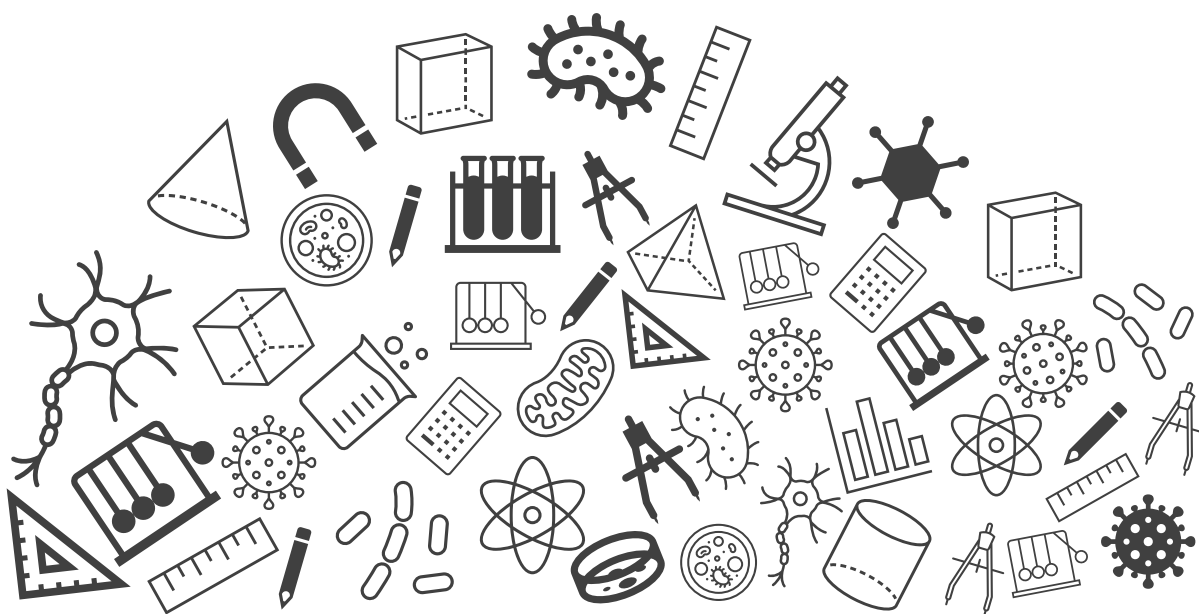




Grade 09: Maths

Exam Important Questions



Number System

Topic : Exam Important Questions

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

[3 marks]

Solution:

A number is said to be rational if it can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

[1 mark]

0 can be expressed in the form $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}$, and so on.

[1 mark]

It satisfies the definition of rational numbers and can be expressed in the form $\frac{p}{q}$, where $q \neq 0$.

[1 mark]

Hence, 0 is a rational number.

[NCERT]

Number System

2. Without actual division, find which of the following rational numbers have terminating decimals.

(i) $\frac{13}{80}$ (ii) $\frac{7}{24}$ (iii) $\frac{5}{12}$

[3 marks]

Solution:

(i) $\frac{13}{80}$

Denominator $80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$

Since denominator doesn't have any factor other than 2 or 5, so the number is terminating.

[1 Mark]

(ii) $\frac{7}{24}$

Denominator $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Since denominator has factor other than 2 or 5, so the number is non-terminating.

[1 Mark]

(iii) $\frac{5}{12}$

Denominator $12 = 2 \times 2 \times 3 = 2^2 \times 3$

Since denominator has factor other than 2 or 5, so the number is non-terminating.

[1 Mark]

[RS Aggarwal]

Number System

3. Insert three rational numbers between $\frac{4}{5}$ and $\frac{6}{5}$.

[2 Marks]

Solution:

Given,

Rational numbers = $\frac{4}{5}, \frac{6}{5}$.

To find the rational numbers between $\frac{4}{5}$ and $\frac{6}{5}$.

Since, the denominators of the rational numbers are same,

Let's multiply the numerator and denominator of each rational number by $(n + 1)$ where 'n' is the number of rational numbers to be inserted in

between $\frac{4}{5}, \frac{6}{5}$.

[1 Mark]

Since, $n = 3$

$$n + 1 = 3 + 1 = 4$$

So,

$$\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$$

Also,

$$\frac{6}{5} \times \frac{4}{4} = \frac{24}{20}$$

Therefore, the three rational numbers between $\frac{4}{5}, \frac{6}{5}$ are $\frac{17}{20}, \frac{18}{20}, \frac{19}{20}$.

