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# Memory Based Answers & Solutions

# JEE (Main)-2024 (Online) Phase-1

(Physics, Chemistry and Mathematics)

### **IMPORTANT INSTRUCTIONS:**

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
- (3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.
- (4) **Section A :** Attempt all questions.
- (5) **Section B**: Attempt any 05 questions out of 10 Questions.
- (6) Section A (01 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) Section B (21 30) contains 10 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.



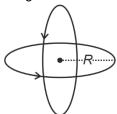
# **PHYSICS**

# **SECTION - A**

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

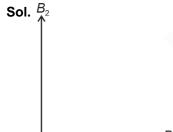
### Choose the correct answer:

 Two rings of equal radius R arranged perpendicular to each other with common centre at C, carrying equal current I. Find magnetic field at C.



- (1)  $\frac{\mu_0}{2R}$
- $(2) \ \frac{\mu_0 I}{R}$
- (3)  $\sqrt{2} \frac{\mu_0 I}{R}$
- (4)  $\frac{\mu_0 I}{\sqrt{2}R}$

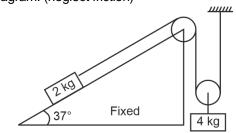
# Answer (4)



$$\vec{B}_1 = \frac{\mu_0 I}{2R} \hat{i}, \vec{B}_2 = \frac{\mu_0 I}{2R} \hat{j}$$

$$B_{\rm C} = \frac{\mu_0 I}{\sqrt{2}R}$$

2. Find the acceleration of 2 kg block shown in the diagram. (neglect friction)



- (1)  $\frac{4g}{15}$
- (2)  $\frac{29}{15}$
- (3)  $\frac{g}{15}$

(4)  $\frac{2g}{3}$ 

# Answer (1)

Sol. For 2 kg block

 $T - 2g \sin 37 = 2a$  ...(i)

For 4 kg block

$$4g-2T=\frac{4a}{2}$$

2g - T = a ...(ii)

T = (2g - a)

 $2g-a-2g\times\frac{3}{5}=2a$ 

 $3a = 2g \times \frac{2}{5}$ 

 $a = \frac{4g}{15}$ 

- 3. A particle of mass *m* is projected with speed *v* at an angle of 30° with the horizontal, find its angular momentum about point of projection when it reaches its maximum height.
  - (1)  $\frac{mv^3}{16a}$
- $(2) \quad \sqrt{3} \frac{mv^3}{16g}$
- (3)  $\frac{mv^3}{3a}$
- $(4) \quad \sqrt{3} \frac{mv^3}{8g}$

### Answer (2)

**Sol.** Velocity at maximum height = *v*coss30°

 $\therefore L = m(v\cos 30) H$ 

$$= mv \left(\frac{\sqrt{3}}{2}\right) \times \frac{v^2 \sin^2 30}{2g}$$

$$=\sqrt{3}\frac{mv^3}{16\ g}$$

- 4. The ratio of kinetic energy & potential energy in 5<sup>th</sup> excited state of Hydrogen atom is
  - (1) -2

- (2) 2
- (3)  $-\frac{1}{2}$
- $(4) \frac{1}{2}$

Answer (3)

**Sol.** Kinetic energy: Potential energy = 1:-2



5.  $700 \Omega$  $3.5 \text{ k}\Omega$  $200 \Omega$ 

> In given circuit find potential difference across 700  $\Omega$  resistance (i.e.  $V_0$ ).

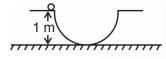
- (1) 2 V
- (2) 0.5 V
- (3) 1.1 V
- (4) Zero

# Answer (3)

**Sol.** 
$$i = \frac{7}{3.5 \ k + 0.9 \ k\Omega} = \frac{7}{4.4 \ k}$$

$$V_0 = i \times 700 \Omega = \frac{7}{44k} \times .7 k = \frac{4.9}{44} = 1.1 \text{ V}$$

A ball is released from a height of 1 m on a smooth hemispherical surface as shown. Find its velocity when it is at a height of 0.5 m. (Take  $q = 10 \text{ m/s}^2$ )



- (1) 20 m/s
- (2) 10 m/s
- (3)  $\sqrt{10}$  m/s
- (4) 5 m/s

# Answer (3)

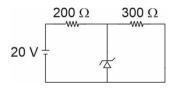
**Sol.** By conservation of mechanical energy

$$mg(1) = \frac{1}{2} \, mv^2 + mg(0.5)$$

$$v^2 = 10$$

$$v = \sqrt{10}$$
 m/s

Find current through zener diode if its breakdown 7. voltage is 5 V.



- (1) 58.33 mA
- (2) 25 mA
- (3) 28.33 mA
- (4) 20.23 mA

# Answer (1)

**Sol.** 
$$i_{\text{battery}} = \frac{(20-5)}{200} = \frac{15}{200} \text{ A}$$

$$i_{300\Omega} = \frac{5}{300} \text{ A}$$

$$i_{\text{zener}} = \frac{15}{200} - \frac{5}{300}$$

= 58.33 mA

- Ball released from height 10 m strikes ground and rebounds height 5 m. Find impulse imparted by ground while collision, given mass of ball is 100 g. (Take  $g = 10 \text{ m/s}^2$ )
  - (1)  $(\sqrt{2}-1)$  Ns
- (2)  $(\sqrt{2} + 2)$  Ns
- (3)  $(2\sqrt{2}-1)$  Ns (4)  $(\sqrt{2}+1)$  Ns

# Answer (4)

**Sol.** 
$$v_1 = \sqrt{2g10}$$

$$v_2 = \sqrt{2g5}$$

$$\vec{I} = \Delta \vec{p}$$

$$I = 0.1\{\sqrt{2g10} + \sqrt{2g5}\}$$

$$=0.1\{10\sqrt{2}+10\}$$

$$=(\sqrt{2}+1)$$
 Ns

- Potential due to electric dipole on axial position at distance r from dipole is proportional to (assume r >>length of dipole)
  - $(1) \frac{1}{-}$

(4) r

# Answer (3)

Sol. 
$$q \xrightarrow{-q} q$$

$$\left|E\right| = \frac{2kP}{r^3}$$

$$E = -\frac{dv}{dr}, \ v \propto \frac{1}{r^2}$$

- 10. Maximum wavelength of light source such that photoelectron can be ejected from material of work function 3 eV is
  - (1) 2133.3 Å
- (2) 3133.3 Å
- (3) 4133.3 Å
- (4) 313.3 Å

