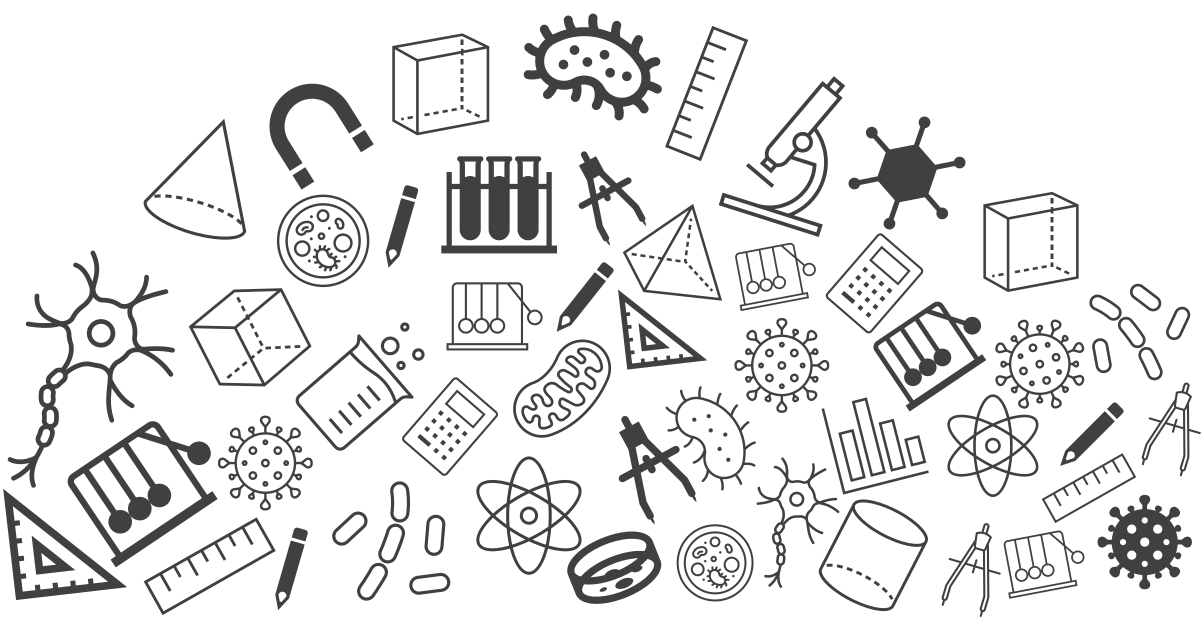
BYJU'S

Grade 10

Mathematics

Exam Important Questions



Topic : Exam Important Questions

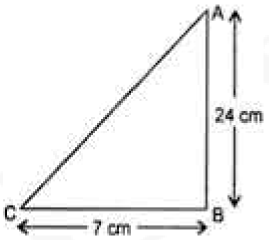
1. In a ΔABC, right angled at B, AB=24 cm, BC=7 cm. Determine

(i) sin A, cos A

(ii) sin C, cos C
[3 Marks]

In ΔABC, Using Pythagoras theorem, we have
$AC^2 = AB^2 + BC^2$
$= (24\ cm)^2 + (7\ cm)^2$
$= 576\ cm^2 + 49\ cm^2$
$= 625\ cm^2$

So, $AC = 25\ cm$ (1 mark)



Now,
$(i)\ \sin\ A\ = \frac{BC}{AC} = \frac{7}{25}$ (0.5 mark)

$\cos\ A\ = \frac{AB}{AC} = \frac{24}{25}$ (0.5 mark)

$(ii)\ \sin\ C\ = \frac{AB}{AC} = \frac{24}{25}$ (0.5 mark)

$\cos\ C\ = \frac{BC}{AC} = \frac{7}{25}$ (0.5 mark)

