Welcome to

Aakash + BYJU'S NOTES

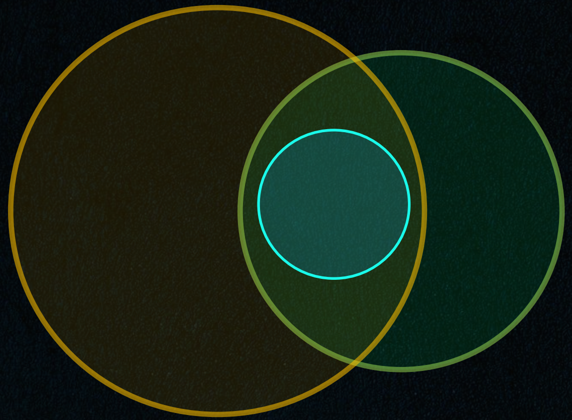
Oscillations

Periodic Motion

PERIODIC MOTION:

A body repeats its motion along a definite path after regular intervals of time.

e.g., Hands of a Clock.



HARMONIC MOTION:

The motion of a particle trapped between two extremes, being both periodic and oscillatory in nature.

OSCILLATORY MOTION:

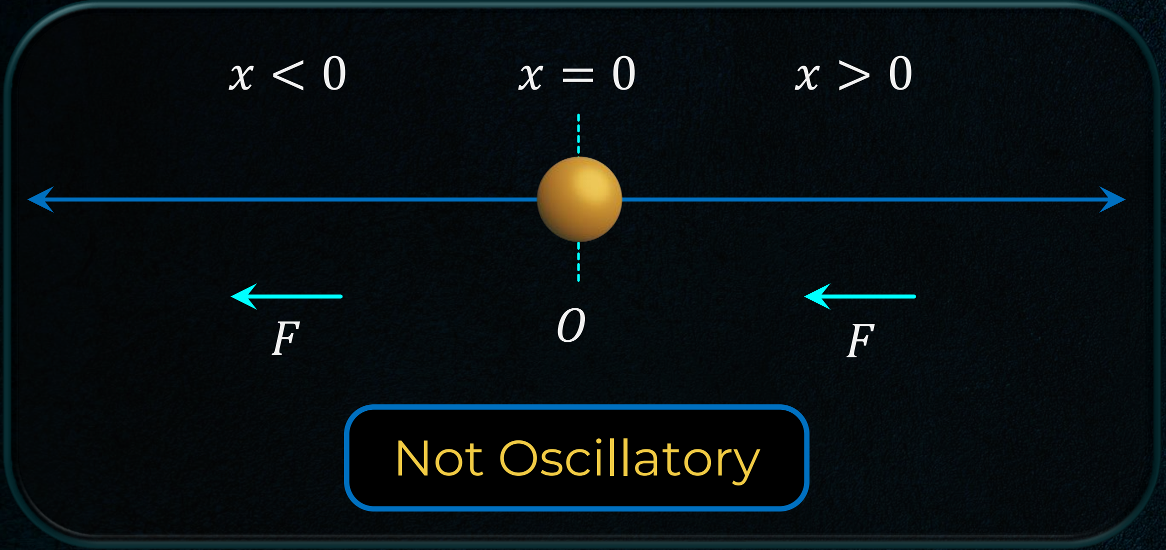
The body undergoes to and fro motion along the same path about a fixed point.

e.g., Simple Pendulum.

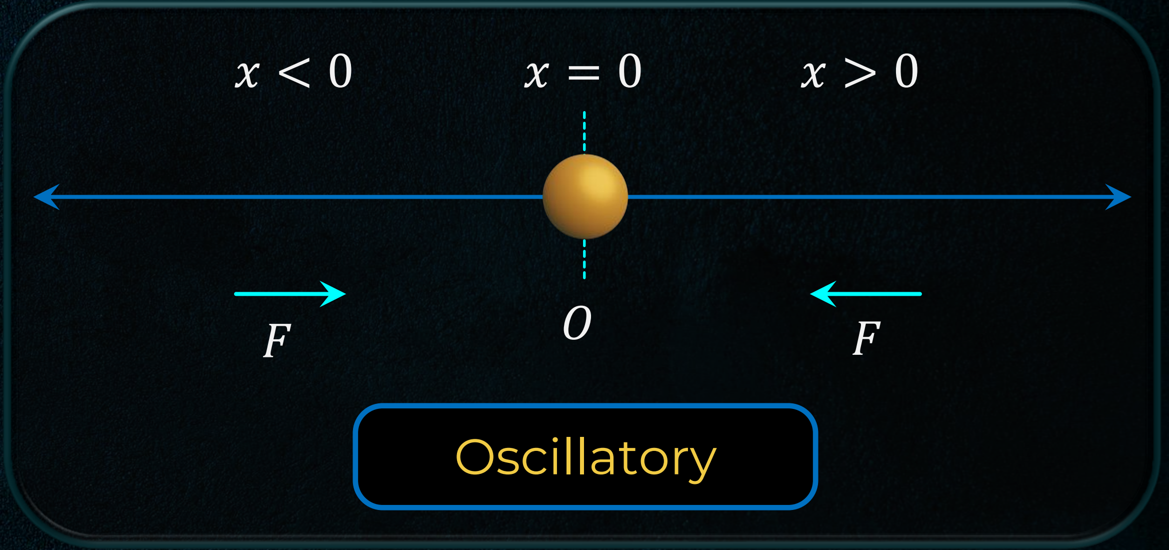
- Consider a particle moving along x-axis subjected to a force,

$$F = -k(x)^n$$

$n =$ Even $(2, 4, 6 \dots)$

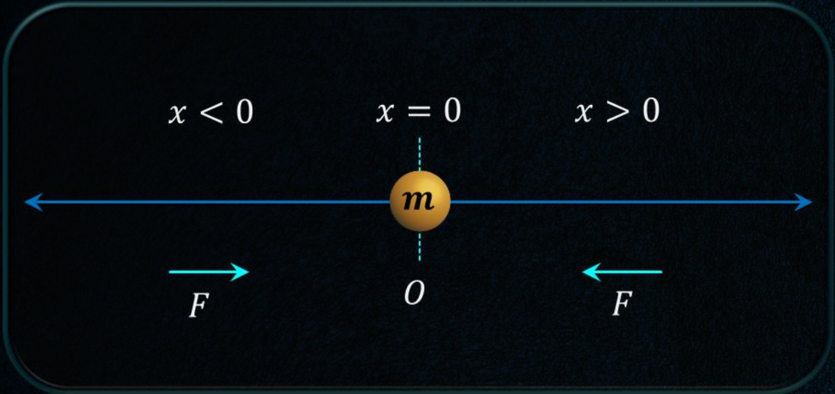


$n =$ Odd $(1, 3, 5 \dots)$



Simple Harmonic Motion

- SHM: To-and-fro motion about a fixed point with the restoring force, $F \propto -x$. [Put $n = 1$ in $F = -kx^n$]



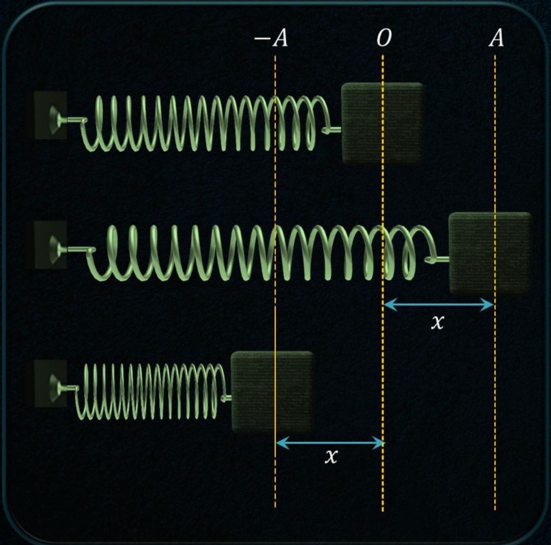
$\vec{F} \propto -\vec{x}$

From Newton's second law of motion:

$m\vec{a} \propto -\vec{x}$

$$\boxed{\vec{a} \propto -\vec{x}}$$

SHM in Spring Mass System

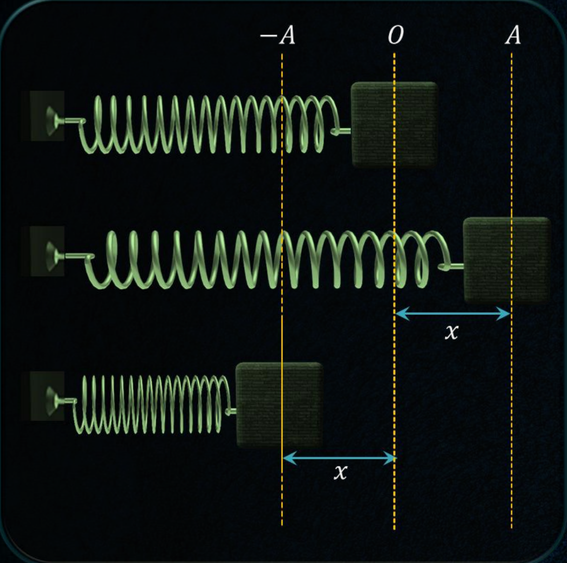


- According to Hooke's law: Spring Force

$$\boxed{\vec{F} = -k\vec{x}}$$

Here, k = Spring constant

Simple Harmonic Motion

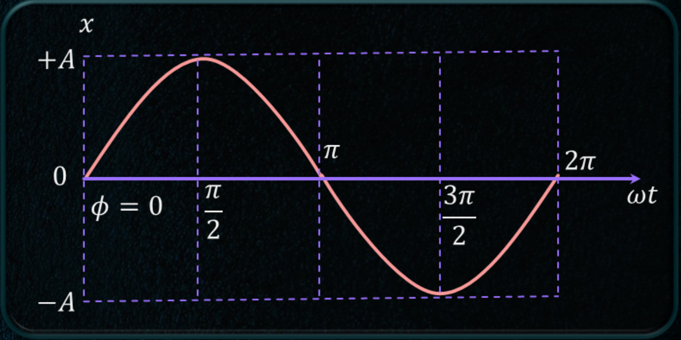


- If the force is linear & restoring, $F \propto -x$

$$\boxed{\Rightarrow \quad v = \omega\sqrt{A^2 - x^2}}$$

$$\boxed{\Rightarrow \quad x = A\sin(\omega t + \phi)}$$

- Graph of sine function.



Extreme	Mean	Extreme
$-A$	O	A
$v = 0$ $\lvert a_{max}\rvert = \omega^2 A$	$v_{max} = A\omega$ $a = 0$	$v = 0$ $\lvert a_{max}\rvert = \omega^2 A$

$$\vec{a} = -\frac{k}{m}\vec{x} = -\omega^2\vec{A}$$

Time Period $= \dfrac{2\pi}{\omega}$

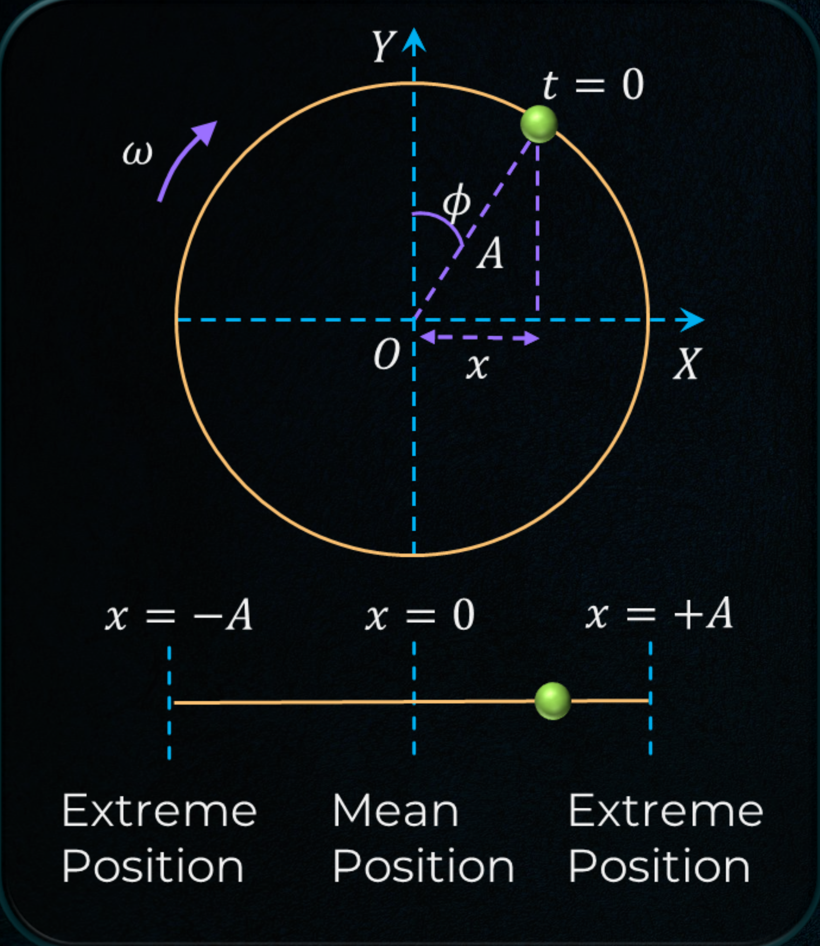
$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

$$x = A\sin(\omega t + \phi) \qquad x_{max} = A \ \& \ x_{min} = -A$$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi) \qquad v_{max} = A\omega \ \& \ v_{min} = -A\omega$$

$$a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t + \phi) \qquad a_{max} = \omega^2 A \ \& \ a_{min} = -\omega^2 A$$

SHM as a projection of UCM



Position of the particle is:

$$x = A\sin(\omega t + \phi)$$

Velocity of the particle is:

$$v = \frac{dx}{dt}$$

$$v = A\omega\cos(\omega t + \phi)$$

Acceleration of the particle is:

$$a = \frac{dv}{dt}$$

$$a = -\omega^2 x$$

- Amplitude(A) : Magnitude of maximum displacement from mean position.
- Time Period(T) : The smallest time interval after which motion gets repeated.

$$T = 2\pi/\omega$$

- Initial phase/Epoch(ϕ) : Determines the status of particle in SHM at $t = 0$.

$$\omega t + \phi \; : Phase(\delta)$$

- Angular Frequency(ω) : Rate of change of the phase (δ).

$$\omega = \frac{d\delta}{dt}$$

The equation of a particle executing simple harmonic motion is $x = 5\sin\left(\pi t + \frac{\pi}{3}\right) m$. Write down the amplitude, time period and maximum speed of the particle. Also find the velocity at $t = 1\ s$.

