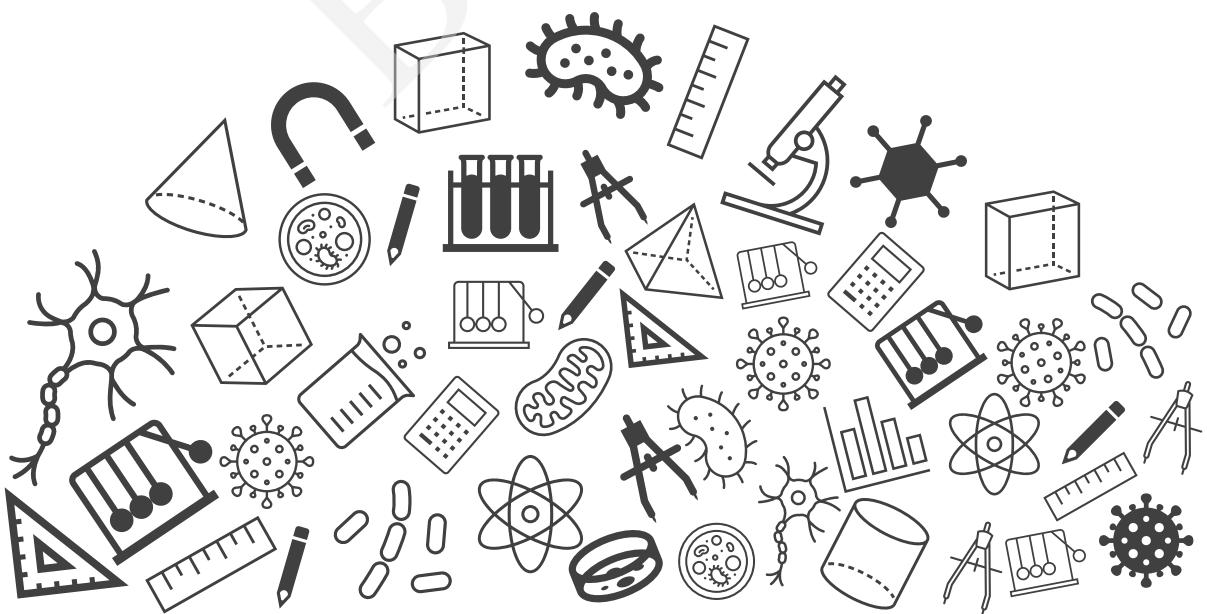




Grade 10

Mathematics Chapter Notes



M A T H E M A T I C S



Introduction to Trigonometry





Topics



1. Trigonometric Ratios

2. Trigonometric Ratios of
standard angles

3. Trigonometric Identities

25° 30°



Trigonometric Ratios

SOH

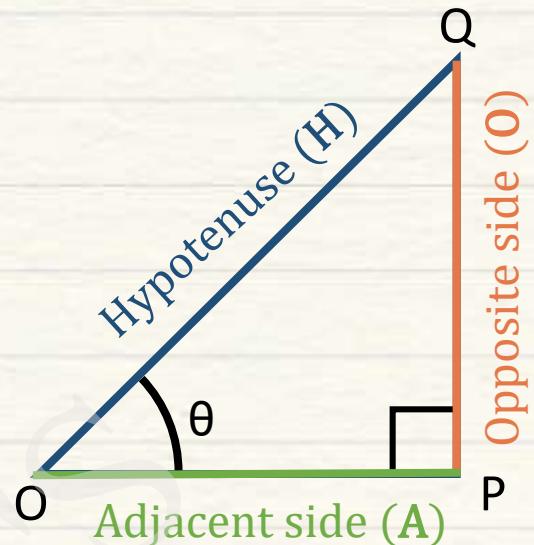
$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

CAH

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

TOA

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{Opposite}}{\text{Adjacent}}$$



i. $\sin \theta$

ii. $\cos \theta$

iii. $\tan \theta$

MULTIPLICATIVE
INVERSE

i. $\cosec \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$

ii. $\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$

iii. $\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Adjacent}}{\text{Opposite}}$



Trigonometric Ratios of Standard Angles

- With just the values of $\sin \theta$, we can calculate all other trigonometric ratios for standard angles.



