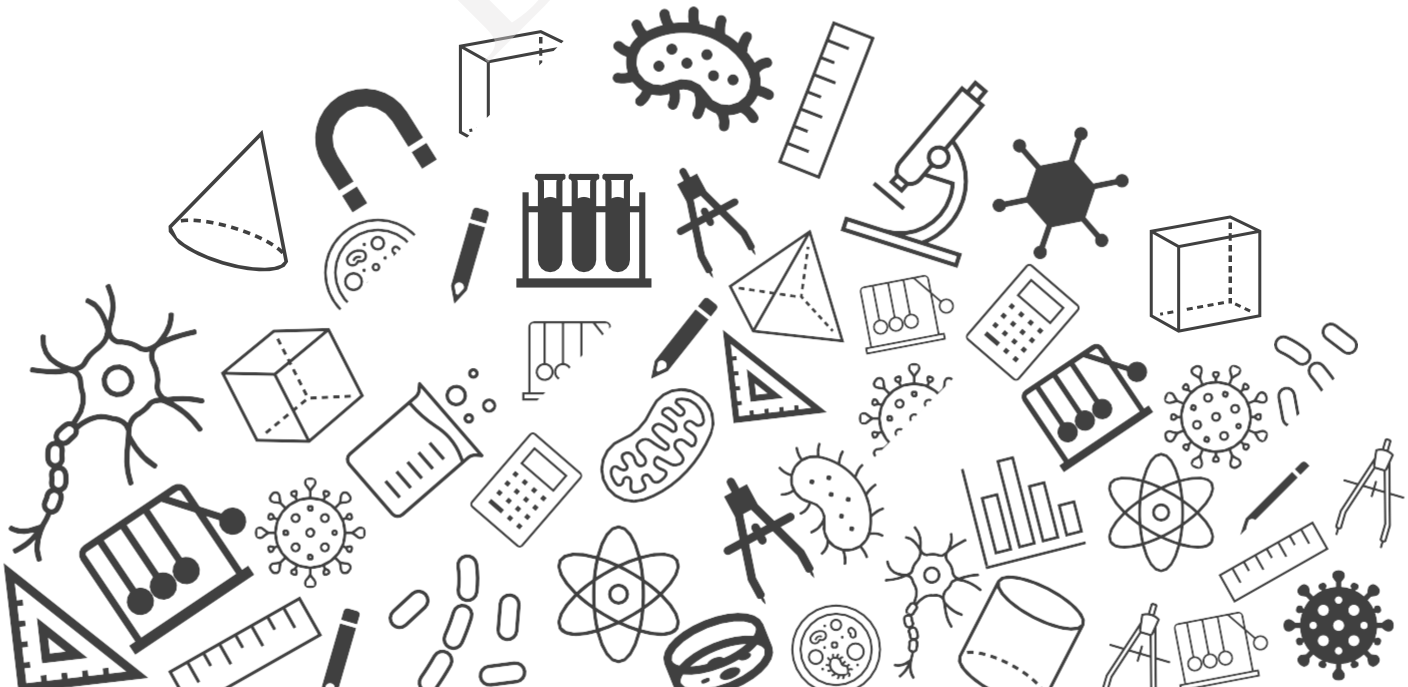
Grade 08
Maths Chapter Notes



BYJU'S

Classes

Chapter Notes

Squares and Square Roots

Grade 08

Topics to be Covered

1. Triangular Numbers

2. Square Numbers

2.1. Adding Triangular Numbers

3. Properties of Square Numbers

3.1. Unit Digit of Perfect Squares
3.2. Numbers between square numbers
3.3. Adding Consecutive Odd Numbers
3.4. Sum of Consecutive Natural Numbers
3.5. Product of two Consecutive Even or Odd Natural Numbers
3.6. Some More Patterns

