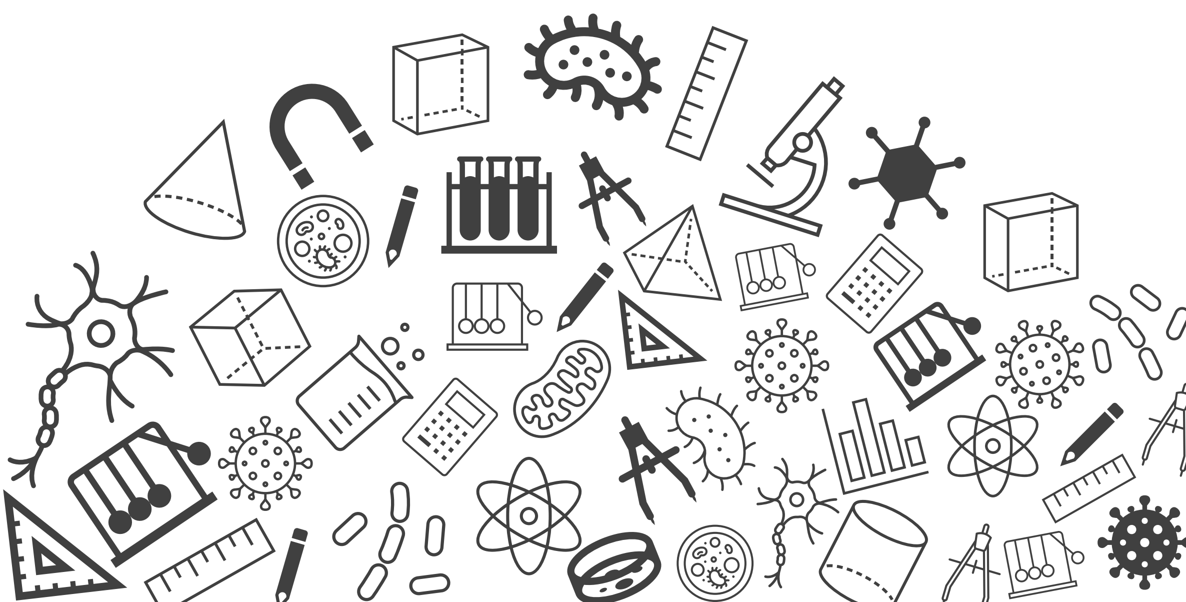
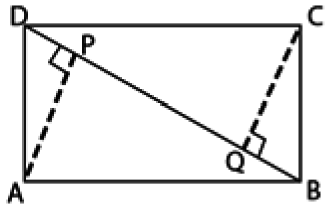
Grade 09: Maths
Exam Important Questions



Topic : Exam Important Questions

1. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD(See the given figure). Show that AP = CQ.



[2 marks]

In $\Delta APB \cong \Delta CQD$
$\angle APB = \angle CQD$ ($Each\ 90^\circ$)

$AB = CD$
(Opposite sides of parallelogram ABCD)

$\angle ABP = \angle CDQ$
(Alternate interior angles for AB || CD)

(1 mark)

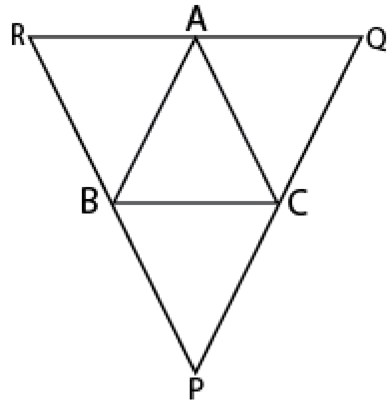
$\therefore \Delta APB \cong \Delta CQD$
(By AAS congruency)

By using this result
$\Delta APB \cong \Delta CQD$, we obtain

AP = CQ (By CPCT)

(1 mark)

2. Through A, B and C lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a ΔABC as shown in figure. Show that $BC = \frac{1}{2}QR$.