[1 Mark]

[RS Aggarwal]

Number System

4. The sum of $0.\overline{3}$ and $0.\overline{4}$ is

(a) $\frac{7}{10}$

(b) $\frac{7}{9}$

(c) $\frac{7}{11}$

(d) $\frac{7}{99}$

[3 marks]

Solution:

The correct answer is the option (b).

[3 marks]

$$\text{Let } x = 0.\overline{3} = 0.333\ldots \text{---(1)}$$

Multiplying both sides by 10, we get

$$10x = 3.3333\ldots \text{---(2)}$$

Subtracting (1) from (2), we get

$$10x - x = 3.333\ldots - 0.333\ldots$$

$$\Rightarrow 9x = 3$$

$$\Rightarrow x = \frac{3}{9}$$

$$\therefore 0.\overline{3} = \frac{3}{9}$$

$$\text{Let } y = 0.\overline{4} = 0.444\ldots \text{---(3)}$$

Multiplying both sides by 10, we get

$$10y = 4.444\ldots \text{---(4)}$$

Subtracting (3) from (4), we get

$$10y - y = 4.4444 - 0.4444$$

$$\Rightarrow 9y = 4$$

$$\Rightarrow y = \frac{4}{9}$$

$$\therefore 0.\overline{4} = \frac{4}{9}$$

$$\text{Sum of } 0.\overline{3} \text{ and } 0.\overline{4} = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$$

[RS Aggarwal]

5. Classify the following as rational or irrational numbers:

A. $\frac{33}{5}$

B. $\sqrt{3}$

C. 3.303003000...

D. $\sqrt{30-5}$

[2 marks]

Solution:

(a) $\frac{33}{5}$ Type equation here.

Since 33 and 5 both are integers, $\frac{33}{5}$ is a rational number.

[0.5 marks]

(b) $\sqrt{3}$

$\sqrt{3} = 1.7320508075688772 \dots$ and it keeps extending. Since it does not terminate or repeat after the decimal point, $\sqrt{3}$ is an irrational number.

[0.5 marks]

(c) 3.303003000 ...

There is no recurring part in this decimal expansion. Hence, it is a non-terminating and non-recurring decimal. Hence, it is an irrational number.

[0.5 marks]

(d) $\sqrt{30-5}$

$$\sqrt{30-5} = \sqrt{25} = 5$$

$\sqrt{30-5}$ is a rational number.

[0.5 marks]

[RS Aggarwal]

Number System

6. Represent geometrically the following numbers on the number line

(i) $\sqrt{4.5}$

(ii) $\sqrt{8.1}$

(iii) $\sqrt{2.3}$

[3 Marks]

Number System

Solution:

(i) Firstly, we draw a line segment $AB = 4.5$ units and extend it to C such that $BC = 1$ unit. Let O be the mid-point of AC .

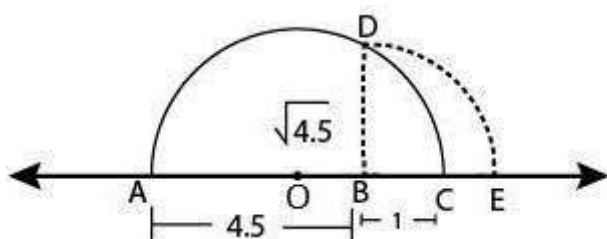
Now, draw a semi-circle with centre O and radius OA . Let us draw BD

perpendicular to AC Hence, the distance BD is $\sqrt{4.5}$ units

[0.5 Marks]

Draw an arc with centre B and radius BD , meeting AC produced at E , then

$BE = BD = \sqrt{4.5}$ units



[0.5 Marks]

(ii) Firstly, we draw a line segment $AB = 8.1$ units and extend it to C such that $BC = 1$ unit, let O be the midpoint of AC .

Now, draw a semicircle with centre O and radius OA .

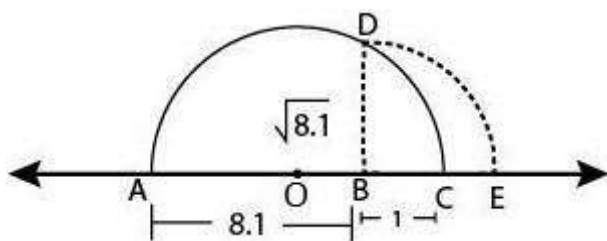
Let us draw BD perpendicular to AC passing through the point B intersecting the semi-circle at point D .

Hence, the distance BD is $\sqrt{8.1}$ units.