2. If cot $\theta = \frac{1}{\sqrt{3}}$,show that $\frac{1-cos^2\theta}{2-sin^2\theta} = \frac{3}{5}$[2 Marks]

Cot x = $\frac{1}{\sqrt{3}}$

x = 60 degrees

cos (x) = cos 60 = $\frac{1}{2}$

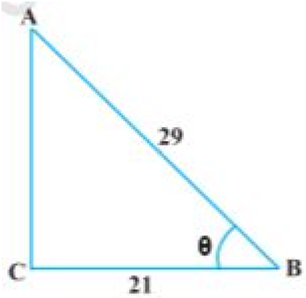
sin x = sin 60 = $\frac{\sqrt{3}}{2}$ (1 mark)

1 - cos² x = 1 - $\frac{1}{4}$= $\frac{3}{4}$

2 - sin²x = 2 - $\frac{3}{4}$ = $\frac{5}{4}$

$\frac{1-cos^2\theta}{2-sin^2\theta} = \frac{3}{5}$ (1 mark)

3. From the figure, determine the value of $sin^2\theta$ + $cos^2\theta$



[3 Marks]

[Trigonometric Ratios]

Solution:

Given, AB = 29 units and BC = 21 units

By pythagoras theorem,

$AB^2 = AC^2 + BC^2$

$29^2 = AC^2 + 21^2$

$29^2 = AC^2 + 21^2$

$841 = AC^2 + 441$

$841 - 441 = AC^2$

$400 = AC^2$

$AC = 20\ units$

[1 Mark]

Now, $sin\ \theta = \frac{AC}{AB} = \frac{20}{29}$

$sin^2\theta = (\frac{20}{29})^2$.......(i)

$cos\ \theta = \frac{BC}{AB} = \frac{21}{29}$

$cos^2\theta = (\frac{21}{29})^2$......(ii)

[1 Mark]

(i) + (ii)

$sin^2\theta + cos^2\theta = \frac{20^2+21^2}{29^2} = \frac{400+441}{841} = \frac{841}{841} =$ 1

$\therefore sin^2\theta + cos^2\theta$ = 1

[1 Mark]

4. If $\sec A = \frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos\ A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

[3 Marks]

We have,sec $A = \frac{5}{4}$

$\Rightarrow\ cosA = \frac{4}{5}[\frac{Base}{Hypotenuse}]$

By pythagoras theorem,

$(Perpendicular)^2 = (Hypotenuse)^2 - (Base)^2$

⇒Perpendicular = $\sqrt{25-16}$

⇒Perpendicular = 3

Then, sin A = $\frac{Perpendicular}{Hypotenuse} = \frac{3}{5}$

tan A = $\frac{Perpendicular}{Base} = \frac{3}{4}$

(1 mark)

Now, we will prove that

$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

LHS = $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$

$= \frac{3\times\frac{3}{5} - 4(\frac{3}{5})^3}{4(\frac{4}{5})^3 - 3\times\frac{4}{5}}$

$\frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$

$= \frac{\frac{225-108}{125}}{\frac{256-300}{125}}$

$= \frac{117}{125} \times \frac{125}{-44}$

$= \frac{117}{-44}$

(1 mark)

RHS = $\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$= \frac{3\times\frac{3}{4} - (\frac{3}{4})^3}{1 - 3(\frac{3}{4})^2}$

$= \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}}$

$= \frac{\frac{117}{64}}{\frac{-11}{16}}$

$= \frac{-117}{44}$

$\therefore\ LHS = RHS$

(1 mark)

5. Evaluate the following:
$sin60^\circ cos30^\circ + sin30^\circ cos60^\circ$
[2 Marks]
[Trigonometric Ratios of Specific Angles]

Solution:

$sin60^\circ cos30^\circ + sin30^\circ cos60^\circ$
$= (\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}) + (\frac{1}{2} \times \frac{1}{2})$

[1 mark]

$= \frac{3}{4} + \frac{1}{4}$
$= \frac{4}{4} = 1$

[1 mark]

