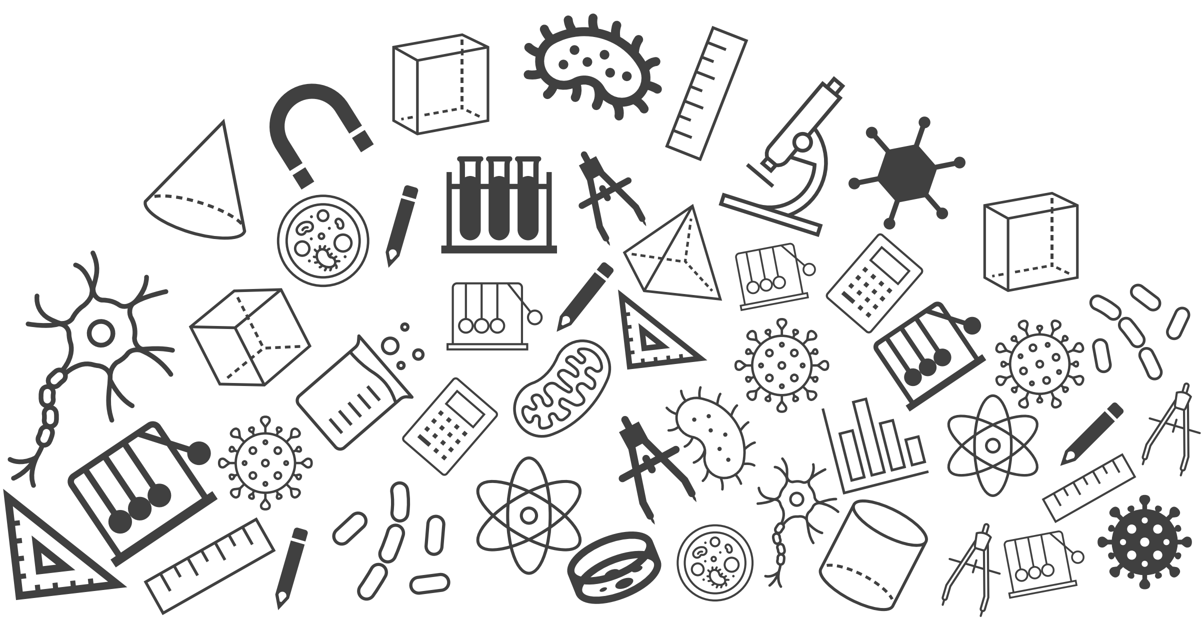
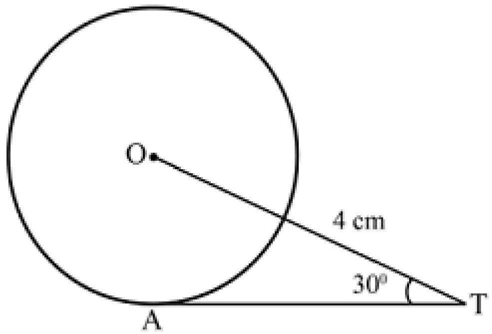
Grade 10
Mathematics
Exam Important Questions



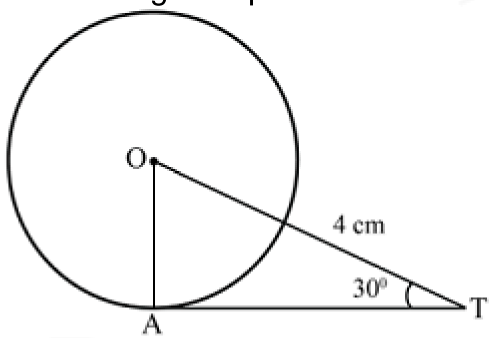
Circles

Topic : Exam Important Questions

1. In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30^\circ$. Then , AT is equal to ? $[2 \text{ Marks}]$



Join OA.
We know that, the tangent at any point of a circle is perpendicular to the radius trough the point of contact.



