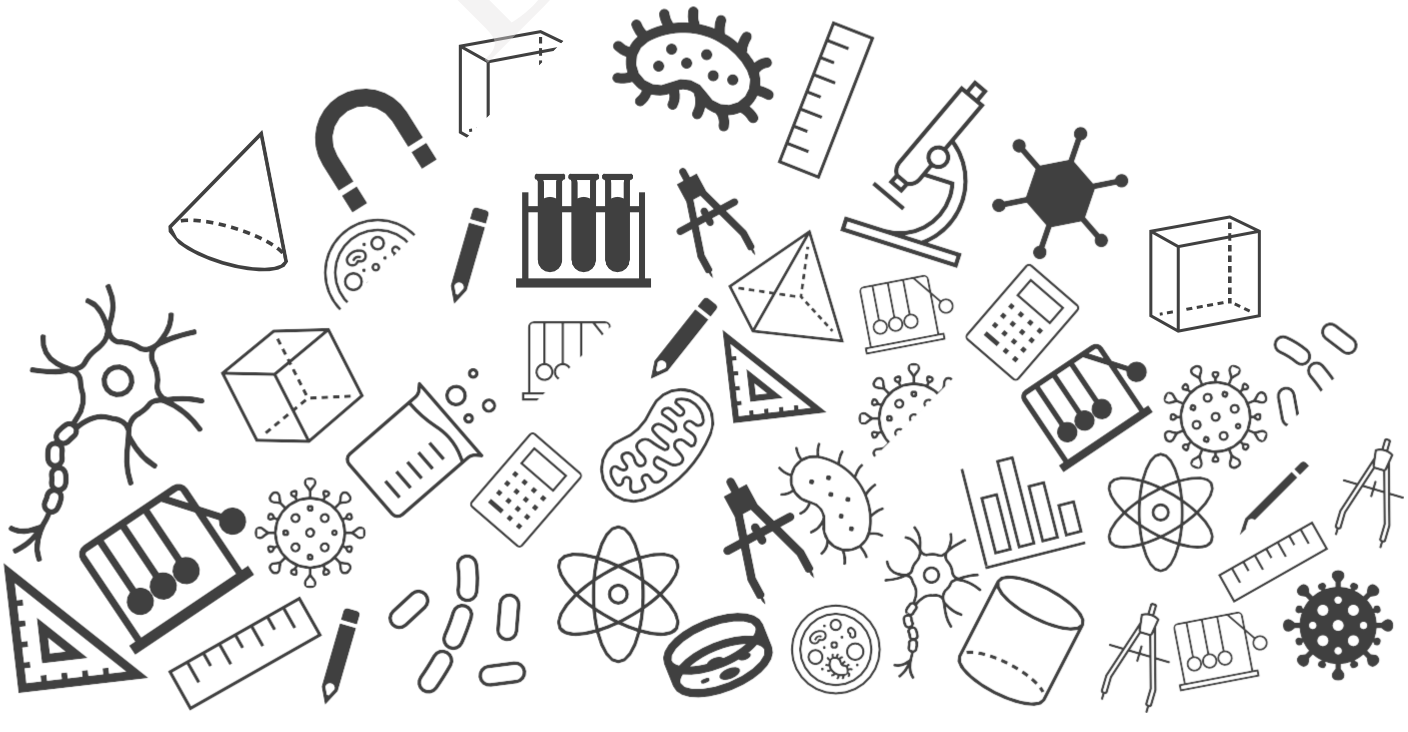
BYJU'S

Grade 08
Maths Chapter Notes



BYJU'S Classes

Chapter Notes

Cubes and Cube Roots

Grade 08

Topics to be Covered

1. Cube Numbers

2. Properties of Cube Numbers

