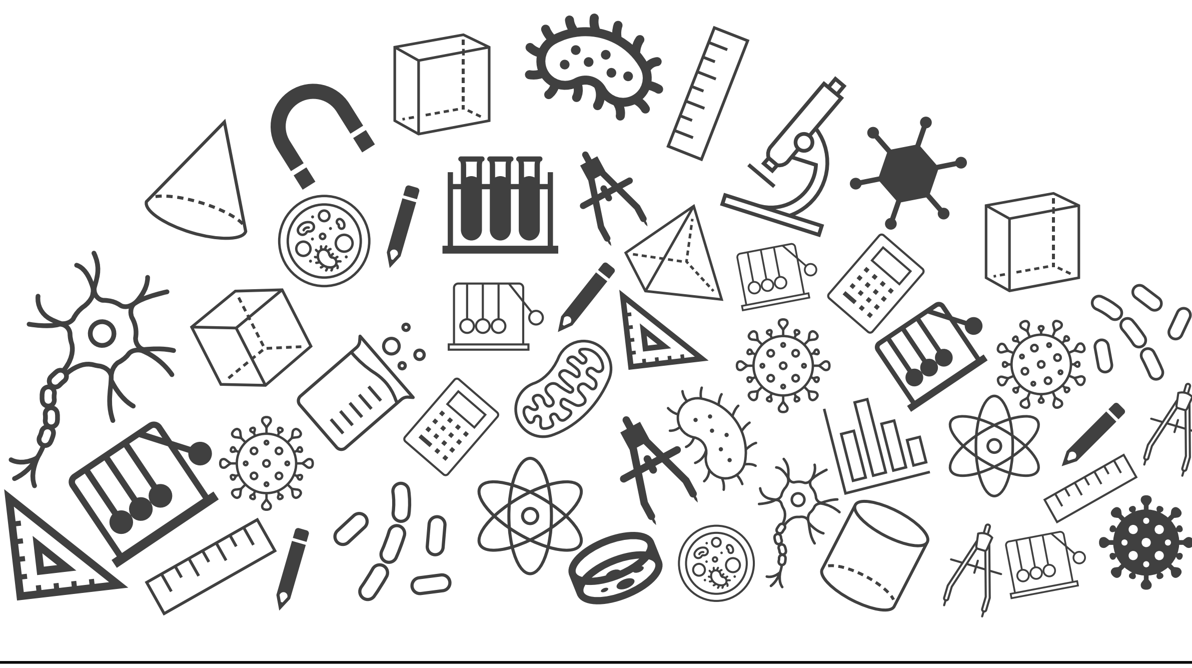
Grade 10

Mathematics

Exam Important Questions



Topic : Exam Important Questions

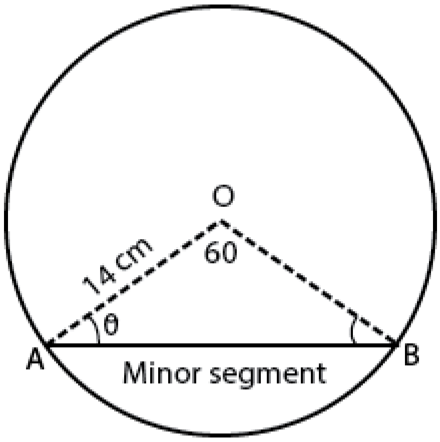
1. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.
$(Use\ \pi = 3.14)$
[2 Marks]

Distance over which light spread i.e. radius, r = 16.5 km
Angle made by the sector $= 80^\circ$
Area of the sector making angle θ
$= (\frac{\theta}{360^\circ}) \times \pi r^2$
[0.5 Marks]

Area of the sea over which the ships are warned
= Area of the sector
$= (\frac{80^\circ}{360^\circ}) \times \pi r^2\ km^2$
[0.5 Marks]

$= \frac{2}{9} \times 3.14 \times (16.5)^2\ km^2$
$= 189.97\ km^2$
[1 Mark]
[Area of sector of a circle]

2. Find the area of the minor segment of a circle of radius 14 cm , when the angle of the corresponding sector is 60°.
[3 marks]



Given that , radius of circle (r) = 14 cm

Angle of the corresponding sector. i.e central angle
$(\theta) = 60^\circ$

$\because$ In Δ AOB
$OA = OB = Radius\ of\ circle$
$\therefore$ Δ AOB is isosceles.

