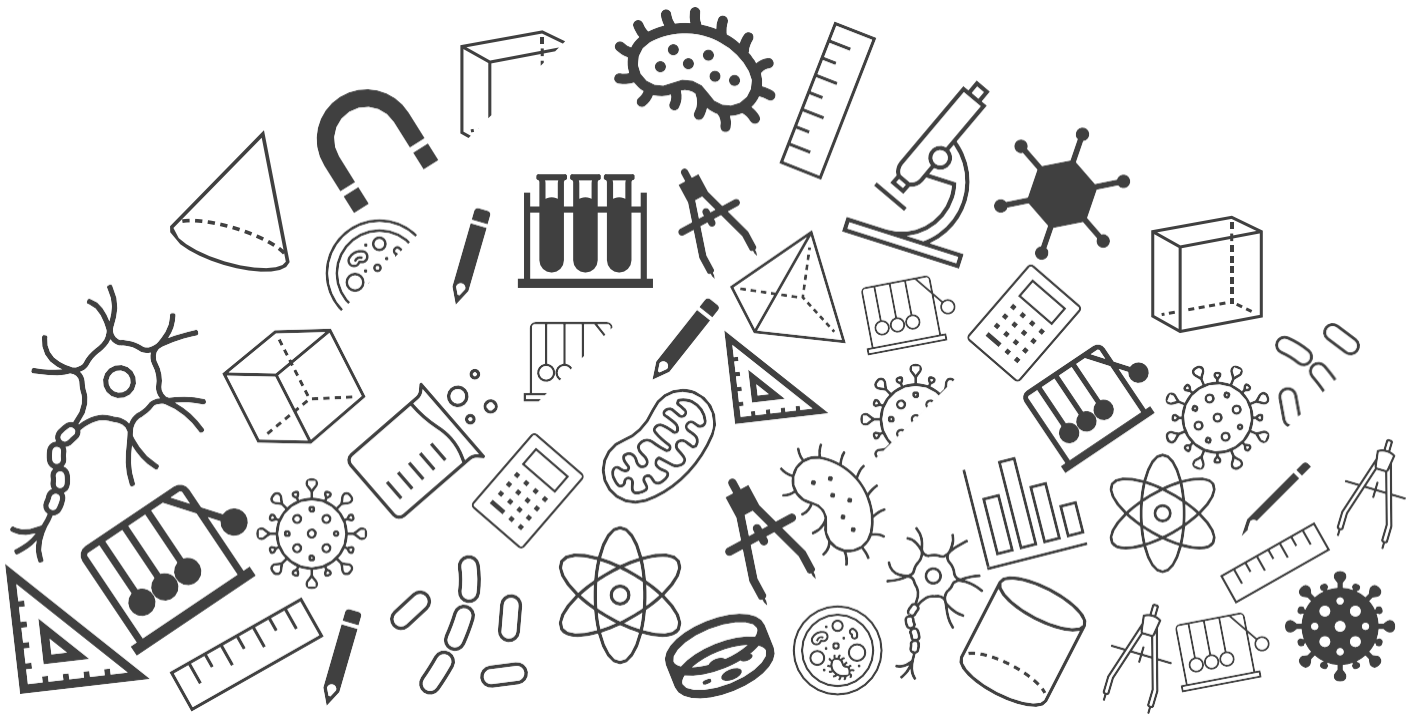
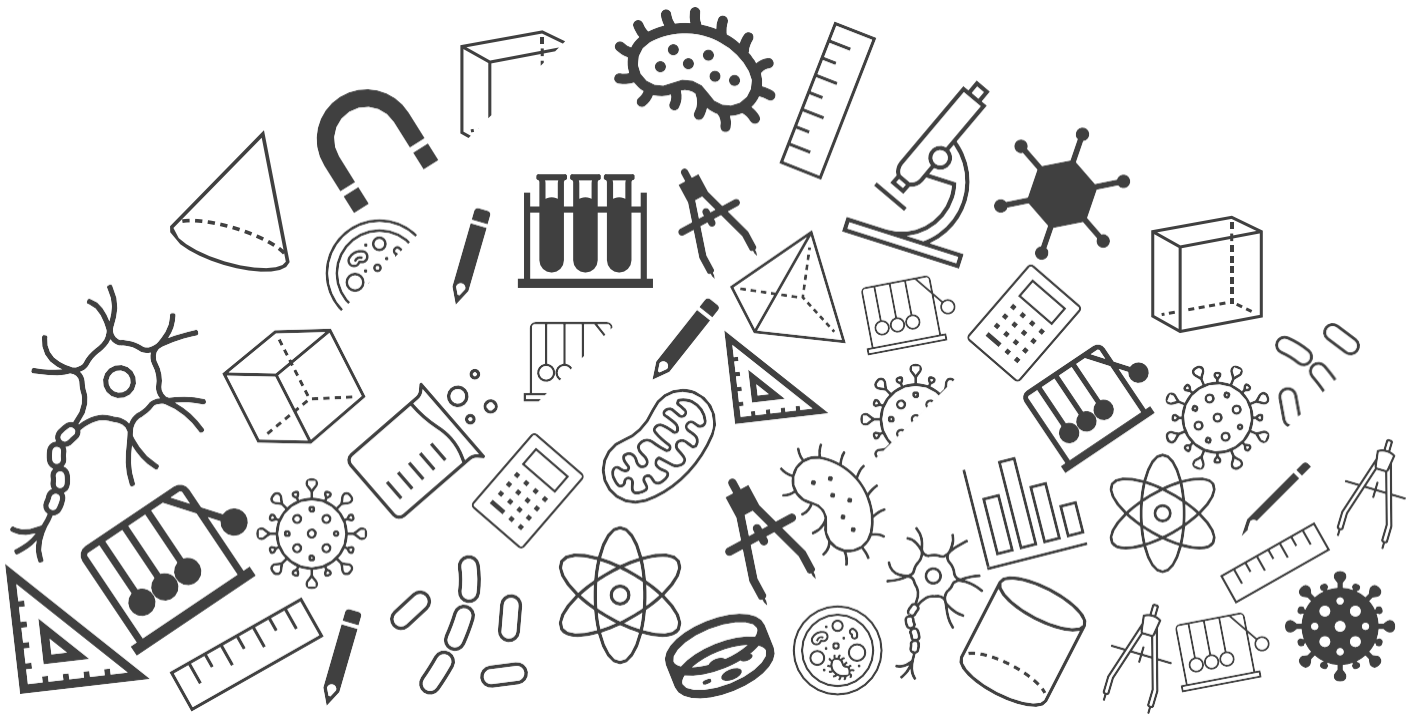
Grade 08: Maths
Exam Important Questions



