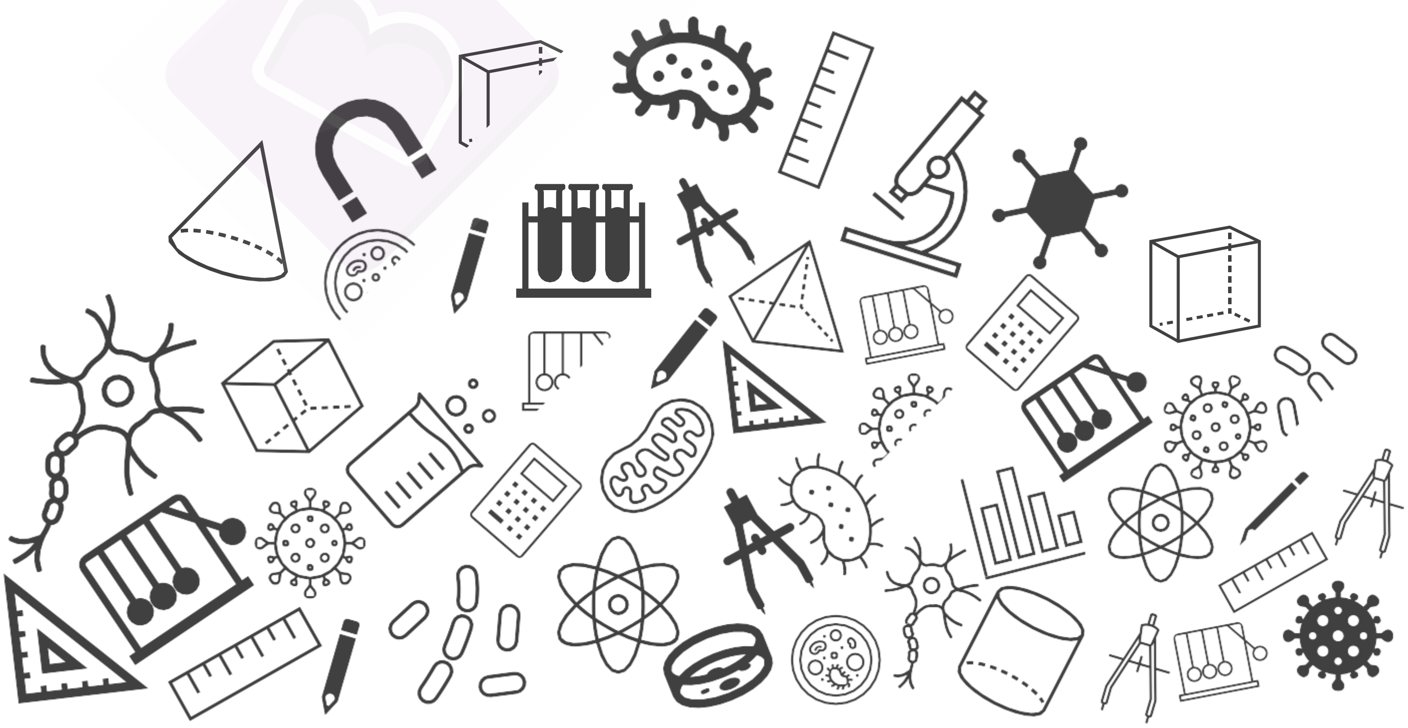
Grade 07: Maths
Chapter Notes

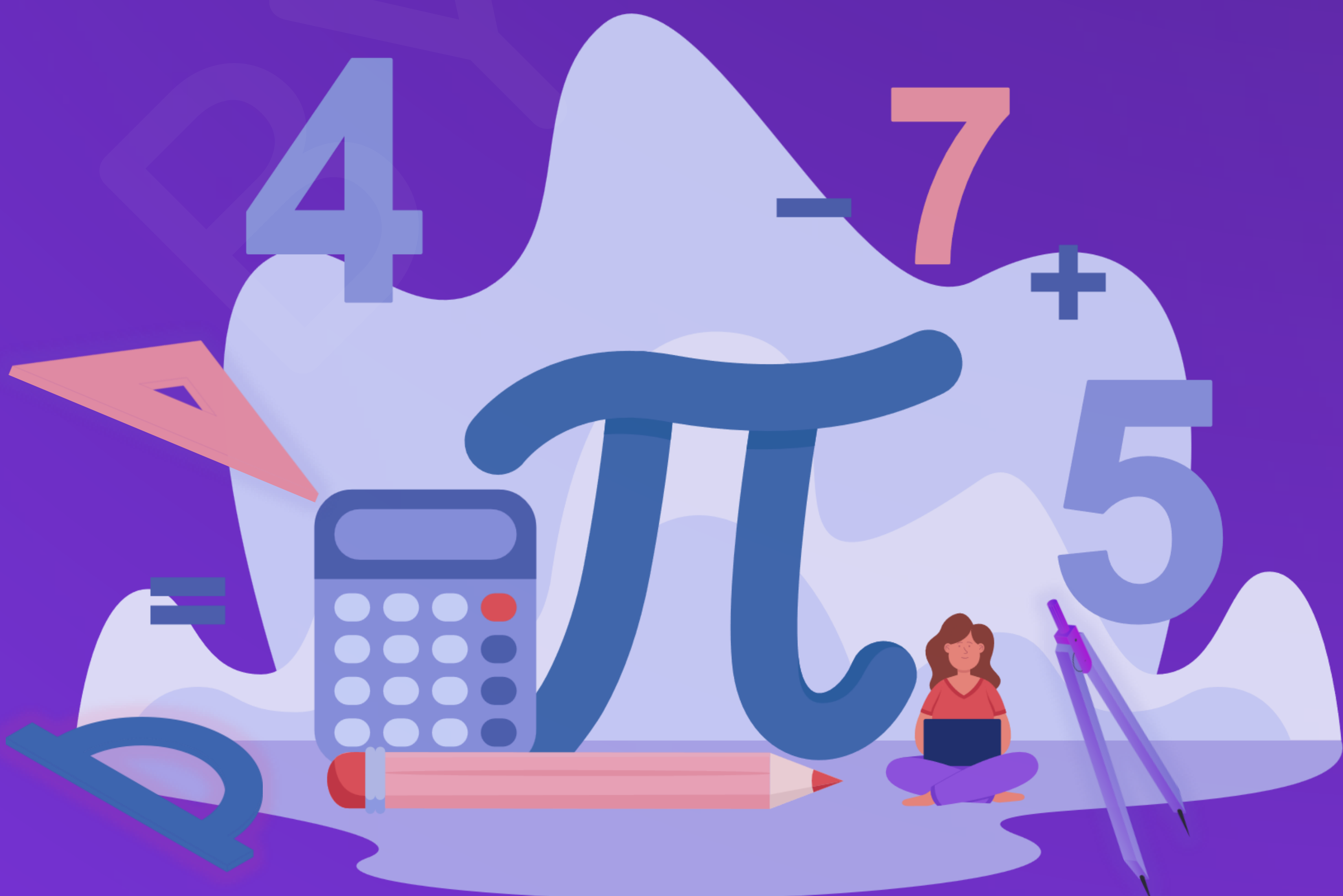


BYJU'S Classes

Chapter Notes

Rational Numbers

Grade 07



Topics to be Covered

1. Introduction

1.1. Family of Rational Numbers

2. Rational Numbers on a Number line

3. Equivalent Rational Numbers

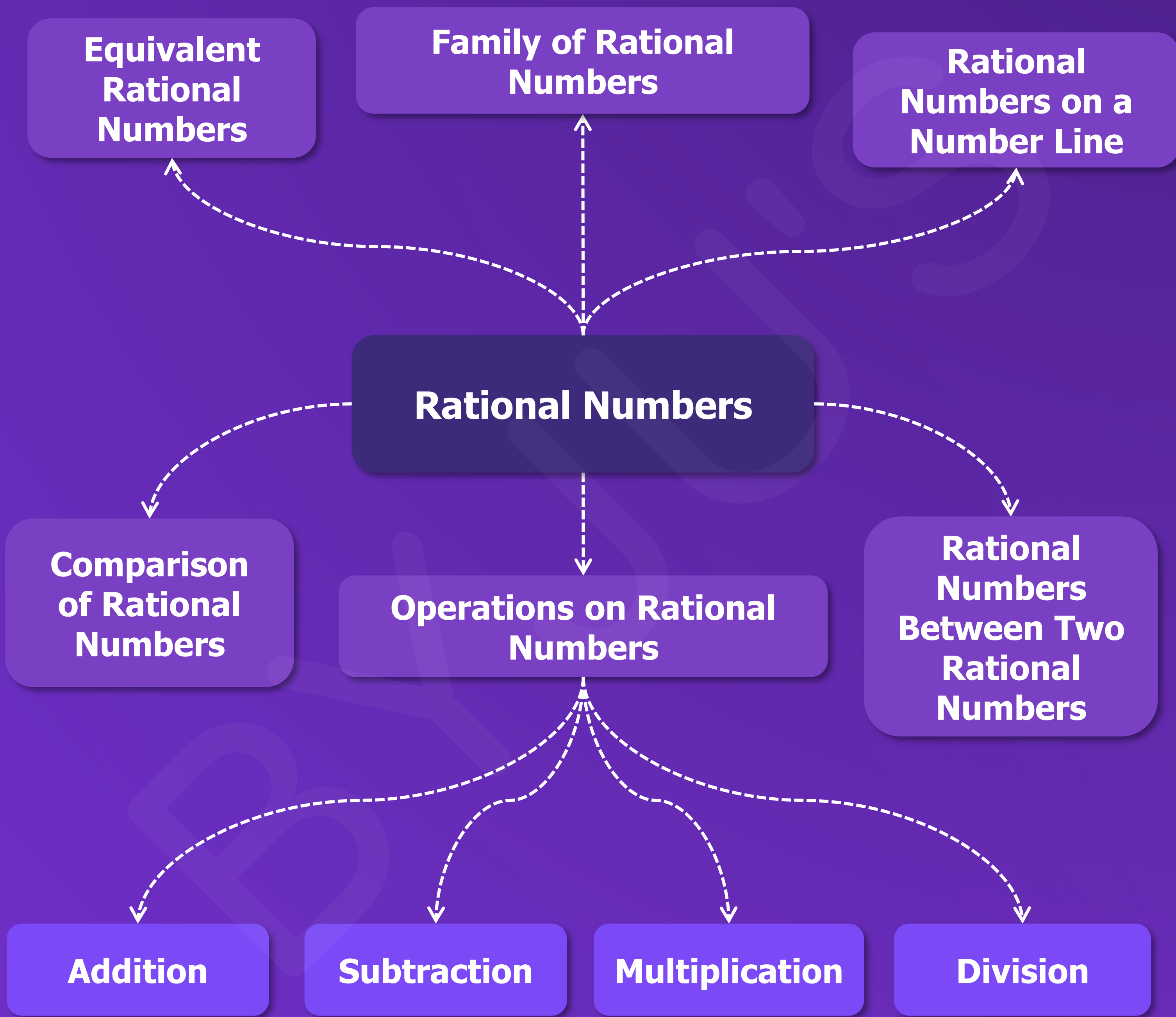
4. Comparison of Rational Numbers

5. Rational Numbers Between Two Rational Numbers

6. Operations on Rational numbers

6.1. Addition
6.2 Subtraction
6.3 Multiplication
6.4 Division

Mind Map



Equivalent Rational Numbers
Family of Rational Numbers
Rational Numbers on a Number Line
Rational Numbers
Comparison of Rational Numbers
Operations on Rational Numbers
Rational Numbers Between Two Rational Numbers
Addition
Subtraction
Multiplication
Division

1. Introduction

A rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Here p is called the Numerator and q is called the denominator.

Every fraction is a rational number, but every rational number might not be a fraction.

Fractions → $\frac{a}{b}$ — a and b are positive integers and $b \neq 0$

Rational numbers → $\frac{p}{q}$ — p and q are integers and $q \neq 0$

Positive and Negative Rational Numbers

If the numerator and denominator of a rational number are both positive integers or are both negative integers, then it is a positive rational number.