$\Rightarrow \angle OAB = \angle OBA = \theta$ (0.5 marks)

Now, in Δ OAB
$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

[Since , sum of interior angles of any triangle is 180°]

$\Rightarrow 60^\circ + \theta + \theta = 180^\circ \quad [given, \angle AOB = 60^\circ]$

$\Rightarrow 2\theta = 120^\circ$

$\Rightarrow \theta = 60^\circ$ (0.5 marks)

i.e $\angle AOB = \angle OBA = 60^\circ = \angle AOB$

Since, all angles of ΔAOB are equal to 60°. So, ΔAOB is an equilateral triangle.

Also, area of ΔAOB
$= \frac{\sqrt{3}}{4} \times (14)^2$
$[\because Area\ of\ an\ equilateral\ triangle = \frac{\sqrt{3}}{4}(side)^2]$

$= \frac{\sqrt{3}}{4} \times 196$
$= 49\sqrt{3}\ cm^2$ (0.5 marks)

Area of sector OBAO
$= \frac{\pi r^2}{360^\circ} \times \theta$
$= \frac{22}{7} \times \frac{14 \times 14}{360} \times 60^\circ$
$= \frac{22 \times 2 \times 14}{6}$
$= \frac{22 \times 14}{3}$
$= \frac{308}{3} cm^2$ (1 mark)

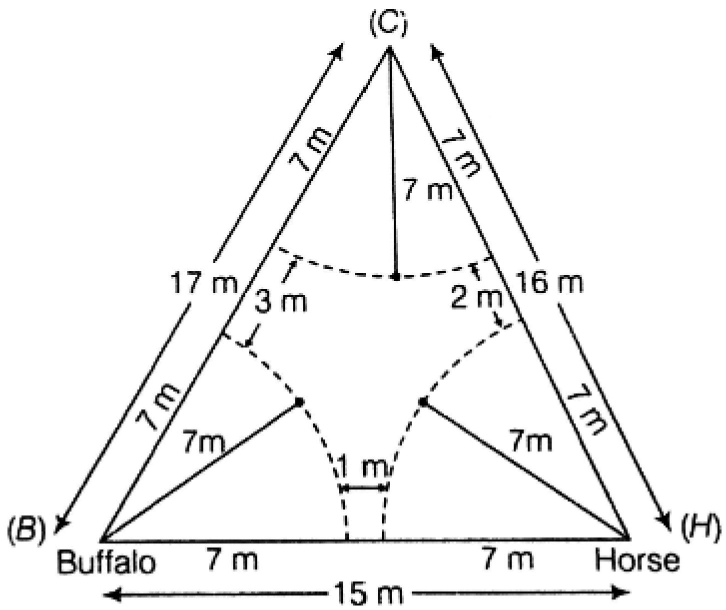
$\therefore$ The area of the minor segment
= Area of sector OBAO - Area of the equilateral triangle
$= (\frac{308}{3} - 49\sqrt{3})\ cm^2$ (0.5 marks)
[Area of segment of a circle]

3. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7m each to graze in the field.
Find the area of the field which cannot be grazed by the three animals.
[5 Marks]

Solution:

Given that, a triangular field with the three corners of the filed a cow, a buffalo an a horse are tied separately with ropes. So, each animal grazed the filed in each corner of triangular field as a sectorial form.

Radius of each sector = (r)=7m



[1 mark]

Now, area of sector with $\angle C$
$= \frac{\angle C}{360^\circ} \times \pi r^2 = \frac{\angle C}{360^\circ} \times \pi \times (7)^2\ m^2$

Area of the sector with $\angle B$
$= \frac{\angle B}{360^\circ} \times \pi r^2 = \frac{\angle B}{360^\circ} \times \pi \times (7)^2\ m^2$

And area of the sector with $\angle H$
$= \frac{\angle H}{360^\circ} \times \pi r^2 = \frac{\angle H}{360^\circ} \times \pi \times (7)^2\ m^2$
[1 Mark]
Therefore, sum of the areas (in cm^2) of the three sectors.

