

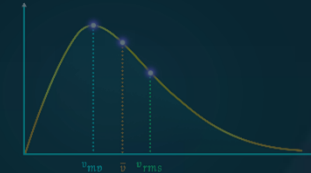
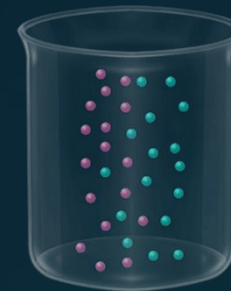
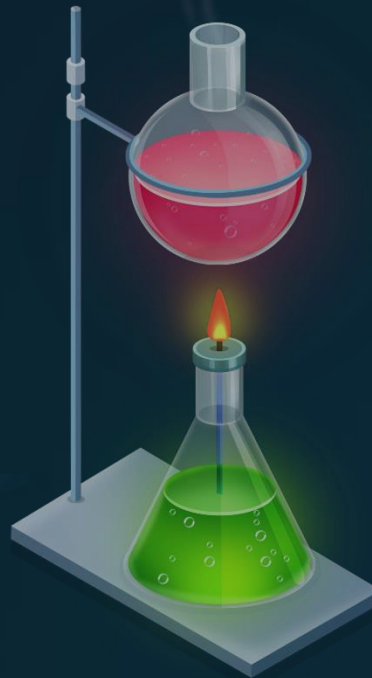
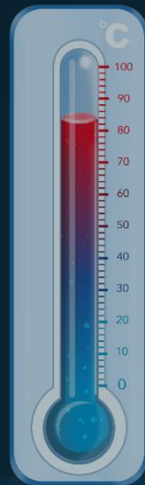
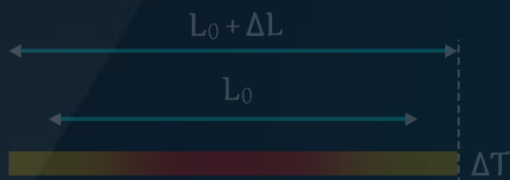
Welcome to



# Aakash

## + BYJU'S NOTES

### Thermal properties of matter





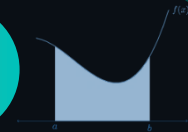
# Heat & Thermodynamics

Heat



Energy in **transit** by virtue of **temperature** difference

Thermodynamics



Heat becomes a **cause of motion** and does mechanical **work**



# Temperature

Temperature may be defined as the **degree of hotness or coldness** of a body



# Thermometer

A device used to **measure the temperature** of an object.



Bulb  
(Filled with  
Thermometric fluid )

Glass Body



# Temperature Scales



Lord Kelvin (William Thomson)

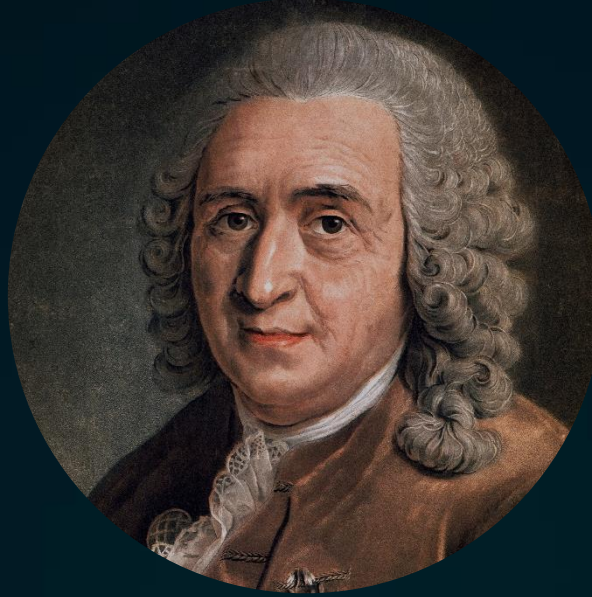


Kelvin Scale

Boiling point of water: 373  $K$

Freezing point of water: 273  $K$

Carolus Linnaeus

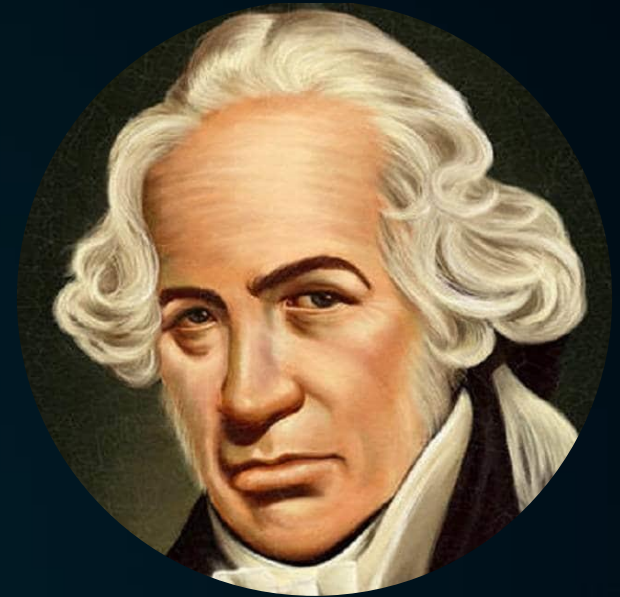


Celsius Scale

Boiling point of water: 100  $^{\circ}C$

Freezing point of water: 0  $^{\circ}C$

Daniel Gabriel Fahrenheit



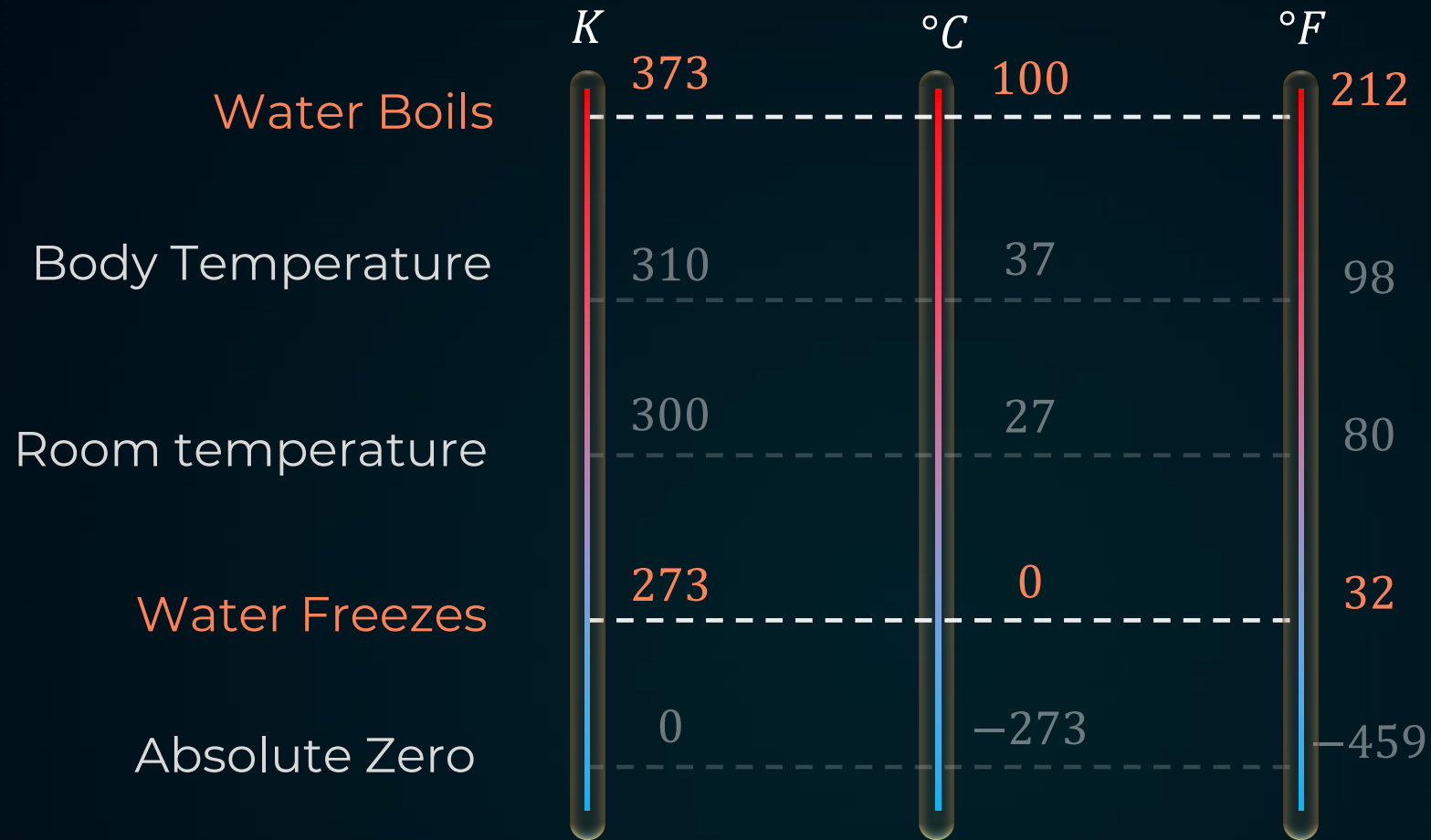
Fahrenheit Scale

Boiling point of water: 212  $^{\circ}F$

Freezing point of water: 32  $^{\circ}F$



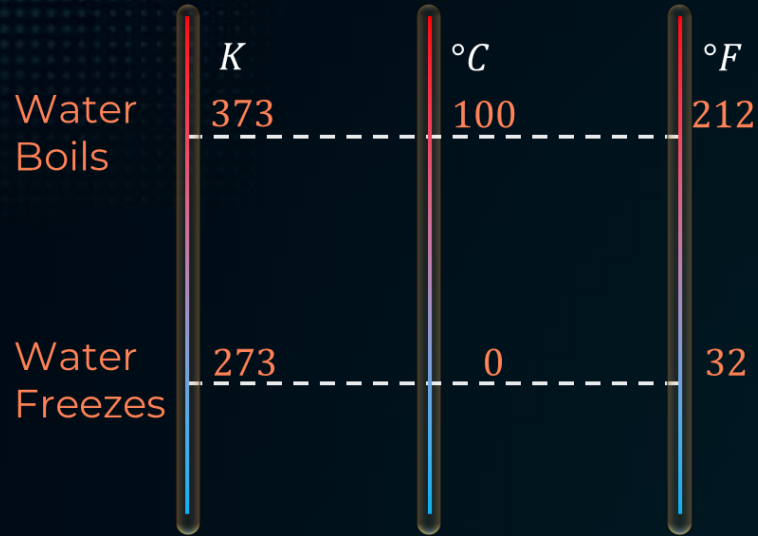
## Comparing Scales



Why **only two** values are important?



# Comparing Scales



For any scale,

Reading on scale – Lower fixed point

Upper fixed point – Lower fixed point

= Constant

$$\frac{K - 273}{100} = \frac{C}{100} = \frac{F - 32}{180} = \text{Constant}$$

For relating  $F$  &  $C$ ,

$$\frac{F - 32}{180} = \frac{C}{100}$$

$$F = \frac{9C}{5} + 32 \quad \text{or} \quad C = \frac{5}{9}(F - 32)$$

For relating  $K$  &  $C$ ,

$$\frac{K - 273}{100} = \frac{C}{100}$$

$$K = C + 273$$

For relating  $F$  &  $K$ ,

$$K = \frac{5}{9}(F - 32) + 273$$



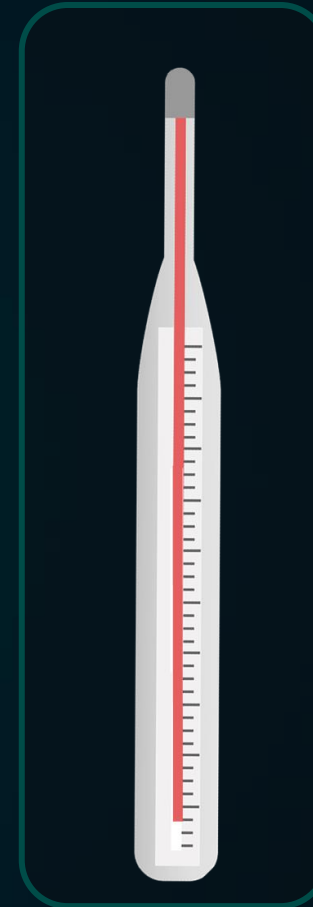
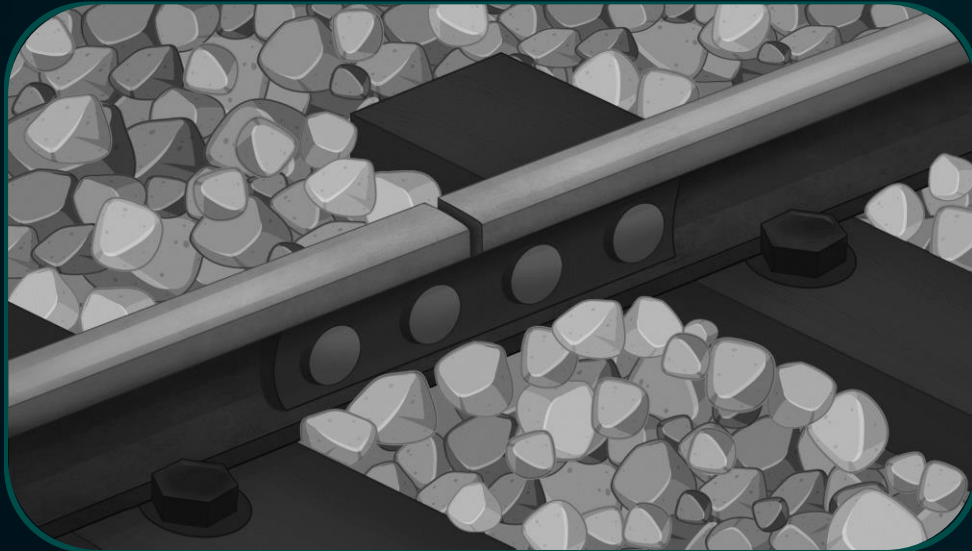
# Thermal Expansion



The tendency of matter to **change** in shape, volume, area, and **configuration** in response to a change in **temperature**.

## Real-Life Examples

- Expansion of Mercury
- Railway buckling





# Thermal Expansion

Linear

Areal

Volumetric



## Linear Expansion



The change in **one-dimension** (length) measurement of an object due to thermal expansion.

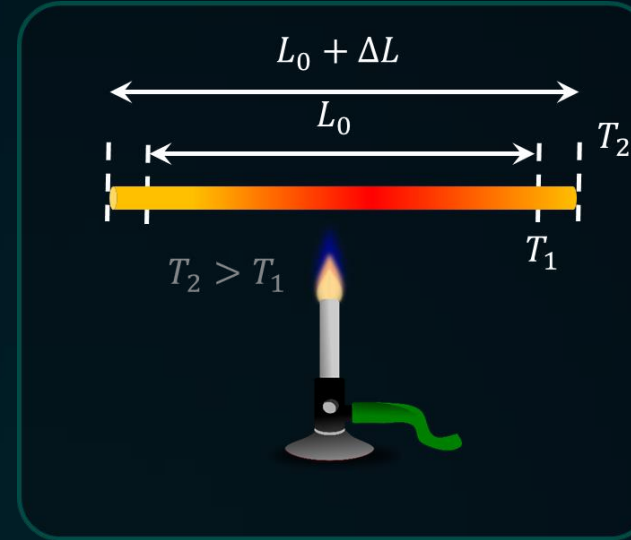
$$\Delta L \propto \Delta T$$

$$\Delta L \propto L_0$$

$$\Delta L \propto L_0 \Delta T$$

$$\Delta L = \alpha L_0 \Delta T \quad (\alpha - \text{coefficient of linear expansion})$$

$$L = L_0(1 + \alpha \Delta T)$$



## Coefficient of linear expansion

The ratio of increase in length to original length for **1° rise in temperature** is defined as the coefficient of linear expansion.

$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$

Unit of  $\alpha$  is  $^{\circ}\text{C}^{-1}$  or  $^{\circ}\text{K}^{-1}$

?

What is the percentage change in length of  $1\text{ m}$  iron rod, if its temperature changes by  $100\text{ }^{\circ}\text{C}$ ? ( $\alpha_{\text{Iron}} = 2 \times 10^{-5}\text{ }^{\circ}\text{C}^{-1}$ )

Given:  $L_0 = 1\text{ m}$ ,  $\Delta T = 100\text{ }^{\circ}\text{C}$ ,  $\alpha_{\text{Iron}} = 2 \times 10^{-5}\text{ }^{\circ}\text{C}^{-1}$

To find: Percentage change in length

Solution: Percentage change in length is given by

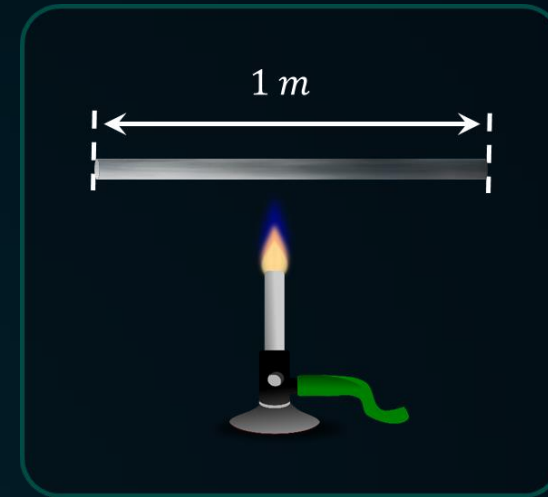
$$= \frac{\Delta L}{L_0} 100\%$$

$$= \frac{\alpha L_0 \Delta T}{L_0} 100\%$$

$$= 2 \times 10^{-5} \times 10^2 \times 100$$

$$= 2 \times 10^{-1}$$

The percentage change in  $L_0$  is  $0.2\%$



a 0.2 %

b 0.1 %

c 0.3 %

d 0.4 %

?

An isosceles triangle is formed with a thin rod of length  $l_1$  and coefficient of linear expansion  $\alpha_1$  as the base and two thin rods each of length  $l_2$  and coefficient of linear expansion  $\alpha_2$  as the two sides. If the distance between the apex and the midpoint of the base remains unchanged as the temperature is varied, show that  $\frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$ .

