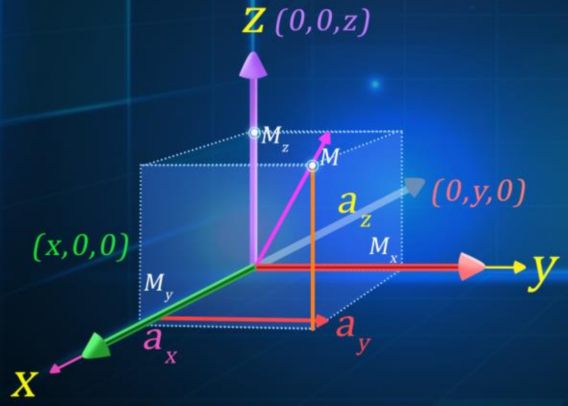
Welcome to

Aakash + BYJU'S LIVE

3D Geometry



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Introduction to three dimensional geometry

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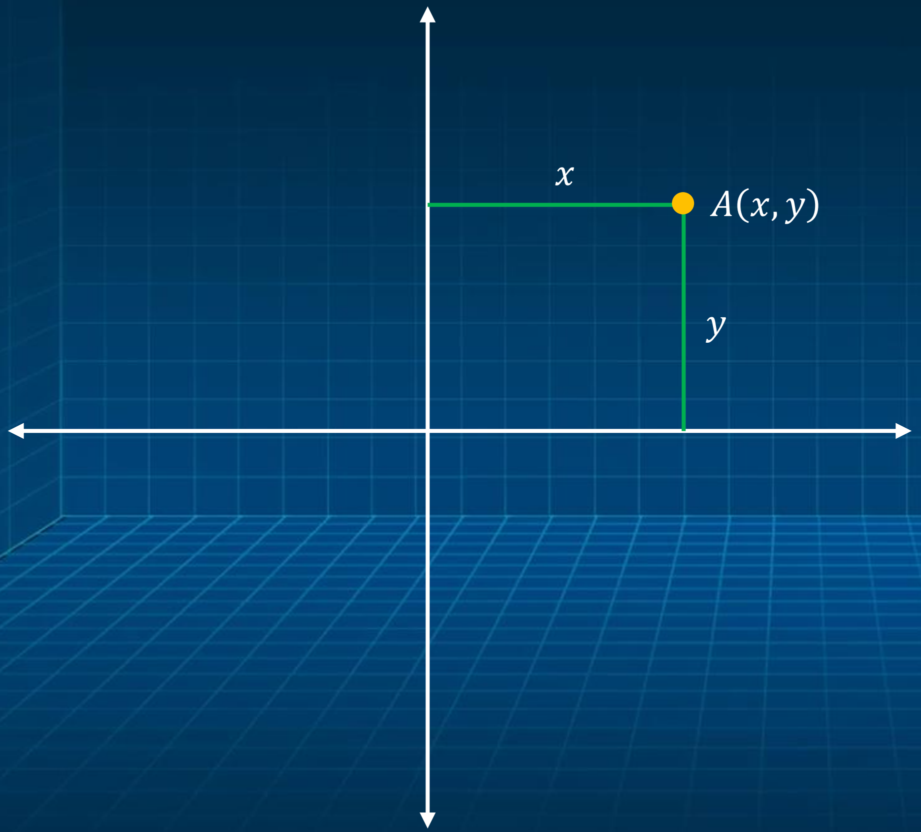

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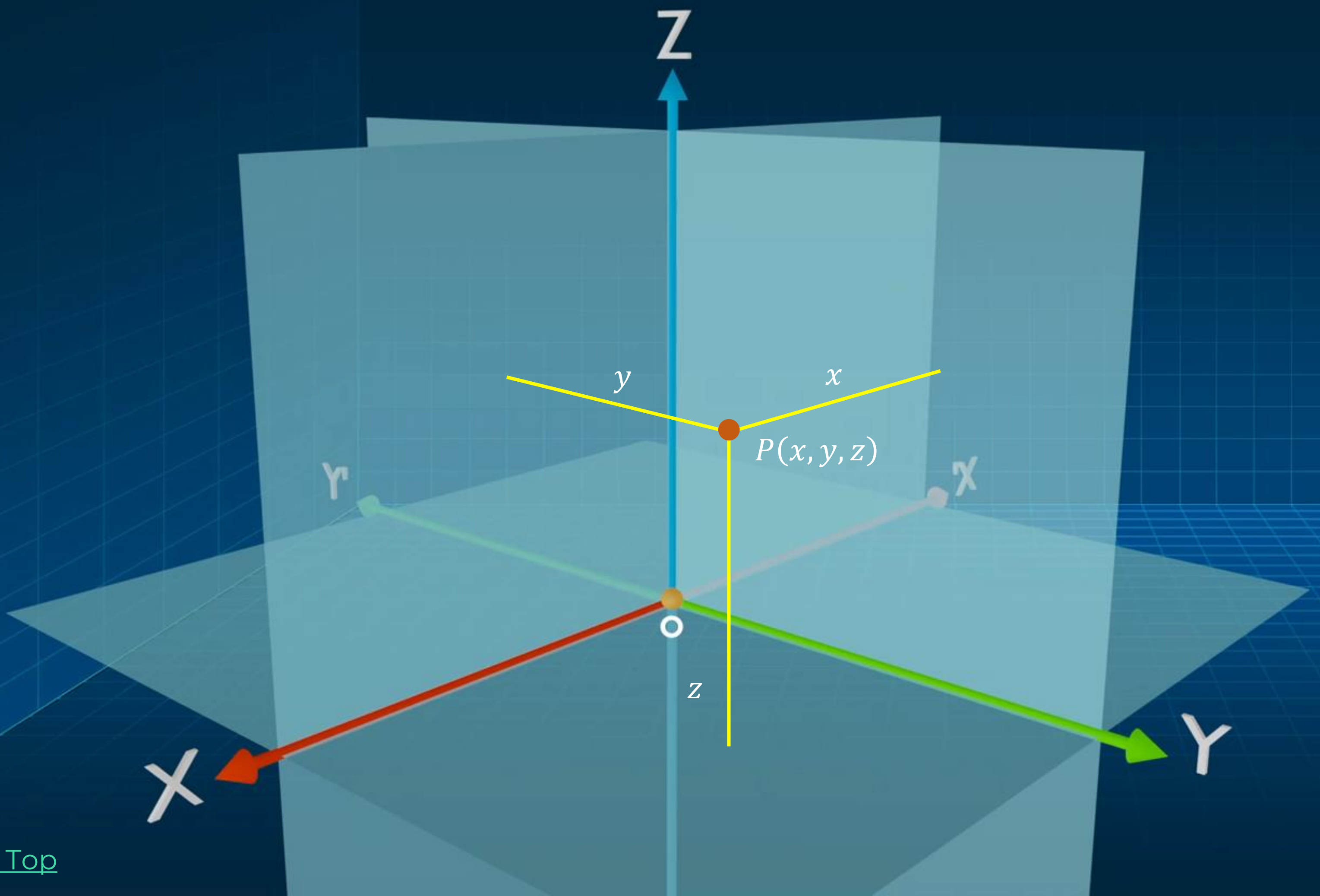

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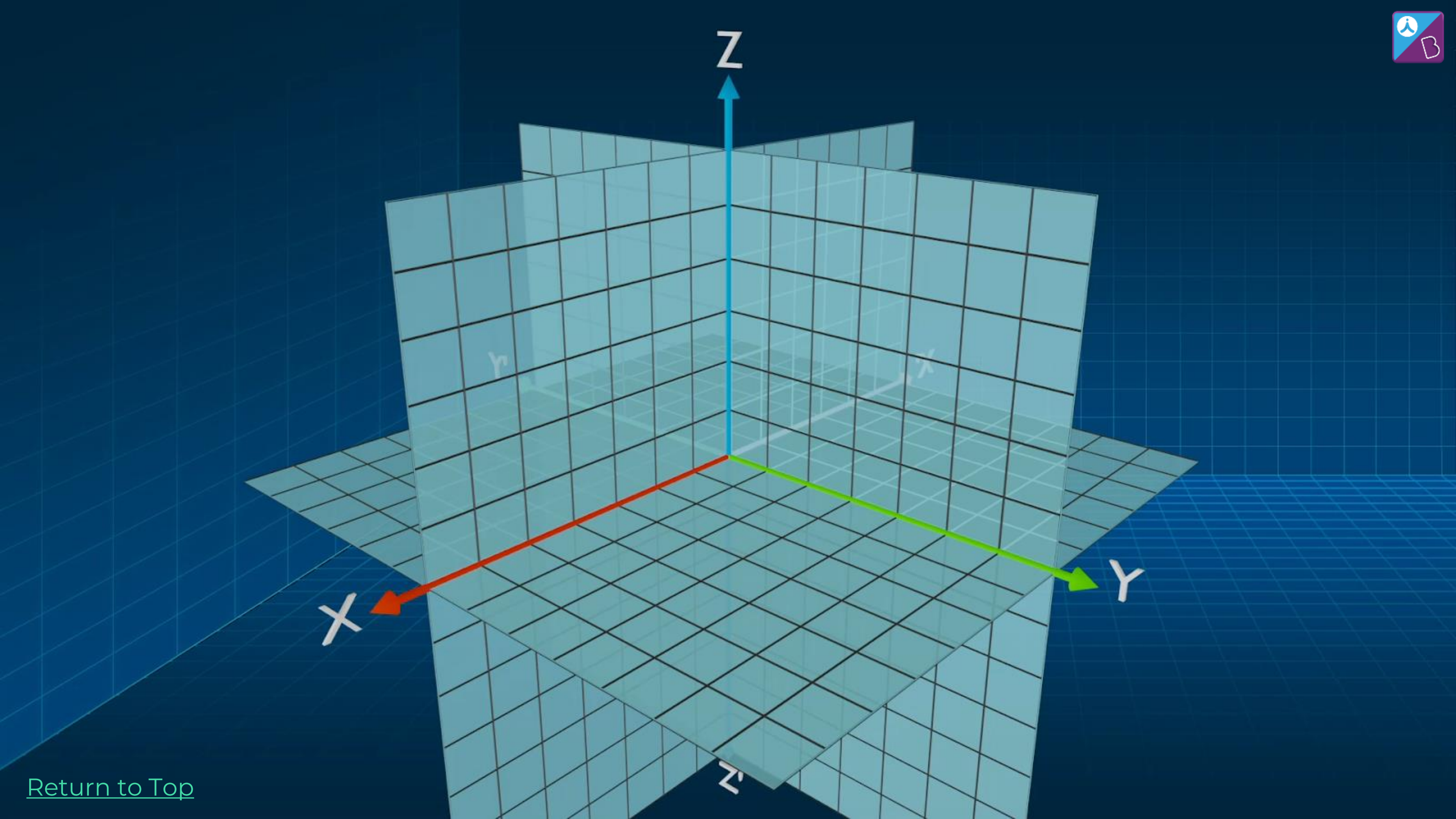

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KEY TAKEAWAYS

Three Dimensional Geometry:

Definition:

It is a geometric setting, in which three different parameters (dimensions) x, y, z are required to determine position of a point.

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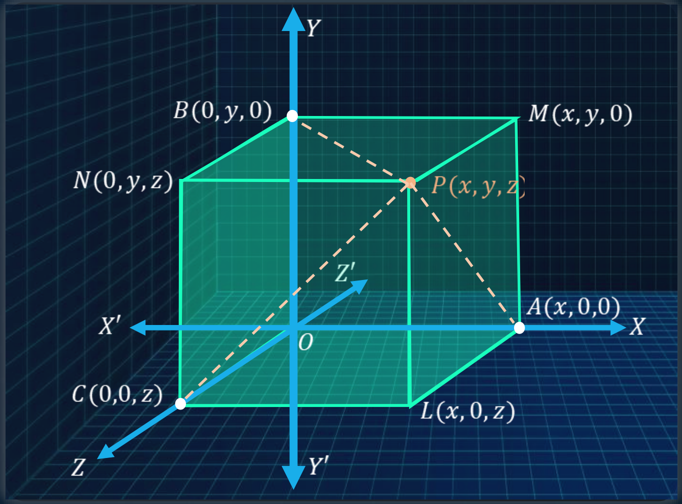
KEY TAKEAWAYS

Coordinate and Position Vector of a point:

$X'X, Y'Y, Z'Z$ are the three coordinate axes.



Note :

Points A, B, C are orthogonal projections of P on the X, Y & Z axes.

Here,

- Point M is in xy plane
- Point N is in yz plane
- Point L is in xz plane

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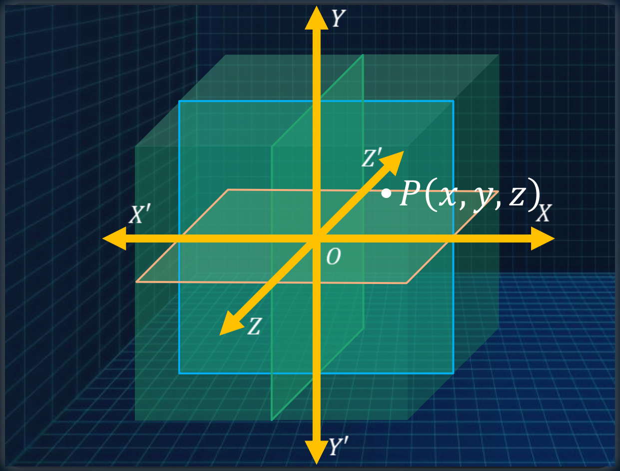
KEY TAKEAWAYS

Octant / Co-ordinate	$OXYZ$	$OX'YZ$	$OXY'Z$	$OXYZ'$	$OX'Y'Z$	$OX'YZ'$	$OXY'Z'$	$OX'Y'Z'$
x	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$
y	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$-$
z	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$



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One of the vertices of a cuboid is $(0, 2, -1)$ and edges from this vertex are along positive x, y and $z -$ axis respectively and are of lengths $2, 2$ & 3 respectively. Then, the coordinates of other vertices are :

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One of the vertices of a cuboid is $(0, 2, -1)$ and edges from this vertex are along positive x, y and $z-$ axis respectively and are of lengths $2, 2$ & 3 respectively. Then, the coordinates of other vertices are :

$P \equiv (0, 2, -1)$

Length of edges are $2, 2, 3$

Other vertices are :

$A(0+2, 2, -1+3) \equiv A(2, 2, 2)$

$B(0, 2, -1+3) \equiv B(0\ 2, 2)$

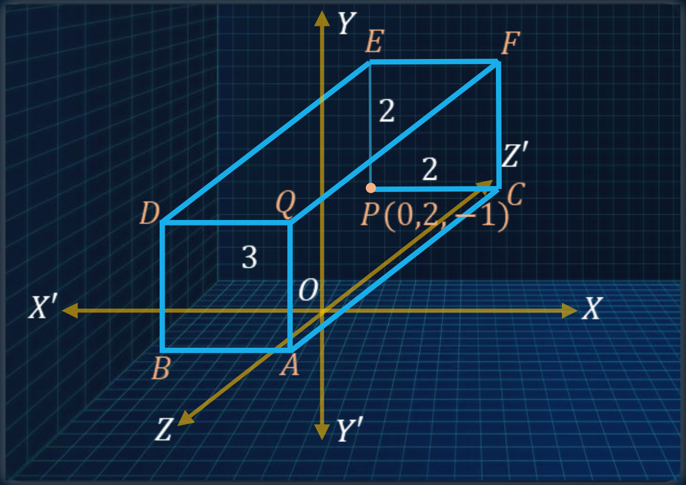
$C(0, +2, 2, -1) \equiv C(2\ 2, -1)$

$D(0, 2+2, -1+3) \equiv D(0\ 4, 2)$

$E(0, 2+2, -1) \equiv E(0\ 4, -1)$

$F(0+2, 2+2, -1) \equiv F(2\ 4, -1)$

$Q \equiv (2\ 4, 2)$



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Planes are drawn parallel to the coordinate planes through the points $(1, 2, 3)$ and $(2, 4, 7)$. Find the length of edges of cuboid so formed,

A $1, 2, 3$

B $1, 2, 4$

C $2, 2, 3$

D $2, 2, 4$

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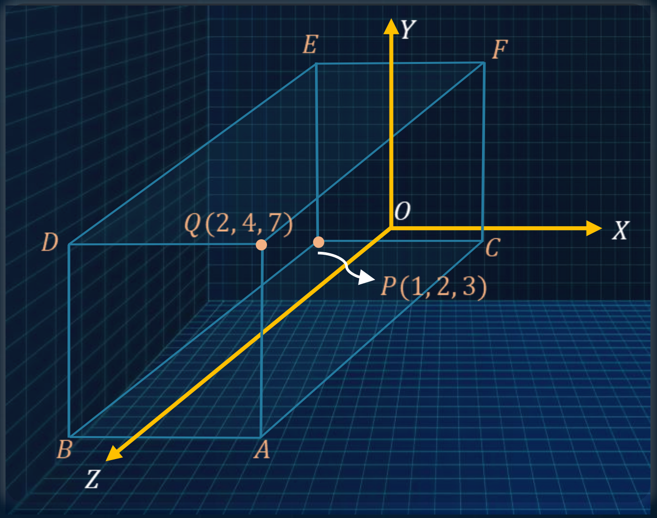
Planes are drawn parallel to the coordinate planes through the points $(1, 2, 3)$ and $(2, 4, 7)$. Find the length of edges of cuboid so formed,

$$P = |2 - 1| = 1$$

$$PE = |4 - 2| = 2$$

$$PB = |7 - 3| = 4$$

$\therefore$ Length of edges are $1, 2, 4$



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A $1, 2, 3$

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C $2, 2, 3$

D $2, 2, 4$

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KEY TAKEAWAYS

Position Vector of a Point:

Let O be origin, then the position vector of a point P is the vector $\overrightarrow{OP}$

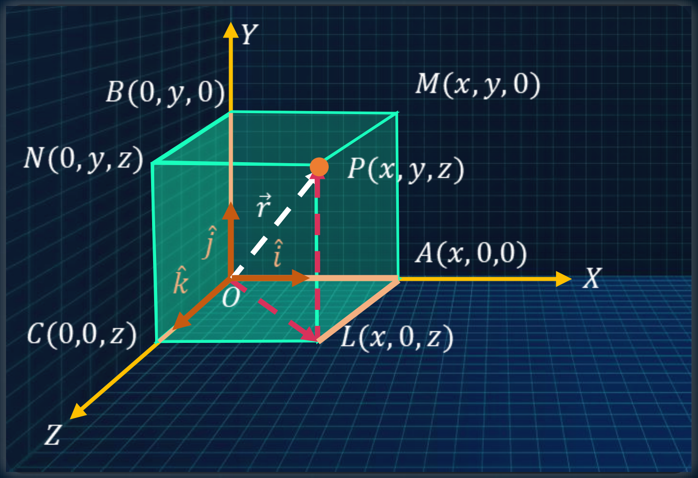
$$\vec{r} = \overrightarrow{OP} = \overrightarrow{OL} + \overrightarrow{LP}$$

$$= \left(\overrightarrow{OA} + \overrightarrow{AL}\right) + \overrightarrow{LP}$$

$$= \left(\overrightarrow{OA} + \overrightarrow{OC}\right) + \overrightarrow{OB}$$

$$= x\hat{\imath} + z\hat{k} + y\hat{\jmath}$$

$\vec{r}$ (position vector of P) $= x\hat{\imath} + y\hat{\jmath} + z\hat{k}$



$$\overrightarrow{OM} = x\hat{\imath} + y\hat{\jmath}$$

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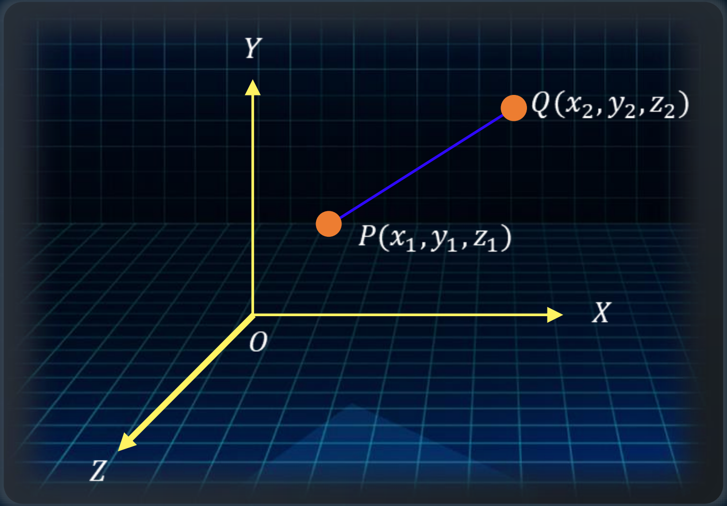
KEY TAKEAWAYS

Distance formula between two points :

$$\text{Distance} = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\overrightarrow{PQ} = (x_2 - x_1)\,\hat{\imath} + (y_2 - y_1)\,\hat{\jmath} + (z_2 - z)\,\hat{k}$$



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The locus of a point P which moves such that $PA^2 - PB^2 = 5$, where A and B are $(3, 4, 5)$ and $(-1, 3, -7)$ respectively, is :

A $8x + 2y + 24z - 9 = 0$

B $8x + 2y + 24z + 14 = 0$

C $8x - 2y + 24z - 5 = 0$

D $8x - 2y - 24z + 13 = 0$

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The locus of a point P which moves such that $PA^2 - PB^2 = 5$, where A and B are $(3, 4, 5)$ and $(-1, 3, -7)$ respectively, is :

Let $P \equiv (x, y, z)$, $PA^2 - PB^2 = 5$

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$$

$$PA^2 - PB^2 = 5 \quad \Rightarrow \left((x-3)^2 + (y-4)^2 + (z-5)^2\right)$$
$$- \left((x+1)^2 + (y-3)^2 + (z+7)^2\right) = 5$$

$$\Rightarrow (\, x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 102 + 25)$$
$$- (x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49) = 5$$

$$\Rightarrow -8x - 2y - 24z - 9 = 5$$

$\therefore$ Locus of $P : 8x + 2y + 24z + 14 = 0$

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C $8x - 2y + 24z - 5 = 0$

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KEY TAKEAWAYS

Distance of a Point from Co-ordinate Axis:

Distance of P from x –axis = PA

$$PA = \sqrt{(x-x)^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$$

Distance of P from y – axis = PB

$$PB = \sqrt{x^2 + (y-y)^2 + z^2} = \sqrt{x^2 + z^2}$$

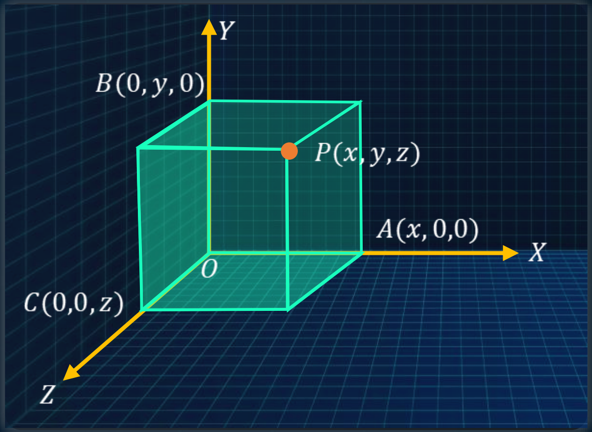
Distance of P from z – axis = PC

$$PC = \sqrt{x^2 + y^2 + (z-z)^2} = \sqrt{x^2 + y^2}$$

Projection of point on x – axis ≡ A

Projection of point on y – axis ≡ B

Projection of point on z – axis ≡ C



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If the sum of the squares of the distances of a point from the three coordinate axes be 36, then its distance from origin is :

A. 6

B. $3\sqrt{2}$

C. $6\sqrt{2}$

D. $2\sqrt{3}$

Let $P \equiv (x, y, z)$

$$PA^2 + PB^2 + PC^2 = 36$$

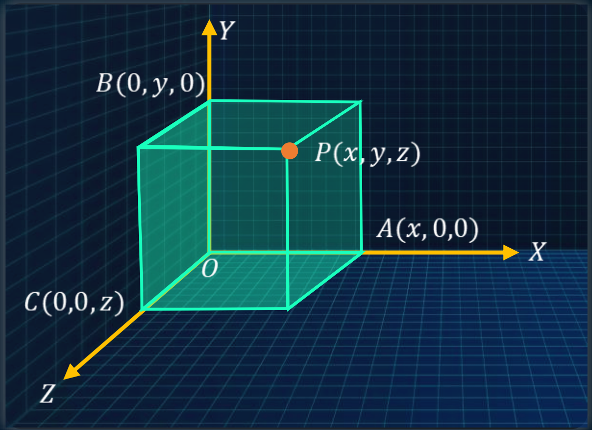
$$(y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2) = 36$$

$$\Rightarrow 2(x^2 + y^2 + z^2) = 36$$

$$\Rightarrow x^2 + y^2 + z^2 = 18$$

$$\Rightarrow OP = \sqrt{x^2 + y^2 + z^2} = \sqrt{18}$$

$$= 3\sqrt{2}$$



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If the sum of the squares of the distances of a point from the three coordinate axes be 36, then its distance from origin is :

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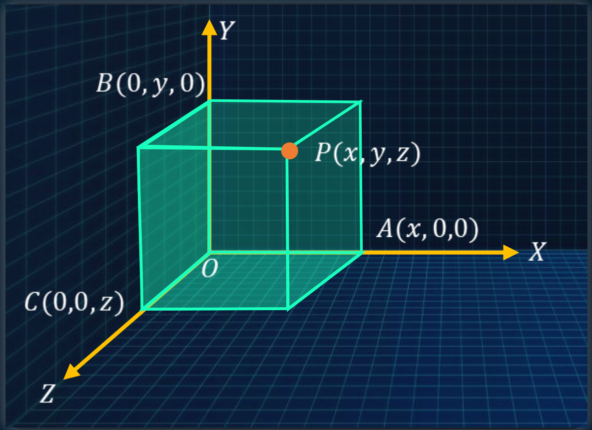

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A point moves so that the sum of the squares of its distances from the six faces of a cube given by $x = \pm 1, y = \pm 1, z = \pm 1$ is 10 units. Then the locus of the point is :

A $x^2 + y^2 + z^2 = 1$

B $x + y + z = 1$

C $x^2 + y^2 + z^2 = 2$

D $x + y + z = 2$

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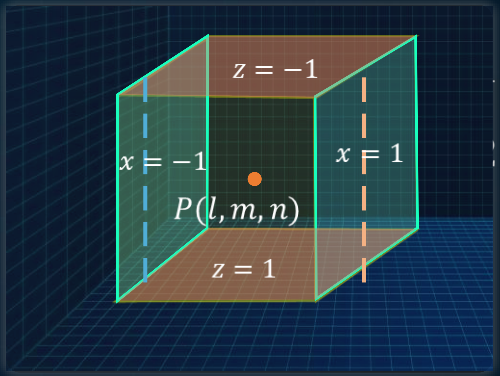
A point moves so that the sum of the squares of its distances from the six faces of a cube given by $x = \pm 1, y = \pm 1, z = \pm 1$ is 10 units. Then the locus of the point is :

Let $P \equiv (l, m, n)$

Distance of P from $x = 1 \Rightarrow |l - 1|$



$$\underbrace{\Rightarrow (l+1)^2 + (m+1)^2 + (n+1)^2 + (l-1)^2 + (m-1)^2 + (n-1)^2}_{=10}$$

$(l+1)^2 + (m+1)^2 + (n+1)^2 \rightarrow x = -1, y = -1, z = -1$

$(l-1)^2 + (m-1)^2 + (n-1)^2 \rightarrow x = 1, y = 1, z = 1$

$$\Rightarrow l^2 + 2l + 1 + m^2 + 2m + 1 + n^2 + 2n + 1 + l^2 - 2l + 1$$
$$+ m^2 - 2m + 1 + n^2 - 2n + 1 = 10$$

$$\Rightarrow 2\,(l^2 + m^2 + n^2) + 6 = 10 \Rightarrow 2\,(l^2 + m^2 + n^2) = 4$$

Generalise, $l \rightarrow x, m \rightarrow y, n \rightarrow z$

$$x^2 + y^2 + z^2 = 2$$

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A point moves so that the sum of the squares of its distances from the six faces of a cube given by $x = \pm 1, y = \pm 1, z = \pm 1$ is 10 units. Then the locus of the point is :

A $x^2 + y^2 + z^2 = 1$

B $x + y + z = 1$

C $x^2 + y^2 + z^2 = 2$

D $x + y + z = 2$

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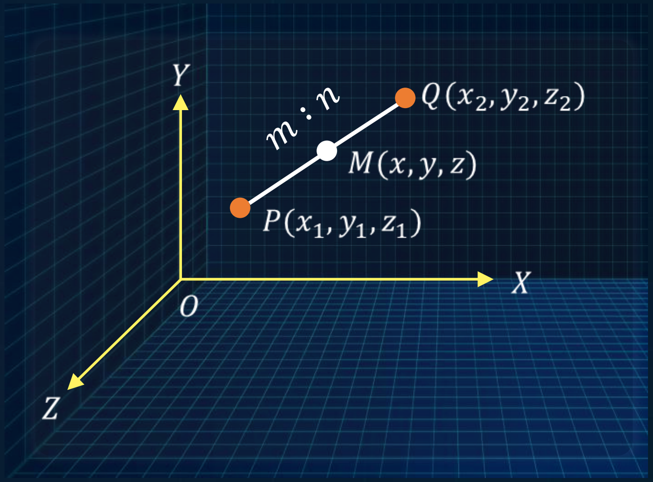
KEY TAKEAWAYS

Section Formula :

Coordinate of a point M which divides the line segment joining points P & Q in $m : n$, is :

$$M \equiv (x, y, z)$$

$$M \equiv \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_1+nz_1}{m+n}\right)$$



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If a point $R\ (4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from origin is :

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A) 6

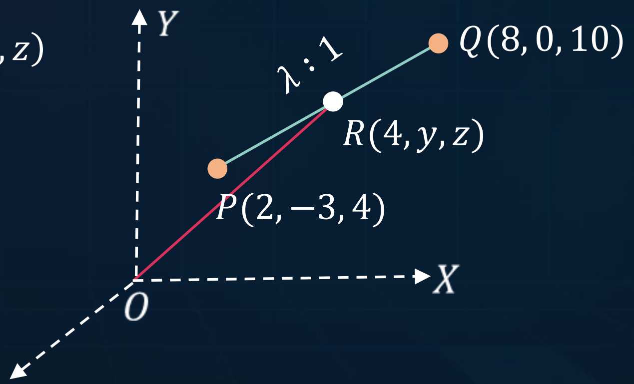
B) $\sqrt{53}$

C) $2\sqrt{14}$

D) $2\sqrt{21}$

Let $\frac{PR}{RQ} = \frac{\lambda}{1}$ (internally)

Section Formula $R \equiv \left(\frac{8\lambda+2}{\lambda+1}, \frac{0+(-3)}{\lambda+1}, \frac{10\lambda+4}{\lambda+1}\right) \equiv (4, y, z)$



$\therefore \frac{8\lambda+2}{\lambda+1} = 4 \Rightarrow 8\lambda + 2 = 4\lambda + 4$

$4\lambda = 2 \Rightarrow \lambda = \frac{1}{2}$

Put λ in $R\ (4, -2, 6)$

$OR = \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14}$

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If a point $R\ (4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from origin is :

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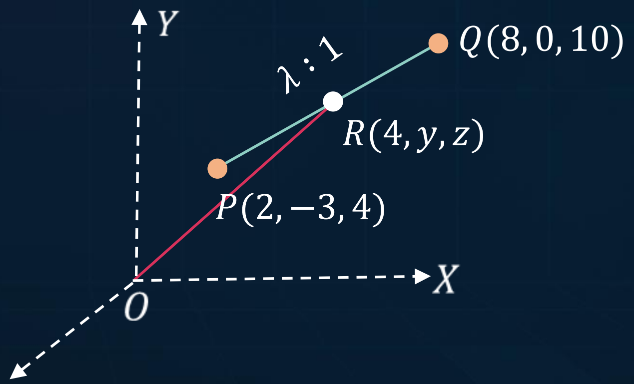

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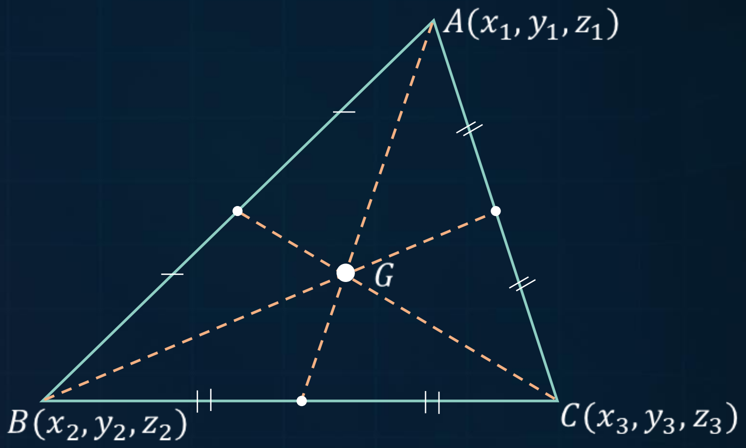
KEY TAKEAWAYS

Centroid of a Triangle

Coordinate of centroid G is :

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$



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Let $A(3,0,-1), B(2,10,6)$ & $C\ (1,2,1)$ be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio 2 : 1, then $\cos(\angle GOA)$, where O is the origin, is equal to

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A $\frac{1}{\sqrt{15}}$

B $\frac{1}{6\sqrt{10}}$

C $\frac{1}{\sqrt{30}}$

D $\frac{1}{2\sqrt{15}}$

G is the centroid

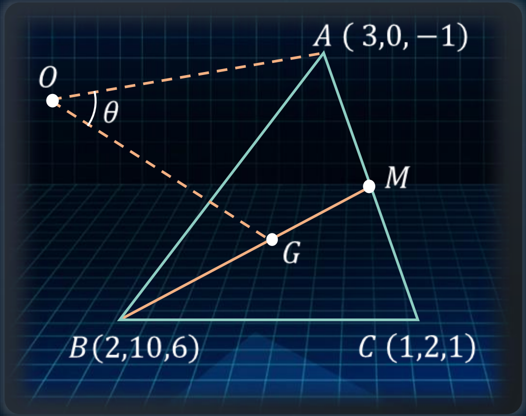
$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$G \equiv \left(\frac{3+2+1}{3}, \frac{10+0+2}{3}, \frac{-1+6+1}{3}\right) \Rightarrow G \equiv (2,4,2)$$

$$\cos\theta = \widehat{OA}\cdot\widehat{OG} = \frac{\overrightarrow{BA}\cdot\overrightarrow{OG}}{|\overrightarrow{OA}|\cdot|\overrightarrow{OG}|}$$

$$\overrightarrow{OA} = 3\hat{\imath} - \hat{k}, \overrightarrow{OG} = 2\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$$

$$\cos\theta = \frac{6-2}{\sqrt{10}\cdot\sqrt{24}} = \frac{4}{4\sqrt{15}} \quad \therefore \cos\theta = \frac{1}{\sqrt{15}}$$



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A $\frac{1}{\sqrt{15}}$

B $\frac{1}{6\sqrt{10}}$

C $\frac{1}{\sqrt{30}}$

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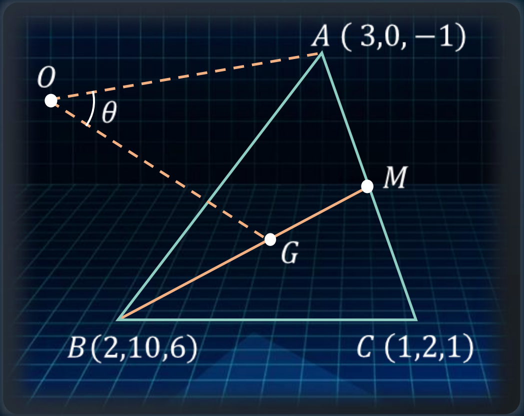

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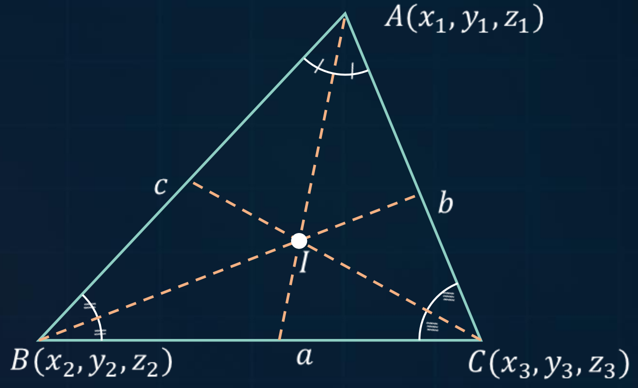
Incentre of a Triangle

Coordinate of incentre I is :

$$G \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$



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The vertices of a triangle are $A\ (1,1,2), B(4,3,1)$ and $C\ (2,3,5)$. Then vector representing internal bisector of the angle A is :

A $\hat{\imath} + \hat{\jmath} + 2\hat{k}$

B $2\hat{\imath} - 2\hat{\jmath} + \hat{k}$

C $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

D $2\hat{\imath} + 2\hat{\jmath} + \hat{k}$

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The vertices of a triangle are $A\ (1,1,2), B(4,3,1)$ and $C\ (2,3,5)$. Then vector representing internal bisector of the angle A is :

$$AB = \sqrt{3^2+2^2+1^2} = \sqrt{14}$$

$$AC = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$\Rightarrow ABC$ is an isosceles triangle.

$\therefore$ Median acts as an angle bisector for angle A.

