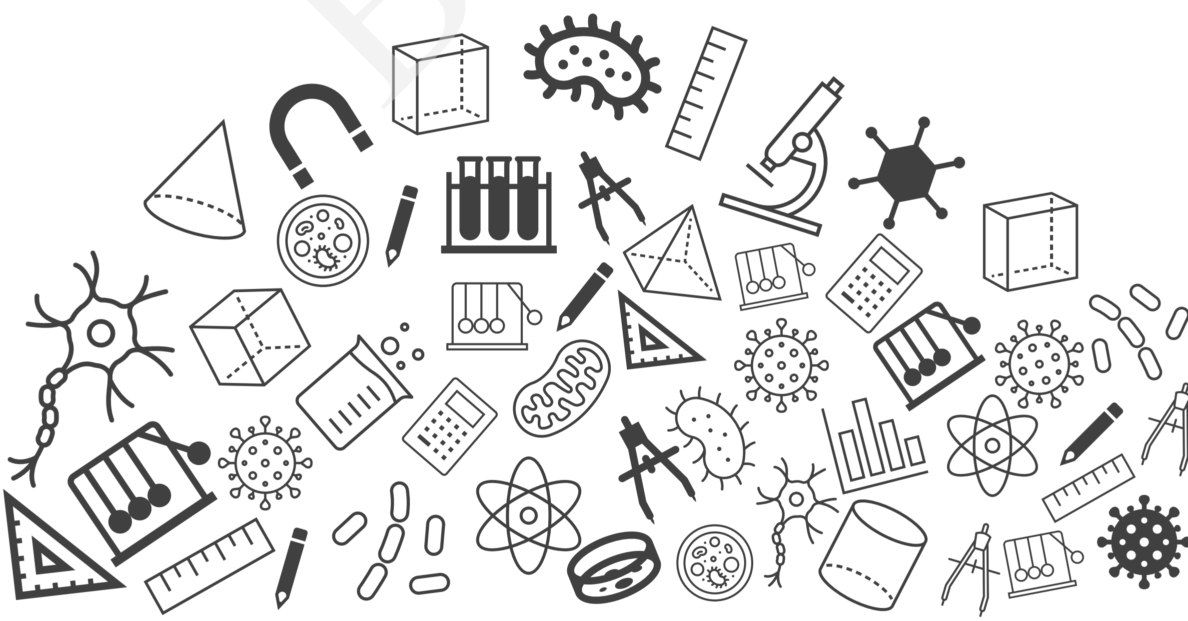
Grade 09
Mathematics Chapter Notes

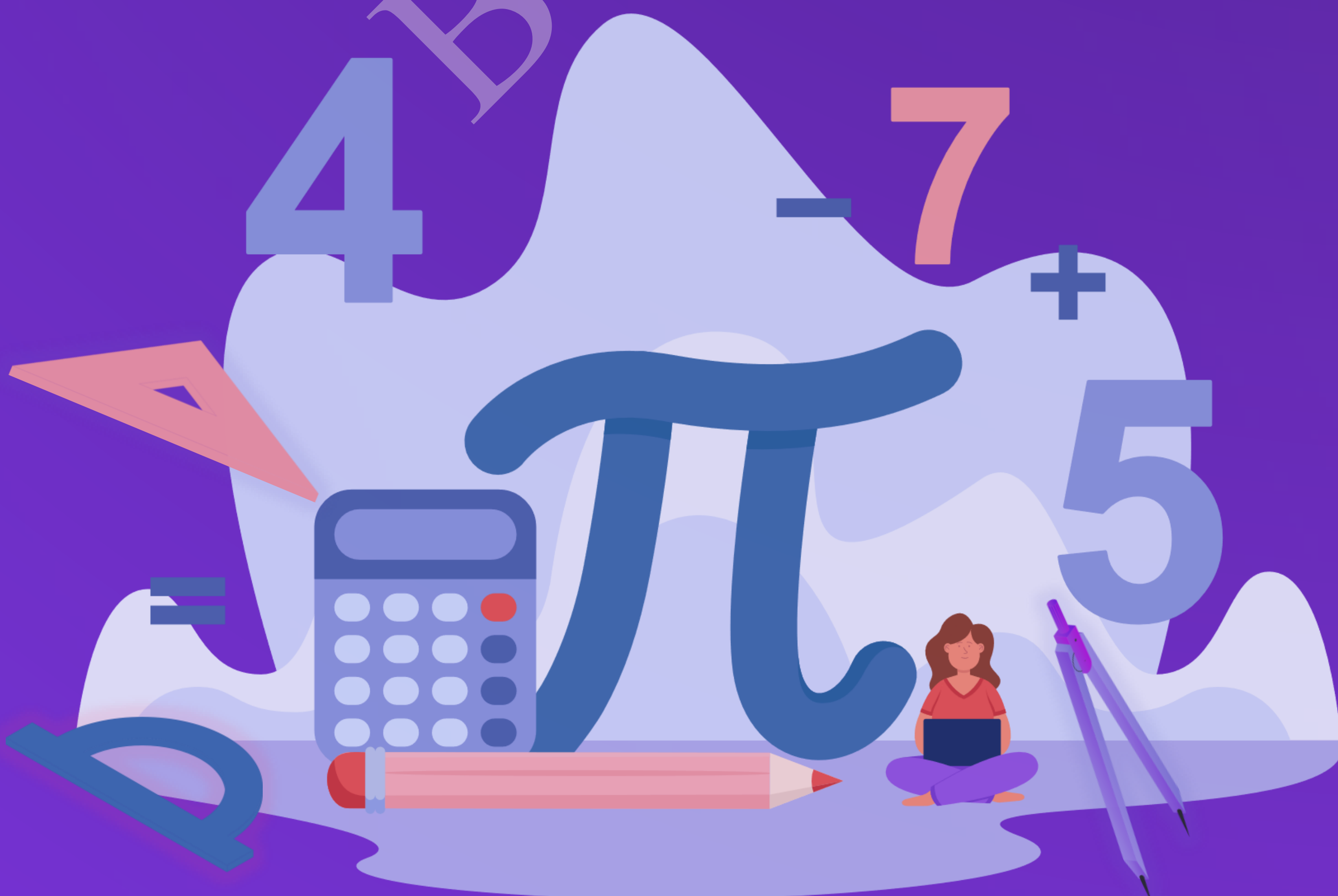


BYJU'S Classes

Chapter Notes

Triangles

Grade 09



Topics to be Covered

1. Introduction to Congruence

- 1.1. Congruency Vs Similarity
- 1.2. Congruence of Triangles

2. Criteria for Congruence

- 2.1. SAS Congruence Rule
- 2.2. ASA Congruence Rule
- 2.3. AAS Congruence Rule
- 2.4. SSS Congruence Rule
- 2.5. RHS Congruence Rule

3. Properties of Triangle

- 3.1. Angles opposite to equal sides of an isosceles triangle are equal