Solution:

$$AD^2 = l^2 = l_2^2 - \frac{l_1^2}{4} \quad \text{and} \quad AD^2 = l^2 = l_2'^2 - \frac{l_1'^2}{4}$$

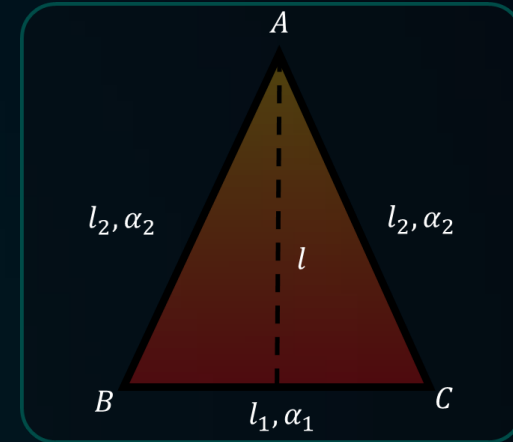
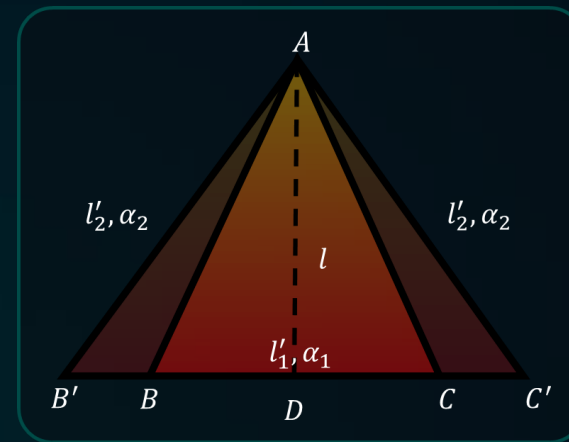
$$l_1' = l_1(1 + \alpha_1\Delta T) \quad \text{and} \quad l_2' = l_2(1 + \alpha_2\Delta T)$$

$$l_2^2 - \frac{l_1^2}{4} = l_2^2(1 + \alpha_2\Delta T)^2 - \frac{l_1^2}{4}(1 + \alpha_1\Delta T)^2$$

$$(1 + x)^n = 1 + nx \text{ if } |x| \ll 1$$

$$l_2^2 - \frac{l_1^2}{4} = l_2^2(1 + 2\alpha_2\Delta T) - \frac{l_1^2}{4}(1 + 2\alpha_1\Delta T)$$

$$\frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$





# Areal Expansion



The **expansion** in the **area** of an object due to the increase in **temperature**.

$$\Delta A = \beta A_0 \Delta T \quad (\beta - \text{coefficient of areal expansion})$$

Final Area,

$$A = A_0 + \Delta A$$

$$A = A_0 + A_0 \beta \Delta T$$

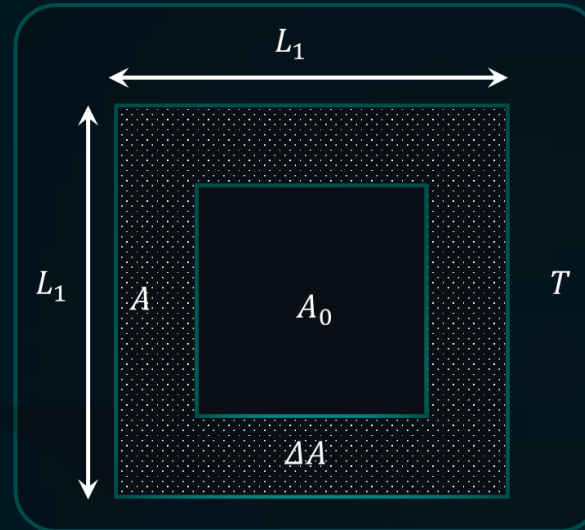
$$A = A_0(1 + \beta \Delta T)$$

$$A = L_0^2(1 + \alpha \Delta T)^2$$

$$(1 + x)^n = 1 + nx \text{ if } nx \ll 1$$

$$A = A_0(1 + 2\alpha \Delta T)$$

$$\beta = 2\alpha$$



Change in Area

$$\Delta A = A_0 \beta \Delta T$$

Final Area of plate

$$A = A_0(1 + \beta \Delta T)$$

Coefficient of areal expansion

$$\beta = \frac{\Delta A}{(A_0 \Delta T)}$$

Relation b/w  $\alpha$  &  $\beta$

$$\beta = 2\alpha$$

?

A rectangular plate has a **circular cavity** as shown. If we **increase** its **temperature**, then

**Solution:**

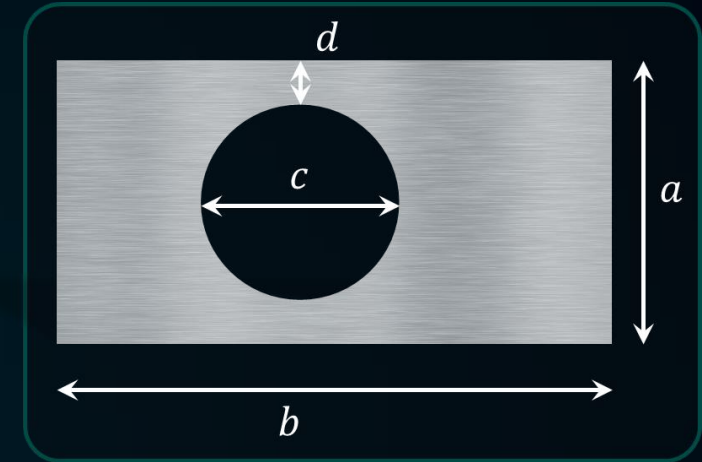
a, b, d increases because of expansion, c decreases because of expansion in all direction.



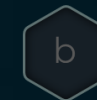
Distance between **any two** points will increase.  
So, all the shown length should increase.



**a, b, c and d will increase**



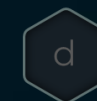
Only a and b increases



Only a, b and d increases



All a, b, c and d increases



Only a, b and c increases



# Volume Expansion



The increase in volume of a solid/liquid due to rise in temperature.

Change in volume

$$\Delta V = \gamma V_0 \Delta T$$

Final volume

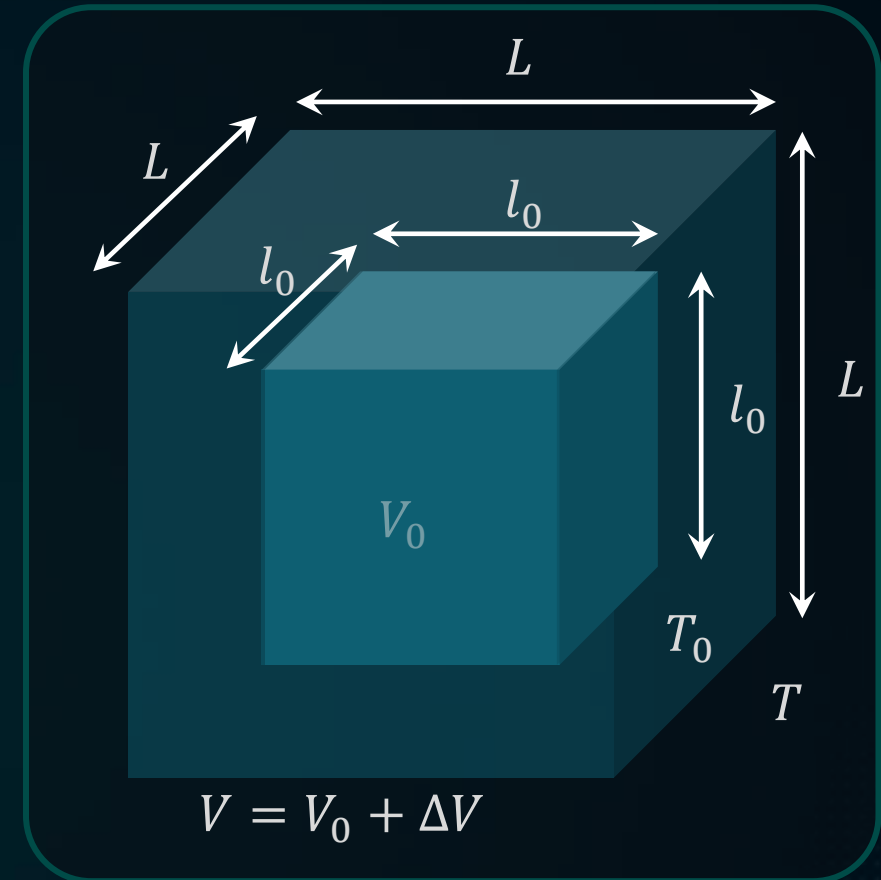
$$V = V_0(1 + \gamma \Delta T)$$

Coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V_0 \Delta T}$$

Relation b/w  $\alpha$  &  $\gamma$

$$\gamma = 3\alpha$$



This type of expansion is also known as Cubic expansion.



# Thermal Expansion



## Linear Expansion

$$\Delta L = L_0 \alpha \Delta T$$

$$L = L_0 (1 + \alpha \Delta T)$$

$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$

$$\alpha$$

## Areal Expansion

$$\Delta A = A_0 \beta \Delta T$$

$$A = A_0 (1 + \beta \Delta T)$$

$$\beta = \frac{\Delta A}{A_0 \Delta T}$$

$$\beta = 2\alpha$$

## Volume Expansion

$$\Delta V = \gamma V_0 \Delta T$$

$$V = V_0 (1 + \gamma \Delta T)$$

$$\gamma = \frac{\Delta V}{V_0 \Delta T}$$

$$\gamma = 3\alpha$$



## Negative Thermal coefficient

Negative thermal coefficient of expansion.

Material will contract on an increase in temperature i. e supply of heat

$$L = L_0 (1 + \alpha \Delta T)$$

$$A = A_0 (1 + \beta \Delta T)$$

$$V = V_0 (1 + \gamma \Delta T)$$

If  $\alpha, \beta, \gamma$  are negative

$$L < L_0$$

$$A < A_0$$

$$V < V_0$$



## Directional Properties

**Isotropic materials:** The materials whose physical properties are **independent** of the **orientation** of the system.

$$\beta = \alpha + \alpha = 2\alpha$$

$$\gamma = \alpha + \alpha + \alpha = 3\alpha$$

**Anisotropic materials:** The materials whose physical properties are **dependent** on the **orientation** of the system.

$$\beta = \alpha_x + \alpha_y$$

$$\gamma = \alpha_x + \alpha_y + \alpha_z$$



# Apparent Expansion of Liquid in a Container



Before Heating

Volume of container ( $V_c$ ) = Volume of liquid ( $V_L$ )

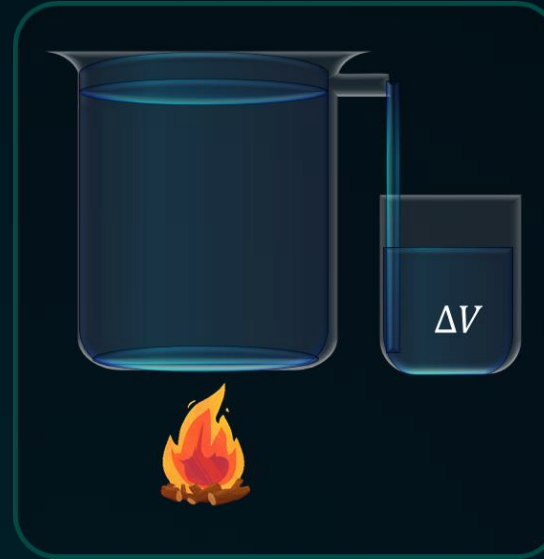
After Heating:

$$V'_L = V_0(1 + \gamma_L \Delta T), \quad V'_c = V_0(1 + \gamma_C \Delta T)$$

Overflow volume of liquid relative to container:

$$\Delta V = V_0(\gamma_L - \gamma_C)\Delta T$$

$$\gamma_{\text{apparent}} = \gamma_L - \gamma_C$$



$$\gamma_{\text{apparent}} = \gamma_L - \gamma_C$$

$$\gamma_L > \gamma_C$$

Liquid level appears to have raised

$$\gamma_L < \gamma_C$$

Liquid level appears to have fallen

?<sub>T</sub>

The volume of a glass vessel is  $1000\text{cm}^3$  at  $20^\circ\text{C}$ . What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature?

$$\gamma_{\text{mercury}} = 1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1} \text{ and } \gamma_{\text{glass}} = 9.0 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$$

Given:  $(V_0)_{\text{glass}} = 1000\text{cm}^3$  at  $20^\circ\text{C}$ ,  $\gamma_{\text{mercury}} = 1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ ,  $\gamma_{\text{glass}} = 9.0 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$

To find:  $(V_0)_{\text{Hg}}$

Solution: • The volume of mercury to be poured into the vessel should be such that the change in the volume of mercury and that of the glass vessel with temperature are equal.

$$(\Delta V)_{\text{Hg}} = (\Delta V)_{\text{glass}}$$

$$\Rightarrow (V_0)_{\text{Hg}} \gamma_{\text{Hg}} \Delta T = (V_0)_{\text{glass}} \gamma_{\text{glass}} \Delta T$$

$$\Rightarrow (V_0)_{\text{Hg}} = \frac{(V_0)_{\text{glass}} \gamma_{\text{glass}}}{\gamma_{\text{Hg}}} = 50 \text{ cm}^3$$

$$(\gamma_{\text{mercury}} = 1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1} \text{ and } \gamma_{\text{glass}} = 9.0 \times 10^{-6} \text{ }^\circ\text{C}^{-1})$$



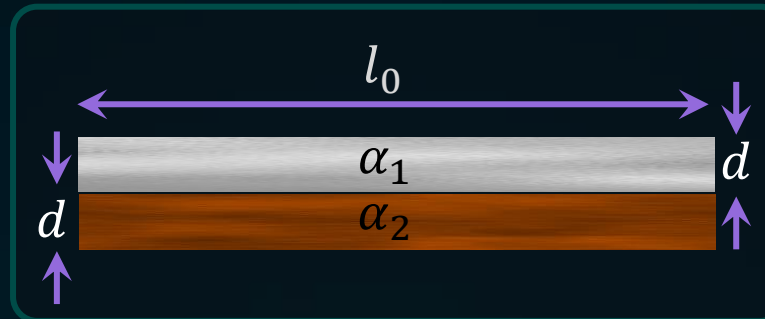


## Bimetallic strip

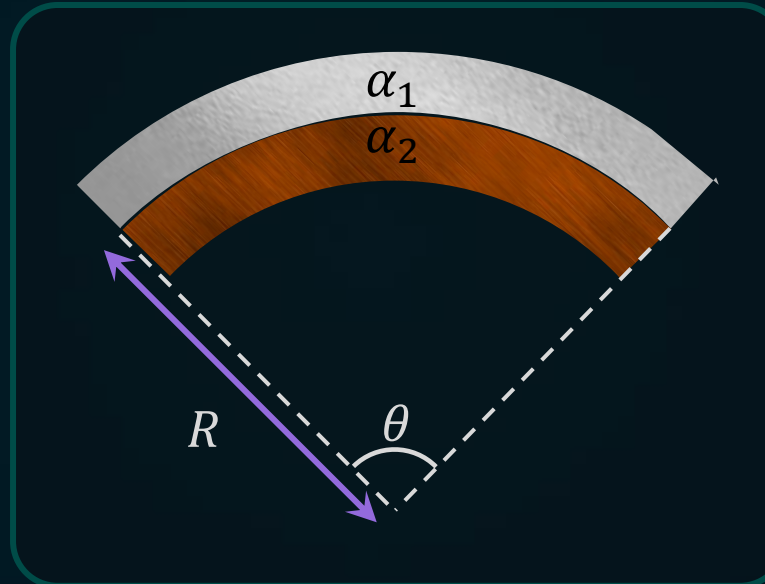


- Two strips of different materials welded together.

At temperature  $T_0$ :



At temperature  $T$ :



$$R = \frac{d}{(\alpha_1 - \alpha_2)\Delta T}$$

$$T > T_0$$

$$l_1 = l_0(1 + \alpha_1\Delta T)$$

$$l_2 = l_0(1 + \alpha_2\Delta T)$$

$$\text{If } \alpha_1 > \alpha_2$$

$$\Rightarrow l_1 > l_2$$



## Density and Temperature



- Density **before heating** (at  $T_0$ ):  $\rho_0 = \frac{m}{V_0}$

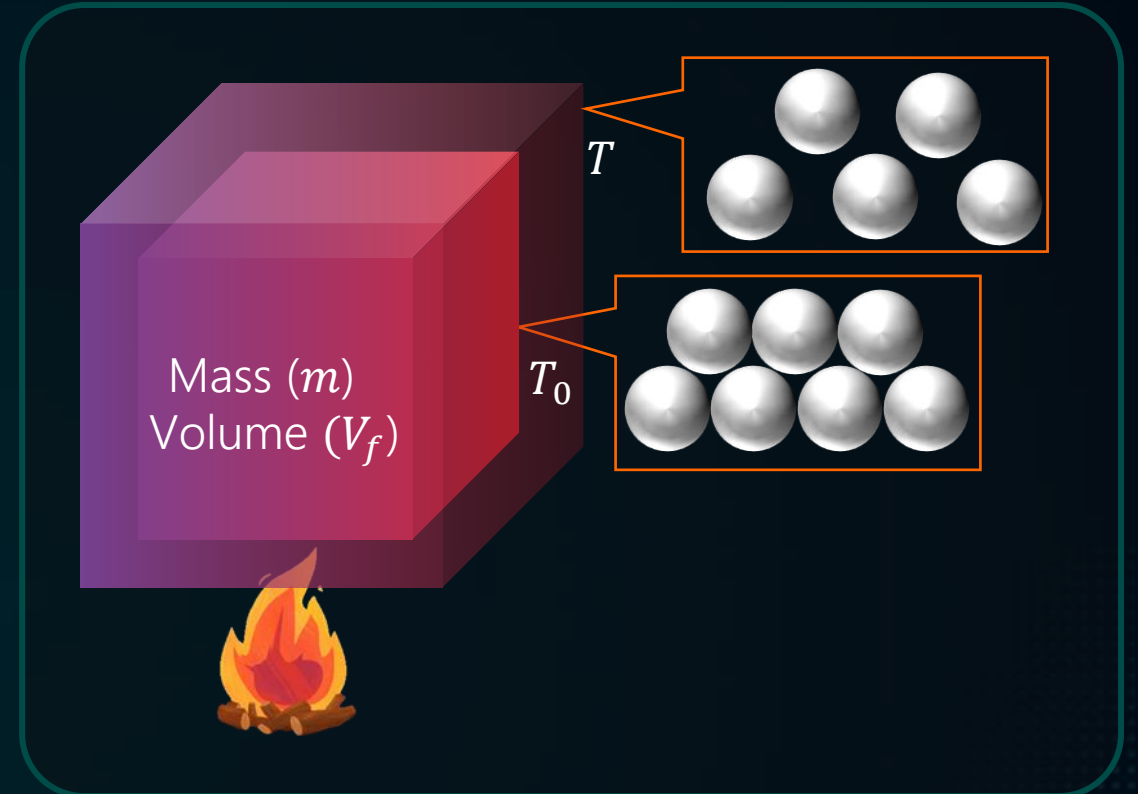
- Density **after heating**:

$$(\rho_f) = \frac{m}{V_f} = \frac{m}{V_0(1 + \gamma\Delta T)}$$

$$\rho_f = \frac{\rho_0}{(1 + \gamma\Delta T)}$$

- For solids,  $\gamma \ll 1$

$$\rho_f = \rho_0(1 - \gamma\Delta T)$$



?

The densities of wood and benzene at  $0^\circ\text{C}$  are  $880\text{ kg/m}^3$  and  $900\text{ kg/m}^3$  respectively.  $\gamma_w = 1.2 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$  and  $\gamma_b = 1.5 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ . At what temperature will a piece of wood just sink in benzene?