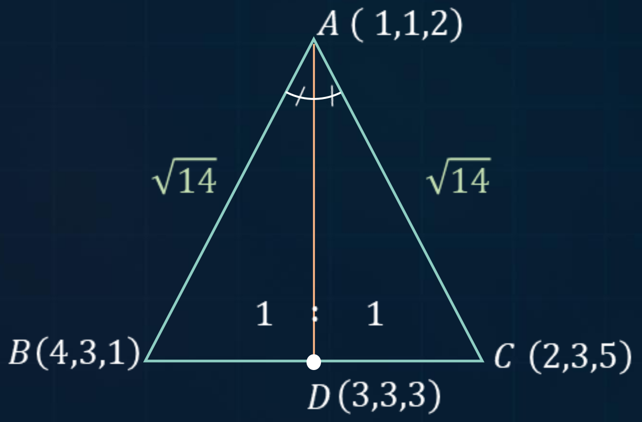
D divides BC in ratio of $AB : AC$

$\Rightarrow D$ is mid point

$$D \equiv (3,3,3) \Rightarrow \overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$= (3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}) - (\hat{\imath} + \hat{\jmath} + 2\hat{k})$$

$$\therefore \overrightarrow{AD} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$$



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The vertices of a triangle are $A\,(1,1,2), B(4,3,1)$ and $C\,(2,3,5)$. Then vector representing internal bisector of the angle A is :

A $\hat{i}+\hat{j}+2\hat{k}$

B $2\hat{i}-2\hat{j}+\hat{k}$

C $2\hat{i}+2\hat{j}-\hat{k}$

D $2\hat{i}+2\hat{j}+\hat{k}$

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Session 02

Direction ratios and direction cosines of a line

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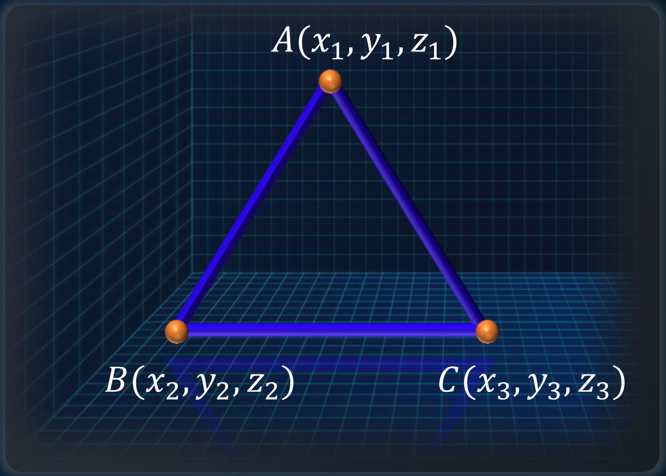
Area of a triangle

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be vertices of a triangle, then

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$\boxed{\text{Area} = \left|\frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}\right|}$$



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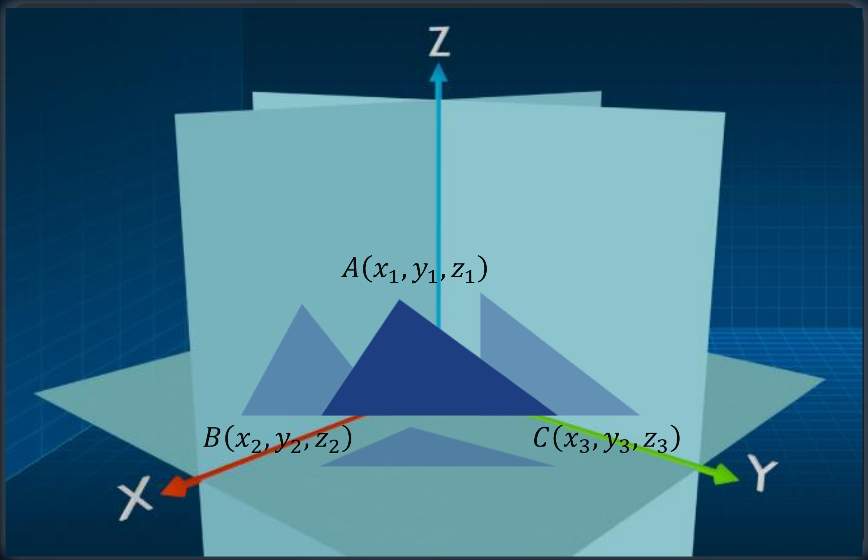
Area of a triangle

Let Δ_x, Δ_y and Δ_z be the area of the projections of the triangle to the YZ, XZ, XY planes respectively.



$$\text{Area of triangle } (\Delta) = \sqrt{{\Delta_x}^2 + {\Delta_y}^2 + {\Delta_z}^2}$$

$$\Delta_x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \Delta_y = \frac{1}{2}\begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}$$

$$\Delta_z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

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The area of triangle formed by joining points $(2,-1,1),(1,-3,-5)$ & $(3,-4,-4)$ is :

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The area of triangle formed by joining points $(2,-1,1),(1,-3,-5)$ & $(3,-4,-4)$ is :

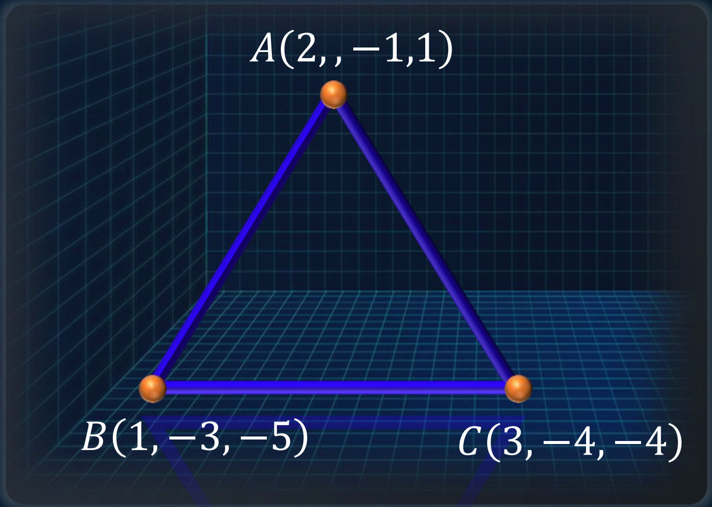
Solution:

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}\right|$$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -2 & -6 \\ 1 & -3 & -5 \end{vmatrix}\right|$$

$$= \frac{\sqrt{210}}{2} \text{ square unit}$$



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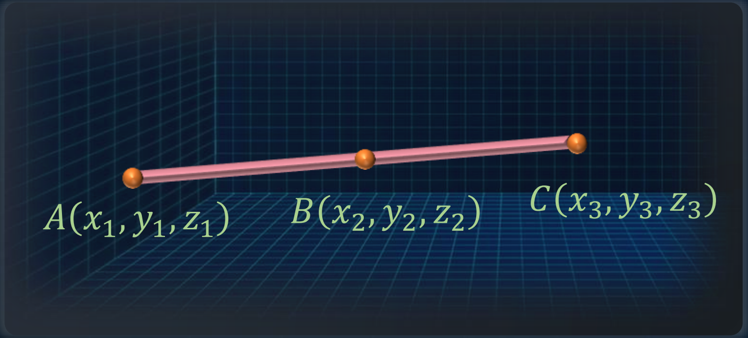
KEY TAKEAWAYS

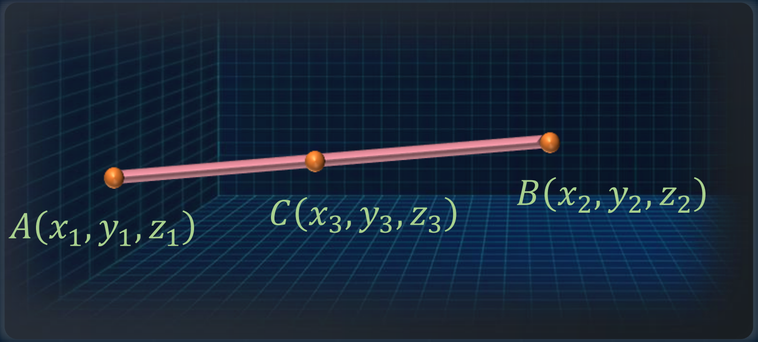
Condition of collinearity

The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear if :

Using Distance formula :



i.e. $AB + BC = AC$



i.e. $AB - BC = AC$

$$\boxed{AB \pm BC = AC}$$

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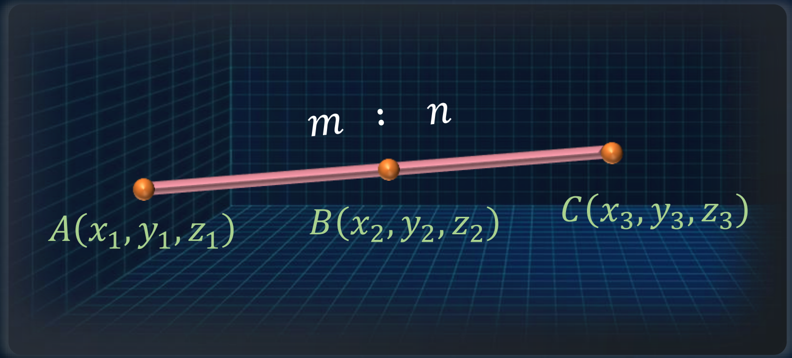
KEY TAKEAWAYS

Condition of collinearity

The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear if :

Using section formula :



Point B divides A & C in ration $m : n$

$$\boxed{x_2 = \frac{mx_3 + nx_1}{m+n}, \quad y_2 = \frac{my_3 + ny_1}{m+n}, z_2 = \frac{mz_3 + nz_1}{m+n}}$$

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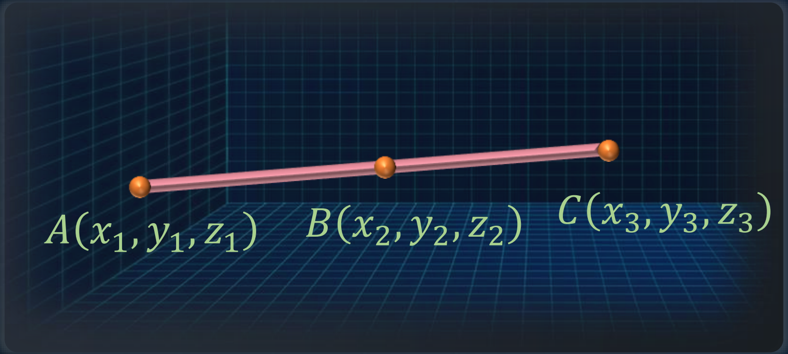
KEY TAKEAWAYS

Condition of collinearity

The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear if :

Using area of triangle :



$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} \right| = 0$$

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KEY TAKEAWAYS

Condition of collinearity

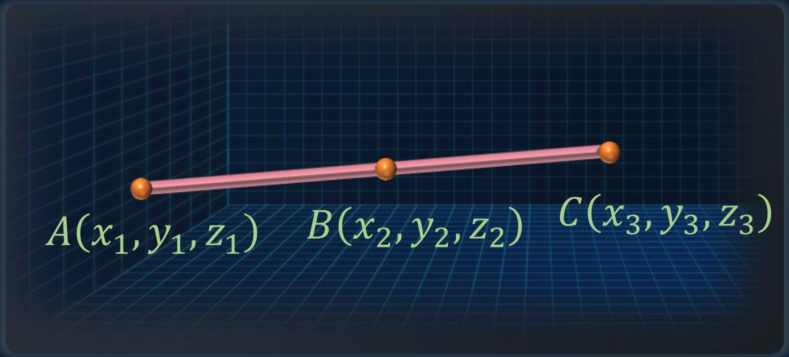
The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear if :

Using vectors :

$$\overrightarrow{AC} \parallel \overrightarrow{AB}$$

$$\overrightarrow{AC} = \lambda \overrightarrow{AB}$$

$$\boxed{\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}}$$



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If the points $(4, 5, 1), (3, 2, 4)$ & $(-1, -10, p)$ are collinear, then value of p is:

A 14

B 15

C 16

D 17

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If the points $(4, 5, 1), (3, 2, 4)$ & $(-1, -10, p)$ are collinear , then value of p is:

Solution:

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$$

$$\frac{-1 - 4}{3 - 4} = \frac{-10 - 5}{2 - 5} = \frac{p - 1}{4 - 1}$$

$$\Rightarrow 5 = 5 = \frac{p - 1}{3}$$

$$\Rightarrow p - 1 = 3 \times 5$$

$$\Rightarrow p = 16$$

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If the points $(4, 5, 1), (3, 2, 4)$ & $(-1, -10, p)$ are collinear, then value of p is:

A. 14

B. 15

C. 16

D. 17

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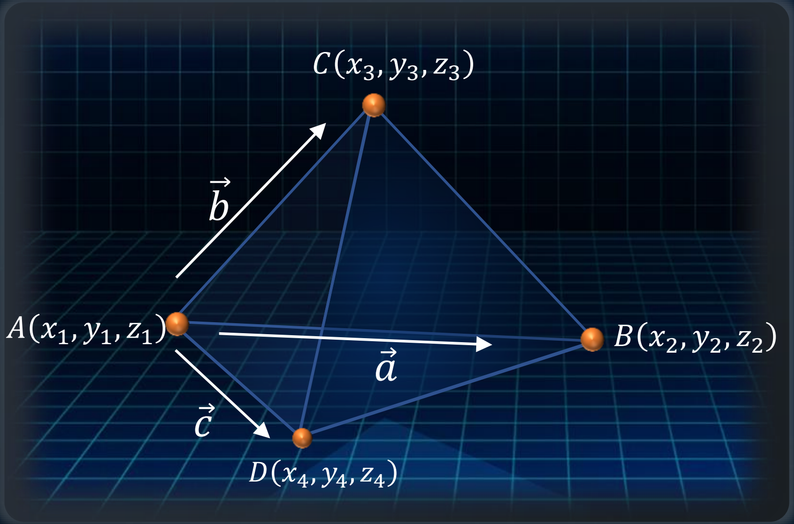
Volume of Tetrahedron

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ be vertices of a tetrahedron, then



$$V = \frac{1}{6}\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$$

$$\boxed{V = \left|\frac{1}{6}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix}\right|}$$

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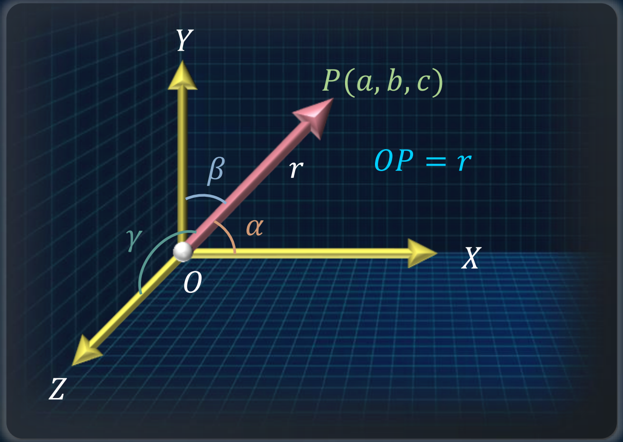
KEY TAKEAWAYS

Direction Cosines of a line

Let α, β, γ be the angles which the directed line makes with the positive directions of the axes of x, y & z respectively, then $\cos\alpha$, $\cos\beta$ & $\cos\gamma$ are called the direction cosines of the line (D.C.'s).



They are usually denoted by l, m, n.

$$\cos\alpha = \frac{a}{r} = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$\cos\beta = \frac{b}{r} = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$\cos\gamma = \frac{c}{r} = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Note $\alpha + \beta + \gamma \neq 2\pi$

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Direction Cosines of a line

The D.C.'s are usually denoted by $l\,, m\,, n$.

$$l = \cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

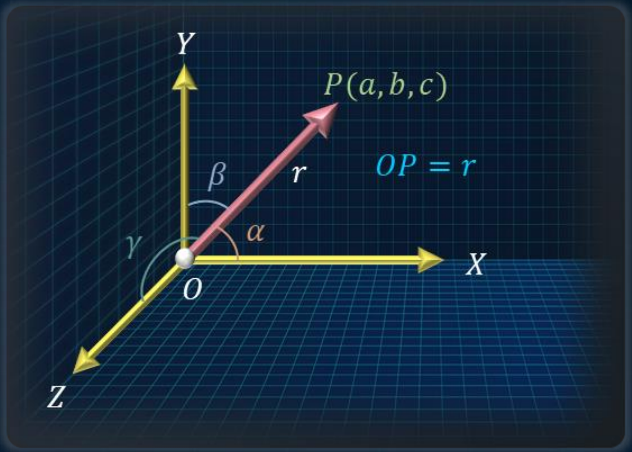
$$m = \cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l^2 + m^2 + n^2 = \frac{a^2}{a^2+b^2+c^2} + \frac{b^2}{a^2+b^2+c^2} + \frac{c^2}{a^2+b^2+c^2}$$

$$= \frac{a^2+b^2+c^2}{a^2+b^2+c^2}$$

$$\boxed{\therefore\ l^2 + m^2 + n^2 = 1}$$



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Direction Cosines of a line

The D.C.'s are usually denoted by l, m, n.

- $OP = r$

  D.C.'s $= l, m, n$

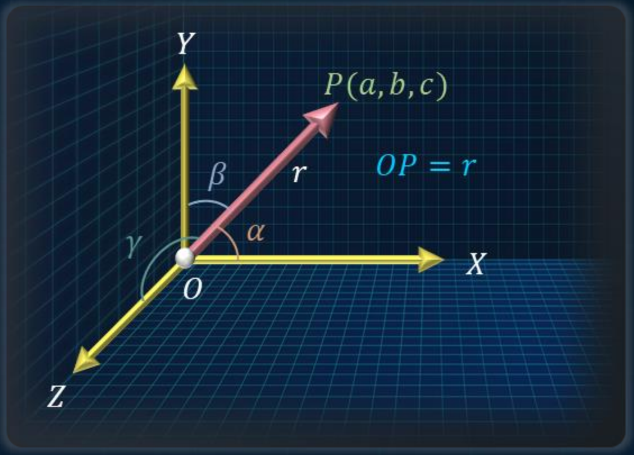
  $$\boxed{\Rightarrow P \equiv (lr, mr, nr)}$$

- $PQ = r$

  D.C.'s $= l, m, n$

  $P(x_1, y_1, z_1)$

  $$\boxed{Q \equiv (x_1 + lr, y_1 + mr, z_1 + nr)}$$



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Direction cosines (D.C.'s) of a line equally inclined with the positive direction of the coordinate axes, is _____.

A $1, 1, 1$

B $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

C $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

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Direction cosines (D.C.'s) of a line equally inclined with the positive direction of the coordinate axes, is _____.

A $1, 1, 1$

B $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

C $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Solution:

$\alpha = \beta = \gamma$ $\quad l = \cos\alpha\ , m = \cos\beta\ , n = \cos\gamma$

$\boxed{l^2 + m^2 + n^2 = 1}$ $\quad l = \cos\alpha = m = n$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}} = l$

Thus , direction cosines : $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

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Direction cosines (D.C.'s) of a line equally inclined with coordinate axes, is ____.

A $1, 1, 1$

B $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

C $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

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If a line makes angles α, β, γ with positive x, y, z axes respectively, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is :

A 1

B 2

C 3

D 4

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If a line makes angles α, β, γ with positive x, y, z axes respectively, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is :

Solution:

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

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If a line makes angles α, β, γ with positive x, y, z axes respectively, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is :

A) 1

B) 2

C) 3

D) 4

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KEY TAKEAWAYS

Direction Ratios of a line

If a, b, c be proportional to the direction cosines (D.C.'s) l, m, n , then a, b, c are called direction ratios (D.R.'s).

Example | Let the D.C.'s of a line be : $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$, then

DRs can be : $2\,, -2\,, 1$

or $-6\,, 6\,, -3$

or $2\sqrt{7}\,, -2\sqrt{7}\,, \sqrt{7}$

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KEY TAKEAWAYS

Direction Ratios of a line

Let (a, b, c) be the D.R.'s and l, m, n be the D.C's of a line, then

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda \Rightarrow l = \frac{a}{\lambda}, m = \frac{b}{\lambda}$$

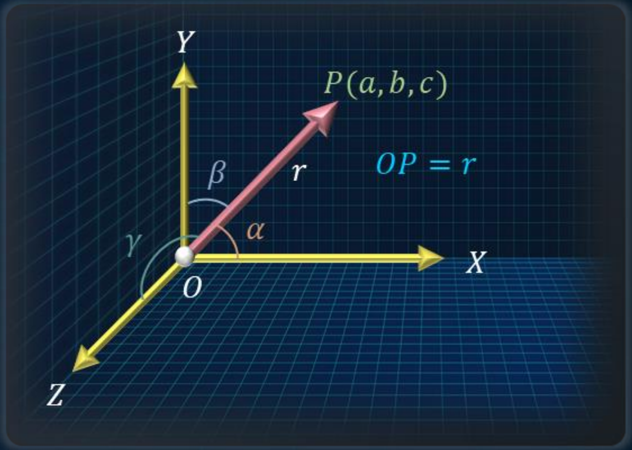
$$l^2 + m^2 + n^2 \Rightarrow \frac{a^2}{\lambda^2} + \frac{b^2}{\lambda^2} + \frac{c^2}{\lambda^2} = 1 \Rightarrow \lambda^2 = (a^2 + b^2 + c^2)$$

$$\Rightarrow \lambda = \pm\sqrt{a^2 + b^2 + c^2}$$

$$\boxed{l, m, n \equiv \left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)}$$

or

$$\boxed{l, m, n \equiv \left(-\frac{a}{\sqrt{a^2+b^2+c^2}}, -\frac{b}{\sqrt{a^2+b^2+c^2}}, -\frac{c}{\sqrt{a^2+b^2+c^2}}\right)}$$



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KEY TAKEAWAYS

Direction Ratios and Direction Cosines of a line

- If a, b, c be the D.R.'s of any line L, then

  $a\hat{i} + b\hat{j} + c\hat{k}$ will be a vector parallel to the line .

- If l, m, n be the D.C.'s of any line L, then

  $l\hat{i} + m\hat{j} + n\hat{k}$ will be a unit vector parallel to the line .

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KEY TAKEAWAYS

Direction Ratios and Direction Cosines of a line

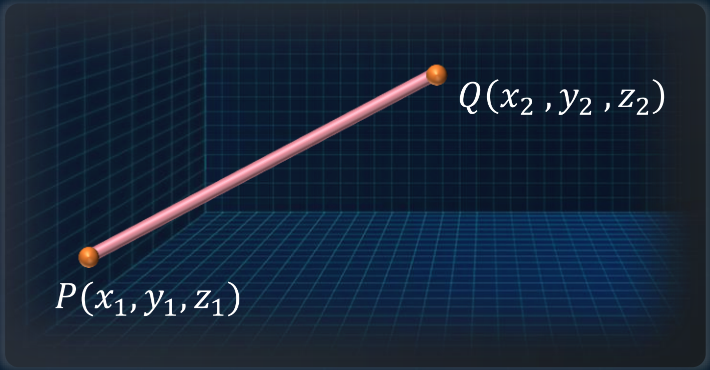
If $P \equiv (x_1, y_1, z_1)$ & $Q \equiv (x_2, y_2, z_2)$, then

- The D.R.'s of line PQ will be

$$\boxed{a = x_2 - x_1\ ,\ b = y_2 - y_1\ ,\ c = z_2 - z_1}$$

- The D.C.'s of line PQ will be

$$\boxed{l = \frac{x_2 - x_1}{|PQ|}\ ,\ m = \frac{y_2 - y_1}{|PQ|}\ ,\ n = \frac{z_2 - z_1}{|PQ|}}$$



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Consider a cube whose edges are parallel to coordinate axes. Then the direction ratios (D.R.'s) and direction cosines (D.C.'s) of its body diagonals, is :

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Consider a cube whose edges are parallel to coordinate axes. Then the direction ratios (D.R.'s) and direction cosines (D.C.'s) of its body diagonals, is :

Solution:

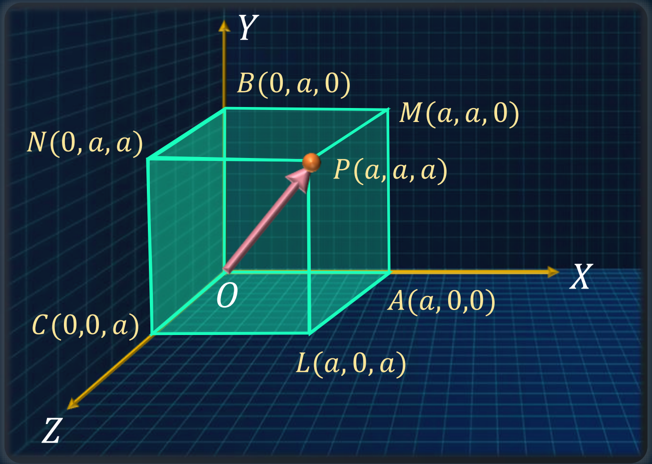
Let side of cube be a

OP : D.R.'s : $(1,1,1)$

D.C.'s : $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

BL : D.R.'s : $(1,-1,1)$

D.C.'s : $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$



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Consider a cube whose edges are parallel to coordinate axes. Then the direction ratios (D.R.'s) and direction cosines (D.C.'s) of its body diagonals, is :

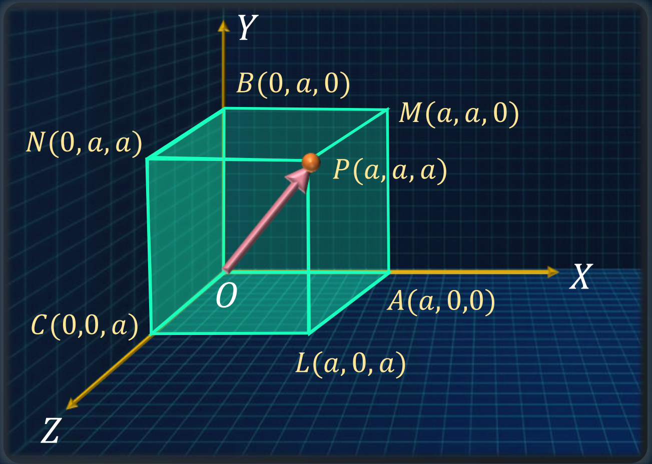
Solution:

AN : D.R.'s : $(-1,1,1)$

D.C.'s : $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

CM : D.R.'s : $(1,1,-1)$

D.C.'s : $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$



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Session 03

Equation of a straight line in 3 –D form

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The direction cosines (D.C.'s) l, m, n of a line which are connected by the relations $l+m+n=0;\ 2lm+2mn-nl=0$, is:

A $-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

C $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

B $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

D $-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

$l+m+n=0 \quad \&;\ 2lm+2mn-nl=0$

Put $n=-l-m$

$\Rightarrow 2lm+(2m-l)(n)=0$

$\Rightarrow 2lm+(2m-l)(-l-m)=0$

$\Rightarrow 2lm-2lm-2m^2+l^2+lm=0$

$\Rightarrow l^2+lm-2m^2=0$

$\Rightarrow (l+2m)(l-m)=0$

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The direction cosines (D.C.'s) l, m, n of a line which are connected by the relations $l + m + n = 0$; $2lm + 2mn - nl = 0$, is:

$$l + m + n = 0 \quad \&; 2lm + 2mn - nl = 0$$

$$\Rightarrow (l + 2m)(l - m) = 0$$

$l = -2m$	$l = m$
$\Rightarrow n = -l - m$	$\Rightarrow n = -l - m$
$\Rightarrow n = m$	$\Rightarrow n = -2m$
$l : m : n :: -2m : m : m$	$l : m : n :: m : m : -2m$
$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{1}$	$\Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{-2}$
DRS $\propto (-2, 1, 1)$	DRS $\propto (1, 1, -2)$

$\therefore$ D.C.'s can be: $-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$ or $-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

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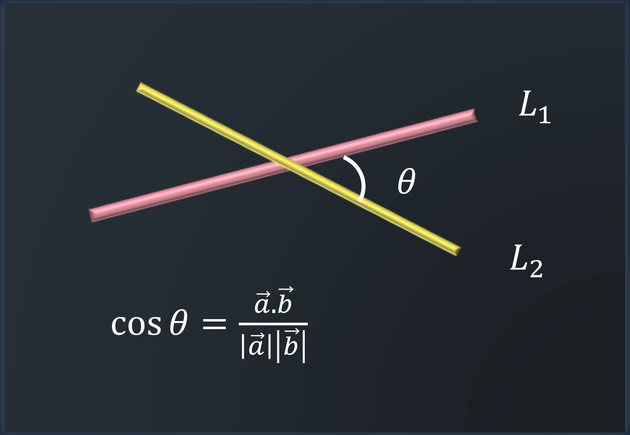
KEY TAKEAWAYS

Angle between two lines

If two lines have D.R.'s a_1, b_1, c_1 and a_2, b_2, c_2 respectively (parallel vectors will be $a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$ and $a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ respectively).
Let θ is the angle between them, then



$$\theta = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$

Lines will be parallel, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Lines will be perpendicular, if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

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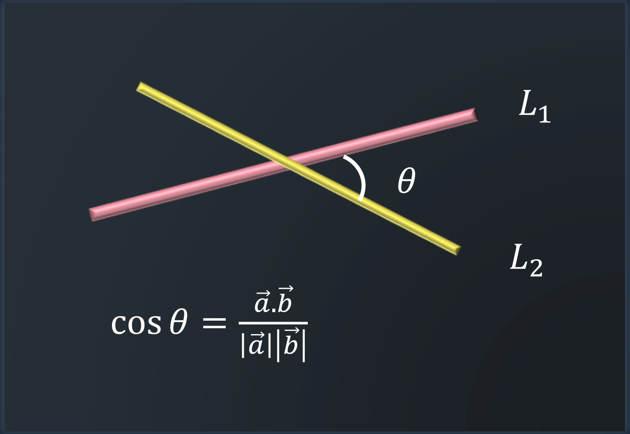
KEY TAKEAWAYS

Angle between two lines

If two lines have D.C.'s l_1, m_1, n_1 and l_2, m_2, n_2 respectively (parallel unit vectors will be $l_1\hat{\imath} + m_1\hat{\jmath} + n_1\hat{k}$ and $l_2\hat{\imath} + m_2\hat{\jmath} + n_2\hat{k}$ respectively). Let θ is the angle between them, then

$$\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$$



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The angle between any two body diagonals of a cube, is :

A $\cos^{-1}\left(\frac{4}{9}\right)$

B $\cos^{-1}\left(\frac{1}{3}\right)$

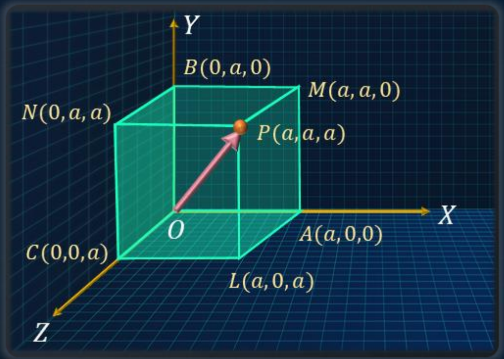
C $\cos^{-1}\left(\frac{2}{3}\right)$

D $\cos^{-1}\left(\frac{2}{9}\right)$

$$\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$$

OP : Direction cosines : $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

BL : Direction cosines : $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$



$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

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The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ & $l^2 = m^2 + n^2$, is:

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A $\frac{\pi}{3}$

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{2}$

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The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ & $l^2=m^2+n^2$, is:

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$$l+m+n=0 \ \ \& \ \ l^2=m^2+n^2$$

$$\Rightarrow l=-(m+n)\cdots(i)$$

Squaring (i),

$$\Rightarrow l^2=m^2+n^2+2mn$$

$$\Rightarrow l^2=l^2+2mn$$

$$\Rightarrow 2mn=0$$

$$\Rightarrow m=0 \text{ or } n=0 \begin{cases} \text{For } m=0,\ l=\frac{1}{\sqrt{2}}, n=-\frac{1}{\sqrt{2}} \\ \text{For } n=0,\ l=\frac{1}{\sqrt{2}}, m=-\frac{1}{\sqrt{2}} \end{cases}$$

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The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ & $l^2=m^2+n^2$, is:

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$$\Rightarrow m=0 \text{ or } n=0 \begin{cases} \text{For } m=0,\ l=\frac{1}{\sqrt{2}}, n=-\frac{1}{\sqrt{2}} \\ \text{For } n=0,\ l=\frac{1}{\sqrt{2}}, m=-\frac{1}{\sqrt{2}} \end{cases}$$

$\therefore$ D.C.'s will be : $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ or $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

$$\theta=\cos^{-1}(l_1l_2+m_1m_2+n_1n_2)$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

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The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ & $l^2 = m^2 + n^2$, is:

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A $\frac{\pi}{3}$

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{2}$

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The coordinates of points A, B, C, D are $(4, \alpha, 2), (5, -3, 2), (\beta, 1, 1)$ & $(3, 3, -1)$ respectively. Line AB would be perpendicular to line CD when :

A $\alpha = -1, \beta = -1$

B $\alpha = 2, \beta = -1$

C $\alpha = 1, \beta = 2$

D $\alpha = 2, \beta = 2$

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The coordinates of points A, B, C, D are $(4, \alpha, 2), (5, -3, 2), (\beta, 1, 1)$ & $(3, 3, -1)$ respectively. Line AB would be perpendicular to line CD when :

Solution:

D.R.'s of line AB : $1, -3-\alpha, 0$

D.R.'s of line CD : $3-\beta, 2, -2$

Lines will be perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 3 - \beta - 6 - 2\alpha = 0$

$\Rightarrow 2\alpha + \beta = -3$

Possible when, $\alpha = -1, \beta = -1$

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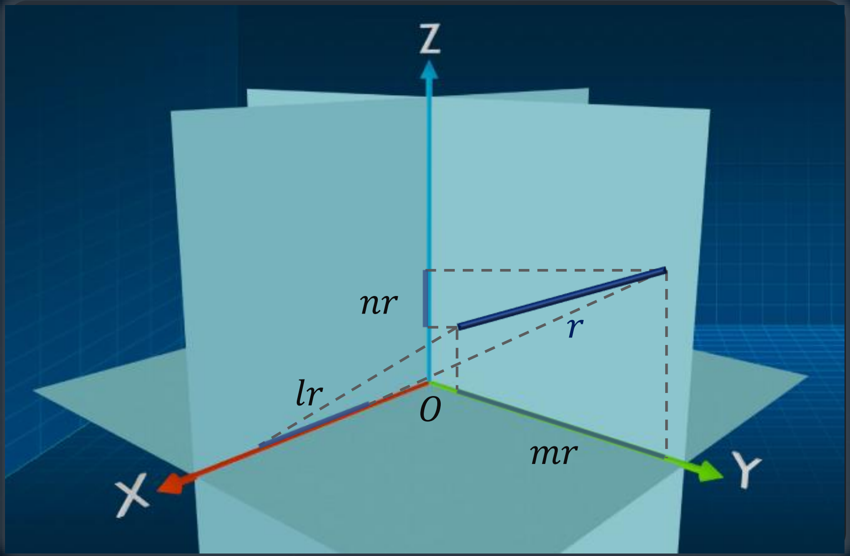
Projection of a Line Segment on Coordinate Axes:

Let a line segment has length r and has direction cosines l, m, n, then its projection on coordinate axes will be lr, mr, nr.



