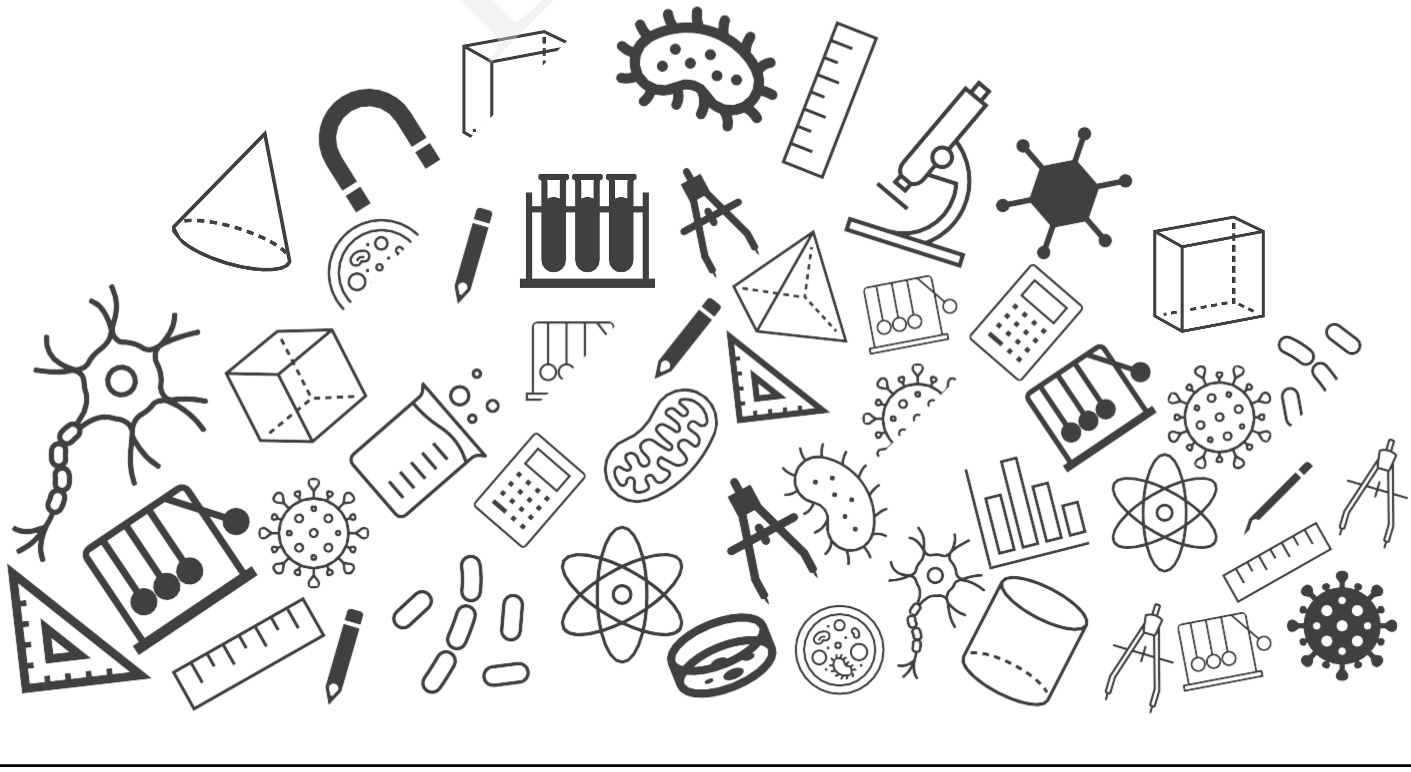
Grade 08
Maths Chapter Notes



BYJU'S Classes

Chapter Notes

Rational Numbers

Grade 08

Topics to be Covered

1. Introduction

1.1 Family of Rational Numbers

2. Properties of Rational Numbers

2.1. Closure
2.2. Commutativity
2.3. Associativity
2.4. Additive Identity
2.5. Multiplicative Identity
2.6. Additive Inverse
2.7. Multiplicative Inverse
2.8. Distributivity