Solution:

$x = A\sin(\omega t + \phi) \ldots\ldots (1)$

$x = 5\sin\left(\pi t + \frac{\pi}{3}\right) \ldots\ldots (2)$

Comparing equation (1) & (2):

Amplitude: $A = 5\ m$

Time period: $T = 2\pi/\omega$

$\omega = \pi$

$T = 2\ s$

Maximum Speed:

$|v_{max}| = |A\omega| = 5\pi\ ms^{-1}$

Velocity at $t = 1\ s$:

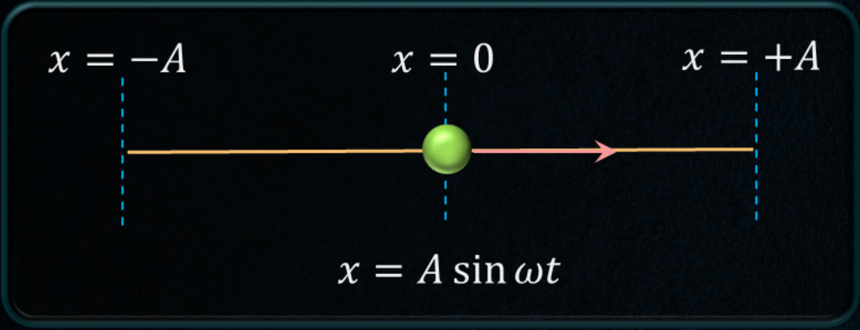
$x = 5\sin\left(\pi t + \frac{\pi}{3}\right)$

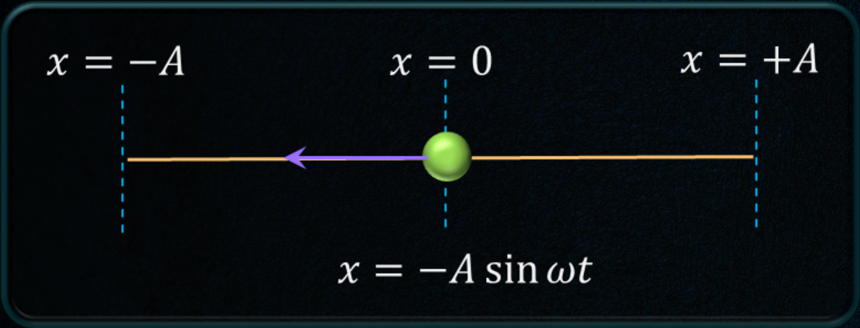
$v = \frac{dx}{dt} = 5\pi\cos\left(\pi t + \frac{\pi}{3}\right)$

At $t = 1\ s$

$v = 5\pi\cos\left(\pi + \frac{\pi}{3}\right) = -5\pi\cos\left(\frac{\pi}{3}\right) = -\frac{5\pi}{2}\ ms^{-1}$

Standard Equation of SHM





$v = A\omega\cos(\omega t + \phi)$

At $t = 0$

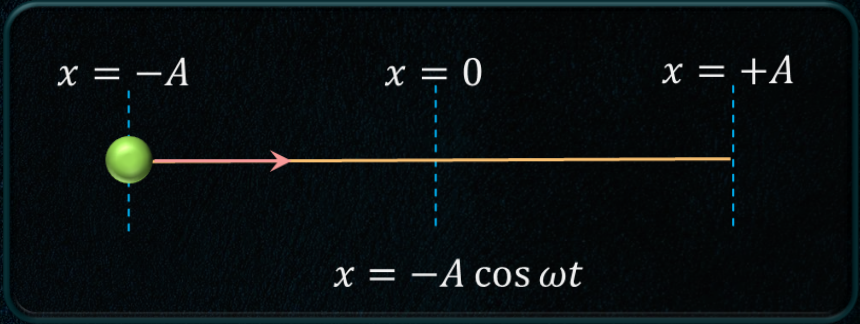
$v = A\omega\cos(\phi)$

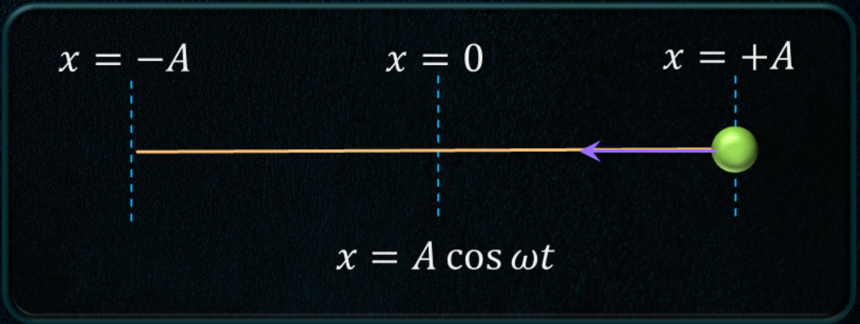
For $\phi = 0$, $v = A\omega$

For $\phi = \pi$, $v = -A\omega$

- If the particle starts from the mean position, the equation will be, $x = \pm A\sin(\omega t)$.

$\phi = 0$





- If the particle starts from the extreme position, the equation will be, $x = -A\cos(\omega t)$.
- If the particle starts from the extreme position, the equation will be, $x = +A\cos(\omega t)$.

Write the equation of SHM for the given situation.

Solution:

At $t = 0\ s$, $\quad x = +\dfrac{A}{2}$

$$\frac{A}{2} = A\sin(\omega \times 0 + \phi)$$

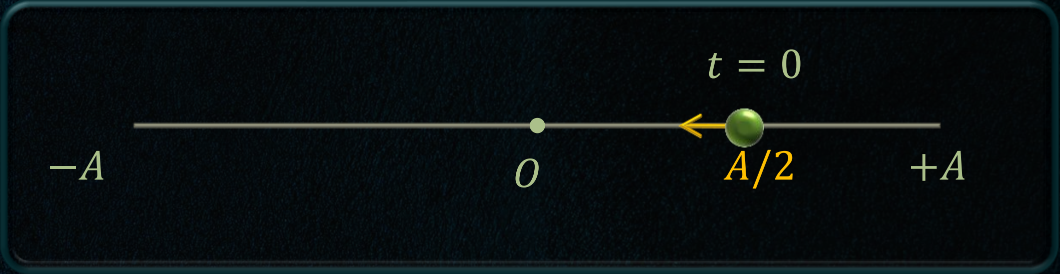
$$\Rightarrow \phi = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

Similarly, at $t = 0\ s$, $v = A\omega\cos(\phi)$

Since particle starts moving to the left, v should be negative.

$\cos\phi$ would be negative only for $\phi = \frac{5\pi}{6}$.

Hence, the equation of SHM for the given configuration will be, $x = A\sin\left(\omega t + \frac{5\pi}{6}\right)$.



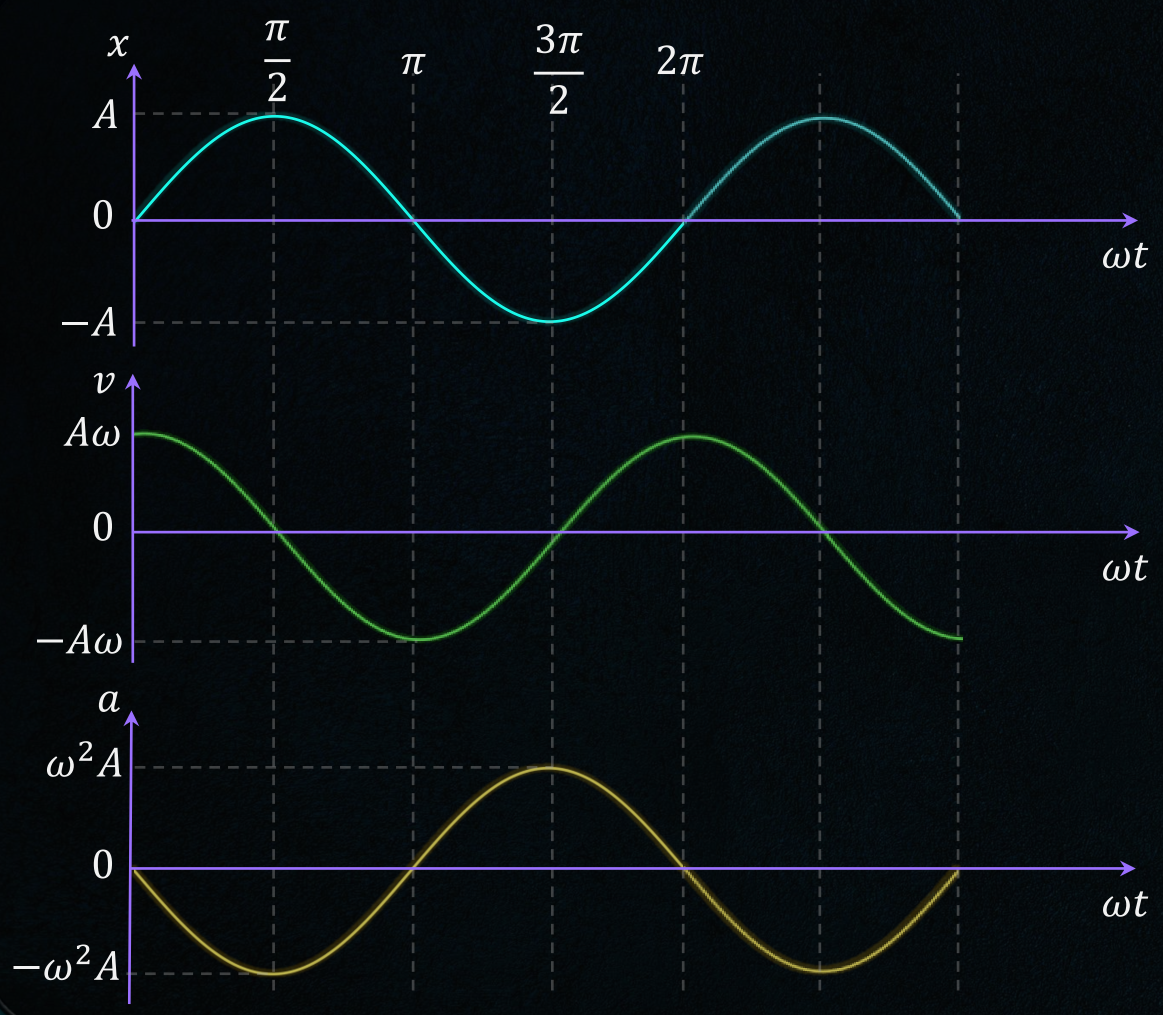
A) $x = A\sin\left(\omega t + \dfrac{5\pi}{6}\right)$