[0.5 Marks]

Draw an arc with centre B and radius BD , meeting AC produced at E , then

$BE = BD = \sqrt{8.1}$ units



[0.5 Marks]

(iii) Firstly, we draw a line segment $AB = 2.3$ units and extend it to C such that $BC = 1$ unit. Let O be the mid-point of AC .

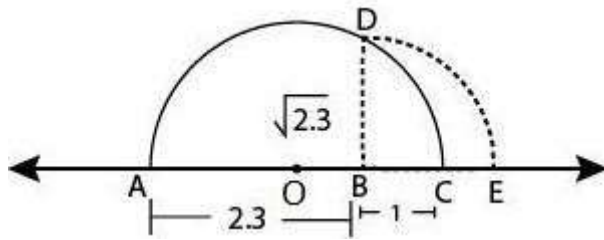
Now, draw a semi-circle with centre O and radius OA .

Let us draw BD perpendicular to AC passing through point B and intersecting the semi-circle at point D .

Hence the distance BD is $\sqrt{2.3}$ units

[0.5 Marks]

$$E = BD = \sqrt{2.3} \text{ units.}$$



[NCERT Exemplar]

[0.5 Marks]

7. Insert a rational number and an irrational number between the following

(i) 2 and 3

(ii) 0 and 0.1

(iii) $\frac{1}{3}$ and $\frac{1}{2}$

(iv) $\frac{-2}{5}$ and $\frac{1}{2}$

(v) 0.25 and 0.26

(vi) $\sqrt{2}$ and $\sqrt{3}$

(vii) 2.357 and 3.121

(viii) .0001 and .001

(ix) 3.623623 and 0.484848

(x) 3.375289 and 6.375738

[5 Marks]

Number System

Solution:

We know that, there are infinitely many rational and irrational values between any two numbers.

(i) A rational number between 2 and 3 is 2.1

To find an irrational number between 2 and 3 . Find a number which is non-terminating non-recurring lying between them,

\therefore Such number will be 2.040040004.....

[0.5 Marks]

(ii) A rational number between 0 and 0.1 is 0.05 and an irrational number between 0 and 0.1 is 0.007000700007.....

[0.5 Marks]

(iii) A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ i.e $\frac{4}{12}$ and $\frac{6}{12}$ is $\frac{5}{12}$. An irrational number between $\frac{1}{3}$ and $\frac{1}{2}$ i.e between 0.3 and 0.5 is 0.4141141114.....

[0.5 Marks]

(iv) A rational number between $-\frac{2}{5}$ and $\frac{1}{2}$ i.e between -0.4 and 0.5 is 0.1 and an irrational between $-\frac{2}{5}$ and $\frac{1}{2}$ i.e between -0.4 and 0.5 is 0.151151115

[0.5 Marks]

(v) A rational number between 0.25 and 0.26 is 0.251. An irrational number between 0.25 and 0.26 is 0.2515515551.....

[0.5 Marks]

(vi) A rational number between $\sqrt{2}$ and $\sqrt{3}$ i.e between 1.4142and 1.7320.....is 1.5

An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is 1.585585558.....

[0.5 Marks]

(vii) A rational number between 2.357 and 3.121 is 3. An irrational number between 2.357 and 3.121 is 3.101101110.....

[0.5 Marks]

(viii) A rational number between 0.0001 and 0.001 is 0.00011. An irrational number between 0.0001 and 0.001 is 0.0001131331333.....

[0.5 Marks]

Number System

[0.5 Marks]

(x) A rational number between 6.375289 and 6.375738 is 6.3753. An irrational number between 6.375289 and 6.375738 is 6.375414114111.....

[0.5 Marks]

[NCERT Exemplar]

8. Given, $\sqrt{2} = 1.4141$ and $\sqrt{6} = 2.449$, find the value of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$ correct to 3 places of decimal.

[3 Marks]

$$\sqrt{6} = 2.449$$

$$\sqrt{3} \times \sqrt{2} = 2.449$$

[1 Mark]

$$\sqrt{3} = \frac{2.449}{1.414} = 1.731$$

[1 Mark]

$$\frac{1}{\sqrt{3}-\sqrt{2}-1} = \frac{1}{1.731-1.414-1}$$

$$= \frac{1}{-0.683}$$

$$= \frac{-1}{0.683}$$

$$= -1.464$$

[1 Mark]

[RD Sharma]

9. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

[3 Marks]

Let $\sqrt{3} + \sqrt{5}$ be a rational number.

