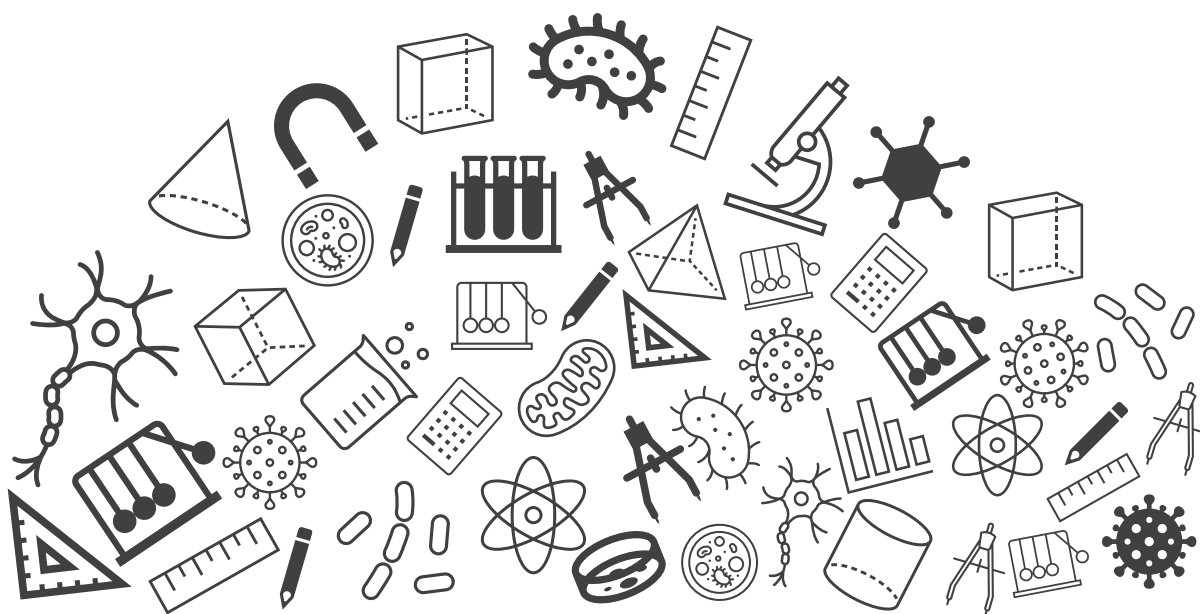




Grade 10

Mathematics

Exam Important Questions



Some Applications of Trigonometry: Application of trigonometry

Topic : Exam Important Questions

BYJU'S

Some Applications of Trigonometry:

