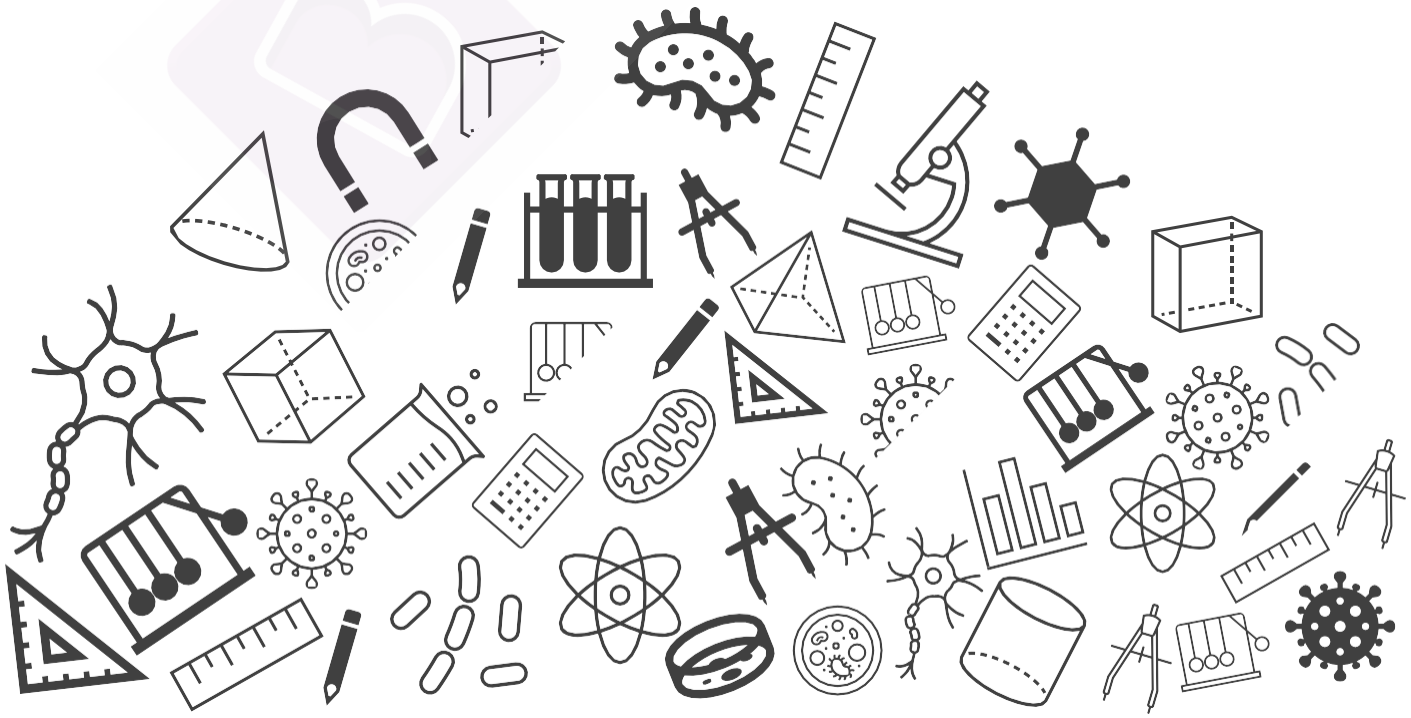
Grade 07: Maths
Chapter Notes

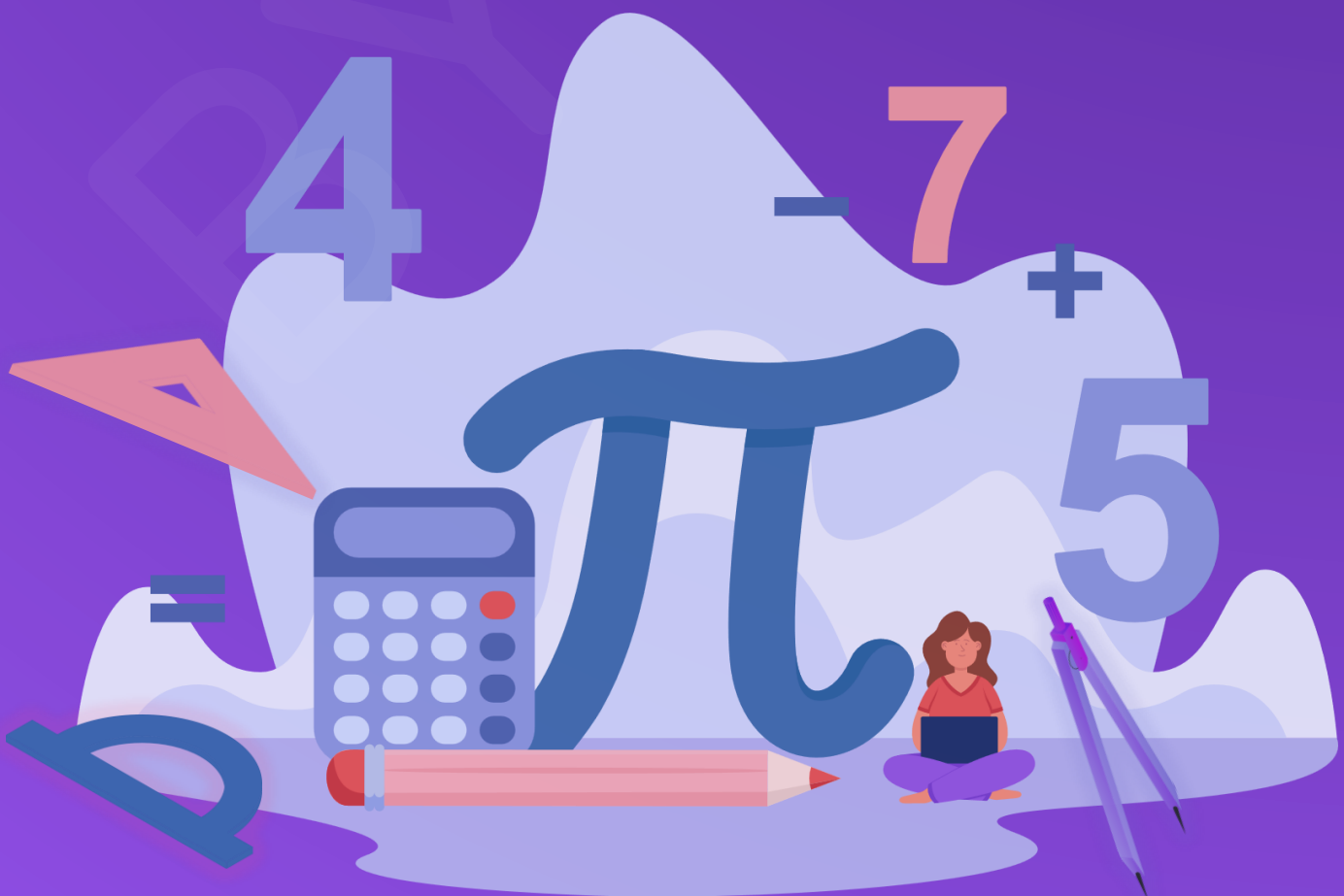


BYJU'S Classes

Chapter Notes

The Triangle and Its Properties

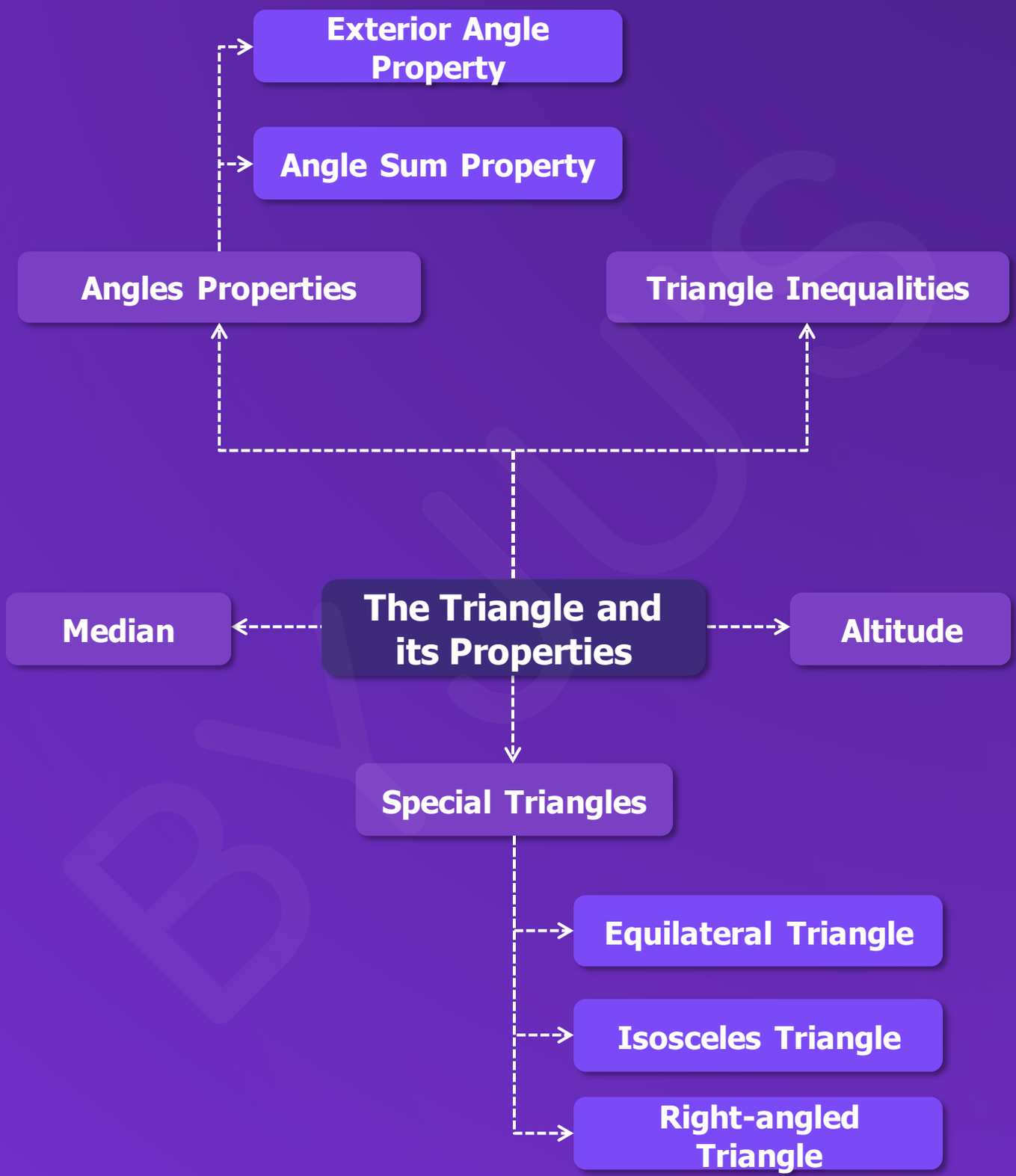
Grade 06



Topics to be Covered

1. Introduction
2. Median
3. Altitude
4. Angle Properties
   - Exterior angle property
   - Angle sum property
5. Triangle Inequalities
6. Special Triangles