Squares and Square Roots



Squares and Square Roots

1. (i) If the number at the unit's place is 2 or 8, what will be the unit's place of the square of that number?

   (ii) What is the square of 12 and find its unit place?

   [2 marks]

   (i) By rule, we know that:
   The square numbers end in $0, 1, 4, 5, 6$ or 9 only.
   and,
   If a number ends with 2 or 8, its square ends with 4.
   [1 mark]

   (ii) $12^2 = 144$
   The number at its unit place is 4.
   [1 mark]

2. Express the following as the sum of two consecutive integers.
$(i)\ 15^2$
$(ii)\ 19^2$
[3 marks]

By the rule we know that:

The sum of two consecutive integers $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ is equals to n^2.

[1 mark]
Therefore, according to the question,
$i)$ For 15^2 :

$$\frac{n^2-1}{2}=\frac{15^2-1}{2}=\frac{225-1}{2}=\frac{224}{2}=112$$

and

$$\frac{n^2+1}{2}=\frac{15^2+1}{2}=\frac{225+1}{2}=\frac{226}{2}=113$$

Thus, $15^2=112+113=225$.
[1 mark]
Similarly,
$ii)$ For 19^2 :

$$\frac{n^2-1}{2}=\frac{19^2-1}{2}=\frac{361-1}{2}=\frac{360}{2}=180$$

and

$$\frac{n^2+1}{2}=\frac{19^2+1}{2}=\frac{361+1}{2}=\frac{362}{2}=181$$

Thus, $19^2=180+181=361$.
[1 mark]

3. Find the square root of 625 by using repeated subtraction method.
[3 marks]

Finding square root of 625:
Now, let's find the square root of 625.
$625 - 1 = 624 - - - - - (1)$
$624 - 3 = 621 - - - - - (2)$
$621 - 5 = 616 - - - - - (3)$
$616 - 7 = 609 - - - - - (4)$
$609 - 9 = 600 - - - - - (5)$
$600 - 11 = 589 - - - - - (6)$
$589 - 13 = 576 - - - - - (7)$
$576 - 15 = 561 - - - - - (8)$
$561 - 17 = 544 - - - - - (9)$
$544 - 19 = 525 - - - - - (10)$
$525 - 21 = 504 - - - - - (11)$
$504 - 23 = 481 - - - - - (12)$
$481 - 25 = 456 - - - - - (13)$
$456 - 27 = 429 - - - - - (14)$
$429 - 29 = 400 - - - - - (15)$
$400 - 31 = 369 - - - - - (16)$
$369 - 33 = 336 - - - - - (17)$
$336 - 35 = 301 - - - - - (18)$
$301 - 37 = 264 - - - - - (19)$
$264 - 39 = 225 - - - - - (20)$
$225 - 41 = 184 - - - - - (21)$
$184 - 43 = 141 - - - - - (22)$
$141 - 45 = 96 - - - - - (23)$
$96 - 47 = 49 - - - - - (24)$
$49 - 49 = 0 - - - - - (25)$
[2.5 marks]

Here we performed repeated subtraction 25 times.
Therefore, $\sqrt{625} = 25$
[0.5 mark]

4. If $(15, a, b)$ form a pythagorean triplet, then find (a, b)?
[4 marks]

Solution:

For an integer $m > 1$ the numbers $m^2 - 1, 2m$ and $m^2 + 1$ form a Pythagorean triplet.
[0.5 marks]