Given:

Parameter	Wood	Benzene
$\rho \text{ at } 0^\circ\text{C} \text{ (kg/m}^3\text{)}$	880	900
$\gamma \text{ (}\times 10^{-3} \text{ }^\circ\text{C}^{-1}\text{)}$	1.2	1.5

To find:  $T$  at which the wood just sink

Solution:

The wood just start sinking when  $\rho_w = \rho_b$

$$\frac{880}{(1 + \gamma_w \Delta T)} = \frac{900}{(1 + \gamma_b \Delta T)}$$

$$\Rightarrow \Delta T = \frac{2}{88\gamma_b - 90\gamma_w} = 83.3^\circ\text{C}$$

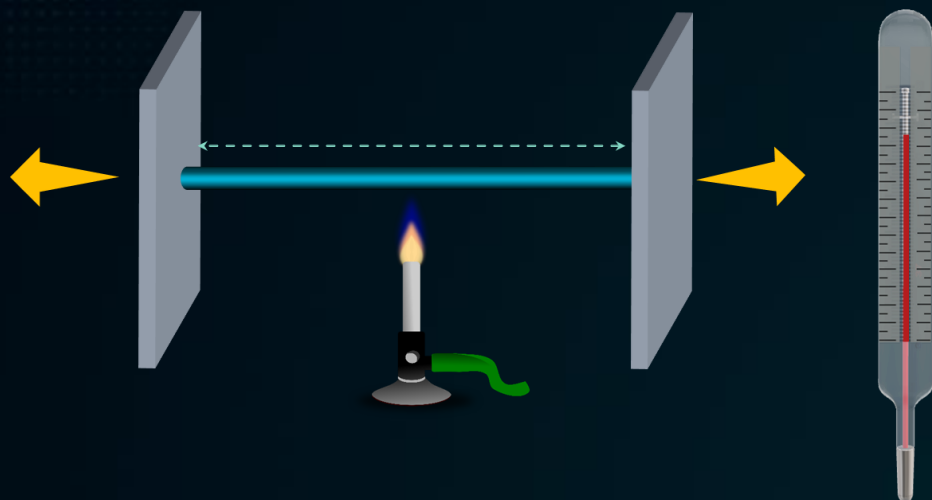
$$\Delta T = T - 0 = 83.3^\circ\text{C}$$

$$T = 83.3^\circ\text{C}$$

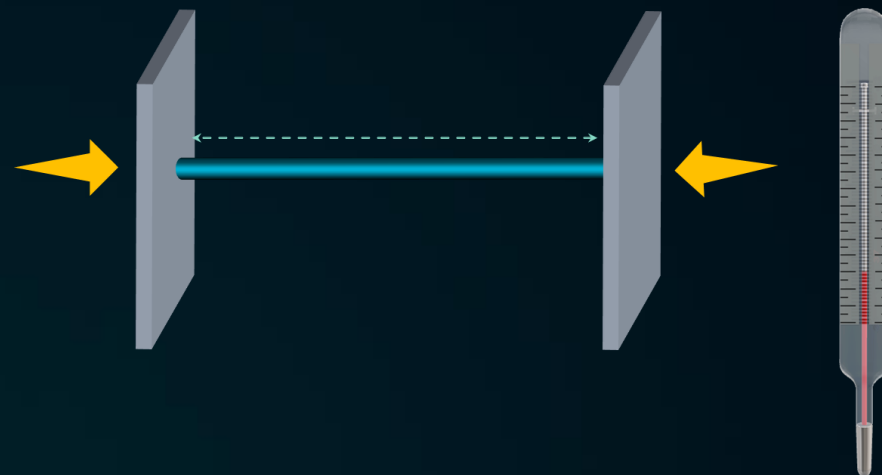




# Thermal Stress

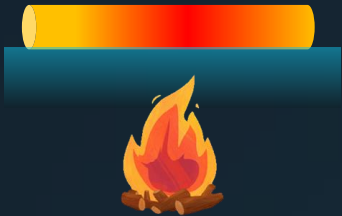




Heating



Cooling

$$\text{Thermal Strain} = \frac{\text{Prevented Change in Dimension}}{\text{Original Dimension}}$$

Case	 <p>No Free expansion prevented</p>	 <p>Entire free expansion prevented</p>	 <p>Partial expansion prevented</p>
Strain ( $\epsilon$ )	0	$\frac{L - L_0}{L} = \frac{\alpha \Delta T}{1 + \alpha \Delta T} \approx \alpha \Delta T$	$\frac{L' - L_0}{L}$
Stress ( $\sigma$ )	0	$\frac{F}{A} = Y \left( \frac{L - L_0}{L} \right) \approx Y \alpha \Delta T$	$\frac{F}{A} = Y \left( \frac{L' - L_0}{L} \right)$
Reaction Force ( $F$ )	0	$Y \alpha \Delta T A$	$F = Y \left( \frac{L' - L_0}{L} \right) A$

?

A steel rod is clamped at its two ends and rests on a fixed horizontal surface. The rod is in natural length at  $20^{\circ}\text{C}$ . Find the longitudinal strain developed in the rod if the temperature rises to  $50^{\circ}\text{C}$ .

$$(\alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1})$$

Given:  $T_0 = 20^{\circ}\text{C}, T = 50^{\circ}\text{C}, \alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$

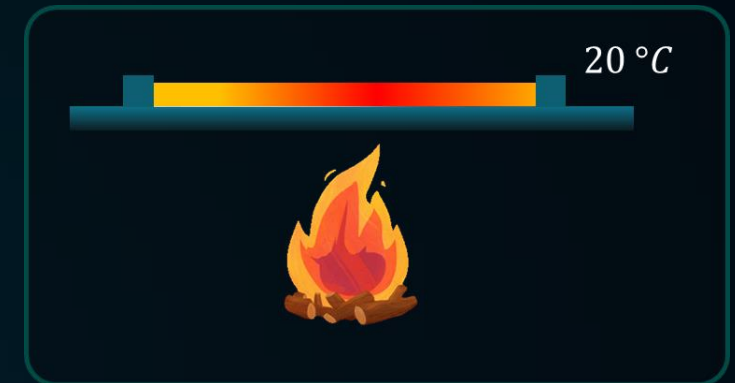
To find:  $\epsilon$

Solution:

Thermal Strain,  $|\epsilon| \approx |\alpha \Delta T|$

$$|\epsilon| \approx 1.2 \times 10^{-5} \times (50 - 20)$$

$$|\epsilon| \approx 3.6 \times 10^{-4}$$



a  $3.6 \times 10^{-3}$

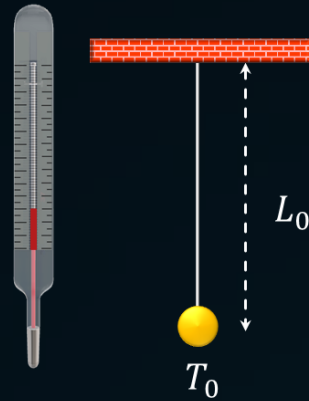
b  $3.6 \times 10^{-4}$

c  $1.8 \times 10^{-4}$

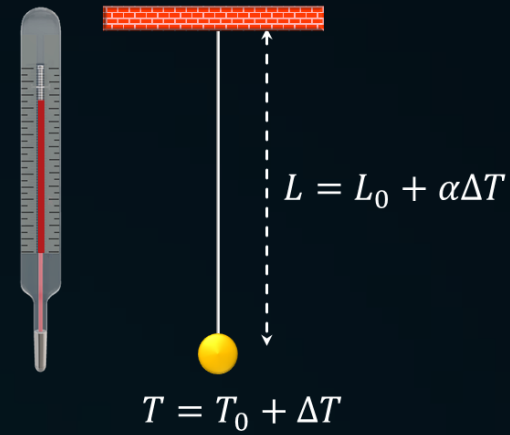
d  $1.8 \times 10^{-3}$



# Time Period of Simple Pendulum



$$t_0 = 2\pi \sqrt{\frac{L_0}{g}}$$



$$t = 2\pi \sqrt{\frac{L_0(1 + \alpha \Delta T)}{g}}$$

$$\frac{t}{t_0} = \sqrt{\frac{L}{L_0}} = \sqrt{\frac{L_0[1 + \alpha \Delta T]}{L_0}} \approx 1 + \frac{1}{2} \alpha \Delta T$$

- Change in time per unit time lapsed:  $\frac{t - t_0}{t_0} \approx \frac{1}{2} \alpha \Delta T$



## Gain and Loss in Time



$$\Delta t \approx \frac{1}{2} \alpha \Delta T t'$$

$$T < T_0$$

$$t < t_0$$

Clock becomes fast  
and gains time

$$T > T_0$$

$$t > t_0$$

Clock becomes slow  
and loses time

?

A pendulum clock consists of an iron rod connected to a small heavy bob. If it is designed to keep correct time at  $20^{\circ}\text{C}$ , how fast or slow will it go in 24 hours at  $40^{\circ}\text{C}$ ? ( $\alpha_{\text{iron}} = 1.2 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ )

Given:  $T_0 = 20^{\circ}\text{C}$ ,  $T = 40^{\circ}\text{C}$ ,  $\alpha_{\text{iron}} = 1.2 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ ,  $t' = 24 \text{ hr}$

To find: Gain or loss in time

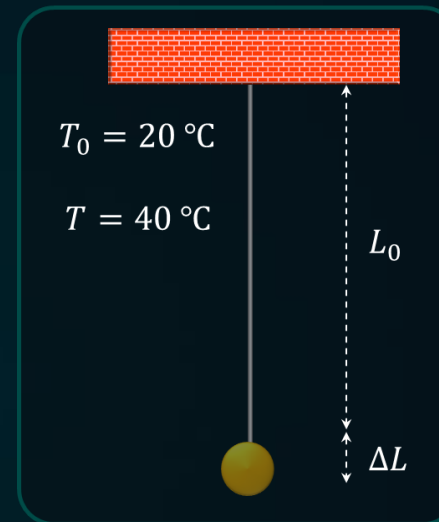
Solution:

Gain/Loss in time:

$$\Delta t \approx \frac{1}{2} \alpha \Delta T t'$$

$$\Delta t \approx \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 24 \times 60 \times 60$$

$$\Delta t \approx 1.04 \text{ s}$$

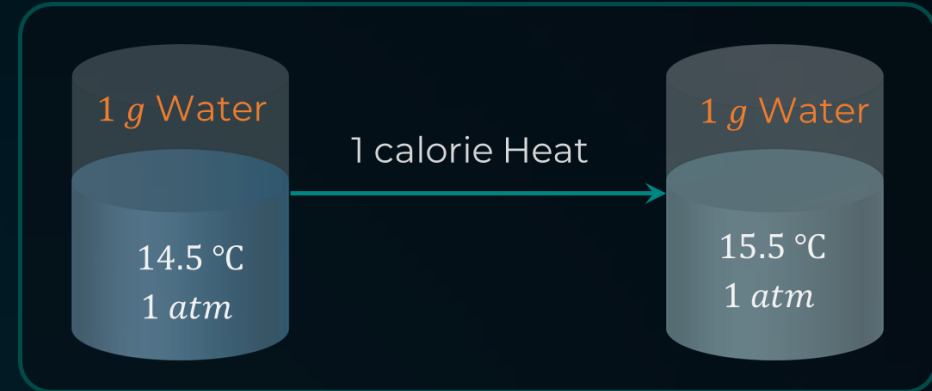




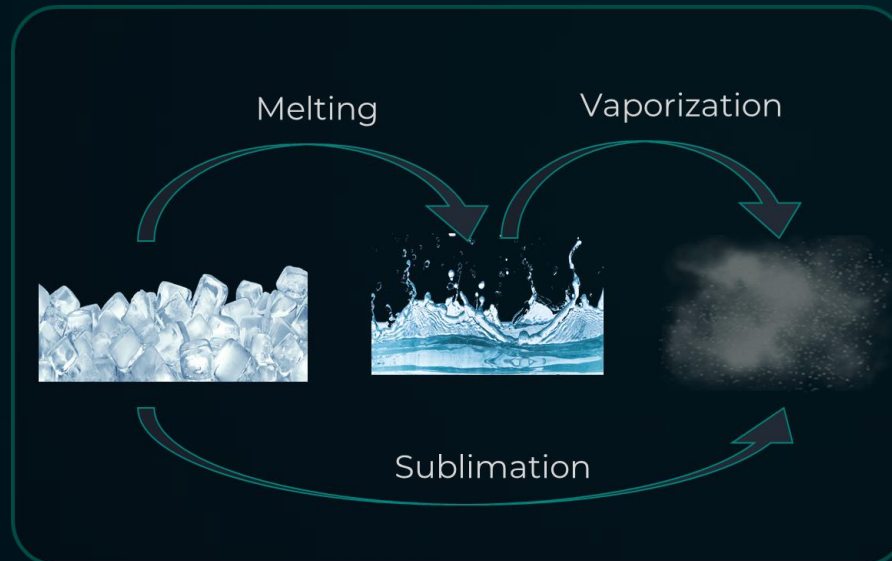
# Heat



- Energy transfer due to temperature difference
- Transfer from **high to low temperature** object
- Unit: *joule (J)*, *calorie (Cal)*
- $1 \text{ calorie} = 4.18 \text{ joule}$



## Results of Heating





## Mechanical Equivalent of Heat



- (Heat produced in system)  $\propto$  (Mechanical work done on it)
- If  $W$  produces same temperature change as  $H$ ,

$$W = JH$$

- $J$ : Mechanical equivalent of heat
- $J$  represents the amount of work required to raise temperature of 1 g of water by 1 °C
- Heat and work are equivalent

## Specific Heat

- The amount of heat ( $\Delta Q$ ) required by a unit mass of substance to raise its temperature by 1°C
- $\Delta Q \propto m$
- $\Delta Q \propto \Delta T$

$$Q = ms\Delta T$$

$s \equiv$  specific heat constant

$$Q = \int m s dT$$

- SI Unit:  $Jkg^{-1}K^{-1}$
- CGS Unit:  $calg^{-1}K^{-1}$
- For Adiabatic Process ( $\Delta Q = 0$ ),
- For Isothermal Process ( $\Delta T = 0$ ),

$$s = \frac{1}{m} \frac{\Delta Q}{\Delta T} = \infty$$

$$s = \frac{1}{m} \frac{\Delta Q}{\Delta T} = 0$$

?

Find the heat required to increase the temperature of **1 kg** water by **20 °C (in kcal)**

Given:  $m = 1 \text{ kg}, \Delta T = 20 \text{ }^{\circ}\text{C}, s_w = 1 \text{ cal g}^{-1} \text{ K}^{-1}$

To find:  $\Delta Q$

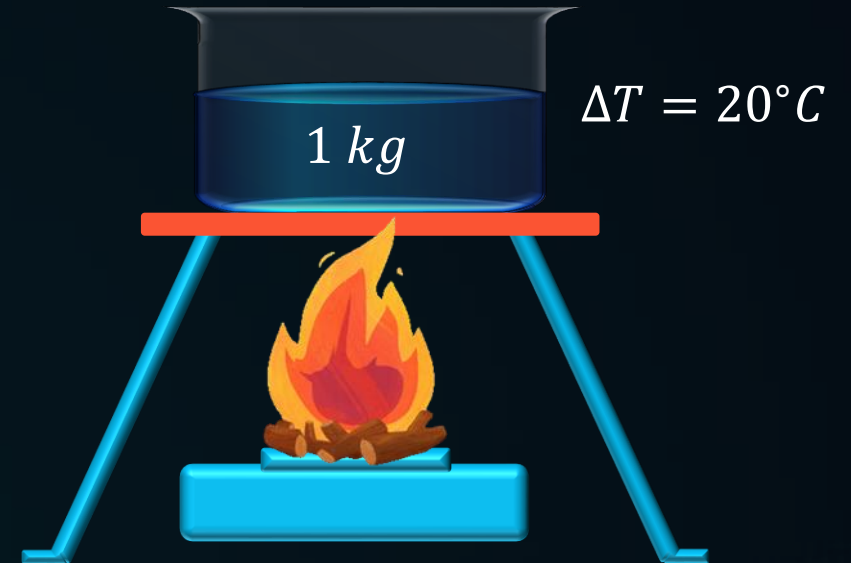
Solution:

Heat Required,

$$\Delta Q = ms\Delta T$$

$$\Delta Q = 1000 \text{ g} \times 1 \text{ cal g}^{-1} \text{ K}^{-1} \times 20 \text{ }^{\circ}\text{C}$$

$$\Delta Q = 20 \text{ kcal}$$





# Heat Capacity

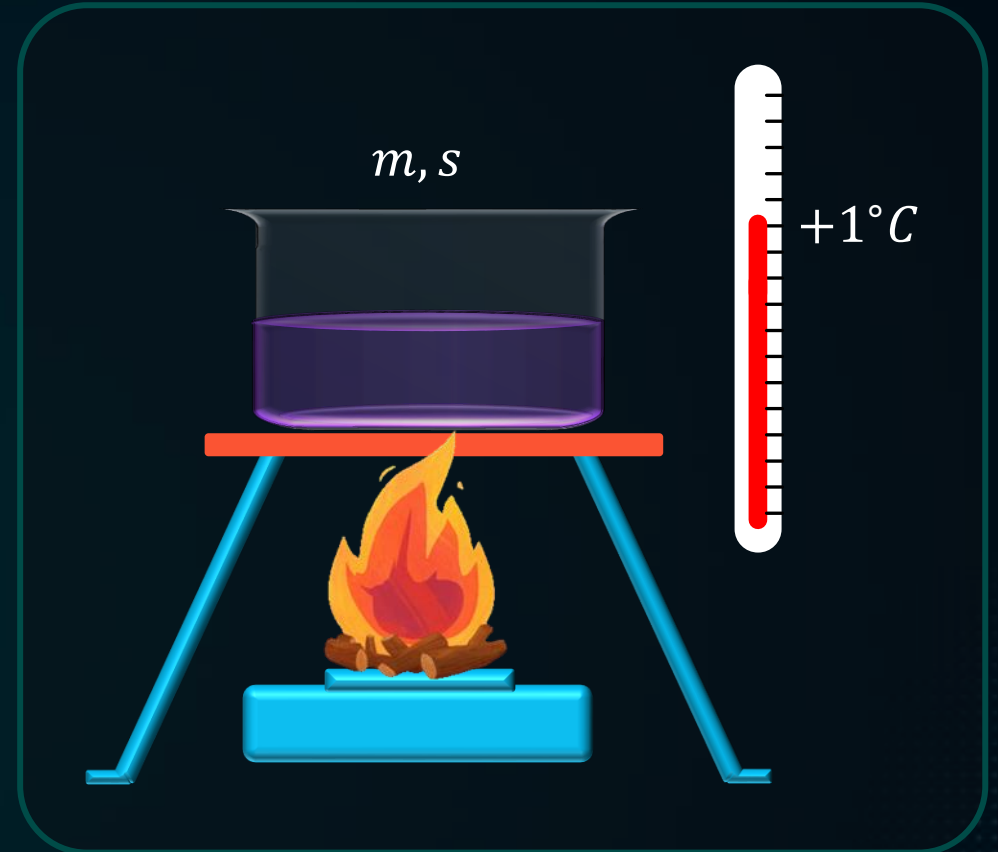


- The quantity of **heat** necessary to produce a **unit change in temperature** for the given mass of a material.

$$\text{Heat capacity } (C) = ms$$

S.I unit: *joule/kelvin* (J/K)