$\angle C = \frac{\angle C}{360^\circ} \times \pi \times (7)^2 + \frac{\angle B}{360^\circ} \times \pi \times (7)^2 + \frac{\angle H}{360^\circ} \times \pi \times (7)^2$

$= \frac{\angle C + \angle B + \angle H}{360^\circ} \times \pi \times 49$

$= \frac{180^\circ}{360^\circ} \times \frac{22}{7} \times 49 = 11 \times 7 = 77 cm^2$
[1 Mark]
Given that, sides of triangle are a = 15, b = 16 and c = 17

Now, semi-perimeter of triangle $S = \frac{a+b+c}{2}$

$\Rightarrow = \frac{15+16+17}{2} = \frac{48}{2} = 24$

$\therefore\ Area\ of\ trianglular f\ ield = \sqrt{s(s-a)(s-b)(s-c)}$ [by Heron's formula]

$= \sqrt{24 \times 9 \times 8 \times 7}$

$= \sqrt{64 \times 9 \times 21}$

$= 8 \times 3\sqrt{21} = 24\sqrt{21}\, m^2$
[1 Mark]
So, area of the field which cannot be grazed by the three animals.

=Area of triangular field - Area of each sectorial field

$= 24\sqrt{21} - 77m^2$

Hence, the required area of the filed which can not be grazed by the three animals is $(24\sqrt{21} - 77)m^2$.
[1 Mark]

4. The area of an equilateral triangle ABC is $17320.5\ cm^2$. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region.
$(Use\ \pi = 3.14\ and\ \sqrt{3} = 1.73205)$

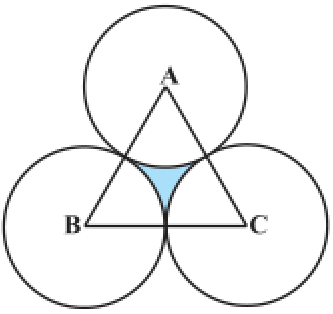


Fig. 12.28

(5 Marks)

ABC is an equilateral triangle.
$\therefore \angle A = \angle B = \angle C = 60^\circ$
There are three sectors each making 60°.

Area of equilateral Δ
$= \frac{\sqrt{3}}{4} \times (side)^2$

$Area\ of\ \Delta ABC = 17320.5\ cm^2$
$\Rightarrow \frac{\sqrt{3}}{4} \times (side)^2 = 17320.5$ [2 Mark]
$\Rightarrow (side)^2 = 17320.5 \times \frac{4}{1.73205}$
$\Rightarrow (side)^2 = 4 \times 10^4$
$\Rightarrow side = 200\ cm$

Radius of the circles
$= \frac{200}{2} cm = 100\ cm$

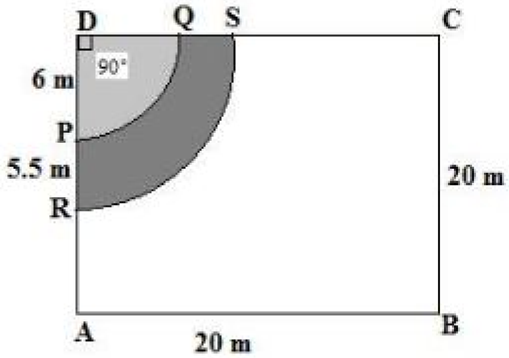
Area of the sector making angle θ
$= (\frac{\theta}{360^\circ}) \times \pi r^2$

Area of the sector
$= (\frac{60^\circ}{360^\circ}) \times \pi r^2\ cm^2$
$= \frac{1}{6} \times 3.14 \times (100)^2\ cm^2$
$= \frac{15700}{3} cm^2$ [1 Mark]

Area of 3 sectors
$= 3 \times \frac{15700}{3} = 15700\ cm^2$ [1 Mark]

Area of the shaded region = Area of equilateral triangle ABC - Area of 3 sectors
$= 17320.5\ cm^2 - 15700\ cm^2 = 1620.5\ cm^2$ [1 Mark]

5. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze.
(3 Marks)



Given: A calf is tied with a rope of length 6m at the corner of a square grassy lawn of side 20m.

