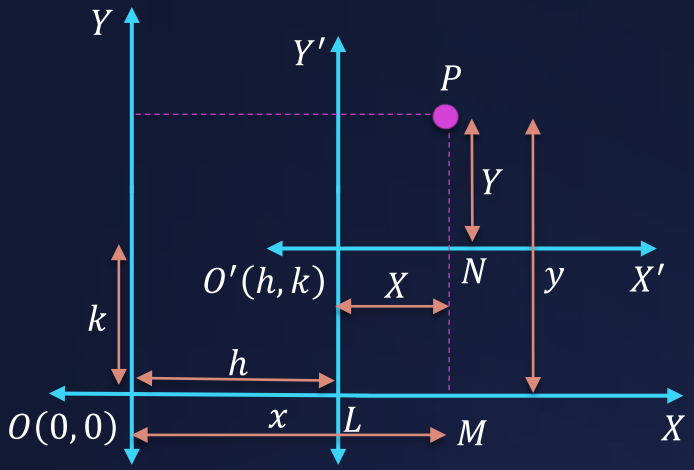
Now, $OM = OL + LM$

$$x = h + X$$

And, $\mathrm{PM} = PN + NM$

$$y = k + Y$$

$$\boxed{P(x, y) = P(X + h, Y + k)}$$



[Return to Top](#)

Key Takeaways

Shifting of Origin:

When the origin is shifted at the point (h, k) then substitute $x = X + h$, $y = Y + k$.

The coordinates of the old origin referred to the new axes are $(-h, -k)$.

Return to Top

At what point the origin be shifted if the coordinates of a point $(4, 5)$, becomes $(-3, 9)$.

A $(7, -4)$

B $(1, 14)$

C $(8, -5)$

D $(-1, -4)$

Return to Top

At what point the origin be shifted if the coordinates of a point $(4, 5)$, becomes $(-3, 9)$.

Let origin be shifted to $O'(h, k)$

Original coordinates = $(4, 5)$

Shifted coordinates = $(-3, 9) \Rightarrow X = -3, Y = 9$

$x = X + h$	$y = Y + k$
$\Rightarrow 4 = -3 + h$	$\Rightarrow 5 = 9 + k$
$\Rightarrow h = 7$	$\Rightarrow k = -4$

Hence the origin is to be shifted to $(7, -4)$

A $(7, -4)$

B $(1, 14)$

C $(8, -5)$

D $(-1, -4)$

Return to Top

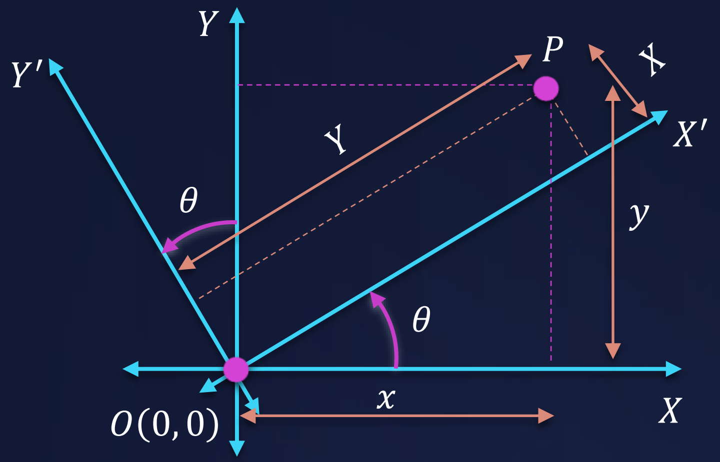
Key Takeaways

Rotation of Axis:

OX & OY → Original Coordinate Axes
$O'X'$ & $O'Y'$ → Rotated Coordinate Axes

$P(x, y)$ coordinate of point P with respect to O.
$P(X, Y)$ coordinate of point P with respect to O'.

	X	Y
x	$\cos\theta$	$\sin\theta$
y	$-\sin\theta$	$\cos\theta$



[Return to Top](#)

Key Takeaways

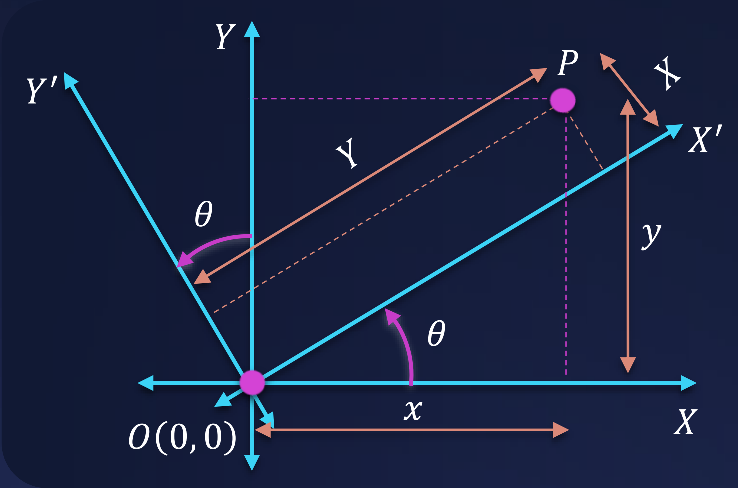
Rotation of Axis:

$$x = X\cos\theta - Y\sin\theta$$

$$y = X\sin\theta + Y\cos\theta$$

$$X = x\cos\theta + y\sin\theta$$

$$Y = -x\sin\theta + y\cos\theta$$



[Return to Top](#)

If the axes are turned through an angle 45^0 in the anti-clockwise direction then the equation $3x^2 + 3y^2 + 2xy = 2$ transforms to:

A $3X^2 + Y^2 = 2$

B $2X^2 + Y^2 = 1$

C $X^2 + 3Y^2 = 2$

D $3X^2 + 3Y^2 = 1$

Return to Top

If the axes are turned through an angle 45^0 in the anti-clockwise direction then the equation $3x^2 + 3y^2 + 2xy = 2$ transforms to:

The axes are turned through an angle 45^0

$\theta = 45^0$

$x = X\cos\theta - Y\sin\theta$ $\quad$ $y = X\sin\theta + Y\cos\theta$

$\Rightarrow \frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}} = x$ $\quad$ $\Rightarrow \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}} = y$

Substituting the value of x and y in the equation:

$3x^2 + 3y^2 + 2xy = 2$

$\Rightarrow 3\left(\frac{X-Y}{\sqrt{2}}\right)^2 + 3\left(\frac{X+Y}{\sqrt{2}}\right)^2 + 2\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) = 2$

$\Rightarrow 3(X^2 - 2XY + Y^2) + 3(X^2 + 2XY + Y^2) + 2(X^2 - Y^2) = 4$

$\Rightarrow 8X^2 + 4Y^2 = 4$

$\Rightarrow 2X^2 + Y^2 = 1$

A $3X^2 + Y^2 = 2$

B $2X^2 + Y^2 = 1$

C $X^2 + 3Y^2 = 2$

D $3X^2 + 3Y^2 = 1$

Return to Top

Session 04

Locus & slope concept and its applications

Return to Top

Key Takeaways

Locus:

- When a point moves in a plane under certain geometric conditions, the point traces out a path. This path of the moving point is called its locus.

Example: Locus of all the points equidistant from a fixed point on a plane is a Circle.

Return to Top

Key Takeaways

To find Locus of a point

- Let (h, k) be the coordinate of the moving point say P.
- Write the given condition in mathematical form involving (h, k).
- Transformation variable(s) if any.
- Replace h by x and k by y.
- Equation obtained is the locus of the point.

Return to Top

The ends of the hypotenuse of a right-angled triangle are $(6,0)$ and $(0,6)$, then the locus of the third vertex is:

A. $x^2 + y^2 - 6x - 6y = 2$

B. $x^2 + y^2 - 6x - 6y = 2$

C. $x^2 + y^2 + 6x + 6y = 2$

D. $x^2 + y^2 + 6x + 6y = 0$

Return to Top

The ends of the hypotenuse of a right-angled triangle are $(6,0)$ and $(0,6)$, then the locus of the third vertex is:

A. $x^2 + y^2 - 6x - 6y = 2$

B. $x^2 + y^2 - 6x - 6y = 0$

C. $x^2 + y^2 + 6x + 6y = 2$

D. $x^2 + y^2 + 6x + 6y = 0$

Let $P(h,k)$ be the third vertex of ΔPAB

In ΔPAB, using Pythagoras theorem:

$|AB|^2 = |AP|^2 + |BP|^2$

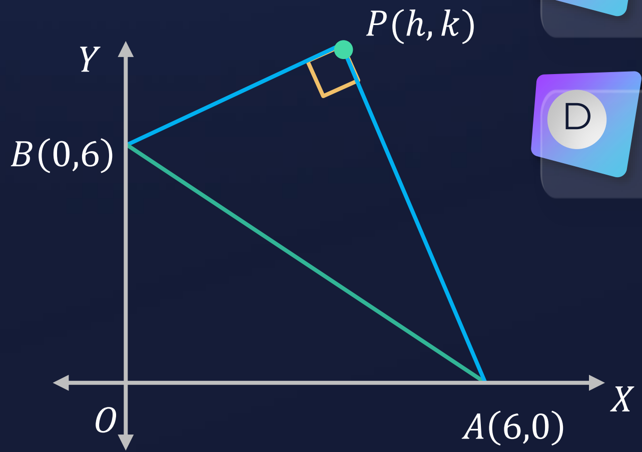
$\Rightarrow (6-0)^2 + (0-6)^2 = (h-6)^2 + (k-0)^2 + (h-0)^2 + (k-6)^2$

$\Rightarrow 36 + 36 = h^2 + 36 - 12h + k^2 + h^2 + k^2 + 36 - 12k$

$\Rightarrow 2h^2 + 2k^2 - 12h - 12k = 0$

$\Rightarrow h^2 + k^2 - 6h - 6k = 0$

$\therefore$ Locus of (h,k) is : $x^2 + y^2 - 6x - 6y = 0$



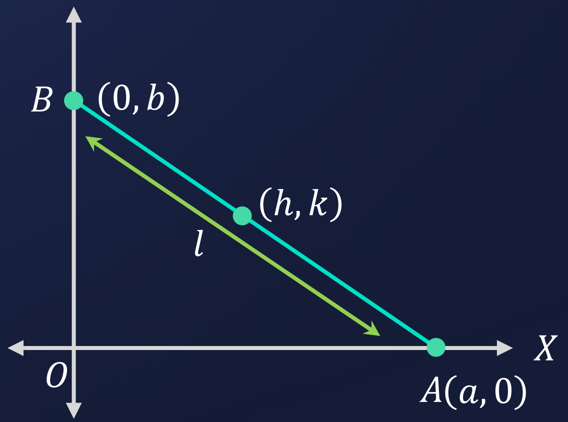
Return to Top

A rod of length l slides with its ends on two perpendicular lines, then the locus of its midpoint is :

a) $x^2 + y^2 = 4l^2$ b) $4x^2 + 4y^2 = l^2$

c) $x^2 + y^2 = l^2$ d) $x^2 + 4y^2 = 4l^2$

Solution :



Let AB be a rod of length l & coordinates of A and B be $(a, 0)$ and $(0, b)$ respectively,

Let $P(h, k)$ be locus of the mid point of the rod AB

$\Rightarrow h = \frac{a+0}{2}$ & $k = \frac{0+b}{2}$

$\Rightarrow h = \frac{a}{2}$ & $k = \frac{b}{2} \Rightarrow a = 2h$ & $b = 2k$

In ΔOAB, Using Pythagoras theorem $\Rightarrow AB^2 = OA^2 + OB^2$

$\Rightarrow l^2 = a^2 + b^2 \Rightarrow l^2 = (2h)^2 + (2k)^2$

$\therefore$ Locus of (h, k) is : $\boxed{4x^2 + 4y^2 = l^2}$

Return to Top

The locus of the point $(t^2 + t + 1, t^2 - t + 1), t \in \mathbb{R}$ is :

A $x^2 + y^2 + 2xy + 2y + 4 = 0$

B $x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$

C $x^2 + y^2 + 2xy + 2x + 2y + 4 = 0$

D $x^2 + y^2 - 2xy + 2x + 4 = 0$

Return to Top

The locus of the point $(t^2+t+1, t^2-t+1), t \in \mathbb{R}$ is :

A $x^2+y^2+2xy+2y+4=0$

B $x^2+y^2-2xy-2x-2y+4=0$

C $x^2+y^2+2xy+2x+2y+4=0$

D $x^2+y^2-2xy+2x+4=0$

Let $(h,k) \equiv (t^2+t+1, t^2-t+1)$

$\Rightarrow h = t^2+t+1, k = t^2-t+1$

$\Rightarrow h-k = 2t \Rightarrow t = \frac{h-k}{2}$

$\Rightarrow h+k = 2t^2+2 \Rightarrow h+k-2 = 2t^2$

$\Rightarrow h+k-2 = 2\left(\frac{h-k}{2}\right)^2$

$\Rightarrow h+k-2 = 2 \times \frac{h^2+k^2-2hk}{4}$

$\Rightarrow h^2+k^2-2hk-2h-2k+4=0$

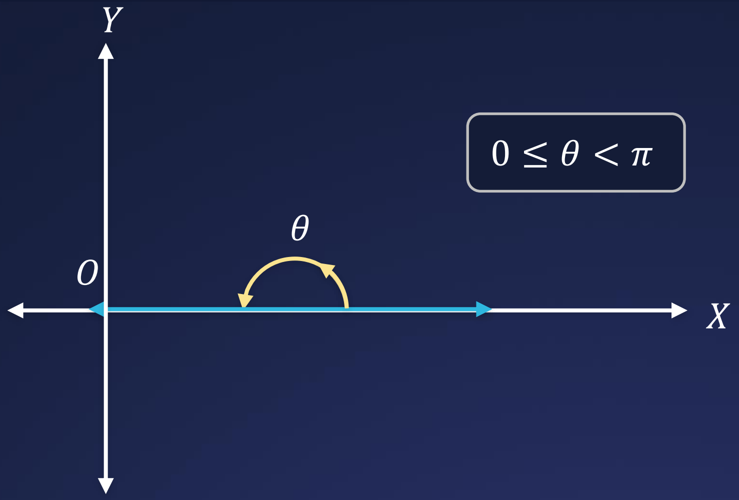
$\therefore$ Locus of (h,k) is : $x^2+y^2-2xy-2x-2y+4=0$

Return to Top

Key Takeaways

Angle of inclination

- Angle $'\theta'$ which a line makes with positive direction of x-axis measured in the anticlockwise sense.



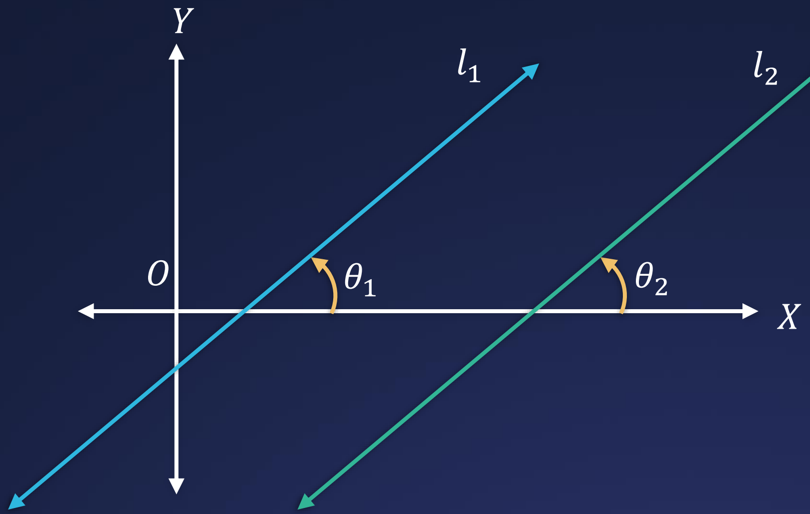
$$0 \leq \theta < \pi$$

[Return to Top]

Key Takeaways

Note

- Parallel lines have same angle of inclination.



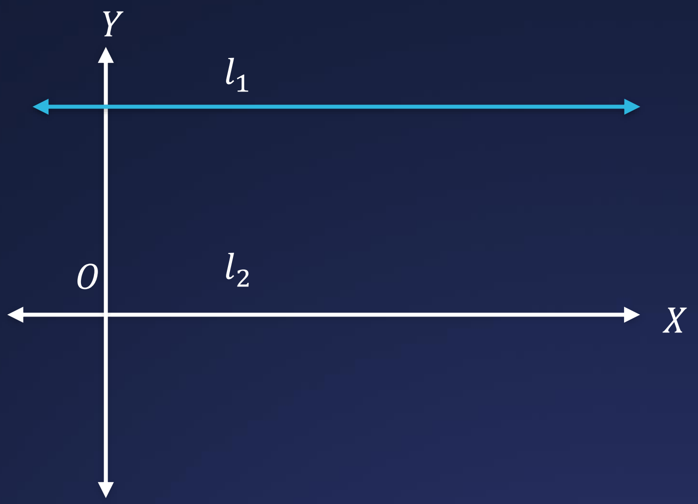
$$\boxed{\theta_1 = \theta_2}$$

Return to Top

Key Takeaways

Note

- Angle of inclination of a line parallel or coincident with X – axis is 0.



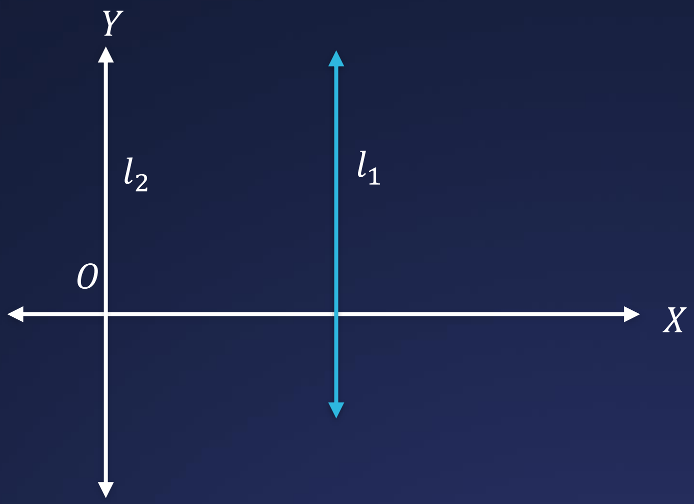
$\theta = 0$

Return to Top

Key Takeaways

Note

- Angle of inclination of a line parallel or coincident with Y – axis is $90°$.



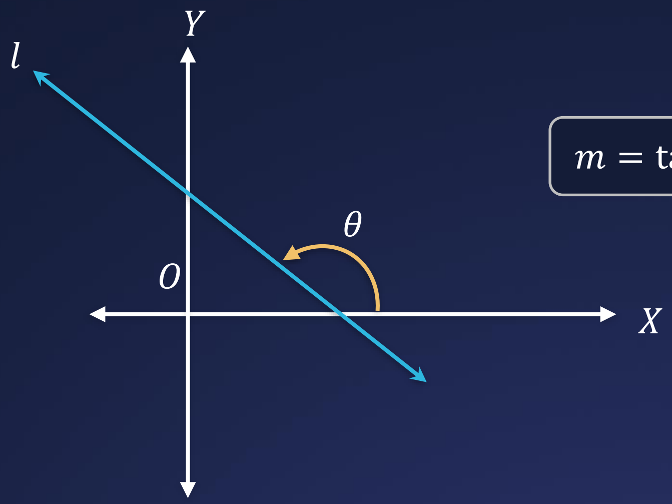
$\theta = 90°$

Return to Top

Key Takeaways

Slope of a line

- If the angle of inclination of a given line $'l'$ is θ then, the slope $'m'$ of that line is given by $\tan\theta$.

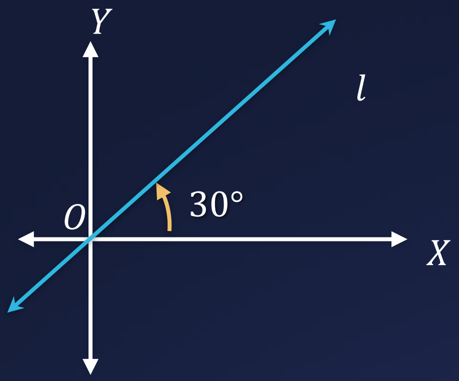


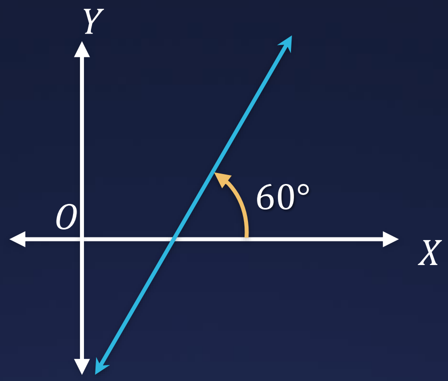
$$m = \tan\theta$$

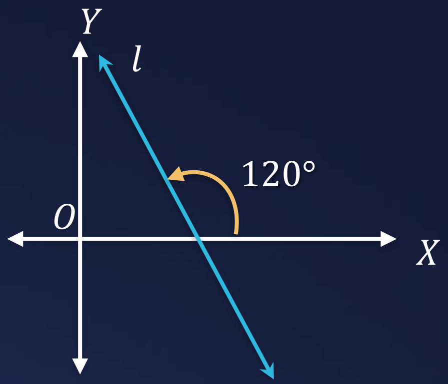
Return to Top

Key Takeaways

Examples







$$m = \tan 30° = \frac{1}{\sqrt{3}}$$

$$m = \tan 60° = \sqrt{3}$$

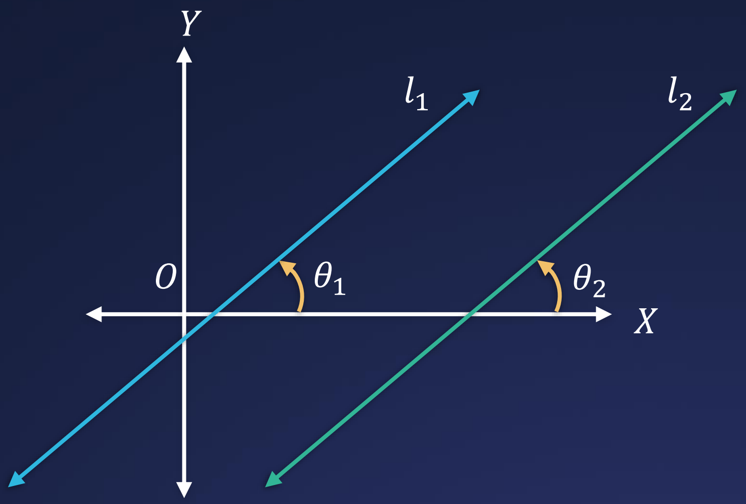
$$m = \tan 120° = -\sqrt{3}$$

Return to Top

Key Takeaways

Note:

- Two parallel lines have same slope.