6. Without using trigonometric tables, evaluate:

$(i) cos\ 48^\circ - sin\ 42^\circ$

$(ii) cosec\ 31^\circ - sec\ 59^\circ$

$(iii) cot\ 34^\circ - tan\ 56^\circ$

$(iv) cos^2\ 13^\circ - sin^2\ 77^\circ$

$(i) cos\ 48^\circ - sin\ 42^\circ = cos(90^\circ - 42^\circ) - sin\ 42^\circ$

$= sin\ 42^\circ - sin\ 42^\circ = 0$

$[\because cos(90^\circ - \theta) = sin\ \theta]$ (1 mark)

$(ii) cosec\ 31^\circ - sec\ 59^\circ = cosec(90^\circ - 59^\circ) - sec\ 59^\circ$

$= sec\ 59^\circ - sec\ 59^\circ = 0$

$[\because cosec(90^\circ - \theta) = sec\ \theta]$ (1 mark)

$(iii) cot\ 34^\circ - tan\ 56^\circ = cot(90^\circ - 56^\circ) - tan\ 56^\circ$

$= tan\ 56^\circ - tan\ 56^\circ = 0$

$[\because cot(90^\circ - \theta) = tan\ \theta]$ (1 mark)

$(iv) cos^2\ 13^\circ - sin^2\ 77^\circ = cos^2\ (90^\circ - 77^\circ) - sin^2\ 77^\circ$

$= sin^2\ 77^\circ - sin^2\ 77^\circ = 0$

$[\because cos(90^\circ - \theta) = sin\ \theta]$ (1 mark)

Introduction to Trigonometry

7. In a ΔABC, right-angled at C, if tan A= $\frac{1}{\sqrt{3}}$, then find the value of (sinA)(sinB) + (cosA)(cosB).

[3 Marks]
[Trigonometric Ratios of Specific Angles]

Solution:

Step 1:

- **First we need to determine the values of angles A and B and determine the value of their sine and cosine.**
- **We are given that tan A = $\frac{1}{\sqrt{3}}$ and tan θ is $\frac{1}{\sqrt{3}}$ for θ = 30º. Therefore, ∠A = 30º**
- Also by angle sum property of a triangle,

∠A + ∠B + ∠C = 180º

[1 Mark]

Substituting ∠A = 30º and ∠C = 90º, We get ∠B = 60º

- **Now, therefore,**
- **Sin A = sin 30º =** $\frac{1}{2}$
- **Sin B = sin 60º =** $\frac{\sqrt{3}}{2}$
- **Cos A = Cos 30º =** $\frac{\sqrt{3}}{2}$
- **Cos B = Cos 60º =** $\frac{1}{2}$

[1 Mark]

Step 2:

- **For the final step, we need to substitute the corresponding values in the given expression.**
- **The given expression is sinA sinB + cosA cosB.**

On substituting the respective values, we get

$= \frac{1}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\times\frac{1}{2}$

$= \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

[1 Mark]

8. If $tan2A = cot(A - 18^\circ)$, where 2A is an acute angle, find the value of A.
[2 marks]

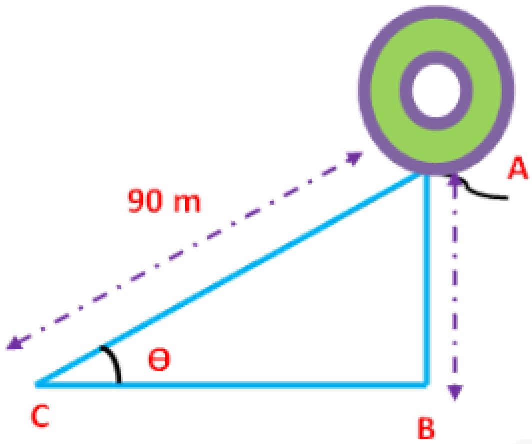
$tan2A = cot(A - 18^\circ)$
$\Rightarrow cot(90^\circ - 2A) = cot(A - 18^\circ)$
(As tan A = cot (90° - A))
[1 mark]

Equating angles,

$\Rightarrow 90^\circ - 2A = A - 18^\circ$
$\Rightarrow 108^\circ = 3A$
$\Rightarrow A = 36^\circ$

[1 mark]

9. The length of a string between a kite and a point on the ground is 90 m. If the string makes an angle θ with the ground level such that tan θ = 15/8, how high will the kite be?