$\therefore \angle OAT = 90^\circ \quad [1 \text{ Mark}]$

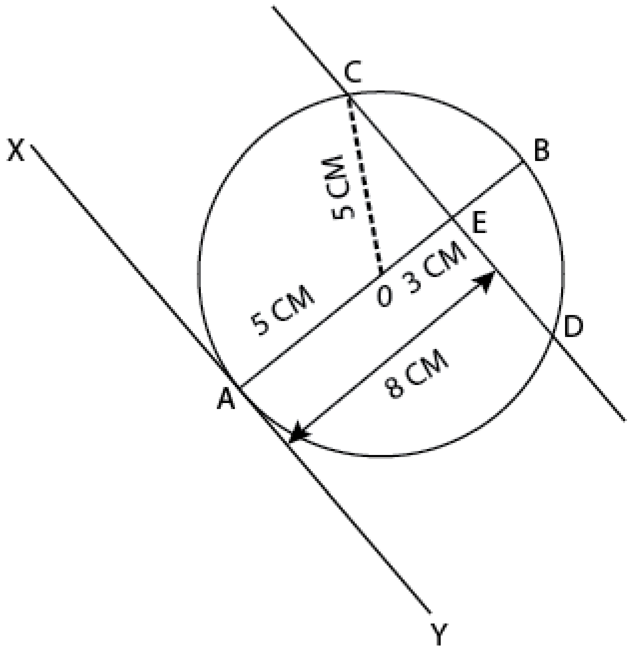
$\text{In } \Delta OAT, \ Cos \ 30^\circ = \frac{AT}{OT}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4}$

$\Rightarrow AT = 2\sqrt{3} \ cm$
$[1 \text{ Mark}]$

2. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. Find the length of the chord CD which is parallel to XY and at a distance of 8 cm from A. [3 marks]

First draw a circle of radius 5 cm having centre O. A tangent XY is drawn at point A



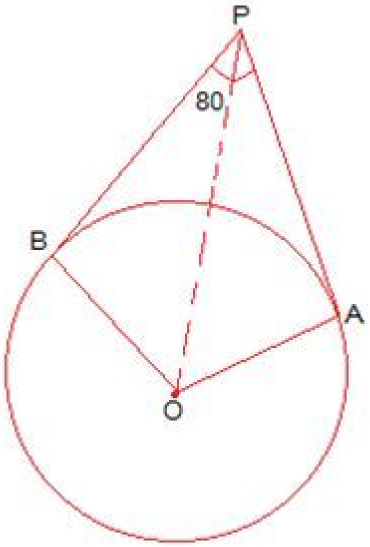
-- (1 Mark)

A Chord CD is drawn which is parallel to XY and at a distance of 8 cm from A.
Now , $\angle OAY = 90^\circ$
[Tangent at any point of a circle is perpendicular to the radius through the point of contact]
$\angle OAY + \angle OED = 180^\circ\ [sum\ of\ cointerior\ angle\ is\ 180^\circ]$
$\Rightarrow\ \angle OED = 90^\circ$

-- (1 Mark)

$Also,\ AE = 8cm.$
$Join\ OC$
$Now,\ in\ right\ angled\ \Delta OEC.,$
$OC^2 = OE^2 + EC^2 [by\ Pythagoras\ theorem]$
$\Rightarrow EC^2 = OC^2 - OE^2$
$= 5^2 - 3^2$
$[\because OC = radius = 5cm, OE = AE - AO = 8 - 5 = 3cm]$
$= 25 - 9 = 16$
$\Rightarrow EC = 4cm$
Hence, length of chord CD = 2 CE = 2 $\times$ 4 = 8 cm
[Since perpendicular from centre to the chord bisects the chord]
-- (1 Mark)

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then find the value of $\angle POA$. [3 marks]



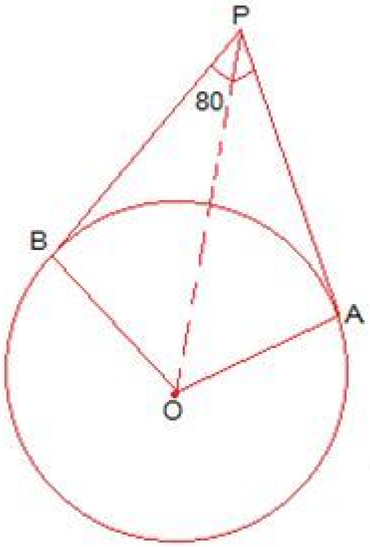
OA and OB are radii of the circle to the tangents PA and PB respectively.
The line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

$\therefore OA \perp PA$ and $OB \perp PB$

$$\angle OBP = \angle OAP = 90^\circ$$

In quadrilateral AOBP,
Sum of all interior angles = 360°

$\angle AOB + \angle OBP + \angle OAP + \angle APB = 360^\circ$



$\Rightarrow \angle AOB + 90^\circ + 90^\circ + 80^\circ = 360^\circ$

$\Rightarrow \angle AOB = 100^\circ$

-- (1 Mark)