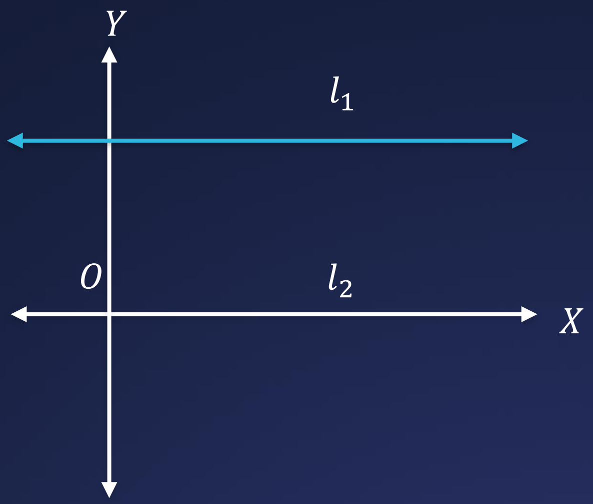
$$\boxed{m_{l_1} = m_{l_2}}$$

Return to Top

Key Takeaways

Note:

- Two parallel lines have same slope ($\theta = 0$).



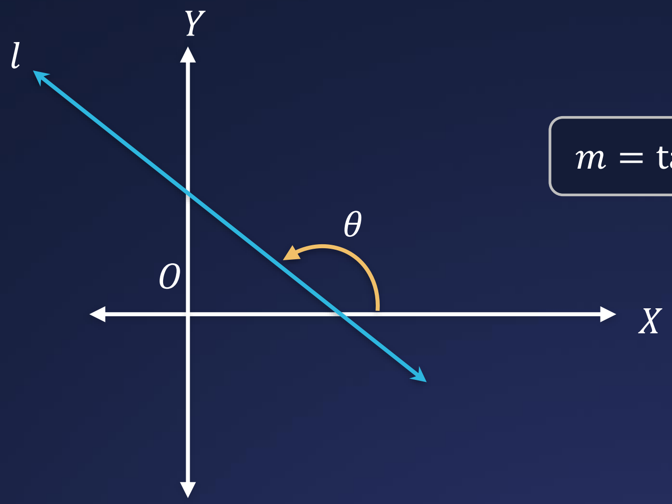
$$\boxed{m_{l_1} = m_{l_2} = \tan\theta = 0}$$

Return to Top

Key Takeaways

Note:

- $\frac{\pi}{2} < \theta < \pi.$



$\boxed{m = \tan\theta < 0}$

Return to Top

Find the angle of inclination of the line whose slope is $-\frac{1}{\sqrt{3}}$:

Given: Slope of the line $'m'$

$$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = -\tan\frac{\pi}{6}$$

$$\Rightarrow \tan\theta = \tan\left(\pi - \frac{\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

Hence angle of inclination (θ) of the line $150°$

Return to Top

Key Takeaways

Calculation of Slope

- In ΔPQN,

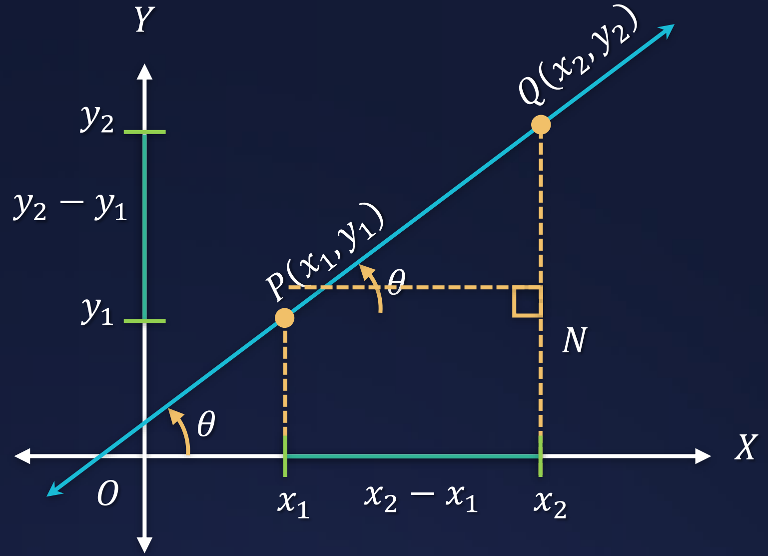
  $\angle QPN = \theta$

  $\Rightarrow$ Slope of $PQ = \tan\theta$

  $\Rightarrow \tan\theta = \frac{QN}{PN}$

  $\Rightarrow \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

$$\boxed{\therefore\ m = \frac{y_2 - y_1}{x_2 - x_1} \text{ OR } \frac{y_1 - y_2}{x_1 - x_2}}$$



Return to Top

The slope of a line joining the points $(2,1)$ & $(0,-3)$ is

A 1

B -2

C -1

D 2

Return to Top

The slope of a line joining the points $(2,1)$ & $(0,-3)$ is

Let $(2,1) = (x_1, y_1)$ & $(0,-3) = (x_2, y_2)$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow m = \frac{-3-1}{0-2}$$

$$\Rightarrow m = -\frac{4}{-2}$$

$$\Rightarrow m = 2$$

A. 1

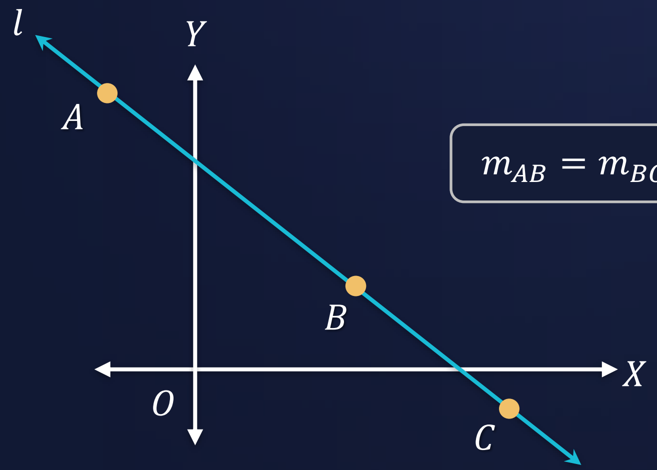
B. -2

C. -1

D. 2

Return to Top

Condition for collinearity



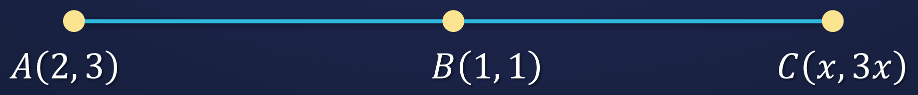
$$m_{AB} = m_{BC} = m_{AC}$$

[Return to Top]

Find x if the points $A(2,3), B(1,1), C(x, 3x)$ are collinear.

Solution :

Given: $A(2,3), B(1,1), C(x, 3x)$ are collinear



$\Rightarrow$ Slope of AB = Slope of BC

$$\Rightarrow \frac{1-3}{1-2} = \frac{3x-1}{x-1}$$

$$\Rightarrow \frac{-2}{-1} = \frac{3x-1}{x-1}$$

$$\Rightarrow 2(x-1) = (3x-1)$$

$$\Rightarrow 2x - 2 = 3x - 1$$

$$\boxed{\Rightarrow x = -1}$$

Return to Top

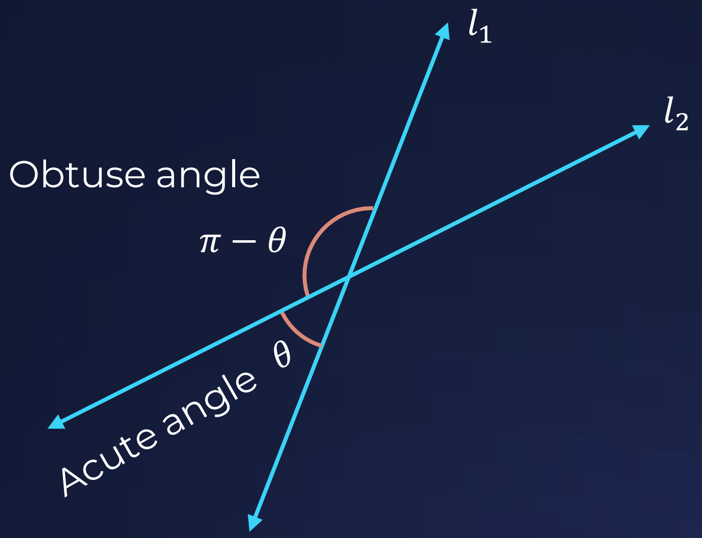
Session 05

Various Forms of Equation of Straight Line

Return to Top

Key Takeaways

Angle between two lines:



Return to Top

Key Takeaways

Angle between two lines:

$$m_1 = \tan\theta_1$$

$$m_2 = \tan\theta_2$$

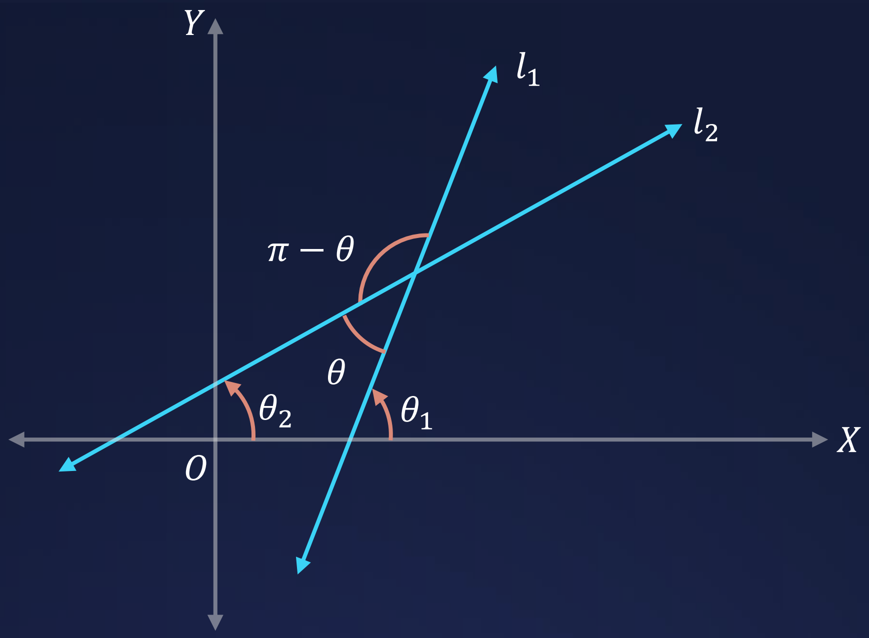
Also, $\theta_1 = \theta + \theta_2$

$$\Rightarrow \theta = \theta_1 - \theta_2$$

$$\Rightarrow \tan\theta = \tan(\theta_1 - \theta_2)$$

$$\Rightarrow \tan\theta = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

$$\Rightarrow \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$



Return to Top

Key Takeaways

Angle between two lines:

$\therefore$ Acute angle θ between two lines:

$$\boxed{\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$$

Case 1:

$l_1 \parallel l_2$

$\theta = 0^\circ$

$\tan\theta = 0$

$\therefore m_1 = m_2$

Case 2:

$l_1 \perp l_2$

$\theta = 90^\circ$

$\cot\theta = 0$

$\therefore m_1 m_2 = -1$

Return to Top

?

The angle between the two lines is 45^0 and the slope of one of them is $\frac{1}{2}$, then the slope of the other line is :

A $\frac{1}{3}$

B $-3, -\frac{1}{3}$

C $3, -\frac{1}{3}$

D $\frac{2}{3}$

Return to Top

The angle between the two lines is 45^0 and the slope of one of them is $\frac{1}{2}$, then the slope of the other line is :

Given, $\theta = 45^\circ$ & $m_1 = \frac{1}{2}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan 45^\circ = \left|\frac{\frac{1}{2} - m_2}{1 + \frac{1}{2}\cdot m_2}\right|$$

$$\Rightarrow 1 = \pm\left(\frac{1 - 2m_2}{2 + m_2}\right)$$

Case I

$$1 = \left(\frac{1 - 2m_2}{2 + m_2}\right)$$

$$\Rightarrow 2 + m_2 = 1 - 2m_2$$

$$\therefore m_2 = -\frac{1}{3}$$

Case II

$$1 = -\left(\frac{1 - 2m_2}{2 + m_2}\right)$$

$$\Rightarrow 2 + m_2 = -1 + 2m_2$$

$$\therefore m_2 = 3$$

A) $\frac{1}{3}$

B) $-3, -\frac{1}{3}$

C) $3, -\frac{1}{3}$

D) $\frac{2}{3}$

Return to Top

The angle between the two lines is 60° and the slope of one of them is $\frac{1}{\sqrt{3}}$, then the slope of the other line is :

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Given: $\theta = 60^\circ, m_1 = \frac{1}{\sqrt{3}}$

$$\Rightarrow \tan 60^\circ = \left|\frac{\frac{1}{\sqrt{3}} - m_2}{1 + \frac{1}{\sqrt{3}} \cdot m_2}\right|$$

$$\Rightarrow \sqrt{3} = \pm\left(\frac{1 - \sqrt{3}m_2}{\sqrt{3} + m_2}\right)$$

Case I

$$\sqrt{3} = \frac{1 - \sqrt{3}m_2}{\sqrt{3} + m_2}$$

$$\Rightarrow 3 + \sqrt{3}m_2 = 1 - \sqrt{3}m_2$$

$$\Rightarrow m_2 = -\frac{1}{\sqrt{3}}$$

$$\boxed{\therefore m_2 = -\frac{1}{\sqrt{3}}}$$

Case II

$$-\sqrt{3} = \frac{1 - \sqrt{3}m_2}{\sqrt{3} + m_2}$$

$$\Rightarrow -3 - \sqrt{3}m_2 = 1 - \sqrt{3}m_2$$

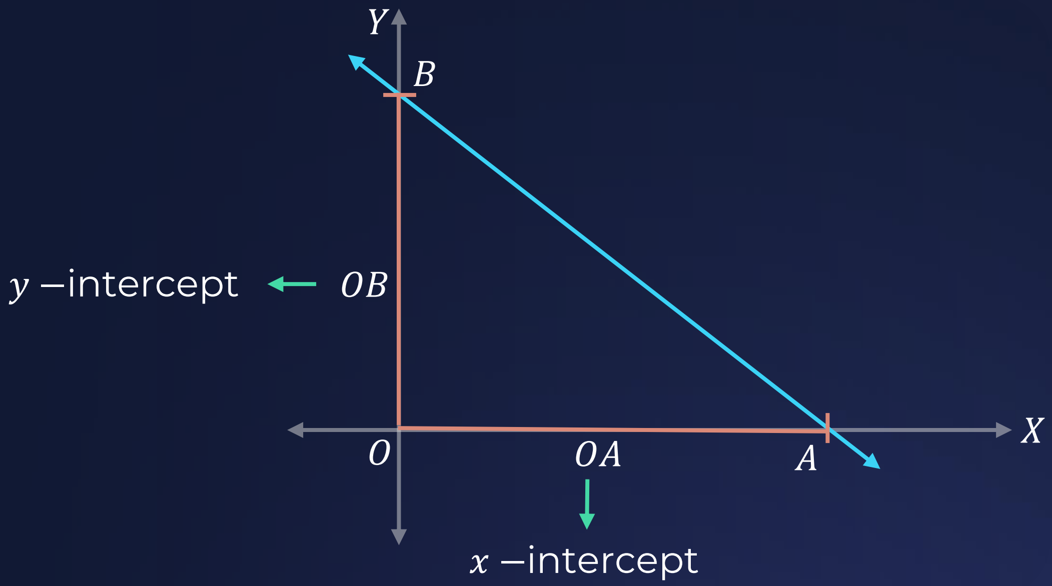
$\Rightarrow m_2$ is undefined.

Return to Top

Key Takeaways

Intercepts of a Line

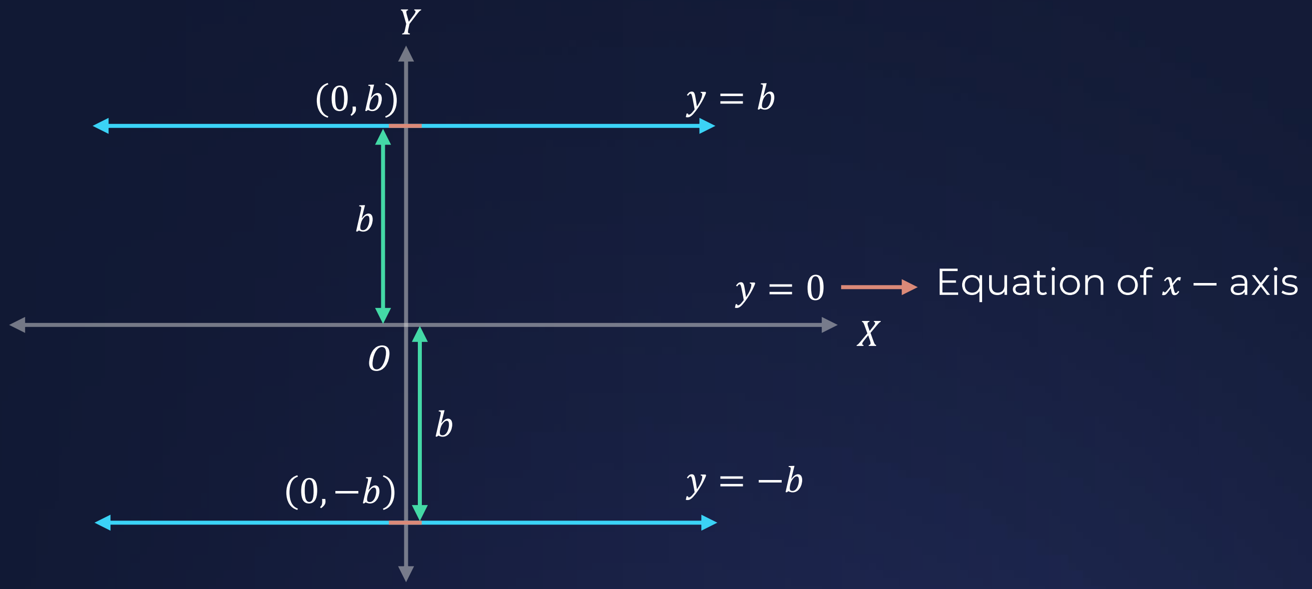
The intercept of a line is the point at which it crosses either the x or y axis .