B) $x = A\cos\left(\omega t + \dfrac{5\pi}{6}\right)$

C) $x = A\sin\left(\omega t + \dfrac{\pi}{6}\right)$

D) $x = A\cos\left(\omega t + \dfrac{\pi}{6}\right)$

Graphical Representation of Position, Velocity & Acceleration



Displacement: $x = A\,sin\omega t$

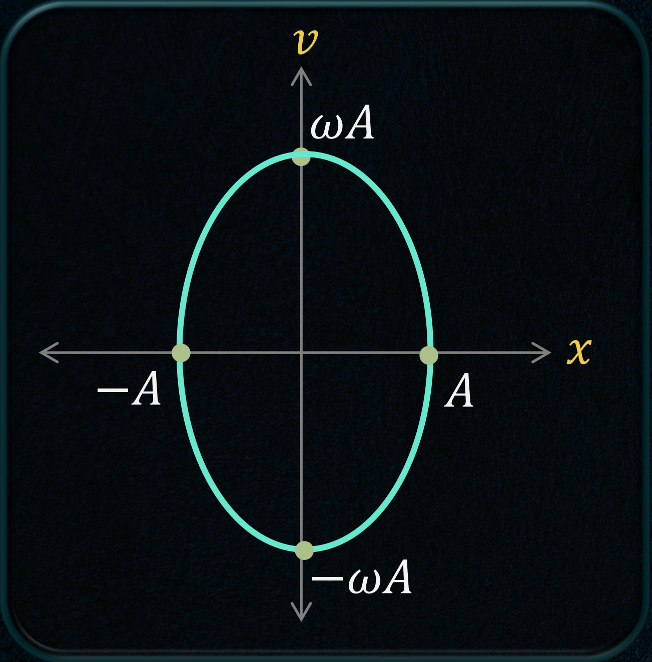
Velocity: $v = A\omega\cos(\omega t)$

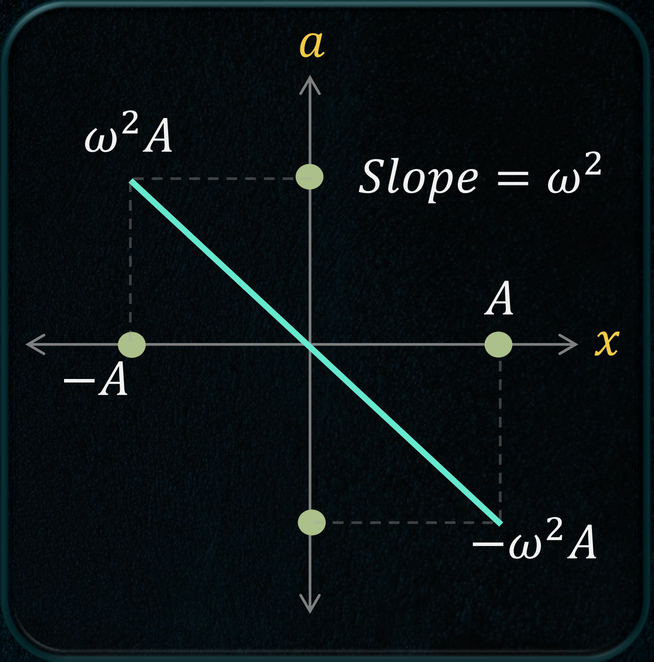
$$v = A\omega\sin(\omega t + \frac{\pi}{2})$$

Acceleration: $a = -A\omega^2\sin(\omega t)$

$$a = A\omega^2\sin(\omega t + \pi)$$

Velocity vs Displacement & Acceleration vs Displacement





Velocity: $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2$$

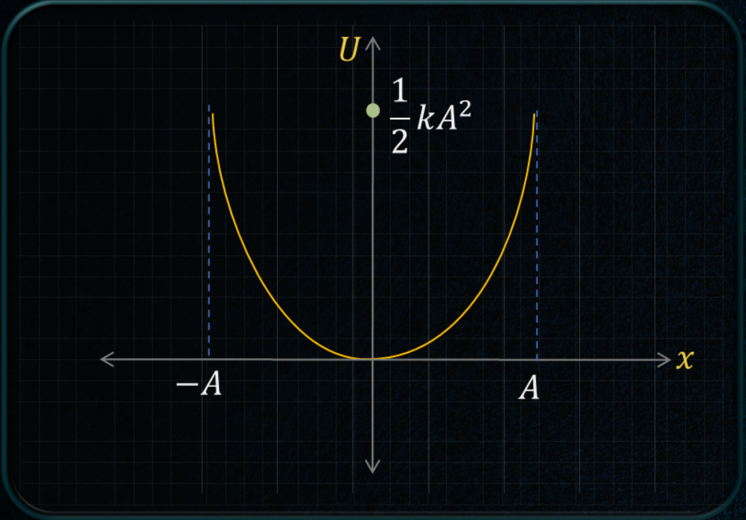
$$\Rightarrow \boxed{\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1}$$

Acceleration:

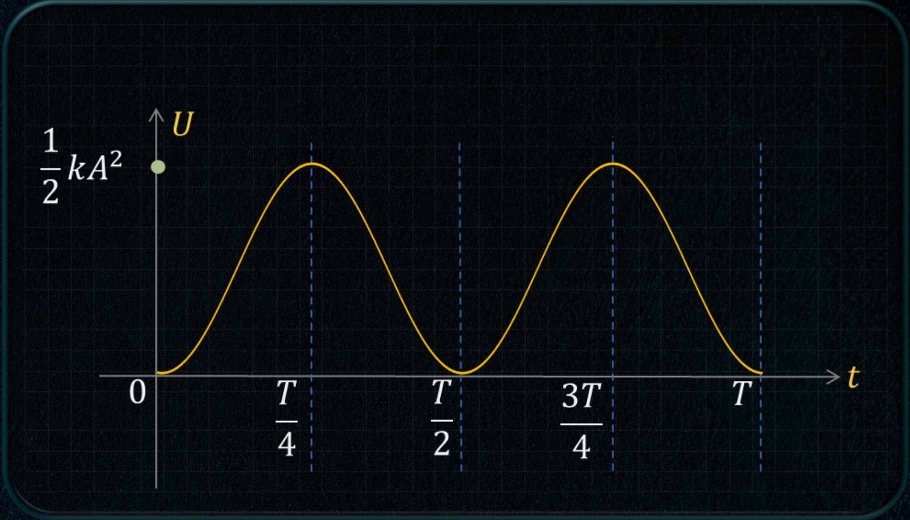
$$a = -\omega^2 A \sin \omega t$$

$$\Rightarrow \boxed{a = -\omega^2 x}$$

Potential Energy of Particle Performing SHM

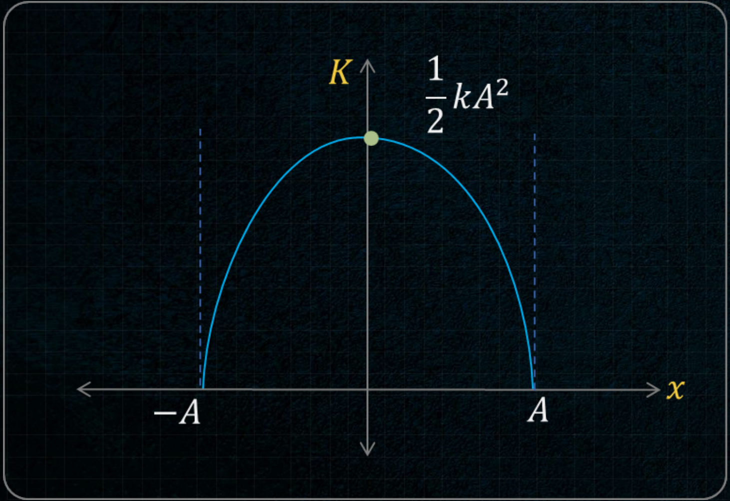


$$U = \frac{1}{2}kx^2$$

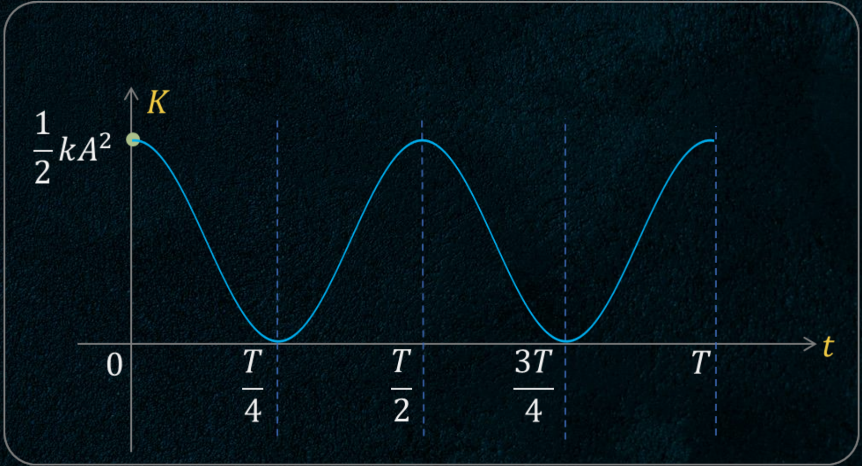


$$U = \frac{1}{2}kA^2 \sin^2(\omega t)$$

Kinetic Energy of Particle Performing SHM



$$K = \frac{1}{2}k(A^2 - x^2)$$

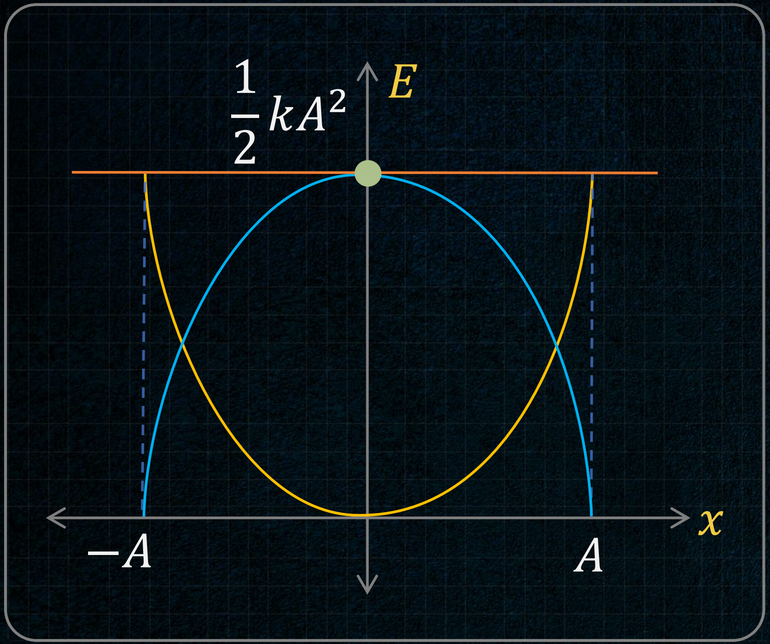


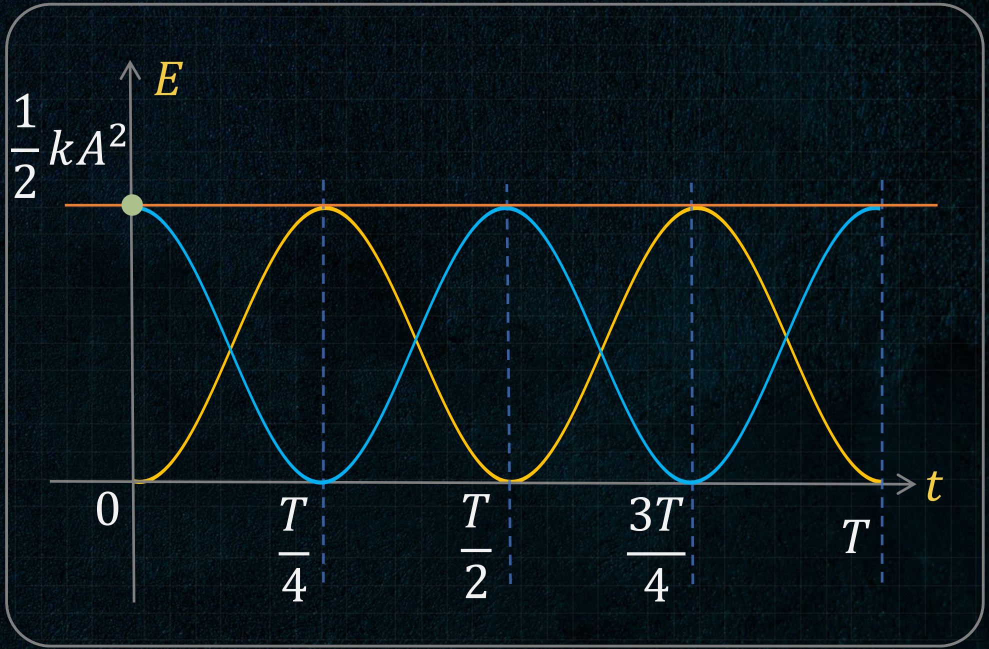
$$K = \frac{1}{2}kA^2 \cos^2(\omega t)$$

Total Energy of SHM

Total Mechanical Energy,

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \Rightarrow \boxed{E = \frac{1}{2}kA^2 = constant}$$





A particle of mass $2\ kg$ is moving on a straight line under the action of force $F = (8 - 2x)\ N$. Is the particle performing simple harmonic motion? If yes, find the equilibrium position of the particle.