[3 marks]
[Properties of a parallelogram]

Given in ΔABC, PQ ∥ AB and PR ∥ AC and RQ ∥ BC

To show $BC = \frac{1}{2}QR$.

Proof in Quadrilateral BCAR, BR ∥CA and BC ∥ RA

So, quadrilateral , BCAR is a parallelogram.

$\therefore BC = AR$ ….(i)

(1 mark)

Now, in quadrilateral BCQA, BC ∥AQ

And AB ∥ QC

So, quadrilateral BCQA is a parallelogram.

BC = AQ …..(ii)

(1 mark)

On adding Eqs (i) and (ii) we get

2BC = AR + AQ

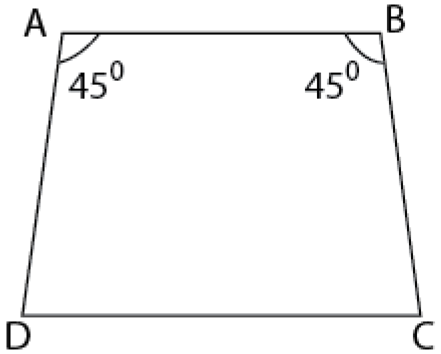
$\Rightarrow 2BC = RQ$

$\Rightarrow BC = \frac{1}{2}QR$. Hence proved

(1 mark)

3. ABCD is a trapezium in which AB|| DC and $\angle A=\angle B=45^{\circ}$. Find angles C and D of the trapezium.
[2 marks]

Given, ABCD is a trapezium and whose parallel sides in the figure are AB and DC. Since, AB || CD and BC is transversal, then sum of two cointerior angles is 180°.



$\therefore \angle B+\angle C=180^{\circ}$

$\Rightarrow \angle C=180^{\circ}-\angle B=180^{\circ}-45^{\circ}$

$\Rightarrow \angle C=135^{\circ}$ (1 mark)

Similarly, $\angle A+\angle D=180^{\circ}$

$\Rightarrow \angle D=180^{\circ}-45^{\circ}[\because \angle A=45^{\circ} given]$

$\Rightarrow \angle D=135^{\circ}$

Hence, angles C and D are 135° each.
(1 mark)

4. Solve the following question:

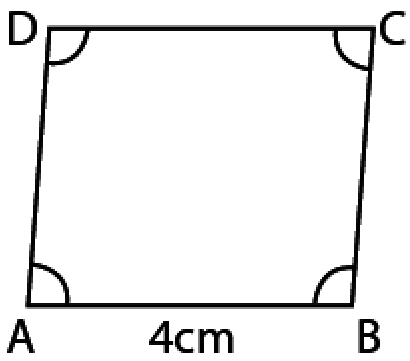
(i) Opposite angles of a quadrilateral ABCD are equal. If AB = 4cm, determine CD. (1 mark)

(ii) Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3cm and OD = 2cm, determine the lengths of AC and BD. (2 marks)

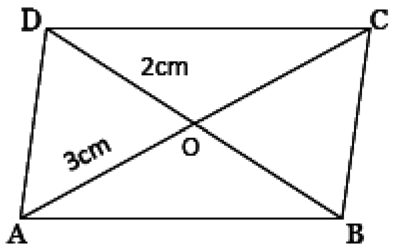
[Properties of a parallelogram]

(i) Given, opposite angles of a quadrilateral are equal. So, ABCD is a parallelogram and we know that in a parallelogram opposite sides are also equal.

CD=AB = 4cm. (1 mark)



(ii) Given, ABCD is a parallelogram OA = 3cm and OD = 2cm



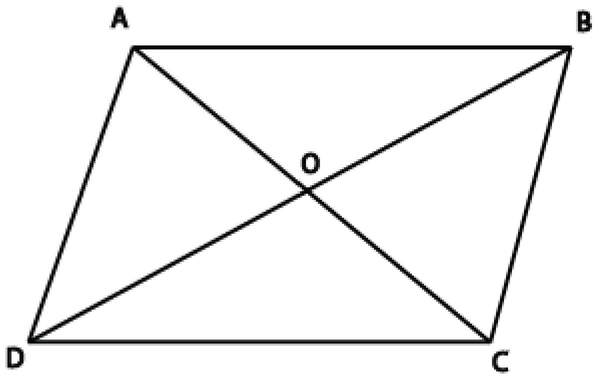
We know that, diagonals of a parallelogram bisect each other.