Return to Top

Key Takeaways

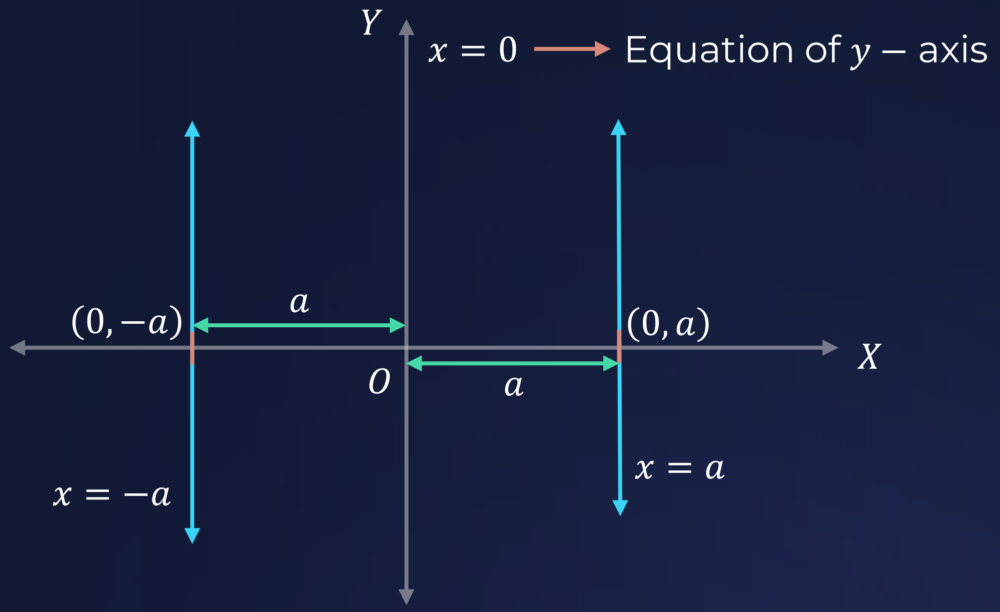
Eqn. of line parallel to X – axis



Return to Top

Key Takeaways

Eqn. of line parallel to Y – axis

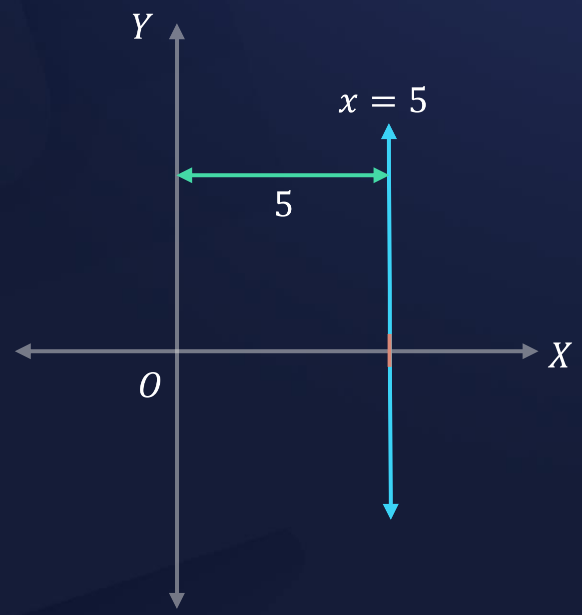


Return to Top

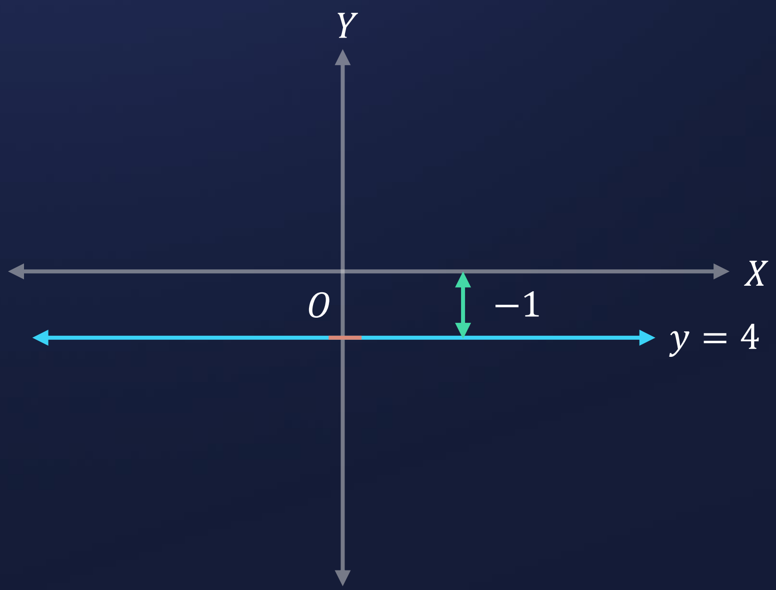
Draw the graph of :

$(i)\ x = 5 \quad (ii)\ y = -1$

A. $x = 5$



D. $y = -1$



Return to Top

Equation of a line parallel to y–axis and passing through $(-4, 3)$ is :

A. $y = -3$

B. $x = -4$

C. $x = 4$

D. $y = 3$

Return to Top

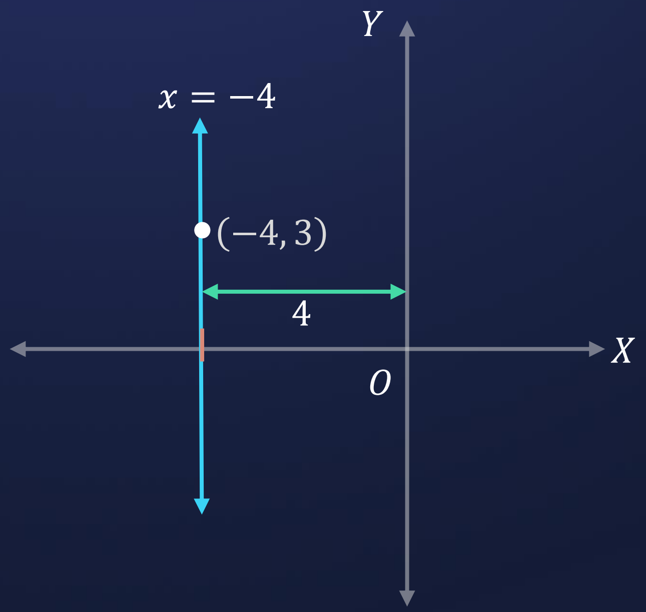
Equation of a line parallel to y–axis and passing through $(-4, 3)$ is :

Given: Line is parallel to y – axis

i.e., its equation is $x = a$

Also, line is passing through $(-4, 3)$

$\therefore$ Equation of the line is $x = -4$.



A $y = -3$

B $x = -4$

C $x = 4$

D $y = 3$

Return to Top

Key Takeaways

Slope - Intercept Form

Equation:

Slope of PA = Slope of $l = m$

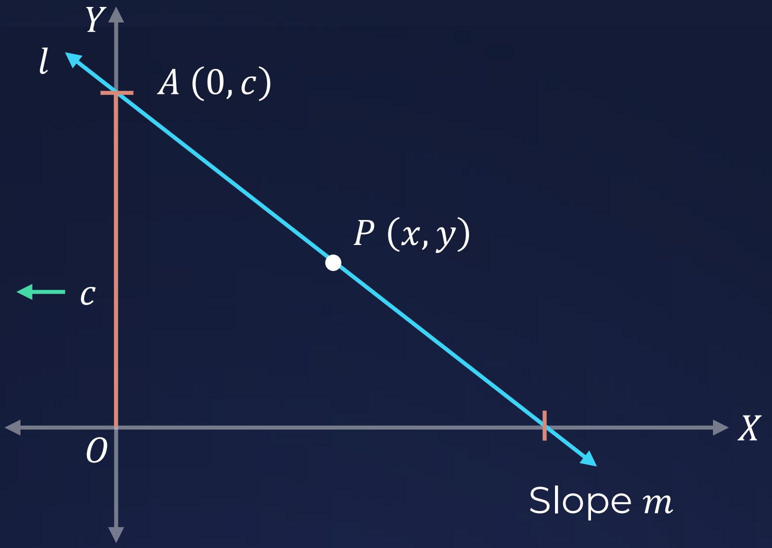
$$\Rightarrow \frac{y-c}{x-0} = m$$

$$\boxed{y = mx + c}$$

Intercept

Slope

y −intercept $\leftarrow c$

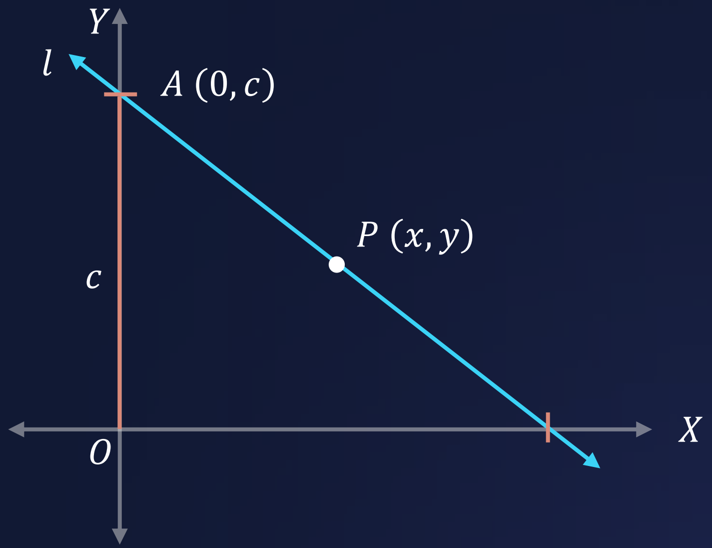


Return to Top

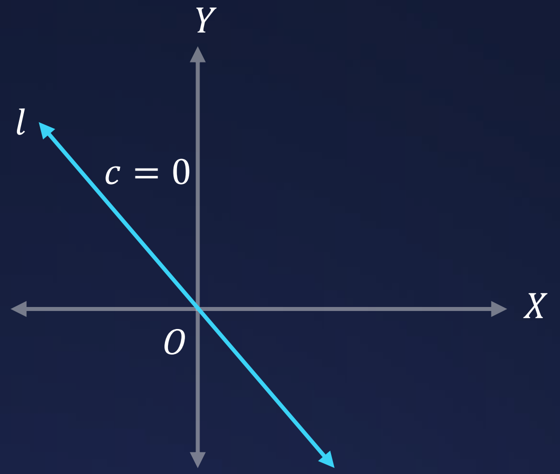
Key Takeaways

Slope - Intercept Form

Note: $y = mx + c$



Note: $y = mx$



[Return to Top](#)

Find the equation of a line which cuts off an intercept of 5 units on negative direction of y –axis and makes an angle of 120° with the positive direction of x– axis.

Given: y – intercept $'c' = -5$

Also, $\theta = 120^\circ$

$\therefore m = \tan\theta = \tan 120^\circ$

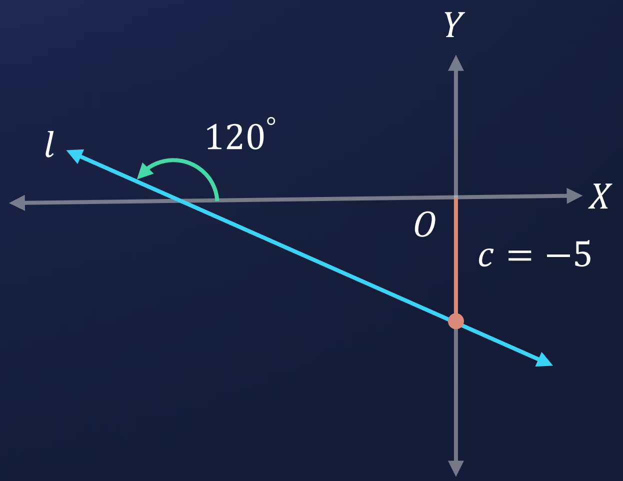
$\Rightarrow m = -\sqrt{3}$

Hence, using Slope-intercept form :

$l : y = mx + c$

$\Rightarrow y = -\sqrt{3}x - 5$

$\Rightarrow y + \sqrt{3}x + 5 = 0$



Return to Top

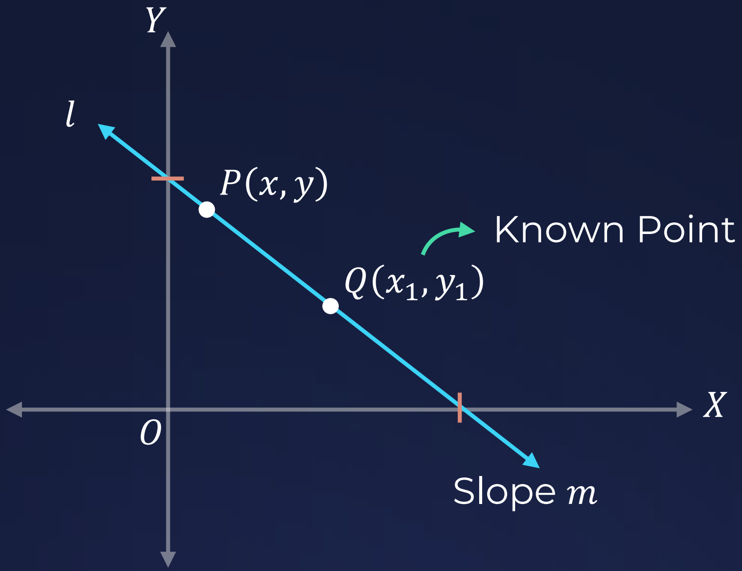
Key Takeaways

Slope Point Form

Equation:

Slope of $PQ = m = \frac{y-y_1}{x-x_1}$

$$\boxed{y - y_1 = m(x - x_1)}$$



Return to Top

Key Takeaways

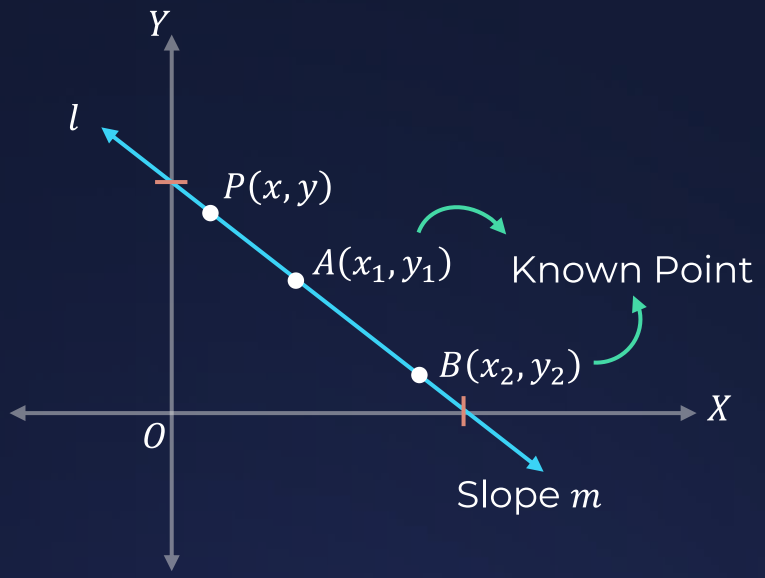
Two Point Form

Equation:

Slope of PA = Slope of AB

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\boxed{y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)}$$



Return to Top

Key Takeaways

Two Point Form

Note:

Equation of a line passing through (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Or

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Return to Top

Key Takeaways

Double Intercept Form

Equation:

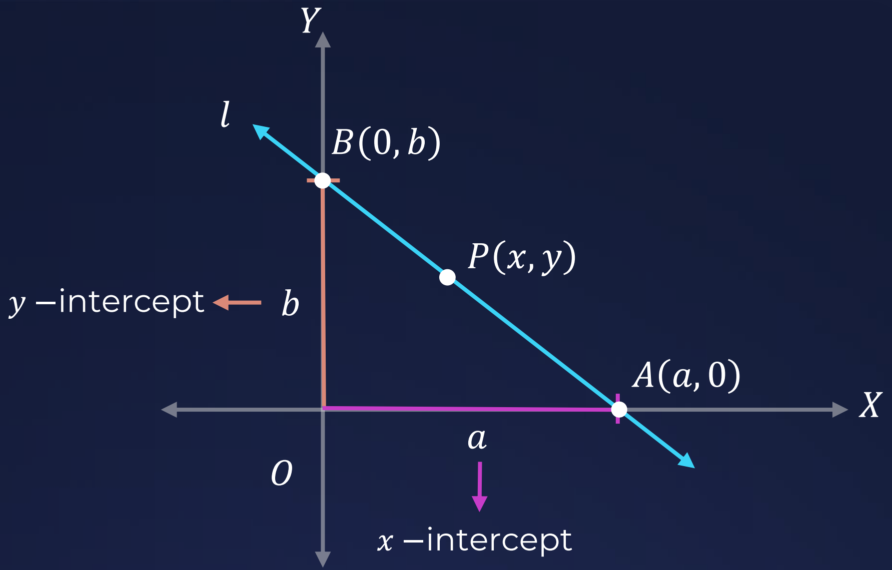
Slope of PA = Slope of AB

$$\frac{y-0}{x-a} = \frac{0-b}{a-0}$$

$$y = -\frac{b}{a}(x-a)$$

$$\frac{bx}{a} + y = b$$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



Return to Top

Area of a triangle formed by the axes and the line $e^{-\alpha}x + e^{\alpha}y = 2$ in square units is :

A. 1

B. 3

C. 2

D. 4

Return to Top

Area of a triangle formed by the axes and the line $e^{-\alpha}x + e^{\alpha}y = 2$ in square units is :

Given: $e^{-\alpha}x + e^{\alpha}y = 2$

Converting the equation into double intercept form :

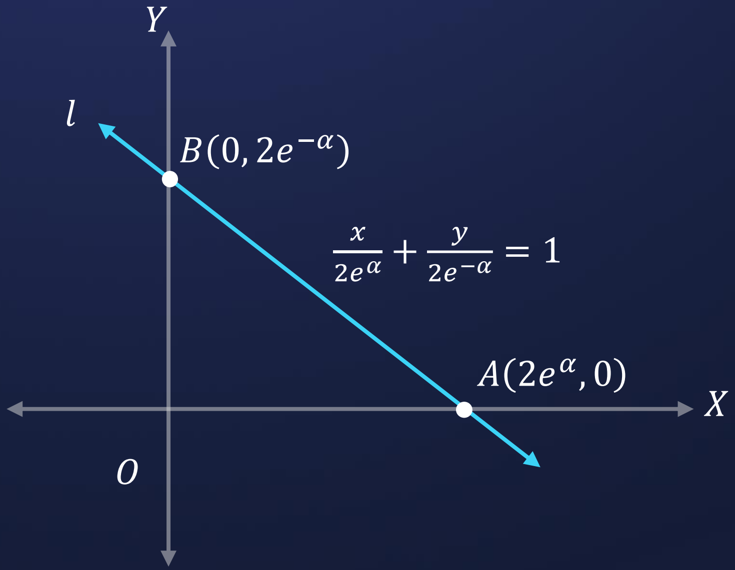
$$\frac{x}{2e^{\alpha}} + \frac{y}{2e^{-\alpha}} = 1$$

Area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 2e^{\alpha} \times 2e^{-\alpha}$

$= 2$

$\therefore$ Area $= 2$ sq. units



A 1

B 3

C 2

D 4

Return to Top

Find the equation of the line which passes through the point $(3,4)$ and the sum of its intercepts on the axes is 14.

Let the equation of line be : $\frac{x}{a} + \frac{y}{b} = 1$

This passes through $(3,4)$

$\therefore \frac{3}{a} + \frac{4}{b} = 1 \quad \cdots (i)$

Given : $a + b = 14 \Rightarrow b = 14 - a$

Putting $b = 14 - a$ in $(i) \Rightarrow \frac{3}{a} + \frac{4}{14-a} = 1$

$$\Rightarrow a^2 - 13a + 42 = 0$$

$$\Rightarrow (a - 7)(a - 6) = 0$$

Return to Top

Find the equation of the line which passes through the point $(3,4)$ and the sum of its intercepts on the axes is 14.

$$\therefore \frac{3}{a}+\frac{4}{b}=1 \quad \cdots (i)$$

$$\Rightarrow (a-7)(a-6)=0$$

$\Rightarrow a=7$	$\Rightarrow a=6$
$b=14-a$	$b=14-a$
$\Rightarrow b=7$	$\Rightarrow b=8$

Putting values of a and b in (i)

$\frac{x}{7}+\frac{y}{7}=1$ And $\frac{x}{6}+\frac{y}{8}=1$

$\boxed{x+y=7}$ And $\boxed{4x+3y=24}$

Return to Top

Session 06

Normal form & parametric form of line

Return to Top

A straight line through the point $A(3,4)$ is such that its intercept between the axes is bisected at A. Its equation is :

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A. $x + y + 7 = 0$

B. $3x - 4y + 7 = 0$

C. $4x + 3y = 24$

D. $3x + 4y = 25$

Return to Top

A straight line through the point $A(3,4)$ is such that its intercept between the axes is bisected at A. Its equation is :

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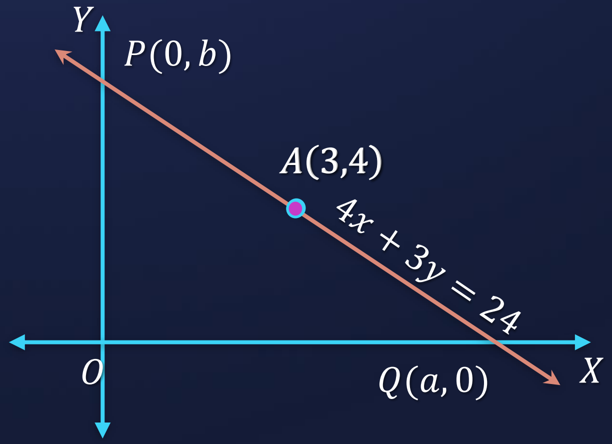
$A(3,4)$ is the mid-point of PQ

$\frac{a+0}{2} = 3$ & $\frac{0+b}{2} = 4$

$a = 6$ & $b = 8$

$\therefore$ Equation is $\frac{x}{6} + \frac{y}{8} = 1$

$4x + 3y = 24$



Return to Top

A straight line through the point $A(3,4)$ is such that its intercept between the axes is bisected at A. Its equation is :

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A. $x + y + 7 = 0$

B. $3x - 4y + 7 = 0$

C. $4x + 3y = 24$

D. $3x + 4y = 25$

Return to Top

Key Takeaways

Normal Form

Equation: Using Intercept form,

Eqn. of $l \equiv \frac{x}{OA} + \frac{y}{OB} = 1$

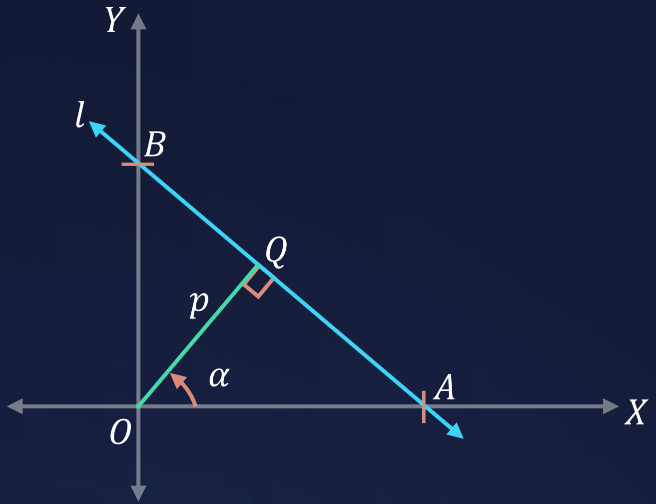
In ΔOAQ

$$\cos\alpha = \frac{p}{OA} \qquad \Rightarrow OA = \frac{p}{\cos\alpha}$$

Similarly, In ΔOBQ

$$\cos(\angle BOQ) = \frac{OQ}{OB} \qquad \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right) = \frac{OQ}{OB}$$

$$\Rightarrow \sin\alpha = \frac{p}{OB} \qquad \Rightarrow OB = \frac{p}{\sin\alpha}$$



