Instructions:
1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagrams and justification should be given wherever necessary.
4. Before starting to answer, fill in the FACE SLIP completely.
5. Your ‘rough work’ should be done in the answer sheet itself.
6. Maximum time allowed is THREE hours.

Question 1:
ABCD is an isosceles trapezium as shown in the figure, in which AB = DC, \( \angle DAP = 20^\circ \), DP is perpendicular to AP, \( \angle C = 70^\circ \), QR is the bisector of \( \angle BQD \) and PS \( \perp \) QR. Calculate \( \angle SPQ \) and \( \angle SRA \). Justify each of the steps in calculation.

Solution:
In the figure,
\( \angle ABC = 70^\circ \) (Base of isosceles trapezium are equal)
Now, Let \( \angle BQR \) be \( x \),
Then, \( \angle SQP = x \)
And \( \angle SPQ = 90^\circ — x \)
Also, \( \angle APQ = 90^\circ \)
In quadrilateral APQR,
\( \angle APQ + \angle PQR + \angle QRA + \angle RAP = 360^\circ \)
\( 90^\circ + x + \angle QRA + 90^\circ = 360^\circ \)
\( \angle QRA = 180^\circ — x \)  

In Triangle RBQ,
\( \angle QRB + \angle RBQ + \angle BQR = 180^\circ \)
\( \angle QRB + 70 + x = 180^\circ \)
\[ \angle QRB = 110 \quad \text{--- Eq(2)} \]

Now, from Eq(1) and Eq(2)
Since ARB is a straight line,
\[ ARQ + \angle QRB = 180^\circ \]
\[ 180^\circ = x + 110 - x = 180^\circ \]
\[ 2x = 180^\circ \]
\[ x = 55^\circ \]

Hence, \[ \angle SPQ = 90^\circ - x = 90^\circ - 55^\circ = 35^\circ \]
And \[ \angle SRA = 180^\circ - x = 180^\circ - 55^\circ = 125^\circ \]

**Question 2:**
Ramanujan is a sixth-grade student. His mathematics teacher gave a problem sheet in maths as home task for the Puja holidays. Ramanujan wants to complete it in 4 days and wants to enjoy the holidays for the remaining 6 days.

On the first day, he worked out one-fifth the number of problems plus 12 more problems.

On the second day, he worked out one-fourth the remaining problems plus 15 more problems.

On the third day, he solved one-third of the remaining problems plus 20 more problems.

The fourth day, he worked out successfully the remaining 60 problems and completed the work.

Find the total number of problems given by the teacher and the number of problems solved by Ramanujan on each day.

**Solution:**

Let the total number of problems be \( P \).

Number of Problems solved by Ramanujan on the first day = \( \frac{P}{5} + 12 \)

Remaining number of problems after the first day

\[
P - \left( \frac{P}{5} + 12 \right) = \frac{P}{1} - \frac{P}{5} - \frac{12}{1}
\]

\[
= \frac{5P - P - 60}{5} = \frac{4P - 60}{5}
\]
Number of problems solved by Ramanujan on the second day
\[
\frac{1}{4} \text{ of } \frac{4P-60}{5} + 15
\]
\[
= \frac{4P-60}{20} + 15
\]
\[
= \frac{4P - 60 + 300}{20}
\]
\[
= \frac{4P + 240}{20}
\]

Now, let us find the total number of problems solved on first and the second day:
\[
\frac{P}{5} + 12 + \frac{4P + 240}{20}
\]
\[
= \frac{8P + 480}{20}
\]

Remaining problems after the second day:
\[
P - \frac{8P + 480}{20}
\]
\[
= \frac{20P - 8P - 480}{20}
\]
\[
= \frac{12P - 480}{20}
\]
\[
= \frac{12P - 480 + 1200}{20}
\]
\[
= \frac{12P + 720}{20}
\]