   - Equilateral triangle
   - Isosceles triangle
   - Right-angled triangle

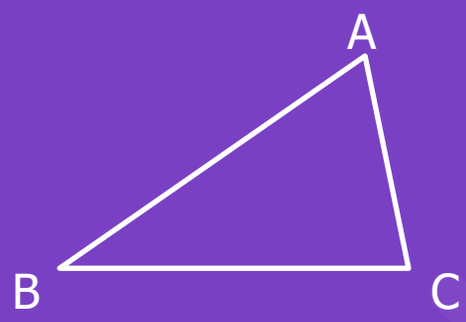
Mind Map


Exterior Angle Property
Angle Sum Property
Angles Properties
Triangle Inequalities
Median
The Triangle and its Properties
Altitude
Special Triangles
Equilateral Triangle
Isosceles Triangle
Right-angled Triangle

1. Introduction

A triangle is a simple closed curve made of three line segments.

- It has three vertices, three sides and three angles.



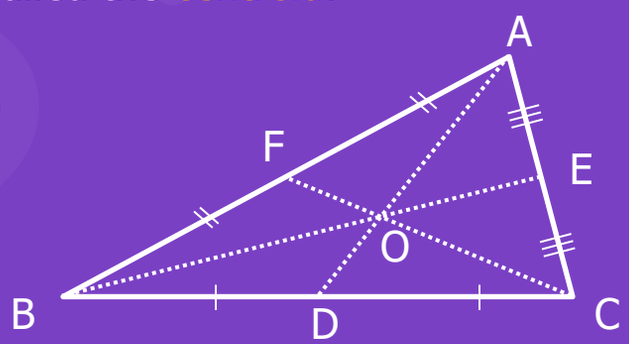
In ΔABC:

- Sides: AB, BC, CA
- Angles: ∠ABC, ∠BCA, ∠CAB
- Vertices: A, B, C

2. Median

A median of a triangle is a line segment that joins a vertex to the mid-point of the side that is opposite to that vertex.

- A triangles has only 3 medians, which always intersect at point called the centroid.