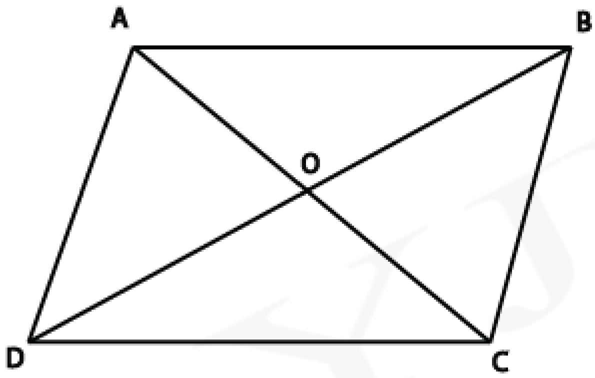
$\therefore$ Diagonal AC = 2 OA = 6cm $[\because AO = OC]$

And Diagonal BD = 2OD = 4 cm $[\because BO = OD]$

Hence the length of the diagonals AC and BD are 6cm and 4cm, respectively (2 marks)

5. Given below is a parallelogram. AC and BD are diagonals. If AO = x + y, OC = 5y, DO = 3x, OB = 12, they find x and y.





In the parallelogram ABCD, AC and BD are diagonals.
Since the diagonals of the parallelogram bisect each other,
AO = OC and BO = OD.

AO = x + y, Oc = 5y (Given)
x + y = 5y ------- (i)

DO = 3x, OB = 12
3x = 12 ---------- (ii)

On solving (ii), we get, $x = \frac{12}{3}$ = 4.
x = 4

Using x = 4 in (i), we get, 4 + y = 5y
On solving, we get, 4 = 5y - y = 4y
4y = 4
$y = \frac{4}{4}$ = 1

x = 4 and y = 1

6. The angle of quadrilateral are in the ratio 3 : 5 : 9 :13. Find the all the angles of the quadrilateral.
   [3 marks]

   Let the common ratio between the angle be x, therefore, the angles will be 3x,5x, 9x, and 13x respectively.
   As the sum of all interior angles of a quadrilateral is 360°,
   $\therefore$ 3x+5x+9x+13x=360°
   [0.5 marks]

   $30x = 360^\circ$
   $x = 12^\circ$
   [0.5 marks]

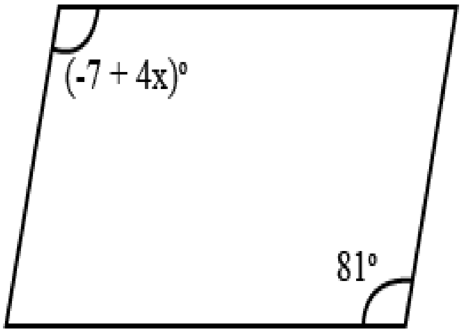
   Hence, the angles are
   $3x = 3 \times 12 = 36^\circ$
   [0.5 marks]

   $5x = 5 \times 12 = 60^\circ$
   [0.5 marks]

   $9x = 9 \times 12 = 108^\circ$
   [0.5 marks]

   $13x = 13 \times 12 = 156^\circ$
   [0.5 marks]

7. In the following parallelogram, find the value of x.



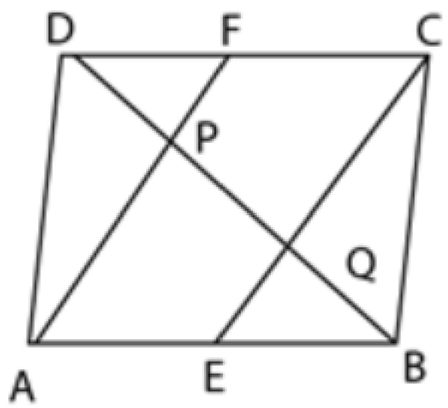
[2 marks]