Return to Top

Key Takeaways

Normal Form

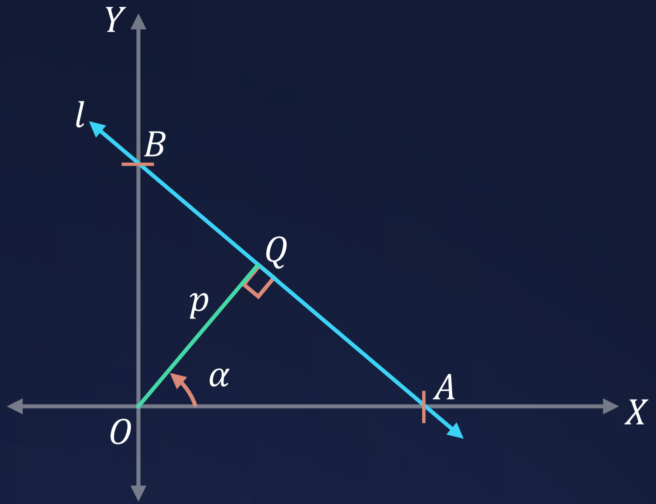
$$\frac{x}{OA} + \frac{y}{OB} = 1$$

$$OA = \frac{p}{\cos\alpha}$$

$$OB = \frac{p}{\sin\alpha}$$

$$\Rightarrow \frac{x}{p} \times \cos\alpha + \frac{y}{p} \times \sin\alpha = 1$$

$$x\cos\alpha + y\sin\alpha = p$$



Return to Top

A line forms a triangle of area $54\sqrt{3}$ sq. units with the coordinate axes. If the perpendicular drawn from the origin to the line makes an angle of 60° with the positive x –axis, then the equation of the line is :

Now, equation of l in Normal Form is :

$x\cos\alpha + y\sin\alpha = p$

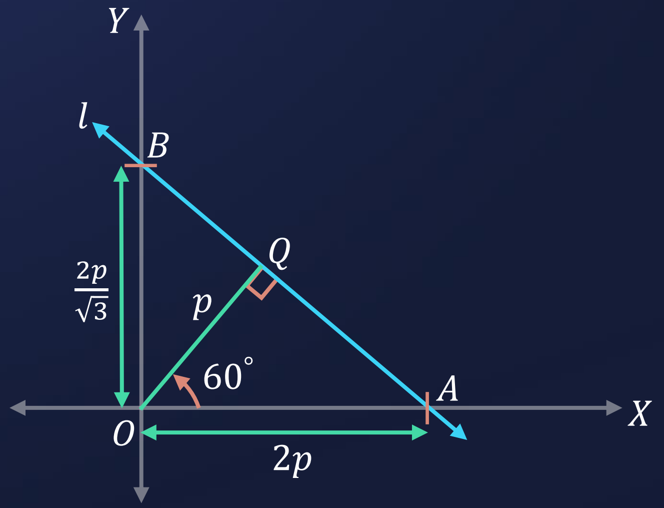
$\Rightarrow x\cos 60^{\circ} + y\sin 60^{\circ} = p$

$\Rightarrow \frac{x}{2} + \frac{\sqrt{3}}{2}y = p$

$\Rightarrow x + \sqrt{3}y = 2p$

$\Rightarrow \frac{x}{2p} + \frac{\sqrt{3}y}{2p} = 1$

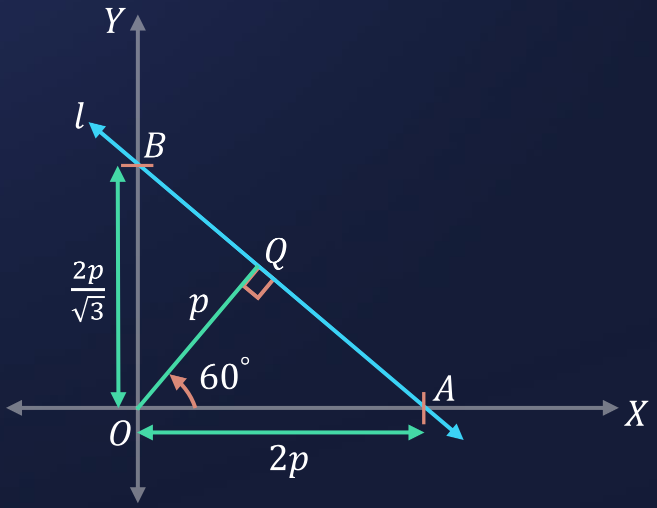
$\Rightarrow \frac{x}{2p} + \frac{y}{\left(\frac{2p}{\sqrt{3}}\right)} = 1$



Return to Top

A line forms a triangle of area $54\sqrt{3}$ sq. units with the coordinate axes. If the perpendicular drawn from the origin to the line makes an angle of 60° with the positive x –axis, then the equation of the line is :

$$\Rightarrow \frac{x}{2p} + \frac{y}{\left(\frac{2p}{\sqrt{3}}\right)} = 1$$



Given : Area $= 54\sqrt{3}$ sq. units

$$\Rightarrow \frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = 54\sqrt{3}$$

$$\Rightarrow p^2 = 81 \Rightarrow p = \pm 9$$

$\Rightarrow p = 9 \because$ length is always positive

Now equation of line is :

$$x + \sqrt{3}y = 2p$$

$$\boxed{\Rightarrow x + \sqrt{3}y = 18.}$$

Return to Top

The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150° with the positive direction of y –axis. Find the equation of the line.

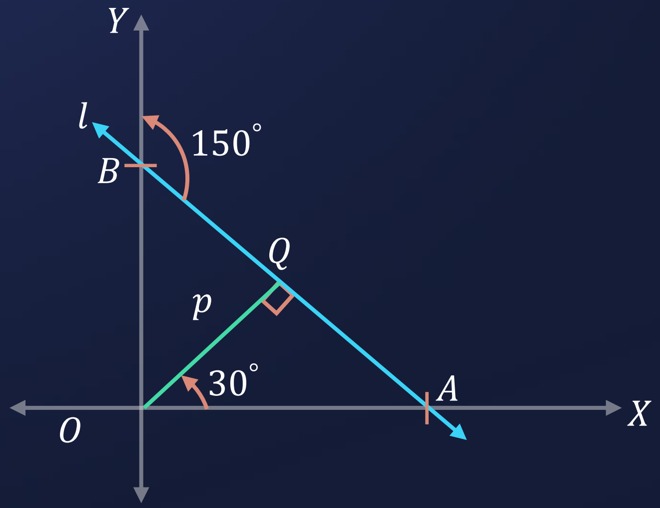
Given : $p = 7$

$l \equiv x\cos\alpha + y\sin\alpha = p$

$\alpha = 30^\circ$

$\Rightarrow x \cdot \frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = 7$

$\boxed{\therefore \sqrt{3}x + y = 14}$



Return to Top

Key Takeaways

Parametric Equation

In $\triangle PQN$,

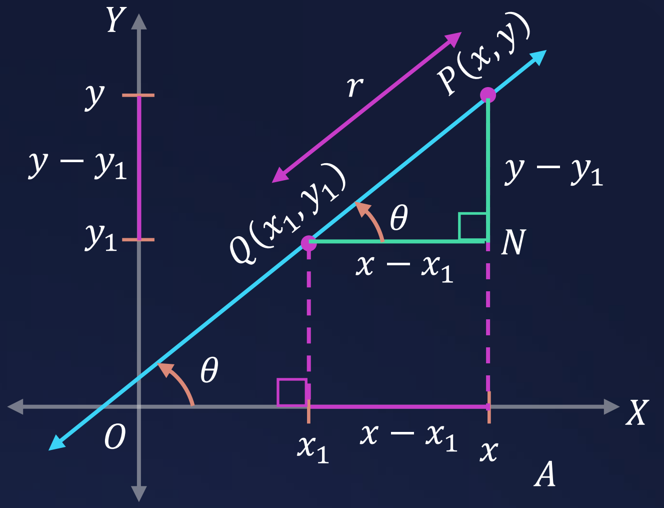
$$\cos\theta = \frac{QN}{PQ} = \frac{x - x_1}{r}$$

$$\Rightarrow r = \frac{x - x_1}{\cos\theta}$$

$$\sin\theta = \frac{PN}{PQ} = \frac{y - y_1}{r}$$

$$\Rightarrow r = \frac{y - y_1}{\sin\theta}$$

Thus, $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$



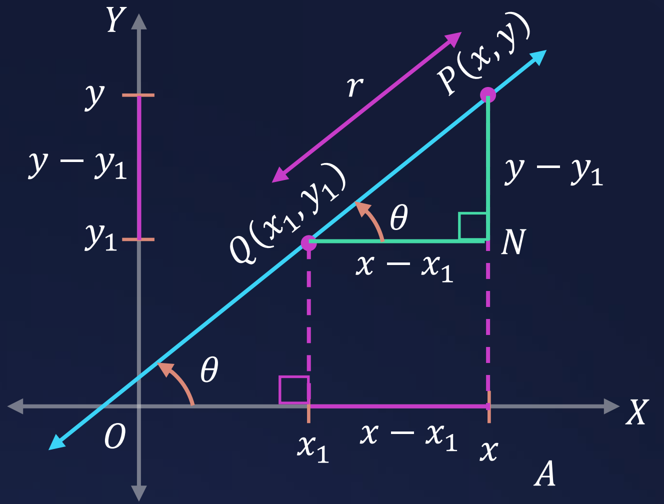
Return to Top

Key Takeaways

Note

$$\boxed{\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r}$$

$x = x_1 + r\cos\theta$ and $y = y_1 + r\sin\theta$ represent coordinates of any point on the line at a distance r from (x_1, y_1).



At a given distance r from (x_1, y_1) on the line $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$ there will be two points i.e., $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ and $(x_1 - r\cos\theta, y_1 - r\sin\theta)$.

Return to Top

A straight line is drawn through the point $P(2,3)$ and is inclined at an angle of 30° with the x –axis in anti-clockwise direction. Find the equation of the line and the coordinates of two points on it at a distance of 4 units from P.

Here $(x_1, y_1) = (2,3), \theta = 30^\circ$

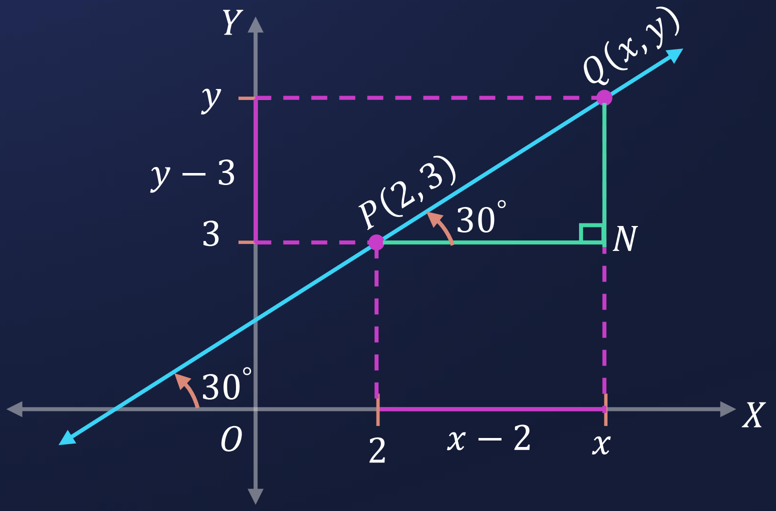
The equation of the line is :

$$\frac{x-2}{\cos 30^\circ} = \frac{y-3}{\sin 30^\circ}$$

$$\Rightarrow \frac{x-2}{\frac{\sqrt{3}}{2}} = \frac{y-3}{\frac{1}{2}}$$

$$\Rightarrow x - 2 = \sqrt{3}(y-3)$$

$$\Rightarrow x - \sqrt{3}y = 2 - 3\sqrt{3}$$



Points on the line at a distance 4 from $(2,3)$:

$$(x_1 \pm r\cos\theta\, , y_1 \pm r\sin\theta)$$

Return to Top

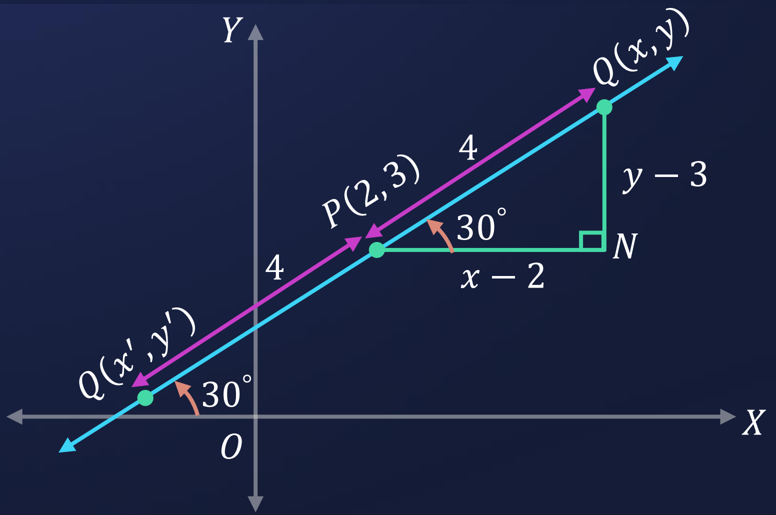
A straight line is drawn through the point $P(2,3)$ and is inclined at an angle of 30° with the x –axis in the anti-clockwise direction. Find the equation of the line and the coordinates of two points on it at a distance of 4 units from P.

$$(x_1 \pm r\cos\theta\,, y_1 \pm r\sin\theta)$$

$$\Rightarrow \left(2 \pm 4\cos 30^\circ\,, 3 \pm 4\sin 30^\circ\right)$$

$$\Rightarrow \left(2 \pm 2\sqrt{3}, 3 \pm 2\right)$$

$$\boxed{\Rightarrow \left(2 + 2\sqrt{3}, 5\right) \text{ and } \left(2 - 2\sqrt{3}, 1\right)}$$



Return to Top

?

The distance of the point $(2, 3)$ from the line $2x - 3y + 9 = 0$ measured along the line $2x - 2y + 5 = 0$ is :

A $\sqrt{2}$

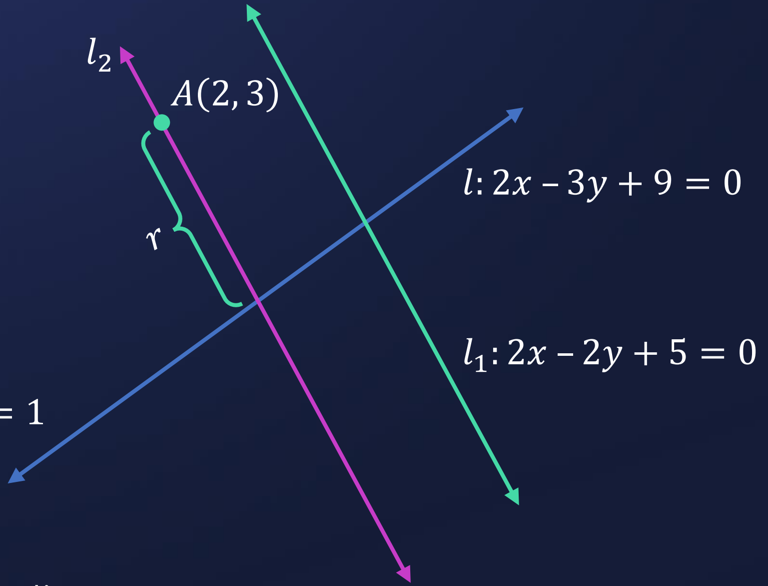
B $2\sqrt{2}$

C $3\sqrt{2}$

D $4\sqrt{2}$

Return to Top

The distance of the point $(2, 3)$ from the line $2x - 3y + 9 = 0$ measured along the line $2x - 2y + 5 = 0$ is :



Given, $l_1: 2x - 2y + 5 = 0$

$\Rightarrow 2y = 2x + 5$

$\Rightarrow y = x + \frac{5}{2}$

$\therefore$ Slope of $l_1 = 1 \Rightarrow \tan\theta = 1$

$\therefore$ l_2 is passing through $A(2, 3)$ with slope $= 1$

$\therefore$ Equation of l_2: $\frac{x-2}{\cos\frac{\pi}{4}} = \frac{y-3}{\sin\frac{\pi}{4}} = r$

Hence, any point lying on l_2 will have coordinates,

$\left(2 + r\cos\frac{\pi}{4}, 3 + r\sin\frac{\pi}{4}\right)$

i.e. $\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$

Return to Top

The distance of the point $(2, 3)$ from the line $2x - 3y + 9 = 0$ measured along the line $2x - 2y + 5 = 0$ is :

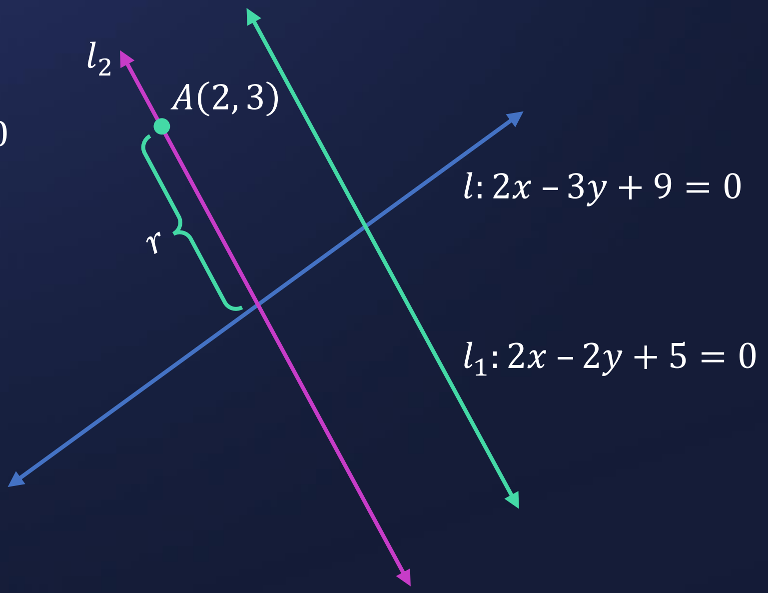
This point lies on the line $l: 2x - 3y + 9 = 0$

$$\therefore 2\left(2 + \frac{r}{\sqrt{2}}\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9 = 0$$

$$\Rightarrow 4 + \frac{2r}{\sqrt{2}} - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$$

$$\Rightarrow \frac{r}{\sqrt{2}} = 4$$

$\therefore$ Distance of $(2, 3)$ from the line $2x - 3y + 9 = 0$ along $2x - 2y + 5 = 0$ is $4\sqrt{2}$ units.



Return to Top

The distance of the point $(2, 3)$ from the line $2x - 3y + 9 = 0$ measured along the line $2x - 2y + 5 = 0$ is :

A $\sqrt{2}$

B $2\sqrt{2}$

C $3\sqrt{2}$

D $4\sqrt{2}$

Return to Top

Two adjacent vertices of a square are $(1, 2)$ and $(-2, 6)$.

Find the coordinates of other vertices.

$$AB = \sqrt{(6-2)^2 + (-2-1)^2}$$

$$= \sqrt{16+9}$$

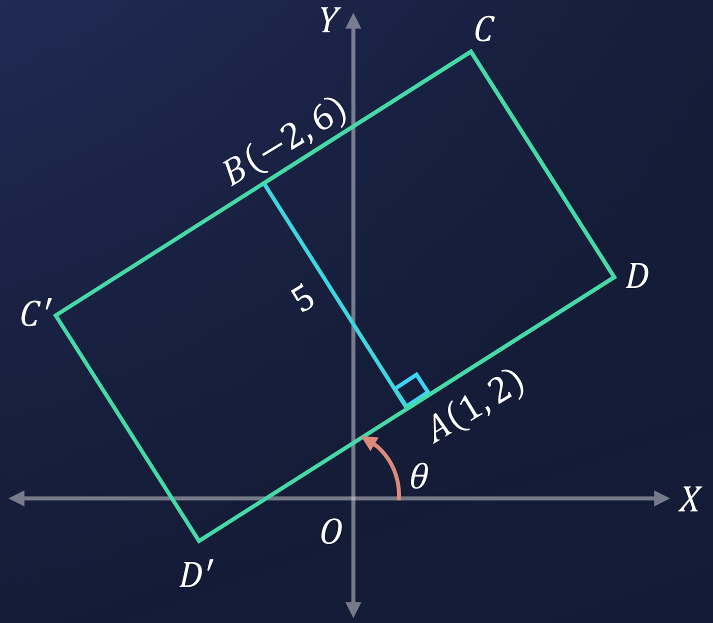
$$= \sqrt{25} = 5 \text{ units.}$$

$$m_{AB} = \frac{6-2}{-2-1} = -\frac{4}{3}$$

Now, $DD' \perp AB \Rightarrow$ Slope of $DD' = \frac{3}{4}$

$$\Rightarrow \tan\theta = \frac{3}{4}$$

$$\Rightarrow \sin\theta = \frac{3}{5}; \cos\theta = \frac{4}{5}$$



Return to Top

Two adjacent vertices of a square are $(1, 2)$ and $(-2, 6)$.

