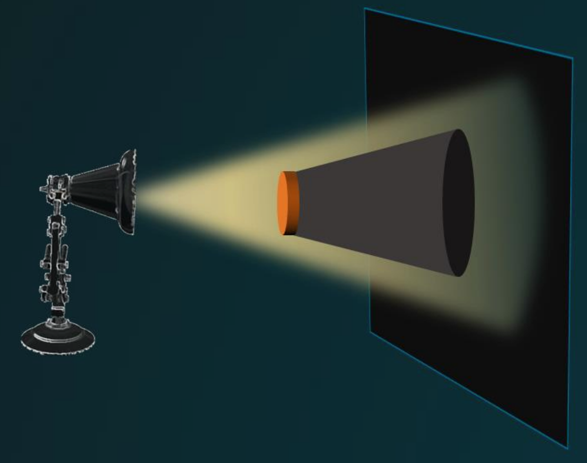
Welcome to

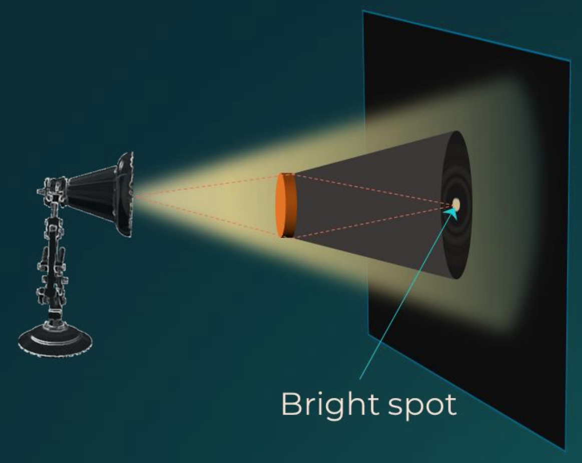
Aakash + BYJU'S NOTES

Wave optics

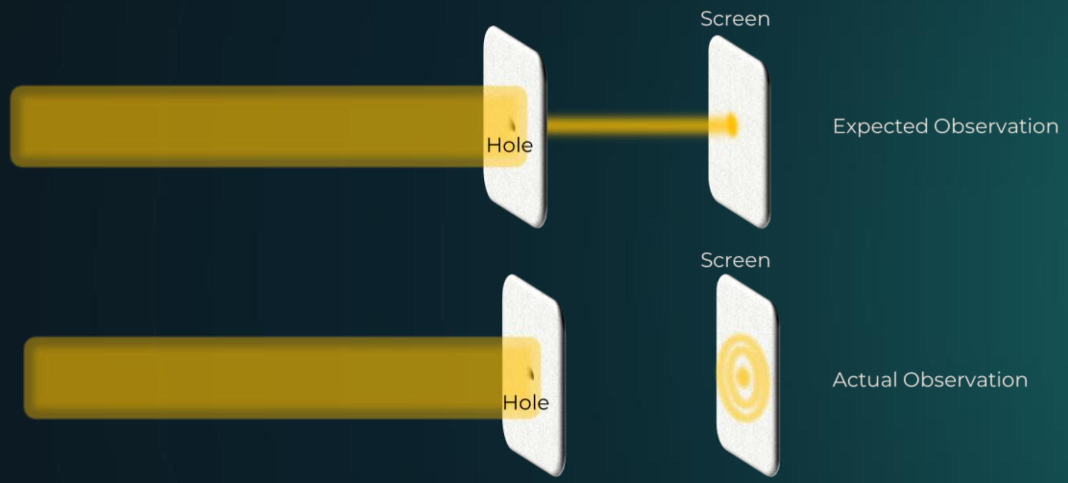
Does Light always exhibit Rectilinear Propagation?



Expected Observation



Actual Observation



Huygens' Principle

Huygens

Wave theory of light

Explanation for bending of light

↓

Wave theory of light

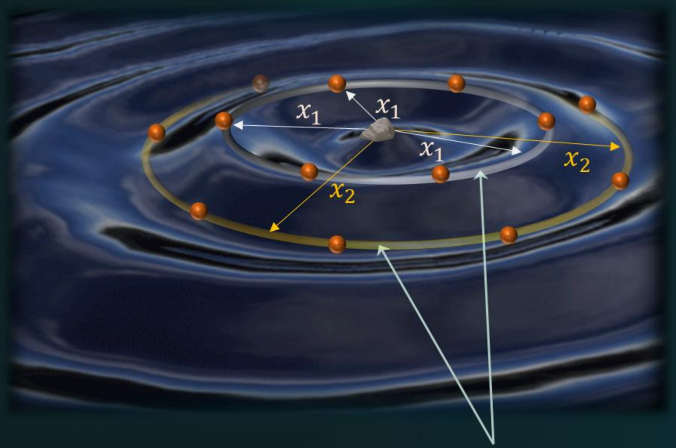
↓

Christiaan Huygens

↓

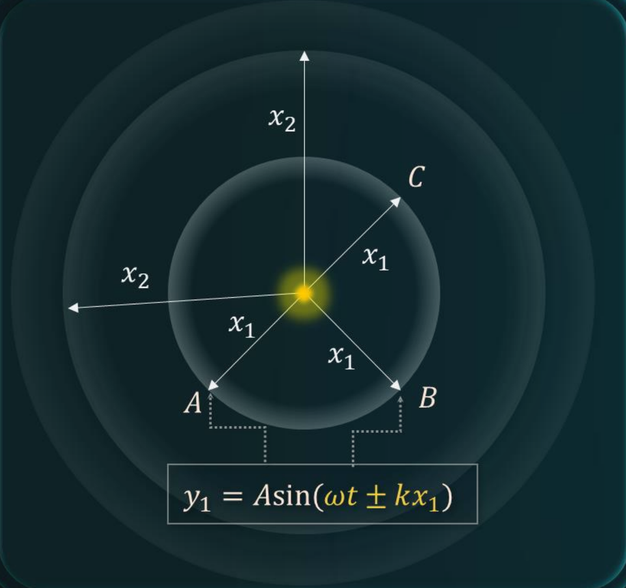
Wavefront

Wavefront



Circular Wavefront

- All points on a particular circle will oscillate with the same phase.
- Wavefront is the locus of all points at which the wave disturbance is in the same phase.

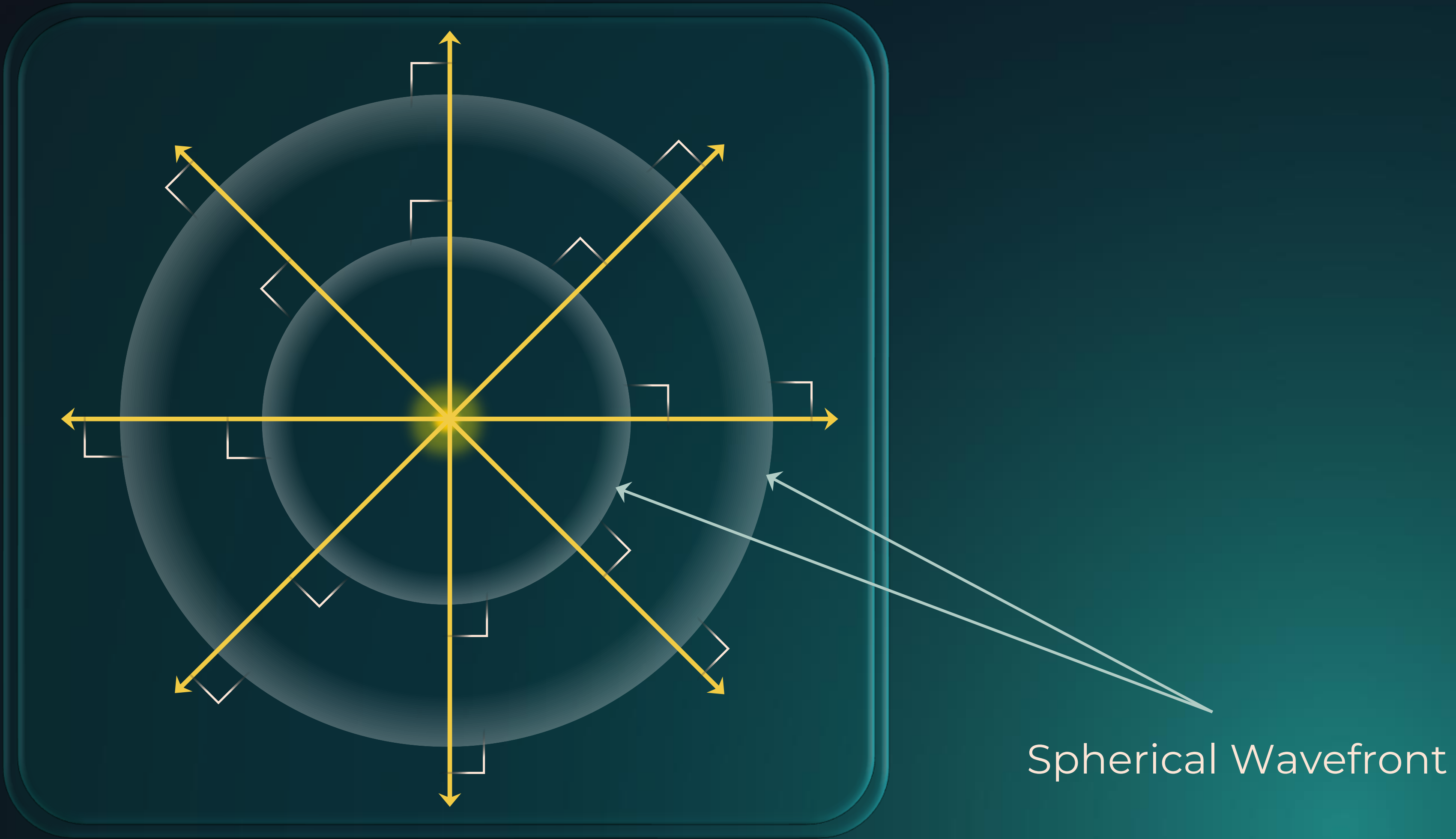


- Point source creates wave traveling in all directions according to equation.

$$y = A\sin(\omega t \pm kx)$$

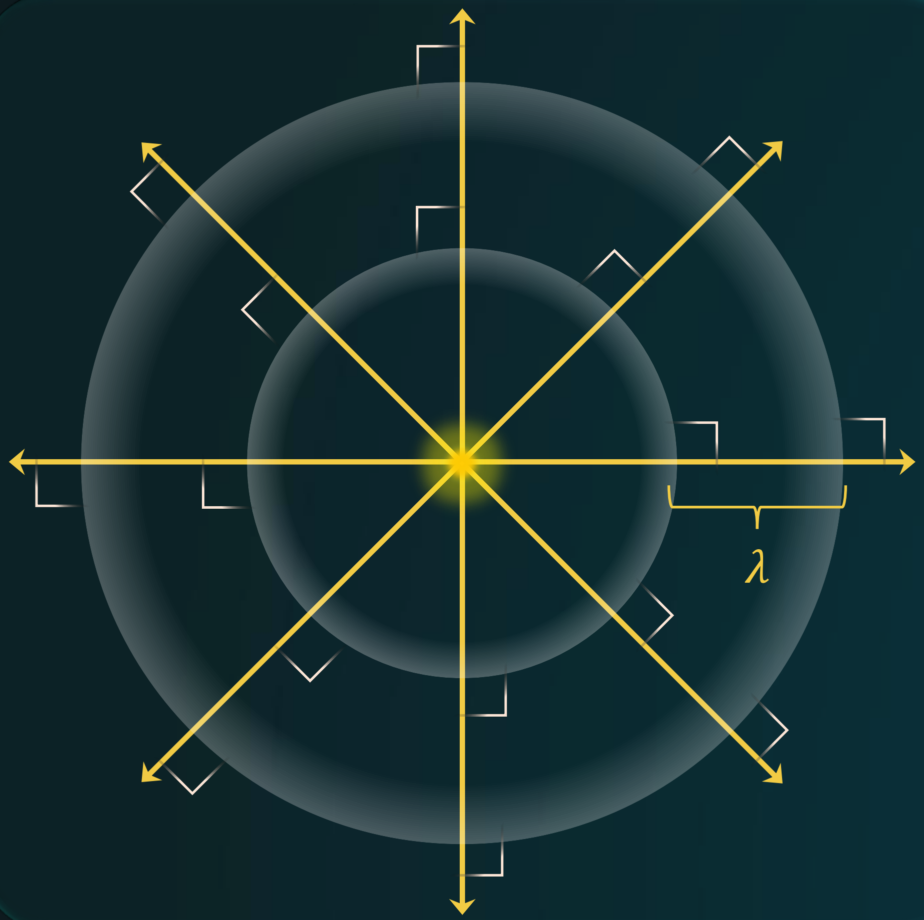
- Points A, B, C have same phase $\omega t \pm kx_1$.
- In $3D$, for a point source, the wavefronts are spherically symmetric.

Direction of Propagation of Light



The direction of propagation of light wave is perpendicular to the wavefront at any given point.

Properties of Wavefront

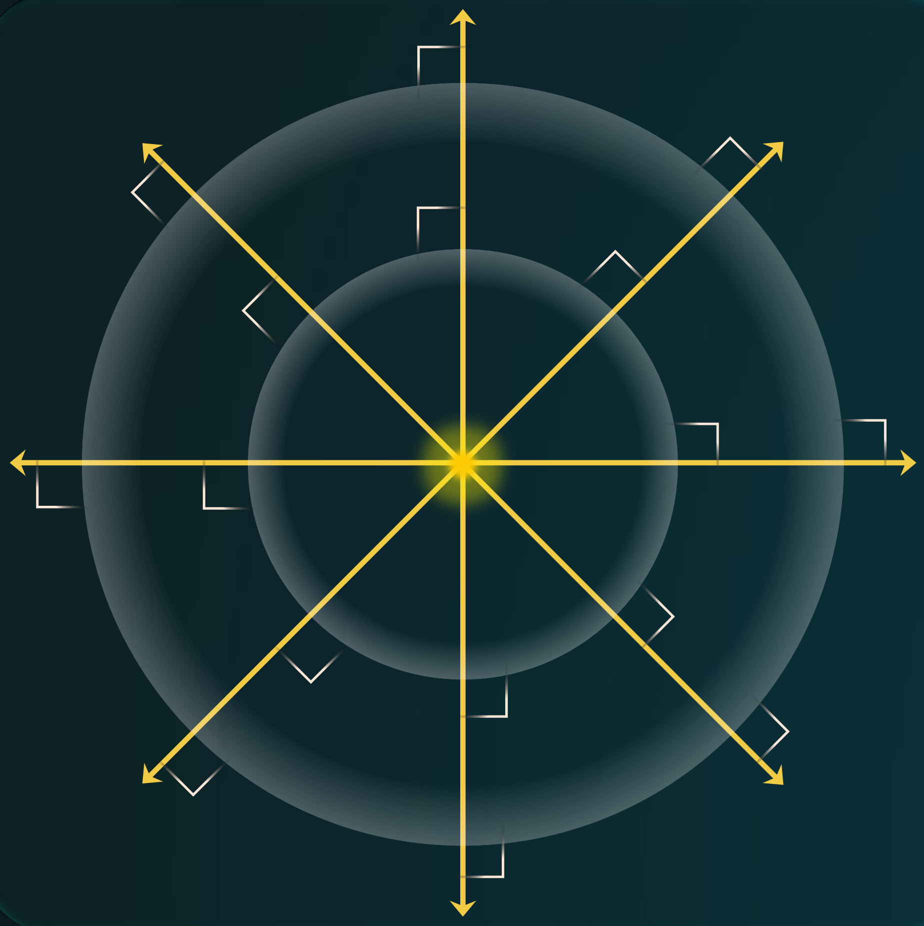


- Time taken by each ray to propagate from one wavefront to the next wavefront in any medium remains same.
- Refractive index, $n = \frac{c}{v}$

  When medium changes, $\frac{n_1}{n_2} = \frac{v_2}{v_1}$

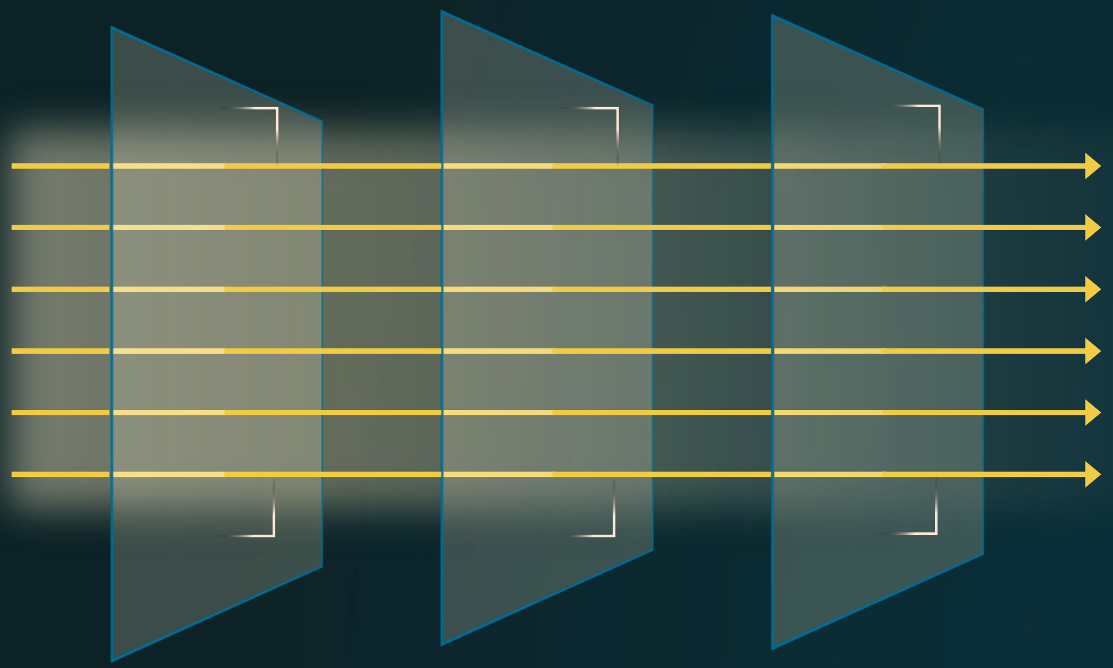
  $\Rightarrow$ Speed of light changes.
- Hence, distance covered by the light in each time interval also changes.
- Distance between two consecutive wavefronts $= \lambda$.

Spherical Wavefront



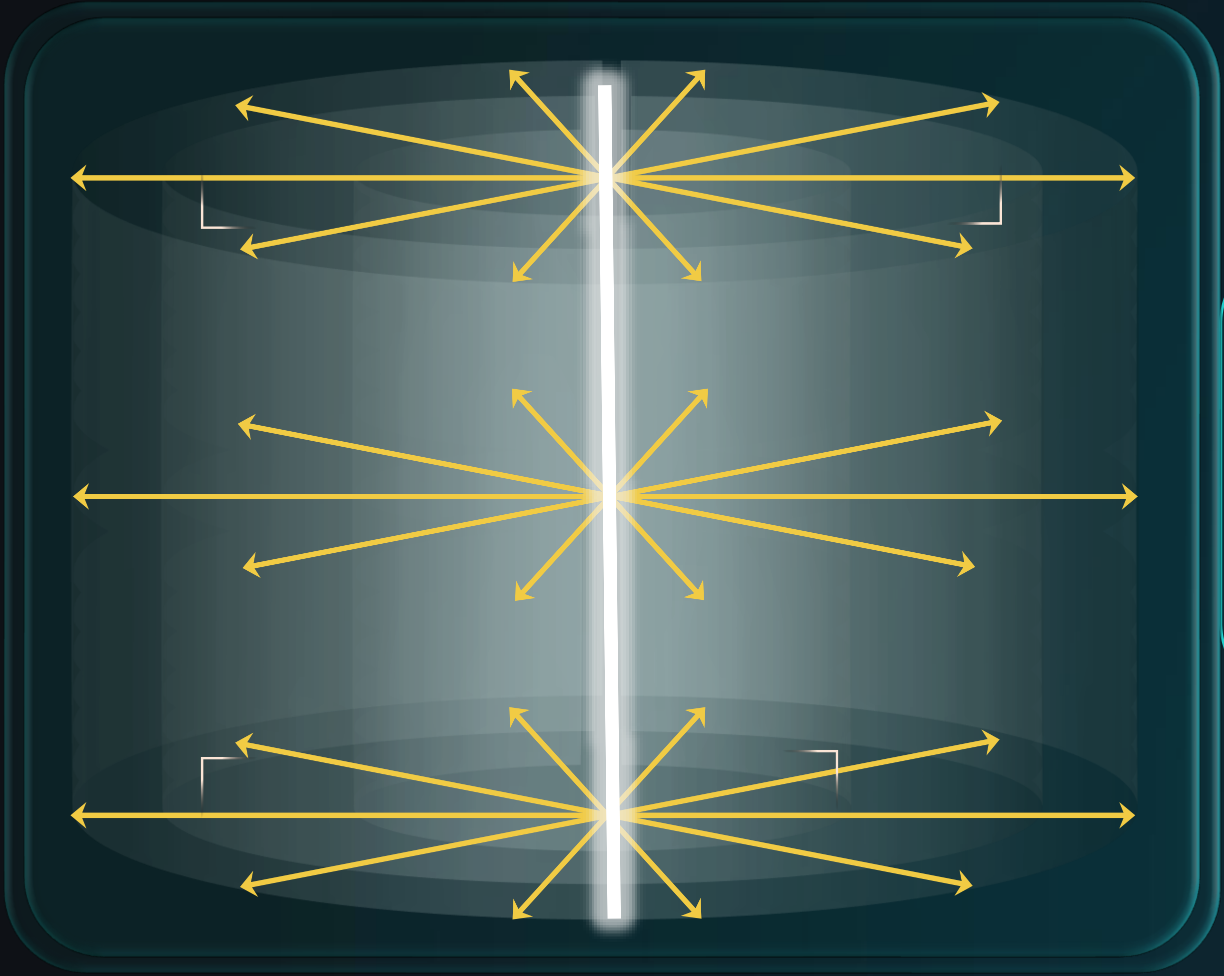
- Spherical wavefront is observed for a point source.
- Direction of propagation of light: Radially outward and perpendicular to the wavefront at any given point.

Planar Wavefront



- Planar wavefronts are observed when the source is at infinity.
- Direction of propagation of light: Perpendicular to the plane.

Cylindrical Wavefront



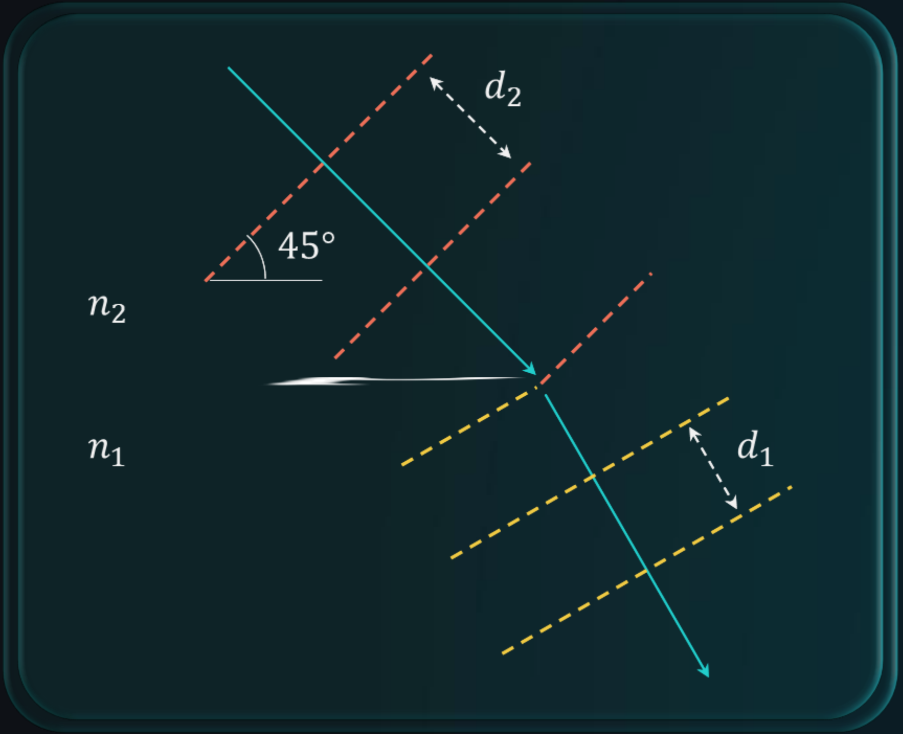
- Cylindrical wavefront is observed for a line source.
- Direction of propagation of light: Radially outward and perpendicular to the wavefront at any given point.

If the distance between the wavefronts in the medium with refractive index n_2 is d_2, then what will be the distance d_1 between the wavefronts in the medium with refractive index n_1?

We known that,

$$n = \frac{c}{v} \quad \Rightarrow \quad {}^{n_2}/_{n_1} = {}^{v_1}/_{v_2}$$

$$\lambda = {}^{v}/_{f} \quad \Rightarrow \quad {}^{\lambda_1}/_{\lambda_2} = {}^{v_1}/_{v_2} = {}^{n_2}/_{n_1}$$



Here, $\lambda_1 = d_1$, and $\lambda_2 = d_2$

Hence,

$$\frac{d_1}{d_2} = \frac{n_2}{n_1}$$

$$\boxed{\Rightarrow d_1 = d_2 \frac{n_2}{n_1}}$$

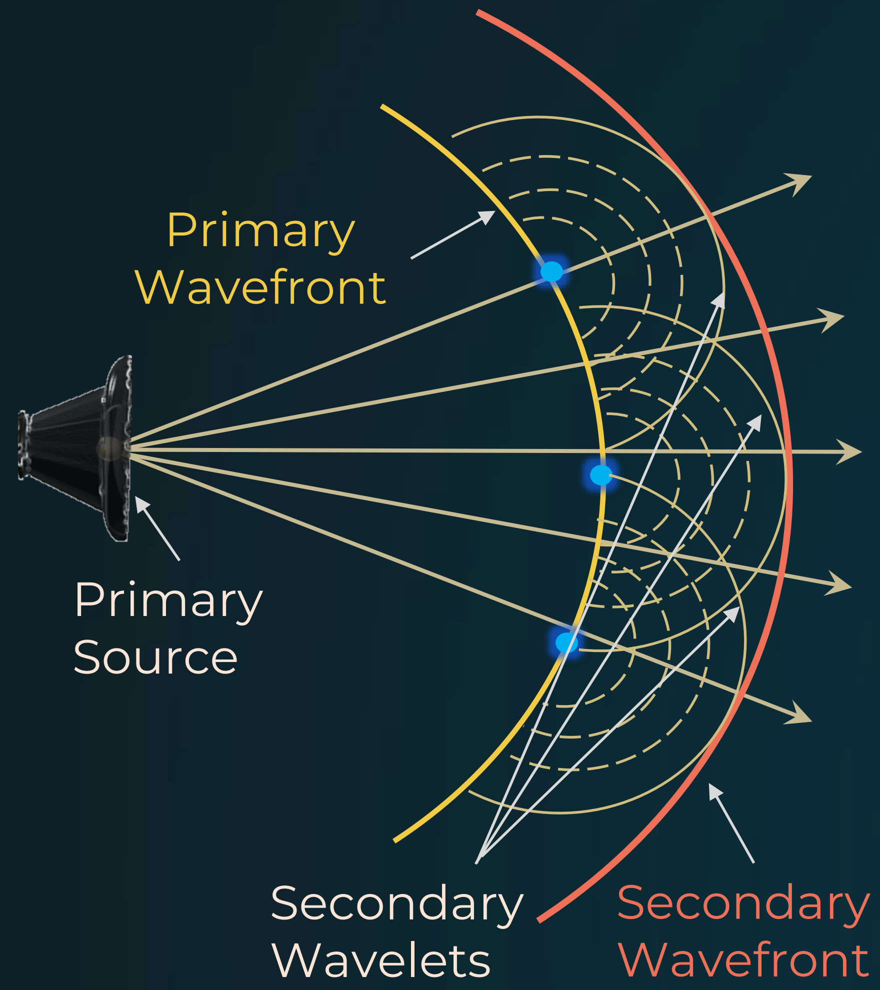
A. $d_1 = d_2 \frac{n_1}{n_2}$

B. $d_1 = d_2 \frac{n_2}{n_1}$

C. $d_1 = d_2$

D. $d_1 = \frac{n_1}{d_2 n_2}$

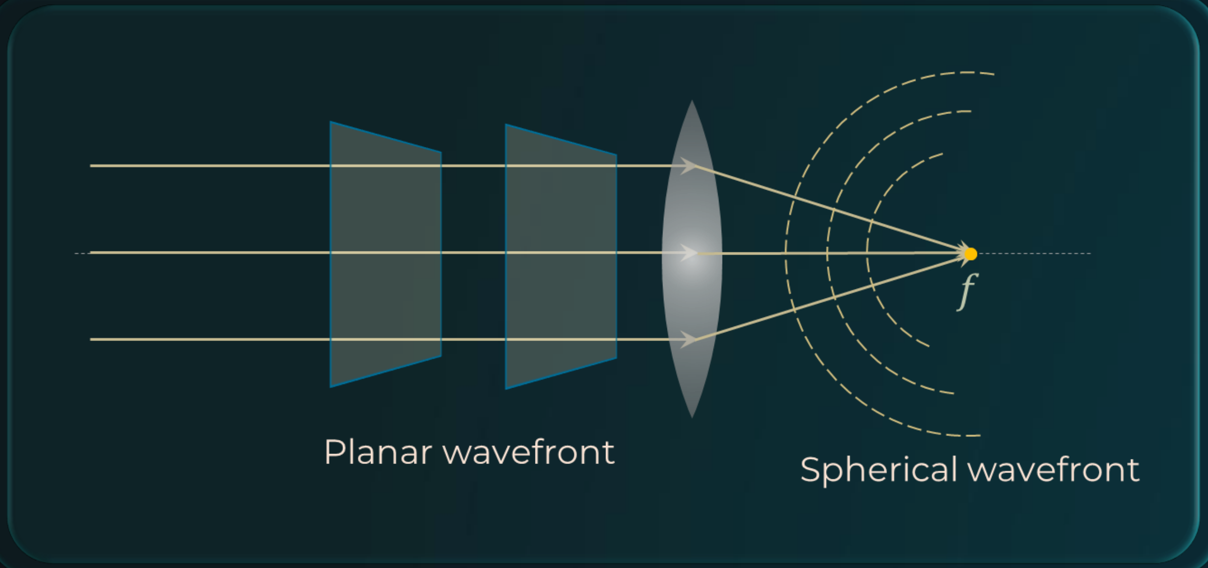
Huygens’ Principle



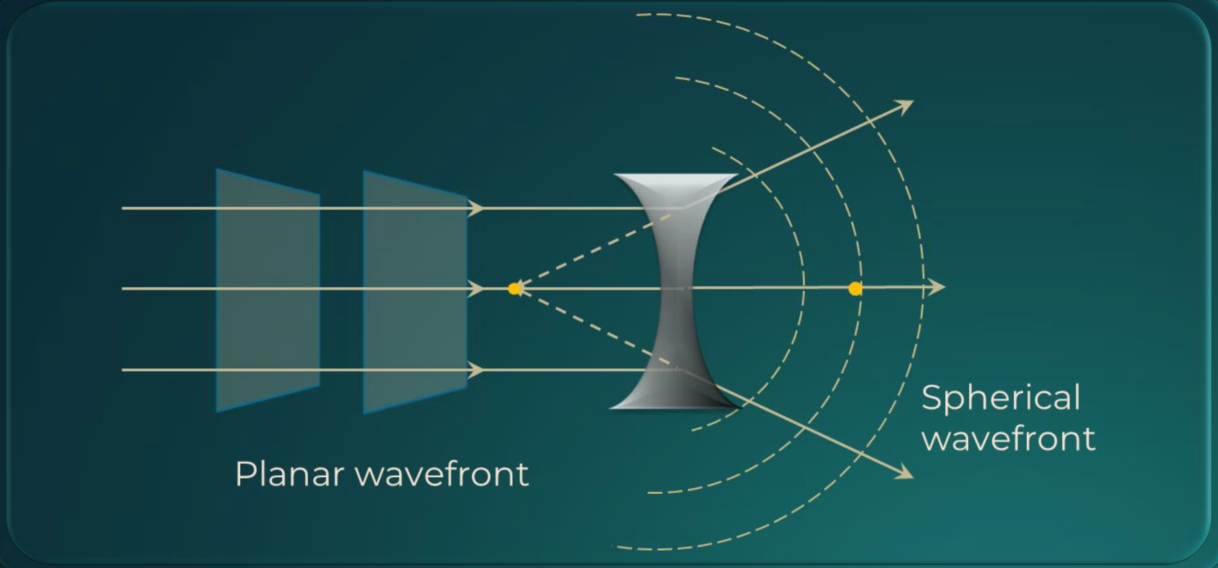
- Every point on the wavefront acts as a point source called secondary wave source and generates secondary wavelets.
- The common tangent to the secondary wavelets in the forward direction gives the secondary wavefront.
- Intensity is maximum in forward direction and zero in backward direction.

Shape of Wavefront

Object at infinity for a convex lens

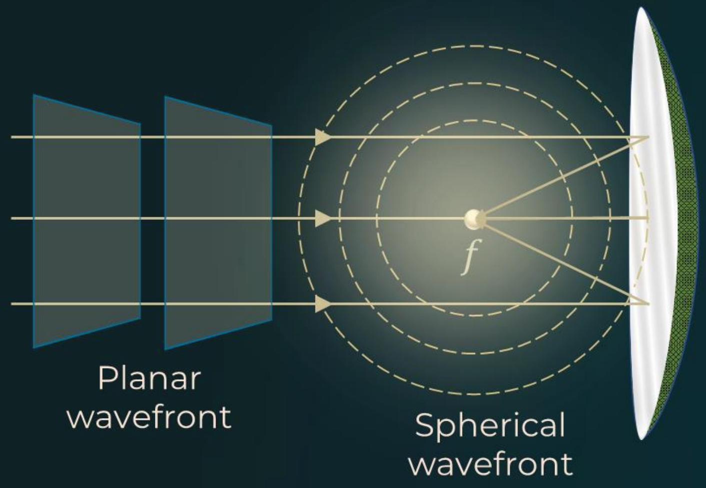


Object at infinity for a concave lens

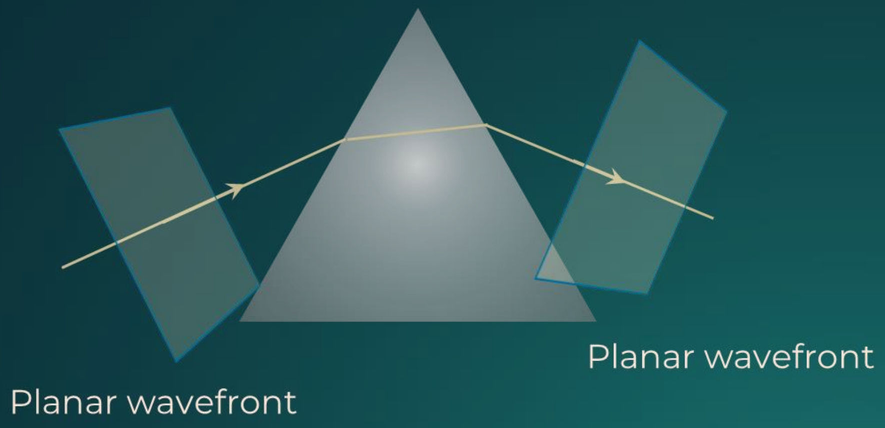


Shape of Wavefront

Object at infinity for a concave mirror

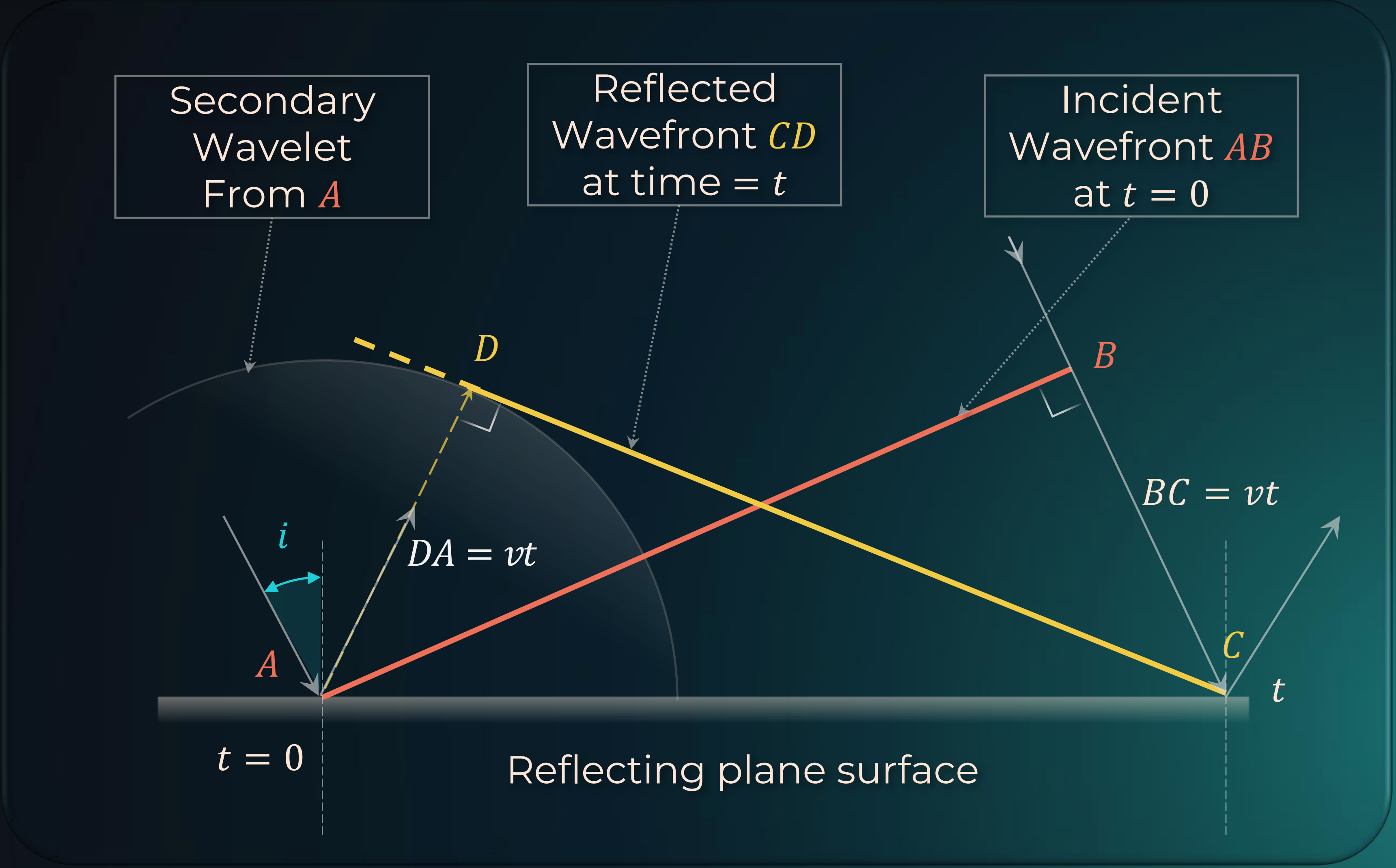


Object at infinity for a prism



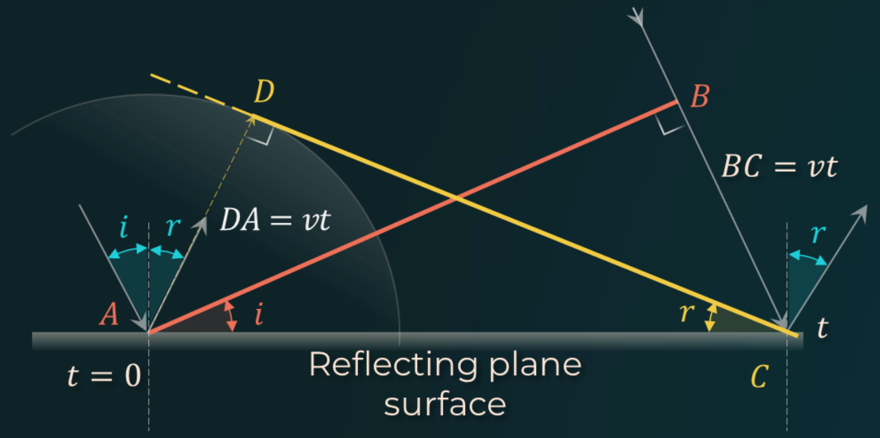
Huygens' Principle - Law of Reflection

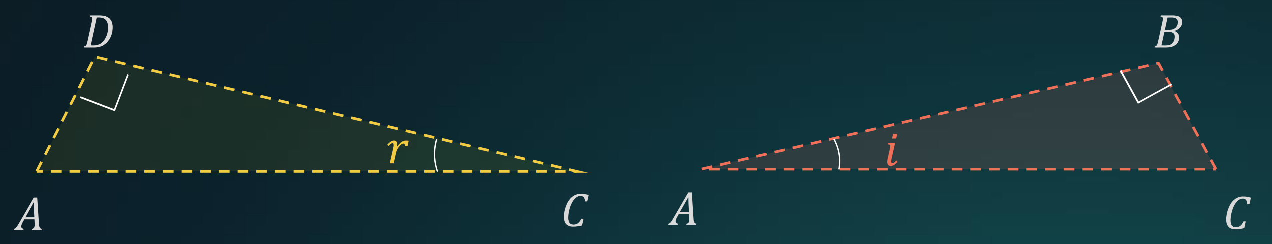
Reflection of a plane wave by a plane surface



Huygens' Principle - Law of Reflection

Reflection of a plane wave by a plane surface





- Angle of incidence is the angle between the incident wavefront and the reflecting surface.

$$\angle BAC = i \rightarrow \text{Angle of incidence}$$

- Angle of reflection is the angle between the reflected wavefront and the reflecting surface.

$$\angle DCA = r \rightarrow \text{Angle of reflection}$$

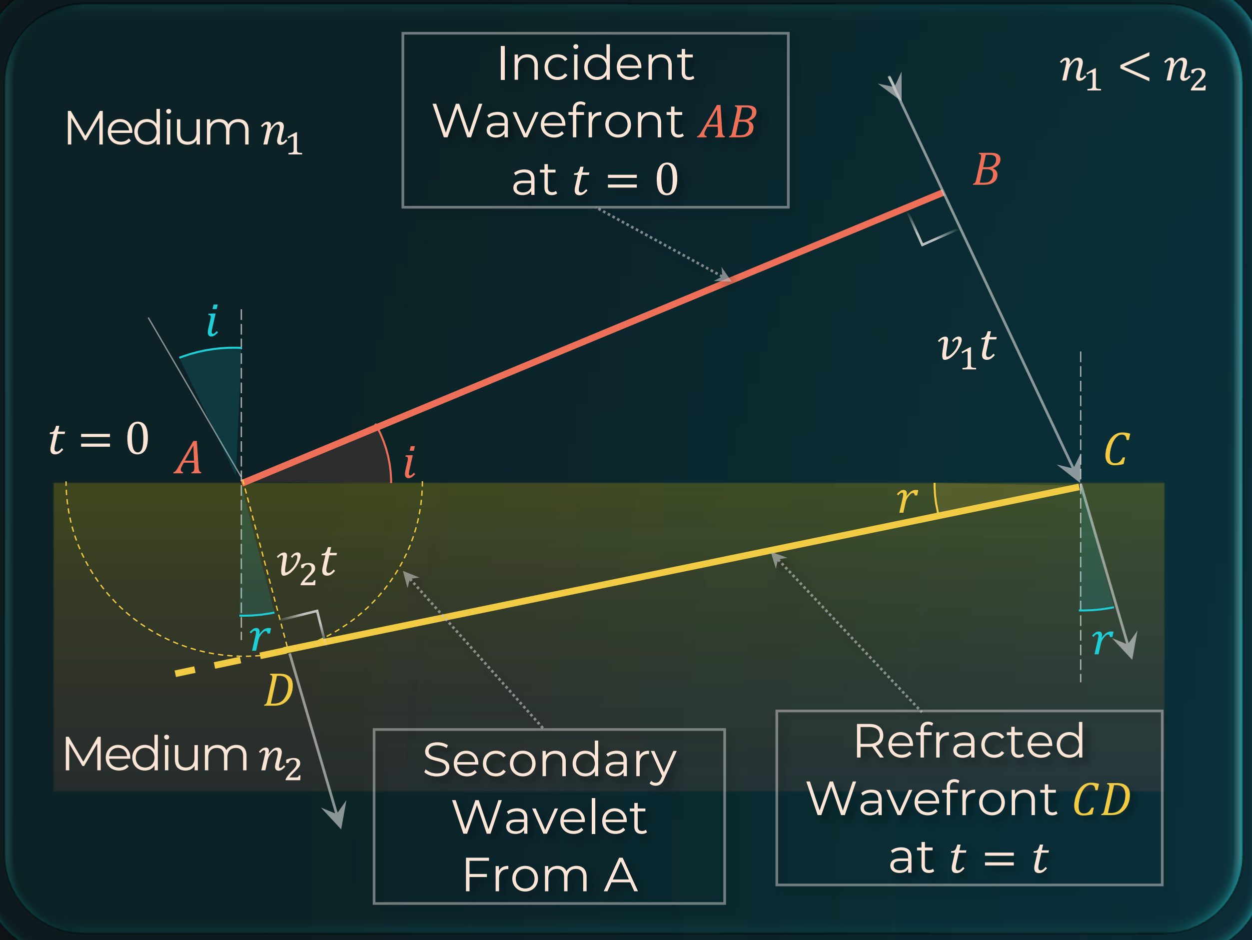
From ΔADC and ΔCBA:

$\angle ADC = \angle ABC = 90°$ $\quad AD = BC = vt$ $\quad AC = AC$

$\therefore \Delta ADC \cong \Delta CBA$ $\quad$ [$R.H.S$ Congruency]

$\Longrightarrow \quad \angle i = \angle r$

Huygens' Principle - Law of Refraction



From ΔABC:

$$\sin i = \frac{BC}{AC} = \frac{v_1\, t}{AC}$$

From ΔADC:

$$\sin r = \frac{AD}{AC} = \frac{v_2\, t}{AC}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = \text{Constant}$$

Light waves travel in vacuum, along the x – axis. Which of the following may represent the wavefronts?