$\therefore (m^2 - 1, 2m, m^2 + 1) = (15, a, b)$
[0.5 marks]

$\Rightarrow m^2 - 1 = 15$

$\Rightarrow m^2 = 16 = 4^2$

$\Rightarrow m = 4$
[1 mark]

$\therefore 2m = 2 \times 4 = 8$
[0.5 marks]

And, $m^2 + 1 = 4^2 + 1 = 16 + 1 = 17$
[1 mark]

Hence, $(15, a, b)$ form a pythagorean triplet
So, $a = 8$ and $b = 17$.
$\therefore (a, b) = (8, 17)$
[0.5 marks]

5. Find the least number which must be added to 1825 so as to get a perfect square. Also, find the square root of the perfect square so obtained.
[4 marks]

According to the question,
Given number is 1825

	42
4	$\overline{18}\ \overline{25}$
	-16
82	225
	$-\ 164$
	61

After the square root operation, the obtained remainder is 61.
[2 marks]

$\therefore$ We can conclude that, $42^2 < 1825$

Next possible perfect square number is $43^2 = 1849$

Hence, number to be added $= 1849 - 1825 = 24$
[1.5 marks]

$\therefore 1825 + 24 = 1849$

And the square root of the perfect square 1849 is 43.
[0.5 mark]

6. Find the least number which must be subtracted from 1989 so as to get a perfect square. Also, find the square root of the perfect square so obtained.
[4 marks]

Given number is 1989.

	44
4	$\overline{19}\ \overline{89}$
	-16
84	389
	-336
	53

[2 marks]

We know that, if we subtract the remainder from the number, we get a perfect square.
Therefore, 53 must be subtracted from 1989 to get a perfect square.
[1 mark]

⇒ 1989 - 53 = 1936
And square root of 1936 is 44.
[1 mark]

7. For 2028 find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also, find the square root of the square number so obtained.
[4 marks]

Solution:
According to the question,

2	2028
2	1014
3	507
13	169
13	13
	1

$2028 = 2 \times 2 \times 3 \times 13 \times 13$
[2 marks]

Here, prime factor 3 has no pair.
Therefore, 2028 must be multiplied by 3 to make it a perfect square.
[1 mark]

$\therefore 2028 \times 3 = 6084$
And $\sqrt{6084} = 2 \times 3 \times 13 = 78$

Thus, 78 is the square root of the square number so obtained.
[1 mark]

8. Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.
[3 marks]

Solution:

3	3645
3	1215
3	405
3	135
3	45
3	15
5	5
	1

$3645 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times 5$
To make 3645 as a product of factor pairs, it has to be divided by 5.
$3645 \div 5 = (3 \times 3) \times (3 \times 3) \times (3 \times 3)$
[2 marks]

$\therefore \sqrt{(3645 \div 5)} = 3 \times 3 \times 3 = 27$
[1 mark]

9. $1,024$ trees are planted in a grid fashion such that the number of rows is equal to the number of columns. If one row and one column of trees are cut, then find the total number of trees that were cut?
[5 marks]

Squares and Square Roots

Solution:
Given,
Number of trees $= 1,024$
Since the number of rows is equal to the number of columns, it implies that the number of trees is a perfect square.
$\therefore$ Number of rows/columns = Square root of 1024 (number of trees)
[1 mark]

Square root of 1024 can be found using the prime factorisation method.

Prime factorization of 1024 is,

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

[1 mark]

$\therefore 1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
[1 mark]

Making pairs of 2:
$1024 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

$\Rightarrow \sqrt{1024} = 2 \times 2 \times 2 \times 2 \times 2$
$\Rightarrow \sqrt{1024} = 32$
[1 mark]

Therefore, the initial number of rows and columns $= 32$

After cutting one row and one column,
Number of rows/columns $= 32 - 1$
$= 31$

The remaining number of trees $= 31^2$

Number of trees that were cut,
$= 32^2 - 31^2$
$= (32 + 31)(32 - 31)$

$= 63 \times 1$
$= 63$

$\therefore$ The total number of trees that were cut is 63.
[1 mark]

10. Find the smallest square number that is divisible by each of the numbers 8, 9 and 10.

[4 marks]

Solution:
The number that will be perfectly divisible by each of the numbers 8, 9, and 10 will be their LCM.

2	$8,\ 9,\ 10$
2	$4,\ 9,\ 5$
2	$2,\ 9,\ 5$
3	$1,\ 9,\ 5$
3	$1,\ 3,\ 5$
5	$1,\ 1,\ 5$
	$1,\ 1,\ 1$

LCM of 8, 9, and $10 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$
[2 marks]

Here we see that prime factors 2 and 5 do not have their respective pairs.

Therefore, 360 is not a perfect square.

If 360 is multiplied by 2×5, then the number obtained will be a perfect square.

Therefore, $360 \times 2 \times 5 = 3600$

Thus, the perfect square number is 3600 which is completely divisible by 8, 9, and 10.
[2 marks]