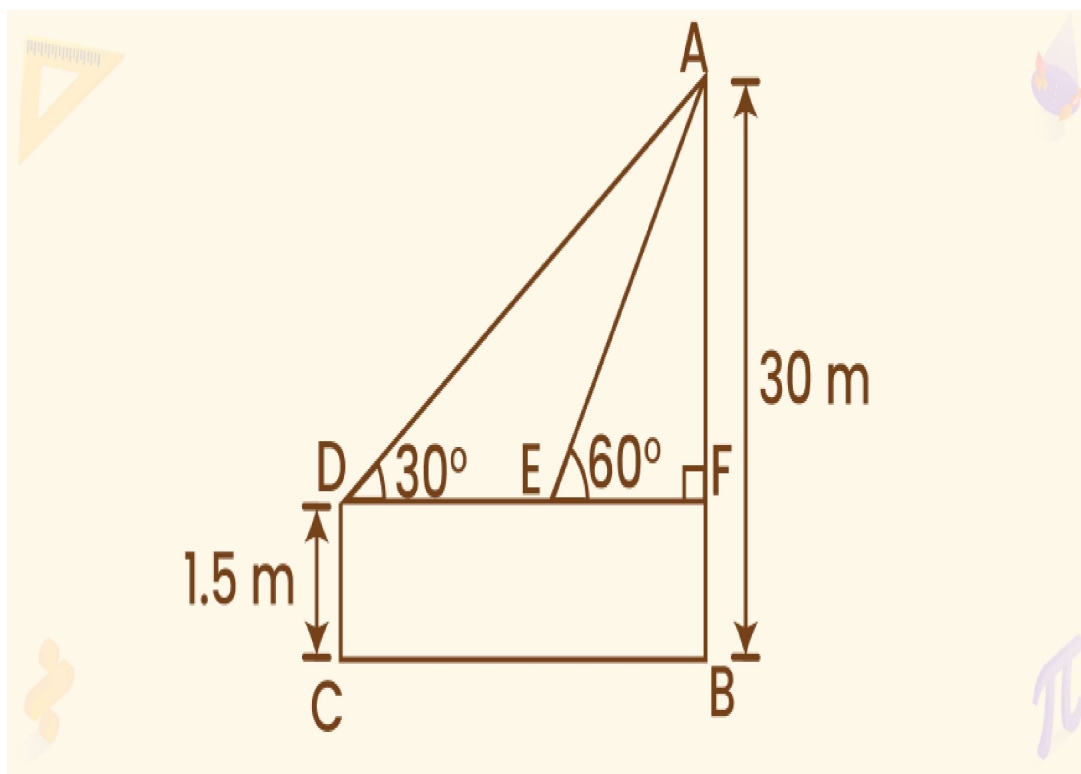
Application of trigonometry

1. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
[4 marks]

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The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. This can be shown as:



Given:

Height of boy (DC) = 1.5 m

Height of building (AB) = 30 m

In $\triangle AEF$,

$$\angle AEF = 60^\circ$$

In $\triangle ADF$,

$$\angle ADF = 30^\circ$$

We are asked to find the distance walked, which is shown by DE .

To calculate DE , we will have to subtract EF from DF .

Now,

$$\begin{aligned} AF &= 30 - 1.5 \\ &= 28.5 \text{ m} \end{aligned}$$

[1 mark]

Step 1: Finding the length EF .

$$\tan 60^\circ = \frac{AF}{EF}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{EF}$$

$$\Rightarrow EF = \frac{28.5}{\sqrt{3}} \quad \dots (1)$$

[1 mark]

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Step 2: Finding the length DF

From $\triangle AFD$,

$$\tan 30^\circ = \frac{AF}{DF}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{DF}$$

$$\Rightarrow DF = 28.5\sqrt{3} \quad \dots (2)$$

[1 mark]

Step 3: Finding the distance walked by the boy towards the building

$$DE = DF - EF$$

$$= \left(28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} \right) \text{ m} = \frac{85.5 - 28.5}{\sqrt{3}} = 19\sqrt{3} \text{ m}$$

Thus, the distance walked by the boy towards the building is $19\sqrt{3}$ m.

[1 mark]

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2. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the heights of the poles and the distances of the point from the poles.
[4 marks]

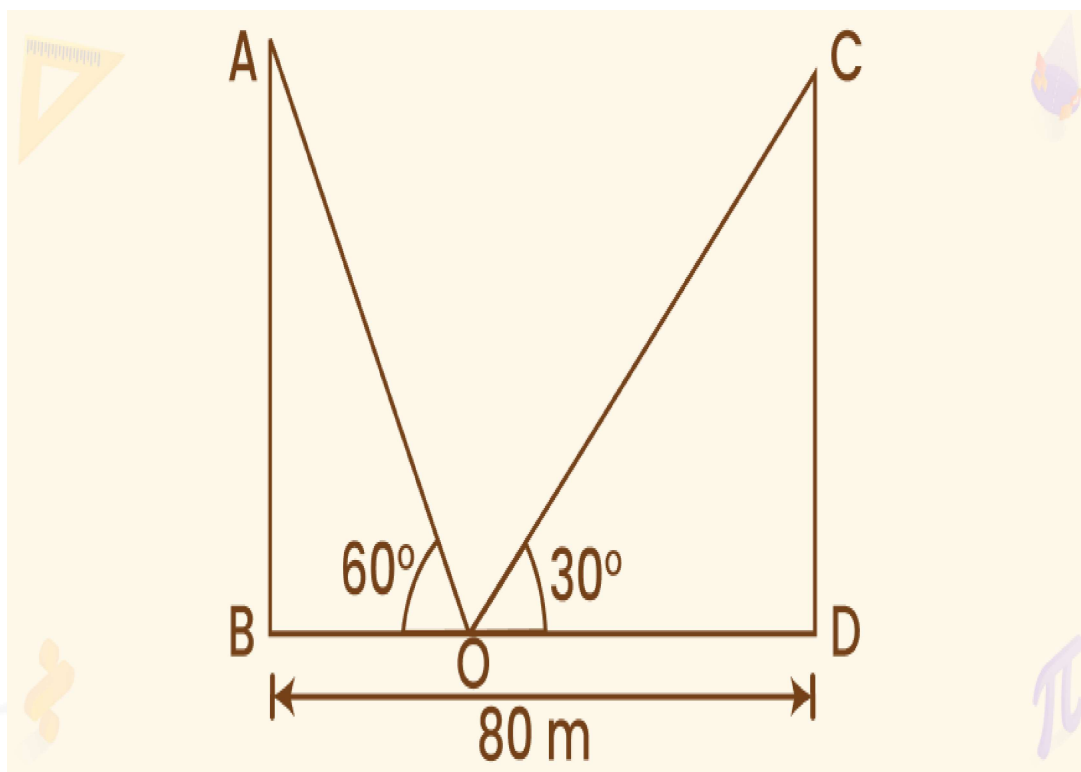
Some Applications of Trigonometry: Application of trigonometry