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The projection of a vector on three coordinate axes are $6, -3$ & 2 respectively. The direction cosines of the vector are :

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A $6, -3, 2$

B $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

C $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$

D $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

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The projection of a vector on three coordinate axes are $6, -3$ & 2 respectively. The direction cosines of the vector are :

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$lr = 6 \;\; ; mr = -3 \;\; ; nr = 2 \qquad l^2 + m^2 + n^2 = 1$

$l^2r^2 + m^2r^2 + n^2r^2 = 6^2 + 3^2 + 2^2$

$r^2(l^2 + m^2 + n^2) = 49$

$\Rightarrow r = 7$

$l \cdot 7 = 6 \Rightarrow \frac{6}{7} \; ; m = -\frac{3}{7} \; ; n = \frac{2}{7}$

Thus, direction cosines : $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

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The projection of a vector on three coordinate axes are $6, -3$ & 2 respectively. The direction cosines of the vector are :

AIEEE 2009

A $6, -3, 2$

B $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

C $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$

D $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

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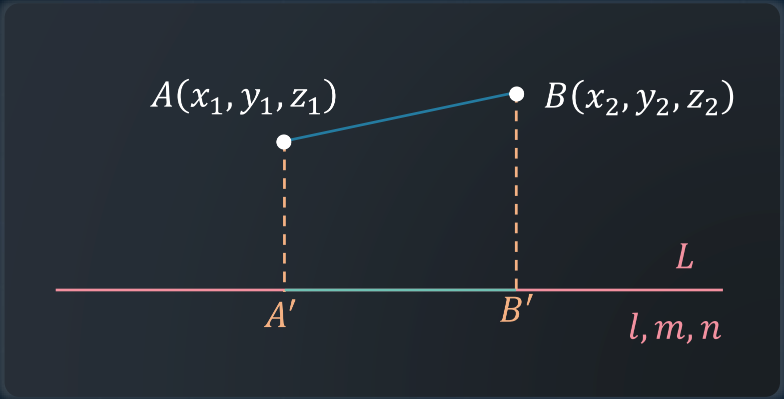
Projection of a Line Segment on Another Line



Projection of $\vec{a}$ on $\vec{b}$ is : $\vec{a} \cdot \dfrac{\vec{b}}{|\vec{b}|}$

Projection of a line segment joining points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ on a line L having direction cosines l, m, n, is :

$$A'B' = (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$$

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The projection of a line segment joining the points $(1, -1, 3)$ and $(2, -4, 11)$ on the line joining the points $(-1, 2, 3)$ and $(3, -2, 10)$ is:

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Solution:

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The projection of a line segment joining the points $(1, -1, 3)$ and $(2, -4, 11)$ on the line joining the points $(-1, 2, 3)$ and $(3, -2, 10)$ is:

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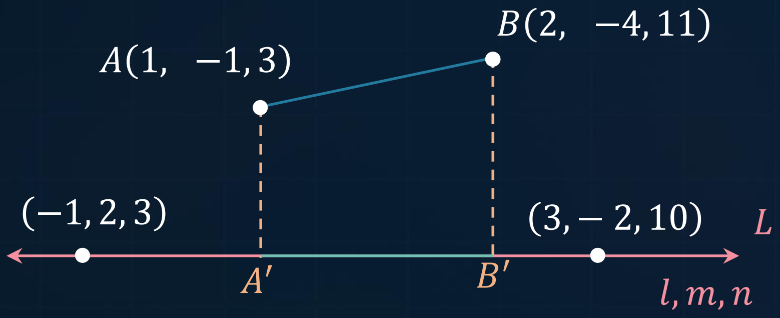
Solution:



The DRs of line L with points $(-1, 2, 3)$ & $(3, -2, 10)$: $4, -4, 7$

$$\therefore l = \frac{4}{9} \ ; \ m = -\frac{4}{9} \ ; n = \frac{7}{9}$$

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The projection of a line segment joining the points $(1,-1,3)$ and $(2,-4,11)$ on the line joining the points $(-1,2,3)$ and $(3,-2,10)$ is:

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$\therefore\ l=\frac{4}{9}\ \ ;\ m=-\frac{4}{9}\ \ ;n=\frac{7}{9}$

$$A'B'=(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n$$

$$=\frac{4}{9}(2-1)\ -\frac{4}{9}(-4+1)\ \ +\frac{7}{9}(11-3)$$

$$=\frac{4}{9}+\frac{12}{9}+\frac{56}{9}$$

$$=8$$

$\therefore$ Projection $=8$

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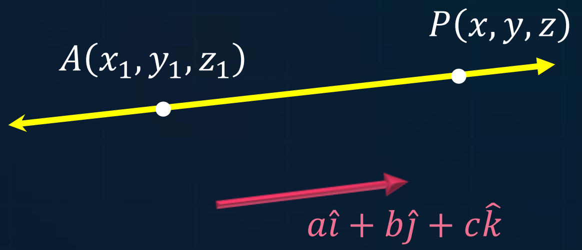
KEY TAKEAWAYS

Equation of a Straight Line

(i) Equation of a line passing through a point $A(x_1, y_1, z_1)$ and having direction ratios a, b, c, is :



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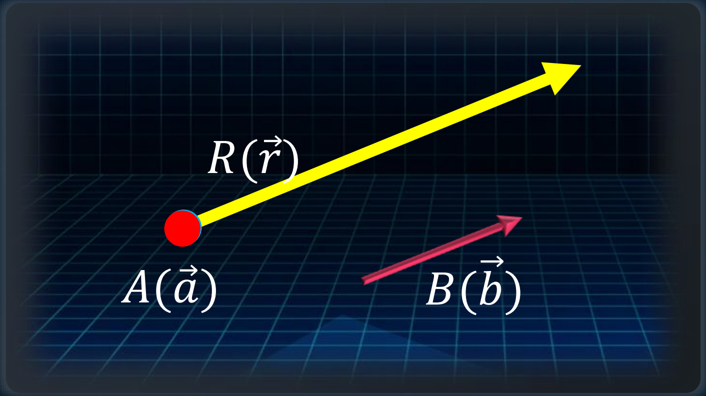
KEY TAKEAWAYS

Parametric Vector Equation of a Straight Line

Vector equation of a straight line passing through a given point $A(\vec{a})$ and parallel to a given vector $B(\vec{b})$

$$\vec{r} = \vec{a} + \lambda\vec{b}$$



where λ is a scalar and for different values of λ, we get different positions of point R.

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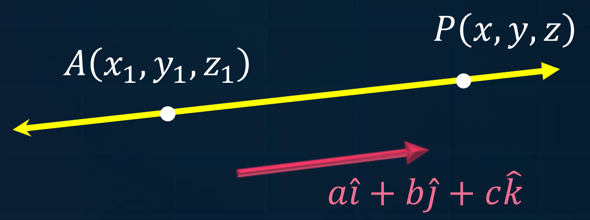
KEY TAKEAWAYS

Equation of a Straight Line

(i) Equation of a line passing through a point $A(x_1, y_1, z_1)$ and having direction ratios a, b, c, is:



$$\underbrace{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda}$$

symmetric form of line

General point on a line:

General point P on this line can be taken as : $x = x_1 + a\lambda$

$$y = y_1 + b\lambda$$

$$z = z_1 + c\lambda$$

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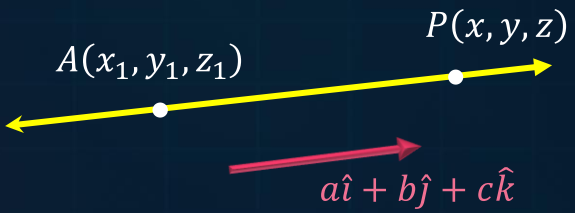
KEY TAKEAWAYS

Equation of a Straight Line

Symmetric form

or $\quad : \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Cartesian form



Vector form : $\quad \vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

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KEY TAKEAWAYS

Equation of a Straight Line

Straight line	Equation
(i) Through origin	$y = mx, z = nx$
(ii) x−axis	$y = 0 \;\&\; z = 0$
(iii) y−axis	$x = 0 \;\&\; z = 0$
(iv) z−axis	$x = 0 \;\&\; y = 0$
(v) Parallel to x−axis	$y = p\,, z = q$
(vi) Parallel to y−axis	$x = h\,, z = q$
(vii) Parallel to z−axis	$x = h, y = p$

$$: \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\vec{r} = \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right) + \lambda\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$$

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The equation of a straight line passing through the point $(3, -6, \ 8)$ and parallel to the line $\frac{x-2}{1} = \frac{y+12}{4} = \frac{-z-7}{5}$, is :

Solution:

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The equation of a straight line passing through the point $(3, -6, 8)$ and parallel to the line $\frac{x-2}{1} = \frac{y+12}{4} = \frac{-z-7}{5}$, is :

Solution:

Given line : $\frac{x-2}{1} = \frac{y+12}{4} = \frac{z+7}{-5}$

DRs of required line will be : $1, 4, -5$

Thus, equation of the line: $\frac{x-3}{1} = \frac{y+6}{4} = \frac{z-8}{-5}$

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The equation of a straight line passing through the point $(-5, 2, 4)$ and parallel to vector $2\hat{i} - 3\hat{j} + \hat{k}$, is :

A $\frac{x+5}{2} = \frac{y-2}{-3} = \frac{z-4}{1}$

B $\frac{x-5}{2} = \frac{y+2}{3} = \frac{z-4}{1}$

C $\frac{x-5}{2} = \frac{y+2}{-3} = \frac{z-4}{2}$

D $\frac{x+5}{1} = \frac{y-2}{-3} = \frac{z-4}{2}$

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The equation of a straight line passing through the point $(-5, 2, 4)$ and parallel to vector $2\hat{i} - 3\hat{j} + \hat{k}$, is :

A $\frac{x+5}{2} = \frac{y-2}{-3} = \frac{z-4}{1}$

B $\frac{x-5}{2} = \frac{y+2}{3} = \frac{z-4}{1}$

C $\frac{x-5}{2} = \frac{y+2}{-3} = \frac{z-4}{2}$

D $\frac{x+5}{1} = \frac{y-2}{-3} = \frac{z-4}{2}$

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The equation of a straight line passing through the point $(-5, 2, 4)$ and parallel to vector $2\hat{i} - 3\hat{j} + \hat{k}$, is :

Solution:

Equation of the line : $\frac{x+5}{2} = \frac{y-2}{-3} = \frac{z-4}{1}$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

OR

Equation of the line in vector form:

$$\vec{r} = \left(-5\hat{i} + 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$$

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If the lines $x = ay + b, z = cy + d$ and $x = a'z + b', y = c'z + d'$ are perpendicular, then:

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A $ab' + bc' + 1 = 0$

B $bb' + cc' + 1 = 0$

C $cc' + a + a' = 0$

D $aa' + c + c' = 0$

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If the lines $x = ay + b, z = cy + d$ and $x = a'z + b', y = c'z + d'$ are perpendicular, then:

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Solution:

Lines can be written as :

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} \cdots (i)$$

$$\frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1} \cdots (ii)$$

For perpendicular lines $\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

$$\Rightarrow aa' + c' + c = 0$$

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If the lines $x = ay + b, z = cy + d$ and $x = a'z + b', y = c'z + d'$ are perpendicular, then:

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A $ab' + bc' + 1 = 0$

B $bb' + cc' + 1 = 0$

C $cc' + a + a' = 0$

D $aa' + c + c' = 0$

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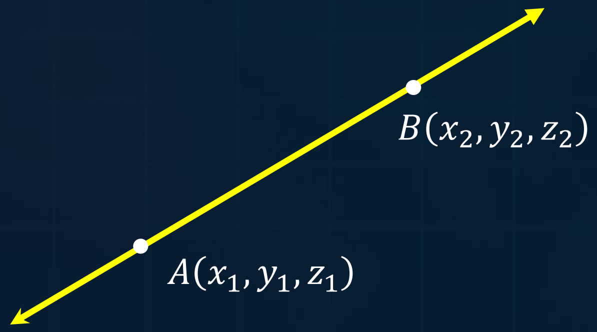
Straight Line

(ii) Equation of a line passing through points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$



DRs of the line will be : $x_2 - x_1 \, , y_2 - y_1 \, , z_2 - z_1$

Equation of the line : $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$

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Straight Line

Example:

The equation of a straight line passing through the points $(1, -2, 7)$ and $(5, 3, -1)$, is :

Equation of the line : $\frac{x-1}{4} = \frac{y+2}{5} = \frac{z-7}{-8}$

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Which of the following does not represent equation of line passing through the points $(2, 1, 3)$ & $(-1, 3, 1)$?

A $\quad \frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$

B $\quad \vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

C $\quad \vec{r} = 8\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

D $\quad \frac{x-5}{-3} = \frac{y+3}{2} = \frac{z-5}{-2}$

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Which of the following does not represent equation of line passing through the points $(2, 1, 3)$ & $(-1, 3, 1)$?

Vector form : $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Cartesian equation : $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$

Vector form: $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

General point on this line is : $\left((2 + 3\lambda), (1 - 2\lambda), (3 + 2\lambda)\right)$

$2 + 3\lambda = 5$

Thus, another point will be: $(5, -1, 5)$

Thus , equation can also be written as: $\frac{x-5}{-3} = \frac{y+1}{2} = \frac{z-5}{-2}$

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Which of the following does not represent equation of line passing through the points $(2,1,3)$ & $(-1,3,1)$?

Vector form : $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Cartesian equation : $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$

Vector form: $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

General point on this line is : $\left((2+3\lambda), (1-2\lambda), (3+2\lambda)\right)$

$2 + 3\lambda = 8 \Rightarrow \lambda = 2$

Point on this line is $(8, -3, 7)$

$\therefore$ Equation can also be : $\vec{r} = 8\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

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Which of the following does not represent equation of line passing through the points $(2, 1, 3)$ & $(-1, 3, 1)$?

A $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$

B $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

C $\vec{r} = 8\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$

D $\frac{x-5}{-3} = \frac{y+3}{2} = \frac{z-5}{-2}$

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The line passing through the points $(5, 1, a)$ & $(3, b, 1)$ crosses the $y - z$ plane at point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$, then:

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A $a = 2, b = 8$

B $a = 4, b = 6$

C $a = 6, b = 4$

D $a = 8, b = 2$

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The line passing through the points $(5, 1, a)$ & $(3, b, 1)$ crosses the $y - z$ plane at point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$, then:

AIEEE 2008

Line passing through $(5, 1, a)$ & $(3, b, 1)$

Cartesian equation : $\frac{x-5}{2} = \frac{y-1}{1-b} = \frac{z-a}{a-1}$

$$\left(2r + 5, 1 + r(1 - b), a + r(a - 1)\right) \equiv \left(0, \frac{17}{2}, -\frac{13}{2}\right)$$

$$r = -\frac{5}{2}$$

$$\left(0, 1 - \frac{5}{2}(1 - b), a - \frac{5}{2}(a - 1)\right)$$

$$1 - \frac{5}{2}(1 - b) = \frac{17}{2} \quad \& \quad a - \frac{5}{2}(a - 1) = -\frac{13}{2}$$

$$\frac{5b}{2} = \frac{17}{2} + \frac{3}{2} \quad \& \quad \frac{-3a}{2} = -\frac{18}{2}$$

$$\frac{5b}{2} = 10 \quad \& \quad \frac{-3a}{2} = -\frac{18}{2}$$

$$b = 4 \quad \& \quad a = 6$$

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The line passing through the points $(5, 1, a)$ & $(3, b, 1)$ crosses the $y-z$ plane at point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$, then:

AIEEE 2008

A $a = 2, b = 8$

B $a = 4, b = 6$

C $a = 6, b = 4$

D $a = 8, b = 2$

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Session 04

Equation of angular bisectors of lines

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Angle θ between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$ is :

A $\cos^{-1}\left(\frac{2\sqrt{3}}{\sqrt{26}}\right)$

B $\cos^{-1}\left(\frac{\sqrt{13}}{2\sqrt{7}}\right)$

C $\cos^{-1}\left(\frac{\sqrt{6}}{2\sqrt{7}}\right)$

D $\cos^{-1}\left(\frac{\sqrt{21}}{2\sqrt{29}}\right)$

Solution:

Direction ratios of lines are: $(1, 2, 3)$ & $(3, -1, 4)$

$(a_1, b_1, c_1)\ (a_2, b_2, c_2)$

$$\boxed{\theta = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}\right)}$$

$$\therefore \theta = \cos^{-1}\left(\frac{3-2+12}{\sqrt{14}\sqrt{26}}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{13}{\sqrt{14}\sqrt{26}}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{13}}{2\sqrt{7}}\right)$$

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KEY TAKEAWAYS

Equation of Angle Bisector of Two Lines :

Let the lines be :

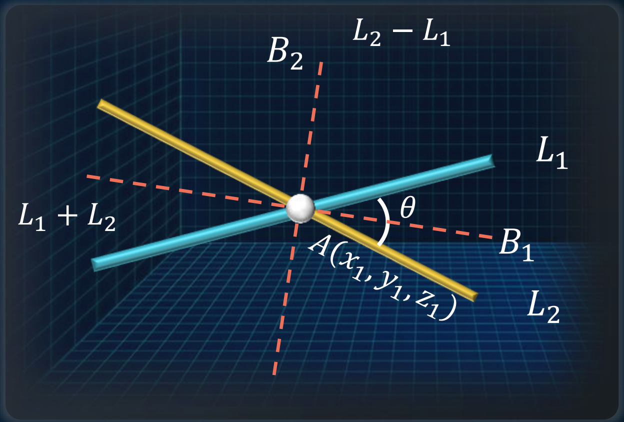
$$L_1 : \frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \rightarrow \text{Through } (x_1, y_1, z_1)$$

$$L_1 = l_1 i + m_1 j + n_1 k$$

$$L_2 : \frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2} \rightarrow \text{Through } (x_2, y_2, z_3)$$

$$L_1 = l_2 i + m_2 j + n_2 k$$

where l_1 , m_1, n_1 and l_2, m_2, n_2 are direction cosines



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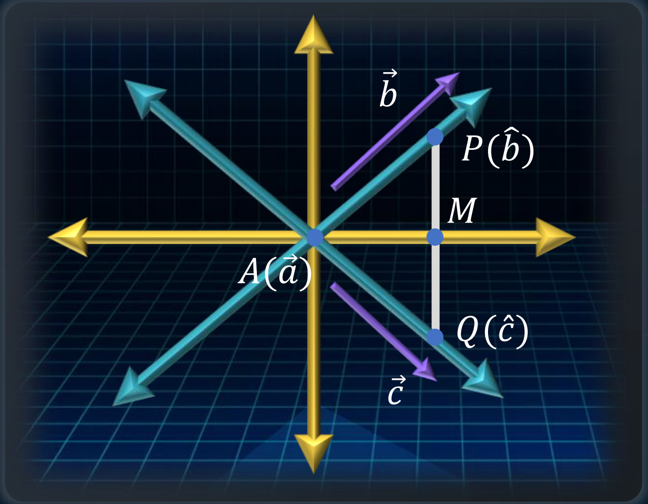
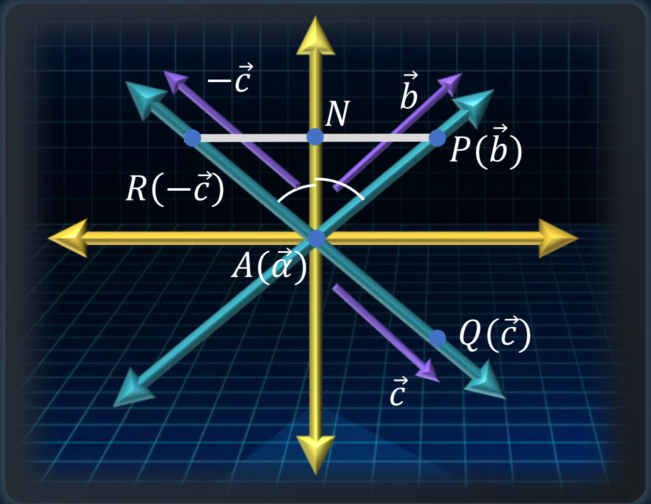
Vector Equation of Angle Bisector Between Two Straight Lines :

Line 1: $\vec{r} = \vec{a} + \lambda\vec{b} \cdots (i)$

Line 2: $\vec{r} = \vec{a} + \mu\vec{c} \cdots (ii)$





Internal angle bisector :

$$\vec{r} = \vec{a} + s(\hat{b}+\hat{c})$$

External angle bisector :

$$\vec{r} = \vec{a} + s(\hat{b}-\hat{c})$$

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KEY TAKEAWAYS

Equation of Angle Bisector of Two Straight Lines :

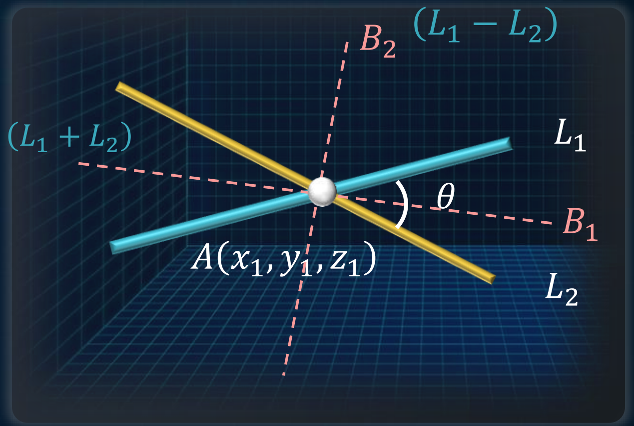
Let the lines be :

$$L_1 : \frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \rightarrow L_1 = l_1 i + m_1 j + n_1 k$$

$$L_2 : \frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2} \rightarrow L_2 = l_2 i + m_2 j + n_2 k$$

$$L_1 + L_2 = (l_1 + l_2)i + (m_1 + m_2)j + (n_1 + n_2)k$$

$$\rightarrow DR's\ of\ B_1\ \alpha\ (l_1 + l_2), (m_1 + m_2), (n_1 + n_2)$$



where l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines

∴ Equation of bisectors will be :

$$\boxed{\frac{x-x_1}{l_1+l_2} = \frac{y-y_1}{m_1+m_2} = \frac{z-z_1}{n_1+n_2}}\ \&\ \boxed{\frac{x-x_1}{l_1-l_2} = \frac{y-y_1}{m_1-m_2} = \frac{z-z_1}{n_1-n_2}}$$

B_1 $\qquad$ B_2

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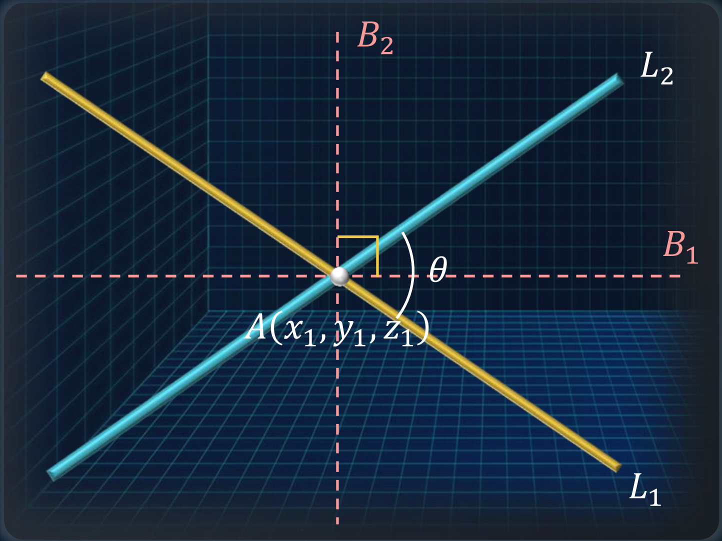
KEY TAKEAWAYS

Equation of Angle Bisector of Two Straight Lines :

Acute and obtuse angle bisectors :



$$\cos\theta = (l_1 l_2 + m_1 m_2 + n_1 n_2)$$

$$B_1 : \frac{x-x_1}{l_1+l_2} = \frac{y-y_1}{m_1+m_2} = \frac{z-z_1}{n_1+n_2}$$

$$B_2 : \frac{x-x_1}{l_1-l_2} = \frac{y-y_1}{m_1-m_2} = \frac{z-z_1}{n_1-n_2}$$

If $\cos\theta > 0$

$\Rightarrow B_1$ is acute angle bisector and B_2 is obtuse bisector.

If $\cos\theta < 0$

$\Rightarrow B_2$ is acute angle bisector and B_1 is obtuse bisector.

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Equation of the angle bisector of the angle between the lines $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$ and $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{-1}$ is :

A $x=1; \frac{y-2}{1}=\frac{z-3}{1}$

B $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

C $\frac{x-1}{2}=\frac{y-2}{2}; z=3$

D $\frac{x-1}{2}=\frac{y-2}{3}; z=3$

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Equation of the angle bisector of the angle between the lines $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$ and $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{-1}$ is :

Solution:

$$\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1} \text{ and } \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{-1}$$

$L_1 = i + j + k$, $\quad L_2 = i + j - k$

$\hat{L}_1 = \frac{i+j+k}{\sqrt{3}} \quad \hat{L}_2 = \frac{i+j-k}{\sqrt{3}}$

→ DR's of bisector $B_1\ \alpha\ (\hat{L}_1 + \hat{L}_2)\ \alpha\ \left(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, 0\right)$

→ DR's of bisector $B_2\ \alpha\ (\hat{L}_1 - \hat{L}_2) \alpha \left(0, 0, \frac{2}{\sqrt{3}}\right)$

→ DR's of bisector $B_1\ \alpha\ (2, 2, 0)$

→ DR's of bisector $B_2\ \alpha (0, 0, 2)$

The equation of bisector is :

$$\Rightarrow \frac{x-1}{2} = \frac{y-2}{2}; z = 3$$

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Equation of the angle bisector of the angle between the lines $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$ and $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{-1}$ is :

A $x=1; \frac{y-2}{1}=\frac{z-3}{1}$

B $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

C $\frac{x-1}{2}=\frac{y-2}{2}; z=3$

D $\frac{x-1}{2}=\frac{y-2}{3}; z=3$

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The direction cosines of the lines bisecting the angle between the lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, and the angle between these lines is θ, are :

A $\dfrac{l_1+l_2}{\cos\left(\frac{\theta}{2}\right)}, \dfrac{m_1+m_2}{\cos\left(\frac{\theta}{2}\right)}, \dfrac{n_1+n_2}{\cos\left(\frac{\theta}{2}\right)}$

B $\dfrac{l_1-l_2}{\sin\left(\frac{\theta}{2}\right)}, \dfrac{m_1-m_2}{\sin\left(\frac{\theta}{2}\right)}, \dfrac{n_1-n_2}{\sin\left(\frac{\theta}{2}\right)}$

C $\dfrac{l_1+l_2}{2\cos\left(\frac{\theta}{2}\right)}, \dfrac{m_1+m_2}{2\cos\left(\frac{\theta}{2}\right)}, \dfrac{n_1+n_2}{2\cos\left(\frac{\theta}{2}\right)}$

D $\dfrac{l_1-l_2}{2\sin\left(\frac{\theta}{2}\right)}, \dfrac{m_1-m_2}{2\sin\left(\frac{\theta}{2}\right)}, \dfrac{n_1-n_2}{2\sin\left(\frac{\theta}{2}\right)}$

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The direction cosines of the lines bisecting the angle between the lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, and the angle between these lines is θ, are :

Solution: $\boxed{\dfrac{x-x_1}{l_1+l_2}=\dfrac{y-y_1}{m_1+m_2}=\dfrac{z-z_1}{n_1+n_2}}$ & $\boxed{\dfrac{x-x_1}{l_1-l_2}=\dfrac{y-y_1}{m_1-m_2}=\dfrac{z-z_1}{n_1-n_2}}$

DRs of bisectors are : $l_1+l_2, m_1+m_2, n_1+n_2$ & $l_1-l_2, m_1-m_2, n_1-n_2$

Now, $(l_1+l_2)^2+(m_1+m_2)^2+(n_1+n_2)^2$

$= l_1^2+m_1^2+n_1^2+l_2^2+m_2^2+n_2^2+2(l_1l_2+m_1m_2+n_1n_2)$

$= 2+2\cos\theta$

$\Rightarrow (l_1-l_2)^2+(m_1-m_2)^2+(n_1-n_2)^2$

$= l_1^2+m_1^2+n_1^2+l_2^2+m_2^2+n_2^2-2(l_1l_2+m_1m_2+n_1n_2)$

$= 2-2\cos\theta$

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The direction cosines of the lines bisecting the angle between the lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, and the angle between these lines is θ, are :

Solution:

DCs of bisectors are :

$$\frac{l_1+l_2}{\sqrt{(l_1+l_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}}, \frac{m_1+m_2}{\sqrt{(l_1+l_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}}, \frac{n_1+n_2}{\sqrt{(l_1+l_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}}$$

and

$$\frac{l_1-l_2}{\sqrt{(l_1-l_2)^2+(m_1-m_2)^2+(n_1-n_2)^2}}, \frac{m_1-m_2}{\sqrt{(l_1-l_2)^2+(m_1-m_2)^2+(n_1-n_2)^2}} \quad \frac{n_1-n_2}{\sqrt{(l_1-l_2)^2+(m_1-m_2)^2+(n_1-n_2)^2}}$$

,

$$\frac{l_1+l_2}{\sqrt{2+2\cos\theta}}, \frac{m_1+m_2}{\sqrt{2+2\cos\theta}}, \frac{n_1+n_2}{\sqrt{2+2\cos\theta}} \Rightarrow \frac{l_1+l_2}{2\cos\left(\frac{\theta}{2}\right)}, \frac{m_1+m_2}{2\cos\left(\frac{\theta}{2}\right)}, \frac{n_1+n_2}{2\cos\left(\frac{\theta}{2}\right)}$$

and

$$\frac{l_1-l_2}{\sqrt{2-2\cos\theta}}, \frac{m_1-m_2}{\sqrt{2-2\cos\theta}}, \frac{n_1-n_2}{\sqrt{2-2\cos\theta}} \Rightarrow \frac{l_1-l_2}{2\sin\left(\frac{\theta}{2}\right)}, \frac{m_1-m_2}{2\sin\left(\frac{\theta}{2}\right)}, \frac{n_1-n_2}{2\sin\left(\frac{\theta}{2}\right)}$$

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The direction cosines of the lines bisecting the angle between the lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, and the angle between these lines is θ, are :

A $\dfrac{l_1+l_2}{\cos\left(\frac{\theta}{2}\right)} = \dfrac{m_1+m_2}{\cos\left(\frac{\theta}{2}\right)} = \dfrac{n_1+n_2}{\cos\left(\frac{\theta}{2}\right)}$

B $\dfrac{l_1-l_2}{\sin\left(\frac{\theta}{2}\right)} = \dfrac{m_1-m_2}{\sin\left(\frac{\theta}{2}\right)} = \dfrac{n_1-n_2}{\sin\left(\frac{\theta}{2}\right)}$

C $\dfrac{l_1+l_2}{2\cos\left(\frac{\theta}{2}\right)} = \dfrac{m_1+m_2}{2\cos\left(\frac{\theta}{2}\right)} = \dfrac{n_1+n_2}{2\cos\left(\frac{\theta}{2}\right)}$

D $\dfrac{l_1-l_2}{2\sin\left(\frac{\theta}{2}\right)} = \dfrac{m_1-m_2}{2\sin\left(\frac{\theta}{2}\right)} = \dfrac{n_1-n_2}{2\sin\left(\frac{\theta}{2}\right)}$

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Foot of Perpendicular from a Point to a Lines :

Let point $A(x_1, y_1, z_1)$ and Line $L : \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Let P is the foot of perpendicular from point A

on the line L.

So, $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = \lambda$

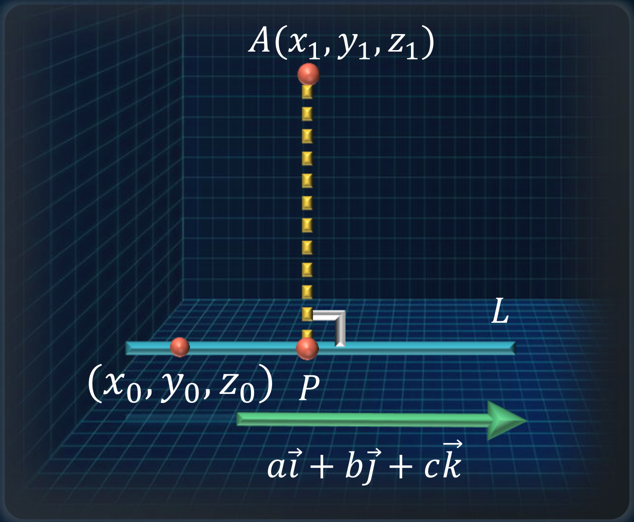
$\therefore P \equiv (x_0 + a\lambda\, , y_0 + b\lambda\, , z_0 + c\lambda)$

DRs of AP: $x_0 + a\lambda - x_1, y_0 + b\lambda - y_1, z_0 + c\lambda - z_1$

DRs of L : a, b, c

$\because AP$ is $\perp$ to L

$a(x_0 + a\lambda - x_1) + b(y_0 + b\lambda - y_1) + c(z_0 + c\lambda - z_1) = 0$



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Foot of Perpendicular from a Point to a Lines :

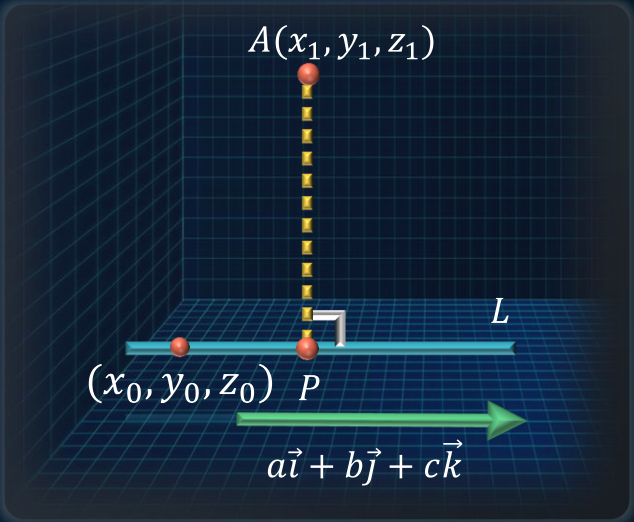
Line L : $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

$P \equiv (x_0 + a\lambda\,, y_0 + b\lambda\,, z_0 + c\lambda)$

$a(x_0 + a\lambda - x_1) + b(y_0 + b\lambda - y_1) + c(z_0 + c\lambda - z_1) = 0$

$$\Rightarrow \lambda = \frac{a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)}{a^2 + b^2 + c^2}$$

Substitute value of λ to get point P



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The foot of perpendicular from the point (1,6,3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is :

A $(0, 1, 2)$

B $(4, 9, 14)$

C $(1, 3, 5)$

D $(-2, -3, -4)$

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The foot of perpendicular from the point (1,6,3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is :

Solution:

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$$

$P \equiv (\lambda\,, 1+2\lambda\,, 2+3\lambda)$

DRs of $AP\ \alpha\ (\lambda\ -1, 2\lambda-5\,, 3\lambda-1)$

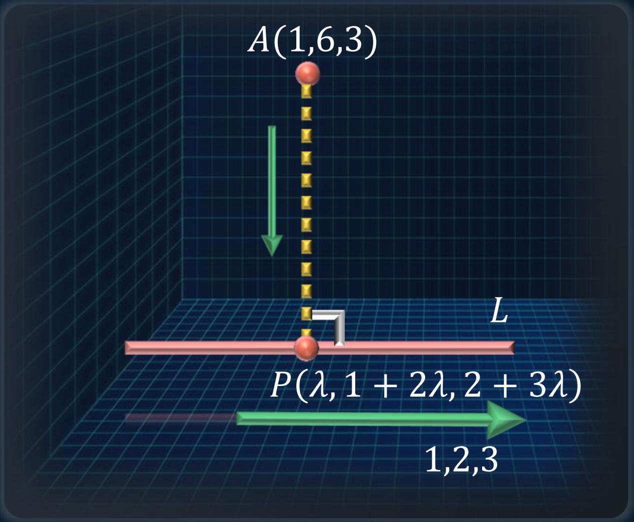
DRs of $L\ \alpha\ (1,2,3)$

$\because\ AP$ is $\perp$ to L

$\Rightarrow 1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow \lambda = 1$

$\therefore P \equiv (1,3,5)$



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The foot of perpendicular from the point (1,6,3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is :

A $(0, 1, 2)$

B $(4, 9, 14)$

C $(1, 3, 5)$

D $(-2, -3, -4)$

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If foot of perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to :

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A 4

B 3

C 2

D 1

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If foot of perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to :

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A 4

B 3

C 2

D 1

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If foot of perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to :

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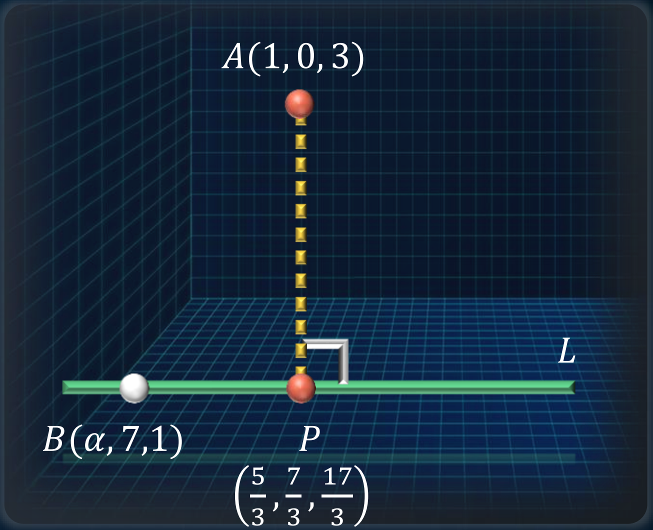
Solution:



DRs of AP α $\left(\frac{5}{3} - 1, \frac{7}{3} - 0, \frac{17}{3} - 3\right) \alpha \left(\frac{2}{3}, \frac{7}{3}, \frac{8}{3}\right)$

DRs of L α $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right) \alpha \left(\alpha - \frac{5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$

$\because AP$ is $\perp$ to L

$\Rightarrow \left(\frac{5}{3} - 1\right)\left(\alpha - \frac{5}{3}\right) + \left(\frac{7}{3} - 0\right)\left(7 - \frac{7}{3}\right) + \left(\frac{17}{3} - 3\right)\left(1 - \frac{17}{3}\right)$

$= 0$

$\Rightarrow \frac{2}{3}\left(\alpha - \frac{5}{3}\right) + \frac{7}{3} \times \frac{14}{3} + \left(\frac{8}{3} \times -\frac{14}{3}\right) = 0$

$\Rightarrow 3\alpha - 5 + 49 - 56 = 0$

$\Rightarrow 3\alpha - 12 = 0 \ \Rightarrow \alpha = 4$

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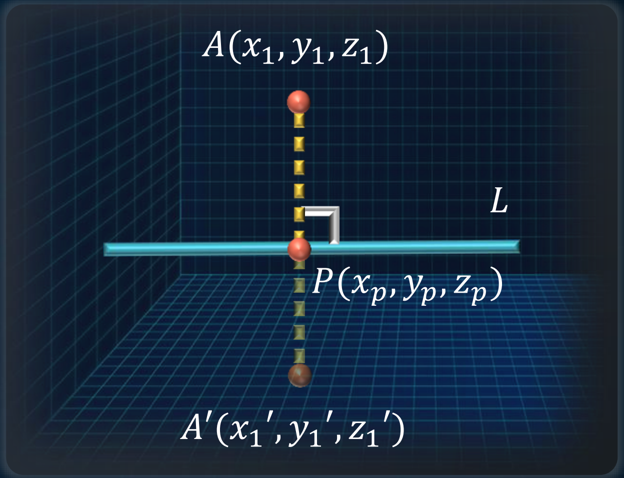
KEY TAKEAWAYS

Image of a Point with Respect to a Line :

Let point $A(x_1, y_1, z_1)$ & Line $L : \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Let $A'(x', y', z')$ is image of point A with respect to line L

and, P is the mid point of the line segment AA' as well as the foot of perpendicular from the point A on the line L



P is mid point of AA'
To get $A' \rightarrow$ find P

Then apply mid point formula

To find point $P(x_p, y_p, z_p)$, apply mid point formula

$$x_p = \frac{x_1+x'}{2} \qquad y_p = \frac{y_1+y'}{2} \qquad z_p = \frac{z_1+z'}{2}$$

$$\therefore A'(x', y', z') \equiv \left((2x_p - x_1), (2y_p - y_1), (2z_p - z_1)\right)$$

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If (a, b, c) is the image of the point $(1, 2, -3)$ in the line,

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equal to :

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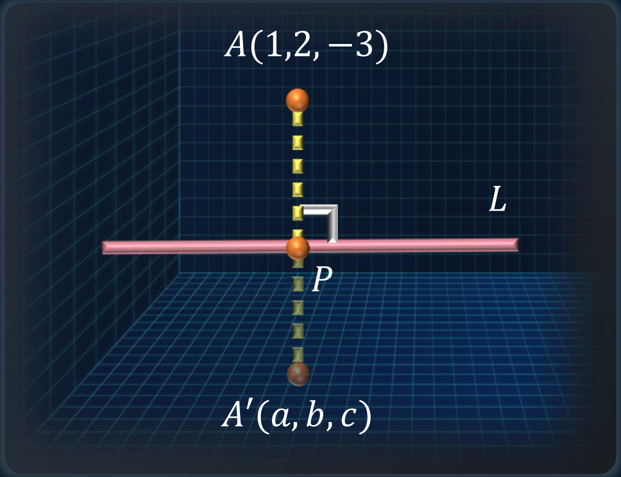
A -1

B 2

C 3

D 1

Solution:



P is a point on the foot of perpendicular of the line L

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$$

$$\Rightarrow P \equiv (-1 + 2\lambda , 3 - 2\lambda , -\lambda)$$

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If (a, b, c) is the image of the point $(1, 2, -3)$ in the line,

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equal to :

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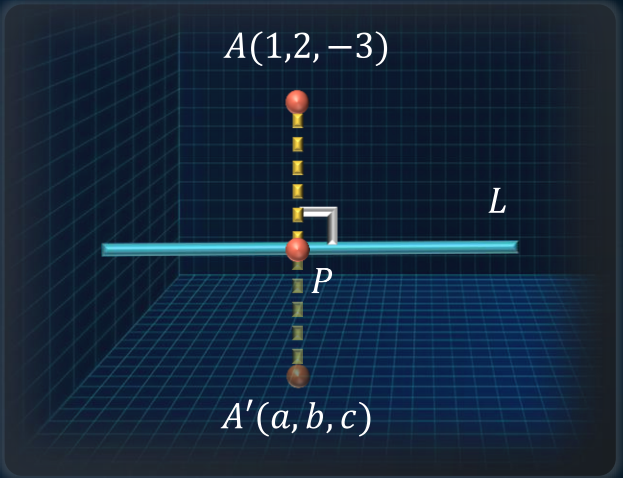
Solution:

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ $\quad P \equiv (-1 + 2\lambda, 3 - 2\lambda, -\lambda)$



DRs of AP α $(2\lambda - 2, 1 - 2\lambda, 3 - \lambda)$

DRs of L $\alpha (2, -2, -1)$

$\because$ AP is $\perp$ to L $\quad \therefore \cos\theta = 0$

$\Rightarrow 2(2\lambda - 2) - 2(1 - 2\lambda) - (-\lambda + 3) = 0$

$\Rightarrow$ Put $\lambda = 1$ $\quad \therefore P \equiv (1, 1, -1)$

Use mid point formula,

$\frac{a+1}{2} = 1 \qquad \frac{b+2}{2} = 1 \qquad \frac{c-3}{2} = -1$

$\Rightarrow a = 1 \qquad \Rightarrow b = 0 \qquad \Rightarrow c = 1 \qquad \Rightarrow a + b + c = 2$

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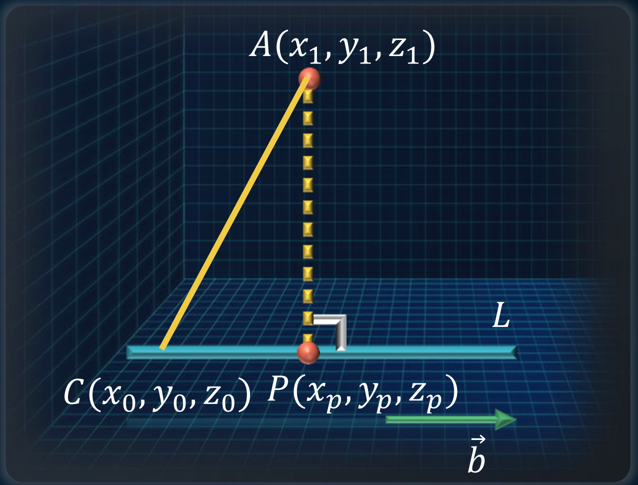
KEY TAKEAWAYS

Perpendicular Distance of a Point from a Line :

Let point $A(x_1, y_1, z_1)$ & Line $L : \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Let P is the foot of perpendicular from point A.