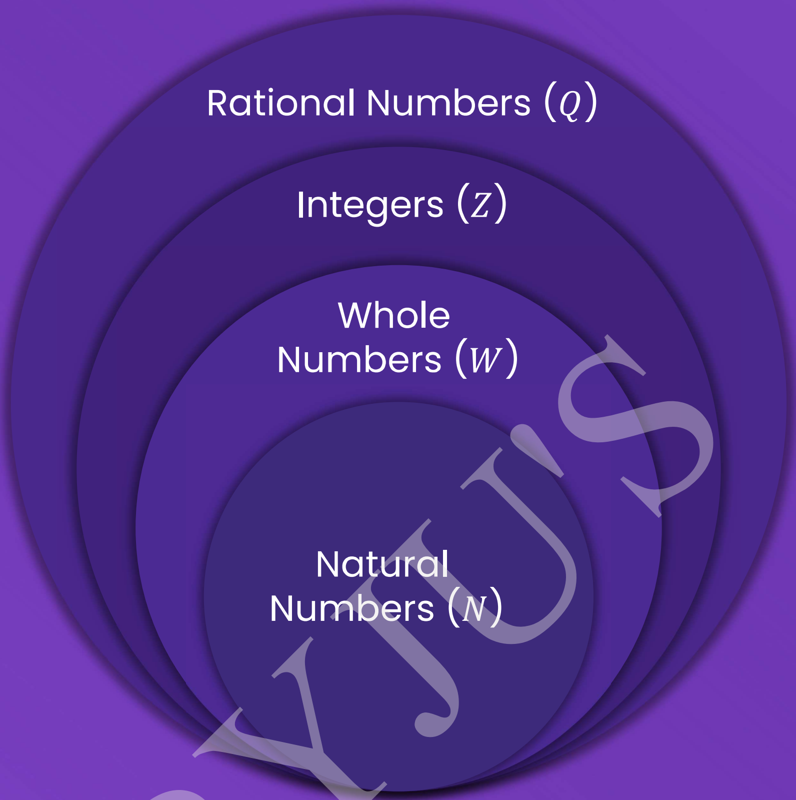
1. Introduction

Rational numbers are the numbers that can be represented in the form of $\frac{p}{q}$, where p, q are integers, and $q \neq 0$.

- Rational numbers can be either positive, negative or zero.
- While specifying a negative rational number, the negative sign is either in front or with the numerator of the number, which is the standard mathematical notation. For example, we denote the negative of $\frac{5}{2}$ as $\frac{-5}{2}$ or $-\frac{5}{2}$.

1. Introduction

1.1. Family of Rational Numbers



- Rational numbers contains all the natural numbers, whole numbers and integers. For example: -5, -4, 0, 1, 2, 5, etc.
- Rational numbers also contains all other numbers which are in the form of $\frac{p}{q}$, where p, q are integers, and q ≠ 0. For example $-\frac{2}{3}$, $\frac{5}{7}$ etc.

2. Properties of Rational Numbers

2.1. Closure

When a set of numbers are closed under any arithmetic operation such as addition, subtraction, multiplication, and division, the answer will belong to the set itself.

Addition

$\frac{3}{8} + \frac{(-5)}{7} = \frac{21+(-40)}{56} = \frac{19}{56}$ (rational number)

So, rational numbers are closed under addition.

Subtraction

$\frac{-5}{7} - \frac{2}{3} = \frac{-5 \times 3 - 2 \times 7}{21} = -\frac{29}{21}$ (rational number)

So, rational numbers are closed under subtraction.

Multiplication

$\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15}$ (rational number)

So, rational numbers are closed under multiplication.

Division

$\frac{-5}{3} \div 0$, which is not defined.

So, rational numbers are not closed under division.

2. Properties of Rational Numbers

2.2. Commutativity

It is being a property of a mathematical operation on numbers in which the result does not depend on the order of the numbers.

Addition

$\frac{-2}{3} + \frac{5}{7} = \frac{1}{21}$ and $\frac{5}{7} + \frac{-2}{3} = \frac{1}{21}$

As both result are same, rational numbers are commutative under addition.

Subtraction

$\frac{-5}{3} - \frac{4}{3} = -\frac{9}{3}$ and $\frac{4}{3} - \frac{5}{3} = -\frac{1}{3}$

As $-\frac{9}{3} \neq -\frac{1}{3}$, rational numbers are not commutative under subtraction.