### Answer (3)

**Sol.** 
$$\lambda = \frac{12400}{3} = 4133.3 \,\text{Å}$$

- 11. A long wire carrying current  $\sqrt{2}A$  is placed in uniform magnetic field of  $3 \times 10^{-5}$  T. If magnetic field is perpendicular to wire, find the magnetic force on unit length of wire.
  - (1)  $3 \times 10^{-4} \text{ N}$
- (2)  $3\sqrt{2} \times 10^{-5} \text{ N}$
- (3)  $3 \times 10^3 \text{ N}$
- (4) Zero

# Answer (2)

Sol.  $\sqrt{2}A$ 

$$F = i L B \sin\theta$$

$$= \sqrt{2} \times 1 \times 3 \times 10^{-5} \times \sin 90$$

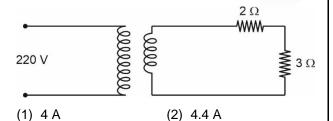
$$F = 3\sqrt{2} \times 10^{-5} \text{ N}$$

- 12. If the area of cross-section is halved and length of wire having young's modulus Yis doubled, then its young's modulus will become
  - (1) Y

 $(3) \frac{Y}{2}$ 

# Answer (1)

- Sol. Young's modulus is property of material of wire and it is independent of geometrical factors.
- 13. In an electric transformer, 220 V is applied on primary coil having number of turn 100. Find output current through 3  $\Omega$  resistance if number of secondary turn is 10.



- (3) 2 A

- (4) 2.2 A

# Answer (2)

**Sol.**  $\frac{V_l}{V_0} = \frac{N_l}{N_0} \Rightarrow \frac{220}{V_0} = \frac{100}{10}$ 

$$V_0 = 22 \text{ V}$$

$$I_0 = \frac{22}{5} = 4.4 \text{ A}$$

- 14. Find the temperature of H<sub>2</sub> gas at which its *rms* speed is equal to that of O<sub>2</sub> at 47°C.
  - (1) 20°C
- (2) -20°C
- (3) -253°C
- (4) 17°C

# Answer (3)

**Sol.**  $V_{rms} = \sqrt{\frac{3RT}{M}}$ 

$$\frac{T}{2} = \frac{320}{32}$$

T = 20 K

∴ *T* = -253°C

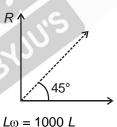
15. In AC circuit with source voltage  $\varepsilon = 20 \sin 1000 t$  is connected to series L - R circuit whose power factor

is  $\frac{1}{\sqrt{2}}$ . If  $E = 25 \sin 2000 t$ , the new power factor is

Answer (2)

Sol.

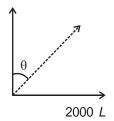
Old



 $\Rightarrow L\omega = R$ .

### New

$$R = 1000 L$$



$$\cos \theta = \frac{R}{Z}$$

$$= \frac{1000 L}{\sqrt{(1000 L)^2 + (2000 L)^2}}$$

$$= \frac{1}{\sqrt{1+4}} = \frac{1}{\sqrt{5}}$$



- 16. In an electromagnetic wave the electric field is given as  $\vec{E} = E_0 \sin(\omega t kz)\hat{i}$  the corresponding magnetic field will be
  - (1)  $E_0 C \sin(\omega t kz) \hat{j}$
  - (2)  $\frac{E_0}{C}\sin(\omega t kz)\hat{j}$
  - (3)  $\frac{E_0}{C}\cos(\omega t kz)\hat{i}$
  - $(4) \quad \frac{E_0}{C}\sin(\omega t kz)\hat{i}$

# Answer (2)

**Sol.**  $\vec{E} \times \vec{B}$  is along +z axis

$$B_0 = \frac{E_0}{C}$$

$$\therefore B = \frac{E_0}{C} \sin(\omega t - kz)\hat{j}$$

- 17. At a point away from planet of radius 6400 km, the gravitational potential and field are  $-6.4 \times 10^7$  SI units and 6.4 SI units respectively. Find height of that point above surface of planet.
  - (1) 3000 km
  - (2) 6400 km
  - (3) 3600 km
  - (4) 9400 km

# Answer (3)

**Sol.** 
$$\frac{GM}{r} = 6.4 \times 10^7$$
 ...(i)

$$\frac{GM}{r^2} = 6.4$$
 ...(ii)

$$r = \frac{6.4 \times 10^7}{6.4}$$

$$= 10^7 \, \text{m}$$

$$= 10,000 \text{ km}$$

$$R + h = 10,000$$

$$h = 10,000 - 6400 = 3600 \text{ km}$$

- 18. A wire has resistance of 60  $\Omega$  at temperature 27°C. When it is connected to a 220 V dc supply, a current 2.75 A flows through it at a certain temperature. Find the value of temperature, if coefficient of thermal resistance ( $\infty$ ) is 2 × 10<sup>-4</sup>/°C.
  - (1) 1694°C
  - (2) 1500°C
  - (3) 1000°C
  - (4) 1200°C

# Answer (1)

**Sol.** Final resistance 
$$(R) = \frac{V}{I} = 80 \Omega$$

then, 
$$R = R_0(1 + \infty \Delta T)$$

$$80 = 60 \left( 1 + 2 \times 10^{-4} \Delta T \right)$$

$$\Delta T = 1666.67$$

$$T - 27$$

$$T = 1693.66$$

19. Match the two columns.

		Column 1		Column 2			
	P.	Surface tension	1.	[ML <sup>2</sup> T <sup>-2</sup> ]			
9	Q.	Viscosity	2.	[ML <sup>2</sup> T <sup>-1</sup> ]			
	R.	Angular momentum	3.	[ML <sup>-1</sup> T <sup>-1</sup> ]			
	S.	Rotational kinetic energy	4.	$[ML^0T^{-2}]$			

- (1) P-1, Q-2, R-3, S-4
- (2) P-4, Q-3, R-2, S-1
- (3) P-1, Q-3, R-4, S-2
- (4) P-4, Q-2, R-1, S-3

### Answer (2)

Sol. 
$$S = \frac{F}{I}$$

$$\Rightarrow$$
 [S]=[MT<sup>-2</sup>]

$$F = nA \frac{dv}{dx}$$

$$\Rightarrow \eta = \frac{MLT^{-2} \cdot T}{L^2} = ML^{-1}T^{-1}$$



$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow L = [ML^2T^{-1}]$$

$$KE = \frac{1}{2}I\omega^2$$

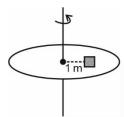
$$\Rightarrow KE = ML^2T^{-2}$$

20.

### **SECTION - B**

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A block of mass 2 kg is placed on a disc which is rotating at constant angular velocity 4 rad/s. Find the friction force (in N) between block and disc if block is not sliding.