Method 1 :

Find point $P(x_p, y_p, z_p)$, and then evaluate distance AP

Method 2 :

$$CP = \left|(\vec{a} - \vec{c}).\hat{b}\right|$$

$$AP = \sqrt{AC^2 - CP^2} = \sqrt{|\vec{a} - \vec{c}|^2 - |\vec{a} - \vec{c}|^2 \cos^2\theta}$$

$$AP = |\vec{a} - \vec{c}|\sqrt{1 - \cos^2\theta}$$

$$AP = |\vec{a} - \vec{c}| \sin\theta \qquad AP = \left|(\vec{a} - \vec{c}) \times \hat{b}\right|$$

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KEY TAKEAWAYS

Computing Distance between two parallel Lines :

$L_1 : \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2 : \vec{r} = \vec{c} + \mu\vec{b}$

Area of $\triangle\, ABC = \frac{1}{2}\left|(\vec{a} - \vec{c}) \times \vec{b}\right|$

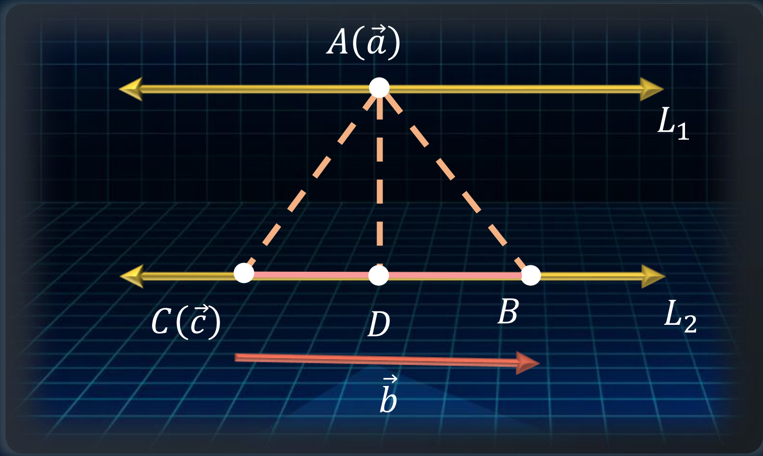
$= \frac{1}{2}\left|\vec{b}\right|.\, AD$

AD = Shortest Distance $= \frac{\left|(\vec{a}-\vec{c})\times\vec{b}\right|}{\left|\vec{b}\right|}$

Get $CD = \left|\ (\vec{a} - \vec{c}).\,\hat{b}\ \right|$

Use Pythagoras to find AD

AD = Shortest Distance



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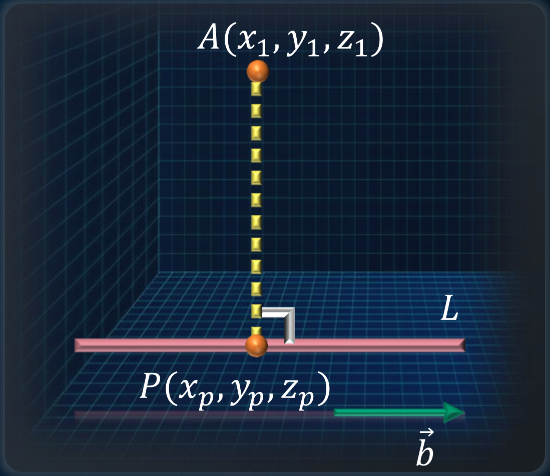
KEY TAKEAWAYS

Perpendicular Distance of a point from a Line:

Let point $A(x_1, y_1, z_1)$ and Line $L : \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Let P is the foot of perpendicular from point A.



Method 1

Find point $P(x_p, y_p, z_p)$, and then evaluate distance AP

Method 2

Let point $A(\vec{a})$ and Line $L : \vec{r} = \vec{c} + \lambda\vec{b}$

Using formula $AP = \left|\frac{(a-\vec{c})\times\vec{b}}{|\vec{b}|}\right|$ $\left\{\begin{array}{l} \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \\ \vec{c} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k} \\ \vec{b} = a\hat{i} + b\hat{j} + c\hat{k} \end{array}\right.$

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The length of perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

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A Greater than 3 but less than 4

B Greater than 2 but less than 3

C Greater than 4

D Less than 2

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The length of perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

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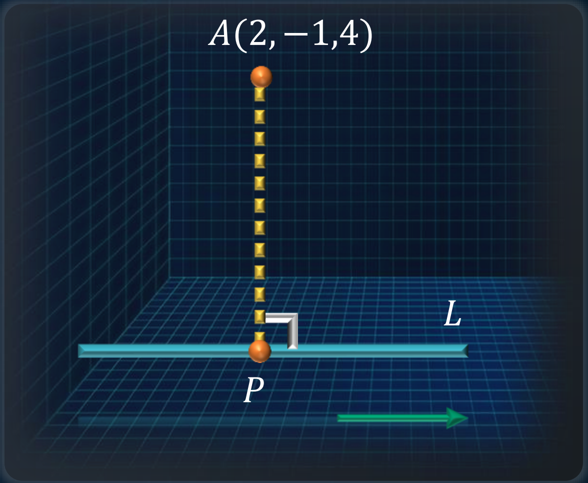

Solution:

$$\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$$

$$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k} \qquad \vec{b} = 10\hat{i} - 7\hat{j} + \hat{k} \qquad \vec{c} = -3\hat{i} + 2\hat{j}$$

$$\vec{a} - \vec{c} = 5\hat{i} - 3\hat{j} + 4\hat{k}$$

$$AP = \left|(\vec{a} - \vec{c}) \times \hat{b}\right| \qquad \hat{b} = \frac{(10\hat{i} - 7\hat{j} + \hat{k})}{\sqrt{150}}$$

$$\left|(\vec{a} - \vec{c}) \times \vec{b}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -3 & 4 \\ 10 & -7 & 1 \end{vmatrix}$$

$$AP = \frac{\left|(\vec{a} - \vec{c}) \times \hat{b}\right|}{|\vec{b}|} = \frac{\sqrt{25^2 + 35^2 + 5^2}}{\sqrt{150}} = \frac{5}{\sqrt{2}}$$

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The length of perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

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A Greater than 3 but less than 4

B Greater than 2 but less than 3

C Greater than 4

D Less than 2

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?

The vertices B and C of ΔABC lie on the line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$, such that $BC = 5$units . Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is :

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A $5\sqrt{17}$

B $\sqrt{34}$

C $2\sqrt{34}$

D 6

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The vertices B and C of ΔABC lie on the line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$, such that $BC = 5$units . Then the area (in sq. units) of this triangle, given that the point $A(1,-1,2)$, is :

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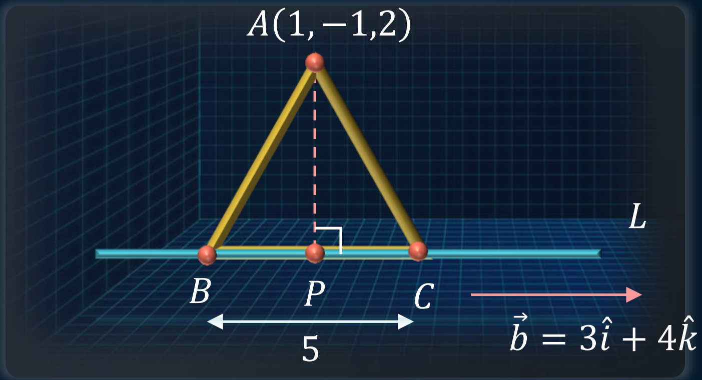
Solution:

$$\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$$

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = 3\hat{i} + 4\hat{k}$$

$$\vec{c} = -2\hat{i} + \hat{j}$$



$$AP = \left|\frac{\left((\hat{i}-\hat{j}+2\hat{k})-(-2\hat{i}+\hat{j})\right)\times(3\hat{i}+4\hat{k})}{|3\hat{i}+4\hat{k}|}\right|$$

$$\boxed{AP = \left|\frac{(\vec{a}-\vec{c})\times\vec{b}}{|\vec{b}|}\right|}$$

$$AP = \left|\frac{(3\hat{i}-2\hat{j}+2\hat{k})\times(3\hat{i}+4\hat{k})}{|3\hat{i}+4\hat{k}|}\right|$$

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The vertices B and C of ΔABC lie on the line $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$, such that $BC=5$units . Then the area (in sq. units) of this triangle, given that the point $A(1,-1,2)$, is :

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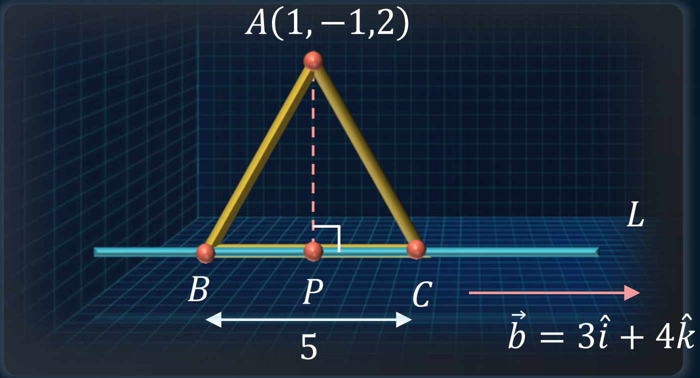
Solution:



$$AP=\left|\frac{(3\hat{\imath}-2\hat{\jmath}+2\hat{k})\times(3\hat{\imath}+4\hat{k})}{|3\hat{\imath}+4\hat{k}|}\right|$$

$$=\left|\frac{(-8\hat{\imath}-6\hat{\jmath}+6\hat{k})}{5}\right|$$

$$=\frac{2\sqrt{34}}{5}$$

$$\therefore \text{Area}=\frac{1}{2}\cdot 5\cdot\frac{2\sqrt{34}}{5}$$

$$=\sqrt{34}$$

$$(3\hat{\imath}-2\hat{\jmath}+2\hat{k})\times(3\hat{\imath}+4\hat{k})$$

$$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & -2 & 2\\ 3 & 0 & 4\end{vmatrix}$$

$$=\hat{\imath}(-8-0)-\hat{\jmath}(12-6)+\hat{k}(0+6)$$

$$=-8\hat{\imath}-6\hat{\jmath}+6\hat{k}$$

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The vertices B and C of ΔABC lie on the line $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$, such that $BC=5$units . Then the area (in sq. units) of this triangle, given that the point $A(1,-1,2)$, is :

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A $5\sqrt{17}$

B $\sqrt{34}$

C $2\sqrt{34}$

D 6

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Session 05

Introduction to plane in 3 –D

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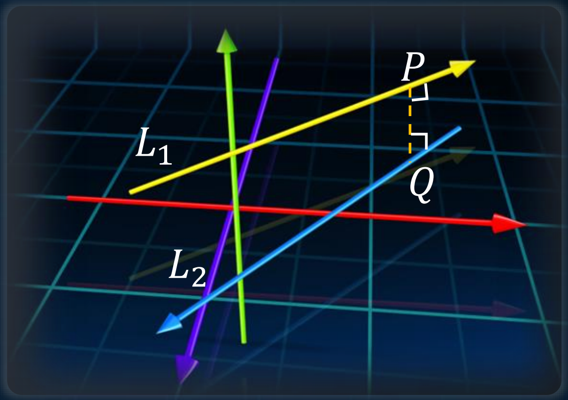
KEY TAKEAWAYS

Skew lines:

Neither parallel nor intersecting straight lines.

Non – coplanar

PQ ($\perp^r$ to both L_1 & L_2) is the shortest distance between lines L_1 & L_2.



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KEY TAKEAWAYS

Shortest distance between 2 skew lines:

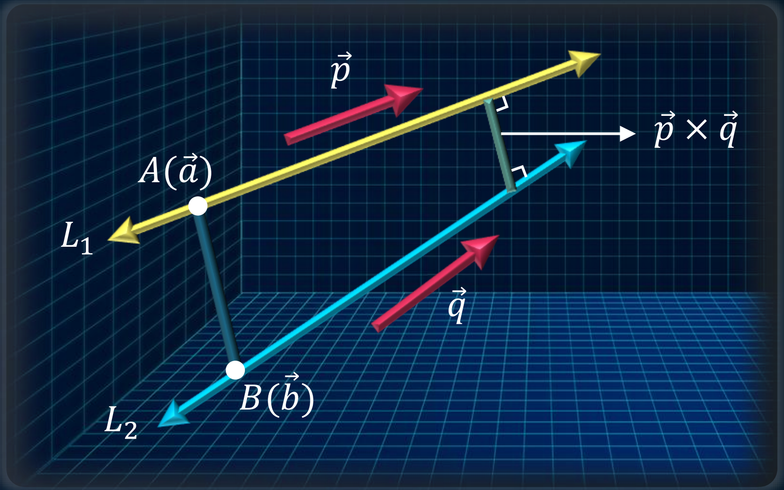
$$L_1 : \vec{r} = \vec{a} + \lambda\vec{p}$$

$$L_2 : \vec{r} = \vec{b} + \mu\vec{q}$$

Shortest distance = | Projection of $\overrightarrow{AB}$ on $\vec{n}$ |

$$= \left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right|$$

$$= \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$



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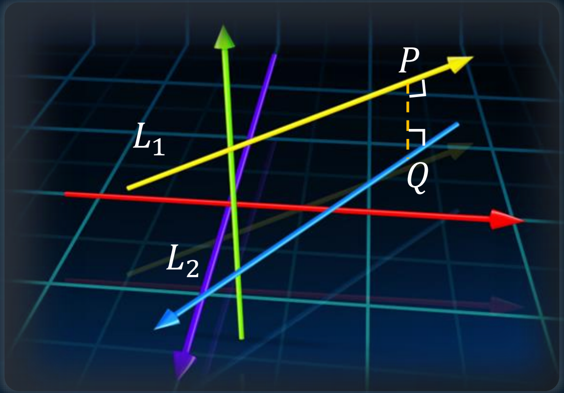
KEY TAKEAWAYS



Shortest distance between 2 skew lines:

Distance PQ is the shortest distance between lines L_1 & L_2.

Let the lines be:

$$L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$$L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

$$\vec{b}-\vec{a} = (x_2-x_1)\hat{\imath} + (y_2-y_1)\hat{\jmath} + (z_2-z_1)\hat{k}$$

$$\boxed{\therefore PQ = \left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum(b_1c_2-b_2c_1)^2}}\right|}$$

Note: If lines are skew, $\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum(b_1c_2-b_2c_1)^2}} \neq 0$

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The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is:

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A $2\sqrt{30}$

B $\frac{7}{2}\sqrt{30}$

C $3\sqrt{30}$

D 3

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The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is:

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Solution:

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$$

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

$$\boxed{\therefore PQ=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum(b_1c_2-b_2c_1)^2}}\right|}$$

$$\therefore PQ=\left|\frac{\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-6)^2+(15)^2+(3)^2}}\right|$$

$$\Rightarrow PQ=\frac{270}{\sqrt{270}}$$

$$\Rightarrow PQ=3\sqrt{30}$$

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The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is:

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A $2\sqrt{30}$

B $\frac{7}{2}\sqrt{30}$

C $3\sqrt{30}$

D 3

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Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

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Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

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$$L_1: \frac{x-\lambda}{1}=\frac{y-\frac{1}{2}}{\frac{1}{2}}=\frac{z}{-\frac{1}{2}} \quad \lambda \in \mathbb{I}$$

$$L_2: \frac{x}{1}=\frac{y+2\lambda}{1}=\frac{z-\lambda}{1}$$

$$\therefore PQ = \left| \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum(b_1c_2-b_2c_1)^2}} \right|$$

$$PQ = \left| \frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}}{\sqrt{1^2+\left(\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}} \right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = \lambda\left(\frac{1}{2}+\frac{1}{2}\right)-\left(\frac{1}{2}+2\lambda\right)\left(1+\frac{1}{2}\right)-\lambda\left(1-\frac{1}{2}\right)$$

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Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

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$$\Rightarrow \begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = \lambda-\left(\frac{1}{2}+2\lambda\right)\left(\frac{3}{2}\right)-\frac{\lambda}{2}=-\frac{5\lambda}{2}-\frac{3}{4}$$

$$\Rightarrow \frac{\left|\frac{-5\lambda}{2}-\frac{3}{4}\right|}{\sqrt{\frac{7}{2}}}=\frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow |-10\lambda-3|=7$$

$$\Rightarrow -10\lambda-3=\pm 7$$

$$\Rightarrow \lambda=\frac{2}{5},-1$$

$$\therefore |\lambda|=1$$

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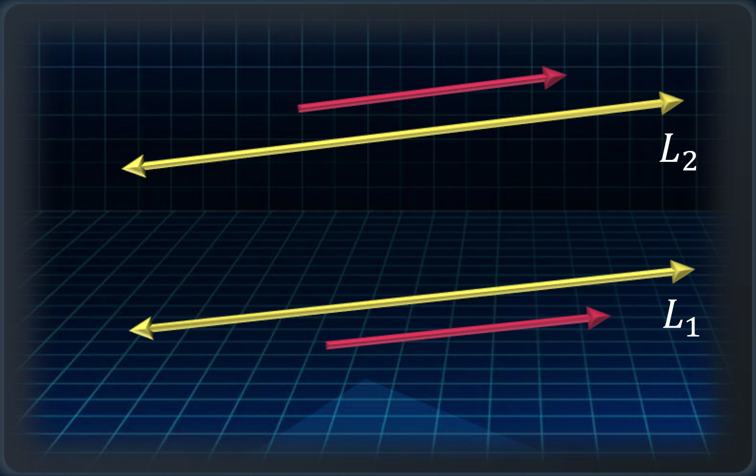
KEY TAKEAWAYS

Condition for lines to be Coplanar:

Two lines which are either intersecting or parallel, are always coplanar (lying in the same plane).



Let lines be:

$$L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

If lines are parallel, they have same direction cosines.

If lines are intersecting, shortest distance between them is 0.

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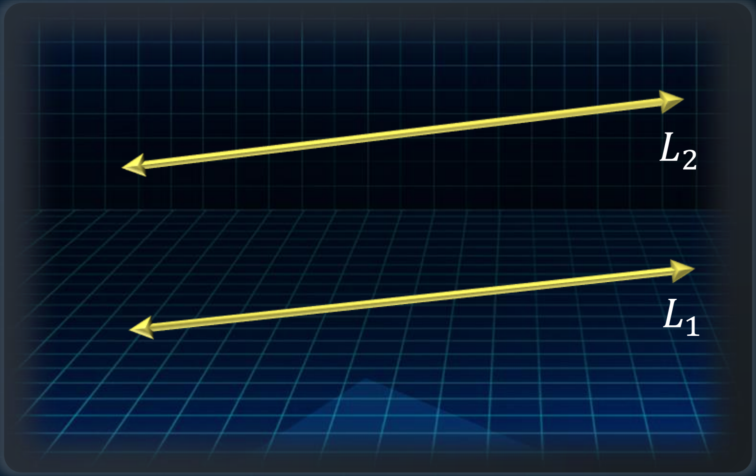
KEY TAKEAWAYS

Condition for lines to be Coplanar:

Condition for co planar lines :

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$



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If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

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A $(-2, 10, 2)$

B $(10, 2, 2)$

C $(10, -2, -2)$

D $(2, -10, -2)$

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If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

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Solution:

$$\left.\begin{aligned} L_1 &: \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1} \\ L_2 &: \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1} \end{aligned}\right\} \text{coplanar}$$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$$\boxed{\because \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

$$\Rightarrow 1(-1-(5-\alpha)) - 3(2-\alpha) + 2(2(5-\alpha)+\alpha) = 0$$

$$\Rightarrow \alpha = -4$$

$$\therefore L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

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If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

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Solution:

$$\therefore L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

Any point on line L_2 can be $(-4\lambda - 2, 9\lambda - 1, \lambda - 1)$

For $\lambda = -1$, it passes through $(2, -10, -2)$.

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If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

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A $(-2, 10, 2)$

B $(10, 2, 2)$

C $(10, -2, -2)$

D $(2, -10, -2)$

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If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}, \frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent, then:

A $h=-2, k=-6$

B $h=\frac{1}{2}, k=-2$

C $h=6, k=2$

D $h=2, k=\frac{1}{2}$

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If the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}, \frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$ and $\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}$ are concurrent, then:

Solution:

$$\underbrace{L_1 : \frac{x}{1} = \frac{y}{2} = \frac{z}{3} = \lambda \qquad L_2 : \frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = \mu \qquad L_3 : \frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}}_{\text{concurrent}}$$

Point on L_1 $(\lambda, 2\lambda, 3\lambda)$

Point on L_2 $(3\mu + 1, -\mu + 2, 4\mu + 3)$

$$\left.\begin{aligned} \lambda &= 3\mu + 1 \\ 2\lambda &= -\mu + 2 \\ 3\lambda &= 4\mu + 3 \end{aligned}\right\} \Rightarrow \lambda = 1, \mu = 0$$

Point of intersection is $(1, 2, 3)$

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If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}, \frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent, then:

Solution:

Point of intersection is $(1,2,3)$

L_3 passes through $(1,2,3)$

Putting in L_3 : $\frac{1+k}{3}=\frac{2-1}{2}=\frac{3-2}{h}$

$\Rightarrow h=2, k=\frac{1}{2}$

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If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}, \frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent, then:

A $h=-2, k=-6$

B $h=\frac{1}{2}, k=-2$

C $h=6, k=2$

D $h=2, k=\frac{1}{2}$

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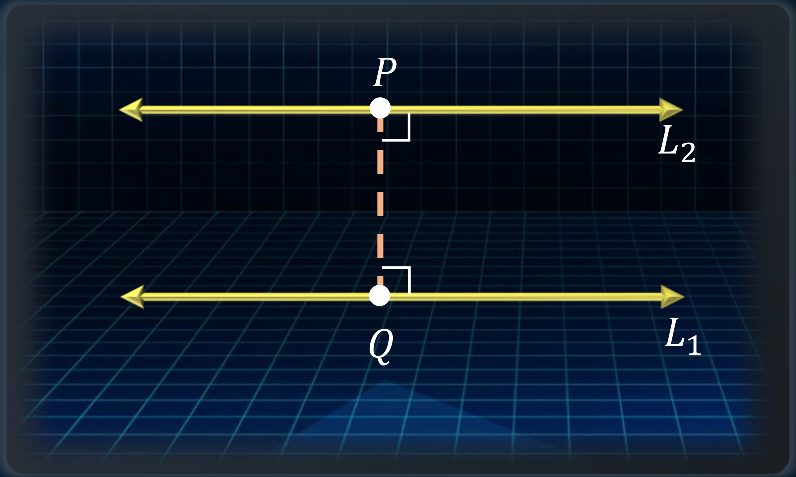
KEY TAKEAWAYS

Shortest Distance between Parallel Lines:

Distance PQ is the shortest distance between lines between lines L_1 & L_2.



Let the lines be:

$$L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

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KEY TAKEAWAYS

Computing distance between two parallel lines:

$$L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

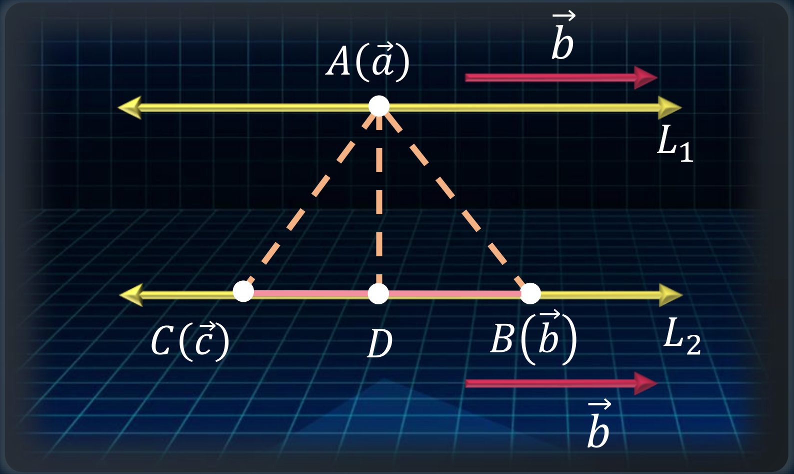
$$L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

Area of $\triangle ABC = \frac{1}{2}\left|(\vec{a}-\vec{c}) \times \vec{b}\right|$

$= \frac{1}{2}\left|\vec{b}\right|. AD$

AD = Shortest Distance$= \frac{\left|(\vec{a}-\vec{c})\times\vec{b}\right|}{\left|\vec{b}\right|}$

$CD = \left|(\vec{a}-\vec{c}) \cdot \hat{b}\right|$ $\quad AD = \sqrt{AC^2 - CD^2} = \left|(\vec{a}-\vec{c}) \times \hat{b}\right|$



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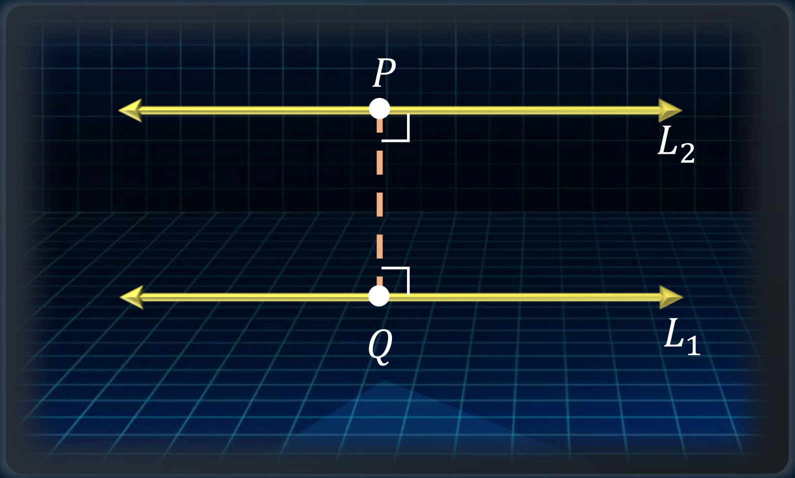
KEY TAKEAWAYS

Shortest Distance between Parallel Lines:

Distance PQ is the shortest distance between lines between lines L_1 & L_2.



Let the lines be:

$$L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

$$\boxed{PQ = \left| \frac{\left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{matrix} \right|}{\sqrt{a_1^2+b_1^2+c_1^2}} \right|}$$

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The shortest distance between the lines $L_1 : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{2}$ and $L_2 : \frac{x-2}{4} = \frac{y}{-2} = \frac{z+1}{4}$, is:

A $\sqrt{26}$

B $\frac{\sqrt{26}}{3}$

C $\sqrt{3}$

D 5

Solution:

$$L_1 : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{2}$$

$$L_2 : \frac{x-2}{4} = \frac{y}{-2} = \frac{z+1}{4}$$

$$\boxed{PQ = \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix}}{\sqrt{a_1^2+b_1^2+c_1^2}} \right|}$$

$$PQ = \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix}}{\sqrt{2^2+(-1)^2+2^2}} \right| = \left| \frac{\hat{\imath}-4\hat{\jmath}-3\hat{k}}{3} \right| = \frac{\sqrt{26}}{3}$$

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KEY TAKEAWAYS

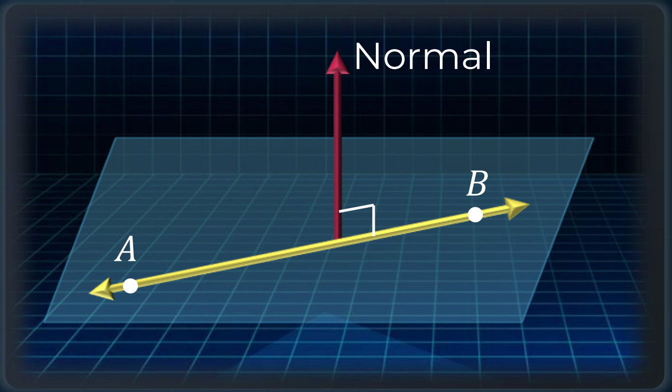
Plane:

If a line joining any two points on a surface lies completely on it, then the surface is a plane.

Or

If the line joining any two points on a surface is perpendicular to some fixed straight line.

Then, the surface is called a plane and a fixed straight line is called normal to the plane.



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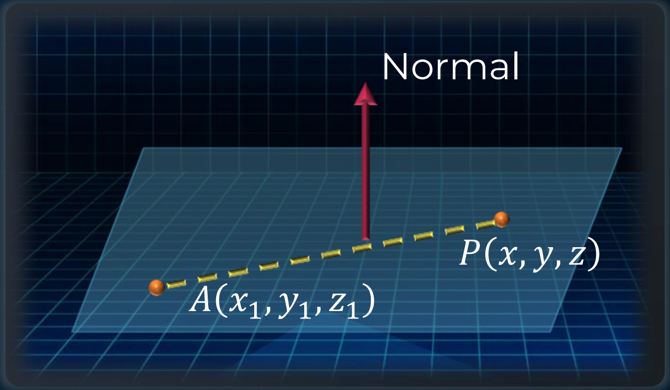
KEY TAKEAWAYS

Equation of plane passing through a point:

Given: Direction ratio of normal of plane a, b, c and a point $A(x_1, y_1, z_1)$ on it.

Equation: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$



$AP \perp$ Normal

DRs of Normal $\propto (a, b, c)$

DRs of AP $\propto (x - x_1, y - y_1, c - c_1)$

$\Rightarrow \cos\theta$

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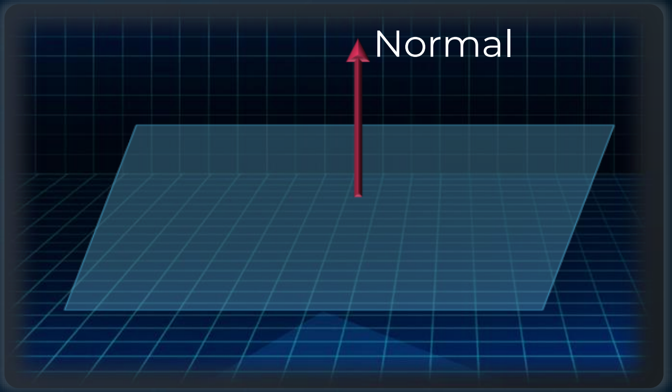
General form of Equation of Plane:



Let direction ratio of normal of plane be a, b, c.

Equation of plane: $ax + by + cz = d$

Plane	Equation
yz plane	$x = 0$
xz plane	$y = 0$
xy plane	$z = 0$
Parallel to x–axis	$by + cz = d$
Parallel to y–axis	$ax + cz = d$
Parallel to z–axis	$ax + by = d$

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Consider the three planes : $P_1 : 3x + 15y + 21z = 9;\ P_2 : x - 3y - z = 5;$ $P_3 : 2x + 10y + 14z = 5$. Then, which one of the following is true ?

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A. P_1 and P_3 are parallel

B. P_2 and P_3 are parallel

C. P_1 and P_2 are parallel

D. P_1, P_2 and P_3 are parallel

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Consider the three planes : $P_1 : 3x + 15y + 21z = 9;\ P_2 : x - 3y - z = 5;$ $P_3 : 2x + 10y + 14z = 5$. Then, which one of the following is true ?

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A. P_1 and P_3 are parallel

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C. P_1 and P_2 are parallel

D. P_1, P_2 and P_3 are parallel

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Consider the three planes : $P_1: 3x + 15y + 21z = 9$; $P_2 : x - 3y - z = 5$; $P_3: 2x + 10y + 14z = 5$. Then, which one of the following is true ?

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Solution:

$$P_1: x + 5y + 7z = 3$$

$$P_2 : x - 3y - z = 5$$

$$P_3: x + 5y + 7z = \frac{5}{2}$$

P_1 and P_3 are parallel.