C.G.S unit: *cal/°C*





## Water Equivalent

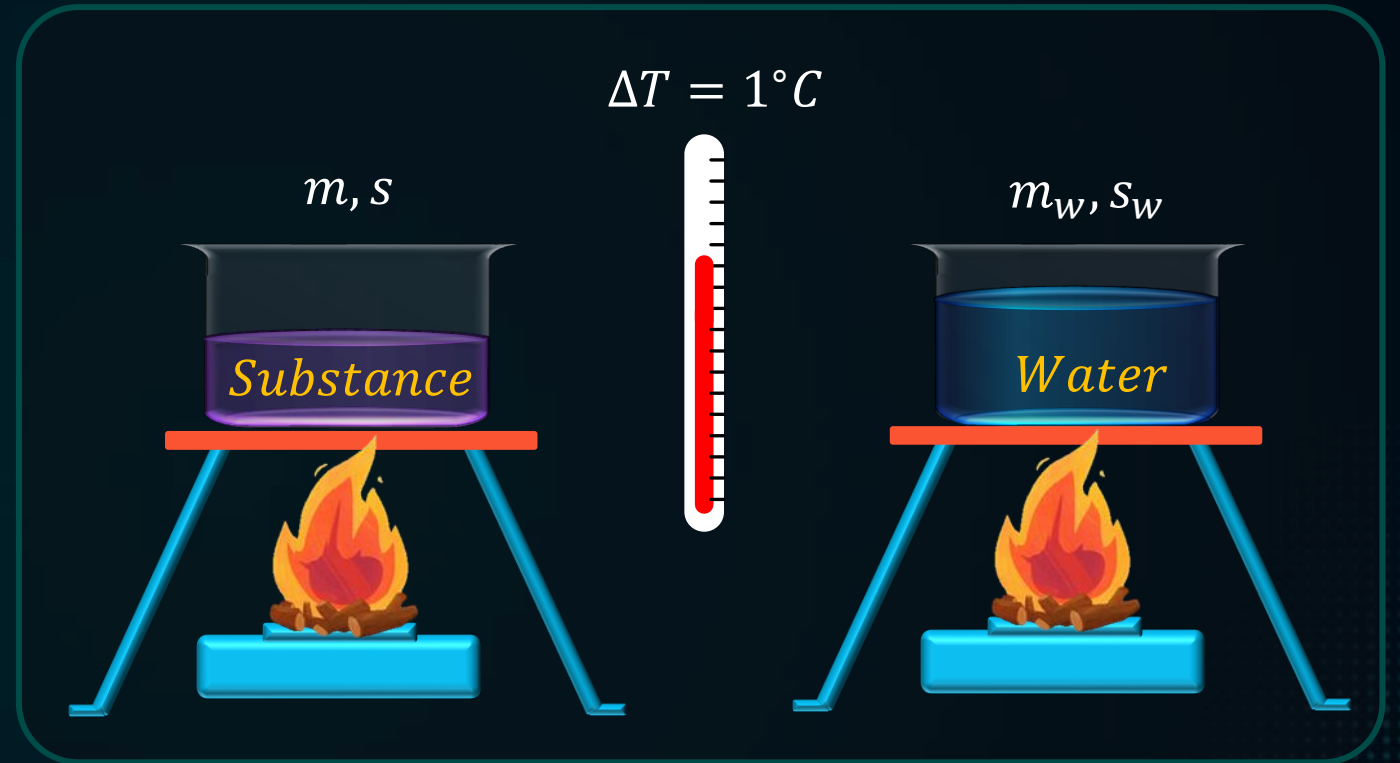


- Amount of water that is required to consume the same quantity of heat as the substance does for a unit rise in temperature

$$ms\Delta T = m_w s_w \Delta T$$

$$m_w = \frac{ms}{s_w}$$

- $m_w$  is the water equivalent of the substance

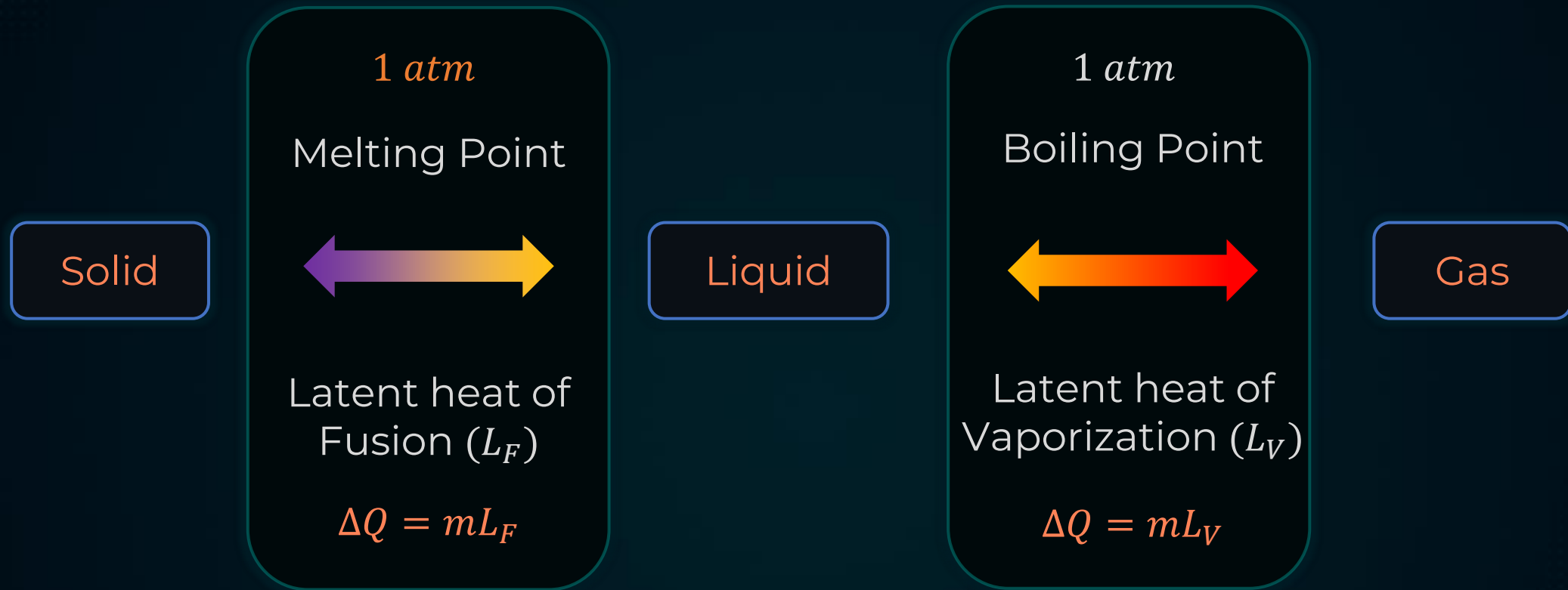




# Phase Change



- **Latent Heat:** Changes the phase of substance at constant temperature





# Phase Change: Water



Ice Cube



$1 \text{ atm}$   
 $0 \text{ }^{\circ}\text{C}$

$L_F = 80 \text{ kcal/kg}$

Water



$1 \text{ atm}$   
 $100 \text{ }^{\circ}\text{C}$

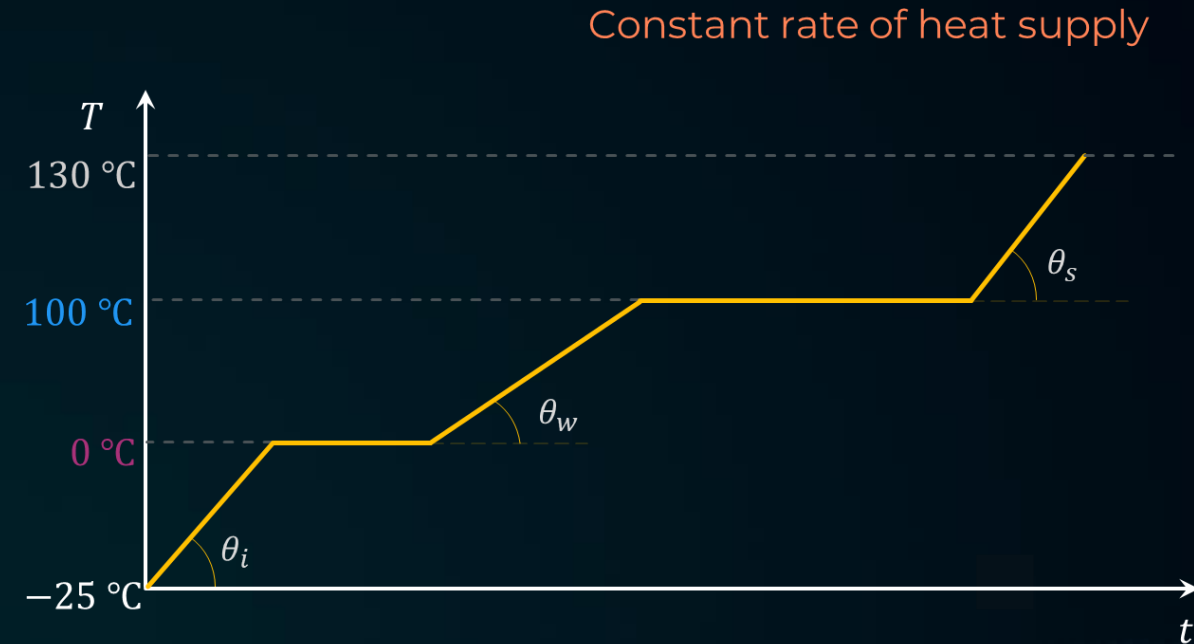
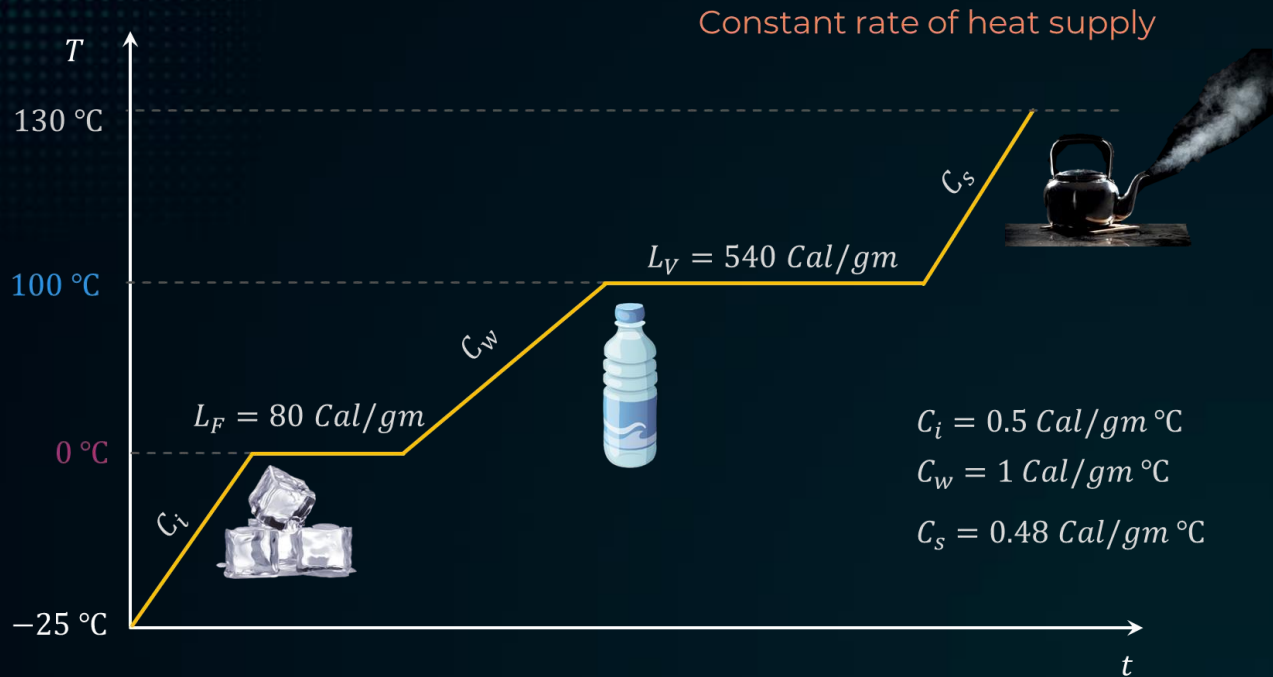
$L_V = 540 \text{ kcal/kg}$

Steam





# Temperature-Heat curve for Water



- Specific heat

$$C_w > C_i > C_s$$

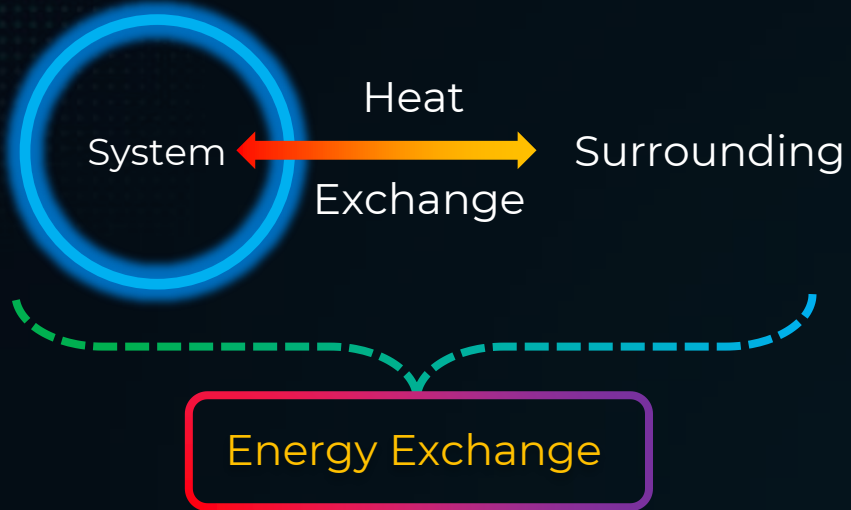
$$\theta_s > \theta_i > \theta_w$$

- Latent heat:

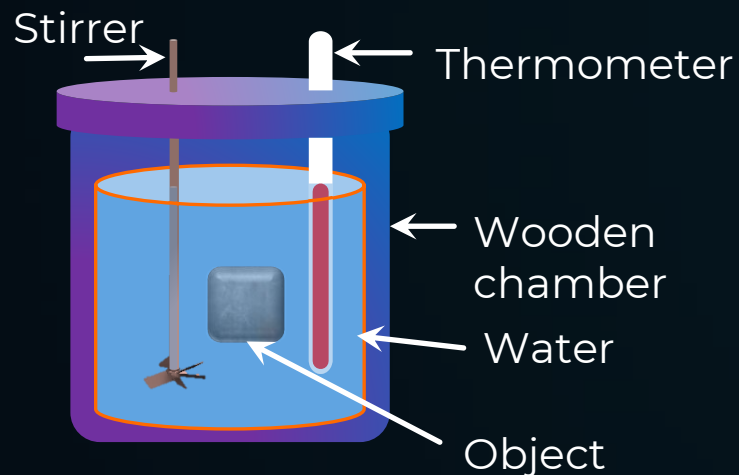
$$L_V > L_F$$



# Calorimetry



- The science associated with determining the **changes in energy** of a system by measuring the heat exchanged with the surroundings.



- For an insulated system,

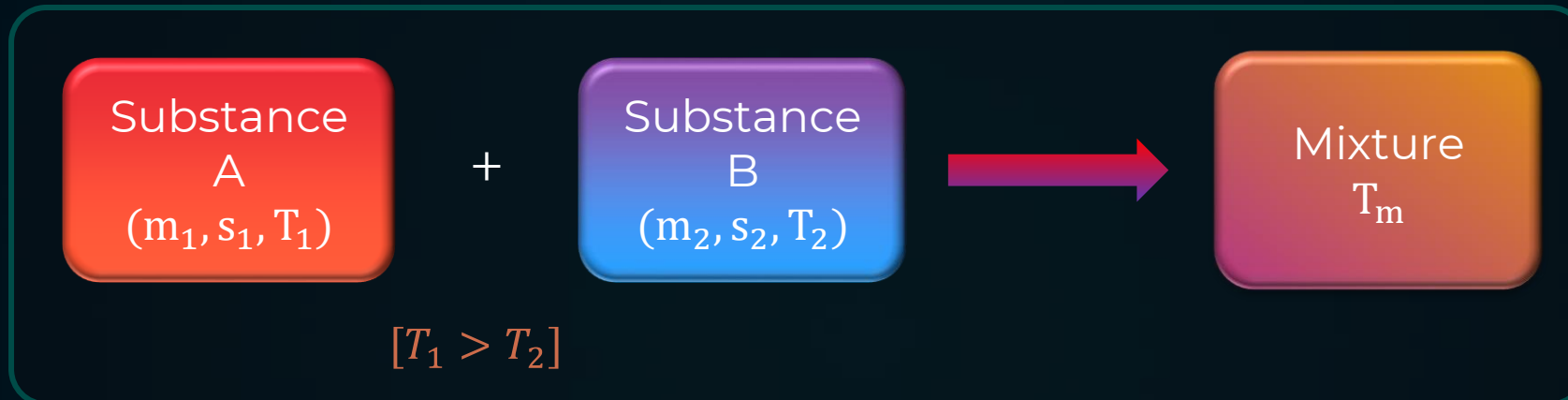
$$\text{Heat Lost by hotter substance} = \text{Heat Gain by cooler substance}$$



## Law of Mixture



- **Assumptions:** i) No phase change is involved.  
ii) there is no heat lost to the surroundings.
- The exchange of heat continues to take place **till** the **temperature** of substances become **equal**.



Heat Lost by hotter substance = Heat Gain by cooler substance

$$m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$

$$\Rightarrow T_1 > T_m > T_2$$



?

The temperature of three different liquids  $A$ ,  $B$  and  $C$  of equal masses are  $10^\circ\text{C}$ ,  $15^\circ\text{C}$  and  $20^\circ\text{C}$  respectively. The temperature when  $A$  and  $B$  are mixed is  $13^\circ\text{C}$  and when  $B$  and  $C$  are mixed is  $16^\circ\text{C}$ . What will be the temperature when  $A$  and  $C$  are mixed?

Solution:



$A$ ,  $B$  are mixed

$$mS_A\Delta T = mS_B\Delta T$$

$$mS_A(13 - 10) = mS_B(15 - 13)$$

$$3S_A = 2S_B \Rightarrow S_A = \frac{2}{3}S_B$$

$B$ ,  $C$  are mixed

$$mS_B\Delta T = mS_C\Delta T$$

$$mS_B(16 - 15) = mS_C(20 - 16)$$

$$S_B = 4S_C \Rightarrow S_C = \frac{1}{4}S_B$$

$A$ ,  $C$  are mixed

$$mS_A\Delta T = mS_C\Delta T$$

$$mS_A(T_m - 10) = mS_C(20 - T_m)$$

$$\frac{2}{3}S_B(T_m - 10) = \frac{S_B}{4}(20 - T_m)$$

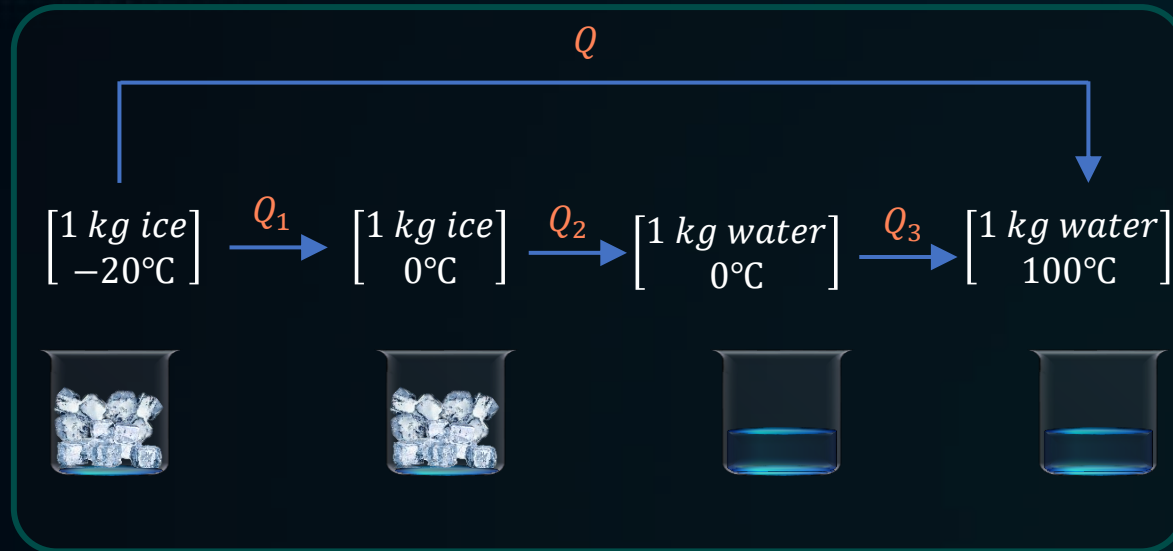
$$8(T_m - 10) = 3(20 - T_m)$$

$$\Rightarrow T_m = \frac{140}{11} \approx 12.72^\circ\text{C}$$

?<sub>T</sub>

1 kg ice at  $-20^{\circ}\text{C}$  is converted to 1 kg water at  $100^{\circ}\text{C}$ . Find the heat  $Q$  required to change the state of the substance?