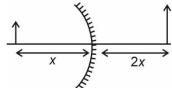
# Answer (32)

Sol. Block is not slipping, so

$$f = mr\omega^2 = 2 \times 1 \times 16 = 32$$

22. Distance between virtual image, which is of twice of size of object placed in front of mirror and object is45 cm. Magnitude of focal length of mirror is\_\_\_\_\_ cm.

### Answer (30)



Sol.

$$|m| = \left| \frac{v}{u} \right| = 2$$

$$|v| = |2u|$$

$$n + 2n = 45$$

$$n = 15 \text{ cm}$$

$$u = -15$$

$$v = 30$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{30} + \frac{1}{-15} = \frac{1}{f}$$

$$\frac{1-2}{30} = \frac{-1}{30} = \frac{1}{f}$$

 $\Rightarrow f = 30 \text{ cm}$ 

23. A particle is having uniform acceleration. If its displacement from t to (t + 1) second is 120 m and change in velocity is 50 m/s. Find its displacement (in m) in (t + 2) second.

# **Answer (170)**

∴ 
$$a = 50 \text{ m/s}^2$$
  
 $s = u + \frac{a}{2}(2(t+1) - 1)$ 

**Sol.**  $\Delta V = a(t+1-t)$ 

$$120 = u + \frac{50}{2}(2t+1)$$

$$u = 120 - 25(2t+1)$$

In (t+2)<sup>th</sup> second

$$s' = u + \frac{a}{2}(2(t+2)-1)$$

$$= u + 25(2t + 3)$$

$$= 120 - 25(2t) - 25 + 25(2t) + 75$$

$$s' = 170 \text{ m}$$

24. A uniform disc of mass 5 kg and radius 2 m is rotating with 10 rad/s. Now another identical disc is gently placed on first disc. Because of friction, both disc acquire common angular velocity. Loss of kinetic energy in process is \_\_\_\_\_\_\_ J.

### Answer (250)

**Sol.** COAM gives  $I\omega_0 = 2I\omega$ 

$$\omega = \frac{\omega_0}{2}$$

Loss in KE = 
$$\frac{1}{2}I\omega_0^2 - \frac{1}{2}(2I)\left(\frac{\omega_0}{2}\right)^2$$

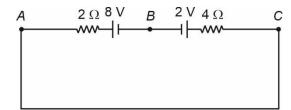
$$=\frac{1}{4}I\omega_0^2$$

$$=\frac{1}{4}\times5\times\frac{2}{2}\times100=250 \text{ J}$$



25. Two cell one of emf 8 V, internal resistance 2  $\Omega$  and other of emf 2 V and internal resistance 4  $\Omega$  are connected as shown in figure.

Find potential difference (in V) across point AC.



# Answer (0)

**Sol.** Current in circuit (
$$I$$
) =  $\frac{8-2}{6}$  = 1 A.

So, 
$$V_C - 4(1) - 2 + 8 - 2(1) = V_A$$

$$V_C - 6 - 2 + 8 = V_A$$

$$V_C - V_A = 0 \text{ V}$$

26. Electron in an hydrogen atom is excited to an energy level having energy –0.85 eV. Find the number of possible transitions it can make while deexcitation.

# Answer (6)

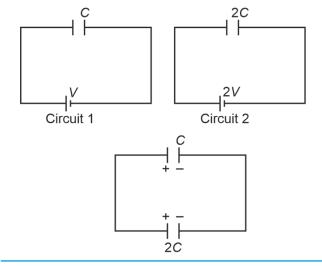
**Sol.** 
$$-0.85 = \frac{-13.6}{n^2}$$

$$n = 4$$

$$\therefore \text{ Number of transitions} = \frac{4 \times 3}{2} = 6$$

27. Energy stored in circuit 1 is *E*. If capacitors in circuit 1 and circuit 2 are connected in parallel as shown,

the energy stored becomes  $\frac{xE}{6}$ , find x.



# Answer (50)

# **Sol.** Charge on $C_1 = CV$

Charge on 
$$C_2 = 4CV$$

When connected in parallel

$$V_c = \frac{5V}{3}$$

$$\therefore Q_1' = \frac{5}{3}CV, Q_2' = \frac{10}{3}CV$$

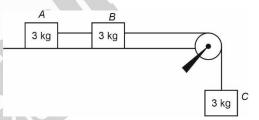
$$:: E = \frac{1}{2}CV^2$$

$$E' = \frac{25}{18}CV^2 + \frac{25}{9}CV^2$$

$$\frac{25}{6}CV^2 = \frac{50E}{6}$$

$$\therefore x = 50$$





If wire *BC* has Young's modulus of  $Y = 2 \times 10^{11}$  N/m<sup>2</sup> and cross section area of  $5 \times 10^{-4}$  cm<sup>2</sup>. Find strain in wire *BC* (in unit of  $10^{-4}$ )

### Answer (20)

**Sol.** 
$$a = \frac{3}{9}g$$
, For C,  $3g - T = 3a = (3)\frac{3}{9}g$ 

$$T = 2q = 20 \text{ N}$$

$$\frac{\sigma}{\in} = Y$$

$$\frac{\sigma}{Y} = \in \Rightarrow \frac{20}{5 \times 10^{-8} \times 2 \times 10^{11}}$$

$$= 2 \times 10^{-3}$$

$$= 20 \times 10^{-4} \Rightarrow 20$$

29.

30.



# **CHEMISTRY**

### **SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

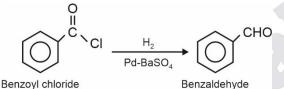
### Choose the correct answer:

1. What is the name of given reaction?

- (1) Etard reaction
- (2) Stephen's reaction
- (3) Wolff Kishner reduction
- (4) Rosenmund reaction

### Answer (4)

**Sol.** Acyl chloride is hydrogenated over catalyst, palladium or barium sulphate. This reaction is called Rosenmund reaction.

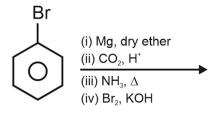


- Which of the given compound will not give Fehling test?
  - (1) Lactose
  - (2) Maltose
  - (3) Sucrose
  - (4) Glucose

### Answer (3)

**Sol.** Sucrose is non-reducing sugar. It does not reduce Fehling solution.

3. Find final product of reaction given below



$$(1) \bigcirc (2) \bigcirc (2) \bigcirc (3) \bigcirc (4) \bigcirc (4) \bigcirc (4) \bigcirc (4) \bigcirc (5) \bigcirc (5)$$

Answer (1)

Sol.

