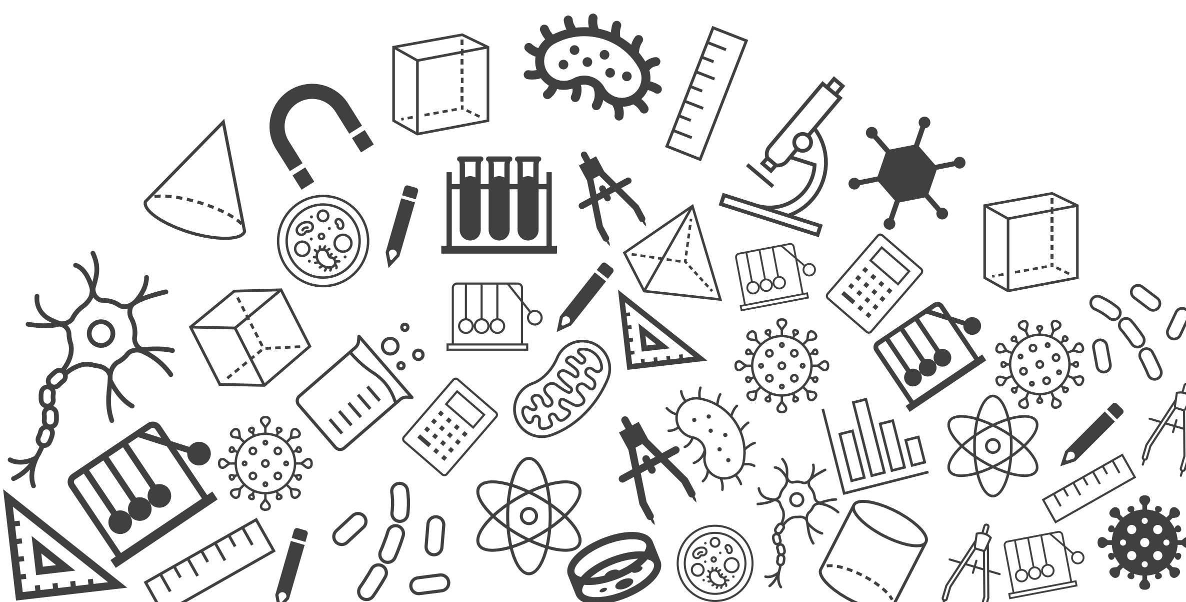
Grade 09: Maths
Exam Important Questions



Topic : Exam Important Questions

1. Write the coefficient of x^2 in each of the following.

(i) 17 - 2x + 7x^2

(ii) 9 - 12x + x^3

(iii) $\frac{\pi}{6}x^2$ - 3x + 4

(iv) $\sqrt{3}$ x - 7

[2 Marks]

Coefficient of x^2,

in (i) is 7 (0.5 mark)

in (ii) is 0 as there is no term of x^2 i.e. 0 x^2 (0.5 mark)

in (iii) is $\frac{\pi}{6}$ (0.5 mark)

in (iv) is 0 (0.5 mark)

2. Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer:

(2 marks)

(i) $3x^2$ - 4x + 15

(ii) y^2 + 2 $\sqrt{3}$

(iii) $3\sqrt{x}+\sqrt{2}x$

(iv) $x-\frac{4}{x}$

(v) $x^{12}+y^3+t^{50}$

(i) $3x^2$ - 4x + 15, (ii) y^2 + 2 $\sqrt{3}$

are polynomial is one variable. Others are not polynornial or polynomials in one variable.

(2 marks)

3. Verify 5 and 0 are the zeros of the polynomial x^2-5x.
[2 marks]

Solution:
Here we have to verify whether 5 and 0 are the zeros of the polynomial x^2-5x.
Let $p(x)=x^2-5x$
⇒ $p(5)=(5)^2-5(5)$
⇒ $p(5)=25-25$
⇒ $p(5)=0$ [1 mark]
Now similarly $p(0)=0^2-5(0)=0-0=0$.
Hence both 5 and 0 are zeros of the polynomial $p(x)=x^2-5x$. [1 mark]

4. Verify whether the following are true or false.

(i) -3 is a zero of $x-3$

(ii) $-\frac{1}{3}$ is a zero of $3x+1$

(iii) $-\frac{4}{5}$ is a zero of $4-5y$

[3 marks]

(i) Let $p(x) = x - 3$

So for $p(x) = 0$ we get zero of $p(x)$

$\Rightarrow \ x - 3 = 0$

$\Rightarrow \ x = 3$

So $x = 3$ is a zero of $(x - 3)$ only.

Hence, -3 is not a zero of $(x - 3)$

Hence, the statement is false. (1 mark)

(ii) Let $p(x) = 3x + 1$

So for $p(x) = 0$ we get zero of $p(x)$

$\Rightarrow \ 3x + 1 = 0$

$\Rightarrow \ x = \frac{-1}{3}$

So $x = \frac{-1}{3}$ is a zero of $(3x + 1)$ only.

Hence, $\frac{-1}{3}$ is a zero of $(3x + 1)$

Hence, the statement is true. (1 mark)

(iii) Let $p(y) = 4 - 5y$

So for $p(y) = 0$ we get zero of $p(y)$

$\Rightarrow \ 4 - 5y = 0$

$\Rightarrow \ y = \frac{4}{5}$

So $y = \frac{4}{5}$ is a zero of $(4 - 5y)$ only.