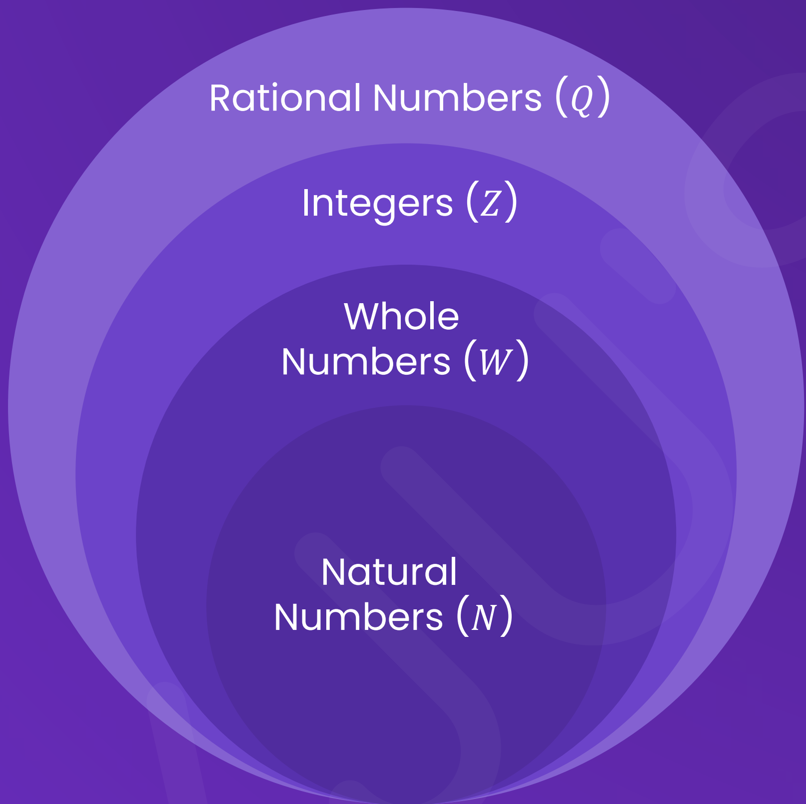
Example: $\frac{2}{3}, \frac{-3}{-5}$

If either the numerator or the denominator of a rational number is a negative integer, then it is a negative rational number.

Example: $\frac{-3}{5}, \frac{8}{-5}$

1. Introduction

1.1. Family of Rational Numbers



- Rational numbers contains all the natural numbers, whole numbers and integers.
  For example: -5, -4, 0, 1, 2, 5 etc.

- Rational numbers also contains all other numbers which are of the form $\frac{p}{q}$, where p, q are integers, and q ≠ 0.
  For example $-\frac{2}{3}, \frac{5}{7}, 0.5$ etc.

2. Rational Numbers on a Number Line

Follow these steps to locate a rational number on the number line:

Step 1:

- Find the two integers between which the given rational number will lies.

$$\frac{x}{y} = a\frac{p}{q}$$

p ⟶ Part to be picked

q ⟶ Total number of parts

Step 2:

- Divide the segment between those integers in q equal parts

Step 3:

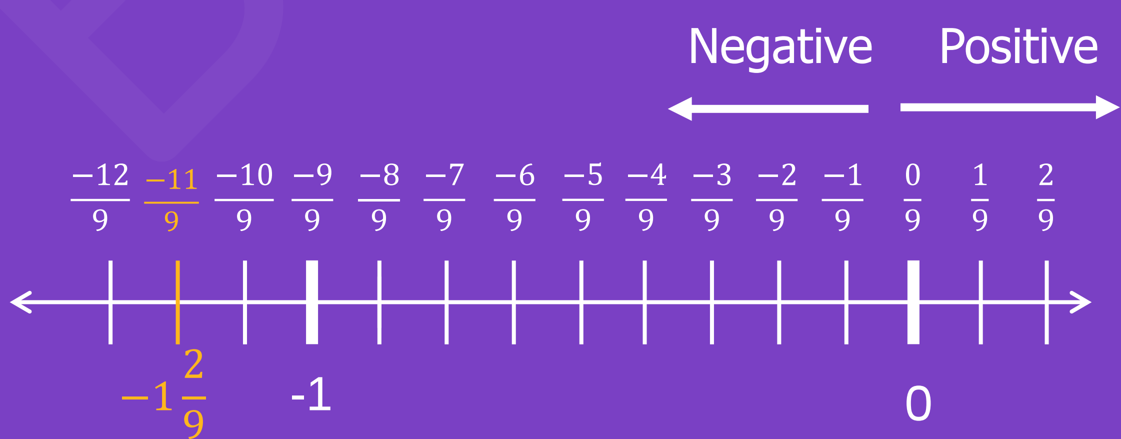
- Represent the rational number on the p^{th} division.

Example: $\frac{-11}{9} = -1\frac{2}{9}$

2 ⟶ Part to be picked

9 ⟶ Total number of parts

Number lies between -1 and -2.



3. Equivalent Rational Numbers

By multiplying the numerator and denominator of a rational number by the same non zero integer, we obtain another rational number equivalent to the given rational number.