Solution:

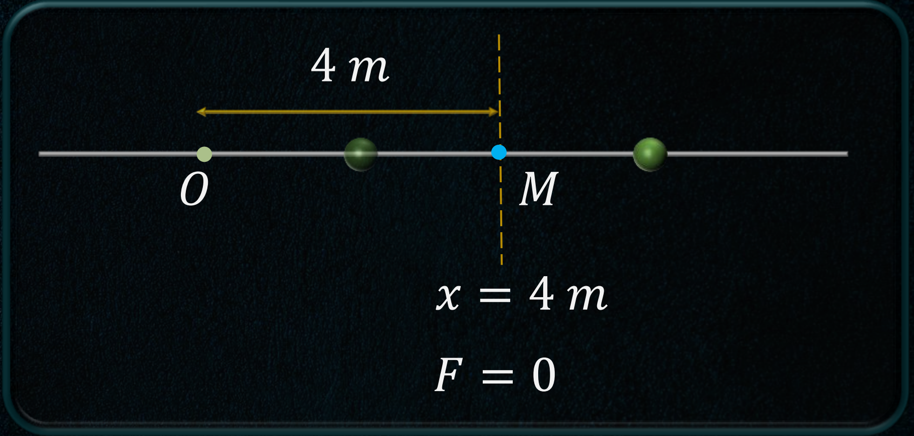
$$F = -kx + c$$

$$F = -k\left(x - \frac{c}{k}\right) = -kx^*$$

So, the particle will perform SHM under force $F = -kx + c$ provided $x = \frac{c}{k}$ is the new mean position.

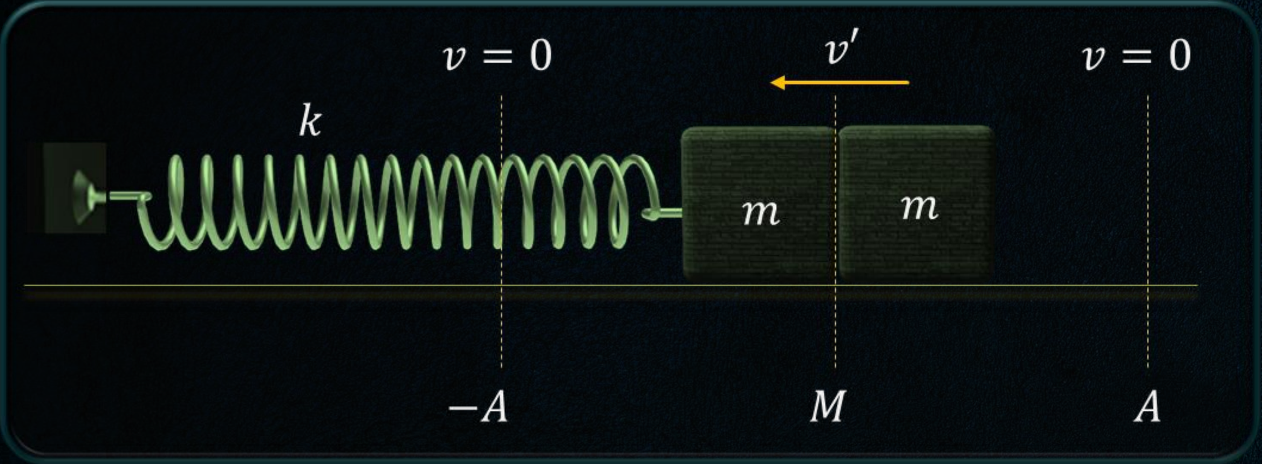
For, $F = 8 - 2x$

Mean position, $x = 4\ m$



A block of mass m moving with a velocity v collides inelastically with an identical block attached to a spring and sticks to it. Find the amplitude of the resulting simple harmonic motion. Consider all the surfaces to be frictionless.

Solution:



As the collision is perfectly inelastic, both block stick together

Conservation of momentum: $p_i = p_f$

$$mv = (m+m)v'$$

$$\Rightarrow v' = \frac{v}{2}$$

After the masses stick together,

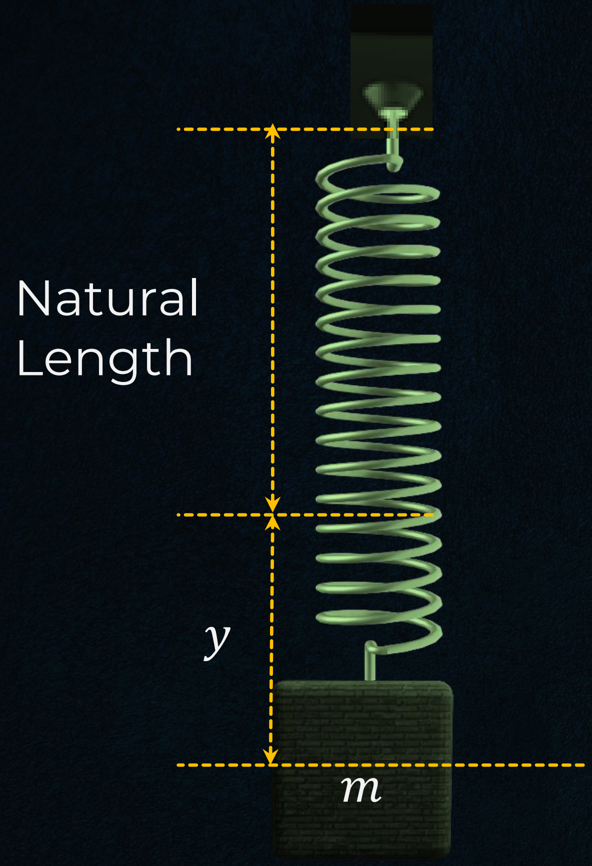
$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Amplitude (A),

$$v' = \frac{v}{2} = A\omega$$

$$\boxed{\Rightarrow A = v\sqrt{\frac{m}{2k}}}$$

Time Period of a Spring Mass System in Vertical Plane



- Time period of SHM of spring in vertical plane is same as time period of SHM of spring in horizontal plane.

Time Period, $$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

Time Period of a Spring Mass System in an Accelerating Lift

Due to upward acceleration of lift,

$g_{eff} = g + a$

Net force along y direction,

$$F_{net} = m(g + a) - k(y + y_0)$$

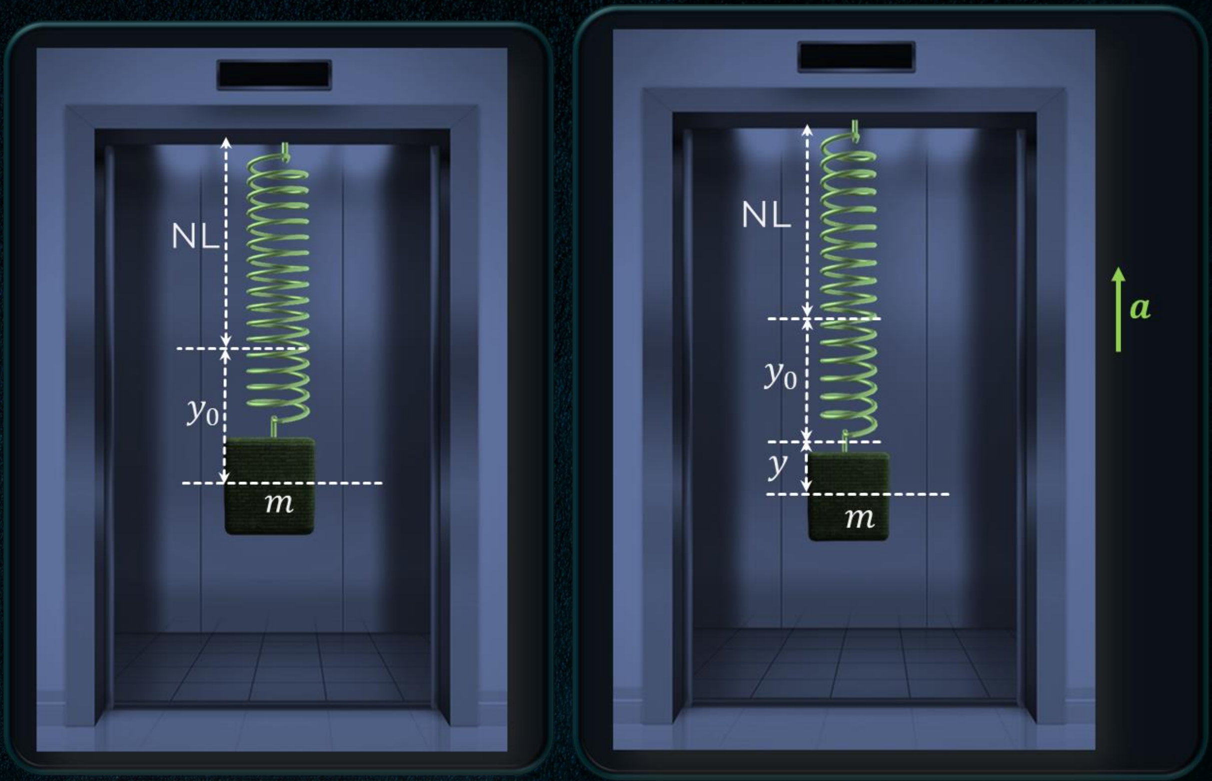
$$= mg + ma - ky - ky_0$$

At equilibrium, $ky_0 = mg$

$$= ma - ky$$

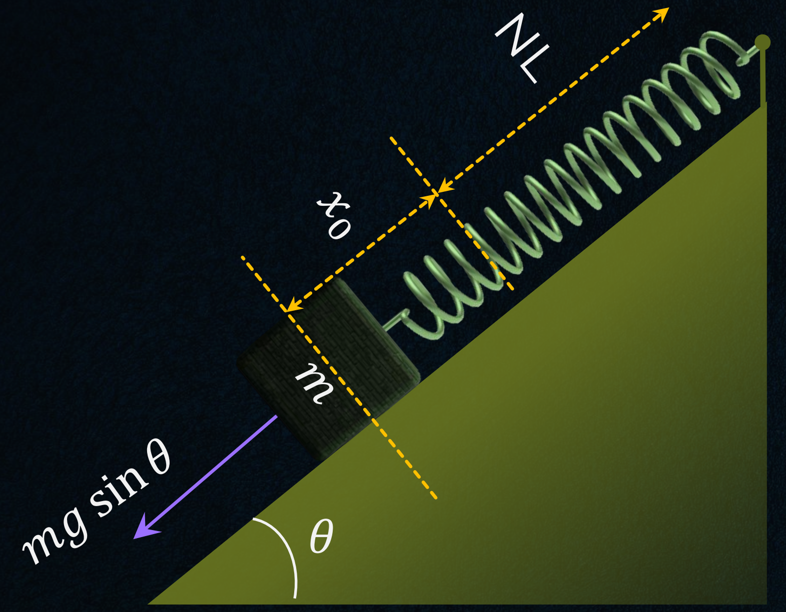
$$= -ky + c \qquad (\because ma = constant = c\text{(say)})$$

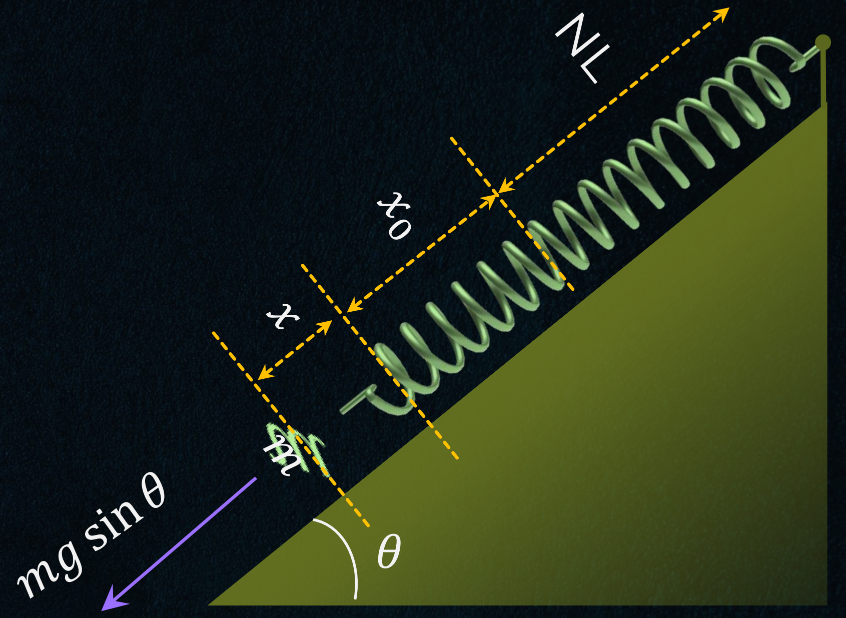
Time Period, $\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$