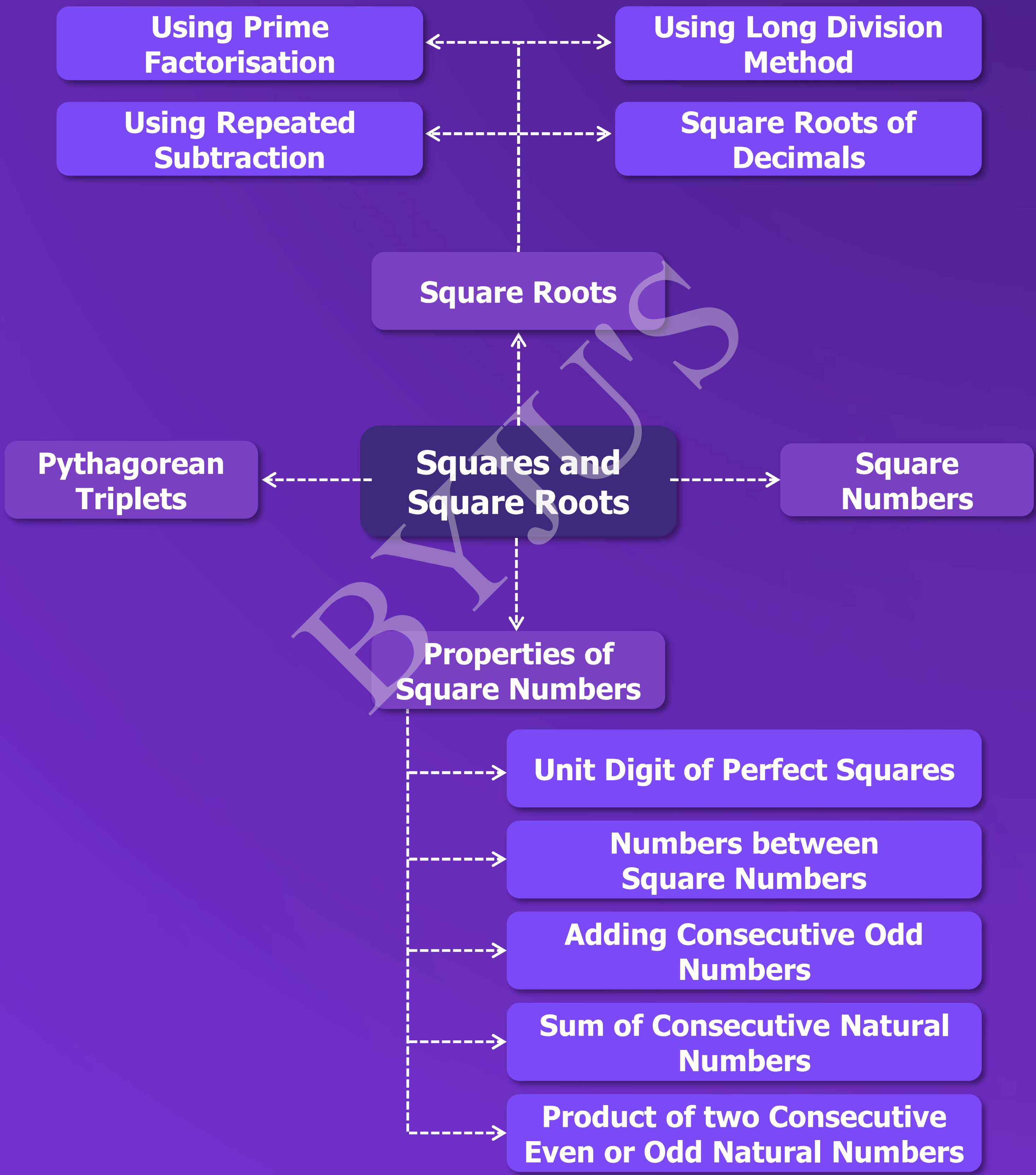
4. Finding the Square of a Number with Unit Digit 5

5. Pythagorean Triplet

6. Square Roots

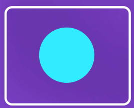
6.1. Using Inverse Operation of Squaring
6.2. Using Repeated Subtraction
6.3. Using Prime Factorisation
6.4. Using Long Division Method
6.5. Square Roots of Decimals

Mind Map


Using Prime Factorisation
Using Long Division Method
Using Repeated Subtraction
Square Roots of Decimals
Square Roots
Pythagorean Triplets
Squares and Square Roots
Square Numbers
Properties of Square Numbers
Unit Digit of Perfect Squares
Numbers between Square Numbers
Adding Consecutive Odd Numbers
Sum of Consecutive Natural Numbers
Product of two Consecutive Even or Odd Natural Numbers

1. Triangular Numbers

A triangular number is the one whose dot pattern can be arranged as triangles.