A. $x = c$

B. $y = c$

C. $z = c$

D. $x + y + z = c$

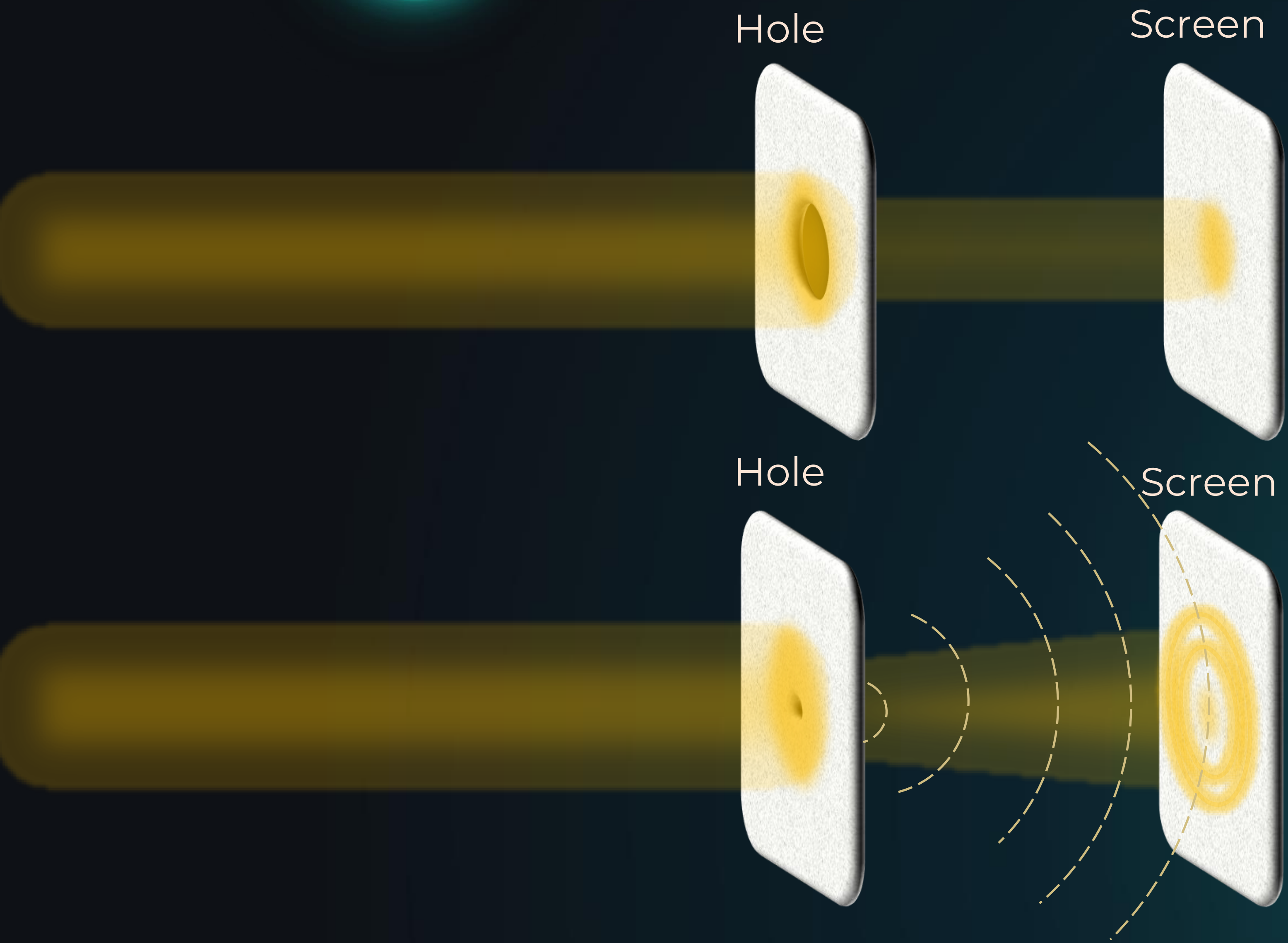
Solution:

The direction of propagation $\rightarrow \hat{\imath}$

$\Rightarrow$ The direction of wavefront $\rightarrow \perp$ *to* $\hat{\imath}$

In the given options, the plane $\perp$ to $\hat{\imath}$ is represented by $x = c$.

Does Light always exhibit Rectilinear Propagation?



Light behaves like a ray (i.e., exhibits rectilinear propagation) for:

Obstacle dimensions $\gg \lambda_{light}$

Light behaves like a wave for:

Obstacle dimensions $\approx \lambda_{light}$

OPTICS

Study of the behavior and properties of light

GEOMETRICAL OPTICS

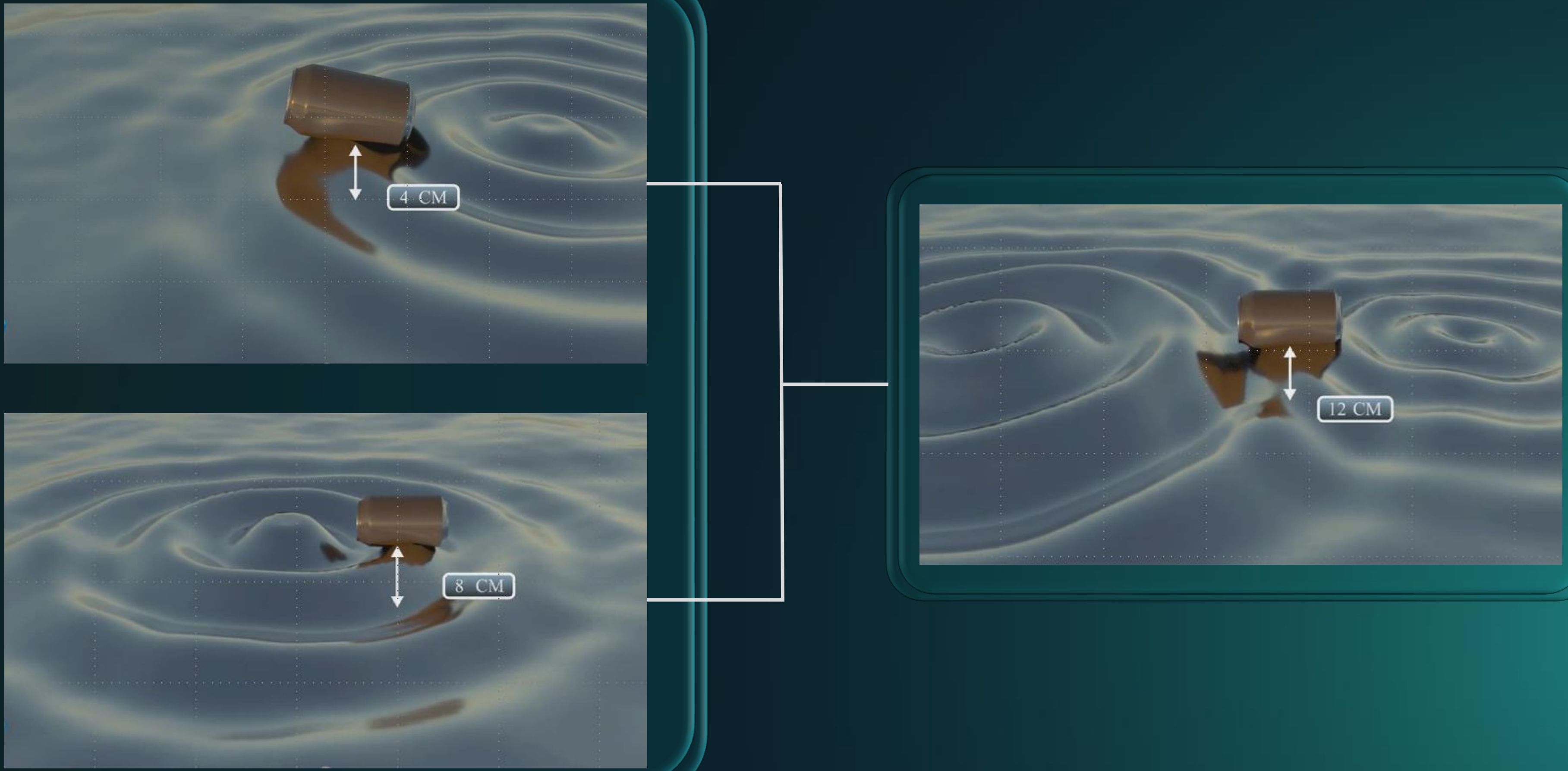
Explains refraction and reflection, but not interference, diffraction and polarization

WAVE OPTICS

Explains reflection and refraction, as well as interference, diffraction and polarization

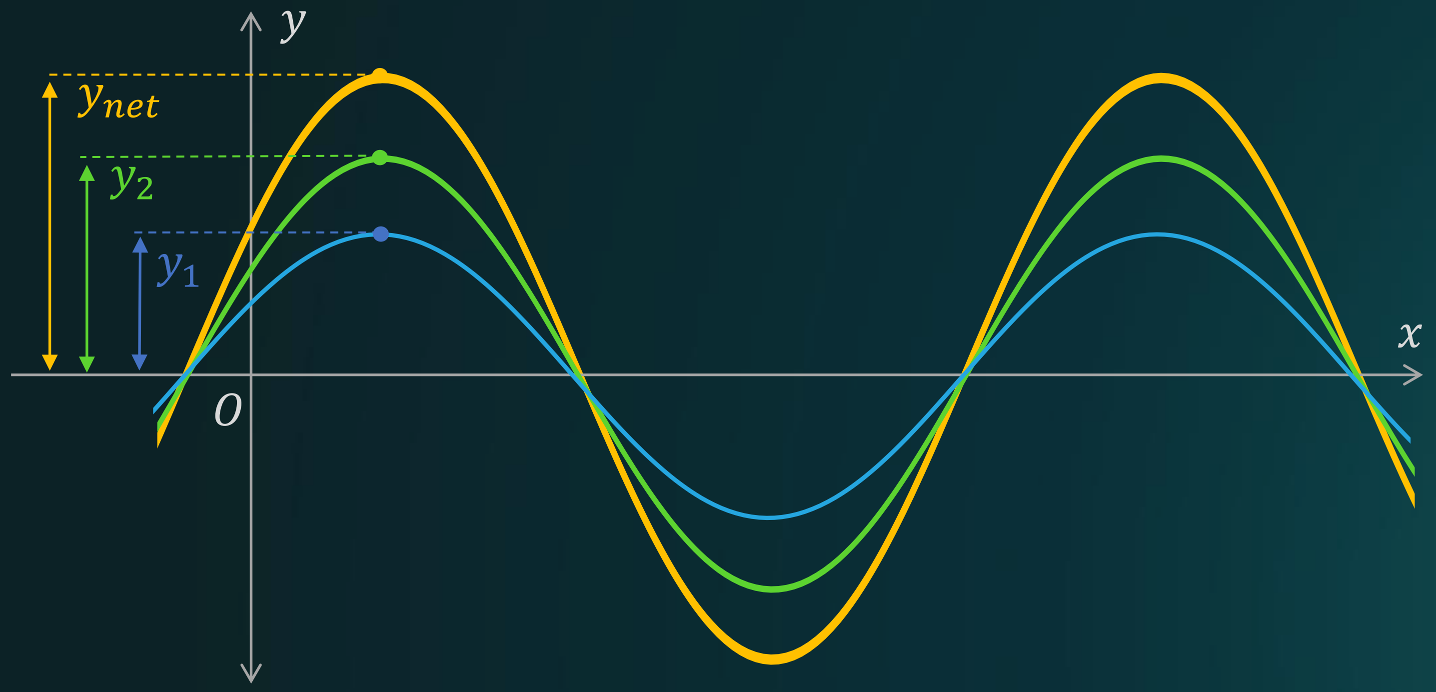
Interference

Interference is the phenomenon in which two waves superpose to form the resultant wave of lower, higher or same amplitude.



Superposition Principle

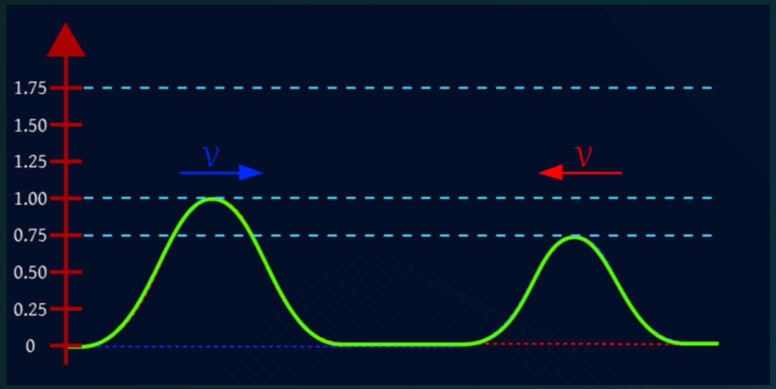
"When two or more waves cross at a point, the displacement at that point is equal to the vector sum of the displacements of individual waves"

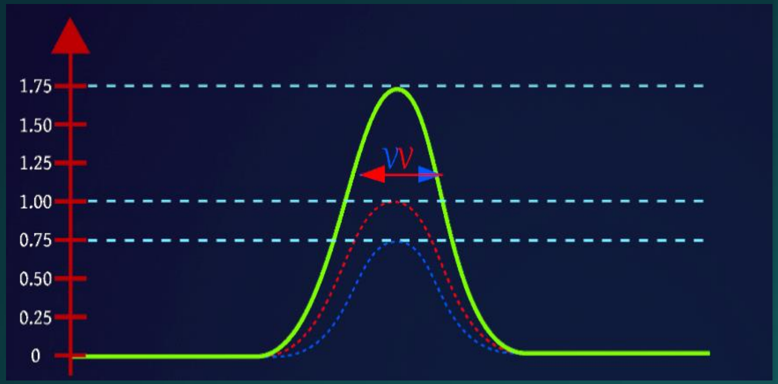


$$\boxed{\vec{y}_{net} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 \ldots \ldots + \vec{y}_n}$$

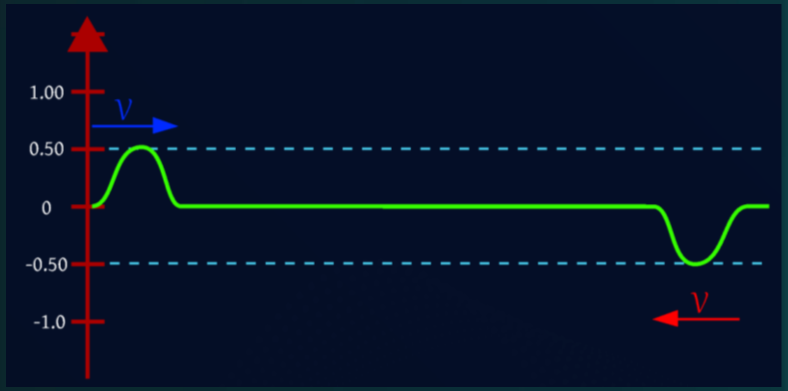
Resultant of Waves

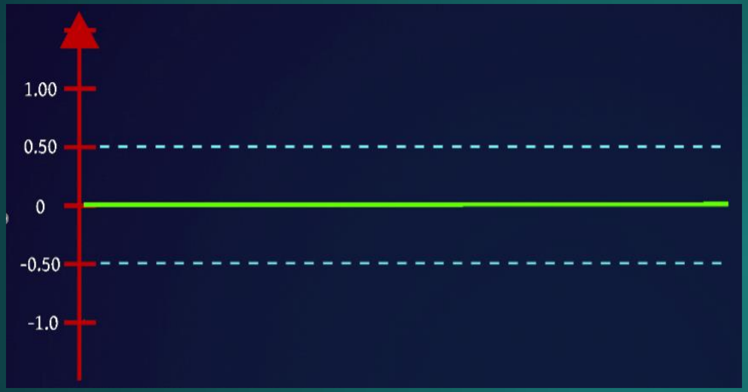
Case 1: When two crest meet



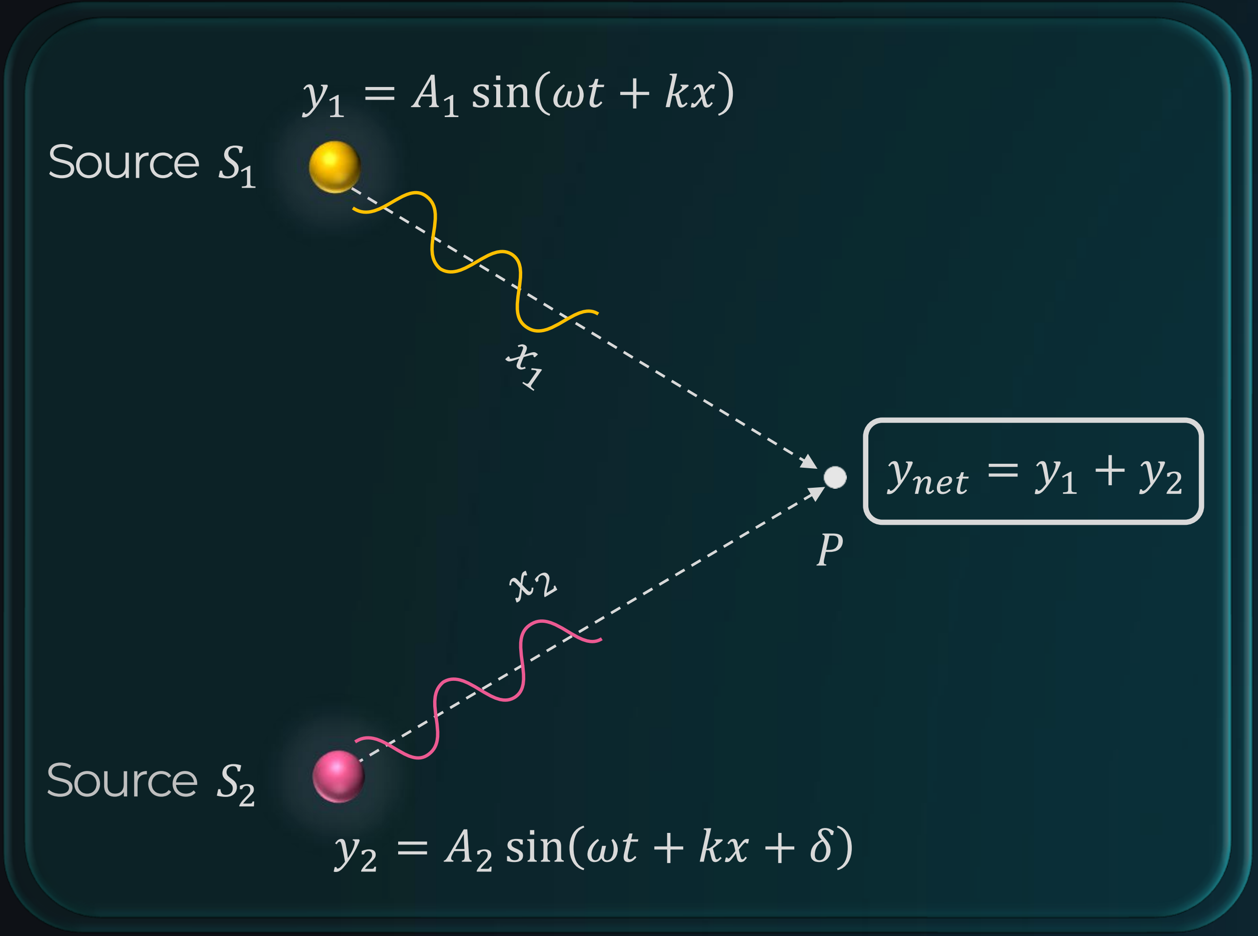


Case 2: When crest and trough meet





Phase Difference and Path Difference



- **Path difference (Δx)**: Difference in the path traversed by the two waves.
- **Phase difference (δ)**: Difference in the phase angle of the two waves.

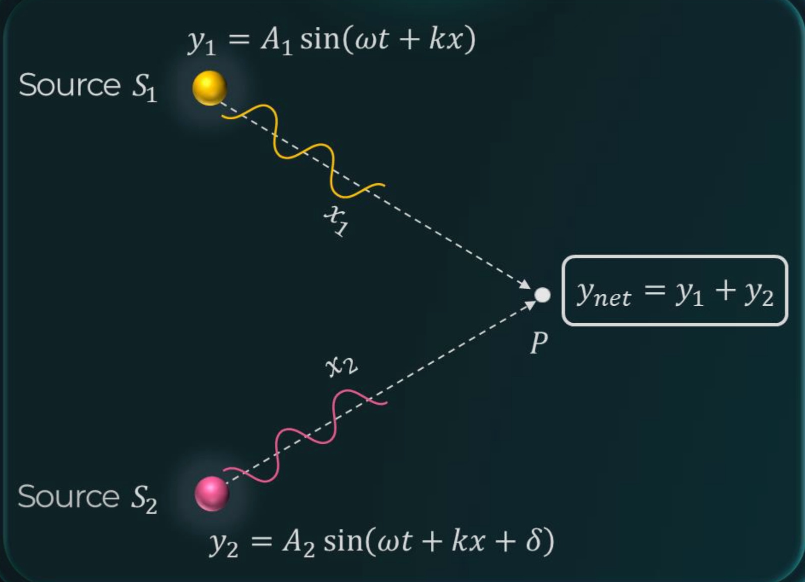
We know that,

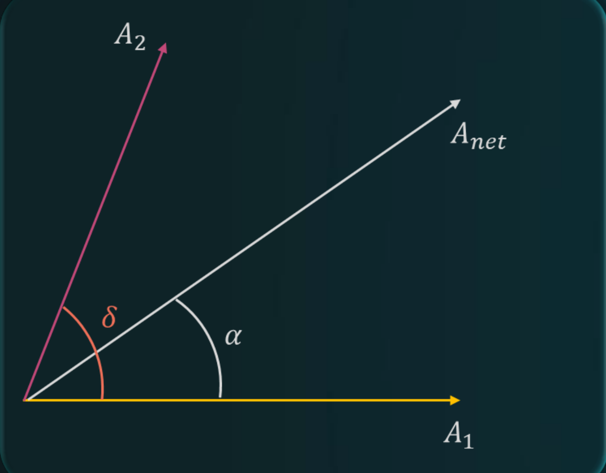
For a path difference λ, phase difference $= 2\pi$

So, for path difference Δx, phase difference $= \frac{2\pi}{\lambda}\Delta x$.

$$\boxed{\delta = \frac{2\pi}{\lambda}\Delta x}$$

Combination of Waves





$$y_{net} = A_1 \sin(\omega t + kx) + A_2 \sin(\omega t + kx + \delta)$$

$$\boxed{y_{net} = A_{net} \sin(\omega t + kx + \alpha)}$$

$$\boxed{A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\delta}}$$

$$\boxed{\tan\alpha = \frac{A_2 \sin\delta}{A_1 + A_2 \cos\delta}}$$

If $A_1 = A_2 = A$

$$\boxed{A_{net} = 2A \cos\left(\frac{\delta}{2}\right)}$$

Constructive Interference

Interference that produces maximum possible amplitude (or maximum intensity) is called constructive interference.

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

For maximum amplitude:

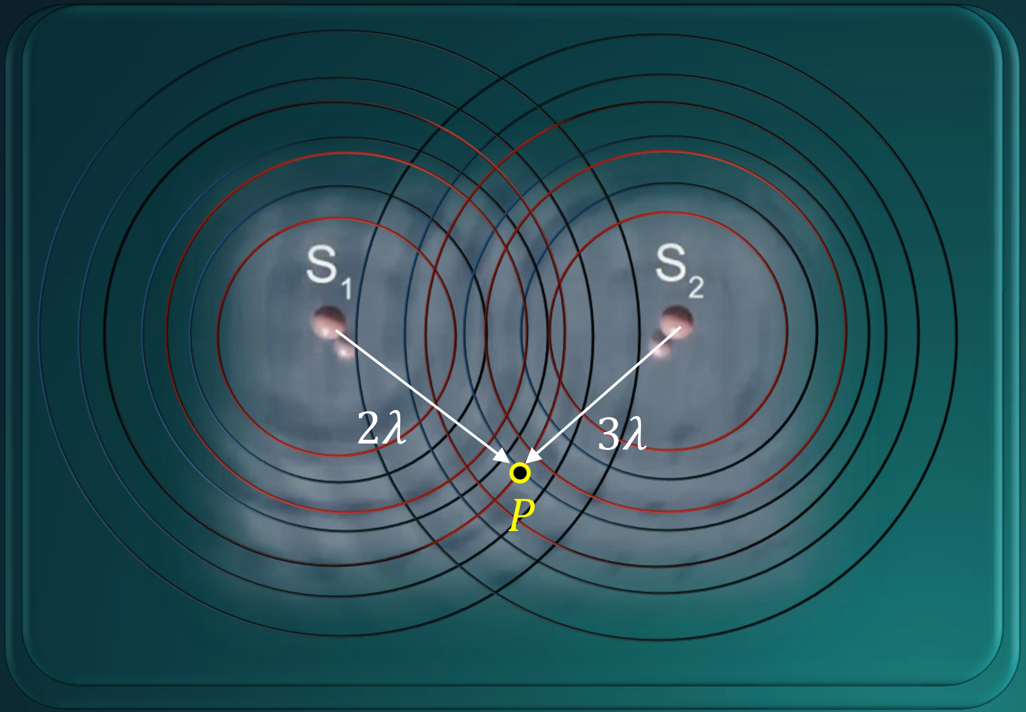
$$\cos\delta = 1 \Rightarrow \delta = 2n\pi$$

$$A = A_{max} = A_1 + A_2$$

$$\delta = \frac{2\pi}{\lambda}\Delta x$$

$$2n\pi = \frac{2\pi}{\lambda} \times (\Delta x)$$

$$\boxed{\text{Path difference} = \Delta x = n\lambda}$$

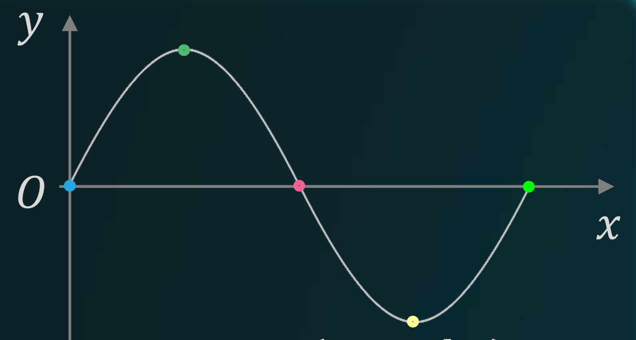


- Path Difference $= 3\lambda - 2\lambda = \lambda$

Constructive Interference

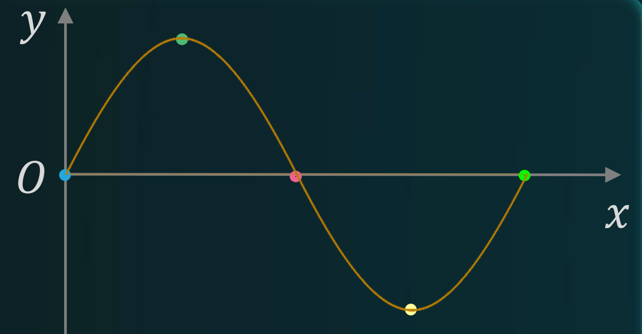
Constructive Interference

Constructive interference occurs when the crest and trough of one wave overlaps with the crest and trough of another wave.



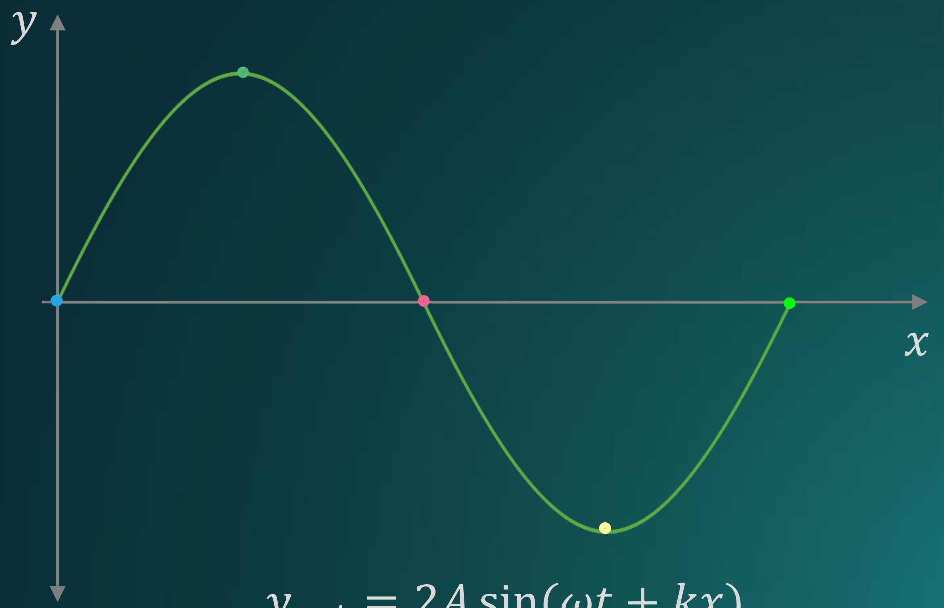
$$y_1 = A\sin(\omega t + kx)$$

$+$



$$y_2 = A\sin(\omega t + kx)$$

$\equiv$



$$y_{net} = 2A\sin(\omega t + kx)$$

Destructive Interference

Interference that produces minimum possible amplitude (or minimum intensity) is called destructive interference.

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

For minimum amplitude:

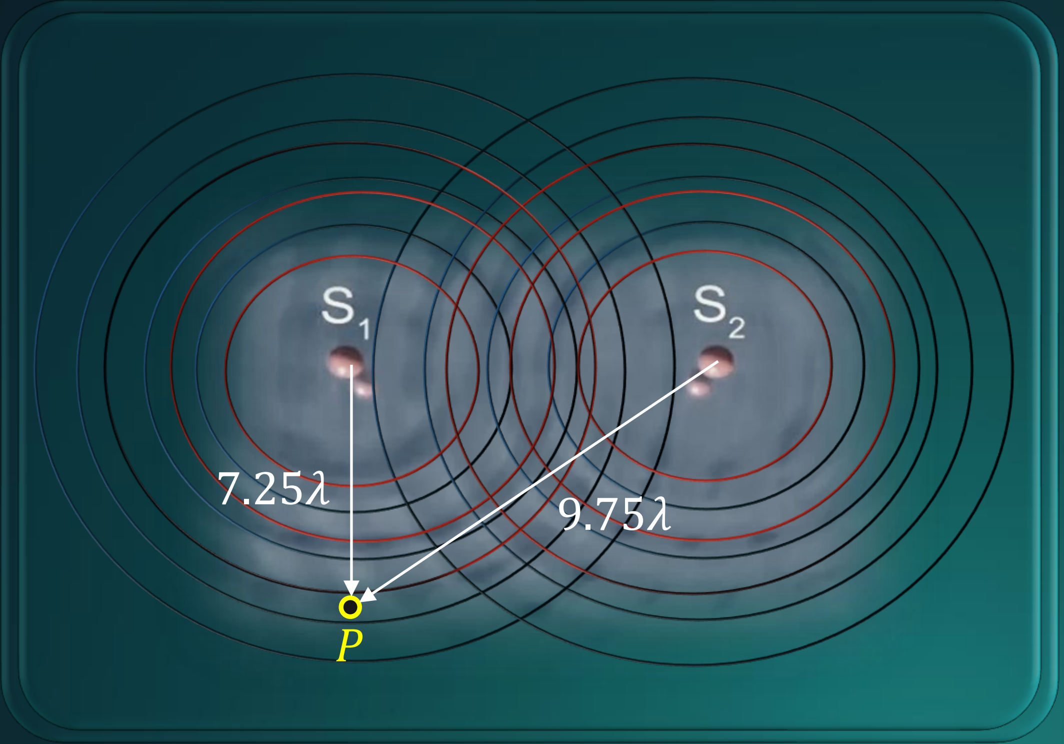
$$\cos\delta = -1 \Rightarrow \delta = (2n+1)\pi$$

$$A = A_{min} = A_1 - A_2$$

$$\delta = \frac{2\pi}{\lambda}\Delta x$$

$$(2n+1)\pi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\boxed{\text{Path difference} = \Delta x = \frac{(2n+1)\lambda}{2}}$$

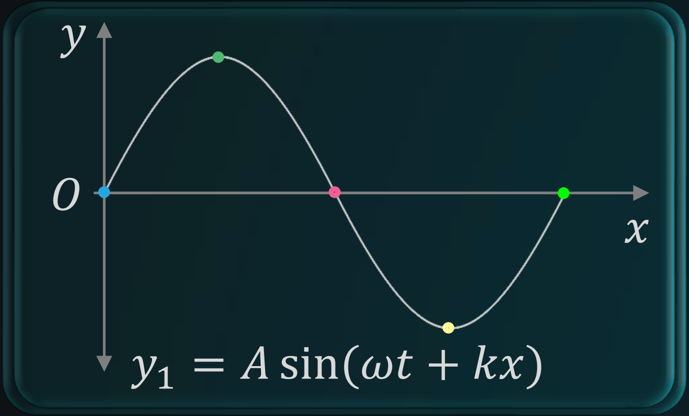


- Path Difference $= 9.75\lambda - 7.25\lambda = 2.5\lambda$

Destructive Interference

Destructive Interference

Destructive interference is resulted when the crest of one wave overlaps with the trough of another wave.



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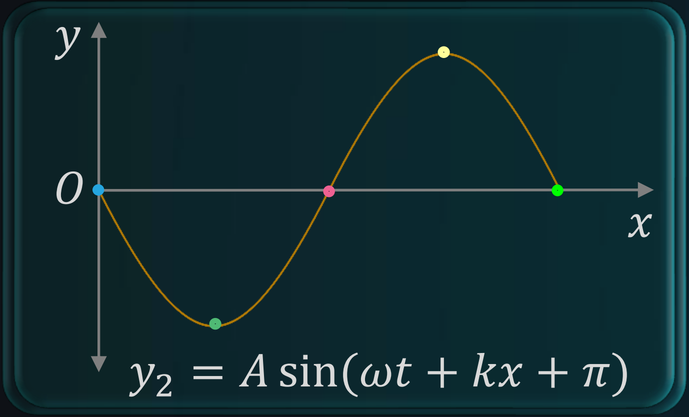
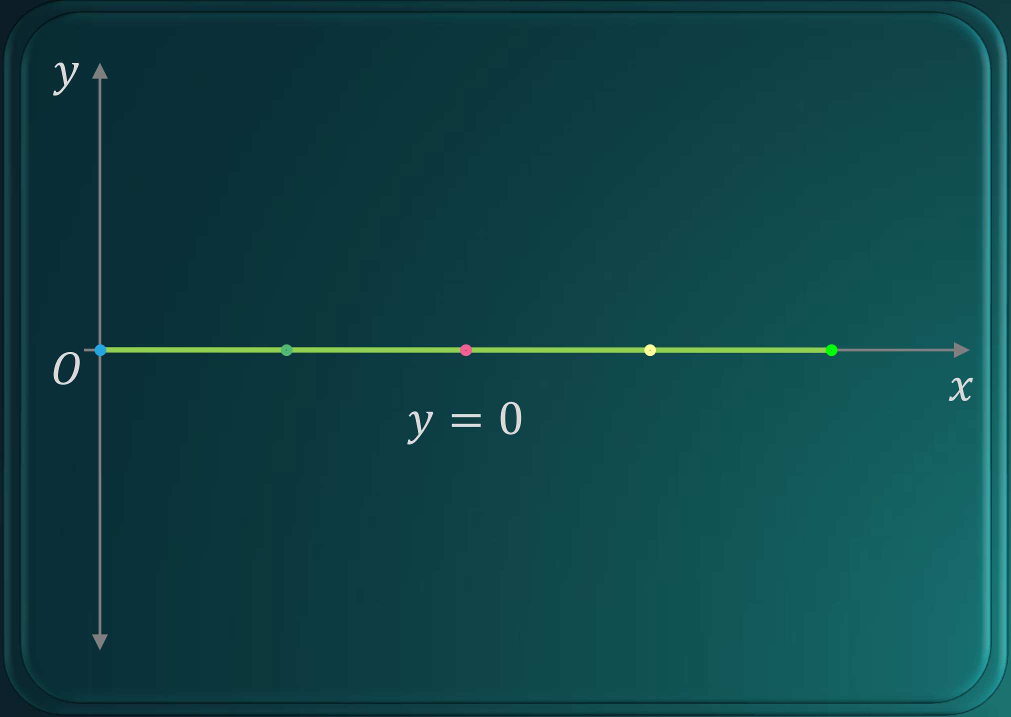
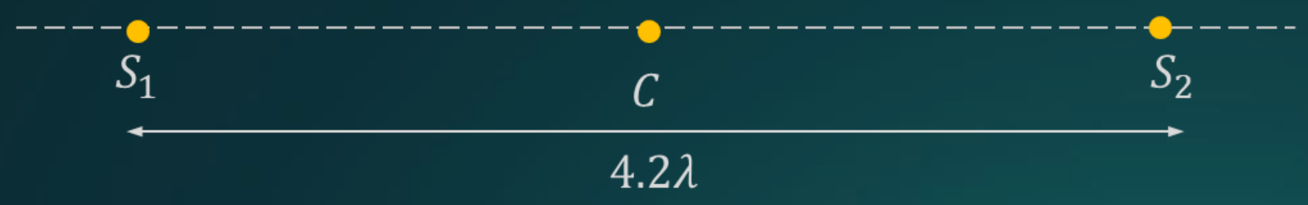


$\equiv$



Two-point light sources S_1 and S_2 are separated by a distance of 4.2λ. If an observer standing at the centre C of the two sources starts moving towards S_2, then find the minimum distance travelled by the observer to meet the first maxima.