Now,
In Δ OPB and Δ OPA,
AP = BP (Tangents from a point are equal)
OA = OB (Radii of the circle)
OP = OP (Common side)
$\therefore OPB \cong \Delta OPA$ (by SSS congruence condition)
-- (1 Mark)

Thus, $\angle POB = \angle POA$

$\angle AOB = \angle POB + \angle POA$

$\Rightarrow 2\angle POA = \angle AOB$

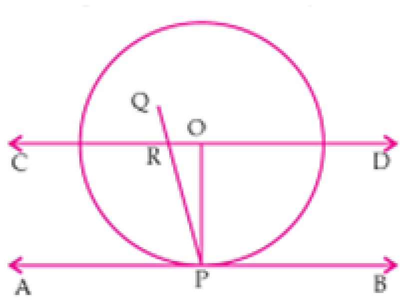
$\Rightarrow \angle POA = 100^\circ/2 = 50^\circ$

$\angle POA = 100^\circ/2 = 50^\circ$

-- (1 Mark)

4. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

$[3\ Marks]$



Let AB be the tangent to the circle at point P with centre O.
We have to prove that PQ passes through the point O.
Suppose that PQ doesn't pass through point O. Join OP.
Through O, draw a straight line CD parallel to the tangent AB.
PQ intersect CD at R and also intersect AB at P.
As, CD || AB, PQ is the line of intersection,
∠ ORP = ∠ RPA (Alternate interior angles)

$[1\ Mark]$

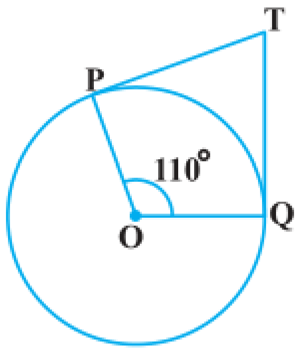
but also,
$\angle RPA = 90^\circ (PQ \perp AB)$
$\Rightarrow \angle ORP = 90^\circ$
$\angle ROP + \angle OPA = 180^\circ$ (Co-interior angles)
$\Rightarrow \angle ROP + 90^\circ = 180^\circ$
$\Rightarrow \angle ROP = 90^\circ$

$[1\ Mark]$

Thus, the Δ ORP has 2 right angles i.e., ∠ ORP and ∠ ROP which is not possible.
Hence, our supposition is wrong.
∴ PQ passes through the point O.

$[1\ Mark]$

5. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle \mathrm{POQ} = 110^\circ$, $\angle \mathrm{PTQ}$ is equal to



$[3\ Marks]$

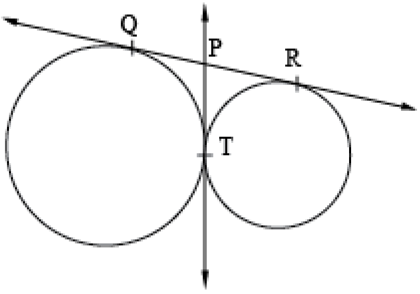
OP and OQ are radii of the circle to the tangents TP and TQ respectively.
Since, the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.
$[1\ Mark]$

$\therefore$ OP $\perp$ TP and OQ $\perp$ TQ.
$\angle OPT = \angle OQT = 90^\circ$

In quadrilateral POQT,
Sum of all interior angles = 360°
$\angle PTQ + \angle OPT + \angle POQ + \angle OQT = 360^\circ$
[1 Mark]
$\Rightarrow \angle PTQ + 90^\circ + 110^\circ + 90^\circ = 360^\circ$
$\Rightarrow \angle PTQ = 70^\circ$

$\angle PTQ$ is equal to 70° [1 Mark]

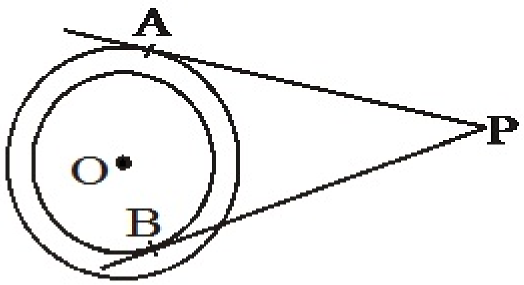
6. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm then the length of QR is



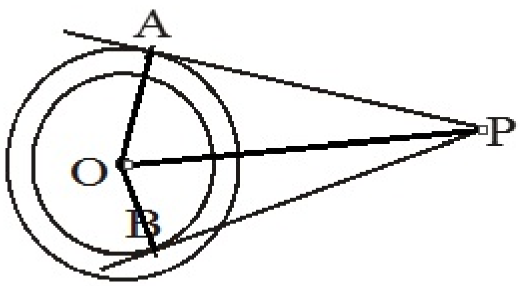
[2 Marks]