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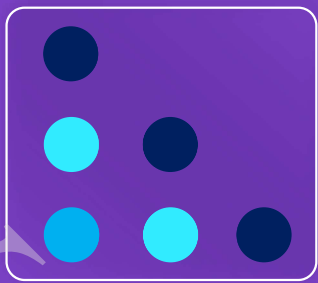
$1 + 2 = 3$



$1 + 2 + 3 = 6$

Here, $1, 3$ and 6 are triangular numbers.

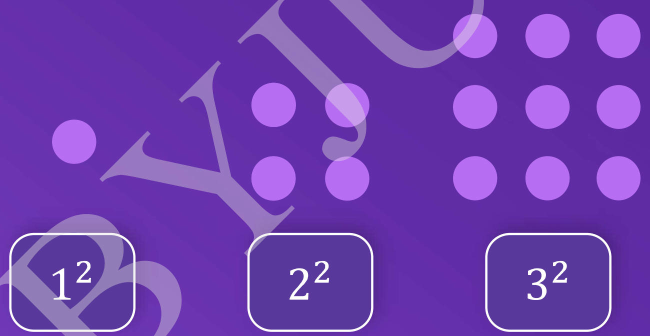
2. Square Numbers

Square numbers are obtained when a number is multiplied to itself.

$$a \times a = a^2$$

$$3 \times 3 = 3^2$$

$1^2 \quad 2^2 \quad 3^2$



The numbers $1, 4, 9, 16, \ldots$ are square numbers, also called as perfect squares.

The square of an odd number will always be an odd number and the square of an even number will always be an even number.

2. Square Numbers

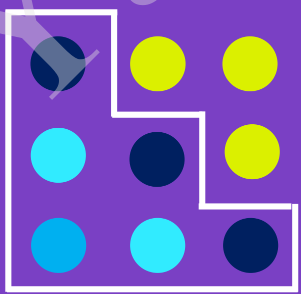
2.1. Adding Triangular Numbers

If we add two consecutive triangular numbers, we get a square number.

Let's consider first two triangular numbers: 1 and 3
$1 + 3 = 4 = 2^2$

$1 + 3 = 2^2 = 4$

Similarly for two consecutive triangular numbers 3 and 6:
$3 + 6 = 9 = 3^2$



$3 + 6 = 9 = 3^2$

3. Properties of Square Numbers

3.1. Unit Digit of Perfect Squares

- All perfect squares end with $0, 1, 4, 5, 6$ or 9 only.
- If a number has $2, 3, 7$ or 8 at its unit place, it is not a perfect square.
- If the unit digit of any number is 0, then the digit at the unit place of its square is 0.
- If the unit digit of any number is 5, then the digit at the unit place of its square is 5.
- If the unit digit of any number is 3, then the digit at the unit place of its square is 9.
- If the unit digit of any number is 2 or 8, then the digit at the unit place of its square is 4.
- If the unit digit of any number is 4 or 6, then the digit at the unit place of its square is 6.
- If the unit digit of any number is 1 or 9, then the digit at the unit place of its square is also 1.

Unit digit of a number	Unit digit of its square
0	0
3	9
5	5
2 or 8	4
4 or 6	6
1 or 9	1

3. Properties of Square Numbers

3.2. Numbers between Square Numbers

Between n^2 and $(n+1)^2$, there are $2n$ non square numbers.
Example: Between 9 and 16 there lies 6 numbers.
$9 = 3^2$ and $16 = 4^2$.
So, between 3^2 and 4^2 there lies 2 × 3 = 6 numbers.

$$9 \quad \boxed{10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15} \quad 16$$

3^2 4^2

3.3. Adding Consecutive Odd Numbers

The sum of **first n odd natural numbers** is n^2.

