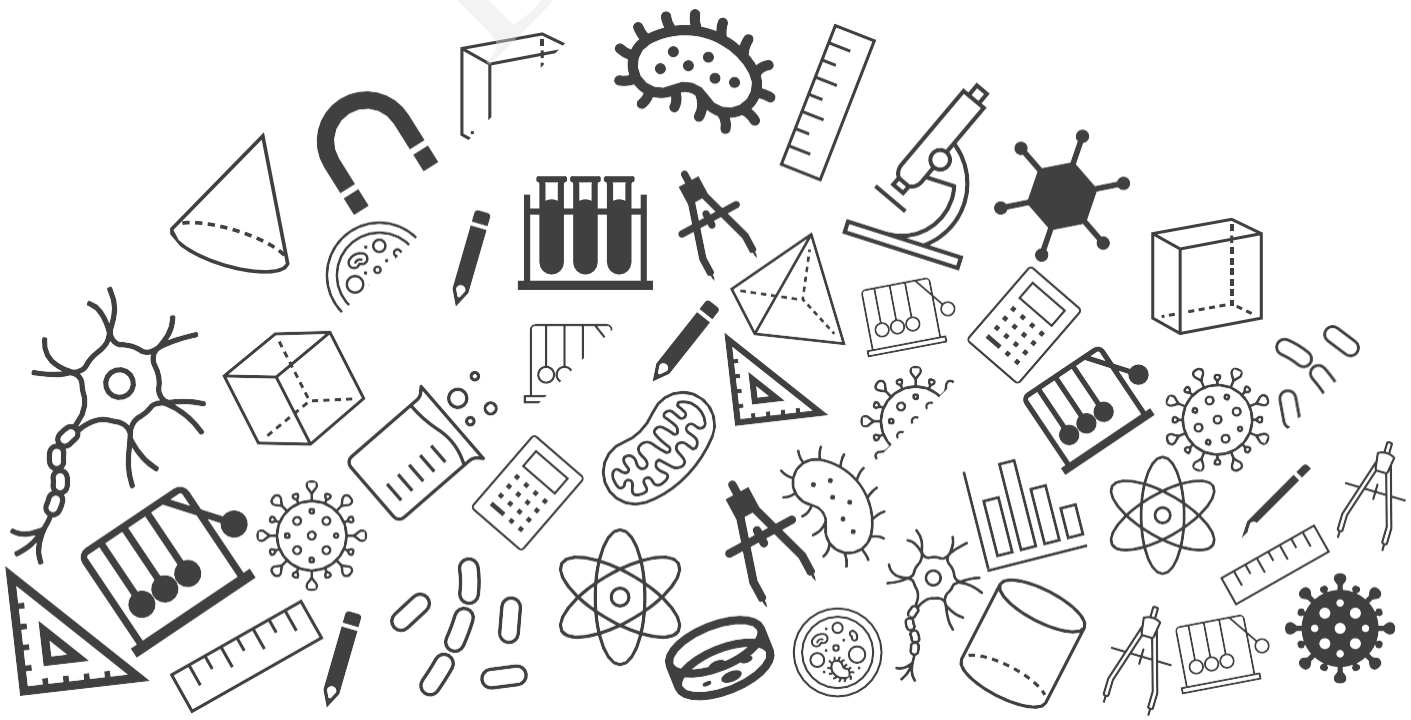
Grade 08
Maths Chapter Notes



BYJU'S Classes

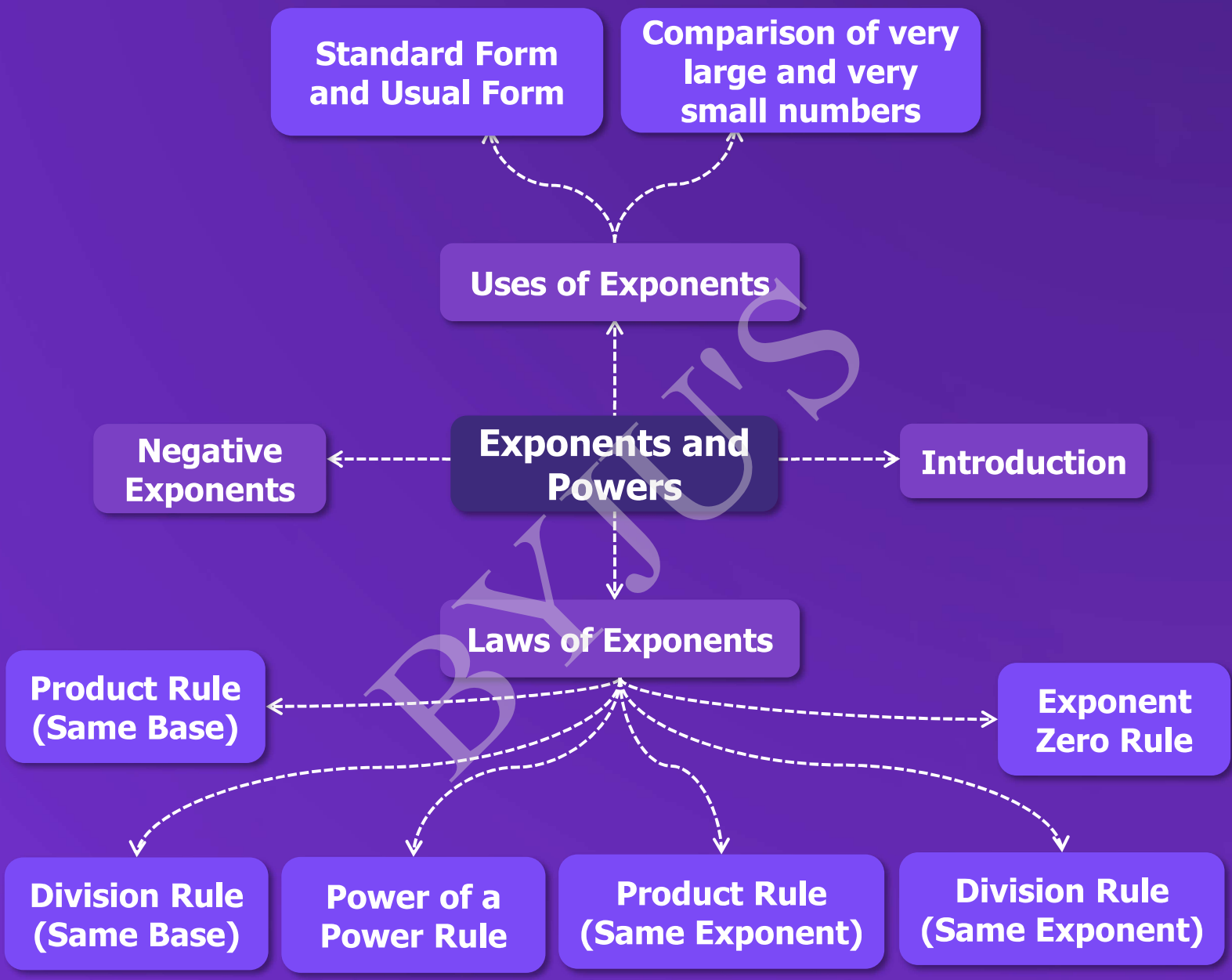
Chapter Notes

Exponents and Powers

Grade 08

Topics to be Covered

Mind Map



1. Introduction

Large numbers can be read, understood and compared easily using exponents.

Let us understand how to write large numbers in a shorter form using exponents by taking the number 10,000.

- Firstly, 10,000 = $10 \times 10 \times 10 \times 10$
- Here 10 is the factor which is multiplied 4 times by itself.
- So, 10,000 can be written in exponential form as 10^4.
- In 10^4, 10 is called the base and 4 is called the exponent.

10000 = **10 x 10 x 10 x 10**

= **10^4** (4 → Exponent; 10 → Base)

10^4 We say: 10 raised to the power 4

2. Powers with Negative Exponents

- A positive exponent defines the number of times the base number is multiplied by itself.
- Whereas, a negative exponent tells us how many times we have to divide the base number.

- If a is any non-zero integer and m is an integer, then a^{-m} will be power with a negative exponent.

$$\Rightarrow \frac{1}{a^m} = a^{-m}$$

For example:

$$\frac{1}{1000} = \frac{1}{10^3}$$

$$= 10^{-3}$$

-3 → Exponent

10 → Base

Multiplicative Inverse

- If a is any non-zero integer and m is an integer, then a^{-m} is the **multiplicative inverse** of a^m.
- Generally, it is also called the reciprocal of a number.

For example:

- Multiplicative inverse of 2^4 is 2^{-4}

3. Laws of Exponents

Laws of exponents are useful in simplifying the mathematical operations on exponents.
These laws hold true for positive exponents as well as negative exponents.

3.1. Product Rule (Same Base)

If a is a non-zero integer and m and n are any integers, then,

$$a^m \times a^n = a^{(m+n)}$$

Product of two powers with the same base and different exponents results in having the same base and exponents added.

