Some Applications of Trigonometry:
Application of trigonometry

Topic : Exam Important Questions
Some Applications of Trigonometry: Application of trigonometry

1. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

[4 marks]
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The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. This can be shown as:

![Diagram](image)

Given:
Height of boy \((DC)\) = 1.5 m
Height of building \((AB)\) = 30 m

\[\text{In } \triangle AEF, \ \angle AEF = 60^\circ\]

\[\text{In } \triangle ADF, \ \angle ADF = 30^\circ\]

We are asked to find the distance walked, which is shown by DE. To calculate DE, we will have to subtract EF from DF.

Now,
\[AF = 30 - 1.5 = 28.5 \text{ m}\]

[1 mark]

Step 1: Finding the length \(EF\).
\[
\tan 60^\circ = \frac{AF}{EF} \Rightarrow \sqrt{3} = \frac{28.5}{EF}
\]

\[EF = \frac{28.5}{\sqrt{3}} \quad \cdots (1)\]

[1 mark]
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Step 2: Finding the length \( DF \)
From \( \triangle AFD \),
\[
\tan 30^\circ = \frac{AF}{DF}
\]
\[
\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{DF}
\]
\[
\Rightarrow DF = 28.5\sqrt{3} \quad \cdots (2)
\]
[1 mark]

Step 3: Finding the distance walked by the boy towards the building
\( DE = DF - EF \)
\[
= \left( 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} \right) \text{ m} = \frac{85.5 - 28.5}{\sqrt{3}} = 19\sqrt{3} \text{ m}
\]

Thus, the distance walked by the boy towards the building is \( 19\sqrt{3} \) m.
[1 mark]
2. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the heights of the poles and the distances of the point from the poles.

[4 marks]
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Given:
Width of road = 80 m.
The heights of the poles are equal.

From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. This can be shown as:

Let $BD$ be 80 m wide road and $AB$ and $CD$ are the two poles of equal heights.

Let the heights of the poles as $x$ meters.
⇒ $AB = CD = x$

From figure,
$BD = BO + OD$ ...(1)
[1 mark]
Step 1: Finding $BO$ in terms of $x$.
In $\Delta AOB$,
$\tan 60^\circ = \frac{AB}{BO}$
⇒ $\sqrt{3} = \frac{x}{BO}$
⇒ $BO = \frac{x}{\sqrt{3}}$ m ...(2)
[1 mark]
Step 2: Finding $DO$ in terms of $x$.
In $\Delta COD$,
$\tan 30^\circ = \frac{CD}{DO}$
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\[ \frac{1}{\sqrt{3}} = \frac{x}{DO} \]

\[ DO = x\sqrt{3} \text{ m} \quad \ldots(3) \]

[1 mark]

Step 3: Replace (2) and (3) in (1).

\[ \frac{x}{\sqrt{3}} + x\sqrt{3} = 80 \text{ m} \]

\[ x\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 80 \text{ m} \]

\[ x\left(\frac{4}{\sqrt{3}}\right) = 80 \text{ m} \]

\[ x = 20\sqrt{3} \text{ m} \]

Therefore, height of both the poles = \(20\sqrt{3}\) m.

[1 mark]
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3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45°, respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)
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Step 1: Assume the height of tower and height of the flagstaff.

Let the height of tower and the flagstaff be BC and AB, respectively.
[1 Mark]

Step 2: Write the trigonometric ratio for the height of the tower.

The trigonometric ratio for the height of the tower.

\[ \tan 30^\circ = \frac{BC}{CD} \]
\[ \Rightarrow CD = \frac{BC}{\tan 30^\circ} \]
\[ \Rightarrow CD = \frac{BC}{\frac{1}{\sqrt{3}}} \]
\[ \Rightarrow CD = \sqrt{3}BC \ldots (1) \]
[1 Mark]

Step 3: Write the trigonometric ratio for the height of both tower and the flagstaff.

\[ \tan 45^\circ = \frac{AC}{CD} \]
\[ \tan 45^\circ = \frac{AB + BC}{CD} \]
\[ \Rightarrow CD \tan 45^\circ = AB + BC \]
\[ \Rightarrow CD \times 1 = AB + BC \]
\[ \Rightarrow CD = AB + BC \]
\[ \Rightarrow CD = 6 + BC \ldots (2) \]

Equations obtained:

\[ CD = \sqrt{3}BC \ldots (1) \]
\[ CD = 6 + BC \ldots (2) \]
[1 Mark]

Step 4: Find the height of the tower by equating both equations.

\[ CD = \sqrt{3}BC \]
\[ CD = 6 + BC \]
\[ \Rightarrow \sqrt{3}BC = 6 + BC \]
\[ \Rightarrow \sqrt{3}BC - BC = 6 \]
\[ \Rightarrow (\sqrt{3} - 1)BC = 6 \]
\[ \Rightarrow BC = \frac{6}{\sqrt{3} - 1} \]
\[ \Rightarrow BC = \frac{6}{0.73} \approx 8.22 \text{ m} \]

So height of the tower is 8.22 m.
[1 Mark]
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4. The lower window of a house is at a height of 2m above the ground and its upper window is 4m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.

[5 Marks]
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Solution:

Let the height of the balloon from above the ground is $H$.