Given:

Width of road = 80 m.

The heights of the poles are equal.

From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. This can be shown as:



Let BD be 80 m wide road and AB and CD are the two poles of equal heights.

Let the heights of the poles as x meters.

$$\Rightarrow AB = CD = x$$

From figure,

$$BD = BO + OD \dots(1)$$

[1 mark]

Step 1: Finding BO in terms of x .

In $\triangle AOB$,

$$\tan 60^\circ = \frac{AB}{BO}$$

$$\Rightarrow \sqrt{3} = \frac{x}{BO}$$

$$\Rightarrow BO = \frac{x}{\sqrt{3}} \text{ m} \dots(2)$$

[1 mark]

Step 2: Finding DO in terms of x .

In $\triangle COD$,

$$\tan 30^\circ = \frac{CD}{DO}$$

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$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{DO}$$

$$\Rightarrow DO = x\sqrt{3} \text{ m} \dots(3)$$

[1 mark]

Step 3: Replace (2) and (3) in (1).

$$\frac{x}{\sqrt{3}} + x\sqrt{3} = 80 \text{ m}$$

$$\Rightarrow x\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 80 \text{ m}$$

$$\Rightarrow x\left(\frac{4}{\sqrt{3}}\right) = 80 \text{ m}$$

$$\Rightarrow x = 20\sqrt{3} \text{ m}$$

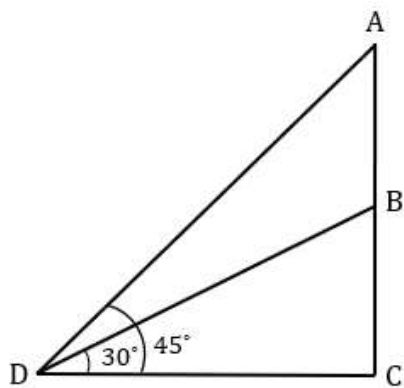
Therefore, height of both the poles = $20\sqrt{3}$ m.

[1 mark]

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3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° , respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)



[4 marks]

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Step 1: Assume the height of tower and height of the flagstaff.

Let the height of tower and the flagstaff be BC and AB, respectively.

[1 Mark]

Step 2: Write the trigonometric ratio for the height of the tower.

The trigonometric ratio for the height of the tower.

In $\triangle BDC$

$$\tan 30^\circ = \frac{BC}{CD}$$

$$\Rightarrow CD = \frac{BC}{\tan 30^\circ}$$

$$\Rightarrow CD = \frac{BC}{\frac{1}{\sqrt{3}}}$$

$$\Rightarrow CD = \sqrt{3}BC \dots (1)$$

[1 Mark]

Step 3: Write the trigonometric ratio for the height of both tower and the flagstaff.