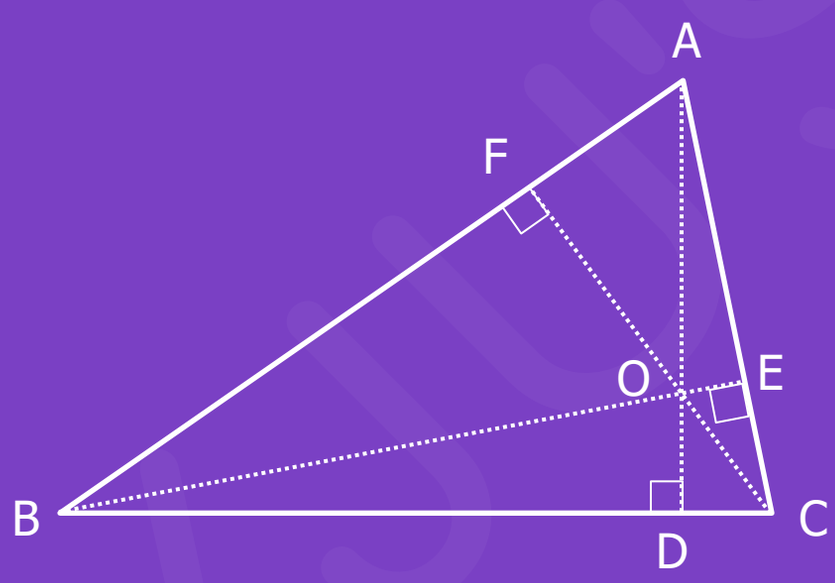
In ΔABC:

- AD is the median that bisects BC.
- BE is the median that bisects AC.
- CF is the median that bisects AB.
- O is the centroid.

3. Altitude

An altitude of a triangle is a line segment that starts from the vertex and meets the opposite side at right angles.

- The altitude is the shortest distance from the vertex to its opposite side.
- Every triangle has 3 altitudes, one from each vertex.
- The 3 altitudes always meet at a single point, no matter what the shape of the triangle is, called the orthocentre.



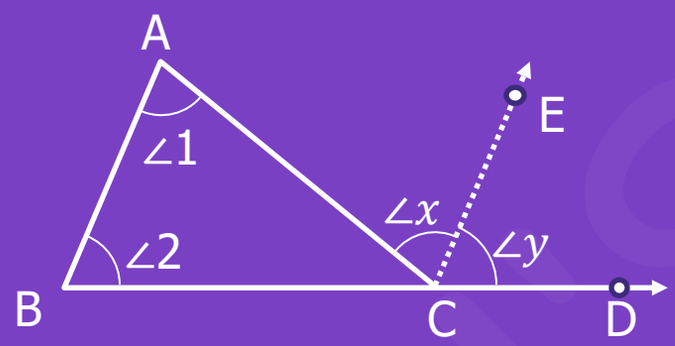
In ΔABC:

- AD is the altitude to side BC.
- BE is the altitude to side AC.
- CF is the altitude to side AB.
- O is the orthocentre.

4. Angle Properties

4.1. Exterior Angle Property

An exterior angle of a triangle is equal to the sum of its interior opposite angles.



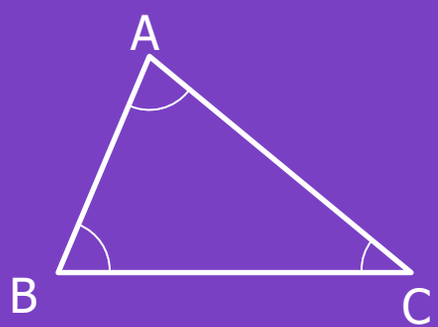
Consider ΔABC:
∠ACD is an exterior angle.

To show: $m\angle ACD = m\angle A + m\angle B$
Construction: Draw a $\overline{CE}$ parallel to $\overline{AB}$
Justification:

- $\angle 1 = \angle x$ [$\overline{CE} \parallel \overline{AB}$ and $\overline{AC}$ is the transversal]
- $\angle 2 = \angle y$ [$\overline{CE} \parallel \overline{AB}$ and $\overline{BD}$ is the transversal]
- $\angle 1 + \angle 2 = \angle x + \angle y = m\angle ACD$
- Hence, $m\angle A + m\angle B = m\angle ACD$