So, radius of quadrant DPQD (*r*) = length of rope = 6 m

Therefore,

$$Area\ of\ sector\ DPQD = \frac{\pi r^2 \theta}{360^0}$$

$$= \frac{3.14 \times (6)^2 \times 90^0}{360^0}$$

$$= 0.785 \times 36 = 28.26 m^2$$
$$(1\ Mark)$$

Now, if the length of the rope is increased by 5.5 m

So, total length of the rope (R) = 6 + 5.5 = 11.5 m

$$Area\ of\ sector\ DRSD = \frac{\pi R^2 \theta}{360^0}$$

$$= \frac{3.14 \times (11.5)^2 \times 90^0}{360^0}$$

$$= 0.785 \times 132.25 = 103.81625 m^2$$

$$(1\ Mark)$$

Therefore,

Increased area = Area of sector DRSD – Area of sector DPQD

= 103.81625 – 28.26

= 75.55625 m^2

= 75.56 m^2

Hence, the increase in area of the grassy lawn is 75.56 m^2.
$(1\ Mark)$

6. A sector is cut from a circle of radius 21 cm. The angle of the sector is 150°. Find the length of the arc and the area of the sector.

[2 Marks]

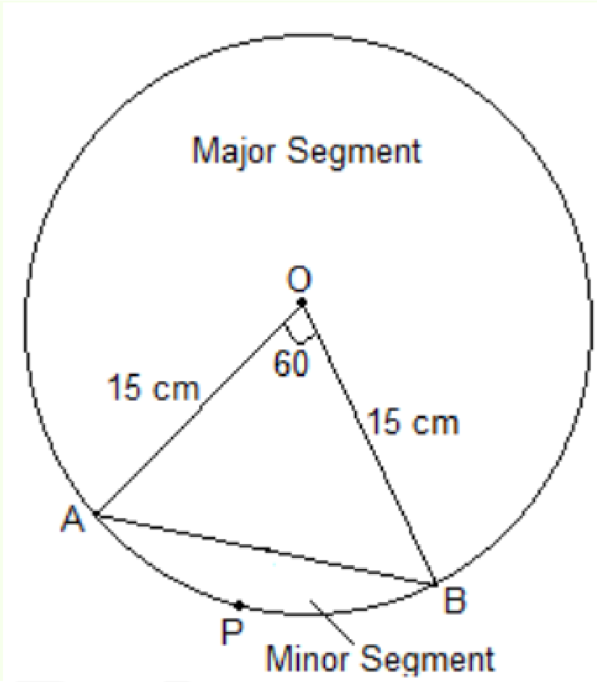
Solution:

Length of the arc:
$= 2\pi r \times \frac{\theta}{360^o}$
$= 2 \times \frac{22}{7} \times 21 \times \frac{150^o}{360^o}$
$= 55\ cm$
[1 Mark]

Area of the sector:
$= \pi r^2 \times \frac{\theta}{360^o}$
$= \frac{22}{7} \times 21 \times 21 \times \frac{150^o}{360^o}$
$= 577.5\ cm^2$

[1 Mark]

7. In the figure given, the radius of the circle is 15 cm. The angle subtended by the chord AB at the centre O is 60°. Find the area of the major and minor segments. [3 marks]



Radius of the circle = 15 cm
ΔAOB is an isosceles triangle as two sides are equal.
$\therefore \angle A = \angle B$
Now, sum of all angles of triangle $= 180^\circ$
$\Rightarrow \angle A + \angle B + \angle O = 180^\circ$
$\Rightarrow 2\angle A = 180^\circ - 60^\circ$
$\Rightarrow \angle A = \frac{120^\circ}{2}$
$\Rightarrow \angle A = 60^\circ$ (0.5 marks)

Hence, the triangle is equilateral as $\angle A = \angle B = \angle C = 60^\circ$
$\therefore OA = OB = AB = 15\ cm$
Area of equilateral $\Delta AOB = \frac{\sqrt{3}}{4} \times (OA)^2$
$= \frac{\sqrt{3}}{4} \times 15^2$
$= (225\frac{\sqrt{3}}{4})\ cm^2 = 97.3\ cm^2$ (0.5 marks)

Angle subtended at the centre by minor segment $= 60^\circ$
Area of the sector making angle $\theta = (\frac{\theta}{360^\circ}) \times \pi r^2$
Area of minor sector making angle $60^\circ = (\frac{60^\circ}{360^\circ}) \times \pi r^2\ cm^2 =$
$(\frac{1}{6}) \times 15^2 \pi\ cm^2 = \frac{225}{6}\pi\ cm^2 = (\frac{225}{6}) \times 3.14\ cm^2 = 117.75\ cm^2$ (0.5 marks)