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The equation of a plane which passes through $(2, -3, 1)$ and is perpendicular to the line joining points $(3, 4, -1)$ & $(2, -1, 5)$, is :

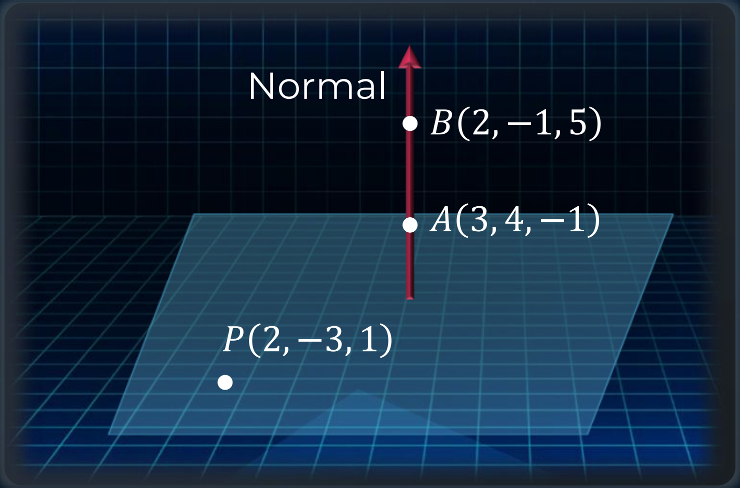
A $x + 5y - 6z + 19 = 0$

B $x - 5y + 6z - 19 = 0$

C $x + 5y + 6z + 19 = 0$

D $x - 5y - 6z - 19 = 0$

Solution:



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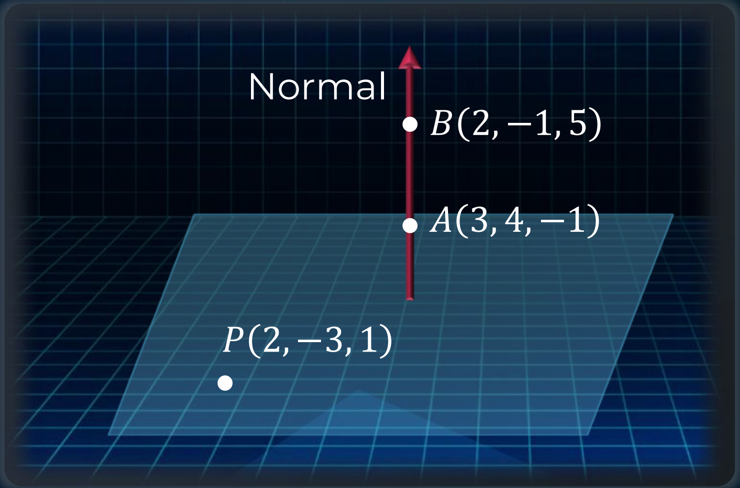
The equation of a plane which passes through $(2, -3, 1)$ and is perpendicular to the line joining points $(3, 4, -1)$ & $(2, -1, 5)$, is :

Solution:



DRs of the line joining AB: $-1, -5, 6$

DRs of the plane will be: $-1, -5, 6$

So, the equation of plane is:

$$-(x-2) - 5(y+3) + 6(z-1) = 0$$

$$\Rightarrow x + 5y - 6z + 19 = 0$$

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Find the vector and cartesian equations of the plane which passes through the points $(5, 2, -4)$ and perpendicular to the line with direction ratios $2, 3, -1$

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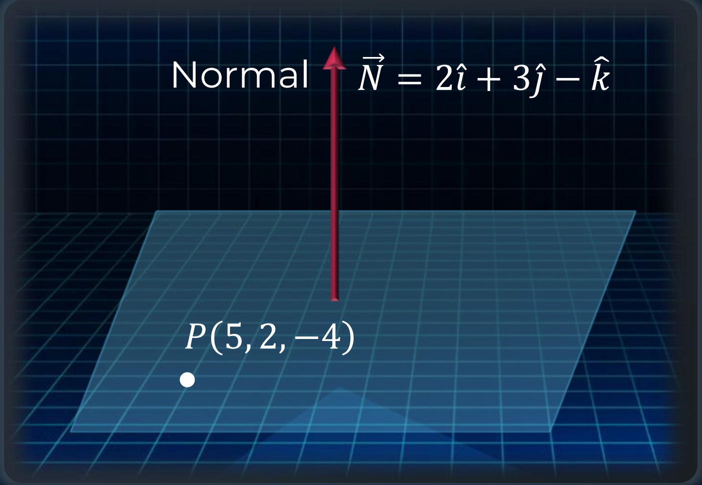
Find the vector and cartesian equations of the plane which passes through the points $(5,2,-4)$ and perpendicular to the line with direction ratios $2,3,-1$

Solution:

We have the position vector of point $(5,2,-4)$ as $\vec{a}=5\hat{\imath}+2\hat{\jmath}-4\hat{k}$ and the normal vector $\vec{N}$ perpendicular to the plane as $\vec{N}$ $=2\hat{\imath}+3\hat{\jmath}-\hat{k}$



Therefore, the vector equation of the plane is given by $(\vec{r}-\vec{a})\cdot\vec{N}=\theta$

or $\left[\vec{r}-(5\hat{\imath}+2\hat{\jmath}-4\hat{k})\right]\cdot\left(2\hat{\imath}+3\hat{\jmath}-\hat{k}\right)=0\cdots(1)$

Transforming (1) into cartesian form, we have

$\left[(x-5)\hat{\imath}+(y-2)\hat{\jmath}+(z+4)\hat{k}\right]\cdot\left(2\hat{\imath}+3\hat{\jmath}-\hat{k}\right)=0$

or $2(x-5)+3(y-2)-(z+4)=0$

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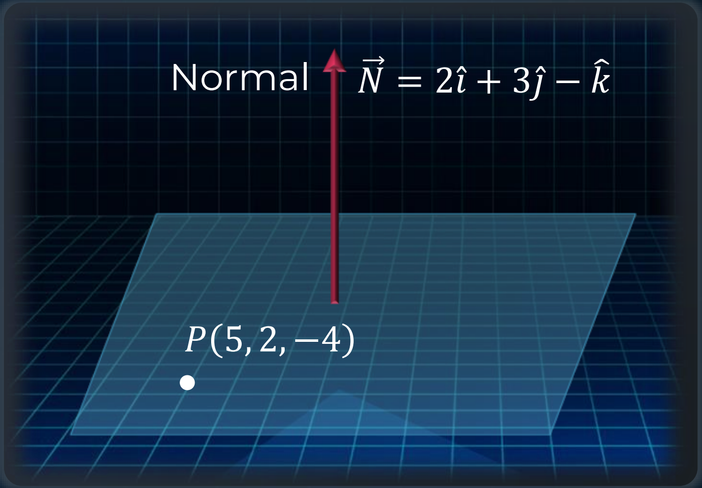
Find the vector and cartesian equations of the plane which passes through the points $(5, 2, -4)$ and perpendicular to the line with direction ratios $2, 3, -1$

Solution:

or $2(x-5)+3(y-2)-(z+4)=0$

i.e. $2x+3y-z=20$

Which is the cartesian equation of the plane



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Session 06

Representation of equation of plane

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The equation of the plane which contains y-axis and passes through the point $(1, 2, 3)$ is :

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A $3x + z = 6$

B $3x - z = 0$

C $x + 3z = 10$

D $x + 3z = 0$

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The equation of the plane which contains y-axis and passes through the point $(1, 2, 3)$ is :

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Solution: Let the equation of plane $ax + by + cz = d$

Then point must pass thru $(0, 0, 0)$

$0 + 0 + 0 = d \Rightarrow d = 0$

Equation of the plane passing through $(1, 2, 3)$ is:

$a + 2b + 3c = 0$

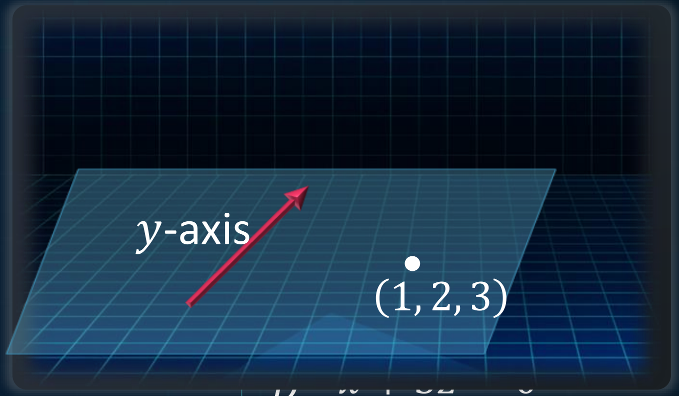
(a, b, c) normal $\perp$ $y -$ axis $(0, 1, 0)$

$\Rightarrow \cos\theta = 0 \Rightarrow a \cdot 0 + b \cdot 1 + c \cdot 0 = 0$

$\Rightarrow b = 0$

$\Rightarrow a + 3c = 0 \Rightarrow a = -3c$

$\therefore$ Equation of the plane is : $ax + cz = 0$



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The equation of the plane which contains y-axis and passes through the point $(1, 2, 3)$ is :

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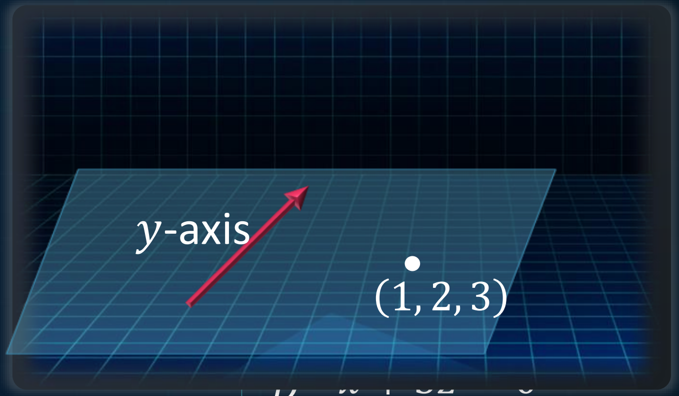
Solution: $\Rightarrow a = -3c$

$\therefore$ Equation of the plane is: $ax + cz = 0$

$$\Rightarrow -3cx + cz = 0$$

$\therefore$ Equation of the plane is: $3x - z = 0$



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The equation of the plane which contains y-axis and passes through the point $(1, 2, 3)$ is :

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A $3x + z = 6$

B $3x - z = 0$

C $x + 3z = 10$

D $x + 3z = 0$

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Let the plane $ax + by + cz + d = 0$ bisects the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at right angles. If a, b, c, d are integers, then the minimum value $(a^2 + b^2 + c^2 + d^2)$ is :

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Solution:

DRs of normal to plane $\equiv$ DRs of $PQ \equiv (2, -6, 6) \equiv (1, -3, 3)$

Let A be the midpoint of P & Q and lie on the plane.

$\therefore A \equiv (3, 0, -2)$

$\min(a^2 + b^2 + c^2 + d^2) = ?\,; a, b, c, d \in \mathbb{I}$

DRs of $PQ : (1, -3, 3) \propto (a, b, c)$

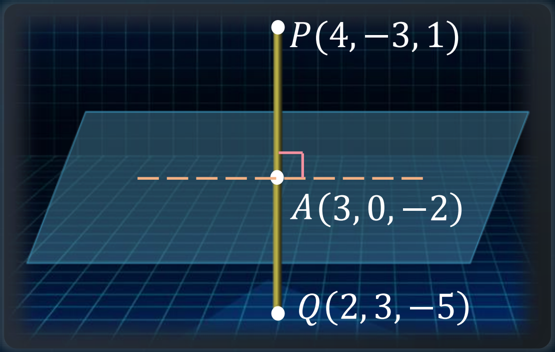
$\Rightarrow x - 3y + 3z + d = 0$

It passes through the point $(3, 0, -2)$

$\Rightarrow 3 - 0 - 6 + d = 0$

$\Rightarrow d = 3$

$\therefore$ Equation of the plane is : $x - 3y + 3z + 3 = 0$



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Let the plane $ax + by + cz + d = 0$ bisects the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at right angles. If a, b, c, d are integers, then the minimum value $(a^2 + b^2 + c^2 + d^2)$ is :

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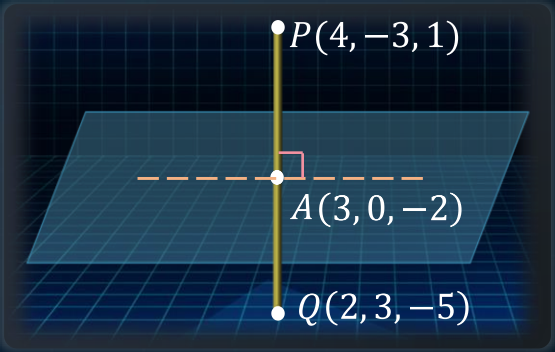
<u>Solution:</u>

DRs of normal to plane $\equiv$ DRs of $PQ \equiv (2, -6, 6) \equiv (1, -3, 3)$

$\Rightarrow d = 3$

$\therefore$ Equation of the plane is : $x - 3y + 3z + 3 = 0$

Minimum value of $(a^2 + b^2 + c^2 + d^2) = 28$



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Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \left(\frac{4\lambda}{11}\right) - 4$ is equal to ____

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Solution:

DRs of PQ : $-1, -5, 6$

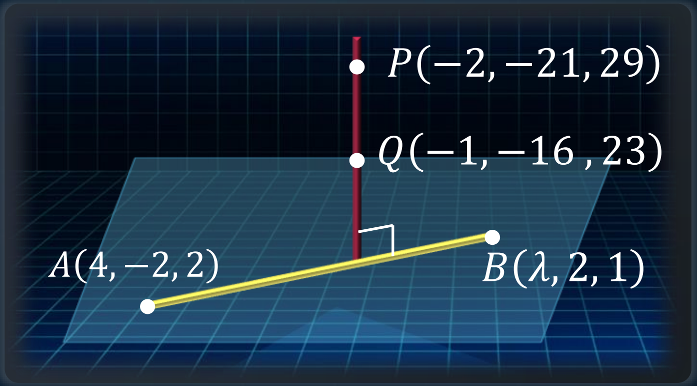
DRs of AB : $4 - \lambda, -4, 1$

AB is perpendicular to PQ

$\Rightarrow (-1)(4 - \lambda) + (-5)(-4) + (6)(1) = 0$

$\Rightarrow \lambda = -22$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \left(\frac{4\lambda}{11}\right) - 4 = 8$



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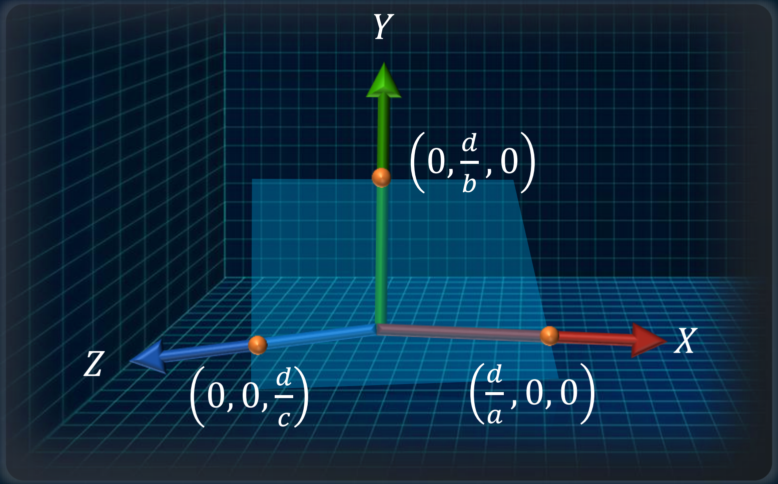
Intercept form of equation of plane:

General form of equation of plane is : $ax + by + cz = d$

$$\Rightarrow \frac{ax}{d} + \frac{by}{d} + \frac{cz}{d} = 1 \Rightarrow \frac{x}{d/a} + \frac{y}{d/b} + \frac{z}{d/c} = 1$$

$$X_{int} = \frac{d}{a} \quad , \; Y_{int} = \frac{d}{b} \quad , \; Z_{int} = \frac{d}{c}$$



Thus, intercept form, is :

$$\frac{x}{X_{int}} + \frac{y}{Y_{int}} + \frac{z}{Z_{int}} = 1$$

DRs of normal is : $\frac{1}{X_{int}}, \frac{1}{Y_{int}}, \frac{1}{Z_{int}}$

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The equation of a plane parallel to $x + 5y - 4z + 5 = 0$ and cutting intercepts on the axes whose sum is 38, is:

A $x + 5y - 4z = 0$

B $x + 5y - 4z = 5$

C $x + 5y - 4z = 10$

D $x + 5y - 4z = 40$

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The equation of a plane parallel to $x + 5y - 4z + 5 = 0$ and cutting intercepts on the axes whose sum is 38, is:

Solution:

As the plane are parallel ⇒ DRs of normal remains same ⇒ coeff of x, y, z

Equation of parallel plane: $x + 5y - 4z = d$ $\quad \frac{x}{d} + \frac{y}{\left(\frac{d}{5}\right)} + \frac{z}{\left(-\frac{d}{4}\right)} = 1$

$X_{int.} = d \qquad Y_{int.} = \frac{d}{5} \qquad Z_{int.} = -\frac{d}{4}$

Given: $X_{int} + Y_{int} + Z_{int} = 38$

Sum = $d + \frac{d}{5} - \frac{d}{4} = 38$

$\Rightarrow d = 40$

Equation of plane : $x + 5y - 4z = 40$

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The equation of a plane parallel to $x + 5y - 4z + 5 = 0$ and cutting intercepts on the axes whose sum is 38, is:

A $x + 5y - 4z = 0$

B $x + 5y - 4z = 5$

C $x + 5y - 4z = 10$

D $x + 5y - 4z = 40$

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If (x, y, z) be an arbitrary point lying on a plane P which passes through the points $(42, 0, 0), (0, 42, 0)$ & $(0, 0, 42)$, then the value of the expression $3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to:

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A) 3

B) 0

C) 39

D) -45

Solution: By intercept form, Equation of plane $P : x + y + z = 42$

$$\Rightarrow \underbrace{(x-11)}_{p} + \underbrace{(y-19)}_{q} + \underbrace{(z-12)}_{r} = 0 \quad \Rightarrow p + q + r = 0$$

$$= 3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

$$= 3 + \frac{p}{(q)^2(r)^2} + \frac{q}{(p)^2(r)^2} + \frac{r}{(p)^2(q)^2} - \frac{p+q+r+42}{14(p)(q)(r)}$$

$$= 3 + \frac{(p)^3+(q)^3+(r)^3}{(p)^2(q)^2(r)^2} - \frac{42}{14(p)(q)(r)} \qquad p + q + r = 0$$

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If (x, y, z) be an arbitrary point lying on a plane P which passes through the points $(42, 0, 0), (0, 42, 0)$ & $(0, 0, 42)$, then the value of the expression $3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to:

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$$= 3 + \frac{(p)^3+(q)^3+(r)^3}{(p)^2(q)^2(r)^2} - \frac{42}{14(p)(q)(r)}$$

$$p + q + r = 0$$

$$= 3 + \frac{3pqr}{(p)^2(q)^2(r)^2} - \frac{3}{(p)(q)(r)}$$

$$\Rightarrow (p)^3+(q)^3+(r)^3 = 3pqr$$

$$= 3$$

A 3

B 0

C 39

D -45

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A plane P meets the coordinate axes at A, B & C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

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A $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

B $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

C $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

D $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

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A plane P meets the coordinate axes at A, B & C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

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Solution:

Centroid of ΔABC: $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$\Rightarrow a = 3, b = 3, c = 6$

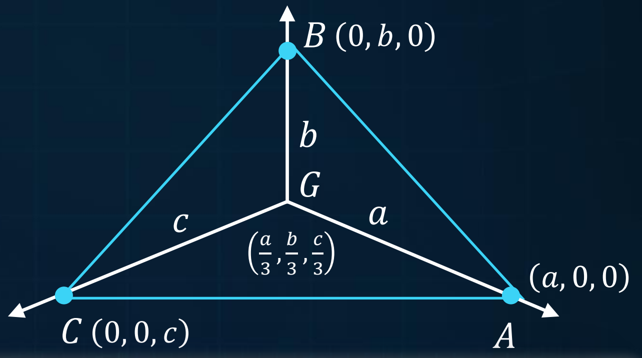
Equation of plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

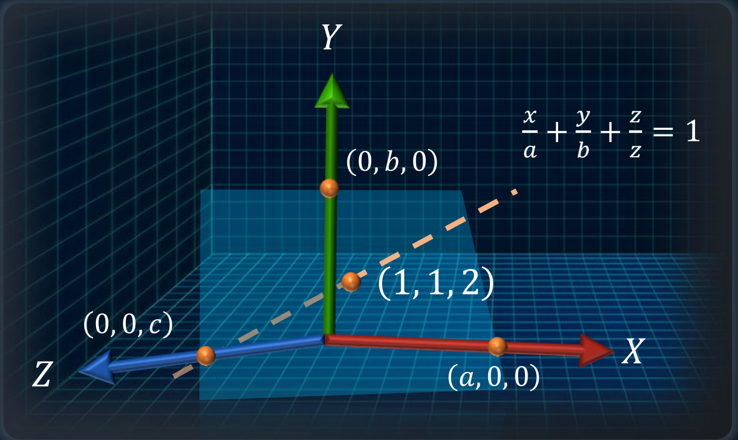
$\Rightarrow 2x + 2y + z = 6$

DRs of line perpendicular to the plane : 2 , 2 , 1

Point on line is: (1 , 1 , 2)

Thus, equation of line is: $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$





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A plane P meets the coordinate axes at A, B & C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

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A $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

B $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

C $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

D $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

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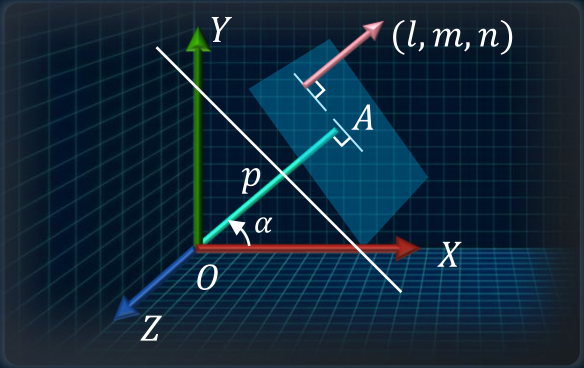
KEY TAKEAWAYS

Normal Form of Plane:

$$lx + my + nz = p \quad \begin{cases} l, m, n \text{ are DCs of normal.} \\ p = \text{distance of plane from origin.} \end{cases}$$



Conversion of general form to normal form:

General form : $ax + by + cz = d$

Divide both sides by $\sqrt{a^2 + b^2 + c^2}$

Normal form : $\dfrac{ax}{\sqrt{a^2+b^2+c^2}} + \dfrac{by}{\sqrt{a^2+b^2+c^2}} + \dfrac{cz}{\sqrt{a^2+b^2+c^2}} = \dfrac{d}{\sqrt{a^2+b^2+c^2}}$

Note | Constant term on right side should be positive .

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Equation of plane upon which the length of normal from origin is 10 and direction ratios of this normal are $3, 2, 6$, is :

A $3x + 2y + 6z = 70$

B $3x + 2y - 6z = 70$

C $3x - 2y - 6z = 70$

D $3x + 2y + 6z = -70$

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Equation of plane upon which the length of normal from origin is 10 and direction ratios of this normal are $3, 2, 6$, is :

Solution:

DRs of normal are: $(3, 2, 6)$

DCs of normal are: $\left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7}\right)$

Equation of plane : $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z = 10$

$lx + my + nz = p$

$3x + 2y + 6z = 70$

$p = 10$

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Equation of plane upon which the length of normal from origin is 10 and direction ratios of this normal are $3, 2, 6$, is :

A $3x + 2y + 6z = 70$

B $3x + 2y - 6z = 70$

C $3x - 2y - 6z = 70$

D $3x + 2y + 6z = -70$

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KEY TAKEAWAYS

Equation of plane passing through three points:

Equation of plane passing through points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is :

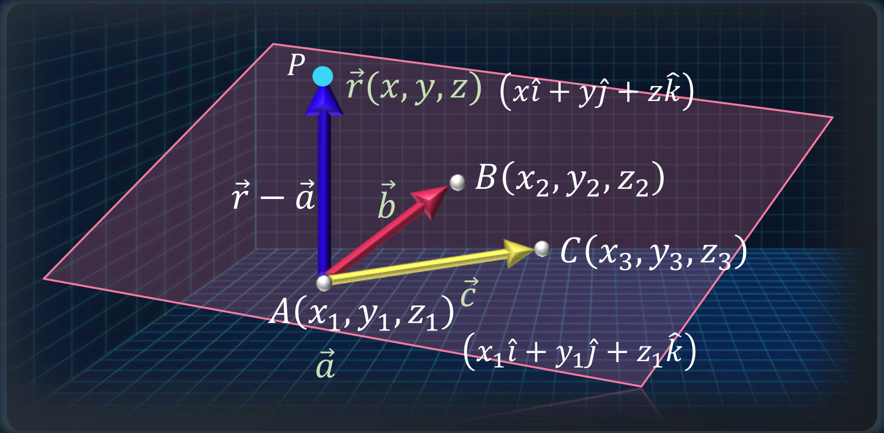
$P(x, y, z)$ is the general point on plane

$\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}$, are coplanar

$$[\overrightarrow{AP} \quad \overrightarrow{AB} \quad \overrightarrow{AC}] = 0$$

$$[\vec{r} - \vec{a} \quad \vec{b} - \vec{a} \quad \vec{c} - \vec{a}] = 0$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$



Equation of plane : $[\vec{r} - \vec{a} \;\; \vec{b} \;\; \vec{c}] = 0$

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Equation of plane passing through the points $(1,1,1),(2,1,-1)$ & $(3,3,0)$ is:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$(x_1, y_1, z_1) \equiv (1,1,1)$

$(x_2, y_2, z_2) \equiv (2,1,-1)$

$(x_3, y_3, z_3) \equiv (3,3,0)$

Equation of plane :

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 0 & -2 \\ 2 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(4) - (y-1)(3) + (z-1)(2) = 0$$

$$\Rightarrow 4x - 4 - 3y + 3 + 2z - 2 = 0$$

$$4x - 3y + 2z = 3$$

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KEY TAKEAWAYS

Condition for four points to be coplanar:

Given points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$

$\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$, are coplanar

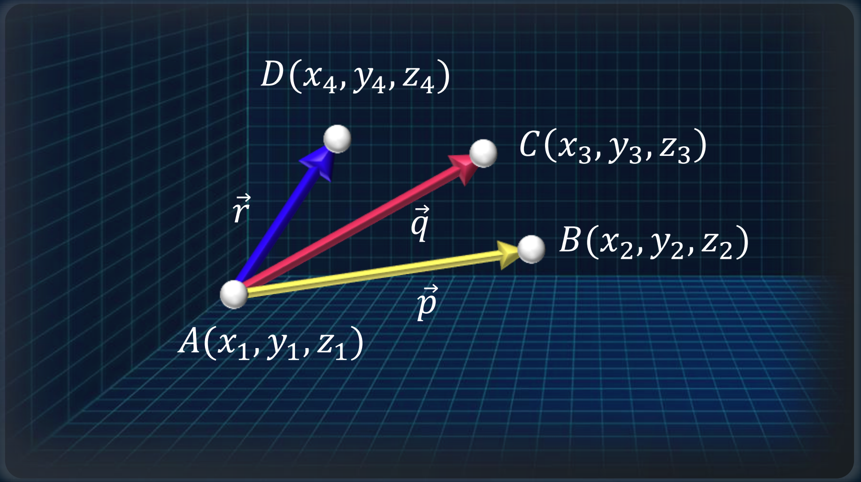
$$\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

$$\overrightarrow{AC} = (x_3 - x_1)\hat{\imath} + (y_3 - y_1)\hat{\jmath} + (z_3 - z_1)\hat{k}$$

$$\overrightarrow{AD} = (x_4 - x_1)\hat{\imath} + (y_4 - y_1)\hat{\jmath} + (z_4 - z_1)\hat{k}$$

Condition for them to lie in a plane :

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0$$



Condition: $[\vec{p}\ \vec{q}\ \vec{r}] = 0$

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If $(1,5,35),(7,5,5),(1,\lambda,7)$ & $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is:

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$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0 \qquad \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$\Rightarrow 5\lambda^2 - 44\lambda + 39 = 0$ $\Rightarrow$ Sum of values of $\lambda : \frac{44}{5}$

A $-\frac{44}{5}$

B $\frac{39}{5}$

C $-\frac{39}{5}$

D $\frac{44}{5}$

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If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ & $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to:

JEE MAINS Jan 2019

A 5

B 7

C 17

D 12

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If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ & $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to:

JEE MAINS Jan 2019

Solution: $2\alpha - 3\beta = ?$

Normal vector to the plane : $\vec{n} = \vec{a} \times \vec{b}$

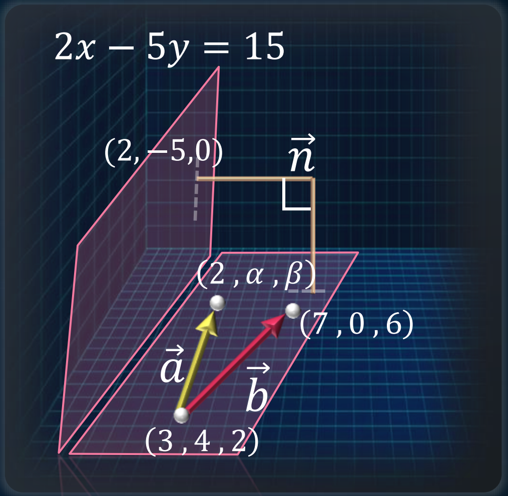
$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & \alpha - 4 & \beta - 2 \\ 4 & -4 & 4 \end{vmatrix}$$

$$\vec{n} = 4(\alpha + \beta - 6)\hat{i} + 4(\beta - 1)\hat{j} + 4(-\alpha + 5)\hat{k}$$

$\because$ it is perpendicular to the plane $2x - 5y = 15$

$$\Rightarrow 8(\alpha + \beta - 6) - 20(\beta - 1) = 0$$

$$\Rightarrow 2\alpha - 3\beta = 7$$



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If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ & $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to:

JEE MAINS Jan 2019

A. 5

B. 7

C. 17

D. 12

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Session 07

A point and a plane

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The equation of the plane passing through the point $(1, 2, -3)$ and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

JEE MAINS FEB 2021

A $3x - 10y - 2z + 11 = 0$

B $11x + y + 17z + 38 = 0$

C $6x - 5y - 2z - 2 = 0$

D $6x - 5y + 2z + 10 = 0$

Solution: P_1: $3x + y - 2z = 5$ $\quad \overrightarrow{n_1} : 3\hat{i} + \hat{j} - 2\hat{k}$

P_2: $2x - 5y - z = 7$ $\quad \overrightarrow{n_2} : 2\hat{i} - 5\hat{j} - \hat{k}$

plane passing through the point $(1, 2, -3)$

Let normal vector to the plane be $\vec{n} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$$

So, equation of the plane: $-11(x - 1) - (y - 2) - 17(z + 3) = 0$

$$\Rightarrow 11x + y + 17z + 38 = 0$$

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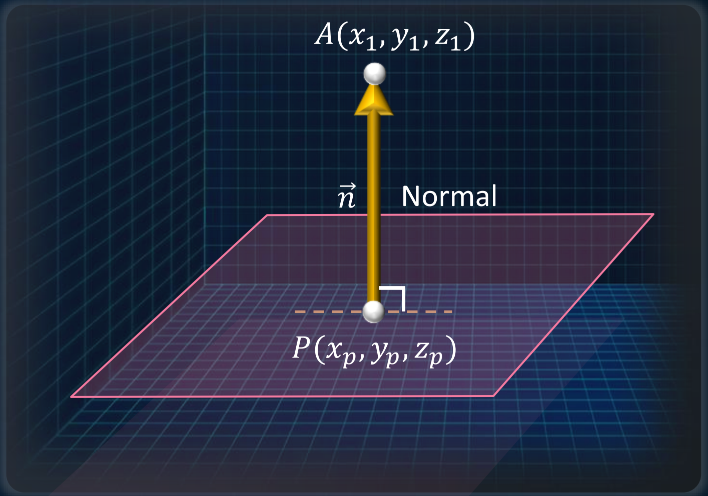
KEY TAKEAWAYS

Foot of perpendicular from a point to a plane:

Let the equation of the plane : $ax + by + cz = d$

AP is parallel to normal to the plane,

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = \lambda \cdots (i)$$



$$\Rightarrow x_p = x_1 + a\lambda\,; \qquad y_p = y_1 + b\lambda\,; \qquad z_p = z_1 + c\lambda$$

Since, P lies on plane

$$a(x_1 + a\lambda) + b(y_1 + b\lambda) + c(z_1 + c\lambda) = d \Rightarrow \lambda = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

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KEY TAKEAWAYS

Foot of perpendicular from a point to a plane:

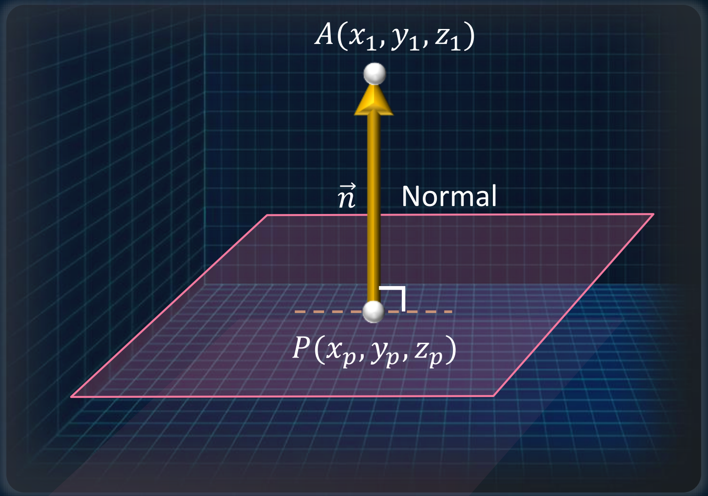
Let the equation of the plane : $ax + by + cz = d$

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = \lambda \cdots (i)$$

$$\Rightarrow \lambda = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

Substituting the value in (i)

$$\boxed{\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}}$$



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The foot of perpendicular of point $(1,0,2)$ to the plane $2x + y + z = 5$, is:

A $\left(\frac{5}{3}, \frac{1}{3}, \frac{4}{3}\right)$

B $\left(\frac{1}{6}, \frac{4}{3}, \frac{10}{3}\right)$

C $\left(\frac{4}{3}, \frac{1}{6}, \frac{13}{6}\right)$

D $(2,0,1)$

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The foot of perpendicular of point (1,0,2) to the plane $2x + y + z = 5$, is:

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

$$\frac{x_p - 1}{2} = \frac{y_p}{1} = \frac{z_p - 2}{1} = -\frac{(2(1) + 0 + 2 - 5)}{(6)}$$

$$x_p = \frac{4}{3};\, y_p = \frac{1}{6};\, z_p = \frac{13}{6}$$

Thus foot of perpendicular is: $\left(\frac{4}{3}, \frac{1}{6}, \frac{13}{6}\right)$

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The foot of perpendicular of point $(1,0,2)$ to the plane $2x + y + z = 5$, is:

A $\left(\frac{5}{3}, \frac{1}{3}, \frac{4}{3}\right)$

B $\left(\frac{1}{6}, \frac{4}{3}, \frac{10}{3}\right)$

C $\left(\frac{4}{3}, \frac{1}{6}, \frac{13}{6}\right)$

D $(2,0,1)$

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KEY TAKEAWAYS

Image of point with respect to a plane :

Let the equation of the plane : $ax + by + cz = d$

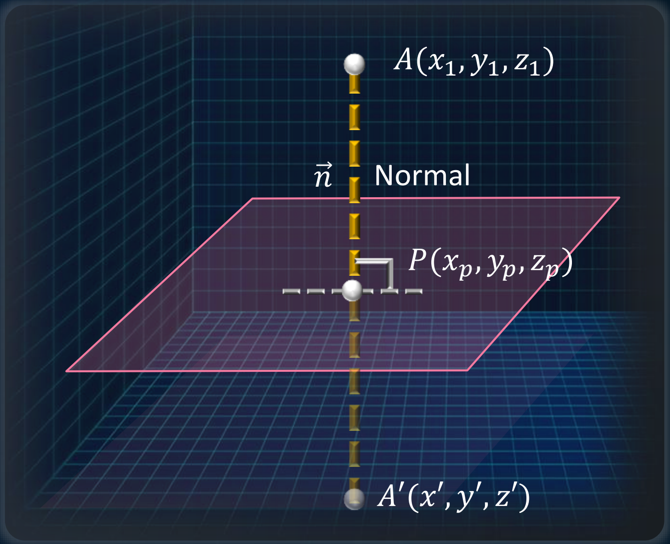
$$x' = 2x_p - x_1; y' = 2y_p - y_1; z' = 2z_p - z_1$$

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = \lambda \cdots (i)$$

$$\lambda = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

$$\frac{x' - x_1}{a} = \frac{2x_p - 2x_1}{a} = 2\lambda$$

$$\boxed{\frac{x' - x_1}{a} = \frac{y' - y_1}{b} = \frac{z' - z_1}{c} = -2\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}}$$



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If the mirror image of the point $(1, 3, 5)$ with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals: ______.