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2$

3. Properties of Square Numbers

3.4. Sum of Consecutive Natural Numbers

The square of any odd number can be written as the sum of two consecutive natural numbers.
Example: 9 can be written as the sum of 4 and 5, where 4 = $\frac{3^2-1}{2}$ and 5 = $\frac{3^2+1}{2}$

$$\frac{3^2-1}{2}$$

$$3^2 = 9 = 4 + 5$$

$$\frac{3^2+1}{2}$$

3.5. Product of Two Consecutive Even or Odd Natural Numbers

The product of two consecutive odd or even natural numbers is equal to the square of the natural number in between them minus 1.
Example:

- $11 \times 13 = (12-1) \times (12+1) = 12^2 - 1$
- $12 \times 14 = (13-1) \times (13+1) = 13^2 - 1$

So, in general $(a+1) \times (a-1) = a^2 - 1$.

3.6. Some More Patterns

Some more interesting patterns related to squares of numbers are as follows:

$$1^2 = 1$$
$$11^2 = 1\ 2\ 1$$
$$111^2 = 1\ 2\ 3\ 2\ 1$$
$$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$$
$$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$$

$$7^2 = 49$$
$$67^2 = 4489$$
$$667^2 = 444889$$
$$6667^2 = 44448889$$
$$66667^2 = 4444488889$$

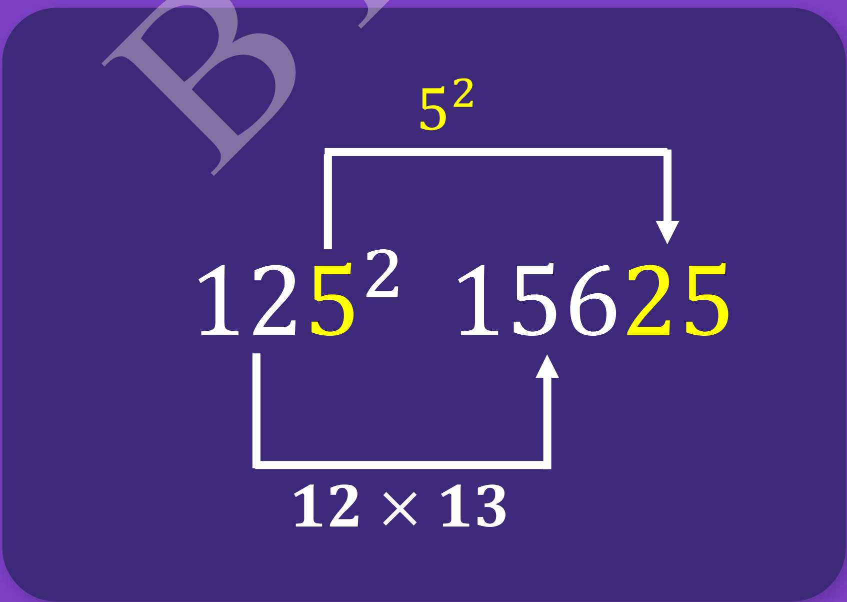
4. Finding the Square of a Number with Unit digit 5

Step 1: Find the square of 5 which is 25 and place it in the units and the tens place.

Example: 125^2
The square of 5 is 25.
Hence, the digit in the tens and the units place in the square of 125 will be 2 and 5, respectively.

Step 2: Multiply the number remaining after removing 5 with its successor and place it before 25.

Now, in 125 the number other than 5 is 12.
The successor of 12 is 13.
12 × 13 = 156
So, the remaining digits in the square of 125 are 1, 5, and 6.