In $\triangle ACD$

$$\tan 45^\circ = \frac{AC}{CD}$$

$$\tan 45^\circ = \frac{AB+BC}{CD}$$

$$\Rightarrow CD \tan 45^\circ = AB + BC$$

$$\Rightarrow CD \times 1 = AB + BC$$

$$\Rightarrow CD = AB + BC$$

$$\Rightarrow CD = 6 + BC \dots (2)$$

Equations obtained:

$$CD = \sqrt{3}BC \dots (1)$$

$$CD = 6 + BC \dots (2)$$

[1 Mark]

Step 4: Find the height of the tower by equating both equations.

$$CD = \sqrt{3}BC$$

$$CD = 6 + BC$$

$$\therefore \sqrt{3}BC = 6 + BC$$

$$\Rightarrow \sqrt{3}BC - BC = 6$$

$$\Rightarrow (\sqrt{3} - 1)BC = 6$$

$$\Rightarrow BC = \frac{6}{(\sqrt{3}-1)}$$

$$\Rightarrow BC = \frac{6}{(1.73-1)}$$

$$\Rightarrow BC = \frac{6}{0.73} = 8.22 \text{ m}$$

So height of the tower is 8.22 m.

[1 Mark]

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4. The lower window of a house is at a height of 2m above the ground and its upper window is 4m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° , respectively. Find the height of the balloon above the ground.

[5 Marks]

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$$\tan 30^\circ = \frac{BQ}{w_1Q}$$

$$\tan 30^\circ = \frac{H-6}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \sqrt{3}(H - 6) \dots\dots (ii)$$

[1 Mark]

From Eq. (i) and (ii),

$$\sqrt{3}(H - 6) = \frac{(H-2)}{\sqrt{3}}$$

$$\Rightarrow 3(H - 6) = H - 2$$

$$\Rightarrow 3H - 18 = H - 2$$

$$\Rightarrow 2H = 16 \Rightarrow H = 8$$

So, the height of the balloon above the ground is 8 m.

[1 Mark]

Some Applications of Trigonometry: Application of trigonometry

5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

[3 Marks]

Let AB be the height of statue.

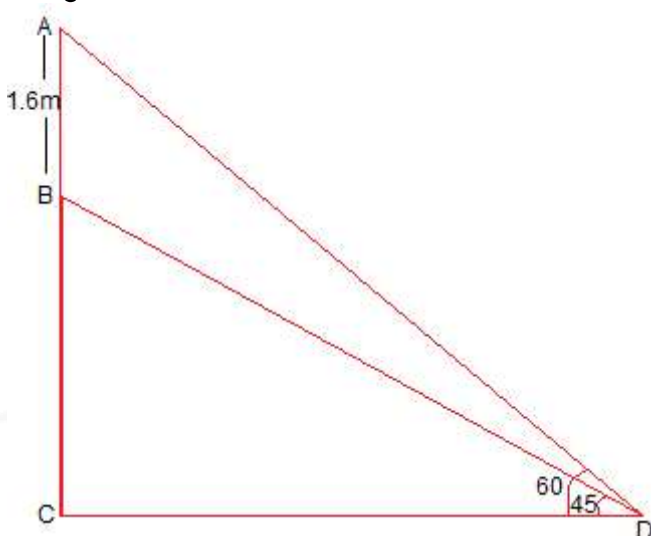
D is the point on the ground from where the elevation is taken.

Height of pedestal = $BC = AC - AB$

[1 Mark]

As per question,

In right $\triangle BCD$,



$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow 1 = \frac{BC}{CD}$$

$$\Rightarrow BC = CD$$

[1 Mark]

Also,

In right $\triangle ACD$,

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{AB+BC}{CD}$$

$$\Rightarrow \sqrt{3}CD = 1.6 + BC$$

$$\Rightarrow \sqrt{3}BC = 1.6 + BC$$

$$\Rightarrow \sqrt{3}BC - BC = 1.6$$

$$\Rightarrow BC(\sqrt{3} - 1) = 1.6$$

$$\Rightarrow BC = \frac{1.6}{(\sqrt{3}-1)}m$$

Rationalising we get,

$$\Rightarrow BC = \frac{1.6}{(\sqrt{3}-1)} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}m$$

$$\Rightarrow BC = 0.8(\sqrt{3} + 1)m$$

Thus, the height of the pedestal is $0.8(\sqrt{3} + 1)m$.