Solution:
Given that the quadrilateral is a parallelogram.
We know that, in a parallelogram the opposite angles are equal.
[1 mark]
$\therefore (-7+4x)^o = 81^o$
$\Rightarrow (4x-7)^o = 81^o$
$\Rightarrow 4x = 81^o + 7^o = 88^o$
$\Rightarrow 4x = 88^o$
$\Rightarrow x = \frac{88^o}{4}$
$\Rightarrow x = 22^o$

Hence the value of x is 22^o.
[1 mark]

8. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.
[3 Marks]
[Mid Point Theorem]

Solution:



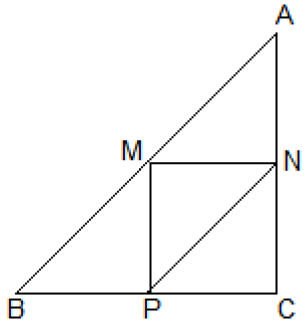
ABCD is a parallelogram.
$AB||CD \Rightarrow AE||FC$
Now, AB = CD, (Opposite sides of a parallelogram)
$\frac{1}{2}AB = \frac{1}{2}CD$
$\Rightarrow AE = FC$ (E and F are the mid-points of AB and CD)
AECF is a parallelogram.
(AE and CF are parallel and equal to each other)
Then, AF || EC (Opposite sides of a parallelogram)
(1 Mark)

Now, in $\triangle DQC$,
F is the mid-point of side DC and FP || CQ(as AF || EC)
P is the mid-point of DQ (Converse of mid-point theorem)
$\Rightarrow DP = PQ$...(i) (0.5 Marks)

In $\triangle APB$,
E is the id-point of side AB and EQ || AB (as AF || EC).
Q is the mid-point of PB.
(By converse of mid-point theorem)
$\Rightarrow PQ = QB$ ...(ii) (0.5 Marks)

Now, we can conclude that,
DP = PQ = BQ (From (i) and (ii))
Hence, the line segments AF and EC trisect the diagonal BD.
(1 Mark)

9. In the given figure, M, N, and P are the mid-points of AB, AC, and BC, respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, find the length of BC, AB, and AC.



[3 Marks]

Here, MN is a line which joins the mid-point M of AB and N of AC.

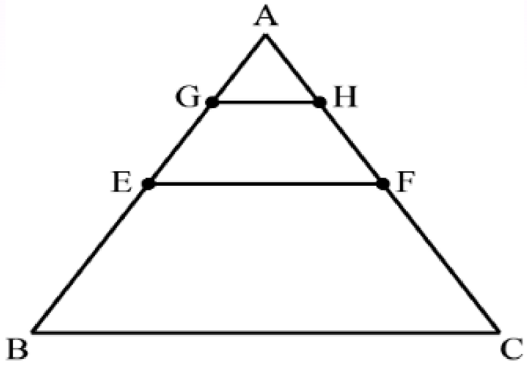
Then, by mid-point theorem, $MN \parallel BC$ and $MN = \frac{1}{2}BC$
$\Rightarrow BC = 2MN = 6\ cm$
[1 Mark]

Similarly, $MP \parallel AC$ and $MP = \frac{1}{2}AC$
$\Rightarrow AC = 2MP = 5\ cm$
[1 Mark]

And, $NP \parallel AB$ and $NP = \frac{1}{2}AB$
$\Rightarrow AB = 2NP = 7\ cm$
[1 Mark]

So, the length of the sides BC, AB, and AC are $6\ cm$, $7\ cm$, and $5\ cm$, respectively.

10.



In figure, E and F are mid - points of the sides AB and AC respectively of the ΔABC. G and H are mid-points of the sides AE and AF respectively of the ΔAEF. If GH =1.8 cm, find BC.