Number of problems solved by Ramanujan on the third day:
\[
\frac{1}{3} \text{ of } \frac{12P - 480}{60} + 20
\]
\[
= \frac{12P - 480}{60} + 20
\]
\[
= \frac{12P - 480 + 1200}{60}
\]
\[
= \frac{12P + 720}{60}
\]

Now let us find the total number of problems solved in three days:
Day 1 (problem solved) + Day 2 (problem solved) + Day 3 (problem solved)
\[
= \frac{8P + 480}{20} + \frac{12P + 720}{60}
\]

Solving the expression above, we get:
\[
= \frac{36P + 2160}{60}
\]

Now let us find the remaining problems after third day:
\[
P - \frac{36P + 2160}{60}
\]
\[
= \frac{60P - 36P + 2160}{60}
\]
\[
= \frac{24P - 2160}{60}
\]

According to the question, the number of problems remaining to be solved after the third day is 60.
Therefore,
\[
\frac{24P - 2160}{60} = 60
\]
\[24P - 2160 = 60 \times 60\]
\[24P = 3600 + 2160\]
\[24P = 5760\]
\[P = \frac{5760}{24}\]
\[P = 240\]
Therefore, the total number of Problems solved by Ramanujan is 240.

Now let us find the number of Problems solved by Ramanujan on each day respectively.

Number of Problems solved by Ramanujan on the first day = \(\frac{P}{5} + 12 = 48 + 12 = 60\)
Number of Problems solved by Ramanujan on the second day = \(\frac{4P + 240}{20} = 60\)
Number of Problems solved by Ramanujan on the third day = \(\frac{12P + 720}{60} = 60\)

**Question 3:**

There are 4 cards and, on each card, a whole number is written. All the numbers are different from one another. Two girls of grade six, Deepa and Dilruba play a game.

Deepa takes 3 cards at a time leaving a card behind. She multiplies the numbers and gets an answer. In the same way, again, she leaves one different card and selects the other three and multiplies the numbers.

She got the answers 480, 560, 420 and 336.

Now, Dilruba has to find the numbers in each card. Dilruba worked out and found the correct numbers.

What are they? Work out systematically and find the numbers.

**Solution:**

Let us assume the cards to be “a”, “b”, “c” and “d” such that a, b, c and d are distinct whole numbers.

Now Deepa picks 3 cards at a time leaving one behind such that she gets 480, 560, 420 and 336.

Now we can write
\[a \times b \times c = 560\]
\[b \times c \times d = 480\]
a \times b \times d = 420 \\
c \times d \times a = 336 \\

Now if were to prime factorize these numbers we get,
480 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \\
560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7 \\
420 = 2 \times 2 \times 3 \times 5 \times 7 \\
336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 \\

But knowing that \( c \times d \times a = 336 \), c, d and a cannot be a multiple of 5 as 336 does not end with a 0 or 5. Therefore b is definitely the only multiple of 5.

Now using \( 336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 \), let us take the factors as 8, 6 and 7. We can do this as through prime factorization we can get many factor pairs and triplets.

Now \( c = 8, d = 6, a = 7 \) using these numbers in the remaining relations that we have
\[
\begin{align*}
 b \times c \times d &= 480 \\
 \Rightarrow b \times 8 \times 6 &= 480 \\
 \Rightarrow b \times 48 &= 480 \\
 \Rightarrow b &= 10 
\end{align*}
\]

Therefore, now our whole numbers are 6, 7, 8 and 10 and we can see that these numbers satisfy all the relations mentioned above.

Therefore, the correct solution for the question is 6, 7, 8, 10.