- 3.2. The sides opposite to equal angles of a triangle are equal

1. Introduction to Congruence

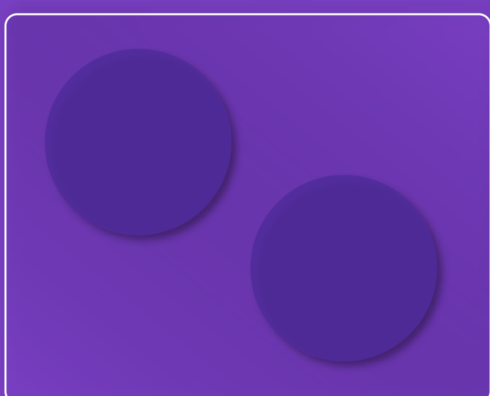
1.1. Congruency Vs Similarity

Same shape
Same size

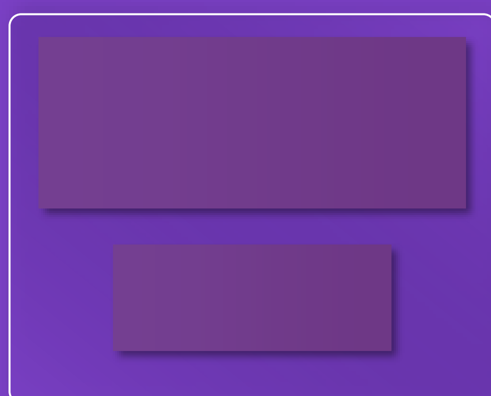
Same shape
Different size

In the above pictures we can observe that in Picture 1 we have two bottles of same shape and same size that means bottle are identical and in Picture 2 we have bottles with same shape but different size, that means bottles are not identical.