2.1. Ones Digit of Cube Numbers

2.2. Cube Numbers as the Sum of Consecutive Odd Numbers

3. Prime Factorisation

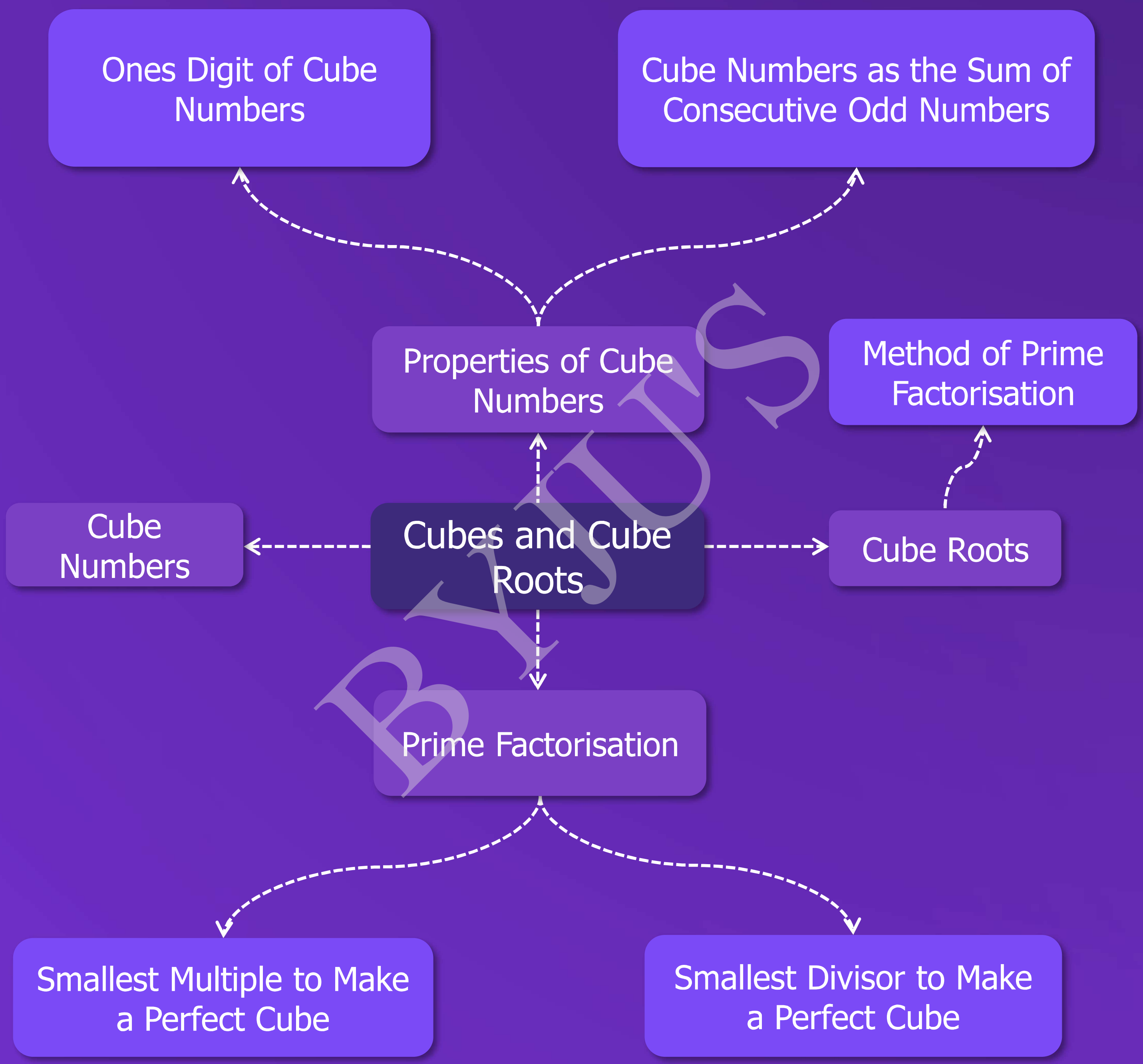
3.1. Smallest Multiple to make a Perfect Cube