Question 4:
An angle is divided into 3 equal parts by two straight lines; such lines are called trisectors. ABCD is a square. The lines (AP, AS) are trisectors of \( \angle BAD \). Similarly, we have the trisectors (BP, BQ), (CQ, CR) and (DR, DS). Prove that PQRS is a square.
Solution:

Since AP and AS are trisectors then the value of angle PAS will be 30°
And AP = AS
Similarly, ∠ PBQ = 30° and PB = BQ
Now triangle APS is congruent to triangle BPQ.
⇒ PS = PQ
Similarly, we can prove QR=RS
Now all four sides are equal. So PQRS could be either square or rhombus.
In triangle APS ∠ P = ∠ S = 75°
In triangle ASD, ∠ S = 120°
In triangle SDR ∠ S = 75°
Now at point S the total sum of angles should be 360°.
∠ PSR = 360° - ∠ ASD - ∠ DSR - ∠ ASP
∠ PSR = 360° - 120° - 75° - 75°
∠ PSR = 90°.
Since all four sides are equal and an included angle is 90° hence it is proved that PQRS is a square.

Question 5:
Five squares of different dimensions are arranged in two ways as shown in the following diagrams. The numbers inside each square represent its area in square units.
Calculate the perimeter $A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_{12}A_1$ and the corresponding perimeter of figure 2. Are they same? If they are different, which is greater?

**Solution:**

In Figure 1,
Area of 1st Square = 9
So, side of 1st square = $\sqrt{9} = 3$
Similarly, side of 2nd Square = 4
Side of 3rd square = 9
Side of 4th square = 8
Side of 5th square = 5
So perimeter $A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_{12}A_1$
$= 3 + 3 + 1 + 4 + 5 + 9 + 1 + 8 + 3 + 5 + 5 + 29$
$= 76$

Similarly,
In Figure 2,
Perimeter $B_1B_2B_3B_4B_5B_6B_7B_8B_9B_{10}B_{11}B_{12}B_1$
$= 4 + 4 + 4 + 8 + 1 + 9 + 4 + 5 + 2 + 3 + 3 + 29$
$= 76$

So, the perimeter for both figures is the same.

**Question 6:**

i) In a book, a problem on fractions is given as

$$\frac{1}{3} - \frac{2}{3} + \frac{3}{8} + \frac{4}{7}a$$

The denominator of the third term is not printed. The answer is given to be 2. What is the missing denominator? Let it be $a$.

ii) Simplify: $\frac{1}{3} - \frac{1}{2} - \frac{1}{(5/7)}$. Let the answer be of the form $\frac{p}{q}$ where $p,q$ have no common factors. Let $p,q$ have no common factors. Let $b = \frac{p}{q}$. 
iii) Find the value of \( \frac{1}{a^2} + b \)

Solution:

i) Given that,

\[
\frac{1}{3} + \frac{9}{5} + \frac{3}{a} + \frac{4}{7} = 2
\]

\[
\frac{1}{16} + \frac{9}{4} + \frac{29}{8a} + \frac{7}{3} = 2
\]

\[
\frac{5}{16} + \frac{1}{4} + \frac{29}{8a} + \frac{1}{8} = 2
\]

\[
\frac{5}{16} + \frac{4}{16} + \frac{58}{16a} + \frac{2}{16} = 2
\]

\[
\frac{3}{16} + \frac{58}{16a} = 2
\]

\[
\frac{58}{16a} = 2 - \frac{3}{16}
\]

\[
\Rightarrow \frac{58}{16a} = \frac{29}{16}
\]

\[
\Rightarrow \frac{2}{16a} = \frac{1}{16}
\]

\[
a = 2
\]

ii) Simplifying,

\[
\frac{1}{3} - \frac{1}{2} = \frac{1}{(5/7)}
\]
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\[ \frac{1}{3} - \frac{1}{7} = \frac{1}{10} \]

\[ \frac{1}{3} - \frac{1}{10} = \frac{1}{5} \]

\[ \frac{1}{3} - \frac{5}{3} = \frac{-2}{3} \]

\[ \frac{1}{9} - \frac{5}{3} = \frac{-16}{9} \]

iii) \((\frac{1}{a^2} + b) = \frac{3}{4}\)

\[ a = 2, b = \frac{3}{4} = (\frac{1}{2^2} + \frac{3}{4}) = \frac{1+3}{4} = \frac{4}{4} = 1 \]

\[ (\frac{1}{a^2} + b) = 1 \]

**Question 7:**

a and b are two integers, find all pairs such that \(\frac{1}{a} + \frac{1}{b} = \frac{1}{2}\). Arrive at your result logically.