Multiplication

$\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15}$ and $\frac{4}{5} \times \frac{-2}{3} = \frac{-8}{15}$

As both result are same, rational numbers are commutative under multiplication.

Division

$\frac{-5}{4} \div \frac{3}{7} = \frac{-35}{12}$ and $\frac{3}{7} \div \left(\frac{-5}{3}\right) = \frac{-12}{35}$

As $-\frac{35}{12} \neq -\frac{12}{35}$, rational numbers are not commutative under division.

2. Properties of Rational Numbers

2.3. Associativity

It is a property of operations on numbers, which means that rearranging the parentheses in an expression will not change the result.

Addition

$\frac{-2}{3} + \left\{\frac{3}{5} + \left(\frac{-5}{6}\right)\right\} = \frac{-2}{3} + \left\{\frac{-7}{30}\right\} = \frac{-9}{10}$ and

$\left\{\frac{-2}{3} + \frac{3}{5}\right\} + \left(\frac{-5}{6}\right) = \left\{\frac{-1}{15}\right\} + \left(\frac{-5}{6}\right) = \frac{-9}{10}$. As results are same, rational numbers are associative under addition.

Subtraction

$\frac{-2}{3} - \left\{\frac{-4}{5} - \left(\frac{-1}{2}\right)\right\} = \frac{-2}{3} - \left\{\frac{-3}{10}\right\} = \frac{-11}{30}$ and

$\left\{\frac{-2}{3} - \left(\frac{-4}{5}\right)\right\} - \left(\frac{-1}{2}\right) = \left\{\frac{2}{15}\right\} - \left(\frac{-1}{2}\right) = \frac{19}{30}$. As $\frac{-11}{30} \neq \frac{19}{30}$, rational numbers are not associative under subtraction.

Multiplication

$\frac{1}{2} \times \left(\frac{1}{3} \times \frac{1}{4}\right) = \frac{1}{2} \times \left(\frac{1}{12}\right) = \frac{1}{24}$ and $\left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{1}{4} = \left(\frac{1}{6}\right) \times \frac{1}{4} = \frac{1}{24}$

As results are same, rational numbers are associative under multiplication.

Division

$\left\{\frac{1}{2} \div \frac{-1}{3}\right\} \div \frac{2}{5} = \left\{\frac{-3}{2}\right\} \div \frac{2}{5} = \frac{-15}{4}$ and

$\frac{1}{2} \div \left\{\frac{-1}{3} \div \frac{2}{5}\right\} = \frac{1}{2} \div \left\{\frac{-5}{6}\right\} = \frac{-6}{10}$. As $\frac{-15}{4} \neq \frac{-6}{10}$, rational numbers are not associative under division.

2. Properties of Rational Numbers

2.4. Additive Identity

Additive identity is the value when added to a number, results in the original number. Zero is the additive identity for rational numbers.

Example: $\frac{-7}{8} + 0 = \frac{-7}{8}$

2.5. Multiplicative Identity

Multiplicative identity is the value when multiplied to a number, results in the original number. One is the multiplicative identity for rational numbers.

Example: $\frac{-7}{8} \times 1 = \frac{-7}{8}$

2.6. Additive Inverse

Additive inverse is a number which on getting added to the original number results in zero. Negative of rational number is the additive inverse of that number.

Example: $-\frac{5}{7} + \frac{5}{7} = 0.$

So, additive inverse of $-\frac{5}{7}$ is $\frac{5}{7}$.

2. Properties of Rational Numbers

2.7. Multiplicative Inverse

Multiplicative inverse of a number is a value which when multiplied by the original number results in 1.

Example: $\frac{8}{21} \times \frac{21}{8} = 1$

So, multiplicative inverse of $\frac{8}{21}$ is $\frac{21}{8}$.

Also, $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$.

2.8. Distributivity

The distributive property states, if a, b and c are three rational numbers, then;

$a \times (b + c) = (a \times b) + (a \times c)$

The same holds true when we are subtracting instead of adding.

Example: $\frac{2}{3}\left(\frac{1}{2} + \frac{3}{4}\right) = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$ and

$\frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{3}{4} = \frac{2}{6} + \frac{6}{12} = \frac{10}{12}$.

So, $\frac{2}{3}\left(\frac{1}{2} + \frac{3}{4}\right) = \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{3}{4}$.