4.2. Angle Sum Property

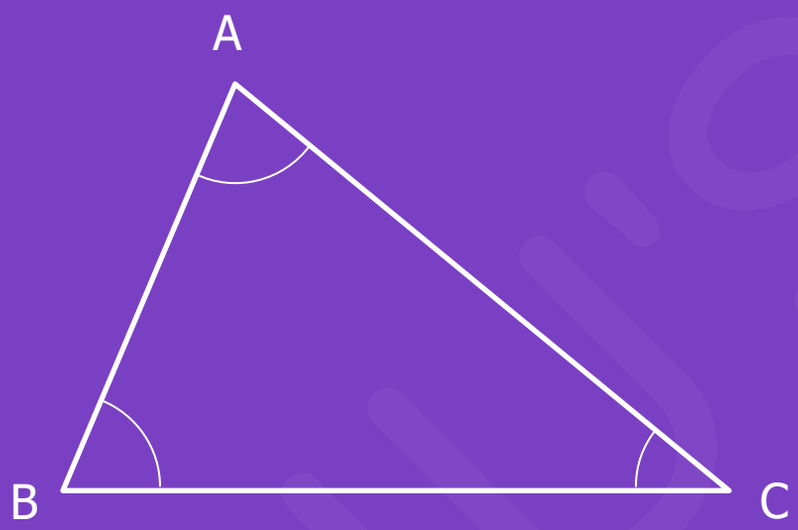
The total measure of the three angles of a triangle is 180°.



Consider ΔABC:
Here, $m\angle A + m\angle B + m\angle C = 180°$

5. Triangle Inequalities

- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- The difference between the lengths of any two sides of a triangle is smaller than the length of the third side.



Consider ΔABC:

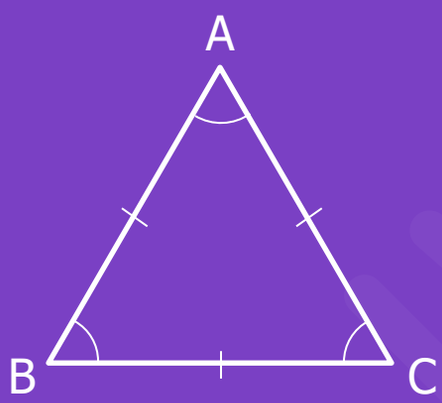
- AB + BC > AC
- AC + BC > AB
- AB + AC > BC

- BC - AB < AC
- AC - BC < AB
- AC – AB < BC

6. Special Triangles

6.1. Equilateral Triangle

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

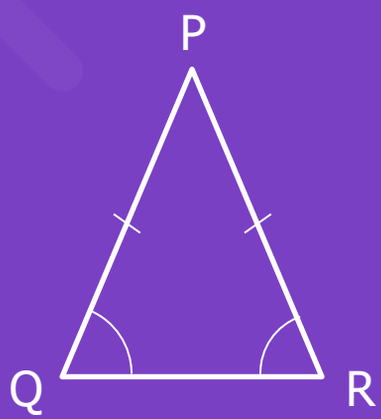


Consider ΔABC which is an equilateral triangle:

- AB = BC = CA
- ∠A = ∠B = ∠C = 60°

6.2. Isoceles Triangle

A triangle in which two sides are of equal lengths is called an isosceles triangle.



Consider ΔPQR which is an equilateral triangle:

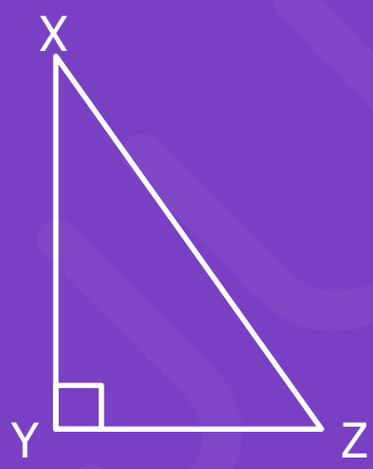
- PQ = QR
- ∠Q = ∠R [i.e., base angles opposite to the equal sides are equal]

6. Special Triangles

6.3. Right-angled Triangle

A triangle in which one of its angle is called a right-angled triangle.

- The side opposite to the right angle is called the hypotenuse and the other two sides are known as the legs of the right-angled triangle.
- In a right-angled triangle, the square on the hypotenuse = sum of the squares on the legs. This is known as Pythagoras' Theorem.



Consider ΔXYZ which is a right-angled triangle:

- XZ is the hypotenuse
- $\angle Y = 90^\circ$
- $XZ^2 = XY^2 + YZ^2$

- In an equilateral triangle the medians and the altitudes are the same.
- In an isosceles triangle the median from the vertex joining the two equal sides bisects the base at 90°.
- In a right-angled triangle the legs of the triangle are two of the altitudes.