3.2. Smallest Divisor to make a Perfect Cube

4. Cube Roots

4.1. Method of Prime Factorisation

Mind Map


Ones Digit of Cube Numbers
Cube Numbers as the Sum of Consecutive Odd Numbers
Properties of Cube Numbers
Method of Prime Factorisation
Cube Numbers
Cubes and Cube Roots
Cube Roots
Prime Factorisation
Smallest Multiple to Make a Perfect Cube
Smallest Divisor to Make a Perfect Cube

1. Cube Numbers

A cube number is obtained by multiplying a number with itself three times. A cube number is also known as a perfect cube.

$$1^3 = 1 \times 1 \times 1 = 1$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$4^3 = 4 \times 4 \times 4 = 64$$

2. Properties of Cube Numbers

2.1. Ones Digit of Cube Numbers

- Cubes of the numbers ending in digit 1, 4, 5, 6 and 9 are the numbers ending in the same digit.
- A number whose units digit is 2 has a cube whose units digit is 8 and vice versa.
- A number whose units digit is 3 has a cube whose units digit is 7 and vice versa.

The following are the cubes of the numbers from 1 to 9:

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

Cubes of odd numbers are odd and cubes of even numbers are even.

2. Properties of Cube Numbers

2.2. Cube Numbers as the Sum of Consecutive Odd Numbers

Express cube of n as the sum of consecutive odd numbers:

Case I: When n is odd, its middle term is n^2

So, the $\frac{n-1}{2}$ consecutive odd numbers comes before n^2

and $\frac{n-1}{2}$ odd consecutive numbers comes after n^2.

Case II: When n is even, its middle terms are n^2 - 1 and n^2 + 1

So, $\frac{n-2}{2}$ odd consecutive numbers comes before n^2 -1 and $\frac{n-1}{2}$ odd consecutive numbers comes after n^2 + 1.

Example: 10^3 as the sum of 10 consecutive odd numbers

$10^3 = 1000$

We can find the middle most odd number by taking the mean:

$\frac{1000}{10} = 100$

But 100 won't be the middle term, as it's an even number.

There will be two middle terms for 10 consecutive odd numbers.

Hence, we take 99 and 101 as two middle terms.

We can write four odd numbers each before 99 and after 101 as:

91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109

So, we can write the sum as:

10^3 = 91 + 93 + 95 97 + 99 +101 + 103 + 105 + 107 + 109

3. Prime Factorisation of a Number

- Each prime factor of a number appears three times in the cube of the number.
- If each prime factor of a number appears three times in the prime factorisation, then the number is a perfect cube.

Let's consider the prime factorisation of 216

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

3. Prime Factorisation

3.1. Smallest Multiple to Make a Perfect Cube

Step 1: Find the prime factorisation of the given number.

For example, consider the number 72

Prime factorisation of 72:

$72 = 2 \times 2 \times 2 \times 3 \times 3$

The prime factors can also be grouped as $2^3 \times 3^2$. Here, 3 is not obtained in triplets.

Step 2: Multiply the given number by prime factors, which are not in triplets.

The smallest number by which 72 must be multiplied to obtain a perfect cube is 3.

3.2. Smallest Divisor to Make a Perfect Cube

Step 1: Find the prime factorisation of the given numbers.

For example, consider a number 53240

Prime factorisation of 53240:

$53240 = 2 \times 2 \times 2 \times 11 \times 11 \times 11 \times 5$

The prime factors can also be grouped as $2^3 \times 11^3 \times 5$. Here, 5 is not obtained in triplets.

Step 2: Divide the given number by prime factors, which are not in triplets.

The smallest number by which 53240 must be divided to obtain a perfect cube is 5.

4. Cube Roots

- The symbol $\sqrt[3]{\ }$ denotes 'cube root'.
- Finding a cube root is the inverse operation of finding the cube.

Cube of numbers	Cube root of numbers
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$

4.1. Cube Roots through Method of Prime Factorisation

For example, let's find cube root of the number 216

Step 1: Resolve the given numbers into prime factors.

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Step 2: Group them in triplets.

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

Step 3: Choose only one prime factor from each group.

$$\sqrt[3]{216} = 2 \times 3$$

Step 4: Multiply the chosen prime factors.

$$\sqrt[3]{216} = 6$$