Solution:



$$Q = Q_1 + Q_2 + Q_3$$

$$L_f = 80 \text{ kcal}$$

$$S_w = 1 \text{ cal/g}^{\circ}\text{C}$$

$$Q_1 = m_{ice} S_{ice} \Delta T = 1000 \times 0.5 \times [0 - (-20)]$$

$$Q_1 = 10 \text{ kcal}$$

$$Q_2 = m_{ice} L_f = 1000 \times 80$$

$$Q_2 = 80 \text{ kcal}$$

$$Q_3 = m_w S_w \Delta T = 1000 \times 1 \times (100 - 0)$$

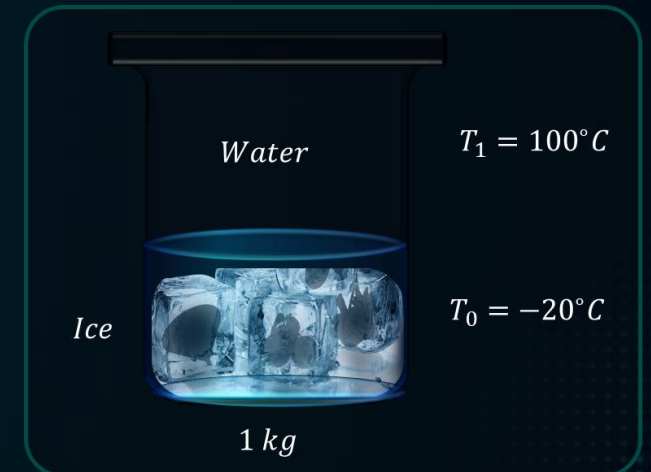
$$Q_3 = 100 \text{ kcal}$$

Total heat required:

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = (10 + 80 + 100) \text{ kcal}$$

$$Q = 190 \text{ kcal}$$

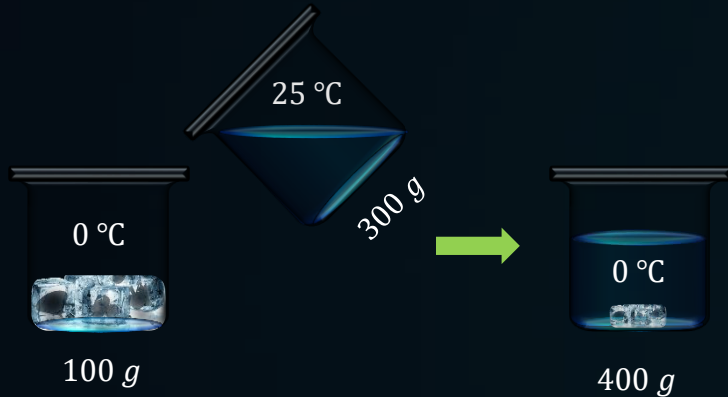




300 g of water at 25 °C is added to 100 g of ice at 0 °C. The final temperature of the mixture is

**Solution:**

**Assumption:** final temperature = 0 °C



$$s_{H_2O} = 1 \frac{\text{cal}}{\text{g } ^\circ\text{C}} \quad L_{\text{fusion}} = 80 \frac{\text{cal}}{\text{g}}$$

Heat required for melting ice =  $H_{\text{required}}$

$$H_{\text{required}} = mL = 100 \times 80 \text{ cal} = 8 \text{ kcal}$$

Heat released from water =  $H_{\text{released}}$

$$H_{\text{released}} = ms \Delta T = 300 \times 1 \times (25 - 0)$$

$$H_{\text{released}} = 7500 \text{ cal} = 7.5 \text{ kcal}$$

$$H_{\text{required}} > H_{\text{released}} \quad (\because 8 \text{ kcal} > 7.5 \text{ kcal})$$

⇒ So some part of ice remains not melted.

And water and ice co-exist at only one temperature i.e. at 0 °C.

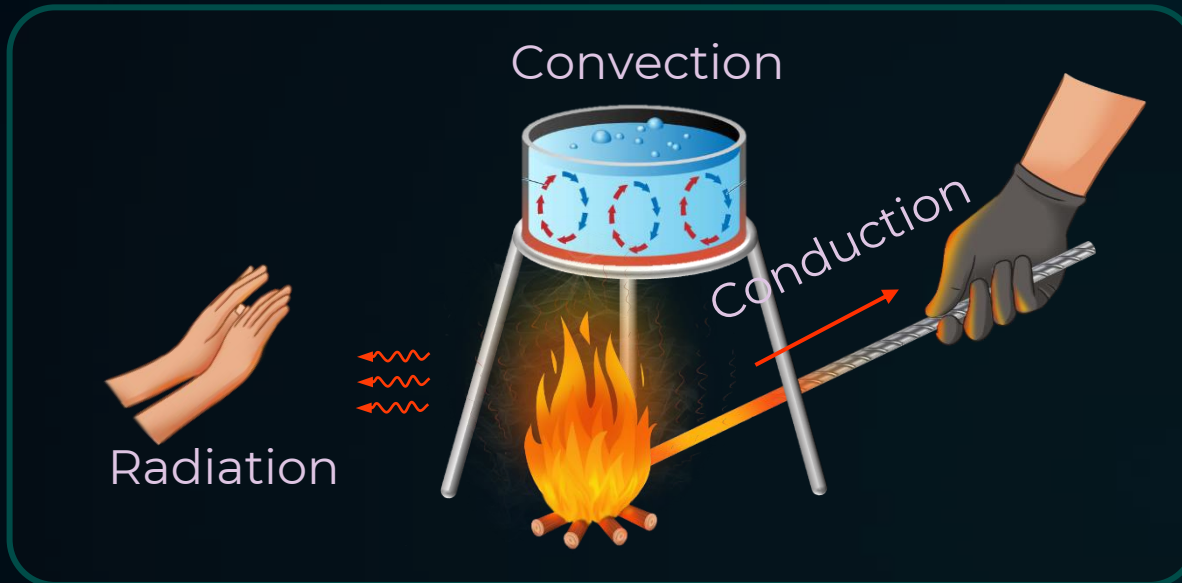
∴ final temperature = 0 °C



# Heat Transfer



- Heat transfer refers to the **flow** of heat(thermal energy) due to **temperature differences** and the subsequent temperature changes.



Heat Transfer

Conduction

Convection

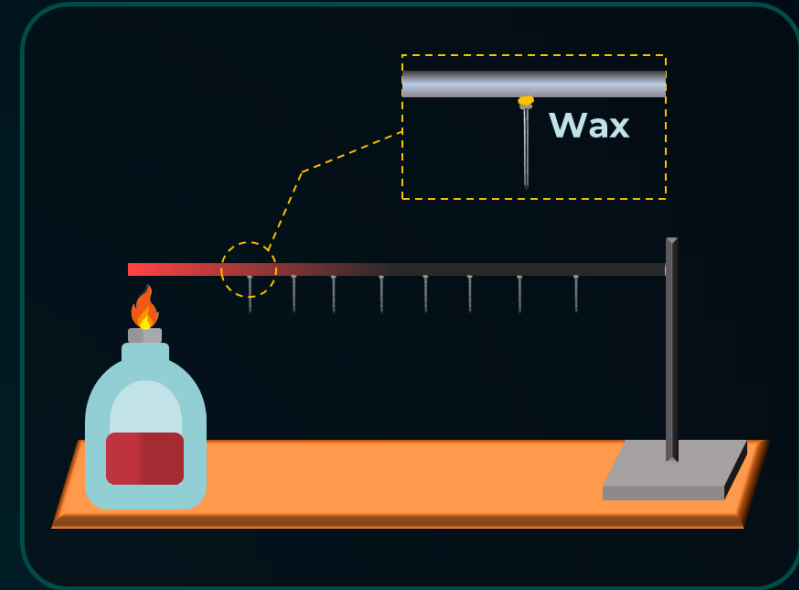
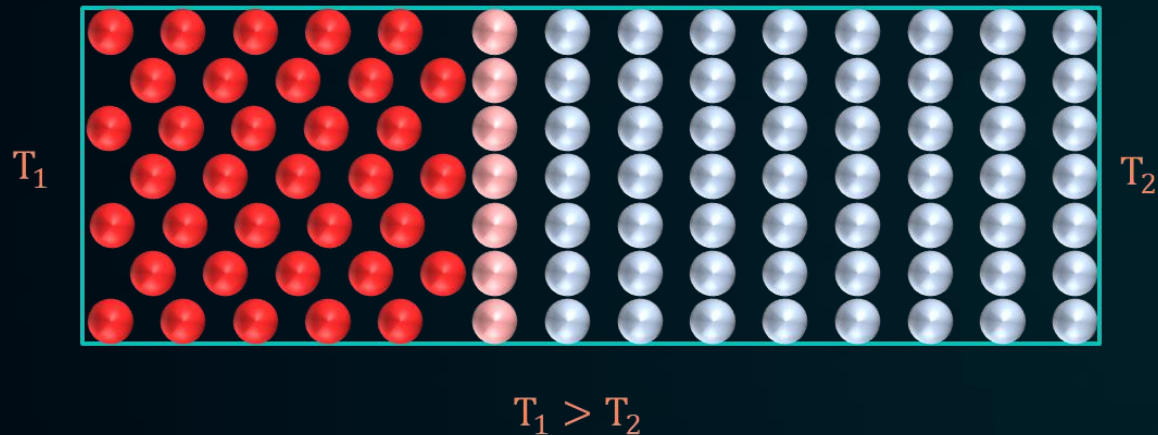
Radiation



# Conduction



- Heat conduction is the flow of internal energy from a region of higher temperature to one at lower temperature by the **interaction** of adjacent particles in the intervening space **without the actual transfer of particles**.
- Conduction involves **heat transfer** but **not mass transfer**.



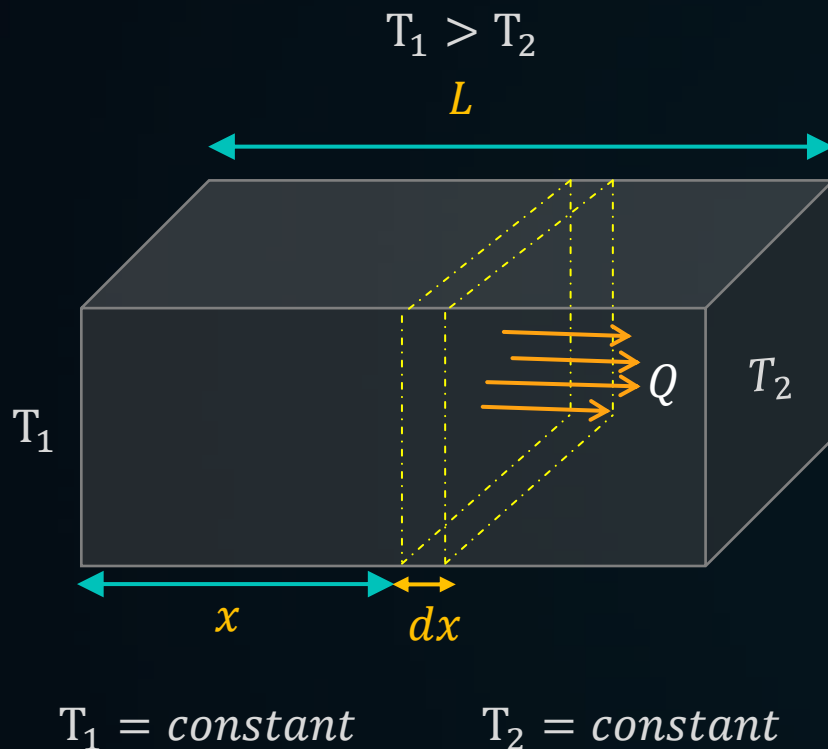
- Heat transfer due to **molecular collisions**.
- Kinetic energy gained is shared between **adjacent molecules**.
- Average position** of a molecule does **not** change.



## Steady State Conduction



- **Steady state:** If the **temperature** of a cross-section at any position  $x$  in the slab remains **constant with time**. It is different from thermal equilibrium.



- Consider a slab of face area  $A$ , Lateral thickness  $L$ , whose faces have temperatures  $T_1$  and  $T_2$ .
- For **Steady state conduction**, rate of heat transfer,

$$H = \frac{dQ}{dt} \propto \frac{AdT}{dx}$$

Where,  $dQ$  is the amount of heat transferred through any cross section in time  $dt$ .

$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

→ Fourier's law



## Thermal Conductivity( $K$ )



$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

Thermal conductivity

$$K_{solid} > K_{liq} > K_{gas}$$

$$K_{metals} > K_{non-metals}$$

- $\frac{dQ}{dt}$  is called the Rate of heat flow.
- $\frac{dT}{dx}$  is called the temperature gradient.
- $K$  is a constant for the material of the slab and is called **Thermal Conductivity** of material.
- Thermal conductivity refers to the **ability** of a given material to **conduct/transfer heat**.
- The **greater** the value of  $K$  for a material, the **more rapidly** will it **conduct heat**.

?

A hollow tube has a length  $l$ , inner radius  $r_1$  and outer radius  $r_2$ . The material has a thermal conductivity  $K$ . Find the rate of heat flow through the walls of the tube if the flat ends are maintained at temperature  $T_1$  and  $T_2$  ( $T_2 > T_1$ ).

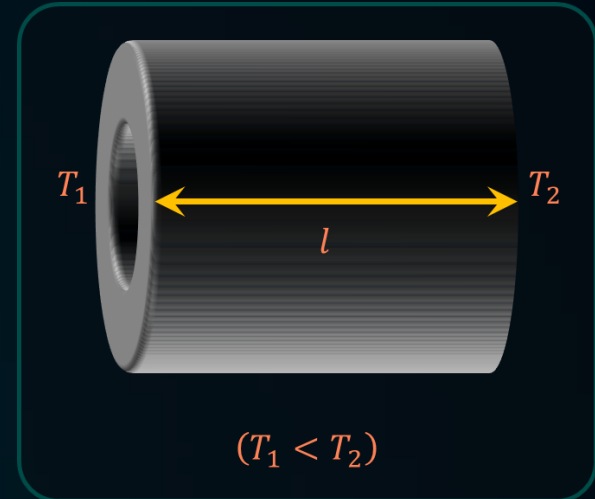
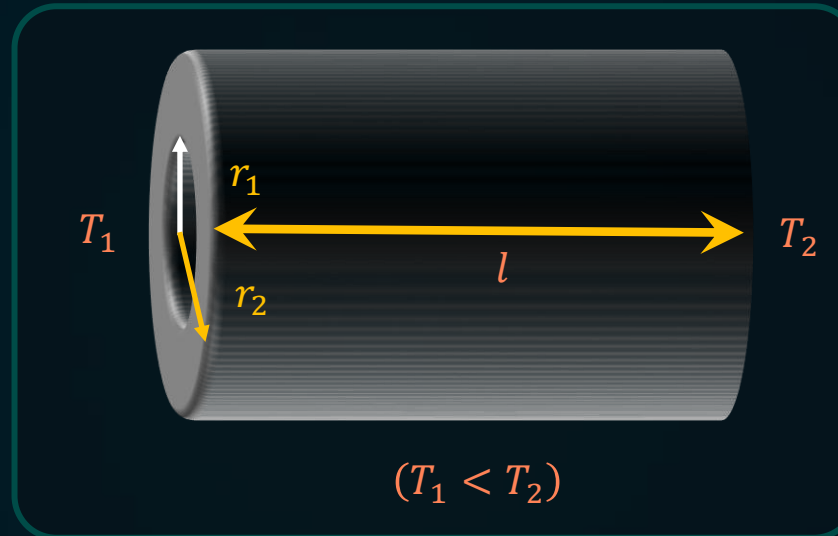
Solution:

Steady state has been maintained i.e.  $T_1, T_2$  are constant.

$$\frac{\Delta Q}{\Delta t} = \frac{-KA(\Delta T)}{l}$$

$$\frac{\Delta Q}{\Delta t} = -\frac{K}{l}(\pi r_2^2 - \pi r_1^2)(T_1 - T_2)$$

$$\frac{\Delta Q}{\Delta t} = \frac{K}{l}\pi(r_2^2 - r_1^2)(T_2 - T_1)$$

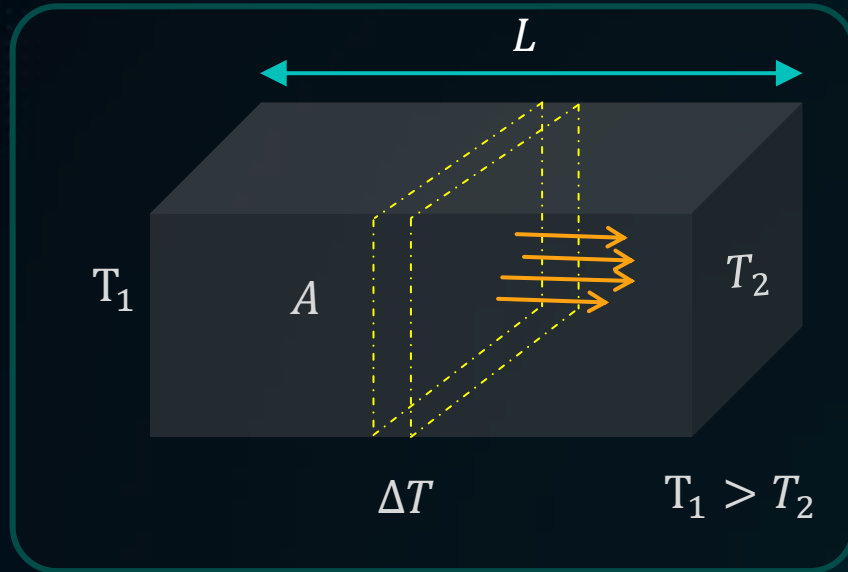




## Analogy b/w Fourier's law and Ohm's law



### Fourier's Law:



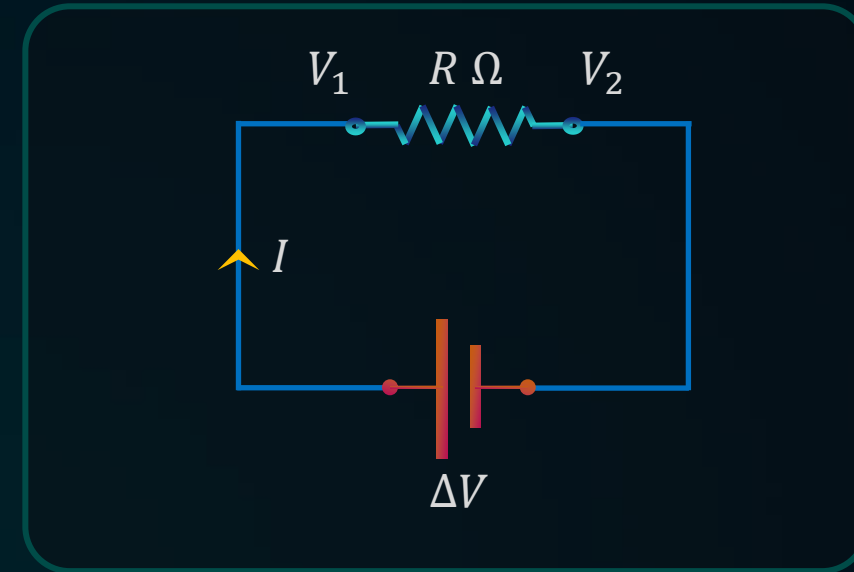
- Rate of Heat transfer:

$$H = \frac{\Delta T}{R_{th}}$$

- Thermal Resistance:

$$R_{th} = \frac{L}{KA}$$

### Ohm's Law:



- Rate of Charge transfer:

$$I = \frac{\Delta V}{R}$$

- Resistance:  $R$

?

Consider the situation shown in the figure. The frame is made of the same material and has a uniform cross-sectional area everywhere. Calculate the amount of heat flowing per second through a cross section of the bent part if the total heat taken out per second from the end at  $100^{\circ}\text{C}$  is  $130\text{ J}$ .

Given:  $H = 130\text{ J/s}$

To find:  $H_1$

Solution:

$$H = H_1 + H_2$$

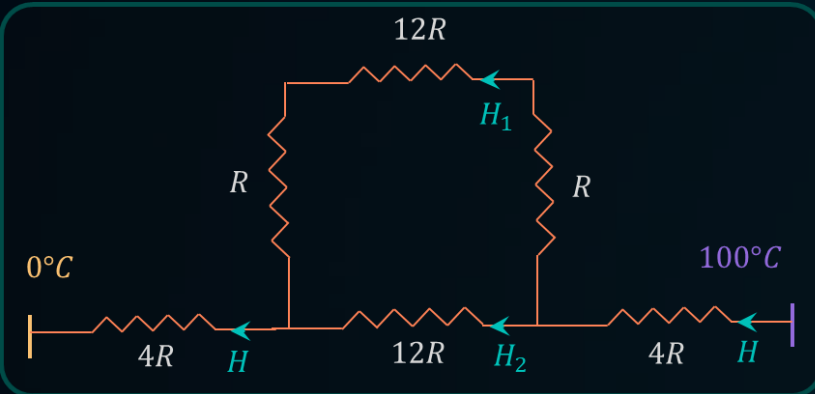
$$H_1 = \frac{\Delta T}{14R}, \quad H_2 = \frac{\Delta T}{12R}$$

$$\frac{H_1}{H_2} = \frac{12}{14} = \frac{6}{7}$$

$$H_2 = \frac{7H_1}{6}$$

$$H = H_1 + \frac{7H_1}{6}$$

$$H_1 = 60\text{ J/s}$$

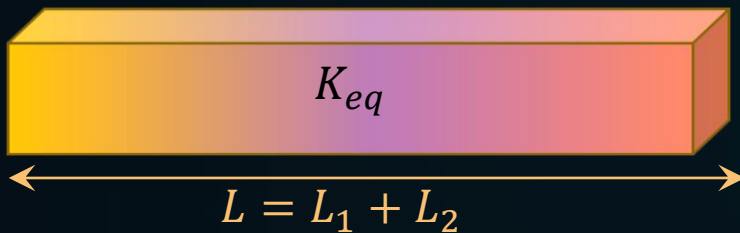


?