Find the coordinates of other vertices.

$\therefore$ Coordinates of C and C'

$$\equiv (-2 \pm 5\cos\theta, 6 \pm 5\sin\theta)$$

$$\equiv \left(-2 \pm 5 \times \frac{4}{5}, 6 \pm 5 \times \frac{3}{5}\right)$$

$$\equiv (-2 \pm 4, 6 \pm 3)$$

$\boxed{C(2, 9) \text{ and } C'(-6, 3)}$

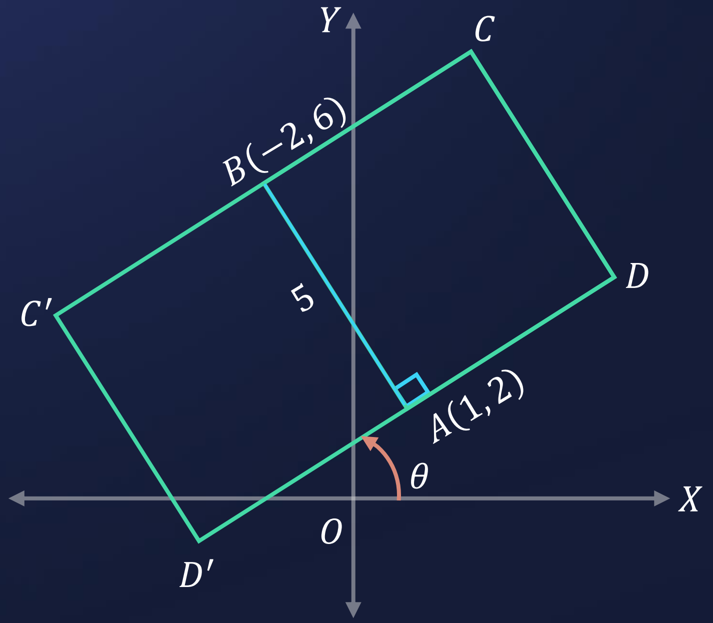
$\therefore$ Coordinates of D and D'

$$\equiv (1 \pm 5\cos\theta, 2 \pm 5\sin\theta)$$

$$\equiv \left(1 \pm 5 \times \frac{4}{5}, 2 \pm 5 \times \frac{3}{5}\right)$$

$$\equiv (1 \pm 4, 2 \pm 3)$$

$\boxed{D(5, 5) \text{ and } D'(-3, -1)}$



Return to Top

Key Takeaways

General Equation:

Every first degree equation in x, y represents a straight line.

$$ax + by + c = 0\,; a, b, c \in \mathbb{R}$$

Example:

$$x + y + 2 = 0$$

$$2x - 3y + 7 = 0$$

Return to Top

Key Takeaways

Slope Intercept Form:

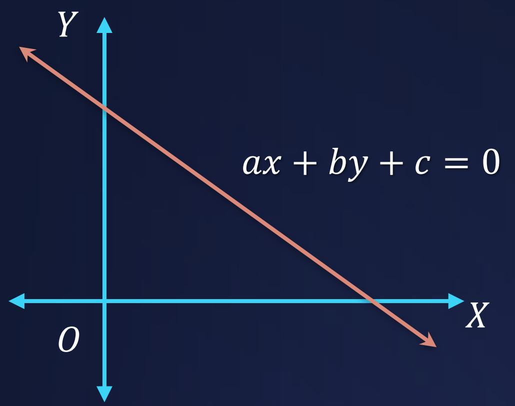
Every first degree equation in x, y represents a straight line.

$$ax + by + c = 0\,;\, a, b, c \in \mathbb{R} \Rightarrow by = -ax - c$$

$$\Rightarrow y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$$

$$\Rightarrow y = mx + c$$

Slope $= m = \left(-\frac{a}{b}\right)$, Y-intercept = c = $\left(-\frac{c}{b}\right)$



Return to Top

Key Takeaways

Intercept Form:

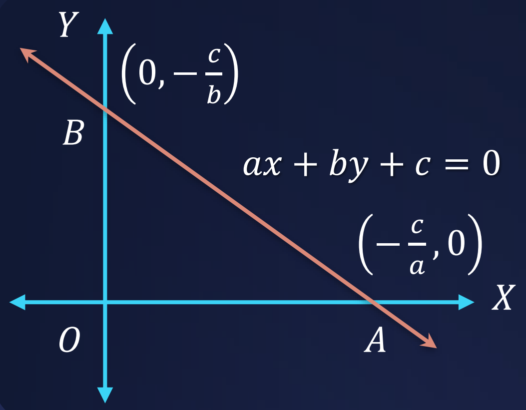
Every first degree equation in x, y represents a straight line.

$$ax + by + c = 0\,;\, a, b, c \in \mathbb{R} \Rightarrow ax + by = -c$$

$$\Rightarrow \frac{ax}{-c} + \frac{by}{-c} = 1$$

$$\Rightarrow \frac{x}{\left(-\frac{c}{a}\right)} + \frac{y}{\left(-\frac{c}{b}\right)} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

X-intercept = $a = \left(-\frac{c}{a}\right)$, Y-intercept = $b = \left(-\frac{c}{b}\right)$



Return to Top

Session 07

Distance evaluation

between line & point, lines.

Return to Top

Reduce $x + \sqrt{3}\,y + 4 = 0$ into

(a) Slope intercept form (b) Intercept form

(a) Slope intercept form:

Given equation, $x + \sqrt{3}\,y + 4 = 0$

$\Rightarrow \sqrt{3}y = -x - 4$

$\Rightarrow y = -\frac{1}{\sqrt{3}}x - \frac{4}{\sqrt{3}}$

$\because y = mx + c$

$\therefore$ Slope $'m' = -\frac{1}{\sqrt{3}}$

Y –Intercept $'\mathrm{c}' = -\frac{4}{\sqrt{3}}$

Return to Top

Reduce $x+\sqrt{3}\,y+4=0$ into

(a) Slope intercept form (b) Intercept form

(b) Intercept form:

Given equation, $x+\sqrt{3}\,y+4=0$

$\Rightarrow x+\sqrt{3}y=-4$

$\Rightarrow \dfrac{x}{-4}+\dfrac{y}{(-4/\sqrt{3})}=1$

$\because \dfrac{x}{a}+\dfrac{y}{b}=1$

$\therefore X-$ intercept $=-4$ & $Y-$ intercept $=-\dfrac{4}{\sqrt{3}}$

Return to Top

If the x-intercept of some line L is double as that of the line $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is :

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A -3

B $\frac{-3}{2}$

C $\frac{-3}{8}$

D $\frac{-3}{16}$

Return to Top

If the x-intercept of some line L is double as that of the line $3x + 4y = 12$ and the Y-intercept of L is half as that of the same line, then the slope of L is :

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Consider $L_1: 3x + 4y = 12 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

X-intercept of $L_1 = 4$

$\therefore X$-intercept of $L = 8$

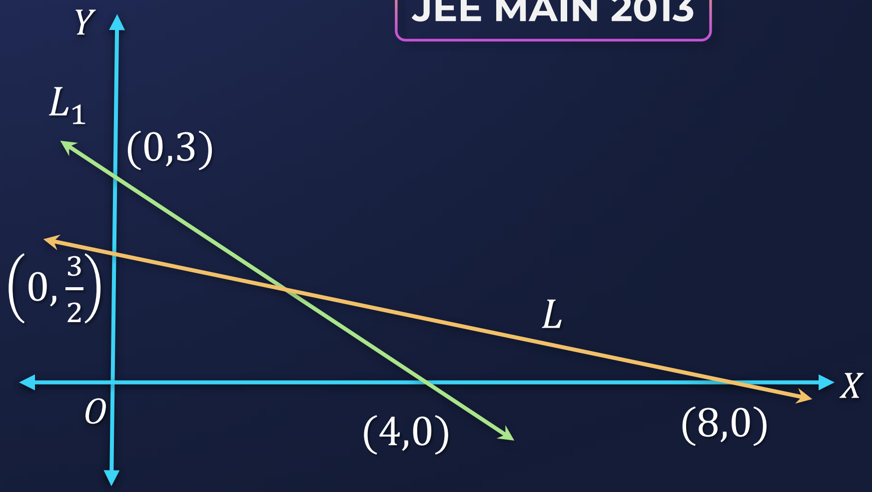
Y-intercept of $L_1 = 3$

$\therefore Y$-intercept of $L = \frac{3}{2}$

$\therefore L \equiv \frac{x}{8} + \frac{y}{\frac{3}{2}} = 1$

$\Rightarrow \frac{x}{8} + \frac{2y}{3} = 1$

Slope$= -\frac{\frac{1}{8}}{\frac{2}{3}} = -\frac{3}{16}$



Return to Top

If the x-intercept of some line L is double as that of the line $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is :

JEE MAIN 2013

A -3

B $\frac{-3}{2}$

C $\frac{-3}{8}$

D $\frac{-3}{16}$

Return to Top

If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7,17)$ and $(15, \beta)$, then β equals :

A $\frac{35}{3}$

B -5

C $\frac{-35}{3}$

D 5

Return to Top

If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7,17)$ and $(15, \beta)$, then β equals :

$$2x - 3y + 17 = 0$$

$\Rightarrow$ Slope $= \frac{2}{3}$

Slope of line passing through the points $(7,17)$ and $(15, \beta) = \frac{\beta - 17}{15 - 7}$

$$= \frac{\beta - 17}{8}$$

Since lines are perpendicular to each other :

$$\Rightarrow \frac{2}{3} \times \frac{\beta - 17}{8} = -1$$

$$\boxed{\beta = 5}$$

Return to Top

If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7,17)$ and $(15, \beta)$, then β equals :

A $\frac{35}{3}$

B -5

C $\frac{-35}{3}$

D 5

Return to Top

?

Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is:

A. $2x + 9y + 7 = 0$

B. $2x - 9y - 11 = 0$

C. $4x + 7y + 3 = 0$

D. $4x + 7y - 3 = 0$

Return to Top

Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is:

PS is the median $\Rightarrow S$ is the midpoint of Q and R

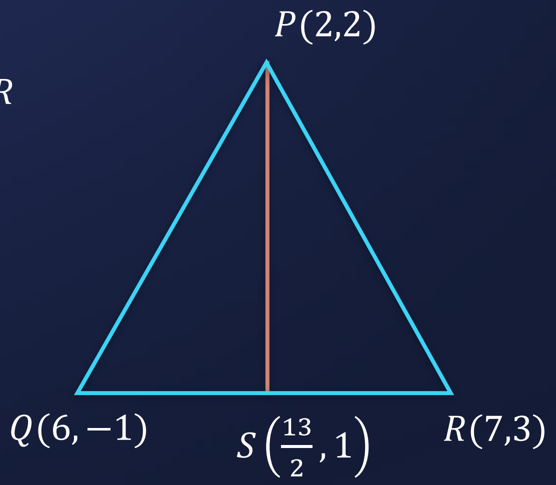
$$\Rightarrow S \equiv \left(\frac{6+7}{2}, \frac{-1+3}{2}\right)$$

$$\Rightarrow S \equiv \left(\frac{13}{2}, 1\right)$$

Now, Slope of $PS = \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{2-1}{2-\frac{13}{2}} = -\frac{1}{9} \times 2 = -\frac{2}{9}$$

Let l be the line parallel to PS passing through $(1,-1)$



Return to Top

Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is:

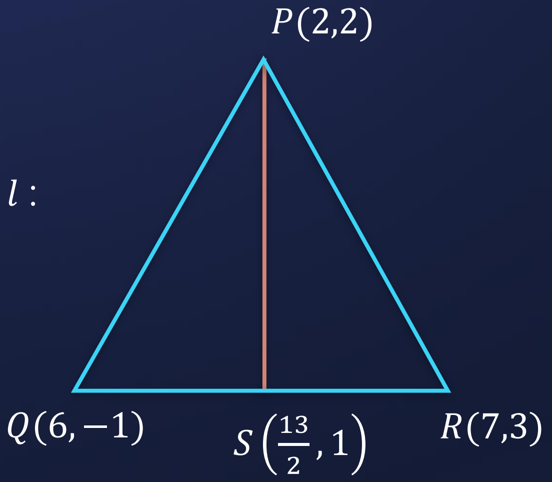
$\therefore$ Slope of $l = -\frac{2}{9}$

$\therefore$ By Point Slope Form, Equation of l :

$$\frac{y-(-1)}{x-1} = -\frac{2}{9}$$

$$\Rightarrow y + 1 = -\frac{2}{9}(x-1)$$

$$\Rightarrow 2x + 9y + 7 = 0$$



Return to Top

Let the orthocenter and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcenter of this triangle, then the radius of the circle having line segment AC as diameter is :

A. $2x + 9y + 7 = 0$

B. $2x - 9y - 11 = 0$

C. $4x + 7y + 3 = 0$

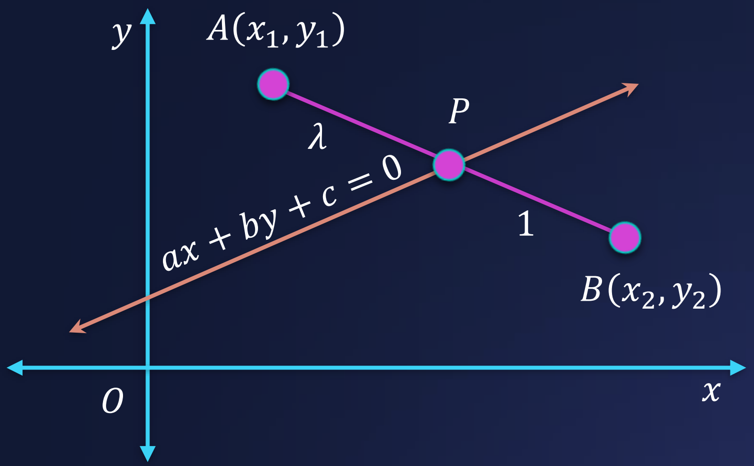
D. $4x + 7y - 3 = 0$

Return to Top

Key Takeaways

Internal Division:

If the straight line $ax + by + c = 0$ divides the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio $\lambda : 1$



$$\frac{AP}{BP} = \frac{\lambda}{1} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$$

Return to Top

Key Takeaways

Internal Division:

If A and B are on the opposite side of the line $ax + by + c = 0$

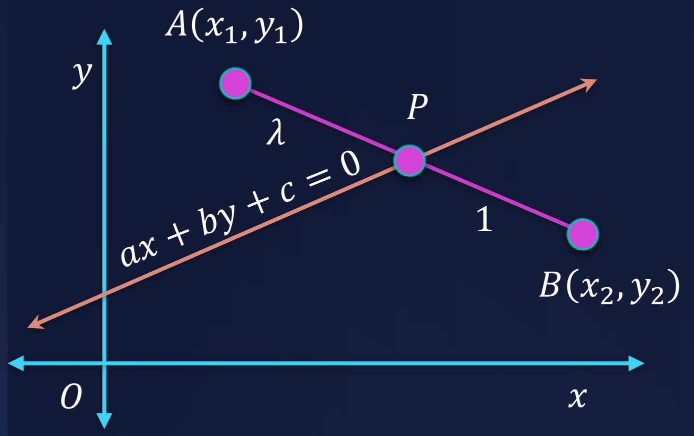
$\Rightarrow ax + by + c = 0$ divides AB internally

$\Rightarrow \lambda > 0$

$$-\frac{ax_1+by_1+c}{ax_2+by_2+c} > 0$$

$$\frac{ax_1+by_1+c}{ax_2+by_2+c} < 0$$

$\therefore ax_1 + by_1 + c \;\&\; ax_2 + by_2 + c$ are of opposite sign.



Return to Top

Key Takeaways

External Division:

If A and B are on the same side of the line $ax+by+c=0$

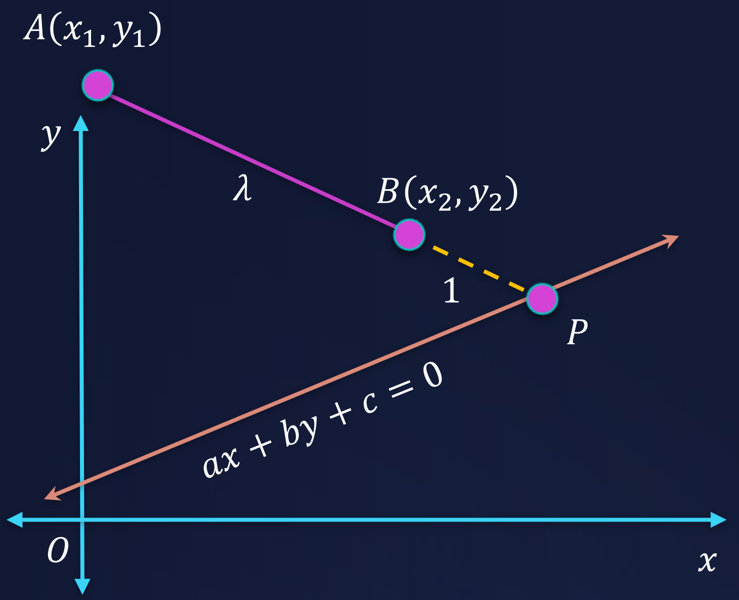
$\Rightarrow ax+by+c=0$ divides AB externally

$\Rightarrow \lambda < 0$

$$-\frac{ax_1+by_1+c}{ax_2+by_2+c} < 0$$

$$\frac{ax_1+by_1+c}{ax_2+by_2+c} > 0$$

$\therefore\ ax_1+by_1+c\ \ \&\ \ ax_2+by_2+c$ are of same sign.



Return to Top

In what ratio is the line segment joining the points (−1,1) and (5,7) divides the line $x + y = 4$.

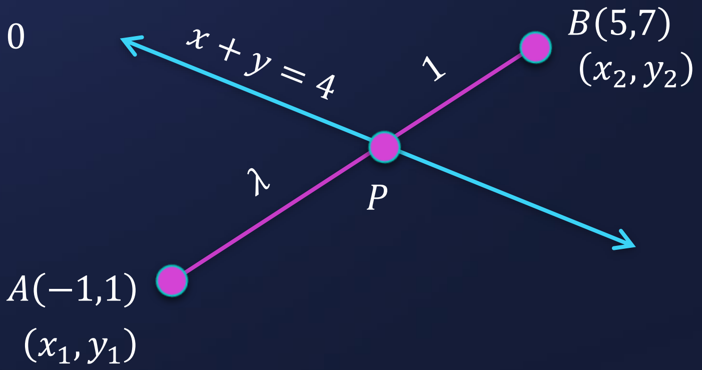
On comparing $x + y = 4$ with $ax + by + c = 0$

$a = 1, b = 1 \;\&\; c = -4$

$$\frac{AP}{PB} = \frac{\lambda}{1} = -\frac{ax_1+by_1+c}{ax_2+by_2+c}$$

$$\frac{AP}{PB} = \frac{\lambda}{1} = -\frac{-1+1+(-4)}{5+7+(-4)} = \frac{4}{8} = \frac{1}{2}$$

$AP : PB = 1 : 2$

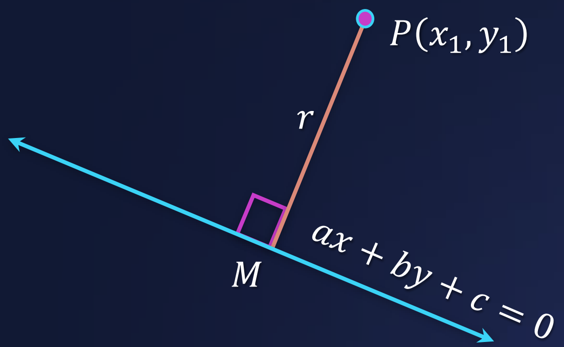


Return to Top

Key Takeaways

Distance of a point from a line:

The length of the perpendicular from a point (x_1, y_1) to the line $ax + by + c = 0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$



$$\boxed{r = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|}$$

Key Takeaways

Distance of a point from a line:

Proof:

We have, $l : ax + by + c = 0$

$\therefore$ Slope of $l = -\frac{a}{b}$

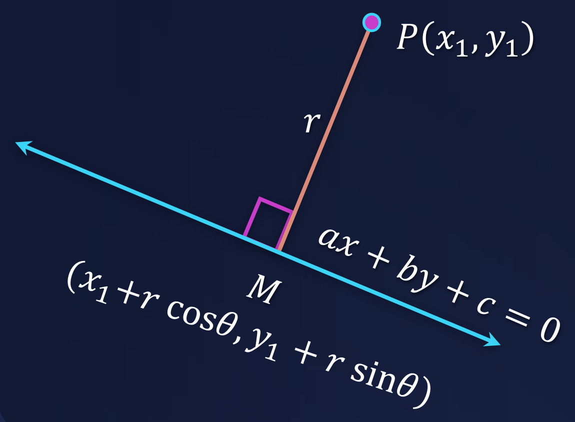
Then, Slope of $PM = \frac{b}{a}$

$\Rightarrow \tan\theta = \frac{b}{a}$

$\Rightarrow \cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ & $\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

M lies on $l : ax + by + c = 0$

$\Rightarrow a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$



Key Takeaways

$$\Rightarrow a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$$

$$\Rightarrow a\,r\cos\theta + b\,r\sin\theta + ax_1 + by_1 + c = 0$$

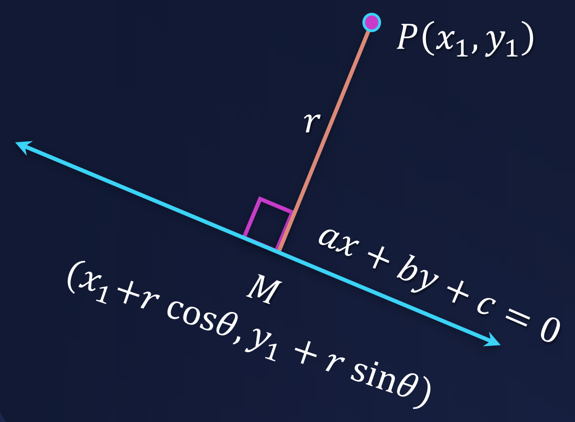
$$\Rightarrow r(a\cos\theta + b\sin\theta) = -(ax_1 + by_1 + c)$$

$$\Rightarrow r = -\left(\frac{ax_1+by_1+c}{a\cos\theta+b\sin\theta}\right)$$

$$\Rightarrow r = -\left(\frac{ax_1+by_1+c}{a\times\frac{a}{\sqrt{a^2+b^2}}+b\times\frac{b}{\sqrt{a^2+b^2}}}\right)$$

But $r \geq 0$

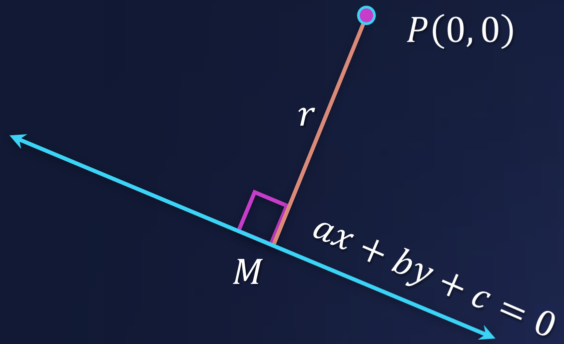
$$\Rightarrow r = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$



Key Takeaways

Distance of a point from a line:

The length of the perpendicular from the origin to the line $ax + by + c = 0$ is $\frac{|c|}{\sqrt{a^2+b^2}}$



$$r = \left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

Find the points on Y –axis whose perpendicular distance from the line $4x - 3y - 12 = 0$ is 3.