In similar way, in geometry, when we have two identical shapes, i.e., with same shape and same size then we say that the figures are **congruent**.
If two shapes have same shape but different size then the figures are said to be **similar**.



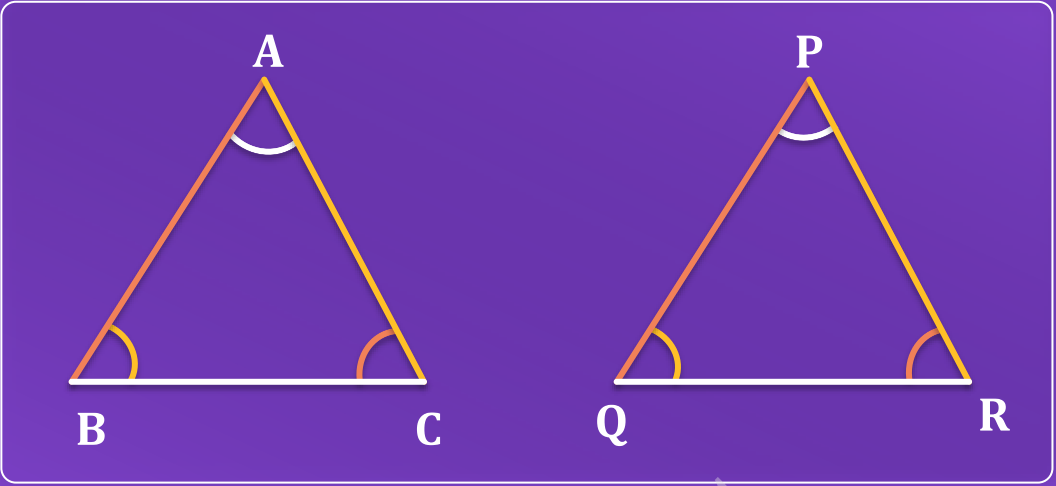
Congruent shapes



Similar shapes

1. Introduction to Congruence

1.2. Congruency of Triangles



Consider two triangles with

- $AB = PQ$
- $BC = QR$
- $AC = PR$
- $\angle A = \angle P$
- $\angle B = \angle Q$
- $\angle C = \angle R$

Thus, the corresponding sides and angles of the given two triangles are equal.

Hence, we say that the given triangles are congruent.

The symbol used to denote congruence of triangles is $\cong$.

For the given case we write the congruence as

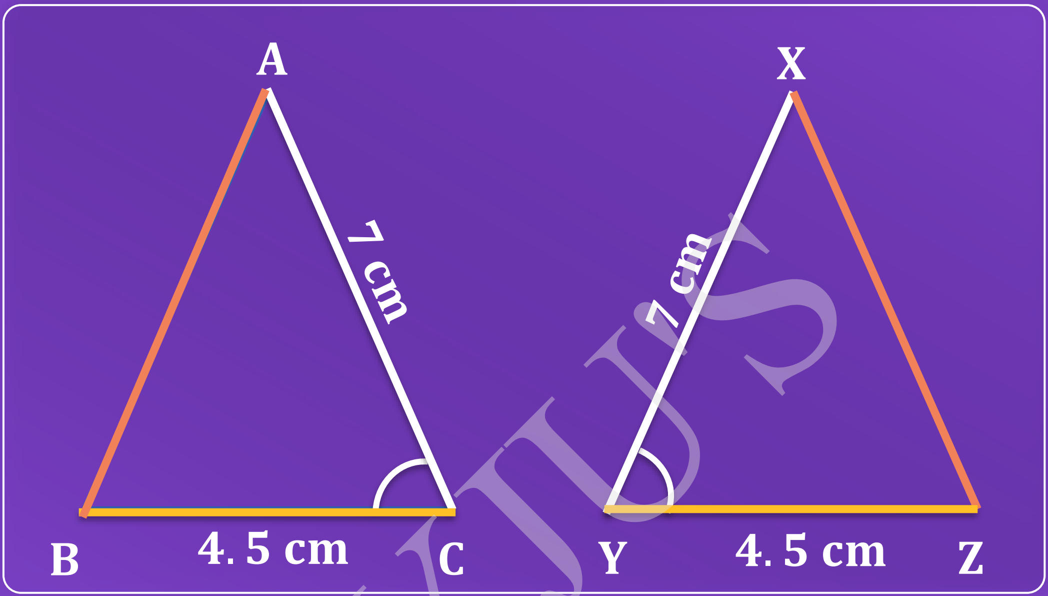
$$\Delta ABC \cong \Delta PQR$$

- The order of vertices is important while denoting congruence of triangles
- In congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