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Solution: $\frac{x\prime - x_1}{a} = \frac{y\prime - y_1}{b} = \frac{z\prime - z_1}{c} = -2\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$

$$\Rightarrow \frac{\alpha - 1}{4} = \frac{\beta - 3}{-5} = \frac{\gamma - 5}{2} = -2\frac{(4(1) - 5(3) + 2(5) - 8)}{(4^2 + (-5)^2 + 2^2)}$$

$$\Rightarrow \alpha = \frac{13}{5}; \beta = 1; \gamma = \frac{29}{5}$$

$$\Rightarrow 5(\alpha + \beta + \gamma) = 13 + 5 + 29 = 47$$

A 47

B 43

C 39

D 41

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The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

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A $(1, -1, 1)$

B $(-1, -1, 1)$

C $(1, 1, 1)$

D $(-1, -1, -1)$

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The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

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Solution:

Mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

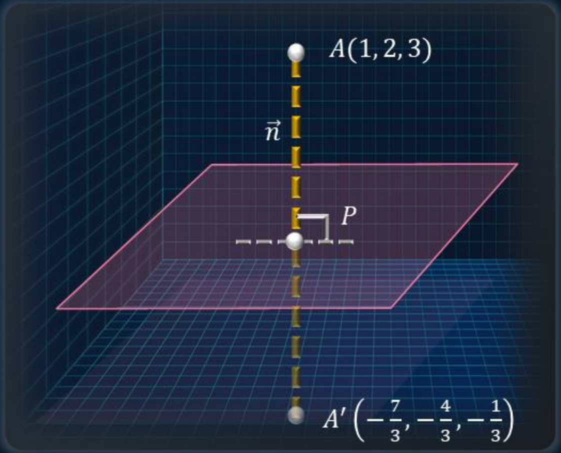
DRs of normal to the plane is:

$$\left(1+\frac{7}{3}, 2+\frac{4}{3}, 3+\frac{1}{3}\right) \equiv (1, 1, 1)$$

Point P is : $\left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Equation of plane :

$$\Rightarrow 1\cdot\left(x+\frac{2}{3}\right)+1\cdot\left(y-\frac{1}{3}\right)+1\cdot\left(z-\frac{4}{3}\right)=0$$



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The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

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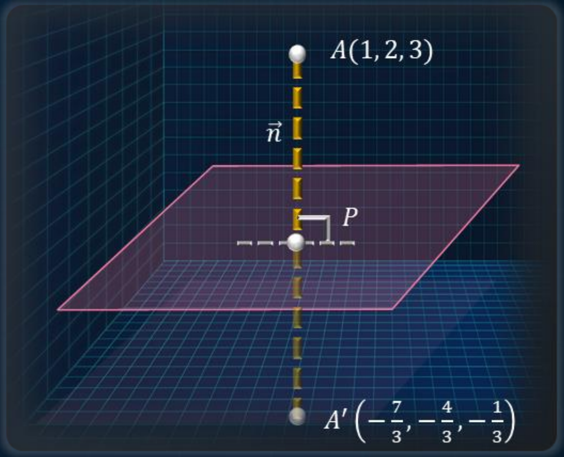
Equation of plane :

$$\Rightarrow 1 \cdot \left(x + \frac{2}{3}\right) + 1 \cdot \left(y - \frac{1}{3}\right) + 1 \cdot \left(z - \frac{4}{3}\right) = 0$$

$$\Rightarrow x + y + z = 1$$

Thus, point $(1, -1, 1)$ lies on the plane



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The mirror image of the point $(1,2,3)$ in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane?

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A $(1,-1,1)$

B $(-1,-1,1)$

C $(1,1,1)$

D $(-1,-1,-1)$

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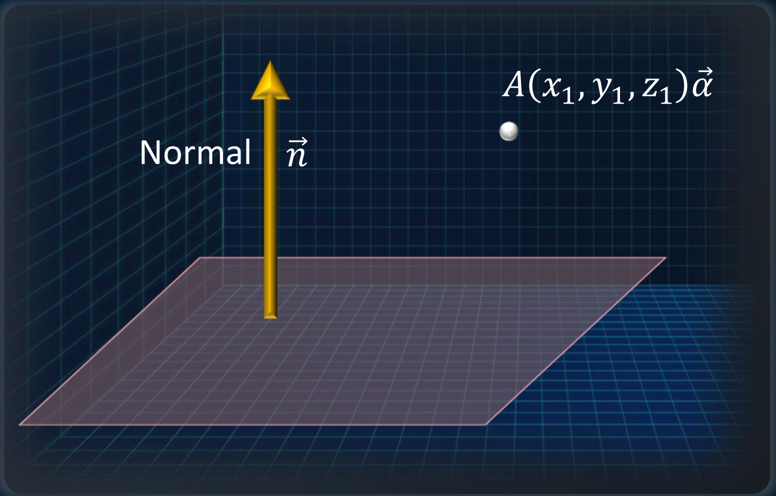
Distance of a Point from a Plane:

Let equation of plane: $ax + by + cz = d$

where a, b, c are DRs of normal.

$$D = \frac{|\vec{\alpha}.\vec{n} - d|}{|\vec{n}|}$$

$$\boxed{D = \left|\frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|}$$



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The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at a unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is____.

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Solution:

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The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at a unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is____.

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Solution:

Let equation of required plane : $x - 2y + 2z + d = 0$

$$\left|\frac{1 - 2(2) + 2(3) + d}{\sqrt{1^2 + (-2)^2 + 2^2}}\right| = 1$$

$$D = \left|\frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$\Rightarrow d = 0, -6$

$(b - d) = -2$ or 4 , $(c - a) = 1$

$\Rightarrow K = -2$ or 4

$\therefore K = 4$

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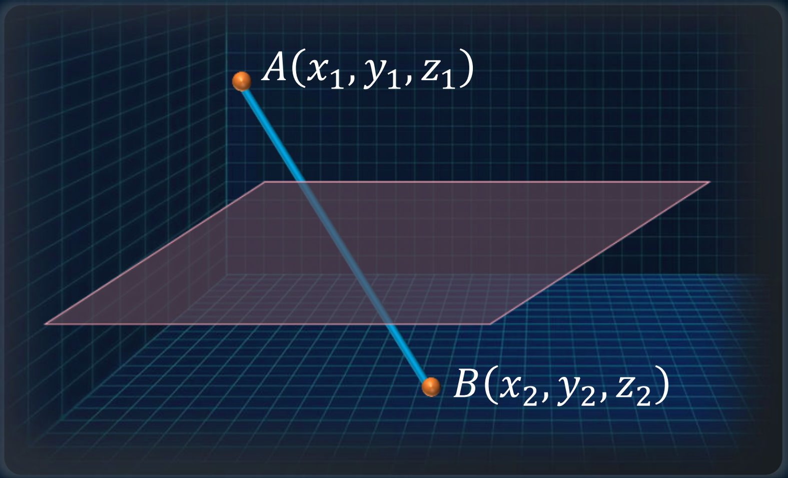
KEY TAKEAWAYS

Relative Position of Two Points with Respect to a Plane:

Let equation of plane : $ax + by + cz - d = 0$

where a, b, c are DRs of normal.

Two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ are on:



Ratio in which the plane divides line joining points A & B is :

$$-\frac{(ax_1 + by_1 + cz_1 - d)}{(ax_2 + by_2 + cz_2 - d)}$$

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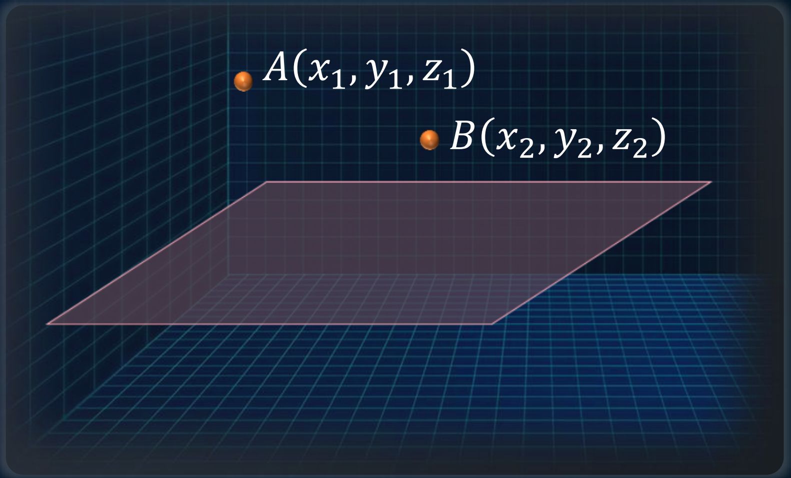
KEY TAKEAWAYS

Relative Position of Two Points with Respect to a Plane:



Ratio in which the plane divides line joining points A & B is :

$$-\frac{(ax_1 + by_1 + cz_1 - d)}{(ax_2 + by_2 + cz_2 - d)}$$

(i) Same side of plane,

$$-\frac{(ax_1 + by_1 + cz_1 - d)}{(ax_2 + by_2 + cz_2 - d)} < 0$$

$$\Rightarrow \frac{(ax_1 + by_1 + cz_1 - d)}{(ax_2 + by_2 + cz_2 - d)} > 0$$

the signs of $ax_1 + by_1 + cz_1 - d$ and $ax_2 + by_2 + cz_2 - d$ are same.

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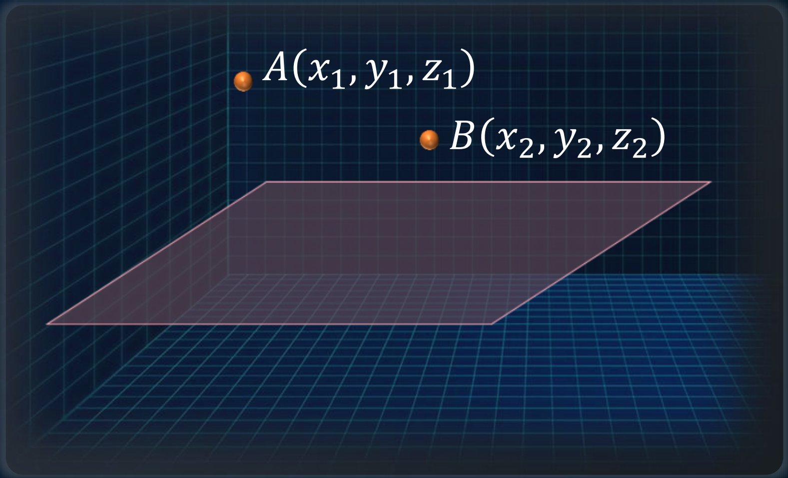
KEY TAKEAWAYS

Relative Position of Two Points with Respect to a Plane:

(ii) Opposite side of plane,

the signs of $ax_1 + by_1 + cz_1 - d$ and $ax_2 + by_2 + cz_2 - d$ are opposite.



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Ratio in which the plane $2x - y + 3z + 4 = 0$ divides the line joining the points $(1, 2, -4)$ & $(-3, 1, -7)$ is:

A $2:3$

B $-1:3$

C $3:4$

D $3:1$

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Ratio in which the plane $2x - y + 3z + 4 = 0$ divides the line joining the points $(1, 2, -4)$ & $(-3, 1, -7)$ is:

Solution:

$$\text{Ratio} = -\frac{(2(1) - (2) + 3(-4) + 4)}{(2(-3) - 1 + 3(-7) + 4)}$$

$$\text{Ratio} : -\frac{(ax_1 + by_1 + cz_1 - d)}{(ax_2 + by_2 + cz_2 - d)}$$

$$= -\frac{1}{3}$$

Division is $1:3$ external.

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Ratio in which the plane $2x - y + 3z + 4 = 0$ divides the line joining the points $(1, 2, -4)$ & $(-3, 1, -7)$ is:

A $2:3$

B $-1:3$

C $3:4$

D $3:1$

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Points $(1,2,3)$ & $(2,-1,4)$ with respect to the plane $x + 4y + z - 3 = 0$ lie on:

A. Opposite side

B. The plane

C. Same side

D. One lie on plane and other doesn't

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Points $(1, 2, 3)$ & $(2, -1, 4)$ with respect to the plane $x + 4y + z - 3 = 0$ lie on:

A. Opposite side

B. The plane

C. Same side

D. One lie on plane and other doesn't

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Points $(1,2,3)$ & $(2,-1,4)$ with respect to the plane $x+4y+z-3=0$ lie on:

Solution:

Let $A(1,2,3)$ & $B(2,-1,4)$

Equation of plane: $x+4y+z-3=0$

For point A: $1+4(2)+3-3 \quad > 0$

For point B: $2+4(-1)+4-3 \quad < 0$

$\therefore$ Points A & B lie on opposite side.

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KEY TAKEAWAYS

Angle between a Line and a Plane:

Let equation of plane: $ax + by + cz = d$

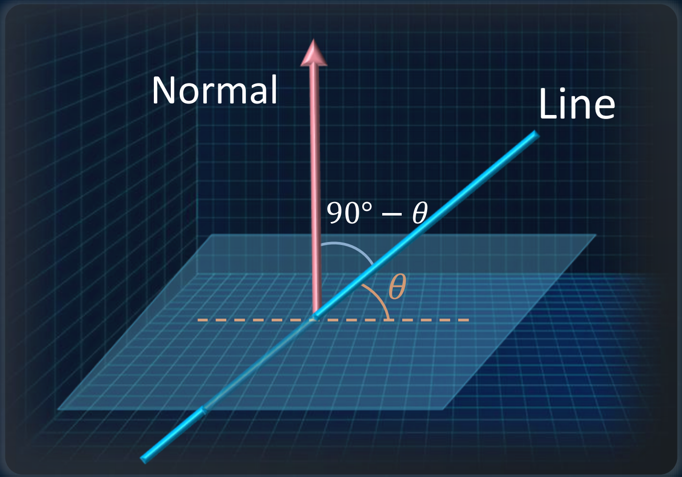
where a, b, c are DRs of normal.

Let equation of line : $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1}$

where a_1, b_1, c_1 are DRs of line.

$$\cos(90° - \theta) = \frac{aa_1 + bb_1 + cc_1}{\sqrt{a^2 + b^2 + c^2}\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}}$$

$$\theta = \sin^{-1}\left(\frac{aa_1 + bb_1 + cc_1}{\sqrt{a^2 + b^2 + c^2}\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}}\right)$$



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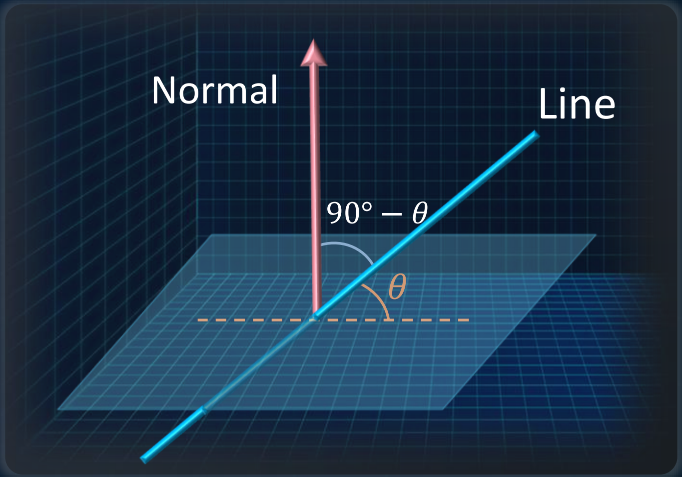
KEY TAKEAWAYS

Angle between a Line and a Plane:

Let equation of plane: $ax + by + cz = d$

Let equation of line : $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1}$



$$\theta = \sin^{-1}\left(\frac{aa_1 + bb_1 + cc_1}{\sqrt{a^2 + b^2 + c^2}\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}}\right)$$

(i) Condition for line to be parallel to plane:

$$\boxed{aa_1 + bb_1 + cc_1 = 0}$$

(ii) Condition for line to be perpendicular to plane:

$$\boxed{\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}}$$

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If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is :

JEE MAINS JAN 2019

A $-\frac{3}{5}$

B $\sqrt{\frac{3}{5}}$

C $\sqrt{\frac{5}{3}}$

D $-\frac{5}{3}$

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If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is :

JEE MAINS JAN 2019

Solution:

Let angle $\theta = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$$\Rightarrow \theta = \sin^{-1}\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\theta = \sin^{-1}\left(\frac{aa_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}}\right)$$

$$\Rightarrow \frac{1}{3} = \frac{2(1)+1(-2)-2(-k)}{\sqrt{2^2+1^2+(-2)^2}\sqrt{1^2+(-2)^2+(-k)^2}}$$

$$\Rightarrow \frac{1}{3} = \frac{2k}{3\sqrt{5+(k)^2}} \quad \Rightarrow \sqrt{5+(k)^2} = 2k \quad \text{Squaring}$$

$$\Rightarrow 3k^2 = 5 \quad \Rightarrow k = \pm\sqrt{\frac{5}{3}}$$

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If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is :

JEE MAINS JAN 2019

A $-\frac{3}{5}$

B $\sqrt{\frac{3}{5}}$

C $\sqrt{\frac{5}{3}}$

D $-\frac{5}{3}$

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Session 08

A line and a plane

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KEY TAKEAWAYS

Condition for a Line to Lie in a Plane

Let equation of plane: $ax + by + cz = d$

where a, b, c are DRs of normal.

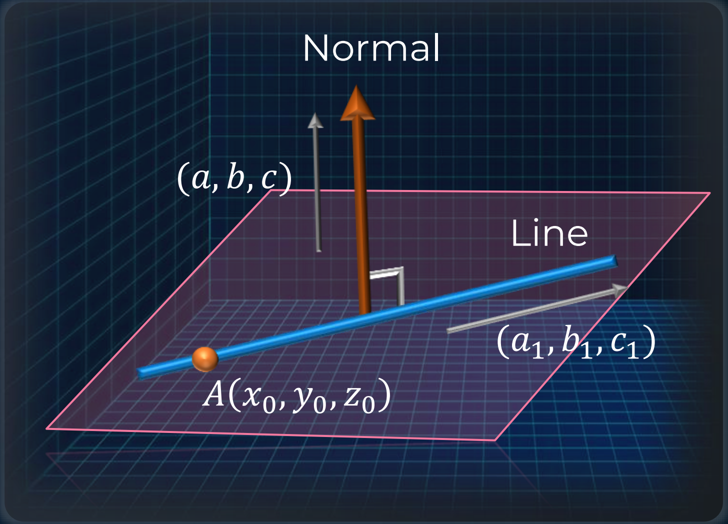
and equation of line : $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1}$

where a_1, b_1, c_1 are DRs of line.

For, line to lie in a plane :

$(i)\ ax_0 + by_0 + cz_0 = d$

$(ii)\ aa_1 + bb_1 + cc_1 = 0$, (Line $\perp^r$ to the normal to the plane)



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If the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$, lies in the plane $lx + my - z = 9$, then $l^2 + m^2$ is equal to :

JEE Main 2016

A 18

C 26

B 5

D 2

Line $\frac{x-3}{2} = \frac{y-(-2)}{-1} = \frac{z-(-4)}{3}$

Line passes through a point $(3, -2, -4)$ & DRs of line $\alpha(2, -1, 3)$

DRs of normal to plane $\propto (l, m, -1)$

1) Point A $(3, -2, -4)$ lies on $lx + my - z = 9$

$\Rightarrow 3l - 2m + 4 = 9 \Rightarrow 3l - 2m = 5$

Line $\perp^r$ to normal $\Rightarrow 2l - m - 3 = 0 \Rightarrow 2l - m = 3$

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If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$, lies in the plane $lx+my-z=9$, then l^2+m^2 is equal to :

JEE Main 2016

$$\Rightarrow 3l-2m+4=9 \Rightarrow 3l-2m=5$$

Line $\perp^r$ to normal $\Rightarrow 2l-m-3=0 \Rightarrow 2l-m=3$

$$l=1, m=-1 \Rightarrow l^2+m^2=1^2+(-1)^2=2$$

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Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If the plane divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$, then the value of k is:

JEE Main Feb 2021

A 1.5

B 2

C 4

D 3

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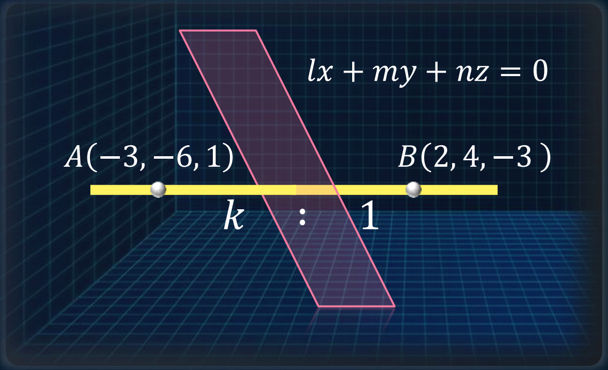
Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If the plane divides the line segment AB joining points $A(-3,-6,1)$ and $B(2,4,-3)$ in ratio $k:1$, then the value of k is:

JEE Main Feb 2021

Solution:

Equation of line: $\frac{x-1}{-1} = \frac{y-(-4)}{2} = \frac{z-(-2)}{3}$ lies on the plane



Point $A'(1,-4,-2)$ lies on $lx + my + nz = 0$

$l - 4m - 2n = 0$

DRs of line $\propto$ $(-1,2,3)$

DRs of normal $\propto (l,m,n)$

Line perpendicular to plane $\Rightarrow -l + 2m + 3n = 0$

$\Rightarrow -2m + n = 0 \quad \Rightarrow n = 2m$

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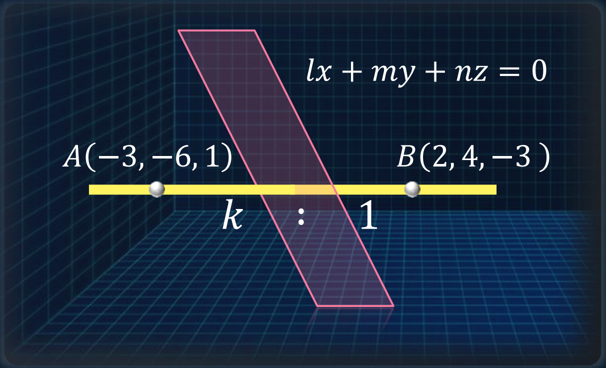
Let P be a plane $lx+my+nz=0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If the plane divides the line segment AB joining points $A(-3,-6,1)$ and $B(2,4,-3)$ in ratio $k:1$, then the value of k is:

JEE Main Feb 2021

Solution:

Equation of line: $\frac{x-1}{-1}=\frac{y-(-4)}{2}=\frac{z-(-2)}{3}$



Line perpendicular to plane

$\Rightarrow -l+2m+3n=0$

$\Rightarrow -2m+n=0 \ \Rightarrow n=2m$

Put $n=2m$ in $-l+2m+3n=0$

$\therefore l=8m$

$\therefore$ Equation of plane : $8mx+my+2mz=0$

$8x+y+2z=0$

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Let P be a plane $lx+my+nz=0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If the plane divides the line segment AB joining points $A(-3,-6,1)$ and $B(2,4,-3)$ in ratio $k:1$, then the value of k is:

JEE Main Feb 2021

Solution: Equation of line: $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$

$\therefore$ Equation of plane : $8x+y+2z=0$

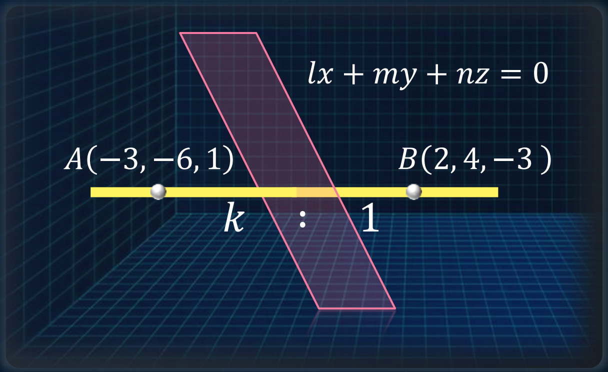
Ratio $=\frac{k}{1}$

$\Rightarrow -\frac{(8(-3)+(-6)+2(1))}{(8(2)+(4)+2(-3))}=\frac{k}{1}$

Ratio $=-\frac{(ax_1+by_1+cz_1-d)}{(ax_2+by_2+cz_2-d)}$

$\Rightarrow \frac{28}{14}=\frac{k}{1}$

$\Rightarrow k=2$



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Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If the plane divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$, then the value of k is:

JEE Main Feb 2021

A 1.5

B 2

C 4

D 3

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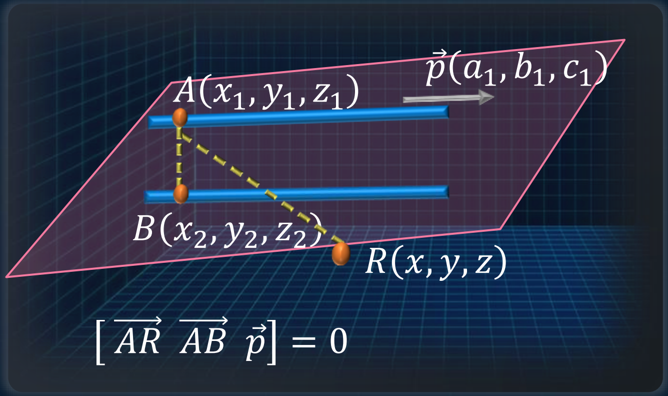
Equation of Plane Containing Two Parallel Lines

Equation of lines : $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

$$\frac{x-x_2}{a_1} = \frac{y-y_2}{b_1} = \frac{z-z_2}{c_1}$$



$\overrightarrow{AR}, \overrightarrow{AB}$ & $\vec{p}$ are coplanar

$$\Rightarrow [\overrightarrow{AR}\ \ \overrightarrow{AB}\ \ \vec{p}] = 0$$

$$\Rightarrow [\vec{r} - \vec{a}\ \ \ \vec{b} - \vec{a}\ \ \ \vec{p}] = 0$$

So, equation of plane is:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

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The equation of plane containing the lines, $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$ and $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z-0}{5}$, is :

Solution:

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The equation of plane containing the lines, $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$ and $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z-0}{5}$, is :

Solution:

$L_1 || L_2 \Rightarrow \vec{p} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

L_1 passes through point $A = (4, 3, 2)$

L_2 passes through point $B = (3, -2, 0)$

The equation of plane :

$$\begin{vmatrix} x-4 & y-3 & z-2 \\ -1 & -5 & -2 \\ 1 & -4 & 5 \end{vmatrix} = 0 \qquad \boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0}$$

$\Rightarrow (x-4)(-25-8) - (y-3)(-5+2) + (z-2)(4+5) = 0$

$\Rightarrow -33x + 3y + 9z + 105 = 0$

$\Rightarrow 11x - y - 3z = 35$ $\therefore$ Equation of plane : $11x - y - 3z = 35$

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KEY TAKEAWAYS

Equation of Plane Containing Two Lines

Equation of lines: $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

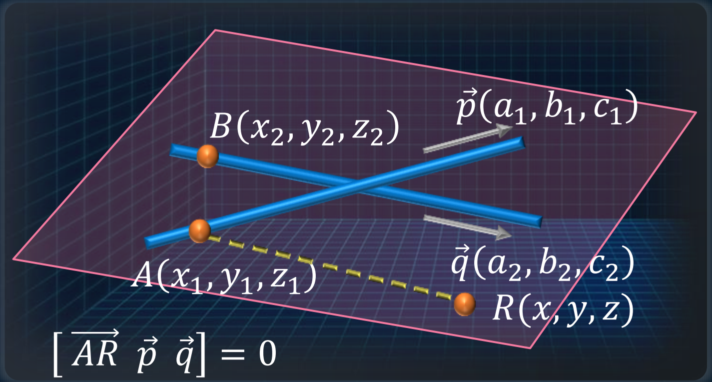
$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

$[\overrightarrow{AR}\ \vec{p}\ \vec{q}]$ are coplanar

$$\Rightarrow [\overrightarrow{AR}\ \vec{p}\ \vec{q}] = 0$$

So, equation of plane is:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$



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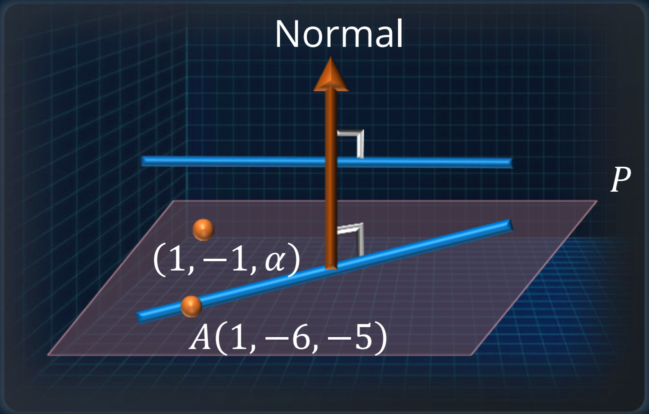
Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ____

JEE Main March 2021

Solution:

L_1 passes through point $(1,-6,-5)$

$L_1 \equiv 3\hat{\imath}+4\hat{\jmath}+2\hat{k}$, $L_2 \equiv 3\hat{\imath}+4\hat{\jmath}+2\hat{k}$



Equation of plane is :

$$\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$$

$\Rightarrow (1,-1,\alpha)$ lies on it

$$\Rightarrow \begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5(13)+25(\alpha+5)=0$$

$$\Rightarrow 5\alpha+38=0$$

$$\Rightarrow |5\alpha|=38$$

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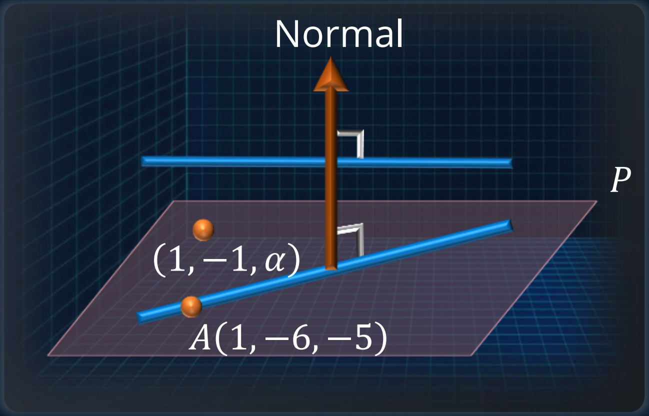
Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ____

JEE Main March 2021

Solution:



Equation of plane is :

$$\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$\Rightarrow (1,-1,\alpha)$ lies on it

$$\Rightarrow \begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$\Rightarrow 5(13) + 25(\alpha+5) = 0$

$\Rightarrow 5\alpha + 38 = 0$

$\Rightarrow |5\alpha| = 38$

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Let a plane P contains two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn form the point $M(1,0,1)$ to P, then $3(\alpha + \beta + \gamma)$ equals ____

JEE Main Sep 2020

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Let a plane P contains two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn form the point $M(1,0,1)$ to P, then $3(\alpha + \beta + \gamma)$ equals ____

JEE Main Sep 2020

Solution:

Equation of plane is

$[\vec{r} - \vec{a} \quad \vec{p} \quad \vec{q}] = 0$ where $\vec{p} = \hat{\imath} + \hat{\jmath}$ and $\vec{q} = \hat{\jmath} - \hat{k}$

Equation of plane is: $\begin{vmatrix} x-1 & y & z \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0$

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

$$\Rightarrow x - y - z = 1$$

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

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Let a plane P contains two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn form the point $M(1,0,1)$ to P, then $3(\alpha + \beta + \gamma)$ equals ____

JEE Main Sep 2020

Solution:

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

$$\Rightarrow \frac{\alpha - 1}{1} = \frac{\beta - 0}{-1} = \frac{\gamma - 1}{-1} = -\frac{(1 - 0 - 1 - 1)}{(1^2 + (-1)^2 + (-1)^2)} = \frac{1}{3}$$

$$\Rightarrow \alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$$

$$\Rightarrow 3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} + \left(-\frac{1}{3}\right) + \frac{2}{3}\right) = 5$$

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A plane passing through the point $(3,1,1)$ contains two lines whose direction ratios are $1,-2,2$ and $2,3,-1$ respectively . If this plane also passes through the point $(\alpha,-3,5)$, then α is equal to :

JEE Main Sep 2021

A -10

C 5

B 10

D -5

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A plane passing through the point (3,1,1) contains two lines whose direction ratios are $1, -2, 2$ and $2, 3, -1$ respectively . If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to :

JEE Main Sep 2021

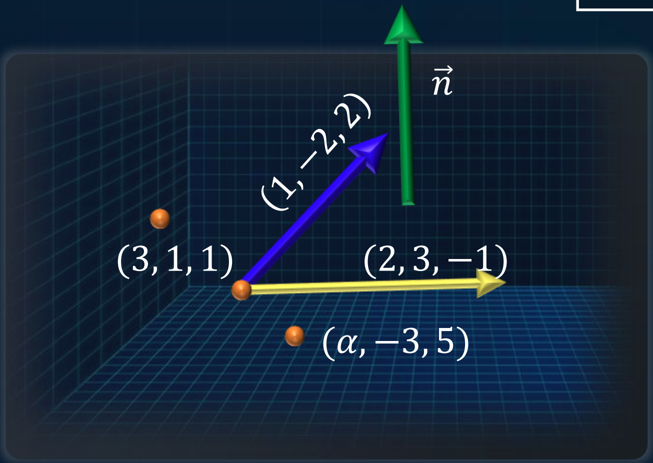
Solution:

DRs of line L_1 : $(1, -2, 2)$

DRs of line L_2 : $(2, 3, -1)$

DRs of line L_1 : $(1, -2,\ \ 2) \equiv \overrightarrow{L_1}.$

DRs of line L_1 : $(2,\ \ 3, -1) \equiv \overrightarrow{L_2}.$



$\overrightarrow{AR}, \overrightarrow{L_1}, \overrightarrow{L_2}$ are coplanar

$$\left[\overrightarrow{AR}\ \ \overrightarrow{L_1}\ \ \overrightarrow{L_2}\right] = 0$$

$$\begin{vmatrix} x-3 & y-1 & z-1 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

Point $(\alpha, -3, 5)$ lies on above plane

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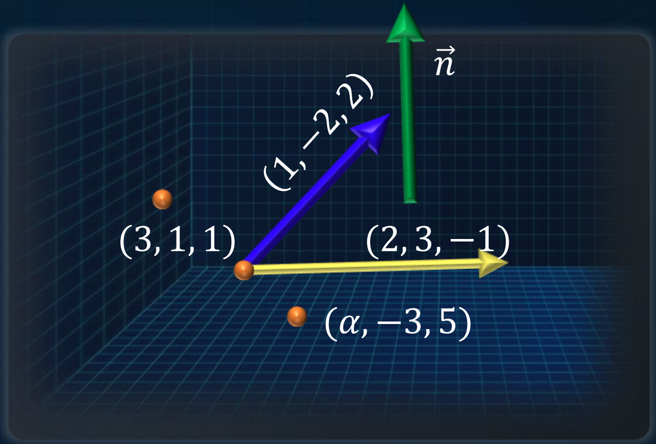
A plane passing through the point (3,1,1) contains two lines whose direction ratios are $1, -2, 2$ and $2, 3, -1$ respectively . If this plane also passes through the point $(\alpha, -3,5)$, then α is equal to :

JEE Main Sep 2021

Solution: $\begin{vmatrix} x-3 & y-1 & z-1 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$



Point $(\alpha, -3, 5)$ lies on above plane

$$\begin{vmatrix} \alpha-3 & -4 & 4 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

$$R_1 \rightarrow \frac{R_1}{2} \Rightarrow \begin{vmatrix} \frac{\alpha-3}{2} & -2 & 2 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{\alpha-3}{2} = 1 \Rightarrow \alpha - 3 = 2$$

$$\Rightarrow \alpha = 5$$

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A plane passing through the point $(3,1,1)$ contains two lines whose direction ratios are $1,-2,2$ and $2,3,-1$ respectively. If this plane also passes through the point $(\alpha,-3,5)$, then α is equal to :

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A -10

C 5

B 10

D -5

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KEY TAKEAWAYS

Intersection point of a line and a plane

Let equation of plane: $ax + by + cz = d$

where a, b, c are direction ratios of normal.

and equation of line: $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1} = \lambda$

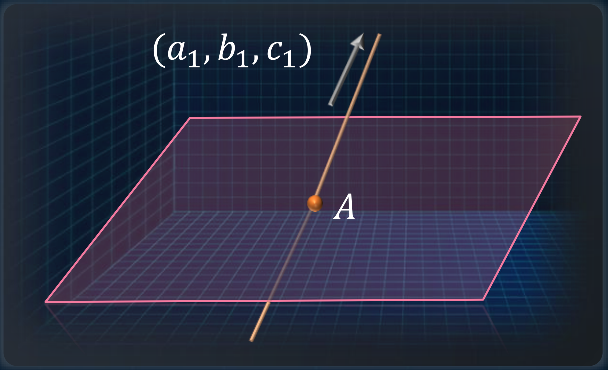
where a_1, b_1, c_1 are direction ratios of the line.

Let A is the point on the line

$\Rightarrow A \equiv (x_0 + a_1\lambda\,, y_0 + b_1\lambda, z_0 + c_1\lambda) \cdots (i)$

A also lies on plane,

$\Rightarrow a(x_0 + a_1\lambda) + b(y_0 + b_1\lambda) + c(z_0 + c_1\lambda) = d$



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KEY TAKEAWAYS

Intersection point of a line and a plane

Let equation of plane: $ax + by + cz = d$

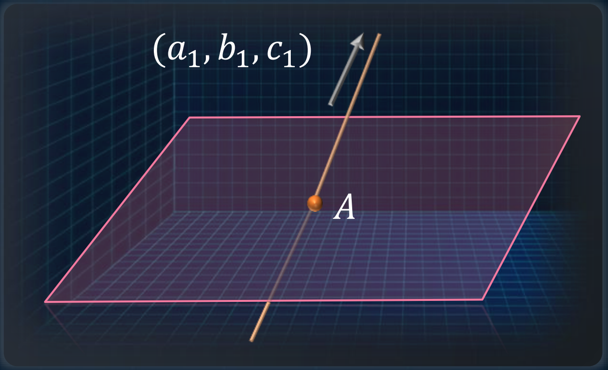
and equation of line: $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1} = \lambda$

$\Rightarrow A \equiv (x_0 + a_1\lambda\,, y_0 + b_1\lambda, z_0 + c_1\lambda) \cdots (i)$

$\Rightarrow a(x_0 + a_1\lambda) + b(y_0 + b_1\lambda) + c(z_0 + c_1\lambda) = d$

$$\therefore \lambda = \frac{d - ax_0 - by_0 - cz_0}{aa_1 + bb_1 + cc_1}$$

Substitute value of λ in (i) to get point A.