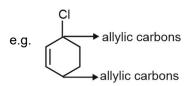
4. Which of the following has allylic halogen?

(1) 
$$CH = CH - CH_2 - CH_2 - CI$$
(2) 
$$CI$$
(3) 
$$CI$$
(4) 
$$CH_3 - CH - CH_3$$

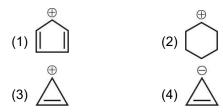
Answer (3)



**Sol.** The carbon next to an alkene is known as allylic carbon and halogen attached to allylic carbon is known as allylic halogen.



5. Which of the following compound or ion is most stable?



# Answer (3)

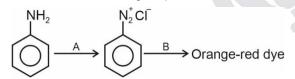
**Sol.**  $\triangle$  is most stable due to aromatic character. It has  $2\pi e^-$  and follow  $(4n + 2)\pi e^-$  Huckel rule.

- 6. Which of the following set contains both diamagnetic ions?
  - (1)  $Ni^{2+}$ ;  $Cu^{2+}$  (2)  $Eu^{3+}$ ;  $Gd^{3+}$  (3)  $Cu^+$ ;  $Zn^{2+}$  (4)  $Ce^{4+}$ ;  $Pm^{3+}$

# Answer (3)

**Sol.** Cu: 4s<sup>1</sup>3d<sup>10</sup>; Cu<sup>+</sup>: 4s<sup>0</sup>3d<sup>10</sup> Zn: 4s<sup>2</sup>3d<sup>10</sup>; Zn<sup>2+</sup>: 4s<sup>0</sup>3d<sup>10</sup>

7. Consider the following sequence of reactions

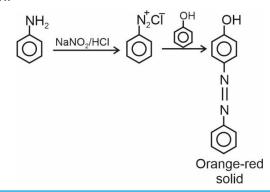


Select the option with correct A and B respectively.

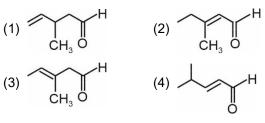
- (1) HNO<sub>3</sub>, Phenol (2) NaNO<sub>2</sub>/HCl, Phenol
- (3) HNO<sub>3</sub>, Aniline (4) NaNO<sub>2</sub>/HCl, Aniline

### Answer (2)

Sol.



8. Which of the following is the correct structure for the given IUPAC name "3-Methylpent-2-enal"



# Answer (2)

**Sol.** 5 4 3 2 1 H

3-Methylpent-2-enal

Functional group should get lowest possible number.

- 9. The group number of Unununium is
  - (1) 11
     (2) 12

     (3) 6
     (4) 14

### Answer (1)

**Sol.** Group number = 11 (Atomic number = 111)

- 10. What is the Geometry of Aluminium chloride in aqueous solution?
  - (1) Square planar(2) Octahedral(3) Tetrahedral(4) Square pyramidal
- Answer (2)

Sol. AlCl<sub>3</sub> exists as

[Al(H<sub>2</sub>O)<sub>6</sub>]Cl<sub>3</sub> in aqueous solution.

11. **Statement-I:** For hydrogen atom, 3*p* and 3*d* are degenerate.

**Statement-II:** Degenerate orbitals have same energy.

- (1) Both statement-I and II are correct
- (2) Both statement-I and II are incorrect
- (3) Statement-I is correct, statement-II is incorrect
- (4) Statement-I is incorrect, statement-II is correct

### Answer (1)

**Sol.** For hydrogen atom energy of orbitals only depends on value of principal quantum number

$$1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f$$

Degenerate orbitals have same energy.



12. Consider the following sequence of reactions

$$\mathsf{CH}_3 - \mathsf{C} \equiv \mathsf{CH} \xrightarrow{\quad \mathsf{Na} \quad} \mathsf{A} \xrightarrow{\quad \mathsf{B} \quad} \mathsf{CH}_3 - \mathsf{C} \equiv \mathsf{C} - \mathsf{CH}_2 - \mathsf{CH}_2 - \mathsf{CH}_3$$

Select A and B respectively

(1) 
$$CH_3 - CH = CH_2$$
,  $CH_3 - CH_2 - CI$ 

(2) 
$$CH_3 - C \equiv CNa$$
,  $CH_3 - CH_2 - CI$ 

(3) 
$$CH_3 - C \equiv CNa, CH_3 - CH_2 - CH_2 - CI$$

(4) 
$$CH_3 - C \equiv C - CH_3$$
,  $CH_3 - CH_2 - CH_2 - CI$ 

### Answer (3)

Sol. 
$$CH_3 - C \equiv CH \xrightarrow{Na} CH_3 - C \equiv CNa + H_2(g)$$

$$\downarrow CH_3 - CH_2 - CH_2 - CI (B)$$

$$CH_3 - C \equiv C - CH_2 - CH_2 - CH_3$$

13. Choose the correct option.

### 

# (1) a(iv), b(iii), c(ii), d(i) (2) a(iv), b(iii), c(i), d(ii) (3) a(iii), b(iv), c(ii), d(i) (4) a(iii), b(iv), c(i), d(ii)

# Answer (1)

**Sol.** BrF<sub>5</sub> - Square pyramidal

 $H_2O$  – Bent  $CIF_3$  – T-shape  $SF_4$  – See-saw

14. Assertion (A): While moving from N to P covalent radius increases significantly but from As to Bi only a small increase is observed.

Reason (R): For a particular oxidation state covalent radii and ionic generally radii increases down the group.

- (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (3) (A) is correct but (R) is incorrect
- (4) (A) is incorrect but (R) is correct

# Answer (2)

**Sol.** Covalent and ionic (in a particular state) radii increases in size down the group. There is a considerable increase in covalent radius from N to P. However, from As to Bi only a small increase in covalent radii is observed. This is due to the presence of completely filled *d* and *f*-orbitals in heavier elements. (lanthanoid contraction)

15. Match the following and select the correct option.

# List I a. $Mn^{2+}$ b. $V^{+}$ c. $Cr^{+}$ d. $Fe^{2+}$ (i) $3d^{3} 4s^{1}$ (ii) $3d^{5} 4s^{0}$ c. $Cr^{+}$ (iii) $3d^{6} 4s^{0}$ d. $Fe^{2+}$ (iv) $3d^{4} 4s^{1}$ (1) $a \rightarrow (i), b \rightarrow (ii), c \rightarrow (iii), d \rightarrow (iv)$ (2) $a \rightarrow (iv), b \rightarrow (iii), c \rightarrow (ii), d \rightarrow (i)$ (3) $a \rightarrow (ii), b \rightarrow (i), c \rightarrow (ii), d \rightarrow (iii)$