Equivalent rational numbers

$$\boxed{\frac{12}{18}} = \frac{12 \times 5}{18 \times 5} = \boxed{\frac{60}{90}}$$

Equivalent rational numbers

$$\boxed{\frac{-4}{6}} = \frac{-4 \times -3}{6 \times -3} = \boxed{\frac{-12}{18}}$$

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

4. Comparison of Rational Numbers

- If the rational numbers to be compared have the same denominator, then the number with the greater numerator is greater of the two.

$$\frac{-4}{5} < \frac{6}{5}$$

- If the rational numbers to be compared have different denominators, then convert them into equivalent rational numbers having the same denominator and compare the two.

Compare $-\frac{2}{3}$ and $-\frac{3}{4}$

Conversion to equivalent rational numbers

L.C.M of 3 and 4 is 12

$$-\frac{2}{3} = -\frac{2 \times 4}{3 \times 4} = -\frac{8}{12}, \quad -\frac{3}{4} = -\frac{3 \times 3}{4 \times 3} = -\mathbf{\frac{9}{12}}$$

Comparison of numerators

As -8 > -9,

$$-\frac{8}{12} > -\frac{9}{12} \Rightarrow -\frac{2}{3} > -\frac{3}{4}$$

5. Rational Numbers Between Two Rational Numbers

We can find unlimited number of rational numbers between any two rational numbers.

Let us understand how to find rational numbers between two rational numbers, say $\frac{1}{5}$ and $\frac{1}{2}$.

Step 1:

- Make the denominators of both the rational numbers same.

$$\frac{1 \times 2}{5 \times 2} = \frac{2}{10}, \qquad \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Step 2:

- Check the integers lying between the two numerators.

$$2 < 3 < 4 < 5$$

$$\text{So, } \frac{2}{10} < \frac{3}{10} < \frac{4}{10} < \frac{5}{10}$$

6. Operations on Rational Numbers

6.1. Addition

- If the rational numbers to be added have the same denominator, then add the numerators and keep the denominator same to get the result.

$$\frac{7}{5} + \left(\frac{-6}{5}\right) = \frac{7 + (-6)}{5} = \frac{1}{5}$$

- If the rational numbers to be added have different denominators, then first convert them into equivalent rational numbers with the same denominator and add.

$$\left(\frac{-3}{4}\right) + \frac{2}{5}$$

Conversion to equivalent rational numbers
L.C.M of 4 and 5 is 20

$$\frac{-3}{4} = \frac{-3 \times 5}{4 \times 5} = \frac{-15}{20}, \quad \frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

Addition of equivalent rational numbers

$$\frac{-15}{20} + \frac{8}{20} = \frac{(-15) + 8}{20} = \frac{-7}{20}$$

Additive inverse of a number is defined as the number, which on adding with the original number gives zero.

For example, $\frac{-4}{7}$ is the additive inverse of $\frac{4}{7}$, because $\left(\frac{-4}{7}\right) + \frac{4}{7} = 0$

6. Operations on Rational Numbers

6.2. Subtraction

- If the rational numbers to be subtracted have the same denominator, then subtract the numerators and keep the denominator same to get the result.

$$\left(\frac{-2}{5}\right) - \frac{6}{5} = \frac{-2-6}{5} = \frac{-8}{5}$$

- If the rational numbers to be subtracted have different denominators, then first convert them into equivalent rational numbers with the same denominator and subtract.

$$\frac{2}{7} - \left(\frac{-5}{6}\right)$$

Conversion to equivalent rational numbers
L.C.M of 7 and 6 is 42

$$\frac{2}{7} = \frac{2 \times 6}{7 \times 6} = \frac{12}{42}, \quad \frac{-5}{6} = \frac{-5 \times 7}{6 \times 7} = \frac{-35}{42}$$

Subtraction of equivalent rational numbers

$$\frac{12}{42} - \left(\frac{-35}{42}\right) = \frac{12-(-35)}{42} = \frac{47}{42}$$

6.3. Multiplication

Rational numbers can be multiplied in the following way:

Step 1:
- Multiply the numerators of the two rational numbers

Step 2:
- Multiply the denominators of the two rational numbers

Step 3: Write the product as $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

$$\frac{-3}{5} \times \frac{2}{7} = \frac{-3 \times 2}{5 \times 7} = \frac{-6}{35}$$

6. Operations on Rational Numbers

6.4. Division

Rational numbers can be divided in the following way:

Step 1:

- Find the reciprocal of the second rational number

Step 2:

- Multiply the first rational number with the reciprocal of the second rational number.

$$\frac{-4}{9} \div \frac{5}{7} = \frac{-4}{9} \times \boxed{\frac{7}{5}} = \frac{-28}{45}$$

Reciprocal of $\frac{5}{7}$