Solution:

For maxima at P:

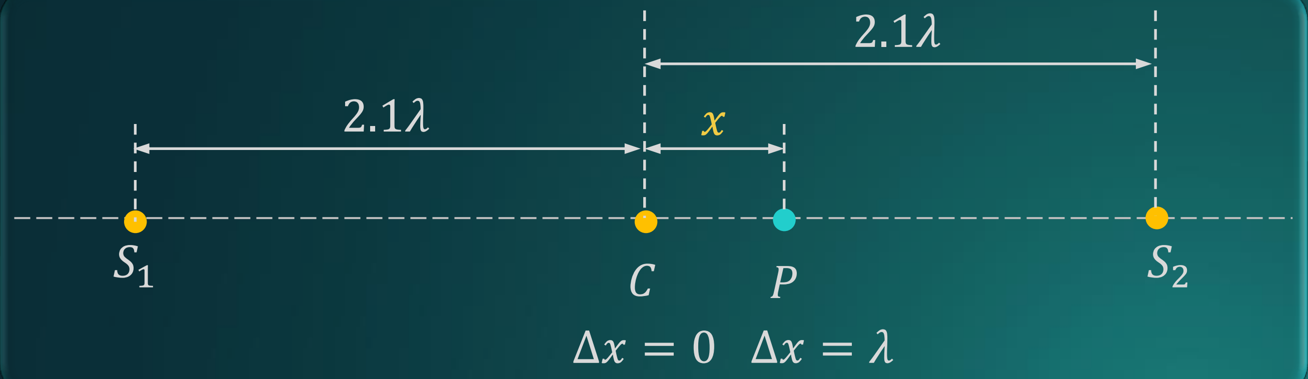
$\Delta x = \lambda$

$\Rightarrow S_1P - S_2P = \lambda$

$\Rightarrow (2.1\lambda + x) - (2.1\lambda - x) = \lambda$

$\Rightarrow 2x = \lambda$

$\boxed{x = 0.5\lambda}$



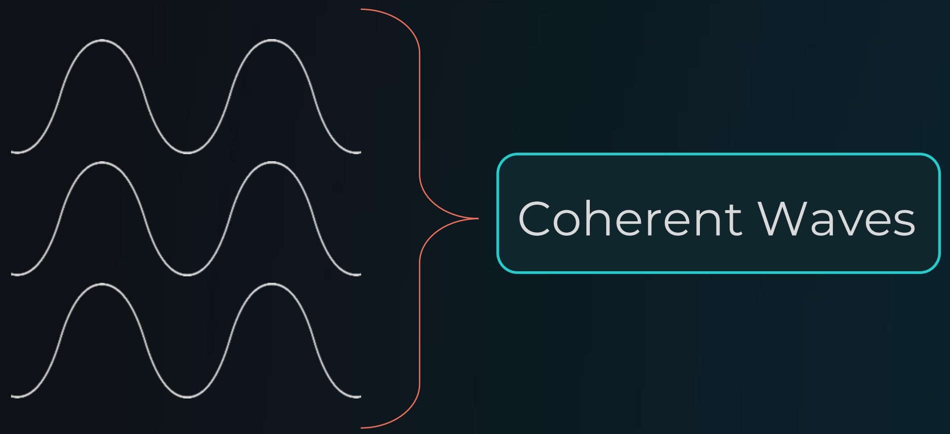
Coherent and Incoherent Sources

Coherent Sources

- Same wavelength
- Same frequency
- Constant phase difference

Incoherent Sources

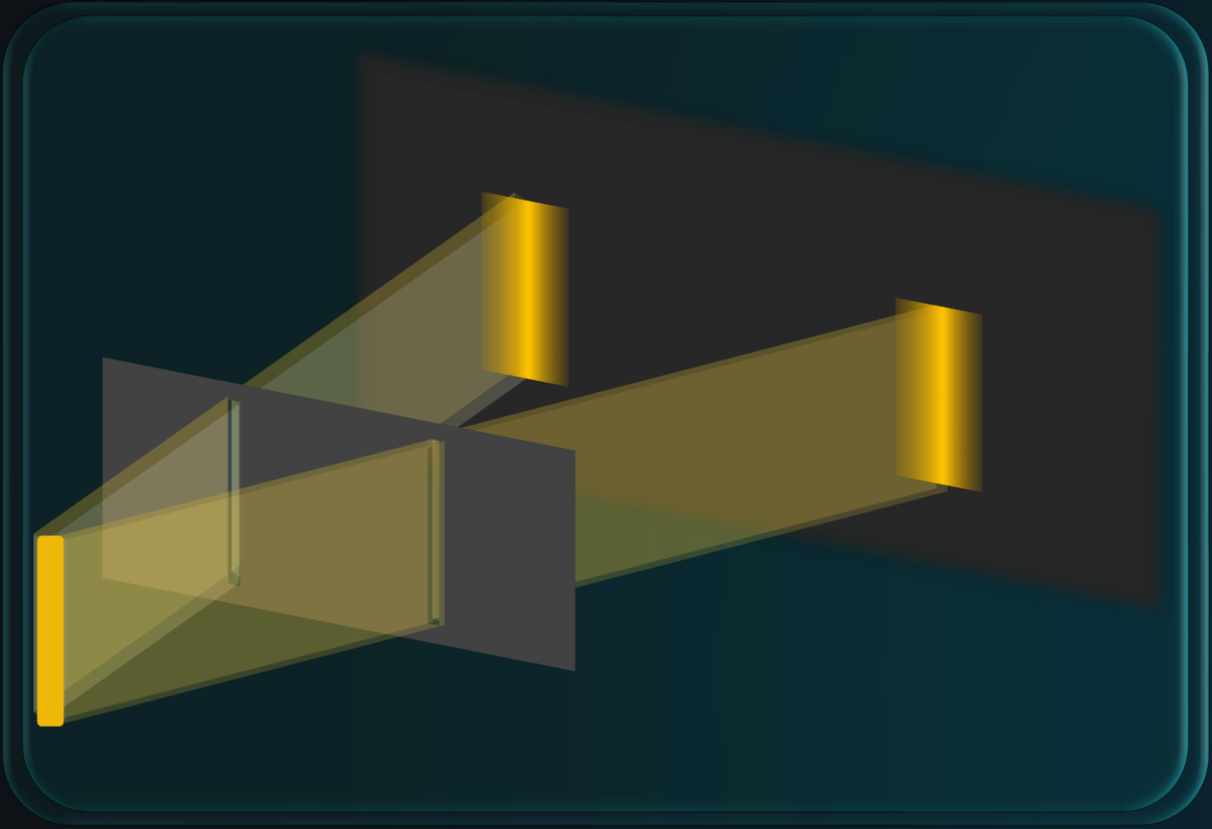
- Different wavelength
- Different frequency
- Varying phase difference



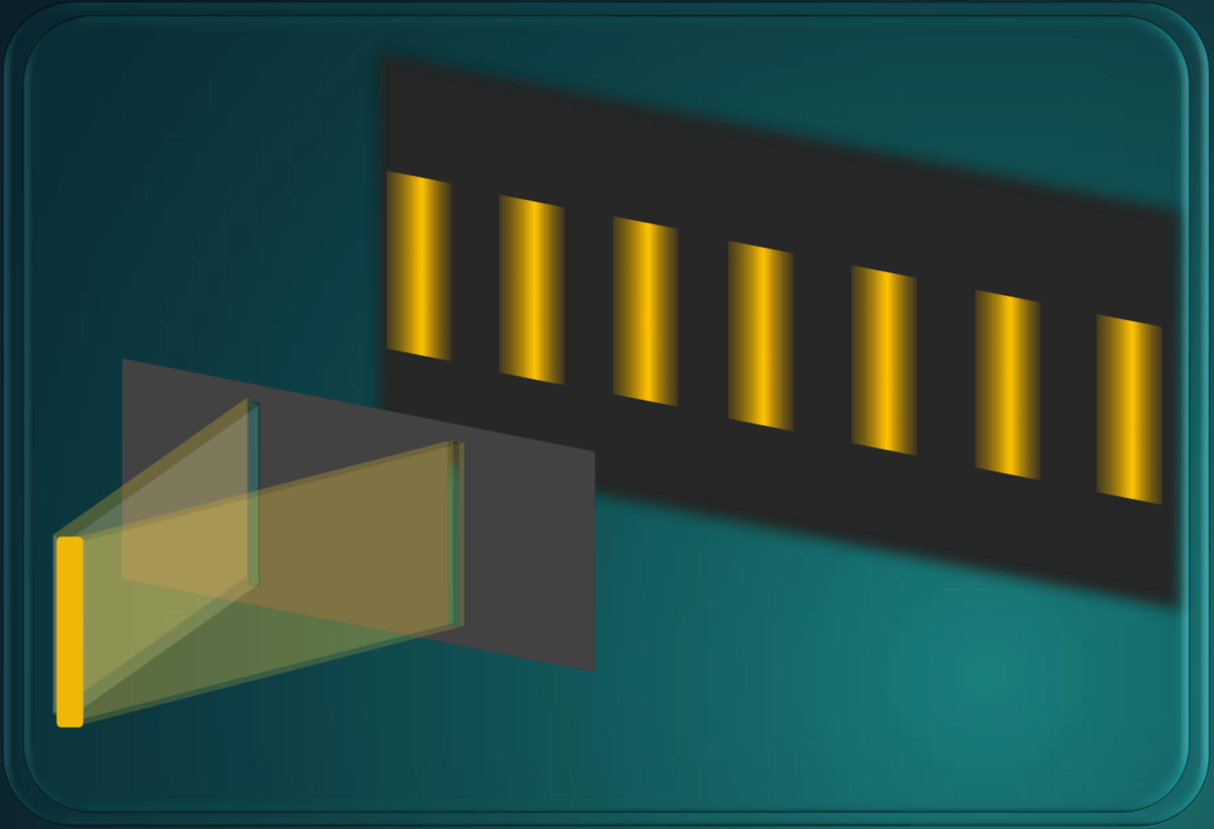
Note: We will always consider coherent sources in our discussion.

Young's Double Slit Experiment

Particle nature of light

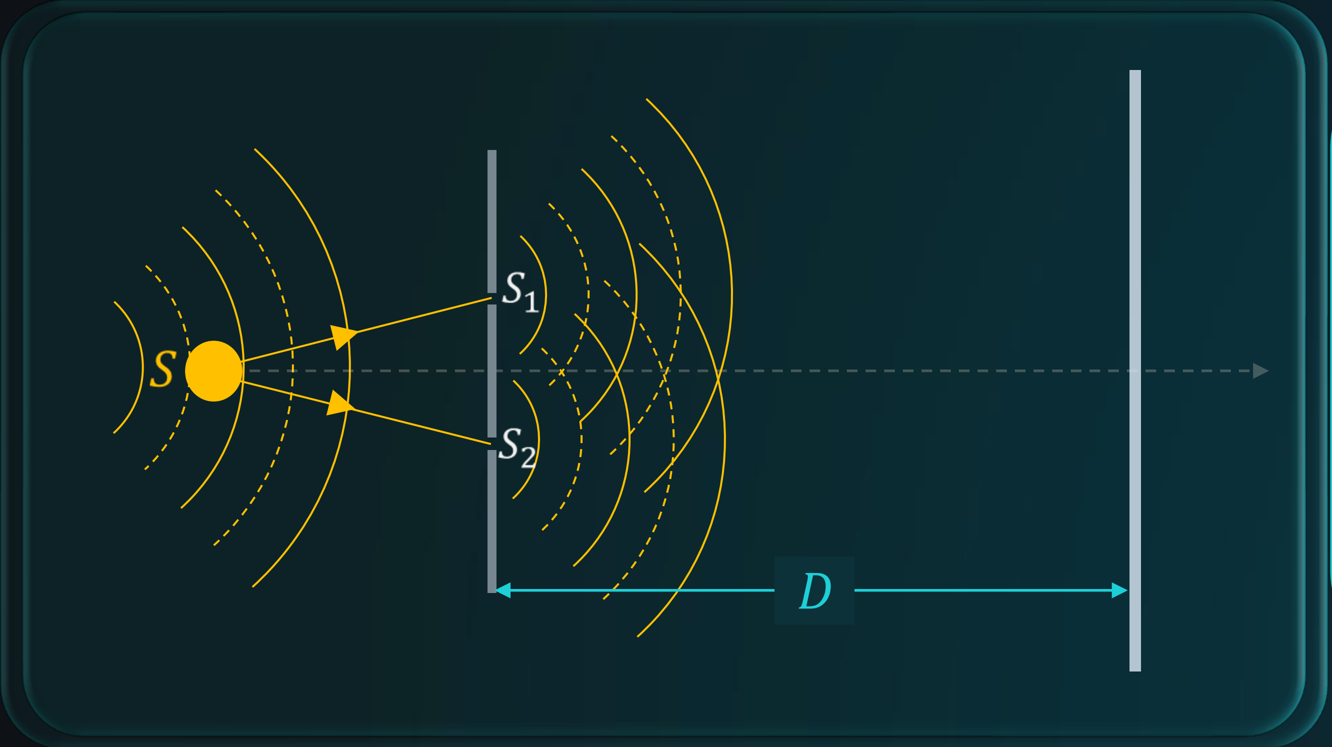


Wave nature of light



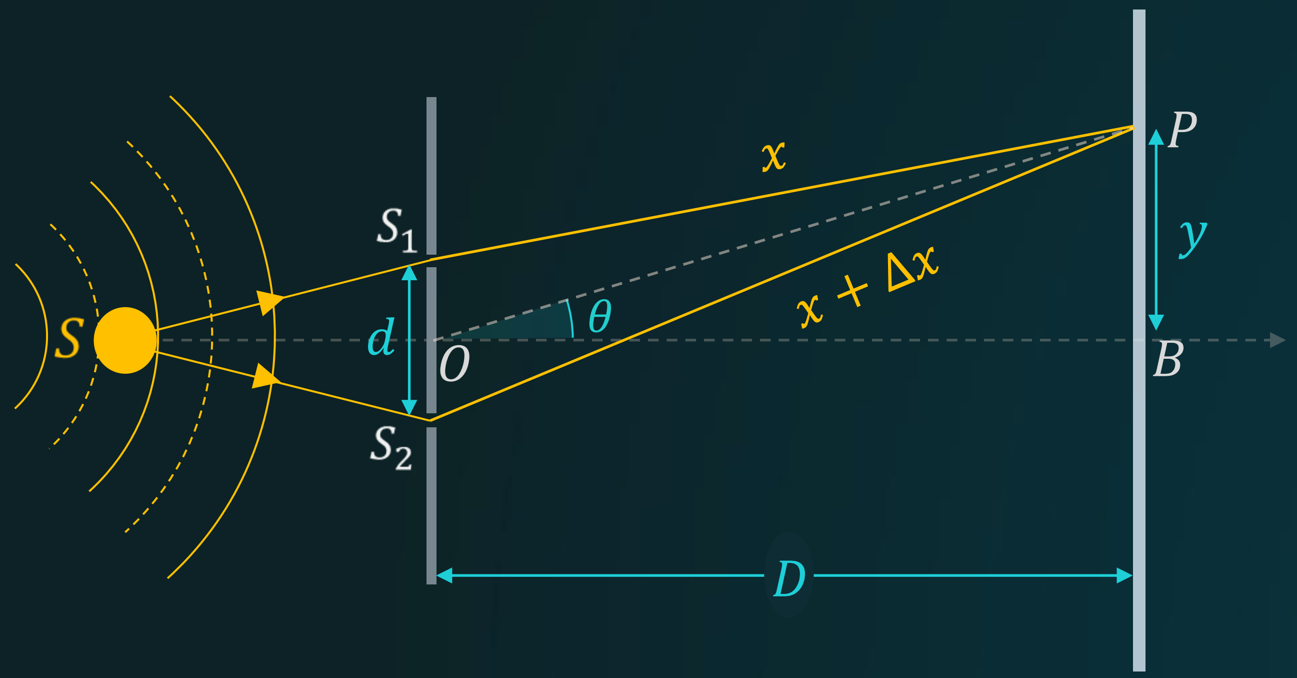
Young's Double Slit Experiment

According to Huygens' principle, the sources S_1 and S_2 will behave as independent sources.



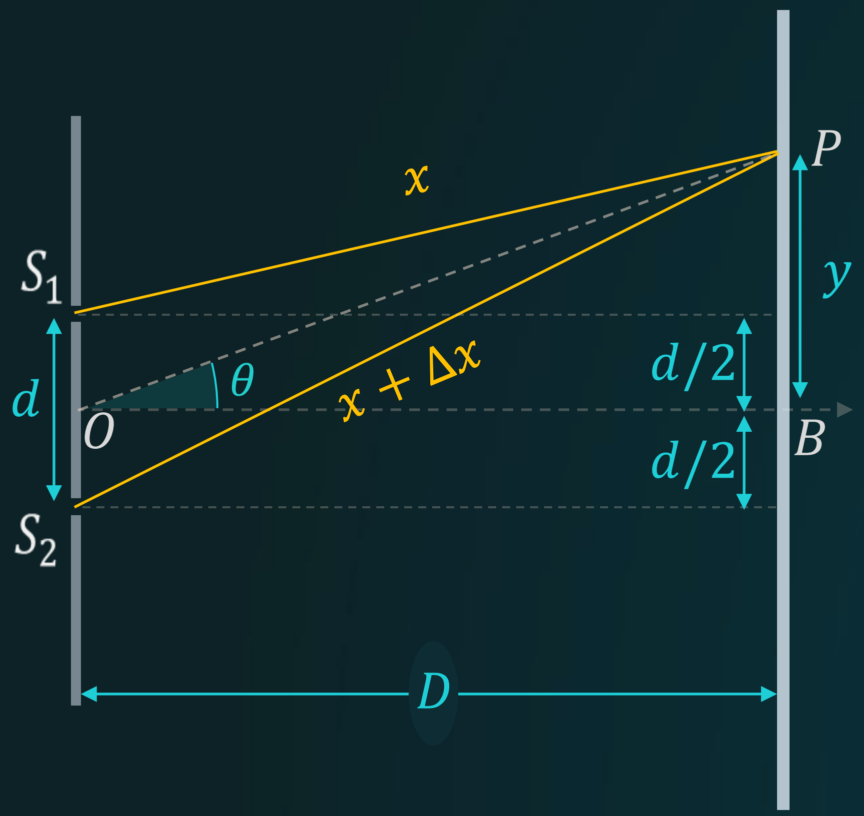
- Light source must be monochromatic.
- Sources S_1 and S_2 must be coherent.
- Width of the slit is comparable to the wavelength of light.
- Waves coming from sources S_1 and S_2 will interference and obtain different interference pattern on the screen.

Young's Double Slit Experiment



- $SS_1 = SS_2$ ($\because$ No path difference till slits.)
- $'d'$ is distance between slits.
- $'D'$ is distance between screen and slits plane.
- OB is the central line.
- Consider point P at a distance y from central line OB.
- $\angle POB = \theta$.
- $S_2P > S_1P \quad \rightarrow \quad S_2P - S_1P = \Delta x$
- Δx is the path difference.

Young's Double Slit Experiment



- Path difference at any general point P,

$\Delta x = S_2P - S_1P$

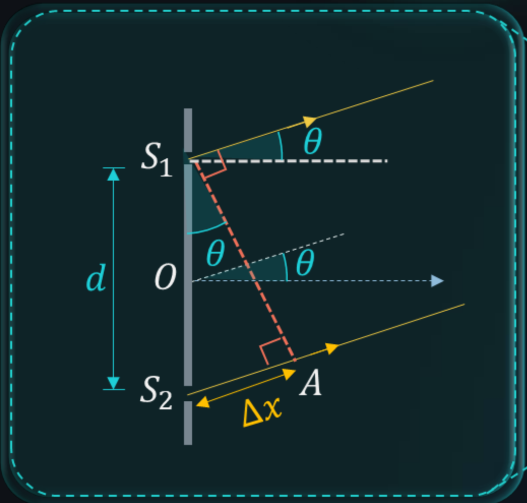
$$\Delta x = \sqrt{\left(y + \frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y - \frac{d}{2}\right)^2 + D^2}$$

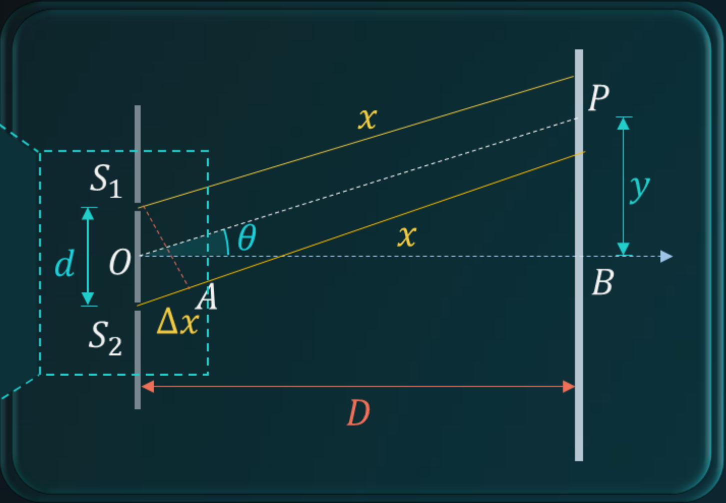
Approximation 1: $D \gg d$

Approximation 2: θ is very small

$$\Delta x \approx \frac{yd}{D}$$

Young's Double Slit Experiment





$$\Delta x = S_2A = d\sin\theta$$

θ is very small $\rightarrow$ $\sin\theta \approx \tan\theta$

$$\Delta x \approx d\tan\theta = \frac{dy}{D}$$

$$\boxed{\Delta x \approx \frac{yd}{D}}$$

- Path difference at any general point P,

  Approximation 1: $D \gg d$

  Approximation 2: θ is very small
- S_1P and S_2P are parallel.
- $S_1P = AP = x$
- $S_2A = \Delta x$ $\rightarrow$ Path difference

Condition for Constructive Interference

- For Maxima (Constructive interference):

$$\Delta x = n\lambda \quad \Rightarrow \frac{dy}{D} = n\lambda$$

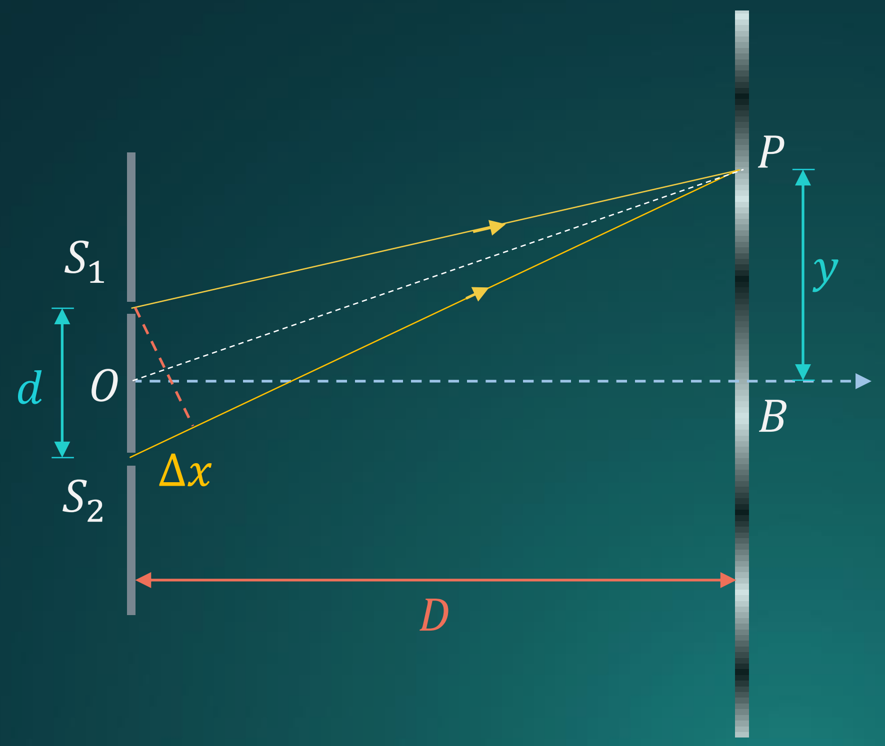
Where, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$$\boxed{y = \frac{n\lambda D}{d}}$$

$n = 0$ corresponds to the central maxima. $\rightarrow$ $y = 0$

$n = \pm 1$ correspond to the 1^{st} maxima. $\rightarrow$ $y = \pm \frac{\lambda D}{d}$

$n = \pm 2$ correspond to the 2^{nd} maxima. $\rightarrow$ $y = \pm \frac{2\lambda D}{d}$



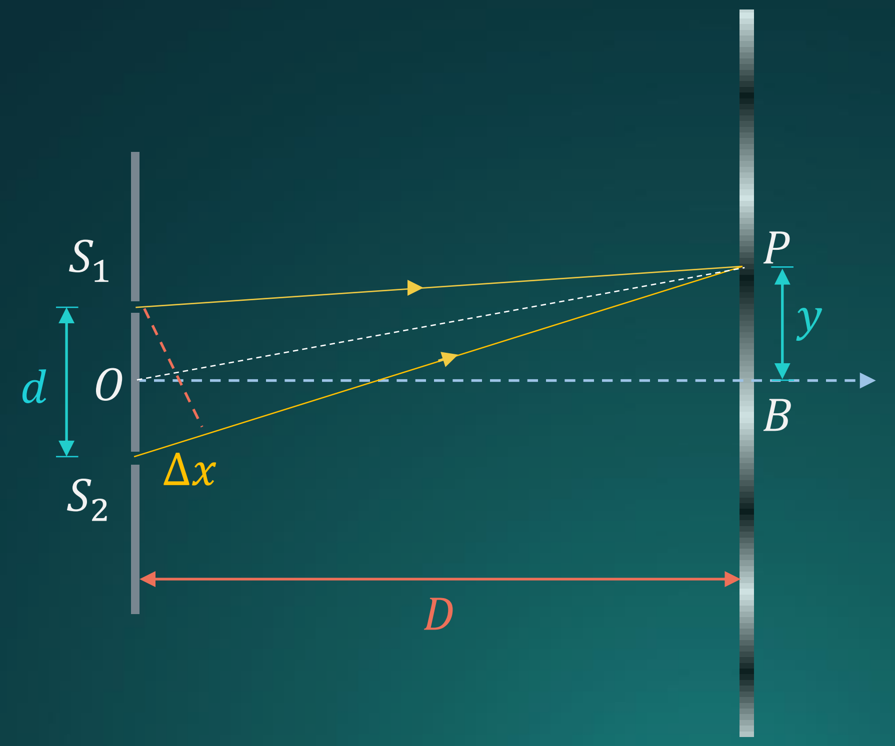
Condition for Destructive Interference

- For Minima (Destructive interference):

$$\Delta x = \left(n + \frac{1}{2}\right)\lambda \quad \Rightarrow \frac{dy}{D} = \left(n + \frac{1}{2}\right)\lambda$$

Where, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$$\boxed{y = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}}$$



$n = 0, -1$ correspond to the 1^{st} minima. $\rightarrow$ $y = \pm \frac{\lambda D}{2d}$

$n = 1, -2$ correspond to the 2^{nd} minima. $\rightarrow$ $y = \pm \frac{3\lambda D}{2d}$

In YDSE, white light is passed through the double slit and interference pattern is observed on a screen $2.5\ m$ away. The separation between the slits is $0.5\ mm$. The first violet and red maxima are formed at distances of $2\ mm$ and $3.5\ mm$ away from the central white maxima, respectively. The wavelengths of red and violet light, respectively, are:

Given: $D = 2.5\ m, d = 0.5\ mm, y_{violet} = 2\ mm, y_{red} = 3.5\ mm$

To find: $\lambda_{violet}, \lambda_{red}$

Solution:

Distance of first maxima from the central maxima is given by:

$$\boxed{y = \frac{D\lambda}{d}}$$

For violet light, $y = 2\ mm$

$$\Rightarrow 2 \times 10^{-3} = \frac{2.5 \times \lambda_{violet}}{0.5 \times 10^{-3}}$$

$$\Rightarrow \lambda_{violet} = \frac{2}{5} \times 10^{-6}\ m$$

$$\boxed{\lambda_{violet} = 400\ nm}$$

For red light, $y = 3.5\ mm$

$$\Rightarrow 3.5 \times 10^{-3} = \frac{2.5 \times \lambda_{red}}{0.5 \times 10^{-3}}$$

$$\Rightarrow \lambda_{red} = 0.7 \times 10^{-6}\ m$$

$$\boxed{\lambda_{red} = 700\ nm}$$

A) $800\ nm$ and $400\ nm$

B) $350\ nm$ and $200\ nm$

C) $750\ nm$ and $350\ nm$

D) $700\ nm$ and $400\ nm$

Resultant Amplitude of the Wave

Electric field of the wave from $S1$:

$$E_1 = E_{01}\sin(kx - \omega t)$$

Electric field of the wave from $S2$:

$$E_2 = E_{02}\sin(kx - \omega t + \delta)$$

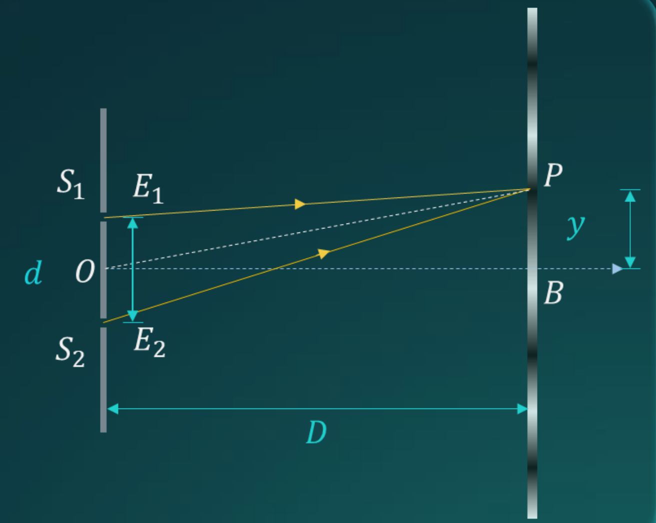
E_{01}, E_{02} *or* $A_{01}, A_{02} \rightarrow$ Electric Field Amplitude.

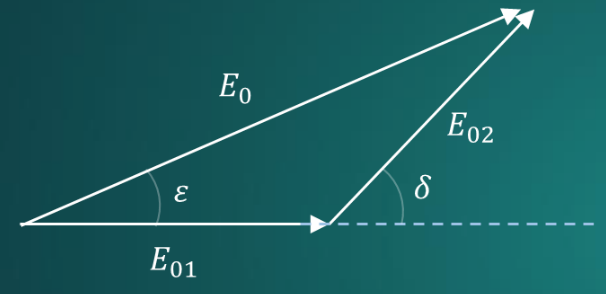
Net electric field after the interference:

$$E_{net} = E_1 + E_2 = E_0\sin(kx - \omega t + \delta)$$

$$\boxed{E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\delta}$$

$$\boxed{\tan\varepsilon = \frac{E_{02}\sin\delta}{E_{01} + E_{02}\cos\delta}}$$





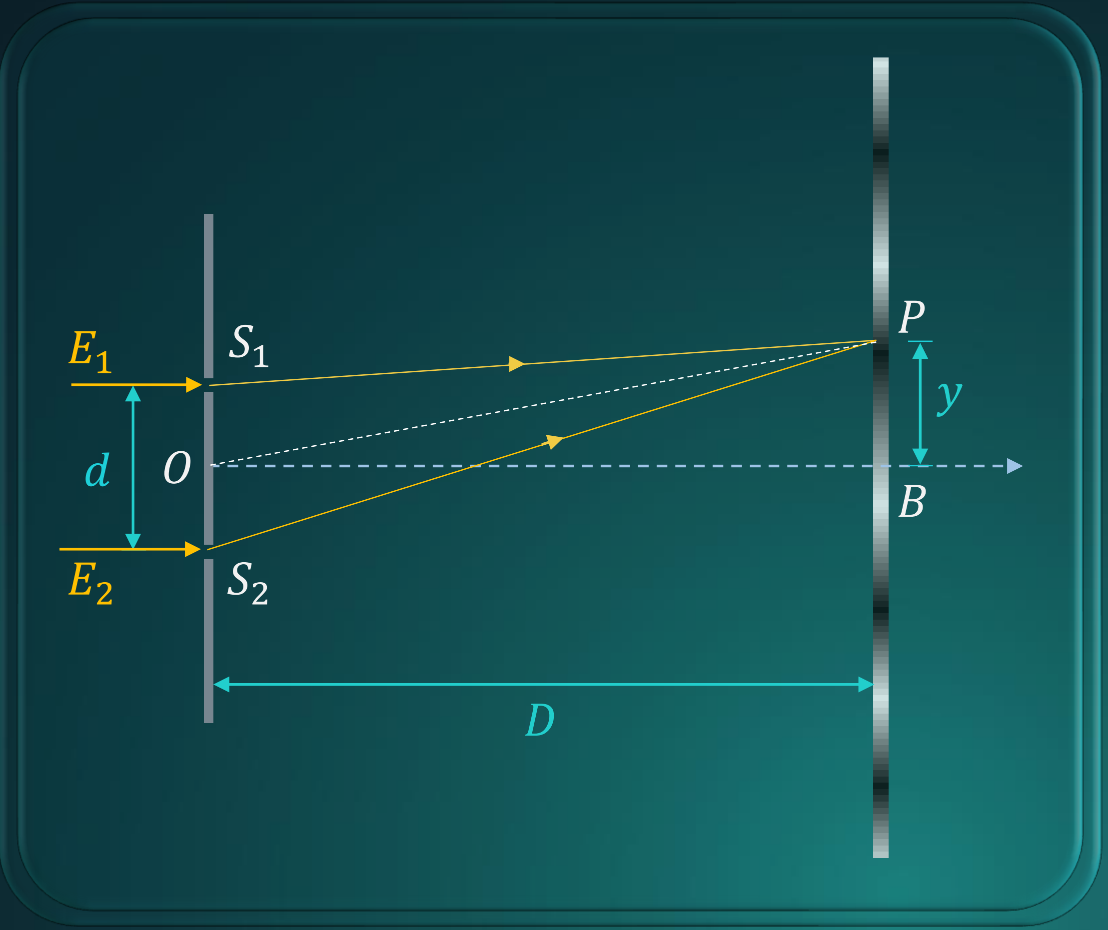
Intensity of Waves

$$I = 2\pi^2 f^2 \rho v E_0^2$$

$$I \propto E_0^2$$

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\delta$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$$



Intensity for Identical Waves

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$$

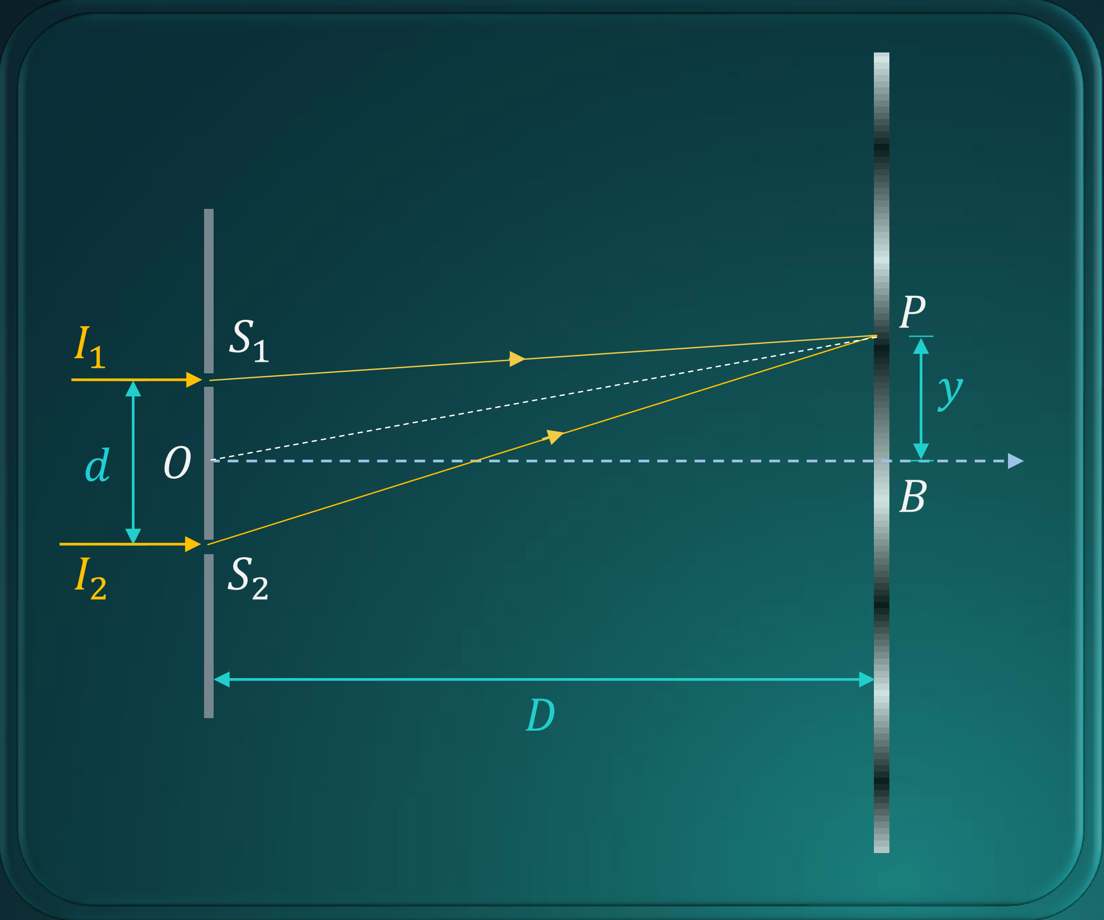
- For identical slits: $I_1 = I_2 = I$

$$I_{net} = I + I + 2\sqrt{I}\sqrt{I}\cos\delta$$

$$I_{net} = 2I + 2I\cos\delta = 2I(1+\cos\delta)$$

$$I_{net} = 2I \times 2\cos^2\left(\frac{\delta}{2}\right)$$

$$\boxed{I_{net} = 4I\cos^2\left(\frac{\delta}{2}\right)}$$



Intensity of Waves

- For constructive interference: $(Maxima)$

$$E_0^{max} = E_{01} + E_{02}$$

- For destructive interference: $(Minima)$

$$E_0^{min} = |E_{01} - E_{02}|$$

$$I \propto E_0^2 \quad \Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{E_0^{max}}{E_0^{min}}\right)^2 \qquad \boxed{\frac{I_{max}}{I_{min}} = \left(\frac{E_{01} + E_{02}}{E_{01} - E_{02}}\right)^2}$$

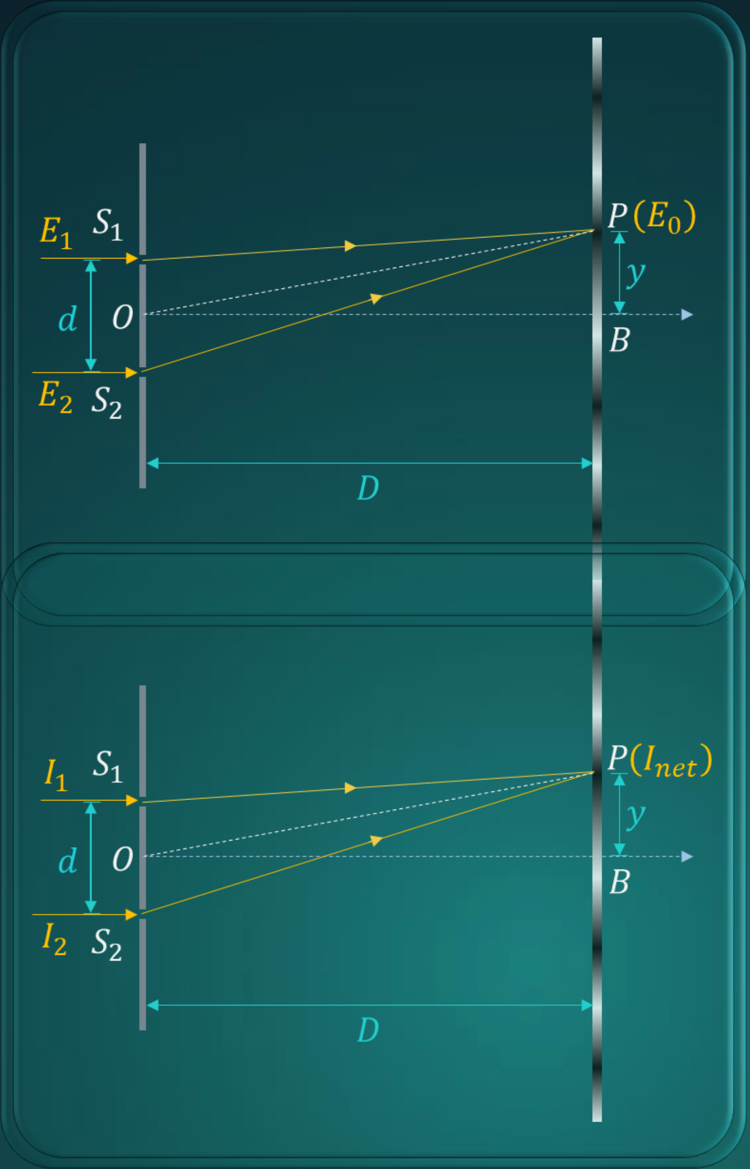
- When $\cos\delta = 1, \delta = 2n\pi$ $(Maxima)$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \Rightarrow \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

- When $\cos\delta = -1, \delta = (2n+1)\pi$ $(Minima)$

$$I_{net} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} \Rightarrow \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I = \begin{cases} \left(\sqrt{I_1} + \sqrt{I_2}\right)^2, & \text{constructive interference} \\ \left(\sqrt{I_1} - \sqrt{I_2}\right)^2, & \text{destructive interference} \end{cases}$$



Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

A. 16 : 9

B. 5 : 3

C. 25 : 9

D. 4 : 1

Given: $\frac{I_{max}}{I_{min}} = 16$

To find: $\frac{I_1}{I_2}$

Solution: $\frac{I_{max}}{I_{min}} = 16 \Rightarrow \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{16}{1}$

$$\Rightarrow \sqrt{I_1}+\sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2} \qquad \Rightarrow 3\sqrt{I_1} = 5\sqrt{I_2}$$

$$\Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

Ratio of intensities: $\frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2$

$$\boxed{\frac{I_1}{I_2} = \frac{25}{9}}$$

Intensity Variation

- Constructive Interference:

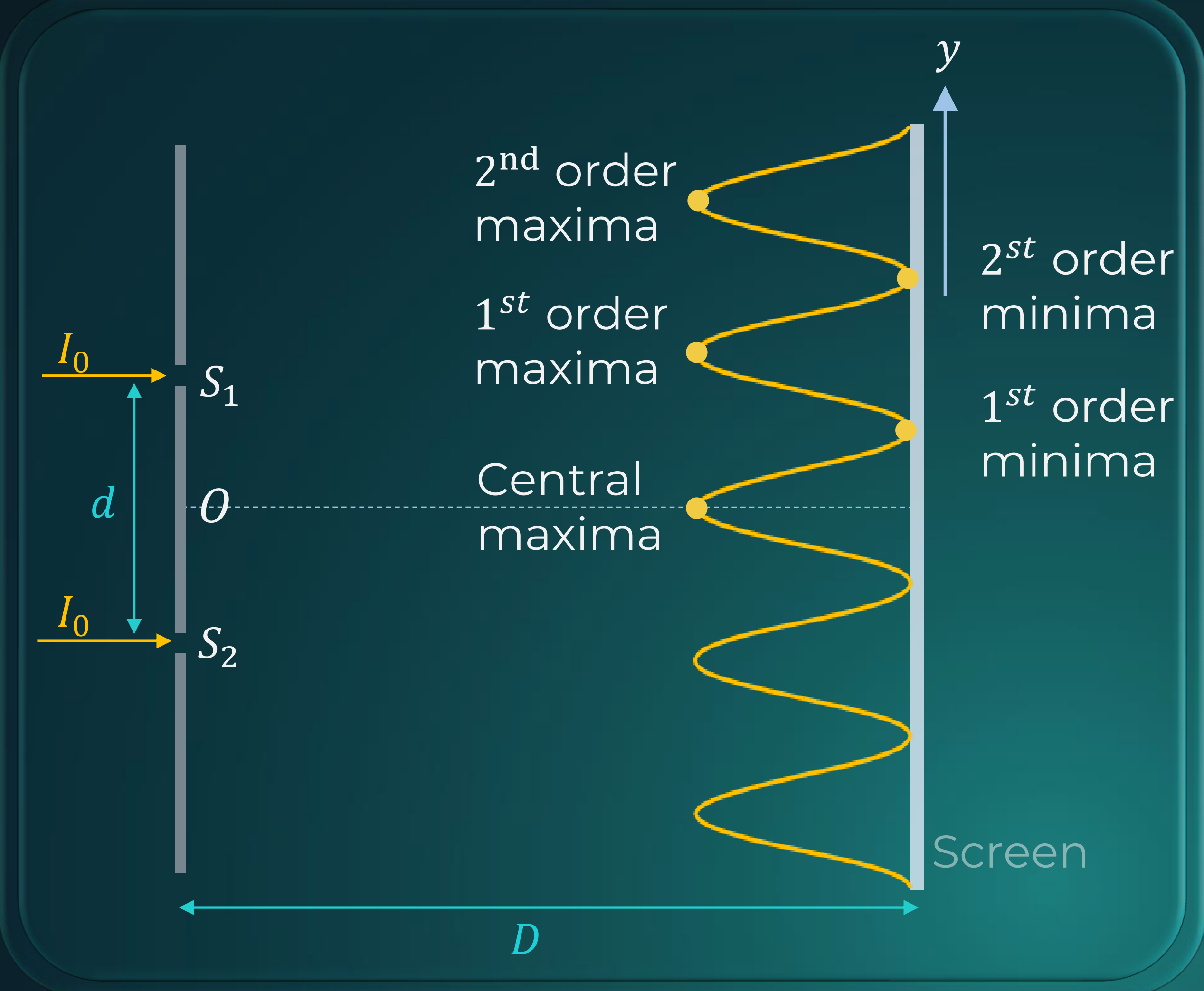
$$\boxed{y = \frac{n\lambda D}{d}}$$

$$y = \pm\frac{\lambda D}{d}, \pm\frac{2\lambda D}{d}, \pm\frac{3\lambda D}{d} \ldots\ldots.$$

- Destructive Interference:

$$\boxed{y = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}}$$

$$y = \pm\frac{\lambda D}{2d}, \pm\frac{3\lambda D}{2d}, \pm\frac{5\lambda D}{2d} \ldots\ldots.$$



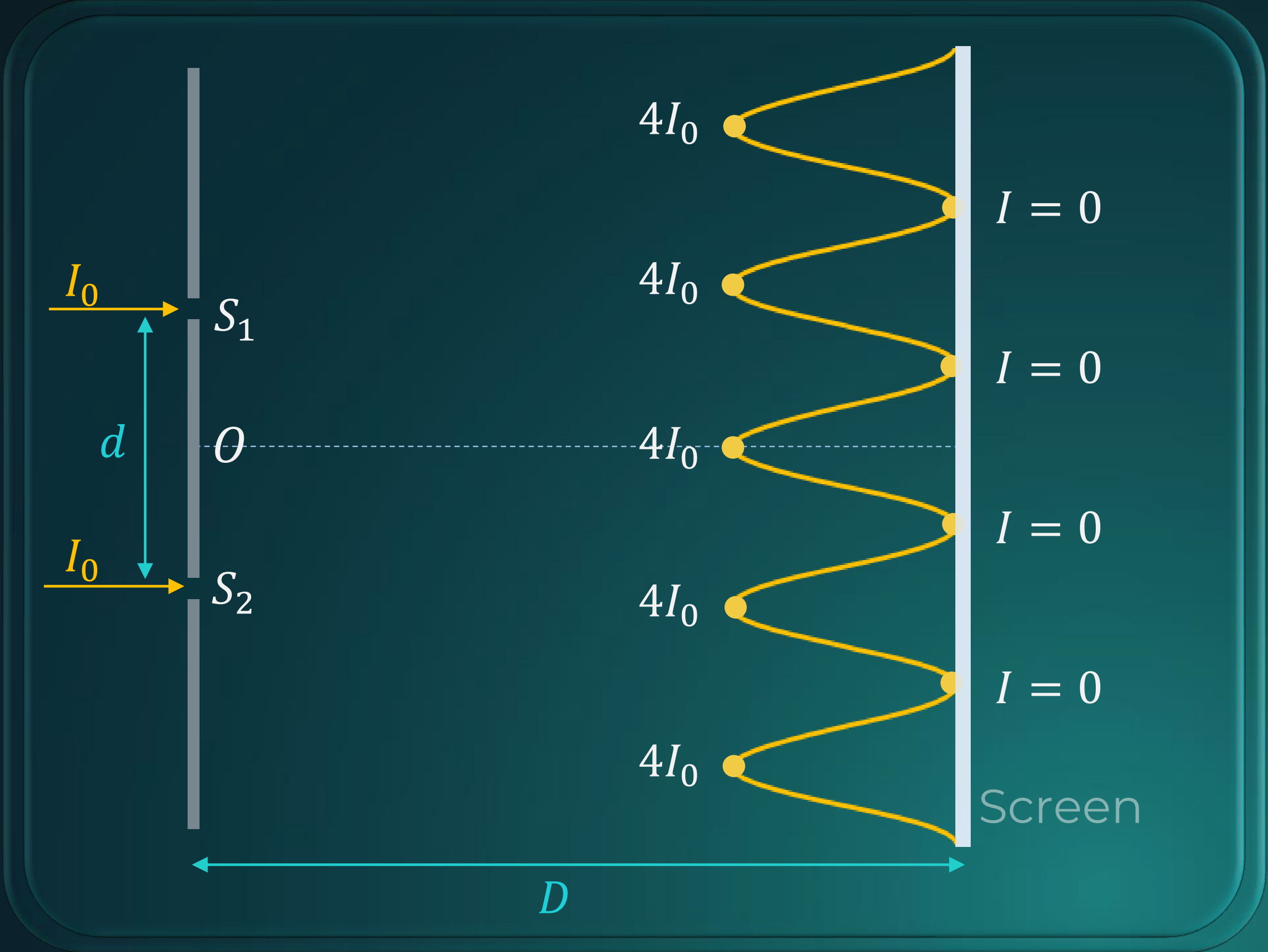
Intensity Variation

- Intensity at any point:

$$\boxed{I = 4I_0 \cos^2 \frac{\delta}{2}}$$

δ = Phase difference between the two waves from S_1 and S_2.