At equilibrium, $ky_0 = mg$

Time Period of a Spring Mass System over an Inclined Plane





At equilibrium,

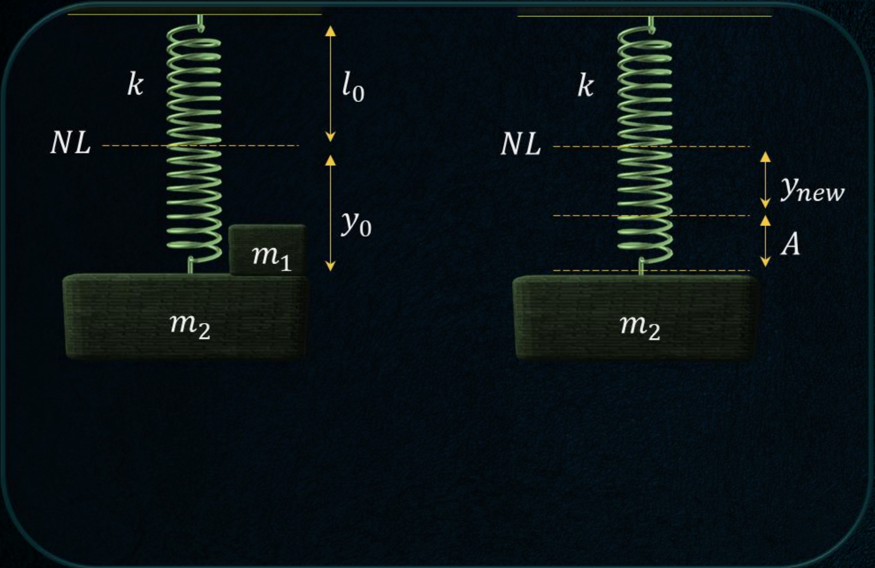
$$mg\sin\theta = kx_0$$

Time Period, $\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$

$$F_{net} = mg\sin\theta - k(x + x_0)$$

$$F_{net} = -kx$$

The system is initially in equilibrium and at rest. When the mass m_1 is abruptly removed from m_2, find the time period and amplitude of resultant motion.



Solution:

Initial elongation at steady state: y_0

$$\Rightarrow (m_1 + m_2)g = ky_0 \quad \text{... (1)}$$

After mass m_1 is removed,

Time Period, $$\boxed{T = 2\pi\sqrt{\frac{m_2}{k}}}$$

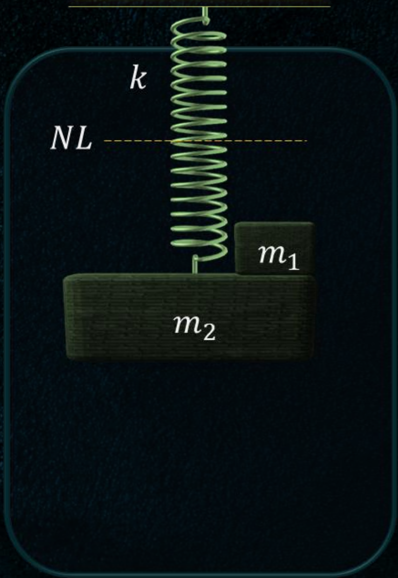
Elongation at mean position: y_{new}

$$\Rightarrow m_2 g = ky_{new} \quad \text{... (1)}$$

Amplitude, $A = y_0 - y_{new}$

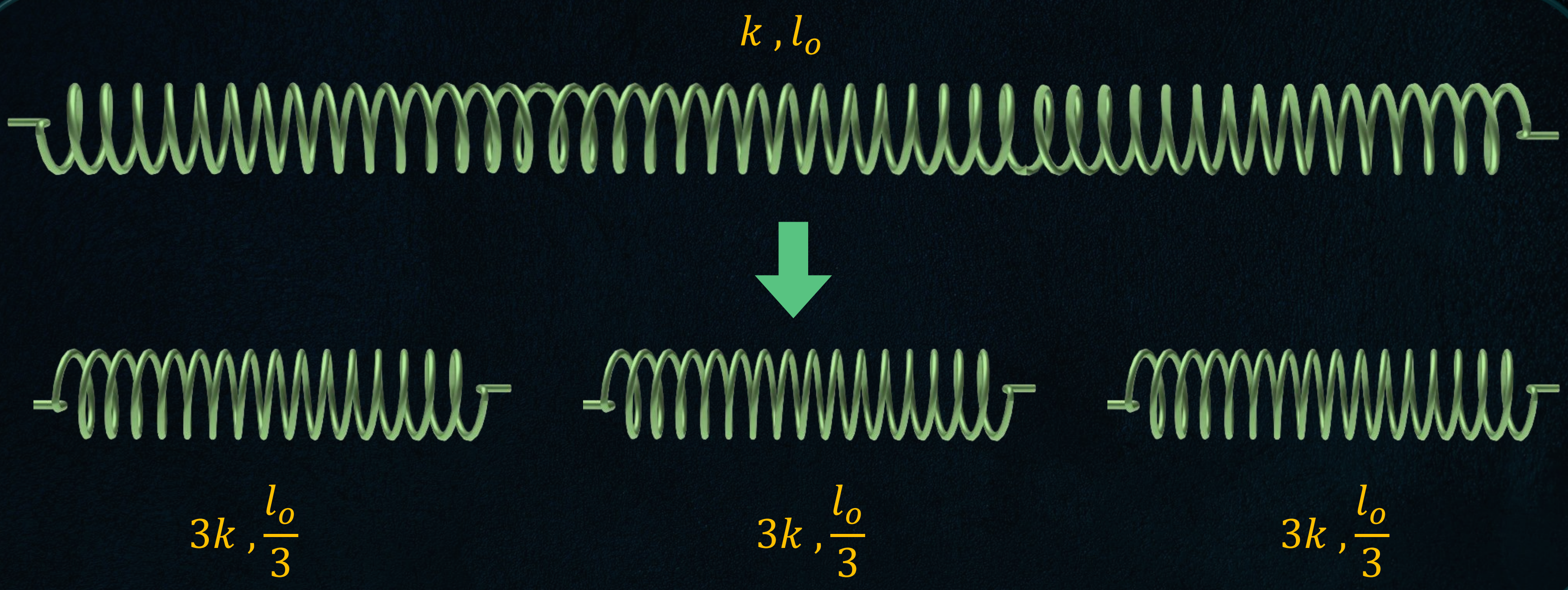
$$= \frac{(m_1 + m_2)g}{k} - \frac{m_2 g}{k}$$

$$\boxed{A = \frac{m_1 g}{k}}$$



Cutting of Springs

- Spring constant of a spring is inversely proportional to its natural length.



- If a spring is cut into 'n' equal pieces, then spring constant of one piece will be nk.

The time period of SHM of a spring mass system is T. The spring is now cut into two equal halves and the same mass is suspended vertically from one of the halves. Find the new time period of vertical oscillation.

Solution:

Since $k \propto \frac{1}{l}$

And $l' = \frac{l}{2}$

$$k' = 2k$$

Time period:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow T \propto \frac{1}{\sqrt{k}}$$

$$\frac{T}{T'} = \sqrt{\frac{k'}{k}}$$

$$\frac{T}{T'} = \sqrt{\frac{2k}{k}} \Rightarrow \boxed{T' = \frac{T}{\sqrt{2}}}$$

A T

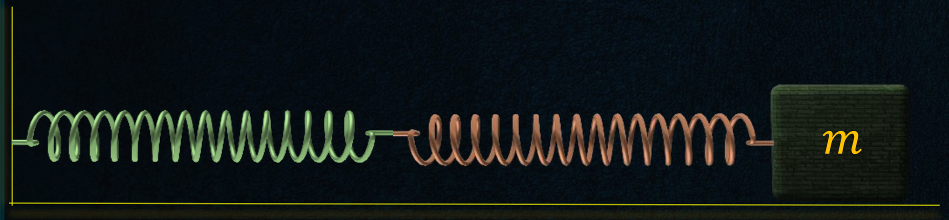
C $\sqrt{2}T$

B $T/\sqrt{2}$

D $4T$

Combination Of Springs

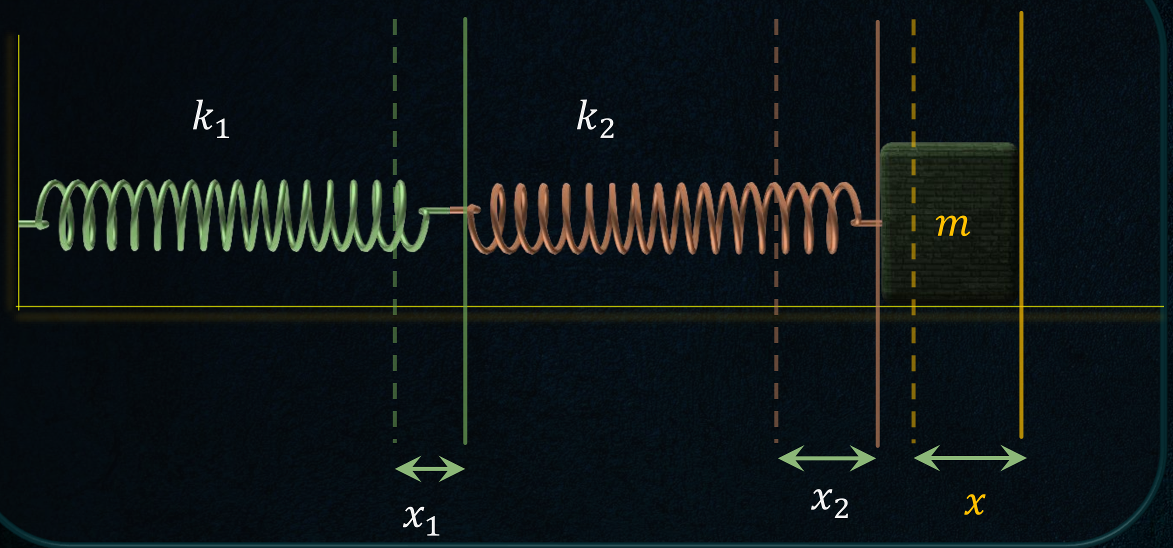
Series



Parallel

Combination Of Springs

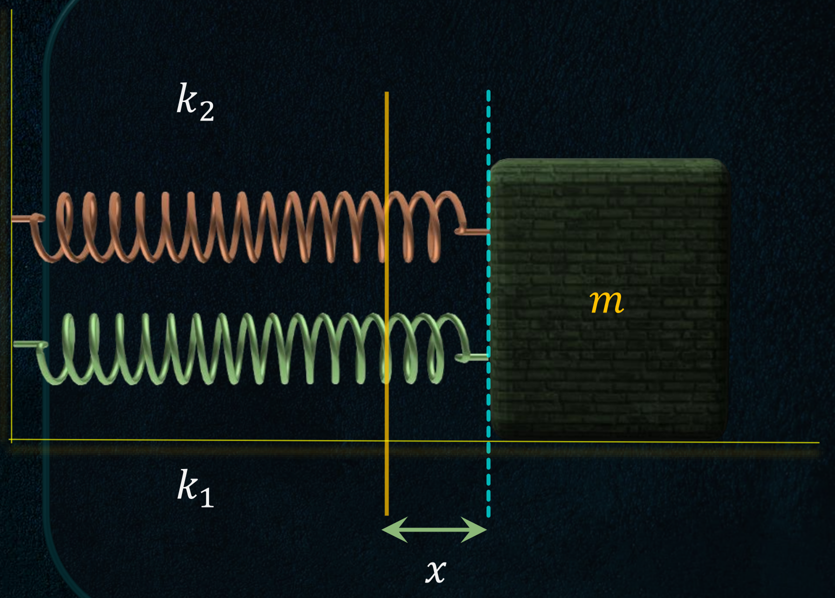
- Restoring force in each spring is the same.



$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots \dots \dots + \frac{1}{k_n}$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

- Extension/Compression in every spring is same.



$$k_{eq} = k_1 + k_2 + k_3 \dots \dots \dots + k_n$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

Find the time period of oscillation of the system shown in the given figure.