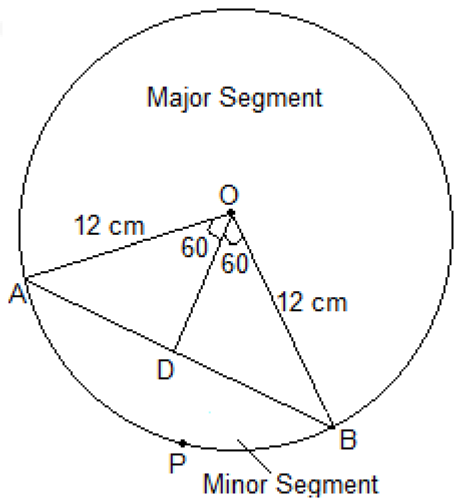
Area of the minor segment= Area of minor sector - Area of equilateral ΔAOB
$= 117.75\ cm^2 - 97.3\ cm^2 = 20.4\ cm^2$ (0.5 marks)

Angle made by major sector $= 360^\circ - 60^\circ = 300^\circ$
Area of the sector making angle 300°
$= (\frac{300^\circ}{360^\circ}) \times \pi r^2\ cm^2$
$= (\frac{5}{6}) \times 15^2 \pi\ cm^2 = \frac{1125}{6}\pi\ cm^2$
$= (\frac{1125}{6}) \times 3.14 cm^2 = 588.75\ cm^2$ (0.5 marks)

Area of major segment
= Area of major sector + Area of equilateral ΔAOB
$= 588.75\ cm^2 + 97.3\ cm^2 = 686.05\ cm^2$
(0.5 marks)
[Area of segment of a circle]

8. A chord of a circle of radius 12 cm subtends an angle of 120° at the center. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14 \; and \; \sqrt{3} = 1.73$)

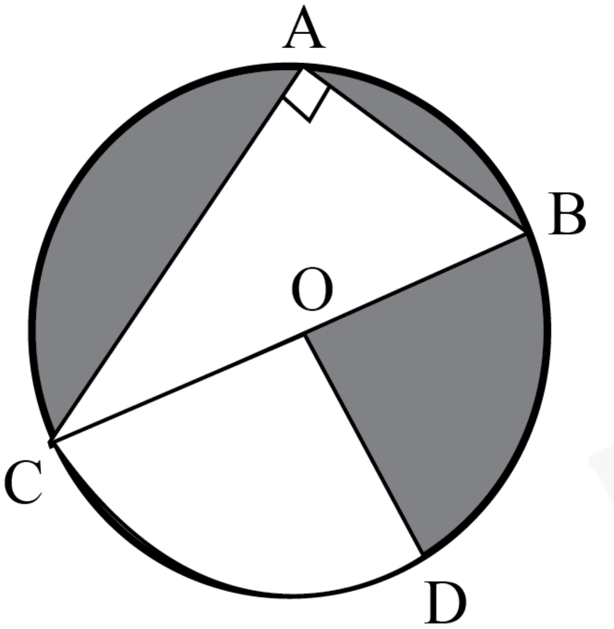
Radius of the circle, r = 12 cm
Draw a perpendicular OD to chord AB. It will bisect AB.
$\angle A = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$
$cos \; \theta^\circ = \frac{Base}{Hypotenuse}$
$cos \; 30^\circ = \frac{AD}{OA}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{12}$
$\Rightarrow AD = 6\sqrt{3} \; cm$ (0.5 marks)
$\Rightarrow AB = 2 \times AD = 12\sqrt{3} \; cm$
$sin \; 30^\circ = \frac{OD}{OA}$
$\Rightarrow \frac{1}{2} = \frac{OD}{12}$
$\Rightarrow OD = 6 \; cm$ (0.5 marks)
$Area \; of \; \Delta AOB = \frac{1}{2} \times base \times height$
$= \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3} \; cm$
$= 36 \times 1.73 = 62.28 \; cm^2$ (0.5 marks)



Angle made by Minor sector $= 120^\circ$
Area of the sector making angle θ
$= (\frac{\theta}{360^\circ}) \times \pi r^2$
Area of the sector making angle 120°
$= (\frac{120^\circ}{360^\circ}) \times \pi r^2 \; cm^2$ (0.5 marks)