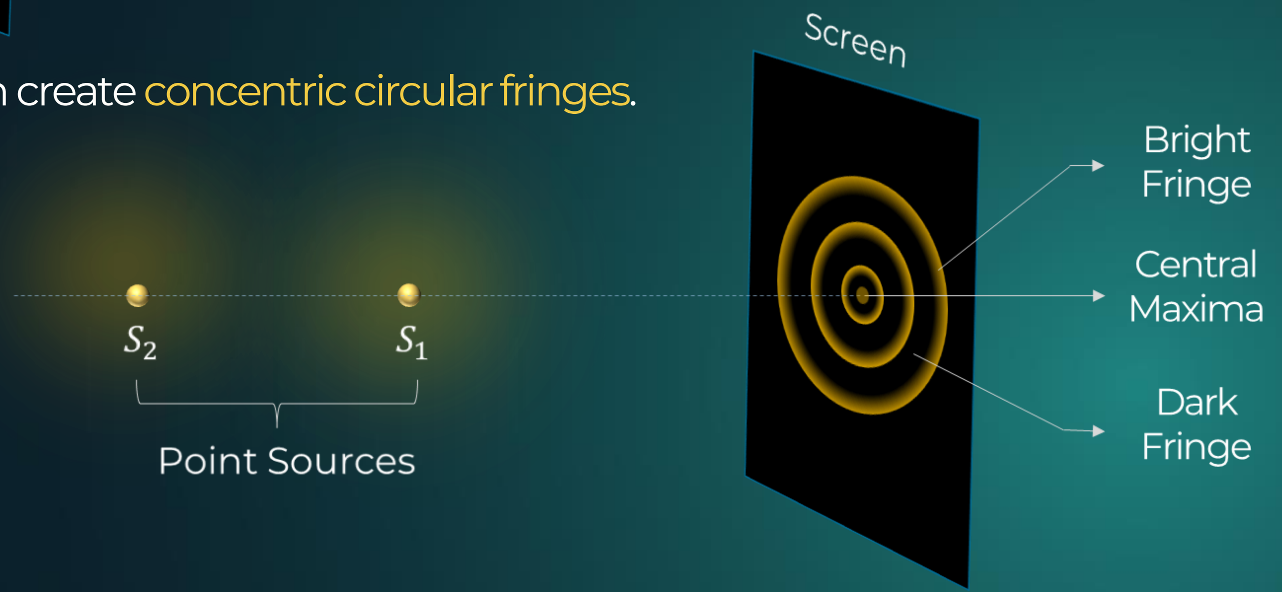
- $I_{max} = 4I_0, I_{min} = 0$



Shape of Fringes on Screen

If we replace slits by pin holes in YDSE, then we will see a Hyperbolic Fringe pattern.

Point sources placed on the perpendicular axis to the screen create concentric circular fringes.

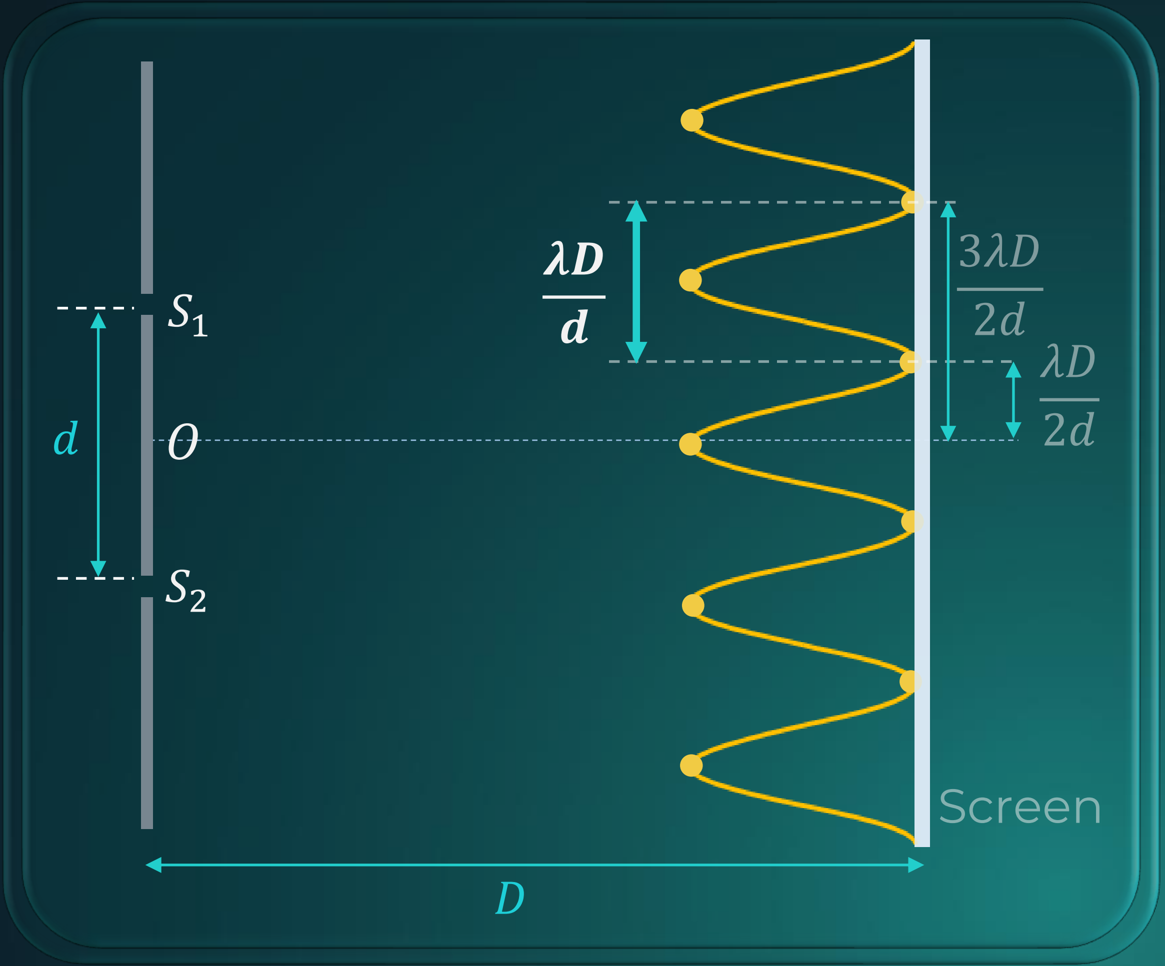


Fringe Width

Fringe width is the distance between two consecutive maxima/minima.

$$\beta = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$



Fringe Width when the setup is inside a Medium

Fringe Width $\beta = \frac{\lambda D}{d}$

Refractive index

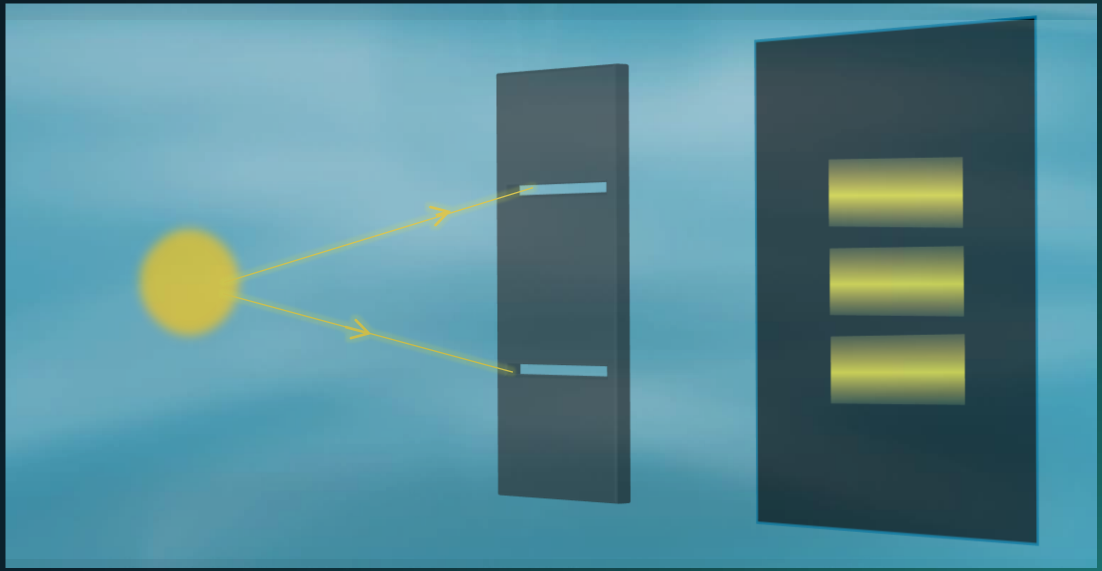
$$\mu = \frac{c}{v}$$

c is speed of light in vacuum

v is speed of light in liquid

$$\mu = \frac{f\lambda_{vacuum}}{f\lambda_{medium}} \Rightarrow \lambda_{medium} = \frac{\lambda_{vacuum}}{\mu}$$

If experimental setup is dipped in liquid



Fringe Width inside liquid, $\beta = \frac{\lambda_{medium} D}{d} \Rightarrow \beta = \frac{\lambda_{vacuum} D}{\mu d}$

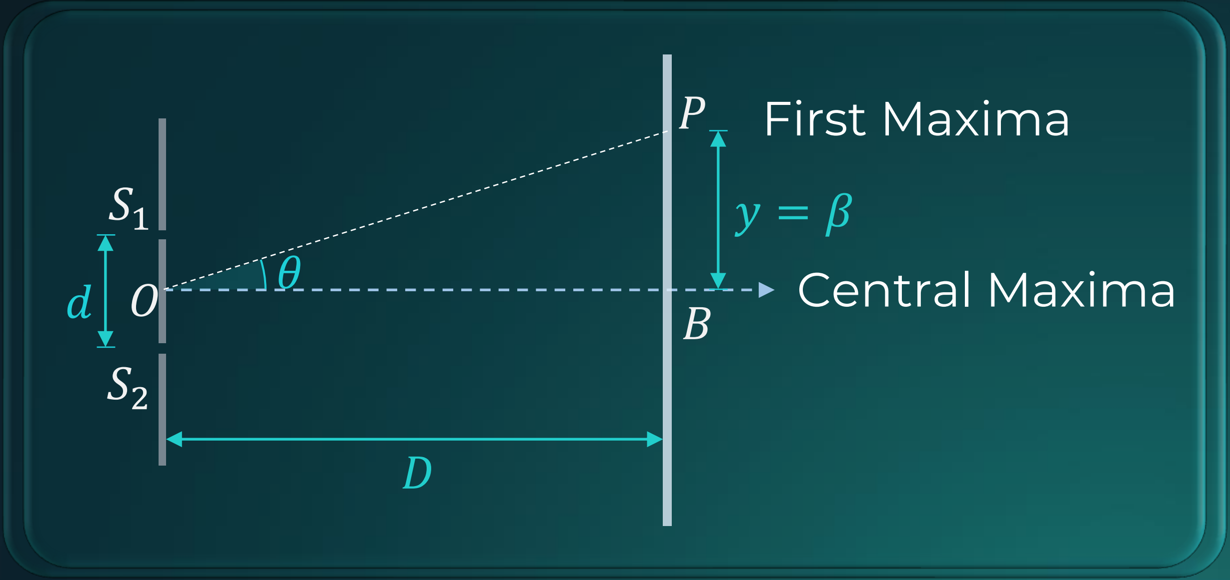
Angular Fringe Width

$\tan\theta \approx \theta = \frac{\beta}{D}$

We know $\beta = \frac{\lambda D}{d}$

After putting β value in $\theta = \frac{\beta}{D}$

$$\boxed{\theta = \frac{\lambda}{d}}$$



Position of Maxima/Minima

- Constructive Interference:

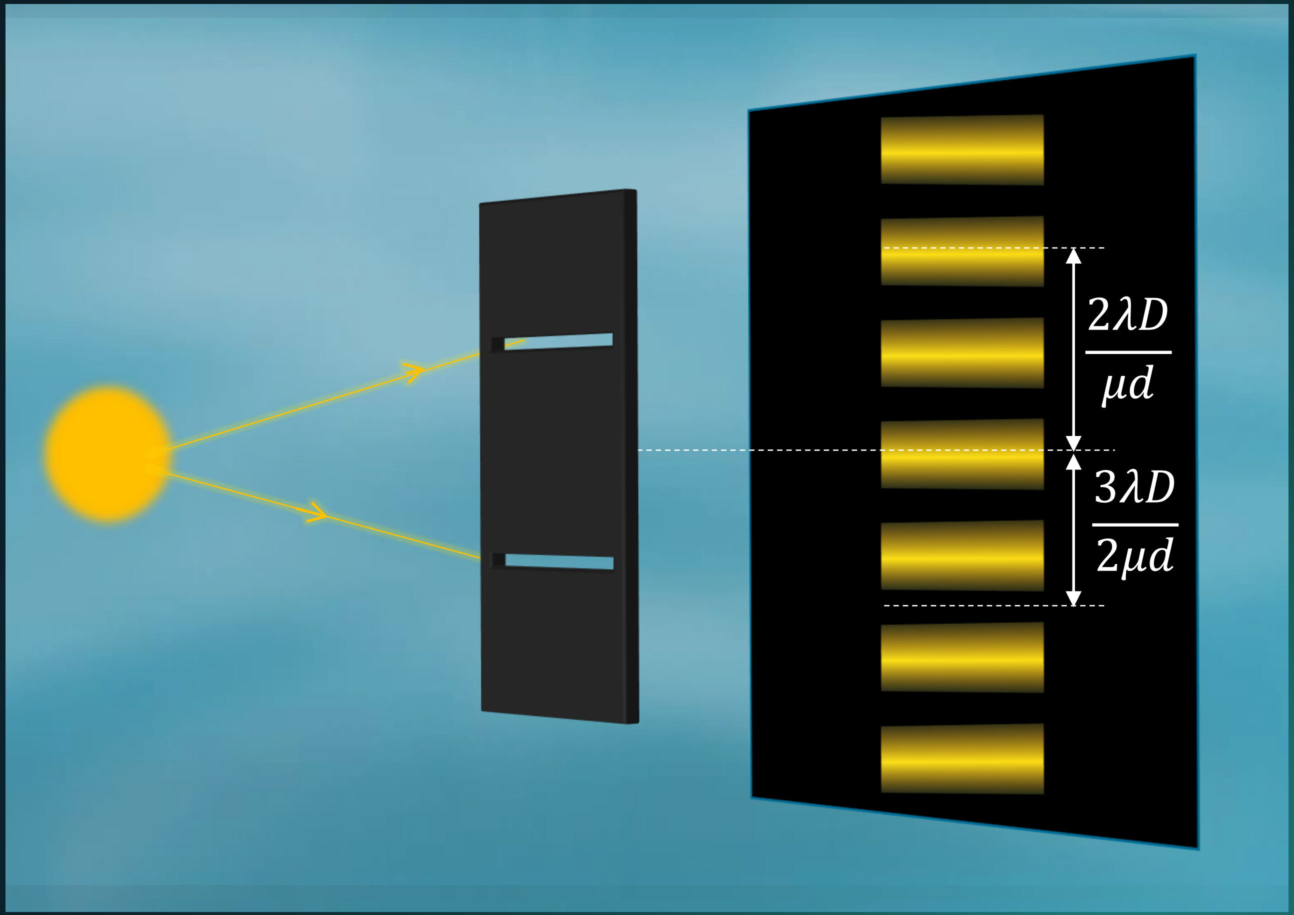
$$\boxed{y = \frac{n\lambda D}{\mu d}}$$

$$y = \pm\frac{\lambda D}{\mu d}, \pm\frac{2\lambda D}{\mu d}, \pm\frac{3\lambda D}{\mu d} \ldots\ldots.$$

- Destructive Interference:

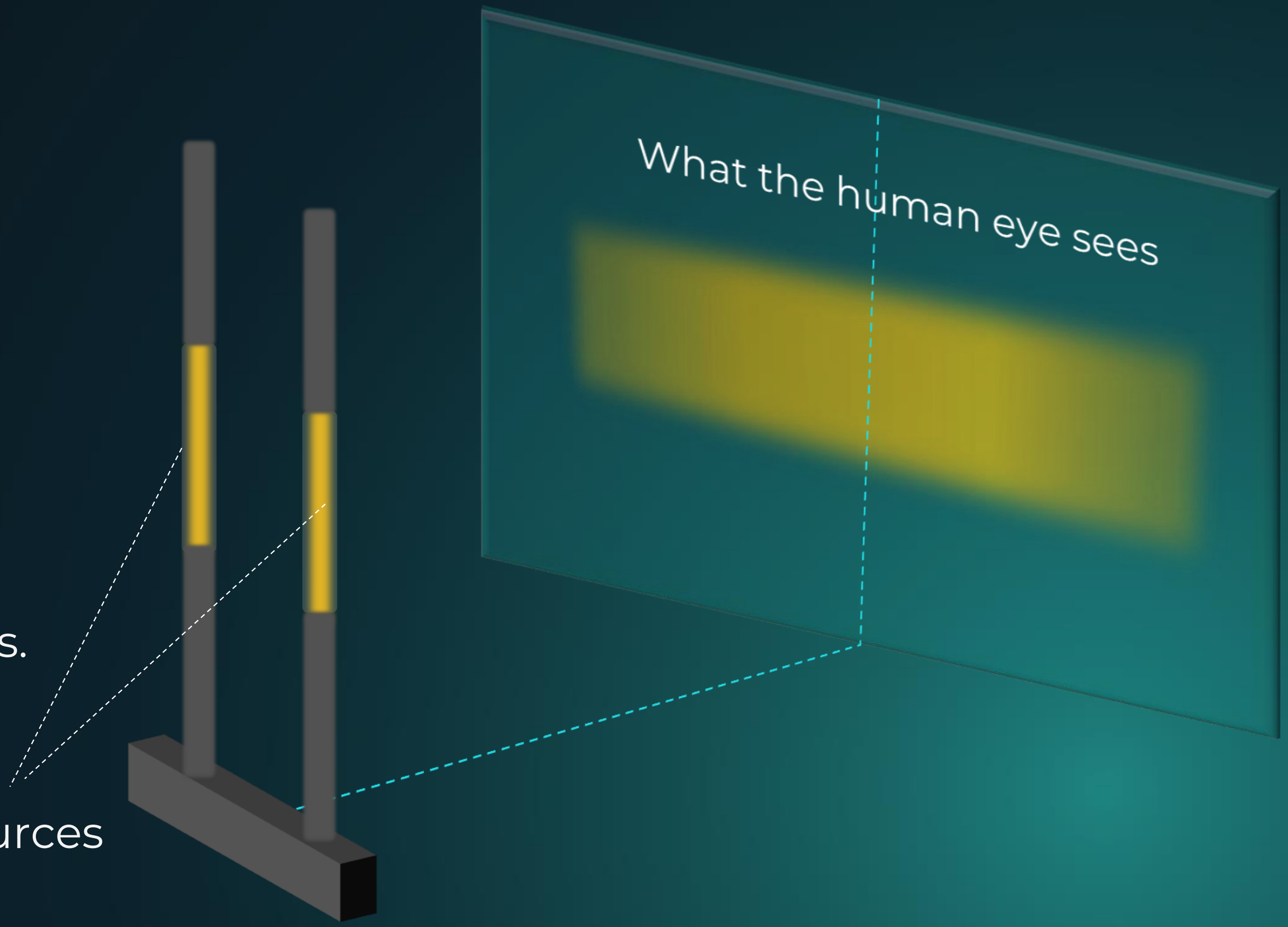
$$\boxed{y = \left(n + \frac{1}{2}\right)\frac{\lambda D}{\mu d}}$$

$$y = \pm\frac{\lambda D}{2\mu d}, \pm\frac{3\lambda D}{2\mu d}, \pm\frac{5\lambda D}{2\mu d} \ldots\ldots.$$



Incoherent Light Sources

- In ultra slow motion, the fringes on the screen flicker.
- However, the human eye cannot the capture the rapidly changing bright and dark fringes.
- So, the eyes see a continuous band of light.
- Net Intensity on the screen is the sum of intensity from two sources.



Incoherent Light Sources

In a Young's double slit interference experiment, the fringe pattern is observed on a screen placed at a distance D from the slits. The slits are separated by a distance d and are illuminated by monochromatic light of wavelength λ. Find the distance from the central point B where the intensity falls to half the maximum.

Solution:

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

Intensity is half the maximum, $4I_0 \cos^2 \frac{\delta}{2} = \frac{1}{2}(4I_0)$

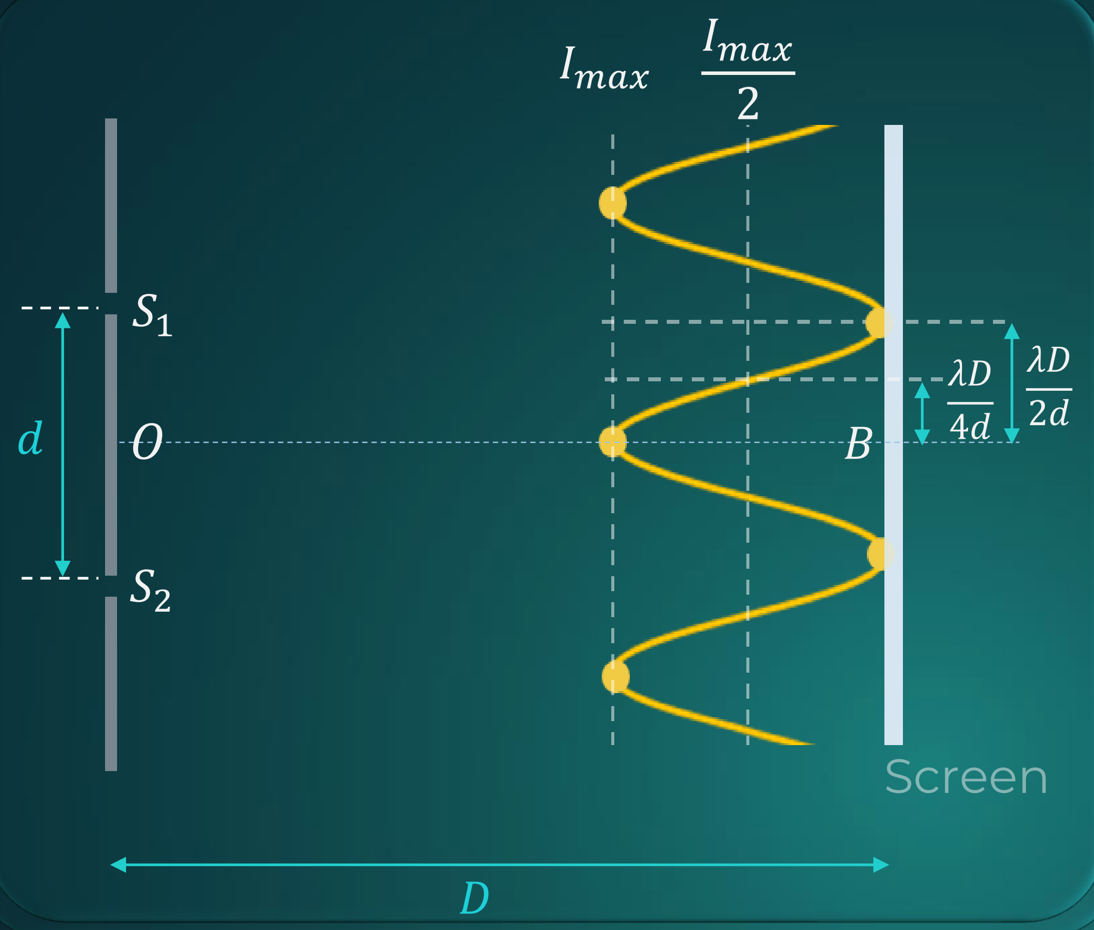
$$\Rightarrow \cos\frac{\delta}{2} = \frac{1}{\sqrt{2}} \Rightarrow \delta = \frac{\pi}{2}$$

Phase difference $(\delta) = \frac{2\pi}{\lambda}\Delta x$

$$\frac{2\pi}{\lambda}\Delta x = \frac{\pi}{2} \Rightarrow \Delta x = \frac{\lambda}{4}$$

Path difference $(\Delta x) = \frac{yd}{D}$

$$\frac{\lambda}{4} = \frac{yd}{D} \Rightarrow \boxed{y = \frac{\lambda D}{4d}}$$



Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. Consider a line passing through S_2 and perpendicular to the line S_1S_2. What is the smallest distance from S_2 where the intensity is minimum?

Given: $d_{S_1S_2} = 2\lambda$

To find: x

Solution: $\sqrt{(2\lambda)^2 + x^2} - x = \left(n + \frac{1}{2}\right)\lambda$

$$\boxed{x = \frac{16\lambda - (2n+1)^2\lambda}{4(2n+1)}}$$

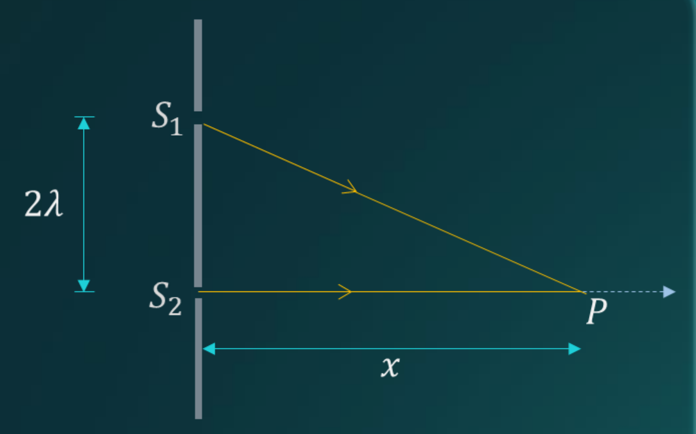
When $x > 0,\ 16\lambda - (2n+1)^2\lambda > 0$

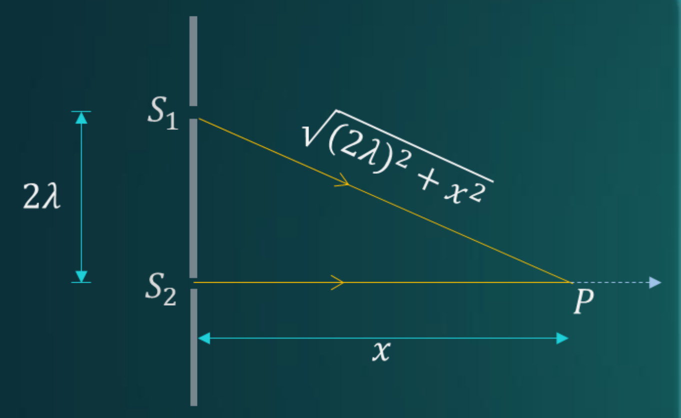
So, $2n + 1 < 4$

$$n < \frac{3}{2}$$

$$\therefore n = 1$$

$$x = \frac{16\lambda - (2n+1)^2\lambda}{4(2n+1)} \Rightarrow \frac{16\lambda - 9\lambda}{12} \qquad \boxed{x = \frac{7\lambda}{12}}$$





A $\frac{\lambda}{2}$

B $\frac{2\lambda}{5}$

C $\frac{3\lambda}{4}$

D $\frac{7\lambda}{12}$

Figure shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let $BP_0 - AP_0 = \lambda/3$ and $D \gg \lambda$. Show that in this case $d = \sqrt{2\lambda D/3}$.

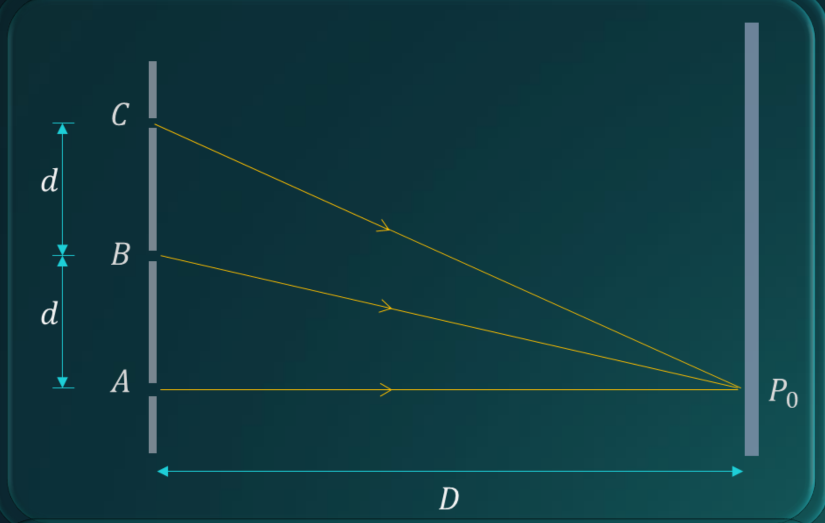
Solution: Optical path difference between BP_0 and AP_0 :

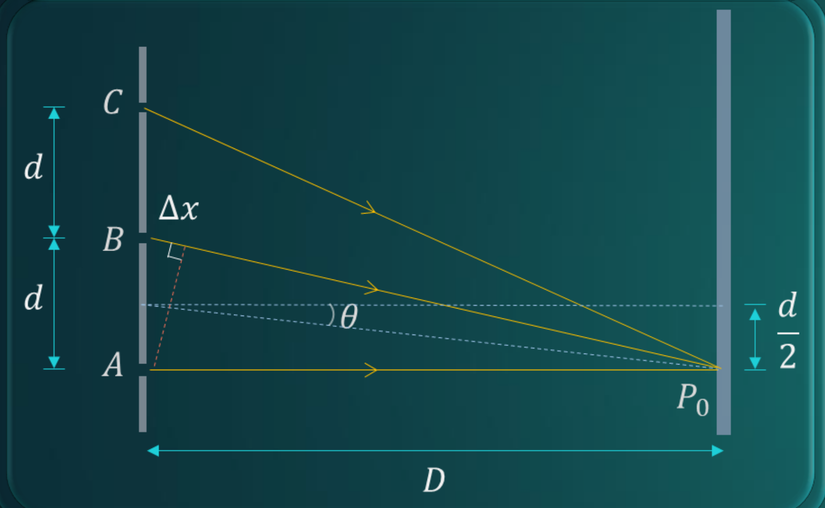
$$\Delta x = d\sin\theta \approx d\tan\theta$$

$$\Delta x = d\left(\frac{d}{2D}\right) = \frac{d^2}{2D}$$

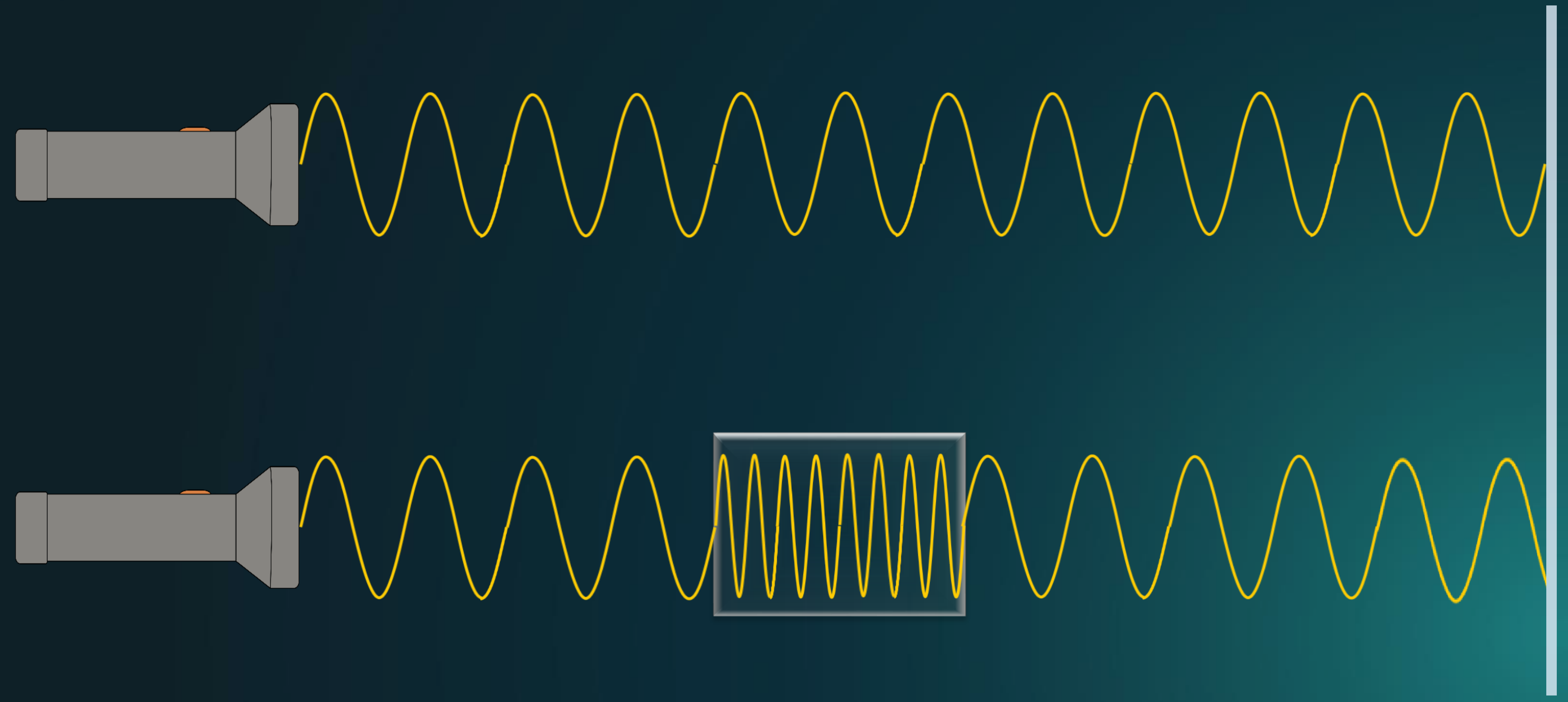
$$\Delta x = \frac{\lambda}{3} = \frac{d^2}{2D}$$

$$\boxed{d = \sqrt{\frac{2\lambda D}{3}}}$$

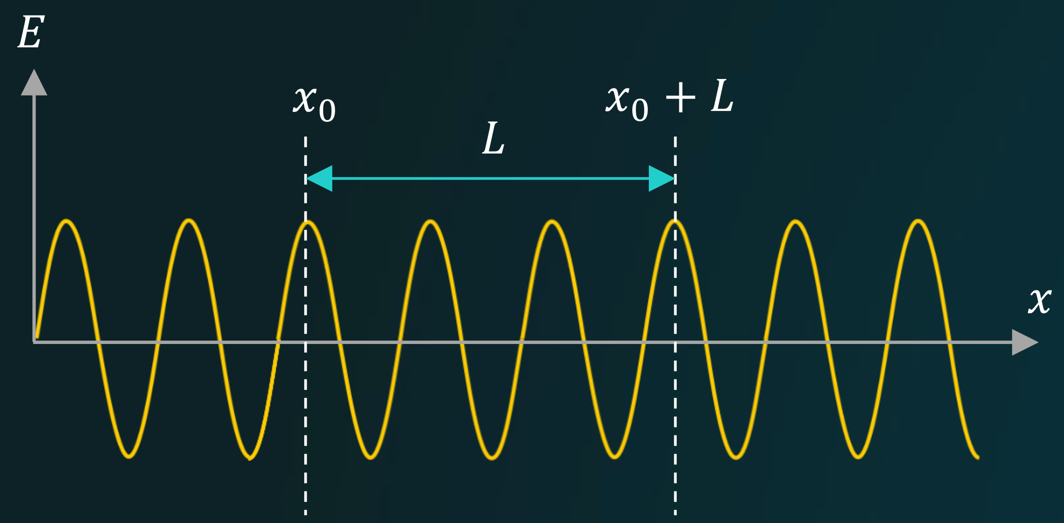




Path Difference between the Two Waves



Optical Path length



Original wave equation:

$$E = E_0 \sin(kx_0 - \omega t)$$

Equation of a wave when it is ahead by a length L:

$$E = E_0 \sin(k(x_0 + L) - \omega t)$$

$$E = E_0 \sin(kx_0 - \omega t + kL)$$

Phase difference $\Delta\phi$

$$k = \frac{2\pi}{\lambda}$$

So, $\Delta\phi = \dfrac{2\pi}{\lambda} L$

$\therefore$ Two points on a wave separated by a path length of L will have a phase difference of $\dfrac{2\pi}{\lambda} L$.

Optical Path length

Optical Path Length in a medium is the corresponding path that light travels in vacuum to undergo the same phase difference.