Let the required point be $P(0, \alpha)$.

Length of the perpendicular from $P(0, \alpha)$ on $4x - 3y - 12 = 0$ is 3

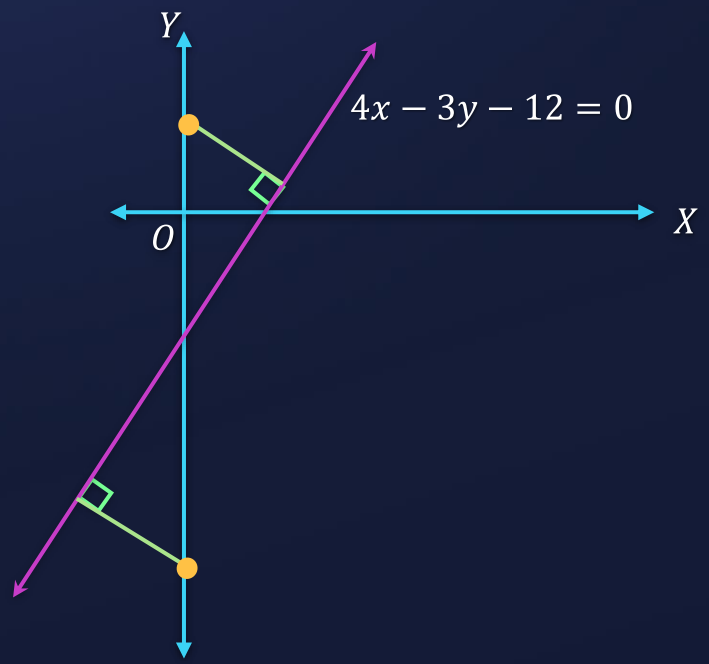
$$\Rightarrow \left|\frac{4(0) - 3\alpha - 12}{\sqrt{4^2 + (-3)^2}}\right| = 3$$

$$\Rightarrow |3\alpha + 12| = 15$$

$$\Rightarrow \alpha + 4 = \pm 5$$

$$\Rightarrow \alpha = 1, -9$$

$\therefore$ Required points are $(0,1)$ and $(0, -9)$



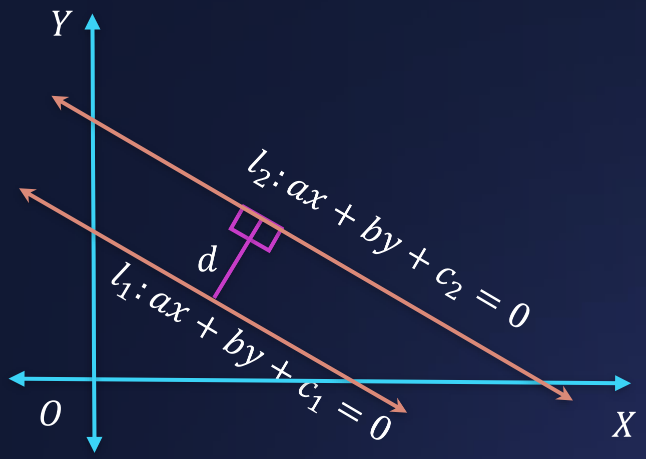
[Return to Top](#)

Key Takeaways

Distance between parallel lines:

Distance between parallel lines

$ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$



Key Takeaways

Distance between parallel lines:

Proof:

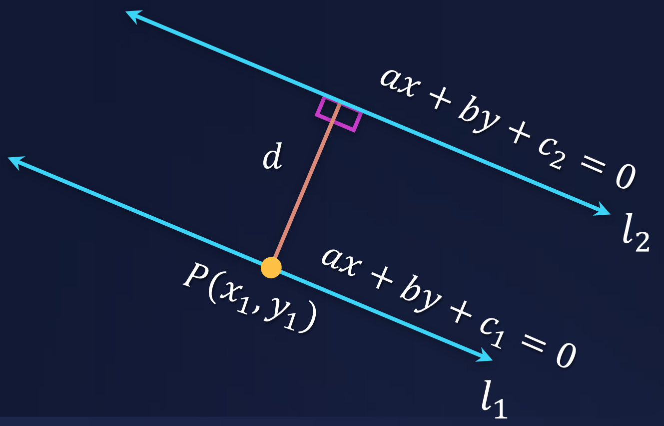
Perpendicular distance of $P(x_1, y_1)$ from l_2 is $d = \left|\frac{ax_1+by_1+c_2}{\sqrt{a^2+b^2}}\right|$

Also, $P(x_1, y_1)$ lies on l_1

$\Rightarrow ax_1 + by_1 + c_1 = 0$

$\Rightarrow ax_1 + by_1 = -c_1$

$\therefore d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$ or $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$



Distance between the lines given by $x - 2y = 1$ and $3x + 15 = 6y$ is :

A. $\frac{16}{\sqrt{3}}$

B. $\frac{16}{\sqrt{5}}$

C. $\frac{8}{\sqrt{3}}$

D. $\frac{6}{\sqrt{5}}$

Return to Top

Distance between the lines given by $x - 2y = 1$ and $3x + 15 = 6y$ is :

Let , $l_1 : x - 2y = 1$

$l_2 : 3x + 15 = 6y$

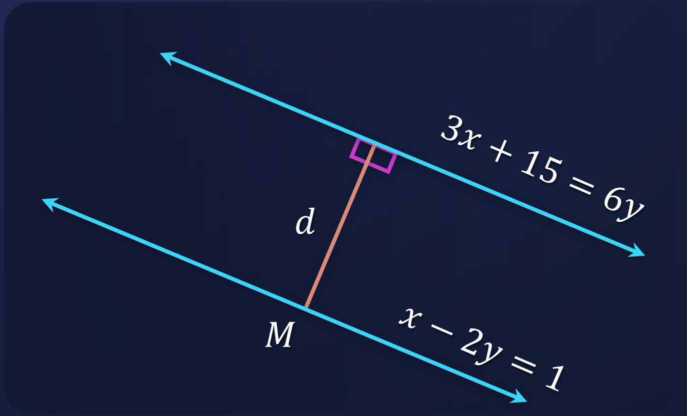
Or $l_2 : x - 2y = -5$

$\because l_1 \parallel l_2$

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$$

$$\Rightarrow d = \left|\frac{-5 - 1}{\sqrt{1^2+(-2)^2}}\right|$$

$\Rightarrow \frac{6}{\sqrt{5}}$ units.



Return to Top

Distance between the lines given by $x - 2y = 1$ and $3x + 15 = 6y$ is :

A $\frac{16}{\sqrt{3}}$

B $\frac{16}{\sqrt{5}}$

C $\frac{8}{\sqrt{3}}$

D $\frac{6}{\sqrt{5}}$

Return to Top

The coordinates of the point on $x + y + 3 = 0$, whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$ units, is

A. $(-8, 5)$

B. $(1, -4)$

C. $(9, -6)$

D. $(1, 4)$

Return to Top

The coordinates of the point on $x + y + 3 = 0$, whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$ units, is

Let the points on the line $x + y + 3 = 0$ be $(a, -3 - a)$

Length of the perpendicular from $(a, -3 - a)$ to $x + 2y + 2 = 0$ is

$$\Rightarrow \left|\frac{a - 6 - 2a + 2}{\sqrt{1^2 + 2^2}}\right| = \sqrt{5}$$

$$\Rightarrow \quad a = -9, 1$$

$\therefore$ The points are $(-9, 6), (1, -4)$.

Return to Top

The coordinates of the point on $x + y + 3 = 0$, whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$ units, is

A $(-8, 5)$

B $(1, -4)$

C $(9, -6)$

D $(1, 4)$

Return to Top

Session 08

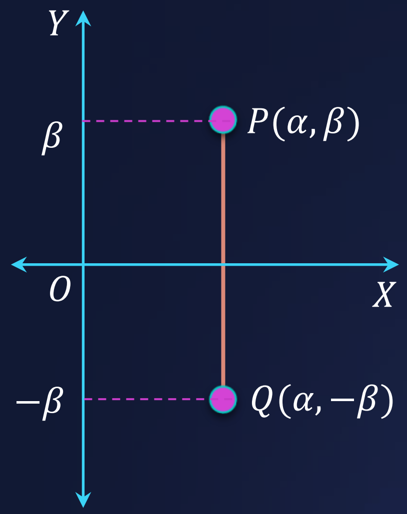
Image of a point and Concurrency of lines

Return to Top

Key Takeaways

Image of a point:

The image of a point $P(\alpha, \beta)$ with respect to x −axis is $Q(\alpha, -\beta)$.

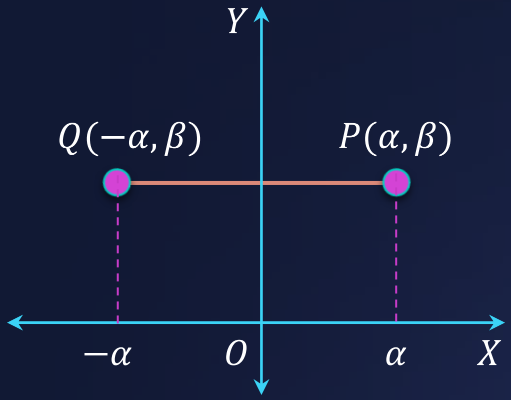


Return to Top

Key Takeaways

Image of a point:

The image of a point $P(\alpha, \beta)$ with respect to y −axis is $Q(-\alpha, \beta)$.

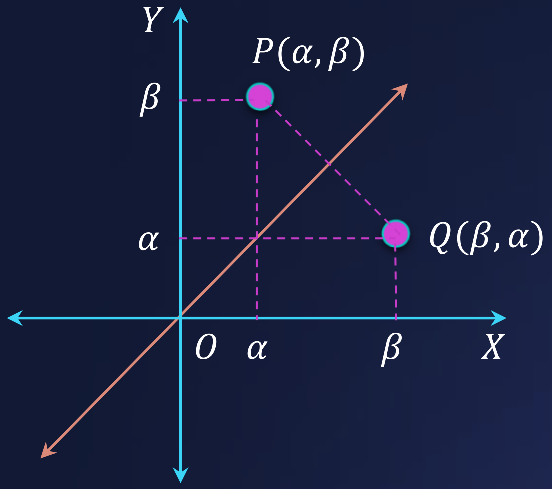


Return to Top

Key Takeaways

Image of a point:

The image of a point $P(\alpha, \beta)$ with respect to $y = x$ is $Q(\beta, \alpha)$.



Return to Top

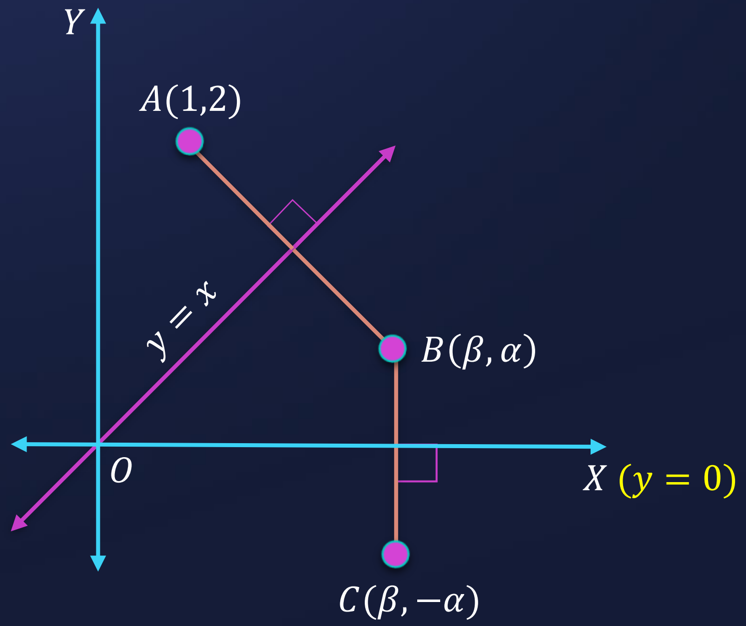
The image of the point $A(1,2)$ by the mirror $y = x$ is the point B and the image of B by the line mirror $y = 0$ is the point $C(\alpha, \beta)$, then $\alpha =$ ____ , $\beta =$ ____.

The image of a point $A(\alpha, \beta)$ with respect to $y = x$ is $B(\beta, \alpha)$.

$\Rightarrow B \equiv (2, 1)$

The image of a point $B(\beta, \alpha)$ with respect to x –axis $(y = 0)$ is $C(\beta, -\alpha)$.

$\Rightarrow C \equiv (2, -1)$

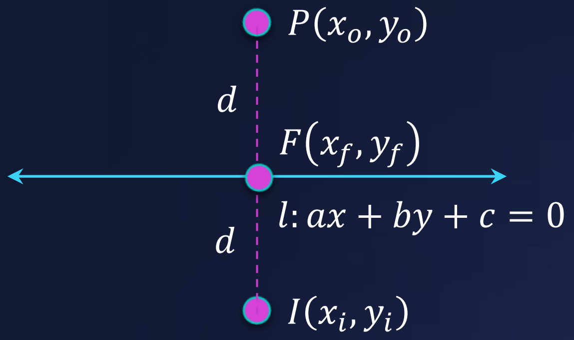


Return to Top

Key Takeaways

Reflection of a point about a Line:

$F(x_f, y_f)$ is the foot of perpendicular.

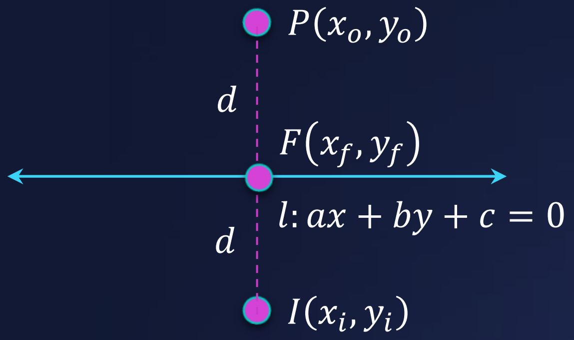


$$\boxed{\frac{x_f - x_o}{a} = \frac{y_f - y_o}{b} = -\frac{ax_o + by_o + c}{a^2 + b^2}}$$

Key Takeaways

Reflection of a point about a Line:

$I(x_i, y_i)$ is the reflection of point $P(x_o, y_o)$ about the line $ax + by + c = 0$.



$$\boxed{\frac{x_i - x_o}{a} = \frac{y_i - y_o}{b} = -2\,\frac{ax_o + by_o + c}{a^2 + b^2}}$$

The coordinates of the foot of the perpendicular and the image of the point $(8, 2)$ about the line $3x - y = 2$ are:

Consider $l : 3x - y = 2$ compare with $ax + by + c = 0$

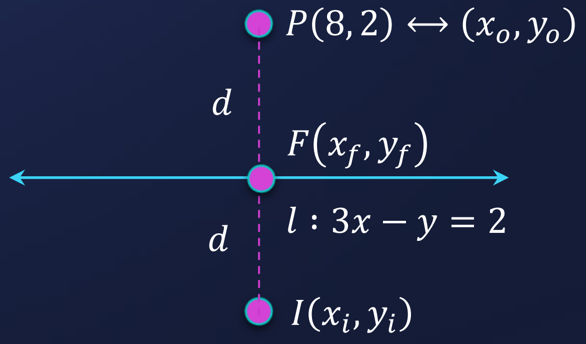
$a = 3, b = -1, c = -2$

$$\frac{x_f - x_o}{a} = \frac{y_f - y_o}{b} = -\frac{ax_o + by_o + c}{a^2 + b^2}$$

$$\Rightarrow \frac{x_f - 8}{3} = \frac{y_f - 2}{-1} = -\frac{24 - 2 - 2}{(3)^2 + (-1)^2}$$

$$\Rightarrow \frac{x_f - 8}{3} = \frac{y_f - 2}{-1} = -2$$

$\frac{x_f - 8}{3} = -2$	$\frac{y_f - 2}{-1} = -2$
$\Rightarrow x_f - 8 = -6$	$\Rightarrow y_f - 2 = 2$
$\Rightarrow x_f = 2$	$\Rightarrow y_f = 4$



Return to Top

The coordinates of the foot of the perpendicular and the image of the point $(8, 2)$ about the line $3x - y = 2$ are:

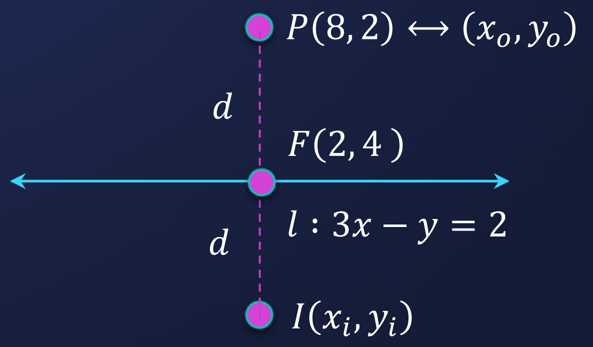
$x_f = 2, y_f = 4$

$PF = IF$

$\Rightarrow F$ is mid point of PI.

$\frac{x_i+8}{2} = 2$	$\frac{y_i+2}{2} = 4$
$\Rightarrow x_i + 8 = 4$	$\Rightarrow y_i + 2 = 8$
$\Rightarrow x_i = -4$	$\Rightarrow y_i = 6$

So, $I \equiv (-4, 6)$



Return to Top

Find a point P on $y = x$ such that $PA + PB$ is minimum where $A \equiv (1, 3), B \equiv (5, 2)$.

$l: y = x \Rightarrow x - y = 0$

$l_A: l_{(1,3)} = 1 - 3 < 0$ and $l_B: l_{(5,2)} = 5 - 2 > 0$

$\Rightarrow A \,\&\, B$ lies on opposite sides of l

$\therefore$ For $AP + BP$ to be minimum,

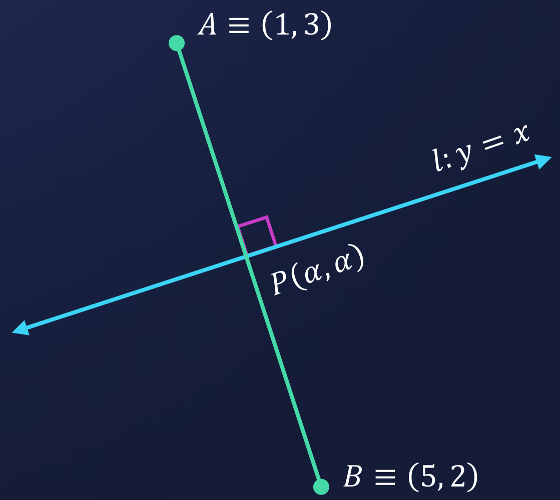
$A, P \,\&\, B$ must be colinear.

$\therefore m_{AP} = m_{AB}$

$\Rightarrow \frac{\alpha-3}{\alpha-1} = \frac{2-3}{5-1}$

$\Rightarrow \alpha = \frac{13}{5}$

$\therefore P \equiv \left(\frac{13}{5}, \frac{13}{5}\right)$



Return to Top

Point R on X –axis such that $PR + RQ$ is minimum when $P = (1, 1)$ and $Q = (3, 2)$ is:

A. $\left(\frac{5}{3}, 0\right)$

B. $(2, 0)$

C. $\left(\frac{3}{2}, 0\right)$

D. $\left(\frac{5}{4}, 0\right)$

Return to Top

Point R on X –axis such that $PR + RQ$ is minimum when $P = (1, 1)$ and $Q = (3, 2)$ is:

P & Q lies on the same sides of X – axis.