2. Criteria for Congruence

2.1. SAS Congruence Rule

If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent.



We can observe that two adjacent sides of one triangle are equal to two adjacent sides of another triangle, i.e.,

$AC = XY$ (Side)

$BC = ZY$ (Side)

Further, the included angle of first triangle is equal to included angle of second triangle. i.e.,

$\angle C = \angle Y$ (Included Angle)

In such cases as per the SAS Congruence Criterion, the triangles are said to be congruent.

$$\Delta ABC \cong \Delta XZY$$

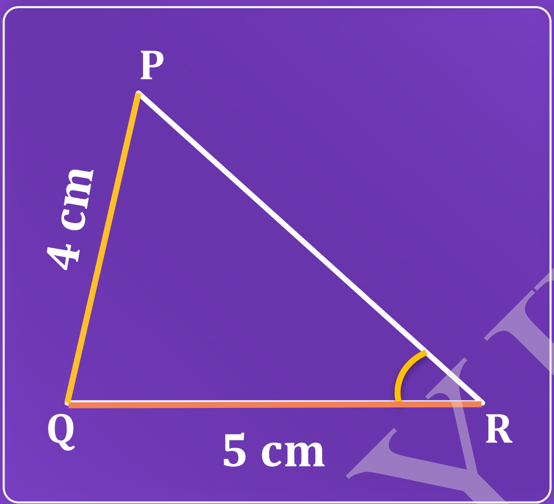
Then by CPCT it follows that

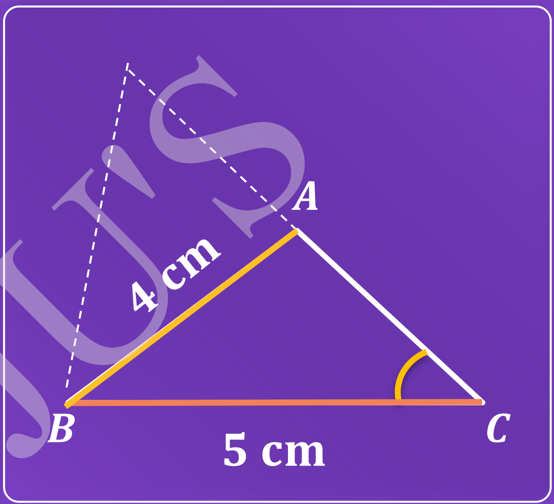
$$AB = XZ$$
$$\angle A = \angle X$$
$$\angle B = \angle Z$$

2. Criteria for Congruence

Why SSA or ASS doesn't work?

SSA stand for side - side – angle, where A is not the included angle.
ASS stand for angle - side – side, where A is not the included angle.