Phase Difference between points A and B on the wave, travelling in air:

$$\Delta\phi_{air} = \frac{2\pi}{\lambda_a} L_{air}$$

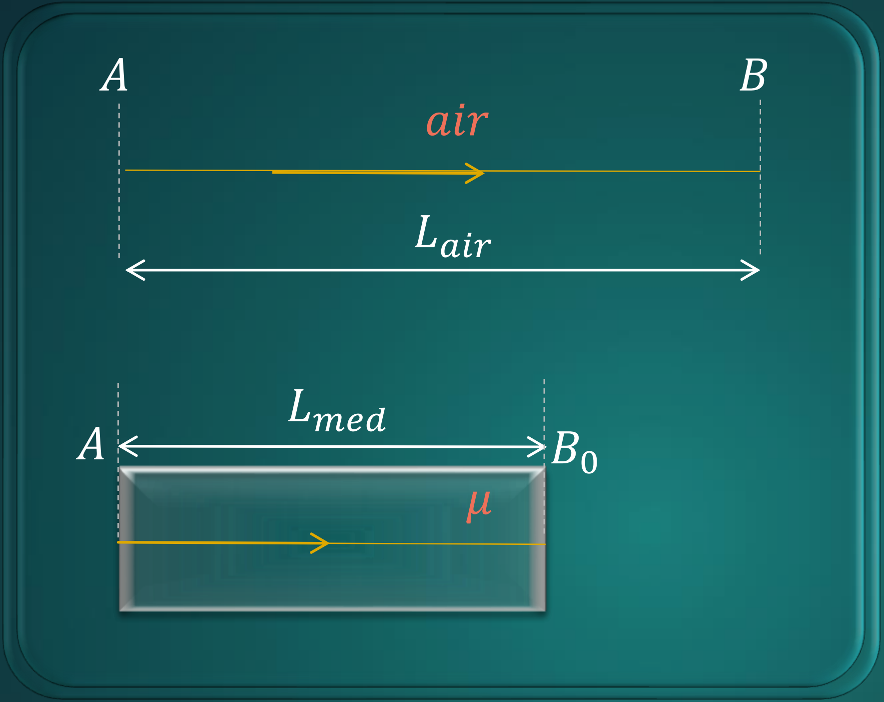
Phase Difference between points A and B_0 on the wave, travelling in medium:

$$\Delta\phi_{med} = \frac{2\pi}{\lambda_{med}} L_{med} = \frac{2\pi}{\left(\frac{\lambda_a}{\mu}\right)} L_{med} \qquad \left\{\lambda_{med} = \frac{\lambda_a}{\mu}\right\}$$

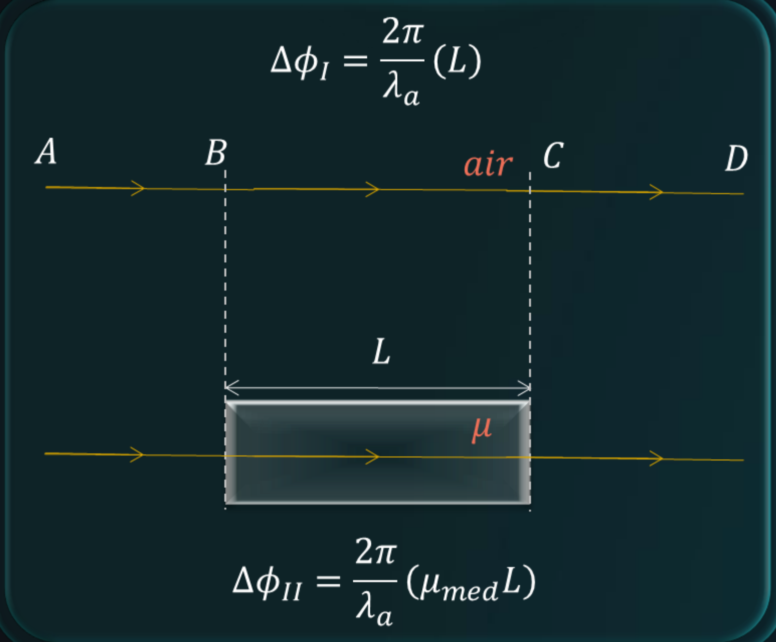
$$\Delta\phi_{med} = \frac{2\pi}{\lambda_a} \mu L_{med}$$

If, $\Delta\phi_{air} = \Delta\phi_{med}$

$$\frac{2\pi}{\lambda_a} L_{air} = \frac{2\pi}{\lambda_a} \mu L_{med} \Rightarrow \boxed{L_{air} = \mu L_{med}}$$



Optical Path Difference



Optical Path Difference:

$$OPD = OPL_{II} - OPL_I = \boxed{(\mu_{med} - 1)L}$$

Phase Difference:

$$\delta = \Delta\phi_{II} - \Delta\phi_I = \frac{2\pi}{\lambda_a}[(\mu_{med} - 1)L]$$

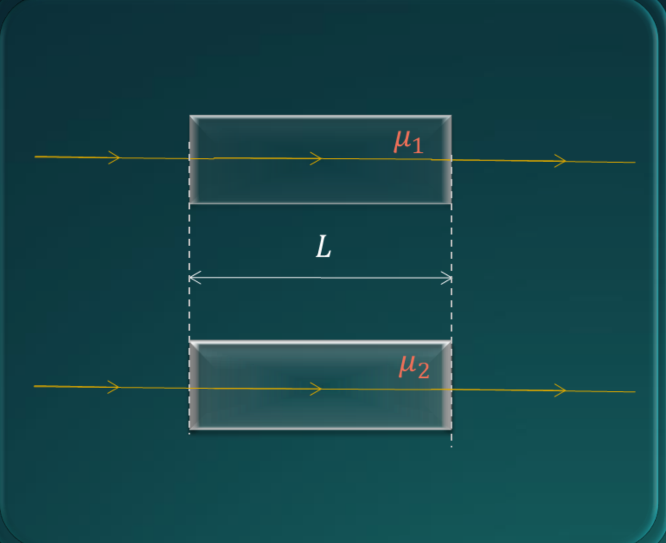
$$\boxed{\delta = \frac{2\pi}{\lambda_a}[OPD]}$$

Optical Path Length in air:

$$OPL_I = AB + \mu_{air}L + CD = AB + L + CD$$

Optical Path Length in medium:

$$OPL_{II} = AB + \mu_{med}L + CD$$



Optical Path Difference:

$$OPD = |OPL_{II} - OPL_I| = \boxed{|\mu_2 - \mu_1|L}$$

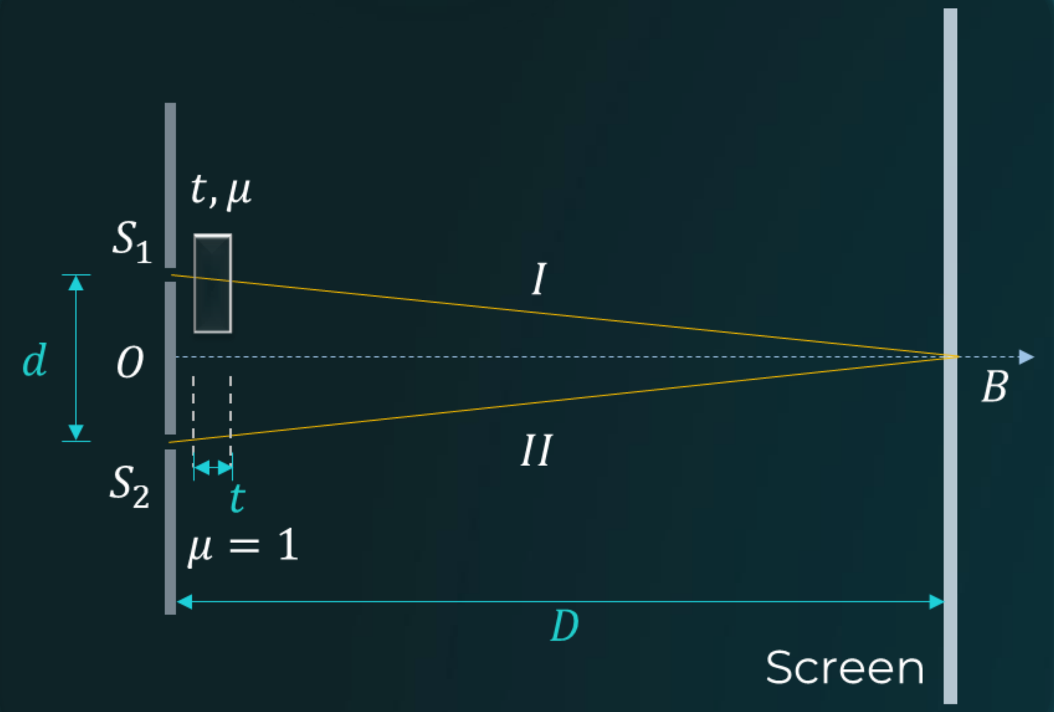
Phase Difference in medium 1 and 2:

$$\Delta\phi_I = \frac{2\pi}{\lambda_a}(\mu_1 L) \quad \text{and} \quad \Delta\phi_{II} = \frac{2\pi}{\lambda_a}(\mu_2 L)$$

Phase Difference:

$$\delta = |\Delta\phi_{II} - \Delta\phi_I| \qquad \boxed{\delta = \frac{2\pi}{\lambda_a}(|\mu_2 - \mu_1|L)}$$

Thin Transparent Film in YDSE



Optical Path Travelled by wave I: $\mu t + (S_1B - t)$

Optical Path Travelled by wave II: S_2B

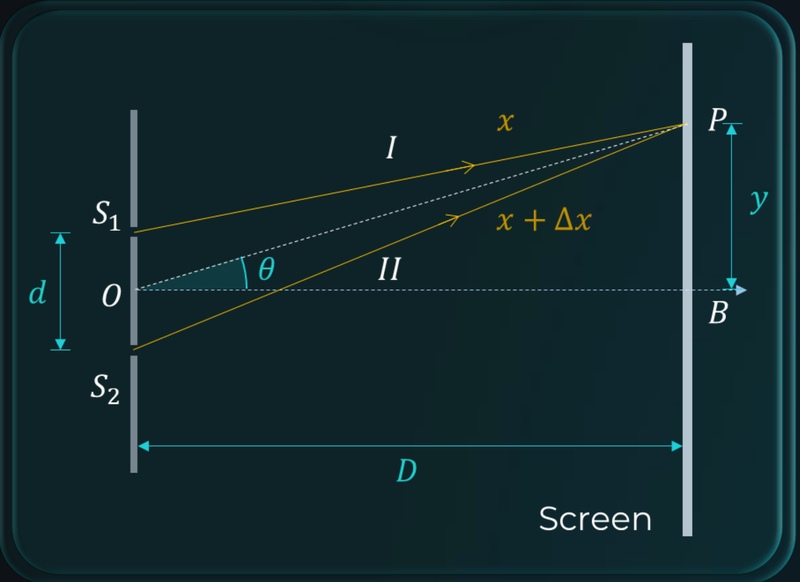
Optical Path Difference:

$$OPD = I - II = \mu t + (S_1B - t) - S_2B$$

$$OPD = \mu t - t = \boxed{(\mu - 1)t} \quad \{S_1B = S_2B\}$$

- Introduction of slab causes change in OPD by $(\mu - 1)t$.
- Path length of the ray S_1B increases when it encounters the thin film.
- Now the central maxima does not lie at B.

Thin Transparent Film in YDSE

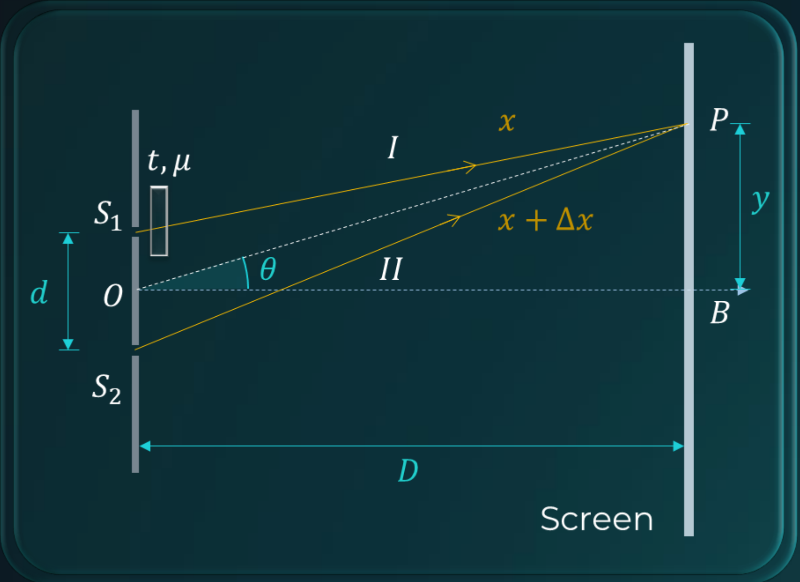


Optical Path Difference:

$\Delta x = II - I$

$\Delta x = S_2P - S_1P$

$$\boxed{\Delta x = \frac{yd}{D}}$$



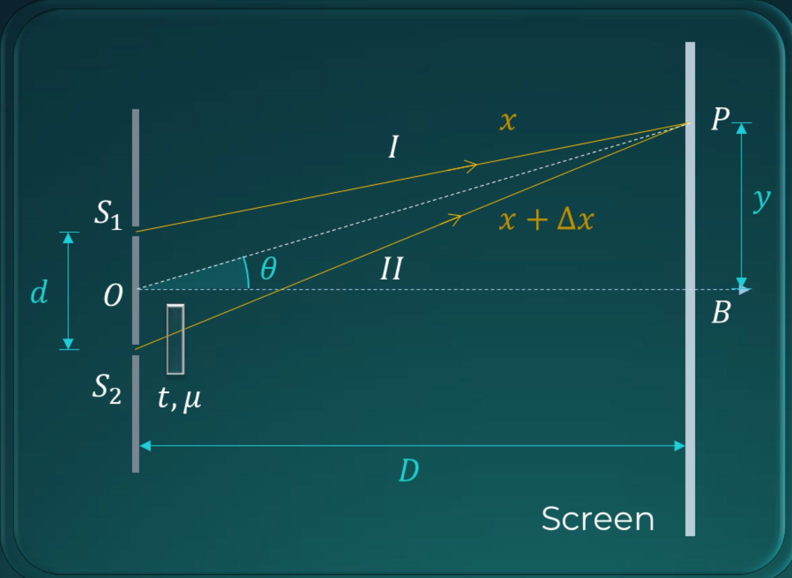
- After inserting thin film, light ray S_1P has travelled an extra path of $(\mu - 1)t$.

Total Optical Path Difference at point P:

$\Delta x = II - I$

$\Delta x = S_2P - \left(S_1P + t(\mu - 1)\right)$

$$\boxed{\Delta x = \frac{yd}{D} - t(\mu - 1)}$$



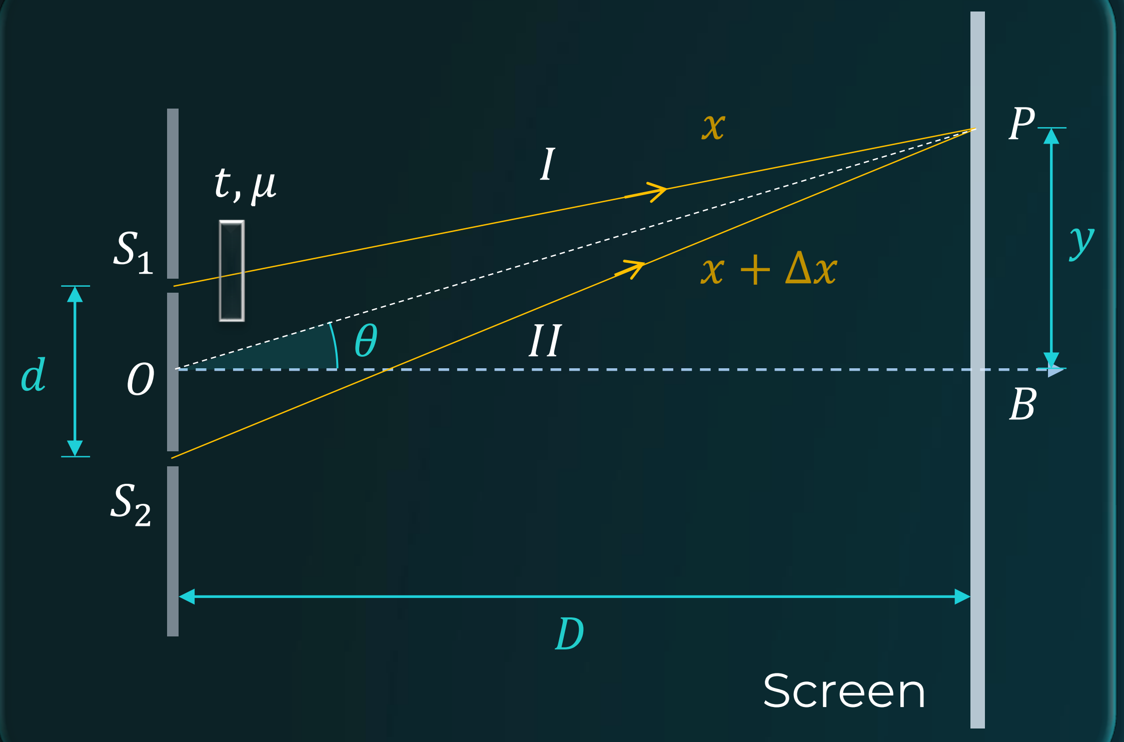
- After inserting thin film, light ray S_2P has travelled an extra path of $(\mu - 1)t$.

Total Optical Path Difference at point P: $\Delta x = II - I$

$\Delta x = \left(S_2P + t(\mu - 1)\right) - S_1P$

$$\boxed{\Delta x = \frac{yd}{D} + t(\mu - 1)}$$

Shift in Central Maxima



At central maxima,

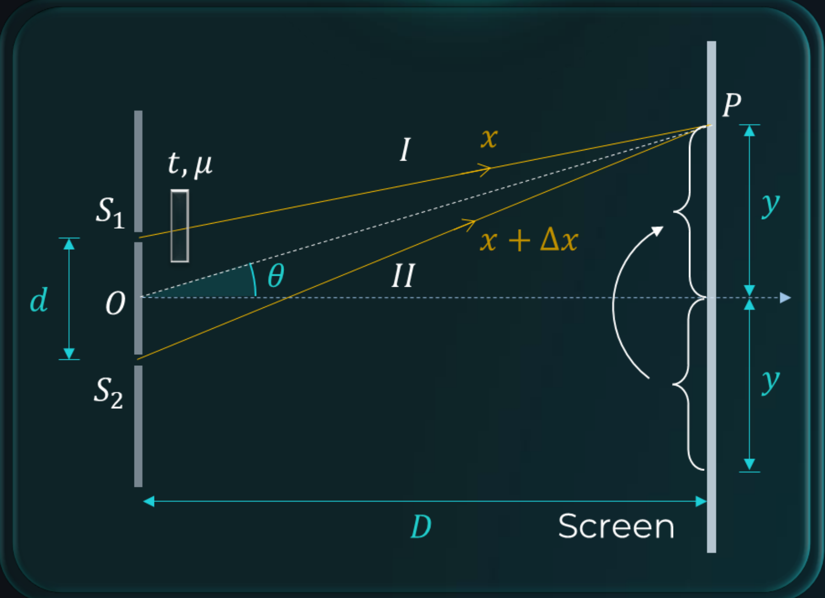
$OPD = 0$

$$\frac{yd}{D} - (\mu - 1)t = 0$$

$$y = \frac{(\mu - 1)tD}{d}$$

Shift in central maxima = $\boxed{\dfrac{(\mu - 1)tD}{d}}$

Number of Fringes Shifted



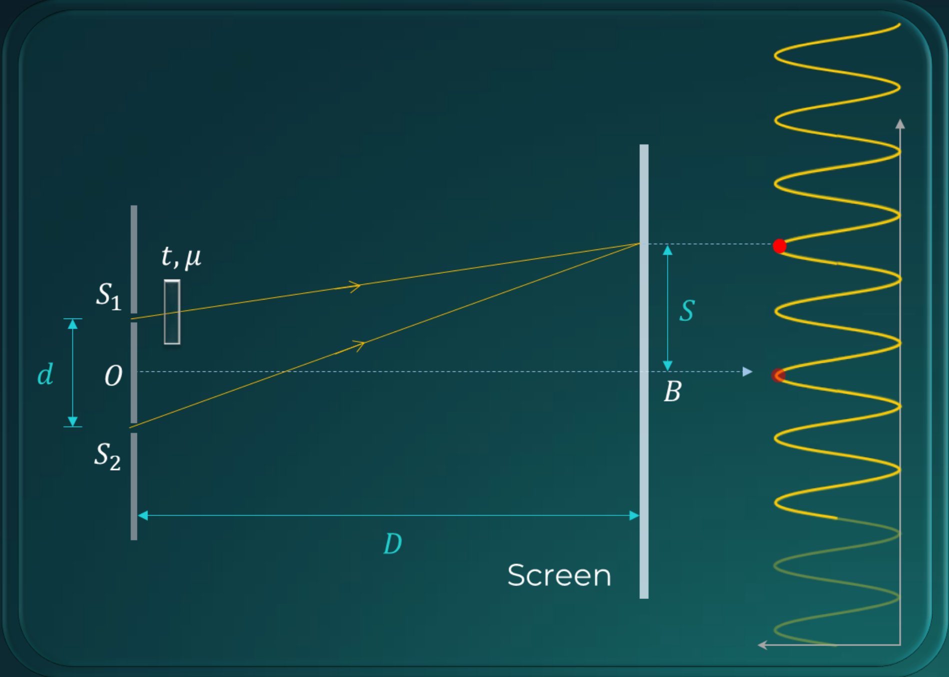
Shift in central maxima:

$$\boxed{\frac{(\mu - 1)tD}{d}}$$

Number of Fringes shifted $= \dfrac{\textit{shift in central maxima}}{\textit{fringe width}}$

$$n = \frac{y}{\beta} = \frac{(\mu - 1)tD}{d} \times \frac{d}{\lambda D}$$

$$\boxed{n = \frac{(\mu - 1)t}{\lambda}}$$



Shift in central maxima:

$$\boxed{S = \frac{(\mu - 1)tD}{d}}$$

Number of fringes shifted:

$$\boxed{n = \frac{(\mu - 1)t}{\lambda}}$$

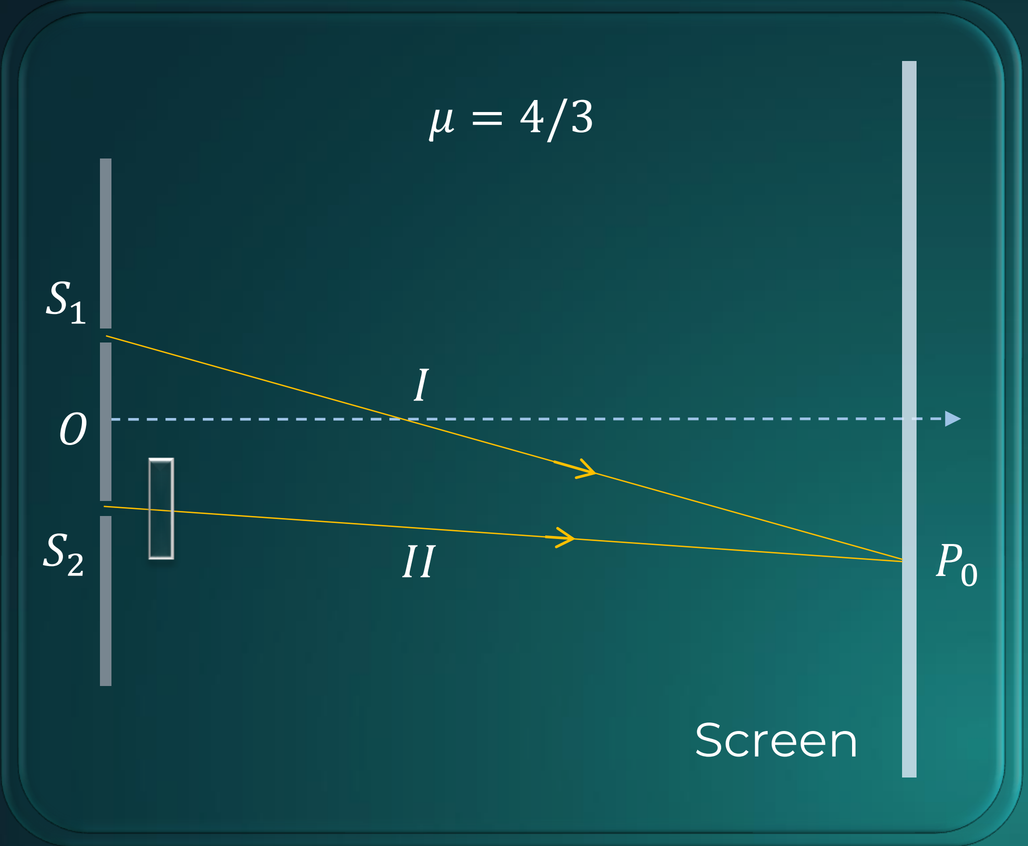
The Young's double slit experiment is done in a medium of refractive index $4/3$. A light of $600\ nm$ wavelength is falling on the slits having $0.45\ mm$ separation. The lower slit S_2 is covered by a thin glass sheet of thickness $10.2\ \mu m$ and refractive index 1.5. The interference pattern is observed on a screen placed $1.5\ m$ from the slits. Find the location of the central maximum on the y –axis.

A. $7.5\ mm$

B. $10.2 mm$

C. $4.25\ mm$

D. $1.25\ mm$



Given: $\mu_{med} = \frac{4}{3}, d = 0.45\ mm, D = 1.5\ m, \lambda = 600\ nm, t = 10.2\ \mu m, \mu_t = 1.5$

To find: Position of central maxima

Solution: $OPD = I - II$

$$OPD = (S_1P_0)\mu_{med} - \left(\mu_g t + [(S_2P_0)\mu_{med} - \mu_{med}t]\right)$$

$$OPD = (S_1P_0 - S_2P_0)\mu_{med} - t\left(\mu_g - \mu_{med}\right)$$

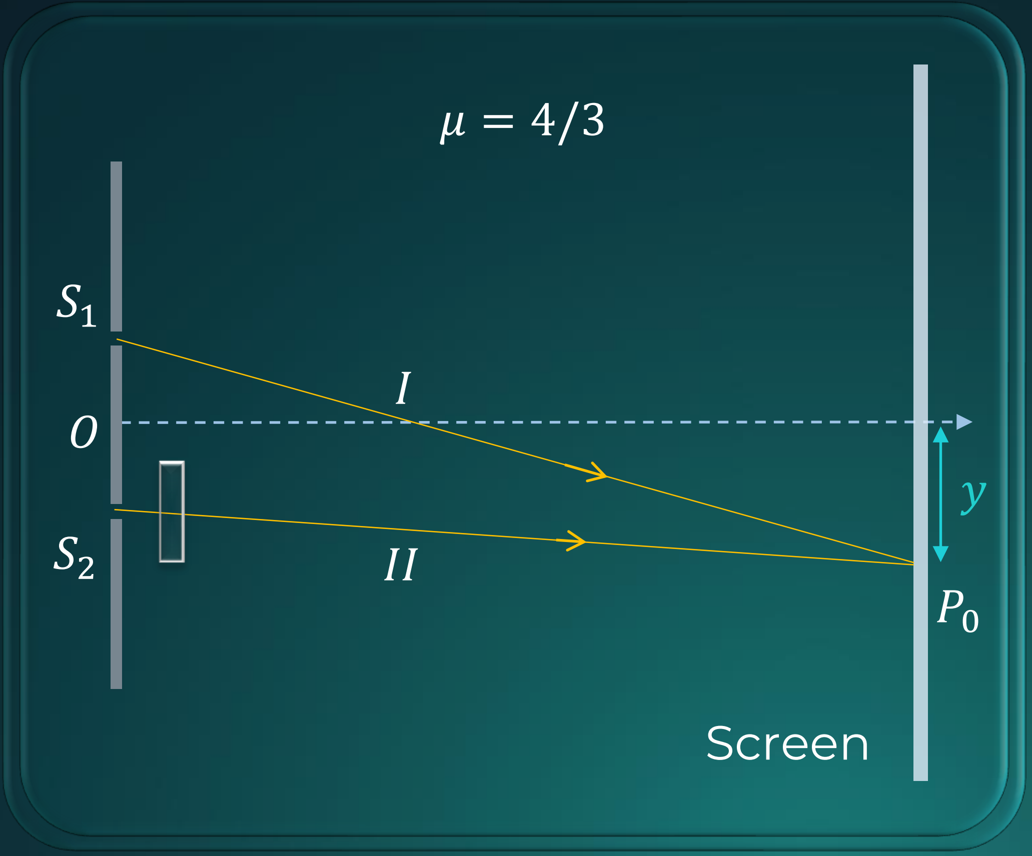
$$OPD = \left(\frac{yd}{D}\right)\mu_{med} - t\left(\mu_g - \mu_{med}\right)$$

At central maxima $OPD = 0$

$$\left(\frac{yd}{D}\right)\mu_{med} - t\left(\mu_g - \mu_{med}\right) = 0$$

$$y = \frac{tD}{d\mu_{med}}\left(\mu_g - \mu_{med}\right)$$

Substituting all the values we get, $\boxed{y = 4.25\ mm}$



In YDSE, find the thickness of a glass slab $(\mu = 1.5)$ which should be kept in front of upper slit (S_1) so that the central maxima is formed at a place where 5^{th} bright fringe was lying earlier (before inserting slab). $(\lambda = 5000\ \dot{A})$

Given: $\lambda = 5000\ \dot{A}, \quad \mu = 1.5,$

To find: t

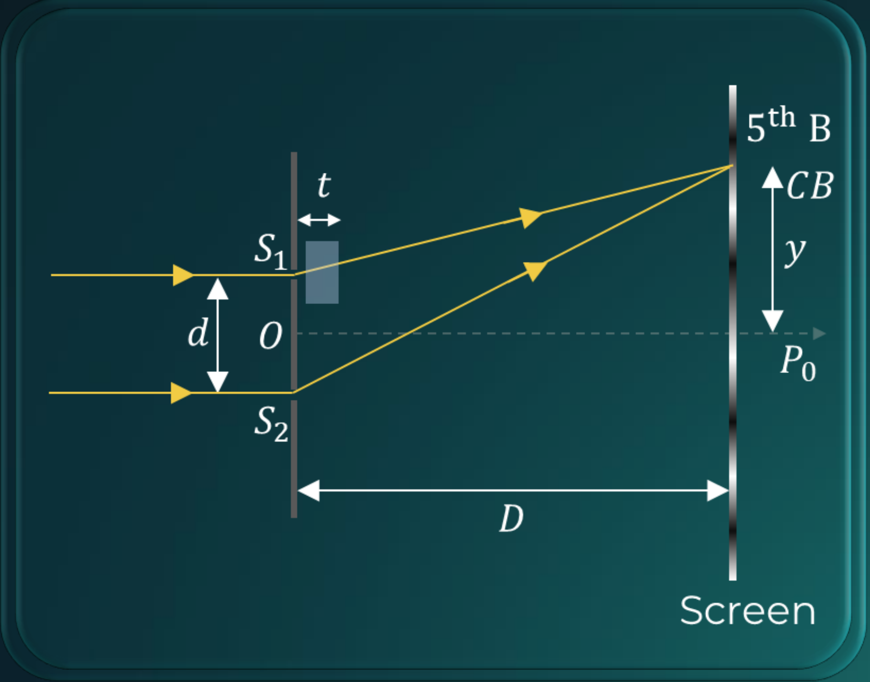
Solution: $y_{5B} = \dfrac{5D\lambda}{d} \quad \left(y_{nB} = \dfrac{nD\lambda}{d}\right)$

Shift in central maxima: $y = \dfrac{(\mu - 1)tD}{d}$

$$\frac{(\mu - 1)tD}{d} = \frac{5D\lambda}{d}$$

$$t = \frac{5\lambda}{\mu - 1} = 5 \times \frac{5000 \times 10^{-10}}{1.5 - 1}$$

$$\boxed{t = 5\ \mu m}$$



A $5\ \mu m$

B $10\ \mu m$

C $1\ \mu m$

D $0.5\ \mu m$

Two transparent slabs, having equal thickness but different refractive indices μ_1 and μ_2 $(\mu_1 > \mu_2)$, are pasted side by side to form a composite slab. This slab is placed just after the double slits in a Young's experiment so that the light from one slit goes through one material and light through other slit goes through other material. What should be the minimum thickness of the slab so that there is a minimum at point P_0 which is equidistant from the slits?

Given: Wavelength = λ, Refractive index = μ_1 & μ_2

To find: $t_1 = t_2 = t = ?$

Solution: Path difference due to both the slabs at P_0:

$$(\mu_1 - 1)t - (\mu_2 - 1)t = (\mu_1 - \mu_2)t$$

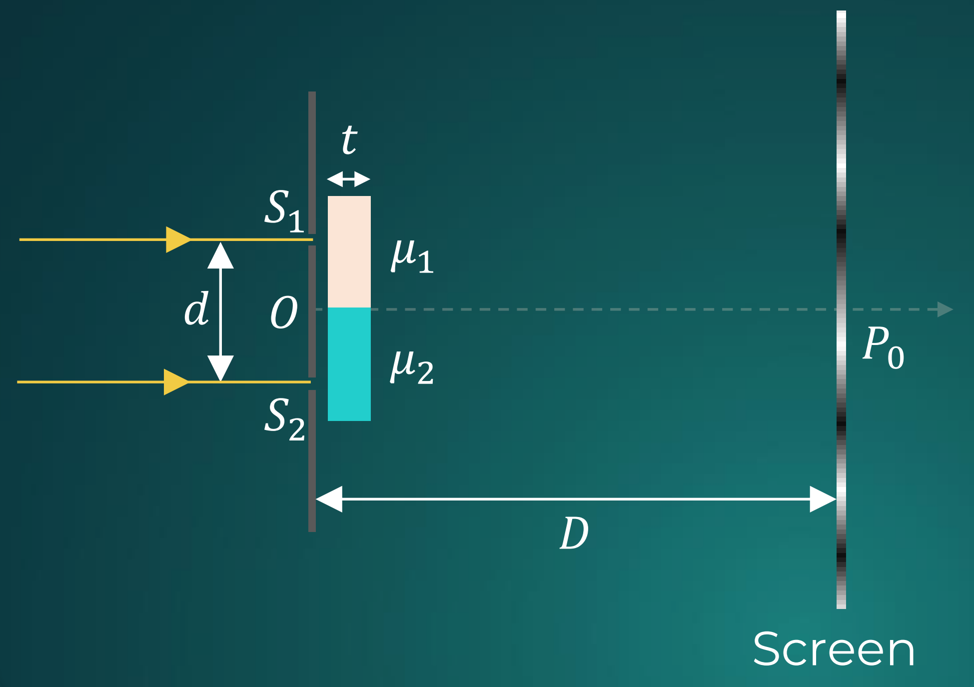
For minima at P_0, we know,

$$\Delta x = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow (\mu_1 - \mu_2)t = \left(n + \frac{1}{2}\right)\lambda$$

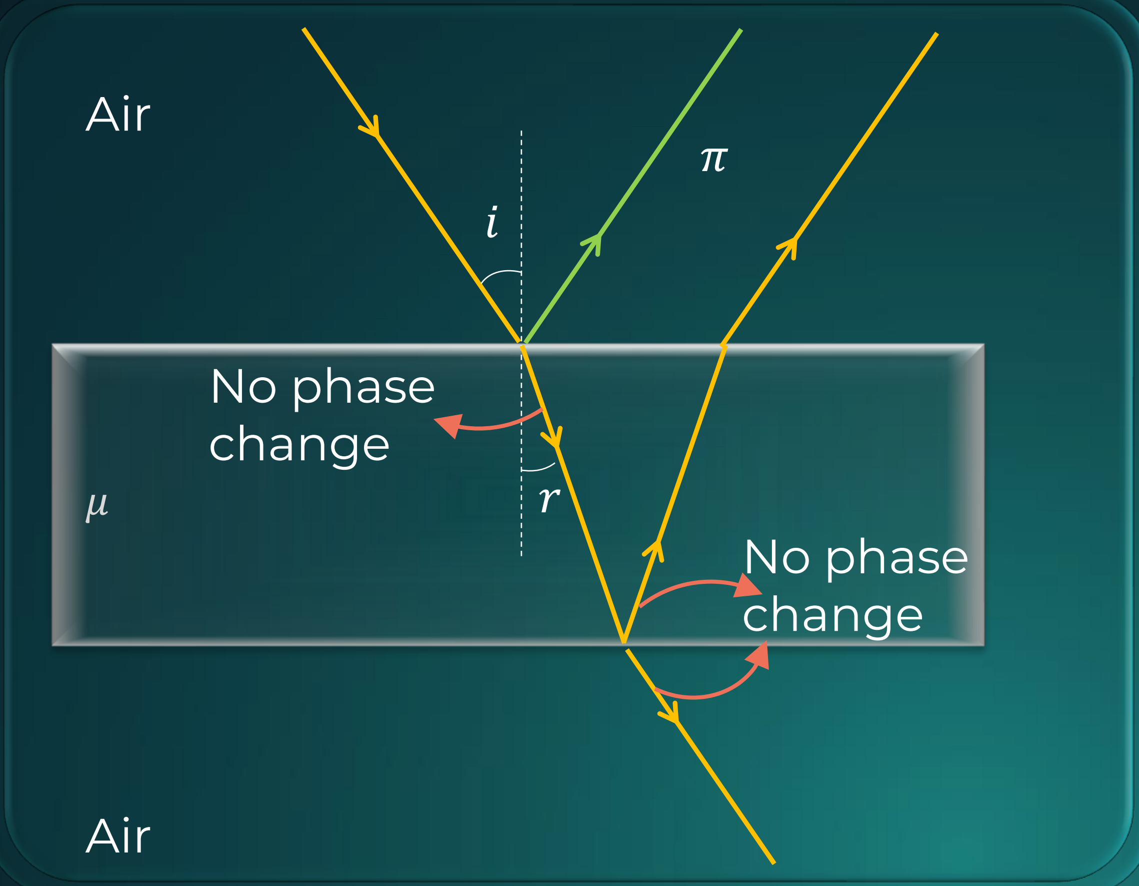
For t to be minimum, put $n = 0$,

$$\boxed{t = \frac{\lambda}{2(\mu_1 - \mu_2)}}$$



Phase change in Reflection & Refraction

Reflected from	Phase change
Denser Medium	Yes
Rarer Medium	No



A narrow slit S transmitting light of wavelength λ is placed at a distance d above a large plane mirror. The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance D from the slit.

(a) What will be the intensity at a point just above O.

Given: Wavelength $= \lambda$, Distance of screen $= D$, Distance of source from the mirror $= d$

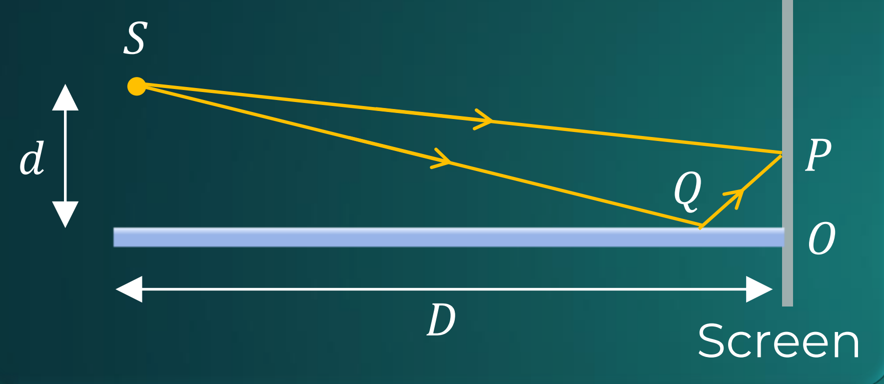
To find: Intensity just above O

Solution: $SP \approx SQP \rightarrow$ path difference $= 0$ (when P is just above O)

$\delta = \pi$ (due to reflection of light from a denser medium)

If phase difference is odd integral multiple of π, destructive interference will take place.

$$\boxed{I_{net} = 0}$$



A narrow slit S transmitting light of wavelength λ is placed at a distance d above a large plane mirror. The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance D from the slit.

(b) At what distance from O does the first maximum occur?

Given: Wavelength $= \lambda$, Distance of screen $= D$, Distance of source from the mirror $= d$

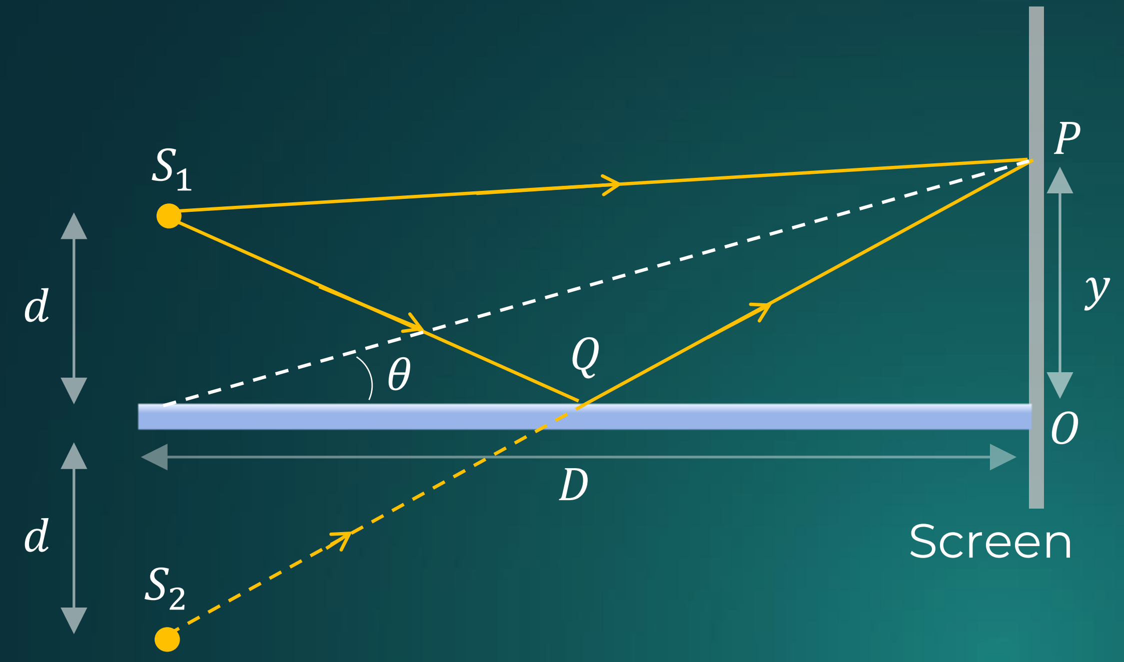
To find: $y_{1st\ maxima}$

Solution: $\Delta x = 2d \sin\theta = 2d \tan\theta = \dfrac{2dy}{D}$

$$\Delta x_{net} = \frac{2dy}{D} + \frac{\lambda}{2} = n\lambda = (1)\lambda$$

$$\Rightarrow \frac{2dy}{D} = \frac{\lambda}{2}$$

$$\boxed{y = \frac{\lambda D}{4d}}$$



A narrow slit S transmitting light of wavelength λ is placed at a distance d above a large plane mirror. The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance D from the slit.

(c) If $d = 1\ mm$, $D = 1\ m$ and $\lambda = 700\ nm$, then find the fringe width.

Given: $\lambda = 700\ nm,\ D = 1\ m,\ d = 1\ mm$

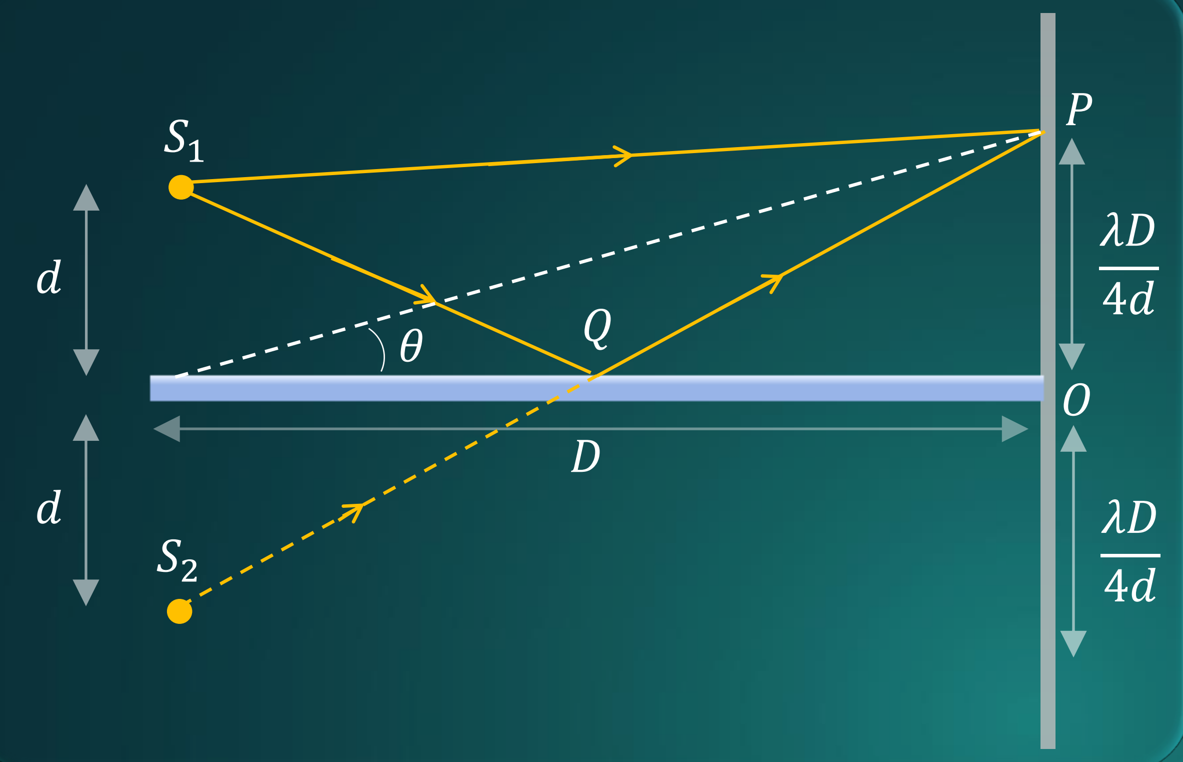
To find: Fringe width $= \beta$

Solution:

$$\beta = 2 \times \left(\frac{\lambda D}{4d}\right) = \frac{\lambda D}{2d}$$

$$\beta = \frac{700 \times 10^{-9} \times 1}{2 \times 10^{-3}} = 3.5 \times 10^{-4}\ m$$

$$\boxed{\beta = 0.35\ mm}$$



A narrow slit S transmitting light of wavelength λ is placed at a distance d above a large plane mirror. The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance D from the slit.

(d) If the mirror reflects only $64\ \%$ of the light energy falling on it, what will be the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen?