(4)  $a \rightarrow (ii), b \rightarrow (i), c \rightarrow (iii), d \rightarrow (iv)$ 

# Answer (3)

**Sol.** Mn<sup>2+</sup> :  $3d^5 4s^0$ V<sup>+</sup> :  $3d^3 4s^1$ Cr<sup>+</sup> :  $3d^5 4s^0$ Fe<sup>2+</sup> :  $3d^6 4s^0$ 

- 16. What happen to freezing point of benzene, when small amount of naphthalene is added to benzene?
  - (1) Increases
  - (2) Decreases
  - (3) Remains unchanged
  - (4) First decreases and then increases

### Answer (2)

- **Sol.** When small amount of naphthalene is added to benzene, depression in freezing point takes place and freezing point of benzene decreases.
- 17. A mixture is heated with dilute H<sub>2</sub>SO<sub>4</sub> and the lead acetate paper turns black by the evolved gas. The mixture contains
  - (1) Sulphite
- (2) Sulphide
- (3) Sulphate
- (4) Thiosulphate

### Answer (2)

**Sol.** Sulphide 
$$\xrightarrow{\text{dil H}_2SO_4} H_2S(g)$$

$$\mathsf{Pb}(\mathsf{CH}_3\mathsf{CO}_2)_2 \xrightarrow{\quad \mathsf{H}_2\mathsf{S} \quad} \mathsf{PbS}_{\mathsf{Black}} + \mathsf{CH}_3\mathsf{CO}_2\mathsf{H}$$

18.

19.

20.

# **SECTION - B**

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find out sum of coefficients of all the species involved in balance equation

$$2MnO_4^- + I^- \xrightarrow{Alkaline \atop Medium}$$

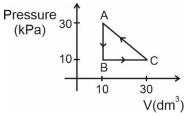
Answer (9)



**Sol.**  $I^- + H_2O + 2MnO_4^- \rightarrow 2MnO_2 + 2OH^- + IO_3^-$ 

Sum of coefficients = 9

22. Find work done in following cyclic process (in J).



# **Answer (200)**

Sol. Work done in cyclic process

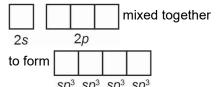
= area inside the figure

$$=\frac{1}{2} \times 20 \times 20 = 200 \text{ J}$$

23. Maximum number of hybrid orbitals formed when 2s and 2p orbitals of a single atom are mixed.

### Answer (4)

**Sol.** When 2s and 2p orbitals are mixed, maximum 4 hybrid orbitals are formed



4 sp<sup>3</sup> hybrid orbitals are formed of same energy.

24. The rate of first order reaction is 0.04 mol lit<sup>-1</sup> sec<sup>-1</sup> at 10 sec and 0.03 mol lit-1 sec-1 at 20 sec. Calculate half-life of first order reaction (in sec).

### Answer (24)

**Sol.** 
$$\frac{0.04}{0.03} = \frac{k \times C_0 e^{-k(10)}}{k \times C_0 e^{-k(20)}} = e^{10k}$$

$$10k = \ln\left(\frac{4}{3}\right)$$

$$k = \frac{1}{10} \ln \left( \frac{4}{3} \right)$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{k}$$

$$=\frac{\ln 2}{\ln \left(\frac{4}{3}\right)} \times 10$$

= 24 sec.

25. The number of atoms in a silver plate having area  $0.05 \text{ cm}^2$  and thickness 0.05 cm is \_\_\_\_\_ ×  $10^{19}$ 

Density of silver is 7.9 g/cm<sup>3</sup>

### Answer (11)

**Sol.** Volume = Area × Thickness  
= 
$$0.05 \times 0.05 \text{ cm}^3$$
  
=  $0.0025 \text{ cm}^3$ 

Mass of silver =  $7.9 \times 0.0025$  q

Moles of silver = 
$$\frac{7.9 \times 0.0025}{108}$$

Number of silver atoms

$$=\frac{7.9\times0.0025}{108}\times6.022\times10^{23}$$

Number of silver atoms =  $0.001101 \times 10^{23}$  $= 11.01 \times 10^{19}$ 

26. The ratio of magnitude of potential energy and kinetic energy for 5th excited state of hydrogen atom

# Answer (2)

**Sol.** According to Bohr model, PE = -2KE

27. 250 mL solution of CH<sub>3</sub>COONa of molarity 0.35 M is prepared. What is the mass of CH<sub>3</sub>COONa required in grams? (Nearest integer)

### Answer (7)

**Sol.** Molarity = 
$$\frac{\text{Number of moles of solute}}{\text{Volume of solution in litre}}$$

Moles of solute = 
$$\frac{\text{Weight}}{\text{Molecular weight}}$$

$$0.35 = \frac{W}{MW(CH_3COONa)} \times \frac{1000}{25}$$

$$W = \frac{0.35 \times 82 \times 250}{1000}$$

$$W = \frac{7175}{1000} = 7.175 \text{ g}$$

Mass of CH<sub>3</sub>COONa required to prepare 250 mL of 0.35 M solution is 7.175 g.

28. The  $K_{sp}$  of  $Mg(OH)_2$  is 1 × 10<sup>-12</sup>, 0.01 M  $Mg^{2+}$  ion will precipitate at the limiting pH equal to (at 25°C).

# Answer (9)

**Sol.** 
$$Mg(OH)_2(s) \rightleftharpoons Mg^{2+}(aq) + 2OH^{-}(aq)$$

$$K_{sp} = [Mg^{2+}](OH^{-}]^{2}$$
  
 $K_{sp} = 0.01 [OH^{-}]^{2}$ 

$$K_{sp} = 0.01 [OH^{-}]^{2}$$

$$\frac{1 \times 10^{-12}}{0.01} = [OH^-]^2$$

$$[OH^-] = \sqrt{1 \times 10^{-10}}$$

$$[OH^{-}] = 10^{-5}$$

$$pOH = 5$$

$$pH = 14 - 5 = 9$$

29.

30.