Solution:

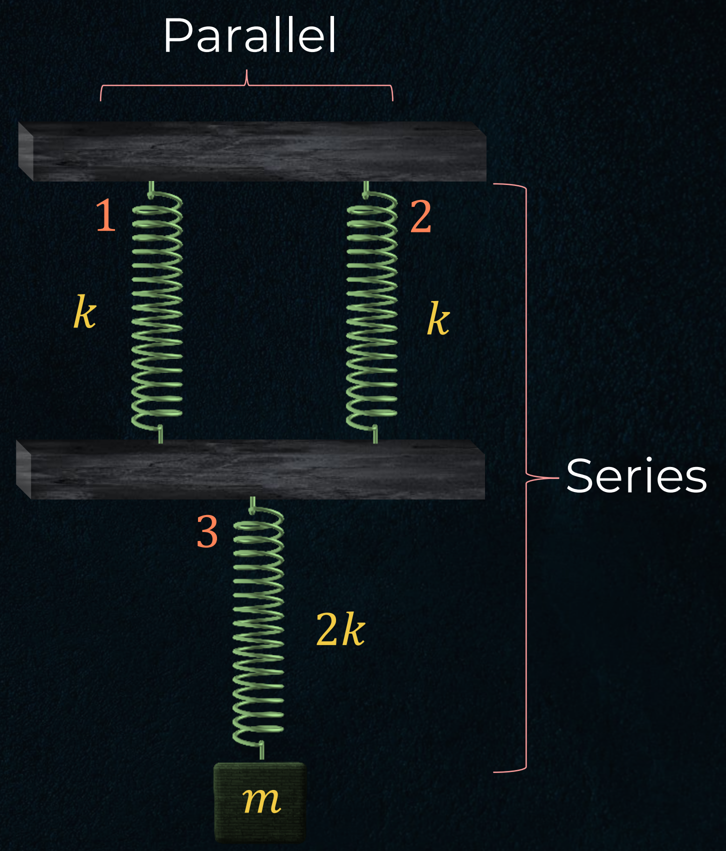
For 1 and 2 connected in parallel,

$$k_p = k + k = 2k \qquad \left[k_p = \sum k\right]$$

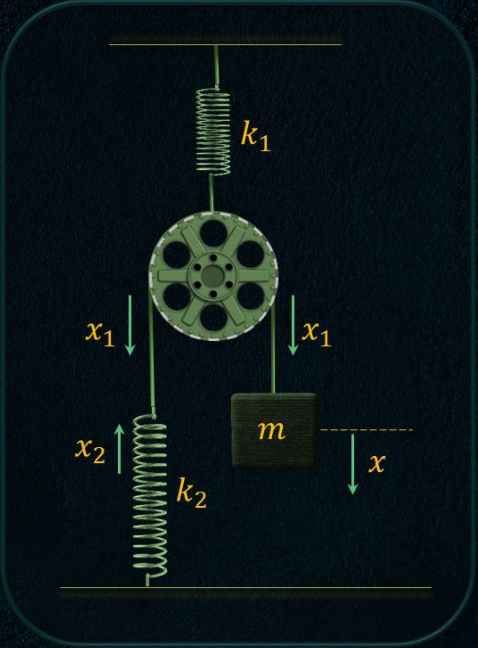
For 1,2 and 3 connected in series,

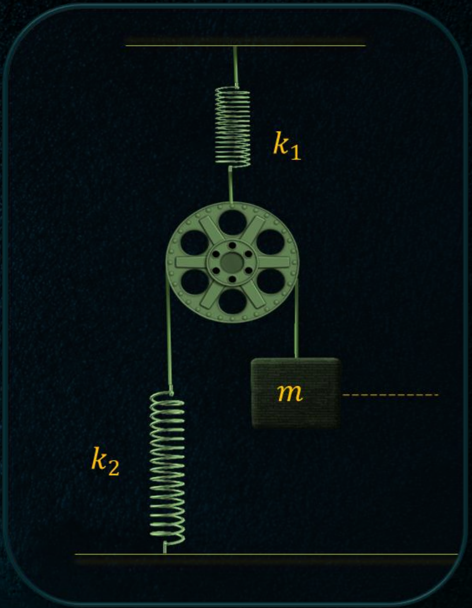
$$\frac{1}{k_{eff}} = \frac{1}{2k} + \frac{1}{2k} = \frac{1}{k} \qquad \left[\frac{1}{k_s} = \sum \frac{1}{k}\right]$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$



Find the time period of small oscillations of mass m about equilibrium position for a given spring mass system as shown in the figure. Neglect friction and masses of springs.





Solution: Let F be the extra tension in the string.

When the block is displaced by x from the mean position,

extension in spring−2, $x_2 = \dfrac{F}{k_2}$

extension in spring−1, $x_1 = \dfrac{2F}{k_1}$

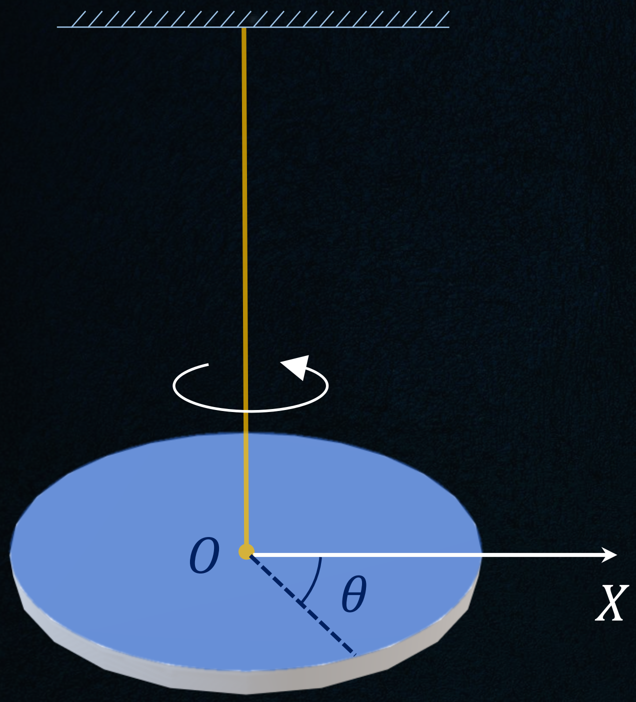
Total extension $x = 2x_1 + x_2 = \dfrac{4F}{k_1} + \dfrac{F}{k_2}$

Extra tension F will be the restoring force on the block.

$$F = -\left[\frac{1}{\frac{4}{k_1} + \frac{1}{k_2}}\right]x = -\left[\frac{k_1 k_2}{4k_2 + k_1}\right]x \Rightarrow k_{eff} = \frac{k_1 k_2}{4k_2 + k_1}$$

Time period of oscillation, $T = 2\pi\sqrt{\dfrac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\dfrac{m(4k_2 + k_1)}{k_1 k_2}}}$

Torsional Pendulum



- An extended body suspended by a light string, rotated by a small angle θ with thread as the axis of rotation.

$$\tau = -C\theta = I\alpha$$

$\tau \rightarrow$ Restoring torque generated by the thread of the pendulum.

$C \rightarrow$ Torsional constant/ Twisting coefficient.

$$\alpha = -\frac{C}{I}\theta \Rightarrow \omega = \sqrt{\frac{C}{I}}$$

- The extended body executes torsional oscillations with time period,

$$\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

A uniform rod of length l and mass m is pivoted at the center. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid supports as shown in the figure and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillations is

Solution:

For small angular displacement θ,

Displacement of spring = arc length

$$x = \frac{l}{2}\theta$$

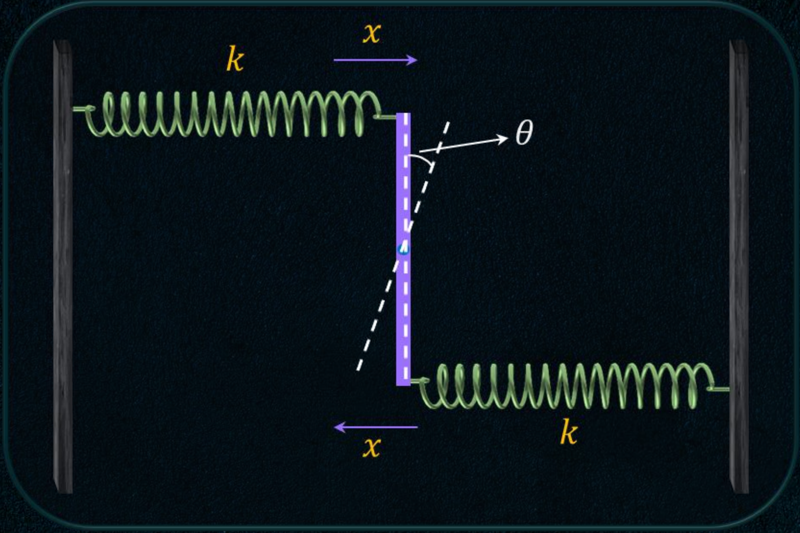
Force in each spring,

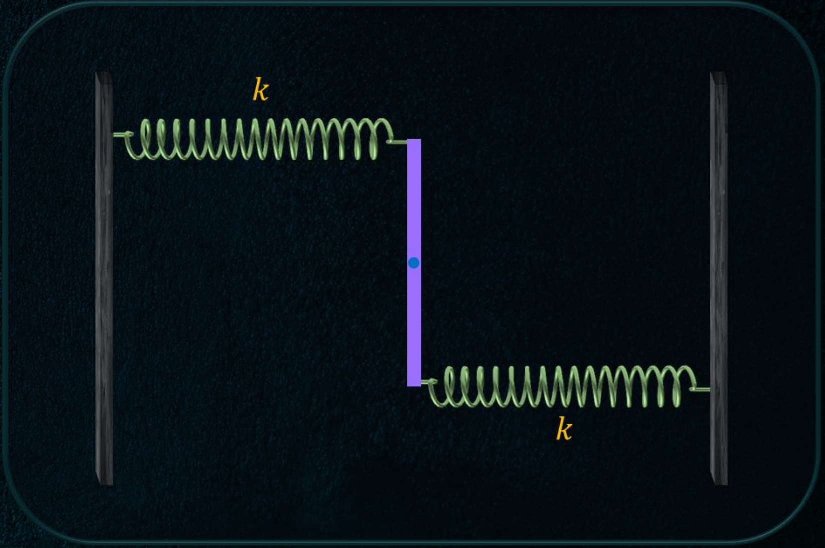
$$F = -k\frac{l}{2}\theta$$

Restoring torque $= I\alpha$

$$\Rightarrow -2 \times k\frac{l}{2}\theta \times \frac{l}{2} = I\alpha$$

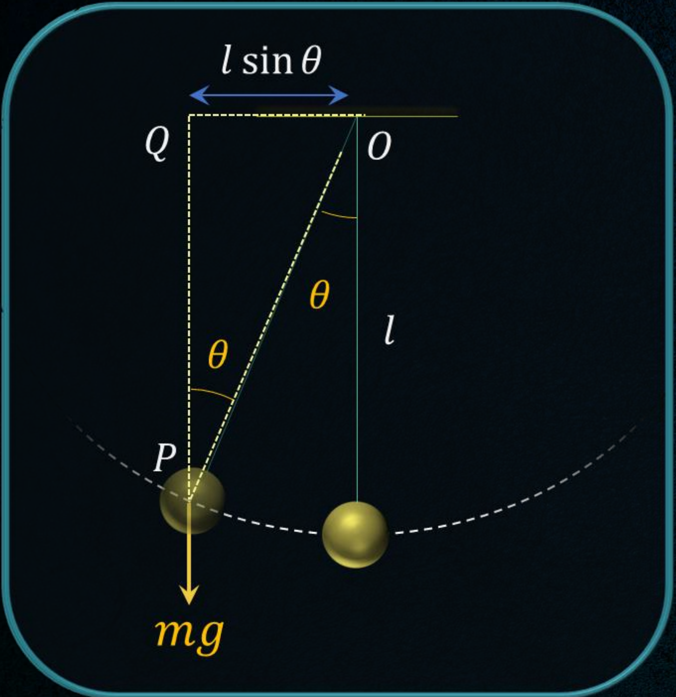
$$\Rightarrow \alpha = -\frac{\frac{kl^2}{2}}{\frac{ml^2}{12}}\theta \Rightarrow \quad \alpha = -\frac{6k}{m}\theta$$





Frequency, $f = \frac{\omega}{2\pi} \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}}$

Simple Pendulum



Net torque on point mass about point 0,

$$(\tau_{net})_o = mglsin\theta = I_o\alpha$$

$$\Rightarrow mglsin\theta = ml^2\alpha$$

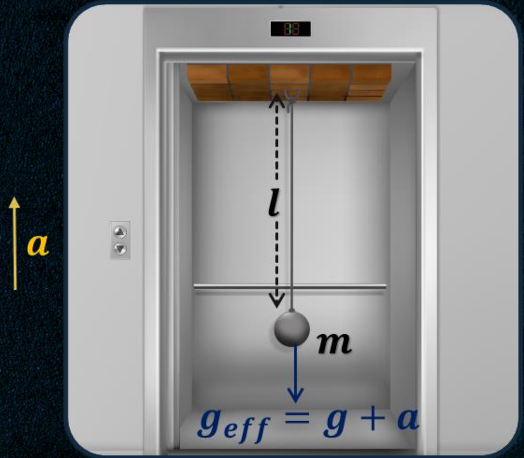
$$\Rightarrow \alpha = \left(\frac{g}{l}\right) sin\theta$$

For small oscillations, $sin\theta \approx \theta$

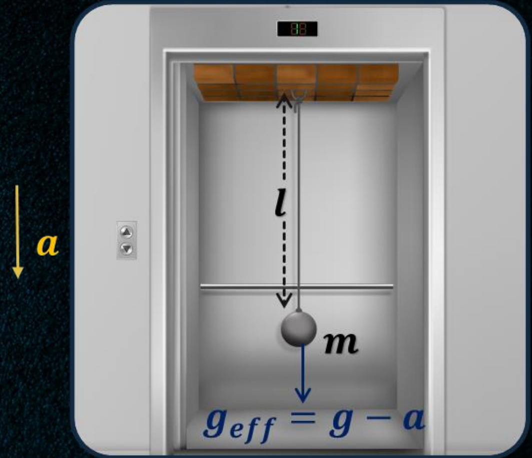
$$\alpha \approx \left(\frac{g}{l}\right)\theta$$

Hence ,Pendulum executes SHM with time period,

$$T = 2\pi\sqrt{\frac{l}{g}}$$



$$T = 2\pi\sqrt{\frac{l}{g+a}}$$



$$T = 2\pi\sqrt{\frac{l}{g-a}}$$

Note- From ground the motion of the pendulum is neither oscillatory nor periodic.