$= (\frac{1}{3}) \times 12^2 \pi cm^2 = \frac{144}{3}\pi \; cm^2$
$= 48 \times 3.14 \; cm^2 = 150.72 \; cm^2$ (0.5 marks)
$\therefore$ Area of the corresponding Minor segment = Area of the Minor sector - Area of ΔAOB
$= 150.72 \; cm^2 - 62.28 \; cm^2$
$= 88.44 \; cm^2$ (0.5 marks)

9. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\triangle$ BOD = 90°. Find the area of shaded region. [Use π = 3.14]



(3 Marks)

In right triangle ABC

$BC^2 = AB^2 + AC^2$
$= 7^2 + 24^2$
$= 49 + 576$
$= 625$
$\therefore, BC^2 = 625$
$\Rightarrow BC = 25$
(1 mark)

Now,$\angle COD + \angle BOD = 180^o$ (Linear pair angles)

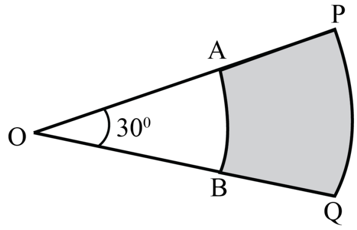
$\Rightarrow \angle COD = 180^o - 90^o = 90^o$
(1 mark)

Now, the area of the shaded region = Area of the sector having the central angle $360^o - 90^o$ – Area of triangle ABC

$= \frac{270^o}{360^o} \times \pi (\frac{BC}{2})^2 - \frac{1}{2} \times AB \times AC$
$= \frac{3}{4} \times 3.14 (\frac{25}{2})^2 - \frac{1}{2} \times 7 \times 24$
$= 367.97 - 84 = 283.97 \ cm^2$
(1 mark)

Hence, the area of the shaded region is $283.97 \ cm^2$

10. In the given figure, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm with centre O. If $\angle POQ = 30^{\circ}$, find the area of the shaded region.



[3 Marks]

Solution:

$\angle$AOB = 30°
Smaller Radius OC (r) = 7 cm
Bigger Radius OB(R)= 3.5cm

Angle made by sectors of both concentric circles $\theta = 30^{\circ}$

Area of the larger sector AOB =$\frac{30^{\circ}}{360^{\circ}} \times \pi R^2 cm^2$

= $\frac{1}{12} \times \frac{22}{7} \times 7^2$

= $12.83 cm^2$
[1 Mark]

Area of the smaller sector COD=$\frac{30^{\circ}}{360^{\circ}} \times \pi r^2 cm^2$

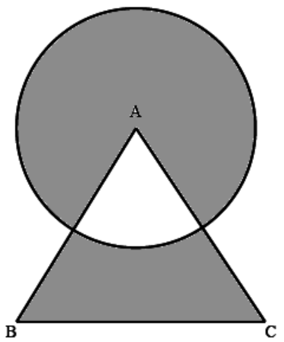
= $\frac{1}{12} \times \frac{22}{7} \times (3.5)^2 cm^2$

= $3.20 cm^2$
[1 Mark]

Area of shaded region= area of sector AOB - area of sector COD

Area of the shaded region = $69.621 cm^2$
[1 Mark]

11. Find the area of a shaded region in the Fig. where a circular arc of radius 7 cm has been drawn with vertex A of an equilateral triangle ABC of side 14 cm as centre. (Use $pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$)



[2 Marks]

Solution:

In equilateral traingle all the angles are of 60°
∴ ∠BAC = 60°
Area of the shaded region = (Area of triangle ABC − Area of sector having central angle 60°) + Area of sector having central angle (360° − 60°)
[1 Mark]

$= \frac{3^{\frac{1}{2}}}{4} \times AB^2 - \frac{60^0}{360} \times \frac{\pi}{7^2} + \frac{300^0}{360} \times \frac{\pi}{7^2}$

$= \frac{3^{\frac{1}{2}}}{4} \times 14^2 - \frac{1}{6} \times \frac{22}{7 \times 7^2} + \frac{5}{6} \times \frac{22}{7 \times 7^2}$

$= 84.77 - 25.67 + 128.35$

=187.45 cm^2

Hence, the area of shaded region is 187.45 cm^2
[1 Mark]