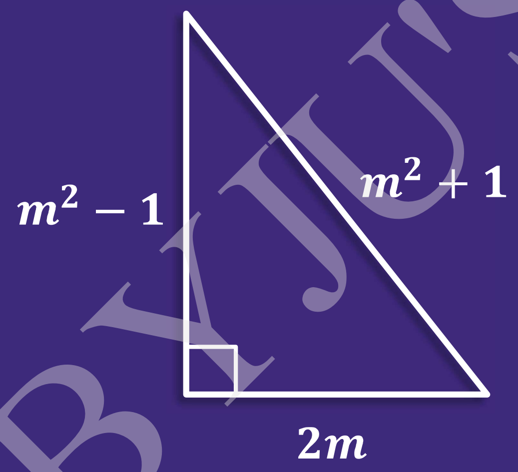


5. Pythagorean Triplet

A Pythagorean triplet consists of three numbers $(a, b$ and $c)$ that can be written in the form $a^2 + b^2 = c^2$

For any natural number m $(m > 1)$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$

Hence, $2m, m^2 - 1,$ and $m^2 + 1$ form a Pythagorean triplet.

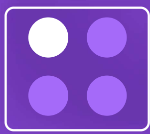


6. Finding Square Roots

6.1. Using Inverse Operation of Squaring

- Finding a square root is the inverse operation of squaring.
- The positive square root of a number is denoted by the symbol $\sqrt{}$.

$2^2 = 2 \times 2 = 4$

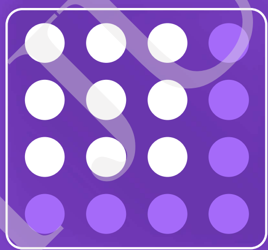


$\sqrt{4} = 2$

$3^2 = 3 \times 3 = 9$

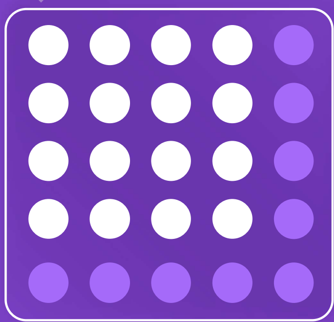
$\sqrt{9} = 3$

$4^2 = 4 \times 4 = 16$



$\sqrt{16} = 4$

$5^2 = 5 \times 5 = 25$



$\sqrt{25} = 5$

If a perfect square is of n-digits then, its square root will have:

- $\frac{n}{2}$ digits if n is even.
- $\left(\frac{n+1}{2}\right)$ digits if n is odd.

6. Finding Square Roots

6.2. Using Repeated Subtraction

Square root of a perfect square can be found by repeatedly subtracting consecutive odd numbers till we get 0. The number of odd numbers subtracted is the square root of the given number.

Example: $\sqrt{25} = 5$

$25 - 1 = 24$ ——— 1
$24 - 3 = 21$ ——— 2
$21 - 5 = 16$ ——— 3
$16 - 7 = 9$ ——— 4
$9 - 9 = 0$ ——— 5

6.3. Using Prime Factorisation

- Express the number as the product of its prime factors.
  Example: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
- Pair these prime factors such that both the numbers in a pair are the same.
  $144 = (2 \times 2) \times (2 \times 2) \times (3 \times 3)$
- Take one number from each pair and multiply them to get the square root.
  $\sqrt{144} = 2 \times 2 \times 3 = 12$