Given: Wavelength $= \lambda$, Distance of screen $= D$, Distance of source from the mirror $= d$

$I_2 = 0.64\ I_0$

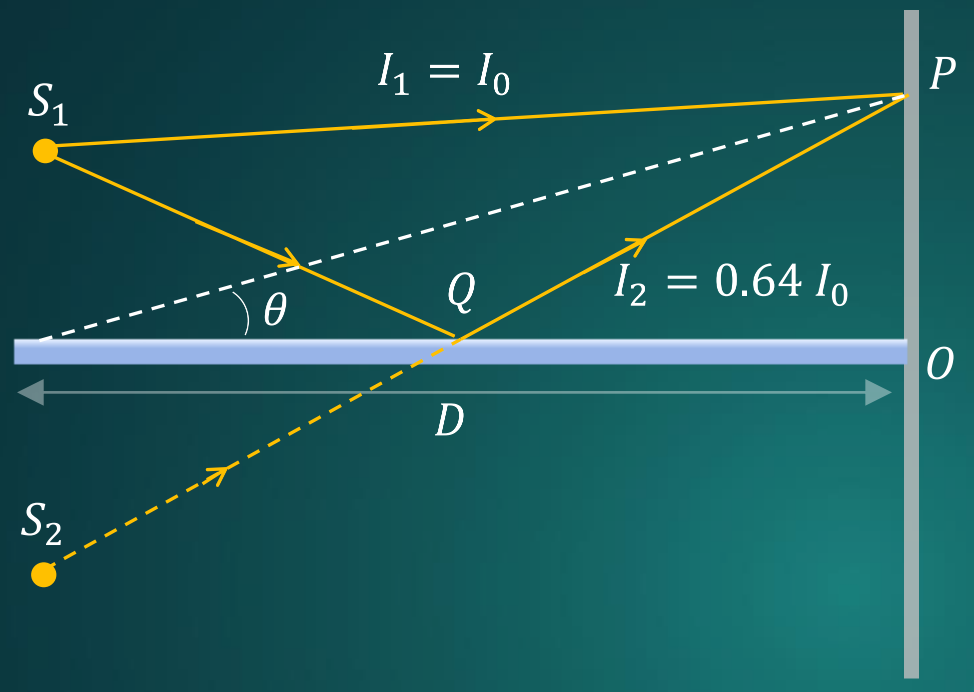
To find: $\dfrac{I_{max}}{I_{min}}$

Solution: $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} \Rightarrow \frac{\left(\sqrt{I_0} + \sqrt{0.64\ I_0}\right)^2}{\left(\sqrt{I_0} - \sqrt{0.64\ I_0}\right)^2}$$

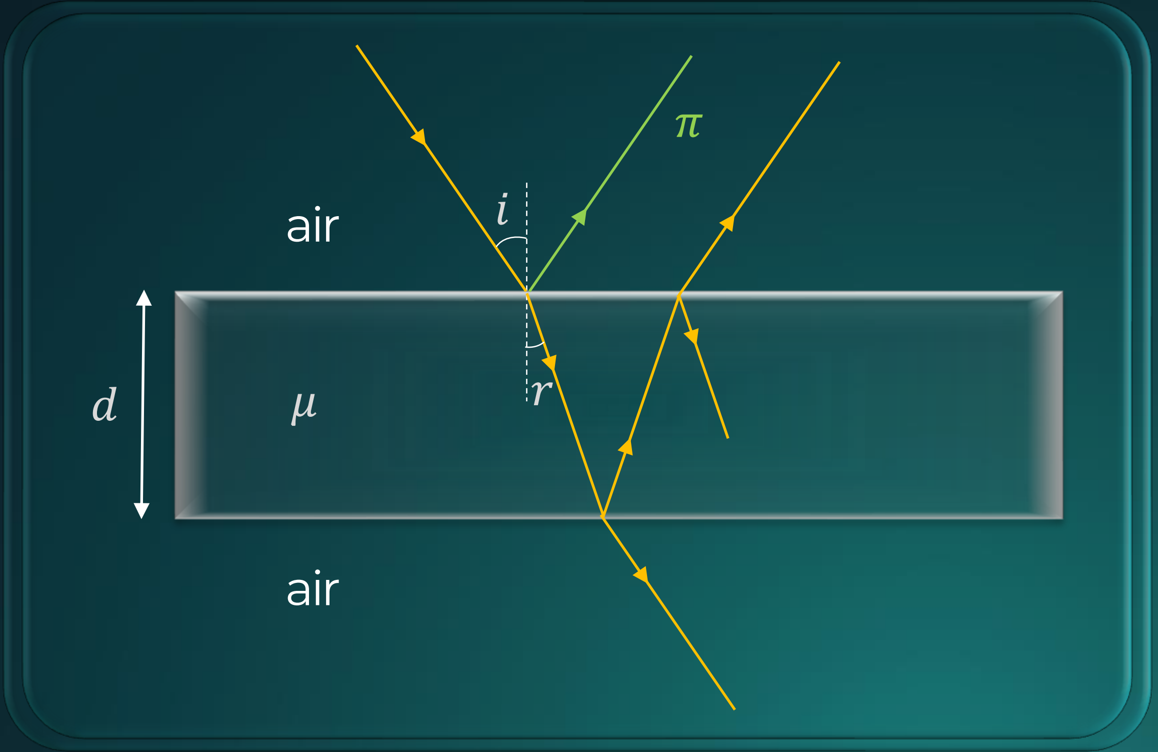
$$= \frac{(1 + 0.8)^2}{(1 - 0.8)^2} = \frac{(1.8)^2}{(0.2)^2}$$

$$\boxed{\frac{I_{max}}{I_{min}} = 81}$$



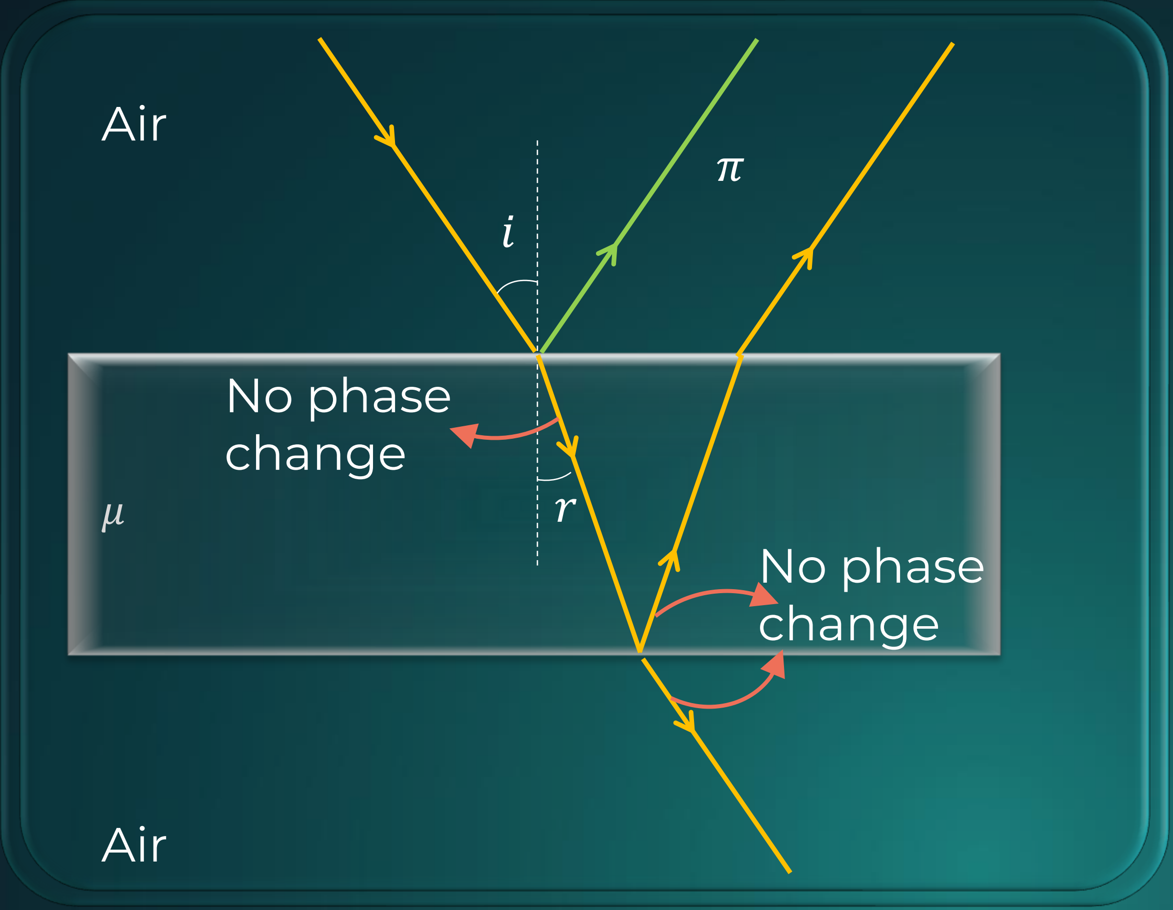
Condition for Thin Film Interference

- The film thickness should be comparable to the wavelength of light $(d \approx \lambda)$.
- The angle of incidence should be small $i \approx 0.$
- The incident light should be white (non - monochromatic).

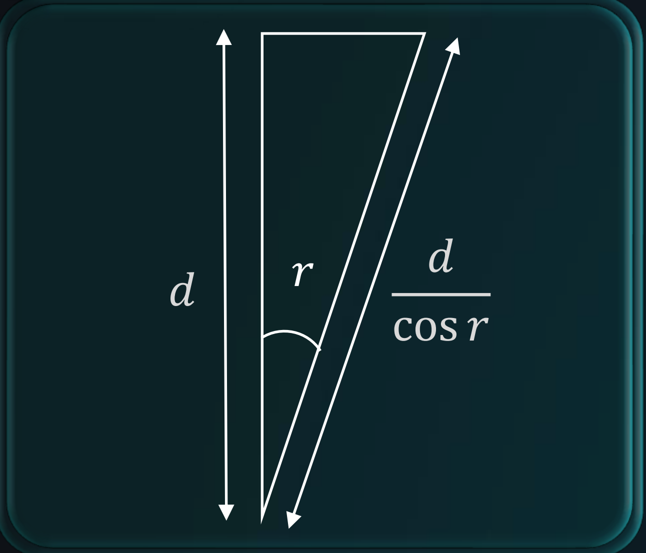


Thin Film Interference

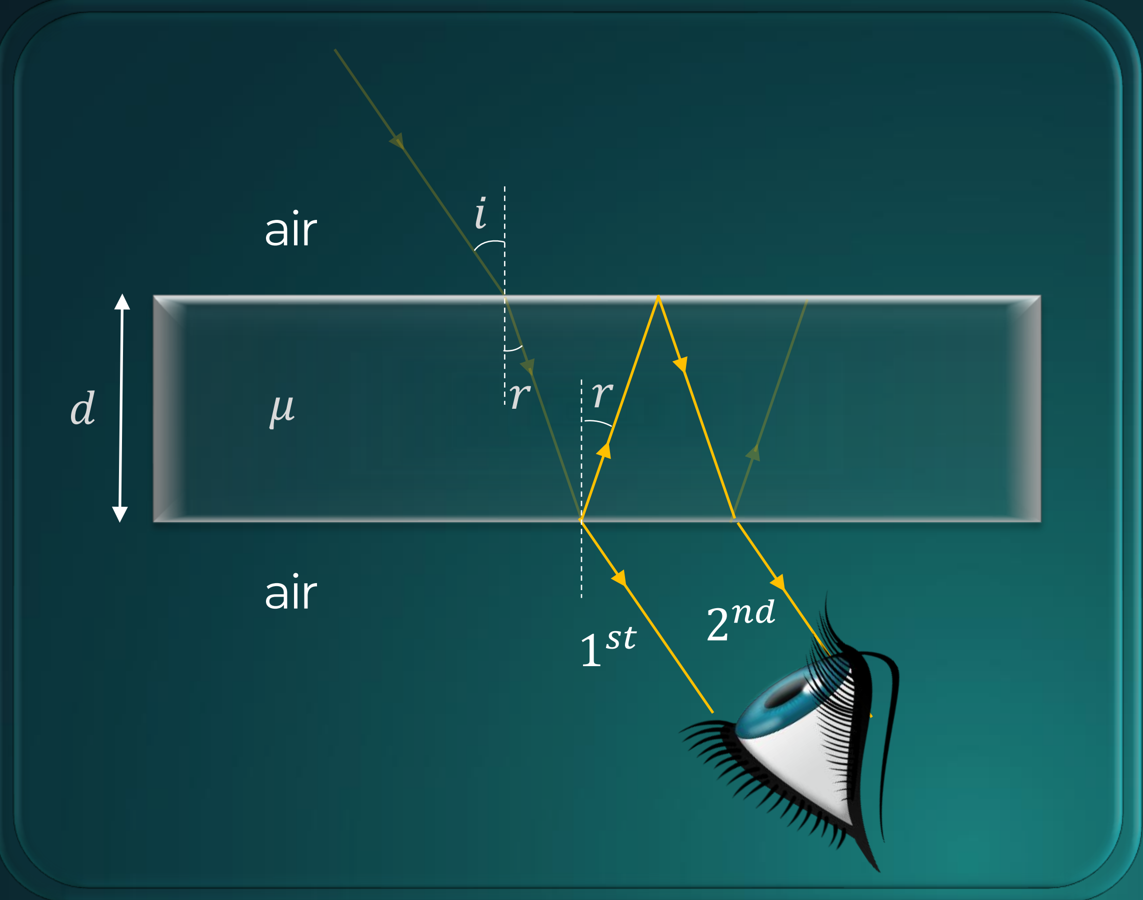
- Interference of light wave being reflected off two surfaces that are at a distance comparable to its wavelength is known as thin film "Thin film interference".
- Inside the film, when a particular colour's path difference is even integral multiple of the wavelength, it undergoes constructive interference, so these colours appear bright.
- When a particular colour's wavelength is odd integral multiple of the path difference inside the film, it undergoes destructive interference, so these colours don't appear at all.



Interference due to Thin Film from Transmitted Light



(angles are very small)



Path difference of 1^{st} and 2^{nd} transmitted light wave $= 2\left(\frac{d}{\cos r}\right)\mu$

When angle is very small, $r \approx 0$

$\cos r \approx 1$

Path difference $\boxed{\Delta x = 2\mu d}$

Interference due to Thin Film from Transmitted Light

For constructive interference:

$$\delta = 2n\pi$$

Or

$$2\mu d = n\lambda$$

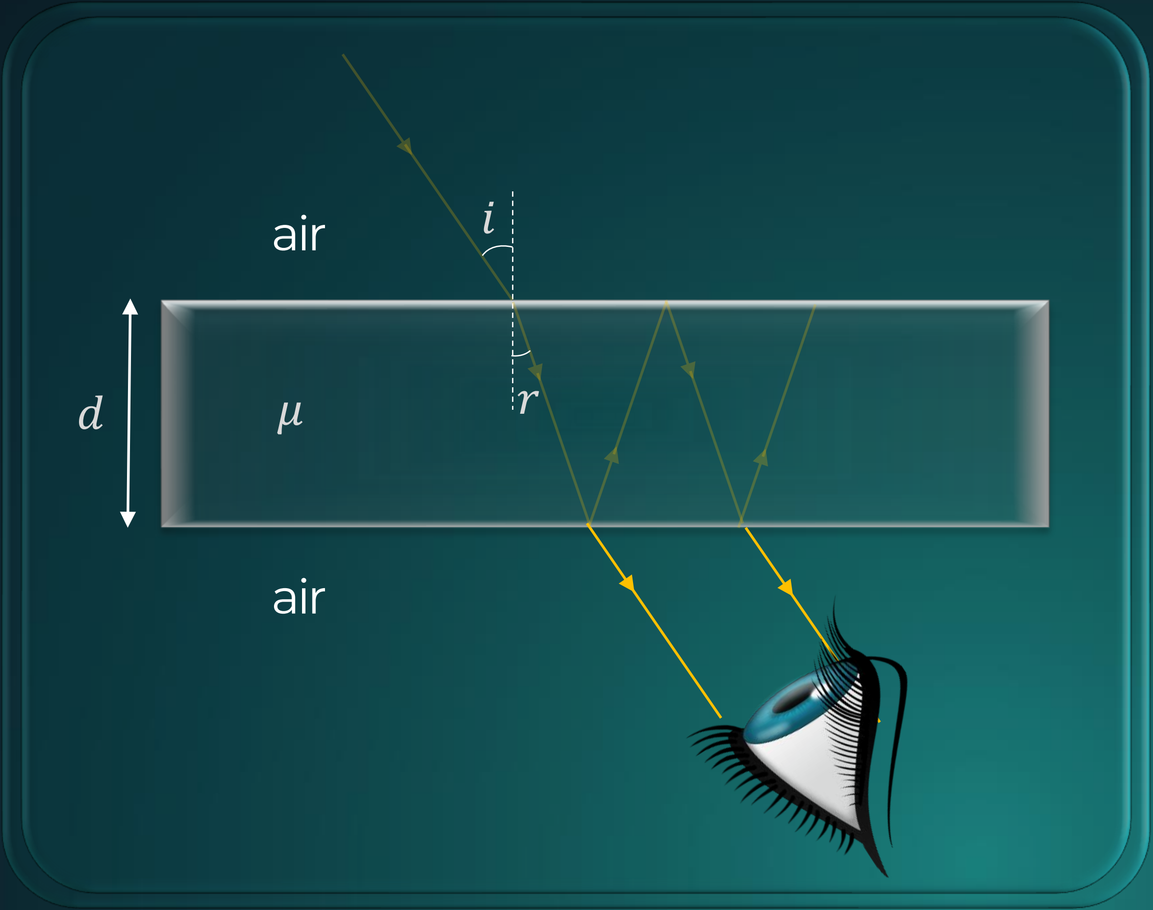
For destructive interference:

$$\delta = (2n+1)\pi$$

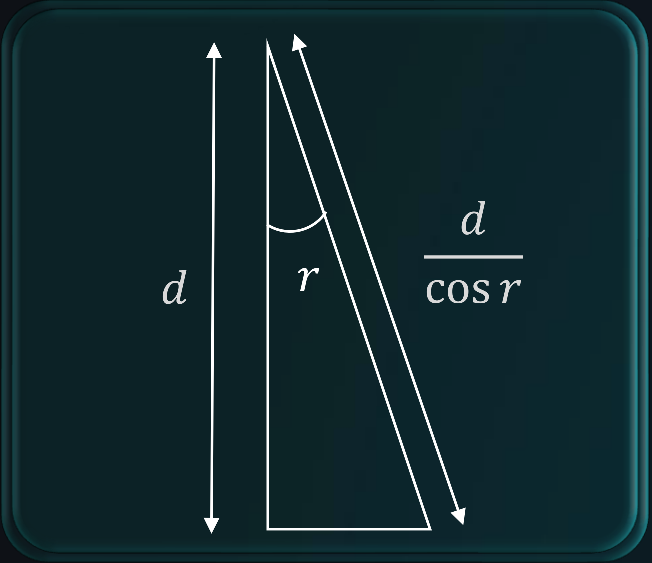
Or

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda$$

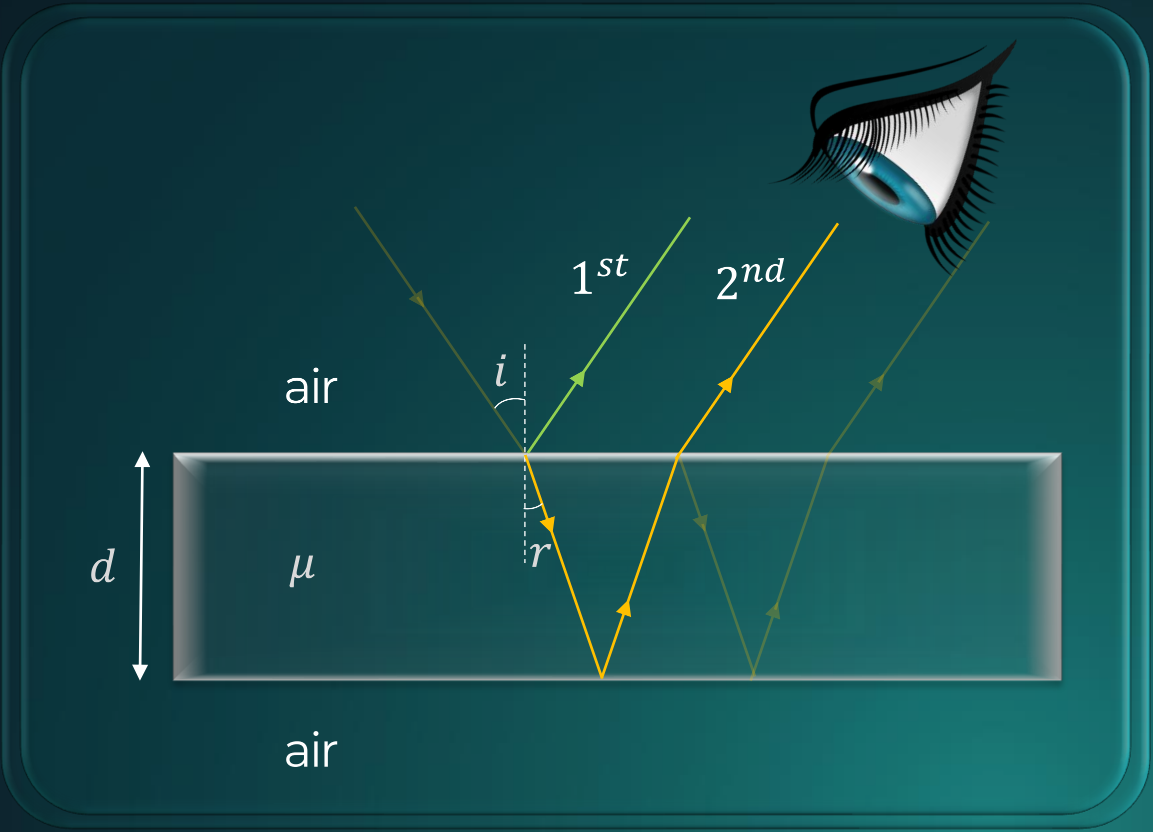
(angles are very small)



Interference due to Thin Film from Reflected Light



(angles are very small)



Path difference of 1^{st} and 2^{nd} reflected light wave $= 2\left(\frac{d}{\cos r}\right)\mu \approx 2\mu d$

Because phase change of π after reflection of 1^{st}, path difference $= -\frac{\lambda}{2}$

Total path difference $\boxed{\Delta x = 2\mu d - \frac{\lambda}{2}}$

Interference due to Thin Film from Reflected Light

For constructive interference:

$$2\mu d - \frac{\lambda}{2} = n\lambda$$

$$\boxed{2\mu d = \left(n + \frac{1}{2}\right)\lambda}$$

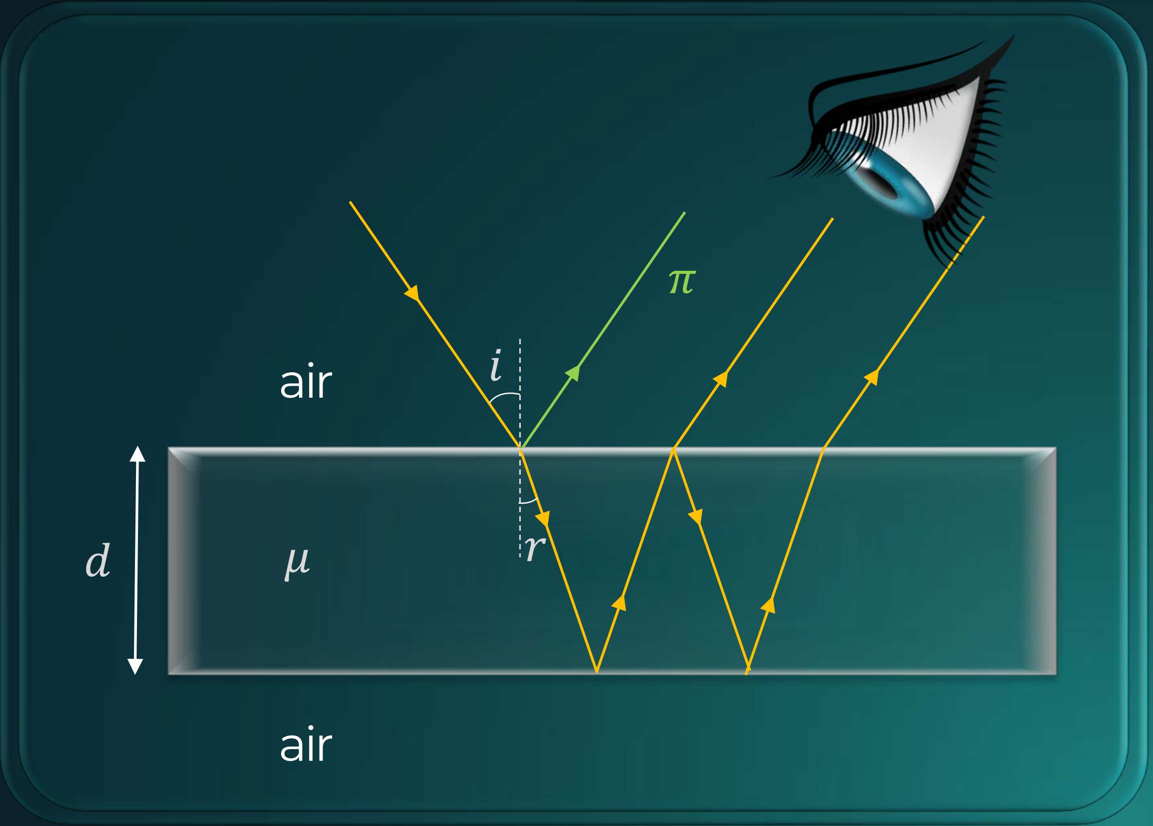
For destructive interference:

$$2\mu d - \frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda \quad \Rightarrow 2\mu d = (n+1)\lambda$$

$$\boxed{2\mu d = n\lambda}$$

n is an integer.

(angles are very small)



Interference due to Thin Film

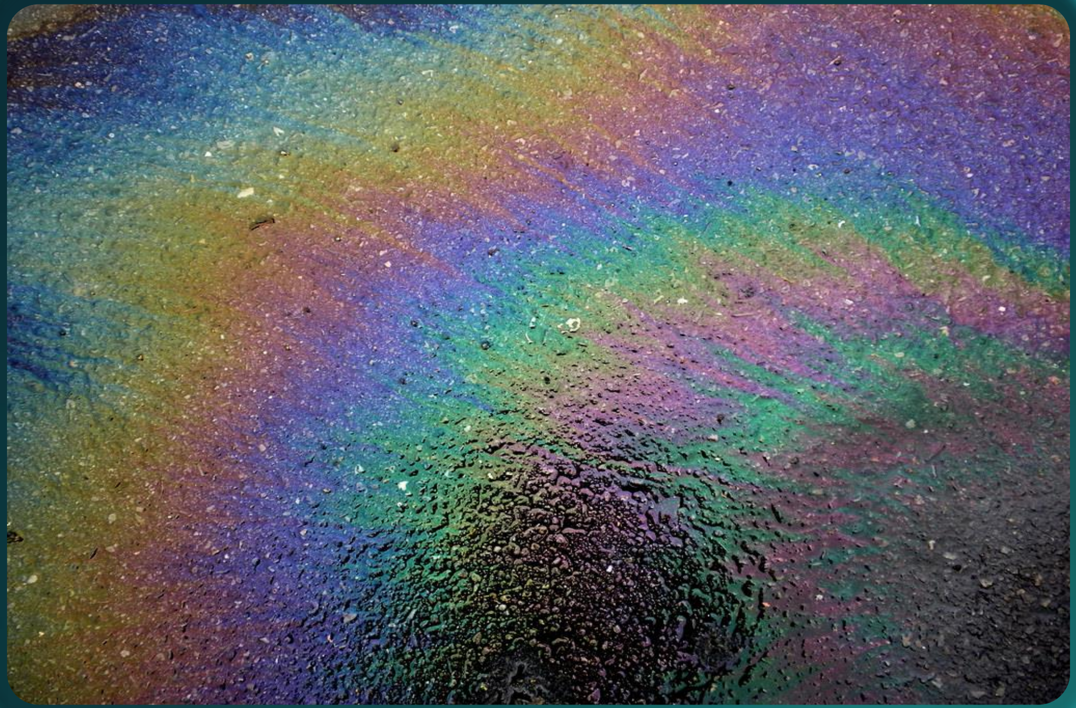
For constructive interference:

- Colours will be strongly reflected/transmitted.

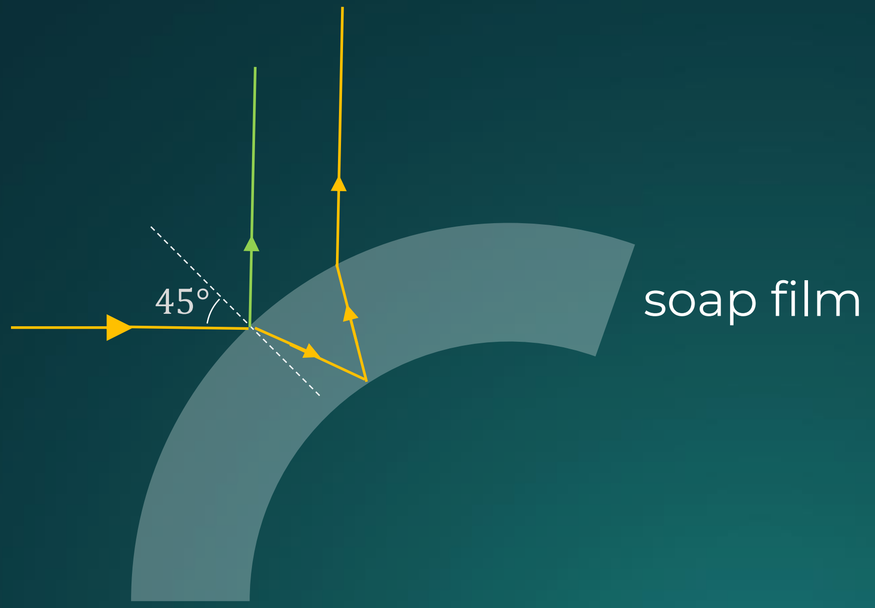
Destructive interference:

- Colours will be poorly reflected/transmitted.

This gives coloured appearance of the film.



A soap film of refractive index 1.33 is illuminated by the light of wavelength $400\ nm$ at an angle of $45°$. If there is complete destructive interference then, find the thickness of the film.



Given: $\mu = 1.33, i = 45°, \lambda = 400\ nm$

To find: t

Solution: $\mu = \dfrac{\sin i}{\sin r} \Rightarrow 1.33 = \dfrac{\sin 45}{\sin r}$

$$\sin r = \frac{3}{4\sqrt{2}} \Rightarrow \cos r = 0.85\ \left\{\cos r = \sqrt{1-\sin^2 r}\right\}$$

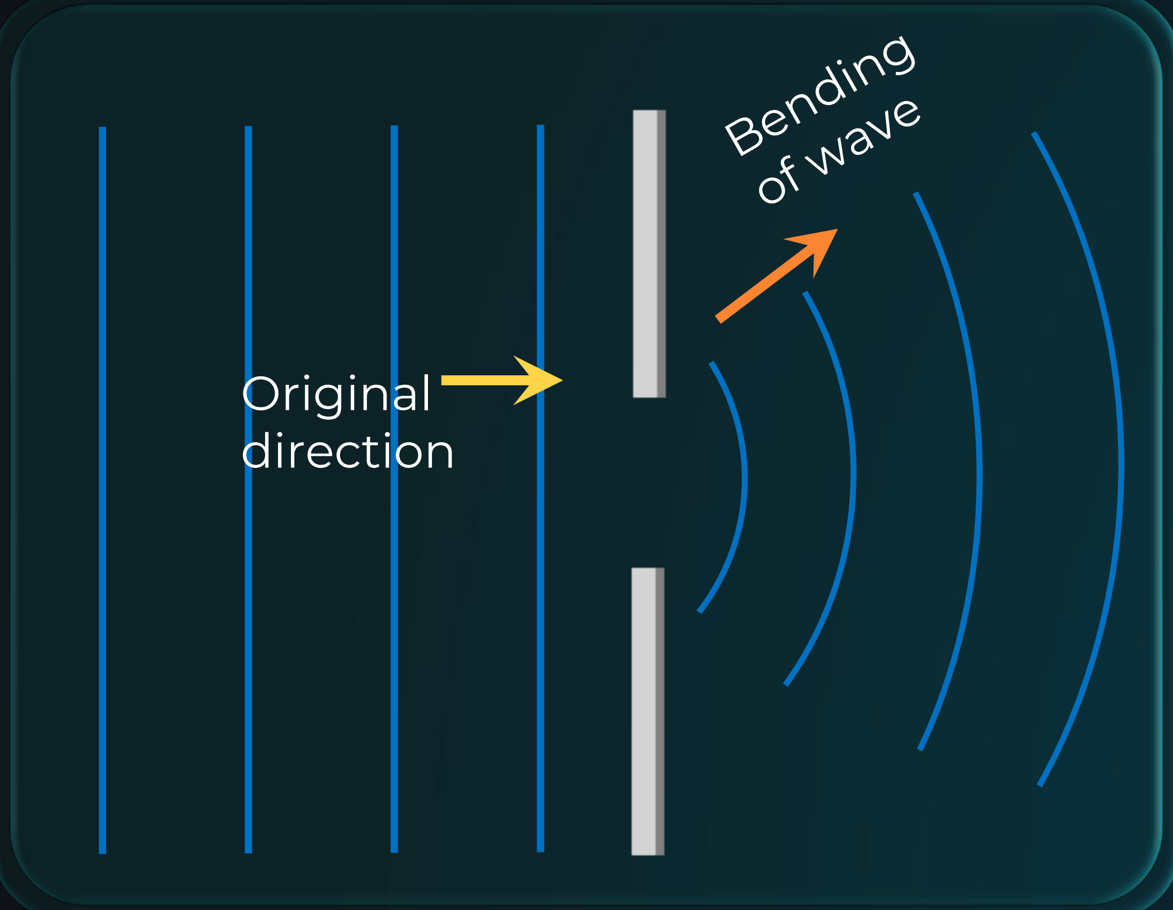
For Destructive interference:

$$\frac{2\mu t}{\cos r} = n\lambda$$

$$\Rightarrow \frac{2(1.33)t}{0.85} = 1(400 \times 10^{-9})$$

$$\boxed{\Rightarrow t = 1.27\ \times 10^{-7}\ m}$$

Diffraction

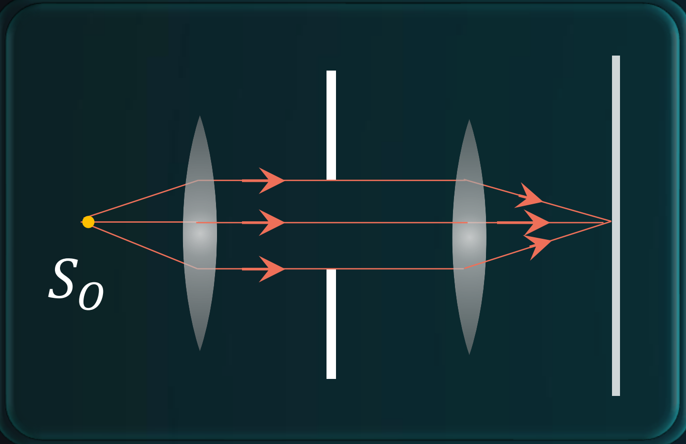


- Bending of a wave or its deviation from its original direction of propagation while passing through a small obstruction is known as diffraction.
- Every point on the wavefront makes a secondary wave according to Huygens Principle.
- Condition for bending: size of obstacle $\approx \lambda$
- Diffraction is explained by wave nature of light.

Diffraction of Light Waves

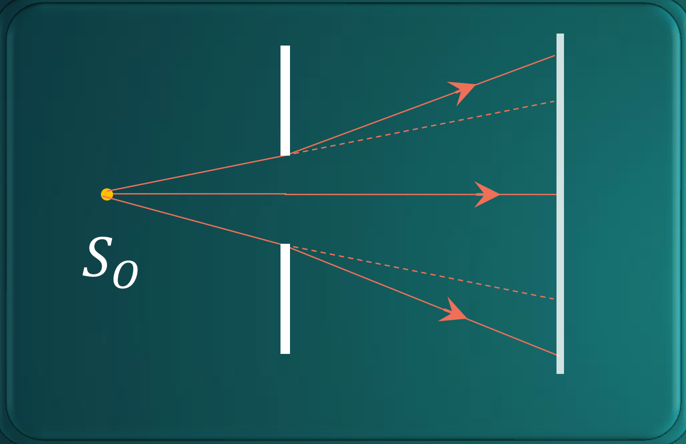
Fraunhofer Diffraction

- Source and screen are at infinite distance from diffraction element.
- High intense interference pattern is observed on screen.

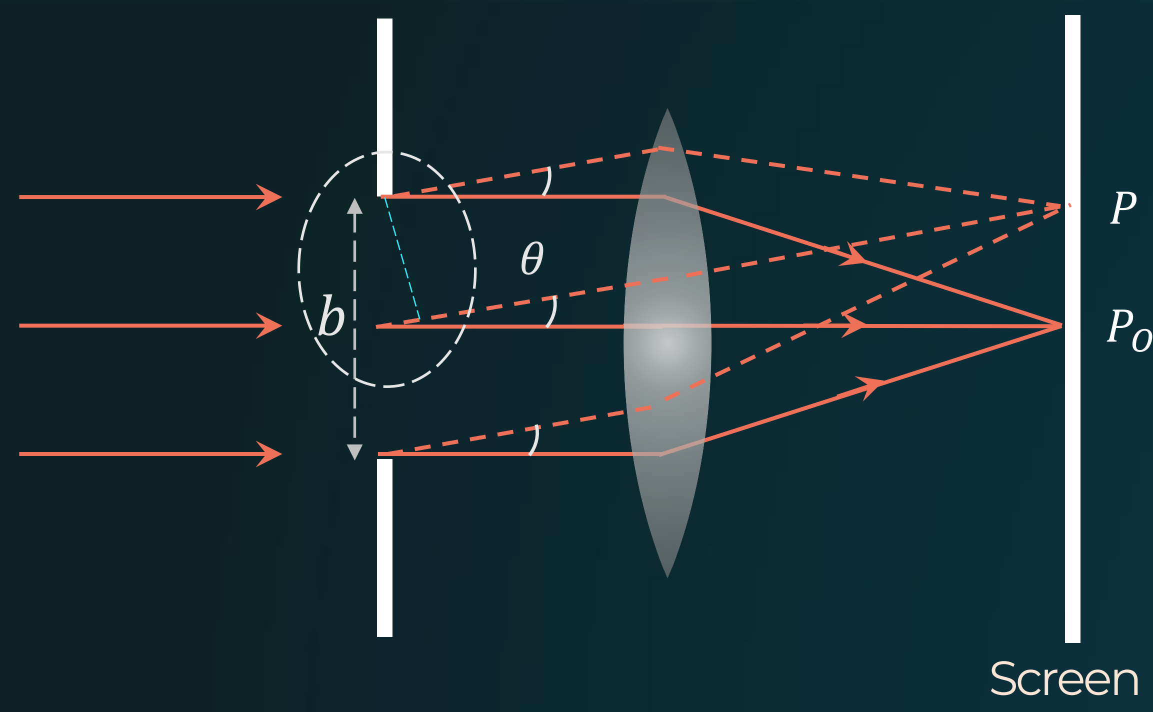


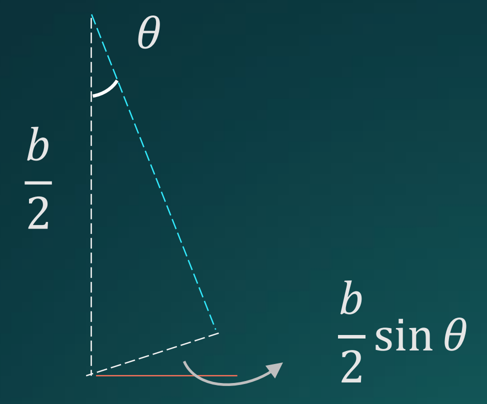
Fresnel Diffraction

- Source and screen are at finite distance from diffraction element.
- Diffraction was first demonstrated by this experiment.