# **MATHEMATICS**

### **SECTION - A**

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

### Choose the correct answer:

- 1. In an arithmetic progression if sum of 20 terms is 790 and sum of 10 terms is 145, then  $S_{15} S_5$  is (when  $S_n$  denotes sum of n terms)
  - (1) 400
- (2) 395
- (3) 385
- (4) 405

# Answer (2)

**Sol.** 
$$S_{20} = \frac{20}{2}[2a+19d] = 790$$
  
 $2a+19d=79$  ...(1)

$$S_{10} = \frac{10}{2} [2a + 9d] = 145$$

$$2a + 9d = 29$$

...(2)

from (1) and (2) a = -8, d = 5

$$S_{15} - S_5 = \frac{15}{2} [2a + 14d] - \frac{5}{2} [2a + 4d]$$

$$=\frac{15}{2}[-16+70]-\frac{5}{2}[-16+20]$$

= 395

2. If the foot of perpendicular from (1, 2, 3) to the line

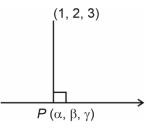
$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z-4}{1}$$
 is  $(\alpha, \beta, \gamma)$  then find  $\alpha + \beta + \gamma$ 

(1) 6

- (2) 5.8
- (3) 4.8
- (4) 5

### Answer (2)

Sol.



$$(\alpha - 1) \times 2 + (\beta - 2) \times 5 + (\gamma - 3) \times 1 = 0$$

$$2\alpha + 5\beta + \gamma - 15 = 0$$

Also, P lie on line

$$\Rightarrow \alpha + 1 = 2\lambda$$

$$\beta - 2 = 5\lambda$$

$$\gamma - 4 = \lambda$$

$$\Rightarrow$$
 2(2 $\lambda$  - 1) + 5(5 $\lambda$  + 2) +  $\lambda$  + 4 - 15 = 0

$$\Rightarrow 4\lambda + 25\lambda + \lambda - 2 + 10 + 4 - 15 = 0$$

$$30\lambda - 3 = 0$$

$$\Rightarrow \lambda = \frac{1}{10}$$

$$\Rightarrow \alpha + \beta + \gamma = (2\lambda - 1) + (5\lambda + 2) + (\lambda + 4)$$

$$= 8\lambda + 5 = \frac{8}{10} + 5 = 5.8$$

3. 
$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$$

(1) 
$$\frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$$

(2) 
$$\frac{\pi}{2\sqrt{3}} + \frac{\pi}{8}$$

(3) 
$$\frac{\pi}{2} - \frac{\pi}{\sqrt{3}}$$

(4) 
$$\frac{\pi}{\sqrt{3}} - \frac{\pi}{4}$$

# Answer (1)

**Sol.** 
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n^3}{n^4 \left(1 + \frac{k^2}{n^2}\right) \left(1 + \frac{3k^2}{n^2}\right)}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\left(1 + \frac{k^2}{n^2}\right) \left(1 + \frac{3k^2}{n^2}\right)}$$

$$= \int_{0}^{1} \frac{dx}{3(1+x^2)(\frac{1}{3}+x^2)}$$

$$= \int_{0}^{1} \frac{1}{3} \times \frac{3}{2} \frac{\left(x^{2}+1\right) - \left(x^{2}+\frac{1}{3}\right)}{\left(1+x^{2}\right)\left(x^{2}+\frac{1}{3}\right)} dx$$

$$= \frac{1}{2} \int_{0}^{1} \left| \frac{1}{x^{2} + \left(\frac{1}{\sqrt{3}}\right)^{2}} - \frac{1}{1 + x^{2}} \right| dx$$

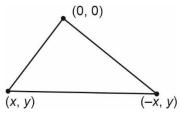
$$= \frac{1}{2} \left[ \sqrt{3} \tan^{-1} \left( \sqrt{3} x \right) \right]_0^1 - \frac{1}{2} \left( \tan^{-1} x \right)_0^1$$

$$=\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$$

- The value of maximum area possible of a  $\triangle ABC$ such that A(0, 0) and B(x, y) and C(-x, y) such that  $y = -2x^2 + 54x$  is (in sq. unit)
  - (1) 5800
- (2) 5832
- (3) 5942
- (4) 6008

# Answer (2)

Sol.



Area of  $\Delta$ 

$$= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \left| \frac{1}{2} (xy + xy) \right| = |xy|$$

Area (
$$\Delta$$
) =  $|xy| = |x(-2x^2 + 54x)|$ 

$$\frac{d(\Delta)}{dx} = |(-6x^2 + 108x)| \Rightarrow \frac{d\Delta}{dx} = 0$$
 at  $x = 0$  and 18

 $\Rightarrow$  at x = 0, minima

and at x = 18 maxima

Area (
$$\Delta$$
) =  $|18(-2(18)^2 + 54 \times 18)| = 5832$ 

- The range of r for which circles  $(x + 1)^2 + (y + 2)^2 =$  $r^2$  and  $x^2 + y^2 - 4x - 4y + 4 = 0$  coincide at two distinct points
  - (1) 3 < r < 7
- (2) 5 < r < 9
- (3)  $\frac{1}{2} < r < 4$  (4) 0 < r < 3

### Answer (1)

Sol. If two circles intersect at two distinct points

$$\Rightarrow |r_1 - r_2| < C_1 C_2 < r_1 + r_2$$

$$|r-2| < \sqrt{9+16} < r+2$$

|r - 2| < 5and r + 2 > 5

-5 < r - 2 < 5

r > 3

...(2)

-3 < r < 7

...(1)

From (1) and (2)

3 < r < 7

- 6. An ellipse whose length of minor axis is equal to half of length between foci, then eccentricity is
  - (1)  $\frac{7}{2}$

- (2)  $\sqrt{17}$
- (3)  $\frac{2}{\sqrt{5}}$
- $(4) \frac{3}{\sqrt{7}}$

# Answer (3)

**Sol.** : ae = 2b

$$\therefore \frac{4b^2}{a^2} = e^2$$

Or 
$$4(1 - e^2) = e^2$$

$$\therefore 4 = 5e^2 \Rightarrow e = \frac{2}{\sqrt{5}}$$

7. If 
$$g'\left(\frac{3}{2}\right) = g'\left(\frac{1}{2}\right)$$
 and

$$f(x) = \frac{1}{2} [g(x) + g(2 - x)]$$
 and  $f'(\frac{3}{2}) = f'(\frac{1}{2})$  then

- (1) f''(x) = 0 has exactly one root in (0, 1)
- (2) f''(x) = 0 has no root in (0, 1)
- (3) f''(x) = 0 has at least two roots in (0, 2)
- (4) f''(x) = 0 has 3 roots in (0, 2)

### Answer (3)

Sol.

$$f'(x) = \frac{g'(x) - g'(2 - x)}{2}, f'\left(\frac{3}{2}\right) = \frac{g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)}{2} = 0$$