In the above two triangles, ΔPQR and ΔABC

$PQ = AB = 4$ cm

$QR = BC = 5$ cm

$\angle R = \angle C$

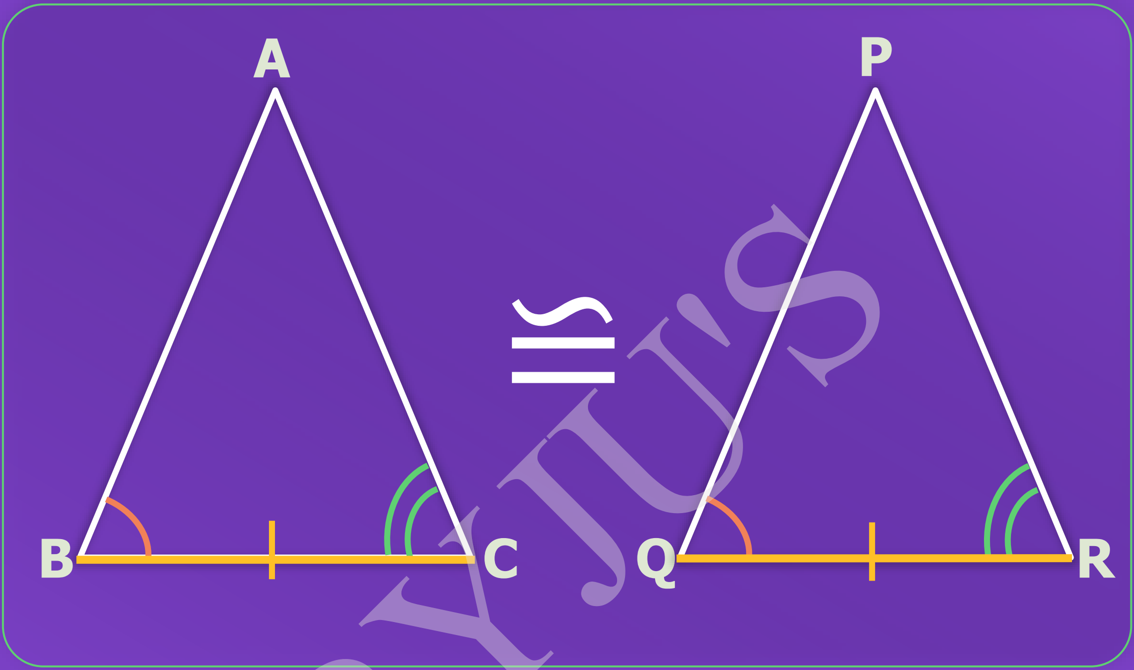
Both the triangles have two sides and one angle equal, where $\angle R$ and $\angle C$ are not included angles, we can see that the triangles are not congruent.

Hence, we can say that SSA or ASS does not work always.

2. Criteria for Congruence

2.2. ASA Congruence Rule

If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent by ASA rule.



From the figure we can observe the congruent pairs,

$\angle B = \angle Q$ (Angle)

$BC = QR$ (Included Side)

$\angle C = \angle R$ (Angle)

In such cases as per the ASA Congruence Criterion, the triangles are said to be congruent.

$$\Delta ABC \cong \Delta PQR$$

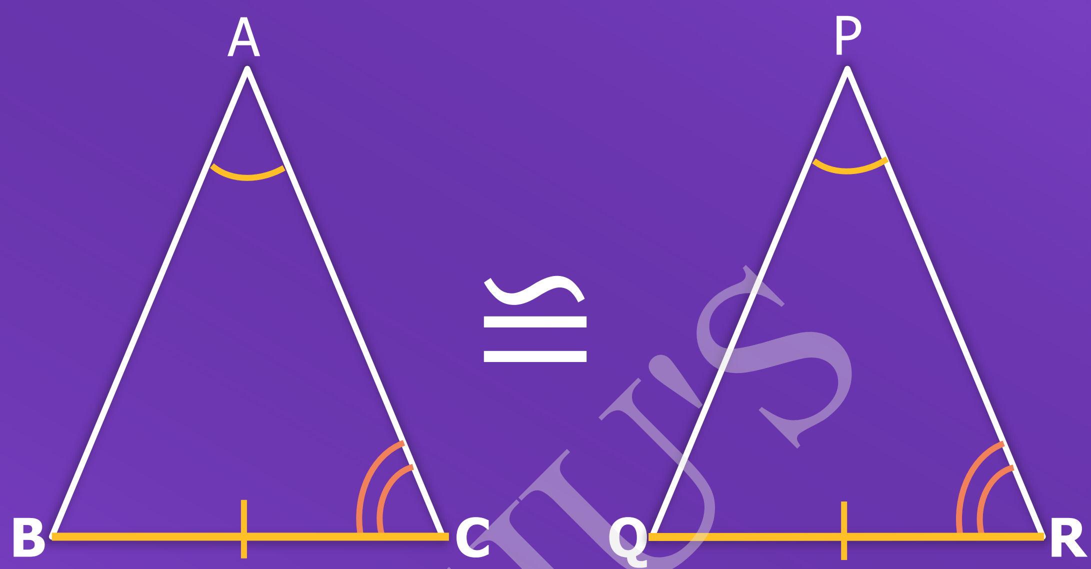
Then by CPCT it follows that

$$AB = PQ$$
$$\angle A = \angle B$$
$$AC = PR$$

2. Criteria for Congruence

2.3. AAS Congruence Rule

Two triangles are congruent by AAS if any two pairs of angles and one pair of corresponding sides are equal.