Fraunhofer Diffraction – Path Difference

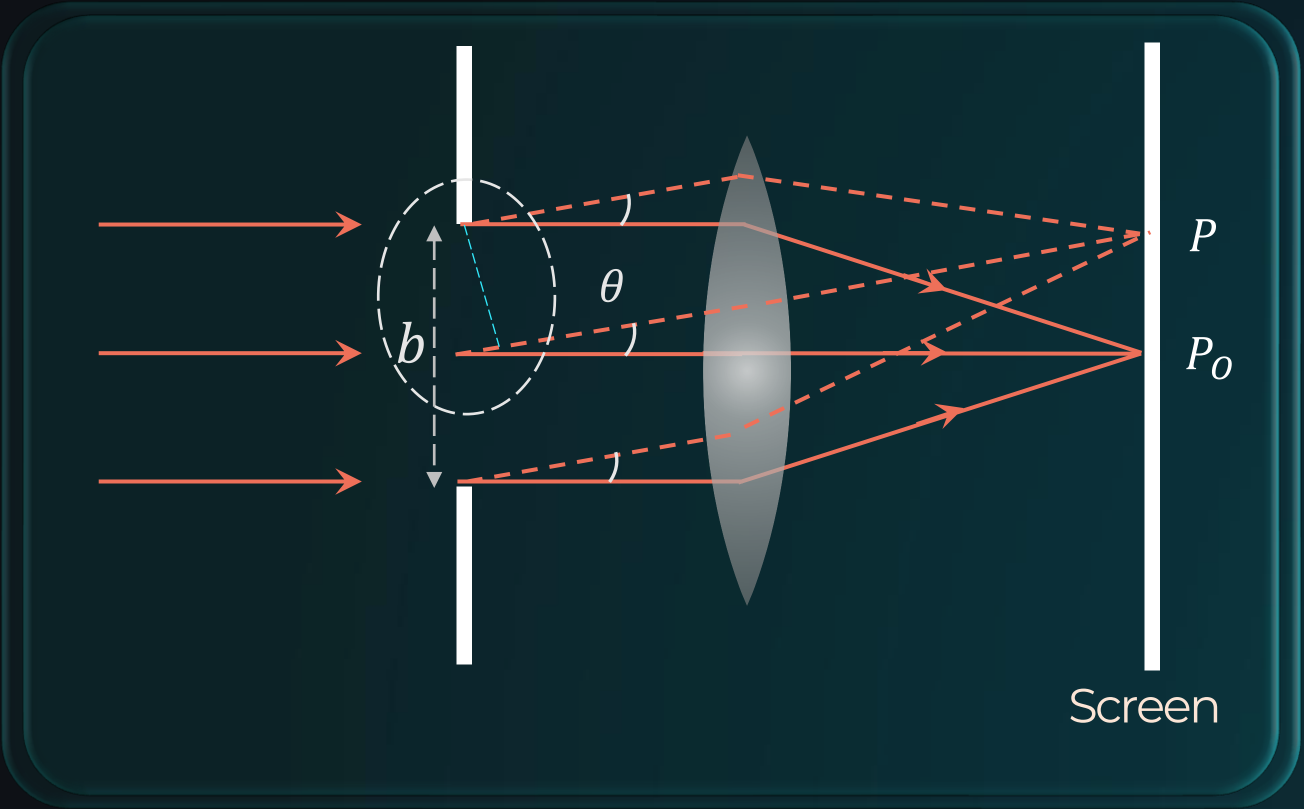




- Path difference between the waves,

$$\Delta x = \frac{b}{2}\sin\theta$$

Fraunhofer Diffraction – first Dark Fringe



- The condition for 1^{st} dark fringe formed at point P:

$$\frac{b}{2}\sin\theta = \frac{\lambda}{2}$$

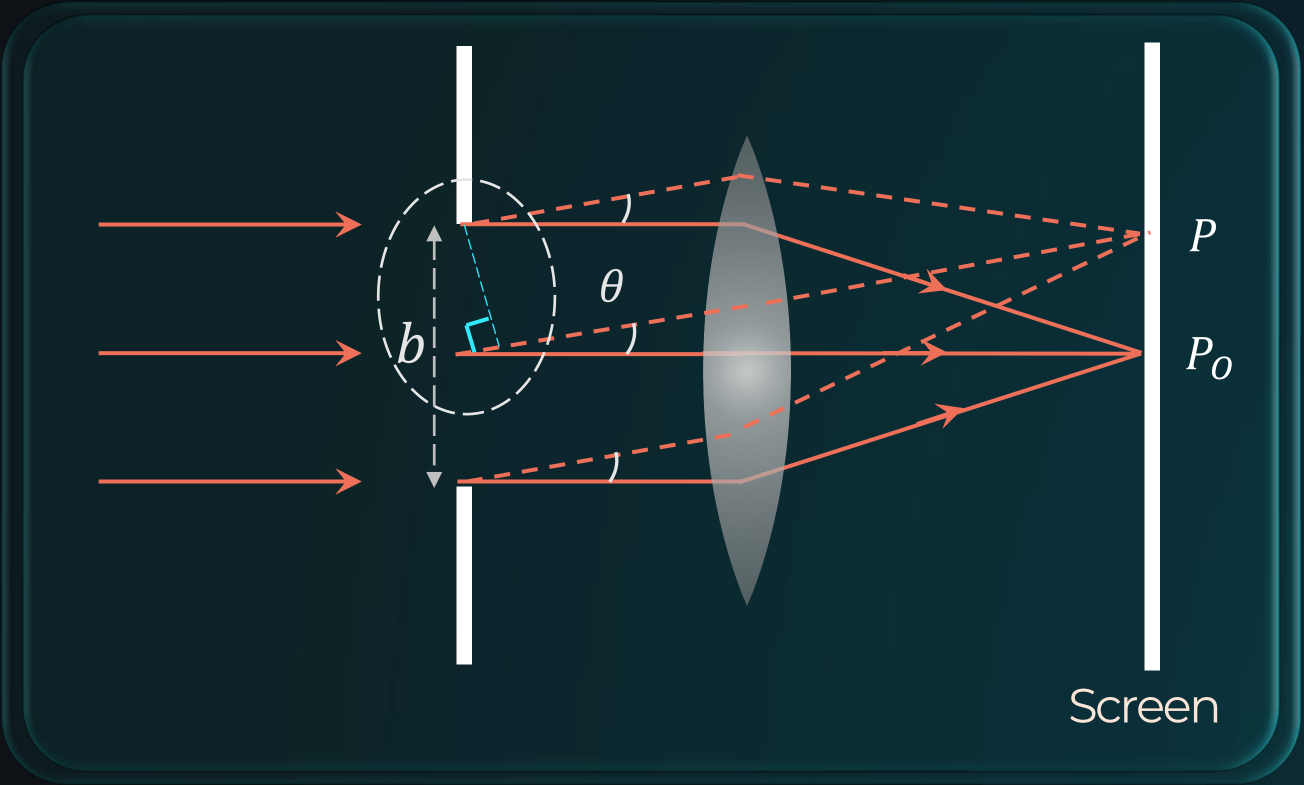
$$\boxed{b\sin\theta = \lambda}$$

- The condition for n^{th} dark fringe:

$$\boxed{b\sin\theta = n\lambda}$$

- Central Maxima is at P_0, where $\theta = 0$

Fraunhofer Diffraction – Intensity at general point



- Electric Field Amplitude at P:

$$\boxed{E' = \frac{E_0 \sin\beta}{\beta}}$$

β = phase difference

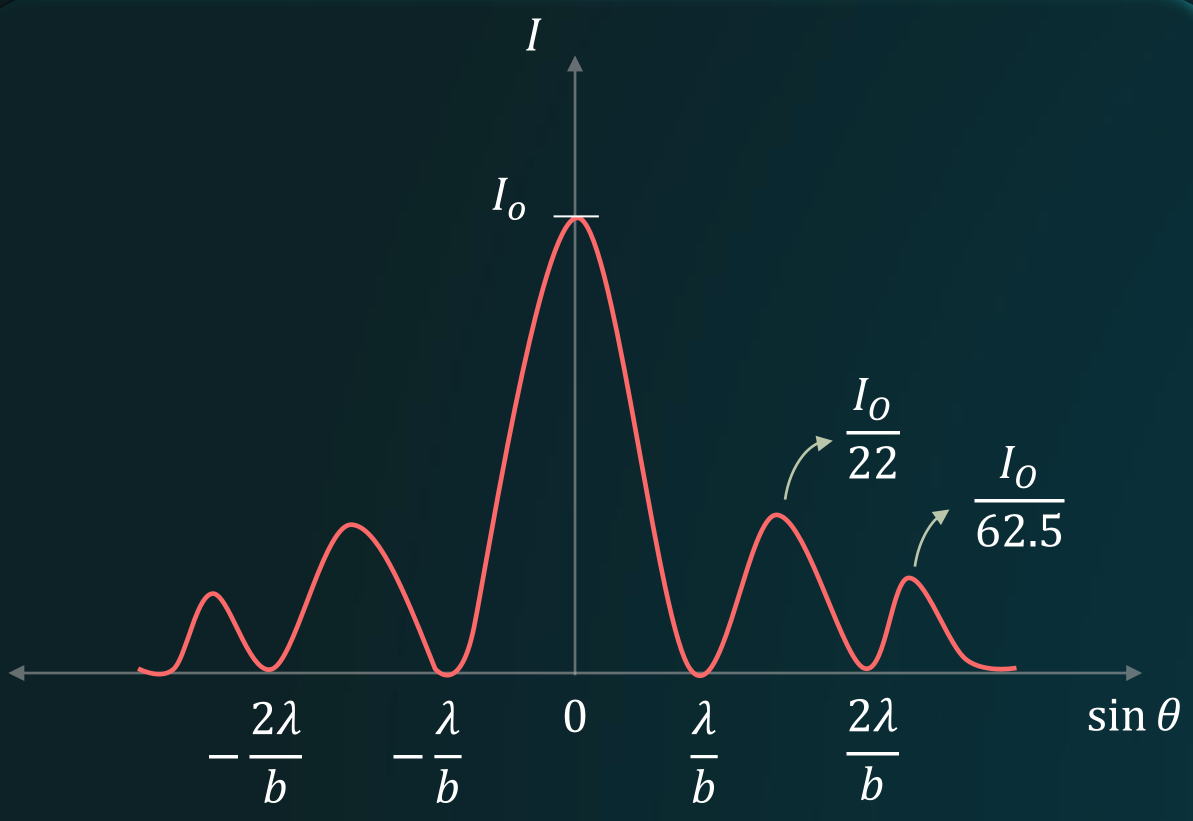
Where $\beta = \frac{2\pi}{\lambda}\left(\frac{b}{2}\sin\theta\right)$

- When $\theta = 0, \beta = 0, E' = E_0$
- Intensity $\propto E^2$
- Intensity at a general point:

$$\boxed{I = \frac{I_O \sin^2\beta}{\beta^2}}$$

Where I_0 is the intensity at Central Maxima

Fraunhofer Diffraction – Graph of Intensity

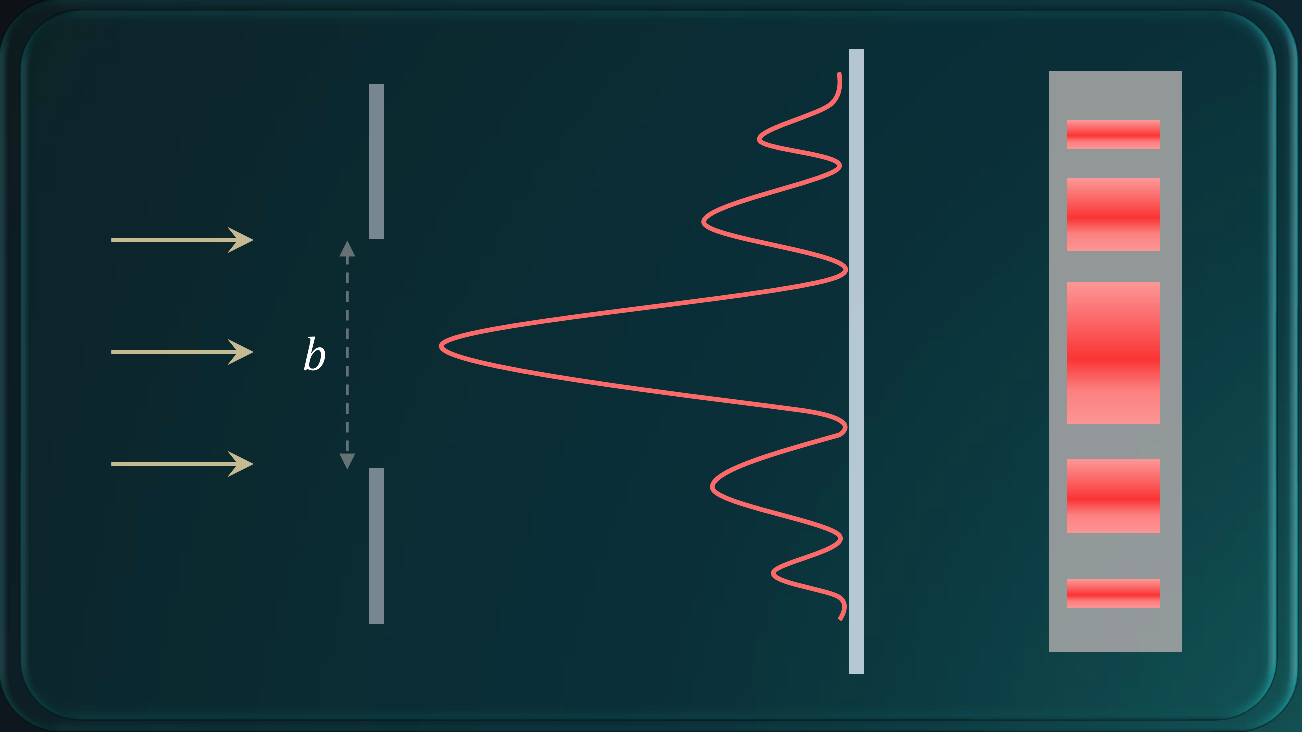


$$\boxed{I = \frac{I_O \sin^2\beta}{\beta^2}} \qquad \beta = \frac{\pi b \sin\theta}{\lambda}$$

- $\sin\theta = 0$; Central Maxima
- $\sin\theta = \frac{n\lambda}{b}$; Minima
- $\sin\theta = \pm\frac{\lambda}{b}, \pm\frac{2\lambda}{b}, \pm\frac{3\lambda}{b} \ldots\ldots\ldots\ldots$
- Maximum intensity is distributed between $-\frac{\lambda}{b}$ and $\frac{\lambda}{b}$.
- If b is large, $\frac{\lambda}{b} \to 0$, then

  Single fringe is observed and no diffraction is seen.

Fraunhofer Diffraction – Graph of Intensity



- Fraunhofer Diffraction ⟶ Intensity decreases away from the centre.
- I_0 → Intensity at the central bright fringe

$$I = \frac{I_0 \sin^2 \beta}{\beta^2}$$

- YDSE ⟶ Intensity is same for all fringes.
- I_0 → Intensity from a single slit.

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Difference between Interference and Diffraction

Interference	Diffraction
It is the phenomenon of superposition of two waves coming from two different coherent sources.	It is the phenomenon of superposition of two waves coming from two different parts of the same wavefront.
In interference pattern, all bright fringes are equally bright and equally spaced.	All bright fringes are not equally bright and equally wide. Brightness and width decreases with the angle of diffraction.
All dark fringes are perfectly dark.	All dark fringes are perfectly dark, but their contrast with bright fringes and width decreases with angle of diffraction.
In interference, bright and dark fringes are large in number for a given field of view.	In diffraction, bright and dark fringes are fewer for a given field of view.

A beam of light of wavelength $600\ nm$ from a distant source falls on a single slit $1\ mm$ wide and a resulting diffraction pattern is observed on a screen $2\ m$ away. The distance between the first dark fringes on either side of central bright fringe is

Solution: First maxima is formed at $\frac{\lambda}{b}$ distance away on both side of central maxima.

$\sin\theta \approx \theta$ (θ is small)

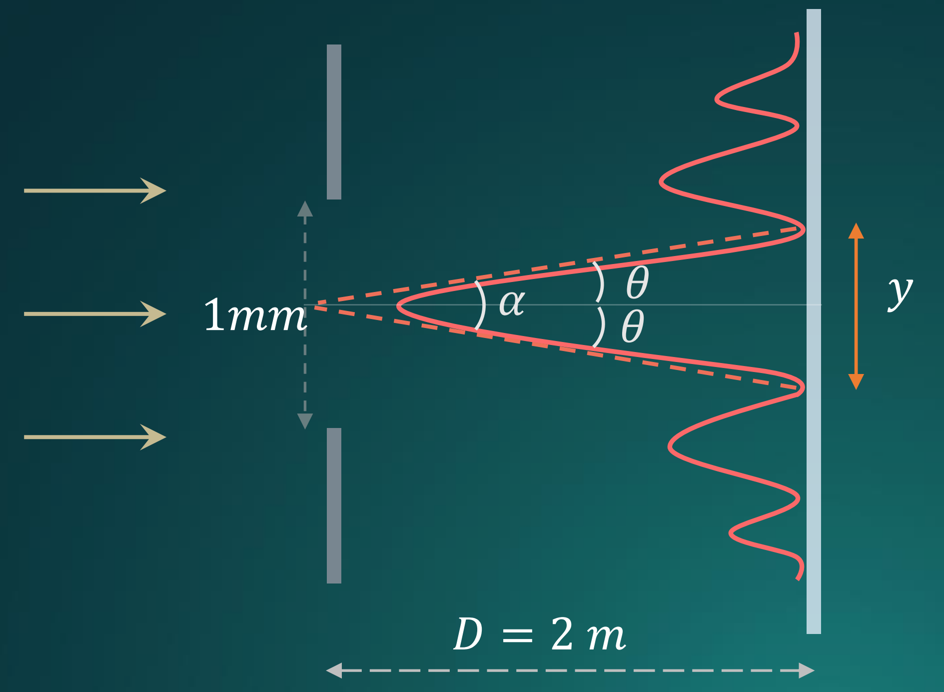
$$\theta = \frac{\lambda}{b}$$

$$\alpha = 2\theta = \frac{2\lambda}{b}$$

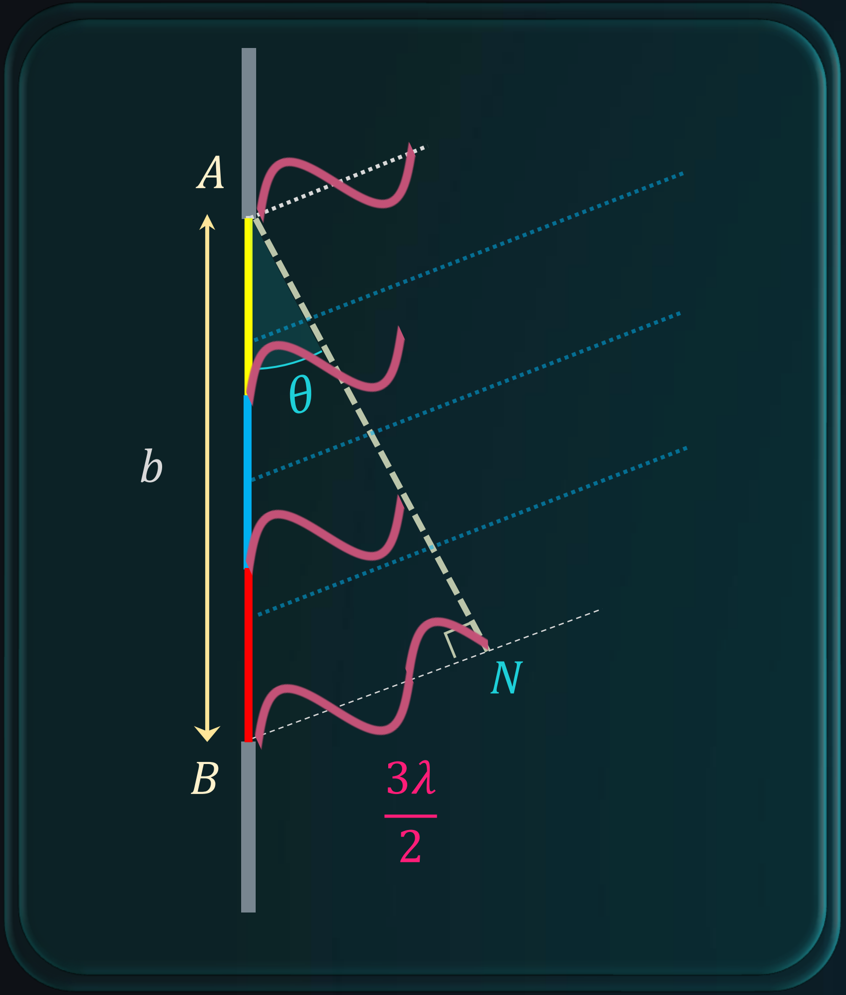
$$\alpha = \frac{y}{D} = \frac{2\lambda}{b} \implies y = \frac{2\lambda D}{b}$$

$$y = \frac{2 \times 600 \times 10^{-9} \times 2}{10^{-3}} = 24 \times 10^{-4}\ m$$

$$\boxed{y = 2.4\ mm}$$



Single Slit Diffraction – 1^{st} Secondary Maxima



- Condition for 1^{st} secondary maxima (bright):

$$b \sin\theta = \frac{3\lambda}{2}$$

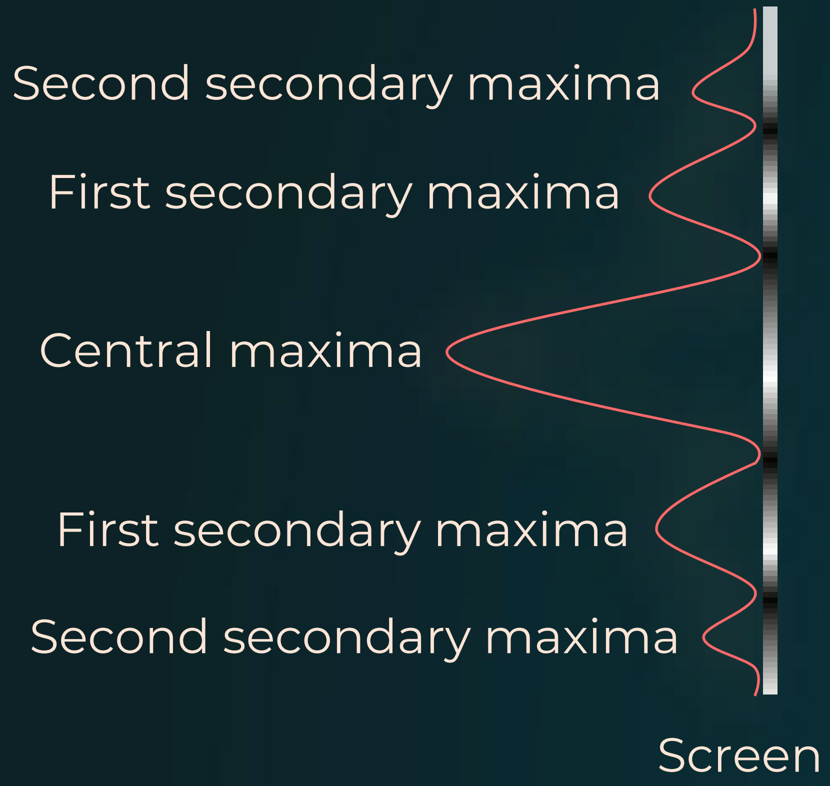
- The angle of diffraction (θ_{1B}) for 1^{st} maximum (bright) is:

$$\boxed{\theta_{1B} = \frac{3\lambda}{2b}}$$

- The position of 1^{st} maximum (bright) from the centre of the screen is:

$$\boxed{y_{1B} = \frac{3D\lambda}{2b}}$$

Single Slit Diffraction – n^{th} Secondary Maxima



- The angle of diffraction (θ_n) for n^{th} maxima (bright) is:

$$\theta_{nB} = \frac{(2n+1)\lambda}{2b}$$

- The position of n^{th} maxima (bright) from the centre of the screen is:

$$y_{nB} = \frac{(2n+1)D\lambda}{2b}$$

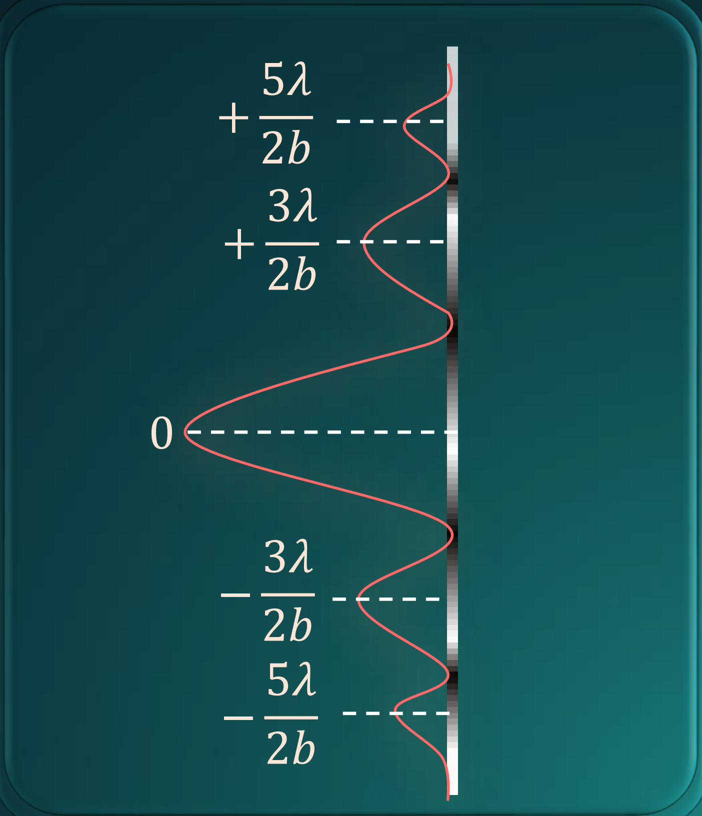
Where, $n = 1, 2, 3, 4$ ... ...

Single Slit Diffraction

- Angular position for minima

$$\sin\theta = \pm\frac{\lambda}{b}, \pm\frac{2\lambda}{b}, \dots\dots, \pm\frac{n\lambda}{b}$$

- Angular position for maxima



$$\sin\theta = \pm\frac{3\lambda}{2b}, \pm\frac{5\lambda}{2b}, \dots\dots, \pm\frac{(2n+1)\lambda}{2b}$$

In a diffraction pattern due to a single slit of width a, the first minima is observed at an angle $30°$ when the light of wavelength $5000\ \dot{A}$ is incident on the slit. The first secondary maxima is observed at an angle of:

Given: $\theta_{1D} = 30°; \lambda = 5000\ \dot{A}$

To find: Angular position of first secondary maximum (θ_{1B})

Solution: Angular position of first minima is given by,

$$\sin\theta_{1D} = \frac{\lambda}{a} \Longrightarrow \frac{1}{2} = \frac{\lambda}{a}$$

Angular position of first secondary maxima is given by,

$$\sin\theta_{1B} = \frac{3\lambda}{2a} \Longrightarrow \sin\theta_{1B} = \frac{3}{4}$$

$$\boxed{\theta_{1B} = \sin^{-1}\frac{3}{4}}$$

A $\sin^{-1}\left(\frac{1}{2}\right)$

B $\sin^{-1}\left(\frac{3}{4}\right)$

C $\sin^{-1}\left(\frac{1}{4}\right)$

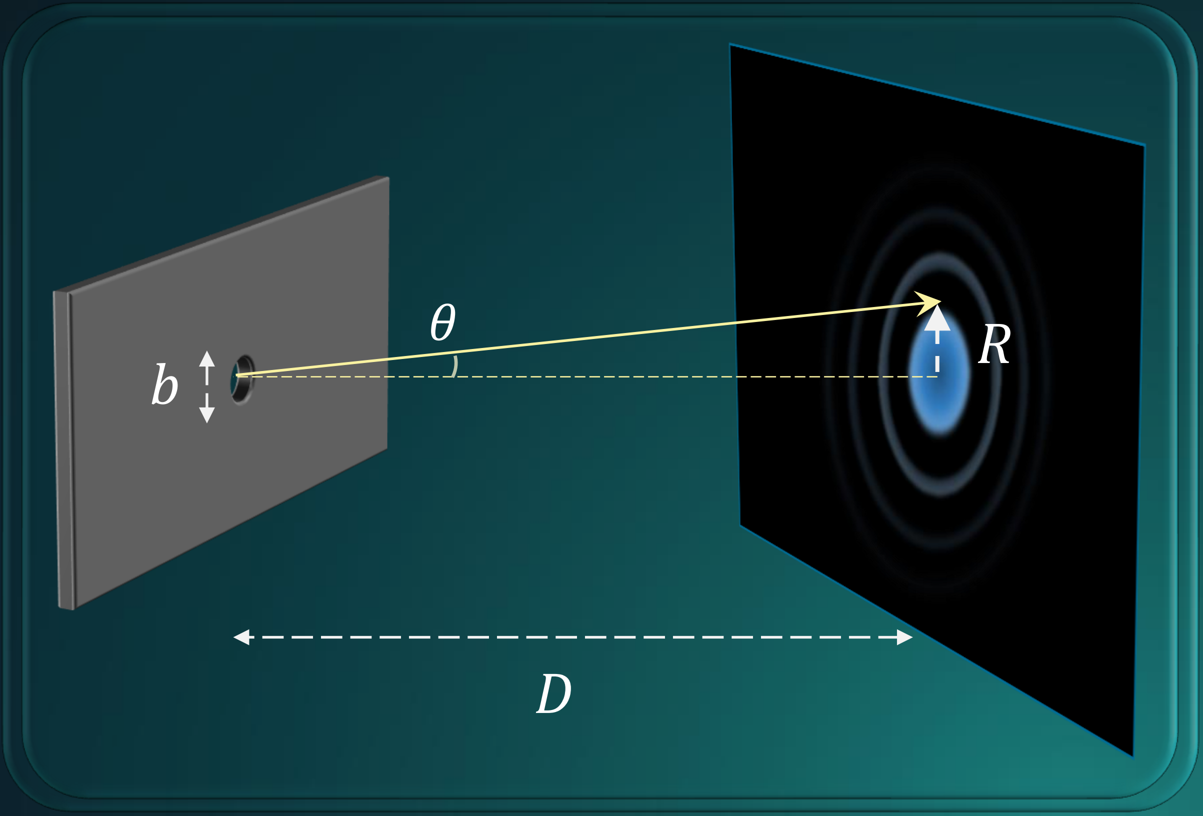
D $\sin^{-1}\left(\frac{2}{3}\right)$

Fraunhofer Diffraction for Hole

- When a monochromatic light is incident on the hole we see concentric circular bright and dark spots on the screen.
- The size of hole should be comparable to the wavelength of incident light.
- Central bright spot contains the most energy.
- Brightness of the rings decreases as we move away from the centre.
- For the first dark ring,

$$\sin\theta = \frac{1.22\lambda}{b}$$

$$R = \frac{1.22\lambda D}{b}$$



A convex lens of diameter 8.0 cm is used to focus a parallel beam of light of wavelength 6200 Å. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot?

Given: $D = 20\ cm,\ b = 8\ cm$ and $\lambda = 6200$ Å

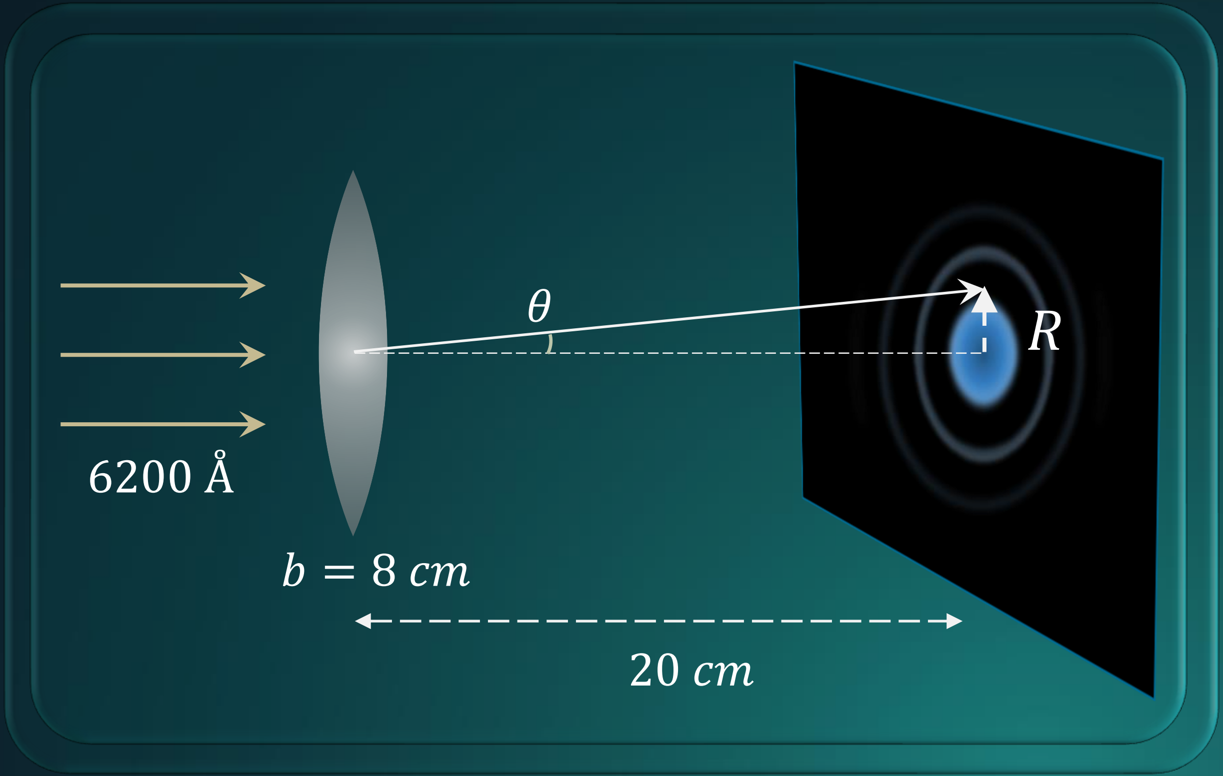
To Find: R

Solution: Radius of central bright spot,

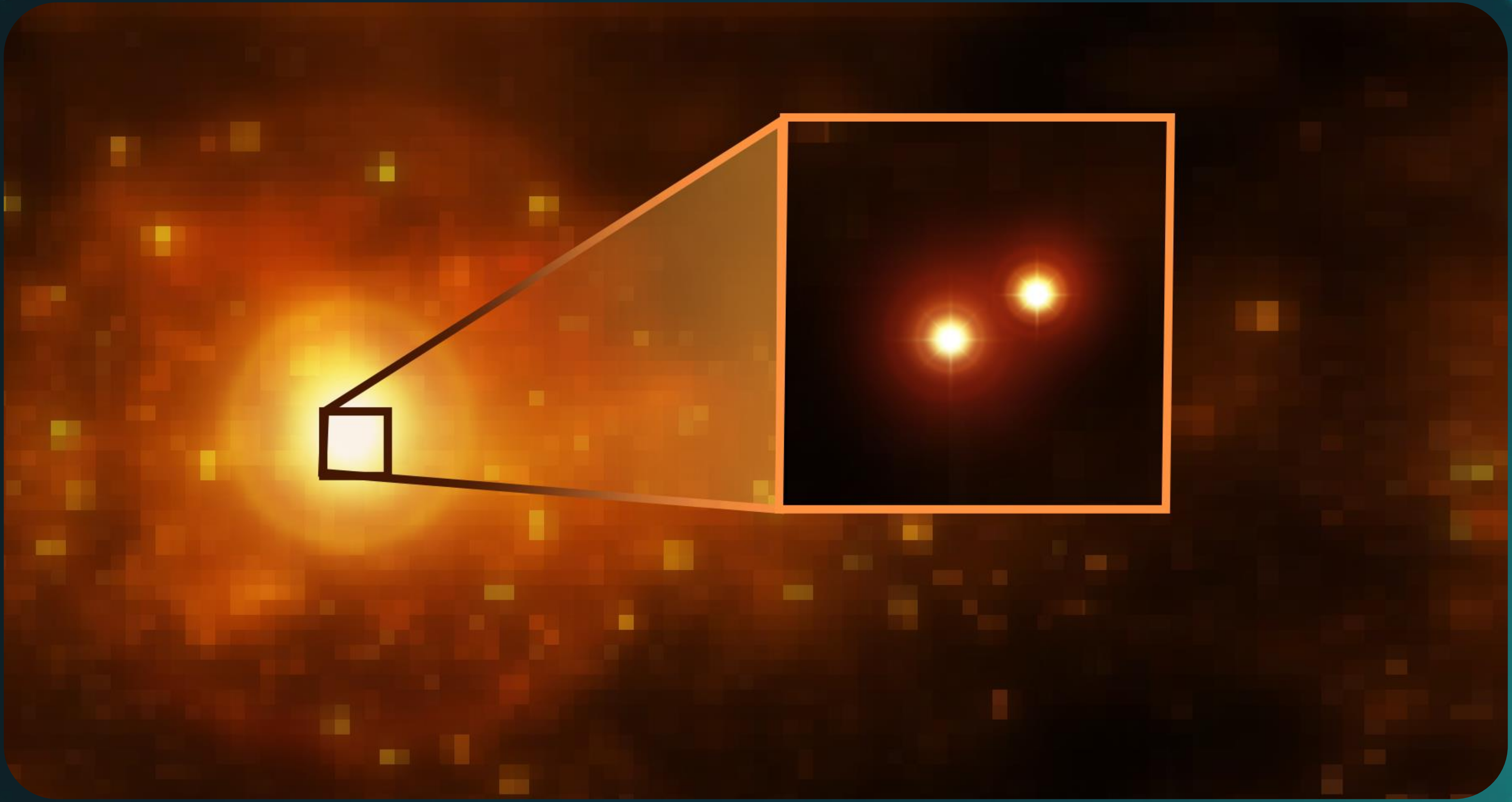
$$R = \frac{1.22\lambda D}{b}$$

$$\Rightarrow R = \frac{1.22 \times 6200 \times 10^{-10} \times 20 \times 10^{-2}}{8 \times 10^{-2}}$$

$$\boxed{R = 1.89 \times 10^{-6}\ m}$$



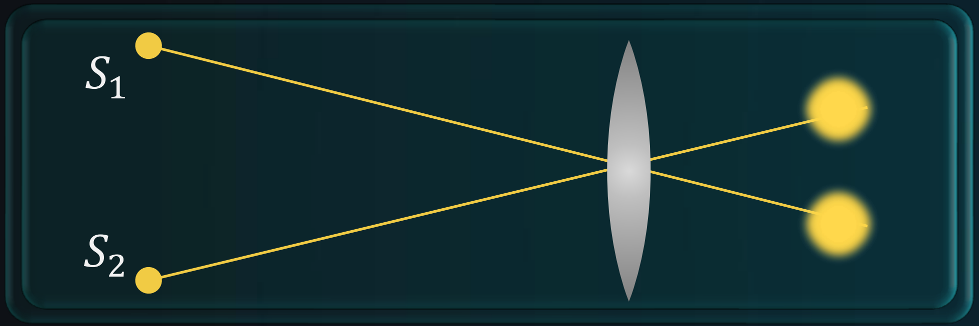
Binary Star



Limit of Resolution

Unresolved: Diffraction discs from both sources overlap.

Just resolved: The periphery of the diffraction disc of one object touches the centre of the diffraction disc of the other object.

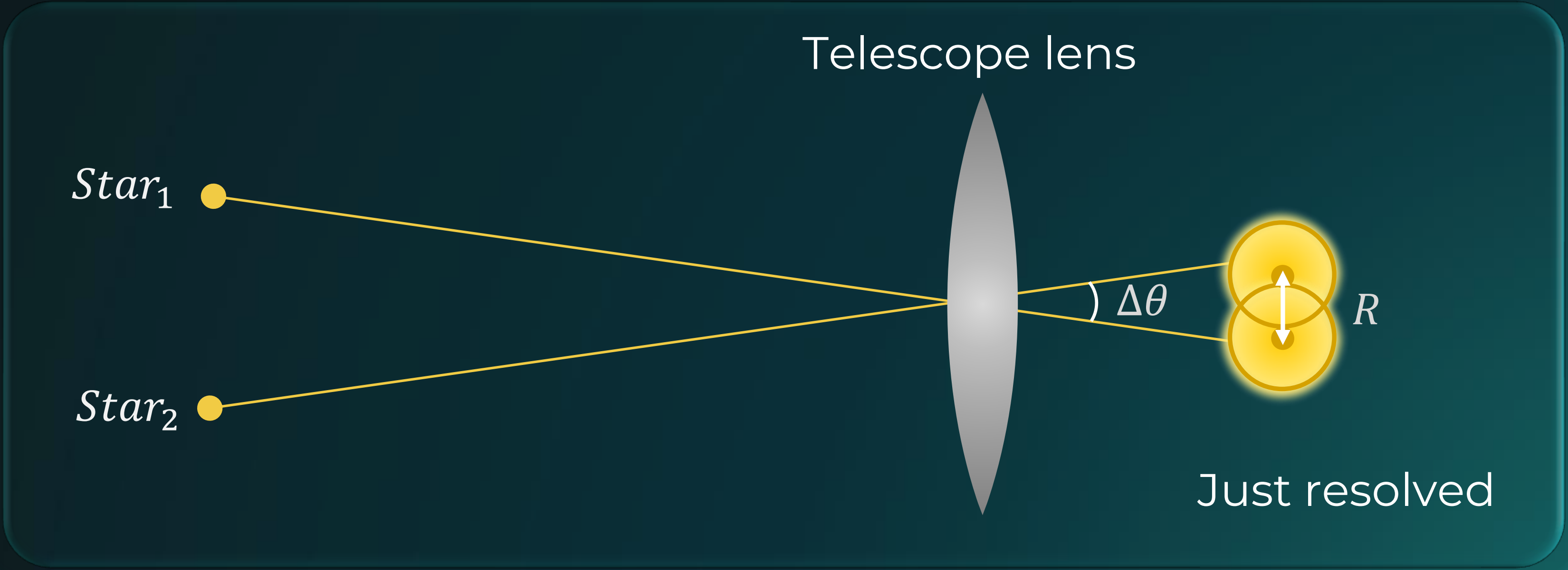


Well resolved: The diffraction discs formed by two objects are well separated from each other.

Rayleigh criterion

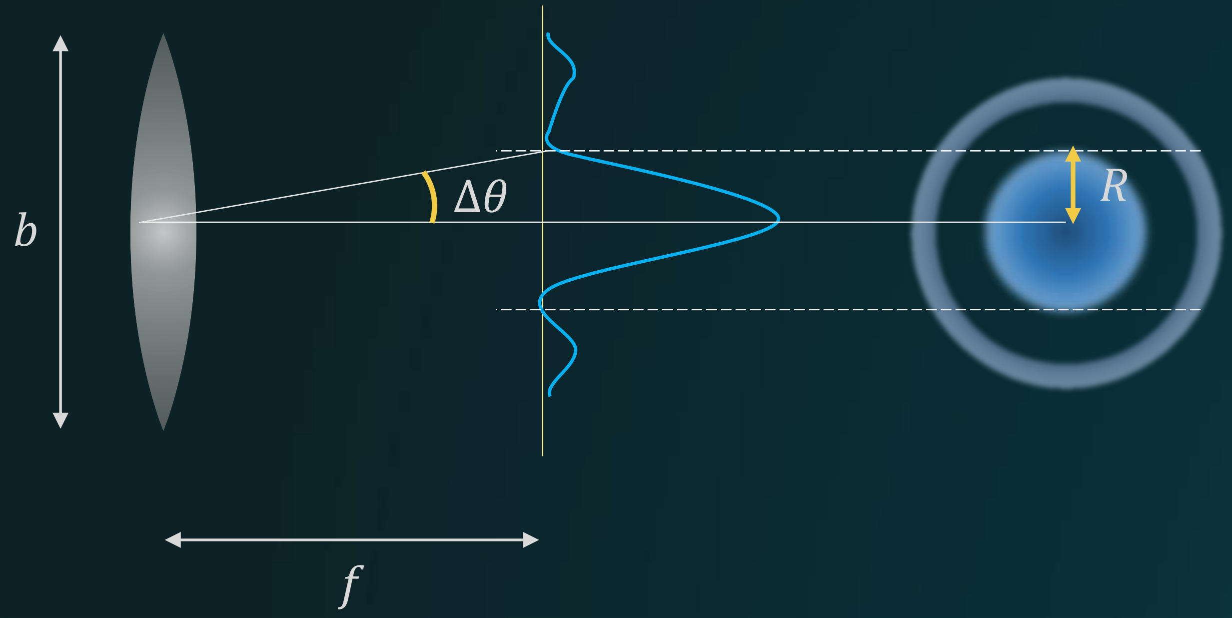
The Rayleigh criterion specifies the minimum separation between two light sources that may be resolved into distinct objects.

Limit of Resolution of a Telescope



- Clear image is formed when the diffraction discs from two sources is just resolved.
- Distance between diffraction disc $> R \Rightarrow$ Well resolved
- Distance between diffraction disc $< R \Rightarrow$ Unresolved

Resolving Power of Telescope



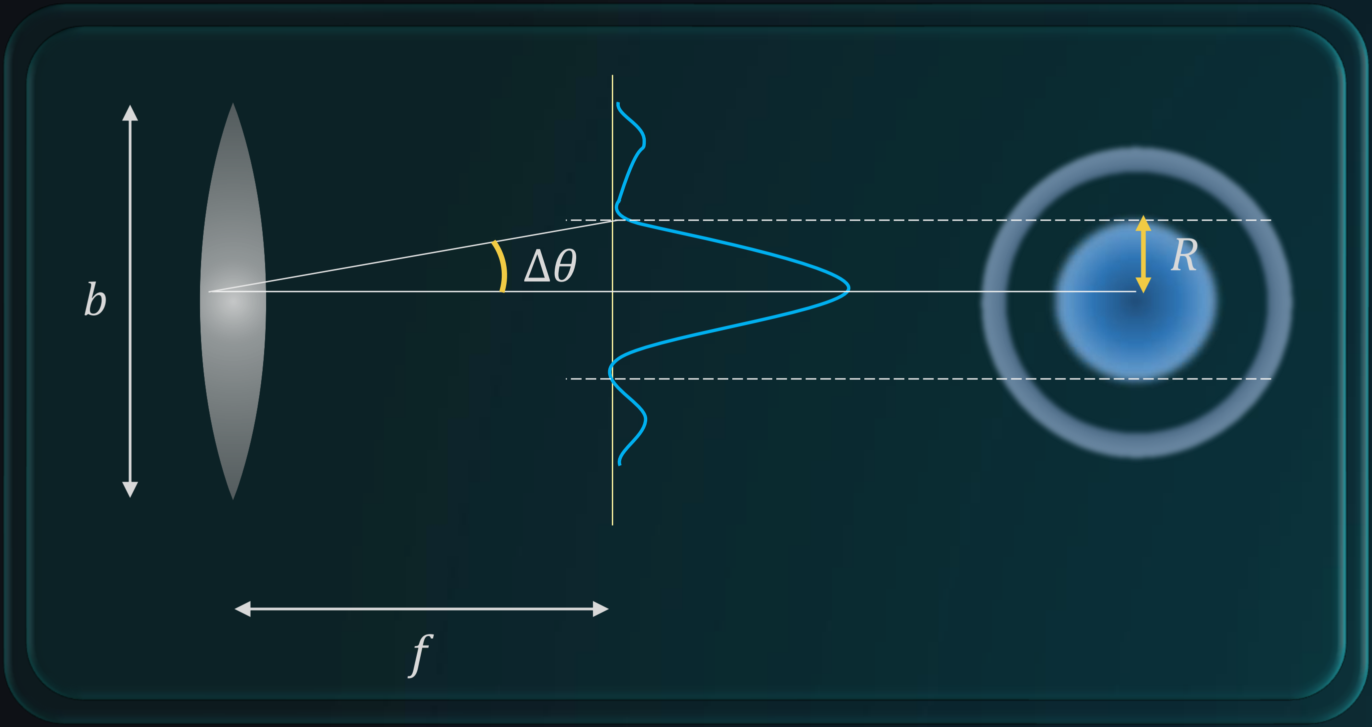
- Angular limit of resolution of telescope:

$$\boxed{\Delta\theta = \frac{1.22\lambda}{b}}$$

$\Delta\theta$ = Limit of resolution

- Angle subtended by the first dark fringe $> \Delta\theta \Rightarrow$ Well resolved
- Angle subtended by the first dark fringe $< \Delta\theta \Rightarrow$ Unresolved

Radius of the Central Bright Spot



- Radius of central bright region is:

$R = f\Delta\theta$

$$\boxed{R = \frac{1.22\lambda f}{b}}$$

- Resolving power of a telescope:

$R.P. = \frac{1}{\Delta\theta}$

$$\boxed{R.P. = \frac{b}{1.22\lambda}}$$

- Bigger lens ⇒ larger b ⇒ smaller Limit of Resolution ($\Delta\theta$) ⇒ higher Resolving Power($R.P.$)

Disadvantages of using Lens

1. Difficult and expensive to build large lenses.

2. Providing mechanical support to large lenses require large and complex machinery.

3. Chromatic aberration (light rays passing through a lens focus at different points, depending on their wavelength).

Fact: The largest lens objective in use has diameter of 40 inch ($\sim 1.02\ m$). It is at the Yerkes Observatory in Wisconsin, USA.