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The equation of line passing through the point of intersection of line $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane $x + y + z - 2 = 0$ is

JEE Main Jan 2019

A $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

C $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

B $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

D $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

Equation of line: $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1} = \lambda$

Let A be a point on the line $\Rightarrow A \equiv (4 + 2\lambda\,, 5 + 2\lambda, 3 + \lambda)$

A also lies on plane $x + y + z - 2 = 0$

$\Rightarrow 4 + 2\lambda + 5 + 2\lambda + 3 + \lambda - 2 = 0$

$\Rightarrow \lambda = -2$

$\therefore A \equiv (0\,, 1\,, 1)$ So, point $A(0, 1, 1)$ lies on the line $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

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The point of intersection of line $\frac{x-3}{4} = \frac{y+2}{-1} = \frac{z-6}{-2}$ and the plane $x - 7y + 3z = 15$ is:

A $(-13, 2, -14)$

B $(-13, 2, 14)$

C $(3, 2, -14)$

D $(13, 12, 14)$

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The point of intersection of line $\frac{x-3}{4} = \frac{y+2}{-1} = \frac{z-6}{-2}$ and the plane $x - 7y + 3z = 15$ is:

A $(-13, 2, -14)$

B $(-13, 2, 14)$

C $(3, 2, -14)$

D $(13, 12, 14)$

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The point of intersection of line $\frac{x-3}{4}=\frac{y+2}{-1}=\frac{z-6}{-2}$ and the plane $x-7y+3z=15$ is:

Solution:

Any given on the line $\frac{x-3}{4}=\frac{y+2}{-1}=\frac{z-6}{-2}$ can be taken as

$\Rightarrow (x, y, z)=(4t+3,-t-2,-2t+6)$

Now for the intersection with the given plane, $(4t+3,-t-2,-2t+6)$ must lie on the plane $x-7y+3z=15$

$\Rightarrow (4t+3)-7(-t-2)+3(-2t+6)=15$

$\Rightarrow 5t+35=15$

$\Rightarrow 5t=-20$

$\Rightarrow t=-4$

Hence, the point of intersection is $(3-16,4-2,8+6)=(-13,2,14)$

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The distance of point $(1, 1, 9)$ from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z - 17 = 0$ is:

JEE Main Feb 2021

A $2\sqrt{19}$

B $19\sqrt{2}$

C $\sqrt{38}$

D 38

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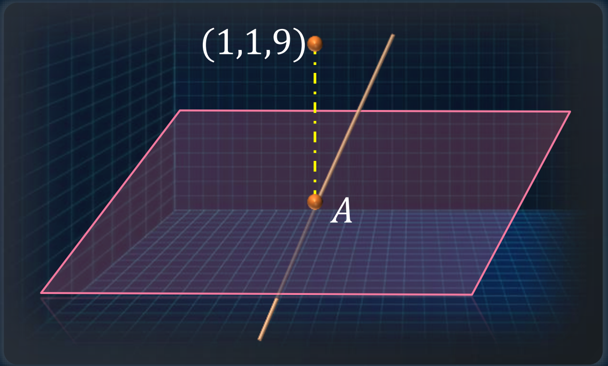
The distance of point $(1,1,9)$ from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x+y+z-17=0$ is:

JEE Main Feb 2021

Solution:



Equation of line: $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

Let A be a point on the line

$\Rightarrow A \equiv (3+\lambda, 4+2\lambda, 5+2\lambda)$

A also lies on plane,

$3+\lambda+4+2\lambda+5+2\lambda-17=0$

$\Rightarrow \lambda = 1 \qquad A \equiv (4,6,7)$

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The distance of point $(1, 1, 9)$ from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z - 17 = 0$ is:

JEE Main Feb 2021

Solution:

$$A \equiv (4, 6, 7)$$

$$\text{Distance} = \sqrt{(4-1)^2 + (6-1)^2 + (9-7)^2}$$

$$= \sqrt{3^2 + 5^2 + 2^2}$$

$$= \sqrt{38}$$

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The distance of point $(1, 1, 9)$ from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z - 17 = 0$ is:

JEE Main Feb 2021

A $2\sqrt{19}$

B $19\sqrt{2}$

C $\sqrt{38}$

D 38

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A plane has equation $x - y + z - 5 = 0$ and a line has direction ratios as $(2, 3, -6)$, then the distance of point $P(1, 3, 5)$ along the line from the given plane is:

JEE Main Aug 2021

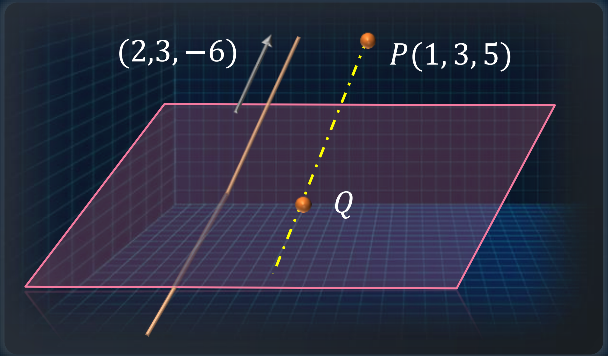
A) 2 unit

B) $3\sqrt{2}$ unit

C) $2\sqrt{3}$ unit

D) 3 unit

Solution:



Equation of line PQ:

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-5}{-6} = \lambda$$

$$Q \equiv (1 + 2\lambda, 3 + 3\lambda, 5 - 6\lambda)$$

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A plane has equation $x - y + z - 5 = 0$ and a line has direction ratios as $(2, 3, -6)$, then the distance of point $P(1, 3, 5)$ along the line from the given plane is:

JEE Main Aug 2021

Solution:

Q also lies on plane: $x - y + z - 5 = 0$

$$\Rightarrow 1 + 2\lambda - (3 + 3\lambda) + 5 - 6\lambda - 5 = 0$$

$$\Rightarrow \lambda = -\frac{2}{7}$$

$$PQ = \sqrt{(1 + 2\lambda - 1)^2 + (3 + 3\lambda - 3)^2 + (5 - 6\lambda - 5)^2}$$

$$= \sqrt{4\lambda^2 + 9\lambda^2 + 36\lambda^2}$$

$$\Rightarrow PQ = \sqrt{49\lambda^2}$$

$$= \sqrt{49 \cdot \frac{4}{49}} = 2$$

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Session 09

Angle bisector of two planes

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The distance of point $P(3,8,2)$ from the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+17=0$ is:

A. 2 unit

B. 3 unit

C. 5 unit

D. 7 unit

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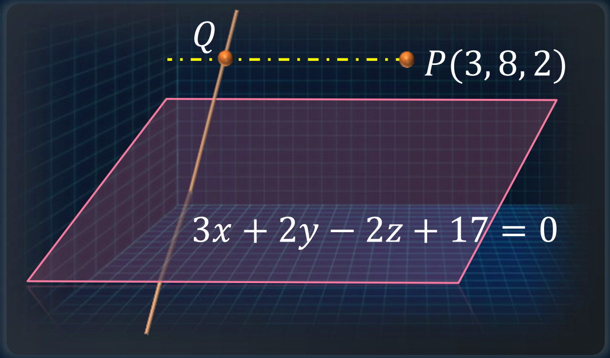
The distance of point $P(3,8,2)$ from the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+17=0$ is:

Solution:

Equation of line: $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}=\lambda$

$PQ=?$

Point $Q\equiv(1+2\lambda, 3+4\lambda, 2+3\lambda)$



Direction ratios of PQ: $2\lambda-2, 4\lambda-5, 3\lambda$

$\because PQ$ is parallel to plane

$\Rightarrow 3(2\lambda-2)+2(4\lambda-5)-2(3\lambda)=0$

$\Rightarrow \lambda=2$

$\Rightarrow Q\equiv(5,11,8)$

$PQ=\sqrt{(5-3)^2+(11-8)^2+(8-2)^2}=7$

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The distance of point $P(3,8,2)$ from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z + 17 = 0$ is:

A 2 unit

B 3 unit

C 5 unit

D 7 unit

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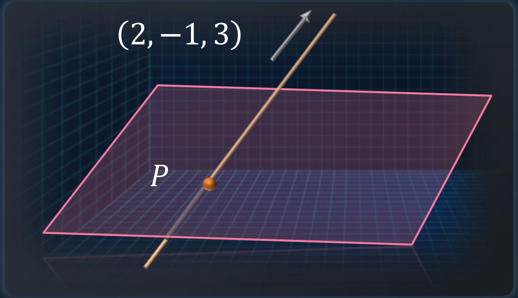
Perpendiculars are drawn form points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line:

JEE Adv 2013

Equation of line: $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3} = \lambda$



Any point P on the given line is

$(2\lambda - 2, -\lambda - 1, 3\lambda)$

The point P lies on the given plane for some λ.

$\Rightarrow (2\lambda - 2) + (-\lambda - 1) + 3\lambda = 3$

$\Rightarrow 4\lambda = 6$

$\Rightarrow \lambda = \frac{3}{2}$

$\Rightarrow P \equiv \left(1, -\frac{5}{2}, \frac{9}{2}\right)$

A $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$

B $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$

C $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$

D $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

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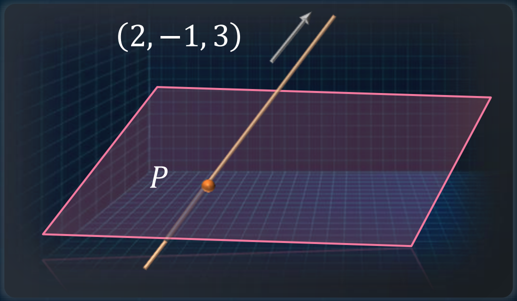
Perpendiculars are drawn form points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$. The feet of perpendiculars lie on the line:

JEE Adv 2013

$\Rightarrow P \equiv \left(1,-\frac{5}{2},\frac{9}{2}\right)$



The foot of the perpendicular from the point $(-2,-1,0)$ on the plane is the point Q.

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{z_2-z_1}{c}=-\frac{(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

$$\Rightarrow \frac{x_1+2}{1}=\frac{x_2+1}{1}=\frac{x_3-0}{1}=-\frac{(1(-2)+1(-1)+1(0)-3)}{1^2+1^2+1^2}=2$$

$Q \equiv (0,1,2)$

The direction ratio of PQ: $\left(-1,\frac{7}{2},-\frac{5}{2}\right)=(2,-7,5)$

Hence, the equation of the line is $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

A $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$

B $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$

C $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

D $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

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The image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$ is:

A $\frac{x-4}{9} = \frac{y}{-1} = \frac{z+3}{-3}$

B $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

C $\frac{x}{9} = \frac{y}{-1} = \frac{z}{-3}$

D $\frac{x+2}{9} = \frac{y-5}{-1} = \frac{z}{-3}$

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The image of the line $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$ in the plane $3x-3y+10z=26$ is:

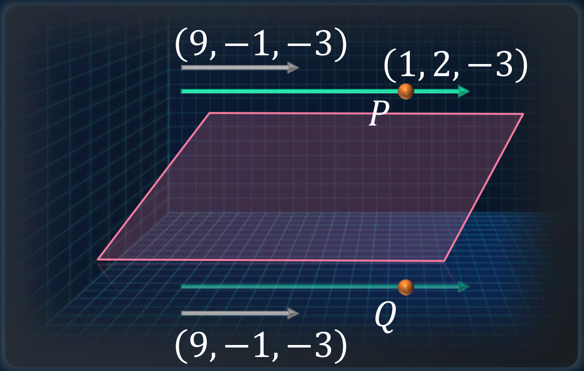
Solution:

Plane : $3x-3y+10z=26$

Line : $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$

$9\cdot 3+(-1)\cdot(-3)+(-3)\cdot 10=0$

∴ Line is parallel to the plane.



Let image of point P with respect to plane is Q.

$$\frac{x-1}{3}=\frac{y-2}{-3}=\frac{z+3}{10}=-2\left(\frac{3(1)-3(2)+10(-3)-26}{118}\right)$$

$$\Rightarrow \frac{x-1}{3}=\frac{y-2}{-3}=\frac{z+3}{10}=1$$

$$\Rightarrow Q\equiv(4,-1,7)$$

∴ Image: $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$

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The image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$ is:

A $\frac{x-4}{9} = \frac{y}{-1} = \frac{z+3}{-3}$

B $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

C $\frac{x}{9} = \frac{y}{-1} = \frac{z}{-3}$

D $\frac{x+2}{9} = \frac{y-5}{-1} = \frac{z}{-3}$

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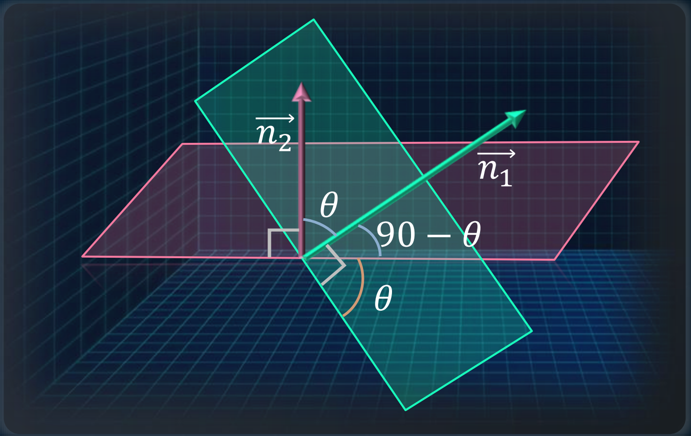
KEY TAKEAWAYS

Angle between two planes:

Let equations of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$



Angle between planes is same as angle between their normals

Let angle between planes is θ, then

$$\cos\theta = \frac{(a_1a_2+b_1b_2+c_1c_2)}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$$

(i) Planes are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(ii) Planes are parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

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The direction ratios of normal to the plane through the points $(0,-1,0)$ and $(0,0,1)$ and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$ are:

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A	$2, \sqrt{2}, -\sqrt{2}$
B	$2, -1, 1$
C	$\sqrt{2}, 1, -1$
D	$2\sqrt{3}, 1, -1$

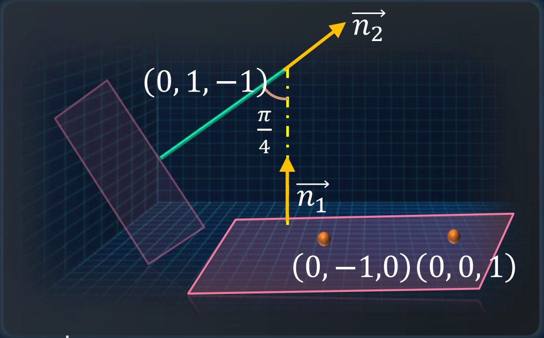
Solution: Let equation of plane be $a(x-0)+b(y+1)+c(z-0)=0$

passes through $(0,0,1)$

$\Rightarrow a(0)+b(1)+c(1)=0$

$\Rightarrow b+c=0$

$$\left.\begin{aligned}\overrightarrow{n_1} &= a\hat{i}+b\hat{j}+c\hat{k}\\ \overrightarrow{n_2} &= \hat{j}-\hat{k}\end{aligned}\right\}\ \theta=\frac{\pi}{4}$$



$$\cos\frac{\pi}{4}=\widehat{n_1}\cdot\widehat{n_2}=\frac{b-c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$\Rightarrow \sqrt{a^2+b^2+c^2}=b-c=2b$

$\Rightarrow a^2+2b^2=4b^2$

$\Rightarrow a=\pm\sqrt{2}b$ and $c=-b$

Direction ratios: $(\sqrt{2},1,-1)$ or $(2,\sqrt{2},-\sqrt{2})$

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A tetrahedron has vertices $P(1,2,1), Q(2,1,3), R(-1,1,2)$ and $O(0,0,0)$. The angle between the faces OPQ and PQR is:

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A $\cos^{-1}\left(\frac{17}{31}\right)$

B $\cos^{-1}\left(\frac{7}{31}\right)$

C $\cos^{-1}\left(\frac{9}{35}\right)$

D $\cos^{-1}\left(\frac{19}{35}\right)$

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A tetrahedron has vertices $P(1,2,1), Q(2,1,3), R(-1,1,2)$ and $O(0,0,0)$. The angle between the faces OPQ and PQR is:

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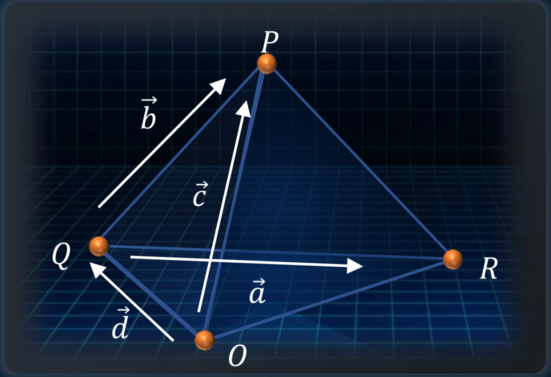
Solution: Angle between the faces OPQ & PQR is same as angle between their normal.



Let normal vector to the face $PQR = \overrightarrow{n_1}$

$\vec{b} = -\hat{i} + \hat{j} - 2\hat{k} \qquad \vec{a} = -3\hat{i} - \hat{k}$

$$\overrightarrow{n_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$\Rightarrow \overrightarrow{n_1} = \hat{i} - 5\hat{j} - 3\hat{k}$

Let normal vector to the face $OPQ = \overrightarrow{n_2}$

$\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{d} = 2\hat{i} + \hat{j} + 3\hat{k}$

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A tetrahedron has vertices $P(1,2,1), Q(2,1,3), R(-1,1,2)$ and $O(0,0,0)$. The angle between the faces OPQ and PQR is:

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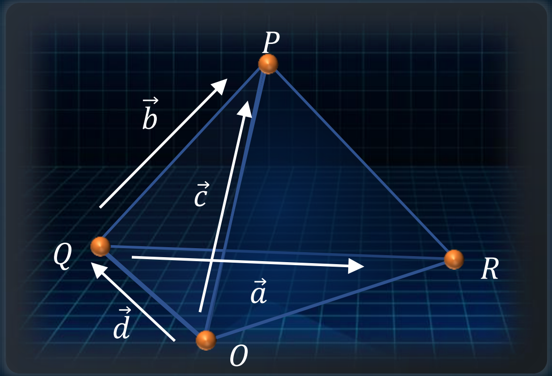
Solution:

$$\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{d} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$$\Rightarrow \overrightarrow{n_2} = 5\hat{i} - \hat{j} - 3\hat{k}$$



$$\theta = \cos^{-1}\left(\frac{(\hat{i} - 5\hat{j} - 3\hat{k})\cdot(5\hat{i} - \hat{j} - 3\hat{k})}{\sqrt{35}\cdot\sqrt{35}}\right)$$

$$\cos\theta = \frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

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A tetrahedron has vertices $P(1, 2, 1), Q(2, 1, 3), R(-1, 1, 2)$ and $O(0, 0, 0)$. The angle between the faces OPQ and PQR is:

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A tetrahedron has vertices $P(1,2,1), Q(2,1,3), R(-1,1,2)$ and $O(0,0,0)$. The angle between the faces OPQ and PQR is:

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A $\cos^{-1}\left(\frac{17}{31}\right)$

B $\cos^{-1}\left(\frac{7}{31}\right)$

C $\cos^{-1}\left(\frac{9}{35}\right)$

D $\cos^{-1}\left(\frac{19}{35}\right)$

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KEY TAKEAWAYS

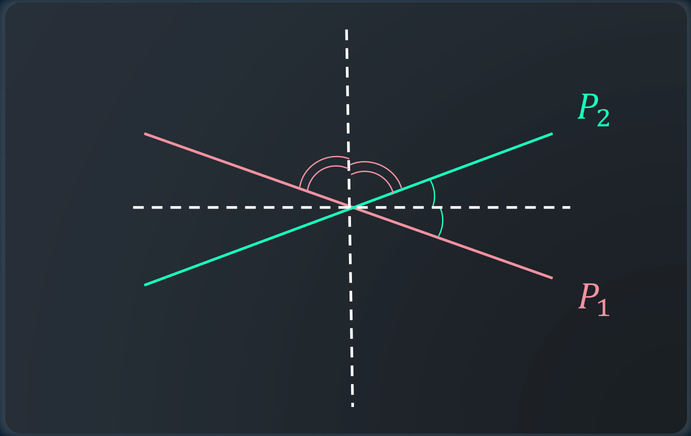
Equation of angle bisector of two planes:

Let equation of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$

Equation of angle bisector planes:

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}}\right) = \pm\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$



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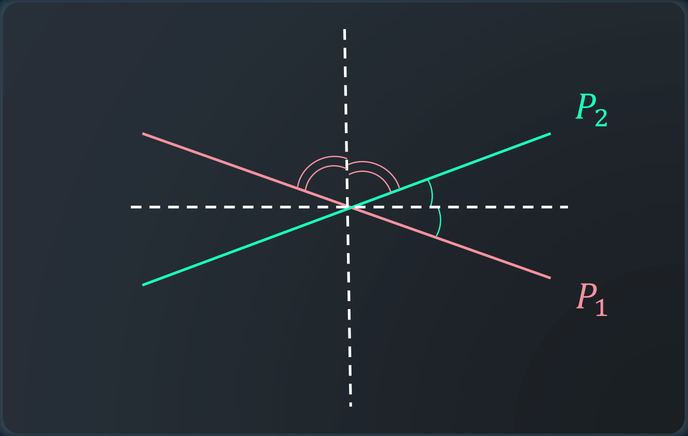
KEY TAKEAWAYS

Equation of angle bisector of two planes containing a point:

Let equation of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$

(i) If sign of $a_1\alpha + b_1\beta + c_1\gamma - d_1$
and $a_2\alpha + b_2\beta + c_2\gamma - d_2$ is same, then equation of bisector containing point (α, β, γ) will be :



$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}}\right) = +\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}\right)$$

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KEY TAKEAWAYS

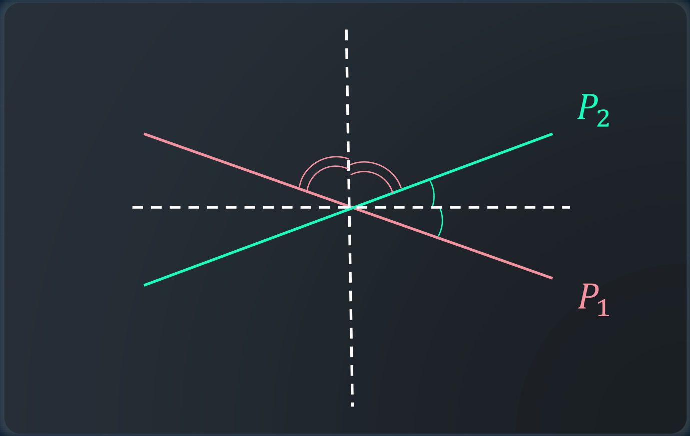
Equation of angle bisector of two planes containing a point:

Let equation of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$

(ii) If sign of $a_1\alpha + b_1\beta + c_1\gamma - d_1$ and $a_2\alpha + b_2\beta + c_2\gamma - d_2$ is opposite, then equation of bisector containing point (α, β, γ) will be :

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}}\right) = -\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}\right)$$



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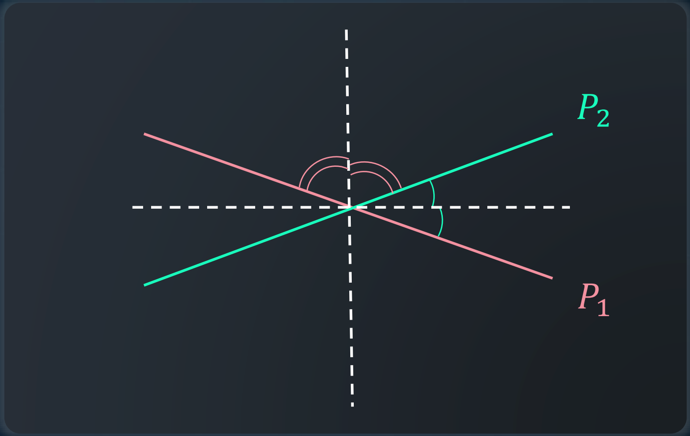
KEY TAKEAWAYS

Equation of acute/obtuse angle bisector of two planes:

Let equation of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$



(i) If $a_1a_2 + b_1b_2 + c_1c_2 > 0$,

Then equation of acute angle bisector

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}}\right) = -\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$

and equation of obtuse angle bisector

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}}\right) = +\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$

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KEY TAKEAWAYS

Equation of acute/obtuse angle bisector of two planes:

Let equation of planes be: $a_1x + b_1y + c_1z = d_1$

and $a_2x + b_2y + c_2z = d_2$

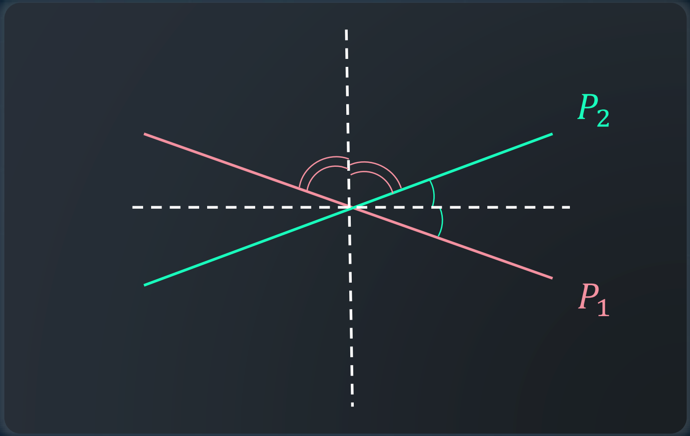
(ii) If $a_1a_2 + b_1b_2 + c_1c_2 < 0$,

Then equation of acute angle bisector

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}}\right) = +\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$

and equation of obtuse angle bisector

$$\left(\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}}\right) = -\left(\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$



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KEY TAKEAWAYS

Distance between parallel Planes:

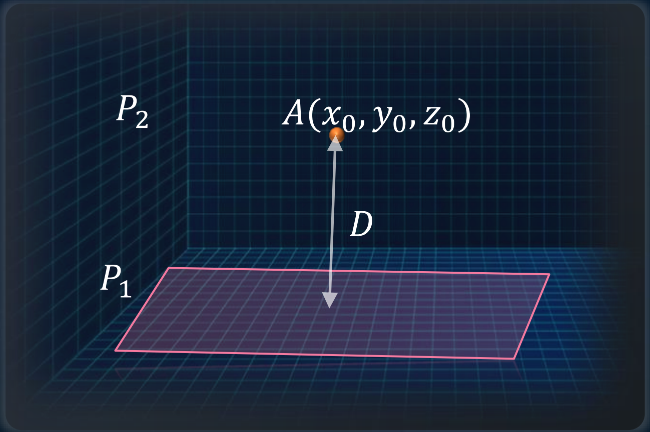
Let equation of planes be: $P_1 : ax + by + cz = d_1$ and $P_2 : ax + by + cz = d_2$

Let A lies on P_2

$$D = \left| \frac{a_1x_0 + b_1y_0 + c_1z_0 - d_1}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}} \right|$$

$$ax_0 + by_0 + cz_0 = d_2$$

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$



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If the plane, $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to:

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A	13
B	9
C	5
D	15

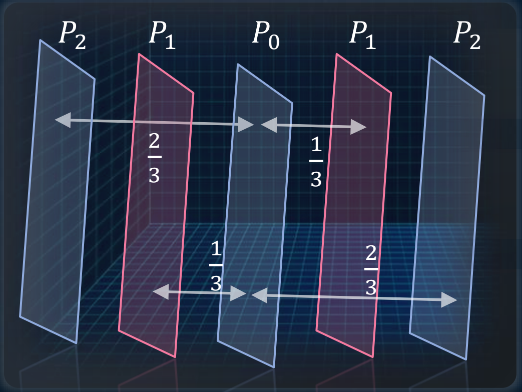
Solution:

$P_0 : 2x - y + 2z + 3 = 0$

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$P_1 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_2 : 2x - y + 2z + \mu = 0$



$$\frac{1}{3} = \left| \frac{\frac{\lambda}{2} - 3}{\sqrt{2^2 + (-1)^2 + (2)^2}} \right|$$

$$\frac{2}{3} = \left| \frac{\mu - 3}{\sqrt{2^2 + (-1)^2 + (2)^2}} \right|$$

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If the plane, $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to:

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A 13

B 9

C 5

D 15

Solution:

$$\frac{1}{3} = \left|\frac{\frac{\lambda}{2} - 3}{\sqrt{2^2 + (-1)^2 + (2)^2}}\right| \qquad \frac{2}{3} = \left|\frac{\mu - 3}{\sqrt{2^2 + (-1)^2 + (2)^2}}\right|$$

$$\Rightarrow 1 = \left|\frac{\lambda}{2} - 3\right|$$

$$\Rightarrow \lambda = 8\,,4$$

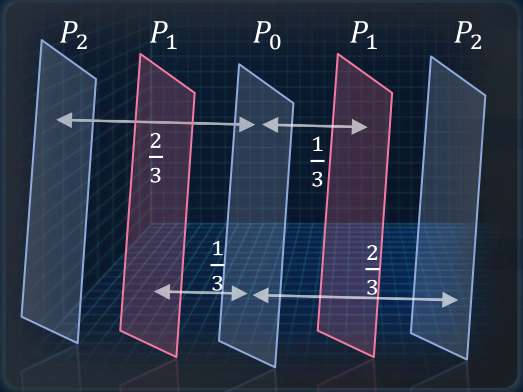
$$\frac{2}{3} = \left|\frac{\mu - 3}{\sqrt{2^2 + (-1)^2 + (2)^2}}\right|$$

$$\Rightarrow 2 = |\mu - 3|$$

$$\Rightarrow \mu = 1\,,5$$

$$(\lambda + \mu)_{max} = 13$$

$(\lambda + \mu)_{\text{max}} = ?$



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If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in \mathbb{R})$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to:

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If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in \mathbb{R})$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to:

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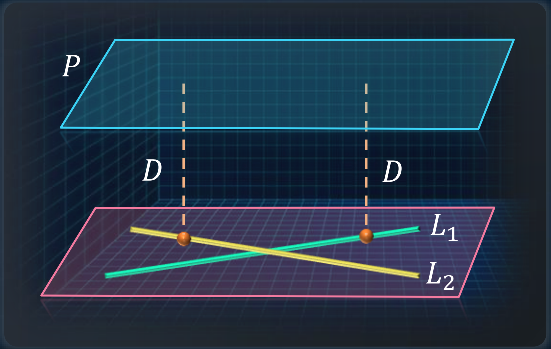
Solution:

Required distance = Perpendicular distance between plane $23x - 10y - 2z + 48 = 0$ either from point $(-1, 3, -1)$ or $(-3, -2, 1)$

$$D = \left|\frac{23(-1) - 10(3) - 2(-1) + 48}{\sqrt{23^2 + (-10)^2 + (-2)^2}}\right| = \frac{3}{\sqrt{529+100+4}}$$



$$\Rightarrow D = \frac{3}{\sqrt{633}}$$

$$\therefore k = 3$$

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A plane which bisects the angle between the two planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

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A $(1, -4, 1)$

B $(1, 4, -1)$

C $(2, 4, 1)$

D $(2, -4, 1)$

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A plane which bisects the angle between the two planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

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A $(1, -4, 1)$

B $(1, 4, -1)$

C $(2, 4, 1)$

D $(2, -4, 1)$

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Session 10

Family of planes and equation of sphere

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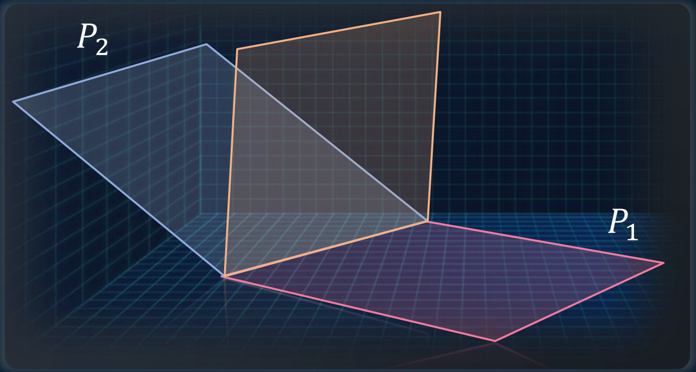
KEY TAKEAWAYS

Family of Planes :

Equation of a plane passing through the line of intersection of non – parallel planes P_1 and P_2, is:

$$\boxed{P_1 + \lambda P_2 = 0, \lambda \epsilon R}$$



Let equation of planes be: $P_1 : a_1x + b_1y + c_1z = d_1$

and $P_2 : a_2x + b_2y + c_2z = d_2$

So, equation of required plane:

$$\boxed{(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0}$$

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If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0, 3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is:

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If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0, 3x-5y+4z+11=0$ and the point $(-2,1,3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is:

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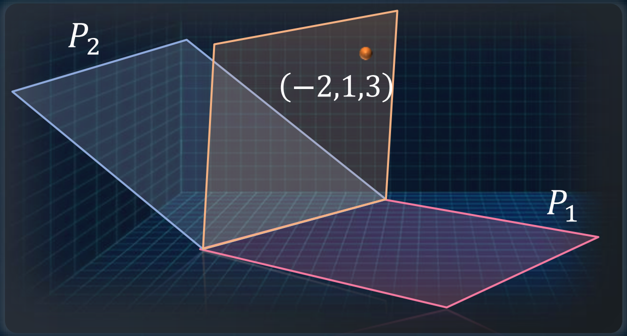
Solution:



Required plane has equation:

$$2x-7y+4z-3+\lambda(3x-5y+4z+11)=0$$

$$x(2+3\lambda)-y(7+5\lambda)+4z(1+\lambda)-3+11\lambda=0 \cdots (i)$$

It passes through the point $(-2\,,1\,,3)$,

$$(-2)(2+3\lambda)-1(7+5\lambda)+12(1+\lambda)-3+11\lambda=0$$

$$\Rightarrow \lambda=\frac{1}{6}$$

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If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0, 3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is:

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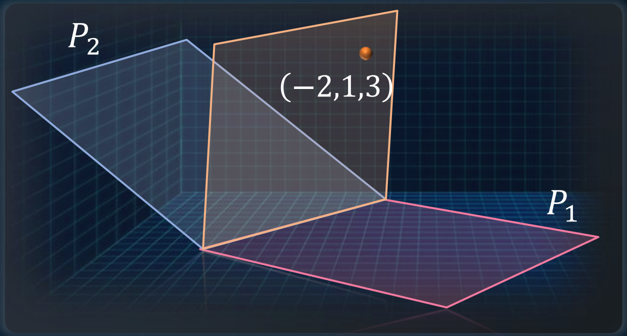
Solution:



$$x(2 + 3\lambda) - y(7 + 5\lambda) + 4z(1 + \lambda) - 3 + 11\lambda = 0 \cdots (i)$$

$$\Rightarrow \lambda = \frac{1}{6}$$

Substituting in (i)

Thus, the plane: $15x - 47y + 28z - 7 = 0$

$$a = 15,\ b = -47,\ c = 28$$

$$\Rightarrow 2a + b + c - 7 = 4$$

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If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0, 3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is:

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If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z - 8 = 0$ is $ax + by + 6z - 15 = 0$, for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ form the plane P is:

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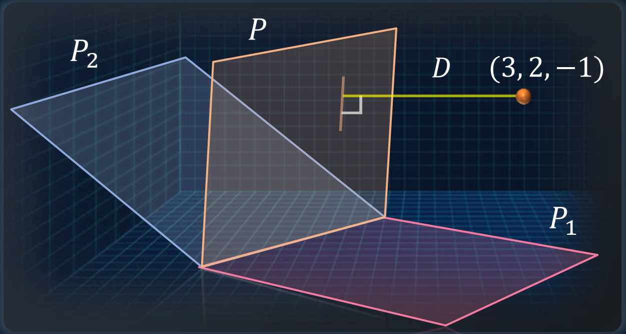
Required plane has equation:

$$x + 4y - z + 7 + \lambda(3x + y + 5z - 8) = 0$$

$$x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + 7 - 8\lambda = 0 \cdots (i)$$



Comparing with the given equation:

$$ax + by + 6z - 15 = 0$$

$$\frac{6}{(-1+5\lambda)} = \frac{-15}{7-8\lambda} \Rightarrow 14 - 16\lambda = 5 - 25\lambda$$

$$\Rightarrow 9\lambda = -9 \Rightarrow \lambda = -1$$

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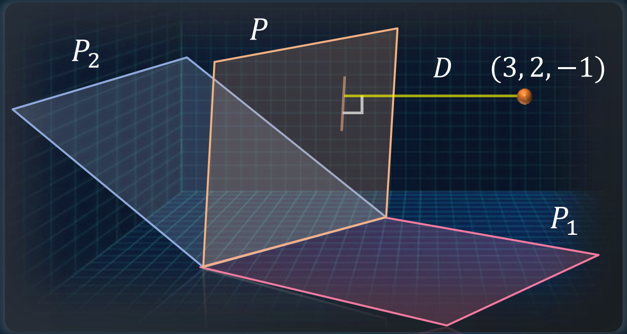
If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z - 8 = 0$ is $ax + by + 6z - 15 = 0$, for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ form the plane P is:

JEE MAINS Sept 2020

$$x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + 7 - 8\lambda = 0 \cdots (i)$$



$$\Rightarrow \lambda = -1$$

Substituting in (i)

Thus, the plane: $-2x + 3y - 6z + 15 = 0$

$$\text{D} = \left| \frac{-6 + 6 + 6 + 15}{\sqrt{(-2)^2 + 3^2 + (-6)^2}} \right|$$

$$= \left| \frac{21}{\sqrt{49}} \right| = 3$$

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The vector equation of the plane through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y + 4z - 5 = 0$ which is perpendicular to the plane $x - y + z = 0$, is:

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A $\vec{r} \cdot (\hat{i} - \hat{k}) - 2 = 0$

B $\vec{r} \times (\hat{i} + \hat{k}) + 2 = 0$

C $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$

D $\vec{r} \times (\hat{i} + \hat{k}) - 2 = 0$

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The vector equation of the plane through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y + 4z - 5 = 0$ which is perpendicular to the plane $x - y + z = 0$, is:

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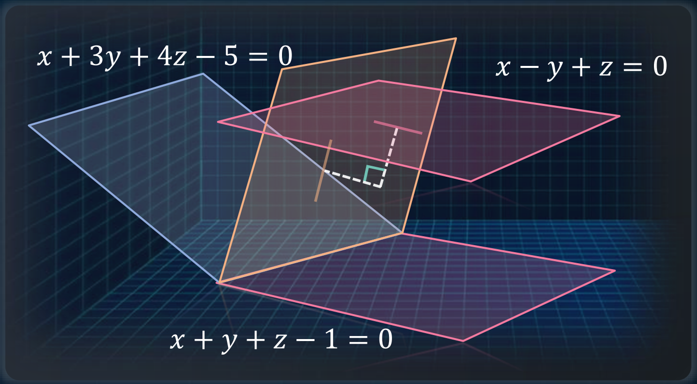
Solution:



Required plane has equation:

$$x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$$

$$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + 4\lambda) - 1 - 5\lambda = 0 \cdots (i)$$

Since it is perpendicular to the plane:

$$x - y + z = 0$$

$$1(1 + 2\lambda) - (1 + 3\lambda) + (1 + 4\lambda) = 0$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

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The vector equation of the plane through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y + 4z - 5 = 0$ which is perpendicular to the plane $x - y + z = 0$, is:

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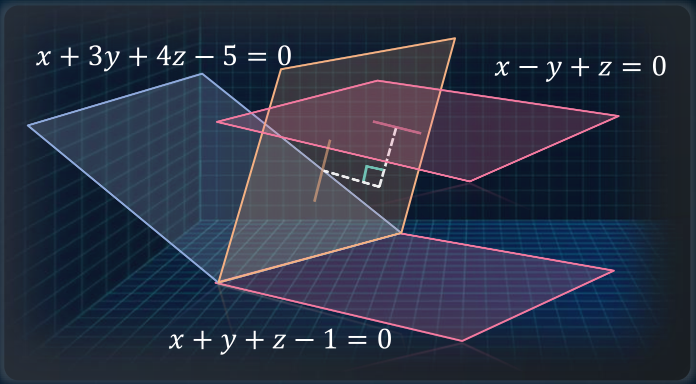
Solution:



$$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + 4\lambda) - 1 - 5\lambda = 0 \cdots (i)$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

Substituting in (i)

$$\frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

$$\Rightarrow x - z + 2 = 0$$

Thus, vector equation of plane: $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$

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The vector equation of the plane through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y + 4z - 5 = 0$ which is perpendicular to the plane $x - y + z = 0$, is:

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A $\vec{r} \cdot (\hat{i} - \hat{k}) - 2 = 0$

B $\vec{r} \times (\hat{i} + \hat{k}) + 2 = 0$

C $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$

D $\vec{r} \times (\hat{i} + \hat{k}) - 2 = 0$

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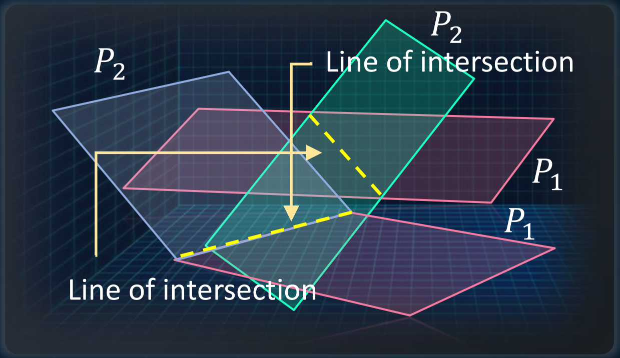
KEY TAKEAWAYS

Non–Symmetrical Form of Line

A straight line in space is characterized by intersection of two planes, which are not parallel.