The mirror image of Q with respect to X – axis is Q'.

$\Rightarrow PR + RQ = PR + RQ'$

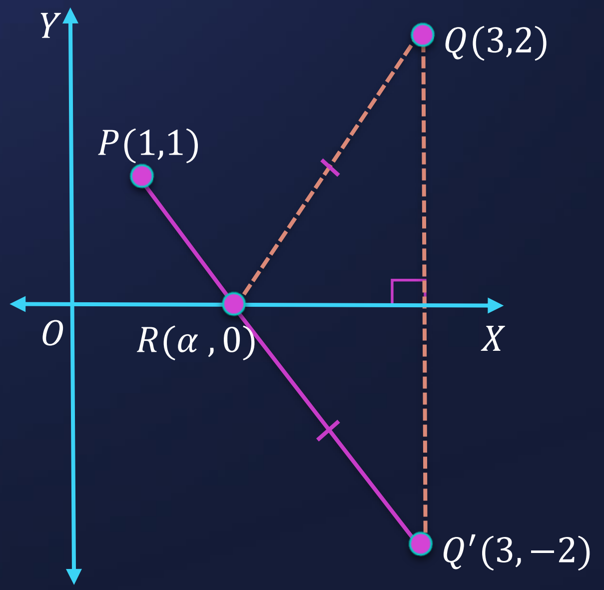
$\therefore$ For $PR + RQ$ to be minimum, when P, R & Q' must be collinear.

$m_{PQ'} = m_{RQ'}$

$\frac{1+2}{1-3} = \frac{0+2}{\alpha-3}$

$\Rightarrow \alpha = \frac{5}{3}$

$\therefore R \equiv \left(\frac{5}{3}, 0\right)$



Return to Top

Point R on X –axis such that $PR + RQ$ is minimum when $P = (1, 1)$ and $Q = (3, 2)$ is:

A $\left(\frac{5}{3}, 0\right)$

B $(2, 0)$

C $\left(\frac{3}{2}, 0\right)$

D $\left(\frac{5}{4}, 0\right)$

Return to Top

Key Takeaways

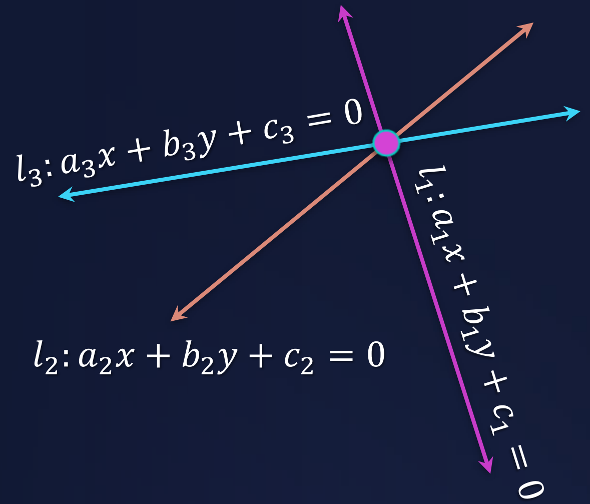
Concurrency of Three Lines:

The three lines are concurrent if any one of the lines passes through the point of intersection of other two lines.

$$a_1(b_2c_3 - c_2b_3) + b_1(c_2a_3 - a_2c_3) + c_1(a_2b_3 - b_2a_3) = 0$$

or

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$



Find the value of λ, if the lines $3x - 4y - 13 = 0$, $8x - 11y - 33 = 0$ and $2x - 3y + \lambda = 0$ are concurrent.

The given lines are concurrent if :

$$\begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 3(-11\lambda - 99) + 4(8\lambda + 66) - 13(-24 + 22) = 0$$

$$\Rightarrow -\lambda - 7 = 0$$

$$\Rightarrow \lambda = -7$$

Return to Top

A ray of light coming from the point $(1,2)$ is reflected at a point A on the x −axis and then passes through the point $(5,3)$.Then the coordinates of the point A are:

Let the coordinate of A be $(a,0)$

Slope of reflected ray AC is,

$$m_{AC} = \frac{3-0}{5-a} = \frac{3}{5-a} = \tan\theta$$

Slope of incident ray AB is,

$$m_{AB} = \frac{2-0}{1-a} = \tan(\pi-\theta) = -\tan\theta$$

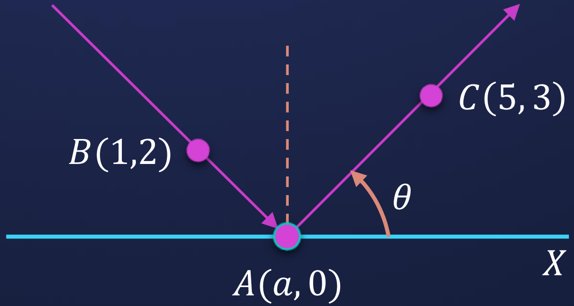
Slope of AB + Slope of $AC = 0$

$$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0$$

$$\Rightarrow 10-2a+3-3a = 0$$

$$\Rightarrow 5a = 13$$

$$\Rightarrow a = \frac{13}{5}$$



A $\left(\frac{13}{5},0\right)$

B $\left(\frac{5}{13},0\right)$

C $(-7,0)$

D None of these

Return to Top

A beam of light is sent along the line $x - y = 1$, which after refracting from the x –axis enters the opposite side by turning through 30^0 away from the normal at the point of incidence on the x –axis. Find the equation of the refracted ray.

Given Equation:

$y = x - 1;$

Slope of line II' is 1.

$\Rightarrow \tan\alpha = 1$

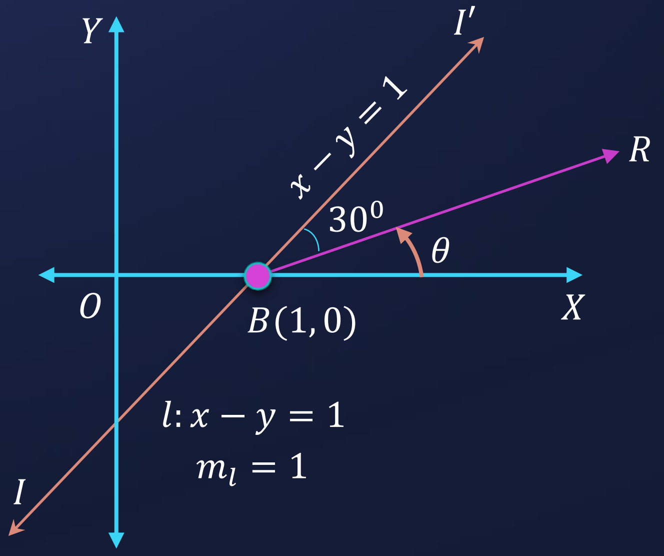
$\Rightarrow \alpha = 45^0$

$\theta = \alpha - 30^0$

$\Rightarrow \theta = 45^0 - 30^0$

$\Rightarrow \theta = 15^0$

$\Rightarrow \tan\theta = \tan 15^0 = (2 - \sqrt{3})$



Equation of BR:

$y - 0 = (2 - \sqrt{3})(x - 1)$

[Return to Top](#)

Session 09

Family of Lines and Angle bisector between Lines

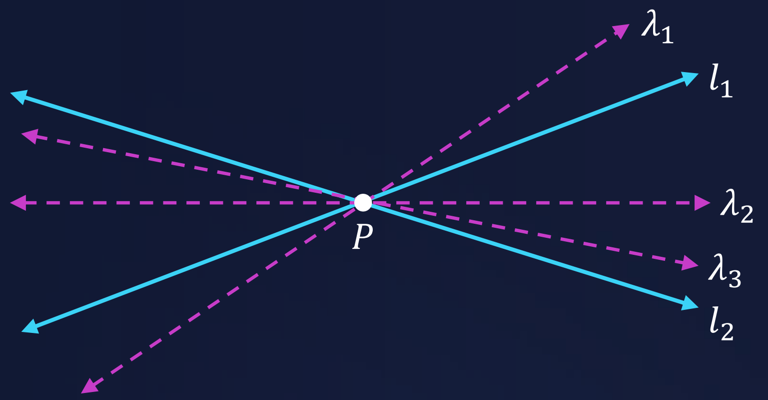
Return to Top

Key Takeaways

Family of Line:

The equation for the family of lines $'L'$ passing through the point of intersection of lines l_1 and l_2 is

$l_1 + \lambda l_2 = 0, \quad \lambda \in \mathbb{R}$



Where λ is a parameter and can be determined from imposed condition.

For $l_1 : a_1x + b_1y + c_1 = 0$

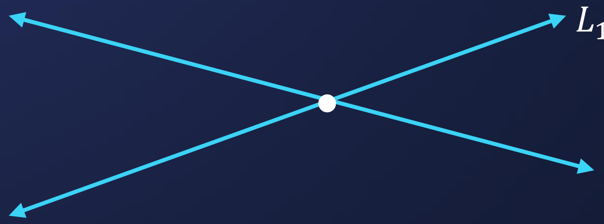
$l_2 : a_2x + b_2y + c_2 = 0$

$$\boxed{(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0}$$

Return to Top

Fixed point passes through which each member of the family of lines $(1+\lambda)x+(2-\lambda)y+5=0$ passes for all values of λ is

$$(1+\lambda)x+(2-\lambda)y+5=0\ \forall\ \lambda \in \mathbb{R}$$



$$L_1 + \lambda L_2 = 0$$

$$\Rightarrow (x+2y+5)+\lambda(x-y)=0$$

$$\left.\begin{array}{l} L_1 \equiv x+2y+5=0 \\ L_2 \equiv x-y=0 \end{array}\right\} \text{Fixed point or intercept point}$$

Put $y = x$ in L_1

$$\Rightarrow 3x+5=0$$

$$\Rightarrow x = -\frac{5}{3}$$

$$\boxed{\text{Intercept point } \left(-\frac{5}{3}, -\frac{5}{3}\right)}$$

Return to Top

If a, b, c are variables such that $21a + 40b + 56c = 0$, then find the fixed point through which each member of the family of the lines $ax + by + c = 0$ passes is

Given: $21a + 40b + 56c = 0$ and $ax + by + c = 0$ passes through the fixed point.

Dividing the equation by 56

$$\Rightarrow \left(\frac{21}{56}\right)a + \left(\frac{40}{56}\right)b + \left(\frac{56}{56}\right)c = 0$$

$$\Rightarrow \left(\frac{3}{8}\right)a + \left(\frac{5}{7}\right)b + c = 0$$

Now, $ax + by + c = 0$ passes through the same point.

$$\Rightarrow x = \frac{3}{8}, y = \frac{5}{7}$$

So, fixed point $\equiv \left(\frac{3}{8}, \frac{5}{7}\right)$

Return to Top

If the straight lines cuts intercepts on the coordinate axes such that the sum of their reciprocals is 3, then the fixed point through which all these lines passes is

A $\left(1, \frac{1}{3}\right)$

B $\left(\frac{2}{3}, \frac{4}{3}\right)$

C $\left(\frac{1}{3}, \frac{1}{3}\right)$

D $\left(\frac{2}{3}, \frac{1}{3}\right)$

Return to Top

If the straight lines cuts intercepts on the coordinate axes such that the sum of their reciprocals is 3, then the fixed point through which all these lines passes is

Let the intercept on x and y axes be a and b respectively.

Given, $\frac{1}{a} + \frac{1}{b} = 3$

$\Rightarrow \frac{1}{3a} + \frac{1}{3b} = 1$

$\Rightarrow \frac{\left(\frac{1}{3}\right)}{a} + \frac{\left(\frac{1}{3}\right)}{b} = 1$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

On comparing we get, $x_0 = \frac{1}{3}, y_0 = \frac{1}{3}$

$\therefore$ Fixed Point $\rightarrow \left(\frac{1}{3}, \frac{1}{3}\right)$.

Return to Top

If the straight lines cuts intercepts on the coordinate axes such that the sum of their reciprocals is 3, then the fixed point through which all these lines passes is

A $\left(1, \frac{1}{3}\right)$

B $\left(\frac{2}{3}, \frac{4}{3}\right)$

C $\left(\frac{1}{3}, \frac{1}{3}\right)$

D $\left(\frac{2}{3}, \frac{1}{3}\right)$

Return to Top

Find the straight line of the family $(x+y)+\lambda(2x-y+1)=0, \lambda \in \mathbb{R}$ that is:

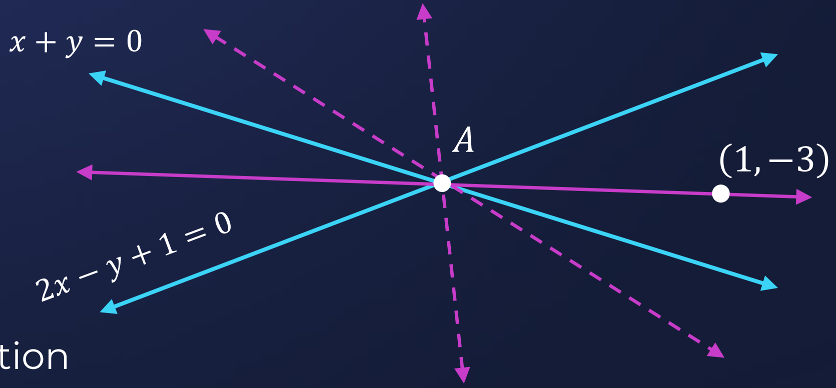
(i) Nearest from the point $(1,-3)$

(ii) Farthest from the point $(1,-3)$

$(x+y)+\lambda(2x-y+1)=0 \quad \cdots(i)$

$L_1+\lambda L_2=0$

Passes through intersection of lines $x+y=0$ and $2x-y+1=0$



(i) Put $(1,-3)$ in (i) family of lines equation

$\Rightarrow (1-3)+\lambda(2+3+1)=0$

$\Rightarrow \lambda=\frac{1}{3}$

Put $\lambda=\frac{1}{3}$ in (i)

$\Rightarrow (x+y)+\frac{1}{3}(2x-y+1)=0$

$\boxed{\Rightarrow 5x+2y+1=0}$

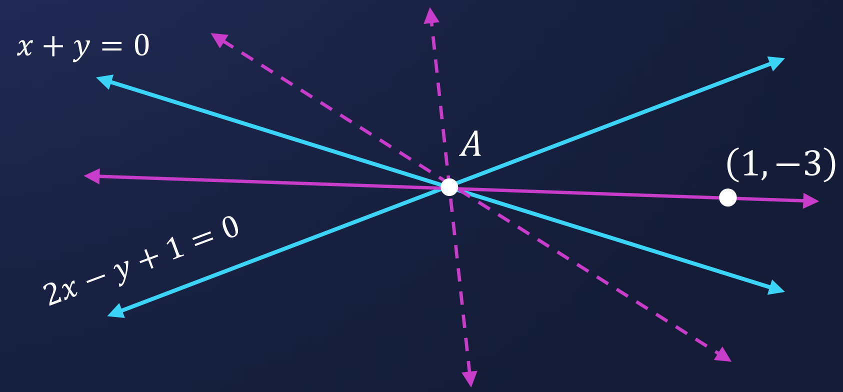
Return to Top

Find the straight line of the family $(x+y)+\lambda(2x-y+1)=0, \lambda \in \mathbb{R}$ that is:

(i) Nearest from the point $(1,-3)$

(ii) Farthest from the point $(1,-3)$

(ii) The line furthest from point $(1,-3)$ will be perpendicular to the line passing through $(1,-3)$ and point A, where A is the point of intersection of lines $x+y=0$ and $2x-y+1=0$



$$(x+y)+\lambda(2x-y+1)=0$$

$$\Rightarrow (2\lambda+1)x+y(1-\lambda)+\lambda=0 \cdots (ii)$$

Slope of line perpendicular to $5x+2y+1=0$ is $\frac{2}{5}$.

$$\therefore \frac{2}{5}=-\frac{(2\lambda+1)}{(1-\lambda)} \Rightarrow 2-2\lambda=-10\lambda-5 \Rightarrow \lambda=-\frac{7}{8}$$

Putting $\lambda=-\frac{7}{8}$ in (i)

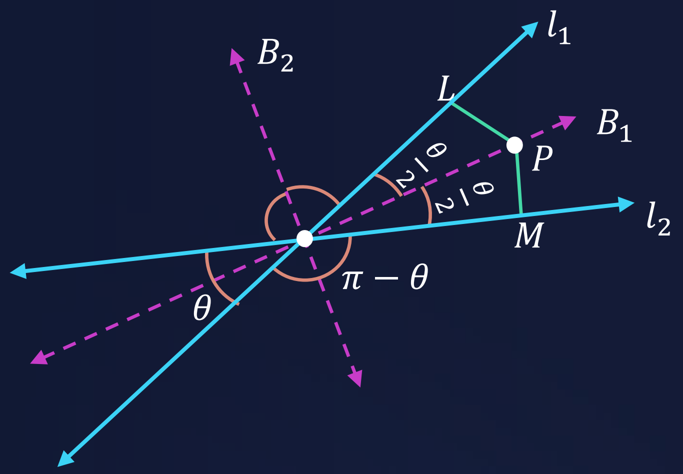
$$(x+y)-\frac{7}{8}(2x-y+1)=0 \Rightarrow \boxed{6x-15y+7=0}$$

Return to Top

Key Takeaways

Angle Bisectors between the lines:

Angle Bisector:
Locus is a moving point equidistant from the two intersecting lines.



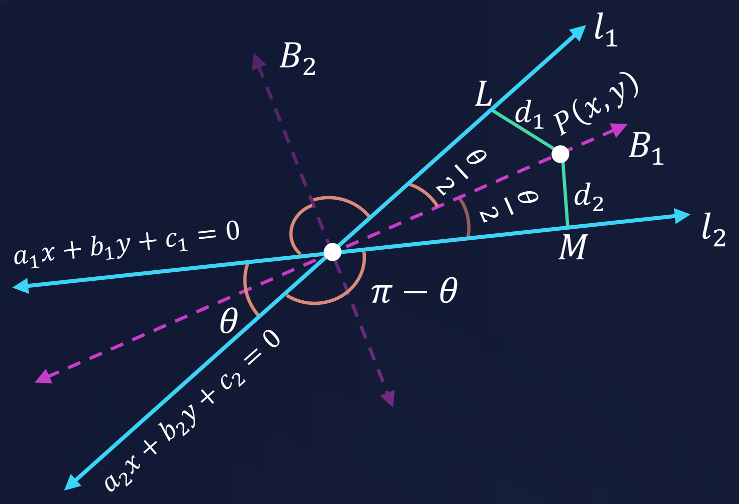
Note:

- The bisectors are orthogonal to each other.
- B_1 and B_2 are always perpendicular to each other.

Return to Top

Key Takeaways

Angle Bisector between the lines:



Equation:

Here, $d_1 = d_2$

$$\left|\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right| = \left|\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right|$$

$a_2^2 + b_2^2 = p$ and $a_1^2 + b_1^2 = q$

$(p = q)$ or $(-p = -q)$ and $(p = -q)$ or $(-p = q)$

$\Rightarrow p = q$ and $\Rightarrow p = -q$

$$\boxed{\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}}$$

Return to Top

Key Takeaways

Angular Bisectors of Acute and Obtuse angles:

Let revised equations

$$\left.\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}\right\} c_1, c_2 > 0$$

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

Step I: Make constants of both lines $(+ve)$.

Step II: Using equation of modified lines find $a_1a_2 + b_1b_2$.

Return to Top

Key Takeaways

Angular Bisectors of Acute and Obtuse angles:

Condition	Acute Angle Bisector	Obtuse Angle Bisector
$a_1a_2 + b_1b_2 > 0$	$-$	$+$
$a_1a_2 + b_1b_2 < 0$	$+$	$-$

Return to Top

Find the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the:

A. Bisector of the obtuse angle between them

B. Bisector of the acute angle between them

Solution:

$4x + 3y - 6 = 0$

$$\left.\begin{aligned} \Rightarrow \quad & -4x - 3y + 6 = 0 \\ \text{And } & 5x + 12y + 9 = 0 \end{aligned}\right\} \quad c_1, c_2 > 0$$

Now, $a_1 = -4, b_1 = -3$ and $a_2 = 5, b_2 = 12$

$a_1 a_2 + b_1 b_2 = (-4)(5) + (-3)(12)$

$\Rightarrow a_1 a_2 + b_1 b_2 = -56 < 0$

Return to Top

Find the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$,

find the equation of the:

A. Bisector of the obtuse angle between them

B. Bisector of the acute angle between them

A. Obtuse angle bisector: $\Rightarrow -ve$

$$\left(\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}}\right) = -\left(\frac{5x+12y+9}{\sqrt{5^2+12^2}}\right)$$

$$\Rightarrow \frac{-4x-3y+6}{5} = -\frac{5x+12y+9}{13}$$

$$\Rightarrow -52x - 39y + 78 = -25x - 60y - 45$$

$$\Rightarrow 27x - 21y - 123 = 0$$

So, the bisector of the obtuse angle is $9x - 7y - 41 = 0$.