A composite slab is prepared by pasting two plates of thicknesses  $L_1$  and  $L_2$  and thermal conductivities  $K_1$  and  $K_2$ . The slabs have equal cross-sectional area. Find the equivalent conductivity of the composite slab.

To find:  $K_{eq}$

Solution:



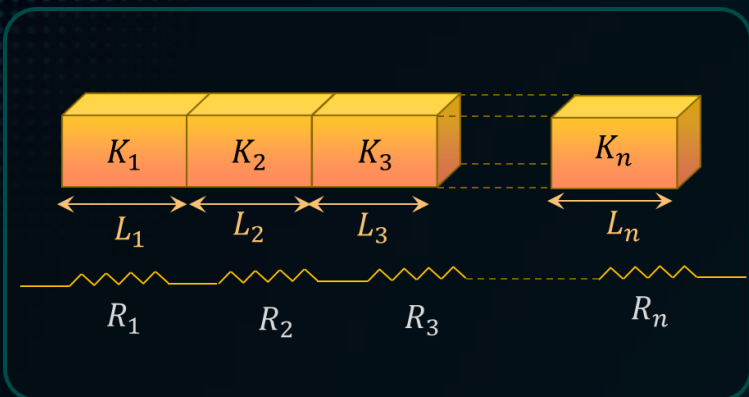
$$R_{eq} = R_1 + R_2$$

$$\frac{(L_1 + L_2)}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$

$$K_{eq} = \frac{K_1 K_2 (L_1 + L_2)}{K_1 L_2 + K_2 L_1}$$



## $n$ identical slabs in series



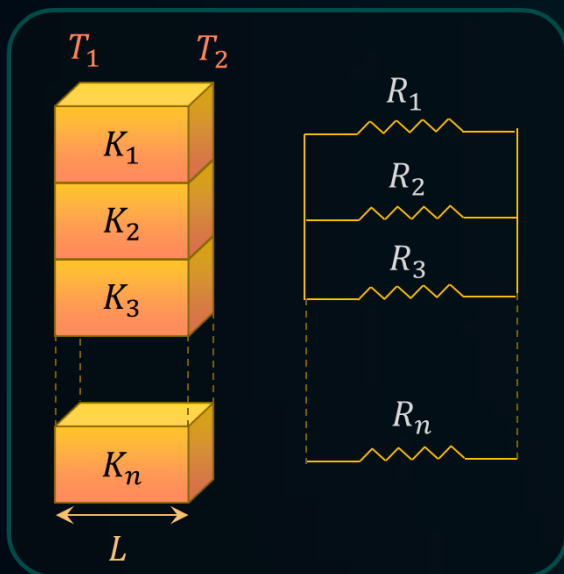
- For  $n$  identical slabs,

$$A_1 = A_2 = \dots A_n \text{ \& } L_1 = L_2 = \dots L_n$$

- The equivalent thermal conductivity is,

$$K_{eff} = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \dots \frac{1}{K_n}}$$

## $n$ identical slabs in parallel



- The net heat current for parallel combination of slabs,

$$q_{eff} = q_1 + q_2 + q_3 + \dots q_n$$

$$q_{eff} = \frac{K_{eff} A (T_1 - T_2)}{L}$$

- For  $n$  identical slabs,

$$A_1 = A_2 = \dots A_n$$

$$K_{eff} = \frac{K_1 + K_2 + K_3 + \dots K_n}{n}$$

$$\frac{K_{eff} A (T_1 - T_2)}{L} = \frac{K_1 A_1 (T_1 - T_2)}{L} + \frac{K_2 A_2 (T_1 - T_2)}{L} + \dots \frac{K_n A_n (T_1 - T_2)}{L}$$

?

Two thin metallic spherical shells of radii  $r_1$  and  $r_2$  ( $r_1 < r_2$ ) are placed with their centres coinciding. A material of thermal conductivity  $K$  is filled in the space between the shells. The inner shell is maintained at temperature  $\theta_1$  and the outer shell at temperature  $\theta_2$  ( $\theta_1 < \theta_2$ ). Calculate the rate at which heat flows radially through the material.

Given:  $\theta_2 > \theta_1$

Solution:

$$H = K(4\pi r^2) \left( \frac{d\theta}{dr} \right) = \text{constant}$$

$$H \frac{dr}{r^2} = 4\pi K d\theta$$

$$H \int_{r_1}^{r_2} \frac{dr}{r^2} = 4\pi K \int_{\theta_1}^{\theta_2} d\theta$$

$$H = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$





The atmospheric temperature is  $-\theta^{\circ}\text{C}$ . A cylindrical drum of height  $h$  made of a bad conductor is completely filled with water at  $0^{\circ}\text{C}$  and is kept outside without a lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is  $K$  and its latent heat of fusion is  $L$ . Neglect expansion of water on freezing. ( $\rho$  is the density of water)

**Given:**  $T_{\text{atm}} = -\theta^{\circ}\text{C}$ ,  $T_{\text{water}} = 0^{\circ}\text{C}$

**Solution:**  $H_{\text{instantaneous}} = \frac{dQ}{dt} = \frac{KA(0 - (-\theta))}{x} \dots \dots (T)$

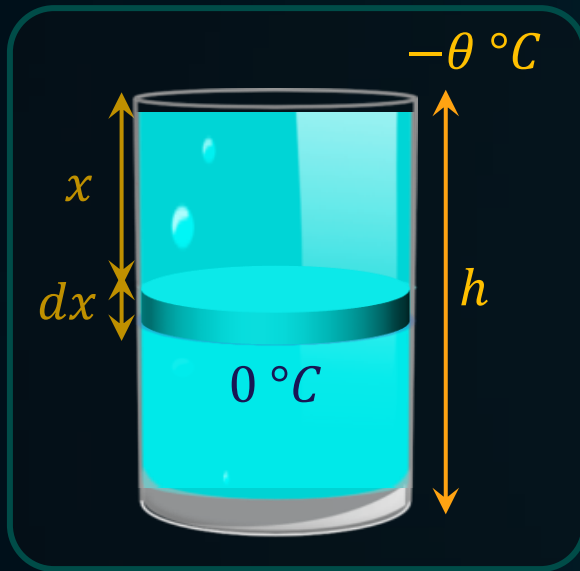
$$\frac{dQ}{dt} = L \frac{dm}{dt} = L \frac{\rho A dx}{dt} \dots \dots (A)$$

From equations (T) & (A),

$$L\rho \frac{dx}{dt} = \frac{K\theta}{x}$$

$$L\rho \int_0^h x dx = K\theta \int_0^{t_0} dt$$

$$t_0 = \frac{\rho L h^2}{2K\theta}$$





Ratio of time taken for the thickness of the ice to grow from 0 to 1 cm, 1 to 2 cm, 2 to 3 cm and so on.

To find:  $\Delta t_1: \Delta t_2: \Delta t_3: \dots \dots \Delta t_n$

Solution: Time taken by the ice to grow a thickness of  $y$  is  $t = \frac{\rho L y^2}{2K\theta}$

The time intervals to change the thickness from 0 to  $y$ ,  $y$  to  $2y$  and so on will be in the ratio:

$$\Delta t_1: \Delta t_2: \Delta t_3 = (1^2 - 0^2) : (2^2 - 1^2) : (3^2 - 2^2)$$

$$\Delta t_1: \Delta t_2: \Delta t_3 = 1 : 3 : 5$$

$$\Delta t_1: \Delta t_2: \Delta t_3: \dots \dots \Delta t_n = 1 : 3 : 5 : \dots \dots$$

?

Figure shows two adiabatic vessels, each containing mass  $m$  of water at different temperatures. The ends of metal rod of length  $L$ , area of cross section  $A$  and thermal conductivity  $K$ , are inserted in water as shown in the figure. Find the time taken for the difference between the temperature in vessels to become half of the original value. The heat capacity of the water is  $s$ . Neglect the heat capacity of rod and the container and any loss of heat to the atmosphere.

To find:  $t$  for  $\Delta T = \frac{\Delta T_0}{2}$

Solution:  $H_{\text{instantaneous}} = \frac{dQ}{dt} = \frac{KA(T_A - T_B)}{L} \dots\dots (T)$

$$\left(-\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$ms \left(-\frac{dT_A}{dt}\right) = ms \left(\frac{dT_B}{dt}\right) \dots\dots (A)$$

$$\frac{dT_A}{dt} - \frac{dT_B}{dt} = \frac{d\Delta T}{dt}$$

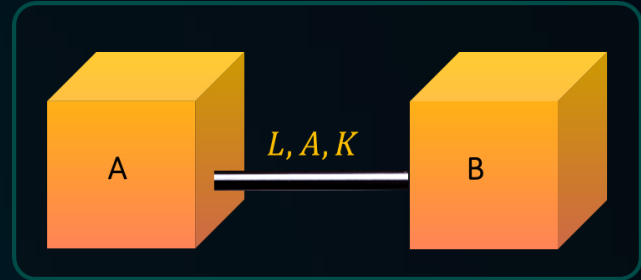
$$-\frac{\left(\frac{dQ}{dt}\right)}{ms} - \frac{\left(\frac{dQ}{dt}\right)}{ms} = \frac{d\Delta T}{dt}$$

$$-\frac{2}{ms} \left(\frac{dQ}{dt}\right) = \frac{d\Delta T}{dt}$$

$$-\frac{2}{ms} \left(\frac{KA}{L} \Delta T\right) = \frac{d\Delta T}{dt}$$

$$\int_{\Delta T_0}^{\frac{\Delta T_0}{2}} -\frac{d\Delta T}{\Delta T} = \frac{2KA}{msL} \int_0^t dt$$

$$t = \frac{msL}{2KA} \ln 2$$





# Convection



Convection is a mode of heat transfer by actual motion of matter.

It is possible only in fluids.

- Natural- Fluid moves due to density difference.
- Forced- Fluid is moved by means of ext. force  
Ex - fan, blower, etc.



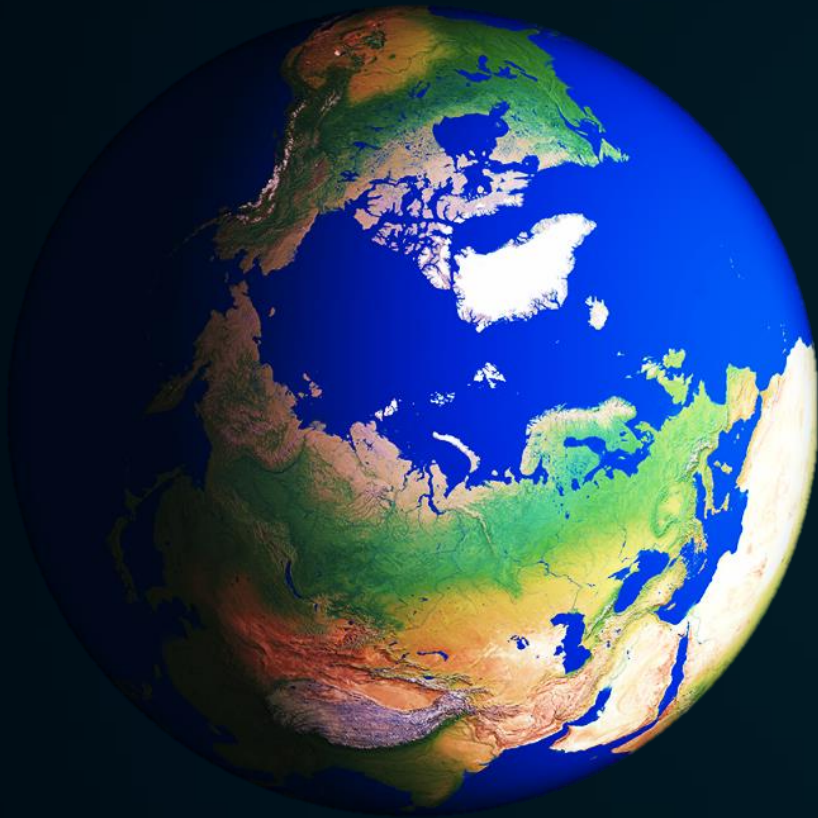
	Conduction	Convection
Energy transfer	✓	✓
Mass transfer	✗	✓



# Radiation



Heat transfer without the need for a material **medium**.





## Prevost Theory of Heat Exchange



It states that **every material body**, at any temperature above absolute zero, **radiates** heat to the surroundings and at the same time **absorbs** heat from the surroundings.

- The rate of thermal radiation emitted per unit time depends on:
  - a) **Surface area** of emitting body.
  - b) **Nature** of emitting surface.
  - c) **Temperature** of emitting surface.
- If a body radiates more amount of heat than it absorbs, its **temperature falls**.
- If a body absorbs more amount of heat than it radiates, its **temperature rises**.
- In thermal equilibrium a body absorbs and radiates the same amount of heat, its **temperature remains constant**.



# Black Body Radiation



- It is a theoretical model which is a **perfect absorber** of radiation over all wavelengths.

Ferry's Black Body



Room temperature

Surface area =  $A$



Reflects more. Absorbs little.  
Emits little.



Reflects little. Absorbs more.  
Emits more.

Good absorbers of Radiation are also good Emitters.



## Kirchhoff's Law



- Ratio of **emissive power** to **absorptive power** is same for all bodies at a given temperature and is equal to emissive power of a blackbody at that temperature.

$$\frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody}) = \text{constant}$$

$$E(\text{body}) \propto a(\text{body})$$

⇒ Good absorbers are good emitters & bad absorbers are bad emitters.



## Energy Spectrum of Black Body

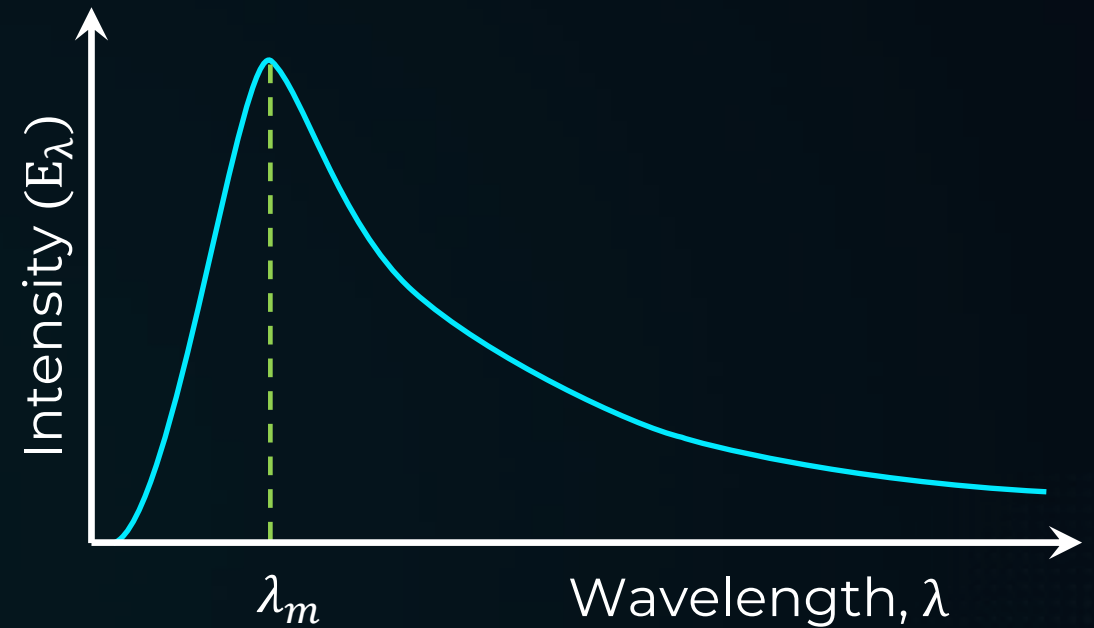


A black body emits radiation of all possible **wavelength**.

At a given temperature:

- Energy is not uniformly distributed over all wavelengths.
- Intensity increases up to a certain maximum value with wavelength, then decreases.

$$\text{Area} = E = \int E_{\lambda} d\lambda = \sigma T^4$$

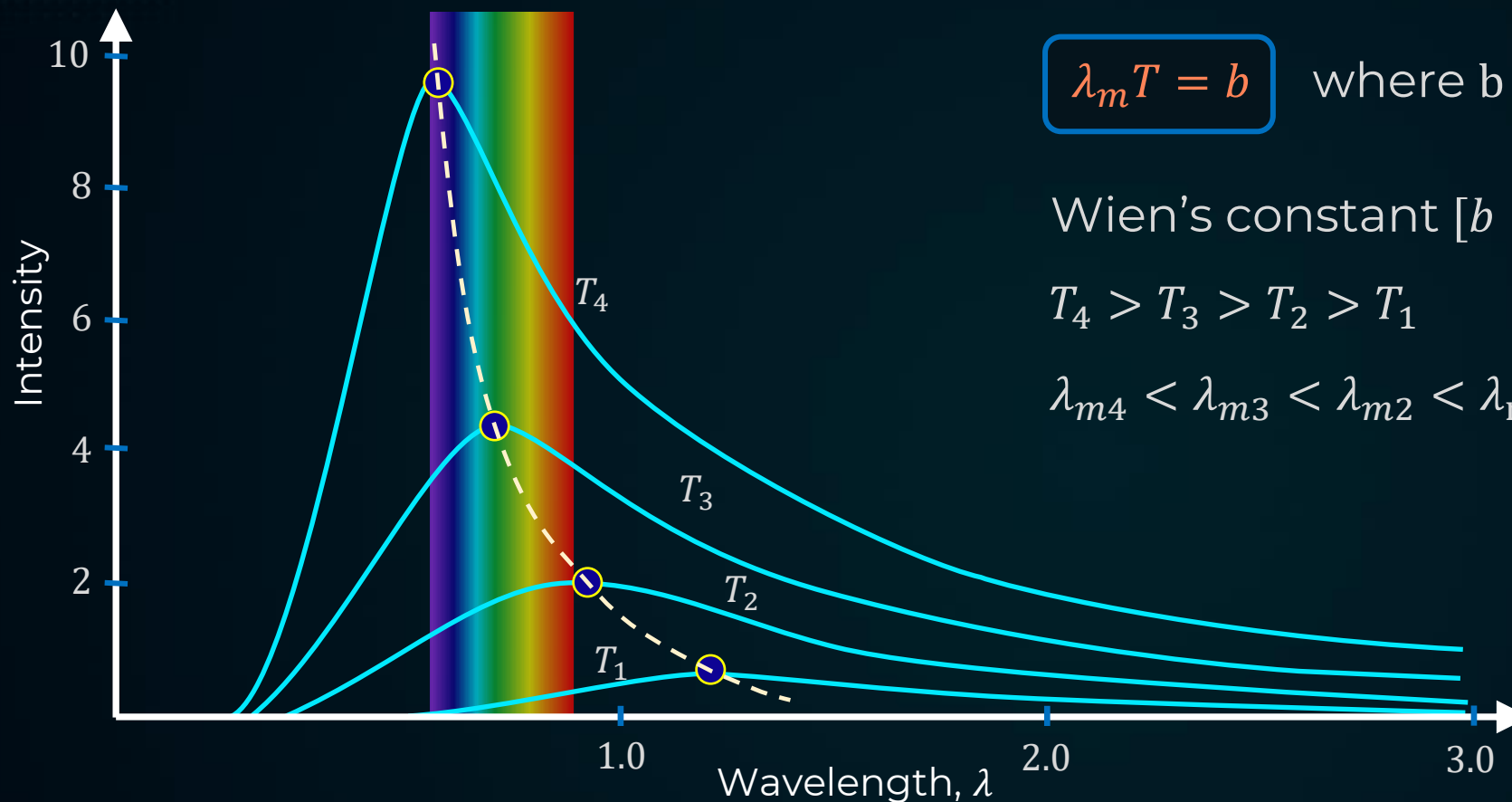




## Wien's Displacement Law



- The **wavelength** of highest intensity ( $\lambda_m$ ) is inversely proportional to the absolute **temperature** of the emitter.