From the figure we can observe the congruent pairs,

$\angle A = \angle P$ (Angle)

$BC = QR$ (Side not included)

$\angle C = \angle R$ (Angle)

In such cases as per the AAS Congruence Criterion, the triangles are said to be congruent.

$$\Delta ABC \cong \Delta PQR$$

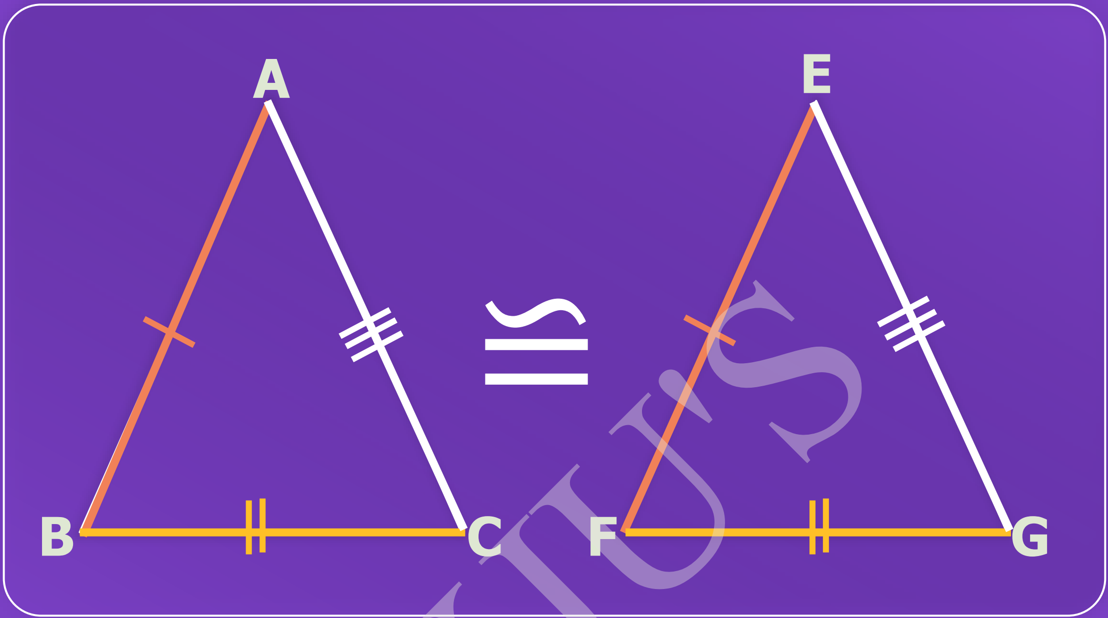
Then by CPCT it follows that

$$AB = PQ$$
$$\angle B = \angle Q$$
$$AC = PR$$

2. Criteria for Congruence

2.4. SSS Congruence Rule

Two triangles are congruent by SSS if all the three pairs of sides are equal in length.



From the figure we can observe the congruent pairs,

$AB = EF$ (Side)

$BC = FG$ (Side)

$AC = EG$ (Side)

In such cases as per the SSS Congruence Criterion, the triangles are said to be congruent.