Lengths of the tangents drawn from an external point to a circle are equal.
∴ QP = PT = 3.8 cm .... (i) (0.5 marks)
PR = PT = 3.8 cm .... (ii) (0.5 marks)
From equations (i) and (ii):
QP = PR = 3.8 cm
But, QR = QP + PR
∴ QR = 3.8 + 3.8
QR = 7.6 cm (1 mark)

7. In the given figure, O is the center of each one of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to outer and inner circle respectively. If PA = 10 cm, find the length of PB(up to two places of decimal). [2 marks]



Join AO, BO, PO



Given that AO = 6 cm, PA = 10 cm, OB = 4 cm

Applying Pythagoras theorem to △OAP

$PO^2 = AO^2 + PA^2$

$PO^2 = 36 + 100$

$PO^2 = 136$ (1 mark)

Applying Pythagoras theorem to ΔOBP

$PB^2 = PO^2 - BO^2$

$PB^2 = 136 - 4^2$

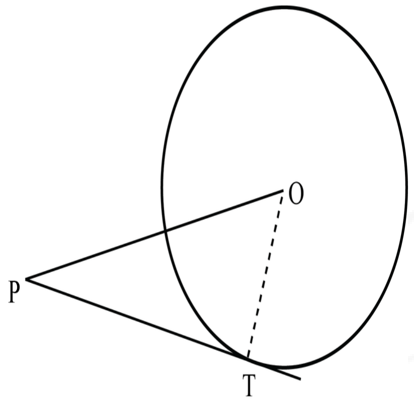
$PB^2 = 120$

PB= 10.95 (1 mark)

8. Write ‘True’ or ‘False’ and justify your answer in each of the following :
(i) The length of tangents from an external point P on a circle is always greater than the radius of the circle.
(ii) The length of tangents from an external point P on a circle with centre O is always less than OP.
(2 marks)

(i) False
Because the length of tangents from an external point P on a circle may or may not be greater than the radius of the circle.
(1 mark)

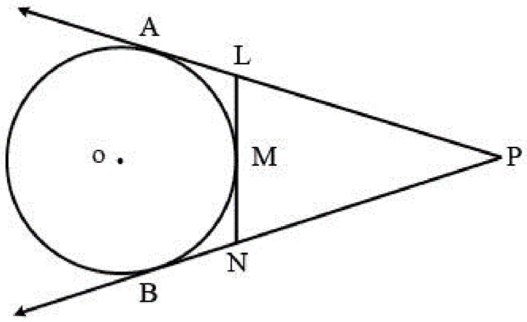
(ii) True



PT is a tangents drawn from external point P . Join OT
$\therefore OT \perp PT$
So, OPT is a right angled triangle formed
In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.
$\therefore$ OP > PT
Or PT < OP (1 mark)

Circles

9. In the given figure, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at M. Prove that PL + ML = PN + MN. [2 Marks]



PA = PB [*the tangents drawn from an external point* to a circle *are equal* ----(1) (1 mark)

PL + AL = PN + BN

AL = ML & BN = MN [same as (1)] (0.5 mark)

From above steps,
Thus, PL + ML = PN + MN (0.5 mark)
Hence, proved.

10. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that ΔAPB is equilateral. [3 Marks]

∠OAP = 90° (PA and PB are the tangents to the circle.)

In ΔOPA,
sin ∠OPA = $\frac{OA}{OP} = \frac{r}{2r}$ [OP is the diameter = 2*radius]

sin ∠OPA = $\frac{1}{2}$ = sin 30º

⇒ ∠OPA = 30°
[1 Mark]

Similarly, ∠OPB = 30°.
∠APB = ∠OPA + ∠OPB = 30° + 30° = 60°
[0.5 Marks]

In ΔPAB,
PA = PB (tangents from an external point to the circle)
⇒ ∠PAB = ∠PBA ....(1) (angles opp. to equal sides are equal)

[0.5 Marks]

⇒ ∠PAB + ∠PBA + ∠APB = 180° [Angle sum property]
⇒ ∠PAB + ∠PAB = 180° – 60° = 120° [Using (1)]
⇒ 2∠PAB = 120°
⇒ ∠PAB = 60° ....(2)
[0.5 Marks]

From (1) and (2)
∠PAB = ∠PBA = ∠APB = 60° (all angles are equal in an equilateral triangle)
[0.5 Marks]

ΔPAB is an equilateral triangle.