Return to Top

Find the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$,

find the equation of the:

A. Bisector of the obtuse angle between them

B. Bisector of the acute angle between them

B. Acute angle bisector: $\Rightarrow +ve$

$$\left(\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}}\right) = \left(\frac{5x+12y+9}{\sqrt{5^2+12^2}}\right)$$

$$\Rightarrow \frac{-4x-3y+6}{5} = \frac{5x+12y+9}{13}$$

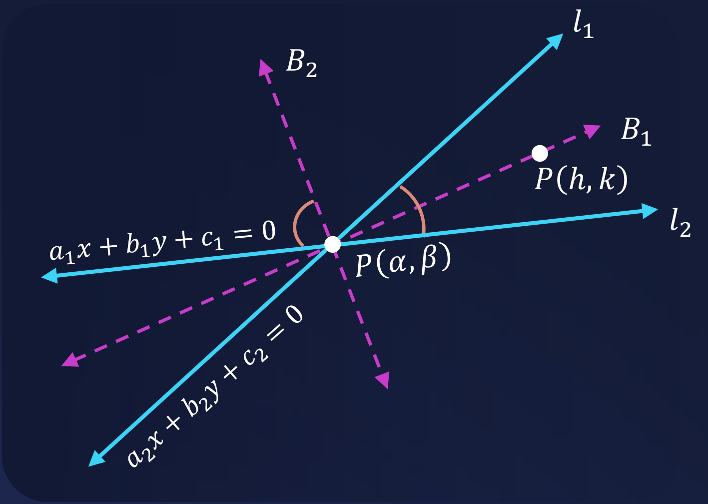
Return to Top

Key Takeaways

Bisector of the angle containing the given point

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

Step I: Substitute the point in the L.H.S of both l_1 & l_2 to get $l_1(P)$ & $l_2(P)$. $a_1\alpha + b_1\beta + c_1$ and $a_2\alpha + b_2\beta + c_2$.



Return to Top

Key Takeaways

Step II: Use the working rule:

Condition	Angle Bisector
$l_1(P) \cdot l_2(P) > 0$	Equation w.r.t (+) is the required bisector
$l_1(P) \cdot l_2(P) < 0$	Equation w.r.t (–) is the required bisector

Return to Top

For the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the bisector of the angle:

(i) Which contains $(1, 2)$

(ii) Which contains the origin

(i) Given $l_1 \equiv 4x + 3y - 6 = 0$ and $l_2 \equiv 5x + 12y + 9 = 0$

$$\left.\begin{array}{l} l_1(1,2) \equiv 4(1) + 3(2) - 6 > 0 \\ l_2(1,2) \equiv 5(1) + 12(2) + 9 > 0 \end{array}\right\} \text{Same sign} \Rightarrow l_1\,(1,2) \cdot l_2(1,2) > 0$$

Hence, equation w.r.t $+ve$ sign is the required bisector.

$$\left(\frac{4x+3y-6}{\sqrt{(4)^2+(3)^2}}\right) = +\left(\frac{5x+12y+9}{\sqrt{5^2+12^2}}\right)$$

$$\Rightarrow 52x + 39y - 78 = 25x + 60y + 45$$

$$\Rightarrow 27x - 21y - 123 = 0$$

So, the bisector of the angle that contains $(1, 2)$ is $9x - 7y - 41 = 0$.

Return to Top

For the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the bisector of the angle:

(i) Which contains $(1, 2)$ (ii) Which contains the origin

(ii) Given $l_1 \equiv 4x + 3y - 6 = 0$ and $l_2 \equiv 5x + 12y + 9 = 0$

$$\left.\begin{array}{l} l_1(0,0) \equiv 4(0) + 3(0) - 6 < 0 \\ l_2(0,0) \equiv 5(0) + 12(0) + 9 > 0 \end{array}\right\} \text{Opposite sign} \Rightarrow l_1\,(0,0) \cdot l_2(0,0) < 0$$

Hence, equation w.r.t $-ve$ sign is the required bisector.

$$\left(\frac{4x+3y-6}{\sqrt{(4)^2+(3)^2}}\right) = -\left(\frac{5x+12y+9}{\sqrt{5^2+12^2}}\right)$$

$$\Rightarrow 52x + 39y - 78 = -25x - 60y - 45$$

$$\Rightarrow 77x + 99y - 33 = 0$$

So, the bisector of the angle that contains the origin is $7x + 9y - 3 = 0$.

Return to Top

Session 10

Family of Lines and Angle bisector between Lines

Return to Top

Key Takeaways

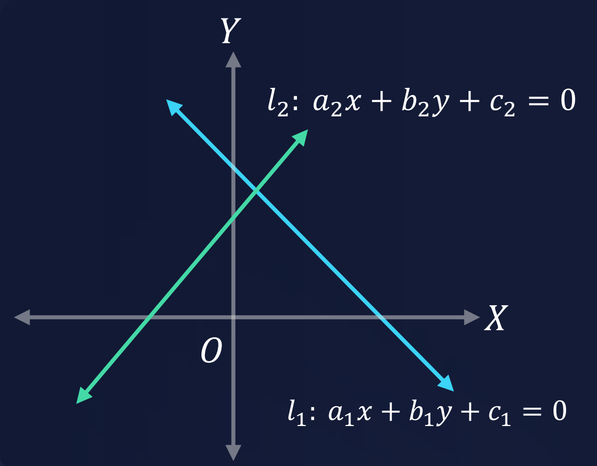
Pair of Straight Lines:

Joint equation of l_1 and l_2 is given by : $l_1 l_2 = 0$

$$\Rightarrow (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$$

$$\Rightarrow a_1a_2x^2 + b_1b_2y^2 + xy(a_1b_2 + a_2b_1) +$$

$$x(a_1c_2 + c_1a_2) + y(b_1c_2 + b_2c_1) + c_1c_2 = 0$$



General Form : $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Return to Top

Key Takeaways

General Equation of Pair of Lines :

General 2^{nd} degree equation in x, y :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Represents a pair of straight lines iff :

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Or

$$\boxed{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$$

Return to Top

Prove that the equation $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ represents two straight lines.

Comparing the given equation $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

We get,

$a = -3, 2h = -8, b = 3, 2g = -29, 2f = 3, c = -18$

We find that

$abc + 2fgh - af^2 - bg^2 - ch^2$

$= (-3)(3)(-18) + 2\left(\frac{3}{2}\right)\left(-\frac{29}{2}\right)(-4) - (-3)\left(\frac{3}{2}\right)^2 - (3)\left(-\frac{29}{2}\right)^2 - (-18)(-4)^2$

$= 0$

Return to Top

Key Takeaways

Homogeneous Equation:

An equation in which combined degree of each term is same is called a homogeneous equation.

Example:

$2x + 3y = 0 \Rightarrow$ Homogeneous Equation of degree 1.

$x^2 - 5xy - 6y^2 = 0 \Rightarrow$ Homogeneous Equation of degree 2.

$x^3 - 6x^2y + 11xy^2 - 6y^3 = 0$ Homogeneous Equation of degree 3.

$2x + 3y + 4 = 0 \Rightarrow$ Not a Homogeneous Equation.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \Rightarrow$ Not a Homogeneous Equation.

Return to Top

Key Takeaways

Homogeneous Equation:

The homogeneous second degree equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin. i.e., $ax^2 + 2hxy + by^2 = 0$ represents two straight lines passing through $(0, 0)$.

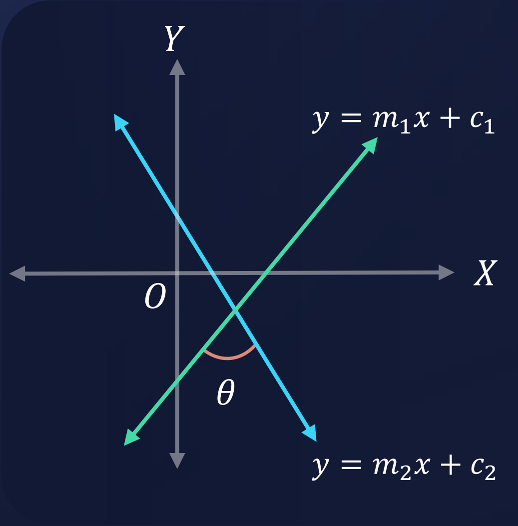
Return to Top

Angle between Pair of Lines for non-homogenous equation:

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents pair of straight lines having slopes m_1 & m_2 then,

$$m_1 + m_2 = -\frac{2h}{b}$$ and $$m_1 \times m_2 = \frac{a}{b}$$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$



Return to Top

Angle between Pair of Lines for non-homogenous equation:

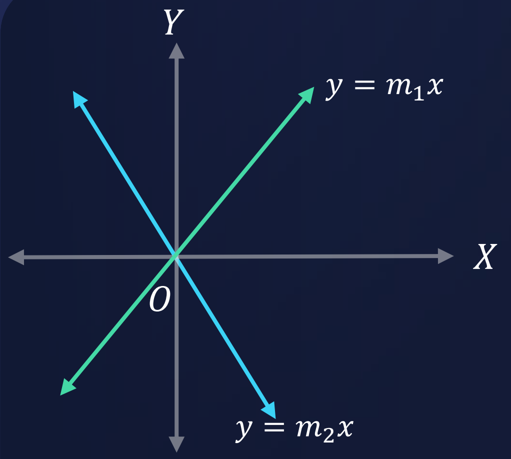
Note:

- If $a + b = 0 \Rightarrow$ Lines are perpendicular i.e. Coeff. of x^2 + Coeff. of $y^2 = 0$
- If $h^2 = ab \Rightarrow$ Lines are coincident.
- If $h^2 > ab \Rightarrow$ Lines are real and distinct.
- If $h^2 < ab \Rightarrow$ Lines are imaginary

Return to Top

Homogeneous Equation

Let $y = mx$ be the 2 straight lines passing through origin represented by $ax^2 + 2hxy + by^2 = 0$ it.



Putting $y = mx$ in $ax^2 + 2hxy + by^2 = 0$,

$$ax^2 + 2hx(mx) + b(mx)^2 = 0$$

$$\Rightarrow ax^2 + 2hmx^2 + bm^2x^2 = 0$$

$$\Rightarrow \underbrace{a + 2hm + bm^2}_{\text{Let the roots be } m_1 \,\&\, m_2} = 0$$

Return to Top

Homogeneous Equation

$$\underbrace{a + 2hm + bm^2}_{} = 0$$

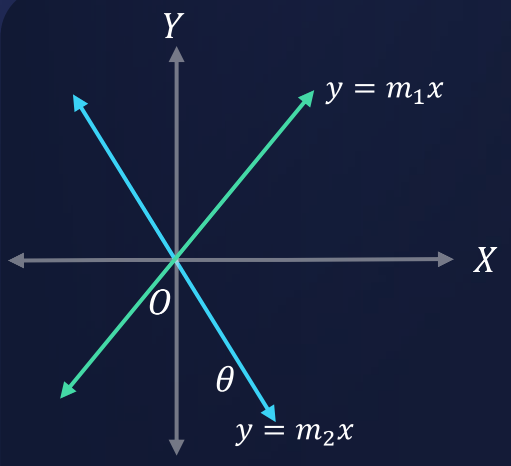
Let the roots be m_1 & m_2

Sum of roots $= m_1 + m_2 = -\frac{2h}{b}$

Product of roots $= m_1 \cdot m_2 = \frac{a}{b}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}$$



Return to Top

The gradient of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, then

A $h^2 = ab$

B $h = a + b$

C $8h^2 = 9ab$

D $h^2 + 9ab = 0$

Return to Top

The gradient of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, then

Given $ax^2 + 2hxy + by^2 = 0$:

Let m and $2m$ be the gradients.

$$\Rightarrow m + 2m = -\frac{2h}{b}$$

$$\Rightarrow m = -\frac{2h}{3b} \quad \cdots (i)$$

Also $m \cdot 2m = \frac{a}{b}$

$$\Rightarrow 2m^2 = \frac{a}{b}$$

$$\Rightarrow m^2 = \frac{a}{2b} \quad \cdots (ii)$$

From (i) and (ii):

$$\left(-\frac{2h}{3b}\right)^2 = \frac{a}{2b}$$

$$\Rightarrow \frac{4h^2}{9b^2} = \frac{a}{2b}$$

$$\Rightarrow 8h^2 = 9ab$$

Return to Top

The gradient of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, then

A $h^2 = ab$

B $h = a + b$

C $8h^2 = 9ab$

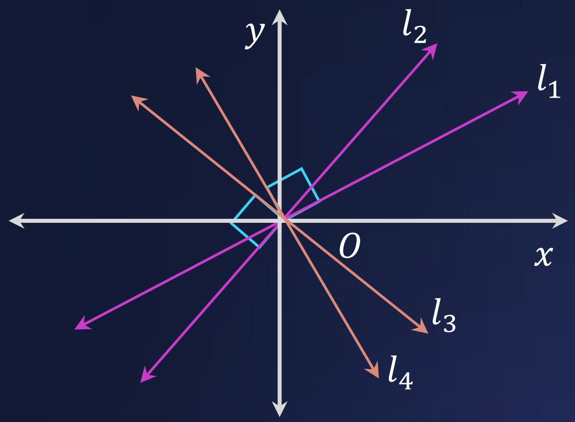
D $h^2 + 9ab = 0$

Return to Top

Key Takeaways

Homogeneous Equation:

If joint equation of l_1 and l_2 is $ax^2 + 2hxy + by^2 = 0$, then joint equation of lines perpendicular to l_1 and l_2 and passing through origin is given by $bx^2 - 2hxy + ay^2 = 0$



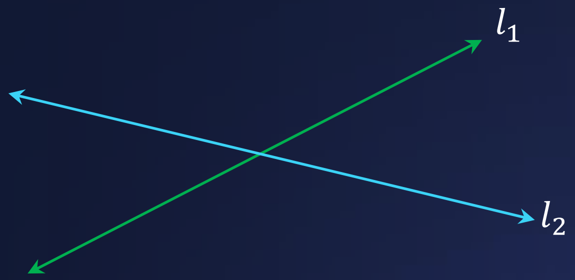
$$l_1 l_2 : ax^2 + 2hxy + by^2 = 0$$

$$l_3 l_4 : bx^2 - 2hxy + ay^2 = 0$$

Return to Top

Key Takeaways

The two lines having joint equation as $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersects at the point $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$, $(h^2 \neq ab)$.



$l_1 l_2 : ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Return to Top

For the pair of lines represented by

$3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$, find:

(i) Point of intersection

(ii) The equation of the lines

(i) Comparing the given equation $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

We get,

$a = -3, 2h = -8, b = 3, 2g = -29, 2f = 3, c = -18$

Point of Intersection: $\left(\dfrac{hf - bg}{ab - h^2}, \dfrac{gh - af}{ab - h^2}\right), (h^2 \neq ab)$

$$= \left(\frac{(-4)\left(\frac{3}{2}\right) - (3)\left(-\frac{29}{2}\right)}{3(-3) - (-4)^2}, \frac{\left(-\frac{29}{2}\right)(-4) - (-3)\left(\frac{3}{2}\right)}{3(-3) - (-4)^2}\right)$$

$$= \left(\frac{-6 + \frac{87}{2}}{-25}, \frac{58 + \left(\frac{9}{2}\right)}{-25}\right) = \left(-\frac{3}{2}, -\frac{5}{2}\right)$$

Return to Top

For the pair of lines represented by

$3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$, find:

(i) Point of intersection

(ii) The equation of the lines

(ii) $3y^2 - 8xy - 3x^2 = (3y + x)(y - 3x)$

Hence, let $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 \equiv (3y + x + p)(y - 3x + q)$

Equating the coefficients of x & y, we get

$-3p + q = -29$ and $p + 3q = 3$

$\Rightarrow p = 9$ and $q = -2$

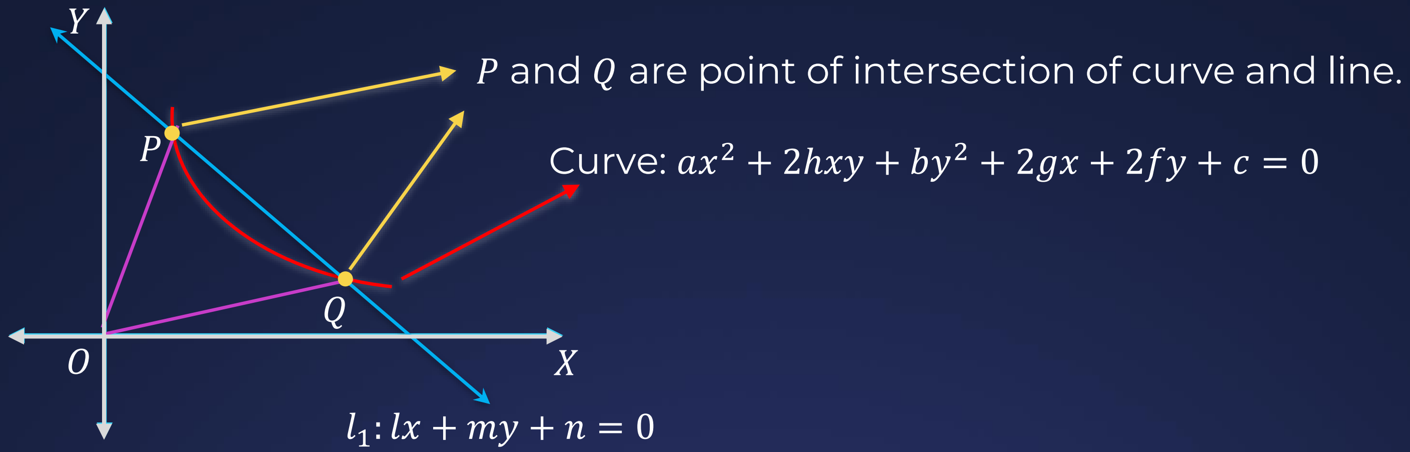
Thus, the equation of the represented lines are $3y + x + 9 = 0$ and $y - 3x - 2 = 0$

Return to Top

Key Takeaways

Homogenization:

It gives the joint equation of Pair of lines joining the Origin and the Points of Intersection of a Line and a Curve. .



Return to Top

Key Takeaways

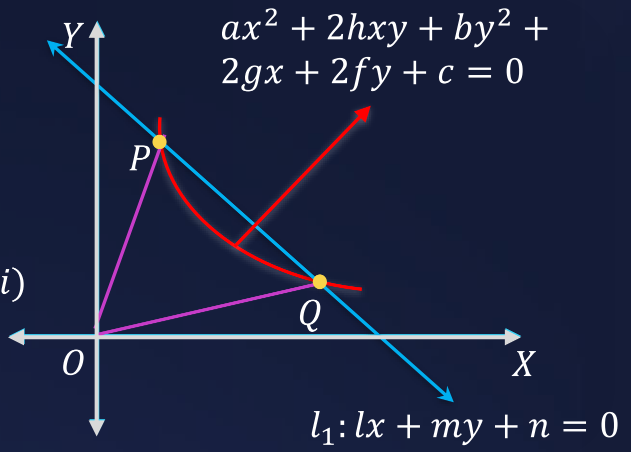
Homogeneous Equation:

Homogenizing equation of curve by using $l_1 = 0$

$$l_1 : lx + my + n = 0 \Rightarrow \frac{lx+my}{-n} = 1 \cdots (i)$$

$$ax^2 + 2hxy + by^2 + 2gx \cdot 1 + 2fy \cdot 1 + c \cdot (1)^2 = 0 \cdots (ii)$$

Substituting (i) in (ii):



$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

This is the joint equation of line OP and OQ.

Return to Top

?

The angle between the lines joining the origin to the points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is:

A $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

B $\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

C $\tan^{-1}\left(\frac{2}{3}\right)$

D $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Return to Top

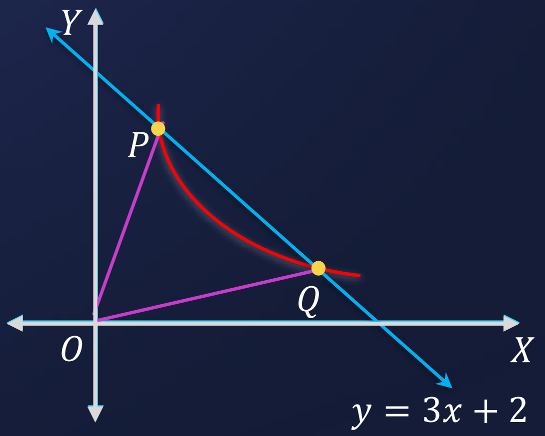
The angle between the lines joining the origin to the points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is:

Given line is $y = 3x + 2$ and curve

$x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$

$\Rightarrow \frac{y-3x}{2} = 1 \quad \cdots (i)$

$x^2 + 2xy + 3y^2 + 4x \cdot 1 + 8y \cdot 1 - 11 \cdot (1)^2 = 0 \quad \cdots (ii)$



Substituting (i) in (ii):

$$x^2 + 2xy + 3y^2 + 4x\left(\frac{y-3x}{2}\right) + 8y\left(\frac{y-3x}{2}\right) - 11\left(\frac{y-3x}{2}\right)^2 = 0$$

$$\Rightarrow 4x^2 + 8xy + 12y^2 + 8xy - 24x^2 + 16y^2 - 48xy - 11y^2 - 99x^2 + 66xy = 0$$

$$\Rightarrow -119x^2 + 17y^2 + 34xy = 0$$

$$\Rightarrow 7x^2 - 2xy - y^2 = 0 \Leftrightarrow ax^2 + 2hxy + by^2 = 0$$

Return to Top

The angle between the lines joining the origin to the points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is:

On comparing : $a = 7, h = -1$ and $b = -1$

Let the required angle be θ.

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{1+7}}{7-1}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Return to Top

The angle between the lines joining the origin to the points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is:

A $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

B $\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

C $\tan^{-1}\left(\frac{2}{3}\right)$

D $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Return to Top

Thank You