A pendulum is suspended from the ceiling of a truck accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of pendulum is l, find the time period of small oscillations about the mean position.

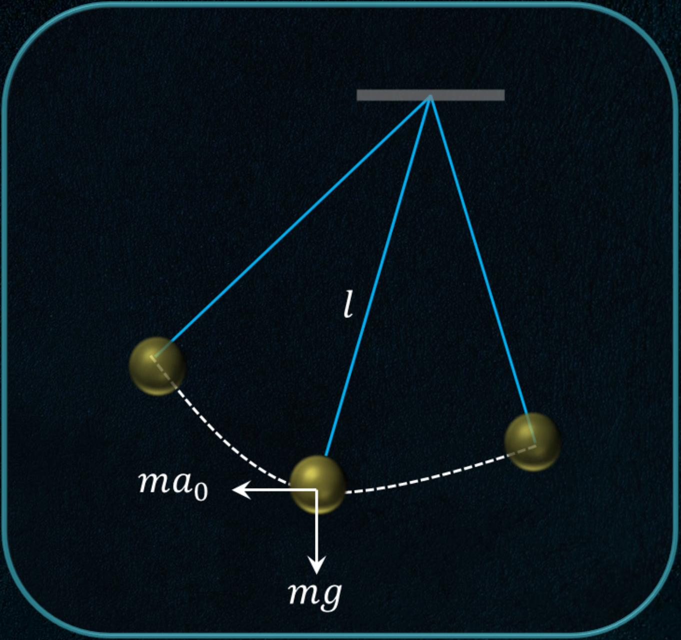
Solution:

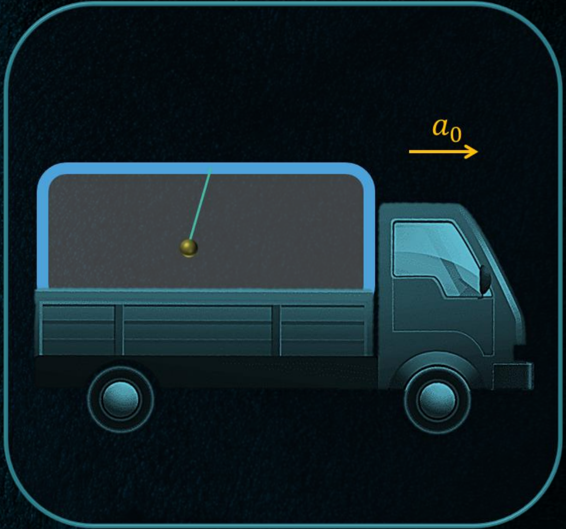
Effective acceleration of bob,

$$\Rightarrow g_{eff} = \sqrt{g^2 + a_o^2 - 2ga_o cos90^\circ} = \sqrt{g^2 + a_o^2}$$

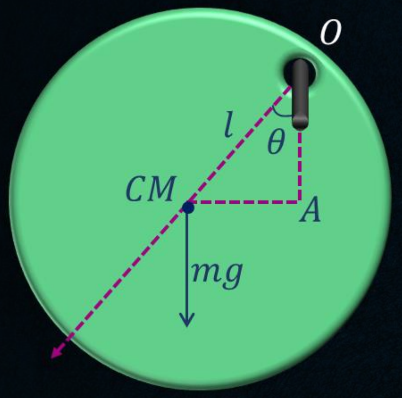
Time period of pendulum will be

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a_0^2}}}}$$





Compound Pendulum



Net torque on point mass about point O,

$$(\tau_{net})_o = mglsin\theta = I_h\alpha$$

$I_h \rightarrow$ Moment of Inertia of the body about hinge axis.

$l \rightarrow$ Distance between point of suspension and CM.

For small oscillations, $sin\theta \approx \theta$

$$\alpha = \left(\frac{mgl}{I_h}\right)\theta = \omega^2\theta$$

Hence , the time period, $T = 2\pi\sqrt{\frac{I_h}{mgl}}$

- Moment of Inertia about hinged point,

$$I_h = I_{CM} + ml^2 \qquad \text{(parallel axis theorem)}$$

$$I_{CM} = mk^2 \qquad (k - \text{radius of gyration about CM})$$

- Time period,

$$\Rightarrow T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}} = 2\pi\sqrt{\frac{\frac{k^2 + l^2}{l}}{g}}$$

- Time period of compound pendulum is same as that of a simple pendulum of length l_{eq}.

$$\frac{k^2 + l^2}{l} = l_{eq}$$

A uniform rod of length l is suspended by an end and is made to undergo small oscillations. Find the length of the simple pendulum having the time period equal to that of the rod as described.

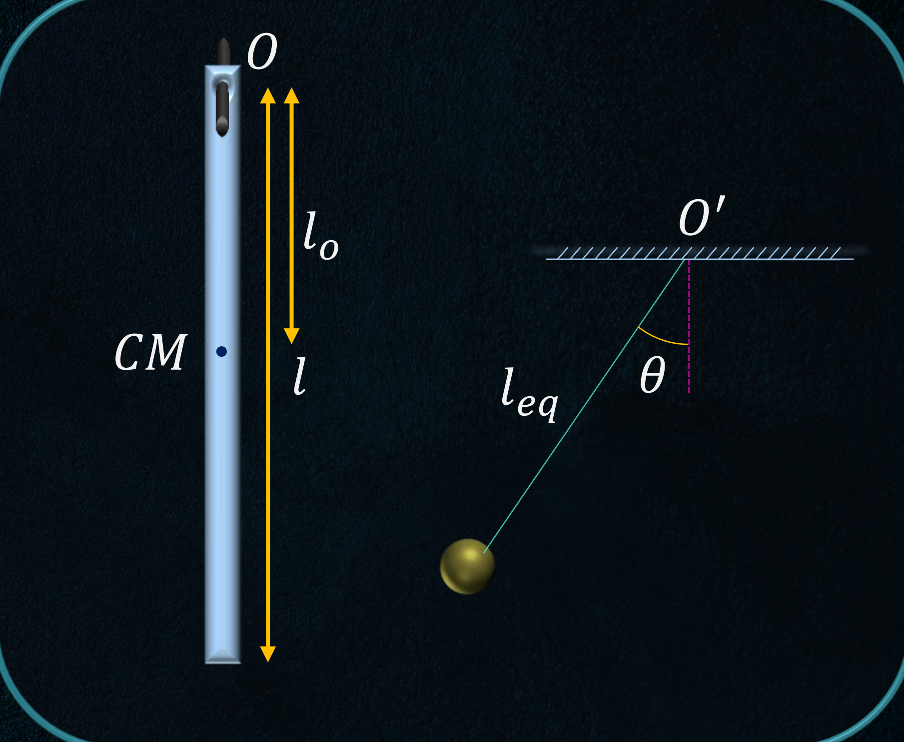
Given: Length of the rod $= l$

To find: l_{eq}

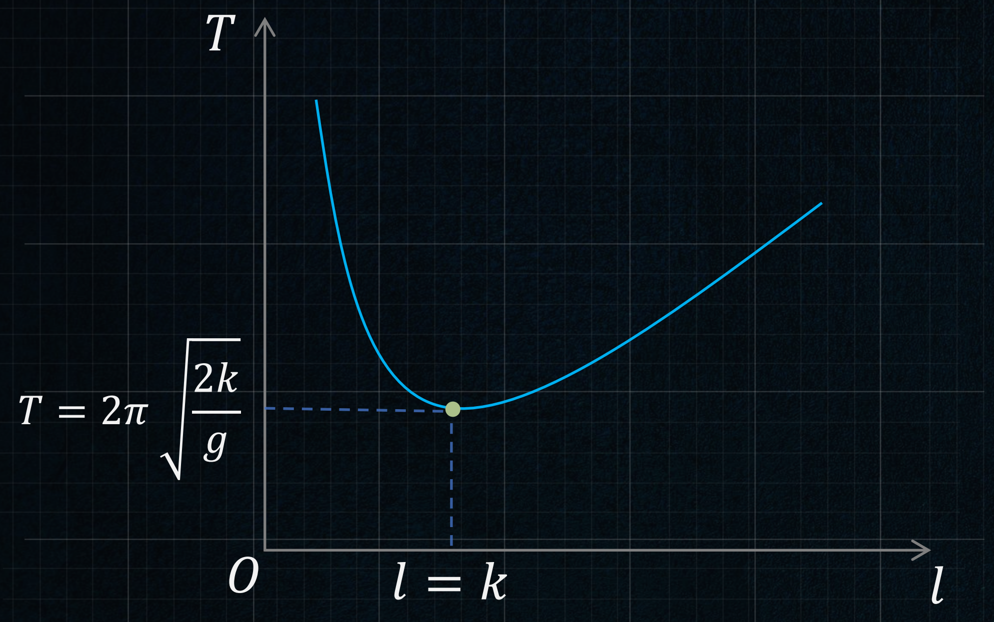
Solution: $$T = 2\pi\sqrt{\frac{I}{mgl_0}} = 2\pi\sqrt{\frac{l_{eq}}{g}} \Rightarrow l_{eq} = \frac{I}{ml_0}$$

$$\Rightarrow l_{eq} = \frac{\frac{ml^2}{3}}{\frac{ml}{2}}$$

$$\boxed{l_{eq} = \frac{2l}{3}}$$



Variation of Time Period with Length



- Time Period of a compound pendulum

$$T = 2\pi\sqrt{\frac{\frac{k^2 + l^2}{l}}{g}}$$

- The time period is minimum at $l = k$ and the minimum value of the time period is,

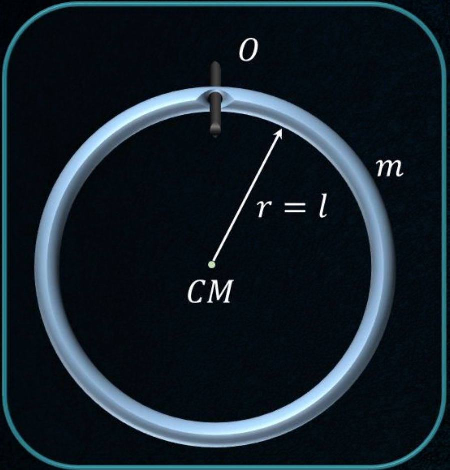
$$T_{min} = 2\pi\sqrt{\frac{2k}{g}}$$

Find the time period of small oscillations of the following systems.

a. A thin ring of mass m and radius r suspended through a point on its periphery.

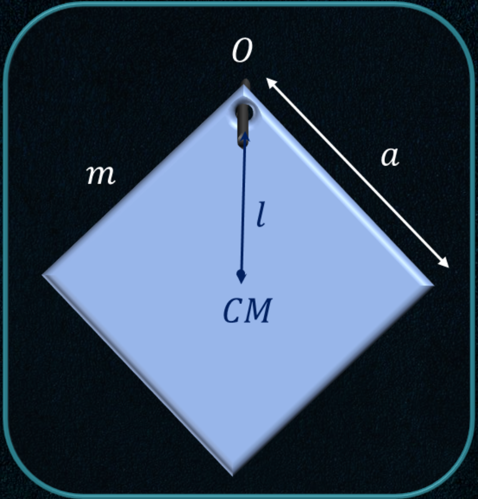
b. A uniform square plate of edge a suspended through a corner.