For example,
$(2)^{-3} \times (2)^{-2} = 2^{-3+(-2)} = 2^{-5}$

3. Laws of Exponents

3.2. Division Rule (Same Base)

If a is a non-zero integer and m and n are any integers, then,

$$a^m \div a^n = a^{(m-n)}$$

Division of two powers with the same base and different exponents results in having the same base and exponents subtracted.

For example,
$(2)^{-3} \div (2)^{-2} = (2)^{-3-(-2)} = (2)^{-1}$

3.3. Power of Power Rule

If a is a non-zero integer and m and n are any integers, then,

$$(a^m)^n = a^{(mn)}$$

Base raised to a power again raised to a power, results in having the same base and exponents multiplied.

For example,
$((2)^{-3})^{-2} = (2)^{(-3) \times (-2)} = 2^6$

3. Laws of Exponents

3.4. Product Rule (Same Exponent)

If a and b are non-zero integers and m is any integer, then,

$$a^m \div b^m = (ab)^m$$

Product of two powers with different bases and same exponent results in having the bases multiplied and exponent kept as it is.

For example,

$(-2)^{-2} \times (-3)^{-2} = ((-2) \times (-3))^{-2}$

$= (6)^{-2}$

3.5. Division Rule (Same Exponent)

If a and b are non-zero integers and m is any integer, then,

$$a^m \div b^m = (ab)^m$$

Division of two powers with different bases and same exponent results in having the bases divided and exponent kept as it is.

For example:

$(-4)^{-2} \div (-2)^{-2} = \left(\frac{-4}{-2}\right)^{-2} = (2)^{-2}$

3. Laws of Exponents

3.6. Exponent Zero Rule

If a is any non-zero integer, then,

$$a^0 = 1$$

If the exponent is zero, then the result is 1, irrespective of the base value (except 0).

For example:
$(-2)^0 = 1$

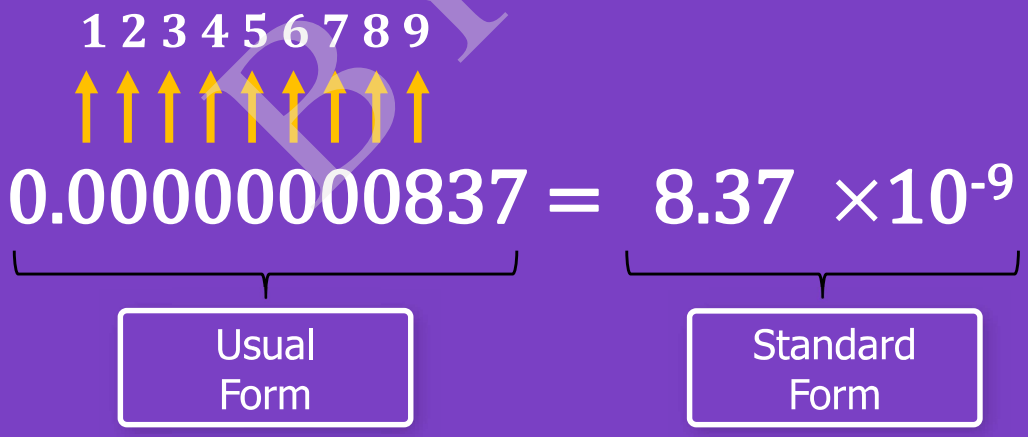
4. Uses of Exponents

4.1. Standard Form

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such form of a number is called its standard form.

Steps to convert a number into its standard form:

- Step 1: Write the first non-zero digit of the given number.
- Step 2: Add a decimal point after the first non-zero digit and write other digits after that.
- Step 3: Find the number of places the decimal point has shifted in the given number and write it to the power of 10. If the decimal point has shifted right, then the exponent is negative. If it has shifted left, then the exponent is positive.



4. Uses of Exponents

4.2. Comparing very large and very small numbers

It is easier to compare very large and very small numbers when they are written in their standard forms.

Let us understand how to compare very large and very small numbers through the following steps:

Step 1: Write the numbers to be compared in their standard forms.

$$\left.\begin{aligned} A &= a \times 10^m \\ B &= b \times 10^n \end{aligned}\right\} \longrightarrow \text{Standard form}$$

Step 2: Take the ratio of their standard forms and divide

$$\frac{A}{B} = \frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}$$

Hence, we can say that A is $\frac{a}{b} \times 10^{m-n}$ times B.

For example:
Diameter of the Sun is 1.4×10^9 m, and diameter of the Earth is 1.2756×10^7 m.
Let's compare the diameter of the Sun and the Earth:

$$\frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 100}{1.2756} \approx 100$$

So, the diameter of the Sun is 100 times the diameter of the earth.