$$\Delta ABC \cong \Delta EFG$$

Then by CPCT it follows that

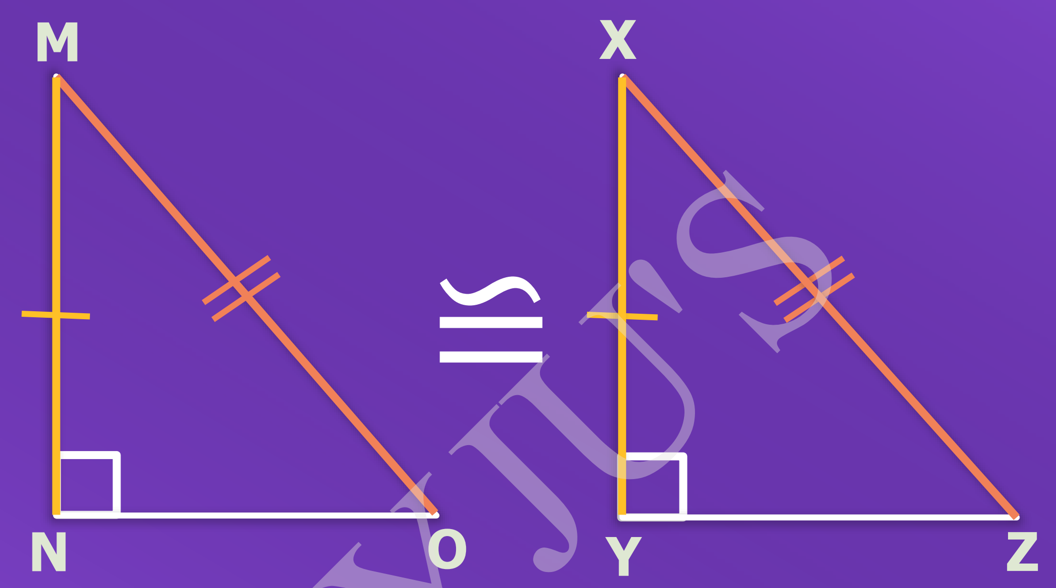
$$\angle A = \angle E$$

$$\angle B = \angle F$$

$$\angle C = \angle G$$

2.5. RHS Congruence Rule

If one side and the hypotenuse of one right-angled triangle are equal to one side and the hypotenuse of another right-angled triangle, then the two triangles are congruent by RHS rule.



From the figure we can observe the congruent pairs,

$MN = XY$ (Side)

$MO = XZ$ (Hypotenuse)

$\angle N = \angle Y$ (Each $90°$)

In such cases as per the RHS Congruence Criterion, the triangles are said to be congruent.

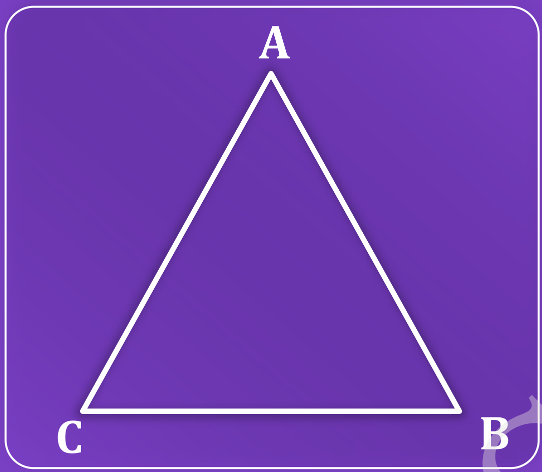
$$\Delta MNO \cong \Delta XYZ$$

Then by CPCT it follows that

$$\angle M = \angle X$$
$$\angle O = \angle Z$$
$$NO = YZ$$

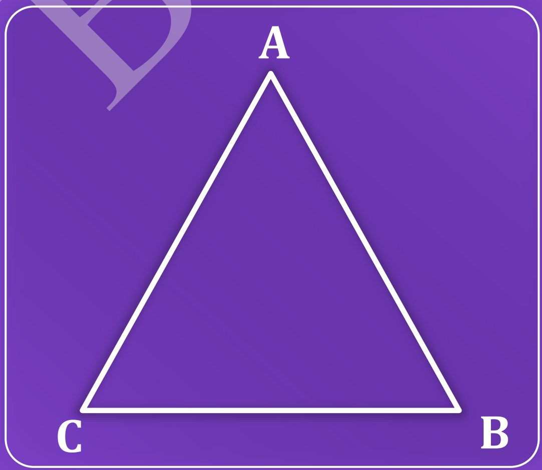
3. Properties of Triangle

Theorem: The angles opposite to equal sides of an isosceles triangle are equal.