[3 Marks]

Here AB represents the height of the balloon from the ground. In the right triangle ABC the side which is opposite to angle θ is known as the opposite side (AB), the side which is opposite to 90° is called the hypotenuse side (AC) and the remaining side is called the adjacent side (BC).

Now we need to find the length of the side AB.

tan θ = $\frac{15}{8}$

cot θ = $\frac{8}{15}$

(1 Mark)

csec θ = $\sqrt{1 + cot^2\theta}$

csec θ = √(1 + $\frac{64}{225}$)

csec θ = √$\frac{(225+64)}{225}$

csec θ = √$\frac{289}{225}$

csc θ = $\frac{17}{15}$

sin θ = $\frac{15}{17}$

(1 Mark)

But, sin θ = $\frac{opposite\ side}{hypotenuse\ side} = \frac{AB}{AC}$.

$\frac{AB}{AC} = \frac{15}{17}$

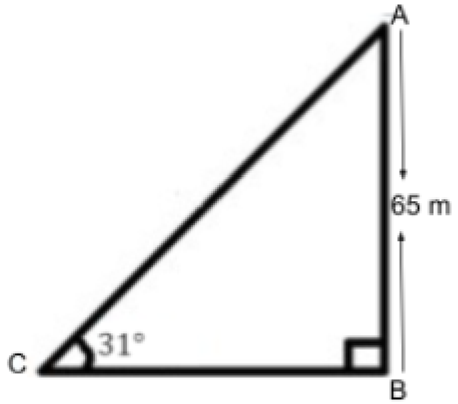
$\frac{AB}{90} = \frac{15}{17}$

AB = 79.41

So, the height of the tower is 79.41 m.

(1 Mark)

10.



A kite is flying at a height of 65 m attached to a string. If the inclination of the string with the ground is 31°, find the length of the string if cos 59° = 0.5150.

(3 Marks)

Here AB represents the height of the kite. In the right triangle ABC the side which is opposite to angle 31° is known as the opposite side (AB), the side which is opposite to 90° is called the hypotenuse (AC) and the remaining side is called the adjacent side (BC).

Now we need to find the length of the string AC.

sinθ = opposite side/hypotenuse side

We know that sin θ**=**$cos(90^{\circ} - \theta\ )$

Given cos 59° = 0.5150

(1 Mark)

∴ **Cos 59° =** $cos(90^{\circ} - 31^{\circ})$

sin 31° = $\frac{AB}{AC}$

0.5150 = $\frac{65}{AC}$

(1 Mark)

AC = $\frac{65}{0.5150}$

AC = 126.2 m

Hence, the length of the string is 126.2 m.

(1 Mark)

11. Evaluate :
$cosec\ 31^{\circ} - sec\ 59^{\circ}$
[2 marks]

$cosec\ 31^{\circ} - sec\ 59^{\circ}$

$= cosec\ (90^{\circ} - 59^{\circ}) - sec\ 59^{\circ}$

$= sec\ 59^{\circ} - sec\ 59^{\circ} = 0$

(As cosec $(90^{\circ}$ - A) = sec A)

[2 marks]

12. If $2\ sin^2\ \theta - cos^2\ \theta = 2$, then find the value of θ.
[2 marks]

Given, $2\ sin^2\ \theta - cos^2\ \theta = 2$

$\Rightarrow 2\ sin^2\ \theta - (1 - sin^2\ \theta) = 2$

$[\because sin^2\ \theta + cos^2\ \theta = 1]$

$\Rightarrow 2\ sin^2\ \theta + sin^2\ \theta - 1 = 2$

[1 mark]

$\Rightarrow 3\ sin^2\ \theta = 3$
$\Rightarrow sin^2\ \theta = 1$
$[\because sin\ 90^\circ = 1]$

$\Rightarrow sin\ \theta = 1 = sin\ 90^\circ$

$\Rightarrow \theta = 90^\circ$

[1 mark]