Also 
$$f'\left(\frac{1}{2}\right) = \frac{g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)}{2} = 0$$
,  $f'(1) = 0$ 

$$\Rightarrow f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow$$
 roots in  $\left(\frac{1}{2},1\right)$  and  $\left(1,\frac{3}{2}\right)$ 

$$\Rightarrow f''(x)$$
 is zero at least twice in  $\left(\frac{1}{2}, \frac{3}{2}\right)$ 

The domain of  $y = \cos^{-1} \left| \frac{2 - |x|}{4} \right| + (\log(3 - x))^{-1}$  is

 $[-\alpha, \beta)$  -{ $\gamma$ }, then value of  $\alpha + \beta + \gamma = ?$ 

(1) 9

(2) 12

(3) 11

(4) 10

Answer (3)

**Sol.** 
$$-1 \le \left| \frac{2 - |x|}{4} \right| \le 1$$

$$\Rightarrow \left| \frac{2-|x|}{4} \right| \leq 1$$

$$\Rightarrow -1 \le \frac{2-|x|}{4} \le 1$$

$$-4 \le 2 - |x| \le 4$$

$$-6 \le -|x| \le 2$$

$$-2 \le |x| \le 6$$

$$|x| \le 6$$

$$\Rightarrow x \in [-6, 6]$$
 ...(1)

Now, 
$$3 - x \neq 1$$

And 
$$x \neq 2$$
 ...(2)

and 
$$3 - x > 0$$

From (1), (2) and (3)

$$\Rightarrow x \in [-6, 3] - \{2\}$$

$$\alpha = 6$$

$$\beta = 3$$

$$\gamma = 2$$

$$\alpha + \beta + \gamma = 11$$

- If y = f(x) is solution of differential equation  $(x^2 1)$  $dy = (x^3 + 1) + \sqrt{1 - x^2} dx$  and y(0) = 2 then find  $y\left(\frac{1}{2}\right)$ .

  - (1)  $\frac{13}{7} \frac{\pi}{2} + \ln 5$  (2)  $\frac{15}{7} + \frac{\pi}{3} + \ln 2$
  - (3)  $\frac{17}{8} + \frac{\pi}{6} \ln 2$  (4)  $\frac{18}{7} \frac{\pi}{6} + \ln 3$

# Answer (3)

Sol. 
$$\frac{dy}{dx} = \frac{(x+1)(x^2 - x + 1) + \sqrt{(1-x)(1+x)}}{(x-1)(x+1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x(x-1) + 1}{(x-1)} + \sqrt{\frac{(1-x)(1+x)}{(x-1)^2(x+1)^2}}$$

$$\frac{dy}{dx} = x + \frac{1}{x-1} + \frac{1}{\sqrt{(1-x)(1+x)}}$$

$$\Rightarrow$$
  $dy = xdx + \frac{1}{(x-1)}dx + \frac{dx}{\sqrt{1-x^2}}$ 

$$\Rightarrow y = \frac{x^2}{2} + \ln|x-1| + \sin^{-1}x + c$$

at 
$$x = 0$$
,  $y = 2 \Rightarrow 2 = c$ 

$$\Rightarrow y = \frac{x^2}{2} + \ln|x - 1| + \sin^{-1} x + 2$$

$$y\left(\frac{1}{2}\right) = \frac{17}{8} + \frac{\pi}{6} - \ln 2$$

- Given  $x^2 70x + \lambda = 0$  with positive roots  $\alpha$  and  $\beta$ where one of the root is less than 10 and  $\frac{\lambda}{2}$  and  $\frac{\lambda}{3}$ are not integers then find value of  $\frac{\sqrt{\alpha-1}+\sqrt{\beta-1}}{|\alpha-\beta|}$  is equal to
  - $(1) \frac{1}{5}$

(2)  $\frac{1}{12}$ 

- $(4) \frac{1}{70}$

# Answer (1)

**Sol.** Given :  $x^2 - 70x + \lambda = 0$ 

 $\Rightarrow$  Let roots be  $\alpha$  and  $\beta$ 

$$\Rightarrow \beta = 70 - \alpha$$

$$\lambda = \alpha (70 - \alpha)$$

 $\lambda$  is not divisible by 2 and 3

$$\Rightarrow \alpha = 5$$
,  $\beta = 65$ 

$$\Rightarrow \frac{\sqrt{5-1}+\sqrt{65-1}}{|60|} = \left|\frac{4+8}{60}\right| = \frac{1}{5}$$

A line passes through (9, 0), making angle 30° with positive direction of x-axis. It is rotated by angle of 15° with respect to (9, 0). Then one of the equation of new line is

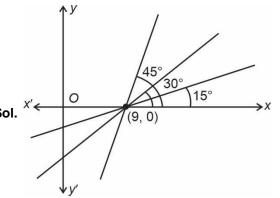
(1) 
$$y = (2 + \sqrt{3})(x-9)$$
 (2)  $y = (2 - \sqrt{3})(x-9)$ 

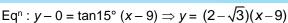
(2) 
$$v = (2 - \sqrt{3})(x - 9)$$

(3) 
$$y = 2(x-9)$$
 (4)  $y = -(x-9)$ 

(4) 
$$y = -(x - 9)$$

# Answer (2)





Eqn:  $y - 0 = \tan 45^{\circ} (x - 9) \Rightarrow y = (x - 9)$ 

Option (B) is correct

- 12. For a non-zero complex number z satisfying  $z^2 + i\overline{z} = 0$ , then value of  $|z|^2$  is
  - (1) 1

(2) 2

(3) 3

(4) 4

# Answer (1)

**Sol.** 
$$z^2 = -i\overline{z}$$

$$|z^2| = |-i\overline{z}|$$

$$|z^2| = |z|$$

$$|z|^2 - |z| = 0$$

$$|z|(|z|-1)=0$$

|z| = 0 (not acceptable)

$$|z| = 1$$

$$|z|^2 = 1$$

13. If 
$$|\vec{a}| = 1$$
.  $|\vec{b}| = 4$ 

$$\vec{a} \cdot \vec{b} = 2$$
 and  $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$ 

Then the angle between  $\vec{b}$  and  $\vec{c}$  is

(1) 
$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$
 (2)  $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 

$$(2) \quad \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(3) 
$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

(3) 
$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$
 (4)  $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$ 

### Answer (1)

**Sol.** Given 
$$|\vec{a}| = 1$$
,  $|\vec{b}| = 4$ ,  $\vec{a} \cdot \vec{b} = 2$ 

$$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$$

Dot product with  $\vec{a}$  on both sides

$$\vec{c} \cdot \vec{a} = -6$$

... (1)