$[2\ Marks]$

Solution:

$EF = \frac{1}{2}BC \ldots\ldots (1)$

$[0.5\ Marks]$

($\because$ E and F are mid-points of sides AB and AC of ΔABC)

$GH = \frac{1}{2}EF \ldots\ldots (2)$

$[0.5\ Marks]$

($\because$ G and H are mid-points of sides AB and AC of ΔAEF)

From (1) and (2), we have,

$GH = \frac{1}{2} \times \frac{1}{2}BC = \frac{1}{4}BC$

$\Rightarrow BC = 4 \times GH = 4 \times 1.8\ cm = 7.2\ cm$

$[1\ Mark]$

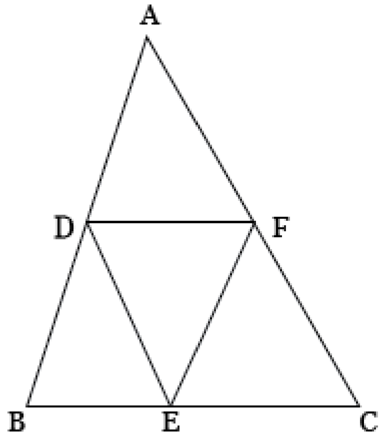
Hence, BC = 7.2 cm

11. D, E and F are respectively the mid-points of the sides AB, BC and CA of a ΔABC. Prove that by joining these mid-points D, E and F, the ΔABC is divided into four congruent triangles.
[4 Marks]

Given in a $\Delta ABC, D, E$ and F respectively the mid-points of the sides AB, BC and CA.

To prove ΔABC is divided into four congruent triangles.

Proof Since, ABC is a triangle and D, E and F are the mid-points of sides AB, BC and CA, respectively



Then, $AD = BD = \frac{1}{2}AB, BE = EC = \frac{1}{2}BC$
And $AF = CF = \frac{1}{2}AC (1 Mark)$

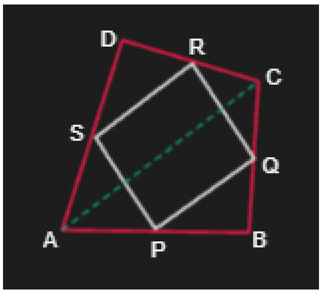
Now, using the mid-point theorem,
$EF \,||\, AB \text{ and } EF = \frac{1}{2}AB = AD = BD$

$ED \,||\, AC \text{ and } ED = \frac{1}{2}AC = AF = CF$

And $DF \,||\, BC \text{ and } DF = \frac{1}{2}BC = BE = CE (1 Mark)$

In $\Delta ADF and \Delta EFD$,
$AD = EF$
$AF = DE$
DF = FD [Common]
$\therefore \Delta ADF \cong \Delta EFD$ [by SSS congruence rule]
(1 Mark)
Similarly, $\Delta DEF \cong \Delta EDB$ (0.5 Marks)
And $\Delta DEF \cong \Delta CFE$ (0.5 Marks)
So, ΔABC is divided into four congruent triangles. Hence proved.

12. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is the diagonal. Show that
(i) SR || AC and $SR = \frac{1}{2}AC$
(ii) PQ = SR
(iii) PQRS is a parallelogram.



[3 Marks]

(i) In $\triangle ADC$, R is the mid-point of DC and S is the mid-point of DA.
Thus, by mid-point theorem, SR||AC and
$SR = \frac{1}{2}AC.$ $(1 Mark)$

(ii) In $\triangle BAC$, P is the mid-point of AB and Q is the mid-point of BC.
Thus, by mid-point theorem, PQ || AC and $PQ = \frac{1}{2}AC.$
Also, $SR = \frac{1}{2}AC$
Hence, PQ = SR. (1 Mark)

(iii) SR || AC ... From Question (i)
PQ || AC ... From Question (ii)
$\Rightarrow PQ||SR$
From (ii), PQ = SR
Since, one pair of opposides of the quadrilateral PQRS is parallel and equal, PQRS is a Parallelogram. (1 Mark)
Hence Proved.