An idea to learn the sin values

| θ | 0° | 30° | 45° | 60° | 90° |
|---|----------------------|----------------------|----------------------|----------------------|----------------------|
| 1. Write numbers from 0 to 4 in order. | 0 | 1 | 2 | 3 | 4 |
| 2. Divide every number by 4 | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ |
| 3. Take the square root of every number | $\sqrt{\frac{0}{4}}$ | $\sqrt{\frac{1}{4}}$ | $\sqrt{\frac{2}{4}}$ | $\sqrt{\frac{3}{4}}$ | $\sqrt{\frac{4}{4}}$ |
| 4. Simplify | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\sin \theta$ | $\sin 0^\circ$ | $\sin 30^\circ$ | $\sin 45^\circ$ | $\sin 60^\circ$ | $\sin 90^\circ$ |



Trigonometric Ratios of Standard Angles

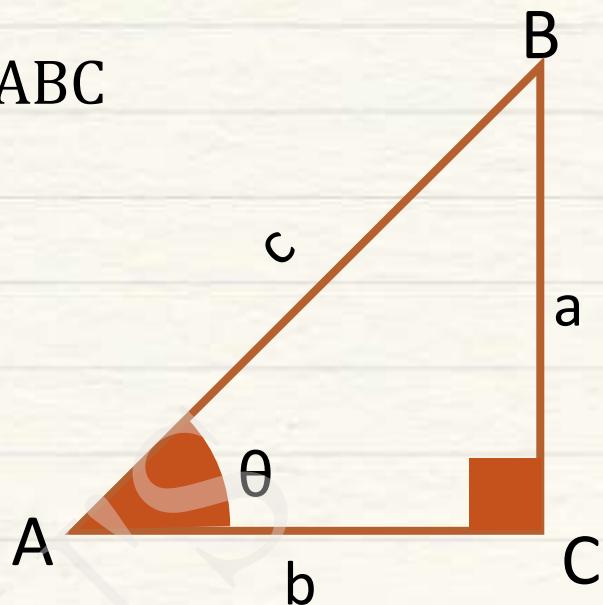
| Angles Ratios | Logic | 0° | 30° | 45° | 60° | 90° |
|------------------|---------------------------------|-------------|----------------------|----------------------|----------------------|-------------|
| $\sin\theta$ | $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos\theta$ | Reverse $\sin\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan\theta$ | $\frac{\sin\theta}{\cos\theta}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not defined |
| cosec θ | $\frac{1}{\sin\theta}$ | Not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| sec θ | $\frac{1}{\cos\theta}$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not defined |
| cot θ | $\frac{1}{\tan\theta}$ | Not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |



Proof of Trigonometric Identities

In a right - Angled Triangle Δ ABC

In $\triangle ABC$ we know that



By Pythagoras Theorem,

$$a^2 + b^2 = c^2$$

Dividing both sides by c^2

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

From 1 and 2,

$$\sin^2 \theta + \cos^2 \theta = 1$$



Proof of $1 + \tan^2\theta = \sec^2\theta$

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing both the sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$



$$\tan^2\theta + 1 = \sec^2\theta$$

Proof of $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing both the sides by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$



$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Three Basic Trigonometric Identities

1

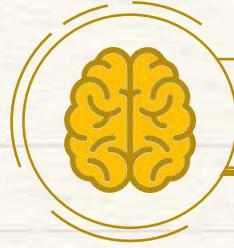
$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta &= 1 - \cos^2\theta \\ \cos^2\theta &= 1 - \sin^2\theta\end{aligned}$$

2

$$\begin{aligned}\sec^2\theta - \tan^2\theta &= 1 \\ 1 + \tan^2\theta &= \sec^2\theta \\ \sec^2\theta - 1 &= \tan^2\theta\end{aligned}$$

3

$$\begin{aligned}\operatorname{cosec}^2\theta - \cot^2\theta &= 1 \\ 1 + \cot^2\theta &= \operatorname{cosec}^2\theta \\ \operatorname{cosec}^2\theta - 1 &= \cot^2\theta\end{aligned}$$



Mind Map