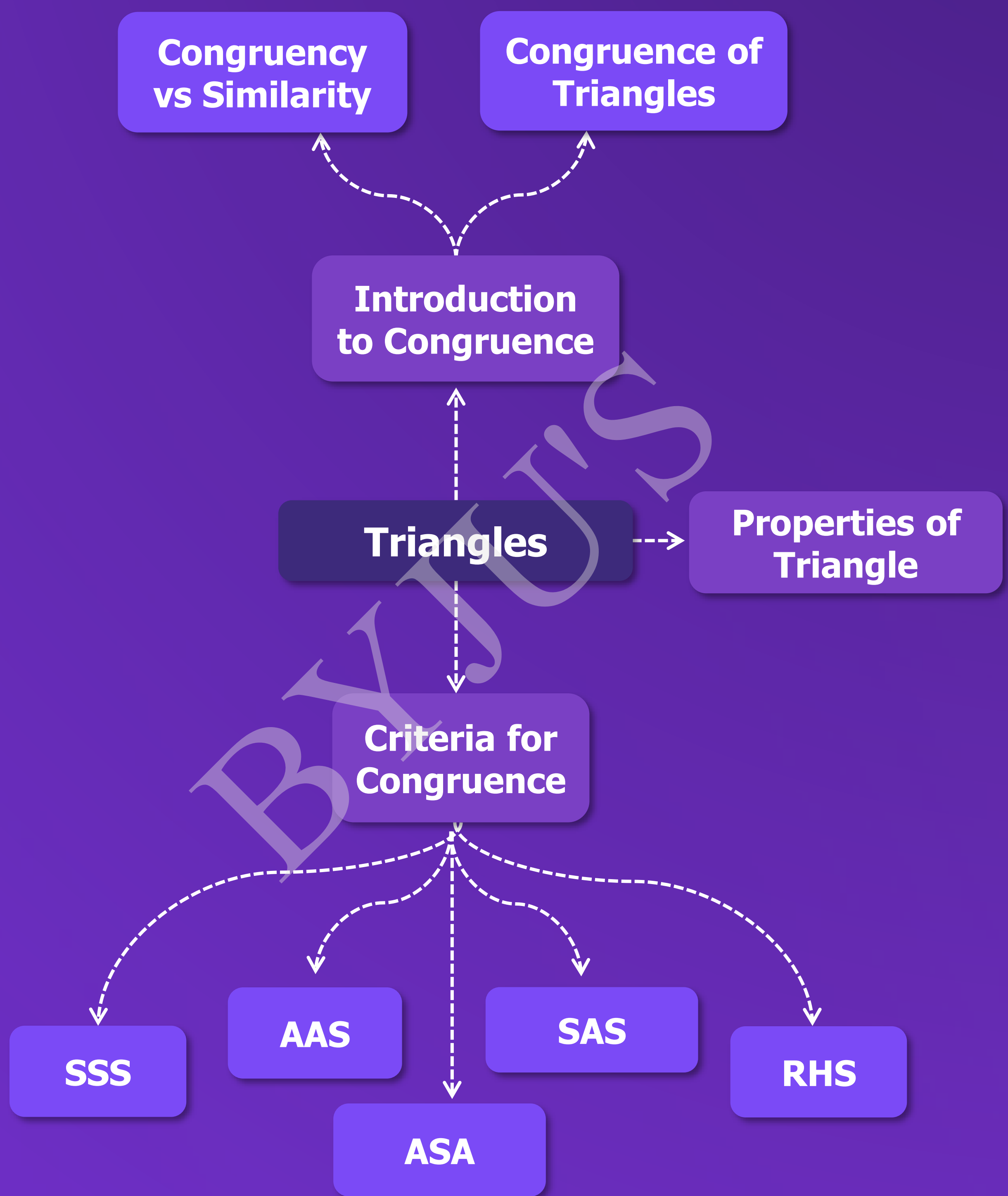
- If $AB = AC$, then $\angle B = \angle C$.
- If $AB = BC$, then $\angle A = \angle C$.
- If $BC = AC$, then $\angle A = \angle B$.

Theorem: The sides opposite to equal angles of a triangle are equal.



- If $\angle B = \angle C$, then $AB = AC$.
- If $\angle A = \angle C$, then $AB = BC$.
- If $\angle A = \angle B$, then $BC = AC$.

Mind Map


Congruency vs Similarity
Congruence of Triangles
Introduction to Congruence
Triangles
Properties of Triangle
Criteria for Congruence
SSS
AAS
ASA
SAS
RHS