2	144
2	72
2	36
2	18
3	9
3	3
	1

6. Finding Square Roots

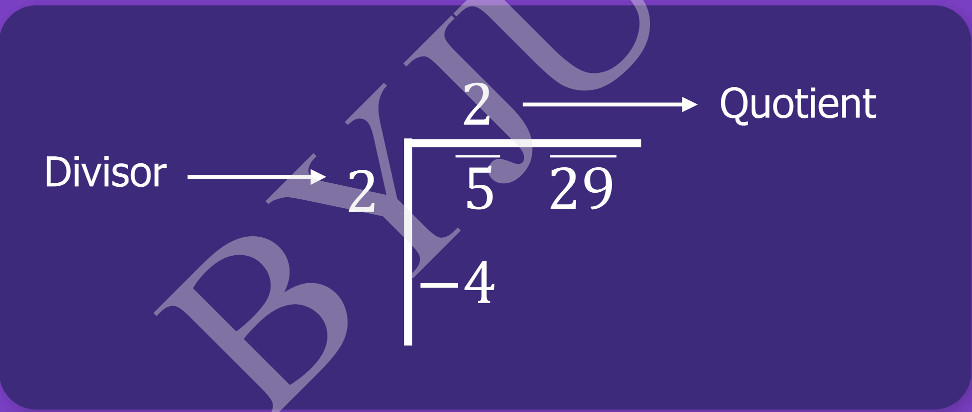
6.4. Using Long Division Method

Step 1: Divide the number into groups of 2 starting from the ones place.

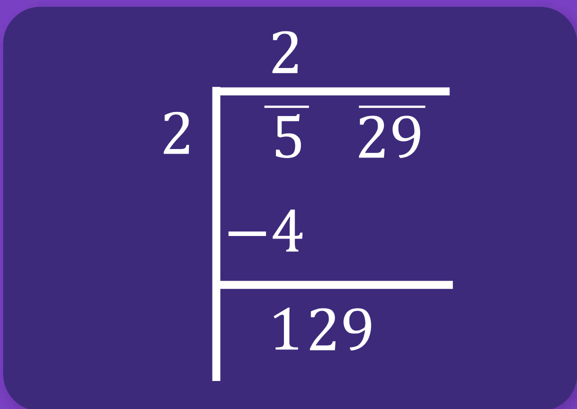
Example: Square root of 529 using long division.

$$529 \Rightarrow \overline{5}\ \overline{29}$$

Step 2: Find the largest number whose square is less than or equal to the number under the extreme left bar. Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend. Divide and get the remainder.



Step 3: Bring down the number under the next bar to the right of the remainder. So, the new dividend is 129.



6. Finding Square Roots

6.4. Using Long Division Method

Step 4: Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \overline{5}\ \ \overline{29} \\ & -4 \\ \hline 4\,\_ & \ \ 1\,29 \end{array}$$

Step 5: Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

$$\begin{array}{r|l} & \quad 23 \\ \hline 2 & \overline{5}\ \ \overline{29} \\ & -4 \\ \hline 43 & \ \ 1\,29 \\ & -1\,29 \\ \hline & \quad 0 \end{array}$$

Step 6: Since the remainder is 0 and no digits are left in the given number, therefore,

$$\sqrt{529} = 23$$

6. Finding Square Roots

6.5. Square Roots of Decimals

Step 1: Put bars on the integral part and place the bars on the decimal part on every pair of digits beginning with the first decimal place.

Example: Square root of 17.64 using long division.

$$17.64 \Rightarrow \overline{17}.\overline{64}$$

Step 2: Find the largest number whose square is less than or equal to the number under the extreme left bar. Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend. Divide and get the remainder.

$$\begin{array}{r|l} & 4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \end{array}$$

Divisor → 4; 4 → Quotient

Step 3: Bring down the number under the next bar to the right of the remainder. So, the new dividend is 164.

$$\begin{array}{r|l} & 4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \\ \hline & \quad 164 \end{array}$$

6. Finding Square Roots

6.5. Square Roots of Decimals

Step 4: Double the quotient and enter it with a blank on its right. Since 64 is the decimal part so put a decimal point in the quotient.

$$
\begin{array}{r|l}
 & \quad 4. \\
\hline
4 & \overline{17}\,.\,\overline{64} \\
 & -16 \\
\hline
8\,\_ & \quad 164
\end{array}
$$

Step 5: Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

$$
\begin{array}{r|l}
 & \quad 4.2 \\
\hline
4 & \overline{17}\,.\,\overline{64} \\
 & -16 \\
\hline
82 & \quad 164 \\
 & -164 \\
\hline
 & \quad 0
\end{array}
$$

Step 6: Since the remainder is 0 and no digits are left in the given number, therefore,

$$\sqrt{17.64} = 4.2$$