A and $OP = w_2R = w_1Q = x$

Given that, height of lower window from above the ground $= w_2P = 2m = OR$

Height of upper window from above the lower window $= w_1w_2 = 4m = QR$

$\therefore BQ = OB - (QR + RO)$

$= H - (4 + 2)$

$= H - 6$

[1 mark]

And, $\angle Bw_1Q = 30^\circ$

$\Rightarrow \angle Bw_2R = 60^\circ$

Now, in $\triangle Bw_2R$,

$\tan 60^\circ = \frac{BR}{w_2R} = \frac{BQ + QR}{x}$

$\Rightarrow \sqrt{3} = \frac{(H-6)+4}{x}$

$\Rightarrow x = \frac{H-2}{\sqrt{3}} \ldots \ldots (i)$

[1 Mark]

And in $\triangle Bw_1Q$, 79
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\[ \tan 30^\circ = \frac{BQ}{WO} \]

\[ \tan 30^\circ = \frac{H-6}{x} = \frac{1}{\sqrt{3}} \]

\[ \Rightarrow x = \sqrt{3}(H - 6) \ldots . . . (ii) \]

[1 Mark]

From Eq. (i) and (ii),

\[ \sqrt{3}(H - 6) = \frac{(H-2)}{\sqrt{3}} \]

\[ \Rightarrow 3(H - 6) = H - 2 \]
\[ \Rightarrow 3H - 18 = H - 2 \]
\[ \Rightarrow 2H = 16 \Rightarrow H = 8 \]

So, the height of the balloon above the ground is 8 m.

[1 Mark]
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5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

[3 Marks]

Let AB be the height of statue.
D is the point on the ground from where the elevation is taken.
Height of pedestal = BC = AC − AB

[1 Mark]
As per question,
In right ΔBCD,

\[
\tan 45° = \frac{BC}{CD}
\]
\[
\Rightarrow 1 = \frac{BC}{CD}
\]
\[
\Rightarrow BC = CD
\]

[1 Mark]
Also,
In right ΔACD,
\[
\tan 60° = \frac{AC}{CD}
\]
\[
\Rightarrow \sqrt{3} = \frac{AC}{\frac{AB + BC}{CD}}
\]
\[
\Rightarrow \sqrt{3}CD = 1.6 + BC
\]
\[
\Rightarrow \sqrt{3}BC = 1.6 + BC
\]
\[
\Rightarrow \sqrt{3}BC − BC = 1.6
\]
\[
\Rightarrow BC(\sqrt{3} - 1) = 1.6
\]
\[
\Rightarrow BC = \frac{1.6}{\sqrt{3} - 1} m
\]
Rationalising we get,
\[
\Rightarrow BC = \frac{1.6 \sqrt{3} + 1}{(\sqrt{3} - 1) \sqrt{3} + 1} m
\]
\[
\Rightarrow BC = 0.8(\sqrt{3} + 1)m
\]
Thus, the height of the pedestal is 0.8(\sqrt{3} + 1)m.

[1 Mark]
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6. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
[ 3 Marks]

Let AB be the lighthouse of height 75 m.
Let C and D be the positions of the ships.
30° and 45° are the angles of depression from the lighthouse.

\[ \text{In right } \triangle ABC, \]
\[ \tan 45^\circ = \frac{AB}{BC} \]
\[ \Rightarrow 1 = \frac{75}{BC} \]
\[ \Rightarrow BC = 75 \text{ m} \quad [1 \text{ mark}] \]

Also,
\[ \text{In right } \triangle ABD, \]
\[ \tan 30^\circ = \frac{AB}{BD} \]
\[ \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD} \]
\[ \Rightarrow BD = 75\sqrt{3} \text{ m} \]

The distance between the two ships
\[ CD = BD - BC = (75\sqrt{3} - 75)m = 75(\sqrt{3} - 1)m. \quad [2 \text{ marks}] \]
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7. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. [3 Marks]

![Diagram of tower and canal]

Here, AB is the height of the tower and BC is the width of the canal.
CD = 20 m
As per question,
In right ΔABD,
\[ \tan 30° = \frac{AB}{BD} \]
\[ \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{(20+BC)} \]
\[ \Rightarrow AB = \frac{(20+BC)}{\sqrt{3}} \quad \text{(i)} \quad [1 \text{ Mark}] \]

Also,
In right ΔABC,
\[ \tan 60° = \frac{AB}{BC} \]
\[ \Rightarrow \sqrt{3} = \frac{AB}{BC} \]
\[ \Rightarrow AB = \sqrt{3}BC \quad \text{(ii)} \quad [1 \text{ Mark}] \]

From equation (i) and (ii)
\[ AB = \sqrt{3}BC = \frac{(20+BC)}{\sqrt{3}} \]
\[ \Rightarrow 3BC = 20 + BC \]
\[ \Rightarrow 2BC = 20 \Rightarrow BC = 10 \text{ m} \]

Putting the value of BC in equation (ii)
\[ AB = 10\sqrt{3} \text{ m} \]

Thus, the height of the tower is \(10\sqrt{3}\) m and the width of the canal is 10 m. [1 Mark]
8. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. [3 marks]

Let AC be the broken part of the tree.
∴ Total height of the tree = $AB + AC$

In right $\triangle ABC$,
$\cos 30^\circ = \frac{BC}{AC}$ [1 mark]

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$
$\Rightarrow AC = \frac{16}{\sqrt{3}}$ [1 Mark]

Also, $\tan 30^\circ = \frac{AB}{BC}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$
$\Rightarrow AB = \frac{8}{\sqrt{3}}$

Therefore, the total height of the tree = $AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$ m. [1 Mark]