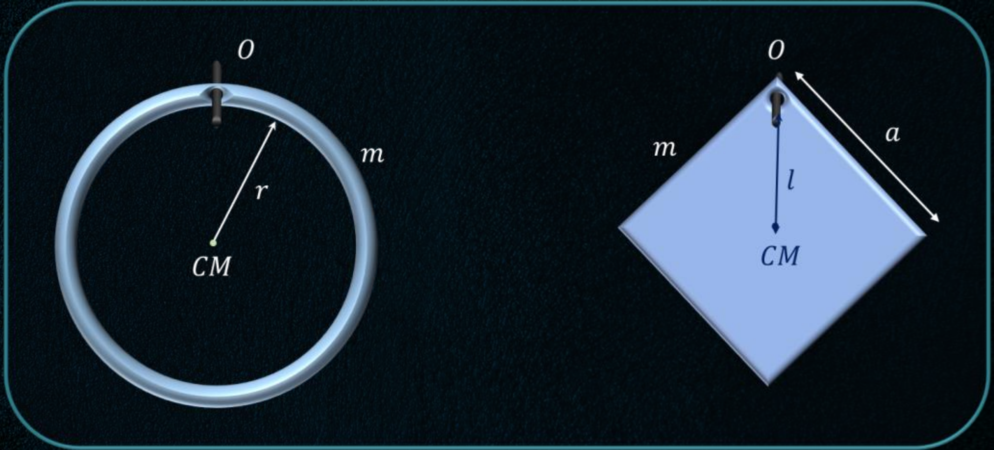
Solution:



$$T = 2\pi\sqrt{\frac{I_h}{mgl}}$$

$$I_h = I_{CM} + ml^2$$





$$I_h = mr^2 + ml^2 = 2mr^2$$

$$T = 2\pi\sqrt{\frac{2r}{g}}$$

$$I_h = \frac{ma^2}{6} + ml^2 = \frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2$$

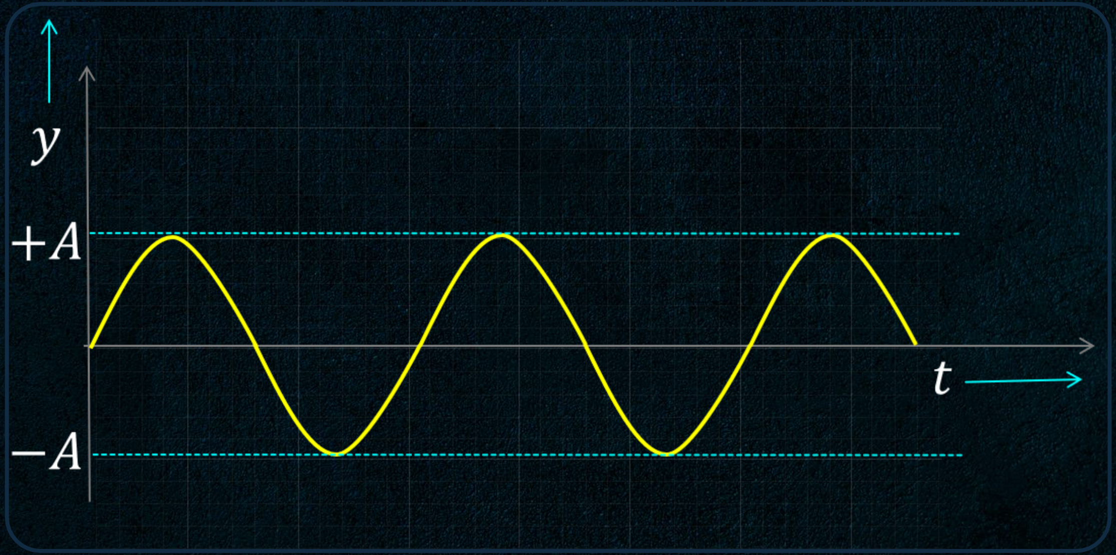
$$T = 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$

Types of Oscillations

- Free Oscillation
- Damped Oscillation
- Forced Oscillation

Free Oscillation

- The oscillation of a particle with fundamental/natural frequency under the influence of restoring force are defined as free Oscillation.
- The amplitude, frequency and energy of oscillation remains constant.



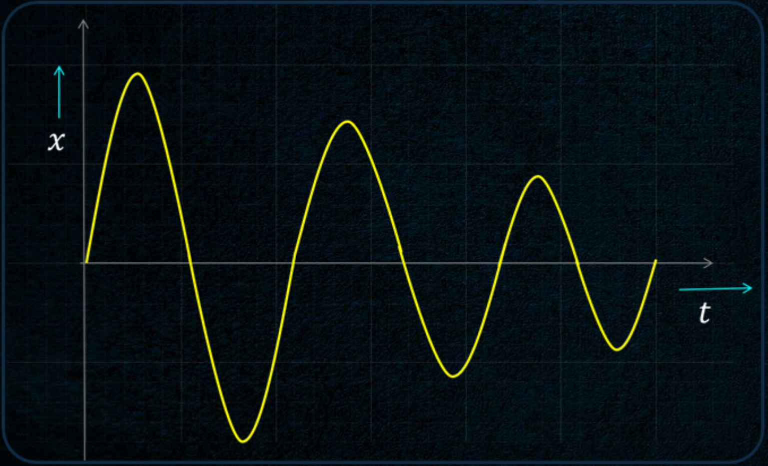
Displacement vs Time graph

Damped Oscillation

- The Oscillation of a body whose amplitude goes on decreasing with time is known as damped oscillation.
- The amplitude of oscillation decreases exponentially due to damping forces like frictional force, air resistance etc.
- The damping force is proportional to the velocity of the oscillator & acts opposite to the direction of velocity.

$\overrightarrow{F_d} = -b\vec{v}$

where, b = damping constant.



Displacement vs Time graph

Damping Force, $F_d = -bv$

Restoring Force, $F_R = -kx$

$$\left.\begin{matrix} F_d = -bv \\ F_R = -kx \end{matrix}\right\} \text{Resultant Force } \vec{F} = -b\vec{v} - k\vec{x}$$

Equation of motion, $\boxed{x = A_0 e^{-bt/2m} \cos(\omega' t + \phi)}$

Angular frequency of the damped oscillator-

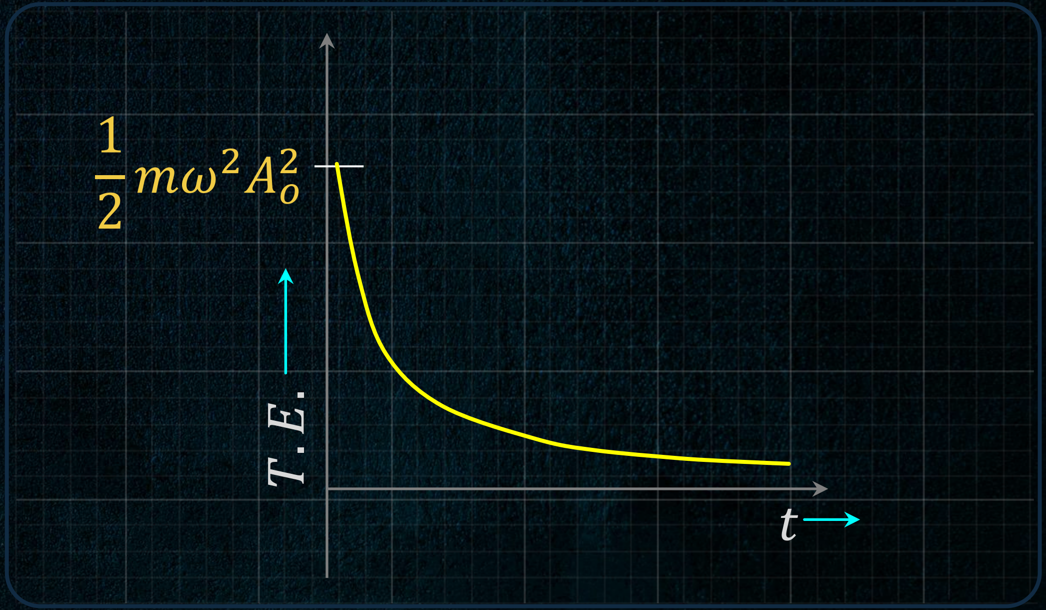
$$\omega' = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2} \quad \text{where, } \omega = \sqrt{\frac{k}{m}} \text{ – Natural frequency}$$

Amplitude of the damped oscillator- $A = A_0 e^{-bt/2m}$

Expression for Total Energy

- For an undamped oscillator - $T.E. = \frac{1}{2}m\omega^2 A_o^2$

- For a damped oscillator - $T.E. = \frac{1}{2}m\omega^2 A_o^2 e^{-bt/m}$



When an oscillator completes 100 oscillations its amplitude is reduced to $\left(\frac{1}{3}\right)^{rd}$ of its initial value. What will be its amplitude when it completes 200 oscillations?

Given: Amplitude after 100 oscillations $= A_1 = \frac{A_0}{3}$

Let the Time period of oscillation be T

To find: Amplitude after 200 oscillations

A) $\left(\frac{1}{6}\right)^{th}$

B) $\left(\frac{1}{9}\right)^{th}$

C) $\left(\frac{2}{3}\right)^{rd}$

D) None of these

Solution:

In the first case,

$$A_1 = \frac{A_0}{3} \quad \text{and } t_1 = 100T$$

$$\Rightarrow \frac{A_0}{3} = A_0 e^{-\frac{b(100T)}{2m}}$$

$$\Rightarrow e^{-\frac{b(100T)}{2m}} = \frac{1}{3}$$

In the second case, $t_2 = 200T$

$$A_2 = A_0 e^{-\frac{bt_2}{2m}} \qquad \Rightarrow A_2 = A_0 e^{-\frac{b(200T)}{2m}}$$

$$\Rightarrow A_2 = A_0 \left(e^{\frac{-b(100T)}{2m}}\right)^2 \qquad \Rightarrow A_2 = A_0 \left(\frac{1}{3}\right)^2$$

$$\boxed{\Rightarrow A_2 = \frac{A_0}{9}}$$

Forced Oscillation

- The oscillation in which a body oscillates under the influence of an external periodic force with an angular frequency ω_d is known as forced oscillation.

Damping Force, $\overrightarrow{F_d} = -b\vec{v}$

Restoring Force, $\overrightarrow{F_R} = -k\vec{x}$

Periodic Force, $F(t) = F_0 \cos \omega_d t$

Resultant Force, $\vec{F} = -b\vec{v} - k\vec{x} + \vec{F}(t)$

$$\downarrow$$

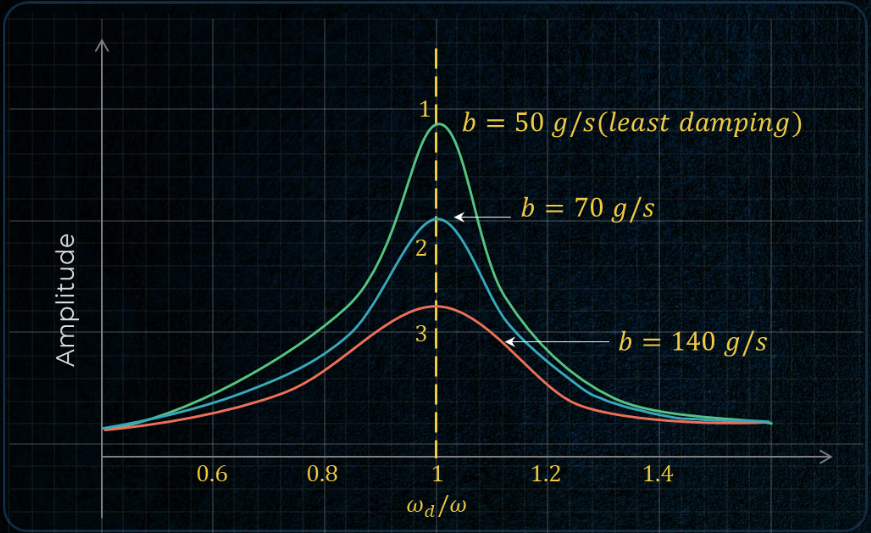
$$x = A_0 \cos(\omega_d t + \phi)$$

Amplitude of the forced oscillator-

$$A_0 = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}}$$

Cases in Forced Oscillation

Case I: Small damping, driving frequency far from natural frequency.



$$\omega_d b \ll m(\omega^2 - \omega_d^2)$$

$$A_0 = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}}$$

$$\Rightarrow A_0 = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$

For smaller damping, the resonance peak is taller and narrower.

Case II: Driving frequency close to natural frequency (Resonance).

- When the frequency of external force is nearly equal to the natural frequency of the oscillator. Then this state is known as the state of resonance and this frequency is known as resonant frequency.

$i.e.\ \omega_d \approx \omega$

$$A_0 = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}}$$

$$\Rightarrow A_0 = \frac{F_0}{\omega_d b}$$ Maximum Amplitude

- At resonance, a forced oscillator oscillates with the maximum amplitude.

Calculate the period of small oscillations of a floating box which was slightly pushed down in vertical direction. The mass of box is m, area of its base is A and the density of liquid is ρ. The resistance of the liquid is assumed to be negligible.

Given: Density of liquid $= \rho$, mass of box $= m$
& area of base of the block $= A$

To find: Time period of oscillation, T

Solution: The net force acting to push the box down-

$$F_{net} = -\Delta B \Rightarrow ma = -Ax\rho g$$ (x – distance moved due to the force)

$$a = -\frac{A\rho g}{m}x$$

∴Time Period, $\boxed{T = 2\pi\sqrt{\frac{m}{A\rho g}}}$