Hence, $-\frac{4}{5}$ is not a zero of $(4 - 5y)$

Hence, the statement is false. (1 mark)

5. Without actual division , prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

[3 marks]

Solution:
Let $p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$
Factorise $x^2 - 3x + 2$.
Now, $x^2 - 3x + 2 = x^2 - 2x - x + 2$ [By splitting middle term]
$= x(x-2) - 1(x-2) = (x-1)(x-2)$
Hence, zeros of $x^2 - 3x + 2$ are 1 and 2. (1 mark)
We have to prove that, $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

i.e. prove that $p(1) = 0$ and $p(2) = 0$
Now. $p(1) = 2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2$
$= 2 - 5 + 2 - 1 + 2 = 6 - 6 = 0$
$p(2) = 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2$
$= 2 \times 16 - 5 \times 8 + 2 \times 4 + 0$
$= 32 - 40 + 8 = 40 - 40 = 0$
Hence, p(x) is divisible by $x^2 - 3x + 2$. (2 marks)

6. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y) = y^2 - y + 1$
(ii) $p(t) = 2 + t + 2t^2 - t^3$

[3 marks]

Solution:
(i)
$p(y) = y^2 - y + 1$
$p(0) = 0^2 - 0 + 1 = 1$ **[0.5 mark]**
$p(1) = 1^2 - 1 + 1 = 1$ **[0.5 mark]**
$p(2) = 2^2 - 2 + 1 = 3$. **[0.5 mark]**

(ii)
$p(t) = 2 + t + 2t^2 - t^3$
$p(0) = 2 + 0 + 2 \times 0^2 - 0^3 = 2$ **[0.5 mark]**
$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 4$ **[0.5 mark]**
$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$.**[0.5 mark]**

7. Factorise:

$(i)\ x^3 - 2x^2 - x + 2$

$(ii)\ x^3 - 3x^2 - 9x - 5$

$(iii)\ x^3 + 13x^2 + 32x + 20$

$(iv)\ 2y^3 + y^2 - 2y - 1$

[5 marks]

Solution:
$(i)\ Given;\ x^3 - 2x^2 - x + 2$

Factorising
$x^3 - x - 2x^2 + 2$

$= x(x^2 - 1) - 2(x^2 - 1)$
$= (x^2 - 1)(x - 2)$
$= [(x)^2 - (1)^2](x - 2)$
$= (x - 1)(x + 1)(x - 2)$

$\because [(a^2 - b^2) = (a + b)(a - b)]$

Conclusion
$Thus,\ x^3 - 2x^2 - x + 2$
$= (x - 1)(x + 1)(x - 2)$

(1 mark)

$(ii)\ Given:\ x^3 - 3x^2 - 9x - 5$

Factorising
$x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$
$= x^2(x + 1) - 4x(x + 1) - 5(x + 1)$
$= (x + 1)(x^2 - 4x - 5)$
$= (x + 1)(x^2 - 5x + x - 5)$
$= (x + 1)[(x(x - 5) + 1(x - 5))]$
$= (x + 1)(x - 5)(x + 1)$

Conclusion
$Thus,\ x^3 - 3x^2 - 9x - 5$
$= (x + 1)(x - 5)(x + 1)$

(1.5 marks)

$(iii)\ Given:\ x^3 + 13x^2 + 32x + 20$

Factorising
$x^3 + 13x^2 + 32x + 20$

$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$
$= x^2(x + 1) + 12x(x + 1) + 20(x + 1)$
$= (x + 1)(x^2 + 12x + 20)$
$= (x + 1)(x^2 + 2x + 10x + 20)$
$= (x + 1)[x(x + 2) + 10(x + 2)]$
$= (x + 1)(x + 2)(x + 10)$

Conclusion
$Thus,\ x^3 + 13x^2 + 32x + 20$
$= (x + 1)(x + 2)(x + 10)$

(1 mark)

$(iv)\ Given:\ 2y^3 + y^2 - 2y - 1$

Polynomials

Factorising
$2y^3 + y^2 - 2y - 1$

$= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$
$= 2y^2(y-1) + 3y(y-1) + 1(y-1)$
$= (y-1)(2y^2 + 3y + 1)$
$= (y-1)(2y^2 + 2y + y + 1)$
$= (y-1)[(2y(y+1) + 1(y+1)]$
$= (y-1)(y+1)(2y+1)$

Conclusion:
$Thus, 2y^3 + y^2 - 2y - 1$
$= (y-1)(y+1)(2y+1)$
(1.5 marks)

8. If $(x^2 - y^2) = 18$ and $(x - y) = 3$, find the value of $16x^2y^2$.

A. 27

B. 81

C. 243

D. 729 ✓

$(x^2 - y^2) = (x-y)(x+y) = 18$

$(x-y) = 3 - - - - (i)$

$(x+y) = \frac{18}{3} = 6 - - - - (ii)$

Adding equation (i) and (ii) we get
$2x = 9$
$x = \frac{9}{2}$

Substituting the value of x in equation (ii) we get $y = \frac{3}{2}$

Now substituting the value of x and y in $16x^2y^2$ we get,

$= 16x^2y^2$
$= 16 \times (\frac{9}{2})^2 \times (\frac{3}{2})^2$
$= 729$

9. The value of $249^2 - 248^2$ is

A. 109

B. 577

C. 487

D. 497 ✓

$249^2 - 248^2 = (249 + 248)(249 - 248)$ Using identity, $[a^2 - b^2 = (a+b)(a-b)]$
$= 497 \times 1$
$= 497$

10. Calculate 103×107 using algebraic identities.

A. 10121

B. 11021 ✓

C. 10021

D. 12011

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$

To split 103×107, we need a square that is easy to calculate.

Hence, $x = 100, a = 3, b = 7$

$\therefore 103 \times 107 = (100+3)(100+7)$
$= 100^2 + (3+7)(100) + (3)(7)$
$= 10000 + 1000 + 21 = 11021$