Let equation of planes be: $P_1: a_1x + b_1y + c_1z = d_1$

and $P_2: a_2x + b_2y + c_2z = d_2$

Equation of line of intersection of planes P_1 and P_2, is:

$$a_1x + b_1y + c_1z - d_1 = 0 = a_2x + b_2y + c_2z - d_2$$

(Non – symmetric form)

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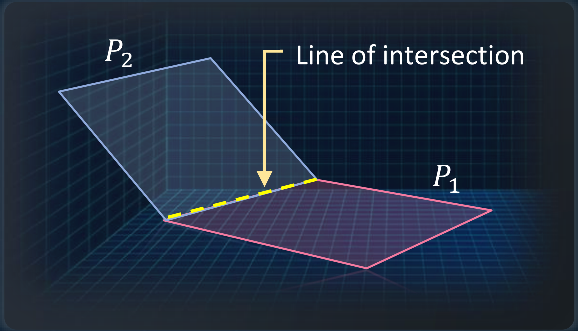
KEY TAKEAWAYS

Non–Symmetrical Form of Line

Equation of line of intersection of planes P_1 and P_2, is:

$$\boxed{a_1x + b_1y + c_1z - d_1 = 0 = a_2x + b_2y + c_2z - d_2}$$

(Non – symmetric form)



To convert to symmetric form of line:

Step 1 : Get direction ratios:

Let a, b, c be the direction ratios

Line of intersection lies on both P_1 & P_2, then

$$a, b, c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

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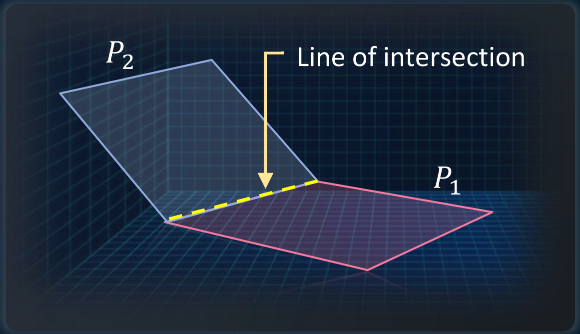
KEY TAKEAWAYS

Non–Symmetrical Form of Line

Equation of line of intersection of planes P_1 and P_2, is:

$$\boxed{a_1x + b_1y + c_1z - d_1 = 0 = a_2x + b_2y + c_2z - d_2}$$

(Non – symmetric form)



To convert to symmetric form of line:

Step 1 : Get direction ratios:

Step 2 : Point on the line: If $a \neq 0$, take a point on $y - z$ plane

i.e. $P(0, y_1, z_1)$, and substitute it in the equation of planes

So, solving the simultaneous equations

$b_1y_1 + c_1z_1 = d_1 \quad b_2y_1 + c_2z_1 = d_2$, to get point P.

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Reduce the equation of line $4x + 4y - 5z - 12 = 0$ & $8x + 12y - 13z - 32 = 0$ in symmetric form:

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Reduce the equation of line $4x + 4y - 5z - 12 = 0$ & $8x + 12y - 13z - 32 = 0$ in symmetric form:

Solution:

Line of intersection of planes:

$$4x + 4y - 5z - 12 = 0 \cdots (i)$$

$$8x + 12y - 13z - 32 = 0 \cdots (ii)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & -5 \\ 8 & 12 & -13 \end{vmatrix} = (8\hat{i} + 12\hat{j} + 16\hat{k})$$

Direction ratio: $a, b, c \;=\; 2, 3, 4$

Putting $z = 0$, in (i) & (ii)

$$x + y = 3$$

$$2x + 3y = 8$$

Point on the line: $x = 1,\ y = 2,\ z = 0$

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Reduce the equation of line $4x + 4y - 5z - 12 = 0$ & $8x + 12y - 13z - 32 = 0$ in symmetric form:

Solution:

Direction ratio: $a, b, c \ = \ 2, 3, 4$

Point on the line: $x = 1, \ y = 2, \ z = 0$

Thus, equation of line: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4}$

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A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?

A $(1, 0, 1)$

B $(2, -1, 1)$

C $(0, 1, 1)$

D $(-1, 1, 2)$

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A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?

A $(1, 0, 1)$

B $(2, -1, 1)$

C $(0, 1, 1)$

D $(-1, 1, 2)$

Solution:

Required plane is a plane passing through the line of intersection of planes

$P_1 \equiv x + 2y + 3z + 1 = 0$

And $P_2 \equiv x - y - z - 6 = 0$

Its equation: $P_1 + \lambda P_2 = 0$

$\Rightarrow (x + 2y + 3z + 1) + \lambda(x - y - z - 6) = 0$

$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + 1 - 6\lambda = 0$

$\because$ Perpendicular to $-2x + y + z + 8 = 0$

$\therefore -2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0$

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A plane P contains the line $x+2y+3z+1=0=x-y-z-6$, and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lies on P?

Solution:

$\therefore -2(1+\lambda)+(2-\lambda)+(3-\lambda)=0$

$\Rightarrow \lambda=\frac{3}{4}$

$\Rightarrow$ Required plane is $7x+5y+9z=14$

Checking the option show that

$(0,1,1)$ Satisfies it.

A $(1,0,1)$

B $(2,-1,1)$

C $(0,1,1)$

D $(-1,1,2)$

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A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?

A $(1, 0, 1)$

B $(2, -1, 1)$

C $(0, 1, 1)$

D $(-1, 1, 2)$

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The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0$ & $2x-y+z+3=0$ is:

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A $\frac{1}{\sqrt{2}}$

B 1

C $\frac{1}{\sqrt{3}}$

D $\frac{1}{2}$

Solution:

Line of intersection of planes:

$x+y+z+1=0 \cdots (i)$

$2x-y+z+3=0 \cdots (ii)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i}+\hat{j}-3\hat{k}$$

Direction ratio: $a, b, c \;=\; 2, 1, -3$

Putting $z=0$, in (i) & (ii)

$x+y+1=0$

$2x-y+3=0$

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The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0$ & $2x-y+z+3=0$ is:

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Solution:

$x+y+1=0 \qquad 2x-y+3=0$

Direction ratio: $a, b, c \;=\; 2, 1, -3$

Point on the line: $x=-\frac{4}{3},\ y=\frac{1}{3},\ z=0$

Thus, equation of line : $\dfrac{x+\frac{4}{3}}{2}=\dfrac{y-\frac{1}{3}}{1}=\dfrac{z}{-3}$

A $\frac{1}{\sqrt{2}}$

B 1

C $\frac{1}{\sqrt{3}}$

D $\frac{1}{2}$

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The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$ & $2x - y + z + 3 = 0$ is:

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A $\frac{1}{\sqrt{2}}$

B 1

C $\frac{1}{\sqrt{3}}$

D $\frac{1}{2}$

Solution:

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1} \qquad \frac{x+\frac{4}{3}}{2} = \frac{y-\frac{1}{3}}{1} = \frac{z}{-3}$$

$$\text{S.D.} = \left|\frac{(b-a).(c \times d)}{|c \times d|}\right|$$

$$c \times d = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 0 & -1 & 1 \end{vmatrix} = -2\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\text{Shortest distance} = \frac{\left\| \begin{matrix} \frac{7}{3} & -\frac{4}{3} & 0 \\ 2 & 1 & -3 \\ 0 & -1 & 1 \end{matrix} \right\|}{\sqrt{(-2)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\left(\frac{7}{3}(1-3) + \frac{4}{3}(2)\right)}{\sqrt{12}} = \frac{1}{\sqrt{3}}$$

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If for some α and β in $\mathbb{R}$, the intersection of the following three planes $x + 4y - 2z - 1 = 0, x + 7y - 5z - \beta = 0$ and $x + 5y + \alpha z = 5$ is a line in $\mathbb{R}^3$, then $\alpha + \beta$ is:

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A 0

B -10

C 10

D 2

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If for some α and β in $\mathbb{R}$, the intersection of the following three planes $x + 4y - 2z - 1 = 0, x + 7y - 5z - \beta = 0$ and $x + 5y + \alpha z = 5$ is a line in $\mathbb{R}^3$, then $\alpha + \beta$ is:

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Solution:

Plane intersect in a line: $\Rightarrow$ there should be infinite solution of the given system of equations for infinite solutions.

$$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \quad \Rightarrow \alpha = -3$$

Also, $\Delta_1 = \begin{vmatrix} 1 & 4 & -2 \\ \beta & 7 & -5 \\ 5 & 5 & -3 \end{vmatrix} = 0$

$$\Rightarrow \beta = 13$$

$$\therefore \alpha + \beta = 10$$

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If for some α and β in $\mathbb{R}$, the intersection of the following three planes $x + 4y - 2z - 1 = 0, x + 7y - 5z - \beta = 0$ and $x + 5y + \alpha z = 5$ is a line in $\mathbb{R}^3$, then $\alpha + \beta$ is:

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A 0

B -10

C 10

D 2

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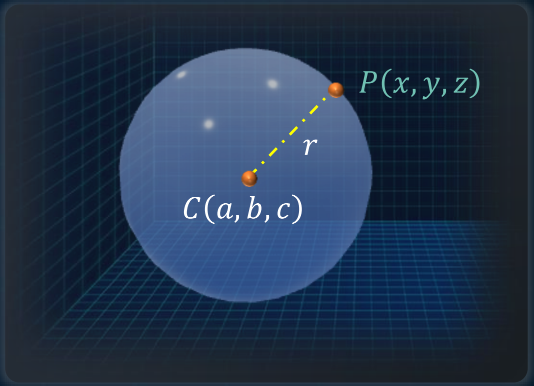
KEY TAKEAWAYS

Sphere

Center radius form: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

General form: $x^2+y^2+z^2+2ux+2vy+2wz+d=0$



Center $\equiv (-u,-v,-w)$

Radius $=\sqrt{u^2+v^2+w^2-d}$

Diametric form:

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$$

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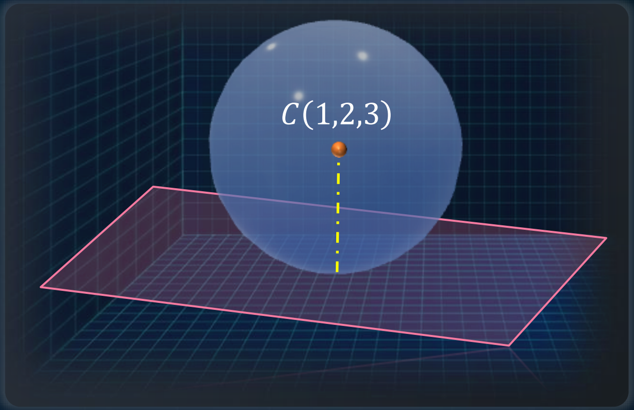
The equation of sphere having center at $(1,2,3)$ and touching the plane $x+2y+3z=0$, is:

Solution:

Radius = distance of center from the plane

$$r = \left|\frac{1+4+9}{\sqrt{1^2+2^2+3^2}}\right|$$

$$\Rightarrow r = \sqrt{14}$$



So, equation: $\boxed{(x-a)^2+(y-b)^2+(z-c)^2=r^2}$

$$(x-1)^2+(y-2)^2+(z-3)^2=14$$

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Plane $x + 2y - z = 4$, cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$. Then the radius of the circle formed is:

A 1 unit

B 2 units

C 3 units

D 4 units

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Plane $x + 2y - z = 4$, cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$. Then the radius of the circle formed is:

A 1 unit

B 2 units

C 3 units

D 4 units

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Session 11

Miscellaneous Questions

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The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions :

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A) 0

B) 168

C) 2

D) $2^9 - 1$

Solution:

Let the matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} a_1x + b_1y + c_1z &= 1 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}$$

Three planes can never intersect at exactly two points .

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If the distance of the point $P(1,-2,1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of perpendicular from P to the plane, is :

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A $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

B $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

C $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$

D $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

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If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of perpendicular from P to the plane, is :

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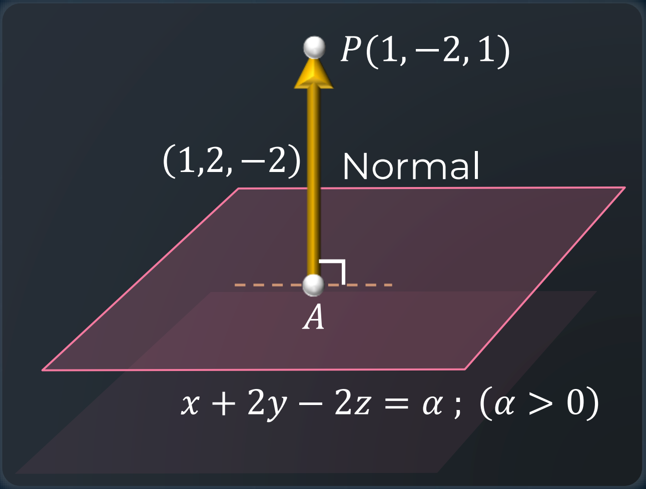
Let A be the foot of the perpendicular.



Distance of P from the plane = 5

$$\Rightarrow \left|\frac{1-4-2-\alpha}{\sqrt{1^2+2^2+(-2)^2}}\right| = \left|\frac{\alpha+5}{3}\right| = 5$$

$\Rightarrow \alpha = 10,\ -20$ (not possible)

$\therefore$ Equation of plane is: $x+2y-2z=10$

$$D = \left|\frac{ax_1+by_1+cz_1-d}{\sqrt{a^2+b^2+c^2}}\right|$$

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If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of perpendicular from P to the plane, is :

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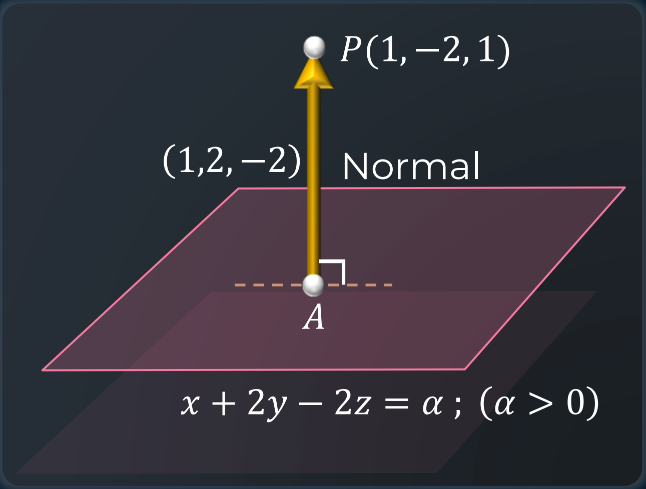
$\therefore$ Equation of plane is: $x+2y-2z=10$

Let the coordinates of A is (p,q,r)

$$\therefore \frac{p-1}{1}=\frac{q+2}{2}=\frac{r-1}{-2}=\frac{-(1-4-2-10)}{9}$$

$$\Rightarrow p=\frac{8}{3}, q=\frac{4}{3}, r=-\frac{7}{3}$$

So, point $A \equiv \left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$



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If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of perpendicular from P to the plane, is :

IIT-JEE 2010

A $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$

B $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$

C $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$

D $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

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Non zero value of α for which the lines $2x - y + 3z + 4 = 0 = \alpha x + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are coplanar is :

Solution:

$$\left.\begin{array}{l} 2x - y + 3z + 4 = 0 = \alpha x + y - z + 2 \\ x - 3y + z = 0 = x + 2y + z + 1 \end{array}\right\} \text{Coplanar, } \alpha \neq 0$$

$$2x - y + 3z + 4 = 0$$

$$\alpha x + y - z + 2 = 0$$

Let $\overrightarrow{n_1}$ is along L_1

$$\therefore \overrightarrow{n_1} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 3 \\ \alpha & 1 & -1 \end{vmatrix} \Rightarrow \overrightarrow{n_1} = -2\hat{\imath} + (2 + 3\alpha)\hat{\jmath} + (2 + \alpha)\hat{k}$$

$$\left.\begin{array}{l} \text{If } x = 0,\ y - z + 2 = 0 \\ -y + 3z + 4 = 0 \end{array}\right\} z = -3,\ y = -5$$

$$\therefore L_1 : \frac{x}{-2} = \frac{y+5}{2+3\alpha} = \frac{z+3}{2+\alpha}$$

A -2

B 4

C 6

D 0

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Non zero value of a for which the lines $2x - y + 3z + 4 = 0 =$ $\alpha x + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are coplanar is :

A -2

B 4

C 6

D 0

Solution:

$$\left.\begin{array}{l} \text{If } x = 0, \quad y - z + 2 = 0 \\ \qquad\qquad -y + 3z + 4 = 0 \end{array}\right\} \quad z = -3\ , y = -5$$

$$\therefore L_1 : \frac{x}{-2} = \frac{y+5}{2+3\alpha} = \frac{z+3}{2+\alpha}$$

$x - 3y + z = 0$

$x + 2y + z + 1 = 0$ Let $\overrightarrow{n_2}$ is along L_2

$$\therefore \overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 2 & 1 \end{vmatrix} \Rightarrow \overrightarrow{n_2} = -5\hat{i} + 5\hat{k}$$

$$\left.\begin{array}{l} \text{If } x = 0, \ -3y + z = 0 \\ \qquad\qquad 2y + z + 1 = 0 \end{array}\right\} \quad y = -\frac{1}{5}\ , z = -\frac{3}{5}$$

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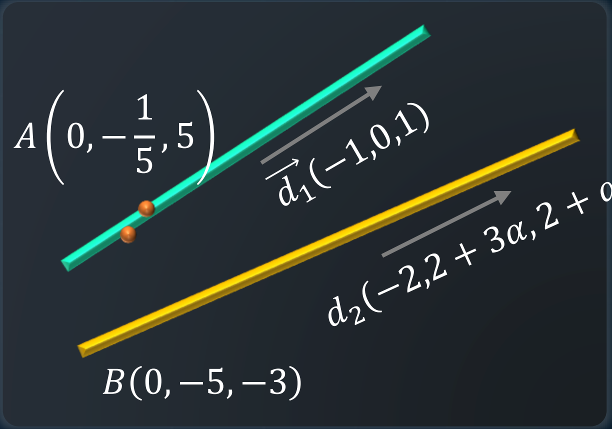
Non zero value of a for which the lines $2x - y + 3z + 4 = 0 = \alpha x + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are coplanar is :

Solution:

$$\left.\begin{aligned} \text{If } x = 0,\ \ -3y + z &= 0 \\ 2y + z + 1 &= 0 \end{aligned}\right\} \quad y = -\frac{1}{5},\ z = -\frac{3}{5}$$

$$\therefore L_2 : \frac{x}{-1} = \frac{y+\frac{1}{5}}{0} = \frac{z+\frac{3}{5}}{1}$$

For 2 lines to be coplanar, $\left[\vec{d_1}\ \ \vec{d_2}\ \ \overrightarrow{AB}\right] = 0$



$$\Rightarrow \begin{vmatrix} -1 & 0 & 1 \\ -2 & 2+3\alpha & 2+\alpha \\ 0 & -5+\frac{1}{5} & -3+\frac{3}{5} \end{vmatrix} = 0$$

$$\Rightarrow -1\left((2+3\alpha)\left(-\frac{12}{5}\right) + (2+\alpha)\left(\frac{24}{5}\right)\right) + 1\left(\frac{48}{5}\right) = 0$$

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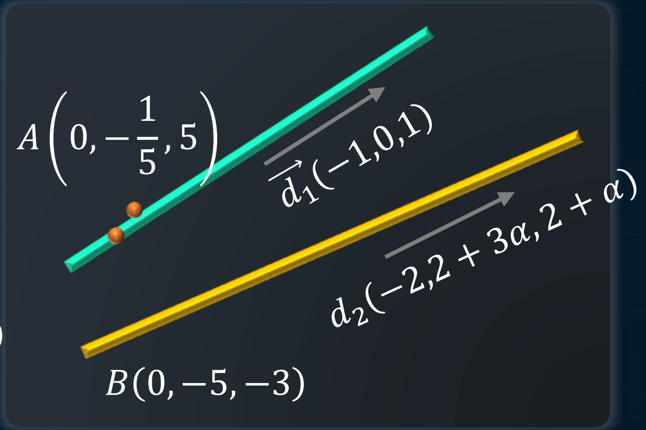
Non zero value of α for which the lines $2x - y + 3z + 4 = 0 = \alpha x + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are coplanar is :

Solution:

For 2 lines to be coplanar, $[\overrightarrow{d_1}\ \ \overrightarrow{d_2}\ \ \overrightarrow{AB}] = 0$

$$\Rightarrow \begin{vmatrix} -1 & 0 & 1 \\ -2 & 2+3\alpha & 2+\alpha \\ 0 & -5+\frac{1}{5} & -3+\frac{3}{5} \end{vmatrix} = 0$$



$$\Rightarrow -1\left((2+3\alpha)\left(-\frac{12}{5}\right) + (2+\alpha)\left(\frac{24}{5}\right)\right) + 1\left(\frac{48}{5}\right) = 0$$

$$\Rightarrow \frac{12}{5}(2 + 3\alpha - 4 - 2\alpha + 4) = 0$$

$$\Rightarrow \frac{12}{5}(2+\alpha) = 0 \quad \Rightarrow \alpha = -2$$

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From the point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/are:

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A 1

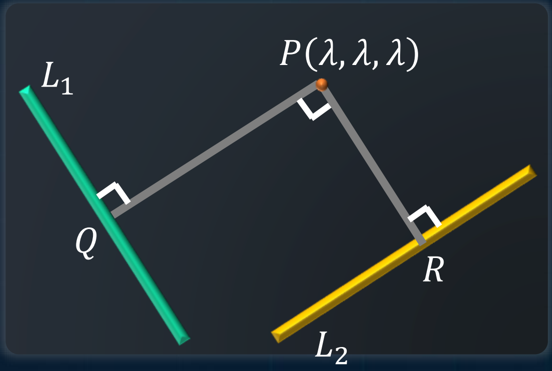
B $\sqrt{2}$

C 6

D 0

Solution:

$L_1 : y = x, z = 1 \qquad L_2 : y = -x, z = -1$



Let $Q \equiv (q, q, 1)$

PQ is perpendicular to the line:

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0}$$

$$(\lambda - q) + (\lambda - q)1 + (\lambda - 1)(0) = 0 \Rightarrow q = \lambda$$

$$\therefore Q \equiv (\lambda, \lambda, 1)$$

Let $R \equiv (r, -r, -1)$

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From the point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn on the lines $y = x, z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/are:

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A 1

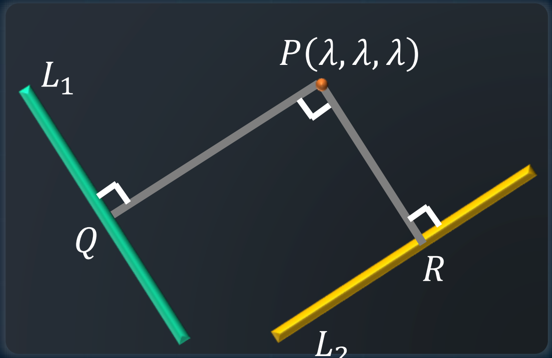
B $\sqrt{2}$

C 6

D 0

Solution:

$L_1 : y = x, z = 1$ $\quad$ $L_2 : y = -x, z = -1$



$\therefore Q \equiv (\lambda, \lambda, 1)$

Let $R \equiv (r, -r, -1)$

PQ is perpendicular to the line :

$$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0}$$

$$(\lambda - r) - (\lambda + r)1 + (\lambda + 1)(0) = 0 \Rightarrow r = 0$$

$\therefore R \equiv (0, 0, -1)$

$PQ \perp PR$

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From the point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn on the lines $y = x, z = 1$ and $y = -x,\ z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/are:

JEE Advanced 2014

A 1

B $\sqrt{2}$

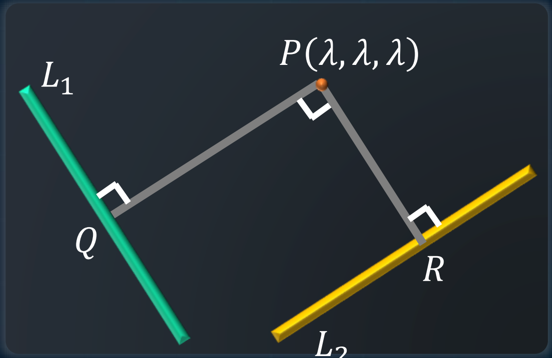
C 6

D 0

Solution:

$L_1 : y = x, z = 1$ $\quad\quad$ $L_2 : y = -x, z = -1$

$\therefore Q \equiv (\lambda, \lambda, 1)$ $\quad\quad$ $\therefore R \equiv (0, 0, -1)$



$PQ \perp PR$

$\Rightarrow 0 \cdot (\lambda - 0) + 0 \cdot (\lambda - 0) + (\lambda + 1)(\lambda - 1) = 0$

$\Rightarrow \lambda^2 - 1 = 0$

$\Rightarrow \lambda = \pm 1$

$\lambda = 1$ is rejected as it will lie on the given line

$\therefore \lambda = -1$

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In R^3 , let L be a straight line passing through origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of feet of perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

JEE Advanced 2015

A $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$

B $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$

C $\left(-\frac{5}{6},0,\frac{1}{6}\right)$

D $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

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In R^3 , let L be a straight line passing through origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x + 2y - z + 1 = 0$ and $P_2: 2x - y + z - 1 = 0$. Let M be the locus of feet of perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

JEE Advanced 2015

L is parallel to the planes P_1 & P_2

Let vector parallel to the line is $\vec{a}$

$$\vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$$

$\therefore$ Direction ratio will be $1, -3, -5$

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In R^3 , let L be a straight line passing through origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of feet of perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

JEE Advanced 2015

$\therefore$ Direction ratio will be $1, -3, -5$

$$L: \frac{x}{1} = \frac{y}{-3} = \frac{z}{-5}$$

Feet of perpendicular of $(0,0,0)$ on the plane P_1 is :

$$\frac{x_p - x_1}{a} = \frac{y_p - y_1}{b} = \frac{z_p - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

$$\frac{x_p - 0}{1} = \frac{y_p - 0}{2} = \frac{z_p - 0}{-1} = -\frac{(1)}{(1^2 + 2^2 + (-1)^2)} = -\frac{1}{6}$$

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In R^3, let L be a straight line passing through origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of feet of perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

JEE Advanced 2015

$$L: \quad \frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}$$

$$\frac{x_p-0}{1}=\frac{y_p-0}{2}=\frac{z_p-0}{-1}=-\frac{1}{6}$$

$$\Rightarrow x_p=-\frac{1}{6}\quad, y_p=-\frac{1}{3}\quad, z_p=\frac{1}{6}$$

Equation of line M : $\dfrac{x+\frac{1}{6}}{1}=\dfrac{y+\frac{1}{3}}{-3}=\dfrac{z_p-\frac{1}{6}}{-5}$

Points $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ and $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$ lie on the line M.

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In R^3 , let L be a straight line passing through origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x + 2y - z + 1 = 0$ and $P_2: 2x - y + z - 1 = 0$. Let M be the locus of feet of perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

JEE Advanced 2015

A. $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$

B. $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

C. $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

D. $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

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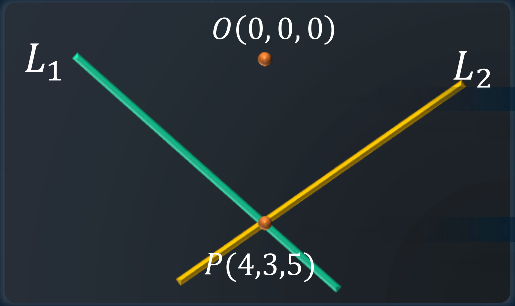
Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point $(0,0,0)$, is :

Solution:

$$L_1 : \frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}=\lambda$$

$$L_2 : \frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\mu$$



A $4x+3y+5z=25$

B $4x+3y+5z=50$

C $3x+4y+5z=49$

D $x+7y-5z=2$

Point on L_1: $(1+3\lambda, 2+\lambda, 3+2\lambda)\cdots(i)$

Point on L_2: $(3+\mu, 1+2\mu, 2+3\mu)\cdots(ii)$

To get intersection point ,

$$\left.\begin{array}{l}1+3\lambda=3+\mu\\ 2+\lambda=1+2\mu\end{array}\right\}\Rightarrow \lambda=\mu=1$$

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Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point $(0,0,0)$, is :

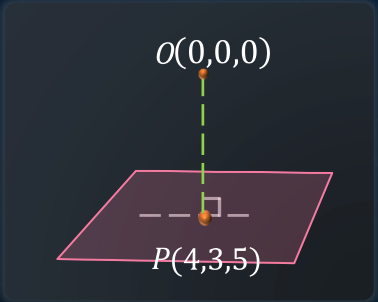
Solution:

$\therefore$ The intersecting point will be $P(4,3,5)$

$OP \geq OQ$

The equation of plane at greatest distance from origin and passing through point $(4,3,5)$ will have normal direction ratios as $4,3,5$.

$\Rightarrow 4(x-4)+3(y-3)+5(z-5)=0$



$\Rightarrow 4x+3y+5z=50$

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let P be a image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is :

JEE Advanced 2016

A $x + y - 3z = 0$

B $3x + z = 0$

C $x - 4y + 7z = 0$

D $2x - y = 0$

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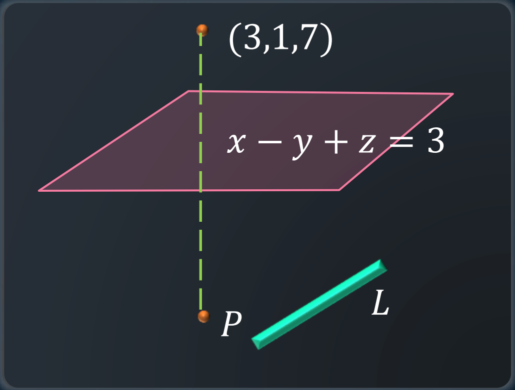
let P be a image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is :

JEE Advanced 2016

$$L : \frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$



Let $P \equiv (x', y', z')$

$$\frac{x' - x_1}{a} = \frac{y' - y_1}{b} = \frac{z' - z_1}{c} = -2\frac{(ax_1 + by_1 + cz_1 - d)}{(a^2 + b^2 + c^2)}$$

$$\frac{x' - 3}{1} = \frac{y' - 1}{-1} = \frac{z' - 7}{1} = -2\frac{(3 - 1 + 7 - 3)}{(1^2 + (-1)^2 + 1^2)}$$

$$= -4$$

$$P \equiv (-1, 5, 3)$$

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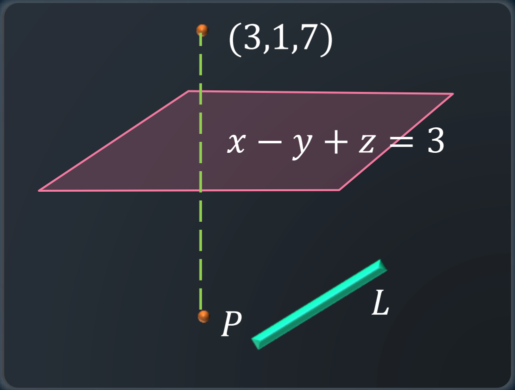
let P be a image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is :

JEE Advanced 2016

$$L : \frac{x}{1} = \frac{y}{2} = \frac{z}{1} \qquad P \equiv (-1\,,5\,,3)$$



Let $\vec{n}$ be the normal vector to the plane

$\vec{n}$ is perpendicular to line $\overrightarrow{OP}$ & given line L

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 5 & 3 \\ 1 & 2 & 1 \end{vmatrix} = -\hat{\imath} + 4\hat{\jmath} - 7\hat{k}$$

$\therefore$ Equation of plane is : $x - 4y + 7z = 0$

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let P be a image of the point (3,1,7) with respect to the plane $x-y+z=3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is :

JEE Advanced 2016

A $x+y-3z=0$

B $3x+z=0$

C $x-4y+7z=0$

D $2x-y=0$

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Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes.
Then which of the following statements(s) is (are) true ?

JEE Advanced 2018

D The line of intersection of P_1 and P_2 has direction ratios $1,2,-1$

B The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2.

C The acute angle between P_1 and P_2 is $60°$.

D If P_3 is the plane passing through the point $(4,2,-2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2,1,1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$.

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Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes.

Then which of the following statements(s) is (are) true ?

JEE Advanced 2018

Solution:

Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes.

Let $\overrightarrow{n_1}$ is along the line of intersection.

$$\Rightarrow \overrightarrow{n_1} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$$

The line of intersection of P_1 and P_2 has direction ratios: $1, -1, 1$

The line $\frac{x - \frac{4}{3}}{3} = \frac{y - \frac{1}{3}}{-3} = \frac{z}{3}$ is parallel to the line of intersection

of P_1 and P_2.

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Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes.

Then which of the following statements(s) is (are) true ?

JEE Advanced 2018

Solution:

Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes.

The line of intersection of P_1 and P_2 has direction ratios: $1, -1, 1$

Let θ be the angle the planes.

$$\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$\Rightarrow \cos\theta = \frac{(2 + 2 - 1)}{\sqrt{2^2 + 1^2 + (-1)^2}\sqrt{1^2 + 2^2 + 1^2}}$$

$$\cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60°$$

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Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes.

Then which of the following statements(s) is (are) true ?

JEE Advanced 2018

Solution:

Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes.

The line of intersection of P_1 and P_2 has direction ratios: $1, -1, 1$

Equation of P_3: $(x - 4) - (y - 2) + (z + 2) = 0$

$\Rightarrow x - y + z = 0$

Distance of the point $(2,1,1) = \left|\dfrac{2 - 1 + 1}{\sqrt{1^2 + (-1)^2 + 1^2}}\right|$

$$= \frac{2}{\sqrt{3}}$$

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Thank You

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