Dot product with  $\vec{b}$  on both sides

$$\vec{b} \cdot \vec{c} = -48$$

... (2)

$$\vec{c} \cdot \vec{c} = 4 |\vec{a} \times \vec{b}|^2 + 9 |\vec{b}|^2$$

$$|\vec{c}|^2 = 4 \left[ |\vec{a}|^2 \cdot |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 \right] + 9 |\vec{b}|^2$$

$$|\vec{c}|^2 = 4[(1)(4)^2 - (4)] + 9(16)$$

$$|\vec{c}|^2 = 4[12] + 144$$

$$|\vec{c}|^2 = 48 + 144$$

$$|\vec{c}|^2 = 192$$

$$\therefore \quad \cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}||\vec{c}|}$$

$$\cos\theta = \frac{-48}{\sqrt{192} \cdot 4}$$

$$\cos\theta = \frac{-48}{8\sqrt{3} \cdot 4}$$

$$\cos\theta = \frac{-3}{2\sqrt{3}}$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$
  $\Rightarrow \theta = \cos^{-1} \left( \frac{-\sqrt{3}}{2} \right)$ 

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

Given set  $S = \{0, 1, 2, 3, ...., 10\}$ . If a random ordered pair (x, y) of elements of S is chosen, then find probability that |x - y| > 5

(1) 
$$\frac{30}{121}$$

(2) 
$$\frac{31}{121}$$

(3) 
$$\frac{62}{121}$$

(4) 
$$\frac{64}{121}$$

# Answer (1)

**Sol.** If 
$$x = 0$$
,  $y = 6$ , 7, 8, 9, 10

If 
$$x = 1$$
,  $y = 7$ , 8, 9, 10

If 
$$x = 2$$
,  $y = 8$ , 9, 10

If 
$$x = 3$$
,  $y = 9$ , 10

If 
$$x = 4$$
,  $y = 10$ 

If 
$$x = 5$$
,  $y = \text{no possible value}$ 

Total possible ways =  $(5 + 4 + 3 + 2 + 1) \times 2$ 

Required probability = 
$$\frac{30}{11 \times 11} = \frac{30}{121}$$

- 15.
- 16.
- 17.
- 18.
- 19.
- 20.



### **SECTION - B**

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Number of integral terms in the binomial expansion of  $(7^{1/2} + 11^{1/6})^{824}$  is

### **Answer (138)**

**Sol.** 
$$T_{n+1} = {}^{n}C_{r} 11^{\frac{r}{6}} \cdot 7^{\frac{824-r}{2}}$$

For integral term

6 should divide r

and 
$$\frac{824-r}{2}$$
 must be integer

- $\Rightarrow$  2 most divide r
- r divisible by 6
- $\Rightarrow$  possible values of  $r \in \{0, 1, 2, ...824\}$
- ⇒ For integer terms

$$r \in \{0, 6, 12, ...822\}$$
 (822 = 0 + ( $n$ -1)6  $\Rightarrow n$  = 138)

= 138 terms

22.  $9 \int_{1}^{9} \sqrt{\frac{10x}{x+1}} dx$  is equal to (where [] represents greatest integer function)

### **Answer (155)**

Sol. 
$$I = 9 \int_{0}^{9} \left[ \sqrt{\frac{10x}{x+1}} \right] dx$$
  

$$= 9 \left[ \int_{0}^{1/9} 0 \ dx + \int_{1/9}^{2/3} dx + \int_{2/3}^{9} 2 dx \right]$$

$$= 9 \left[ \frac{2}{3} - \frac{1}{9} + 2 \left[ 9 - \frac{2}{3} \right] \right]$$

$$= 9 \left[ \frac{5}{9} + 2 \times \frac{25}{3} \right]$$

$$= 5 + 6 \times 25$$

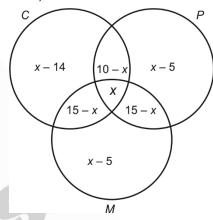
$$= 5 + 150$$

$$= 155$$

23. In a class there are 40 students. 16 passed in Chemistry, 20 passed in Physics, 25 passed in Mathematics. 15 students passed in both Mathematics and Physics. 15 students passed in both Mathematics and Chemistry and 10 students passed in both Physics and Chemistry. Find the maximum number of students that passed in all the subjects.

# Answer (19)

**Sol.** 
$$n(C) = 16$$
,  $n(P) = 20$ ,  $n(M) = 25$   
 $n(M \cap P) = n(M \cap C) = 15$ ,  $n(P \cap C) = 10$ ,  
 $n(M \cap C \cap P) = x$ .



$$n(C \cup P \cup M) \le n(U) = 40$$

$$n(C \cup P \cup M) = n(C) + n(P) + n(M) - n(C \cup M) - n(P \cup M) - n(C \cap P) + n(C \cap P \cap M)$$

$$40 \ge 16 + 20 + 25 - 15 - 15 - 10 + x$$

$$40 \ge 61 - 40 + x$$

$$19 \ge x$$

So maximum number of students that passed all the exams is 19.

# 24. For the following data table

$\boldsymbol{X}_{i}$	$ f_i $
0 – 4	2
4 - 8	4
8 - 12	7
12 - 16	8
16 - 20	6
	1

Find the value of 20 M (where M is median of the data)

### **Answer (245)**

	$X_i$	$f_i$	c.f.
Sol.	0 – 4	2	2
	4 - 8	4	6
	8 - 12	7	13
	12 - 16	8	21
	16 - 20	6	27

$$N = \sum f = 27$$

$$\left(\frac{N}{2}\right) = \frac{27}{2} = 13.5$$

So, we have median lies in the class 12 - 16

$$I_1 = 12$$
,  $f = 8$ ,  $h = 4$ ,  $c.f. = 13$ 

So, here we apply formula

$$M = I_1 + \frac{\frac{N}{2} - c.f.}{f} \times h = 12 + \frac{13.5 - 13}{8} \times 4$$

$$= 12 + \frac{.5}{2}$$

$$M = \frac{24.5}{2} = 12.25$$

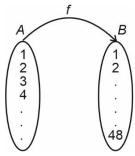
25. Set 
$$A = \{1, 2, 3, 4, 5, 6, 7\}$$

If number of functions from set A to power set of A can be expressed as  $m^n$  (m is least integer). Find m + n.

# Answer (51)



**Sol.** 
$$n P(A) = 2^7 = 128$$



$$f: A \rightarrow B$$

Number of function =  $128 \times 128....128 = 128^7$ 

$$=\left(2^{7}\right)^{7}=2^{49}$$

$$\Rightarrow m^n = 2^{49}$$

$$m + n = 49 + 2 = 51$$

- 26.
- 27.
- 28.
- 29.
- 30.