[1 Mark]

Let $x = \sqrt{3} + \sqrt{5}$

[1 Mark]

Squaring both sides $x^2 = (\sqrt{3} + \sqrt{5})^2 = 3 + 5 + 2 \times \sqrt{3} \times \sqrt{5}$

$$x^2 - 8 = 2\sqrt{15}$$

$\frac{(x^2 - 8)}{2}$ is rational

$\therefore \sqrt{15}$ is rational

But it is not possible as $\sqrt{15}$ is an irrational number

\therefore Our supposition is wrong

Hence $\sqrt{3} + \sqrt{5}$ is an irrational number.

[1 Mark]

[RD Sharma]

10. Simplify :

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$(ii) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

[4 Marks]

Solution:

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$\text{Now, } \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}$$

(Rationalising the denominator)

$$\begin{aligned} &= \frac{(3\sqrt{2}-2\sqrt{3})^2}{(3\sqrt{2})^2-(2\sqrt{3})^2} \\ &= \frac{9 \times 2 + 4 \times 3 - 2 \times 3 \times \sqrt{2} \times 2\sqrt{3}}{9 \times 2 - 4 \times 3} \\ &= \frac{18+12-12\sqrt{6}}{18-12} = \frac{30-12\sqrt{6}}{6} = 5 - 2\sqrt{6} \end{aligned}$$

$$\text{And, } \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{4 \times 3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{2 \times 3 + 2\sqrt{6}}{3-2}$$

$$= \frac{6+2\sqrt{6}}{1} = 6 + 2\sqrt{6}$$

$$\therefore \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = 5 - 2\sqrt{6} + 6 + 2\sqrt{6} = 11$$

[2 Marks]

$$(ii) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$= \frac{(7+3\sqrt{5})(3-\sqrt{5}) - (7-3\sqrt{5})(3+\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{(21-7\sqrt{5}+9\sqrt{5}-3 \times 5) - (21+7\sqrt{5}-9\sqrt{5}-3\sqrt{5} \times \sqrt{5})}{(3)^2-(\sqrt{5})^2}$$

$$= \frac{(21+2\sqrt{5}-15) - (21-2\sqrt{5}-15)}{9-5}$$

$$= \frac{(6+2\sqrt{5}) - (6-2\sqrt{5})}{4}$$

$$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4} = \frac{4\sqrt{5}}{4} = \sqrt{5}$$

[2 Marks]

[RD Sharma]

Number System

11. Which of the following is equal to x ?

A) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$

B) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$

C) $(\sqrt{x^3})^{\frac{2}{3}}$

D) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

[4 Marks]

The answer is C.

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}} = x^{\frac{5}{7}+1} - x^{\frac{5}{7}}$
 $= x^{\frac{5}{7}} \cdot x - x^{\frac{5}{7}} [\because a^{m+n} = a^m a^n]$

[1 Mark]

(b) $\sqrt[12]{(x^4)^{\frac{1}{3}}} = \left((x^4)^{\frac{1}{3}}\right)^{\frac{1}{12}} [\sqrt[n]{a} = a^{1/n}]$
 $= x^{4 \times \frac{1}{3} \times \frac{1}{12}} = x^{\frac{1}{9}} [\because (x^m)^n = x^{mn}]$
 $\neq x$

[1 Mark]

(c) $(\sqrt{x^3})^{\frac{2}{3}} = (x^{\frac{3}{2}})^{\frac{2}{3}} = x^{\frac{3}{2} \times \frac{2}{3}} = x [\because (a^m)^n = a^{mn}]$

[1 Mark]

(d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}} = x^{\frac{12}{7} + \frac{7}{12}} [\because a^m \cdot a^n = a^{m+n}]$
 $= x^{\frac{144+49}{84}} = x^{\frac{193}{84}} \neq x [\because a^m \cdot a^n = a^{m+n}]$

[1 Mark]

[NCERT]

12. The value of x , if $5^{x-3} \times 3^{2x-8} = 225$

[3 marks]

We have

$$5^{x-3} \times 3^{2x-8} = 225$$

Express RHS in terms of $5^m \times 3^n$

$$= 225$$

$$= 5^2 \times 3^2$$

[1 mark]

Equate the power of 5 and 3.

$$x - 3 = 2$$

$$2x - 8 = 2$$

[1 mark]

Solve for x .

$$x - 3 = 2 \Rightarrow x = 5$$

$$2x - 8 = 2 \Rightarrow x = 5$$

[1 mark]

[R. S. Aggarwal]