$$\lambda_m T = b \quad \text{where } b \text{ is called}$$

Wien's constant [ $b = 2.89 \times 10^{-3} \text{ m} \cdot \text{K}$ ]

$$T_4 > T_3 > T_2 > T_1$$

$$\lambda_{m4} < \lambda_{m3} < \lambda_{m2} < \lambda_{m1}$$



## Stefan's Law



- The thermal energy **emitted** by a body of surface area **A** per unit time is given by

$$\frac{dQ}{dt} = e\sigma AT^4$$

$\sigma$  = Stefan-Boltzmann constant

$$[\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}]$$

$T$  = Temperature on Absolute scale

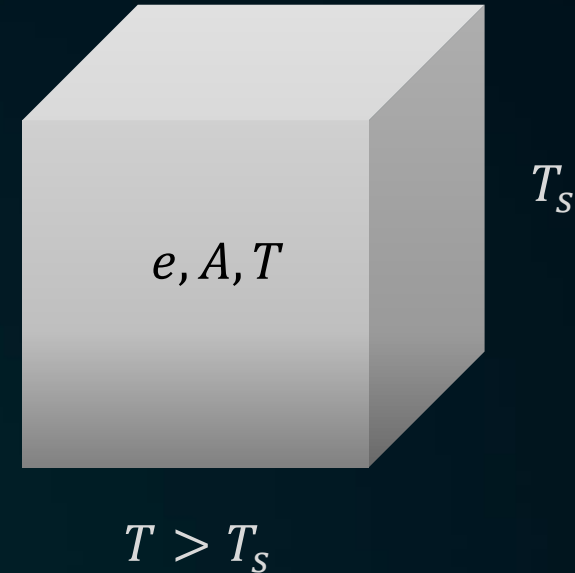
$e$  = emissivity of the surface (constant &  $0 \leq e \leq 1$ )

$$P_{\text{radiation}} = e\sigma AT^4$$

$$P_{\text{incident}} = a\sigma AT_s^4$$

Net rate of heat loss,

$$P_{\text{net}} = e\sigma A(T^4 - T_s^4) = -ms \frac{dT}{dt}$$



?<sub>T</sub>

A copper sphere is kept in a chamber maintained at  $300\text{ K}$ . The sphere is maintained at a constant temperature of  $500\text{ K}$  by heating it electrically. A total of  $210\text{ W}$  of electric power is needed to do it. When the surface of the copper sphere is completely blackened,  $700\text{ W}$  is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

Power required to maintain the temperature of copper sphere,

$$P_{Cu} = e_{Cu} A \sigma (T^4 - T_s^4) = 210\text{ W} \dots \dots (1)$$

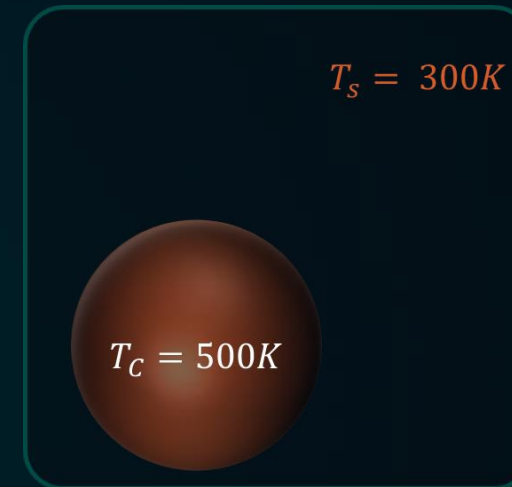
Power required to maintain the temperature of sphere after blackening it,

$$(P_{Cu})_{black} = (1) A \sigma (T^4 - T_s^4) = 700\text{ W} \dots \dots (2)$$

On dividing equation (1) by equation (2), we get,

$$\Rightarrow \frac{e_{Cu} A \sigma (T^4 - T_s^4)}{A \sigma (T^4 - T_s^4)} = \frac{210}{700}$$

$$\Rightarrow e_{Cu} = 0.3$$



a

0.2

b

0.3

c

0.5

d

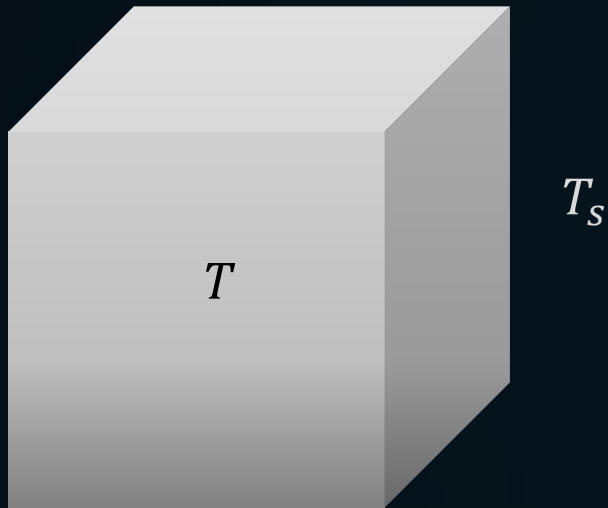
0.6



## Rate of Cooling



- The **rate of loss of heat**,  $\frac{-dQ}{dt}$  of the body is **directly proportional** to the difference of **temperature**  $\Delta T = (T - T_s)$  of the body and the surroundings.
- The law holds good only for small difference in the temperature.



$$\left(\frac{dQ}{dt}\right)_1 = e\sigma AT^4, \quad \left(\frac{dQ}{dt}\right)_2 = e\sigma AT_s^4$$

**(Emitted)                      (Absorbed)**

$$P_{net} = e\sigma A(T^4 - T_s^4) = ms \left(-\frac{dT}{dt}\right)$$

Rate of Cooling,

$$-\frac{dT}{dt} \propto (T^4 - T_s^4)$$



# Newton's Law of Cooling



- Rate of Cooling:

$$-\frac{dT}{dt} \propto (T^4 - T_s^4)$$

- Let  $\Delta T = T - T_s$

$$\Rightarrow T^4 = (T_s + \Delta T)^4$$

$$\Rightarrow T^4 = T_s^4 \left(1 + \frac{\Delta T}{T_s}\right)^4$$

$$\Rightarrow T^4 \approx T_s^4 \left(1 + 4 \frac{\Delta T}{T_s}\right) \quad [\because \Delta T \ll T_s]$$

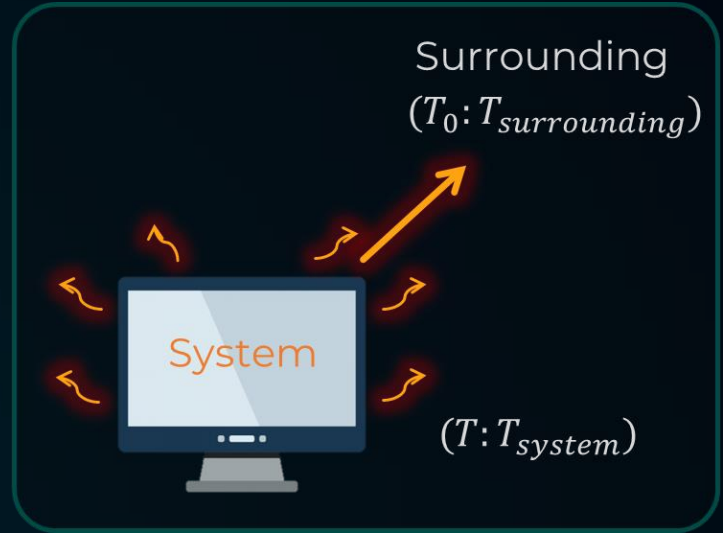
$$\Rightarrow (T^4 - T_s^4) \propto (T - T_s)$$

- From Stephen-Boltzmann law

$$-\frac{dT}{dt} \propto (T - T_s)$$

- In integral form,

$$-\int \frac{dT}{T - T_s} = \int k dt$$



?

The temperature of a body falls from  $40^{\circ}\text{C}$  to  $36^{\circ}\text{C}$  in 5 minutes when placed in a surrounding of constant temperature  $16^{\circ}\text{C}$ . Find the time taken for the temperature of the body to become  $32^{\circ}\text{C}$ .

Solution:

Newton's Law of Cooling,  $-\frac{dT}{dt} = k(T - T_s)$

For small temperature differences, the curve can be assumed to be linear

$$\begin{aligned} T &= T_{avg} \\ \frac{dT}{dt} &= \frac{\Delta T}{\Delta t} \end{aligned}$$



Case 1:

$40^{\circ}\text{C} \xrightarrow{5 \text{ min}} 36^{\circ}\text{C}$

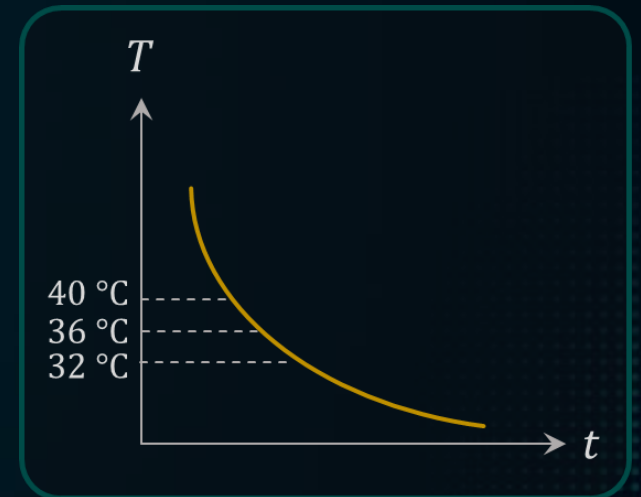
$$\frac{40 - 36}{5} = k \left( \frac{40 + 36}{2} - 16 \right)$$

Case 2:

$36^{\circ}\text{C} \xrightarrow{t} 32^{\circ}\text{C}$

$$\frac{36 - 32}{t} = k \left( \frac{36 + 32}{2} - 16 \right)$$

$t = 6.1 \text{ min}$



?

A hot body placed in the air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being  $k$  times the temperature difference from the surrounding. Starting from  $t = 0$ , find the time in which the body will lose half the maximum heat it can lose.

Solution:  $-\ln \left| \frac{T - T_0}{T_1 - T_0} \right| = kt$

$$\Rightarrow \left| \frac{T - T_0}{T_1 - T_0} \right| = e^{-kt}$$

$$\Rightarrow \Delta T(t) = (\Delta T)_0 e^{-kt}$$

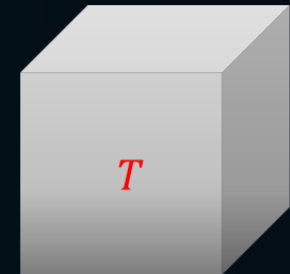
Heat lost by the body:  $Q = ms(\Delta T)$

For  $Q = \frac{Q_{max}}{2}, \quad \Delta T = \left( \frac{\Delta T_0}{2} \right)$

$$\Rightarrow \frac{\Delta T_0}{2} = \Delta T_0 e^{-kt}$$

$$\Rightarrow t = \frac{\ln 2}{k}$$

Surrounding ( $T_0$ )  
 $T > T_0$



$t$	0	$t$
$T$	$T_1$	$T$
$T_s$	$T_0$	$T_0$



## Solar Constant



- Solar electromagnetic radiation per meter square area on Earth's surface.
- Power radiated by sun:

$$P_S = \sigma AT^4 = \sigma 4\pi R^2 T^4$$

Intensity on Earth's surface:

$$I_E = \frac{\sigma 4\pi R^2 T^4}{4\pi d^2}$$

$$\sigma = 5.67 \times 10^{-8}, \quad d = 1.5 \times 10^{11}$$

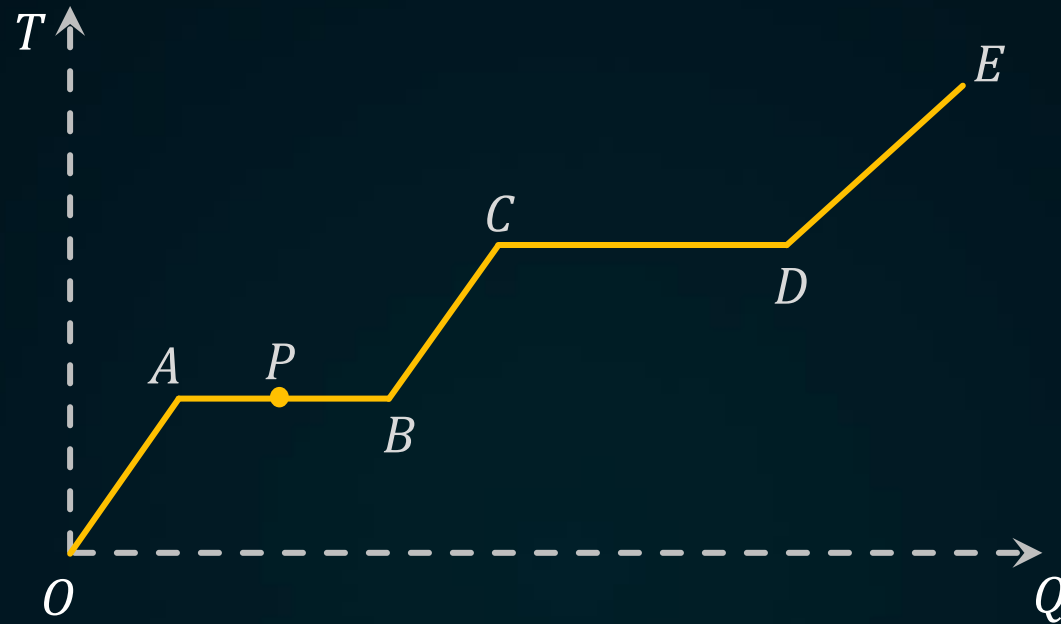
$$R_S = 7 \times 10^8, \quad T = 5778 \text{ K}$$

$$I_E = \frac{5.67 \times 10^{-8} \times 4\pi \times (7 \times 10^8)^2 (5778)^4}{4\pi \times (1.5 \times 10^{11})^2}$$

$$I_E \approx 1400 \text{ W/m}^2$$



The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in **solid state at the point  $O$** . The state of the material at the **point  $P$**  is



**Solution:**

From Point  $A$  to point  $B$ , the solid converts into liquid. At point  $A$ , the phase is completely solid and at point  $B$ , it is completely liquid. Thus, **at point  $P$ , it will be partly solid and partly liquid.**

?

The Earth receives on its surface radiation from the Sun at the rate of  $1400 \text{ W/m}^2$ . The distance of the center of the Sun from the surface of the Earth is  $1.5 \times 10^{11} \text{ m}$  and the radius of the Sun is  $7 \times 10^8 \text{ m}$ . Treating Sun as a block body, it follows from the above data that its surface temperature is *(JEE 1989)*

Given:  $I_E = 1400 \text{ W/m}^2, d = 1.5 \times 10^{11}, R_S = 7 \times 10^8$

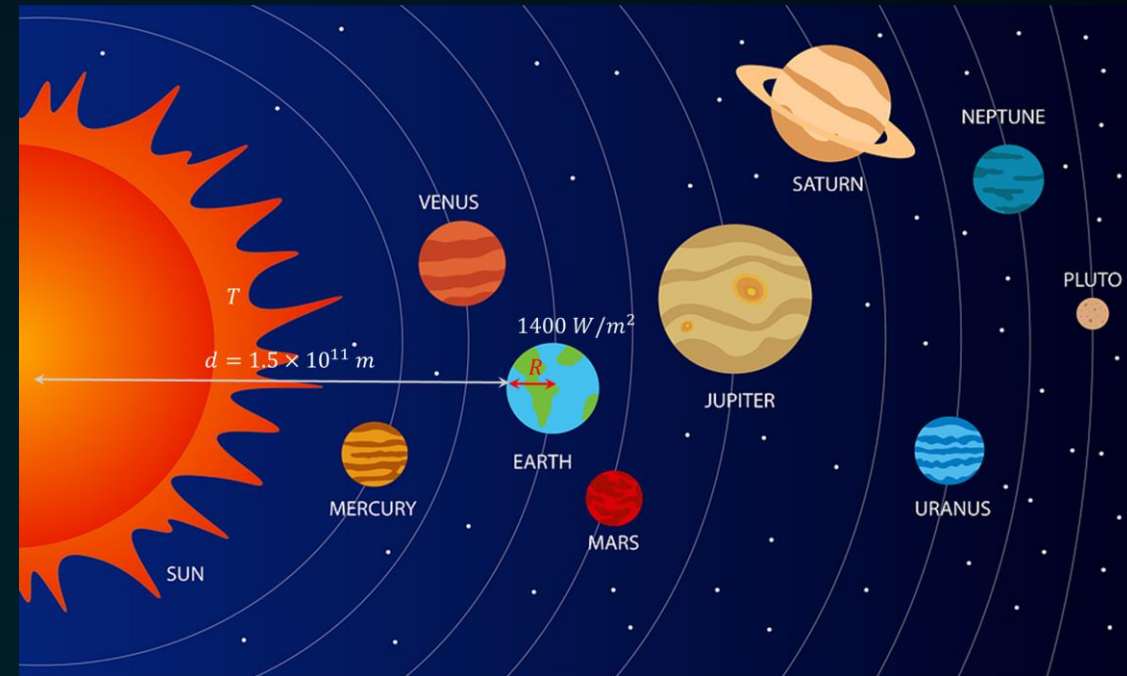
To find:  $T$

Solution:

Intensity at Earth's Surface,  $I_E = \frac{\text{Power radiated by Sun}}{4\pi d^2}$

$$I_E = \frac{\sigma 4\pi R^2 T^4}{4\pi d^2}$$

$$\Rightarrow T = \left[ \frac{1400 \times (1.5 \times 10^{11})^2}{(5.67 \times 10^{-8})^2 (7 \times 10^8)^2} \right]^{1/4}$$



$$T = 5801 \text{ K}$$

?

Earth receives  $1400 \text{ W/m}^2$  of solar power. If all the solar energy falling on a lens of area  $0.2 \text{ m}^2$  is focused onto a block of ice of mass  $280 \text{ g}$ , the time taken to melt the ice will be \_\_\_\_\_ minutes. (Latent heat of fusion of ice =  $3.3 \times 10^5 \text{ J/kg}$ ). (JEE 1997)

Given:  $I_E = 1400 \text{ W/m}^2$ ,  $A = 0.2 \text{ m}^2$

$m = 280 \text{ g}$ ,  $L_F = 3.3 \times 10^5 \text{ J/kg}$

Solution:

Solar power concentrated by the lens melts the ice.

Heat required for melting:

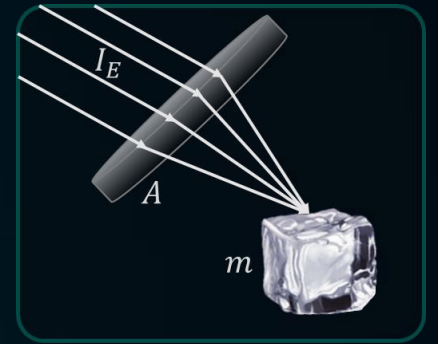
$$Q = m \times L_F$$

Energy from the Sun in time  $t$ :

$$Q = I_E \times A \times t$$

$$0.28 \times 3.3 \times 10^5 = 1400 \times 0.2 \times t$$

$$t = 330 \text{ s} = 5.5 \text{ min}$$



?

Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross section of each rod is  $4 \text{ cm}^2$ . End of copper rod is maintained at  $100^\circ\text{C}$  whereas the ends of brass and steel are kept at  $0^\circ\text{C}$ . Lengths of the copper, brass and steel rods are  $46 \text{ cm}$ ,  $13 \text{ cm}$  and  $12 \text{ cm}$  respectively. The rods are thermally insulated from surrounding except at ends. Thermal conductivities of copper brass and steel are  $0.92$ ,  $0.26$  and  $0.12$  CGS units respectively. Find the rate of heat flow through copper rod. *(JEE Main 2014)*

Given:

Parameter	Copper	Brass	Steel
$L \text{ (cm)}$	46	13	12
$A \text{ (cm}^2\text{)}$	4	4	4
$T \text{ (}^\circ\text{C)}$	100	0	0
$k \text{ (CGS unit)}$	0.92	0.26	0.12

Solution:

Rate of Heat Flow:  $Q = KA \frac{T_1 - T_2}{L}$

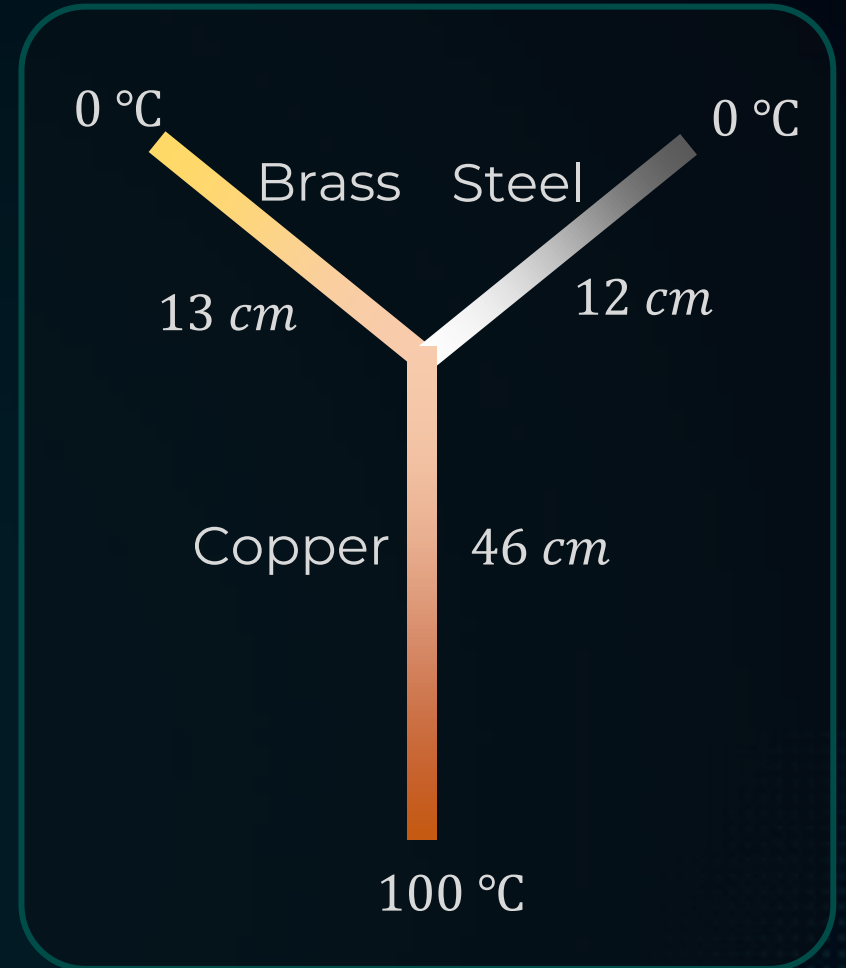
If the junction temperature is  $T$ ,

Conservation of energy:

$$Q_{copper} = Q_{brass} + Q_{steel}$$

$$0.92 \times 4 \times \frac{100 - T}{46} = 0.26 \times 4 \times \frac{T - 0}{13} + 0.12 \times 4 \times \frac{T - 0}{12}$$

$$\Rightarrow T = 40^\circ\text{C}$$





A vessel of volume  $V_0$  contains an ideal gas at pressure  $P_0$  and temperature  $T$ . Gas is contiy pumped out of this vessel at a constant volume-rate  $\frac{dV}{dt} = r$  keeping the temperature constant. The pressure of the gas being taken out equals the pressure inside the vessel. Find  
 (a) The pressure of the gas as a function of time.  
 (b) The time taken before half of the original gas is pumped out.

Given:  $\frac{dV}{dt} = r$

Solution:  $m_t = m_{t+dt}$

To find:  $P(t)$

$$\rho V_0 = (\rho + d\rho)(V_0 + dV)$$

$$\rho(rdt) = -d\rho V_0$$

$$\frac{d\rho}{\rho} = -\frac{r dt}{V_0}$$

$$\frac{d\rho}{\rho} = \frac{dP}{P} \dots \dots \dots \left( \rho = \frac{PM}{RT} \right)$$

$$\int_{P_0}^P \frac{dP}{P} = \int_0^t -\frac{r dt}{V_0}$$

$$P = P_0 e^{-\frac{rt}{V_0}}$$

Solution:

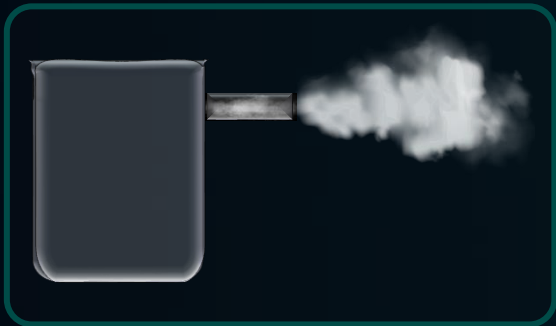
$$t = 0 \quad n_0 \longrightarrow \frac{n_0}{2}$$

$$t = t_0 \quad P_0 \longrightarrow \frac{P_0}{2}$$

$$P = P_0 e^{-\frac{rt}{V_0}}$$

$$\frac{P_0}{2} = P_0 e^{-\frac{rt}{V_0}}$$

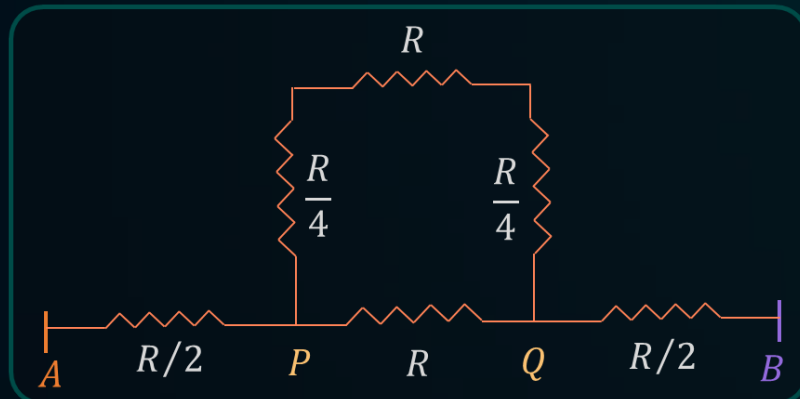
$$t = \frac{V_0 \ln 2}{r}$$



?

The temperature difference of  $120^\circ\text{C}$  is maintained between two ends of a uniform rod  $AB$  of length  $2L$ . Another bent rod  $PQ$ , of same cross-section as  $AB$  and length  $\frac{3L}{2}$ , is connected across  $AB$ . In steady-state, the temperature difference between  $P$  and  $Q$  will be close to:

Solution:



$$(R_{eq})_{PQ} = \frac{3R/2 \times R}{\frac{3R}{2} + R} = \frac{3}{5}R$$

$$R_{eq} = \frac{8}{5}R$$

$$\therefore H_{PQ} = H_{AB}$$

$$\left(\frac{\Delta T}{R_{eq}}\right)_{PQ} = \left(\frac{\Delta T}{R_{eq}}\right)_{AB}$$

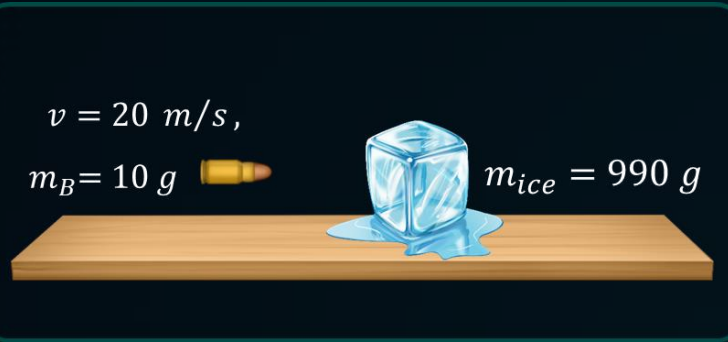
$$\Delta T_{PQ} = \frac{120}{\frac{8}{5}R} \times \frac{3}{5}R$$

$$\left(\frac{\Delta T_{PQ}}{\frac{3}{5}R}\right) = \left(\frac{120}{\frac{8}{5}R}\right)$$

$$\Delta T_{PQ} = 45^\circ\text{C}$$

?

A bullet of mass  $10\text{ g}$  moving with a speed of  $20\text{ m/s}$  hits an ice block of mass  $990\text{ g}$  kept on a frictionless floor and gets stuck in it. How much ice will melt if  $50\%$  of the lost KE goes to ice?



**Solution:** Since No  $F_{ext}$

Using Momentum Conservation,

$$m_B v = (m_{ice} + m_B) V$$

$$V = 0.2\text{ m/s}$$

initial  $K.E.$   $KE_i = \frac{1}{2} m_B v^2 = 2\text{ J}$

Final  $K.E.$   $KE_f = \frac{1}{2} (m_B + m_{ice}) V^2 = 0.02\text{ J}$

Loss of Kinetic Energy,

$$\Delta KE = 1.98\text{ J}$$

50% of Loss of Kinetic Energy is absorbed by the ice for melting,

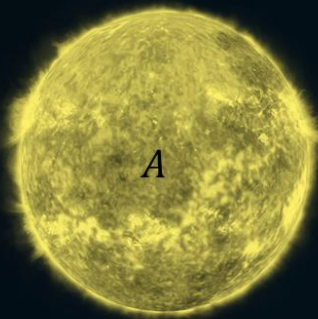
$$50\% \text{ of } KE = m_{ice} L$$

$$\frac{50}{100} \times 1.98\text{ J} = m_{ice-melted} \times 80 \times 4.2$$

$$m_{ice-melted} = 0.003\text{ g}$$

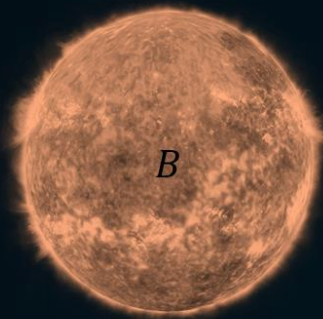
?

Two stars **A** and **B** of same size, have thermal emissivity of **0.2** and **0.64** respectively. Both stars emit total radiant power at same rate. If the temperature of **A** is **5000 K** and the wavelength  $\lambda_A$  corresponding to maximum spectral radiancy in the radiation from **B** is shifted from the wavelength corresponding to maximum spectral radiancy in radiations from **A** by **2.0  $\mu m$** , then find the temperature of star **B** and wavelength  $\lambda_B$ .



$$\epsilon_A = 0.2$$

$$T_A = 5000 \text{ K}$$



$$\epsilon_B = 0.64$$

$$T_B = ?$$

Here,  $\epsilon_A < \epsilon_B$

But  $P_A = P_B$ , &  $A_A = A_B$

$\therefore T_A > T_B$  (Stefan's Law)

$\therefore \lambda_A < \lambda_B$  (Wien's Displacement Law)

$$\therefore \lambda_B - \lambda_A = 2.0 \mu m$$

**Solution:**

$$P = \epsilon \sigma A T^4$$

$$\epsilon_A T_A^4 = \epsilon_B T_B^4 \dots \dots \dots (P_A = P_B, A_A = A_B)$$

$$T_B^4 = \frac{0.2}{0.64} (5000^4)$$

$$T_B = 3738 \text{ K}$$

According to Wien's displacement law,

$$\lambda_A T_A = \lambda_B T_B$$

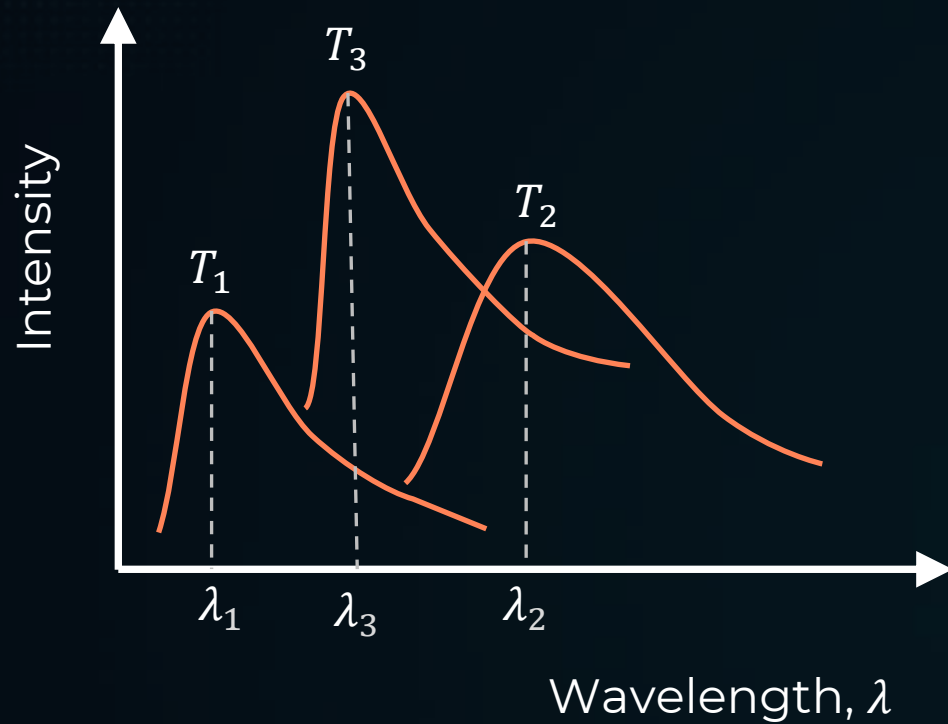
$$\lambda_B = \frac{5000}{3740} \times \lambda_A$$

$$\lambda_B - \lambda_A = 2.0 \mu m$$

$$\lambda_B = 7.93 \mu m$$



The plots of intensity versus wavelength for three black bodies at temperatures  $T_1$ ,  $T_2$  and  $T_3$  respectively are shown in figure. Their temperatures are such that:



**Solution:**

According to Wein's displacement law,

$$\lambda_m T = \text{constant}$$

$$\lambda_2 > \lambda_3 > \lambda_1$$

$$\Rightarrow T_1 > T_3 > T_2$$

a  $T_1 > T_2 > T_3$

b  $T_1 > T_3 > T_2$

c  $T_2 > T_3 > T_1$

d  $T_3 > T_2 > T_1$



?

Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures  $T_1 = 300K$  and  $T_2 = 100K$  as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are  $K_1$  and  $K_2$  respectively. If the temperature at the junction of the two cylinders in the steady state is  $200K$ , then  $\frac{K_1}{K_2} =$

Given:  $T_1 = 300 K$ ,  $T_2 = 100 K$   
 $r_2 = 2r_1$

To find:  $\frac{K_1}{K_2}$

Solution:

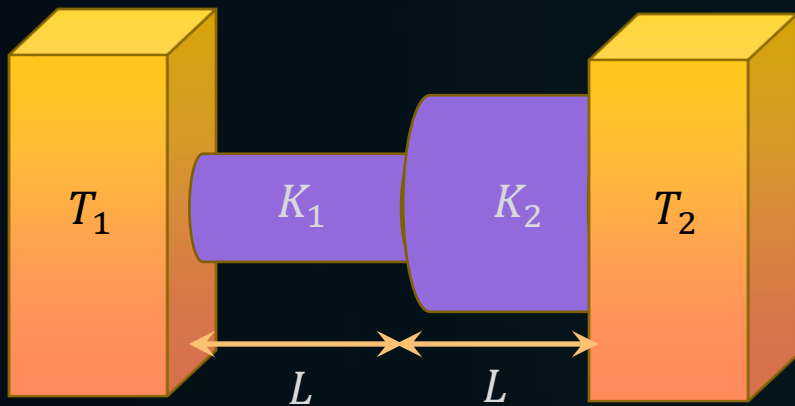
In steady-state heat flow across the two cylinders will be same,

$$H_1 = H_2$$

$$\left( \frac{\frac{\Delta T_1}{L}}{\frac{K_1 A_1}{L}} \right) = \left( \frac{\frac{\Delta T_2}{L}}{\frac{K_2 A_2}{L}} \right)$$

$$\left( \frac{300 - 200}{\frac{L}{K_1 \pi r_1^2}} \right) = \left( \frac{200 - 100}{\frac{L}{K_2 \pi (2r_1)^2}} \right)$$

$$\frac{K_1}{K_2} = 4$$



?

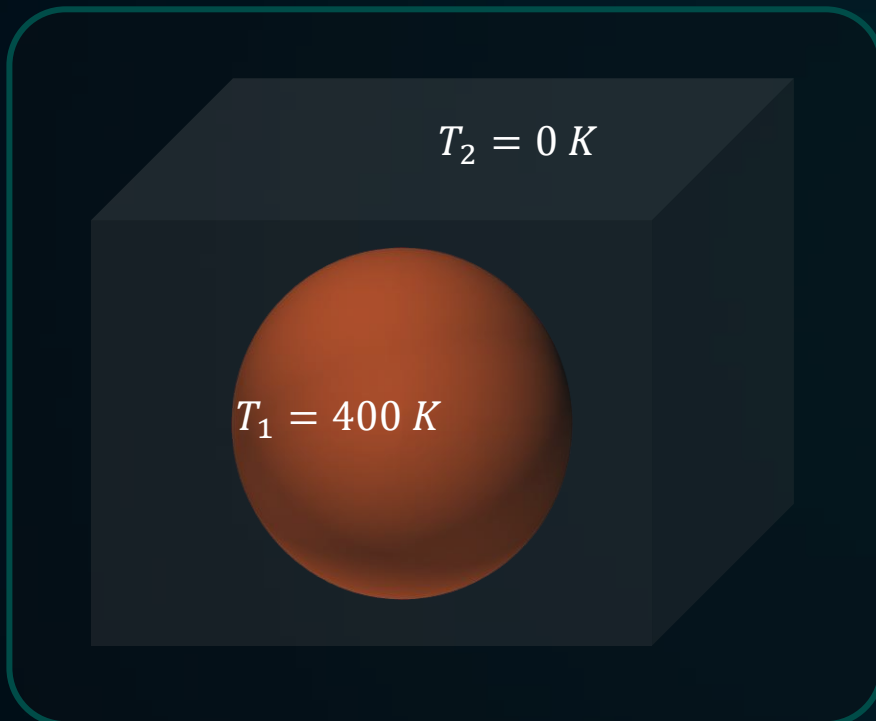
A solid copper sphere of density  $\rho$  and specific capacity  $C$  has radius  $R$ . If it is heated to a temperature of  $400\text{ K}$  is suspended inside a chamber whose walls are at almost  $0\text{ K}$ . The time required for the temperature of the sphere to drop to  $200\text{ K}$  is

Given:  $T_1 = 400\text{ K}$ ,  $T_2 = 0\text{ K}$

To find:  $t$

Solution:

Assuming, Heat loss from the sphere is only due to radiation



$$\epsilon \sigma A T^4 = -mC \frac{dT}{dt}$$

$$\epsilon \sigma \times 4\pi R^2 \times T^4 = -\left(\frac{4}{3}\pi R^3 \times \rho\right) C \frac{dT}{dt}$$

$$\frac{R\rho C}{3\epsilon\sigma} \int_{400}^{200} \frac{dT}{T^4} = -\int_0^t dt$$

$$t = \frac{7 R\rho C}{576 \epsilon\sigma} \times 10^{-6} \text{ s}$$