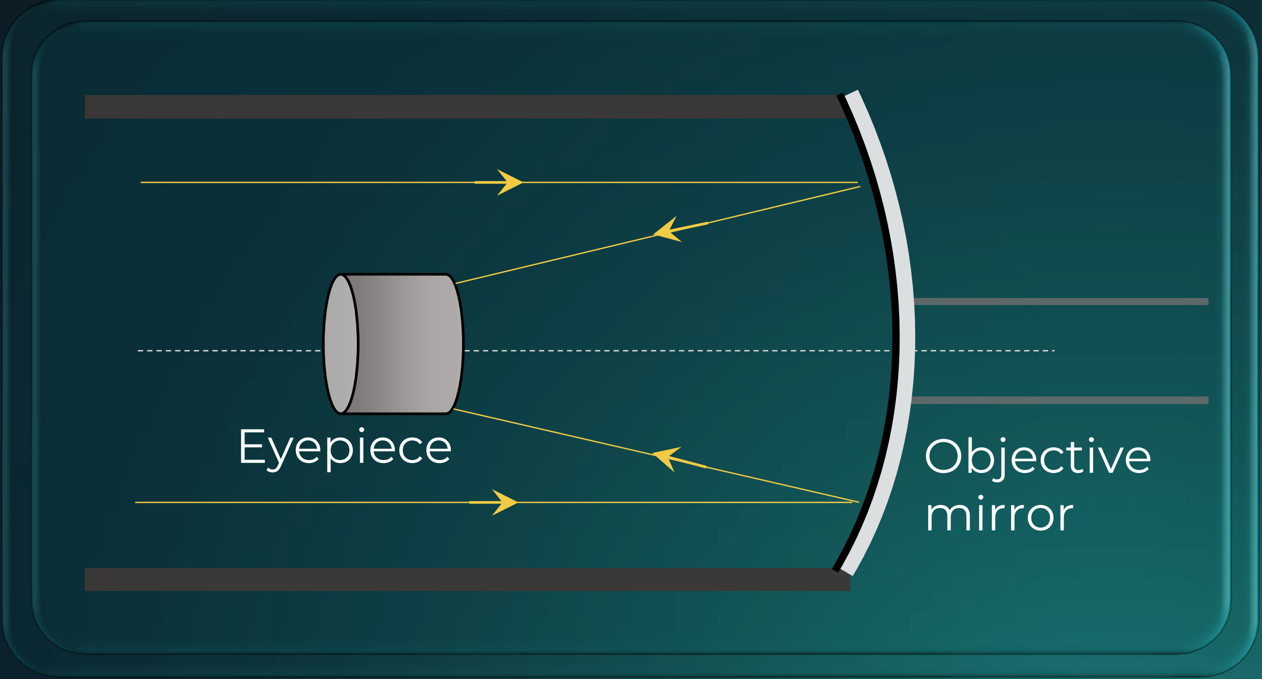
Chromatic aberration

Advantages of using Mirror as the Objective in Telescope

1. No chromatic aberration

2. Parabolic mirror used
   to counter spherical aberration.

3. Large mirrors can be
   supported from the back.



Fact: The viewer sits near the focal point of the mirror, in a small cage.

Calculate the limit of resolution of a telescope objective having a diameter of $200\ cm$, if it has to detect light of wavelength $500\ nm$ coming from a star.

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Given: $b = 200\ cm, \quad \lambda = 500\ nm$

To find: Limit of resolution of telescope

Solution: Limit of resolution of telescope is given by:

$$\boxed{\Delta\theta = \frac{1.22\lambda}{b}}$$

$$\Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$$

$$\boxed{305 \times 10^{-9}\ radian}$$

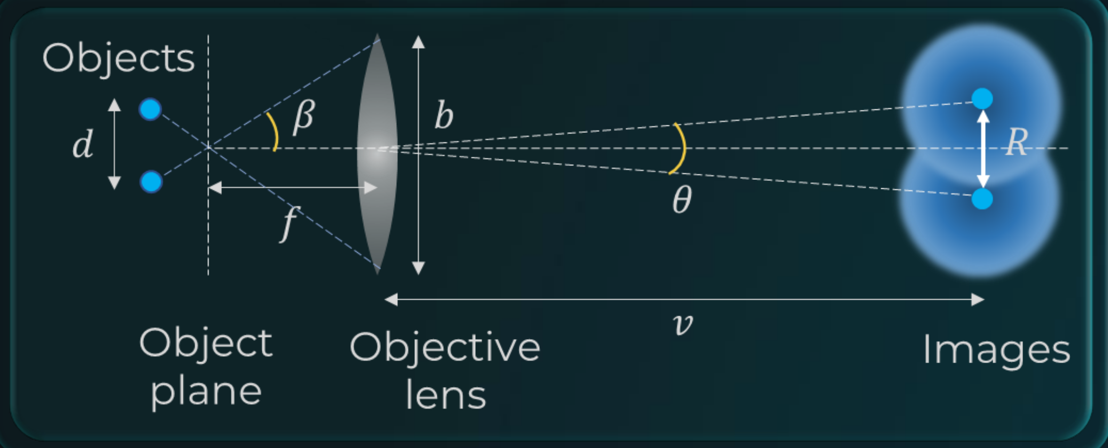
A. $610 \times 10^{-9}\ radian$

B. $152.5 \times 10^{-9}\ radian$

C. $457.5 \times 10^{-9}\ radian$

D. $305 \times 10^{-9}\ radian$

Limit of Resolution of a Microscope



- Clear images can be seen in the microscope if the diffraction discs are just resolved.

Angular limit of resolution of microscope:

$$\boxed{\sin\theta \approx \theta = \frac{1.22\lambda}{b}}$$

$$R = v\theta = v \times \frac{1.22\lambda}{b}$$

Magnification of convex lens: $m = \frac{R}{d} \Rightarrow d = \frac{R}{m}$ | $d = v \times \frac{1.22\lambda}{bm}$

Lens Formula: $\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$

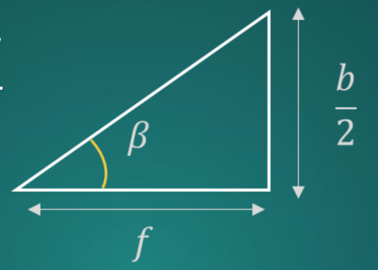
$$1 - \frac{v}{u} = \frac{v}{f} \quad \Rightarrow 1 - m = \frac{v}{f} \qquad \left\{m = \frac{v}{u}\right\}$$

$$m = 1 - \frac{v}{f}$$

$$m \approx -\frac{v}{f} \qquad \left\{v \gg f \Rightarrow \frac{v}{f} \gg 1\right\}$$

We know that: $d = v \times \frac{1.22\lambda}{bm}$

$$d = \frac{1.22\lambda}{bm} \times |-mf| = \frac{1.22\lambda f}{b}$$



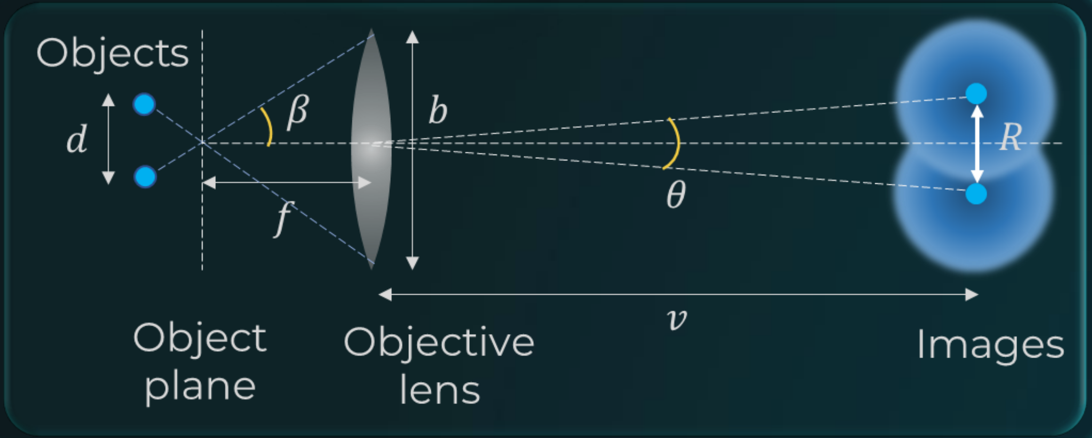
$$\tan\beta = \frac{b}{2f} \quad \Rightarrow b = 2f\tan\beta$$

β is small, $b = 2f\sin\beta$

$$d_{min} = \frac{1.22\lambda f}{b} = \frac{1.22\lambda f}{(2f\sin\beta)}$$

$$\boxed{d_{min} = \frac{1.22\lambda}{2\sin\beta}}$$

Microscope immersed in Oil



Note: The product $\mu\sin\beta$ is called the numerical aperture and is sometimesmarked on the objective.

When the setup is immersed in oil, λ_{med} changes to $\frac{\lambda}{\mu}$

$$d_{min} = \frac{1.22\lambda_{med}}{2\sin\beta} \longrightarrow \boxed{d_{min} = \frac{1.22\lambda}{2\mu\sin\beta}}$$

Resolving power of a microscope:

$$R.P. = \frac{1}{d_{min}} \Rightarrow \boxed{R.P. = \frac{2\mu\sin\beta}{1.22\lambda}}$$

- Resolving power of the microscope increases when it is immersed in a medium.

Validity of Ray Optics

- Consider a single slit of width 'a'.Diffraction pattern will be observed.
- There will be central maxima due to diffraction.
- Angular size of central maximum, $\theta = \frac{\lambda}{a}$
- The width of diffracted beam after it has travelled by z, $y = z \times \theta = \frac{\lambda z}{a}$

If $y \approx a$ $\quad \boxed{z \approx \frac{a^2}{\lambda} = z_F}$ $\quad z_F$ is Fresnel distance.

- If $z < z_F$, the ray optics is valid.
- If $z > z_F$, spreading due to diffraction dominates.

For what distance is ray optics a good approximation when a plane light wave is incident on a circular aperture of width $2\ mm$ having wavelength $600\ nm$?

Given: $a = 2\ mm; \lambda = 600\ nm$

Solution: Ray optics is a good approximation up to Fresnel distance only.

$$\boxed{z \approx \frac{a^2}{\lambda} = z_F}$$

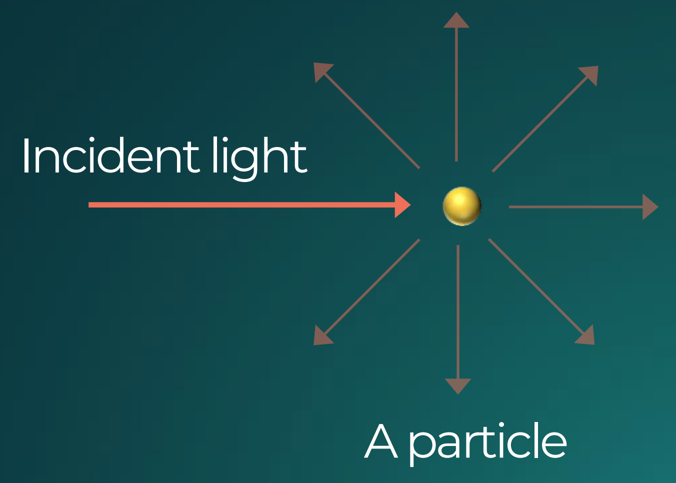
$$z_F = \frac{(2 \times 10^{-3})^2}{600 \times 10^{-9}} = 6.7\ m$$

- If $d < 6.7\ m \longrightarrow$ Ray optics holds.
- If $d > 6.7\ m \longrightarrow$ Spreading due to diffraction dominates.

Scattering

When a parallel beam of light passes through a medium, a part of it appears in directions other than the incident direction. This phenomenon is called scattering of light.

- Light is an EM wave, it oscillates the charged particles in a medium because of its oscillating electric field.
- Oscillating charged particles emit EM waves.
- If the oscillating electric field of incident light has frequency f, the frequency of scattered wave will also have frequency f.



Rayleigh's Law of Scattering

- Intensity of scattering depends on

  1. Wavelength of light

  2. Size of particles causing scattering

- When size of particles $< \lambda$

$$\boxed{\text{Intensity of scattered wave} \propto \frac{1}{\lambda^4}}$$

$\lambda - minimum$ | V | Scatters most

I

B

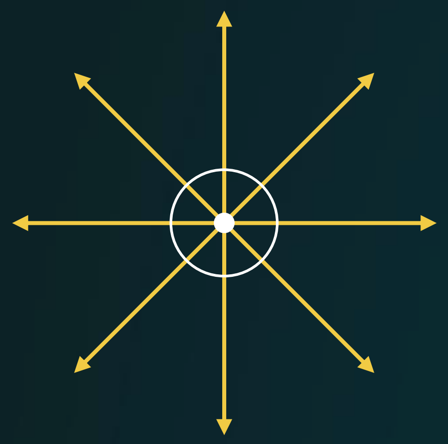
λ increases and Intensity of scattering decreases $\downarrow$ | G

Y

O

$\lambda - maximum$ | R | Scatters least

Unpolarized Light



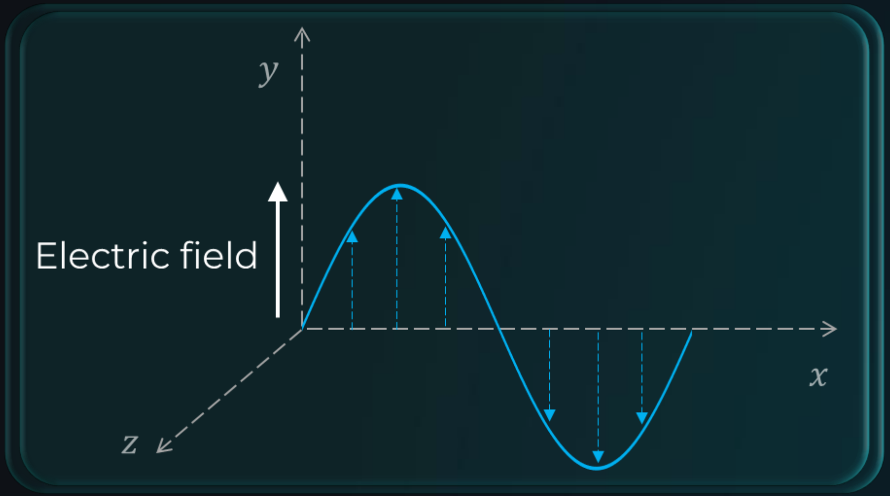
- The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation.
- Examples of source of unpolarized light: Candle, Bulb, Sun etc.

Note: Light wave is coming out of the screen, and arrows show direction of oscillation of electric field.

Plane Polarized Light

Plane Polarized light – When electric field at a point always remains parallel to a fixed direction as time passes.

Plane of polarization – Plane containing electric field and direction of propagation.



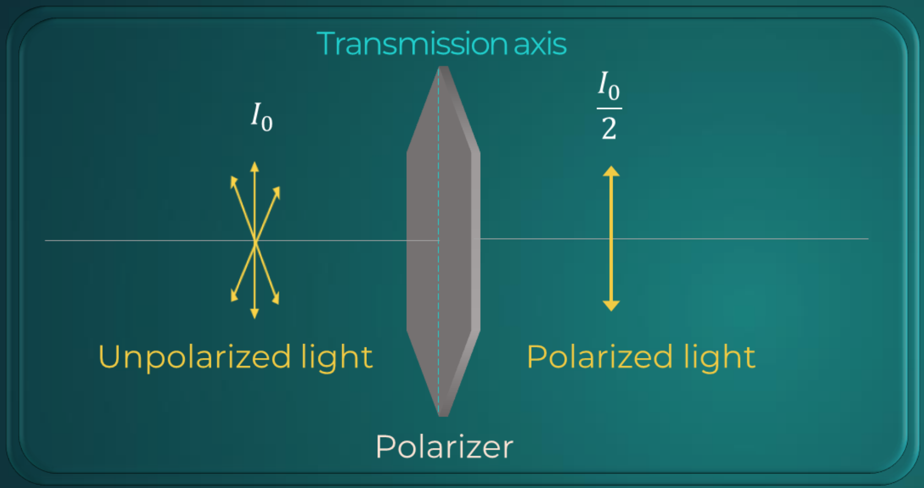
$x - axis$ → Direction of propagation.

$y - axis$ → Oscillation of electric field.

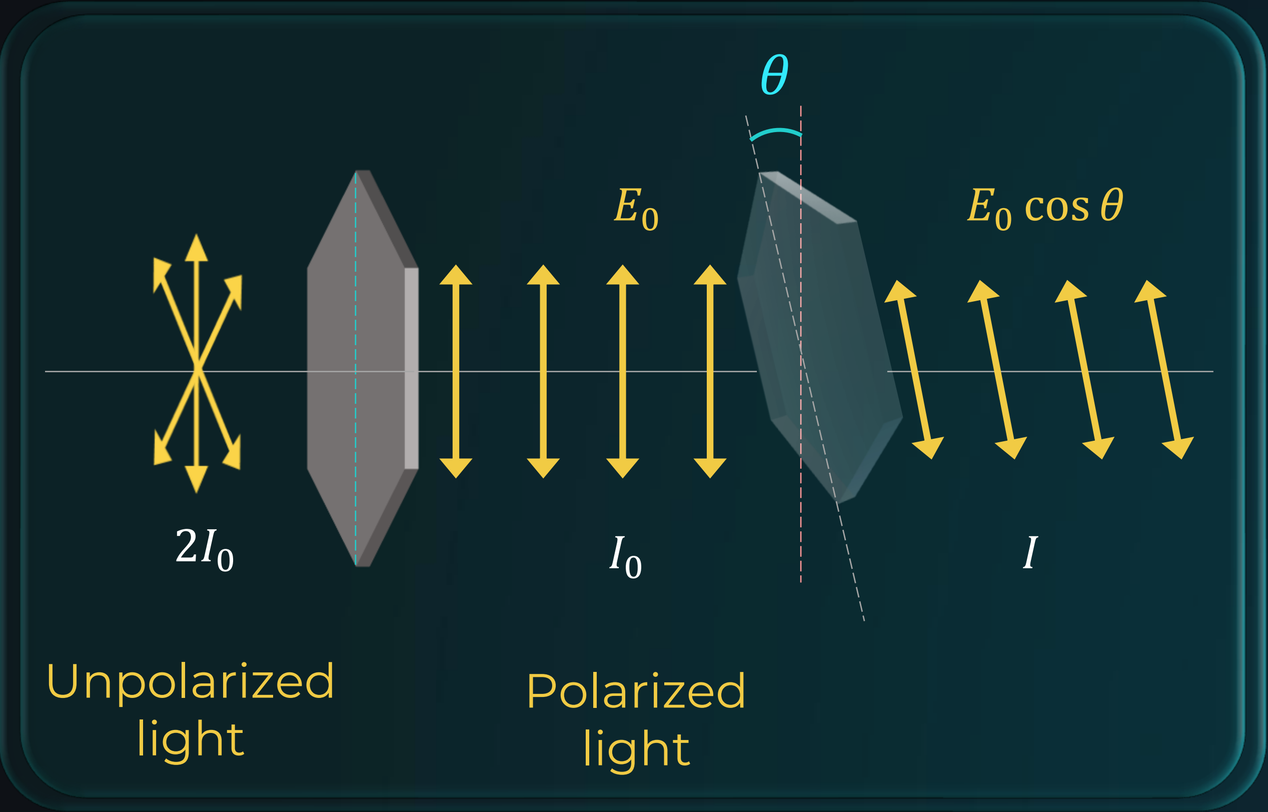
$xy\ plane$ → Plane of polarization.

When the polarizer is placed in the path of unpolarized light, the direction of oscillation of the electric field becomes parallel to the transmission axis.

Note: If any unpolarized light of intensity I_0 is incident on a polarizer, we get a polarized light of intensity $\frac{I_0}{2}$.



Law of Malus



Electric field amplitude of the wave after crossing the polarizer:

$$E = E_0 \cos\theta$$

Intensity of the wave after crossing the polarizer:

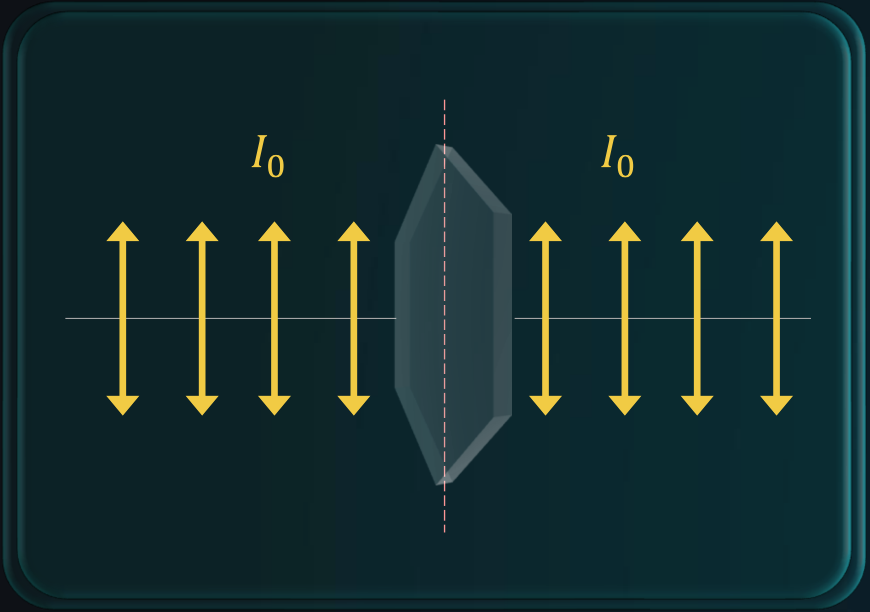
$$I = I_0 \cos^2\theta \qquad \{I \propto E^2\}$$

- I = Intensity of transmitted light
- I_0 = Intensity of incident light

Law of Malus

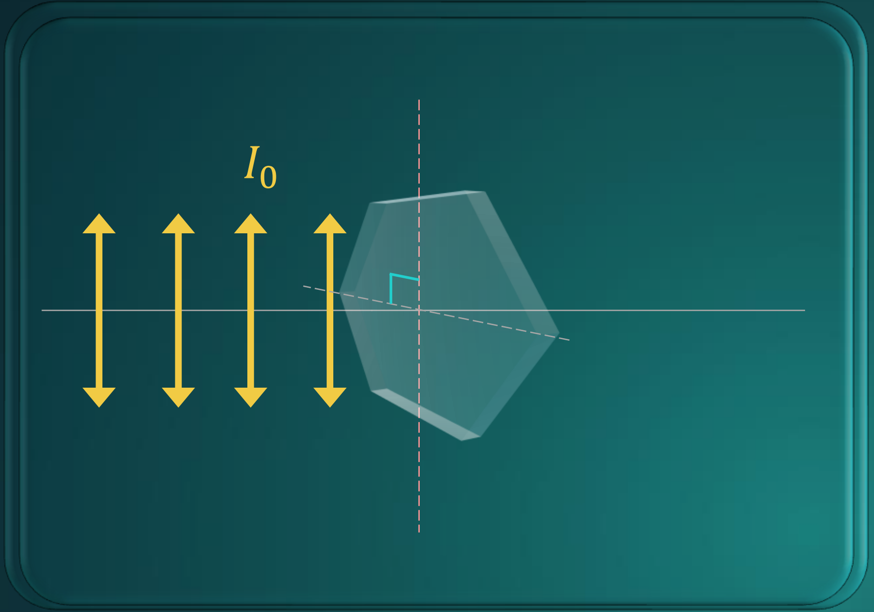
- Case 1 → when $\theta = 0°$:

$$I = I_0 \cos^2 0° = I_0$$

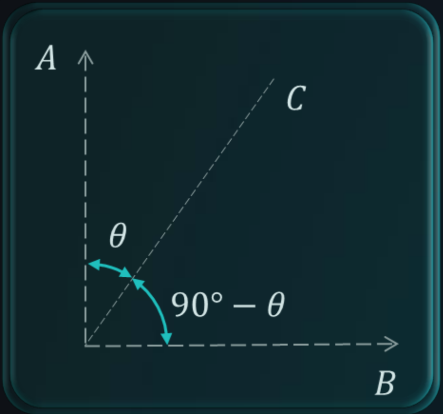


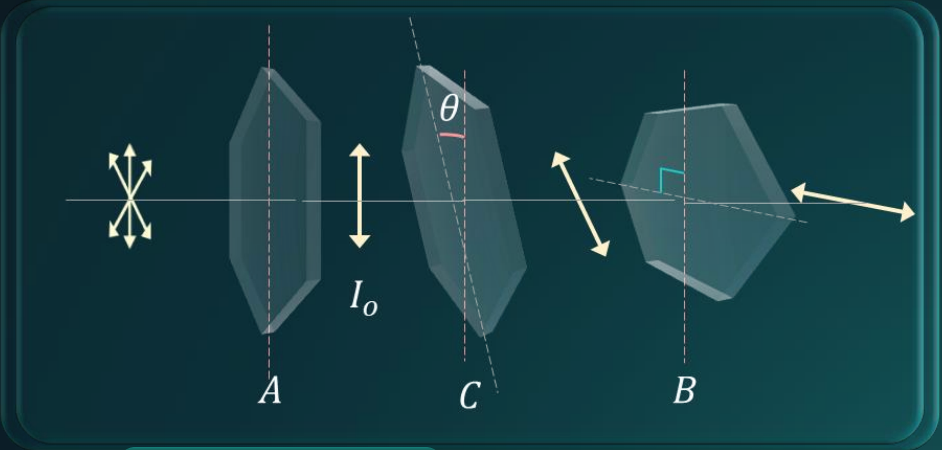
- Case 2 → when $\theta = 90°$:

$$I = I_0 \cos^2 90° = 0$$



Two 'crossed' polaroids A and B are placed in the path of a light beam. In between these, a third polaroid C is placed whose polarization axis makes an angle θ with the polarization axis of the polaroid A. If the intensity of light emerging from the polaroid A is I_o, then the intensity of light emerging from polaroid B will be





Solution: Intensity of light emerging from polaroid C: $\boxed{I_C = I_0 \cos^2\theta}$ {Applying the law of Malus.}

Intensity of light emerging from polaroid B: $I_B = I_C \cos^2(90° - \theta)$

$$I_B = (I_0 \cos^2\theta).\cos^2(90° - \theta)$$

$$I_B = I_0 \cos^2\theta\,.\sin^2\theta = \frac{I_0}{4}(2\sin\theta\cos\theta)^2$$

$$\boxed{I_B = \frac{I_0}{4}\sin^2(2\theta)}$$

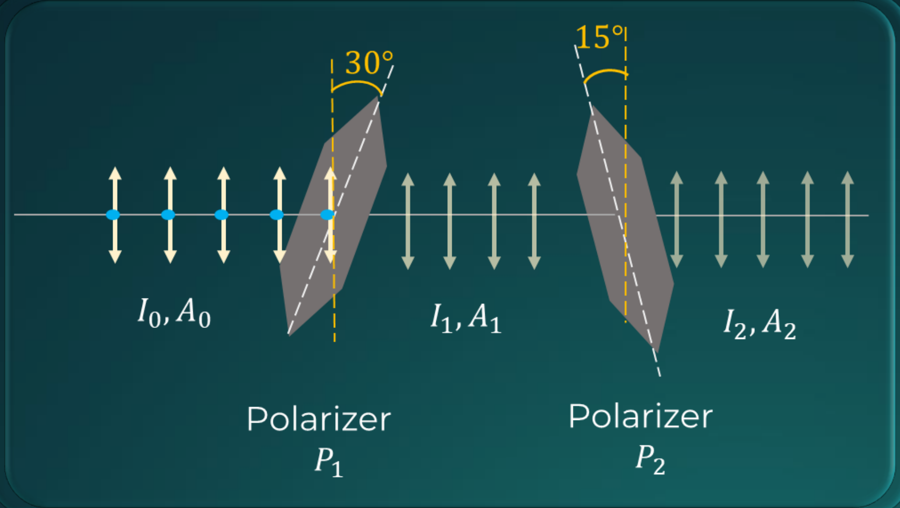
A. $I = \frac{I_o}{4}\sin^2(2\theta)$

B. $I = \frac{I_o}{2}\sin^2(2\theta)$

C. $I = \frac{I_o}{4}\sin^2(\theta)$

D. $I = \frac{I_o}{2}\sin^2(\theta)$

Unpolarized light with amplitude A_0 passes through two polarizers. The first one has an angle of 30° clockwise to vertical and second one has an angle of 15° counter-clockwise to the vertical. What is the amplitude of the light emitted from the second polarizer?



Given: $A = A_0$

To find: A_2

Solution:

We know that: $I \propto A^2$.

$\therefore A \propto \sqrt{I}$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\sqrt{I_1}}{\sqrt{I_2}}$$

$$I_1 = \frac{I_0}{2}$$

$$\frac{A_1}{A_0} = \sqrt{\frac{I_1}{I_0}} = \sqrt{\frac{I_0}{2I_0}} = \frac{1}{\sqrt{2}}$$

$$A_1 = \frac{A_0}{\sqrt{2}}$$

$$A_1 = \frac{A_0}{\sqrt{2}}$$

Again, $\frac{A_1}{A_2} = \frac{\sqrt{I_1}}{\sqrt{I_2}}$

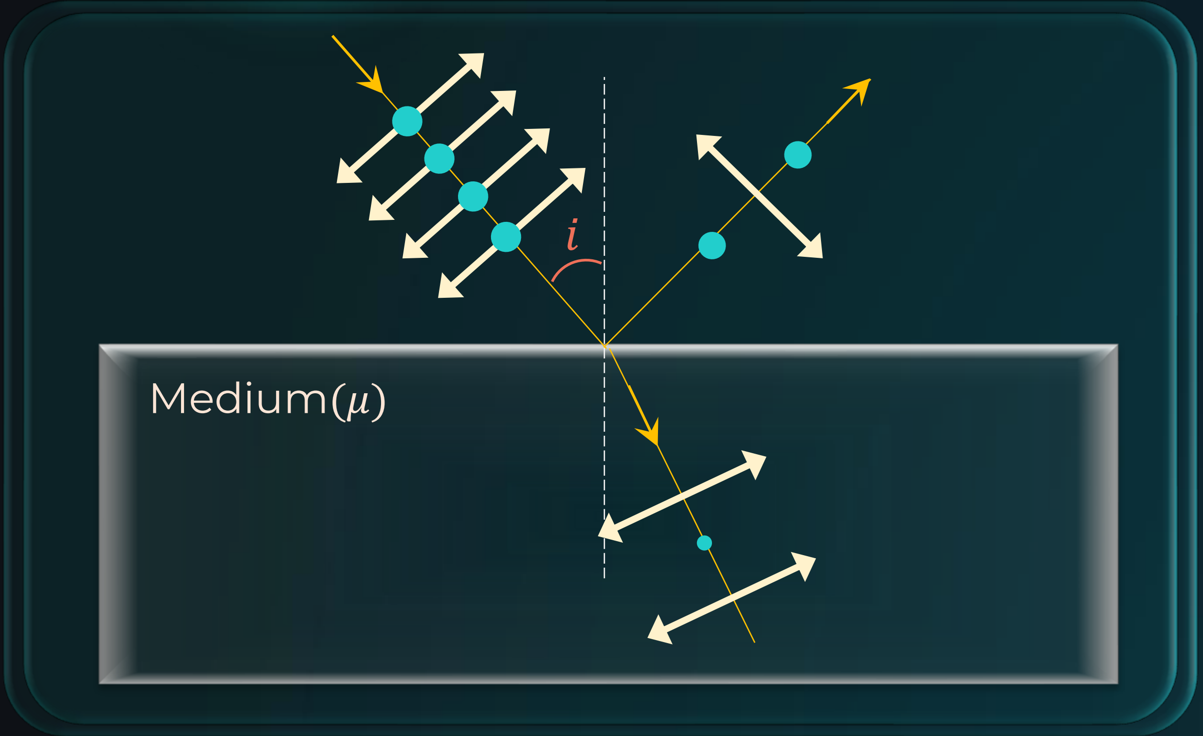
$$I_2 = I_1 \cos^2\theta \quad \left(\theta = 30° - (-15°)\right) = 45°$$

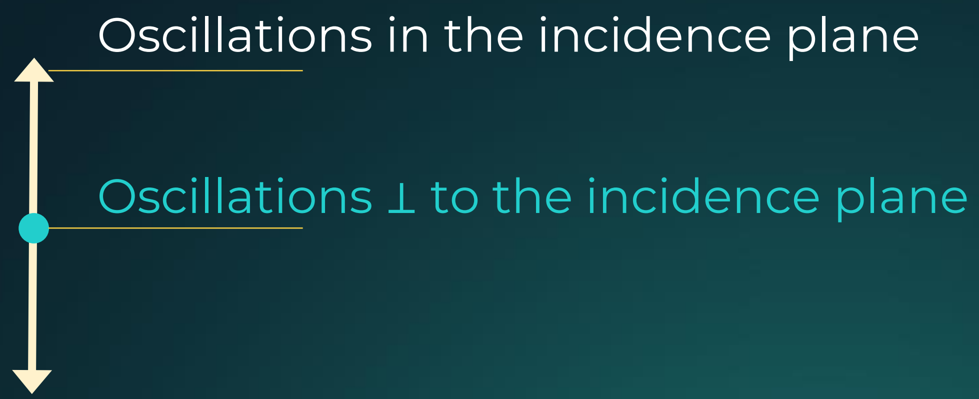
$$I_2 = \frac{I_0}{2}\cos^2 45° = \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

$$\frac{A_2}{A_0} = \sqrt{\frac{I_2}{I_0}} = \sqrt{\frac{I_0}{4I_0}} = \frac{1}{\sqrt{4}}$$

$$\boxed{A_2 = \frac{A_0}{2}}$$

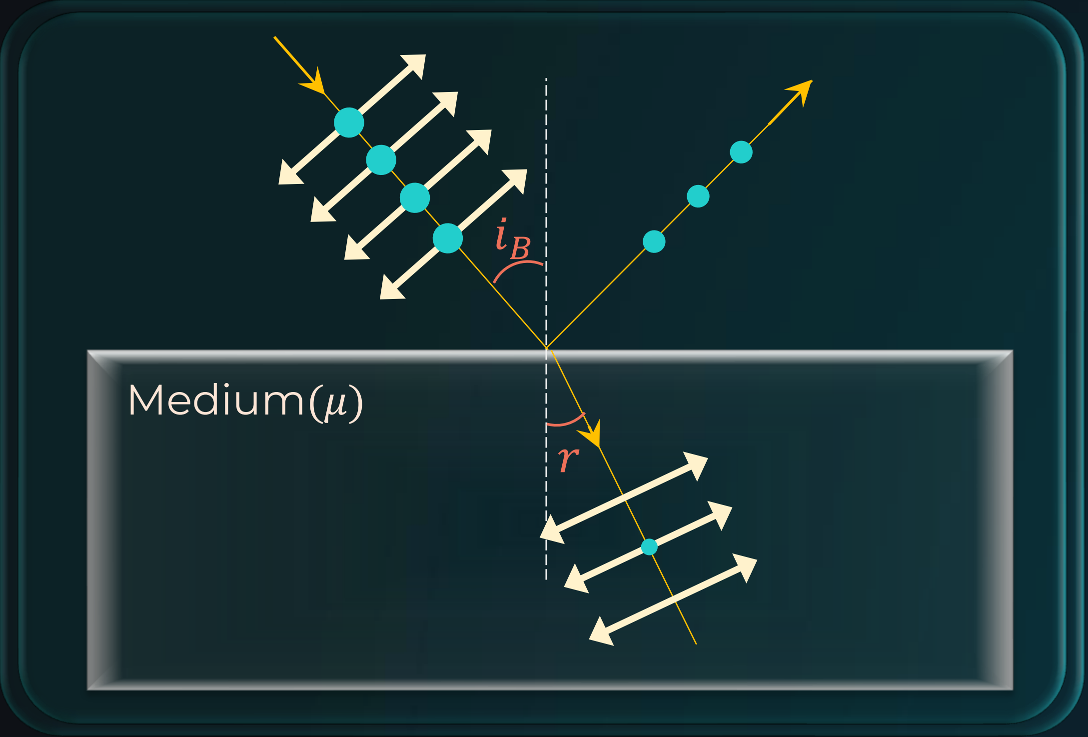
Polarization by Reflection





- The reflected light has more vibrations perpendicular to plane of incidence.
- The refracted light has more vibration parallel to the plane of incidence.
- The percentage of polarization in reflected light changes as we change the angle of incidence i.

Brewster's Law



- For a particular angle of incidence (i_B), the reflected light becomes completely plane polarized.
- The required condition for this purpose is:

$$i_B + r = 90°$$

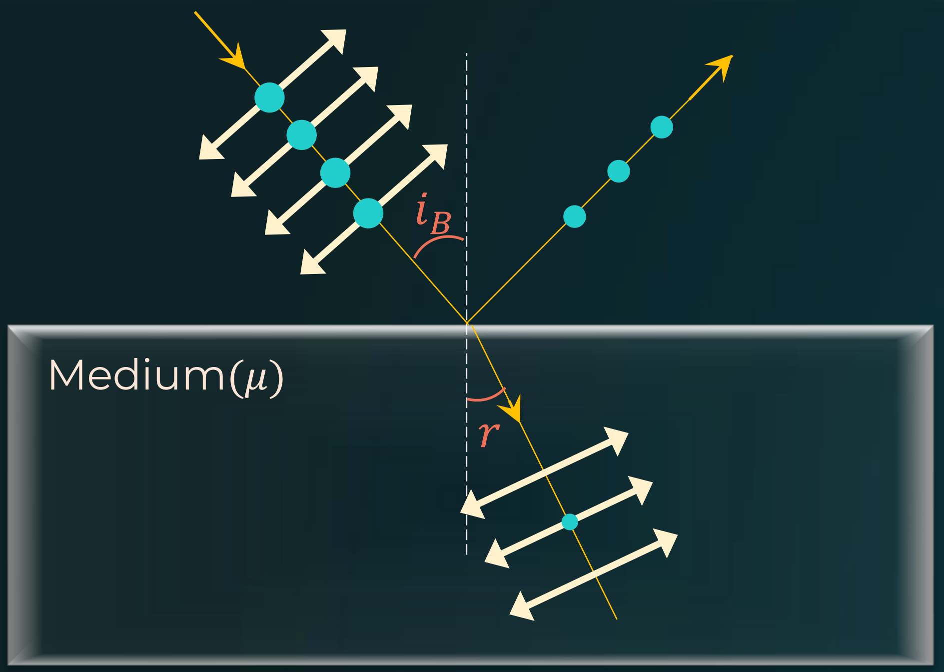
- Brewster's Law $\boxed{\tan i_B = \mu}$

i_B = Brewster angle/polarising angle

- If the light ray travels from one medium to another with refractive μ_1 and μ_2 respectively, then Brewster's law becomes,

$$\boxed{\tan i_B = \frac{\mu_2}{\mu_1}}$$

Relation between Critical Angle and Brewster Angle



- Brewster's Law $\boxed{\tan i_B = \mu}$

  i_B = Brewster angle/polarising angle

- Critical Angle:

$$\sin \theta_C = \frac{1}{\mu}$$

$$\therefore \tan i_B = \frac{1}{\sin \theta_C}$$

$$\boxed{i_B = \tan^{-1}\left(\frac{1}{\sin \theta_C}\right)} \quad \text{And} \quad \boxed{\theta_C = \sin^{-1}\left(\frac{1}{\tan i_B}\right)}$$

The polarizing angle of diamond is 67°. The critical angle of diamond is nearest to: [Given $\tan 67° = 2.36$]

Given: $i_B = 67°$

To find: Critical angle

Solution: Critical angle is given by: $\boxed{\theta_C = \sin^{-1}\left(\frac{1}{\tan i_B}\right)}$

$$\theta_C = \sin^{-1}\frac{1}{\tan 67°}$$

$$\theta_C = \sin^{-1}\frac{1}{2.36}$$

$$\frac{1}{2.36} < \frac{1}{2} \quad \Rightarrow \sin^{-1}\left(\frac{1}{2.36}\right) < \sin^{-1}\left(\frac{1}{2}\right) \quad \Rightarrow \theta_C < 30°$$

∴ Out of the four option only 22° is less than 30°

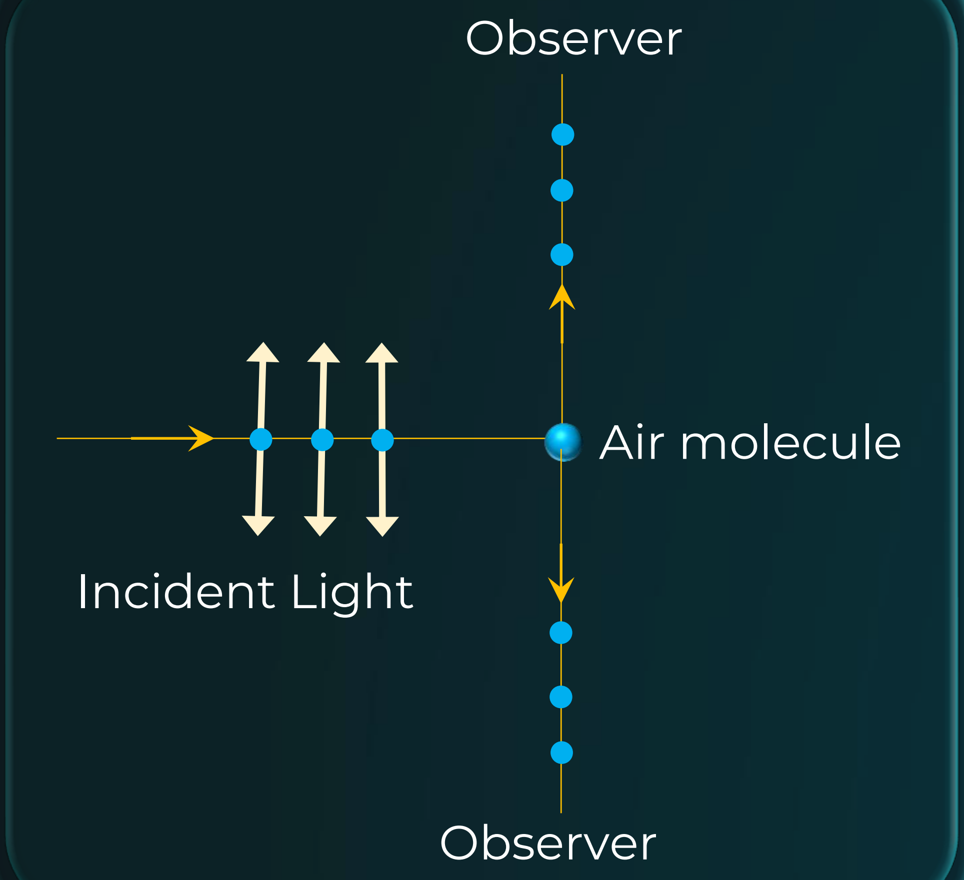
A. 34°

B. 45°

C. 60°

D. 22°

Scattering by Polarization



- If an unpolarized light gets scattered from air molecule, light perpendicular to original ray is plane polarized.
- If we draw a plane perpendicular to the incident light, then from every viewpoint on the plane we can see the plane polarized light.

A ray of light, travelling in air, is incident on a glass slab with angle of incidence $60°$. It is found that the reflected ray is plane polarized. The velocity of light in the glass is:

Given: $i_p = 60°$

To find: Velocity of light in glass (v_g)

Solution:

As reflected light is plane polarized,

$i_p = 60°$

According to Brewster's law,

$\mu = \tan i_p = \tan 60° = \sqrt{3}$

As, $\mu = \frac{v_a}{v_g} \Rightarrow v_g = \frac{v_a}{\mu}$

$$\boxed{v_g = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8\ m/s}$$