[1 Mark]

Some Applications of Trigonometry: Application of trigonometry

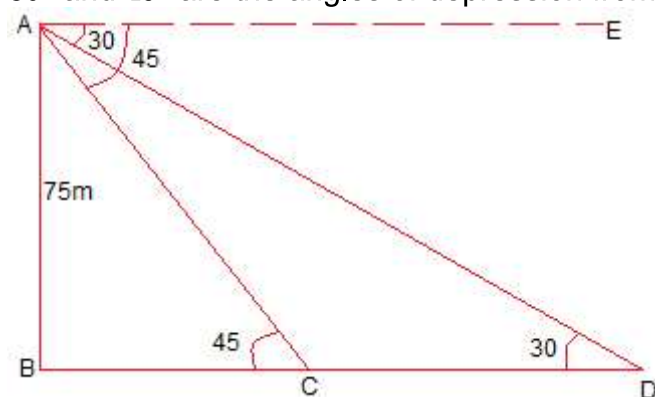
6. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

[3 Marks]

Let AB be the lighthouse of height 75 m.

Let C and D be the positions of the ships.

30° and 45° are the angles of depression from the lighthouse.



As per question,

In right $\triangle ABC$,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{75}{BC}$$

$$\Rightarrow BC = 75m \quad [1 \text{ mark}]$$

Also,

In right $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$$\Rightarrow BD = 75\sqrt{3}m$$

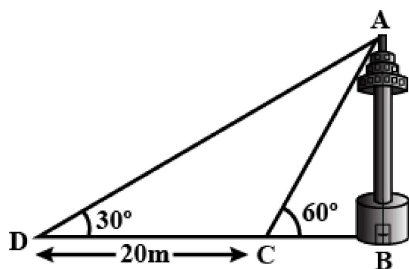
The distance between the two ships

$$= CD = BD - BC = (75\sqrt{3} - 75)m = 75(\sqrt{3} - 1)m.$$

[2 marks]

Some Applications of Trigonometry: Application of trigonometry

7. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal. [3 Marks]



Here, AB is the height of the tower and BC is the width of canal.

CD = 20 m

As per question,

In right $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{(20+BC)}$$

$$\Rightarrow AB = \frac{(20+BC)}{\sqrt{3}} \dots (i) \text{ [1 Mark]}$$

Also,

In right $\triangle ABC$,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$$\Rightarrow AB = \sqrt{3}BC \dots (ii) \text{ [1 Mark]}$$

From equation (i) and (ii)

$$AB = \sqrt{3}BC = \frac{(20+BC)}{\sqrt{3}}$$

$$\Rightarrow 3BC = 20 + BC$$

$$\Rightarrow 2BC = 20 \Rightarrow BC = 10m$$

Putting the value of BC in equation (ii)

$$AB = 10\sqrt{3}m$$

Thus, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m. [1 Mark]

Some Applications of Trigonometry: Application of trigonometry

8. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

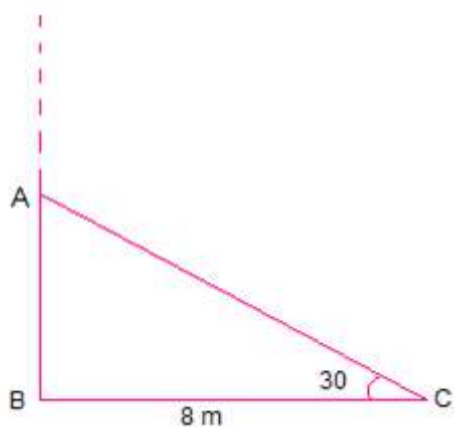
[3 marks]

Let AC be the broken part of the tree.

\therefore Total height of the tree = $AB + AC$

In right $\triangle ABC$,

$$\cos 30^\circ = \frac{BC}{AC} \text{ [1 mark]}$$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$\Rightarrow AC = \frac{16}{\sqrt{3}} \text{ [1 Mark]}$$

$$\text{Also, } \tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\Rightarrow AB = \frac{8}{\sqrt{3}}$$

$$\text{Therefore, the total height of the tree} = AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m.}$$

[1 Mark]