**Solution:**

Given: \(\frac{1}{a} + \frac{1}{b} = \frac{1}{2}\), where a, b are integers
Let's multiply both sides by $2ab$ to get,
\[ ab - 2a - 2b = 0 \]
Add $2^2$ on both the sides
Therefore, $ab - 2a - 2b + 2^2 = 2^2$
On simplifying we get as $(a-2)(b-2) = 2^2$
In general : $(a-n)(b-n) = n^2$ (n is integer)
Therefore the divisors of $n^2$ are $\pm n^2, \pm n, \pm 1$
Also $-n$ is not possible.
So the general solution when n is a prime number is given by
\[ (a, b) = (n-n^2, n-1), (n-1, n-n^2), (n+1, n+n^2), (n+n, n+n), (n+n^2, n+1) \]
Hence for the equation, $(a-2)(b-2) = 2^2$, we get
\[ (a, b) = (-2, 1), (1, -2), (3, 6), (4, 4) \text{ and } (6, 3) \]

**Question 8 :**
A train starts from a station A and travels with constant speed up to 100 kms/hr. After some time, there appeared a problem in the engine and so the train proceeds with \( \frac{3}{4} \) th of the original speed and arrives at station B, late by 90 min. Had the problem in the engine occurred 60 kms further on, then the train would have reached 15 min sooner. Find the original speed of the train and distance between stations A and B.

**Solution:**
Let total distance between AB = x
Original speed of the train = v
Total time taken if no problem appeared = t

In first condition, D is the point where problem occurred in the engine,
Here, time taken by train from point A to D
⇒ \( t_a = \frac{y}{v} \)

Similarly, time taken by train from point D to B
⇒ \( t_b = \frac{x - y}{\frac{3}{4}v} \)

Now \( t_a + t_b = t + \frac{90}{60} \)
⇒ \( \frac{y}{v} + \frac{4(x - y)}{3v} = \frac{x}{v} + \frac{3}{2} \)
⇒ \( \frac{y}{v} + \frac{4x}{3v} - \frac{4y}{3v} = \frac{x}{v} + \frac{3}{2} \)
⇒ \( \frac{x}{3v} - \frac{y}{3v} = \frac{3}{2} \)
⇒ \( x - y = \frac{9}{2}v \) …(1)

Similarly in second condition,
⇒ \( \frac{y + 60}{v} + \frac{4(x - y - 60)}{3v} = \frac{x}{v} + \frac{75}{60} \)
⇒ \( \frac{y}{v} + \frac{60}{v} + \frac{4x}{3v} - \frac{4y}{3v} - \frac{4 \times 60}{3v} = \frac{x}{v} + \frac{75}{60} \)
⇒ \( \frac{x}{3v} - \frac{y}{3v} = \frac{5}{4} \)
⇒ \( \frac{x - y}{3v} = \frac{5}{4} \)
⇒ \( \frac{1}{3v} \times \frac{9v}{2} - \frac{20}{v} = \frac{5}{4} \) [Using equation (1), \( x - y = \frac{9}{2}v \)]
⇒ \( \frac{3}{2} - \frac{20}{v} = \frac{5}{4} \)
⇒ \( \frac{20}{v} = \frac{3}{2} - \frac{5}{4} = \frac{1}{4} \)
\( v = 80 \text{ km/hr} \)

Now \( x - y = \frac{9}{2} \times 80 \) [Putting value of \( v \) in equation (1)]
⇒ \( x - y = 360 \text{ km} \)

**Hence, original speed of the train is 80 km/hr**

With the given information, we cannot find the distance between A and B.
(Insufficient information)